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On the assignment of non-teaching tasks to reduce teachers' work pressure

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On the assignment of non-teaching tasks to reduce teachers' work pressure

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Abstract

Teachers in secondary schools experience high work pressure. One of the causes for this is the amount of non-teaching tasks that they have to do. An unsuccessful task assignment causes teachers to work on tasks they dislike, to work on more tasks than they actually should work on and to work until late in the evening. This paper presents a mixed integer linear programming model to assign non-teaching tasks to teachers, taking into account the preferences of the teachers, their workload targets and possible overwork caused by these tasks.
Keywords: assignment problem, mixed-integer linear programming, non-teaching tasks

1 Introduction

Teaching classes is the most important activity for secondary school teachers. However, besides teaching, there are several non-teaching tasks that teachers have to carry out. Examples of these tasks are organizing field trips, being the department coordinator of a certain subject and arranging the debate club. It has been reported that teachers experience more and more work pressure (CNV Onderwijs, 2013). Part of this work pressure is caused by the high amount of tasks that teachers have to do. These tasks have a certain workload that can exceed the estimated number of hours a teacher gets for these non-teaching tasks. This workload can be concentrated at certain moments during the year or it is spread over a longer period of time. An unsuccessful task assignment can result in a peak in workload for a teacher as the time at which assigned tasks need to be done coincide. Taking into account the schedule of teaching classes, this peak in workload can cause overwork on certain days of the school year. In addition, some teachers may be unsatisfied with the tasks that they are assigned to. All three effects of the assignment of non-teaching tasks can possibly cause a reduction of job satisfaction which is undesirable.

In this paper, a model is constructed that allows secondary schools to assign non-teaching tasks to teachers within a secondary school unit, addressing the issues mentioned in the previous paragraph. This model is tested with data from CSG Reggesteyn, a secondary school in Nijverdal. The resulting task-assignments yield possible improvements on the preferences, the distribution of workload due to non-teaching tasks and the amount of overwork hours. The mathematical model that is presented in this paper can be generalized for usage in other secondary schools.

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This paper is organized as follows. In the next section, an overview of related literature is given. In Section 3, mathematical theory is presented. A mathematical model is presented in Section 4 and Section 5 provides the results of the implementation for CSG Reggesteyn. In Section 6, a discussion on this research and future research suggestions are given. Section 7 provides the conclusion. The Appendix provides an overview of used data and a reflection on the co-operation with CSG Reggesteyn.

2 Literature Review

There are problems that share similarities with the problem observed for secondary school teachers. One of these problems is the teaching assistant task assignment problem (TA-assignment problem). This problem can be described as follows: at universities, teaching assistants are assigned to course sections such as tutorials and guided self-studies to help students solving problems. At the department of mathematics at University of Twente for instance, the TA-assignment is constructed manually. Teaching Assistants hand in their preferences and hope that they are assigned to tasks that they prefer. At other universities, a more systematic approach is used. Examples are the Istanbul Kultur University and the Boğaziçi University. At the Industrial Engineering Department of the latter, the TA-assignment is performed using a mixed integer programming model with multiple objectives that is introduced by Güler et al. (2015). They tried to maximize the utility (satisfaction with the tasks) of the teaching assistants and to minimize the deviation from the target workload for each teaching assistant. They wanted to incorporate preferences in the allocation as the ratio between master teaching assistants and PhD teaching assistants changed by the increased number of PhD students. According to Güler et al. (2015), master teaching assistants do not provide strong preferences on the tasks since they will be leaving the university in 2 years. For PhD students this is not the case: they want to develop teaching skills and knowledge in certain areas. Another reason for their more systematic approach on the TA-assignment was reducing the time to allocate the tasks as it has been done manually until 2015. Their results show better assignments comparing to the manual assignments in all their performance criteria.

In the paper by Üney-Yüksektepe and Karabulut (2011), a mixed integer linear programming model is proposed to solve the assignment problem for the Istanbul Kultur University. Here the model is introduced to minimize the difference between the maximum and minimum workload in order to balance the workloads between the teaching assistants. Moreover, the availability of the teaching assistants is incorporated in the model. The results presented in this paper are, like in the paper of Güler et al. (2015), an improvement in comparison with the manually constructed task assignment.

Another interesting paper on the assignment of tasks to staff, although not in the educational atmosphere, is a paper by Eiselt and Marianov (2008). In this paper, the assignment of tasks within a company department is described. The authors wanted to balance workload and assign tasks that are in correspondence with the skill sets of the employees to keep everyone motivated. Moreover, they tried to reduce the costs for employees working overtime or subcontracting overwork. To achieve a task assignment that takes these three challenges into account, they proposed a mixed integer linear programming model. Their model has been tested in practice with 15 employees and 22 tasks using different parameter values.

The secondary school task assignment problem involves multiple objectives. Firstly, the preferences of the teachers should be maximized. Moreover, the workload needs to be as close as possible to the target workload for all teachers. Finally, it is required to minimize the amount of overwork for the teachers. This shows that multiple-objective optimization is needed to solve the problem. General theory on this subject is given in a book by Cohon (2004).

3 Theoretical background

In this section, mathematical theory that is required to solve the teacher-task assignment problem, is introduced. As we are dealing with a problem that concerns optimizing a task assignment, we start by looking at linear programming models since these models are used in situations that share similarities.

3.1 LP-model

A linear programming model (LP-model) is a tool for solving optimization problems consisting of a linear objective function and a set of linear constraints. A general linear program is given by:

$$
\begin{aligned}
\min \ z = \sum_{j=1}^{n} c_j x_j & & \qquad (1)\\
\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \geq b_j, & \qquad i = 1, \ldots, m \\
x_j \geq 0, & \qquad j = 1, \ldots, n.
\end{aligned}
$$

Here $z = \sum_{j=1}^{n} c_j x_j$ is the objective function with c_j its coefficients and a_{ij} and b_j the coefficients of the constraints. The variables x_j can take on any non-negative value. A linear program is convex as the feasible region is convex (Bertsimas and Tsitsiklis, 1997).

In the case of the teacher-task assignment, the variables cannot be continuous as we are dealing with the decisions whether or not to assign a task to a teacher. Fortunately, there are linear programs that deal with these kind of variables. These linear programs are called integer linear programs and are introduced now.

3.2 ILP-model

For many optimization problems that occur in practice, the variables are integer valued. This turns the general problem as defined in (1) into an integer linear programming model (ILP). When the decision variables are restricted to 0 or 1, the corresponding integer linear program is also called a binary integer program. A general ILP is given by:

$$
\begin{aligned}
\min \ z = \sum_{j=1}^{n} c_j x_j & \\
\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \geq b_i, & \qquad i = 1, \ldots, m \\
x_j \in \mathbb{Z}, & \qquad j = 1, \ldots, n.
\end{aligned}
$$

Solving an integer program is much harder than solving LP problems as the solution space clearly is not convex. An integer program can be viewed as a linear program with one extra constraint which states that the variables should be integer valued. This LP is therefore called the LP-relaxation of the integer program. If the optimal solution of the LP-relaxation is feasible for the integer program, this solution is optimal for both programs. If this is not the case, it can be shown that the optimal solution for the LP-relexation is less or equal (in case of minimization) to the optimal solution of the corresponding integer program if it is feasible at all (Bisschop, 2006).

3.3 Assignment problems

The original assignment problem deals with the problem of assigning n tasks to n agents in the best possible way. The mathematical model for the classical assignment problem is given by the following integer linear program:

$$
\begin{aligned}
\min \quad z = & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, \ldots, n \\
& \sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, \ldots, n \\
& x_{ij} \in \{0, 1\},
\end{aligned}
$$

where $x_{ij} = 1$ if agent i is assigned to task j and 0 if this is not the case. When agent i is assigned to task j, this results in a cost of c_{ij}. The first set of constraints ensures that all tasks are assigned to precisely one agent. The second set of constraints makes sure that every agent is assigned to exactly one task. For the original assignment problem, the binary constraint can be omitted as the structure of the problem is such that there automatically is an optimal linear programming solution in which the binary constraint is satisfied (Pentico, 2007).

In most practical examples it is needed to assign multiple tasks to one agent as there are more tasks than agents. The generalized assignment problem can be used to deal with this situation. In the general assignment problem, every agent has a given capacity. Every assigned task takes on part of this capacity. The mathematical formulation is as follows:

$$
\begin{aligned}
\min \quad z = & \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, \ldots, m \\
& \sum_{j=1}^{n} a_{ij} x_{ij} \leq b_i, \qquad i = 1, \ldots, n \\
& x_{ij} \in \{0, 1\}.
\end{aligned}
$$

Again, $x_{ij} = 1$ if agent i is assigned to task j and 0 if not. The cost of assigning task j to agent i is again given by c_{ij}. The value a_{ij} represents the capacity of agent i that is used if that agent is assigned to task j. The value b_i represents the total capacity of agent i. The first set of constraints ensures the assignment of every task to an agent while the second constraint guarantees that the set of tasks assigned to an agent does not exceed the capacity of the agent (Pentico, 2007).

3.4 Branch and Bound

To solve integer programming problems, one approach is to solve the underlying LP-relaxation and reformulate the model until the optimal solution of the LP-relaxation is integer valued. One method that uses this idea is the branch and bound method. This method is based on the idea that the set of all feasible solutions can be partitioned into smaller subsets of solutions that can be solved systematically using the LP-relaxation (Winston and Goldberg, 2004). Consider the following example:

Example 1. *Suppose we want to solve the following problem:*

$$\begin{aligned} \min \ & z = 7x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 \\ \text{s.t.} \ & -4x_1 - 2x_2 + x_3 - 2x_4 - x_5 \leq -3 \\ & -4x_1 - 2x_2 - 4x_3 + x_4 + 2x_5 \leq -7 \\ & x_i \in \{0,1\}, \ \ i = 1, 2, \ldots, 5. \end{aligned} \tag{2}$$

We first solve the LP-relaxation of this problem. We find the optimal solution $7\frac{2}{3}$ with $x = (\frac{1}{3}, 1, 1, \frac{1}{3}, 1)$. This value is the lower bound for the solution of the integer program. As x_1 is not binary, we introduce two branches. The first branch considers $x_1 = 0$ and the second branch considers $x_1 = 1$. We start by solving the first branch. Here we solve the LP-relaxation with extra constraint $x_1 = 0$. This problem is infeasible and hence will certainly not give a solution for the ILP. The second branch with additional constraint $x_1 = 1$ gives a solution of $8\frac{1}{2}$ with $x = (1, 0, \frac{3}{4}, 0, 0)$. We continue by branching on x_3. Introducing the constraint $x_3 = 0$ in addition to the constraint $x_1 = 1$, we find that the LP-relaxation is not feasible. Therefore, branch 4 needs to be considered. Here we find an optimal value of 9 with $x = (1, 0, 1, 0, 0)$. This solution is feasible for our original problem. We consider now branch 5 to solve the linear program with only additional constraint $x_4 = 0$. This gives a solution of $7\frac{3}{4}$ with $x = (\frac{9}{20}, 1, \frac{4}{5}, 0, 0)$. Branch 6, with $x_4 = 1$ gives a value of 9.5 with $x = (\frac{1}{2}, 1, 1, 1, 0)$, which is higher than the value we already found for the integer program. Hence we cannot improve on this branch. Continuing this way, we see that the final solution of the problem is given by $x = (1, 0, 1, 0, 0)$ with an optimal value of 9. This example is based on an example by (Winston and Goldberg, 2004).

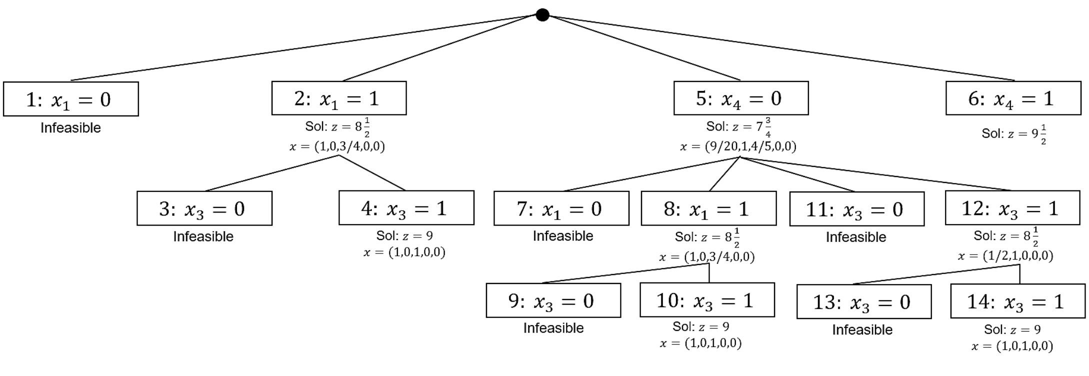


Figure 1: The branching tree of problem 2.

3.5 Cutting planes

Another method that can be used to solve integer linear programs is the method of cutting planes. This method gives a clever way to improve the bounds in the branch and bound method. In order to explain this method, we need a few definitions.

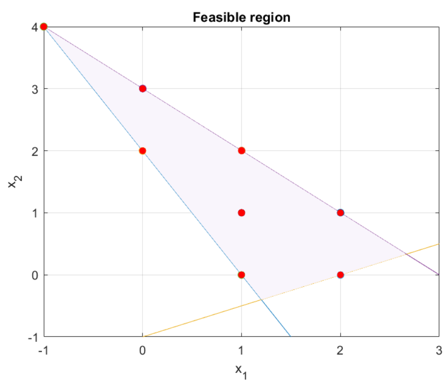
Definition 2 (Valid Inequality)**.** *Let P be the feasible set of some integer programming problem. A valid inequality is an additional inequality, $\sum_{j=1}^{n} a_j x_j \leq b$, that is satisfied for every $x \in P$* (Havas et al., 2013).

We illustrate this definition with the following example:

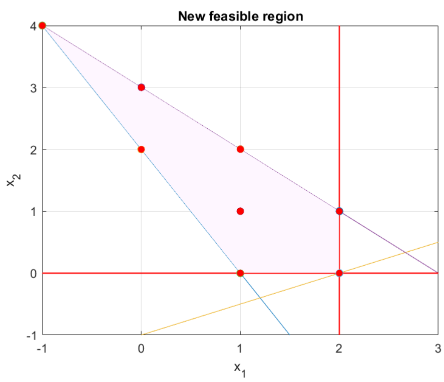
Example 3. Suppose we have the following integer program:

$$\begin{aligned} \min \ z = x_1 + x_2 & \\ \text{s.t. } 2x_1 + x_2 &\geq 2 \\ x_1 - 2x_2 &\geq 2 \\ x_1 + x_2 &\leq 3 \\ x_1, x_2 &\in \mathbb{Z} \end{aligned}$$

The LP-relaxation of this problem has an optimal solution of $\frac{4}{5}$ at the point $(\frac{6}{5}, -\frac{2}{5})$. Clearly, this solution is infeasible for the integer program. However, by looking at the constraints, one can find that $x_2 \geq -\frac{2}{5}$ which is a valid inequality. However, as we allow only integer solutions, $x_2 \geq 0$ is also a valid inequality as the feasible set for the integer program does not change. The feasible set for the linear program relaxation does change however.



