Improving the portfolio insurance strategy of Binck Bank:

A QUANTITATIVE ANALYSIS OF DYNAMIC MULTIPLIERS

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Abstract

This paper derives and investigates different dynamic proportion portfolio insurance strategies, and compares them to the each other, and the CPPI strategy in terms of risk, return and transaction costs. The strategies and results can be useful to portfolio managers. The performance of the strategies is back-tested with using a rolling analysis. Thereafter, the practical implications, equity allocation, gap risk and validity of the best performing model(s) are assessed. It has been found that the dynamic proportion portfolio insurance strategy with a multiplier based solely on volatility outperforms the other assessed strategies and the original CPPI strategy. Moreover, this strategy has no drawback in terms of gap risk or practical implications.

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Preface

This thesis is my last assignment before obtaining the degree of Bachelor of Industrial Engineering and management. I am lucky to start my master's degree with practical experience in the financial field. This research is a result of ten weeks preparing and ten weeks working on sight at Binck Bank and I am proud to present you the results.

I would like to take this opportunity to thank the people that helped me to get this assignment. First, I would like to thank Treske Heere for getting me in touch with Yaela van Raalte of Binck Bank. Furthermore, I would like to thank Leander Tijssen my supervisor at Binck. It was great to work with you, you were supportive, welcoming and ready to answer any questions. Next, I would like to thank the rest of the investment management team for the pleasant work environment, support and feedback throughout the period I was working at the department. Finally, I would like to thank my UT supervisors Berend Roorda and Wouter van Weeswijk for insightful feedback.

Charly Hunsicker, August 2019

Management summary

Introduction

The following bachelor thesis is an assignment for Binck Bank B.V. (hereafter referred to as Binck), a Dutch online discount broker. The thesis focuses on improving their most risk averse strategy "comfort". Currently Binck uses a constant proportion portfolio insurance strategy (CPPI). Portfolio insurance (PI) strategies are created to help investors control downside risk of the portfolio value while still participating in case of upward market opportunities. To improve the strategy six different dynamic multipliers are introduced dependent on market returns and volatility.

The risk/return characteristics will be discussed, and six different dynamic proportion portfolio insurance strategies will be compared with each other and the original CPPI implementation. The risk/return characteristics of the different strategies were derived by means of a historical back-test on relevant equity and bond markets for the Comfort strategy. Based on the resulting characteristics and additionally taking product positioning in mind implementing the volatility based dynamic multiplier is advised.

Research

The aggregate results show that both the strategy based on volatility only as well the strategy based on a combination of volatility and momentum outperform the original CPPI strategy on key return/risk characteristics.

When (inter)comparing these 2 dynamic multiplier strategies using the restriction of a minimum multiplier of 2 and maximum of 7 the results indicate that the strategies are very similar with the main difference being that the volatility strategy has a lower turnover. Based on these results and taking into account the risk-averse nature of the comfort product it is argued that the volatility strategy is the preferred option. Additional tests regarding equity allocation, gap risk and validity are in line with this statement.

Volatility based dynamic proportion portfolio insurance strategy

The volatility-based multiplier is based on low volatility strategies within portfolio management. The volatility multiplier is computed by: $\frac{1}{Volatility}$. By construction the multiplier goes down (and therefore

also the equity allocation in the portfolio) when the volatility goes up and vice versa. Within this construction there are 2 decision points:

- How to compute the volatility
- Scaling the formula

Volatility computation
Several periods and methodologies were assessed, of which the 128-day exponentially weighted version turned out the most logical.

Scaling
Several scaling factors were tested where, within a certain range, the test-results were similar. Within that specific range we opted for a scaling factor such that the average multiplier over the back-test period was 5 (equal to the original CPPI methodology). This leads to the following 2 interesting properties:

- The average risk taken is the same from a leveraging-the-cushion perspective. Note that the average risk is lower in terms of both volatility as well as drawdown[1]. Therefore, the proposed strategy is not riskier than the original strategy.
- The benchmarks could be left unchanged since the average allocation and multiplier should be the same as the original strategy.

The resulting scaling factor is 0,6. However the results of the validity test show the dangers of overfitting. Therefore, Binck should test what risk factor has the desired results in the market Comfort is active in.

Conclusion/recommendation

Based on the complete research it is recommended to change the static multiplier to a dynamic multiplier and implement the continuous volatility strategy with a risk factor scaled in relevant markets as new multiplier.

$$m_t = a * \frac{1}{\sigma_{\mathrm{t,exp(128)}}}$$

In which the multiplier has a minimum of 2 and a maximum of 7.

Further research

For further research the following is advices; 1) Changing the multiplier ranges and rebalancing weights and check the impact on the KPIs of the different strategies. 2) Use different volatility estimators such as implied volatility to see if there are better performing estimators. 3) Combining the goal based and the risk averse strategies results in better portfolio performance and reduces the workload for Binck. 4) Different sorts of DPPI strategy based on machine learning algorithms such as genetic algorithms.

[1] This results from the backtest but can also be easily understood/explained from the mechanics of this dynamic multiplier.

Contents

1. Introduction

The research is a research for Binck Bank B.V., the following chapter is an introduction to the enterprise, core problem, key concepts, research questions, goal, scope and limitations.

1.1. Binck

The following bachelor thesis is an assignment for Binck Bank B.V. (hereafter referred to as Binck), a Dutch online discount broker. One of their activities is investing for customers using certain strategies. Binck has three different strategies: comfort, forward and select. This research focusses on improving comfort, their most risk averse strategy.

1.2. Portfolio insurance strategies

Portfolio insurance (PI) strategies are created to help investors control downside risk of the portfolio value while still participating in case of upward market opportunities, where portfolio theory refers to reducing unsystematic risk through diversification and covering systematic risk by introducing the beta coefficient as risk indicator (Markowitz, 1952). Portfolio insurance strategies focus solely on managing systematic risk (Agic-Sebeta, 2017).

In practice a wide range of PI strategies are used, and many new strategies are developed regularly by practitioners. The main classification factor lies between option-or-spot strategies and dynamic-or-static approaches. A short list of the most used approaches is shown in Table 1.

Table 1: Classification of portfolio insurance strategies based on Laurent (2003) and Rudolf (1994) and Agic-Sebata (2017)

Option or spot strategies	Static approach	Dynamic approach
Option strategies	Protective put	Protective put with delta hedging
	Long call, covered call	Long call with delta adjustment
Option duplicating strategies		Synthetic protective put
		Synthetic long call
Spot strategies	Stop-loss	Constant-proportion portfolio insurance (CPPI)
		Dynamic proportion portfolio insurance (DPPI)
		Time-invariant portfolio insurance (TIPP)

In Binck's case the comfort strategy executes a constant proportion portfolio strategy (CPPI) with regular floor resets. CPPI was introduced by Perold and Sharpe (1988). The methodology uses a simple strategy to dynamically assign asset allocations to a mix of risky and riskless assets. Binck has researched all of the main strategies listed above and have found indications that two strategies that outperform the rest, namely, the CPPI and dynamic proportion portfolio insurance (DPPI) strategy. The following subgraphs will give introductions into the two strategies.

1.2.1. CPPI

The CPPI strategy is the strategy Binck currently uses, it exists of a floor percentage, floor value, cushion, multiplier, exposure, risk free asset allocation. The following Paragraph will explain these properties of the strategy.

The floor percentage is chosen by the client, the higher the risk tolerance of the client the lower the floor percentage and vice versa. It is defined as the lowest acceptable percentage of the (initial) portfolio value. The floor percentage is multiplied with the initial investment which results in the floor value. The floor percentage will be constant throughout the investment; however, the realized floor value will be reset every year. The floor value is recalculated yearly by multiplying the floor percentage with the portfolio value.

After deciding on the floor value, the cushion is computed. The cushion is the absolute difference between the portfolio value and the floor value.

The allocation of the risky asset is defined by the cushion and a certain multiplier. The cushion is multiplied by a predetermined multiplier and invested into risky assets; this is called the exposure. Because the exposure is a multiplication of the cushion, it is capital that is partly leveraged from the floor value. The remaining equity is invested into risk-free assets.

The multiplier value depends on the risk preference of the client. The higher the multiplier, the more will be invested into risky assets, and therefore the riskier the strategy will be. When the cushion grows exposure will grow and vice versa. When the portfolio value declines, exposure will decline and approach zero. When the portfolio value reaches the floor value, asset allocation should solely be in risk-free investments. If the cushion has grown a substantial amount, portfolio value is invested completely into risky assets. The strategy defines when to rebalance. It is possible to rebalance after a certain time interval, and after the modelled asset allocation weights differ from the realized asset allocation weights by a certain margin. Figure 1 shows the properties of the CPPI in a bar graph.

The mathematical properties of the portfolio value, floor value, exposure, risk-free asset allocation and cushion of the CPPI strategy, described in this Paragraph, are shown below:

$$V_0 = E_0 + R_0 = C_0 + F_0$$
$$E_0 = m * C_0$$
$$F_0 = F^p * II$$

$$V_t = E_t + R_t = C_t + F_t$$
$$E_t = m * C_t$$
$$F_t = V_{t-1} * F^p$$

$$E_t = E_{t-1} * r_{t-1}$$
$$R_t = R_{t-1} * r_{t-1}$$

$Where\ V_t = portfolio\ value\ at\ time\ t,$
$E_t = position\ of\ risky\ asset\ at\ time\ t\ (exposure),$

$R_t = position\ riskless\ asset\ at\ time\ t,$
$C_t = cushion\ at\ time\ t,$
$F_t = floor\ value\ at\ time\ t$
$II = initial\ investment$
$m = constant\ multiplier$

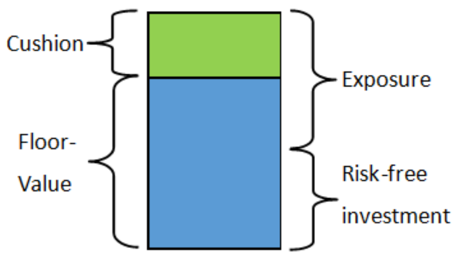

Figure 1: CPPI model

1.2.2. DPPI

DPPI strategies do not have a formal definition, however, they are part of the proportion portfolio insurance strategy family. This means their goal is to minimize downside risk, without necessarily compromising on the returns. DPPI strategies are closely related to the CPPI strategy and have the same mathematical properties as the CPPI strategy. The difference is that the strategy's multiplier is variable dependent on certain market indicators (for example market volatility).

1.3. Rebalancing

The rebalancing strategy influences the performance of the strategies. For this research, rebalancing is done after the modelled asset allocation weights differ from the realized asset allocation weights by a certain margin. Therefore, a distinction between modelled and realized values has to be made. The following formulas show the difference between the realized-and-actual risky-and-riskless asset values:

$$modelled\ exposure = \left(E_{t-1} * r_{1,t-1} + R_{t-1} * r_{2,t-1} - F_t\right)$$
$$modelled\ risk\ free\ asset\ value = (V_{t-1} - modelled\ expore) * r_{2,t-1}$$
$$realized\ exposure = E_{t-1} * r_{1,t-1}$$
$$realized\ risk\ free\ asset\ value = R_{t-1} * r_{2,t-1}$$
$$where, r_{1,t} = risky\ asset\ return\ at\ time\ t, \qquad r_{2,t} = risk\ free\ asset\ return\ at\ time\ t$$

1.4. Risk profiles and the floor

The floor percentage refers to the minimal percentage of the initial investment the investor wants to insure[2]. The floor value is initially calculated by multiplying the initial investment with the floor percentage, thereafter, it is recalculated yearly by multiplying the floor percentage with the portfolio value at that moment. This reduces the number of floor hits. An internal research of Binck has shown that redetermining the floor value yearly improves the overall performance of the CPPI strategy.

Currently all risk profiles have the same multiplier and rebalancing weights. Moreover, in previous researches the impacts of different floor values have not been considered. The floor percentage parameter is F^p. The fraction of clients per risk profile in May 2019 is shown in Table 2. These percentages are used to create a weighted average of the performance of strategies over different floor percentages (and therefore different floor values).

Table 2: Client risk profile percentage

Floor percentage	Fraction of clients
0,95	2%
0,925	4%
0,9	27%
0,875	12%
0,85	21%
0,825	12%
0,8	12%
0,775	7%
0,75	3%
Total	100%

[2] More detailed explanation is in Section 1.2.1.

1.5. Gap risk

One of the main benefits of the portfolio insurance strategies is their downside protection. Therefore, an important concept to take into account is the gap risk. The following Paragraph will explain the theory behind gap risk.

CPPI and DPPIs are often modelled in continuous time, in real market condition however, "because of the existence of market friction factors like transaction cost, rebalancing happens on the discrete adjustment point" (Xing, Xue, Feng, & Wu, 2014). Therefore, it is possible for the portfolio value to drop below the floor value, due to the sharp fall between two adjustment points. Figure 2 displays this event.

