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MASTER THESIS

A dynamic life cycle analysis for a Defined Contribution pension plan

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Management summary

Pension is an important part of our social security. Pension funds are under enormous pressure and pension cuts seem unavoidable without political interference. PME and PMT have already announced that cuts in 2020 are very likely. The current Defined Benefit pension system is under review and the market shifts towards a Defined Contribution pension plan, with more emphasis on individualisation. Such pension plans are still relatively new and therefore more research is needed in this area. This shift increases the importance of life cycle investing. A life cycle investment strategy attempts to determine the most appropriate asset mix for Defined Contribution pension plan participants to balance their risk and return profiles based on the number of years the participants have until retirement. We have found no substantiation for the fact that the current life cycle performs well under the current economic conditions and leads to an optimal pension benefit. We have contributed to the literature because we have compared existing life cycles with optimised linear and dynamic life cycles, while in the current literature one of them is usually taken. The design of a dynamic life cycle has not yet been evaluated in the Dutch pension context. This explains the relevance of this research and the answer on the following main research question:

How should the life cycle be designed for a Defined Contribution pension plan?

In order to answer this question we have developed a method to analyse Defined Contribution life cycles. We started with capital calculations to gain insight into the capital development during the working period of a participant. This is used to calculate the ratio between the accumulated capital and the discounted value of the expected pension benefits, called the coverage ratio. Note that this coverage ratio is not the same as the definition used in a Defined Benefit pension system. The coverage ratio serves as an input for the constant relative risk aversion utility function. The utility is used to compute the certainty equivalents to compare the different life cycle designs. In addition to the assessment framework, we have built a simulation model to model the interest rate and equity returns. We have used the dynamic Nelson-Siegel model in combination with a vector autoregression model to simulate

these interest rates and equity returns. The addition of a Markov regime switching component to the simulation model is of added value because it provided insight in how dynamic life cycles can be designed depending on the state of the economy.

We have showed with the analysis of the traditional, reverse, and constant life cycles that the currently most used life cycle, the traditional life cycle, should not be seen as a guarantee for the optimal pension result. Which life cycle is preferred depends on the risk aversion of a participant. The reverse, constant, and traditional life cycles are preferred for the low, medium, and high risk aversion perception respectively. This finding suggests that determining the risk aversion of a participant is of great importance.

In the second part of the life cycle analysis we have performed the optimisation with the goal to maximise the average utility by changing the life cycle. First, this is done for linear life cycles. We have found that the optimised linear life cycles result in higher utility values than the three existing life cycles. The shapes of these life cycles are completely different than the existing life cycles. In case a participant has a low risk aversion most capital is allocated to the return portfolio with a slight decrease over time. For the medium risk aversion profile the life cycle starts with an allocation of around zero percent to the return portfolio and increases to almost forty percent to the return portfolio at the end. In case a participant has a high risk aversion then the return portfolio allocation starts at zero percent and increases to almost twenty percent. These results, together with the sensitivity analysis, have showed that the linear life cycle design is highly dependent on the risk aversion coefficient, especially for the low risk aversion profile. This research goes beyond a linear life cycle which is only a function of age. The dynamic life cycle does not necessarily have to be a linear function and is state dependent in order to incorporate the market conditions in deciding the return portfolio allocation. It appeared that adding these two elements to a life cycle result in higher utilities and more certainty, in terms of coverage ratio, compared to a linear life cycle.

Altogether, we have showed that the most used life cycle, the traditional life cycle, is outperformed by other linear and dynamic life cycles. This implies that when a pension fund offers, or wants to offer, a Defined Contribution pension scheme, it should not be taken for granted that the traditional life cycle should be used. In fact, our research have showed that using a dynamic life cycle, which is non-linear and dependent on the market conditions, adds value to the concept of life cycle investing. This research can be seen as one of the contributions to highlight the added value of a dynamic life cycle. More research is needed to identify the implications and risks of using a dynamic life cycle in a Defined Contribution pension plan.

Contents

List of acronyms

AOW	State guaranteed pension
CE	Certainty Equivalence
CR	Coverage Ratio
CRRA	Constant Relative Risk Aversion
CVAR	Conditional Value At Risk
DB	Defined Benefit
DC	Defined Contribution
DNB	The Dutch Bank
DNS	Dynamic Nelson-Siegel model
ECB	European Central Bank
EPB	Expected Pension Benefit
EURIBOR	Euro Interbank Offered Rate
FV	Future Value
GDP	Gross Domestic Product
HICP	Harmonised Index of Consumer Prices
LC	Life Cycle
MSCI	Morgan Stanley Capital International
NS	Nelson-Siegel
OLS	Ordinary Least Squares
PME	Pension fund MetalElektro

PMT Pension fund Metaal & Techniek

PV Present Value

RQ Research Question

SBK Strategic Investment Framework

VAR Vector Auto Regression

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Chapter 1

Introduction

In essence, a pension contract is a straightforward product. A premium is paid during working years in exchange for a pension benefit after retirement. A pension fund invests all the collected premiums and strives to get a good investment return while taking the risk into account. However, this long-term process is accompanied by uncertainties and difficulties to design a solid and future-proof pension system. The Dutch pension system is seen as one of the best in the world but is nevertheless currently a subject of the political debate. The current collective pension system is under review and the market shifts more towards an individualised pension plan. Because such pension plans are still relatively new, there is the need to do research into an individualised pension plan.

Currently, the market shifts from a current collective pension (Defined Benefit) plan towards a more individual pension (Defined Contribution) plan. In a Defined Contribution (DC) pension plan the investment strategy is commonly referred to as a life cycle. A life cycle investment strategy attempts to determine the most appropriate asset mix for DC plan participants to balance their risk and return profiles based on the number of years the participants have until retirement. In the traditional life cycle, which is the most used life cycle in a DC pension plan, more capital is allocated to the return portfolio in the beginning of the accumulation phase. This return portfolio is then linearly substituted for a more matching-like portfolio as the retirement date approaches. But is this actually the optimal life cycle? Does this life cycle result in the optimal pension, given the current market conditions and low interest rate environment? The increasing importance of DC plans and their life cycles justify this research.

This opening chapter focusses on the framework surrounding this research. First of all, in Section 1.1 we give a short introduction about the organisation. Subsequently, some background information is provided about the Dutch pension landscape. In Section 1.3 we cover the problem analysis which leads to a problem statement. Section 1.4 and 1.5 are devoted to the research questions and research design respectively. Thereafter, in Section 1.6 we briefly address the scope of this research. Finally, we give a rough thesis outline in Section 1.7.

1.1 Company profile

MN is the fiduciary manager for the Dutch manufacturing industry and the maritime sector. In terms of asset under management they are the third largest in the Netherlands and the largest in the sector. MN manages €135 billion in assets for more than 2 million people and are committed to their future income (MN, n.d.). The client list of MN includes Pension fund Metaal & Techniek (PMT), Pension fund MetalElektro (PME) and Koopvaardij. MN is a large company with close to a thousand employees and several business units.

This research fits to the business unit portfolio management under the Investment Strategy (Dutch: Strategisch Beleggingsbeleid) of the fiduciary advice department. The core task of fiduciary advice is to empower the members of the pension fund board making the right decisions in the field of portfolio management through good research which is then translated into policy advice and product development. This department is also responsible for writing investment strategies and mandates for implementation.

A pension fund board is responsible for the investment policy and makes use of supporting parties for advice and implementation. Central to the MN approach is a modern and effective investment framework. In 2015 this investment framework has been formalised into the ”Strategisch Beleggingskader” (SBK). In the framework the objective of the pension fund has been stated and how it wants to achieve the goal. A unique aspect of this approach is that MN prepares the SBK for all clients in a document in close consultation with the board. It forms the basis for the role as a fiduciary manager.

One of the ambitions of the fiduciary advice business unit is to be able to respond to the changing pension environment. This ambition serves as the perfect starting point for research about how the investment policy can be improved given the changes in the pension system. In the following section we elaborate on the problem that MN is facing.

1.2 Dutch pension landscape

After seven years the Dutch pension system is again the number one pension system in the world according to the Global Pension Index 2018 (Mercer, 2018). Since 2009 Mercer compares the quality of pension systems over thirty countries worldwide and is based on three basic elements: adequacy, future-proofing, and integrity. This ranking indicates that the Dutch pension system is doing really well.

The three pillars

The Dutch pension system consists of three pillars. The first pillar concerns the state-guaranteed pension (AOW) which was introduced in 1957. It is the basic income and everyone who lives or works in the Netherlands will receive AOW as soon as the legal retirement age has been reached. The first pillar is financed through a pay-as-you-go system. This means that the working population pays the cost of the AOW of the current pensioners. The second pillar is a collective pension system organised around a specific industry/company. This is funded by the premiums that people have invested in the past, plus the return on it. If the employer has such a supplementary pension scheme, retired employees will receive an additional benefit on top of the AOW. The third pillar concerns the individual pension products. In particular, employees in sectors without a pension scheme and self-employed make use of this pillar. Our research focusses on the second pillar of the Dutch pension system.

First pillar	State pension	Pay-as-you-go
Second pillar	Occupational pension	Funded
Third pillar	Individual pension	Funded

Table 1.1: Overview Dutch pension system.

Defined Benefit and Defined Contribution

In the current situation the second pillar of the Dutch pension industry offers different kinds of pension contracts. The majority of the sector in the Netherlands uses the Defined Benefit (DB) pension system. With a DB pension system, the pension payment is guaranteed. A partial entitlement of the pension benefit is accrued for each active year of service, which is based on a percentage of the average salary. The pension fund then sets the investment policy that is suitable to fulfil the guaranteed pension pay-outs. This policy is applied collectively and is the same for all participants. Several tools are available for the pension board to adjust the financial position of the fund. These tools include the premiums paid by the active members (premium policy) and the inflation indexation that applies to all participants (indexation policy), with a negative indexation as the ultimate variant. The viability of a DB pension system is questionable and is one of the reasons why the current pension system is under discussion.

A Defined Contribution (DC) pension plan is a bit more straightforward. In a DC, the employee and employer contributions are invested on behalf of the employee (Hull, 2015). However, in a DC it is not known in advance what the exact pension benefit will be after retirement. Of course, this depends on the amount contributed but also on the growth of the capital throughout the years. Once the employee retires, the capital can be converted to a lifetime annuity. Because the pension benefit highly depends on the returns it increases the importance of the investments. This is where the life cycle comes into play. This is an investment policy which is often based on a function of the age of the (active) participant.

The majority of the sector in the Netherlands uses the DB pension system. However, numerous alternatives are the subject of discussion in politics and the pension industry. One of the alternatives is a personal pension account with a collective buffer. This alternative can be seen as a hybrid between a DB and DC.

An important difference between DB and DC is the fact that the latter is a more individual kind of pension system. This can be seen as an individual employee account where the pension benefit is calculated based only on the funds on that account. This is in contrast with a DB pension system where there are no such individual accounts. The contributions are pooled and invested and the pension benefits are paid from the pooled capital. Another big difference is the way the risks are borne. With a DC plan the risk is fully carried by the employee because the pension benefit directly correlates with the total amount of capital of the individual fund. However, an advantage of a DC system is the flexibility. It can be adjusted based on personal characteristics which is not possible with a DB pension system.

1.3 Problem analysis

Although the Dutch pension system is recognised as one of the best internationally, some shortcomings have become increasingly visible in recent years. These are particularly related to the overarching themes such as the transparency of and the trust in the system which are under pressure mainly due to the current market conditions. The historically low interest rate is a good example for the changed circumstances compared to the past. Does the current (DB) pension system still work under these circumstances?

Because of the increasing social and political pressure, MN is interested in the question whether a hybrid pension system, which incorporates good elements of both the DC and DB systems, can meet the objectives. Right now the scope of this question is still quite broad and therefore it is necessary to narrow it down. Based on the preferences of MN, our research zooms in on the life cycle of the DC system. Questioning the current life cycle is enhanced by literature pointing out that more research needs to be done to improve the current life cycle. There are already some studies that evaluated life cycle designs of a DC pension plan, but they are somewhat outdated and therefore not representative for the currently low interest rate environment, for example the study of Blake et al. (2001). However, these papers did not investigate a dynamic life cycle with the same definition as in this study. Next to that, Basu et al. (2011) stated that the LC can be counterproductive when moving away from stocks to low-return assets just when the size of the contributions are growing larger. They tested a dynamic life cycle where it is only allowed to switch between stocks and bonds during the last ten or twenty years. So, our study differs from their research in the definition of the dynamic life cycle. Another article stated that there is room for added value for the one-size-fits-all LC to incorporate classes of investors characteristics such as risk attitude and income (EDHEC-Risk Institute, 2011). In addition to that, the current LC does not incorporate investment results that are very dependent on market behaviour (Arnott et al., 2013). This means that there is no feedback or performance check built in the investment strategy which can have influence on the remaining part of the LC. Poterba et al. (2006) found that the distribution of retirement wealth associated with typical life cycle investment strategies is similar to that from an age-invariant asset allocation strategy. They stated that it might be useful to compare the optimal life cycles with the existing ones. This literature study shows that there is indeed room for improvement and could be seen as an invitation to join the research about the life cycle.

Is it possible to add elements to the life cycle, which is now often a function of age, so that there will be feedback between the investment policy and the desired end goal? Is it also possible to include multiple decision factors such as the current level, remaining life cycle, desired risk, etc.? An interesting question may also be how the investment policy will be adjusted if a participant has accumulated capital that is above the final goal (for example 70% of the average salary in accordance with the current DB ambition) while the participant is only 54 years old. Then you could, for example, take less risk in the remaining time. Figure 1.1 gives an overview of the problems related to the life cycle.

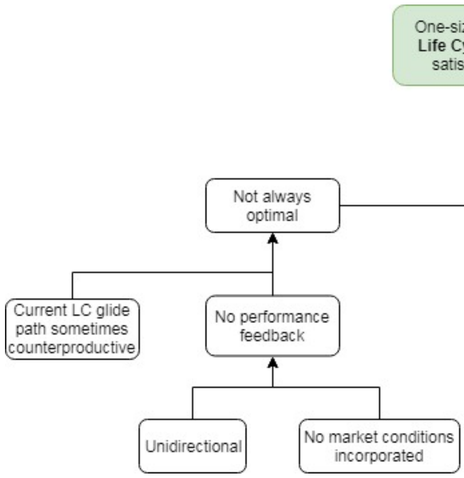

Figure 1.1: Problem cluster.

1.4 Research questions

At the time of writing this report, the Dutch pension system is still under review. As the third biggest fiduciary manager in the Netherlands it is important for MN to build a sustainable pension plan, especially given the changing pension landscape.

One of the aspects being discussed in the review is the shift from a Defined Benefit pension plan towards a Defined Contribution pension plan. This trend stems from the more flexible labour market, longevity and ageing population. In addition to that, low interest rate environments have put more pressure to the coverage ratio (CR) of the majority of Defined Benefit schemes in the Netherlands. With the sustainability of the current DB plan being questioned and the trust in the system diminishing, there is an increasing pressure to look into another system such as Defined Contribution plans.

The shift from DB plans to DC plans presents a new challenge for fiduciary managers such as MN to advise the pension board how to best manage the retirement assets. In addition to the policy regarding employees contribution, the investment strategy/asset allocation decisions play an important role in determining the pension outcome. The increasing importance of DC plans and its life cycle justifies the need for this research.

The main objective of our research is to investigate the optimal investment strategy/life cycle in a DC plan. The investment strategy of a DC plan (life cycle) involves allocating the accumulated wealth/assets to equity-like assets (return portfolio) and bond-like assets (matching portfolio) according to a certain fixed glide path. Currently, this glide path is a function of the participant's age. To achieve the research objective, we have formulated several research questions. These serve as the basis, guideline and structure for this thesis. We define the main research question as follows:

How should the life cycle be designed for a Defined Contribution pension plan?

