Perceiving Directional Feedback via Vibro-Tactile Stimulation in a Virtual Environment

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I am happy to announce that I am now working as a junior researcher at Elitac to continue the current study applied to the Neuroshirt. I also commenced a new masters degree in cognitive science and artificial intelligence in Tilburg.
Abstract

Tactile feedback may benefit numerous applications where other modalities are already highly loaded (e.g. air navigation and neuronavigation). The current study attempted to explore and get insights in directional feedback perception via vibration on the torso. The main goal was to test two models which might explain how people perceive directions through vibrational feedback. The circular model (CM) suggested an internal circle as reference to explain perceived directions, and the ellipse model (EM) suggested an internal ellipse which returns perceived directions with a bias towards the navel and the spine.

Data of fourteen participants was gathered in an experiment where participants had to point to the perceived directions within a sphere in virtual reality, when they felt vibrations on the torso via a shirt with 15 horizontal and nine vertical tactors. The models were compared on goodness of fit on the data. Results show that the EM best fitted the data for every participant. Due to big variability in answers, the average ellipse shape of the EM could not be used to develop a universal model. Future research is recommended to optimize the virtual reality with more visual reference cues and to test whether a similar model fits vertical data.
Samenvatting

Tactiele feedback kan een uitkomst bieden in verschillende toepassingen waar overige zintuigen al hoog belast worden (bijv. in luchtvaart navigatie of neuronavigatie). Deze exploratieve studie onderzoekt het waarnemen van richtingen via vibraties op de torso. Het hoofddoel was om twee modellen te testen die het waarnemen van richtingen via tactiele feedback zouden kunnen beschrijven. Het circulaire model (CM) beschrijft hoe een interne cirkel de basis vormt waaruit mensen richtingen waarnemen. Het ellips model (EM) suggereert een intern ellips waaruit waarnemingen ontstaan met een afwijking naar de navel en de ruggengraat.

Data van veertien deelnemers werd verzameld in een experiment waar men in een virtual reality omgeving waargenomen richtingen moest aanwijzen wanneer hij/zij vibraties voelde op de torso door middel van een shirt met 15 horizontale en negen verticale tactoren. Er werd onderzocht hoe goed de modellen bij de data pasten (goodness of fit) en werden zo met elkaar vergeleken. Uit de resultaten bleek dat voor elke deelnemer het EM beter bij de data paste dan het CM. De variabiliteit van de antwoorden was heel groot. Daardoor kan de gemiddelde vorm van het EM niet worden gebruikt als een universeel model. Het is voor toekomstig onderzoek aanbevolen om meer visuele referentie punten toe te voegen in de virtual reality omgeving en om te testen of er een model bestaat dat bij verticale data past.
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1 Introduction

To understand the world around us, people create cognitive maps. Those maps are abstract models of how things in the world are related to each other. Humans also create cognitive maps of the psychical environment they are in. These are called spatial representations. A spatial representation is formed in the posterior parietal cortex by the input of different sensory signals (Andersen, 1997; Grieves & Jeffery, 2017). The current study aims to help explain how vibro-tactile signals might contribute to the creation of a spatial representation. The aim of the study is to test whether there is a model which explains how directions are perceived via vibrations on the torso.

1.1 Background of the study

1.1.1 Sensory modalities

People have five sensory modalities; vision, audio, touch, smell and taste. Vision is often called the dominant modality in human experience; people would least want to miss this modality (Fiore, 2010; Schifferstein, 2006), and seem to have an attentional bias towards vision, meaning that they more frequently respond to a visual stimulus than to a stimulus from another modality (Colavita, 1974; Hecht & Reiner, 2009; Posner, Nissen, & Klein, 1976). Likewise, this modality is favoured in certain technological products. For example; even though the mobile phone was initially meant for calling (audio), nowadays mobile applications primarily use visual cues to navigate the customer. However, as Wickens (2002) described in the multiple resources theory (MRT), people might get a sensory overload when multiple tasks are executed using the same modality. Activities using different modalities meanwhile, can enhance performance (Wickens, 2002). Thus, if we use our eyes so often throughout the day and for all kinds of purposes, should we not also make use of other sensory modalities?

To prevent a sensory overload, audio is often chosen. To come back to the example of mobile phone applications, visual appearance is often expanded with auditive ‘clicks’, alarms
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or music. But touch is also increasingly used, for example through small vibrations if someone uses the touchpad to type. No attentional bias for either audio or touch is found, which might indicate that people do not favour one of the two (Colavita, 1974; Hecht & Reiner, 2009). In some tasks, the modality of touch is less loaded than audio or vision and might therefore be an interesting choice. The current study will focus on this modality. Lastly, different modalities might also be used together. This is especially effective in highly loaded situations (Baldwin et al., 2012; Spence, 2010).

Within the modality of touch, three subcategories can be distinguished: The perception of pain, temperature and pressure. This article will discuss the perception of pressure. Pressure is perceived differently throughout the body. In certain places people can better differentiate pressure on the skin, than in other places. Body parts are most sensitive to pressure when there is a large amount of receptors that respond to a stimulus (Nolen-Hoeksema, Fredrickson, Loftus, & Wagenaar, 2009). The law of mobility implies that sensitivity also increases when the mobility of body parts increases (Cholewiak, Brill, & Schwab, 2004; Van Erp, 2000; Vierordt, 1870). For example, this implies that sensitivity is better on the fingers compared to the back.

1.1.2 Tactile feedback in navigation

Navigation can be described as a process of controlled movement to reach a goal without getting lost. Environmental cues and artificial assistance may be used during this process (Darken & Sibert, 1993). Willén and colleagues (2008) showed how a visual, an auditory and a tactile device can be used for waypoint navigation for soldiers. Groen and colleagues (2009) used the same three modalities to guide pilots in a mission simulator cockpit. The tactile devices used in these studies were a belt and a vest with various vibrating elements (tactors) around the torso of the participant. Results showed that tactile feedback can be successfully used in navigation and might even be preferred by users over feedback via other modalities (Groen, Cornelisse, & Jansen, 2009; Willén et al., 2008).
Neuronavigation is a form of navigation used to guide a neurosurgeon when drilling through a skull, for example with the aim of operating or removing a tumour. Different tools may be used to differentiate between essential (e.g. the facial nerve) and non-essential structures for neurological functioning (Newell, 2005; Voormolen, 2018). Image-guided neurosurgery uses pre-operative images of the patient’s brain to locate target structures. In combination with a computer workstation and tracking devices, the surgeon perceives a visual, three dimensional and real-time experience of the patient’s anatomy (Newell, 2005). In 2018, Voormolen developed a tool which adds auditive feedback to image-guided neuronavigation (Voormolen, 2018). However, since an operation theatre can be very noisy, the auditive feedback could still cause a sensory overload. Therefore, Elitac is developing a new tool intended to use vibro-tactile feedback to provide surgeons with clear information without distracting him. The goal of Elitac’s Neuroshirt is to reduce the visual workload and increase situational awareness of the surgeon (ELITAC & UMCU, 2017).

An important challenge in the development of the Neuroshirt is the arrangement of vibro-tactors in order to give intuitive indications of directions to the surgeon. Prior to the current research, a literature review was done to determine the current state of knowledge on human perception of directional tactile feedback on the torso.

1.2 Literature review

The literature study extracted ten useful articles after two selection rounds. In appendix II, the flow chart of the literature selection can be found. This appendix also shows a table with summarized findings of the literature review. In the following paragraphs some of those findings are discussed in more detail.

1.2.1 Earlier research in perception of directional stimuli

Most findings of the literature review focus on tactors placed in a horizontal line around the torso, whilst less information is available on directions on the vertical axis. It was observed that
participants in horizontal directional localisation tasks showed a bias towards the sagittal plane (navel and spine). This means that when a participant had to guess the direction of a stimulus presented just on the right or left side of the navel, participants often guessed the direction of mid front (Van Erp, 2005, 2007; Yang, Jang, & Kim, 2002). Figure 1 presents these results in a graph. Furthermore, the variability in answers was least at the frontside and backside of the participant, compared to the sides (Van Erp, 2007; Yang et al., 2002). Van Erp (2007) used a set-up where people were standing in the middle of a circular shaped table, while they indicated a direction by turning a rotary knob in front of them which was connected to a laser pointer projected from the ceiling.