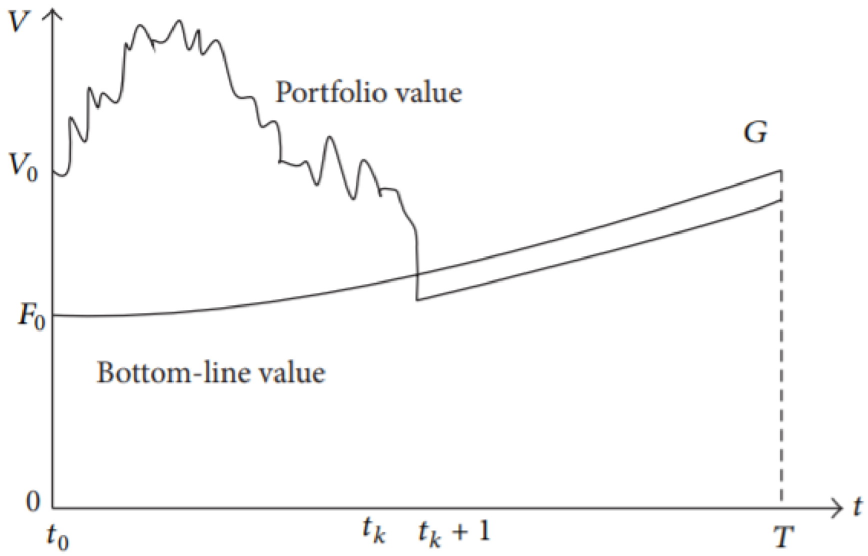

Figure 2: Gap risk, adopted from (Xing, Xue, Feng, & Wu, 2014)

The principle of gap-risk modelling is as follows. The theory is based on the assumption that in discrete PPI strategies, rebalancing (checks) happens on equally spaced time series, defined as $\{t_0^n = 0 < t_1^n \ldots < t_k^n \ldots < \mathrm{t}_\mathrm{n}^\mathrm{n} = \mathrm{T},\ \mathrm{t}_{\mathrm{k+1}}^\mathrm{n} - \mathrm{t}_\mathrm{k}^\mathrm{n} = \frac{\mathrm{T}}{n}\}$ where n denotes the number of rebalances. Risky asset exposure equals to the cushion multiplied with the multiplier therefore, the largest tolerable drop of risky asset is 1/m. The following equations proof that if between two discrete rebalancing points (t_k, t_{k+1}) the risky asset value drops with 1/m, then the portfolio value $V_{t_{k+1}}$ will be less or equal to the floor value at time t_{k+1}, which will further increase with the risk-free rate.

$$
\begin{aligned}
&V_{t_{k+1}} = mC_{t_k}\left(1 - \frac{1}{m}\right) + \left(V_{t_k} - mC\right)(1 + r) \\
&= (V_{t_k} - C_{t_k} + \left(V_{t_k} - mC_{t_k}\right)r \\
&= F_{t_k} + \left(V_{t_k} - mC_{t_k}\right)r \\
&\leq F_{t_{k+1}}
\end{aligned}
$$

$Where, V_t = portfolio\ value\ at\ time\ t,$
$m = multiplier, \qquad C_t = cushion\ at\ time\ t$
$F_t = Floor\ value\ at\ time\ t, \qquad r = risk\ free\ rate$

Therefore, the largest drop of risky asset is $\frac{1}{m}$ in a rebalancing period (t_k, t_{k+1}). The probability of risky asset being more than $\frac{1}{m}$ in a rebalancing period is the measurement of gap-risk. The chance that the risky asset drops with more than $\frac{1}{m}$ in an adjustment period needs to be extremely small. The measurement of gap risk is the following:

$$P\left(x_k > \frac{1}{m}\right) = a$$

$$x_k = -\left(\frac{p_{t_{k+1}} - p_{t_k}}{p_{t_k}}\right)$$

$$where, x_k = the\ drop\ in\ risky\ asset\ between\ time\ k, k+1\ ,$$
$$a = probality\ of\ risky\ asset\ dropping\ more\ than\ \frac{1}{m}$$
$$p_t = asset\ price\ at\ time\ t$$

1.6. Problem description

In 2015 Binck had an internal research about PI strategies. The outcome resulted in the implementation of their current strategy: CPPI with regular floor resets and rebalancing when the weights differ by a certain margin. However, In the same research there were indications that a DPPI strategy might perform better that the CPPI (and all other PIs described in Table 1). Moreover, their research did not consider the impact of different floor percentage values. Therefore, Binck is interested in how different DPPI strategies perform compared with the current strategy also taking into account the different risk profiles.

1.7. Aim, scope and limitations

1.7.1. Goal

Binck wants to know if there are DPPI strategies that systematically perform better in terms of return and transaction costs, without compromising on risk measures[3]. Therefore, the goal is to analyze and describe the performance of different DPPI strategies and to compare them with the original strategy while considering the different floor values.

1.7.2. Scope

As described earlier DPPI strategies do not have a formal definition, however, they are part of the proportion portfolio insurance strategy family. This means their goal is to minimize downside risk, without necessarily compromising on the returns. The difference is that the multiplier is dependent on market indicators. Therefore, the multiplier needs to be dependent on market conditions.

The second part of the requirements is provided by Binck. They cannot implement every strategy due to customer expectations and internal system requirements. They have set the following restrictions for the strategies.

1) The floor percentage is set by the client, the floor value is derived based on the initial investment and the floor percentage. Moreover, the floor value is reset yearly based on the portfolio value at that time.
2) The strategies should not include options. This is because strategies including options are more difficult to explain to the clients.

[3] The KPIs for risk, return and risk are explained in 3.1.3.

3) The portfolio weights are checked daily and rebalancing can be done daily if necessary. This is due to an earlier internal research, and the implementation of the strategy integrated with other strategies.

1.7.3. Limitations

There are numerous limitations and constraints foreseen for this research. First, the research needs to focus on the preferences of Binck as described in the scope. Furthermore, there are time constraints to this research, it must be done within 10 weeks. Therefore, it is not possible to access every model in literature. Filtering strategy of the researches and models is based on the theory of Neftci (2008). Out of scope are dynamic multipliers defined by genetic algorithms, or the likes. Machine learning multiplier construction algorithms are suggested for further research.

1.8. Research questions

After determining the current strategy, the problem, the goal and the scope the following question are identified:

1. Which dynamic proportion portfolio insurance strategies can be identified that are relevant for Binck?
    a. What strategies are in literature?
    b. What strategies can be derived using financial and mathematical knowledge?
    c. How are these strategies modeled?
2. How do strategies identified in RQ1 perform?
    a. What are the KPIs performance is based on?
    b. What research methodology should be used to analyze performance and behavior?
        i. Are the results valid?
    c. How do the strategies perform in different market conditions?
3. Which dynamic portfolio strategy, that can be applied to Binck, performs best in comparison with each other and the original strategy?
    a. What are practical implications that need to be taken into account?
    b. What is the gap risk of this strategy?

2. Dynamic proportion portfolio insurance strategies

In this Chapter the DPPI strategies will first be derived based on different market indicators. Therafter the strategies will be explained and modelled.

2.1. Deriving DPPI strategies

Deriving DPPI strategies is done in two parts. First different market indicators as base for the multiplier are assessed, resulting in different strategies. Thereafter, the mathematical model for dynamic multiplier adjustment is shown.

2.1.1. Market indicators

The DPPI strategy family is closely related to the CPPI strategy. The difference is that the leverage ratio (the multiplier) is variable on predefined market behavior. Algorithms are developed in order to achieve this. The idea is that the multiplier becomes depended on variables that the practitioner finds relevant. In general, there are four different factors the exposure may dependent on (Neftci, 2008), namely:

1. The past behavior of the returns
2. The volatility of the returns

3. The liquidity observed in the market for the underlying asset, since the methodology is heavily dependent on the correct rebalancing
4. The gap-risk

Due to the extra restriction Binck has set for this research, only return and volatility-based multipliers are relevant for this research. The latter two are not of interest because, first, the comfort strategy does not invest in illiquid markets. Second, gap risk will be evaluated separately, therefore there is no need to make the multiplier dependent on gap risk.

2.1.2. Momentum

A general believe in literature is the efficient market hypothesis. It is believed that securities markets are efficient in responding to information regarding stock individual stocks and stock markets as a whole (Malkiel, 2003). Efficient market hypothesis in its weakest form suggests that future prices cannot be predicted by analyzing prices from the past. However, evidence of momentum strategies suggests that future returns are not completely independent of past returns. The effect of momentum has been observed across a large variety of assets and markets (e.g. L. K. Chan, Jegadeesh, and Lakonishok, 1996; Carhart, 1997, Moskowitz and Grinblatt, 1999; Lee and Swaminathan, 2000; Moskowitz, Ooi, and Pedersen, 2012; Berghorn, 2015; Daniel and Moskowitz, 2016; Maheshwari and Dhankar, 2017; Berghorn, Vogl, Schulz, and Otto, 2018)

The momentum strategy is first described by Jegadeesh and Titman (1993). In its simplest form, it suggests that strategies in which buying stocks that have performed well in the past and selling stocks that have performed poorly in the past, generate significant excess returns. “the strategy is built upon the notion that current ascendant and descendant trends continue in the near future” (Tijssen, 2019). That means that investing in assets that are upward trending and selling when assets value is downward trending results in excessive positive returns. It can be explained from the perspective of behavioral finance as the “bandwagon effect”, where individuals are drawn into the market after short term momentum (Makiel 2003; Thaler 2005). The strategy has been observed across different assets and markets, and significant empirical evidence of excessive returns has been found supporting the strategy evaluating different time series (Moskowitz, Ooi, & Pedersen, 2011). However, the strategy has also shown to have drawbacks. First, Maheshwari and Dhankar (2017) find that “during extreme market environment, such as a financial crisis, momentum profitability disappears suggesting dramatic impact of the financial crisis on momentum profits”. Second, Cooper, Jagannathan and Kim (2012), have shown that the moment strategy generates poor results in unstable markets where volatility is high, and better results when markets are stable. The idea of momentum can be used to develop dynamic multiplier strategies. Note that the original CPPI strategy can already be seen as a momentum strategy, since when the cushion decreases due to losses, proportionally more is invested into risk-free assets. Moreover, when the cushion increases, due to increase in asset values, more is invested into risky assets.

Adopting the momentum strategy and its implications, three different strategies are developed. First, trend based (momentum) DPPI adopts the general notion of the momentum strategy and uses past returns to determine the multiplier value. Second, a combination strategy in which the equity allocation follows momentum more when markets are stable and less when the markets are volatile. Finally, a combination strategy that quickly shifts the equity allocation to the risk-free asset when returns are negative in an unstable market. The last can be seen as a crisis control strategy.

2.1.3. Low volatility anomaly

The low volatility anomaly is described as “a provocative long-term connection between future stock returns and various measures of prior stock price variability, including total return volatility, idiosyncratic volatility, and beta" (Li et al. 2016). Researchers have found evidence that in global markets, previously low-return-variability portfolios outperform those of historically highly-return-variability in terms of (risk adjusted) return (see, e.g., Clarke, de Silva, and Thorley 2006; Baker, Bradley, and Wurgler 2011; Li, Sullivan, and Garcia-Feijóo 2014). Therefore, the low volatility anomaly indicates that buying previously low volatility stocks and selling previously high volatility stocks results in substantial abnormal returns. In literature there are two types of explanations for the anomaly, namely, systematic risk and market mispricing due to irrationality of investors. Ang et al. (2009) has found evidence supporting the systematic risk explanation. They have found that the volatility anomaly in numerous countries is highly correlated with that in the U.S. indicating the affect is driven by latent systematic risk. Others have argued that the effect is due to behavioral biases, such as "lottery ticket investing", attention biases of high volatile stocks and managers' bonusses for outperforming the benchmark (Baker, Bradley and Wurgler 2011; Barberis and Huang 2008; Barber and Odean 2008).

The low volatility anomaly notion can be used to derive volatility based DPPI strategies where the strategy should suggest allocating more into risk-free assets when the equity volatility is high and vice versa.

2.1.4. Mathematical model for dynamic multiplier adjustment

When looking at the mathematical derivation of the dynamic strategies out of the normal CPPI model, based on the mathematical model for dynamic multiplier adjustment of Yao & Li (2016), the derivation looks as follows:

Recalling the CPPI strategy:

$$C_0 = V_0 - F_0$$
$$E_0 = m_0\, C_0 = S_0 * n$$
$$V_0 = E_0 + R_0$$
$$Where\ V_t = portfolio\ value\ at\ time\ t,$$
$$E_t = position\ of\ risky\ asset\ at\ time\ t,$$
$$R_t = position\ riskless\ asset\ at\ time\ t,$$
$$C_t = cusion\ at\ time\ t,$$
$$F_t = floor\ value\ at\ time\ t,$$
$$m_t = multiplier\ at\ time\ t,$$
$$S_0 = initial\ risky\ asset\ price,$$
$$n = amount\ of\ risky\ assets\ in\ possesion$$

Combining these formulas results in,

$$V_0 = S_0 * n + R_0$$

Therefore,

$$\frac{\Delta V_0}{\Delta S_0} = n_0\ and\ \frac{\Delta V_t}{\Delta S_t} = n_t, \qquad t = 0,1,\ldots,T$$

When rebalancing is done then the position between risky and riskless asset is $S_1 * n_1 = m\,(V_1 - F_1)$ Which is the result of,

$$(S_0 + \Delta S_0)(n_o + \Delta n_0) = (m_0 + \Delta m_0)[(V_0 + \Delta V_0) - (F_0 + \Delta F_0)]$$

When taking into account the discrete time intervals, the t-th adjustment is the following,

$$\left(S_0 + \sum_{i=1}^{k} \Delta S_{t-1}\right)\left(n_0 + \sum_{t=1}^{k} \Delta n_{t-1}\right) = (\Delta m_0 + \sum_{t=1}^{k} \Delta m_{t-1})[(V_0 + \sum_{t=1}^{k} \Delta V_{t-1}) - (F_0 + \sum_{t=1}^{k} \Delta F_{t-1})]$$

Using the cushion and exposure formula and dividing by the risky asset price results in,

$$n_0 + \sum_{t=1}^{k} \Delta n_{t-1} = (\Delta m_0 + \sum_{t=1}^{k} \Delta m_{t-1}) \frac{[(C_0 + \sum_{t=1}^{k} (\Delta V_{t-1} - \Delta F_{t-1})]}{(S_0 + \sum_{t=1}^{k} \Delta S_{t-1})}$$

The last formula shows that "the accumulative amount of the changes of shares in the risky asset is composed of the accumulative amount of the changed multiplier $\sum_{t=1}^{k} \Delta m_{t-1}$, the accumulative amount of the portfolio value $\sum_{t=1}^{k} \Delta V_{t-1}$, the accumulative amount of the floor $\sum_{t=1}^{k} \Delta F_{t-1}$ and the accumulative amount of the changed price of the risky asset $\sum_{t=1}^{k} \Delta S_{t-1}$" (Yao & Li, 2016). Since the portfolio value is an arrangement of the other variables, the floor value is set and chosen by the client and the price of risky asset cannot be influenced, it indicates that the multiplier is the most flexible parameter. Therefore, the DPPI models focus on the multiplier value, especially on the calculation of Δm_{t-1}.