(a) The original feasible region.



(b) The feasible region after adding valid inequalities.

A valid inequality is also known as a cutting plane or cut. These cuts are desirable to eliminate parts of the LP-feasible region. This brings up the next definition:

Definition 4. *Let $x^1, \ldots, x^k$ be vectors in $\mathbb{R}^n$ and let $\lambda_1, \ldots, \lambda_2$ be nonnegative scalars whose sum is unity.*

1. *The vector $\sum_{i=1}^{k} \lambda_i x^i$ is said to be a **convex combination** of the vectors $x^1, \ldots, x^k$.*

2. *The **convex hull** of the vectors $x^1, \ldots, x^k$ is the set of all convex combinations of these vectors (Bertsimas and Tsitsiklis, 1997).*

Hence, it follows that the convex hull of all integer solutions is the smallest LP-feasible region that contains all feasible integer solutions. As the convex hull has integer points as vertices, the optimal solution of the LP-relaxation with as feasible region the convex hull of all feasible integer solutions equals the optimal solution of the integer program.
Consider Example 5.

Example 5. *Adding the valid inequalities $x_2 \geq 0$ and $x_1 \leq 2$, we find the convex hull of all feasible integer solutions. Solving the LP-relaxation with these additional inequalities gives the optimal integer solution of* 1 *at* $(1, 0)$.

In the example, we were able to determine the convex hull of all integer solutions to find the optimal solution by adding valid inequalities. In general, this is very hard. However, adding valid inequalities makes the feasible region approach the convex hull. This moves the solution of the LP-relaxation closer to the solution of the integer program which is desirable. The cutting plane method adds one or more valid inequalities every iteration to solve the integer program (Havas et al., 2013).

3.6 Absolute value in the objective function

Consider the following model:

$$\begin{aligned} \min \ z = \sum_{j=1}^{n} c_j \, |x_j| & \qquad c_j > 0 \\ \text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \geq b_i, & \qquad i = 1, \ldots, m \end{aligned} \tag{3}$$

The absolute value makes it impossible to use the standard linear programming approach. However, this problem can be tackled by introducing the variables x_j^+ and x_j^- in the following manner (Bertsimas and Tsitsiklis, 1997):

$$\begin{aligned} x_j &= x_j^+ - x_j^- \\ |x_j| &= x_j^+ + x_j^- \\ x_j^+, x_j^- &\geq 0. \end{aligned}$$

This results in the following linear program that is equivalent to (3):

$$\begin{aligned} \min \ z = \sum_{j=1}^{n} c_j (x_j^+ + x_j^-) & \qquad c_j > 0 \\ \text{s.t.} \sum_{j=1}^{n} a_{ij} (x_j^+ - x_j^-) \geq b_i, & \qquad i = 1, \ldots, m \\ x_j^+, x_j^- \geq 0, & \qquad j = 1, \ldots, n. \end{aligned}$$

This technique is used to deal with the deviation of the individual workload of every teacher from the target workload of that teacher.

4 The model

Before the start of every school year, it is common at many secondary school units in the Netherlands to distribute a list of non-teaching tasks to the teachers. Teachers select the tasks that they prefer the most and hope that they are assigned to these tasks. The managers of the units receive these preferences and try to come up with a task assignment that takes the submitted preferences into account. However, assigning the non-teaching tasks is not only a matter of taking into consideration the preferences. All teachers are contracted by the school for a certain amount of hours per year. This is called the 'netto jaartaak'. This 'netto jaartaak' consists of different elements. The core element is teaching classes. Besides teaching classes, part of the 'netto jaartaak' consists of general school activities and increasing educational expertise. After subtracting the hours for teaching, general school activities and increasing educational expertise from the 'netto jaartaak', a number of hours remains available for performing non-teaching tasks. The manager of a school unit should make a fair assignment in order to get as close as possible to this target workload. Making this assignment manually takes a lot of effort and due to the size of the problem it might be possible that the task assignment is far from optimal. Moreover, only two of the three consequences of the task assignment that are mentioned in the introduction are taken into account, namely the deviations from the target workload and preferences. It is possible that the task assignment results in high peaks of workload which is undesirable. Therefore this problem is approached as an assignment problem having multiple objectives.

In this section, a multiple-objective mixed-integer linear programming model is proposed to solve the teacher-task assignment problem. Firstly, a number of assumptions that are is in the model is stated. Secondly, an overview of sets, parameters and variables is given. Finally, we consider the constraints that the task assignment has to satisfy and we give the objective function.

4.1 Assumptions

In order to simplify the model, some assumptions were made. The consequences of these assumptions are discussed in Section 6. The assumptions are the following:

- The first assumption that has been made is that every task can be assigned to teachers after the teaching timetables are already constructed.
- The second assumption is that teachers only work at days they are required to be at school. This means we do not incorporate the weekend in our model.
- A third assumption concerns the types of tasks. We assume that there are two different types of tasks: non-flexible and flexible tasks. Non-flexible tasks are tasks that are pre-scheduled and take place at specific moments during the year. Flexible tasks can be performed during a period of time as they only have a start date and a deadline. When a task consists of both a flexible and a non-flexible part, it will be broken into a flexible and a non-flexible part with the additional constraint that both parts are assigned to the same teacher.
- A flexible task j that starts on day s_j starts at the beginning of that day. A task with deadline d_j should be completed at the end of d_j. That is, at day $d_j + 1$ it is impossible to work on this task, while it is possible to work on the task on day d_j.

- We assume that every task j has a fixed workload w_j in hours which is assumed to be independent of the person who performs the task.

We continue by giving an overview of important sets and input for the model.

4.2 Definition of sets, parameters and the input for the model

An overview of all important sets used in the model is given in Table 1 below.

Set	**Definition**
$L = \{1, 2, \dots, n\}$	Set of teachers in a school unit.
L_j^f	Set of teachers capable of performing flexible task j.
L_j^n	Set of teachers capable of performing non-flexible task j.
$T_f = \{1, 2, \dots, m_1\}$	Set of flexible tasks.
$T_n = \{1, 2, \dots, m_2\}$	Set of non-flexible tasks.
$D = \{1, 2, \dots, d\}$	Set of all days of the school year.
D_s	Set of all days that all teachers have to be available to be at school.
$A_j^{n,n}$	Set of all non-flexible tasks that should be assigned to the same teacher as non-flexible task j.
$A_j^{n,f}$	Set of all flexible tasks that should be assigned to the same teacher as non-flexible task j.
$A_j^{f,n}$	Set of all non-flexible tasks that should be assigned to the same teacher as flexible task j.
$A_j^{f,f}$	Set of all flexible tasks that should be assigned to the same teacher as flexible task j.
E_l^n	Sets of non-flexible tasks that cannot be assigned to the same teacher.
E_l^f	Sets of flexible tasks that cannot be assigned to the same teacher.

Table 1: Sets that are used in the model.

For consistency, the letter i will be used when indexing teachers in the sets L, L_j^f and L_j^n while tasks are indexed with the letter j. Indexing days will be done using the letter k.

In Table 2, parameters that are used in the model are introduced.

Parameter	**Definition**
α	weight of the preference objective
β	weight of the deviation objective
γ	weight of the overwork objective
$\max_o^+$	maximal value for individual overwork
$\max_d^+$	maximal value for positive deviations from the target workload
$\max_d^-$	maximal value for negative deviations from the target workload

Table 2: Parameters that are used for the model.

To construct a useful task assignment for a secondary school unit that takes into account different aspects that the teacher-task assignment should fulfill, specific information about the tasks and teachers is given as input for the model. This data consists of the time-schedules of all teachers, a list of all tasks with their classification and expected workload, the fixed moments that tasks take place and the start-dates and deadlines of flexible tasks. How this data will be stored for the model is explained here.

The first information that is used is the personal schedule of the teachers. This data is stored in a matrix $S = (S_{ik})$. When teacher i^* is working 7 hours on day k^*, the value for $S_{i^*k^*}$ is set to 7. This is done for every teacher i and day k. To account for the available time to work on non-educational tasks, a similar matrix is introduced $C = (C_{ik})$. If teacher i^* has 1 hour available on day k^*, the value for $C_{i^*k^*} = 1$. Final information about the teachers is given by the target workloads. For every teacher, the estimated hours for non-teaching tasks is collected in the vector $\mathbf{t} = [t_1, t_2, \ldots, t_n]$, where t_i denotes the target workload of teacher i.

The tasks are not as easily stored as it is important to take into account what classification every task has. We assumed that for both types of tasks, it is known how much work the tasks take in total. The vectors $\mathbf{w}^f = \left[w_1^f \, w_2^f \, \ldots \, w_{m_1}^f\right]$ and $\mathbf{w}^n = \left[w_1^n \, w_2^n \, \ldots \, w_{m_2}^n\right]$ are used to save the total workload that every task provides. For non-flexible tasks, it is exactly known when an assigned teacher is busy with the task. This information is stored in the matrix $Y^n = (Y_{jk}^n)$. If for task j a meeting of h hours is scheduled on day k, then $Y_{jk}^n = h$. As the total workload over all days of every non-flexible task j should equal w_j^n, we have that:

$$\sum_{k=1}^{d} Y_{jk}^n = w_j \text{ for all } j \in T_n.$$

Such a matrix $Y^f = (Y_{jk}^f)$ cannot be given for flexible tasks, as there are no specific moments on which these tasks have to be carried out. For flexible tasks, only the period in which teachers can work on the task is known. This information is captured by the vectors $\mathbf{s} = [s_1, s_2, \ldots, s_{m_1}]$ and $\mathbf{d} = [d_1, d_2, \ldots, d_{m_1}]$ containing the start dates and deadlines of the flexible tasks respectively.

The final information that is provided as input for the model contains preferences of the teachers for different tasks. All teachers can assign a value of 1 to tasks that they prefer, a value of 3 to the tasks that they would rather not do and a value of 6 to tasks that they really do not prefer. The preferences for the tasks are collected in two matrices $P^f = (P_{ij}^f)$ and $P^n = (P_{ij}^n)$ for the flexible tasks and the non flexible tasks respectively. To make sure that preferences for tasks that have been divided into separate tasks are not counted more than once, various columns of $P^f = (P_{ij}^f)$ are set to zero. For instance, when a task consists of a non-flexible task and two flexible tasks, only the preference for the non-flexible task is collected in the matrix $P^n = (P_{ij}^n)$ and the preferences for the corresponding flexible tasks are set to zero in the matrix $P^f = (P_{ij}^f)$. More details are given in Appendix B.3. An overview of input variables is provided in Table 3.

Variable	Definition
S	n by d matrix containing the working hours of all teachers.
C	n by d matrix containing the available hours for tasks of all teachers.
P^f	n by m_1 matrix containing the preferences of the teachers for flexible tasks.
P^n	n by m_2 matrix containing the preferences of the teachers for non-flexible tasks.
Y^n	$m_1 + m_2$ by d matrix containing the planning of the non-flexible tasks.
$\mathbf{w}^f$	vector containing the workload of all flexible tasks $j \in T^f$.
$\mathbf{w}^n$	vector containing the workload of all non-flexible tasks $j \in T^n$.
$\mathbf{s}$	vector containing the start date of all flexible tasks $j \in T^f$.
$\mathbf{d}$	vector containing the deadlines of all flexible tasks $j \in T^f$.
$\mathbf{t}$	vector containing the target workloads of all teachers $i \in L$.

Table 3: Input for the model.

4.3 Decision variables

In the model, two binary decision variables are used: x_{ij}^n and x_{ij}^f. The variable x_{ij}^n equals 1 when non-flexible task j is assigned to teacher i and 0 otherwise. Similarly, x_{ij}^f equals 1 when flexible task j is assigned to teacher i and 0 otherwise.

The deviation from the target workload is given by the variable δ_i which depends on the decision variables x_{ij}^n and x_{ij}^f. This dependence is formulated as:

$$\delta_i = \sum_{j=1}^{m_1} x_{ij}^f w_j^f + \sum_{j=1}^{m_2} x_{ij}^n w_j^n - t_i \qquad \forall i \in L.$$

As one of the objectives is to minimize the sum of absolute deviations $\sum_{i=1}^{n} |\delta_i|$, this causes the introduction of two variables δ_i^+ and δ_i^- using the method described in Section 3.6.

To account for overwork, the difference between the time available for non-teaching tasks and the workload of assigned tasks for teacher i on day k is given by the variable:

$$o_{ik} = \sum_{j=1}^{m_1} Y_{jk}^f x_{ij}^f + \sum_{j=1}^{m_2} Y_{jk}^n x_{ij}^n - C_{ik}, \qquad \forall i \in L, \forall k \in D.$$

4.4 Heuristic method to deal with the planning of flexible tasks

A flexible task can be planned in many ways. For every teacher, there is an optimal planning of these tasks that results in the least amount of overwork. In the optimization model, we want to incorporate the work flow of these flexible tasks in order to balance the workload for every teacher during the year. To deal with the flexibility of these tasks, we introduce the planning matrix $Y^f = (Y_{jk}^f)$ for the flexible tasks. During the optimization of the teacher-tasks assignment, this matrix is filled as follows:

$$Y_{jk}^f = \begin{cases} 0 & \text{for } k < s_j \\ \frac{w_j^f}{d_j - s_j + 1} & \text{for } s_j \leq k \leq d_j \\ 0 & \text{for } k > d_j. \end{cases} \tag{4}$$

Overwork hours that are a result of the assignment of a flexible task to a teacher are penalized with a relatively small penalty due to this uniform distribution for workload as it is possible that a better planning of these flexible tasks is possible. To get a more realistic view on the impact of flexible tasks on overwork, we use a separate linear program to investigate the amount of overwork caused by these tasks. This model will be introduced after introducing the general model that makes use of the uniform distributed planning Y^f_{jk}.