An important aspect here is the question when the life cycle is optimal. On one hand, it is clear that a higher pension benefit is better than a lower pension benefit. On the other hand, more certainty in terms of the pension payment is better than less certainty. The problem is that these two outcomes are often substitutes: a higher benefit is usually accompanied by more uncertainty. Therefore, it is necessary to look at the trade-off between risk and return to see which life cycle offers the best result. The main research question is broken down into two research questions. These form collectively an answer to the main research question.

Research question 1
Which life cycle design offers the best risk-return trade-off given a certain risk-aversion level of the participant and stochastic interest rates and equity returns?

In this step of the research we introduce a set of scenarios for interest rates and equity returns. The traditional life cycle follows a fixed glide path in which more is allocated to the return portfolio in the beginning of the accumulation path. This return portfolio is then substituted for a more matching-like portfolio as the retirement date approaches. Most of the current DC plans adopt this traditional life cycle as opposed to a reverse glide path and a constant mix. Although several plans vary slightly in terms of rates at which the return portfolio is reduced according to the risk aversion level of the participant, the implementation of a reverse glide path and a constant mix life cycle is minor. Each life cycle produces a wealth distribution at the retirement date. The expected pension payout can then be calculated from this result. The traditional life cycle is the most used strategy. Therefore, in an environment where interest rates and equity returns are stochastic, we expect that a glide path with a decreasing return portfolio over the accumulation phase yields superior pension benefit at the given retirement date in comparison with a reverse glide path and a constant allocation. No distinction is made between the risk aversion of the participants and therefore we expect that the life cycle preference is independent from the risk aversion level of the participant.

Research question 2
What is the impact of adjusting the return versus matching portfolio based on the target pension benefit throughout the working life of the participant?

In this next step of the analysis we propose a new method. Instead of looking at the age of the participant (or the time to retirement) as an anchor point for determining the asset mix, the proposed life cycle defines the asset mix based on the extent to which the target pension benefit is achieved. At each point in time (for example each year) the value of the pension contribution is evaluated against the value of accruals at retirement. As the ratio of the contribution to accruals is higher (i.e. the pension contribution matches the pension payout) a bigger portion of pension benefit is secured by allocating more to the matching portfolio. This is done irrespective of the age of the participant at that point. Based on this idea, we expect that a dynamic life cycle, in which the allocation between return and matching portfolio is managed against the target pension benefit throughout the accumulation phase, generates a better pension result than the traditional life cycle in which the allocation is defined only based on the age of the participant.

Both research questions are related to the design of a life cycle for a DC plan and contribute in their own way to answering the main research question. In the next section we elaborate on the research design.

1.5 Research design

The research design serves as a roadmap for the entire process to achieve the research objective. Here, we break down the research questions into smaller steps. We conduct the analysis through an iterative process, each time with small adjustments. In this way the impact of the incremental changes in the analysis can be isolated. In this research we explain and clearly state all assumptions for replication purpose. The main methods and data sources we use are literature studies, the MN pension database, extern data portals such as Bloomberg and the MN employees. Knowledge about the pension industry is gathered through discussions and conversations with MN specialists. In the following paragraphs we elaborate on the different steps that we take to answer the research questions.

The focus of the first step is to understand how to assess the trade-off between risk and return. There are couple of requirements to be able to assess the different life cycles. These are related to the capital calculations, utility function, and certainty equivalent. First, we discuss the capital calculations because pension contributions are made during the participant's working years in exchange for a pension benefit after retirement. Therefore, it is fundamental to do the capital calculations to see how the total accumulated capital will change over time. Next to that, we use the Constant Relative Risk Aversion (CRRA) utility function which is one of the most commonly used utility functions in the pension industry (Yue, 2014) and is often used as an evaluation measure in the literate on dynamic asset allocation. The capital calculations, the CRRA utility function, and the certainty equivalents form the basis for the analysis of life cycle design. In order to conduct this analysis, data from the MN database is used as input. This includes the mortality table, career path percentages, and pension contribution table.

The goal of the second step is to be able to test the life cycle designs under different economic circumstances. We need to generate stochastic interest rates and equity returns to create a more realistic view of the performance of the life cycles. We do some literature study to gain knowledge about different models such as Vasicek, Nelson-Siegel (NS), Heath-Jarrow-Morton framework (HJM), Geometric Brownian Motion (GBM) and Markov regime. Given the scope of this research, we keep the scenario generation relatively simple. This also reduces the dependency on large amount of data. We gather the input data for scenario generation, for example data about swap rates and indices, using Bloomberg.

We use the results of the previous two steps in the third step to analyse three existing unidirectional life cycles. Once we have modelled the capital calculations, utility function, interest rates and equity returns, we test different life cycles to answer Research Question 1. These are referred as the constant, traditional and reverse life cycles, which we discuss in more detail in Chapter 4.

In the last step in this research we perform the dynamic life cycle optimisation. While in step three the design of the life cycle is constant and defined at the beginning of the accumulation phase, in this step we adjust the life cycle design along the way based on the projected pension payout. So, what is the impact of this continuous adjustment of the life cycle design to the pension benefit at retirement date? The effect of using a dynamic asset allocation approach (dynamic glide path) to incorporate elements such as current level, remaining life cycle, and desired risk is investigated. However, the scope of this research is limited to determining the optimal allocation to the matching and return portfolio. The purpose of the matching portfolio is to hedge away interest rate risks as effectively as possible. The interest rate risk is the risk that the value of these pension liabilities rises faster than the value of the total assets. The goal of the return portfolio is to generate returns above the interest rate based on an optimal risk-return trade-off. We do not investigate which specific financial products should be used to get the corresponding asset allocation mix.

To summarise, we execute several steps to be able to answer the main research question. These steps together serve as a roadmap for this research. This can be represented in the following flowchart:

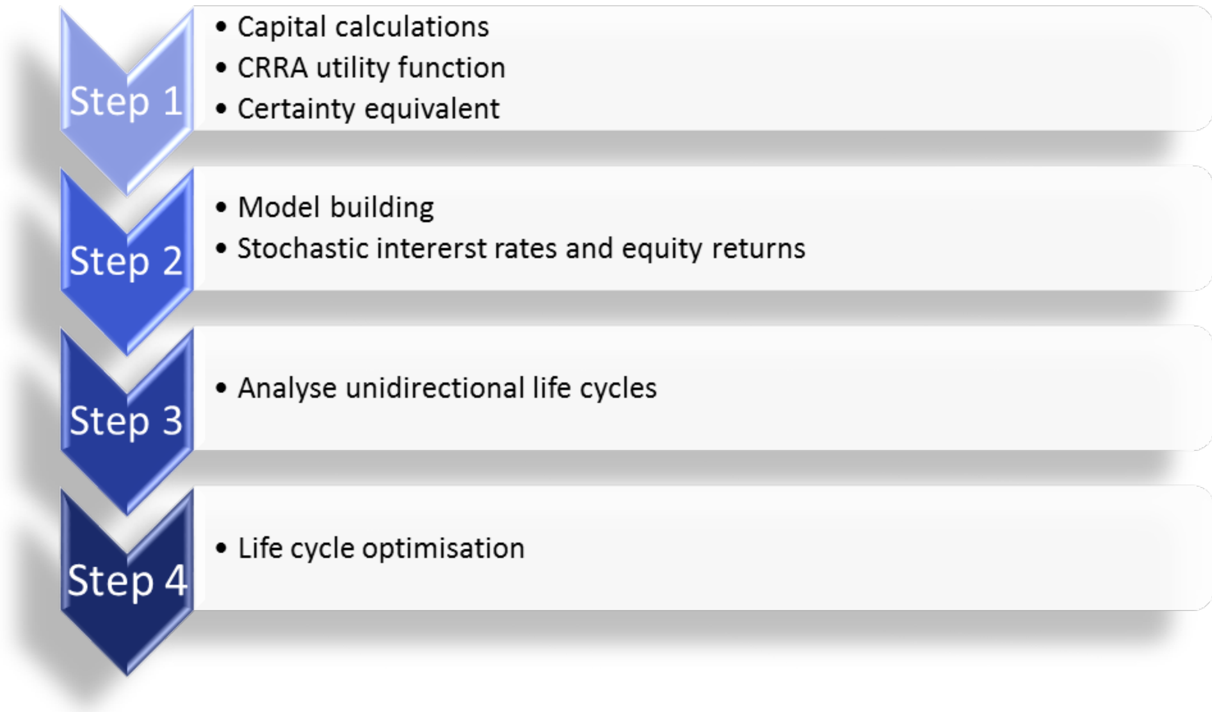

Figure 1.2: Research design.

1.6 Scope

Naturally, in every research it is important to determine the scope. First of all, our research is based on the Dutch pension system and hence not always applicable to pension systems from other countries. If someone wants to reproduce this research, one should think carefully about the underlying pension system which could have different assumptions and legal requirements. In addition, our research will not go into detail about the personal characteristics. This means that personal life events, which could have an impact on the pension, will not be incorporated. Another decision is related to the retirement age. It is impossible to predict what kind of regulatory changes will happen in the future. Therefore, we use the retirement age based on the current regulation. Also, our research does not take into account the changes in demographics. Finally, we do not take taxes, transactions costs, and leverage into account. All this decreases the complexity and narrows down the scope of this research.

1.7 Report outline

From the previous sections, the thesis structure can be derived. In Chapter 2 we discuss the framework to assess the trade-off between risk and return. The assessment framework is built on the basis of two features, the capital calculations and the utility function. We explain the interest rate and equity return model step by step in Chapter 3. This is done to be able to assess the different life cycles with stochastic interest rates and equity returns. In Chapter 4 we test three existing unidirectional life cycle designs to get a first impression of their performance. In addition, we answer Research Question 1. In Chapter 5 we elaborate on the dynamic life cycle design to be able to answer Research Question 2. We use all findings to answer the main research question in Chapter 6.

Chapter 2

Assessment framework

As described in the first chapter our research consists of two research questions with the main objective to see whether a dynamic life cycle outperforms the current unidirectional life cycle. In order to answer the research questions we discuss the mechanism to capture risk-return trade-off first. The goal of this chapter is to come up with a risk-return measure to evaluate each of the life cycle design and to quantify the effect of changing the asset allocation mix. First, it is necessary to understand how the pension payout is calculated and what the pension contributions should be that the participant has to pay to achieve this pension payout objective throughout the accumulation phase. This is referred to as the capital calculations. In this calculation the target pension payout at retirement date is set. We calculate the present value (PV) of the life-long benefit entitlements that are accrued by discounting the annual payout with the interest rates. The annual pension contribution (premium) is then calculated in such a way that at the target retirement date the future value (FV) of these premiums matches the present value of the annual pension payout. The same interest rate structure that is used to discount the pension payouts is used to calculate the future value of the contributions. The next step is related to the utility function. The risk aversion coefficient has to be determined in order to assess the life cycle using the utility function. Finally, the utility serves as an input in the certainty equivalent (CE) calculation. We discuss all steps in more detail in the following sections.

2.1 Capital calculations

The participants pay pension contribution (premium) during their working years in exchange for a pension benefit after retirement. Currently, the retirement age is part of the political discussion, so there is some uncertainty about what the retirement age will be in the future. For the purpose of this research we have assumed that the pension age is fixed. We have used the following inputs/assumptions in the capital calculations:

- The starting age for the wealth accumulation period is at the beginning of 25.
- The retirement age is at the beginning of 67.
- The last evaluated age is 103.
- The starting annual salary is €27,000.
- The franchise is €15,304.
- The payouts are at the beginning of a year (primo).
- The inflation correction is 0.5% per year.
- The career path is given as input.
- The mortality table is given as input.
- The pension accrual is 1.875% per year.
- The spot interest rate as of 29-3-2019 is used.
- Simulated interest rate and equity returns are used.
- The expected inflation term structure as of end March 2019 is used.
- No life events, like divorces or promotions, are incorporated.
- Partner pension is not taken into account.

First, the capital calculation is applied to a deterministic scenario based on the spot interest rate as of end March 2019. We discuss the use of stochastic scenarios in the next chapter. In each scenario the interest rates and equity returns are defined. Figure 2.1 gives an overview of the annual capital growth based on the previously mentioned inputs. In the paragraphs below the figure we explain all calculation steps. The specific formulas can be found in Appendix A.

Age	Salaris	Franchise	Pensioen Grondslag	Premie	Pensioen Uitkering	Rendement aandelen	Rendement rente	Rendement EUR	Kapitaal ultimo
	€ 27.000,00	€ 15.304,00							
25	€ 29.678,85	€ 15.496,25	€ 14.182,60	€ 2.346,65	€ -	€ 0,04	€ 0,01	€ 23,81	€ 2.370,46
26	€ 32.118,59	€ 15.722,48	€ 16.396,11	€ 2.740,42	€ -	€ 0,04	€ 0,01	€ 51,90	€ 5.162,77
27	€ 34.324,24	€ 15.962,42	€ 18.361,82	€ 3.100,13	€ -	€ 0,04	€ 0,01	€ 83,64	€ 8.346,54
28	€ 36.075,60	€ 16.214,96	€ 19.860,64	€ 3.387,12	€ -	€ 0,04	€ 0,01	€ 119,00	€ 11.852,66
29	€ 37.859,00	€ 16.485,78	€ 21.373,22	€ 3.682,05	€ -	€ 0,04	€ 0,01	€ 157,45	€ 15.692,16
30	€ 39.653,85	€ 16.768,71	€ 22.885,14	€ 3.982,48	€ -	€ 0,04	€ 0,01	€ 199,33	€ 19.873,97
31	€ 41.432,61	€ 17.067,23	€ 24.365,37	€ 4.283,02	€ -	€ 0,04	€ 0,01	€ 245,22	€ 24.402,21
32	€ 43.181,03	€ 17.380,30	€ 25.800,73	€ 4.581,37	€ -	€ 0,04	€ 0,01	€ 294,54	€ 29.278,12
33	€ 44.997,12	€ 17.713,73	€ 27.283,39	€ 4.893,88	€ -	€ 0,04	€ 0,01	€ 347,31	€ 34.519,31
34	€ 46.885,92	€ 18.069,53	€ 28.816,40	€ 5.221,39	€ -	€ 0,04	€ 0,01	€ 404,58	€ 40.145,29
35	€ 48.590,13	€ 18.440,16	€ 30.149,97	€ 5.518,65	€ -	€ 0,04	€ 0,01	€ 466,41	€ 46.130,35
36	€ 50.083,61	€ 18.826,62	€ 31.256,99	€ 5.779,71	€ -	€ 0,04	€ 0,01	€ 531,66	€ 52.441,72
37	€ 51.740,72	€ 19.229,46	€ 32.511,25	€ 6.073,21	€ -	€ 0,04	€ 0,01	€ 609,84	€ 59.124,76
38	€ 53.574,29	€ 19.649,35	€ 33.924,93	€ 6.403,33	€ -	€ 0,04	€ 0,01	€ 696,07	€ 66.224,17
39	€ 55.357,89	€ 20.084,66	€ 35.273,23	€ 6.728,55	€ -	€ 0,04	€ 0,01	€ 791,29	€ 73.744,00
40	€ 57.084,28	€ 20.536,82	€ 36.547,46	€ 7.047,23	€ -	€ 0,04	€ 0,01	€ 893,14	€ 81.684,37
41	€ 58.682,36	€ 21.008,00	€ 37.674,36	€ 7.344,83	€ -	€ 0,04	€ 0,01	€ 1.003,88	€ 90.033,08
42	€ 60.524,55	€ 21.496,83	€ 39.027,72	€ 7.694,47	€ -	€ 0,04	€ 0,01	€ 1.137,91	€ 98.865,47
43	€ 62.431,71	€ 22.001,37	€ 40.430,33	€ 8.063,81	€ -	€ 0,04	€ 0,01	€ 1.284,06	€ 108.213,35
44	€ 64.214,35	€ 22.519,03	€ 41.695,32	€ 8.415,98	€ -	€ 0,04	€ 0,01	€ 1.444,45	€ 118.073,77
45	€ 65.887,23	€ 23.049,11	€ 42.838,12	€ 8.753,73	€ -	€ 0,04	€ 0,01	€ 1.617,26	€ 128.444,77
46	€ 67.443,34	€ 23.593,48	€ 43.849,86	€ 9.074,74	€ -	€ 0,04	€ 0,01	€ 1.806,59	€ 139.326,10
47	€ 69.043,17	€ 24.153,14	€ 44.890,03	€ 9.412,04	€ -	€ 0,04	€ 0,01	€ 2.037,81	€ 150.775,94
48	€ 70.690,58	€ 24.729,45	€ 45.961,13	€ 9.768,64	€ -	€ 0,04	€ 0,01	€ 2.277,90	€ 162.822,48
49	€ 72.390,50	€ 25.324,13	€ 47.066,37	€ 10.145,48	€ -	€ 0,04	€ 0,01	€ 2.543,80	€ 175.511,77
50	€ 74.159,21	€ 25.942,87	€ 48.216,34	€ 10.546,22	€ -	€ 0,04	€ 0,02	€ 2.833,76	€ 188.891,74
51	€ 75.997,92	€ 26.586,10	€ 49.411,82	€ 10.972,30	€ -	€ 0,04	€ 0,02	€ 3.162,35	€ 203.026,39
52	€ 77.885,92	€ 27.246,57	€ 50.639,35	€ 11.422,80	€ -	€ 0,04	€ 0,02	€ 3.527,28	€ 217.976,47
53	€ 79.796,75	€ 27.915,03	€ 51.881,72	€ 11.895,53	€ -	€ 0,04	€ 0,02	€ 3.934,32	€ 233.806,32
54	€ 81.697,65	€ 28.580,02	€ 53.117,64	€ 12.387,35	€ -	€ 0,04	€ 0,02	€ 4.368,05	€ 250.561,72
55	€ 83.601,86	€ 29.246,16	€ 54.355,70	€ 12.900,97	€ -	€ 0,04	€ 0,02	€ 4.842,11	€ 268.304,79
56	€ 85.548,00	€ 29.926,97	€ 55.621,03	€ 13.443,90	€ -	€ 0,04	€ 0,02	€ 5.329,81	€ 287.078,50
57	€ 87.536,46	€ 30.622,59	€ 56.913,88	€ 14.016,61	€ -	€ 0,04	€ 0,02	€ 5.878,09	€ 306.973,20
58	€ 89.567,75	€ 31.333,19	€ 58.234,57	€ 14.621,84	€ -	€ 0,04	€ 0,02	€ 6.446,48	€ 328.041,52
59	€ 91.642,47	€ 32.058,98	€ 59.583,49	€ 15.260,42	€ -	€ 0,04	€ 0,02	€ 6.979,89	€ 350.281,83
60	€ 93.761,32	€ 32.800,21	€ 60.961,11	€ 15.930,68	€ -	€ 0,04	€ 0,02	€ 7.597,22	€ 373.809,73
61	€ 95.925,15	€ 33.557,17	€ 62.367,97	€ 16.636,44	€ -	€ 0,04	€ 0,02	€ 8.183,09	€ 398.629,26
62	€ 98.134,95	€ 34.330,22	€ 63.804,73	€ 17.376,39	€ -	€ 0,04	€ 0,02	€ 8.805,30	€ 424.810,94
63	€ 100.391,86	€ 35.119,75	€ 65.272,11	€ 18.152,26	€ -	€ 0,04	€ 0,02	€ 9.468,04	€ 452.431,24
64	€ 102.697,21	€ 35.926,22	€ 66.770,99	€ 18.965,99	€ -	€ 0,04	€ 0,02	€ 10.159,72	€ 481.556,95
65	€ 105.052,07	€ 36.750,01	€ 68.302,05	€ 19.819,02	€ -	€ 0,04	€ 0,02	€ 10.929,89	€ 512.305,86
66	€ 107.457,43	€ 37.591,47	€ 69.865,96	€ 20.714,75	€ -	€ 0,04	€ 0,02	€ 11.826,12	€ 544.846,73
67	€ -	€ 37.779,43	€ -	€ -	€ -33.707,86	€ -	€ -	€ -	€ 511.138,88