![Figure 1](image)

*Figure 1. Bias (difference) between stimulus and indicated direction) per tactor angle (Van Erp, 2007).*

Different theories try to explain how people perceive a vibration on the torso as an external direction. The most basic explanation suggests that people estimate a certain direction by imagining a line which originates in an internal fictional point (within their body) and travels through the place where they feel the vibration. Van Erp (2007) changed this theory or model to an internal point for each body half (left and right), which might explain the found bias towards the sagittal plane. However, it is unclear which point is used to perceive directions very close to the navel and the spine, whilst results suggest that in those places the least bias is found. The current study therefore introduces the circular and ellipse model. The circular model (CM) is similar to the theory with the single internal point, but assumes an internal circle. This model
suggests that people imagine a line originating perpendicular to the same relative location of
the internal circle with unit circumference, as the relative location of where they feel the
vibration on their torso. This circle can be deformed to an ellipse which is flatter on two sides,
to explain a bias towards the sagittal plane (ellipse model, EM). In appendix III a figure is
attached to better illustrate the different models, and formulas concerning the mathematics of
ellipses are presented. The current study attempts to test whether a circular model (CM) or
ellipse model (EM) might be used to explain perceived directions through vibro-tactile cues.

1.2.2 Earlier research in cuing techniques

Examples of studies using a vertical tactile display on the torso, are experiments with pilots
during decreased vision conditions (Groen et al., 2009; Jansen, Wennemers, Vos, & Groen,
2008; Van Erp et al., 2007). Jansen and colleagues (2008) used 21 tactors placed vertically on
the pilot’s back during helicopter landing. When the helicopter was lower than 45.72 meters,
the lowest tactor was activated continuously with a 200ms on/off pattern to indicate the ground.
Simultaneously, the current height of the helicopter was also indicated with a 200ms on/off
pattern, but anti-phased with the reference stimulus. This cuing technique of presenting the
target stimulus together with a reference stimulus indicating the ground, was perceived as very
informative (Jansen et al., 2008). However, this cuing technique was not compared with a
technique of only offering the target stimulus. Besides, no speculations about a model of how
people perceive vertical directions were found. (Groen et al., 2009; Jansen et al., 2008).

1.3 Aim of the Study

The goal of the current study was to test whether the circular (CM) or ellipse model (EM)
best predicts the perceived direction when stimulating a certain location on the torso. This might
in turn be used to explain which location on the torso has to be stimulated in order to indicate a
certain external direction. The study divided external directions into vertical and horizontal
directions. Besides, two tactile cuing techniques were used; a single stimulus technique as used
by Van Erp (2007) where only the target stimulus was used, and a reference stimulus technique similar to the study of Jansen and colleagues (2008). The study explored whether a reference stimulus on the navel might reduce biases to the sagittal plane and which cuing technique was preferred by the participants.

To test the two models, they were fitted to the data and compared. Besides, the optimal shape of the ellipse model was estimated for every participant (see appendix III). In order to measure both horizontal and vertical directions, a virtual reality (VR) environment was created. The expectation was to receive similar findings as Van Erp (2007) in the horizontal condition with the single stimulus technique. Similar findings in bias might suggest that the VR environment is an appropriate way for testing directional feedback. The research questions of the study are as follows:

1. Which model predicts perceived direction best, when stimulating a certain location on the torso:
   a. In the horizontal line?
   b. In the vertical line?

2. Is there a difference in performance and preference between the two tactile cuing techniques?

3. Is the VR environment a valid measurement for external directions via tactile stimulation on the torso?

2 Methods

2.1 Participants

An overall sample of 19 participants joined the current study. Due to technical issues only the data of the last 14 participants was used. Nine of those 14 participants were female. Their age ranged from 18 to 29, with an average of 21.64 (SD= 3.45). All participants were right handed, except for one. Ten Participants had (positive) former experience with virtual
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reality. One participant experienced vibrating tactors on the skin before. None of the participants had a tremor. This information was gathered before the experiment through a questionnaire.

Participants were asked to only sign up for the experiment if they had a waist circumference between 75 and 100cm. Although some participants turned out to be smaller, they fell within the above range whilst wearing the T-shirt. Only one participant with a circumference of 101.5 cm fell out of the range, but was not excluded. The average circumference of participants wearing the shirt, was 87.54 (SD= 7.53). The torso length was measured from the jugular notch to the top of the hips and was in average 40.79 (SD= 3.51). The researcher always checked whether the tactors were placed on the abdomen, even when they were not placed within the determined torso length.

Participants were recruited using the Sona system and oral communication amongst students of the University of Twente. Participants were rewarded with two university credits for their involvement in this study. People participated voluntarily.

2.2 Apparatus

A shirt was made to fit all horizontal and vertical tactors in one place. Photo 1 (left) (see Appendix I - Photos) shows how the strings were placed within the T-shirt. The tactors were worn on the inside, over the shirt of the participant. The vertical string contained nine tactors and the horizontal string 16, of which only the first 15 were used. Tactors were sewn into the shirt with a distance of five centimetres in between. The shirt had a circumference of 105 cm and was lightly stretchable. This meant there was a space on the back, in between both ends of the horizontal string, of 35 centimetres. Elastic bands were used to fit the tactors closer to the skin and the shirt was tightened on the back and shoulders if needed, see photo 1 (right) (see Appendix I - Photos). Depending on the body shape, participants had different tactor locations.
Each string had to be connected separately, given the condition. Tactors vibrated on the highest intensity level which is clearly detectable.

For the set-up of the experiment, a virtual environment was created in Unity. The environment contained a gridded sphere. In the middle of the sphere, a platform was placed. The participant point of view was as if he or she was sitting on the platform. The platform was designed to prevent a feeling of falling. To indicate the starting position, a canvas was placed in front of the platform. The canvas also gave information about the trial and condition, and was used by the participant to start the trials self-paced. Photo 2 (see Appendix I - Photos) shows the view of the participant. The study used the VIVE HTC VR glasses and controllers.

A controller (which is also shown in photo 2) was connected to the environment. With a laser, the participant could see where he or she was pointing to. The controller was used to save the perceived direction of the participant. Through a link with the string in the shirt, the answers were saved along with the stimulus location. No instructions were given regarding which hand to use for the controller.

A headset with white noise was used to prevent localisation by sound. Lastly, two short surveys were made and used to ask participants about personal characteristics and their preference for a cuing technique (see Appendix V – Surveys).

2.3 Design

People’s perception of direction by vibro-tactile feedback on the torso was measured using two different cuing techniques: single vibration (SS) and vibration with a reference stimulus (RS). Two different direction axes (horizontal and vertical) were measured. Each participant went through four conditions or blocks; HSS (horizontal single stimulus), HRS (horizontal reference stimulus), VSS (vertical single stimulus) and VRS (vertical reference stimulus). Accordingly, the current study was designed as a 2X2 within-subject-study to explore the effect of two types of vibration cuing to provide information about stimuli on two different
direction lines. The tactors vibrated 300ms in the single stimulus conditions. In the reference conditions, the reference tactors (tactor five for vertical and tactor eight for the horizontal condition) vibrated 100ms, and after a pause of 50ms, the target stimulus vibrated 300ms.

2.4 Procedure

Before the start of the experiment, each participant read the study information and consent to take part in the study (see Appendix IV - Study information and informed consent). When the informed consent was signed, the researcher measured the circumference of the waist and the length of the torso of the participant. Afterwards the participant was asked to fill in a survey, which included personal characteristics and demographics (see Appendix V - Surveys). In the end, not all data from the surveys was used for the experiment. The researcher helped participants into the shirt, tracked the exact location of each tactors and noted it down in the score table (see Appendix VI – Score table). The participant was then instructed about the tasks. The virtual reality glasses and headset were placed on the participant’s head. Each participant sat down on a stool fixed to a spot, but able to rotate. This spot was located in the middle of both the lab room and the virtual sphere. Participants were asked to sit straight in front of the canvas at the start of each trial. The experiment started when the experimenter launched the program and stimuli. After each block of stimuli, the connection was readjusted in order to minimize connection errors.

A participant had to point with the controller and click ‘start’ to start a new block and ‘next’ to start each vibration. Only after feeling the vibration, participants could move to point towards the direction which they perceived. They were asked to hold their hands on the side of the chair during the vibration, to exclude tactile perception of the stimulus location through their hands. A virtual laser point showed where the controller pointed within the virtual environment. By clicking the controller, the answer was saved and a red dot appeared. The red dot indicated the last answer, and disappeared with the next ‘shot’. After each indication,
participants had to move back to sit straight in front of the canvas again and click ‘next’ to start the next trial. Photo 3 (see Appendix I - Photos) shows participants during the experiment. After the final block, participants were asked to complete the second survey about their preference in tactile cuing technique. This was the last part of the procedure.

The order of the four blocks was counter balanced over the participants to control for learning and fatigue effects. Participants perceived the trial order within each block in a random order. The stimulated tactor within a trial always had a distance of at least two tactor positions from the trial before. In one block, each stimulus was presented seven times. For the horizontal conditions this meant a total of 105 trials, since 15 different stimuli were presented. In the vertical conditions only nine stimuli were discriminated, so those blocks included 63 trials. Before each block, seven randomly chosen trials were performed to practice with the new condition. In between the blocks and before the first block, the participant had to answer six questions about possible sickness. These questions were taken from the simulator sickness questionnaire (Kennedy, Lane, Berbaum, & Lilienthal, 1993), and are found in Appendix VI. Since nobody showed any sickness symptoms, no further steps were taken. The total experiment took approximately one hour per participant.