4.5 Constraints

In this section, the mathematical constraints that are needed to fulfill the requirements of the task assignment are introduced and explained.

$$\sum_{i=1}^{n} x^n_{ij} = 1, \qquad \forall j \in T_n. \tag{5}$$

(5) Every non-flexible task should be assigned to exactly one teacher.

$$\sum_{i=1}^{n} x^f_{ij} = 1, \qquad \forall j \in T_f. \tag{6}$$

(6) Similarly, every flexible task should be assigned to exactly one teacher.

$$x^n_{ij} = 0, \qquad \forall j \in T_n, i \notin L^n_j. \tag{7}$$

(7) A non-flexible task cannot be assigned to a teacher that does not have the necessary skill set to complete the task.

$$x^f_{ij} = 0, \qquad \forall j \in T_f, i \notin L^f_j. \tag{8}$$

(8) Similarly, a flexible task cannot be assigned to a teacher that does not have the necessary skill set to complete the task.

$$x^n_{ij_r} = x^n_{ij}, \qquad \forall j \in A^{n,n}_{j_r}, \forall j_r \in T_n, \forall i \in L. \tag{9}$$

(9) All non-flexible tasks that should be assigned to the same teacher as non-flexible task j_r are assigned to the same teacher.

$$x^n_{ij_r} = x^f_{ij}, \qquad \forall j \in A^{n,f}_{j_r}, \forall j_r \in T_n, \forall i \in L. \tag{10}$$

(10) All flexible tasks that should be assigned to the same teacher as non-flexible task j_r are assigned to the same teacher.

$$x^f_{ij_r} = x^n_{ij}, \qquad \forall j \in A^{f,n}_{j_r}, \forall j_r \in T_f, \forall i \in L. \tag{11}$$

(11) All non-flexible tasks that should be assigned to the same teacher as flexible task j_r are assigned to the same teacher.

$$x^f_{ij_r} = x^f_{ij}, \qquad \forall j \in A^{f,f}_{j_r}, \forall j_r \in T_f, \forall i \in L. \tag{12}$$

(12) All flexible tasks that should be assigned to the same teacher as flexible task j_r are assigned to the same teacher.

$$\sum_{j \in E_l^n} x_{ij}^n + \sum_{j \in E_l^f} x_{ij}^f \leq 1, \qquad \forall i \in L, \forall l. \tag{13}$$

(13) A teacher can only be assigned to at most one of the tasks of the set $E_l^n \cup E_l^f$ for all sets $E_l^n \cup E_l^f$.

$$\delta_i = \sum_{j=1}^{m_1} x_{ij}^f w_j^f + \sum_{j=1}^{m_2} x_{ij}^n w_j^n - t_i, \qquad \forall i \in L. \tag{14}$$

(14) As defined earlier, the deviation from the target workload t_i for teacher i is the difference between the workload from both flexible as non-flexible assigned tasks and the target workload t_i.

$$\delta_i = \delta_i^+ - \delta_i^-, \qquad \forall i \in L. \tag{15}$$

(15) The difference between the positive deviation δ_i^+ and the negative deviation δ_i^- equals the deviation δ_i.

$$\delta_i^+ \leq \max_d^+ \qquad \forall i \in L. \tag{16}$$

(16) The positive deviation from the target workload for every teacher i is less than or equal to a given maximum deviation $\max_d^+$.

$$\delta_i^+ \leq \max_d^- \qquad \forall i \in L. \tag{17}$$

(17) The negative deviation from the target workload for every teacher i is less than or equal to a given maximum deviation $\max_d^-$.

$$o_{ik} = \sum_{j=1}^{m_1} Y_{jk}^f x_{ij}^f + \sum_{j=1}^{m_2} Y_{jk}^n x_{ij}^n - C_{ik}, \qquad \forall i \in L, \forall k \in D. \tag{18}$$

(18) The difference between the workload of teacher i due to tasks and the teacher's capacity for tasks on day k equals o_{ik}.

$$o_{ik} = o_{ik}^+ - o_{ik}^-, \qquad \forall i \in L, \forall k \in D. \tag{19}$$

(19) The difference between actual overwork o_{ik}^+ and 'underwork' o_{ik}^- equals o_{ik} for every teacher i and every day k. This allows to penalize only actual overwork o_{ik}^+.

$$o_{ik}^+ \leq \max_o^+ \qquad \forall i \in L, \forall k \in D. \tag{20}$$

(20) Overwork due to assigned tasks is less than or equal to $\max_o^+$ hours.

$$x_{ij}^n = 0, \qquad \forall i \in L \text{ if } Y_{jk}^n > 0 \text{ and } S_{ik} = 0, j \in T_n, k \in D \backslash D_S. \tag{21}$$

(21) Non-flexible tasks are not assigned to teachers when the task takes place on a day that they normally have a day off. The exception is formed on certain days that teachers are required to be available for school even if it a teacher normally has a day off. These dates are stored in D_s.

4.6 Objective function

The objective function for this model takes in consideration all three aspects of a successful teacher-task assignment. Together with the constraints that are introduced in Section 4.5, it is possible to find a correct task-assignment that takes the following into account:

- It is desirable to assign tasks to teachers that have affinity with that task. Taking into account their preferences is done by minimizing the total assigned preference value of all teachers $\sum_{i=1}^{n}\left(\sum_{j=1}^{m_1} x_{ij}^f P_{ij}^f + \sum_{j=1}^{m_2} x_{ij}^n P_{ij}^n\right)$ as higher preference for a task means a lower preference value in the matrices P^f and P^n.
- It is desirable to assign the tasks in such a way that the deviations between the target workload and the assigned workload is small. This is done by minimizing the sum of all individual deviations, $\sum_{i=1}^{n}\left(\delta_i^+ + \delta_i^-\right)$.
- Finally, it is desirable to make a task assignment that causes minimal overwork. This is done by minimizing $\sum_{i=1}^{n}\sum_{k=1}^{d} o_{ik}^+$.

The relative importance of these three objectives is incorporated by introducing the weighting constants α, β, γ. Together with these constants, we have the following objective function:

$$\min_{x_{ij}^n,\, x_{ij}^f} \ \alpha\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{m_1} x_{ij}^f P_{ij}^f + \sum_{j=1}^{m_2} x_{ij}^n P_{ij}^n\right)\right] + \beta\left[\sum_{i=1}^{n}\left(\delta_i^+ + \delta_i^-\right)\right] + \gamma\left[\sum_{i=1}^{n}\sum_{k=1}^{d} o_{ik}^+\right]. \quad (22)$$

The whole mixed integer linear program can be viewed in Appendix A.

4.7 Rescheduling the flexible tasks

As stated earlier, a heuristic approach is used regarding the flexible tasks. We will now discuss how we find the matrix $Y^f = (Y_{jk}^f)$ in such a way that the amount of overwork originated from the flexible tasks is as low as possible and how we use this matrix in the model.

To find the optimal planning of the flexible tasks for teachers assigned to these tasks, we take into account the task assignment found using the main model, the vector of start dates and deadlines, the planning and workload of the non-flexible tasks and the schedule of the teachers. The optimal planning of flexible tasks that takes into account the given task assignment can be made using a linear program that will be discussed here. We start again by explaining the constraints.

$$o_{ik} = \sum_{j=1}^{m_1} Y_{jk}^f x_{ij}^f + \sum_{j=1}^{m_2} Y_{jk}^n x_{ij}^n - C_{ik}, \qquad \forall i \in L, \forall k \in D. \quad (23)$$

(23) The difference between the workload of teacher i due to tasks and the teacher's capacity for tasks on day k equals o_{ik}. Note that the workload per day due to non-flexible tasks is already known as the task assignment is already constructed. This is not the case for flexible tasks as these tasks are not scheduled.

$$o_{ik} = o_{ik}^{+} - o_{ik}^{-} \qquad \forall i \in L, \forall k \in D. \tag{24}$$

(24) The difference between actual overwork o_{ik}^{+} and 'underwork' o_{ik}^{-} equals the value for o_{ik} for every teacher i and every day k. This allows to penalize only actual overwork o_{ik}^{+}.

$$o_{ik}^{+} \leq \max_{o}^{+} \qquad \forall i \in L, \forall k \in D. \tag{25}$$

(25) Overwork due to assigned tasks is less than or equal to $\max_{o}^{+}$ hours.

$$\sum_{k=s_j}^{d_j} Y_{jk}^{f} = w_j, \qquad \forall j \in T_f. \tag{26}$$

(26) The total workload over all days between the start of flexible task j and the deadline of flexible task j equals the total workload of flexible task j for all flexible tasks j.

$$\sum_{k=1}^{s_j-1} Y_{jk}^{f} + \sum_{k=d_j+1}^{d} Y_{jk}^{f} = 0, \qquad \forall j \in T_f. \tag{27}$$

(27) It is impossible to work on a task when the start date of the task has not yet passed. Moreover, it is impossible to work on a flexible task after the deadline of the task.

$$Y_{jk}^{f} = 0, \qquad \text{if } x_{ij}^{f} = 1 \text{ and } S_{ik} = 0, j \in T_f, k \in D. \tag{28}$$

(28) When a teacher i is assigned to flexible task j, this teacher cannot work on this task on a free day, i.e. when $S_{ik} = 0$.

The objective function for this linear optimization problem only concerns overwork. Therefore, the objective function is given by:

$$\min_{Y_{jk}} \sum_{i=1}^{n} \sum_{k=1}^{d} o_{ik}^{+}. \tag{29}$$

The complete linear program for the flexible task planning is given in Appendix A.3. Using this program, we replace the uniform matrix $Y^f = (Y_{jk}^{f})$ that was used to investigate the influence of flexible tasks on overwork in the task assignment optimization. That approach is used as it was computationally too hard to both schedule and assign the flexible tasks in the same model. However, we wanted to take into account some workload of flexible tasks in the period that these tasks had to be performed. Hence, when a flexible task has a peak in workload, this has been taken into account by the task assignment. The linear program that is presented here is used to optimize the planning of the flexible tasks when the assignment is already made to make sure that unnecessary overwork that has originated from the uniform planning is not counted in the final task assignment.

After optimizing the planning of the flexible tasks, the matrix $Y^f = (Y^f_{jk})$ is used to evaluate the objective function of the original problem. Hence, when considering the objective value of a given task assignment, we always consider

$$o_{ik} = \sum_{j=1}^{m_1} Y^f_{jk} x^f_{ij} + \sum_{j=1}^{m_2} Y^n_{jk} x^n_{ij} w^n_j - C_{ik}$$

with $Y^f = (Y^f_{jk})$ as result from the linear program that just explained.

4.8 Implementation

For the implementation, MATLAB is used. Solving the mixed integer linear program is done using the Optimization Toolbox that comes with MATLAB. This solver uses the method of cutting planes to tighten the LP relaxation of the mixed integer problem and uses the branch and bound algorithm to search for an optimal solution (MathWorks, 2019). The code can be asked for by the author.[1]

5 Results

In this section, results are determined using the implementation in MATLAB. Firstly, the test data is discussed. Secondly, a general overview of results for different parameters is given. After this, one parameter choice is selected and for this parameter choice, the task assignment is investigated in more detail and compared to the task assignment that was used in 2017-2018 at CSG Reggesteyn. Finally, the effect of changes parameters is discussed.

Our test data is taken from CSG Reggesteyn, a secondary school in Nijverdal. Data from school year 2017-2018 of the unit 'havo bovenbouw' is used. At this unit, 25 teachers from 15 different disciplines are working. There are 94 tasks that are divided into 49 non-flexible tasks and 104 flexible tasks. The argumentation behind this division can be found in Appendix B.1. The rest of the data consists of the preferences of the teachers, [2] their 'norm-jaartaken' and schedules, the annual planning of the school for the year 2017-2018 and finally the actual task assignment that was used that school year based on the information from the 'norm-jaartaken'.

Results are obtained for various values for the parameters α, β and γ in order to investigate how well each objective of a teacher-task assignment is covered. In Table 4, the performance in the three areas is summarized when taking into account only one or two objectives.

Weight on the preference objective	1	0	0	1	1	0
Weight on the deviation objective	0	1	0	0	1	1
Weight on the overwork objective	0	0	1	1	0	1
Total preference value	176	225	233	194	190	226
Total deviation value	705	453	881	947	453	453
Total overwork value	256	268	197	191	291	214

Table 4: Effects of different weights on the objective functions.

[1]Email: w.r.vandermeulen@student.utwente.nl

[2]Unfortunately, not all teachers have given their preferences, see Appendix E

The first thing that has to be noticed is the value of 453 hours for the total hours of deviation that occurs for different parameter choices. The occurrence of this value is not coincidental. The sum of the workload of all tasks that should be assigned to teachers gives a total workload of 3964 hours. Looking at the sum of all target values for the teachers, a total target workload of only 3511 hours is found. The difference between the total workload and the total target workload of the non-teaching tasks is precisely 453 hours. This means that there is more workload due to task that has to be assigned to the teachers than would be desirable following the total target workload. Hence, the total deviation cannot be smaller than 453 hours.

Columns 1, 2 and 3 of Table 4 show that taking into account only one of three objectives causes task assignments that are undesirable. When only taking into account the preferences, it can be seen that the total hours of deviation from the total workload target is high. Moreover, the total overwork sum is high. Taking into account only the deviation results in a task assignment that does not represent the preferences of the teachers and causes even more overwork. Finally, only giving weight to overwork hours causes large deviations and also causes a bad task assignments with respect to the preferences of the teachers. Therefore, it is undesirable to weight only one of the three objectives.

Columns 4, 5 and 6 of Table 4 show results when taking into account two of three objectives of the teacher-task assignment. Here undesirable results occur as well: in column 4, the total deviation from the target workload is very high, in column 5, there are many overwork hours and in column 4, the preferences are very badly represented in the task assignment. Therefore, it is also undesirable to weight only two of the three objectives.

Weight on the preference objective	1	1	1	1	1	1	1	2
Weight on the deviation objective	1	1	1	2	3	4	1	1
Weight on the overwork objective	1	2	3	1	1	1	4	2
Total preference value	204	214	207	206	207	207	216	202
Total deviation value	457	459	475	453	453	453	457	459
Total overwork value	200	192	197	211	192	221	193	195.5

Table 5: Effects of different weights on the objective functions taking into account all three objectives of a teacher-task assignment.

Table 5 shows results when all objectives are taken into account. Choosing different weights for the objective functions causes different task assignments that can be compared in the three general performance areas. It is up to the school to decide how imporant each objective is for their task assignment. More results are given in Appendix C.

Until this point, only the general performance of the task assignments that are created using the optimization program are discussed. These results have not been compared to the task assignment that was used at CSG Reggesteyn in 2017-2018. Moreover, we have not investigated how well the task assignment performs at an individual level. To do this, we use our model to construct a teacher-task assignment using weights $\alpha = 2$, $\beta = 1$ and $\gamma = 2$ and compare this assignment to the given assignment from CSG Reggesteyn on both the general performance as the individual performance.