Furthermore, one of the main concerns foreseen, however not discussed in literature, for DPPI strategies with nature of $m_{t+1} = \mathrm{m}_t + \Delta \mathrm{m}_t$, is that over a longer period the multiplier can diverge into extreme values downwards or upwards. There are three solutions identified: resetting the multiplier value every time interval, applying mean regression or capping the multiplier. The downside of resetting the multiplier every time interval is that the performance of the strategy becomes more dependent on the start date. With a high client base, there will be cases where resetting will be done in unfavorable market conditions, resulting in poor results. Furthermore, for DPPI strategies to perform well, they should respond quickly to changes in market condition (Neftci, 2008). When considering mean regression compromises on the reaction speed are made, since there is a part in the multiplier formula that will always want to regress to the mean[4]. Therefore, a multiplier maximum (cap) is chosen. This way the multiplier will never diverge into extreme values, without losing reaction speed and compromising on the timing of the multiplier value. The downside of this approach is that if the maximum is chosen incorrectly it compromises on upside potential. In this case, taking gap risk and upside potential into account, the maximum allowed multiplier is set on 7. On the other hand, the multiplier normally can be greater than or equal to 0. However, investors expect some return and market participation, therefore the minimum allowed multiplier is set on 2[5].

2.2. Assumptions and modelling restrictions

The original strategy is described in the first Section. The alternative models are split into three categories: market trend, volatility, and combinations of both. The market trend DPPI strategies are based on the direction of market trends. The volatility DPPI strategies are based on market fluctuations. The combination models are strategies based on both market trends and volatility, with different views on what favorable market conditions are.

The assumptions for every model (unless stated otherwise) are the following:

1. The effects of dividends are compromised by using the adjusted closing price

[4] Mean regression in this case would look like: $m_t = (initial\ multiplier * p) + (1 - p)(m_{t-1} + \Delta \mathrm{m}_t)$, $where\ p = regression\ proportion$

[5] Note that the multiplier range affects the results and different ranges are suggested for further research

2. Fractals of stocks can be bought to satisfy model requirements
3. Time t refers to the current day after the market closed, but before the next day has started

The portfolio value, exposure, riskless asset value and cushion calculation for each model is the same as for the original CPPI model. The difference for each strategy is how the multiplier is calculated.

$$V_t = E_t + R_t = C_t + F_t$$
$$E_t = m_t * C_t$$
$$V_t = portfolio\ value, E_t = position\ of\ risky\ asset,$$
$$R_t = position\ of\ riskless\ asset, C_t = \ cushion, a = risk\ preference$$

The restrictions for all models are :

1. The multiplier range is between 2 and 7 (as explained in Paragraph 2.1):
$$2 \leq \ m_t \leq 7$$

2. Risky and riskless asset exposure is between 0%-100%:
$$0 \leq E_t \leq V_t$$
$$0 \leq R_t \leq V_t$$

3. Rebalancing is done when weights differ +-10%:
$$w_{1,t} = \frac{modelled\ exposure}{modelled\ exposure + \ modelled\ riskfree\ asset\ \ value}$$
$$w_2, t = \frac{realized\ exposure}{realized\ exposure + \ realized\ riskfree\ asset\ value}$$
$$if\ abs(w1 - w2) \geq 0{,}2\ then\ rebalancing\ is\ done\ at\ time\ t$$

4. Rebalancing is done when risky or riskless asset exposure is 0 or 100%:
$if\ w_{1,t} = 0\ or\ w_{1,t} = 1\ then\ rebalancing\ is\ done\ at\ time\ t$ (this extra restriction needed because otherwise the weights could get stuck at for example 0,98 due to the previous weight based rebalancing method)

5. Floor values are recalculated every 252 trading days:
$$if\ t\ mod\ 252 = 0\ then$$
$$F_t = V_t * F^p$$
$$F^p = floor\ percentage\ (determined\ by\ client)$$
$$V_t = portfolio\ value\ at\ time\ t$$

2.3. Market trend DPPI strategy

Market trend DPPI strategies depend on market momentum. The notion of the strategy is that the multiplier increases when the price of the risky asset increases and decreases when the prices of the risky asset decreases (Yao and Li 2016; Kunnas et al. 2008). Therefore, the strategy follows past market momentum in line with the momentum strategy and enhances the effects of momentum compared with the original CPPI strategy.

2.3.1. Model 1 – market trend-based

The market trend proportional portfolio insurance strategy that will be evaluated is based on the strategy proposed by Yao & Li (2016). The difference with the CPPI strategy is that the multiplier is conditional on the stock price on a time period and the stock price of the previous time period. Return periods can be daily, weekly or monthly. Recall from the CPPI model that with a higher multiplier the proportional investment in risky assets are higher and therefore the exposure is higher (vice versa for a lower multiplier). The strategy proposed by Yao and Li (2016), enhances the idea that more risk should be taken with a higher cushion and less risk with a lower cushion. When the stock price of the risky asset increases, the multiplier is enlarged ($\Delta \mathrm{m}_{t-1} > 0$) to increase the exposure for upward potential profits. When the stock price of the risky asset decreases, the multiplier decreases ($\Delta \mathrm{m}_{t-1} < 0$) to limit risk and protect the profits. The model simply changes the multiplier according to price fluctuations of the risky asset. The formula used to determine the change in multiplier is the following:

$$when\, T = 0, \qquad then\, m_0 = Initial\ multiplier$$
$$when\, T = 1, \qquad then\ m_{t+1} = m_t + \Delta \mathrm{m}_t,$$

$$where, \qquad \Delta \mathrm{m}_t = a\, ln \frac{S_t}{S_{t-1}}, \qquad a > 0, \qquad t = 0,1,2,3, \dots, T,$$
$$a = risk\ preference\ constant$$
$$s_t = stock\ price\ risky\ asset\ at\ time\ t$$
$$m_t = multiplier\ at\ time\ t$$

The dynamic multiplier is an adjustment of the previous multiplier by the ratio of logarithm return of the current and previous value of the risky asset. Multiplier movement is described below:

$$if\ S_t > S_{t-1}, then\ the\ multiplier\ m_t\ increseases\ to\ m_t + a \ln \frac{S_t}{S_{t-1}}$$
$$if\ S_t = S_{t-1}\ then\ m_{t+1} = m_t$$
$$if\ S_t < S_{t-1}\, , then\ the\ multiplier\ m_t\ decrases\ to\ m_t\ a \ln \frac{S_t}{S_{t-1}}$$
$$S_t = \ closing\ price\ of\ the\ risky\ asset\ \ on\ day\ t$$

2.4. Volatility based DPPI strategy

The volatility-based strategy depends on market fluctuation in a different way than the trend-based strategy. Whereas the trend strategy responds to a higher multiplier if the market fluctuates upwards and vice versa, volatility-based strategies have a lower multiplier in more fluctuating markets and vice versa, disregarding the momentum. The strategies are based on the findings of the low volatility anomaly, that suggest buying in stable markets and selling in unstable markets.

Note that the multiplier values using the volatility-based strategies do not seem dependent on the multiplier value at time = t-1 ($m_t = m_{t-1} + \Delta \mathrm{m}_t$). However, due to the way the volatility is calculated[6] the multiplier at time t is dependent on previous multipliers, since the volatility is still influenced by previous multipliers.

In the follow subgraphs first the measure for volatility will be explained, thereafter the different strategies are modelled.

2.4.1. Volatility

There are many approaches to calculate the volatility of risky assets, most use past data as estimator for current market volatility. For this research we are looking at dynamic proportion portfolio insurance strategies. Therefore, it is important that the volatility of the stock price at time t itself is also dynamic (not constant). Hendriks (1996) evaluates multiple volatility indicators and shows that most indicators give similar results. Moreover, he validated that the exponentially moving average approach (EMWA)

[6] The volatility calculations approach is explained in 2.4.1.

among others is a valid estimate for volatility. The volatility measure for this research is the EMWA approach. The formula for the EWMA approach is:

$$\sigma_t = \sqrt{(1-\lambda)\sum_{s=t-k}^{t-1} \lambda^{t-s-1}\,(x_s-u)^2}$$

Which can also be written, for a more intuitive understanding, as:

$$\sigma_t = \sqrt{\lambda\sigma_{t-1}^2 + (1-\lambda)(x_{t-1}-u)^2}$$

$Where$:
$\sigma_t = volatility\ at\ time\ t$
$\lambda =$ decay factor
$t = day\ t\ (t = 0,1,2,\dots)$
$x\ = log\ return\ of\ portfolio$
$k = number\ of\ days\ included\ in\ the\ moving\ average\ (observation\ period)$
$u = expected\ daily\ return$

The parameter λ is the decay factor, which is the rate at which past observations decay as they become more distant. Bollen (2014) researched what the optimal decay factor should be when using monthly volatility. His empirical results show that a decay factor of 0,97 is the best reflection of reality. Binck uses a similar decay factor, taking both into account the decay factor used for this research is 0,98. Figlewski (1994) tests different methodologies of calculating the expected return as input for the volatility estimation approaches. The empirical results of his research show that assuming a daily return of 0, is the best indicator for the expected return.

The number of observations (k) must be infinite to have the sum of the chances cumulate to 1. The formula above assumes an infinite horizon, in practice, however, the horizon is finite. Therefore, the formula is scaled accordingly to match to compensate. The scaled formula is as follows:

$$\sigma_t = \sqrt{\frac{(1-\lambda)}{(1-\lambda^n)}\sum_{J=0}^{n-1} \lambda^j\left(r_{t-j}-u\right)^2}$$

$Where\ n = number\ of\ days\ included\ in\ moving\ average$

The value for n is chosen because first, the multiplier needs to be a stable, shorter periods result in more multiplier fluctuations. Furthermore, the multiplier needs to respond relatively fast to different market conditions. When the market becomes more volatile the multiplier needs to respond quickly accordingly. When a relatively longer period is used, the multiplier takes too long to respond to changes in market conditions.

At last, the volatility-based multipliers are based on yearly volatility[7], therefore the volatility is annualized. Annualizing is done using the following formula:

$\sigma_T = \sqrt{T} * \sigma_t$
$Where\ T = total\ trading\ days\ in\ a\ year\ (252)$

[7] This makes scaling easier.

To determine how many days should be included in the moving average (n), the annualized volatility progression over the period from 10-12-2001 until 4-1-2019 is computed using the Stoxx 1800 index.

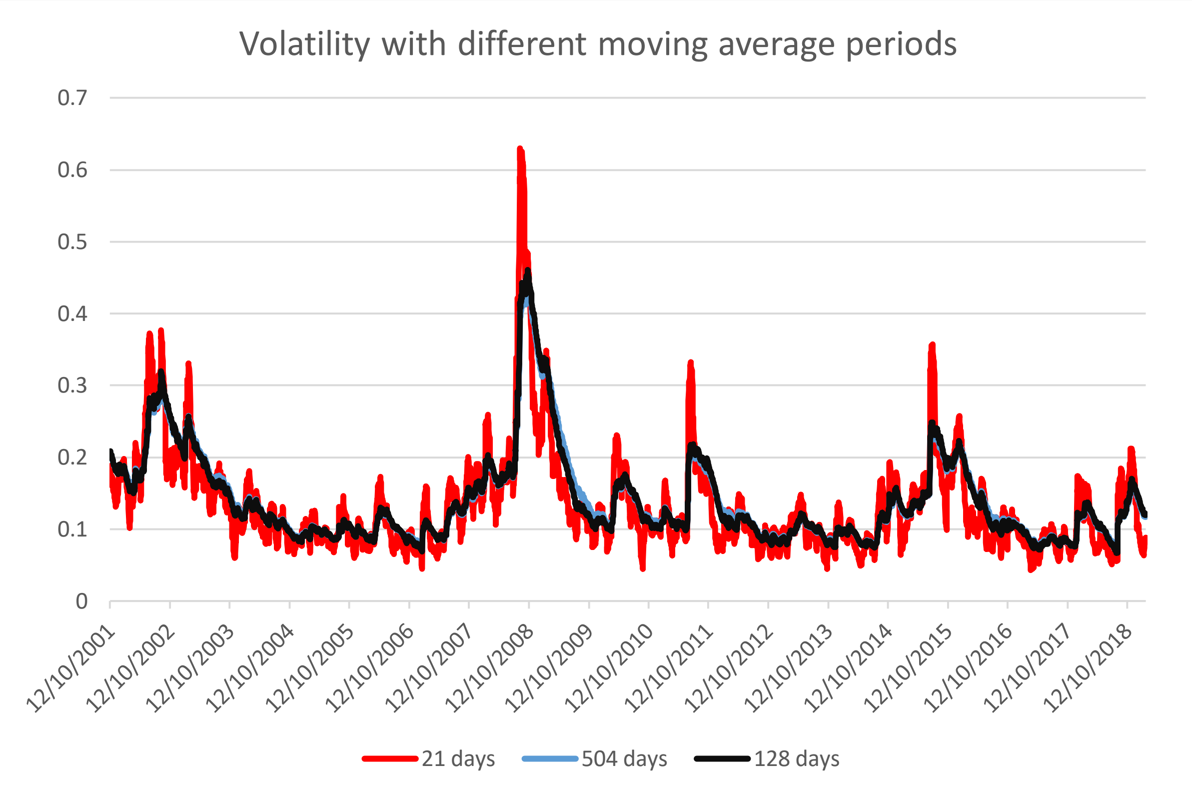

Figure 3: volatility progression over a time period with different moving average periods

Figure 3 shows clearly that a monthly moving average result in an unstable volatility. However, including a 128-day period in the moving average, results in similar stable results as 504 days. Therefore, the moving average period used to calculate volatility is 128[8]. The implications of different moving average period on the final recommended model and can be found in appendix A.