2.5 Data analysis

For each participant and condition, the VR-environment saved the trial number, stimulus name and answers in terms of location on the surface of the sphere (X, Y, Z). The individual locations of the tactors on the participant’s body were matched to the locations indicated by participants on the virtual sphere. For each participant, the relative tactor location was estimated by dividing the tactor location by the person’s circumference (while wearing the shirt) for horizontal conditions, and dividing it by the person’s torso length for the vertical conditions.

Some raw data plots were made to examine outliers. For the data analyses, two-dimensional vectors were calculated per trial per participant: X and Z for azimuth and Z and Y
for elevation. Furthermore, the mean angle in which a participant had pointed after receiving a stimulus was calculated in both degrees and radians.

2.5.1 Model

To explore, in tune with the main aims of the study, whether the data better fits a circular model or an ellipse model, data was used to estimate three scenarios, as follows:

1. Perfect-Fit (PF): This represents the (unlikely) situation in which participants are able to answer (point on the sphere surface) following the exact same angle as where the tactor is placed on their body without any distortion.

2. Circular Model (CM): This represents an estimation assuming that a participant will point on the sphere following a line which initiates from a location of an internal circle which matches the relative tactor location of that participant. This estimation was compared to the real answered direction.

3. Ellipse Model (EM): A comparison of the real answered direction with a direction following a line which initiates from a location on an internal ellipse through the same relative location of the tactor on the participant’s torso. The best fitting shape of this ellipse with unit circumference was estimated with use of Microsoft Excel© spreadsheet and its Solver© tool. A shape was best fitting for a participant (in a certain condition) when the estimations were closest to the real answered locations. The Solver tool concentrated on finding the optimal ‘fraction b’ of an ellipse (see figure 2) using the generalized reduced gradient (GRG) nonlinear algorithm and the constraint to find a value bigger than zero. The current study always used a ‘fraction a’ of 1, thus a circle also has a ‘fraction b’ of 1.
For each participant the following data analysis was performed:

i. The parameters of the three scenarios were estimated and a plot was created to compare both CM and EM with PF.

ii. The goodness of fit of circular data was calculated, where the higher goodness of fit (GF) among the models was considered the best model to explain the data of the participants.

iii. To determine the difference among the three models, either a paired t-test or Wilcoxon Signed-Ranks (S-R) test (depended on whether the data was normally distributed according to the Shapiro-Wilk normality test) was used to see whether GF values of one model differed significantly from the other model. Moreover, the ‘fractions b’ of the EM within the horizontal condition were tested to be significantly different from 1. In fact, in the horizontal condition, if ‘fraction b’ is equal to 1, this will suggest to adopt a circular model.

Those results were also used to test the validity of the VR-environment (research question 3). This was tested by comparing HSS results to the results found in the study of Van Erp (2007) and Yang and colleagues (2002). That is to say, whether the results also show a bias towards the sagittal plane. The shape of the internal model already indicates whether there is a found bias, but this bias was also visualized by plotting the best fitting EM together with the
CM on even direction angles per relative location. The difference between those models illustrates the found bias.

Only data from conditions HSS and HRS were transformed to calculate the CM and EM, because relative tactor locations in these conditions are equal to a location on an imaginary circle with normalized circumference. Conversely, in the vertical conditions it is not possible to estimate with accuracy which part of a virtual circle participants were pointing at. For this reason it was decided to exclude the vertical conditions from the transformations. However, the average scope of directions which might be possible to display on the vertical plane, was determined by taking the mean values of answered angles at a relative tactor location of zero (jugular notch) and one (top of hips) in the linear trend per participant. Also the average relative tactor location at which participants perceived the direction as mid front was calculated.

2.5.2 Cuing technique

The favourite cuing technique was measured both in a subjective and objective way. During the subjective analysis, preference of a cuing technique was measured by counting the responses of the second questionnaire. The objective evaluation consisted out of two ways to explore whether there were differences in how people performed between the two cuing techniques. In one part, performance was explained through constancy. A Wilcoxon S-R test was performed with the median of the mean vector variations per participant, where HSS was grouped with HRS and VSS with VRS. It was studied whether participants answered more consistently during one technique, compared to the other. In the second part, a paired sample t-test or Wilcoxon S-R Test was conducted with the mean goodness of fits per participant. This was used to see whether one cuing technique had a significant better fit compared to the other condition. This was done for both CM and EM.

3 Results
When reviewing the data, it was discovered that eight trials were not recorded: one in HSS, four in HRS and three in VRS. Through raw data plots, trials where the answer was saved in the middle of the sphere were revealed. These were probably mistakes where participants pointed at the instruction canvas, instead of the sphere. Twenty-nine of those cases were excluded: one in HSS, seven in HRS, 10 in VSS and 11 in VRS. It was not clear whether other outliers of the data were the result of errors or answer variation. Besides, deleting those outliers only gave a small difference in the mean answered angle in vector. Because of this reasoning, no other outliers were deleted.

The raw data underwent some transformations. In Appendix VII an example of raw data transformations for one participant in the HSS condition are shown.

3.1 Model

3.1.1 Horizontal model

To study whether the data fitted better within a circular (CM) or ellipse model (EM), a plot as in figure 3 was made for each participant in both the HSS and HRS condition. Figure 3 shows the CM and best EM, compared to the PF for participant number six in condition HSS. The X-axis indicates the relative location of the activated stimuli, where 0 and 1 is middle front and .5 is at the back of the participant. Relative locations on a horizontal line along the surface of the virtual sphere are indicated on the Y-axis, where the mean answers per tactol location are fitted to the models.

These models give insight in individual biases and model fitting. In this specific case, the graph belonging to the data of participant number six shows that the EM lies closer to the PF, compared to the CM. This suggests that for this participant the EM is a better fit. This can also be seen by looking at the goodness of fit (GF) values for this participant, where the GF is highest for the ellipse model. Furthermore, the measured points of this particular participant lie closest to the PF at tactol locations close to the navel.
Appendix VII contains an example of the transformations executed for one participant to calculate the values used in figure 3. Outcomes of these transformations for all participants are summarized in table 1. The goodness of fit (GF) values for each participant in HSS and HRS are shown, which indicate how the models CM and EM fit the found data. For HRS, some GF values turned out negative and were excluded. The ultimate ‘fraction b’ for the EM is also shown. This value was delivered by Microsoft Excel’s Solver© by finding a shape of ellipse which matched the best fitting EM (with the highest GF possible). Since the solver tool could not find an optimal ‘fraction b’ for participant seven in condition HRS, this value is missing.

As table 1 shows, the GF for each participant is higher for EM, which might suggest a better fit than CM. Since the data was not normally distributed for HSS (W=.52, p<.001) and HRS (W=70, p=.001) according to the Shapiro-Wilk normality test, a Wilcoxon S-R test was executed to test whether the EM better fits the data compared to CM. Both in HSS (Z=-3.30, p< .001) and in HRS (Z=-2.80, p< .05), EM fitted the data significantly better compared to CM.
Table 1. GF (goodness of fit) values and ‘fraction b’ values for all participants in the horizontal conditions. Negative GF values are deleted.

*Note.* CM = circular model, EM = ellipse model.

<table>
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<th>Participant</th>
<th>GF CM</th>
<th>GF EM</th>
<th>Best ‘fraction b’ EM</th>
<th>GF with ‘fraction b’ of 1.35</th>
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<td>16.46</td>
</tr>
<tr>
<td>14</td>
<td>0.96</td>
<td>0.98</td>
<td>1.38</td>
<td>0.98</td>
<td>0.78</td>
<td>0.80</td>
<td>1.48</td>
</tr>
<tr>
<td>15</td>
<td>0.94</td>
<td>0.94</td>
<td>1.20</td>
<td>0.94</td>
<td>0.77</td>
<td>0.78</td>
<td>1.35</td>
</tr>
<tr>
<td>16</td>
<td>0.99</td>
<td>0.99</td>
<td>1.08</td>
<td>0.98</td>
<td>0.45</td>
<td>0.68</td>
<td>2.57</td>
</tr>
<tr>
<td>17</td>
<td>0.96</td>
<td>0.97</td>
<td>1.27</td>
<td>0.97</td>
<td>0.96</td>
<td>0.98</td>
<td>1.33</td>
</tr>
<tr>
<td>18</td>
<td>0.86</td>
<td>0.89</td>
<td>1.62</td>
<td>0.89</td>
<td>-</td>
<td>0.65</td>
<td>6.70</td>
</tr>
<tr>
<td>19</td>
<td>0.98</td>
<td>0.98</td>
<td>1.26</td>
<td>0.98</td>
<td>0.93</td>
<td>0.93</td>
<td>1.08</td>
</tr>
<tr>
<td>Mean</td>
<td>0.91</td>
<td>0.94</td>
<td>1.35</td>
<td>0.93</td>
<td>0.77</td>
<td>0.79</td>
<td>3.05</td>
</tr>
<tr>
<td>Median</td>
<td>0.96</td>
<td>0.97</td>
<td>1.26</td>
<td>0.96</td>
<td>0.84</td>
<td>0.79</td>
<td>1.35</td>
</tr>
</tbody>
</table>

To test the benefit of EM in another way, a one-sample t-test was used to see whether the found ‘fractions b’ differed from the ‘fractions b’ of the circular model (‘fraction b’ of one).