	Preference	Deviation	Overwork	Total
Task assignment CSG Reggesteyn	401*	987	244.7	1632.7
Optimized task assignment	202	459	195.5	856.5

Table 6: Comparison of the performance of the task assignment that is used at CSG Reggesteyn in 2017-2018 and a task assignment using the optimization model.

Before comparing the general performance, some attention should be directed at the preference value of the task assignment that was used at CSG Reggesteyn. The value of 401 for the preferences in the given task assignment is not as bad as it might sound as a large part of this preference value is caused by two teachers that are assigned to tasks that they should not be assigned to. Teachers have given a preference value of 100 for tasks that they think they cannot do. This is taken into account in the optimization model. However, in the assignment of CSG Reggesteyn two tasks were assigned to teachers that stated they cannot do these tasks. A more realistic value for the preference objective of CSG Reggesteyn is 213, as we only award a penalty of value 6 for the assignment of these tasks making the assignments of these task only undesirable but not impossible.

When comparing the general performance of the two task assignments, it can be noticed that for all three objectives the task assignment made using our model performs better than the manually constructed task assignment that was used at CSG Reggesteyn. Moreover, this task assignment fulfills all requirements which is not the case for the manual one. Especially the total hours of deviation is much better covered by the task assignment that is made using the mathematical model.

In Tables 7 and 8 on the next page, the performance of the two task assignments at individual levels is summarized.[3]

For the preferences it is impossible to give individual results for every teacher as only nine teachers have handed in their preferences. For the other teachers, all tasks are given a preference value of 3 and hence making a comparison for these teachers is unfair. For the nine teachers that provided their preferences, the preferences of five of them are better captured in the task assignment that is made using our model with on average a decrease of 6.6 in the preference value (taking the original preference value of 104 for teacher 15 equal to 10 as explained earlier). For the other 4, a slight increase can be noticed. The maximum increase is 6, which is limited considering that the assignment of one undesired task contributes a value of 6 to the individual preference value.

The total hours of deviation from the target workload is much improved in the optimized task assignment. For 20 of the 25 teachers, their assigned workload is closer to their target workload than in the assignment that was used in 2017-2018. For 2 teachers, the situation remains the same. For the other 3 teachers, we see an increase of deviation of the target workload of 15, 6 and 8 hours for teachers 1, 4 and 21 respectively. This increase in deviation from the target workload is relatively small when considering that this deviation is distributed over a whole school year.

[3] Teachers with a * have not given their preferences

The total overwork hours decrease for 12 teachers and for 7 teachers, they remain the same. For the other 6 teachers, the optimized task assignment is causing on average an increase of 12.3 hours of overwork which is relatively small when observing that a school year has 41 weeks excluding the holidays.

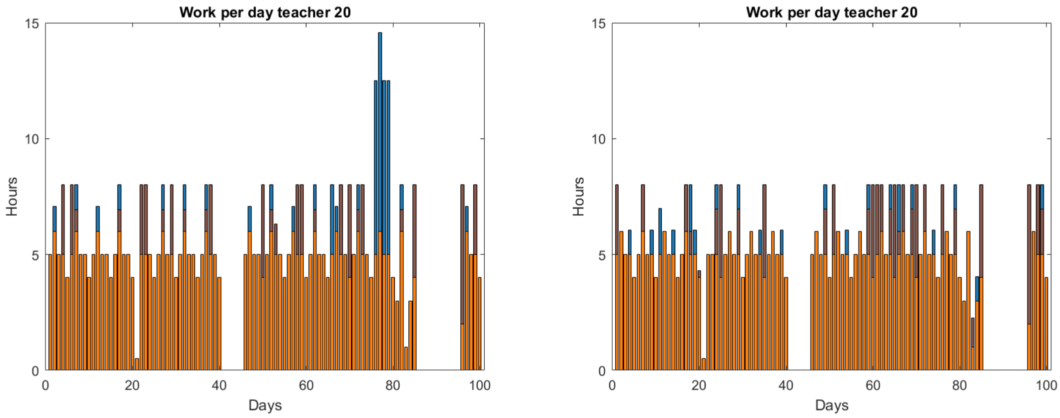
Teacher	Preference	Deviation	Overwork
1*	6	40	34
2	4	33	18.6
3*	3	32	2.1
4	2	2	17.1
5*	6	68	0
6*	9	55	0.1
7*	6	4	2.3
8*	21	24	14.9
9*	9	18	7.6
10*	9	140	7.6
11	16	24	43.1
12	20	7	37.1
13*	6	54	1
14*	18	29	9.1
15	104	103	6
16	10	21	13.1
17*	0	106	0
18*	9	22	0
19	2	34	0
20*	12	15	20.1
21*	3	16	2.1
22*	9	36	1
23*	6	45	7
24	0	20	0
25	111	39	1

Table 7: Results for individual teachers for the given task assignment.

Teacher	Preference	Deviation	Overwork
1	9	55	41.5
2	10	7	0
3	3	32	2.1
4	3	8	0
5	3	33	0
6	6	55	0
7	9	3	2.3
8	24	3	9.6
9	9	3	6.5
10	6	46	0
11	7	1	0
12	15	1	40.1
13	3	47	1
14	18	11	9
15	3	57	0
16	9	16	10
17	9	3	0
18	3	0	0
19	8	6	35.8
20	12	0	0.1
21	6	24	11.5
22	3	14	0
23	12	2	16.6
24	1	19	0
25	11	13	9.5

Table 8: Results of individual teachers for $\alpha = 2$, $\beta = 1$ and $\gamma = 2$.

The model also gives the opportunity to see how the work is distributed over the year and at what days overwork occurs. In Figure 3 below, the amount of work per day for teacher 20 is shown for the first 100 years of the year. The orange bars give the scheduled hours for teacher 20. Blue and brown bars on top of the orange bars show workload due to non-flexible and flexible tasks respectively. In Figure 3a, the situation with the original task assignment is shown. Figure 3b shows the situation with the task assignment that is constructed using our model. Due to the new assignment of tasks, the peak in workload around day 78 has removed.

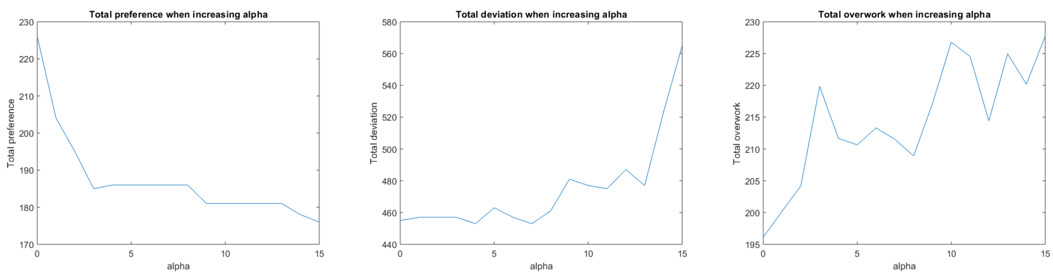


(a) The original distribution of work. (b) The new distribution of work.

Figure 3: The distribution of work for teacher 20 for the first 100 days of the school year.

The task assignment that is constructed using the mixed integer linear programming model gives improvements on all three performance areas in comparison with the manually constructed task assignment of CSG Reggestyen. Also when comparing the objectives at an individual level, it can be seen that the assignment based on the optimization model performs better than the given assignment. In Appendix C, the task assignment that is constructed using the parameter choice $\alpha = 2$, $\beta = 1$ and $\gamma = 2$ is given together with the assignments with other choices of the weighting parameters. Other choices for these parameters give different task assignments that can be compared to the manually constructed task assignment of CSG Reggesteyn. We will now analyze the influences of the parameters α, β and γ on the different objectives of the mixed integer linear programming model.

In different runs, the value of one of the parameters α, β and γ was increased to investigate the influence of these parameters on the three objectives. We start with increasing the weight on the preference value by increasing α from 0 to 15 and keeping $\beta = 1$ and $\gamma = 1$ during the first run. The results are shown in Figure 4.



(a) Total preference. (b) Total hours of deviation. (c) Total hours of overwork.

Figure 4: The results for the three objectives when increasing α from 0 to 15.

In Figure 4a, the total preference value,

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{m_1} x_{ij}^f P_{ij}^f + \sum_{j=1}^{m_2} x_{ij}^n P_{ij}^n \right),$$

is shown for different values of α. As we keep β and γ constant, the relative importance of the preferences of the teachers on the task assignment is increasing. Figure 4a shows that the total preference value decreases from 226 to 176 when the weight α is increased from 0 to 15.

The results for the total hours of deviation from the target value,

$$\sum_{i=1}^{n} \left(\delta_i^+ + \delta_i^-\right),$$

are shown in Figure 4b. The increase in relative importance of the preferences causes an increase of the total hours of deviation. For values for $\alpha \leq 7$, this increase is small. However for larger values for α it can be seen that the total hours of deviation increases from 453 to 565 hours.

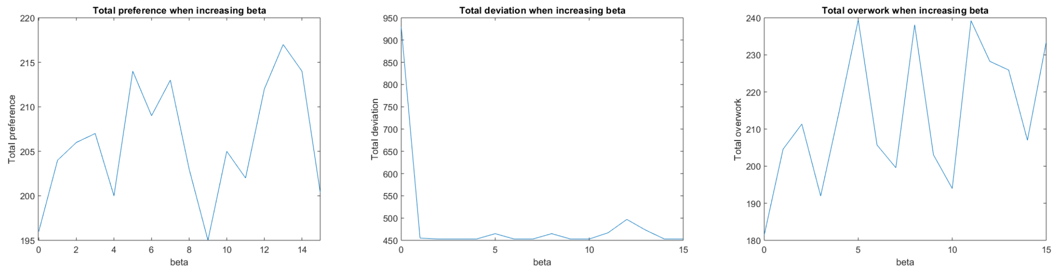
By increasing the relative importance of the preferences, it can be seen in Figure 4c that the total hours of overwork,

$$\sum_{i=1}^{n} \sum_{k=1}^{d} o_{ik}^+,$$

increases from 196 hours to 228 hours in total. Hence, another consequence of increasing the relative importance of the preferences is an increase of overwork. However, this increase is limited when considering it is the total sum of overwork hours of 25 teachers for a whole year.

The decrease of the total preference level and increase of deviation and overwork hours are not monotone. This has two reasons. First of all, the run-time of 6000 seconds was too short to obtain an optimal solution as after 6000 seconds the best feasible solution found by the solver until that moment is returned. Moreover, the heuristic approach on overwork due to flexible tasks can remove more overwork in one run than another due to different task assignments. Because of this, it is possible that the total hours of overwork for $\alpha = 3$ is higher than the total hours of overwork for $\alpha = 4$ for instance. Despite this inconsistency, Figure 4 still shows that increasing the relative importance of the preferences leads to task-assignments in which the preferences are better represented. This comes however at the expense of the total hours of deviation and overwork which is undesirable. A teacher-task assignment that is mainly based on the preferences is therefore undesirable for the work-pressure of secondary school teachers.

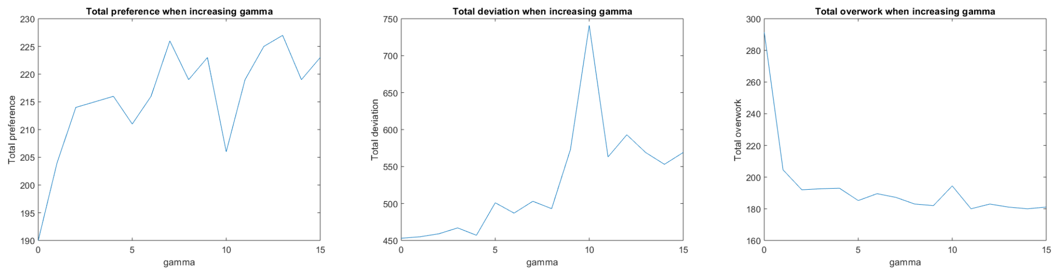
The results for the same procedure for β are slightly surprising. Figure 5 shows that giving different (increasing) relative weights to the deviation objective results in unpredictable objectives of the total preference level and the total hours of overwork. This can be caused by the fact that the objective value of the deviation function is already large at the minimum level in comparison with the objectives for the preferences and overwork. More weight for this objective simply makes the other two objectives irrelevant. To investigate if the results are more predictable for smaller values of β, the same approach has been taken for smaller values of β, see Appendix C.4.



(a) Total preference. (b) Total hours of deviation. (c) Total hours of overwork.

Figure 5: The results for the three objectives when increasing β from 0 to 15.

We continue by analyzing the effects of increasing the relative importance of the overwork part of our objective function, keeping α and β constant at a weight of 1, while increasing the weight γ from 0 to 15. The results are shown in Figure 6.



(a) Total preference. (b) Total hours of deviation. (c) Total hours of overwork.

Figure 6: The results for the three objectives when increasing γ from 0 to 15.

As the relative importance of the overwork objective function increases, Figure 6c shows that the total hours of overwork are decreasing. This decrease of total overwork hours comes at the expense of the preferences and the total hours of deviation. Similarly to Figure 4 it can be seen that the increase in preference value and deviation and the decrease in overwork is not monotone. This is again caused by the run-time and the heuristic method concerning flexible tasks.

What can be taken away from Figures 4 and 6 is that a secondary school can influence the task assignments by changing the weights on the objective in order to create a teacher-task assignment that performs well in the area(s) that is (are) most important for the school.

6 Discussion

We start by taking a look at the assumptions that are stated in Sections 4.1. The first assumption states that all tasks can be assigned to teachers when their timetables are already known. This assumption was needed to incorporate overwork into the model as overwork is given by the influence of assigned tasks on top of the regular schedule. In practice, however, there are some tasks that are assigned before the schedules are known. One of these tasks is the task ‘mentorraad’. When a school would like to use the model that is given in this paper for assigning their non-teaching tasks, they should make a slight change in their task-assigning timing such that the schedules are already known or they should produce their schedules at an earlier moment.

The workload w_j per task is taken as a known and fixed value. This corresponds with the current task-assigning policy of the school. In practice, teachers experience differences in workload. Moreover, it is possible that the workload of a certain task largely increases due to unexpected events during the school year. One example can be for a teacher who is mentor of a student that has problems at home. This teacher will probably be working more on this task than expected. It would be an improvement if the randomness of tasks can be taken into account in the teacher-task assignment model that is presented in this paper.

A heuristic method was used to find the optimal schedule for the flexible tasks given the task assignment found using the model. This was needed as overwork due to flexible of tasks is overestimated by using the uniform distribution of workload during the optimization of the teacher-task assignment. This could potentially influence the task assignment as there might be a better task assignment available when the scheduling of flexible tasks was done simultaneously. In the original idea, scheduling of flexible tasks would take place in the same model as the assignment. However this caused an increase of variables as for every teacher, flexible task and day a variable had to be added. This increase of approximately 600.000 variables had too much influence on the run time of the program.