2.4.2. Model 2 – discrete volatility-based DPPI

This strategy has as basic principle that the multiplier should decrease if the market is very volatile and increase when the market is stable as the low volatility anomaly suggests. The result is that the possibility of making heavy losses when the stock market goes down rapidly is eliminated. The multiplier can jump between values depending on the observed volatility of at a certain time. The strategy is derived and altered from Kunnas et al. (2008).

The model of Kunnas et al. (2008) is as follows:

Table 3: Defining the multiplier with the help of volatility adapted from Kunnas et al. (2008)

$Vol_{t-1}(\%)$	≤ 10	$10 < vol \leq 15$	$15 < vol \leq 20$	$20 < vol \leq 25$	< 25
m_t	5	4	3	2	1

With the following extra restriction: the multiplier will only be increased if the volatility has been in a higher for 5 consecutive business days.

[8] A more detailed explanation can be found in appendix A

When putting these boundaries into the multiplier range [2,7] it results in the following formulas:

$$when\ T = 0, \qquad then\ m_0 = Initial\ multiplier$$
$$when\ T = t, \qquad then$$

$$if\ \sigma_t > \sigma_{t-1}, \qquad then\ m_t = \begin{Bmatrix} 2, if\ \sigma_t > .30 \\ 3, if\ .30 \geq \sigma_t > .25 \\ 4, if\ .25 \geq \sigma_t > .20 \\ 5, if\ .20 \geq \sigma_t > .15 \\ 6, if\ .15 \geq \sigma_t > .10 \\ 7, if\ .10 \geq \sigma_t \end{Bmatrix}, \qquad t = 0,1,2,\dots,T$$

$$if\ \sigma_t = \sigma_{t-1}, \qquad then\ m_t = m_{t-1}$$

$$if\ \sigma_t < \sigma_{t-1}, \sigma_{t-2}, \sigma_{t-3}, \sigma_{t-4}, \qquad then\ m_t = m_{t-1} + 1$$

The reason for the first extra restriction is that the multiplier will not jump to an unfavorable high value when the market is not as volatile for a short time period. However, this behavior is also already countered with the approach for calculating the multiplier which takes the volatility of previous days into account.

2.4.3. Model 3 – continuous volatility based DPPI

The previous strategy has relatively high computational effort, and the volatility intervals are unsubstantiated. The following strategy makes the multiplier continuously dependent on volatility using its inverse. The strategy is that exposure should be less when the market is volatile and exposure is high when the market is stable, which is the same notion as proposed by Kunnas et al. (2008) and the low volatility anomaly. When the market is highly volatile the multiplier is low and vice versa. The multiplier itself is derived and suggested for further research by an internal research of Binck. The function for the multiplier is as follows:

$$when\ T = 0, \qquad then\ m_0 = initial\ multiplier$$

$$when\ T = t, \qquad then\ m_t = a * \frac{1}{\sigma_t}, \qquad a > 0, \qquad m \geq 0 \quad t = 0,1,2,\dots,T$$
$$where, \sigma_t = volatility\ at\ time\ t$$
$$a = risk\ factor$$

The risk factor is an extra input variable. It is introduced for if the volatility values are too extreme, resulting in extreme multiplier values. In that case the risk factor can be less than one, resulting in less extreme and less fluctuating multiplier values. If the multiplier values seem to be too low a risk factor bigger than one can be used to increase the impact of the strategy[9].

2.5. Trend and volatility based DPPI strategy

The issue with the momentum strategy is that it performed well in stable markets, but poorly in unstable markets. Including the volatility in the trend-based strategy, is expected to address this issue. Moreover, results of Kunnas et al. (2008) and Neftci (2008) suggest that combining trend and volatility-based strategies can be beneficial for the performance of the DPPI. Therefore, different combination models are proposed in the following subgraphs.

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[9] Note that the risk factor is an input variable that does not change over time

2.5.1. Model 4 – continuous trend and volatility-based DPPI strategy (momentum)

Findings of the momentum strategy suggest that following momentum results in significantly high returns when markets are stable and low returns when markets are volatile. Therefore, the multiplier should follow market momentum when the volatility is low. This means that the multiplier should increase and decrease in line with upward and downward market movement respectively, resulting a variant of the momentum strategy. When volatility is high the effect of momentum disappears. Therefore, in volatile markets the multiplier should not follow market momentum (resulting in a more static multiplier).

The following model is a combination of the trend-based model of Yao & Li (2016) and the continuous volatility-based model. The strategy proposes that the multiplier should:

- Increase relatively rapid when returns are positive, and volatility is low
- Decrease relatively rapid when returns are negative, and volatility is low
- Increase relatively slow when returns are positive, and volatility is high
- Decrease relatively slow when returns are negative, and volatility is high

The general movement is comparable with the trend-based strategy. When the cushion grows, due to gains in risky asset value, more risk can be taken by increasing the multiplier. When the cushion decreases, exposure decreases due to a decrease in multiplier. However, the amount the multiplier increases or decreases also depends on the volatility. When volatility is relatively high the multiplier increases and decreases slower. When volatility is relatively low the multiplier increases and decrease faster. This has as implications that the multiplier will grow and decline faster when the market is stable. Moreover, fluctuations in multiplier due to market volatility is decreased, creating are more stable multiplier value.

The strategy results in the following formula for the multiplier:

$$When\, T = 0, \qquad then\, m_0 = initial\, multiplier$$

$$When\, T = t, \qquad then\;\; m_{t+1} = m_t + \Delta \mathrm{m_t}$$

$$where, \qquad \Delta \mathrm{m_t} = \mathrm{a} * \frac{1}{\sigma_\mathrm{t}} * \ln \frac{S_t}{S_{t-1}}, \qquad a > 0\,, \qquad t = 0,1,2,\ldots,T$$

The formula illustrates that whether the multiplier grows, or declines is dependent on the return. How much it increases, and decreases is also dependent on the volatility and the risk factor. The risk factor is added as an input variable in case the $\Delta \mathrm{m_t}$ Influences the formula too much or too little. It functions as a risk amplifier. Choosing a, appropriate risk factor depends on how much risk a practitioner wants to take and what their chosen floor value is.

2.5.2. Model 5 – Discrete trend and volatility-based DPPI strategy (crises)

Another concern for the momentum strategy, is that during extreme market environments such as crises, momentum profitability disappears. Financial crises go hand in hand with high market volatility across all markets (Schwert, 2010). The following strategy is introduced to reduce the effects of potential financial crisis on the momentum strategy, by letting the multiplier decrease rapidly when returns are negative in highly volatile market conditions. Moreover, the notion removes overcompensation of the multiplier in due to negative returns in stable markets.

The strategy is a combination of the models proposed by Kunnas et al. (2008) and Yao & Li (2016), and has the following properties:

- Increase relatively rapid when returns are positive, and volatility is low

- Decrease relatively rapid when returns are negative, and volatility is high
- Increase relatively slow when returns are positive, and volatility is high
- Decrease relatively slow when returns are negative, and volatility is low

When the cushion grows exposure in risky asset grows as well, how much depends on the volatility of the market. When the cushion decrease, exposure decreases as well. The benefit of this strategy is that the multiplier does not increase rapidly when its due to market volatility. It only increases rapidly when the market itself is increasing steadily. When making a loss due to a high volatile market the multiplier responds quickly reducing the exposure. Furthermore, when the market decreases slowly with low volatility, risky asses exposure decreases slowly as well. The model of the strategy is as follows:

$$when\, T = 0, \qquad then\, m_0 = Initial\ multiplier$$
$$when\, T = 1, \qquad then\, m_{t+1} = m_t + \Delta \mathrm{m_t}, \qquad t = 0,1,2,3, \ldots T$$

$$if\ S_t > S_{t-1}, \qquad then\ the\ multplier\ increases\ with\ \Delta \mathrm{m_t} = \begin{Bmatrix} 1 \ln \dfrac{S_t}{S_{t-1}}\ if\ \sigma_t > .3 \\ 2 \ln \dfrac{S_t}{S_{t-1}}\ if\ .3 \geq\ \sigma_t > .2 \\ 3 \ln \dfrac{S_t}{S_{t-1}}\ if\ \ .2 \geq \sigma_t \geq\ .1 \\ 4\ \ln \dfrac{S_t}{S_{t-1}}\ if\ \ \sigma_t \geq\ .1 \end{Bmatrix}$$

$$if\ if\ S_t = S_{t-1}\,, \qquad then\, m_{t+1} = \mathrm{m}_t$$

$$if\ S_t < S_{t-1}\,, \qquad then\ the\ multiplier\ decreases\ with\ \Delta \mathrm{m_t} = \begin{Bmatrix} 4 \ln \dfrac{S_t}{S_{t-1}}\ if\ \sigma_{t-1} > .3 \\ 3 \ln \dfrac{S_t}{S_{t-1}}\ if\ .3 \geq\ \sigma_{t-1} > .2 \\ 2 \ln \dfrac{S_t}{S_{t-1}}\ if\ \ .2 \geq \sigma_{t-1} \geq\ .1 \\ 1\ \ln \dfrac{S_t}{S_{t-1}}\ if\ \ .2 \geq \sigma_{t-1} \geq\ .1 \end{Bmatrix}$$

$$S_t =\ closing\ price\ of\ the\ risky\ asset\ \ on\ day\ t$$

2.5.3. Model 6 – continuous trend and volatility-based DPPI strategy (Crises)

The following strategy has the same principle as the discrete trend and volatility based DPPI strategy. However, it is made into a continuous formula. Also, the proportions of direction of $\Delta \mathrm{m_t}$ is different from the discrete trend and volatility based DPPI strategy. The formula was derived of the needs of the strategy, meaning, First the strategy was developed, then a formula satisfying the requirements of the strategy was derived. The strategy goes as follows, the multiplier should:

- Increase rapidly when returns are positive, and volatility is low
- Decrease rapidly when returns are negative, and volatility is high
- Increase slowly when returns are positive, and volatility is high
- Decrease slowly when returns are negative, and volatility is low

The formula derived for this strategy goes as follows:

$$When\, T = 0, \qquad then\, m_0 = initial\ multiplier$$

$$When\, T = t, \qquad then\ \ m_{t+1} = \mathrm{m}_t + \Delta \mathrm{m_t}$$

$$where, \qquad \Delta \mathrm{m_t} = \mathrm{a} * (\sigma_t)^{\frac{-\ln \frac{S_t}{S_{t-1}}}{u}} * \ln \frac{S_t}{S_{t-1}}, \qquad u > 0, \qquad t = 0,1,2,\dots,T$$

$where\ u = high\ expected\ daily\ return$
$a = risk\ factor$

This might be difficult to interpret; therefore, a numerical example is given. The risk factor in this case is 1 and the high expected daily return chosen is 0,02.

Table 4: Model 6 numerical example

Input / strategy	Positive return and low volatility	Negative return and low volatility	Positive return and high volatility	Negative return and high volatility
S_t	100,5	100	100,5	100
S_{t-1}	100	100,5	100	100,5
σ_{t-1}	0,1	0,1	0,5	0,5
$\Delta \mathrm{m_t}$	0,008856	-0,002808	0,005929	-0,004195

Table 4 clearly shows that it satisfies the properties proposed by the strategy. When the returns are positive and the volatility is low, $\Delta \mathrm{m_t}$ is relatively high, resulting in a rapid grow in multiplier value and therefore exposure. When returns are positive and volatility is low, the growth in multiplier value is less. When the returns are negative, and the volatility is low the multiplier decreases slowly. At last, when returns are negative and volatility is high, the multiplier value decreases rapidly.

When looking at how the formula itself behaves, when the returns are negative $\Delta \mathrm{m_t}$ is also negative. Volatility cannot be negative, therefore the direction of $\Delta \mathrm{m_t}$ is solely decided by the return. How much the multiplier increases and decreases however, is influenced by the volatility. When returns are positive, the function in the power becomes negative. Therefore, the trend function is multiplied with an inverse of the volatility. Resulting in that when the volatility is high and the returns are positive, the multiplier will increase slowly. Due to the inverse volatility property when the returns are positive, the multiplier will increase more rapidly when the volatility is low. When the returns are negative, the volatility will not be inversed, resulting in a higher decrease when volatility is high, and a lower decrease when volatility is low. The reasoning for using the high daily return input factor, is to get the function in the power closer to 1, making $\Delta \mathrm{m_t}$ more symmetric in both directions. This formula can be seen as an extension of the previous continuous trend and volatility-based model (model 5). If the high daily return is the expected historical return, then the situation of model 5 is created. This is shown below:

$$\Delta \mathrm{m_t} = \mathrm{a} * (\sigma_t)^{\frac{-\ln \frac{S_t}{S_{t-1}}}{\ln(\frac{S_t}{S_{t-1}})}} * \ln \frac{S_t}{S_{t-1}} = \mathrm{a} * \sigma_t^{-1} * \ln \frac{S_t}{S_{t-1}}$$

3. Empirical research methodology

The empirical research consists of two sections. First the CPPI and DPPI strategies are compared with a mix of risky and riskless assets[10]. The goal is to compare the strategies using a weighted average of the floor values. All the different models will be subjected to a back-test which allows us to calculate the KPIs. Based on the KPIs derived it is possible to select one preferred model. This model will then

[10] The research is conducted with daily returns of different markets and bonds which are described in Chapter 4.

be further investigated on gap risk and equity allocation. Based on the results of these two steps, conclusions and recommendations are derived. Finally, the validity will be evaluated by running all models with different benchmarks and comparing results. If the results of the validity test are similar to the original results, it can be said that the back-test is valid.