The ‘fractions b’ in HSS were normally distributed (W=.94, p=.39). With a test value of one (indicating the ‘fraction b’ of a circular model) it was suggested that the found ‘fractions b’ of the HSS condition, significantly differed from one (t_{13}=3.59, p< .05). The mean ‘fraction b’ in HSS was 1.35 (SD=0.36). This suggests that the found ellipse in this condition has a significantly different shape than a circle which is in line with the results of GF. Figure 4 visualizes the biases with a ‘fraction b’ of 1.35 and shows that these biases are oriented towards the navel and spine. Lastly, the shaded column in table 1 shows how an EM with a ‘fraction b’ of 1.35 fits the data. It is observed that the mean fit here is better than the mean circular fit, but not for every participant. Also a Wilcoxon R-S test showed that this difference in medians is not significant (Z=-1.48, p=.15). To conclude, although the shape of an ellipse with a ‘fraction b’ of 1.35 is significantly different from a circle, this shape does not fit better than a CM for every participant.
Perceiving Directions through Vibration

In HRS however, the ‘fractions b’ were not normally distributed (W=.60, p<.001) and a Wilcoxon S-R test was used where the median of ‘fractions b’ of all participants were paired per model (GM and CM). It was found that the fractions significantly differed from each other (Z=-2.34, p<0.05). This suggests that the found ellipses in this condition has a significantly different shape than circles. It is noteworthy however, how the ‘fractions b’ differ per participant in HRS.

3.1.2 Vertical Model

Although the vertical conditions were not included in the transformations to discover an internal model, a short analysis was conducted to determine the scope of displayed directions. For each participant, in both vertical conditions, a plot was made of the mean answered location on the sphere surface (radians) as a function of the activated relative tactor location. A linear trendline was added to the function. Figure 5 shows an example of such a plot with trendline, for participant number six in VSS. The formula belonging to this trend, was used to predict
answer locations per relative tactor location. Two participants (in VSS and VRS) had a trendline with a very low R-squared value (respectively .03 and .40) and were excluded. All other R-squared values were .64 or above. By using the formulas of each trendline, the average range in the vertical conditions was estimated and later transformed from radians into degrees. Besides, the mean relative tactor location that was perceived as mid front was estimated.

![Perceived direction per relative tactor location with linear trend](image)

*Figure 5.* Mean answered location on the sphere surface (Y) as a function of the activated relative tactor location (X) for participant number 6 during the vertical single stimulus condition (VSS).

On average, participants indicated directions with a maximum angle of 6.57 degrees and a minimum angle of -29.95 degrees in the single stimulus condition (VSS). On average, participants answered the mid front location on the sphere surface when they felt a vibration at the relative location of .13. This is relatively high since zero is at the height of the jugular notch and one at the top of the hips. As seen in table 2, these data did not differ much in the reference cuing condition.

*Table 2. Scope of vertical directions.*

*Note. VSS = vertical single stimulus condition, VRS = vertical reference stimulus condition*

<table>
<thead>
<tr>
<th></th>
<th>VSS</th>
<th>VRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum angle</td>
<td>6.57° (SD=11.79)</td>
<td>7.66° (SD=9.81)</td>
</tr>
<tr>
<td>Minimum angle</td>
<td>-29.95° (SD=14.82)</td>
<td>-29.96° (SD=14.10)</td>
</tr>
<tr>
<td>Perceived middle (relative tactor location)</td>
<td>.13 (SD=.27)</td>
<td>.18 (SD=.25)</td>
</tr>
</tbody>
</table>
3.2 Cuing technique

3.2.1 Subjective evaluation

In the questionnaire, 12 out of 14 participants responded that they preferred the single stimulus (SS) technique. Given arguments were all very similar. Examples of argumentation for the single stimulus technique were: “Less confusing and more easy to focus your attention on the direction”, “Because I only had to focus on one stimulus, the other technique needs more concentration” and similar. One participant suggested: “If the reference technique had a slightly longer delay between the two vibrations, then that one would have been better. They were too fast and sometimes I mixed them up”.

Quotes from participants that preferred the reference technique were: “I had a reference on where the middle was and I could be ready for the stimulus”, and: “For vertical, reference is necessary because of the small length of vertical, and the nodes are really close to each other. So, there is higher chance of confusing between the targets. For horizontal, it is not really necessary, I felt”.

3.2.2 Objective evaluation

The median vector variation in HSS was 0.83, while in HRS it was 0.74. This data was not normally distributed (W=.84, p<.05). A Wilcoxon signed-rank test showed that there was no significant difference between the variations (Z = -1.41, p = .17). This suggests that participants did not answer more consistent in either of the horizontal conditions. Also no significant difference in the vertical blocks between the conditions with and without reference stimuli were found (Z = -0.09, p=.95). The rounded median vector variation was both 0.03 in VSS and VRS.

Table 1 shows that both ellipse and circular means and medians of the GF values for HSS are higher than for HRS. The differences in GF values of HSS and HRS were normally distributed for EM (W=.91, p=.29), but not for CM (W=.84, p<.05). A paired t-test suggested
that the data did not fit better in any cuing condition for EM \( (t_{12}=1.77, p= .10) \). A Wilcoxon signed-rank tests showed that the difference in medians was not significant for CM either \( (Z=-1.78, p= .08) \).

4 Discussion

The goal of the study was to test whether there is a model which explains how directions are perceived via vibrations on the torso and if this model is circular or ellipse shaped. Besides, it was investigated if offering a reference stimulus would benefit or change this model.

4.1 Conclusion

4.1.1 Model (research question 1: Which model predicts perceived direction best, when stimulating a certain location on the torso?)

The experimental outcomes partially answered the question. Results could be summarised as follows:

- **Horizontal axis:** The results concerning the goodness of fit (GF) values show that the (on average) best fittings were obtained with the ellipse model (EM) instead of the circular model (CM). Although the EM is suggested to be a better fitting model than the CM, much variability between participants is shown and an EM with a universally best fitting shape could not be created.

- **Vertical axis:** Due to complexity, a model for the vertical axis turned out to be beyond the scope of the current study. However, the study provides some conclusions about the scope which could be displayed within the vertical directions. In average this scope is quite small and low oriented: from around seven degrees above the horizontal until 30 degrees under the horizon, regardless the cuing condition. Likewise, it is suggested that people perceived a point in the middle of their chest as the middle point of the vertical axis of the sphere.
4.1.2 Cuing technique (research question 2: Is there a difference in performance and preference between the two tactile cuing techniques?)

Many participants mentioned that they were confused when a reference stimulus (indicating the middle tacton) was used, before the target stimulus (indicating the direction). Participants also noted that the pause in between the stimuli was too short (50ms) and that it took a lot of concentration to identify the direction stimulus. Despite their preferences, no significant effect could be found when evaluating the cuing technique in an objective way.

4.1.3 Virtual reality environment (research question 3: Is the VR-environment a valid measurement for external directions via tactile stimulation on the torso?)

To judge whether the method (of using virtual reality to indicate and save external directions with a laser point on a sphere around the participant) is valid, results of the current study (HSS) were compared to results of earlier studies (Van Erp, 2007; Yang et al., 2002). The found mean ‘fraction b’ in HSS of 1.35, might indicate an ellipse which is longer on the left and right, compared to the front and back (similar to the shape in figure 2). Figure 4 showed how an ellipse with such a shape suggests a bias towards the sagittal plane, which is similar to the results of Van Erp (2007) presented in figure 1. However, participants in the current study varied a lot in their answers and this is not shown in the model of figure 4, since the model focusses on the average ‘fraction b’ of all participants together. When looking at individual scores, people sometimes never answered to directions behind them, whilst tactons were indeed placed on their back.

4.2 Limitations

The study encountered several limitations which will be discussed in the current section. Some limitations related to technical aspects such as the virtual environment or the vibrating tactons. Other limitations are probably due to physical limitations or the design of the study.