Figure 2.1: Capital calculations.

The first column is the age of the participant. We have calculated the capital on an annual basis starting with the year when the participant is 25 years old until he/she reaches the age of 103. We have chosen this lifetime range to accommodate the mortality table input; at the age of 103 the life expectancy is close to zero. The pension benefit is paid out at the beginning of the year. The retirement age kicks in when the participant reaches 67 (beginning of the year).

The second column contains the yearly salary information. Recall from the assumptions that the starting salary is €27,000. At any year that the participant is younger than the retirement age (67) he/she earns a salary. The amount of salary changes over time depending on the inflation and the career path. We have assumed that the salary grows with the inflation. The career path shows the percentages with which the salary grows compared to the previous year as a result of a participant's career.

The next column is the franchise computation. This is the part of the salary on which no pension is accrued and therefore no pension contribution (premium) is paid. It also depends on the starting value and is also effected by the inflation.

Next, the pension base (Dutch: pensioengrondslag) is calculated by deducting the franchise from the salary. The pension base is the part of the salary on which pension is accrued and therefore pension contribution is paid.

The next step is to calculate the pension contribution (premium) by multiplying the pension base with the accrual percentage. As you can see in Figure 2.1, premium is only paid when the participant is 66 years old or younger. In addition to the returns on investment, the inflow of premiums is an important source of capital. The premium policy is one of the management tools that a pension board can use if necessary. In our research, however, we have not used the premium as a steering tool. We have used a fixed contribution table, which can be found in Appendix B. The contribution percentages are based on the accrual percentage of 1.875% per year and the spot interest rate of March 31, 2019. In other words, this pension contribution must be paid to build up 1.875% pension per year, assuming all capital is allocated to the matching portfolio. We have found that this results in a pension annuity of €33,707.68 and is used as the pension ambition in our research. This stylised framework offers a clearer insight into the influence of the life cycle designs on the pension result.

In the sixth column the pension benefit is calculated. Note that the participant only receives the benefit when he/she is 67 years or older. The pension benefit depends on the total accumulated capital so far and on the annuity factor (Dutch: koopsom). In this research an annuity factor is an one-time payment to buy a pension entitlement (Pensioen.com, n.d.). The annuity factor represents the amount of money that is now needed to be able to buy an annual of one euro pension entitlement which is distributed from retirement age until the participant dies.

The next column contains information about the annual returns (also called EUR return). The return depends on the accumulated capital so far and the way the capital is invested. The investment policy is defined based on the chosen life cycle. A life cycle is used to determine how much capital is invested in the matching and how much in the return portfolio. In our research the matching portfolio consist of interest rate investments and the return portfolio consist of equities. Therefore, the total annual returns depend on the interest rates and the equity returns.

The last column is the final step to calculate the total accumulated capital. This is called the capital ultimo. It depends on the accumulated capital up to the previous year, the contribution in the current year, the pension benefit paid out this year (if any), and the total annual return on investment. We have used this capital to calculate the pension benefit because it incorporates everything such as premium, pension benefit, and returns from the investment portfolio.

Annuity factor

As mentioned before, we have used the annuity factor to determine the pension benefit based on the accumulated capital. The necessary inputs to calculate the annuity factors are age, mortality rate, and spot interest rates. A mortality table shows, for every age, what the probability is that a person of that age will die. The mortality table can be found in Appendix C. By using the mortality table, it is possible to calculate the conditional probability of survival at a certain age. The probability that someone is still alive up until a certain age, multiplied by one minus the probability that a person will die at that age, gives the conditional probability of survival. In other words, it is the probability that a participant will survive given that the participant survived up to now. We have done this for every year, ranging from 25 until 103 years old, assuming that the participant is now 25 years old. These probabilities are needed to generate the expected pension payout table with two dimensions, the current age of the participant and the time horizon. Given a certain age the expected pension payout is calculated. This is done by multiplying the €1 pension entitlement per year (annuity) with the probability that the participant is still alive each year. So, for example given that a person is now 60 years old (*t = 35*) and the horizon is 7 years (*h = 7*), what is the expect pension payout? We have done this for every age and horizon to generate the expected pension payout table. The expected pension benefit (EPB), dependent on the current age and horizon, is given by:

$$\mathrm{EPB_{t,h}} = \begin{cases} €1 \times \frac{\mathrm{P_{t+h}}}{\mathrm{P_t}} , & \mathrm{t+h} \geq 67 \\ 0 , & \mathrm{t+h} < 67 \end{cases} , \tag{2.1}$$

where *P* is the probability of survival at a current age $t \in \{25, 26, ..., 103\}$ and horizon $h \in \{0, Z^{+}\}$.

Once the expected pension benefit table is generated it should be discounted. We have used the spot interest rate to discount the expected pension benefit. The formula to calculate the annuity factor *A* is given by:

$$A_t = EPB_{t,0} + \sum_{h=1}^{T} \frac{EPB_{t,h}}{(1 + r_h)^h} \,, \tag{2.2}$$

where r_h is the spot interest rate with a horizon *h*.

Now that these annuity factors are calculated we can use them to compute the pension benefit given the accumulated capital so far. This step serves as an input for the capital calculations.

Discounting cash flows

As mentioned in the previous section, the payments of the pension contribution and the pension benefit take place at different moments in time. This means that we need to accrue the contribution payments and to discount the pension benefits to be able to fairly compare the available money and liabilities at the retirement age. This gives an indication about the solvency of the pension fund at the time the participant retires. Figure 2.2 and Figure 2.3 give a schematic overview how the different cash flows are accrued and discounted.

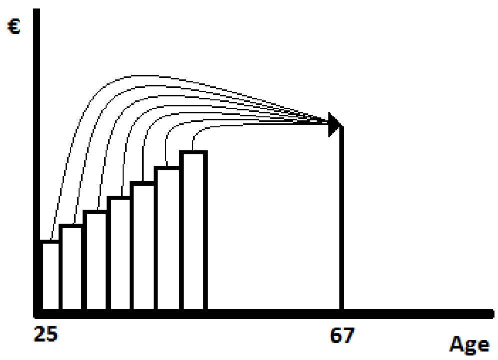

Figure 2.2: Accruing the premiums.

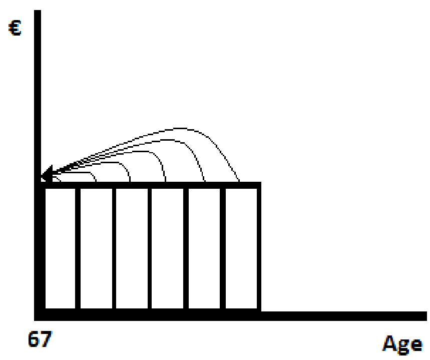

Figure 2.3: Discounting the benefits.

The premiums are paid annually and therefore the number of years until retirement decreases annually. This means that the premium paid by a participant at the age of 25 is invested for a period of 42 years while the premium paid at the age of 26 is invested for a period of 41 years and so on.

The discount rate is a critical input parameter for the outcome of the present value and future value calculations. In the stochastic analysis we have used the simulated yield curves to accrue the pension contributions and to discount the pension benefits. In the deterministic setting, we have used the forward rates and have been derived from the term structure of the spot interest rate. We have used the equation below to calculate the forward rates using the spot interest rate.

$$F_{a+h} = \left(\frac{(1 + r_{a+h})^{a+h}}{(1 + r_h)^h} \right)^{\frac{1}{a}} - 1 \ , \tag{2.3}$$

where *r* is the spot interest rate, *a* is the time to maturity (in years), and *h* is the horizon (in years).

This forward rate can be interpreted as the spot interest rate *h* years into the future with a time to maturity *a*. Using this formula, we have filled a two dimensional forward rates matrix with the dimensions *time to maturity* and *horizon*. The time to maturity, as its name already suggest, is the number of years until the investment is settled. The horizon, on the other hand, can be seen as moving the settlement date further into the future. So, for example when *h = 43* and *a = 1*, it means that the investment is settled at the corresponding forward rate for a period of one year at the beginning of age 68. This is because the payments are made at the beginning of the year. As already mentioned before, the capital calculations start at the beginning of age 25.

Once the forward rates have been calculated, they are used to accrue the premiums and to discount the pension benefits as showed in Figure 2.2 and Figure 2.3. For every payment the time to maturity and horizon will be determined to see which forward rate is applicable.

After accruing the investment portfolio and discounting the benefits, it is possible to see what the total future value of the invested premiums and the present value of the pension benefits is at retirement age. This gives an indication about the coverage ratio. To recall, the coverage ratio is the relationship between the current available capital and the future pension obligations. Note that the term coverage ratio is used in our research loosely and does not correspond to the definition of coverage ratio used in the DB system. The coverage ratio says something about the relationship between the premiums and the expected pension benefit. We have used this ratio as a solvency measure and serves as an input in the utility function. In the following section we explain the relationship between the utility function and the coverage ratio in more detail. To compute the coverage ratio we have used the following equation:

$$x_t = \frac{I_t}{B_t} \ , \tag{2.4}$$

where *x* is the coverage ratio, *I* is the value of the investment portfolio, and *B* is the present value of the benefits at time *t*.

The investment portfolio in the formula above can be seen as the future value of the invested premiums (see the last column of Figure 2.1). The life cycle represents the percentage allocated to the return and matching portfolio and thus has an influence on the investment portfolio. In addition, we have incorporated the probability of survival in the present value calculation of the benefits because there is a chance that the participant dies earlier. The pension benefits are first multiplied by the probability of survival before discounting. It is the probability of survival given that the participant has reached the retirement age. This means that the pension benefit received at the beginning of 67 is multiplied by one because the probability of survival given that the participant reached the retirement age is one. The pension benefit at the beginning of 68 is multiplied by 0.9857 (mortality rate is 1.43% at the age of 67) because that is the probability the participant will receive the pension benefit. This is done up to the age of 103.

2.2 Measures

In economics, the utility function measures the welfare or satisfaction of a consumer as a function of consumption (Investopedia, 2018). In this case, the consumption is in terms of CR because it serves as a solvency indication of their pension. As we have already stated in the introduction chapter, we have used the Constant Relative Risk Aversion (CRRA) utility function, which is according to Yue (2014) one of the most commonly used utility functions in the pension industry. As the name already somewhat implies, risk aversion is the concept of human behaviour of disliking uncertainty. To give an example, if a player gets two options, a guaranteed payment of €50 or a 50% chance on €100 and 50% chance on €0, a highly risk-averse player will choose the guaranteed payment while the expected payouts are both the same. A risk neutral player would be indifferent between the two options. Someone's risk aversion is incorporated in terms of a risk aversion coefficient in the utility function. The CRRA utility function is defined as follows:

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} , \tag{2.5}$$

where x is the coverage ratio and γ is the risk aversion coefficient.

The risk aversion coefficient is an indicator of how much a person wants to avoid risk. Not everyone is the same and therefore it is obvious that the risk aversion coefficient varies per person. However, it is out of scope of our research to investigate what the real risk aversion coefficient is for everyone and to compute the utility function on an individual basis. Instead, we have created three profiles with different risk aversion coefficients, based on the paper of EDHEC (2014). We have used the following risk aversion profiles:

- Low risk aversion (offensive), $\gamma = 2$.
- Medium risk aversion (neutral), $\gamma = 5$.
- High risk aversion (defensive), $\gamma = 10$.

The utility values for every risk profile can be compared in order to determine the life cycle preferences. Interpreting one single utility is difficult because what does an utility of minus one, for example, mean? To get more feeling about the results we have used the certainty equivalent measure. This transforms a distribution of uncertain outcomes into a single value with probability one that has the same utility. It can be interpreted as a guaranteed CR that someone would accept rather than taking a chance on a higher, but uncertain, CR in the future. After determining the expected utilities it is straightforward to determine the certainty equivalent. Ranking alternatives by certainty equivalents is the same as ranking them by their expected utilities. Rewriting Formula 2.5 results into the following certainty equivalent equation:

$$C = \left(E\left(U\right) \times \left(1 - \gamma\right)\right)^{\frac{1}{1-\gamma}} \quad , \tag{2.6}$$

where *E(U)* is the expected utility and $\gamma \in \{2, 5, 10\}$ is the risk aversion coefficient.