3.1. Strategies comparison

3.1.1. Rolling analysis

The method used to simulate and evaluate the strategies is by a rolling analysis. The rolling event windows is five years, with quarterly steps. Ideally steps should be daily, however due to the runtime restrictions of VBA the number of simulations is reduced.

Rolling analyses are often used to back-test financial models on historical data to evaluate stability and predictive accuracy. Since portfolio insurance strategies are path dependent the results differ significantly per start date. Moreover, PI strategies are used for long term investments, and are supposed to be safe, therefore the stability of the strategy over time is important. For the rolling analysis using historical data sequences, there are no statistical assumptions. Moreover, the newly developed strategies are based on the perspective of the momentum strategy and low volatility anomaly. therefore, the historical effects of behavioral finance should be included in the analysis. When computing a rolling analysis compared to simulating market conditions, the effects of behavioral finance are included. Furthermore, when computing a rolling analysis, and the parameters are constant, then the estimates over the rolling window should not significantly differ. If the parameters are not constant and change at some point during the sample, the rolling estimates capture this instability (Zivot & Wang, 2006). Yao & Li (2016) have shown that it is important to include different market conditions (bull market, dear market etc.) in the DPPI analyses, since every strategy performs different under other circumstances. With the rolling time period method, different real market conditions are created. Finally, to back-test portfolio insurance strategies, it is important to have a long enough time series to include extreme events (Xing, Xue, Feng, & Wu, 2014). For this research a twenty-year timeseries is analyzed, including the financial crisis of 2008. Therefore, the rolling analysis covers extreme events and their implications on the strategies.

An effect of this approach, is that if there is one day with characteristics that heavily influence the results, depending on which day, it will occur in multiple runs. On one hand is the event in reality often an anomaly. However, it occurs often in the analysis. Therefore, the data set can be seen as an unreliable reflection of reality. On the other hand, it can be seen as a positive implication since the data sample can be seen as a worst-case scenario sample, and the effects of the event become clear. A deeper analysis of the effects of the rolling analysis on the results can be found in the discussion.

The rolling analysis is used to eliminate strategies that underperform compared to others. The input variables and KPIs will be explained in the following subgraphs.

3.1.2. Input parameters

Rolling analyses are computed for every model with relevant input parameters. The models are defined in Chapter 1 and 2. When recalling Table 2 the client risk profiles, the step was 0,025. Due to runtime restrictions the decision for step 0,05 is made. The floor percentage and client risk profiles are used to calculate a weighted average of the KPIs using the percentage of clients in every floor. Resulting in aggregated results. This way the impact of the different floor percentages is considered. Table 5 shows the floor percentages and their weights:

Table 5: Floor percentage weights

Floor	Weight
0,75	10%
0,8	24%
0,85	33%
0,9	31%
0,95	2%

The models are run over the following input variables:

Table 6: strategy input parameters

Model	Parameter	Range	Total number of runs (for 1 floor)	Total number of runs (for all floors)[11]
CPPI	Constant multiplier	3 to 7	5	25
Trend based	Initial multiplier Risk factor Return period	3 to 7 1 to 4 1, 5, 21 days	60	300
Discrete Volatility	No parameter		1	5
Continuous Volatility	Risk factor	0,5 to 1,5 with step 0,25	5	25
Continuous trend and volatility (momentum)	Initial multiplier Risk factor Return period	3 to 7 2 to 4 with step 0,5 1, 5 and 21 days	75	375
Discrete trend and volatility	Initial multiplier Return period	3 to 7 1, 5, 21 days	15	75
continuous trend and volatility (crisis)	Initial multiplier Risk factor Expected high daily return Return period	3 to 7 0,5 to 2 with step 0,5 0,01 to 0,04 with step 0,01 1, 5, 21 days	240	1200
Total				2005

[11] Note that every floor run consists of 40 runs over different time periods (rolling analysis)

Initial multiplier values around 5 (m = 3, 4,…,7) are chosen, because internal research of Binck about the CPPI concluded that 5 is the optimal multiplier. The risk factor and high daily return values are chosen based on trial and error of one time period. The values outside of the range had unfavorable impacts on the strategies.

3.1.3. Key performance indicators

To compare the performance of the different strategies, key performance indicators are defined. Binck was interested in the profitability, including return and transaction costs, and risk of the strategies compared to the original strategy. Therefore, the following KPIs are defined.

3.1.3.1. Return

Return gives an indication of how profitable the strategy would be, not taking transaction costs into consideration. The average yearly return of a period in this research is calculated the following way:

First, the portfolio daily return is calculated using:

$$\frac{(S_t - S_{t-1})}{S_{t-1}} = r_i$$

Then returns are annualized using by multiplier the average return with the number of trading days per year:

$$r_i \, * 252 = r_t$$
$$Where\ r_t = annualized\ daily\ return$$

At last, average yearly returns of the annualized returns are calculated:

$$\frac{1}{n}\sum_{i=1}^{n} r_t = \bar{r},$$
$$where\ n = amount\ of\ days\ in\ period,\ \ i = 1,2, \dots, n$$
$$\bar{r} = average\ annualized\ \ return$$

Moreover, to give an indication of the skewness of the returns the median of the annualized returns is also calculated.

3.1.3.2. Turnover

Turnover can be defined as the amount traded as a fraction of the total invested amount over a certain time period. It gives a good indication of the impact of transaction costs when comparing the different strategies. The definition of turnover is based on the proposal of DeMiguel et al. (2009), the formula is as follows:

$$Turnover = \frac{1}{T - z - 1}\sum_{t=z}^{T-1}\sum_{j=1}^{N}\left(\left|w_{j,t+1}^{i} - w_{j,t^+}^{i}\right|\right)$$
$$Where, T = total\ number\ of\ returns\ in\ the\ dataset$$
$$z = estimation\ window\ (5\ years, 1260\ days)$$
$$N = amount\ of\ assets\ (2, risky\ and\ risk\ free)$$
$$i = strategy\ i$$
$$w_{j,t} = \ portfolio\ weight\ in\ asset\ j\ at\ time\ t$$
$$w_{j,t+1} = desired\ portfolio\ weight\ at\ time\ t + 1$$
$$w_{j,t^+} = the\ portfolio\ weight\ before\ rebalancing\ but\ at\ time\ t + 1$$

The definition seems complicated but when making a distinction between modelled and real values (the difference is due to the rebalancing method of the strategies) and express them in exposure and risky asset values, it becomes clearer. The real value refers to the realized exposure and riskless asset value of the portfolio at time t. the modeled values refer to what the model calculated to be the exposure and risk-free asset value. These values can differ due to the rebalancing aspect of the strategy. The turnover is only calculated when there is rebalancing done between t and t+1, otherwise turnover is 0. The weights of risky (j=1) and riskless (j=2) asset can be easily calculated with the following formula[12]:

$$w_1 = \frac{exposure}{exposure + \ riskfree\ asset\ \ value}$$
$$w_2 = \frac{riskfree\ asset\ value}{exposure + riskfree\ asset\ value}$$

Furthermore, the moment $t + 1$ can be defined as the modeled values of the adjusted close price after rebalancing. The moment t^+ can be seen as the actual adjusted close price at of day t.

3.1.3.3. Transaction frequency

Transaction frequency refers to the average yearly transaction over the time period. If transactions on time t take place the number of rebalances increase by one. After all transactions are counted, a yearly average over the time period is calculated. Together with turnover, transaction frequency can give an indication of the transaction costs when comparing different strategies.

3.1.3.4. Risk adjusted return

The Risk adjusted return calculates the risk-adjusted return. A greater Risk adjusted return means a better risk adjusted performance[13]. If the Risk adjusted return is negative, either the portfolio return is negative. The risk-free rate (and therefore the Sharpe ratio) is not considered because the aim is to compare different strategies and all the strategies have the same risk-free rate during the evaluated time periods. The formula is as follows:

$$Risk\ adjusted\ return = \frac{r_p}{\sigma_p}$$
$$Where\ r_p = portfolio\ return$$
$$\sigma_p = \ portfolio\ volatility$$

For the return the average yearly returns as described earlier are used. The portfolio volatility is calculated as an annualized standard deviation of the portfolio returns.

3.1.3.5. Maximum drawdown

Maximum drawdown is the maximum loss from a peak until a new peak starts. It is used as a measure of risk, since it calculates the maximum loss of a portfolio when using a strategy. In percentage the maximum drawdown is calculated as[14]:

$$Maximum\ drawdown = \frac{P - L}{p}$$
$$Where, P = peakvalue\ before\ largest\ drop,$$
$$L = Lowest\ value\ before\ new\ peak\ value\ is\ established$$

For calculation in VBA the function “drawdown” from Wes Grey and Jack Vogel is used (2013).

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[12] Note that these are realized, and modelled values as described in 1.3.
[13] Source : https://www.investopedia.com/terms/s/sharperatio.asp
[14]Source : https://www.robeco.com/en/insights/2018/04/the-formula-maximum-drawdown.html

3.1.3.6. *Floor hits*

Hitting the floor value is dissatisfactory for clients since the floor value is indicated to be a guaranteed[15] cashback value, moreover, potential gains are minimized during the period. Hitting the floor value during a date happens when: $V_t < F_t$. The KPI counts the number of floor hits per run.

3.1.3.7. *Portfolio participation*

Portfolio participation is an extra insight in the risk of the strategy. Combined with risk adjusted return and return, portfolio participation can give insights to why strategies perform better or worse. For example, when the strategy allocates equity only in the risky asset, returns will excess the other strategies, however, risk is much higher. Since comfort is a risk averse strategy, lower weight in equity without compromising on other KPIs is more favorable. Portfolio participation shows how the average ratio invested in the risky asset. The formula used to asses portfolio participation is the following:

$$Equity\ weight = \frac{1}{n}\sum_{t=1}^{n}\frac{E_t}{V_t}$$

$$Where\ E_t = exposure\ on\ time\ t,$$
$$V_t = portfolio\ value\ on\ time\ t,$$
$$n = amount\ of\ days\ in\ period$$

3.1.3.8. *Sortino ratio*

The Sortino ratio is similar to the risk adjusted return. However, instead of using the portfolio volatility, only downside volatility is taken into account. Therefore, the returns are only compromised by 'bad' risk, since positive volatility can be seen as a benefit[16]. The formula for the Sortino ratio[17] is given below:

$$Sorino\ ratio = \frac{r_p}{\sigma_d}$$

$$where, \qquad \sigma_d = \sqrt{\frac{1}{n} * \sum_{r_t<0}^{n} r_t^2}$$

$$n = total\ number\ of\ observed\ negative\ returns$$
$$r_t = observed\ negative\ return$$

3.1.3.9. *Modified omega ratio*

The omega ratio favors return distributions that have a positive mean and are skewed to the right. Moreover, it favors strategies which returns have an exponentially decreasing left tail. Therefore, the omega strategy penalizes "dangerous asset behavior which can potentially exist in an edge case" [18]. The omega ratio contains information that adjusted return and Sortino ratio do not, since it also takes into account the skew and kurtosis, in addition to the mean and variance. In short, the ratio favors strategies that have result in a higher number of positive yearly returns, regardless of the value of the returns. It is valuable for non-normal investments such as portfolio insurance strategies. The formulas for the Omega and modified Omega ratios are the following:

$$\Omega = \frac{\sum_{i=1}^{n} r_t^+}{\sum_{i=1}^{n} r_t^-}$$

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[15] Note that the floor value is not actually guaranteed, more about this is explained in the gap risk theory

[16] Source: https://www.investopedia.com/terms/s/sortinoratio.asp

[17] Note that the risk-free rate is taken out of the equation because all strategies have the same risk-free rate

[18] Source: http://investexcel.net/modified-omega-ratio/

$$where\ r_t^+ = postive\ returns^{19}\ of\ year\ t$$
$$r_t^- = negative\ returns\ of\ year\ t$$
$$n = amount\ of\ y\text{ear returns}$$

$$Modified\ \Omega = \frac{E(win)}{E(loss)} * max(\Omega - 1, 0)$$
$$where\ E(win) = the\ average\ positive\ yearly\ \ return$$
$$E(loss) = the\ average\ negative\ yearly\ return$$

The max function favors an Omega Ratio that spends more time winning than losing. This change eliminates Omega Ratios less than 1. The multiplication by the expected win/loss ratio rewards returns distributions that have a positive mean, reducing the impact of low-probability, high impact events[20].

3.2. Equity allocation in different market conditions

The rolling analysis does not give insights in the behavior of the strategies in different market conditions. To understand potential differences or the lack hereof, in regards of (risk adjusted) return and maximum drawdown, the weights in risky assets during a time period can be compared to the returns of the risky asset of that time period. This shows the efficiency of the strategies under different market conditions, since the analysis will show how much is invested in risky assets under these market conditions. For example, transitioning from a bull market into a dear market, the equity allocation should quickly shift from more weight in risky asset to more weight in the risk-free asset and vice versa. Therefore, analyzing equity allocation in different market conditions can give extra insight in understanding the performance of a strategy compared to the original strategy.

3.3. Practical implications

Changing the multiplier within Comfort also impacts the benchmarks which are used to measure the performance of the strategy. The comparison between the performance of the strategy and the benchmark, is an important aspect for customer satisfaction of the comfort strategy. The benchmark refers to a constant mix portfolio, if the weights in equity or average multiplier of the strategy changes, the benchmark must be adjusted accordingly. Moreover, the benchmark should not be changed often since the client has to understand the chosen benchmarks and the reasoning behind it. Furthermore, there are strict legislations and guidelines benchmarks for financial institutes when selecting new benchmarks (GIPS guidance statement on benchmarks, 2019). To not increase risk by investing more in the risky asset and for ease of implementation when choosing a new strategy ideally the equity allocation and average multiplier should be the same as originally with the CPPI strategy.

3.4. Gap risk analysis

The gap risk theory was introduced in Paragraph 1.5., the following Paragraph will describe how the effects of gap risk will be analyzed.