4.2.1 Technical limitations
Six participants mentioned that they sometimes felt two vibrations at the same time in the conditions with single stimulus technique, or three during the reference conditions. These unexpected reports of participants could be due to misalignment within the connection between the Unity program of the VR system and the tactors string. Unfortunately, those specific events were not identified and could therefore not be excluded. This might have played a role in the identified variability of participants’ answers.

As explained in section 4.1.3, some participants almost never answered at the backside of the sphere. This might be caused by the few orientation points within the virtual environment. The participant could only detect his/her position within the sphere by the canvas which indicated mid front and the red dot of their last answer. The lack of more orientation cues might have also contributed to the high variability within the data compared to the studies of Van Erp (2007) or Yang (2002), where participants had more visual cues of orientation. In fact, due to the need of offering a reference point to participants in a virtual immersive system, and because of some technical limitation of the Unity program, the middle of the sphere was a platform. Participants were placed a bit above this platform, so that people had the idea that they were sitting on top of it. Therefore, the viewpoint of the participants was a bit higher positioned on the Y-axis of the sphere, compared to the middle. In other directions (X and Z) the viewpoint was in the centre. Also the shoulder joint, from where people move their arms, was a bit above the middle of the sphere. It can be discussed which point should be defined as the middle of the sphere: the point of view of the participant, the place where people move from (shoulder joint) or the ground (as was done in this study). However, in this study, the relative position might need an adjustment to better interpret vertical data; the vertical range in which people answered (table 2), might be even lower if the participant had a lower viewpoint.

4.2.2 Physical limitations
The biggest uncertainty in this study was the big variability which was found in answers per participant and between participants. For example, in individual plots like in figure 3, the ‘s’-shape as in figure 1 of Van Erp was not found because of the big variety in answers. The mean vector variation was .82 in HSS and .70 in HRS. It can be discussed whether this variability is a consequence of the virtual environment, the connection with the tactor string or whether it is related to another cause such as physical limitations. The current study only controlled for fatigue by balancing conditions over participants. Besides, participants were asked whether they suffered from tremors. However, the study did not control for physical limitation by calibrating the controller per individual or by practicing on a visual cue in order to discover whether participants could point accurately and constantly. Besides, the research did not study whether participants differed in their physical way of pointing. For example, a participant might have point to the back by turning left or right. Participants also differed in the way they held their controller, some with only their dominant hand, others with two hands. It is not studied whether those physical differences influenced accuracy or constancy.

A reason that people perceived a point in the middle of their chest as a direction from the middle point of the sphere, might be that the chest is close to the shoulder joint, which is the point from which people move their arms. If people have to point straight in front, they might move their arms horizontally from that point.

The problem that some people did not respond at the back at all, might also have had a physical cause. As explained in the introduction, pressure is differently perceived on different parts of the body. People experience lower sensitivity in the back than in the abdomen (Van Erp & Werkhoven, 1999).

4.2.3 Design limitations

The sample size of the current study was quite small (14 participants). However, the sample size is similar to the study of Van Erp (2007) (10 participants) and Yang (2002) (20
participants) and fits an explorative study. Nevertheless, a bigger sample size might have covered for the current found variability.

Another issue concerning the participants is that nine participants were female. The results are compared to Van Erp (2007) and Yang (2002), which only used male participants. It was sometimes hard to place the tactors close to the skin on the female chests. One person did not even feel certain tactors in the vertical conditions (this person was one of the first five participants and thus excluded). This difference in gender might also partially explain why the accuracy was lower in the current research compared to earlier studies.

Participants felt vibrations on their torso. These were stimuli presented within the peripersonal space, immediately surrounding the body (di Pellegrino & Làdavas, 2015; Previc, 1998). The directions on the other hand, had to be indicated on the surface of an outer sphere. This sphere lied within the extrapersonal space, outside grasping distance (di Pellegrino & Làdavas, 2015; Previc, 1998). Not every application of tactile feedback will use a similar presentation of direction. For example the Neuroshirt or navigation for pilots, will use tactile feedback in order to move tools within the peripersonal space. It can be discussed whether the current study used the right design for those applications.

Lastly, the current study investigated whether the GM fitted the data better than the CM, while the GM had more parameters than the CM. This might be an unequal benefit and it is questionable if the same results would have been found if the study had controlled for this.

4.3 Future recommendations

To continue research on displaying external directions by vibrations on the torso, the following design recommendations are suggested:

1. The virtual reality environment should be optimized with visual reference cues, such that participants know in which direction they are pointing. It should also be discussed
beforehand which point should be the middle of the sphere, the viewpoint, the shoulder joint or a virtual floor.

2. Alternative cuing techniques might be studied again. For a reference cuing technique, it is recommended to take a longer break in between the stimuli. Variations in the intensity and duration of the stimuli could be made. Besides, the reference stimulus might be given on the chest which people received as their ‘middle point’ in the current study. However, different techniques might differ in usefulness per application.

3. Physical limitations and learning effects should be studied or controlled for. It is also suggested to study whether physical limitations are related to gender or other factors.

4. It should be studied whether there is a model which fits the vertical axis of the torso. This way, predictions might be made on how people perceive three dimensional directions.

4.4 Application of research: Neuroshirt

The current research is useful for many applications. In this section, it will be discussed how the results might be useful for the development of the Neuroshirt (see section 1.1.2) and which topics are advised to be studied more accurately.

The optimal Neuroshirt would base the shape of the EM on the surgeon’s individual biases. However, this would be an expensive and difficult solution. Although the current study does not provide prove that an EM with a ‘fraction b’ of 1.35, fits better than CM for every person, a trend is visible. Therefore it is suggested to use this model instead of the CM.

The results for the vertical axis suggest that the range which can be displayed, is quite small and downwards. Since the skull is quite small, this should not be a problem. To predict stimuli on this axis, more research should be executed.

It is suggested to use a single stimulus technique to display directions. Participants preferred this technique and it is easier to predict how people respond. Besides, presenting a
reference stimulus might be too risky for the precise task of drilling through a skull, since it introduces a feedback delay.

The current results are discussed with neurosurgeons and engineers within the Neuroshirt team. Based on this meeting and the discussion of the current study, some recommendations for future research concerning the Neuroshirt are given:

1. Studying how people link vibrations on the torso with movement of a drill within a skull (peripersonal space). This application might differ from pointing towards a location on the surface of a sphere around the participant (extrapersonal space). It is suggested to continue research with a simulator devise, to simulate the tasks which neurosurgeons have to perform.

2. The task of a neurosurgeon to drill through a skull, is a task which is practised multiple times. Surgeons will also practice with the Neuroshirt before use. Therefore, the learning effects in recognizing directions via vibro-tactile cuing should be studied. There might be a possibility that users could learn directions connected to a vibration, which might mean that it does not have to be completely intuitive.

3. The idea of the Neuroshirt is to use tactile feedback as an addition to visual cues of the microscope and possibly visual navigational warnings. Perceiving directional information via both visual and tactile cues, should be studied together. It is expected that the addition of visual signals will benefit the tactile cues of the Neuroshirt, since multimodal cues usually capture the spatial attention of a practitioner working under highly loaded conditions (Baldwin et al., 2012; Spence, 2010).
Literature


Soesterberg, The Netherlands.


Yang, U., Jang, Y., & Kim, G. J. (2002). Designing a vibro-tactile wear for close range interaction for vr-based motion training. In *International conference on artificial reality*
and telexistence (pp. 4–9).
Appendix I – Photos

Photo 1. T-shirt with tactors.

Photo 2. Screenshot of the participants view within the virtual reality environment.
Perceiving Directions through Vibration

Photo 3. The experimental set-up.
Appendix II – Figures from literature review

Figure 1. Flowchart of literature selection.