In the model that is presented in this paper, the hours that teachers are teaching lessons were taken as timetable. This means that it is possible that a teacher has only 3 lessons on a day, but still has to work until late as the lessons take place at the end of the day. Some tasks can only be carried out after the last lesson and this is now not counted as overwork as the model gives an availability for tasks of 5 hours for this teacher. Hence, it is possible that the hours of overwork are lower than is realistic due to this assumption. Approaching the timetables differently under the assumptions that non-teaching tasks always take place after the last lesson has taken place would give different results that could be compared to the results that are presented in this paper.

We investigated the teacher-task assignment for one unit of a secondary school. Therefore, we have taken all the non-teaching tasks that are done by teachers of this unit. In practice, this division is not as explicit as we have used it for this research as there are tasks that can be assigned to all teachers of a school. One example of such task is the task 'externe sport' which can be organized by all physical education teachers of the school. We assumed that the two teachers for this task should be working in the unit 'havo bovenbouw', however this does not have to be the case in reality. This problem can be solved by looking at the task assignment at the size of a whole school. However, this would lead to an increase of the problem size as more teachers and tasks with certain requirements should be taken into account. Moreover, it requires close collaboration of the team managers of the different units of the school. It is questionable if that is desirable.

The test data that is used in this research was far from complete as interviews with teachers have never taken place due to organizational problems that are discussed in Appendix E. A closer collaboration with a secondary school would provide much more information on both the tasks as well as the preferences. Doing interviews would make sure that the preferences of all teachers are taken into account in the model and that the tasks are much better investigated, resulting in better divisions of flexible and non-flexible tasks and in better schedules of these tasks. Moreover, it would be possible to use the experienced workload in the model instead of the fixed workload as is used by the school management. This would provide a more realistic view on overwork hours as these hours are limited with the usage of data that is used for this research. In addition, it would be possible to individualize the weights on the objective function per teacher by investigating what a teacher thinks is important for his or her task assignment. For instance, a teacher that has no problem with working overtime would give more weight on the preferences and the deviation than other teachers.

To use this model in practice, a user friendly interface should be provided. This has not been done in this research. Therefore, it takes a lot of effort to convert the data into the right format before it can be used in this model to find task assignments. For instance converting the schedules of the teachers into the matrix $S = (S_{ik})$ is now done manually which is time consuming. Moreover it takes time to find all information about the tasks in the 'norm-jaartaken' of the teachers and to convert this information into the input variables that are used in the model.

The program that is written in MATLAB to solve the assignment problem is slow when taking into account the overwork objective. In optimization runs where overwork is taken into account the solution time is too long to find the optimal solution. In that case, the best feasible solution found by the solver within a run-time of 6000 seconds is returned by the program. The speed of the program can be improved by using a CPLEX solver within MATLAB or by transferring the model to the program AIMMS which has a built in CPLEX solver. This would possibly improve the solutions as the CPLEX solver is faster than the built-in solver of MATLAB.

7 Conclusion

Secondary school teachers experience high work pressure. Part of this work pressure comes from non-teaching tasks that they have to carry out during the year. In this paper, a mixed integer linear programming model is proposed to investigate if it is possible to create an assignment of non-teaching tasks for secondary school teachers that takes into account three objectives:

- Teachers should be assigned to the tasks they prefer.
- The workload for each teacher should be as close as possible to a certain target workload.
- Overwork due to assigned tasks should be as low as possible.

Together with three weighting constants, these three objectives are combined into the objective function of a mixed integer linear programming model that takes into account all requirements that a teacher-task assignment has to satisfy. Using data from CSG Reggesteyn, a school in Nijverdal, this model has been tested. Feasible solutions for the teacher-task assignment have been found for different weights on the objectives. This research has shown that it is possible to create a feasible teacher-task assignment that gives an improvement on all three objectives in comparison with the assignment that was used at CSG Reggesteyn. Since the task assignment is based on certain assumptions and as the implementation is not user friendly, the model cannot immediately be used at CSG Reggesteyn. Moreover, it is possible to improve the implementation of this model to speed up the optimization. Finally, it would be possible to improve the task assignments when more detailed data is provided via closer collaboration with a secondary school during a longer period of time.

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Appendix

The Appendix gives a full overview of the mixed integer linear programming model for the teacher-task assignment problem in Section A. In Section A.3, the full linear programming model for the optimal planning of the flexible tasks is given. In Section B, the data from CSG Reggesteyn is discussed in more detail. In Section C, an overview of results for different parameters is given. Section D provides information about the ethical approval of this research. Section E provides a reflection on the collaboration with CSG Reggesteyn.

A The full mixed integer linear program

In this section of the Appendix, the general mixed integer linear programming model for the teacher-task assignment is given consisting of the objective function and all constraints that are explained in Section 4. For convenience, also the tables of sets, parameters and variables are restated here.

A.1 Tables of variables and sets

Sets

Set	**Definition**
$L = \{1, 2, \ldots, n\}$	Set of teachers in a school unit.
L_j^f	Set of teachers capable of performing flexible task j.
L_j^n	Set of teachers capable of performing non-flexible task j.
$T_f = \{1, 2, \ldots, m_1\}$	Set of flexible tasks.
$T_n = \{1, 2, \ldots, m_2\}$	Set of non-flexible tasks.
$D = \{1, 2, \ldots, d\}$	Set of all days of the school year.
D_s	Set of all days that all teachers have to be available to be at school.
$A_j^{n,n}$	Set of all non-flexible tasks that should be assigned to the same teacher as non-flexible task j.
$A_j^{n,f}$	Set of all flexible tasks that should be assigned to the same teacher as non-flexible task j.
$A_j^{f,n}$	Set of all non-flexible tasks that should be assigned to the same teacher as flexible task j.
$A_j^{f,f}$	Set of all flexible tasks that should be assigned to the same teacher as flexible task j.
E_l^n	Sets of non-flexible tasks that cannot be assigned to the same teacher.
E_l^f	Sets of flexible tasks that cannot be assigned to the same teacher.

Table 9: Sets that are used in the model.

Parameters

Parameter	Definition
α	weight of the preference objective
β	weight of the deviation objective
γ	weight of the overwork objective
$\max_o^+$	maximal value for individual overwork
$\max_d^+$	maximal value for positive deviations from the target workload
$\max_d^-$	maximal value for negative deviations from the target workload

Table 10: Parameters that are used for the model.

Input variables

Input variable	Definition
S	n by d matrix containing the working hours of all teachers.
C	n by d matrix containing the available hours for tasks of all teachers.
P^f	n by m_1 matrix containing the preferences of the teachers for flexible tasks.
P^n	n by m_2 matrix containing the preferences of the teachers for non-flexible tasks.
Y^n	$m_1 + m_2$ by d matrix containing the planning of the non-flexible tasks.
$\mathbf{w}^f$	vector containing the workload of all flexible tasks $j \in T^f$.
$\mathbf{w}^n$	vector containing the workload of all non-flexible tasks $j \in T^n$.
$\mathbf{s}$	vector containing the start date of all flexible tasks $j \in T^f$.
$\mathbf{d}$	vector containing the deadlines of all flexible tasks $j \in T^f$.
$\mathbf{t}$	vector containing the target workloads of all teachers $i \in L$.

Table 11: Input for the model.

A.2 Full mixed integer linear programming model

$$\min_{x^n_{ij}, x^f_{ij}} \ \alpha\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{m_1} x^f_{ij}P^f_{ij} + \sum_{j=1}^{m_2} x^n_{ij}P^n_{ij}\right)\right] + \beta\left[\sum_{i=1}^{n}\left(\delta_i^+ + \delta_i^-\right)\right] + \gamma\left[\sum_{i=1}^{n}\sum_{k=1}^{d} o^+_{ik}\right]$$

s.t.

$$
\begin{aligned}
\sum_{i=1}^{n} x^n_{ij} &= 1, && \forall j \in T_n\\
\sum_{i=1}^{n} x^f_{ij} &= 1, && \forall j \in T_f\\
x^n_{ij} &= 0, && \forall j \in T_n, i \notin L^n_j\\
x^f_{ij} &= 0, && \forall j \in T_f, i \notin L^f_j\\
x^n_{ij_r} &= x^n_{ij}, && \forall j \in A^{n,n}_{j_r}, \forall j_r \in T_n, \forall i \in L\\
x^n_{ij_r} &= x^f_{ij}, && \forall j \in A^{n,f}_{j_r}, \forall j_r \in T_n, \forall i \in L\\
x^f_{ij_r} &= x^n_{ij}, && \forall j \in A^{f,n}_{j_r}, \forall j_r \in T_f, \forall i \in L\\
x^f_{ij_r} &= x^f_{ij}, && \forall j \in A^{f,f}_{j_r}, \forall j_r \in T_f, \forall i \in L\\
\sum_{j \in E^n_l} x^n_{ij} + \sum_{j \in E^f_l} x^f_{ij} &\le 1, && \forall i \in L, \forall l\\
\delta_i &= \sum_{j=1}^{m_1} x^f_{ij}w^f_j + \sum_{j=1}^{m_2} x^n_{ij}w^n_j - t_i, && \forall i \in L\\
\delta_i &= \delta_i^+ - \delta_i^-, && \forall i \in L\\
\delta_i^+ &\le \max{}^+_d, && \forall i \in L\\
\delta_i^+ &\le \max{}^-_d, && \forall i \in L\\
o_{ik} &= \sum_{j=1}^{m_1} Y^f_{ijk}x^f_{ij} + \sum_{j=1}^{m_2} Y^n_{jk}x^n_{ij}w^n_j - C_{ik}, && \forall i \in L, \forall k \in D\\
o_{ik} &= o^+_{ik} - o^-_{ik}, && \forall i \in L, \forall k \in D\\
o^+_{ik} &\le \max{}^+_o, && \forall i \in L, k \in D\\
x^n_{ij} &= 0, && \forall i \in L \text{ if } Y_{jk} = 1 \text{ and } S_{ik} = 0, j \in T_n, k \in D \backslash D_S\\
\delta_i^+ &\ge 0\\
\delta_i^- &\ge 0\\
o^+_{ik} &\ge 0\\
o^-_{ik} &\ge 0\\
x^n_{ij} &\in \{0,1\}\\
x^f_{ij} &\in \{0,1\}
\end{aligned}
$$

A.3 Linear program for flexible task planning

$$\min_{Y_{jk}} \sum_{i=1}^{n} \sum_{k=1}^{d} o_{ik}^{+}$$

s.t.

$$
\begin{aligned}
o_{ik} &= \sum_{j=1}^{m_1} Y_{jk}^{f} x_{ij}^{f} + \sum_{j=1}^{m_2} Y_{jk}^{n} w_j^{n} - C_{ik} && \forall i \in L, \forall k \in D. \\
o_{ik} &= o_{ik}^{+} - o_{ik}^{-} && \forall i \in L, \forall k \in D. \\
o_{ik}^{+} &\leq \max_o^{+}, && \forall i \in L, k \in D \\
\sum_{k=s_j}^{d_j} Y_{jk} &= w_j && \forall j \in T_f \\
\sum_{k=1}^{s_j-1} Y_{jk}^{f} + \sum_{k=d_j+1}^{d} Y_{jk}^{f} &= 0 && \forall j \in T_f \\
Y_{jk}^{f} &= 0 && \text{if } x_{ij}^{f} = 1 \text{ and} S_{ik} = 0, j \in T_f, k \in D \\
o_{ik}^{+} &\geq 0 \\
o_{ik}^{-} &\geq 0 \\
Y_{jk}^{f} &\geq 0.
\end{aligned}
$$

B Data from CSG Reggesteyn

In this section of the Appendix, the data from CSG Reggesteyn is discussed in more detail. The data that is used consists of a list of all tasks, the 'norm-jaartaken' of the teachers from which the task assignment is extracted, the schedules of the teachers, the year-planning of the school and the preferences of the teachers. For this research, data from 2017-2018 is used. Due to privacy reasons, the 'norm-jaartaken' and schedules of teachers are not given in the Appendix.

B.1 Tasks for havo-bovenbouw CSG Reggesteyn

In this subsection, we provide detailed information on all tasks that need to be assigned to teachers of 'havo-bovenbouw' from CSG Reggesteyn. We start with a list of tasks and their classifications.

Mentorraad jaar 3

1. 3 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task consists of pre-scheduled mentor-hours and parents-evenings and therefore is non-flexible.

Mentorraad jaar 4

1. 10 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 54 hours.
3. This task not only consists of pre-scheduled mentor-hours and parents-evenings (50 hours) but also has a flexible part of 4 hours that can be distributed over the year. This task will be split in a flexible and a non-flexible part.

Mentorraad jaar 5

1. 10 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 39 hours (due to examination year).
3. The task consists of pre-scheduled mentor-hours and parents-evenings and therefore is non-flexible.

Havo op maat

1. 4 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 40 hours.
3. The task is flexible.

Leerlingenbegeleider

1. 6 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 7 hours.

3. The task is flexible.

Werkweek

1. 9 teachers from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 40 hours.
3. The task is non-flexible as the dates are already known.

Portefeuille-houder

1. 3 teachers from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 100 hours.
3. The task is flexible.

Examen secretaris

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible: it has peaks of workload around test-weeks and this has been done by splitting these tasks into different tasks all concerning one test-period.

Vakgroep coördinator

1. 5 teachers from the unit ‘havo-bovenbouw’ should be assigned to this task for the subjects SK, LO, BIO, ENG and ECO.
2. The total workload of this task is 30 hours.
3. The task both flexible and non-flexible as it consists of (non-flexible) meetings and preparation and evaluation of meetings (flexible). Therefore, these tasks have been split into separate tasks

Datateam

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 60 hours.
3. The task is flexible.

Profielwerkstukken

1. This is a pre-assigned task as it is already known who supervises which student. However, this counts as a task and hence the workload of this task should be taken into account for the task assignment.
2. The total workload of this task is depending on the number of students that a teacher supervises.
3. The task is flexible.

Cambridge

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible.

Wiskunde D

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 100 hours.
3. The task is flexible.

Maatschappelijke stage

1. 6 teachers from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 15 hours.
3. The task has both non-flexible as flexible components.

Technator

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 300 hours.
3. The task is flexible.

Duurzaamheid

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 40 hours.
3. The task is flexible.

Democratiedagen

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 40 hours.
3. The task is non-flexible.

Debatclub

1. 1 teacher from the unit ‘havo-bovenbouw’ should be assigned to this task.
2. The total workload of this task is 10 hours.
3. The task is non-flexible.