The modelling restrictions show that the rebalancing decision for this research is made on the difference in weights between modelled and realized values. Therefore, the probability of breaching the floor value is higher than the probability of risky asset dropping more than $\frac{1}{m}$. Moreover, the multiplier for the dynamic strategies fluctuates between 2 and 7. Note that these complications are taken into account, due to the modelling restrictions, when calculating the floor hits in the rolling analysis.

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[19] Note that these returns differ from the annualized returns described earlier. Every run has 5 year returns and every parameter has 40 runs, therefore 200 year returns per floor are used to calculate the omega ratio
[20] Source : http://investexcel.net/modified-omega-ratio/

When modelling the strategies rebalance weights are assessed daily, the closing price is known, and trades are made at opening for the closing price. This is a simplification of reality. The comfort strategy invests in exchange traded funds (ETF). The information of the closing prices of ETFs can be delayed, sometimes until the closing time of the next day. This impacts the gap risk since there is a delay between price fluctuations and the opportunity to rebalance

To include the delay into the gap risk evaluation. The models are run with two day returns instead of daily returns. This way the shocks in the returns are bigger, representing the worst-case real situation. With these settings, the relevant strategies are rerun. The performance indicator for the gap risk is the number of floor hits, if there are no floor hits, it can be assumed that gap risk is not of importance for the strategy.

Moreover, the maximum realized multipliers of the strategies are compared with the worst two-day return of the benchmark over the full time period.

3.5. Validity

To check the validity of the results. The complete back-test is rerun with different a risky and risk-free asset selection. Since historical data of most of the indices Binck uses only goes back to roughly the year 2000, the time period stays more or less the same.

4. Data

The historical data selected is the Stoxx 1800 index as risky asset and a combination of 40% Bloomberg Barclays Euro Corporate Bonds 1-5 years and 60% Bloomberg Barclays Euro Aggregate Treasury 3-5 Year as risk-free asset. Data from 3-1-2000 to 1-4-2019 is used. The first 262 days are used to calculate the risky asset volatility (EMWA approach Stoxx 1800 index as risky asset). For the validity check the historical data selected as risky asset is the S&P 500 index, and for riskless asset the Barclays US Treasury Index is used. The data used is from 3-1-2000 until 6-6-2019, resulting in the same number of trading days as the original data. For all data the adjusted close price is used.

4.1. Rolling analysis

The rolling analysis is run from 3-1-2001 until 1-4-2019. The rolling window is 1260 trading days (262*5), making steps of 88 trading days. Ideally steps of one day would be taken, due to runtime restriction of VBA however, steps have been made quarterly. The rolling analysis results in 40 time periods (and thus 40 runs) per input parameter. The first run starts on 3-1-2001 and the last run starts on 2-6-2014.

4.2. Equity allocation in different market conditions

For the equity allocation in different market condition analysis, the longest possible time period is computed. The period is from 3-1-2001 until 1-4-2019. The floor percentage used is 85% since this is the risk profile most often selected by clients (as shown in Table 5).

4.3. Gap risk

To simulate the effects of the information delay, explained earlier, on the gap risk. Every second data point from the original input is deleted, resulting in a datasheet representing two-day returns. This is in line with the described information maximum information delay of ETFs. Thereafter the rolling analysis is rerun, and the number of floor hits is counted. The input data for risky and riskless asset stays the same as for the initial analyses.

5. Results

The goal of this research is to investigate whether a DPPI strategy can be an improvement over the current CPPI strategy. Based on the aggregated results from the back-test it is possible to select an outperforming DPPI strategy compared to other DPPI strategies and the original strategy. On this level there is accounted for a single overall strategy which can be used for all floors. This strategy will then be further investigated and the impact on a floor level will also be evaluated.

5.1. Strategy comparison

The results of the back-test are shown in the two Tables below. Table 7 shows the aggregated absolute results of the back-test of the seven different models. Table 7 does not show all 825 runs, but only the best classes per model. Table 8 shows the percentual differences with the original CPPI with set multiplier model. The green markings in Table 8 show the KPI's of the DPPI that outperform the original strategy. For example, the return of the continuous volatility strategy with a risk factor of 0,75 performs best considering annualized average return (6,3%), which is 16,4% better than the return of the CPPI strategy. The results of the all the input parameters per class are in the excel file.

When looking at Tables 7 and 8 the following information can be distilled. First, for all strategies the portfolio value never reaches the floor value, therefore, no floor hits are realized.

Secondly, transaction costs can be reduced when using the trend-based strategy, since the yearly turnover and transaction frequency decrease by 8% and 7% respectively. The trend-based strategy performs best regarding transaction costs out of all models. Considering return, risk adjusted return, drawdown, omega ratio and Sortino ratio, no significant changes between the trend based and original strategy can be identified. Moreover, there are strategies that perform better than the trend-based strategy on every aspect except for transaction costs. Therefore, the trend-based strategy is not seen as the best performing strategy.

The discrete volatility-based strategy has no input parameters. The multiplier is solely based on the volatility intervals modified of Kunnas et al. (2008). With this strategy turnover and transaction frequency increase by 76% and 55% respectively compared to the original CPPI strategy. Even with the extra restrictions added to reduce jumps in multiplier value, the turnover and transaction frequency, and therefore transaction costs, are high compared to all other strategies. Moreover, the strategy does not show convincing improvements on other KPIs, therefore, the discrete volatility-based strategy is not optimal.

The discrete and continuous crisis control combination strategies perform 0,9% and 4,3% respectively better than the original strategy in regards of return. Moreover, the discrete variant slightly outperforms the original strategy in terms of maximum drawdown (31% and 30% respectively). However, these outperformances are not significant. At last, the crisis combination models both perform slightly worse than the original strategy considering transaction costs, risk adjusted return, omega ratio and Sortino ratio.

The results show that the continuous volatility strategy outperforms the original CPPI strategy on every KPI and that the continuous combination strategy outperform the CPPI strategy on every KPI except for rebalance frequency. If these two strategies are compared, the KPI results are similar. There are only two differences with the first one being that the continuous volatility strategy performs better on rebalance frequency. The continuous strategy has on average 0,79 (8,46 – 7,67) rebalances per year less than the momentum based continuous combination.

The second difference is that the continuous combination strategy has a higher omega ratio than the continuous volatility strategy (10,26 and 7,86 respectively). This means that the combination strategy yields more positive returns than negative returns. Based on these results it is difficult to declare one of these strategies as the most optimal strategy. However, the continuous volatility strategy seems to slightly outperform the continuous combination strategy. At last, the continuous volatility strategy is the only strategy that outperforms the original strategy on every KPI[21].

When looking at the results for all input parameters of the excel file the following can be derived:

1. A lower initial multiplier results in lower returns, but in higher risk adjusted return, omega ratio and sortino ratio and a lower maximum drawdown.
2. For the trend and continuous combination strategy (momentum based), when the risk factor increases, returns increase, and transaction costs also increase.
3. For the all past return based strategies, when using monthly returns as input instead of daily or weekly transaction costs decrease, without compromising on other KPIs.

[21] Note that the complete row of the continuous volatility strategy in Table 7 is green

Table 7: absolute KPI results of best models per class

Model	Initial multiplier	Risk factor	High daily return	Return period	Average annualized return	Median annualized return	Average yearly turnover	average yearly rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Weight in equity	Average multiplier	Modified omega ratio	Sortino ratio
CPPI	5				5.4%	5.8%	2.04	7.91	0.84	-31%	0	0.74	5.00	5.94	0.55
Trend	5	4		21	5.6%	5.8%	1.87	7.37	0.84	-30%	0	0.75	5.26	5.77	0.55
Disc Vola	5				5.7%	6.1%	3.60	12.26	0.84	-30%	0	0.79	5.81	6.00	0.55
Cont Vola	5	0.75			6.3%	6.4%	1.93	7.67	0.94	-27%	0	0.78	5.77	7.86	0.62
Cont Combi (1)	4	4		21	6.2%	6.3%	2.01	8.46	0.94	-25%	0	0.76	5.45	10.26	0.61
Disc. Combi	5				5.5%	5.8%	2.10	8.34	0.83	-30%	0	0.74	5.07	5.89	0.55
Cont Combi (crisis)	5	2	0.01	21	5.7%	5.9%	2.13	8.26	0.82	-31%	0	0.78	5.72	5.59	0.54

Table 8: Relative KPI results of best models per class

Model	Initial multiplier	Risk factor	High daily return	Return period	Average annualized return	Median annualized return	Average yearly turnover	average yearly rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Modified omega ratio	Sortino ratio
CPPI	5				5.4%	5.8%	2.04	7.91	0.84	-31%	0	5.94	0.55
Trend	5	4		21	3.6%	0.9%	-8%	-7%	0.3%	-3%	0%	-3%	0%
Disc Vola	3				5.6	5%	76%	55%	-1%	-3%	0%	1%	0%
Cont Vola	5	0.75			16.4%	11.6%	-5%	-3%	7.1%	-13%	0%	32%	12%
Cont Combi (1)	4	4		21	14.5%	9.5%	-1%	7%	7.7%	-19%	0%	73%	11%
Disc. Combi	5				0.9%	1.0%	3%	5%	-0.8%	-3%	0%	-1%	-1%
Cont Combi (crisis)	5	2	0.01	21	4.3%	1.3%	4%	4%	-11.9%	2%	0%	-6%	-3%

5.2. Equity allocation compared to risky asset return

The multiplier within the volatility-based model only depends on the market equity volatility. If the market volatility is high, the multiplier will decrease while the multiplier will increase when the market volatility is low. Thus, in high volatile markets less risk will be taken while in less volatile markets more risk is taken. There are in total four different market states that can occur within the market:

1. High volatility and negative returns
2. High volatility and positive returns
3. Low volatility and negative returns
4. Low volatility and positive returns

If the returns of the volatility model with the original CPPI model are compared, based on the different leverage ratio assignments the following information can be distilled. In state 1, the multiplier of the volatility-based model will be lower than the original CPPI model. This results in a lower equity allocation for the volatility-based model and therefore, less negative returns. This way the model reduces the risk of breaching the floor. However, in state 2 the volatility-based model will have a lower equity allocation while the returns are positive. This will lead to an underperformance compared to the original CPPI model. In state 3 the equity allocation of the volatility-based model will be higher compared to the original CPPI model which will lead to underperformance, in state 4 this results in outperformance.

If the four different market states would manifest with equal probabilities, then the volatility-based model would perform equal to the original model. However, there is evidence which suggests that stock market volatility is higher during recessions than during expansions, exhibiting a pronounced business cycle behavior (see e.g., Officer, 1973; Schwert, 1989; Hamilton and Lin, 1996; Brandt and Kang, 2004; Mele, 2007). This means there is a negative relation between equity market returns and volatility. If this is considered with the above market states, it means that states 1 and 4 are more likely to occur compared to state 2 and 3. Based on this information, the volatility model should outperform the original CPPI model.

To empirically assess the increase in return and decrease in drawdown of the continuous volatility strategy compared to the original CPPI strategy in terms of equity allocation while considering the risky asset returns. Figure 4 shows the equity allocation of the original CPPI strategy, volatility strategy[22] and the risky asset value[23] over the longest available time period[24].

[22] CPPI with multiplier 5, continuous volatility strategy with risk factor 0,75, both with floor percentage 0,85
[23] The risky asset value is scaled by dividing by 140
[24] The specific time period can be found in Paragraph 4.2

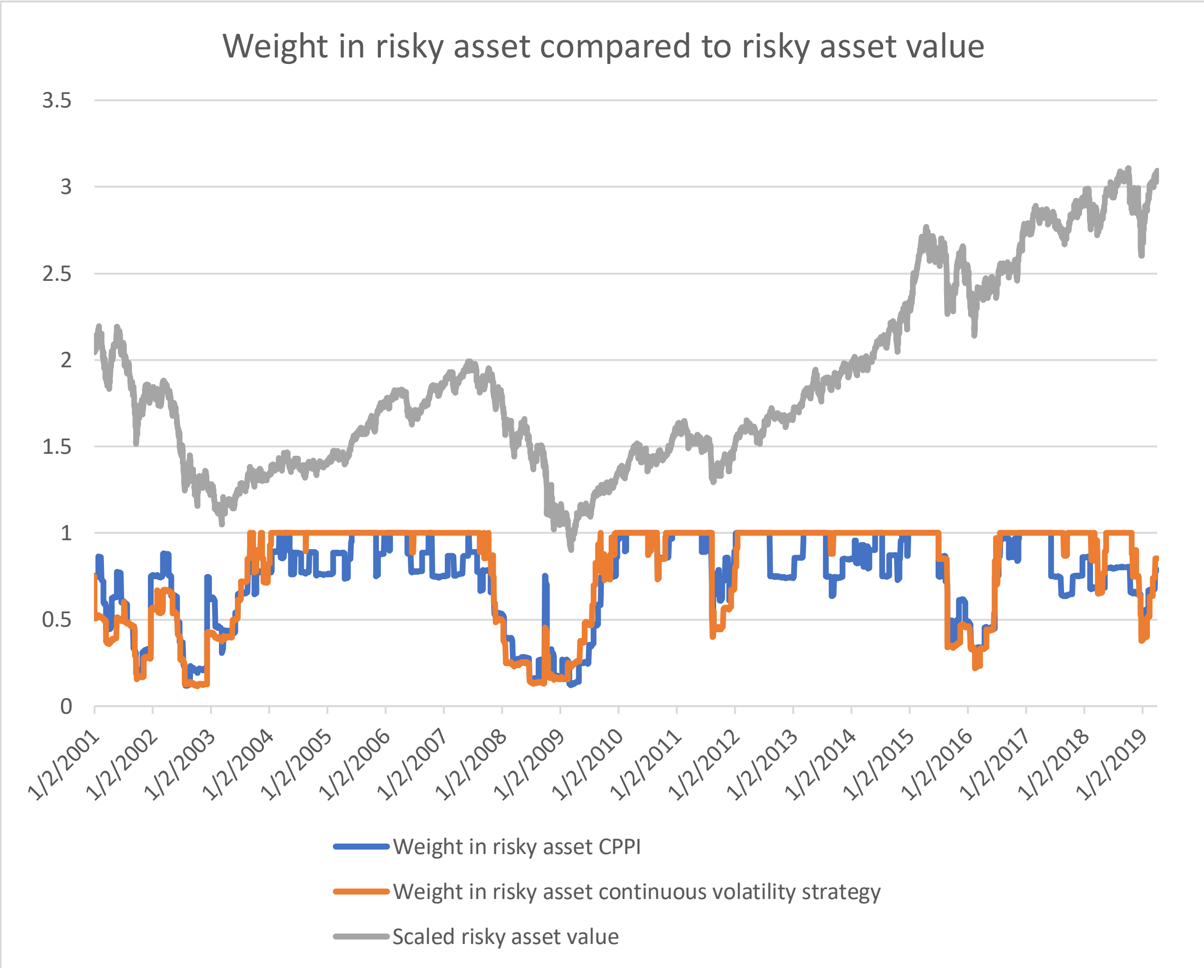

Figure 4: equity allocation of CPPI and continuous volatility-based strategy with floor value 0,85 (initial) multiplier 5 and risk factor 0,75 compared with scaled risky asset value

The first thing that can be distilled from the graph above, is that in stable upward trending markets, the volatility-based strategy has a higher weight in equity, and therefore outperforms the original strategy in this scenario. Examples hereof are the periods 2-1-2003 until 2-1-2007 and 2-6-2012 until 2-4-2015. This is in line with the idea of the volatility-based strategy, where the multiplier high (max 7) in stable markets.