Note. Selection based on title also included exclusion of duplications.
Table 2. Summary of publications in the field of conveying directional feedback into vibro-tactile stimulation

<table>
<thead>
<tr>
<th>Publication</th>
<th>Main topic</th>
<th>Methods</th>
<th>Main findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Van Erp &amp; Werkhoven, 1999, as cited in Van Erp, 2000)</td>
<td>Classification of spatial vibro-tactile resolution on torso</td>
<td>Left/right localisation task with 11 vibro-tactors taped on torso</td>
<td>Higher sensitivity on the ventral side compared to dorsal, higher sensitivity in middle compared to sides</td>
</tr>
<tr>
<td>(Van Erp, 2000)</td>
<td>External horizontal direction determination on torso (2D)</td>
<td>12 Vibro-tactors taped on torso, 1 activated at the time, participant indicated direction with pointer on circle</td>
<td>Answer variability and bias in direction indication, mainly towards the navel and spine (sagittal plane)</td>
</tr>
<tr>
<td>(Van Erp, Van Veen, Jansen, &amp; Dobbins, 2005)</td>
<td>Waypoint direction and distance presented on the waist</td>
<td>Pedestrians, a helicopter pilot and a boat driver used waypoint navigation via 8 tactors on elastic band around waist</td>
<td>Directional waypoint navigation was successful in both experiments, implementation of distance did not have additional value</td>
</tr>
<tr>
<td>(Van Erp et al., 2007)</td>
<td>Detecting presence and location of threads with visual and tactile displays (3D).</td>
<td>9 Pilots had to identify targets in a flight simulator via either a visual display or a visual and tactile display (5x12 tactors)</td>
<td>Tactile cues were more favourably than visual, and RT and chase time were faster with addition of tactile display</td>
</tr>
<tr>
<td>(Van Erp, Kroon, Mioch, &amp; Paul, 2017)</td>
<td>Information overload and processing capabilities when using tactile obstacle detection (direction, distance, height and ID) for visually impaired users</td>
<td>4 Experiments performed with a 3 x 3 tactors display on abdomen</td>
<td>Displayed parameters should be minimalized. Horizontal tactor information is advised. Vertical requires extra tactors, is more complex, expensive and less user friendly</td>
</tr>
<tr>
<td>(Van Erp, 2005)</td>
<td>Horizontal direction determination on torso through internal reference point or torso’s curvature</td>
<td>15 Vibro-tactors taped on torso, 1 activated at the time, participant indicated direction with pointer on circle</td>
<td>Confirms that directional information can be localized with single stimulus activation. Reference</td>
</tr>
<tr>
<td>Reference</td>
<td>Description</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(Van Veen &amp; Van Erp, n.d.)</td>
<td>Horizontal and vertical position feedback to maintain stable hover with helicopter. 12 columns and 5 rows of tactors on chest and tactors on each shoulder and thigh. Tactile display improved performance during reduced and normal vision.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yang et al., 2002)</td>
<td>Designing a 3D vibrotactile display to assist a VR motion training system. Participants felt movement of a virtual object through a 5x12 tactile display on the torso. Six conditions of the object: 1D line, 2D plane, 3D sphere x slow speed, fast speed. Least directional error at a slow moving 1D line. More accurate responses in orthogonal directions. Expect that 8 instead of 12 tactors will be 10% more accurate.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Cholewiak et al., 2004)</td>
<td>Vibro-tactile localization on the abdomen. 4 Experiments performed with a tactor display on torso ranging from 5x12 to 5x6 tactors and a cylindrical response keyboard with the same amount of keys. Localisation bias towards navel and spine (sagittal plane). Performance decreased and RT increased in conditions without stimuli on the spine and navel.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Groen et al., 2009)</td>
<td>Personalized Fighter Aircraft Multimodal Cockpit. Simulation user evaluation of the multimodal cockpit with horizontal and vertical resolution tactor display (5x12) for different purposes (e.g. threat indication). Pilots found direction indications clear, but tactile display could be optimized with smoother coding and increased alerting value during high cognitive workload.</td>
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Appendix III – Extra information about the models

Visualization of the ellipse (EM) and circular model (CM)

Figure 6. Ellipse model (EM) and circular model (CM)

The outer green shape in figure 6 presents the top view of a participant’s waist, with nose on top (looking straight). The small shapes are different internal models (one circle (black) and two ellipses (purple and green)) with the same normalized circumference. The dashes of the ellipses are all equal in length. The straight lines indicate perceived horizontal direction (with equal stimulus location) on an external circle for the ellipse model. It can be seen that the shape of the internal model can cause a bias in perceived direction. The green ellipse for example, has a bias towards the sagittal plane.

Estimation of ellipse circumference:

\[ C \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right] = \pi \left[ 3(a + b) - \sqrt{10ab + 3(a^2 + b^2)} \right] \]

Distance on ellipsoid

\[ m(\varphi) = \frac{a + b}{2} \left( H_0 \varphi + H_2 \sin 2\varphi + H_4 \sin 4\varphi + H_6 \sin 6\varphi + H_8 \sin 8\varphi + \cdots \right), \]

With:

\[ H_0 = 1 + \frac{1}{4} n^2 + \frac{1}{64} n^4 + \cdots, \]
\[ H_2 = -\frac{3}{2} n + \frac{3}{16} n^3 + \cdots, \quad H_0 = -\frac{35}{48} n^3 + \cdots, \]
\[ H_4 = \frac{15}{16} n^2 - \frac{15}{64} n^4 - \cdots, \quad H_8 = \frac{315}{512} n^4 - \cdots. \]

Parametric description of ellipsoid

\[(x, y) = (a \cos t, b \sin t), \quad 0 \leq t < 2\pi.\]

https://en.wikipedia.org/wiki/Ellipse
https://en.wikipedia.org/wiki/Meridian_arc#Meridian_distance_on_the_ellipsoid
https://en.wikipedia.org/wiki/Flattening#First,_second_and_third_flattening
https://en.wikipedia.org/wiki/Ellipse#Ellipse_in_Cartesian_coordinates
Appendix IV – Study information and informed consent

**Study Information for ‘Perceiving Directions through Vibration’**

In collaboration with the department of Cognitive Psychology and Ergonomics (CPE) and Elitac, this research studies how people perceive directional information given by vibration on the torso. The study divides external direction into vertical and horizontal direction. Besides, two tactile cuing techniques are used; a technique with only one stimulus and a technique with a reference stimulus. The goal is to develop a model that explains which location on the torso has to be stimulated in order to indicate a certain external direction. This might be used for different applications, like way point navigation, navigation for the blind, or neuronavigation. Besides, the current study will do an explorative analysis of individual characteristics, in order to find possible correlations with performance or preference for a stimulus technique.

This study is approved by the BMS Ethics Committee. You as a participant will receive credits and has the benefit of experiencing being a participant in scientific research. Since this study is using virtual reality glasses, you may have the risk of experiencing cyber sickness. To minimize this risk, the experiment has breaks in which your feelings of nausea are tested. When you feel sick or nausea, the break can be extended and if it will not get better, the study may be cancelled.

In general participation is voluntary and you may withdraw from the study at any time, without giving a reason.

This study involves body size measurements, a questionnaire survey which has to be completed by the participant and an active experiment with vibrations on the torso and virtual reality. This will take no longer than two hours. All data that will be provided by you as a participant, will be anonymized and archived in the Faculty of Behavioural, Management and Social sciences (BMS) of University of Twente. The anonymized data will be used in the graduation project and might be published in scientific publications.

**Study contact details for further information:**
For further details on this study you can contact Nienke Bierhuizen (n.m.bierhuizen@student.utwente.nl) or Dr Simone Borsci (s.borsci@utwente.nl).

**Contact Information for Questions about Your Rights as a Research Participant:**
If you have questions or concerns about this study that you want discuss with someone other than the researcher(s), please contact the Secretary of the Ethics Committee of the Faculty of Behavioural, Management and Social Sciences at the University of Twente (ethicscommittee-bms@utwente.nl).

UNIVERSITY OF TWENTE.
Consent Form for ‘Perceiving Directions through Vibration’

YOU WILL BE GIVEN A COPY OF THIS INFORMED CONSENT FORM

Please tick the appropriate boxes

Taking part in the study
I understood the study information dated /2018 which I have read or have been read to me. I have been able to ask questions about the study and my questions have been answered to my satisfaction. ○ ○
I consent voluntarily to be a participant in this study and understand that I can refuse to answer questions and I can withdraw from the study at any time, without having to give a reason. ○ ○
I understand that taking part in the study involves completing a survey questionnaire by me, measurements of body sizes and an active experiment with vibrations on the torso and virtual reality. ○ ○

Risks associated with participating in the study
I understand that taking part in the study involves the risk of physical discomfort due to virtual reality sickness and that I may take a break or withdraw from the study at any time. ○ ○

Use of the information in the study
I understand that information I provide will be used in the graduation project and might be published in scientific publications. ○ ○
I understand that personal information collected about me that can identify me, such as [e.g. my name or where I live], will not be shared beyond the study team. ○ ○

Optional: I agree that my information can be quoted in research outputs, without disclosing my identity. ○ ○

Optional: I agree that a photo of the experimental set-up, with me as a participant, can be used in the graduation project and might be published in scientific publications. ○ ○

Future use and reuse of the information by others
I give permission for the survey and experiment data that I provide to be archived in the Faculty of Behavioural, Management and Social sciences (BMS) of University of Twente so it can be used for future research on directional haptic feedback. Data will be used only for the research that is carried out at BMS. ○ ○

Signatures

Name of participant  Signature  Date

I have accurately read out the information about study to the potential participant and, to the best of my ability, ensured that the participant understands to what they are freely consenting.

Nienke Bierhuizen

Researcher name  Signature  Date

UNIVERSITY OF TWENTE.
Study contact details for further information:
For further details on this study you can contact Nienke Bierhuizen
(n.m.bierhuizen@student.utwente.nl) or Dr Simone Borsci (s.borsci@utwente.nl).