2.3 Conclusion

The capital calculations are indispensable because the pension benefit is not known in advance in a DC pension plan. Therefore, we have started with the capital calculations to get some insight in how the capital will develop over time and what the pension benefit will be at retirement. In order to keep the connection with the most used pension system, the DB pension plan, we have used a pension accrual percentage of 1.875% per year to construct the contribution table for the DC pension plan. Together with the other stated assumptions, the CR is 100% when all the capital is allocated to the matching portfolio in the deterministic scenario. This results in a pension benefit of €33,707.68. We have assumed that this is the lifetime annuity regardless of the fact that the asset allocation mix will change later in our research.

In addition, we have used the utility function to assess the trade-off between risk and return and to be able to compare the different life cycles fairly. We discuss the three existing unidirectional life cycles, which are tested later on in this research, in more detail in Section 4.1. They differ in terms of riskiness. Riskier life cycles might result in a higher pension benefit but at the same time have a greater chance on a terrible pension entitlement. In case a participant has already a good pension prospect then it might not be worth it to take extra risk to get an even better pension benefit. Besides that, not everyone is willing to take the same risk. Determining the risk aversion of every individual is not the goal of our research. Based on this reason, we have created three risk aversion profiles (defensive, neutral, and offensive) with their own risk aversion coefficient. The utility function is a useful measure for comparing the life cycles but the values do not say anything in itself. Therefore, we have transformed the utilities into certainty equivalents to be able to better interpret the results.

To conclude this chapter, the capital calculations and the utility function are essential in order to assess the different life cycles. Together with the stochastic interest rates and equity returns, which is the topic of the next chapter, it forms the basis to assess the three existing unidirectional life cycles and the dynamic life cycle.

Chapter 3

Model building

We have built a simulation model to test the life cycles using stochastic interest rates and equity returns as input, which we discuss in this chapter. First, we give a short introduction about interest rates and equity returns. This emphasises the importance to use stochastic interest rates and equity returns. In Section 3.2 we explain the simulation model step by step, which consists of a dynamic Nelson-Siegel and a vector autoregression (VAR) model with a Markov regime switching component. The addition of a Markov regime switching component is not done a lot in the literature and therefore can be seen as an innovative element.

3.1 Interest rate and equity returns

The interest rate has a huge impact on the economy, and also on the pension industry. For the pension industry interest rates are used as input variable for calculating the values of the pension liabilities. Pensions are accrued over a long period and are usually paid for quite a long time. In determining the pension liabilities, pension funds must therefore use a long-term interest rate prescribed by the Dutch bank (DNB). Currently, the interest rates are historically low, around zero or even slightly negative. The interest rate drop is significant as can be seen in Figure 3.1. Because of the low interest rate, pension funds are obliged to have more money in cash than, for example, a few years ago. This puts an enormous pressure on the pension industry and is also one of the reasons why the current pension industry is being discussed.

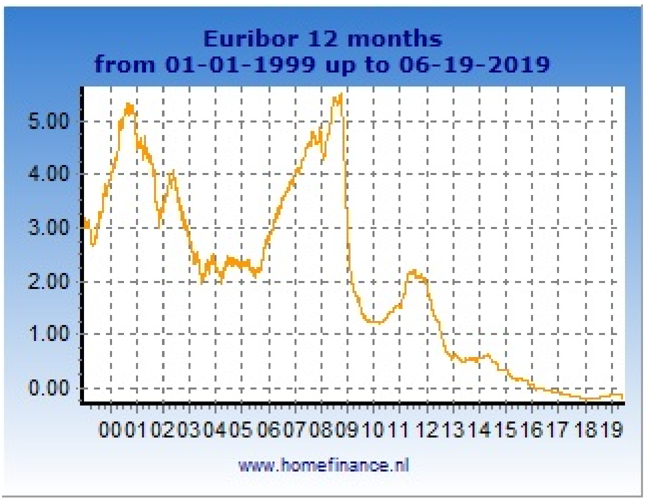

Figure 3.1: Historical EURIBOR interest rate percentages (Home Finance, 2019).

Because of the high importance of the interest rate a lot of research has been conducted to be able to model interest rate movements. These researches can essentially be classified into two frameworks. The first framework is to model the interest rate by modelling the evolution of the short rate. Under a short rate model, the stochastic state variable is taken to be the instantaneous spot rate (Wikipedia, 2019). From the short rate an entire yield curve is built up. If one chooses to fit the resulting interest rates on the current yield curve, the parameters of the model have to be calibrated to be consistent with the current observed prices of interest rate instruments. Well-known one-factor short rate models are for instance:

- Vasicek model (1977).
- Cox-Ingersoll-Ross model (1985).
- Ho-Lee model (1986).
- Hull-White model (1990).

One-factor short rate models are computationally efficient and are often analytically and numerically tractable. The second class of interest rate frameworks are multi-factor short rate models such as the Nelson and Siegel model (1987), the Longstaff-Schwartz model (1992), and the Chen model (1996). The popular yield-curve representation that was introduced by Nelson and Siegel is in 2006 extended by Diebold and Li to a dynamic Nelson-Siegel model (DNS) and is used in our research to model the interest rate. The main reason why we have chosen the Nelson-Siegel model is because central banks extensively use this model to estimate the term structure of interest rates (Bank for International Settlements, 2005). In addition, the model is parsimonious due to its simple functional form (see the formula in the next section) and can be extended to a no-arbitrage model.

In addition to the interest rate, modelling the equity returns is also an important part of this research. The return on equity investments comes in the form of dividend payments or capital gains from the increase in equity prices. A lot of research has been done regarding stock price modelling. Prices can fluctuate considerably as expectation regarding earnings growth and risk premium changes. As investors are faced with high risks they also demand higher returns on equity investments.

We have used the MSCI All Country World Index as a proxy for worldwide equities. Figure 3.2 gives an overview of the annual total returns on equities. As can be seen from the figure below the returns fluctuate quite a lot, which illustrates the difficulty to predict the future returns. However, Sengupta (2004) wrote in his book: ”We talk about simulating stock prices only because future stock prices are uncertain (called stochastic), but we believe they follow, at least approximately, a set of rules that we can derive from historical data and our other knowledge of stock prices. This set of rules is called the model for stock prices”. We discuss our model used to ‘explain’ the interest rate and equity return paths in the next section.

Figure 3.2: Historical equity returns.

3.2 Simulation model

Simulations can be used to show the possible effects of alternative conditions but building a simulation model can be a complex thing to do. In the remaining of this section we explain our model step by step, see Figure 3.3 for an overview of the model. First, we explain the two underlying concepts, which are the dynamic Nelson-Siegel model and the Markov regime switching. Then we discuss some interim results, after which we explain the refinement and calibration of the model. We have used Matlab and Excel to build the simulation model. We do not discuss these codes in detail but they can be found in Appendix F.

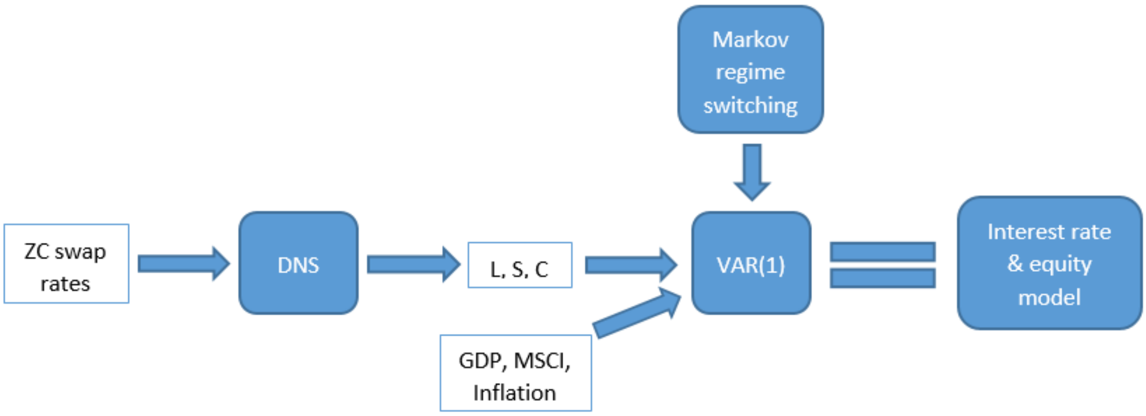

Figure 3.3: Flowchart simulation model.

Dynamic Nelson-Siegel
A yield curve is a compilation of the interest rates with different times to maturity and visually plotted as a curve. A three factor representation of the yield curve was first introduced by Nelson and Siegel in 1987. Since then much research has been conducted to improve the NS model. Well known is the extended model by Svensson (1995). This model includes an extra factor to provide more flexibility. Another extension came from Diebold and Li (2005) who transformed the model into a dynamic version making the parameters time-dependent. Once again, research have been done to improve the DNS model. Examples of such extensions are models that incorporate a time-varying loading parameter, volatility or unconditional mean. Ferguson and Raymar (1998) and Cairns and Pritchard (2001), however, showed that the nonlinear estimators are extremely sensitive to the starting values used and that the probability of getting local optima is high. Taking these drawbacks into account, most researchers have fixed the loading parameter and have estimated a linearised version of the Nelson-Siegel model (Annaert et al., 2012). Therefore, we have used

the following DNS formula to model the yield curve:

$$y_t\left(\tau\right) = \beta_{1,t} + \beta_{2,t} \times \left(\frac{1-e^{-\lambda\times\tau}}{\lambda\times\tau}\right) + \beta_{3,t} \times \left(\frac{1-e^{-\lambda\times\tau}}{\lambda\times\tau} - e^{-\lambda\times\tau}\right), \tag{3.1}$$

where $y_t(\tau)$ is the yield with maturity τ at time *t*, λ is the loading parameter, and β_1,β_2 and β_3 are the level, slope and curvature factors respectively.

As can be seen in the formula above the loading parameter is not dependent on time, which simplifies the underlying assumptions of the model. The loading factor determines the exponential decay of the slope and curvature factor. The other variables, the latent factors β_1, β_2, β_3, are dependent on time. These betas are called level, slope, and curvature respectively and carry some level of economical interpretation (Koopman et al., 2012). The first component equally influences the short and long-term interest rate and can therefore be interpreted as the overall level. The second component converges to one if the maturity goes to zero and converges to zero if maturity goes to infinity. This indicates that this component influences mainly the short term interest rate. The third component is associated with medium term interest rates because it is a concave function which converges to zero if maturity goes to zero and also converges to zero if maturity goes to infinity. The loading parameter influences the moment when the third component reaches its maximum. Figure 3.4 gives an overview of the loading factors in relation to time to maturity. These three loading factors together can capture a lot of different kind of shapes observed in yield data.

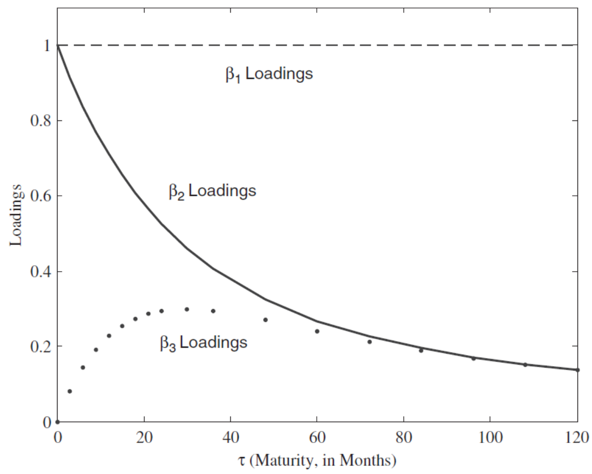

Figure 3.4: Factor loadings DNS model $\gamma = 0.0609$ (Diebold & Li, 2005).

Now that we have introduced the DNS model, the next step is to estimate the parameters. The first step is to find the optimised loading parameter. We have gathered monthly zero coupon swap rates from January 2000 through March 2019 with different time of maturities using Bloomberg. These maturities are 1 up to 10-year, 12, 15, 20, 25, 30, 40, and 50-year. We have estimated the loading parameter using the optimisation toolbox of Matlab and the Ordinary Least Squares (OLS) method. OLS regression is a statistical method that estimates the relationship between a response variable and one or more explanatory variables. The method estimates the relationship by minimising the sum of the squares in the difference between the observed and predicted values of the response variable configured as a straight line, and is also referred as linear regression (Dickey et al., 2001). Initially, a loading parameter is set in order to compute the error in the first iteration. In every iteration a loading parameter is chosen in order to compute the level, slope and curvature factors. With these factors the swap rates are estimated which are then compared to the real data to calculate the error. For the next iteration the optimisation command in Matlab automatically chooses a new loading parameter and again computes the level, slope, and curvature factors to calculate the error. This process is repeated many times to find the optimal loading parameter which is the one with the minimum sum of squares. We have programmed this method in Matlab and the code can be found in Appendix F. The resulting loading parameter is 0.579, which we use in the remainder of our study. Finally, we have used this loading parameter to compute the time series of the level, slope and curvature factors. These time series are used to compute the parameters for the Markov regime switching.

Markov regime switching

Once the loading parameter, and the level, slope and curvature factors have been estimated we have extended the model with a Markov regime switching component. This model is widely applied in both finance and macroeconomics to incorporate regime switches. Financial time series occasionally display dramatic breaks in their behaviour, for example due to a financial crisis or government policy changes. Therefore it makes sense to incorporate regime switches in the DNS model. The idea behind Markov regime switching is that processes can occur in different states, or regimes. As the behaviour of the time series changes, regime switches are assigned to these changes, making the time series alternate between a predetermined number of states. The goal of this step is to evaluate when a regime changes and to estimate the values of the parameters associated with each regime. We have used a Vector Auto Regression (VAR) model to compute the coefficients, the covariance matrix, and the transition probability matrix. VAR is a commonly used model in economic data analysis to simultaneously analyse multiple time series and capture linear interdependencies. An advantage of a VAR model is that this model is easy to use. In a VAR model each response variable has its own equation explaining the evolution based on its own lagged values, the lagged values of other variables, and an error term. The lagged variables are also referred to as the explanatory variables. In this research we have used a VAR model with one lag (abbreviated as VAR(1)). Let's assume that historical behaviour can be described adequately with the following regression:

$$Y_t = \beta_1 \; X_{t-1} \; + \varepsilon_t \; , \tag{3.2}$$

where $\varepsilon_t \sim N(0, \sigma^2)$

Now suppose that an event takes place that changes the level of the response variable Y_t dramatically and cannot be linearly explained by Equation 3.2. Then the formula above does not fit the historical behaviour anymore. The regression formula changes to:

$$Y_t = \beta_2 \; X_{t-1} \; + \varepsilon_t \; , \tag{3.3}$$

where $\varepsilon_t \sim N(0, \sigma^2)$

The formulas above can be rewritten to a Markov regime switching model. If there are *k* states then there are *k* values for the coefficient and volatility. If there is only one state then the formula is the same as a linear regression model. The Markov regime switching formula can be written as follows:

$$Y_t = \beta_{S_t} \; X_{t-1} \; + \varepsilon_{S_t} \; , \tag{3.4}$$

where S_t is State *1,2,...,k* and $\varepsilon_{S_t} \sim N(0, {\sigma_{S_t}}^2)$.