On the other hand, it was expected that the CPPI would outperform the volatility-based strategy in the scenario of stable downward trending markets. However, when considering the historical data, no such market conditions existed in the past 20 years. In the cases of downward trending markets, volatility increased and therefore the multiplier adjusted accordingly. The volatility-based strategy seems to respond quicker and the weight in risky assets seems to be consistently lower in downward trending markets than the CPPI strategy. When looking at the downward trend of 2-1-2001 until 2-1-2003, the equity allocation in risky asset of the volatility-based strategy is consistently lower than the CPPI strategy. However, when transitioning from a stable upward trending market into the financial crisis of 2008, the volatility-based strategy responds slightly slower than the original CPPI strategy. This is due to the lag in volatility calculations using the EMWA approach. In general, downward trending market and financial crises create more volatile markets, therefore, the multiplier should be relatively lower during these periods. Combined with the losses and the lower multiplier the equity allocation shifted from risky to riskless asset.

When considering downward shocks such as around 2-9-2012 and 2-4-2013, it can be concluded that in relatively smaller shocks in stable upward trending markets, the CPPI responds more heavily by shifting weight to the riskless asset, which is not optimal. When the shocks are relatively bigger, the volatility-based strategy responds more heavily.

5.3. Practical implications

There are other aspects besides the KPIs that influence the choice for most optimal strategy, such as the general idea behind the comfort product. Comfort offers clients the ease of mind by focusing on capital preservation. In this light, using a strategy which has a trend component might be less optimal. Given that the continuous volatility strategy is a more focused on risk control than the momentum strategy. Therefore, the idea behind the continuous volatility strategy is in line with the goal of Comfort.

This section will provide additional results for the continuous volatility strategy. First, the strategy will be further tested on the effect of the risk factor on the strategy by allowing it to range between 0.5 and 0.75 with steps of 0.05 (note that in the previous analysis the risk factor was allowed to range between 0,5 to 1,5 with steps 0,25). The reason for this new range will become apparent later in this section. Moreover, the KPIs of the strategy for the different floor values will be evaluated.

Table 9 shows the aggregated results of the continuous volatility strategy for different risk factors. The results show that lowering the risk factor leads to an increase in Sharpe, Sortino and omega and a decrease in return. Furthermore, the average multiplier and the weight in equity decreases. The results show that as the risk factor is reduced, less risk is taken. Based on this analysis is it possible to select a dynamic volatility strategy which has similar average multiplier and weight in equity characteristics as the original CPPI strategy, and therefore takes the same sort of risk, whilst the strategy can be an improvement for the other KPIs. Matching the multiplier and weight in equity characteristics to the original CPPI strategy also has another benefit. Currently Binck uses the multiplier to construct the benchmark. If a risk factor that resembles the CPPI multiplier and weight in equity characteristics is selected, it is possible to leave the current benchmarks unaltered. Based on the results, the risk factor which does precisely that and scores better on the KPIs than the original CPPI strategy equals 0,6. However, there are some compromises made on return and transaction costs. The risk related KPIs risk adjusted return, drawdown, Sortino ratio and omega ratio, however, have better results when using a risk factor of 0,6 compared to 0,75, which is logical since the multiplier is systematically lower with a risk factor of 0,6 compared to 0,75. Table 9 also shows that the client would compromise on return and turnover for the ease of implementation of Binck. This is a tradeoff between risk, return and practical implications. However, since the comfort strategy is a risk averse strategy, slight decreases in return for less risk is allowed, moreover, changing the benchmark too often is not satisfactory for the client and Binck as described in Section 3.3.

Table 10 shows the results for the individual floor for both the CPPI strategy and the continuous volatility strategy with a risk factor of 0.6. The results show that continuous volatility performs better than the original CPPI strategy in terms of (risk adjusted) return, Omega ratio and Sortino ratio. For the two most defensive profiles the turnover and the average weight in equity slightly increases. However, the increase in return outweighs the increase in transaction costs for these floors, therefore, it can be said that the continuous volatility strategy with risk factor 0,6 outperforms the original CPPI strategy on every floor percentage, and therefore for every risk profile.

Table 9: KPIs for the different risk factors of the continuous volatility strategy

Riskfactor	Average annualized return	Median annualized return	Average yearly turnover	average yearly rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Weight in risky asset	Average multiplier	Omega	Sortino
0.5	5.7%	5.8%	2.03	8.32	1.01	-23%	0	0.66	4.18	11.50	0.67
0.55	5.9%	5.9%	2.03	8.31	0.99	-23%	0	0.70	4.58	10.89	0.66
0.6	6.1%	6.2%	2.04	8.31	0.97	-24%	0	0.73	4.95	9.80	0.64
0.65	6.2%	6.1%	1.98	8.05	0.95	-25%	0	0.75	5.27	9.02	0.63
0.7	6.3%	6.2%	1.94	7.81	0.94	-26%	0	0.77	5.55	8.29	0.63
0.75	6.3%	6.4%	1.93	7.67	0.94	-27%	0	0.78	5.77	7.86	0.62

Table 10: The KPIs for both the original CPPI strategy and the continuous volatility strategy with a risk factor of 0,6

Model	Initial multiplier	Risk factor	Floor	Average annualized return	Median annualized return	Average yearly turnover	average yearly rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Weight in risky assety	Average multiplier	Omega	Sortino
CPPI	5		0.75	5.2%	5.5%	1.20	4.22	0.62	-46%	0	0.92	5	1.56	0.40
CPPI	5		0.8	5.6%	5.9%	1.74	6.70	0.70	-39%	0	0.88	5	2.50	0.46
CPPI	5		0.85	5.6%	6.3%	2.39	9.64	0.81	-30%	0	0.77	5	4.24	0.53
CPPI	5		0.9	5.2%	5.3%	2.19	8.35	1.01	-21%	0	0.57	5	8.99	0.66
CPPI	5		0.95	4.8%	4.9%	1.28	4.65	1.56	-12%	0	0.33	5	57.64	1.01
Cont vola		0.6	0.75	6.1%	6.1%	0.93	3.60	0.77	-35%	0	0.88	4.95	2.84	0.51
Cont vola		0.6	0.8	6.4%	6.4%	1.31	5.35	0.86	-30%	0	0.83	4.95	4.38	0.57

Cont vola	0.6	0.85	6.3%	6.4%	2.10	8.79	0.95	-24%	0	0.75	4.95	7.00	0.64
Cont vola	0.6	0.9	5.6%	5.8%	2.88	11.62	1.11	-17%	0	0.59	4.95	14.07	0.72
Cont vola	0.6	0.95	5.1%	5.2%	1.96	7.39	1.63	-10%	0	0.35	4.95	103.57	1.03

5.4. Gap risk

For the gap risk analysis, Table 11 shows the number of floor hits in the gap risk analysis. In this analysis the model is run with two day returns instead of daily returns. This way the shocks in the returns are bigger, representing the worst-case real situation. The results show that the floor is not breached, and the worst two-day loss of the assessed period was ~9,3%.

Table 11: number of floor hits of preferred strategy

Model	Risk Factor	Floor hits
Cont Vola	0.6	0
Cont Vola	0,75	0

5.5. Validity

As a validity check the models were tested on the S&P 500 with the same period. These results can be found in Tables 12 and 13. The results are similar to that of the original data. The conclusions of the original investigation do not change. The continuous volatility strategy is still outperforming the other strategies. However, there are some differences between the original analysis and the validity check.

First, the returns of the validity check are higher than then the returns of the original back-test. The Stoxx 1800 and S&P 500 had an average annualized return of 3% and 5% respectively. Moreover, the average annualized return is similar of the risk-free asset for both runs[25]. Therefore, the difference in returns of the strategies between the two runs is to be expected.

The second difference is the skewness of the return distributions. The original analysis showed a higher median than return for every strategy, indicating the distribution of the returns is left-skewed. However, the validity analysis shows the opposite. In the results of the validity analysis the median is consistently lower than the returns, indicating the returns distribution of the strategies are right-skewed. Therefore, no conclusion can be drawn on the skewness of the return distributions.

When looking at the weight in equity and average multiplier, there is a slight difference between the original results and the results of the validity analysis. Whereas in the original results the risk factor needed to be fitted to meet the benchmark requirements, in the validity analysis the requirements are met with a risk factor of 0,75. Different markets have different volatilities, therefore, the difference is to be expected. However, on one hand the results show the risks of overfitting. On the other hand, since the original weight in equity is systematically lower and therefore less risky, in this case this it has no significant implications. The validity test shows that Binck needs to make a separate analysis for the risk factor for the markets the comfort strategy is active in.

The relative results are similar to for the original back-test and the validity test. The main difference is the omega ratio. The omega ratio of the validity run is significantly higher than original. This means in the validity run there were more positive returns than in the original run for every strategy. This could be due to the higher returns of the S&P 500 compared with the Stoxx1800 as described earlier.

[25] The risk-free asset had for both the original and validity analysis a 4% average annualized return

Table 12: KPI results of best model per class of validity test

Model	Initial multiplier	Risk factor	High daily return	Return period	Average yearly return	median	Average yearly turnover	average daily rebalance frequency	Altered sharpe ratio	Maximum Drawdown	Floor hits	Weight in risky asset	Average m	Omega	Sortino
CPPI	5				6,0%	4,4%	2,22	8,53	0,69	-29%	0	0,74	5,00	9,68	0,50
Trend	5	4		21	6,3%	4,0%	1,99	7,80	0,68	-28%	0	0,74	5,26	10,67	0,49
Disc Vola	3				6,4%	5,1%	5,56	17,82	0,71	-28%	0	0,77	5,38	15,78	0,51
Cont Vola	5	0,75			6,7%	5,7%	2,06	8,37	0,77	-23%	0	0,73	5,05	27,98	0,56
Cont Vola	5	1			6,9%	5,9%	2,04	7,93	0,75	-26%	0	0,79	6,01	16,37	0,54
Cont Combi (1)	4	4		21	6,7%	5,8%	2,07	8,57	0,75	-24%	0	0,74	5,34	23,72	0,54
Cont Combi (1)	5	4		21	6,7%	5,6%	2,12	8,71	0,74	-24%	0	0,75	5,47	20,59	0,53
Disc. Combi	5			21	6,2%	4,1%	2,22	8,75	0,68	-29%	0	0,75	5,17	10,39	0,49
Cont Combi (2)	5	2	0,01	21	6,2%	4,4%	2,33	8,98	0,67	-31%	0	0,77	5,67	8,30	0,48

Table 13: relative KPI results of best model per class of validity test

Model	Initial multiplier	Risk factor	High daily return	Return period	Average yearly return	Average yearly turnover	average daily rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Omega	Sortino	Median
CPPI	5				6,0%	2,22	8,53	0,69	-29,3%	0	9,68	0,50	4,4%
Trend	5	4		21	4%	-10%	-9%	-1%	-3%	0%	10%	-1%	-11%
Disc Vola	3				6%	151%	109%	3%	-4%	0%	63%	3%	15%
Cont Vola	5	0,75			11%	-7%	-2%	12%	-22%	0%	189%	12%	29%
Cont Vola		1			15%	-8%	-7%	8%	-12%	0%	69%	8%	32%
Cont Combi (1)	4	4		21	12%	-6%	0%	9%	-17%	0%	145%	8%	32%
Cont Combi (1)	5	4		21	11%	-4%	2%	7%	-17%	0%	113%	6%	27%
Disc. Combi	5			21	2%	0%	3%	-2%	-2%	0%	7%	-1%	-8%
Cont Combi (2)	5	2	0,01	21	3%	5%	5%	-4%	7%	0%	-14%	-3%	-1%

6. Conclusions/recommendation

In this research 6 momentum and/or risk control based DPPI strategies are back tested and compared with each other and the CPPI strategy. The strategies were back tested using a rolling analysis. The rolling period was 5 years and the data used for the rolling analysis was Stoxx 1800 index as risky asset and a combination of 40% Bloomberg Barclays Euro Corporate Bonds 1-5 years and 60% Bloomberg Barclays Euro Aggregate Treasury 3-5 Year as risk-free asset. Data from 3-1-2000 to 1-4-2019 is used. Thereafter, the best performing strategy was evaluated over the complete period with a floor percentage of 85%. The KPIs to evaluate the strategies were based on (gap) risk, return and transactions costs. For the validity check the historical data selected as risky asset is the S&P 500 index, and for riskless asset the Barclays US Treasury Index is used. The data used is from 3-1-2000 until 6-6-2019, resulting in the same number of trading days as the original data. For all data the adjusted close price is used.