Contact Information for Questions about Your Rights as a Research Participant:
If you have questions or concerns about this study that you want discuss with someone other
than the researcher(s), please contact the Secretary of the Ethics Committee of the Faculty of
Behavioural, Management and Social Sciences at the University of Twente
(ethicscommittee-bms@utwente.nl).
Appendix V – Surveys

Survey 1: personal characteristics and demographics

Perceiving Directional Feedback via Vibro-Tactile Stimulation in a Virtual Environment

Start of Block: Default Question Block

Q1 What is your gender?

- Male (1)
- Female (2)

Q2 What is your hand preference?

- I am left handed (1)
- I am right handed (2)
- I don’t have a hand preference (3)

Page Break

Q7 Do you game a lot?

- A great deal (1)
- A lot (2)
- A moderate amount (3)
- A little (4)
- None at all (5)
Display This Question:
If Q7 = 1
Or Q7 = 2
Or Q7 = 3
Or Q7 = 4

Q8 What type of games do you play?

☐ Action (1)
☐ Adventure (2)
☐ Mental orientation (3)
☐ Puzzle (4)
☐ Shooting games (5)
☐ Simulation (6)
☐ Strategy games (7)
☐ Other (8)

Q3 Do you have any experience with virtual reality glasses?

☐ No I never worn virtual reality glasses before (1)
☐ Yes I have experience with virtual reality (2)
Q4 Did you feel sick or uncomfortable in any way, during your experience with virtual reality?

- A great deal (1)
- A lot (2)
- A moderate amount (3)
- A little (4)
- None at all (5)

Display This Question:
- If Q4 = 1
- Or Q4 = 2
- Or Q4 = 3
- Or Q4 = 4

Q5 Could you describe your discomfort during this experience?

Q6 Did you ever worn clothes with tactile electronics or did you participate in a study where tactors were vibrating on your skin?

- Yes, more than once (1)
- Yes, once (2)
- No, never (3)
Q9 Did you drink coffee in the last 24 hours?

○ No I did not (1)
○ Yes, 1 cup (2)
○ Yes, 2 - 4 cups (3)
○ Yes, 5 or more cups (4)

Q10 Did you consume any alcohol during the last 24 hours?

○ No I did not (1)
○ Yes, 1 - 2 glasses last night (2)
○ Yes, 3 - 6 glasses last night (3)
○ Yes, more than 6 glasses last night (4)
○ Yes, I consumed it this morning (5)

Q11 Did you consume any drugs during the last 24 hours?

○ No I did not (1)
○ Yes, stimulants ('uppers') (2)
○ Yes, depressants ('downers') (3)
○ Yes, both stimulants and depressants (4)
○ Yes, but another type of drugs (5)
Q12 Do you feel tired?

- A great deal (1)
- A lot (2)
- A moderate amount (3)
- A little (4)
- None at all (5)

End of Block: Default Question Block
Perceiving Directions through Vibration: 2/2

Start of Block: Default Question Block

Q1 Participant number (Filled in by researcher)

Page Break

Q2 During the experiment, you experienced two cuing techniques:
   - **Single stimulus technique:** A single stimulus to indicate a direction
   - **Reference stimulus technique:** A reference stimulus to indicate the middle, followed by the stimulus indicating a direction (Both techniques were used in horizontal and vertical direction)

Q7 Which stimulus technique did you prefer?
   - [ ] Single stimulus technique (1)
   - [ ] Reference stimulus technique (2)

**Display This Question:**
*If Q7 = Single stimulus technique*

Q4 Could you describe why you preferred the **single stimulus technique**?
Q5 Could you describe why you preferred the reference stimulus technique?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Page Break

Q6 Do you have any other comments about the study?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

End of Block: Default Question Block
Appendix VI – Score table

Score table

Participant: 

<table>
<thead>
<tr>
<th>Circumference</th>
<th>Torso length</th>
</tr>
</thead>
</table>

Tactor location:
Horizontal: Measured from mid front, in direction of clock
Vertical: Measured from jugular notch, in down wards direction

<table>
<thead>
<tr>
<th>H8</th>
<th>H15</th>
<th>V9</th>
</tr>
</thead>
<tbody>
<tr>
<td>H7</td>
<td>H14</td>
<td>V8</td>
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<tr>
<td>H6</td>
<td>H13</td>
<td>V7</td>
</tr>
<tr>
<td>H5</td>
<td>H12</td>
<td>V6</td>
</tr>
<tr>
<td>H4</td>
<td>H11</td>
<td>V5</td>
</tr>
<tr>
<td>H3</td>
<td>H10</td>
<td>V4</td>
</tr>
<tr>
<td>H2</td>
<td>H9</td>
<td>V3</td>
</tr>
<tr>
<td>H1</td>
<td>V2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V1</td>
<td></td>
</tr>
</tbody>
</table>

Cyber sickness:
None (1) – Slight (2) – Moderate (3) – Severe (4)

<table>
<thead>
<tr>
<th>General discomfort</th>
<th>Round 0</th>
<th>Round 1</th>
<th>Round 2</th>
<th>Round 3</th>
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</thead>
<tbody>
<tr>
<td>Nausea</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Headache</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dizziness</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difficulty focussing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difficulty concentrating</td>
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<tr>
<td>Other?</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Comments:
Appendix VII - Data transformations

Example of raw data transformation of one tactor for one participant (yellow includes data from all tactors):

<table>
<thead>
<tr>
<th>Trial number</th>
<th>Condition</th>
<th>Name stimulus</th>
<th>Tactor number</th>
<th>Location</th>
<th>Relative location</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>HSS</td>
<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-12.2</td>
<td>-6.8</td>
<td>43.2</td>
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<td>HSS</td>
<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-27.6</td>
<td>-4.9</td>
<td>35.2</td>
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<td>HSS</td>
<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-26.2</td>
<td>-1.4</td>
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<td>78</td>
<td>0.91</td>
<td>-25.8</td>
<td>-5.7</td>
<td>36.4</td>
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<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-32.7</td>
<td>-14.9</td>
<td>24.9</td>
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<tr>
<td>96</td>
<td>HSS</td>
<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-33.4</td>
<td>-3.1</td>
<td>29.7</td>
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<td>9</td>
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<td>10tactor_HS</td>
<td>10</td>
<td>78</td>
<td>0.91</td>
<td>-19.7</td>
<td>-8.3</td>
<td>39.9</td>
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</table>

<table>
<thead>
<tr>
<th>Length vector</th>
<th>x'</th>
<th>z'</th>
<th>Control</th>
<th>Mean x'</th>
<th>Mean z'</th>
<th>Length mean vector</th>
<th>Variation</th>
<th>Relative variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.89</td>
<td>-0.27</td>
<td>0.96</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>44.73</td>
<td>-0.62</td>
<td>0.79</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>45.01</td>
<td>-0.58</td>
<td>0.81</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>44.62</td>
<td>-0.58</td>
<td>0.82</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>41.10</td>
<td>-0.80</td>
<td>0.61</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>44.70</td>
<td>-0.75</td>
<td>0.67</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>44.50</td>
<td>-0.44</td>
<td>0.90</td>
<td>1</td>
<td>-0.57</td>
<td>0.62</td>
<td>0.85</td>
<td>0.15</td>
<td>0.19</td>
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<table>
<thead>
<tr>
<th>Angle</th>
<th>Angle in degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.75</td>
<td>-42.96</td>
</tr>
<tr>
<td>-0.75</td>
<td>-42.96</td>
</tr>
<tr>
<td>-0.75</td>
<td>-42.96</td>
</tr>
<tr>
<td>-0.75</td>
<td>-42.96</td>
</tr>
<tr>
<td>-0.75</td>
<td>-42.96</td>
</tr>
<tr>
<td>-0.75</td>
<td>-42.96</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean total x'</th>
<th>Mean total z'</th>
<th>Total length mean vector</th>
<th>Total variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.19</td>
<td>0.19</td>
<td>0.81</td>
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</tbody>
</table>

Legend:

<table>
<thead>
<tr>
<th>Value</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative location</td>
<td>Location / circumference participant</td>
</tr>
<tr>
<td>Length vector</td>
<td>SQRT(X^2 + Z^2)</td>
</tr>
<tr>
<td>x'</td>
<td>X / Length vector</td>
</tr>
<tr>
<td>z'</td>
<td>Z / Length vector</td>
</tr>
<tr>
<td>Control</td>
<td>SQRT(x'^2 + z'^2)</td>
</tr>
<tr>
<td>Mean x'</td>
<td>AVERAGE (All x' from that tactor)</td>
</tr>
<tr>
<td>Mean z'</td>
<td>AVERAGE (All z' from that tactor)</td>
</tr>
<tr>
<td>Length mean vector</td>
<td>SQRT(Mean x'^2 + Mean z'^2)</td>
</tr>
<tr>
<td>Variation</td>
<td>1 - Length mean vector</td>
</tr>
<tr>
<td>Relative variation</td>
<td>Variation / Total variation</td>
</tr>
<tr>
<td>Angle</td>
<td>ATAN(Mean z' / Mean x')</td>
</tr>
<tr>
<td>Angle in degrees</td>
<td>Angle / PIL* 180</td>
</tr>
</tbody>
</table>