With a regime switching model the dynamic nature of the financial markets can be captured more accurately. In this research we have extended the DNS model and VAR(1) model with a two states Markov regime switching component. Markov chain is the underlying concept of Markov regime switches which is a mathematical system that uses probability rules to simulate transitions from one state to another. Predicting traffic flows, queues, and in game theory, Markov chains are commonly used. It differs from a general stochastic process because a Markov chain process must be memory-less. This means that the future state is not dependent upon the steps that led to the present state. This is called the Markov property and can be mathematically represented as follows:

$$P\left(X_n = i_n \mid X_0 = i_0, X_1 = i_1, ..., X_{n-1} = i_{n-1}\right) = P\left(X_n = i_n \mid X_{n-1} = i_{n-1}\right). \quad (3.5)$$

In other words, all that matters to determine the probability of the current state is the knowledge of the previous state. All the information that could influence the future evolution of the process is fully captured by the present state. This information is stored in the transition matrix Π. We have assumed that Π is independent of time *t*. Every row of the transition matrix is a probability vector and must be equal to one because it is absolutely certain that the future state is either state *i* or state *j* (assuming a two state system). The $(i,j)^{th}$ element is given by the following formula:

$$\pi_{i,j} = P\left(X_{t+1} = j \mid X_t = i\right). \quad (3.6)$$

For example, consider a two state system where at a certain time *t* the state is 1. Then there is a probability ($\pi_{1,2}$) of moving from State 1 to State 2 between time *t* and *t+1*. Likewise, there is a probability ($\pi_{1,1}$) of staying in state 1. Mostly, the transition probabilities are assumed to be constant over time, but it is also possible to use changing transition probabilities called a time varying transition probabilities model. In this research we have assumed that the transition probabilities are constant over time. The transition probability matrix can be represented as:

$$\Pi = \begin{pmatrix} \pi_{1,1} & \cdots & \pi_{1,k} \\ \vdots & \ddots & \vdots \\ \pi_{k,1} & \cdots & \pi_{k,k} \end{pmatrix}. \quad (3.7)$$

Now that we have explained the basics of Markov regime switching models, the task remains to incorporate the methodology in the simulation model. We have used Perlin (2015) as a guideline for the implementation of a Markov switching component. The author created a Matlab package for the estimation, simulation and forecasting of a general Markov regime switching model. The technical explanation of the method is provided by Perlin (2015).

As previously explained, we have estimated the time series of the level, slope and curvature. To transform these variables into invariants, we have taken the first differences. The delta level, delta slope, and delta curvature are used as response variables. The reason behind this transformation is the fact that the inference based on the absolute level, slope, and curvature factors does not say anything about the regimes. The delta values are composed and used as input for the Markov switching regime model and provide information about the regimes. Higher deltas indicate a higher volatility regime. The delta level, slope, and curvature values are not the only three response variables. The other three response variables are the Gross Domestic Product (GDP), inflation and equity returns. This means that the VAR(1) model has a total of six equations with level, slope, curvature, inflation, GDP, and equity returns as the response variables and the lagged values of the just mentioned variables as the explanatory variables. We discuss the specification of the data in the next paragraph. The VAR(1) model can be represented by the following equation:

$$Y = {D_{S_t}}^T \, Z + W_{S_t} \ , \tag{3.8}$$

where Y, Z, and W are column vectors with dimension 6x1 representing the six variables. The response variables are represented by Y, the lagged explanatory variables by Z, and the random variables by matrix W $\sim N(\mathbf{0}, \Sigma_{S_t})$, which is state dependent. S_t is modelled with a Markov regime switching process with the help of the MS Regress package of Perlin (2015). The outputs of the Markov regime switching are discussed later in this report. D is the coefficient matrix of State *S* at time *t* (see Table 3.1 and Table 3.2 for the coefficient matrix of State 1 and State 2 respectively).

Both the coefficients and the error term are state dependent. This means that the coefficients and variance are switching according to the transition probabilities. We have built this VAR(1) model with the help of the MS Regress package of Perlin (2015) to compute the coefficient matrices, the covariance matrices, and the transition probability matrix.

Data

In order to simulate the interest rates and equity returns using the DNS model with Markov regime switching it is necessary to gather input data. We have used monthly zero coupon swap rates to compute the level, slope, and curvature factors. This data are one of the inputs for the Markov regime switching. In addition to that, we have used real Gross Domestic Product (GDP), equity returns, and inflation data to incorporate some macroeconomic factors. GDP growth is usually considered to be the most important economic statistic to give a rough guide regarding the level of economic activity. Two consecutive quarters of a fall in GDP indicates a recession and a rising GDP indicates growth. The Eurozone YOY real GDP growth are collected from the Consensus Economics database, and transformed to monthly real GDP percentages. We have chosen to use real GDP as this variable captures the real economic growth excluding inflation. To measure inflation we have gathered the Eurozone seasonally adjusted Harmonised Index of Consumer Prices (HICP) monthly percentages from the European Central Bank (ECB) database. This variable measures the change over time in the prices of consumer goods and services acquired, used or paid for by euro area household. We have chosen to use seasonally adjusted data to provide a more accurate depiction of price movements void of anomalies that can occur during specific seasons. At last, we have used monthly All Country World Index from the MSCI database as a proxy for worldwide equity returns. This index captures the equity markets from 23 developed and 26 emerging markets and is therefore a good proxy for global equity benchmark. All data in this research covers the period from December 2000 to March 2019.

Output Markov regime switching

Up to now we have discussed the DNS model with the Markov switching regime component and the underlying data. In this step we have used the multivariate regression previously described and the MS Regress Matlab package of Perlin (2015) to compute the coefficients, transition probability, and covariance matrices. These are necessary in order to simulate the interest rates and equity returns. The two-state model entails two sets of coefficient matrices and covariance matrices. The coefficient matrices are represented in the following tables:

	Level	**Slope**	**Curvature**	**Inflation**	**GDP**	**MSCI**
Level$_{t-1}$	0.0384	0.4527	0.0045	0.0660	0.0330	1.7158
Slope$_{t-1}$	0.1836	0.2805	-0.4659	0.0152	0.0233	1.8861
Curvature$_{t-1}$	0.0537	0.00098	-0.0201	0.0077	0.0047	-0.2966
Inflation$_{t-1}$	0.1037	-0.0533	-0.2557	0.3176	-0.0040	2.2860
GDP$_{t-1}$	-0.1652	0.1641	0.1268	0.5358	0.9885	0.0106
MSCI$_{t-1}$	0.0053	-0.0085	0.0143	0.0059	0.0006	0.10099

Table 3.1: Coefficient matrix State 1.

	Level	**Slope**	**Curvature**	**Inflation**	**GDP**	**MSCI**
Level$_{t-1}$	-0.0605	0.4294	0.0080	0.2337	-0.0105	-1.0168
Slope$_{t-1}$	-0.0645	0.0098	0.5484	0.0012	-0.0227	-0.9746
Curvature$_{t-1}$	-0.0742	0.0001	0.0771	-0.0175	-0.0005	1.0447
Inflation$_{t-1}$	0.2085	-0.0422	-0.0316	0.2985	-0.0026	-0.1099
GDP$_{t-1}$	0.0679	-0.7152	-0.2121	1.1974	1.0015	-0.0012
MSCI$_{t-1}$	-0.0035	0.0055	-0.0047	0.0052	-0.0005	-0.0884

Table 3.2: Coefficient matrix State 2.

Based on the comparison of Table 3.1 with Table 3.2 we can conclude that there are differences between the coefficients of State 1 and State 2. The frequency of using the corresponding coefficient matrix depends on the probability of being in State 1 or State 2. The bottom graph in Figure 3.5 gives an overview of the probability of being in State 1 or State 2 over time. As can be seen, quite some switches occur between State 1 and State 2 in the beginning. Then the system stays in State 1 for a while, after which it switches again more often to State 2, followed by somewhat longer periods staying in State 1. The smoothing probabilities (bottom graph) can also be reconciled with the behaviour of the corresponding conditional standard deviation (middle graph). Where the higher volatilities of the financial crises of 2001 - 2003 and 2008 - 2009 are evident, throughout observation 0 - 25 and 85 - 100 approximately, the smoothing probabilities of both portfolios demonstrate that State 2 is probably the ‘bear’ regime.

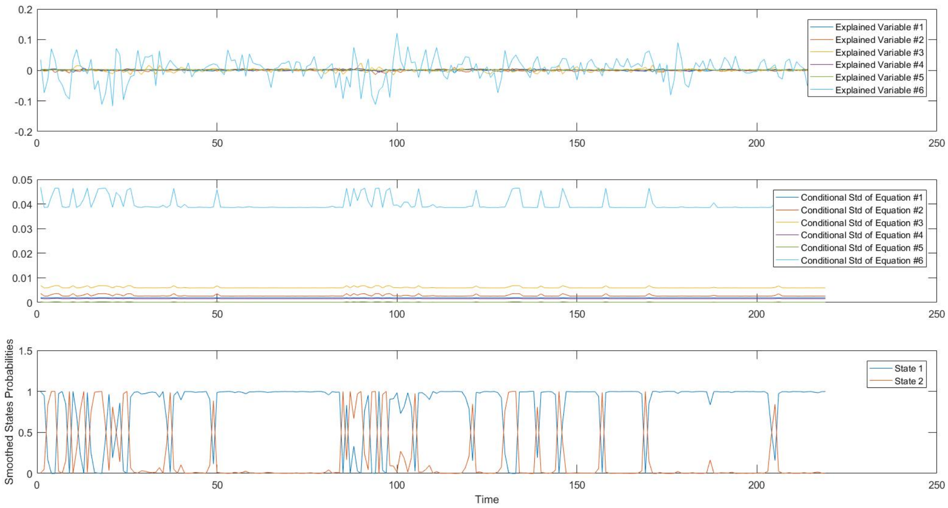

Figure 3.5: Graphs regime switches.

The other Markov regime switching output is the transition probability matrix which gives the probabilities of going from one state to another state. This is given in the following transition probability matrix:

State	**1**	**2**
1	0.878	0.122
2	0.516	0.484

Table 3.3: Transition probability matrix.

Let’s assume the system is currently in State 1. With a probability of 87.8% the system stays in State 1 in the next period. With probability of 12.2%, however, the system moves to State 2. With a probability of 51.6% the process reverts from State 2 to State 1 in the next time period. In addition, the sum of the rows are equal to one because it is certain that the system will either stay in the same state or go to the other state. We have used this transition probability matrix, next to the coefficient and covariance matrices, to simulate the interest rates and equity returns.

Simulation

A simulation model can be used to determine the probabilities of outcomes, which cannot be determined or are difficult to determine analytically, because of the randomness of several input variables. Such a model can be used to simulate the interest rates and equity returns. The outcomes from the earlier analysis are needed to perform the simulation. These include the coefficient matrices, the covariance matrices, the two correlated random variables matrices and the transition probability matrix. The input data covers the period up to March 2019 and is therefore the last period that the data on the level, slope, curvature, inflation, GDP, and equity returns are known. We have used March 2019 data as a starting point of the simulation, which is done on a monthly basis. As previously mentioned, we have used the delta values of the level, slope, and curvature factors to compute the Markov regime switching parameters. These delta values are also used to calculate the new delta values for the simulated periods. Then these new delta values together with the random variables are added to the absolute starting values. The random variables are correlated and therefore we have used the Cholesky decomposition, which can be found in Appendix D. The delta starting point and the absolute starting point are given in the table below. Note that delta and absolute values are the same for the inflation, GDP, and equity return data. This is because these are already month-over-month percentages.

March 2019	Level	Slope	Curvature	Inflation	GDP	MSCI
Delta	-0.27%	0.36%	-0.06%	0.110%	0.113%	2.75%
Absolute	1.31%	-1.00%	-3.54%	0.110%	0.113%	2.75%

Table 3.4: Starting values simulation.

As already mentioned, there are two coefficient matrices and two correlated random variable matrices. To determine in which state the system will be in the first simulated period, we have used the probability of being in State 1 or State 2 at March 2019. Besides the coefficient matrices and transition matrix, the probability of being in State 1 or State 2 at the end of March 2019 is also part of the output of the Markov regime switching package of Perlin (2015).This resulted in the probability of being in State 1 of 99.83% and the probability of being in State 2 of 0.17%. Then a random number is generated from a uniform distribution between zero and one. If this random number is less than 0.9983 then the system is in State 1, otherwise it is in State 2. This procedure is only used to determine the first state. After that we have used the transition probability matrix to determine the next state. Let's say the system is currently in State 1. To check whether the system will stay in State 1 or go to State 2, again a random number from the same distribution is drawn. If this random number is less than 0.878 then the system will stay in State 1 the next period. If the system is currently in State 2 and the random number is less than 0.516 then the system will go to State 1. If that is not the case the system will stay in state 2. So, in line with the Markov chain theory, the next state only depends on the current state and the transition probabilities.

So far it is clear in which state the system is. The next step is to compute the simulated values of level, slope, curvature, inflation, GDP, and equity returns. Based on the information whether the system is in State 1 or State 2, the corresponding coefficient matrix of State 1 or State 2 is used. Then by using Equation 3.8 we have calculated the level, slope, curvature, inflation, GDP, and equity return values for the next period.

As can be seen in the formula, the six response variables are dependent on the coefficient matrix, the values of the previous period, and the correlated random variables matrix. Which coefficient matrix and correlated random variables matrix are used depend on the state of the system. We have used Equation 3.8 to calculate the new level, slope, curvature, inflation, GDP, and equity return values for every simulated period. The number of simulated periods is 504 (42x12) because a 42-year interest rate and equity return forecast is needed for the capital calculations. In total five thousand simulations are done to assess the life cycles with different interest rate paths and equity returns.

The result of the previous step is a 504x6 matrix, which contain the values of the six factors, for every simulation. Now, we have simulated the equity returns and no further steps are required. On the other hand, an additional step is needed for the interest rate. Up to now, we have simulated the level, slope, and curvature values. However, these data are on a monthly basis whereas the capital calculations are on an annual basis. Therefore we have filtered the data first in such a way that only the yearly level, slope, and curvature values remain. Then we have used the DNS formula, see Equation 3.1, to construct the yield curves using the simulated level, slope, and curvature values and the previously determined loading parameter of 0.579. A figure showing the 30-year yield curves can be found in Appendix E.

Calibrating the model

So far the simulation model is not calibrated yet. A calibrated model means that the parameters of the model are consistent with the market observations. Calibrating the interest rate and equity returns is also a necessary step in our research. First, we have calibrated the interest rate based on the forward rates. This means that the average of the yield curves should approximately the same as the forward rates based on the spot interest rate. As is described in Chapter 2 we have used the spot interest rate to calculate the forward rates using Equation 2.3. This will be referred as the forward rate matrix. In addition, the average yield curves of the five thousand simulations are calculated and is also a two dimensional matrix with the axes *horizon* and *maturity*. This matrix is then subtracted from the forward rate matrix. This result is added to the simulated yield curves every time the simulation is executed to make sure that the simulated yield curves are approximately consistent with the market observations.

The equity returns need to be calibrated as well and is done in the same way. This time it is somewhat less clear based on which value the equity returns should be calibrated. This is due to the fact that the current market observation does not necessarily mean that it is representative for the future. Therefore we have calibrated the equity returns based on the historical excess returns of 3.5%, which is based on a long-term return study (1814 - 2014) of the Deutsche Bank (2014). First, we have constructed a 5000x42 simulated equity return matrix with the dimensions *simulation run* and *horizon*. A matrix with the same dimensions is also created but this time with the matching returns. How these matching returns are calculated is discussed in the next paragraph. Subsequently, we have substracted the equity return matrix from the matching return matrix, which results in an excess return matrix. Then for every horizon we have calculated the average excess return resulting in a 1x42 vector. These values should be approximately 3.5%. We have created a vector called delta equities by subtracting 3.5% from the 1x42 vector. This delta equities vector is then added to the simulated equity returns every time the simulation is executed. In this way the average simulated excess return is consistent with the historically observed excess returns.

Matching and equity returns

The matching and equity returns are needed for the assessment of the life cycles. Calculating the equity returns is easier than the matching returns because the simulated equity values are already returns. The only thing that needs to be done is to transform the monthly equity returns to annual equity returns. On the other hand, calculating the matching returns takes more effort because up to now we have modelled the yield curves. A yield curve is a compilation of the interest rates with different times to maturity and can be used to discount the expected cash flows, which are the expected pension benefits in this research. Because the payments of the pension benefits take place at different moments in time, they should be discounted by using the yield curves. The return on the matching portfolio can be seen as a replication of the return on the liabilities because we have calculated the expected pension benefits in such a way that these benefits are achieved when all the capital is allocated to the matching portfolio. Therefore, the present values of the benefits can be seen as a theoretical price of the matching portfolio. Once we have calculated the 'prices', the price movements are computed as a result of the changing interest rates. This is done by calculating the present value changes. We have calculated the present value at the age of 25 *(t = 0)* by using the spot interest rate, which means that it is the same in every run of the simulation. We have calculated the other present values by using the simulated yield curves. This means that at the age of 26 the expected cash flows are discounted with the simulated interest rate curve at $t = 1$ and at the age of 27 the simulated interest rate curve at $t = 2$ is used and so on. In this way all present values are calculated, thus at the age of 25 till 67. The matching return formula is as follows:

$$M_t = \frac{B_{t+1}}{B_t} - 1 \ , \tag{3.9}$$

where M is the return on the matching portfolio and B is the present value of the benefits at time t.