Stagiair

1. This is a pre-assigned task before the start of the school year.
2. The total workload of this task is depending per teacher.
3. The task is flexible.

Identiteit

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 20 hours.
3. The task is flexible.

Externe sport

1. 2 teachers from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task has both flexible as non-flexible components.

Mediatheekcomissie

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 20 hours.
3. The task is flexible.

Rooster

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible.

Nieuwbenoemden begeleiding

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible.

Nieuwbenoemden begeleiding

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible.

Taal coördinator

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 50 hours.
3. The task is flexible.

Reken coördinator

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 70 hours.
3. The task is flexible.

Reken specialist

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 100 hours.
3. The task is non-flexible.

Overnachting uitwisseling

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 30 hours.
3. The task is non-flexible.

ICD

1. 1 teacher from the unit 'havo-bovenbouw' should be assigned to this task.
2. The total workload of this task is 100 hours.
3. The task is flexible.

The list of tasks and their classification is derived from the 'norm-jaartaken' of the teachers, the year-schedule of CSG Reggesteyn, one meeting with the manager of facility organization and one meeting with my supervisor Tom Coenen. In the original plan, better insight in the tasks would be obtained from interviews with all teachers. These interviews have not taken place as it was impossible to arrange them with the team manager of 'havo-bovenbouw'.[4]

[4]See Section E.

B.2 Overview of all tasks as used in the optimization and the assignment of these tasks as used by CSG Reggesteyn in 2017-2018

In this subsection, we consider the tasks as they are used in the model. This includes splitting up tasks in their non-flexible and flexible parts and creating separate tasks when more teachers should be assigned to tasks that are discussed in Section B.1. How these separate tasks are connected in the task assignment is stated in Section B.3. Also the assignment to the teachers as was used by CSG Reggesteyn is given in the last column of these table.

Task	Description	Workload	Start	Deadline	Assignment
1	Mentor 4 (1)	4	1	235	1
2	Mentor 4 (2)	4	1	235	4
3	Mentor 4 (3)	4	1	235	5
4	Mentor 4 (4)	4	1	235	9
5	Mentor 4 (5)	4	1	235	14
6	Mentor 4 (6)	4	1	235	16
7	Mentor 4 (7)	4	1	235	18
8	Mentor 4 (8)	4	1	235	19
9	Mentor 4 (9)	4	1	235	22
10	Mentor 4 (10)	4	1	235	25
11	Havo op maat (1)	40	1	235	1
12	Havo op maat (2)	40	1	235	8
13	Havo op maat (3)	40	1	235	11
14	Havo op maat (4)	40	1	235	14
15	Leerlingenbegeleider (1)	7	1	235	6
16	Leerlingenbegeleider (2)	7	1	235	13
17	Leerlingenbegeleider (3)	7	1	235	15
18	Leerlingenbegeleider (4)	7	1	235	18
19	Leerlingenbegeleider (5)	7	1	235	22
20	Leerlingenbegeleider (6)	7	1	235	25
21	Portefeuille-houder (1)	100	1	235	12
22	Portefeuille-houder (2)	100	1	235	14
23	Examensecretaris TW1	5	49	55	8
24	Examensecretaris TW2	5	106	112	8
25	Examensecretaris TW3	5	157	163	8
26	Examensecretaris CSE1	15	186	196	8
27	Examensecretaris TW4&CSE2	20	211	225	8
28	Vakgroep SK verwerking (1)	3	18	46	8
29	Vakgroep SK verwerking (2)	3	48	61	8
30	Vakgroep SK verwerking (3)	3	63	81	8
31	Vakgroep SK verwerking (4)	3	83	116	8
32	Vakgroep SK verwerking (5)	3	118	156	8
33	Vakgroep SK verwerking (6)	3	158	235	8
34	Vakgroep LO verwerking (1)	3	18	46	12
35	Vakgroep LO verwerking (2)	3	48	61	12
36	Vakgroep LO verwerking (3)	3	63	81	12
37	Vakgroep LO verwerking (4)	3	83	116	12
38	Vakgroep LO verwerking (5)	3	118	156	12

39	Vakgroep LO verwerking (6)	3	158	235	12
40	Vakgroep bio verwerking (1)	3	18	46	15
41	Vakgroep bio verwerking (2)	3	48	61	15
42	Vakgroep bio verwerking (3)	3	63	81	15
43	Vakgroep bio verwerking (4)	3	83	116	15
44	Vakgroep bio verwerking (5)	3	118	156	15
45	Vakgroep bio verwerking (6)	3	158	235	15
46	Vakgroep eng verwerking (1)	3	18	46	16
47	Vakgroep eng verwerking (2)	3	48	61	16
48	Vakgroep eng verwerking (3)	3	63	81	16
49	Vakgroep eng verwerking (4)	3	83	116	16
50	Vakgroep eng verwerking (5)	3	118	156	16
51	Vakgroep eng verwerking (6)	3	158	235	16
52	Vakgroep eco verwerking (1)	3	18	46	23
53	Vakgroep eco verwerking (2)	3	48	61	23
54	Vakgroep eco verwerking (3)	3	63	81	23
55	Vakgroep eco verwerking (4)	3	83	116	23
56	Vakgroep eco verwerking (5)	3	118	156	23
57	Vakgroep eco verwerking (6)	3	158	235	23
58	Datateam	60	1	235	8
59	Cambridge	50	1	235	16
60	Wiskunde D	100	1	235	19
61	Voorbereiding MAS (1)	3	1	26	6
62	Begeleiding MAS (1)	10	28	235	6
63	Voorbereiding MAS (2)	3	1	26	9
64	Begeleiding MAS (2)	10	28	235	9
65	Voorbereiding MAS (3)	3	1	26	14
66	Begeleiding MAS (3)	10	28	235	14
67	Voorbereiding MAS (4)	3	1	26	18
68	Begeleiding MAS (4)	10	28	235	18
69	Voorbereiding MAS (5)	3	1	26	22
70	Begeleiding MAS (5)	10	28	235	22
71	Voorbereiding MAS (6)	3	1	26	25
72	Begeleiding MAS (6)	10	28	235	25
73	Technator	300	1	235	7
74	Duurzaamheid	40	1	235	20
75	Democratiedagen	3	1	96	25
76	Stagiair 1	30	1	235	3
77	Stagiair 2	34	1	235	15
78	Stagiair 3	35	1	235	25
79	Identiteit	20	1	235	25
80	Externe sport (1)	5	1	79	2
81	Externe sport (2)	5	1	79	12
82	Mediatheekcomissie	20	1	235	5
83	Rooster	50	1	235	8
84	Nieuwbenoemdenbegeleider	50	1	235	10
85	Taalcoördinator	50	1	235	14

86	Rekencoördinator	70	1	235	11
87	ICD	100	1	235	2
88	PWS begeleiding 1	40	28	126	1
89	PWS begeleiding 2	20	28	126	2
90	PWS begeleiding 3	5	28	126	6
91	PWS begeleiding 4	5	28	126	4
92	PWS begeleiding 5	105	28	126	5
93	PWS begeleiding 6	10	28	126	7
94	PWS begeleiding 7	30	28	126	13
95	PWS begeleiding 8	85	28	126	15
96	PWS begeleiding 9	20	28	126	17
97	PWS begeleiding 10	15	28	126	19
98	PWS begeleiding 11	35	28	126	20
99	PWS begeleiding 12	15	28	126	21
100	PWS begeleiding 13	10	28	126	22
101	PWS begeleiding 14	30	28	126	23
102	PWS begeleiding 25	15	28	126	11
103	Stagiair 4	35	1	235	2
104	Portefeuille houder 3	100	1	235	20

Table 12: The flexible tasks for the unit havo bovenbouw and their start-dates and deadlines and total workload.

Task	Description	Workload	Assignment
1	Mentorraad 3 (1)	50	3
2	Mentorraad 3 (2)	50	7
3	Mentorraad 3 (3)	50	10
4	Mentorraad 4 (1)	50	1
5	Mentorraad 4 (2)	50	4
6	Mentorraad 4 (3)	50	5
7	Mentorraad 4 (4)	50	9
8	Mentorraad 4 (5)	50	14
9	Mentorraad 4 (6)	50	16
10	Mentorraad 4 (7)	50	18
11	Mentorraad 4 (8)	50	19
12	Mentorraad 4 (9)	50	22
13	Mentorraad 4 (10)	50	25
14	Mentorraad 5 (1)	39	2
15	Mentorraad 5 (2)	39	6
16	Mentorraad 5 (3)	39	8
17	Mentorraad 5 (4)	39	11
18	Mentorraad 5 (5)	39	12
19	Mentorraad 5 (6)	39	13
20	Mentorraad 5 (7)	39	15
21	Mentorraad 5 (8)	39	20
22	Mentorraad 5 (9)	39	21
23	Mentorraad 5 (10)	39	23

24	Werkweek Alps	40	2
25	Werkweek London	40	4
26	Werkweek Krakau	40	8
27	Werkweek London	40	9
28	Werkweek Prague	40	10
29	Werkweek London	40	11
30	Werkweek Alps	40	12
31	Werkweek Krakau	40	14
32	Werkweek Italy	40	16
33	Vakgroep SK	12	8
34	Vakgroep LO	12	12
35	Vakgroep Bi	12	15
36	Vakgroep Eng	12	16
37	Vakgroep Eco	12	23
38	MAS (1)	2	6
39	MAS (2)	2	9
40	MAS (3)	2	14
41	MAS (4)	2	18
42	MAS (5)	2	22
43	MAS (6)	2	25
44	Democratiedagen	32	25
45	Debatclub	10	25
46	Externe sport	45	2
47	Externe sport	45	12
48	Rekenspecialist	100	11
49	Overnachting uitwisseling	30	20

Table 13: Non-flexible tasks and their allocation.

B.3 Specific sets from the data

We will now specify the specific sets that belong to this data set in order to use the data in the MILP model that is introduced in Section 4.

The sets $\mathbf{A_j^{n,n}}$, $\mathbf{A_j^{n,f}}$, $\mathbf{A_j^{f,n}}$ and $\mathbf{A_j^{f,f}}$
The following sets are used to connect the tasks corresponding to Tables 12 and 13.

$A_j^{n,n} = \{j\}\ j = 1 \dots 49$
$A_j^{n,f} = \emptyset\ j = 1, 2, 3$
$A_j^{n,f} = \{j-3\}\ j = 4, 5, \dots 13$
$A_j^{n,f} = \emptyset\ j = 14, 15, \dots, 31, 32$
$A_{33}^{n,f} = \{28, 29, \dots 33\}$
$A_{34}^{n,f} = \{34, 35, \dots 39\}$
$A_{35}^{n,f} = \{40, 41, \dots 45\}$
$A_{36}^{n,f} = \{46, 47, \dots 51\}$
$A_{37}^{n,f} = \{52, 53, \dots 57\}$
$A_{38}^{n,f} = \{61, 62\}$
$A_{39}^{n,f} = \{63, 64\}$
$A_{40}^{n,f} = \{65, 66\}$
$A_{41}^{n,f} = \{67, 68\}$
$A_{42}^{n,f} = \{69, 70\}$
$A_{43}^{n,f} = \{71, 72\}$
$A_{44}^{n,f} = \{75\}$
$A_{45}^{n,f} = \emptyset$
$A_{46}^{n,f} = \{80\}$
$A_{47}^{n,f} = \{81\}$
$A_j^{f,f} = \{j\}\ j = 1, \dots, 22, 58, 59, 60, 73, 74, \dots 104$
$A_{23}^{f,f} = \{23, 24, 25, 26, 27\}\ j = 23, 24, \dots, 27$
$A_j^{f,f} = \{28, 29, 30, 31, 32, 33\}\ j = 28, 29, \dots, 33$
$A_j^{f,f} = \{34, 35, 36, 37, 38, 39\}\ j = 34, 35, \dots, 39$
$A_j^{f,f} = \{40, 41, 42, 43, 44, 45\}\ j = 40, 41, \dots, 45$
$A_j^{f,f} = \{46, 47, 48, 49, 50, 51\}\ j = 46, 47, \dots, 51$
$A_j^{f,f} = \{52, 53, 54, 55, 56, 57\}\ j = 52, 53, \dots, 57$
$A_j^{f,f} = \{j\}\ j = 1, \dots, 22$
$A_j^{f,f} = \{23, 24, 25, 26, 27\}\ j = 23, 24, \dots, 27$
$A_j^{f,f} = \{28, 29, 30, 31, 32, 33\}\ j = 28, 29, \dots, 33$
$A_j^{f,f} = \{34, 35, 36, 37, 38, 39\}\ j = 34, 35, \dots, 39$
$A_j^{f,f} = \{40, 41, 42, 43, 44, 45\}\ j = 40, 41, \dots, 45$
$A_j^{f,f} = \{46, 47, 48, 49, 50, 51\}\ j = 46, 47, \dots, 51$
$A_j^{f,f} = \{52, 53, 54, 55, 56, 57\}\ j = 52, 53, \dots, 57$
$A_j^{f,f} = \{j\}\ j = 58, 59, 60$
$A_j^{f,f} = \{j, j+1\}\ j = 61, 63, 65, 67, 69, 71$
$A_j^{f,f} = \{j-1, j\}\ j = 62, 64, 66, 68, 70, 72$
$A_j^{f,f} = \{j\}\ j = 73, 74, \dots 104$
$A_j^{f,n} = j + 3\ j = 1, 2, \dots, 10$
$A_j^{f,n} = \emptyset\ j = 11, 12, \dots, 27$
$A_j^{f,n} = \{33\}\ j = 28, 29, \dots, 33$
$A_j^{f,n} = \{34\}\ j = 34, 35, \dots, 39$
$A_j^{f,n} = \{35\}\ j = 40, 41, \dots, 45$
$A_j^{f,n} = \{36\}\ j = 46, 47, \dots, 51$
$A_j^{f,n} = \{37\}\ j = 52, 53, \dots, 57$
$A_j^{f,n} = \emptyset\ j = 58, 59, 60$
$A_j^{f,n} = \{38\}\ j = 61, 62$
$A_j^{f,n} = \{39\}\ j = 63, 62$
$A_j^{f,n} = \{40\}\ j = 65, 62$
$A_j^{f,n} = \{41\}\ j = 67, 62$
$A_j^{f,n} = \{42\}\ j = 69, 62$
$A_j^{f,n} = \{43\}\ j = 71, 72$
$A_j^{f,n} = \emptyset\ j = 73, 74$
$A_{75}^{f,n} = \{44\}$
$A_j^{f,n} = \emptyset\ j = 76, 77, 78, 79$
$A_{80}^{f,n} = \{46\}$
$A_{81}^{f,n} = \{47\}$
$A_j^{f,n} = \emptyset\ j = 82, 83, \dots 104$

The sets $\mathbf{E_l^f}$ and $\mathbf{E_l^n}$

These sets are used in combination with constraint (13) to make sure that a teacher can only be assigned to at most one of the tasks per set.

$$\begin{aligned} E_1^n &= \{1, 2, \ldots, 23\} \\ E_1^f &= \emptyset \end{aligned} \tag{30}$$

(30) This set, together with constraint (13) ensure that a teacher can only be mentor of one class.

$$\begin{aligned} E_2^n &= \{24, 25 \ldots, 32\} \\ E_2^f &= \emptyset \end{aligned} \tag{31}$$

(31) This set, together with constraint (13) ensures that a teacher is only assigned to one 'werkweek'.

$$\begin{aligned} E_3^n &= \{38, 39, \ldots 43\} \\ E_3^f &= \emptyset \end{aligned} \tag{32}$$

(32) This set, together with constraint (13) ensures that teachers are only assigned to one 'maatschappelijke stage' task.

$$\begin{aligned} E_4^n &= \{46, 47\} \\ E_4^f &= \emptyset \end{aligned} \tag{33}$$

(33) This set, together with constraint (13) ensures that two different teachers are assigned to the task 'externe sport'.

$$\begin{aligned} E_5^n &= \emptyset \\ E_5^f &= \{11, 12, 13, 14\} \end{aligned} \tag{34}$$

(34) This set, together with constraint (13) ensures that four different teachers are assigned to the task 'havo op maat'.

$$\begin{aligned} E_6^n &= \emptyset \\ E_6^f &= \{15, 16, \ldots, 20\} \end{aligned} \tag{35}$$

(35) This set, together with constraint (13) ensures that six different teachers are assigned to the task 'leerlingenbegleider'.

$$E_7^n = \emptyset \tag{36}$$
$$E_7^f = \{21, 22, 104\}$$

(36) This set, together with constraint (13) ensures that three different teachers are so-called 'portefeuillehouders' during the school year.