It has been found that the continuous volatility and continuous momentum combination strategy outperform the original CPPI strategy and the other tested DPPI strategies. The continuous volatility strategy performs 16,4%, 11,6%, 5%, 3%, 7,1%, 13%, 32%, 12% better than the CPPI in return, median, turnover, rebalance frequency, risk adjusted return, maximum drawdown, modified omega ratio and Sortino ratio respectively. Whereas the momentum based continuous combination strategy performs better than the CPPI strategy considering every KPI except transaction frequency. Table 14 shows the key takeaways from the rolling analysis. Moreover, the volatility-based strategy is more of a risk control strategy where the combination strategy can be seen as a momentum strategy. Therefore, we have argued that the combination strategy is a better fit for the Comfort strategy of Binck.

Table 14: Rolling analysis takeaways

Model	Comparisons
Trend based (1)	Performs best in terms of yearly turnover and transaction frequency compared to all other models (1,87 and 7,37 respectively).
	Performs similar to the CPPI strategy considering all other KPIs.
Discrete volatility based (2)	Performs significantly worse than the CPPI strategy in terms of turnover and transaction costs (an increase of 76,8% and 55% respectively compared to the CPPI strategy).
	Performs 5,6% better in terms of return than the CPPI strategy.
	Performs similar considering the other KPIs compared to the CPPI strategy.
Continuous volatility based (3)	Outperforms the CPPI strategy considering all KPIs.
	performs best considering (median of) returns and Sortino ratio compared to all other strategies.
	Is the best fit for the comfort strategy.
Continuous momentum-based combination (4)	Outperforms the original strategy considering every KPI except for rebalance frequency. Transaction

	frequency increases with 7% compared to the CPPI strategy
	Performs best considering modified Omega ratio, with a value of 10,26, compared to all other strategies
Discrete crisis-based combination (5)	Performs similar to the CPPI strategy
Continuous crisis-based combination (6)	Performs similar to the CPPI strategy

This strategy has been further compared to the original CPPI strategy in terms of equity allocation and seems to outperform the original CPPI strategy in the more common market conditions. Moreover, in terms of crisis situations and drawdown, the volatility strategy seems to outperform the CPPI strategy as well.

When considering the practical implications, in the original back-test average multiplier value and average equity allocations of the continuous volatility strategy are comparable to the original CPPI strategy when the risk factor is 0,6. However the validity test implicates that the risk factor is market specific since every market has a different volatility.

The strategies are back tested with two-day returns, to model the effects of information delays. When back-tested with two-day returns, no floor values were breached for all strategies.

The validity test showed the results in a market with higher risky asset returns. The results were comparable except for the median and omega ratio. The difference in omega can be explained when considering the higher returns. The difference in median shows that we cannot be sure about the skewness of returns of the different strategies. The relative results for both back-tests were comparable, therefore it can be assumed that the findings were valid.

In conclusion a volatility-based strategy with the multiplier: $m_t = a * \frac{1}{volatility}$ is advised. Binck should scale the risk factor in such a way that the weight in equity per floor is similar to the current strategy and the average multiplier is similar to the current multiplier for easy implementation.

7. Discussion

Although the results seem valid, some compromises were made during this research. The following Paragraph will discuss the methodology choices and compare the historical performances to near future expectations.

7.1. Limited data and runtime restrictions

The data used for this research only goes back to the year 2000. Bloomberg has no data prior to 2000 on the Stoxx 1800 index and Barclay bonds used for this research. The decision to use these indices as benchmark was because it was important for scaling to use markets that are relevant for Binck. However, a longer back-test period with more market scenarios was preferred.

Due to runtime restriction of VBA the rolling analysis was computed with quarterly increments instead of daily. This resulted in 40 different paths for the rolling period. This might compromise on the statistical significance of the findings.

7.2. Risk-free rate

The average annualized bond return of the complete back-test period, the period before 2008 and the last 5 years were 3,7; 4,2; 1,8 respectively. The historical risk-free rate is much higher than it is now, and it is not sure if they will increase is the near future. Therefore, the back-test results might differ compared to near future performances.

7.3. Effects of rolling analysis

The back-test is a rolling 5-year analysis beginning with a starting date of 3-1-2001 and ending with a starting date of 02-06-2014. Every quarter (defined as 88 days) a new back-test of the strategy is executed. In total this results in 40 KPI samples per strategy. Figure 5 shows the risky asset over the evaluated period. Figure 6 shows the total number of recurrences of the returns in the rolling analysis (e.g. the returns of the first 88 trading days are included only once).

Based on Figure 5 it is possible to distinguish three different phases of return recurrence. The first phase starts on 3-1-2001 and ends on 22-9-2005. The second phase is from 23-9-2005 until 7-4-2014 and the last period starts on 8-4-2014 and ends on 28-12-2018. From this Figure it becomes clear that the first phase and the third phase have less of an (standalone) impact on the results than phase 2.

Figure 5 shows us that in phase 1 the risky asset was first in a recession and eventually showed a recovery. In phase 2, the most period with the most impact, the market starts with an upward trend followed by the financial crisis of 2008 which results in another recession. The period ends with a recovery. Phase 3 starts with a small downward trend, followed by a recovery and continued upward trend.

The different phases all contain a recession and recovery and/or upward trending market state. The size of the recessions and following upward trends differ per phase, however, the business cycle characteristics are similar for all the phases. This indicated that the effects of the rolling property do not have negative effects on the end results.

Figure 6 shows some fluctuation in return recurrences in the second phase. This is due to the fact that a quarterly step was used instead of daily steps. When using daily steps, every event window includes and removes one trading day. However, when using steps of 88 days and a 5-year period, some days of the quarter are included in 1 more event window than other days. To be precise: in phase 2, the first 29 returns of a quarter (88-day period) are used for one more event window than the last 59 days of that quarter. This is due to the overlap of the end dates of previous 5-year periods and the start dates of recent 5-year periods.

Figure 5: risky asset value

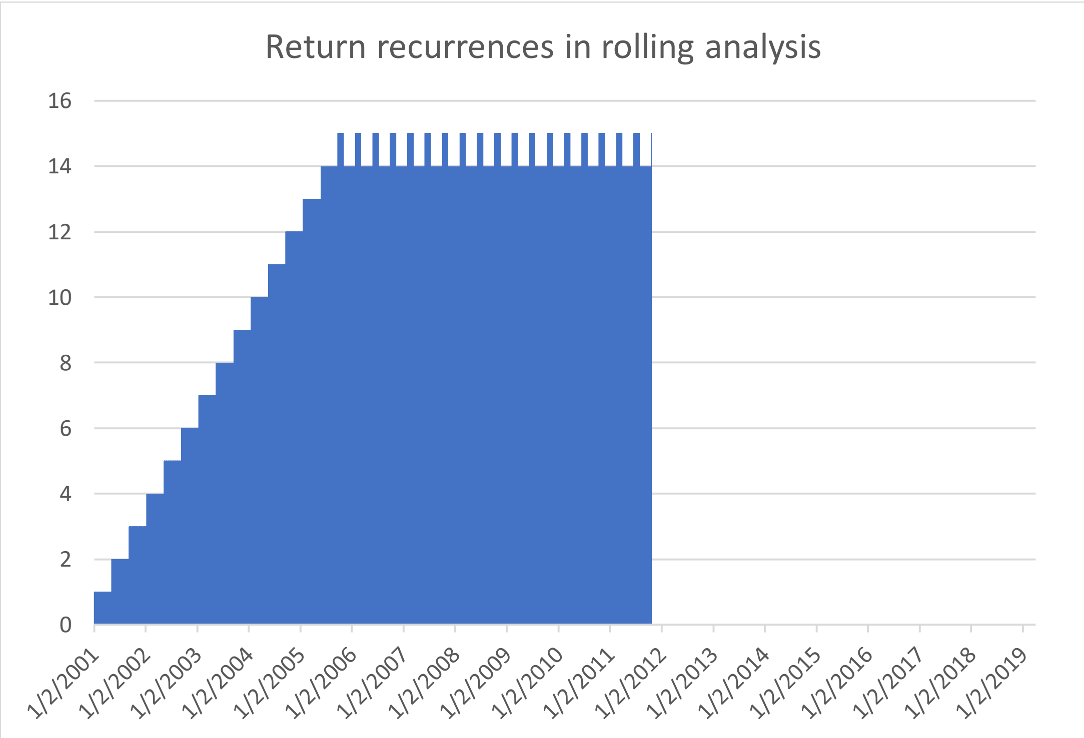

Figure 6: return recurrences in rolling analysis

8. Further research

The current research assessed six different models based on trend, volatility and combinations and compared them with each other and the CPPI method. There were some input parameters out of range such as multiplier range and rebalance weight. Moreover, is the EMWA approach used to calculate the volatility. The scope was dynamic multipliers that depend on past returns and volatility, however there are other methods of computing dynamic multipliers such as machine learning algorithms. At last, does Binck have multiple portfolio investment strategy for different types of clients (risk averse etc.). It might be interesting to see if these strategies can be combined to reduce the workload and improve performances.

8.1. Multiplier range and rebalancing weights

Changing the multiplier range (currently 2-7) and the rebalancing weights (currently +-0,1) was out of scope for this research. However, changing the multiplier ranges can have significant impact on KPIs such as drawdown, return and weight in equity (and therefore the practical implications). Moreover, different strategy might outperform the volatility-based strategy with different multiplier ranges. Changing rebalancing weights influences transaction costs. There might be weights that historically would perform better in terms of turnover and transaction frequency without significantly compromising on the other KPIs.

8.2. Volatility estimation approach

For this research the different input variables for the EMWA approach were tested and evaluated. However, there might be different volatility estimates that give different or better results (e.g. GARCH). Therefore, it is advised to test the different strategies with other volatility estimators.

8.3. Combining Binck Bank strategies

Binck has a goal-based portfolio strategy named “forward”. Chen and Liao (2007) show that it is possible to compute a piecewise nonlinear goal directed CPPI strategy that might outperform the CPPI strategy. This strategy is a combination of the floor protection of the CPPI and the goal-based characteristics of the forward strategy. Integrating the strategies might lead to a lower workload and better results, therefore, an additional research on combining the strategies is recommended.

8.4. Machine learning approaches

The current research only focusses on return and volatility-based indicators for the dynamic multiplier. Out of scope were the machine learning approaches that compute (dynamic) multiplier. For example, Deghanpour & Esfahanipour, 2018 shows a model that computes a dynamic multiplier using genetic programming.

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Appendix A – implications of different moving average periods

Different number of days included in the moving average (EMWA approach) for volatility calculation are evaluated. The following formula is used to calculate the volatility:

$$\sigma_t = \sqrt{\frac{(1-\lambda)}{(1-\lambda^n)}\sum_{J=0}^{n-1}\lambda^j\left(r_{t-j}-u\right)^2}$$

$\sigma_t = volatility\ at\ time\ t,\qquad \lambda = \text{decay factor},\qquad r_t = return\ at\ time\ t,$
$u = expected\ return,$
$n = amount\ of\ days\ included\ in\ moving\ average$

When looking at the formula, generally when using a longer moving average period, volatility is expected to be more stable. However, at some point increasing the moving average period, will not results in significant changes in volatility values, and therefore, only result in unnecessary computational effort. Moreover, when using a longer moving average period, changes in volatility are captured later compared to shorter periods.

Table 15 shows the implications of changing the moving average period considering the KPIs. Seven different input periods are used. The results start stabilizing after a moving average period of 128 days. However, longer periods result in lower turnover and a slightly lower weight in equity and risk adjusted returns, whereas shorter periods perform worse or equal on every KPI except for maximum drawdown. Higher turnover for a shorter period is to be expected since the volatility values, and therefore multiplier values, are more receptive to small changes in market conditions. The differences in drawdown are not significant, but also to be expected. The results clearly show that a period of 21 days, is too unstable, whereas a period longer than 128 days slightly compromises in regards of return, Omega-and-Sortino ratio. When returning to figure 2 it shows that a relatively short period results in unstable volatility values, whereas 128 and 504 days both show a stable volatility progression through the time period.

Note that the results with 128 days included in the moving average differ from the original results of Table 7. This is because originally the first year of data was used to calculate the initial volatility. In this analysis however, the longest period included in the moving average was 2 years (504 days). Therefore, the initial volatility fitting period has to be at least two years. This results in a different rolling period (with 37 runs instead of 40), and therefore, in different results.

Table 15: KPI values of continuous volatility strategy with risk factor 0,6 over different moving average periods

Days included in moving average	Average yearly return	Average yearly turnover	average yearly rebalance frequency	Risk adjusted return	Maximum Drawdown	Floor hits	Weight in risky asset	Average m	Omega	Sortino	Median
21	5,9%	3,68	9,71	0,94	-27%	0	0,75	5,18	7,82	0,62	6,3%
42	6,4%	2,49	8,18	1,00	-25%	0	0,74	5,12	8,82	0,66	6,8%
84	6,4%	2,18	7,70	1,01	-24%	0	0,74	5,04	9,39	0,67	6,7%
128	6,4%	2,07	7,45	1,01	-25%	0	0,74	5,00	9,64	0,67	6,8%
252	6,3%	2,00	7,34	1,01	-25%	0	0,73	4,96	8,84	0,67	6,8%

378	6,3%	2,00	7,83	1,01	-25%	0	0,73	4,95	8,76	0,66	6,7%
504	6,3%	2,00	8,80	1,01	-25%	0	0,73	4,94	8,81	0,66	6,7%