We have done the matching and equity returns calculations for every simulation run. In this way the matching and equity returns are stochastic and can be used in the assessment of the life cycles.

3.3 Conclusion

Currently, the historical low interest rate puts enormous pressure on the feasibility of the current DB pension plan. It is important to assess the life cycle under different circumstances because of the unpredictability of the future. In this research we have generated stochastic interest rates and equity returns because these two have a great impact on the performance of the life cycles. This comes from the fact that the asset allocation mix, the ratio between the matching and return portfolio, determines the exposure to the interest rates and equity returns respectively. With this reason in mind, we have built a simulation model to be able to generate different interest rate and equity return scenarios. The foundations of the model used in this research are widely accepted theories in the financial industry. The underlying theories are the Dynamic Nelson-Siegel model and the Markov regime switching model. The input for the simulation model cover the data from December 2000 to March 2019.

The input data about inflation, equity returns, and GDP are also part of the dataset to incorporate some macroeconomic factors for the Markov regime switching. This is to compute the coefficient matrices, the covariance matrices, and the transition probability matrix. These are used to simulate the interest rates and equity returns. We have calibrated the interest rates based on the spot interest rates as of March 2019 and the equity returns have been calibrated based on the historical observed excess returns. As a result, a set of stochastic matching returns and equity returns are generated. These are then used in the next chapters to test which life cycle design performs best and in the optimisation of a dynamic life cycle design.

Chapter 4

Life cycle analysis

Up to now we have modelled the capital calculations, utility function, interest rates and equity returns. This framework can now be used to test some existing unidirectional life cycles in order to answer Research Question 1. The research question that is central to this chapter is formulated as follows:

Which cycle design offers the best risk-return trade-off given a certain risk-aversion level of the participant and stochastic interest rates and equity returns?

To answer this question we introduce the different kind of life cycle designs first. This is done in Section 4.1. Once we have explained the life cycles we use the assessment framework and simulation model to test the life cycles. We have used Matlab and Excel to perform the analysis. The associated Matlab codes can be found in Appendix F. In Section 4.2 we interpret and discuss the empirical results.

4.1 Life cycle designs

Recall that a life cycle is the asset allocation mix during the accumulation phase that reflects the ratio between the matching and return portfolio. The accumulation phase is the period in which a participant accrues pension by paying a premium. The purpose of the matching portfolio is to replicate the change in value of the liabilities. Commonly used financial products for the matching portfolio are government bonds, corporate bonds and interest rate swaps. The purpose of the return portfolio is to maintain purchasing power by generating excess returns compared to the nominal obligations. One can think of equities, real estate investments, and high yield bonds as financial products used in the return portfolio. According to the matching and return theorem, it is possible to combine the risk-free matching portfolio and the risky return portfolio to replicate any desired risk and return profile for a pension fund. This lowers complexity, promotes transparency, and offers flexibility and control.

Our research starts with the analysis of three unidirectional life cycle designs which are referred as the constant, the traditional and the reverse life cycle designs. These are displayed in Figure 4.1. The life cycle plots show only the return portfolio percentages because the matching portfolio percentage is one minus the return portfolio.

- **Constant life cycle**
  The most basic life cycle is a constant life cycle where the asset allocation ratio between the matching and the return portfolio is kept constant during the accumulation phase of the participant. This implies that a 25 years old participant has the same allocation to equities in their portfolio as a 50 years old participant. The average allocation mix of the individual Dutch pension funds is 52% matching and 48% return (De Nederlandsche Bank, 2019b). Based on this finding, we have chosen to use a constant asset allocation mix of 52% matching and 48% return in this research.

- **Traditional life cycle**
  The traditional life cycle makes use of a standard fixed glide path, which refers to a formula that defines the asset allocation mix, based on the number of years to the target date (Investopedia, 2019). Linear glide paths are used in both the traditional and the reverse life cycle designs. In the traditional life cycle more capital is allocated to the return portfolio in the beginning of the accumulation phase. This return portfolio is then linearly substituted for a more matching-like portfolio as the retirement date approaches. The reduction of the risk of an older age is in line with the seemingly plausible idea that investors are able to take more risk when they are young than when they are approaching retirement. This is because there is less time available to recover in the event of poor investment results. In addition, the high allocation to the matching portfolio ensures that a large part of the pension result is insured as the retirement date approaches. Based on a previous investigation conducted by MN, we have chosen to use a traditional life cycle with 80% return at the beginning of the accumulation phase and linearly decreasing to 20% return at the end.

- **Reverse life cycle**
  In contrast to the traditional life cycle, the reverse life cycle allocates more to the return portfolio as the participant ages. This life cycle profile is based on two ideas. First, the interest rate risk of a young participant is much higher than when the participants gets older. This suggests that a more matching-like portfolio allocation at the beginning makes more sense. On the other hand, more capital has been built up at an older age than at the beginning of a participant's career. A higher allocation to the return portfolio at an older age means that potentially more return can be achieved. This leads to the expectation that this life cycle will provide higher pension results than the traditional life cycle. Arnott et al. (2013) showed that the reverse life cycle is superior to the traditional life cycle through a historical analysis of bond and stock returns from 1871-2011. The reverse life cycle is the opposite of the traditional life cycle. At the beginning 20% be will allocated to the return portfolio which will linearly increase to 80% at the end of the accumulation phase.

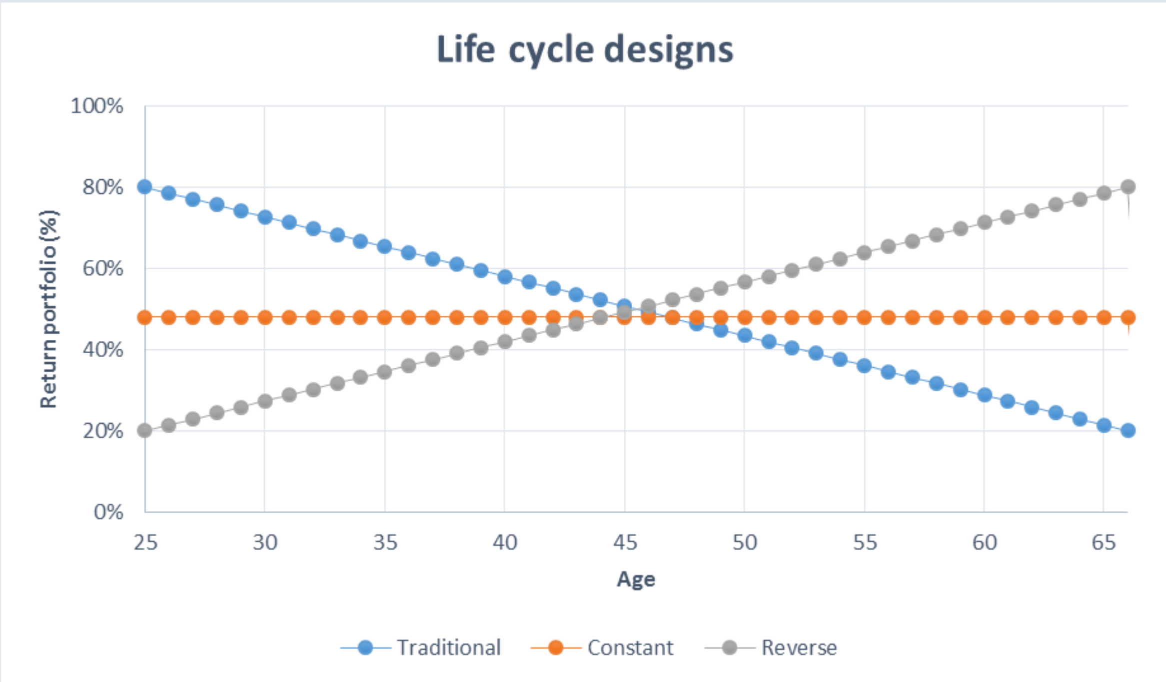

Figure 4.1: Traditional, constant and reverse life cycle.

4.2 Empirical results

In the previous chapters we have discussed the capital calculations, the utility function, and the stochastic interest rate and equity returns model. This means that the foundation have been laid to assess the different life cycles. An optimal life cycle is the life cycle that makes the best trade-off between the expected level and the uncertainty of the pension result. This trade-off is incorporated in the coverage ratio calculation and the utility function. Three profiles are created to test whether the outcome depends on the risk aversion perception. These have the following risk aversion parameters:

- Low risk aversion (offensive), $\gamma = 2$.
- Medium risk aversion (neutral), $\gamma = 5$.
- High risk aversion (defensive), $\gamma = 10$.

We have calculated the utility values for all three life cycle designs based on these risk aversion parameters. The preference for a specific life cycle is assessed based on this utility value. The higher the utility for a certain life cycle, the higher the preference for that life cycle.

In the first analysis, we have tested the three life cycle designs with deterministic interest rates and equity returns to get a first feeling about the performance of the life cycles. This means that in this situation the stochastic interest rates and equity returns are not used yet. We have used the average historical equity return of 3.5% as the annual equity return and we have calculated the matching returns based on the current term structure of interest rate and its derived forward rates. Based on these inputs, together with the other inputs and assumptions stated in Chapter 2, the coverage ratios are:

	Coverage ratio
Constant life cycle	115.66%
Traditional life cycle	113.21%
Reverse life cycle	119.69%

Table 4.1: Coverage ratios given deterministic interest rates and equity returns.

The coverage ratios are given due to the fact that in the deterministic analysis only one scenario is used which results in just one coverage ratio per life cycle. This is not the case when the stochastic interest rates and equity returns are used. Then five thousands coverage ratios are the result of the five thousand scenarios. But what do the results in the table above say about the preference for a certain life cycle design? The reverse life cycles is preferred to the traditional and constant life cycle. This is in line with the results of Arnott et al. (2013). It is also logical that the coverage ratios are above hundred percent because the contribution percentages are composed in such a way that the coverage ratio will be hundred percent when all the capital is invested in the matching portfolio. This means that when some capital is also allocated to the return portfolio (with higher returns on average) more capital will be accumulated and thus will result in a higher coverage ratio. This result is in contradiction with the first intuition that the traditional life cycle would outperform the other two life cycles. Nonetheless, Research Question 1 cannot be answered yet because it is about the performance of the three life cycles given stochastic interest rates and equity returns.

In the second analysis we have used a set of stochastic interest rates and equity returns to test whether this would give the same results. These scenarios are now used to calculate the coverage ratios, the corresponding utility values and certainty equivalents. The goal of this analysis is to test how the existing life cycles perform given stochastic interest rates and equity returns. The resulted certainty equivalents of the analysis are shown in the table below.

	Certainty equivalents		
Risk aversion	**Low**	**Medium**	**High**
Constant life cycle	130.1%	46.7%	16.2%
Traditional life cycle	121.2%	44.8%	16.3%
Reverse life cycle	134.5%	41.2%	14.4%

Table 4.2: Certainty equivalents given stochastic interest rates and equity returns.

The certainty equivalents given in the table above are the coverage ratios that would result in the same utility in case there is no uncertainty involved. It can be interpreted as the coverage ratio a participant would accept in order to avoid the uncertainty of what the coverage ratio will be at retirement. This time the outcomes differ from the previous results. Which life cycle is preferred is dependent on the risk aversion parameter. A reverse life cycle is preferred when a participant has a low risk aversion. This intuitively makes sense because the reverse life cycle can be seen as the riskiest of the three life cycles. Therefore it is a likely outcome that indeed the reverse life cycle is preferred in case a participant has a low risk aversion. The idea behind the reverse life cycle is that potentially more return can be achieved because more capital is accumulated at the end of a participant's career compared to the beginning of the career. Apparently, this indeed results in a higher certainty equivalent in comparison with the other life cycles. Secondly, if a participant has a high risk aversion the traditional life cycle is preferred. This can be explained by the fact that the traditional life cycle can be seen as a risk-reducing life cycle because it starts with an allocation mainly to the return portfolio and ends with a more matching-like allocation. With this asset allocation mix the most risk is taken in the beginning. Finally, in case a participant has a neutral risk aversion then the constant life cycle results in the highest certainty equivalent. This life cycle has a bit of both worlds. Achieving potentially more returns in the beginning and less in the end compared to the reverse life cycle whereas it is the other way around compared to the traditional life cycle. So, based on the CEs it turned out that the life cycle preference depends on the risk aversion of a participant. By inspection the distribution of the coverage ratios, it is possible to gain more insight into the associated risks with different life cycle designs. Different coverage ratio percentiles are given in the table below.

	Coverage ratio distribution				
Percentiles	**5%**	**25%**	**50%**	**75%**	**95%**
Constant life cycle	56%	107%	165%	251%	451%
Traditional life cycle	49%	100%	159%	247%	463%
Reverse life cycle	55%	109%	173%	271%	499%

Table 4.3: Statistics of the existing life cycles.

Looking at the 5% percentile it appears that when using the traditional life cycle the downside risk, as expressed by the coverage ratio at 5^{th} percentile, is higher compared to the constant and reverse life cycle. It seems that the traditional life cycle gives less downside protection compared to the other two life cycles. This observation is also supported by calculating the Conditional Value At Risk (CVAR), which is a measure that quantifies the amount of tail risk (Investopedia, 2019), for the five percent tail which is 42%, 41%, and 36% for the reverse, constant, and traditional life cycle respectively. Another observation is that the 5% and 25% percentiles of the reverse and constant life cycles are almost similar whereas the other percentiles indicate that the reverse life cycle has more upside potential than the constant and traditional life cycle.

Altogether, the distribution of the coverage ratios indeed gives us some insights about the performance and associated risk of the three life cycles.

4.3 Conclusion

The main topic of this chapter is the life cycle analysis to investigate which life cycle design will generate more wealth as expressed by the utility and certainty equivalents. We have explained and discussed the three unidirectional life cycle designs and the empirical results in the previous sections. Every life cycle has its own risk distribution. The constant life cycle keeps the asset allocation ratio between the matching and return portfolio constant during the accumulation phase. A traditional life cycle allocates more capital to the return portfolio at the beginning of the accumulation phase. This return portfolio is then linearly substituted for a more matching-like portfolio as the retirement date approaches. Lastly, the reverse life cycle allocates more capital to the matching portfolio in the beginning of the accumulation phase which is linearly shifting towards the return portfolio as the retirement date approaches. We have tested these three life cycles using the set of scenarios with different interest rates and equity returns. We have executed the capital calculations for every simulation to calculate the coverage ratio and corresponding utilities. The utilities are calculated for three risk aversion coefficients to see if this influences the order of preference. Using the assessment framework and the simulation model, we have tested the three life cycle designs with deterministic interest rates and equity returns in the first case and then by using stochastic interest rates and equity returns.

With the knowledge gained we can answer the first research question. To recall, Research Question 1, which is central to this chapter, is formulated as follows:

Research Question 1: *Which life cycle design offers the best risk-return trade-off given a certain risk-aversion level of the participant and stochastic interest rates and equity returns?*

Evaluating the life cycle designs with the deterministic interest rates and equity returns says nothing about the performance of the life cycles because it only tests the life cycles under one specific circumstance. Research Question 1 aims at gaining insight into the performance of existing life cycles in an environment with stochastic interest rates and equity returns. We have found that the life cycle preference depends on the risk aversion of the participant. In case a participant has a low risk aversion then the reverse life cycle is favoured. Secondly, the constant life cycle is preferred in case a participant has a neutral risk aversion. If the participant is highly risk averse then the traditional life cycle is favoured.