The sets $\mathbf{L_j^n}$ and $\mathbf{L_j^f}$

In this section, sets are introduced that, together with constraint (7) and 8, ensure that tasks are assigned to teachers that actually can perform a task.

$$L_j^f = L\backslash\{10\}\ j = 1, 2, \ldots, 10$$
$$L_j^f = L\backslash\{12\}\ j = 11, 12, 13, 14$$
$$L_j^f = L\backslash\{12, 15, 24, 25\}\ j = 15, 16, \ldots 20$$
$$L_j^f = L\backslash\{19\}\ j = 21, 22$$
$$L_j^f = \{4, 8, 11\}\ j = 23, 24, \ldots 27$$
$$L_j^f = \{8\}\ j = 28, 29, \ldots 33$$
$$L_j^f = \{2, 12\}\ j = 34, 35, \ldots 39$$
$$L_j^f = \{15\}\ j = 40, 41, \ldots 45$$
$$L_j^f = \{4, 16, 22\}\ j = 46, 47, \ldots 51$$
$$L_j^f = \{1, 3, 23\}\ j = 52, 53, \ldots 57$$
$$L_{58}^f = L\backslash\{12, 16, 19\}$$
$$L_{59}^f = \{4, 16, 22\}$$
$$L_{60}^f = \{10, 11, 19\}$$
$$L_j^f = L\ j = 61, 62, \ldots 72$$
$$L_{73}^f = \{7, 11\}$$
$$L_{74}^f = \{2, 12, 19\}$$
$$L_{75}^f = L\backslash\{2, 11, 12, 15, 19, 24\}$$
$$L_{76}^f = \{3\}$$
$$L_{77}^f = \{15\}$$
$$L_{78}^f = \{25\}$$
$$L_{79}^f = L\backslash\{12, 19\}$$
$$L_j^f = \{2, 4, 12, 16, 25\}\ j = 80, 81$$
$$L_{82}^f = L\backslash\{11, 12, 19\}$$
$$L_{83}^f = \{8, 11\}$$
$$L_{84}^f = L\backslash\{12\}$$
$$L_{85}^f = \{4, 14, 16, 18, 21, 22, 24\}$$
$$L_{86}^f = \{10, 11, 19\}$$
$$L_{87}^f = \{12, 16, 19, 24\}$$

$L^f_{88} = \{1\}$
$L^f_{89} = \{2\}$
$L^f_{90} = \{6\}$
$L^f_{91} = \{4\}$
$L^f_{92} = \{5\}$
$L^f_{93} = \{7\}$
$L^f_{94} = \{13\}$
$L^f_{95} = \{15\}$
$L^f_{96} = \{17\}$
$L^f_{97} = \{19\}$
$L^f_{98} = \{20\}$
$L^f_{99} = \{21\}$
$L^f_{100} = \{22\}$
$L^f_{101} = \{23\}$
$L^f_{102} = \{11\}$
$L^f_{103} = \{2\}$
$L^f_{104} = L\backslash\{19\}$

$L^n_j = L\backslash\{1,4,5,11,13,14,19,23,24,25\}\ j = 1,2,3$
$L^n_j = L\backslash\{10\}\ j = 4,5,\dots,13$
$L^n_j = L\backslash\{3,25\}\ j = 14,15,\dots,23$
$L^n_{24} = L\backslash\{12,16,25\}$
$L^n_j = L\backslash\{12\}\ j = 25,25,\dots,29$
$L^n_{30} = L\backslash\{12,16,25\}$
$L^n_j = L\backslash\{12\}\ j = 31,32$
$L^n_{33} = \{8\}$
$L^n_{34} = \{2,12\}$
$L^n_{35} = \{15\}$
$L^n_{36} = \{4,22,16\}$
$L^n_{37} = \{1,3,23\}$
$L^n_j = L\ j = 38,39,\dots,43$
$L^n_{44} = L\backslash\{2,11,12,15,19,24\}$
$L^n_{45} = \{2,11,12,15,16,19\}$
$L^n_j = \{2,4,12,16,25\}\ j = 46,47$
$L^n_{48} = \{10,11,19\}$
$L^n_{49} = \{10,11,19\}$

Preferences
In this part of the Appendix, the preferences of the teachers are discussed. Teachers of CSG Reggesteyn were asked to give their preferences for the list of tasks that is discussed in Section B.1. Teachers give a preference value of 1 for tasks that they prefer, a value of 3 for tasks that they can do and a value of 6 when they would rather not do that task. A teacher that cannot perform a task gives a value of 100.

To ensure that for every task the right preferences is taken into account, the following columns of P^f are set to zero. This is needed as tasks are connected via the sets $A^{n,n}_j$, $A^{n,f}_j$, $A^{f,n}_j$ and $A^{f,f}_j$. Not setting these columns to zero causes that the preferences are counted twice or more which is undesirable.

$$P^f_{ij} = 0,\ \forall i \in L,\ j = 1,2\dots,10,24,25\dots57,61,62,\dots,72,75,76,77,78,80,81,88,89,\dots103.$$

For all other columns of P^f_{ij} and all columns P^n_{ij}, the preferences are given by the values 1, 3, 6 and 100. Due to the constraints it is impossible to assign a task to a teacher when the preferences value of this teacher for this task is 100. For teachers that were not able to give their preferences, a preference value of 3 is awarded to all tasks, see Section E.

Parameters
To obtain our results, we have chosen the following parameters:

$$\max_o^+ = 8$$
$$\max_d^+ = 60$$
$$\max_d^- = 60$$

Target workloads per teacher

Teacher	Target workload t_i
1	94
2	297
3	48
4	101
5	111
6	11
7	364
8	285
9	91
10	0
11	290
12	266
13	22
14	270
15	92
16	153
17	126
18	54
19	203
20	229
21	70
22	50
23	144
24	20
25	120

Table 14: The target workloads per teacher in hours.

The set of days that all teachers have to be available to do non-flexible tasks, $\mathbf{D_s}$

The following days, all teachers are required to be available for school due to parent evenings, activities. Also holidays are excluded as no non-flexible tasks are scheduled on these days and hence no constraints are required for these days.

$$\begin{aligned} D_s = \{&11, 17, 18, 27, 66, 67, 80, 81, 82, 83, 84, 96, 96, 98, 99, 100,\\ &101, 102, 103, 104, 105, 106, 137, 152, 153, 197, 198, 199,\\ &200, 201, 202, 203, 204, 205, 206, 208, 213, 226, 227, 228, 229\} \end{aligned}$$

C Overview of results

This section provides the individual performance of the task assignment of CSG Reggesteyn and multiple task assignments constructed using the model that is presented in this paper.

C.1 Results for the task assignment given by CSG Reggesteyn

In this section, we give an overview of the performance measures for the original task assignment that has been used for CSG Reggesteyn in 2017-2018. Unlike in the 'norm-jaartaken' of the teachers the workload of every assigned task is taken into account in the model. That is, when in the 'norm-jaartaken' certain workload for tasks had been removed to the next year (although the task was assigned to this teacher), this is not done here as we had to take workload for every task that has to be assigned into account to ensure a fair comparison between the original assignment and the task assignments that are found using the MILP model of Section 4. This means that the numbers for the deviation can deviate from the deviation that is found in the 'norm-jaartaken' of the teachers.

Performance

Teacher	Preference	Deviation	Overwork
1*	6	40	34
2	4	33	18.6
3*	3	32	2.1
4	2	2	17.1
5*	6	68	0
6*	9	55	0.1
7*	6	4	2.3
8*	21	24	14.9
9*	9	18	7.6
10*	9	140	7.6
11	16	24	43.1
12	20	7	37.1
13*	6	54	1
14*	18	29	9.1
15	104	103	6
16	10	21	13.1
17*	0	106	0
18*	9	22	0
19	2	34	0
20*	12	15	20.1
21*	3	16	2.1
22*	9	36	1
23*	6	45	7
24	0	20	0
25	111	39	1

Table 15: Results for individual teachers for the given task assignment. Teachers with a star at their name did not hand in their preferences.

C.2 Results for different parameters

In this section, tables for different assignments are given.

C.2.1 Parameter set 1

$\alpha = 2, \beta = 1$ and $\gamma = 2$

Performance

Teacher	Preference	Deviation	Overwork
1	9	55	41.5
2	10	7	0
3	3	32	2.1
4	3	8	0
5	3	33	0
6	6	55	0
7	9	3	2.3
8	24	3	9.6
9	9	3	6.5
10	6	46	0
11	7	1	0
12	15	1	40.1
13	3	47	1
14	18	11	9
15	3	57	0
16	9	16	10
17	9	3	0
18	3	0	0
19	8	6	35.8
20	12	0	0.1
21	6	24	11.5
22	3	14	0
23	12	2	16.6
24	1	19	0
25	11	13	9.5

Table 16: Results of individual teachers for $\alpha = 2$, $\beta = 1$ and $\gamma = 2$.

Flexible task assignment

Task	Assignment
1	19
2	18
3	12
4	6
5	17
6	4
7	22
8	20
9	23
10	2
11	16
12	17
13	20
14	8
15	10
16	23
17	6
18	11
19	8
20	7
21	14
22	12
23	8
24	8
25	8
26	8
27	8
28	8
29	8
30	8
31	8
32	8
33	8
34	12
35	12

Task	Assignment
36	12
37	12
38	12
39	12
40	15
41	15
42	15
43	15
44	15
45	15
46	4
47	4
48	4
49	4
50	4
51	4
52	1
53	1
54	1
55	1
56	1
57	1
58	20
59	16
60	11
61	17
62	17
63	2
64	2
65	25
66	25
67	8
68	8
69	23
70	23

Task	Assignment
71	9
72	9
73	7
74	20
75	12
76	3
77	15
78	25
79	14
80	12
81	25
82	4
83	11
84	8
85	14
86	11
87	2
88	1
89	2
90	6
91	4
92	5
93	7
94	13
95	15
96	17
97	19
98	20
99	21
100	22
101	23
102	11
103	2
104	2

Table 17: Flexible task assignment for $\alpha = 2$, $\beta = 1$ and $\gamma = 2$.

Non-flex task assignment

Task	Assignment
1	3
2	7
3	8
4	19
5	18
6	12
7	6
8	17
9	4
10	22
11	20
12	23
13	2
14	1
15	10
16	24
17	16
18	13
19	9
20	5
21	21
22	11
23	14
24	1
25	23

Task	Assignment
26	21
27	16
28	8
29	25
30	9
31	19
32	14
33	8
34	12
35	15
36	4
37	1
38	17
39	2
40	25
41	8
42	23
43	9
44	14
45	25
46	12
47	25
48	19
49	12

Table 18: Non-flexible task assignment for $\alpha = 2$, $\beta = 1$ and $\gamma = 2$.

C.2.2 Parameter set 2

$\alpha = 4, \beta = 1$ and $\gamma = 1$

Performance

Teacher	Preference	Deviation	Overwork
1	6	15	33
2	7	7	18.6
3	3	32	2.1
4	3	18	16.1
5	3	33	0
6	3	48	0
7	9	3	2.3
8	21	10	9.6
9	9	3	6.5
10	6	46	0
11	7	1	0
12	15	2	40.1
13	3	47	1
14	18	1	9.1
15	3	57	0
16	6	6	11
17	9	3	0
18	3	0	0
19	8	6	35.7
20	9	0	0
21	9	31	11.5
22	3	14	0
23	12	2	3
24	2	59	0
25	11	15	7

Table 19: Results of individual teachers for $\alpha = 4$, $\beta = 1$ and $\gamma = 1$.

Flexible task assignment

Task	Assignment
1	19
2	14
3	23
4	22
5	18
6	12
7	4
8	6
9	17
10	20
11	17
12	24
13	23
14	2
15	21
16	11
17	7
18	23
19	14
20	10
21	20
22	12
23	8
24	8
25	8
26	8
27	8
28	8
29	8
30	8
31	8
32	8
33	8
34	12
35	12

Task	Assignment
36	12
37	12
38	12
39	12
40	15
41	15
42	15
43	15
44	15
45	15
46	16
47	16
48	16
49	16
50	16
51	16
52	1
53	1
54	1
55	1
56	1
57	1
58	8
59	16
60	11
61	8
62	8
63	23
64	23
65	9
66	9
67	14
68	14
69	25
70	25

Task	Assignment
71	17
72	17
73	7
74	20
75	14
76	3
77	15
78	25
79	25
80	12
81	2
82	4
83	11
84	8
85	14
86	11
87	2
88	1
89	2
90	6
91	4
92	5
93	7
94	13
95	15
96	17
97	19
98	20
99	21
100	22
101	23
102	11
103	2
104	14

Table 20: Flexible task assignment for $\alpha = 4$, $\beta = 1$ and $\gamma = 1$.