Mean total x' = AVERAGE (All x' from that participant)
Mean total z' = AVERAGE (All z' from that participant)
Total length mean vector = SQRT(Mean total x'^2 + Mean total z'^2)
Total variation = 1 - Total length mean vector
**Example of data transformations to find the best circular and ellipse model:**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Tactor number</th>
<th>Tactor location (cm)</th>
<th>Relative tactor location</th>
<th>Response angle (rad)</th>
<th>Perfect fit</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS 10</td>
<td>78</td>
<td>0.91</td>
<td>5.53</td>
<td>0.91</td>
<td>0.83-0.55i</td>
<td></td>
</tr>
<tr>
<td>HSS 11</td>
<td>73</td>
<td>0.85</td>
<td>5.34</td>
<td>0.85</td>
<td>0.58-0.81i</td>
<td></td>
</tr>
<tr>
<td>HSS 12</td>
<td>68</td>
<td>0.79</td>
<td>5.18</td>
<td>0.79</td>
<td>0.25-0.97i</td>
<td></td>
</tr>
<tr>
<td>HSS 13</td>
<td>63</td>
<td>0.73</td>
<td>4.63</td>
<td>0.73</td>
<td>-0.11-0.99i</td>
<td></td>
</tr>
<tr>
<td>HSS 14</td>
<td>58.5</td>
<td>0.68</td>
<td>4.25</td>
<td>0.68</td>
<td>-0.42-0.91i</td>
<td></td>
</tr>
<tr>
<td>HSS 15</td>
<td>53</td>
<td>0.62</td>
<td>3.85</td>
<td>0.62</td>
<td>-0.75-0.67i</td>
<td></td>
</tr>
<tr>
<td>HSS 1</td>
<td>36</td>
<td>0.42</td>
<td>2.43</td>
<td>0.42</td>
<td>-0.87+0.49i</td>
<td></td>
</tr>
<tr>
<td>HSS 2</td>
<td>32</td>
<td>0.37</td>
<td>2.47</td>
<td>0.37</td>
<td>-0.69+0.72i</td>
<td></td>
</tr>
<tr>
<td>HSS 3</td>
<td>26.5</td>
<td>0.31</td>
<td>1.55</td>
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<td>-0.36+0.93i</td>
<td></td>
</tr>
<tr>
<td>HSS 4</td>
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<td>1.49</td>
<td>0.25</td>
<td>-3.49E-15+i</td>
<td></td>
</tr>
<tr>
<td>HSS 5</td>
<td>16.5</td>
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<td>0.92</td>
<td>0.19</td>
<td>0.36+0.93i</td>
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<tr>
<td>HSS 6</td>
<td>11</td>
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<td>0.80</td>
<td>0.13</td>
<td>0.69+0.72i</td>
<td></td>
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<tr>
<td>HSS 7</td>
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<td>0.06</td>
<td>0.18</td>
<td>0.06</td>
<td>0.93+0.36i</td>
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</tr>
<tr>
<td>HSS 8</td>
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<td>0</td>
<td>6.10</td>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>HSS 9</td>
<td>82.5</td>
<td>0.96</td>
<td>5.43</td>
<td>0.96</td>
<td>0.97-0.25i</td>
<td></td>
</tr>
</tbody>
</table>

R: 0.16  
var: 0.84  
GF: 1.00

### Circular fit Vector Residual vector
0.88  0.73-0.68i  0.99-0.17i  
0.85  0.59-0.81i  0.99+0.01i  
0.83  0.45-0.89i  0.98+0.21i  
0.74  -0.08-0.99i  0.99+0.03i  
0.68  -0.45-0.90i  0.99-0.02i  
0.61  -0.76-0.65i  0.99-0.02i  
0.39  -0.76+0.66i  0.98-0.20i  
0.39  -0.79+0.62i  0.99+0.14i  
0.25  0.02+0.99i  0.93-0.38i  
0.24  0.08+0.99i  0.99-0.08i  
0.15  0.61+0.80i  0.96-0.28i  
0.13  0.70+0.71i  0.99-0.01i  
0.03  0.98+0.18i  0.98-0.18i  
0.97  0.98-0.19i  0.98-0.19i  
0.86  0.65-0.76i  0.82-0.57i  

R: 0.21  
var: 0.79  
GF: 0.98
Perceiving Directions through Vibration

<table>
<thead>
<tr>
<th>Best ellipse fit</th>
<th>Vector</th>
<th>Residual vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>0.74-0.68i</td>
<td>0.99-0.16i</td>
</tr>
<tr>
<td>0.85</td>
<td>0.60-0.80i</td>
<td>0.99+0.02i</td>
</tr>
<tr>
<td>0.83</td>
<td>0.46-0.89i</td>
<td>0.98+0.22i</td>
</tr>
<tr>
<td>0.74</td>
<td>-0.08-0.99i</td>
<td>0.99+0.03i</td>
</tr>
<tr>
<td>0.68</td>
<td>-0.45-0.89i</td>
<td>0.99-0.03i</td>
</tr>
<tr>
<td>0.61</td>
<td>-0.76-0.65i</td>
<td>0.99-0.03i</td>
</tr>
<tr>
<td>0.39</td>
<td>-0.76+0.65i</td>
<td>0.98-0.20i</td>
</tr>
<tr>
<td>0.39</td>
<td>-0.79+0.61i</td>
<td>0.99+0.14i</td>
</tr>
<tr>
<td>0.25</td>
<td>0.02+0.99i</td>
<td>0.93-0.38i</td>
</tr>
<tr>
<td>0.24</td>
<td>0.08+0.99i</td>
<td>0.99-0.08i</td>
</tr>
<tr>
<td>0.15</td>
<td>0.61+0.79i</td>
<td>0.96-0.29i</td>
</tr>
<tr>
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<td>0.71+0.71i</td>
<td>0.99-0.02i</td>
</tr>
<tr>
<td>0.03</td>
<td>0.98+0.18i</td>
<td>0.98-0.19i</td>
</tr>
<tr>
<td>0.97</td>
<td>0.98-0.19i</td>
<td>0.98-0.19i</td>
</tr>
<tr>
<td>0.87</td>
<td>0.66-0.75i</td>
<td>0.83-0.56i</td>
</tr>
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</table>

R: 0.21
var: 0.79
GF: 0.98

Legend:

<table>
<thead>
<tr>
<th>Value</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response angle (rad)</td>
<td>MOD (Angle + 2<em>PI()) / 2</em>PI()</td>
</tr>
<tr>
<td>Perfect fit</td>
<td>Relative tactor location</td>
</tr>
<tr>
<td>Vector (perfect)</td>
<td>IMEXP((COMPLEX(0, Perfect fit * 2*PI()))</td>
</tr>
<tr>
<td>Circular fit</td>
<td>Response angle (rad) / (2*PI())</td>
</tr>
<tr>
<td>Vector (circular)</td>
<td>IMEXP((COMPLEX(0, Circular fit * 2*PI())))</td>
</tr>
<tr>
<td>Residual vector (circular)</td>
<td>IMEXP((COMPLEX(0, (Circular fit – Perfect fit) * 2*PI())))</td>
</tr>
<tr>
<td>Best geodetic ellipse fit</td>
<td>(a+b) / 2*(1+1/4* Third flattening ^2+1/64* Third flattening ^4)<em>SIN(2</em> Response angle) = (15/16* Third flattening <em>2-15/64</em> Third flattening ^4)<em>SIN(4</em> Response angle) / Circumference (ellipse)</td>
</tr>
<tr>
<td>Vector (geodetic)</td>
<td>IMEXP((COMPLEX(0, Best geodetic ellipse fit * 2*PI())))</td>
</tr>
<tr>
<td>Residual vector (geodetic)</td>
<td>IMEXP((COMPLEX(0, (Best geodetic ellipse fit – Perfect fit) * 2*PI())))</td>
</tr>
</tbody>
</table>

R = IMABS(IMSUM(Vector)) / COUNT(Tactor number)
Var = 1 - R
GF = 1 - Var (residual) / Var

a = 1
b = Estimated by Excel Solver
Third flattening = (a - b) / (a + b)
First flattening = (a - b) / a
Circumference (ellipse) = PI() * (3*(a + b) - SQR(10* a * b + 3*(a ^2 + b ^2)))