Chapter 5

Optimisation life cycle

With the analysis from the previous chapter we have showed that the order of preference of the three existing life cycles depends on the risk aversion. In this chapter we optimise the expected utility by changing the life cycle to test which life cycle design results in the highest expected utility. The optimisation problem is formulated as follows:

$$max_{LC}\ E\left(U\left(x\right)\right). \tag{5.1}$$

We have executed the optimisation by using the FMINCON Matlab command. This is a nonlinear programming solver that tries to find the minimum value of a function. The objective is to maximise the expected utility. We have multiplied the utilities by minus one in order to transform the maximisation problem into a minimisation problem. The Matlab codes used to execute the optimisation can be found in Appendix F.

In the previous chapter we have tested only linear life cycles. This linearity is also the starting point of the first optimisation and is explained in Section 5.1. This means that only the starting point, slope, and end value are determined to compute the entire linear life cycle.

In the second part of this chapter we explain the dynamic life cycle optimisation. This means that the life cycle does not have to be a linear function and is dependent on the market. At various moments in time, the accumulated capital so far forms the basis for making a decision whether more capital should be allocated to the return or matching portfolio.

The different life cycle designs discussed in the previous sections contain certain assumptions. In order to check the robustness of the findings we have conducted some sensitivity analyses. We have performed a risk aversion sensitivity analysis to check whether the risk aversion coefficient has a big impact on the results. In addition, we have executed the optimisations using different excess return calibrations.

5.1 Optimisation linear life cycle

In the previous chapter we have discussed the assessment framework and stochastic interest rates and equity returns. These are now used to optimise the linear life cycle. As explained in Chapter 2, we have composed the contribution percentages used in the capital calculations in such a way that it covers all the pension benefits based on the spot interest rate as of end March 2019. However, it is possible that the premiums paid and the realised returns are not enough to pay for the pension entitlement when stochastic interest rates and equity returns are used as input. In this section we test only linear functions, where the slope and starting point can be changed, because this is the simplest and the most used life cycle design.

To make sure that the optimisation finds only feasible solutions it must satisfy two constraints. The equation does not allow a negative asset allocation (long only constraint) or an allocation of more than hundred percent (budget constraint). We have programmed the constraints into Matlab using the option to define the constraints that the optimisation solution must meet. The two inequality constraints are defined as follows:

$$\begin{aligned} ax + b - 1 &\leq 0 \\ -ax - b &\leq 0 \end{aligned} \tag{5.2}$$

In the optimisation we have drawn two random variables as starting point which represent the slope and level of the linear function. We have used these variables to compute an initial life cycle. This life cycle serves as an input in the capital calculations to compute the coverage ratio and the corresponding utility and certainty equivalent. Subsequently, different values for *a* and *b* are chosen which results in a different life cycle. Again, the life cycle is used to calculate the coverage ratio, utility, and certainty equivalent. This result is then compared with the previous values. Optimising the *a* and *b* is done by using the FMINCON command of Matlab. The default underlying optimisation algorithm is the interior point algorithm and is used as the algorithm to find the optimal level and slope values. A lot of literature is available about the interior point algorithm and is therefore not included in this report. For more technical details see, for example, the book *Interior-Point Algorithm: Theory and Analysis* (Ye, 1996). To prevent from getting stuck in local minima, this process is repeated several times with different starting values. The life cycle with the highest expected utility is chosen and labelled as the optimised life cycle. The optimised life cycle for every risk aversion coefficient is displayed in the figure below.

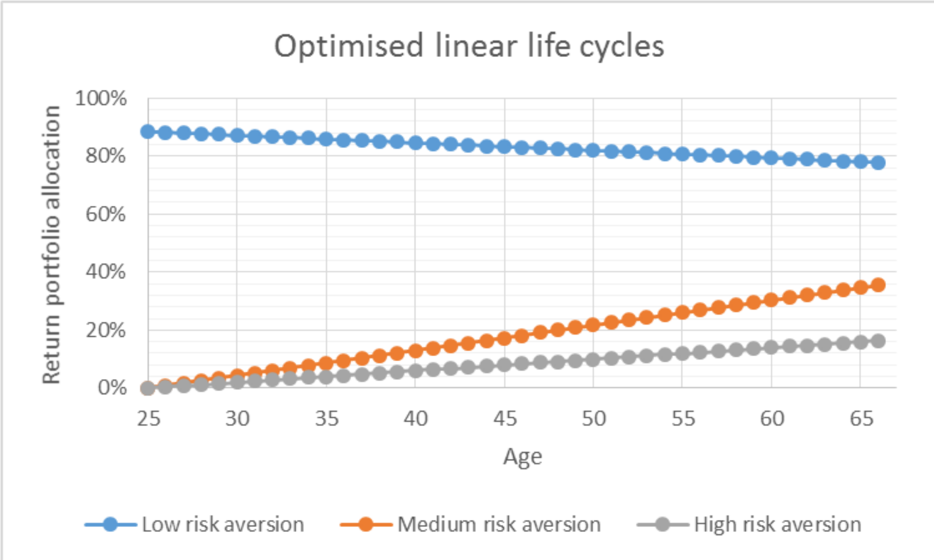

Figure 5.1: Optimised linear life cycles.

One of the observations is that the higher the risk aversion the lower the allocation to the return portfolio generally. This intuitively makes sense because financial products in the return portfolio are riskier compared to those in the matching portfolio. In addition to that, it is surprising to see the big difference between the life cycle represented by the blue line and the life cycles illustrated by the orange and grey lines. The former is completely different compared to the reverse life cycle we have tested in the previous chapter, which resulted in the highest utility. In this life cycle around eighty percent, with a slight decrease over time, is allocated to the return portfolio during a participant's career. The other two life cycles are somewhat similar to the reverse life cycle defined in Section 4.1, but with a much lower allocation to the return portfolio. These life cycles can be explained by the fact that at the beginning of the accumulation phase the interest rate risk is high, meaning that it is wise to allocate more capital to the matching portfolio. As the duration decreases, the portfolio becomes more balanced and more is allocated to the return portfolio. The question still remains whether these optimised life cycles are indeed better than the given life cycles we have analysed in Chapter 4. To answer this question we have calculated the CEs for every risk aversion coefficient. The CEs are given in the table below.

	Certainty equivalents		
Risk aversion	**Low**	**Medium**	**High**
Optimised linear life cycle	141.5%	60.4%	29.2%

Table 5.1: Certainty equivalents of the optimised linear life cycles.

To recall, the highest CEs of the life cycles discussed in the previous chapter are 134.5%, 46.7%, and 16.3% for the low, medium, and high risk aversion respectively. The results from this analysis show that the CEs of the optimised linear life cycles are higher than the CEs of the given life cycles. This means that the optimised linear life cycles are indeed an improvement compared to the traditional, constant, and reverse life cycles. In the remainder of this report, when speaking about a linear life cycle it refers to the optimised linear life cycle and not the life cycles discussed in Chapter 4.

5.2 Optimisation dynamic life cycle

In the previous optimisation we have assumed that the life cycle is a linear function. From this point onwards, we do not use it as a requirement. The life cycle does not necessarily have to be a linear function as was the case in the preceding analysis. This is one aspect of the dynamic life cycle. The other aspect is the market-dependency of the life cycle. As we have explained in Chapter 3, the simulation model has been expanded with a Markov regime switching component. We have assumed that the system has two states, where State 1 and 2 are the low and high volatility states respectively. We have used a random number generator (assuming the uniform distribution) and the transition probability matrix to determine the next state of the simulation model. The different states represent the different market conditions. This regime switching component ensures that the life cycle is state dependent. We have optimised two life cycles, one life cycle for state one and another for state two. These two features together form the definition of the dynamic life cycle.

To find an optimal solution for the dynamic life cycle, we have used the objective function as is defined by Equation 5.1. As a first attempt, we have executed the optimisation without any restrictions. The resulting dynamic life cycle, which is a time and regime dependent asset mix between the return and matching portfolio, was very volatile. The yearly turnover of the return portfolio can reach to ninety percent. Of course, this is not realistic because of the high transaction cost to rebalance the matching and the return portfolios every year. However, there was somewhat of a trend visible corresponding to the linear functions. In order to reduce the variation, we have chosen to introduce a bandwidth around the optimised linear life cycles. Note that the ten percent bandwidth is used as an indication to reduce the life cycle variability. The optimised dynamic life cycles are displayed in Figure 5.2, 5.3, and 5.4.

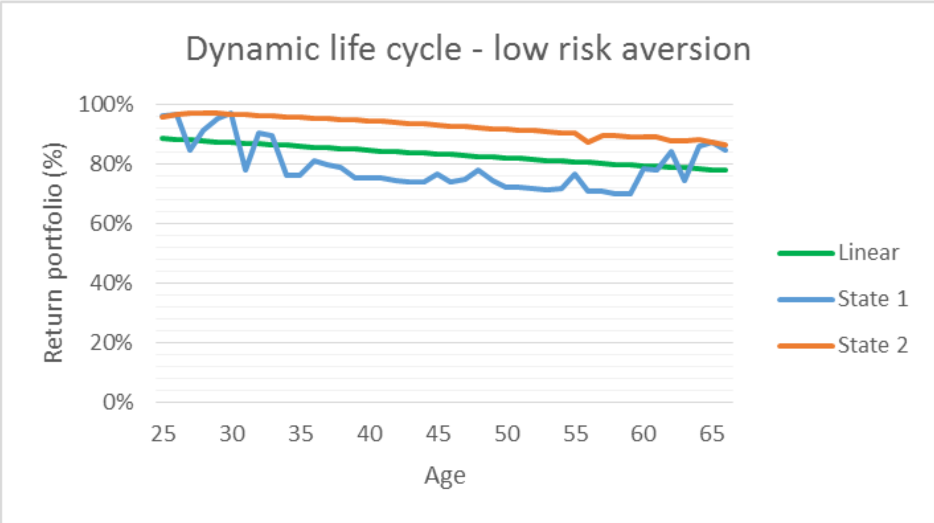

Figure 5.2: Dynamic life cycle low risk aversion.

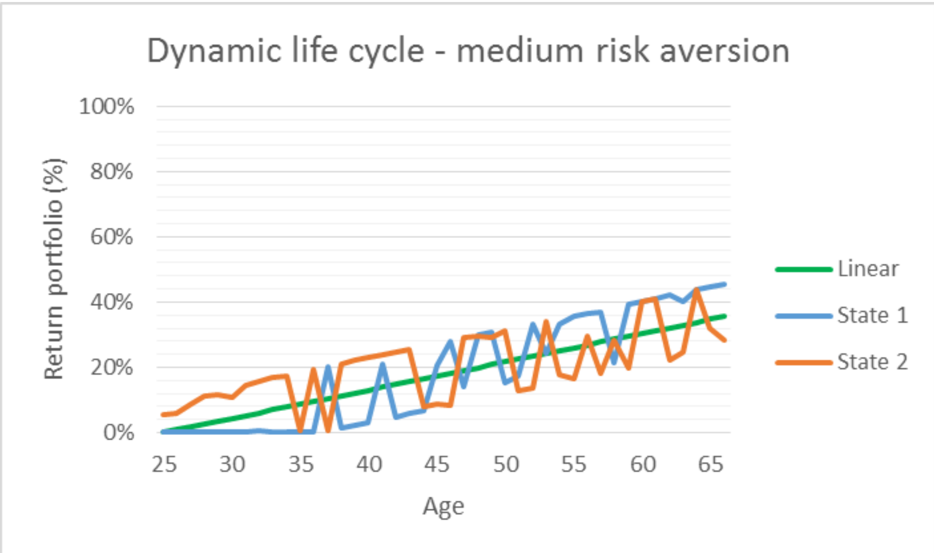

Figure 5.3: Dynamic life cycle medium risk aversion.

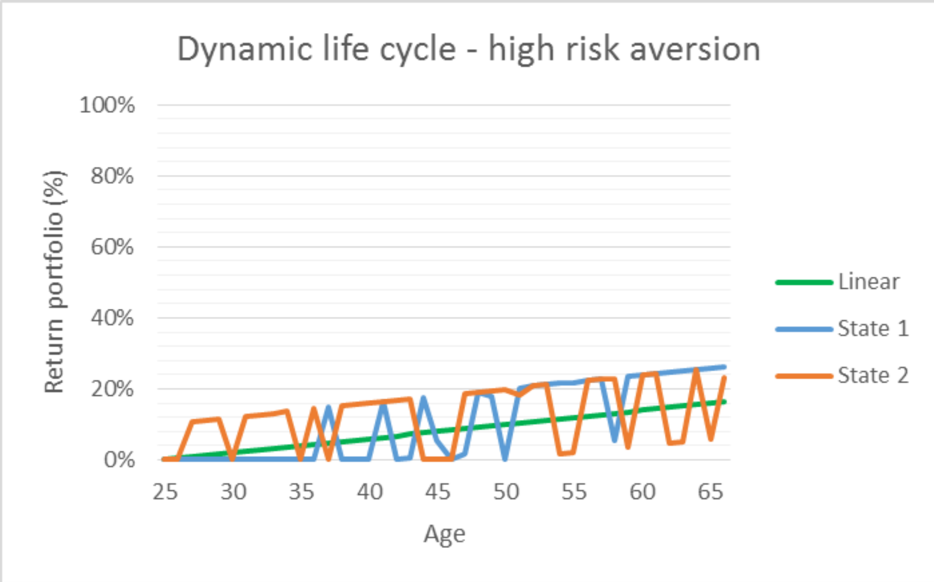

Figure 5.4: Dynamic life cycle high risk aversion.

Based on Figure 3.5 and the transition probability matrix it can be concluded that the model will be in State 1 most of the time. Therefore, the blue lines in the three figures above are the dominant life cycles. This means that the allocation to the return portfolio will be mostly defined according to these blue life cycles. In case the system will shift to State 2, another life cycle will be applied. This life cycle is indicated by the orange lines. But does adding the second dimension to the dynamic life cycle also provide extra added value? This question is related to Research Question 2. In order to answer this question the certainty equivalents and the coverage ratio distributions are given below.

	Certainty equivalents		
Risk aversion	**Low**	**Medium**	**High**
Optimised linear life cycle	144.6%	71.1%	34.6%

Table 5.2: Certainty equivalents of the dynamic life cycles.

The CEs of the dynamic life cycle are higher than the CEs of the optimised linear life cycles. This indicates that a dynamic life cycle is preferred over a linear life cycle, independent of the risk aversion. In order to get some more insight about the performance and the risk trade-off of the different life cycle designs the coverage ratio distributions are given in the tables below for the three risk aversion coefficients.

	Coverage ratio distribution					CVAR(5%)
Percentiles	**5%**	**25%**	**50%**	**75%**	**95%**	
Linear life cycle	51%	109%	173%	271%	499%	37%
Dynamic life cycle	57%	100%	159%	247%	463%	41%

Table 5.3: Statistics of the dynamic life cycles with low risk aversion.

	Coverage ratio distribution					CVAR(5%)
Percentiles	**5%**	**25%**	**50%**	**75%**	**95%**	
Linear life cycle	52%	91%	132%	192%	315%	41%
Dynamic life cycle	55%	94%	133%	190%	307%	44%

Table 5.4: Statistics of the dynamic life cycles with medium risk aversion.

	Coverage ratio distribution					CVAR(5%)
Percentiles	**5%**	**25%**	**50%**	**75%**	**95%**	
Linear life cycle	45%	80%	119%	173%	290%	36%
Dynamic life cycle	50%	85%	121	172%	287%	40%

Table 5.5: Statistics of the dynamic life cycles with high risk aversion.

The 5^{th} and 25^{th} percentiles of the dynamic life cycle are higher than those of the optimised linear life cycle, independent of the risk aversion. This indicates that the dynamic life cycle gives some extra downside protection in comparison to the linear one. This observation is also supported by CVAR of the five percent tail, with a higher CVAR for the dynamic life cycle compared to the optimised linear life cycle. On the other hand, the 75^{th} and 95^{th} percentiles of the dynamic life cycle are somewhat lower than the same percentiles of the optimised linear life cycle, especially when the low risk aversion parameter is used.