Non-flex task assignment

Task	Assignment
1	3
2	7
3	8
4	19
5	14
6	23
7	22
8	18
9	12
10	4
11	6
12	17
13	20
14	1
15	21
16	24
17	13
18	16
19	9
20	5
21	10
22	11
23	2
24	2
25	4

Task	Assignment
26	19
27	16
28	14
29	21
30	9
31	8
32	25
33	8
34	12
35	15
36	16
37	1
38	8
39	23
40	9
41	14
42	25
43	17
44	25
45	25
46	12
47	2
48	19
49	12

Table 21: Non-flexible task assignment for $\alpha = 4$, $\beta = 1$ and $\gamma = 1$.

C.2.3 Parameter set 3

$\alpha = 2, \beta = 0.5$ and $\gamma = 1$

Performance

Teacher	Preference	Deviation	Overwork
1	6	15	33
2	7	7	18.6
3	3	32	2.1
4	3	8	0
5	3	48	0.1
6	6	55	0
7	9	3	2.3
8	18	0	9.6
9	9	3	6.5
10	3	39	0
11	7	16	0.1
12	15	1	40.1
13	3	47	1
14	18	1	8
15	3	57	0
16	12	2	10
17	12	5	0
18	3	0	0
19	8	6	35.7
20	9	0	0
21	6	24	11.5
22	3	14	0
23	12	2	16.6
24	2	59	0
25	11	15	7

Table 22: Results of individual teachers for $\alpha = 2$, $\beta = 0.5$ and $\gamma = 1$.

Flexible task assignment

Task	Assignment
1	22
2	18
3	12
4	19
5	5
6	4
7	20
8	23
9	11
10	6
11	24
12	14
13	20
14	2
15	17
16	16
17	7
18	11
19	23
20	6
21	12
22	20
23	8
24	8
25	8
26	8
27	8
28	8
29	8
30	8
31	8
32	8
33	8
34	12
35	12

Task	Assignment
36	12
37	12
38	12
39	12
40	15
41	15
42	15
43	15
44	15
45	15
46	4
47	4
48	4
49	4
50	4
51	4
52	1
53	1
54	1
55	1
56	1
57	1
58	14
59	16
60	11
61	8
62	8
63	25
64	25
65	17
66	17
67	23
68	23
69	9
70	9

Task	Assignment
71	16
72	16
73	7
74	14
75	12
76	3
77	15
78	25
79	25
80	2
81	12
82	4
83	11
84	17
85	14
86	11
87	2
88	1
89	2
90	6
91	4
92	5
93	7
94	13
95	15
96	17
97	19
98	20
99	21
100	22
101	23
102	11
103	2
104	8

Table 23: Flexible task assignment for $\alpha = 2$, $\beta = 0.5$ and $\gamma = 1$.

Non-flex task assignment

Task	Assignment
1	3
2	7
3	8
4	22
5	18
6	12
7	19
8	5
9	4
10	20
11	23
12	11
13	6
14	2
15	21
16	24
17	17
18	13
19	10
20	16
21	9
22	14
23	1
24	2
25	23
26	14
27	25
28	8
29	16
30	9
31	19
32	21
33	8
34	12
35	15
36	4
37	1
38	8
39	25
40	17
41	23
42	9
43	16
44	25
45	25
46	2
47	12
48	19
49	12

Table 24: Non-flexible task assignment for $\alpha = 2$, $\beta = 0.5$ and $\gamma = 1$.

C.2.4 Parameter set 3

$\alpha = 1, \beta = 1$ and $\gamma = 1$

Performance

Teacher	Preference	Deviation	Overwork
1	6	15	33
2	17	4	6
3	3	32	2.1
4	5	13	1
5	3	48	0.1
6	3	48	0
7	9	3	2.3
8	21	1	9.6
9	6	3	7.6
10	3	39	0
11	7	1	0
12	21	9	41.1
13	3	47	1
14	21	6	9.1
15	3	57	0
16	6	6	11
17	9	3	14.1
18	6	25	9.5
19	11	6	35.6
20	9	10	0
21	6	24	11.5
22	3	14	0
23	12	2	3
24	1	19	0
25	10	20	7

Table 25: Results of individual teachers for $\alpha = 1$, $\beta = 1$ and $\gamma = 1$.

Flexible task assignment

Task	Assignment	Task	Assignment	Task	Assignment
1	6	36	12	71	17
2	17	37	12	72	17
3	23	38	12	73	7
4	9	39	12	74	25
5	8	40	15	75	8
6	2	41	15	76	3
7	22	42	15	77	15
8	5	43	15	78	25
9	14	44	15	79	14
10	20	45	15	80	12
11	8	46	16	81	2
12	2	47	16	82	4
13	14	48	16	83	11
14	23	49	16	84	20
15	14	50	16	85	4
16	8	51	16	86	11
17	2	52	1	87	2
18	23	53	1	88	1
19	11	54	1	89	2
20	7	55	1	90	6
21	14	56	1	91	4
22	12	57	1	92	5
23	8	58	8	93	7
24	8	59	16	94	13
25	8	60	11	95	15
26	8	61	14	96	17
27	8	62	14	97	19
28	8	63	2	98	20
29	8	64	2	99	21
30	8	65	23	100	22
31	8	66	23	101	23
32	8	67	19	102	11
33	8	68	19	103	2
34	12	69	12	104	20
35	12	70	12		

Table 26: Flexible task assignment for $\alpha = 1$, $\beta = 1$ and $\gamma = 1$.

Non-flex task assignment

Task	Assignment
1	3
2	12
3	7
4	6
5	17
6	23
7	9
8	8
9	2
10	22
11	5
12	14
13	20
14	18
15	21
16	10
17	16
18	4
19	19
20	13
21	24
22	11
23	1
24	17
25	9

Task	Assignment
26	18
27	16
28	14
29	25
30	8
31	19
32	21
33	8
34	12
35	15
36	16
37	1
38	14
39	2
40	23
41	19
42	12
43	17
44	25
45	25
46	12
47	2
48	19
49	12

Table 27: Non-flexible task assignment for $\alpha = 1$, $\beta = 1$ and $\gamma = 1$.

C.3 General results for different weights

Weight on the preference objective	0	1	2	3	4	5	6	7
Weight on the deviation objective	1	1	1	1	1	1	1	1
Weight on the overwork objective	1	1	1	1	1	1	1	1
Total preference value	226	204	195	185	186	186	186	186
Total deviation value	455	457	457	457	453	463	457	453
Total overwork value	196	200	204	220	212	211	213	212

Table 28: Effects of different weight on the objective functions.

Weight on the preference objective	8	9	10	11	12	13	14	15
Weight on the deviation objective	1	1	1	1	1	1	1	1
Weight on the overwork objective	1	1	1	1	1	1	1	1
Total preference value	186	181	181	181	181	181	178	176
Total deviation value	461	481	477	475	487	477	523	565
Total overwork value	209	217	227	225	214	225	220	228

Table 29: Effects of different weight on the objective functions (continued).

Weight on the preference objective	1	1	1	1	1	1	1	1
Weight on the deviation objective	1	1	1	1	1	1	1	1
Weight on the overwork objective	0	1	2	3	4	5	6	7
Total preference value	190	204	214	215	216	211	216	226
Total deviation value	453	455	459	467	457	501	487	503
Total overwork value	291	205	192	193	193	185	190	187

Table 30: Effects of different weight on the objective functions (continued).

Weight on the preference objective	1	1	1	1	1	1	1	1
Weight on the deviation objective	1	1	1	1	1	1	1	1
Weight on the overwork objective	8	9	10	11	12	13	14	15
Total preference value	219	223	206	219	225	227	219	223
Total deviation value	493	573	741	563	593	569	553	569
Total overwork value	183	182	194	180	183	181	180	181

Table 31: Effects of different weight on the objective functions (continued).

Weight on the preference objective	1	1	1	1	1	1	1
Weight on the deviation objective	2	3	4	5	6	7	8
Weight on the overwork objective	1	1	1	1	1	1	1
Total preference value	206	207	200	214	209	213	203
Total deviation value	453	453	453	465	453	453	465
Total overwork value	211	192	215	239	206	200	238

Table 32: Effects of different weight on the objective functions (continued).

Weight on the preference objective	1	1	1	1	1	1	1
Weight on the deviation objective	9	10	11	12	13	14	15
Weight on the overwork objective	1	1	1	1	1	1	1
Total preference value	195	205	202	212	217	214	200
Total deviation value	453	453	467	497	473	453	453
Total overwork value	203	194	239	228	226	207	233

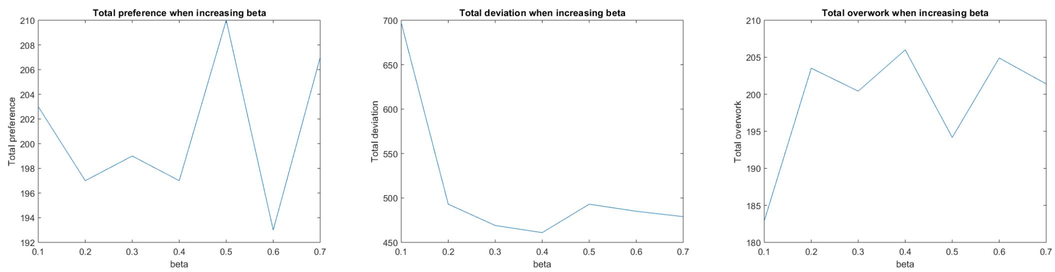
Table 33: Effects of different weight on the objective functions (continued).

Weight on the preference objective	2	2	1	1	1	1	1	1	1
Weight on the deviation objective	1	0.5	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Weight on the overwork objective	2	1	1	1	1	1	1	1	1
Total preference value	202	191	203	197	199	197	210	193	207
Total deviation value	459	459	697	493	469	461	493	485	479
Total overwork value	195.5	202	183	204	200	206	194	205	201

Table 34: Effects of different weight on the objective functions (continued).

C.4 Results for smaller weights on the deviation objective

In Figure 7 that is made using runs of just 600 seconds, is shown that already for very small values the deviation goes to 453 when having α and γ both equal to one. For these smaller values, slightly more predictable behaviour takes place, however it is still not satisfactory. When choosing the relative weights for the objective functions, increasing the value of β is not the way to go, or even smaller values should be considered.



(a) Total preference. (b) Total hours of deviation. (c) Total hours of overwork.

Figure 7: The results for the three objectives when increasing β from 0 to 15.

D Ethical approval

This research has been reported to the Ethics Committee (EC). Besides this, an information brochure was made to give to teachers when interviews would take place. Moreover, an informed consent form was designed and finally the check list for ethics was filled in.

E Collaboration with school and reflection

In this section, a reflection on the collaboration with CSG Reggesteyn is given. Here is explained why the data that is used to test the model was not complete.

E.1 Conference call with Ineke Munter, 17 May 2019

To arrange an appointment, I had a conference call with Ineke Munter, principal of CSG Reggesteyn. She invited me for the meeting on May 21.

E.2 Report visit to CSG Reggesteyn, 21 May 2019

On May 21, I have visited the school CSG Reggesteyn to start the cooperation between me and the school. I was invited to come by during a meeting of Ineke Munter and the so-called unit-leaders (team managers) to explain the project to investigate how we could collaborate in this project. To be there as a professional, I prepared a presentation of approximately 10 minutes making use of a PowerPoint (in UT-lay-out). This presentation consisted of a short introduction, an explanation of the model as it was then and as conclusion the possibilities that were available for this research. I discussed that the time was very limited, however that it would presumably possible to show the school some insight in the assignment of non-educational tasks. The unit-leaders and school director were enthusiast and in the conversation that followed our talk, we decided to work with the data of 2017-2018 of the unit 'bovenbouw havo'.

E.3 Report visit to CSG Reggesteyn, 29 May 2019

On May 29, I visited CSG Reggesteyn for the second time. This time, I had a meeting with Jan Draaijer, who is the manager of facility organization of CSG Reggesteyn. He provided timetables of the 25 teachers of the unit 'bovenbouw havo' and gave me insight in the tasks that are assigned to the teachers. He also explained me how the 'norm-jaartaak' works. The tasks were a little bit different from what I expected, however, I could start with converting data into the right format and see if this was enough information to test the model with.

E.4 Further contact with CSG Reggesteyn

After the meeting with Jan Draaijer, I had to get into the documents and see if this provided enough information to get useful results from the model. Part of the research was looking at the tasks to make a profile of each task. This was difficult and that was the moment I decided to talk with Tom Coenen to solve this problem. We decided that it was necessary to have interviews with the teachers of CSG Reggesteyn in order to get the program working for the overwork and preference objectives. I sent an email the team manager of 'bovenbouw havo' to arrange interviews with the teachers. As more than one week passed without reaction, I contacted Tom Coenen with the request to remind this team manager to reply on my email. However, also this reminder did not help. In the meantime I decided to send an form to get information on the preferences and the workload per task. This form was distributed via Jan Daarijer to the teachers. However, only 4 of 25 teachers did reply on this form. As the deadline was approaching, I decided to call Jan Draaijer and ask for possibilities to distribute the documents on paper. He could only send a reminder to the teachers. So the only possibility to get results faster was via the team manager of 'havo bovenbouw' whom still had not replied on my email. I contacted Tom Coenen again and at 20 June we finally had contact with this team manager. After we had contact, the team manager urged his teachers to fill in the form in order to make sure that I could run my program and get results. It was simply too late to arrange meetings with teachers now. With some help of Tom Coenen this made the responds grow to 9 of 25 teachers. This is still low unfortunately. It can be that the results regarding the preferences would have been much better when having data of all 25 teachers. This shows how busy teachers and team managers of a school are: there was simply not enough time to respond to my request for information.

E.5 Reflection

At the start of this project, I experienced that the management of CSG Reggesteyn was really busy, however willing to take part in this research. After my introduction, a first step in the research was easily made. Together with the manager facility activities, I obtained general data on both teachers, planning and tasks. However, soon I found out that I needed more information. It was at that moment that I experienced how difficult it is to work together with an ex-tern organization. The team manager was so busy that he did not reply on my email. Even a reminder of Tom Coenen did not solve the issue. Moreover, the teachers were also too busy to hand in their preferences. At a certain moment I really needed the data to get working on the results section and the discussion and here it started influencing the results as I had to work with the data of just 9 teachers. If the goal of the project was clear much earlier and the co-operation with the school was already set-up at the start of the project, it would possibly been possible to discuss the project earlier with the school and to schedule interviews with the teachers. Despite this difficulty, I enjoyed working on such an applied project and I hope that this research can contribute in better work pressure for secondary school teachers in the future.

F Matlabcode

The code can be asked for by the author.[5]

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