Altogether, we can conclude that the dynamic life cycle gives some extra downside protection but at the expense of some upside potential. This leads to less variability and gives more certainty. The coverage ratio distributions in combination with the utility values show that the dynamic life cycle is preferred over the linear life cycle.

5.3 Sensitivity analysis

We have conducted the sensitivity analyses for the linear life cycles because these are used as an anchor point for the dynamic life cycle. One component that partly explains the big difference between the three optimised linear life cycles is the risk aversion coefficient. The optimised linear life cycles using different risk aversion coefficient are given in the figure below. We have found that the optimised linear life cycle is highly dependent on the risk aversion coefficient. The path of the life cycle changes significantly in the range of a risk aversion coefficient between 1.5 and 3. The life cycles with higher risk aversion coefficients look more like each other.

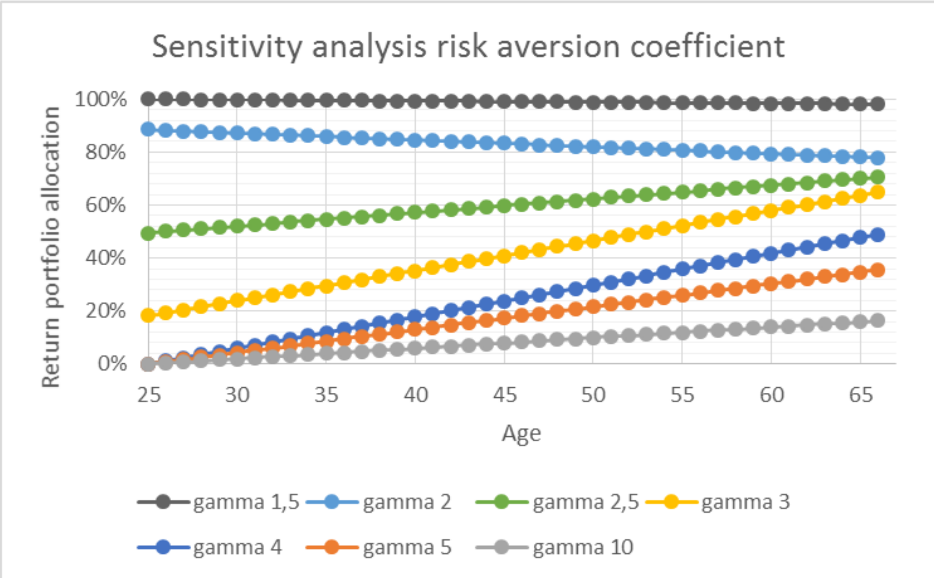

Figure 5.5: Optimised linear life cycles using different risk aversion coefficients.

Another component that might influence the optimised linear life cycle is the calibration of the equity returns. The results we have just described are based on the average excess return calibration of 3.5% annually. In order to investigate the sensitivity of the calibration, we have optimised the life cycles using equity returns calibrated based on other average excess return percentages. We have not done the sensitivity analysis for the matching returns because these are calibrated based on the forwards which are a representation of the current market observation. In the table below the starting and end value are given for the linear life cycles when the equity returns are calibrated on a different average excess return.

Risk aversion	**Low**	**Medium**	**High**
Average Excess return			
5,5%	100% - 100%	0% - 45%	0% - 21%
4,5%	99% - 89%	0% - 40%	0% - 18%
3,5%	88% - 78%	0% - 35%	0% - 16%
2,5%	74% - 66%	0% - 31%	0% - 15%
1,5%	56% - 55%	0% - 27%	0% - 13%
0,5%	36% - 44%	0% - 23%	0% - 12%

Table 5.6: Life cycles with different excess return calibrations.

With this analysis we have showed that the calibration of the equity returns has the most impact on the life cycle with the low risk aversion. Decreasing the average excess returns result in a drastic decrease of the start and end value of the return portfolio percentage when the risk aversion is low. This decline is not as large when the risk aversion is medium or high. In this case, the difference is only a couple of percentages for the end value. In addition, for the medium and high risk aversion coefficients, the return portfolio allocation linearly increases in every case. This is not true for the low risk aversion parameter. In some cases the return portfolio allocation increases while in other cases the return portfolio allocation remains constant or decreases. This leads to the observation that the calibration of the equity returns has the highest impact on the life cycle for a participant with a low risk aversion.

5.4 Conclusion

In this chapter we have explained the optimisations to maximise the average utility by changing the life cycle. In the first section of this chapter we have discussed the optimised linear life cycles. It turned out that the optimised linear life cycles result in higher CEs than those of the traditional, constant, and reverse life cycles. This showed that the current 'old-fashion' life cycles do not give the optimal results. It should be noted that the linear life cycle is dependent on the risk aversion coefficient, especially a low risk aversion coefficient has quite some impact on the shape of the life cycle. The higher the risk aversion parameter the lower the impact on the life cycle. We have used these linear life cycles as a reference point for the dynamic life cycle. The dynamic life cycle has two key features, the non-linearity and the market dependency. Throughout the working life of a participant the allocation to the return portfolio can be changed depending on the market conditions. Transaction costs to rebalance the portfolios are not taken into account but to somewhat limit the life cycle variability we have determined a minimum and maximum value based on the optimised linear life cycle. The results showed that the dynamic life cycle results in higher CEs than the linear life cycle, independent of the risk aversion coefficient. Based on the results it is possible to answer Research Question 2, which is formulated as follows:

Research Question 2: *What is the impact of adjusting the return versus matching portfolio based on the target pension benefit throughout the working life of the participant?*

The results showed that a dynamic life cycle in which the allocation between return and matching portfolio is managed against the target pension benefit throughout the accumulation phase results in higher utility values than the linear life cycle in which the allocation is defined based only on the age of the participant. The dynamic life cycle gives more downside protection, which is an attractive result given the uncertain financial market. But this life cycle comes at the expense of some upside potential. The value of the downside protection is not proportional to the value of the upside potential. This concept is incorporated in the utility function. Therefore it is justified to compare the linear life cycles with the dynamic life cycles and can we conclude that the dynamic life cycle outperforms the linear life cycle.

Chapter 6

Conclusion

We summarise the results from the analysis in this concluding chapter. The first paragraph is devoted to the conclusion, answering the main research question. In Section 6.2 we discuss the method and results. In the next section we cover the limitations and in the last section we address some suggestions for further research.

6.1 Conclusions

In the Netherlands a Defined Benefit pension plan is mostly used but is losing ground to a Defined Contribution pension plan. This increases the importance of the life cycle, which is the investment policy used in the Defined Contribution pension plan. To our knowledge, not much research has been conducted that addresses the added value of an optimised life cycle based on the state of the market in comparison to a linear life cycle. We have developed a method is developed to analyse and compare different life cycle designs given stochastic interest rates and equity returns in a Dutch pension context. In this research we have started with the capital calculations to compute the coverage ratio. Subsequently, we have used the coverage ratio as one of the inputs in the utility function and certainty equivalent calculation, which together form the assessment framework. The other input is the risk aversion parameter. To include different risk aversion perceptions, we have created three profiles with their own risk aversion coefficient; low, medium, and high. The assessment framework together with the interest rates and equity returns simulation model form the basis for analysing life cycle designs and ultimately answering the following main research question:

How should the life cycle be designed for a Defined Contribution pension plan?

First, we have evaluated three existing life cycles to get a first impression about their performance. In the traditional life cycle most of the capital is allocated to the return portfolio in the beginning of the accumulation phase and is linearly transformed to a more matching-like portfolio as the retirement date approaches. The reverse life cycle is the opposite of the traditional life cycle. More capital is allocated to the matching portfolio at the beginning and more to the return portfolio as the participant ages. The third life cycle is the constant life cycle which keeps the return portfolio allocation constant over time. Based on the analysis of the three existing life cycles Research Question 1 is answered. The CEs of the traditional, constant, and reverse life cycles showed that the order of preference depends on the risk aversion. The reverse, constant, and traditional life cycles are preferred in case a participant has a low, medium, or high risk aversion respectively.

We have answered Research Question 2 based on the results of the optimisation of the life cycle. First, we have optimised the life cycle under the linearity constraint. We have found that the risk aversion coefficient has a big impact on the design of the life cycle, in particular for the low risk aversion profile. The life cycle starts with a return portfolio allocation of around ninety percent and slightly decreases over time in case a participant has a low risk aversion. The life cycles of the medium and high risk aversion profiles begin with an allocation to the return portfolio of around zero percent and end around forty and twenty percent respectively. The optimised life cycles resulted in higher CEs than the traditional life cycle. In addition, with the dynamic life cycle analysis we have showed that the usage of the current traditional life cycle is debatable. The traditional life cycle is a linear function of age and ignores the changing market conditions over time. The dynamic life cycle used in this study is market dependent and does not necessarily has to be a linear function. The results of the analysis showed that using a dynamic life cycle leads to more certainty and some extra downside protection than a linear life cycle. The dynamic life cycle gives more freedom to respond better to the market conditions.

All in all, the knowledge obtained from these two research questions form the answer to the main research question. This research substantiates the added value of a dynamic life cycle in comparison to linear life cycles. The results also question the rationale for the traditional life cycle as the prevalent practice among pension funds.

6.2 Discussion

Our results can partly explained by diving into the CRRA utility function and contribution table. We have constructed the contribution table in such a way that the pension ambition of 1.875% per year is achievable by paying the corresponding premiums, assuming all capital is invested in the matching portfolio, when using the spot interest rate at the end March 2019. This ensures that the coverage ratio is hundred percent in the deterministic scenario. When the stochastic interest rates and equity returns are used the average coverage ratio is around 125%. This means that the pension ambition is generally achieved, even without taking any risky investments. The fact that the average coverage ratio is around 125% indicates that the contribution table and/or accrual percentage should be dynamic in order to make the model cost-covering (to get an average coverage ratio of around 100%) when using stochastic interest rates and equity returns. So, for example, a higher accrual percentage can be achieved when the coverage ratio is above the hundred percent. Due to the fact that the coverage ratio is already above the target, given that all capital is allocated to the matching portfolio, it makes more sense that at the beginning of the dynamic life cycle for the medium and high risk aversion profiles most of the capital is still allocated to the matching portfolio. This is because the coverage ratios are used as input in the utility function. As the coverage ratio increases, it is worth less and less to take extra risk trying to get an even higher coverage ratio. Two of the three optimised dynamic life cycles look more like the reverse life cycle so apparently sometimes it is worth taking the extra investment risk in order to get a better coverage ratio and thus a higher utility value.

In addition, the risk aversion parameters we have used in this study are based on the report of EDHEC, which are two, five, and ten for the low, medium, and high risk aversion profiles respectively. It can be discussed whether these parameters have realistic values. The results of the life cycle analysis have showed that risk aversion has a substantial influence on the design of the life cycle. The shapes of the optimised life cycles are quite dependent on the risk aversion coefficient. Also when looking at the CE and coverage ratio distribution of the high risk aversion profile, it seems that a risk aversion coefficient of ten means that a participant is really risk averse because the CE is even lower than the 5^{th} percentile of the coverage ratios. This means that a participant is very keen to avoid the risk while there is a relatively high chance on a better pension result. Apparently, ten is an extreme risk aversion coefficient which has quite some impact on the results. In a recently published report of the DNB risk aversion coefficients of three, five, and seven are used (De Nederlandsche Bank, 2019a). This substantiates our opinion that the main focus should lie on the medium risk aversion profile because the DNB states that a risk aversion parameter of five is a commonly used value in the academic literature. We have used the other two risk aversion parameters to give some insight into the change of the life cycle design as a result of another risk aversion perception of a participant. The specific coefficients of the low and high risk aversion profiles are more subject to discussion but do not undermine the added value of a dynamic life cycle.

Besides, in this research we have incorporated the state of the economy by using a Markov regime switching component in the simulation model. We have assumed that there are two states. But is that also true in reality? It can be discussed that the assumption does not hold in the real world. However, a two state model provides the ability to make the life cycle design market dependent. In order to improve the flexibility of the model one can think of increasing the number of states or performing a multistage optimisation. This might increase the accuracy of the results.

Finally, as already said before, the results of this study contradict the currently used traditional life cycle. An important note here is that we have done the optimisation for just one age cohort. This means that we have tested the dynamic life cycle design for a person that starts working at an age of 25 and retires at an age of 67. As a result, it is not possible to fully compare the results of this study with the existing life cycles. More research needs to be done with different cohorts, which have their own characteristics such as investment horizon, already accumulated capital, and contribution percentages. It is very likely that the life cycle design would be different compared to those in this study when analysing the life cycle design for other cohorts. Therefore, when interpreting the result of this study, it must be kept in mind that this study is about a participant that starts working at an age of 25 and retires at an age 67.

6.3 Limitations

The limitations of this research are mainly related to underlying assumptions. On the one hand, we have made assumptions which serve as inputs for the life cycle analysis, such as the risk aversion coefficient, starting and retirement age, mortality table, contribution table, career path, and so on. The results showed that the life cycle design is highly dependent on the risk aversion coefficient used in the CRRA utility function. Determining risk aversion on a individually basis is an extensive task. To keep focused on the method to analyse different life cycle designs instead of spending a lot of time investigating the individual risk aversion, we have created three risk aversion profiles based on a paper of EDHEC (2014). We have done a sensitivity analysis, but it is still somewhat unclear which risk aversion coefficients should be used. Some other assumptions are related to Dutch pension regulations, for example retirement age, premium, and accrual percentages. Because these topics are still part of the pension debate during the writing of this study, these might be outdated if regulations change. One of the assumptions is that a participant starts working at an age of twenty five. Obviously, this is not always the case in reality, for example if someone starts working at an older age. Because this research is more about evaluating different life cycle designs the results are considered justified.

On the other hand, we have made assumptions in order to simplify the life cycle assessment method. For example, transaction cost, indexation, and leverage are not taken into account. In reality, these do play a role but are probably not the most decisive factors. Also the assumptions behind the interest rate and equity return simulation model falls into this category of assumptions. One of them is the choice for macroeconomic factors such as GDP, inflation, and equity returns. It can be discussed which factors should all be taken into account as an accurate representation of the economy. There are probably more macroeconomic factors but the ones used in this research are certainly three important factors. Other examples of simplifying assumptions are the ignoring of the partner pension and the possibility to continue investing after retirement. This reduced the complexity but they do play a role in reality.

Additionally, one limitation that is not related to the underlying assumptions is the data availability. In this study, we have used zero coupon swap rates from 2000 to 2019 as input for the simulation model. Now the question is whether this period is long enough to be a good representation of the different regimes. This could have influences on the coefficient and covariance matrices.

6.4 Further research

Some limitations just described can be used as inspiration for possible further research topics. Al ready mentioned, the Dutch pension system is under review and therefore it is possible that regulatory changes take place that could have an impact on the performance of the life cycle designs. Therefore it is interesting to monitor the pension debates and regulatory changes.

Next to that, it is useful to do research into the risk aversion of the participants. We have showed that the risk aversion has quite some impact on the life cycle design. In case a DC pension plan becomes reality for MN, it is of great relevance to have some insight into the risk aversion of their participants. This information can be helpful in deciding which risk aversion coefficients should be used and therefore what kind of life cycles are offered.

Another interesting research topic is related to the investment portfolio. In this study we have limited the investments to the return and matching portfolio. An important choice still has to be made within the matching portfolio, namely to what extent the interest rate should be hedged. The percentage allocated to the matching portfolio is not necessarily the same as the interest rate hedge. It is also possible to use leverage to increase the interest rate hedge. This extra decision should be incorporated in the optimisation of the life cycle. Adding this extra decision would increase complexity and is therefore a suggestion for further research.

Additionally, this research can be expanded by analysing what the impact is on the life cycle in case a participant does not start at an age of 25 but at a later age. This would have a big impact on the capital accumulation and the available time to build up a decent pension. This probably has an influence on the decision to allocate the capital to the matching or return portfolio. Therefore we recommend to test the dynamic life cycles designs for other age cohorts as well.

Finally, it is interesting to do more research into the transaction cost of switching between the return and the matching portfolio. In this research we have not taken the transaction costs into account. But does the use of a dynamic life cycle still pay off if the transaction costs are included? Intuitively, yes. Presumably switching will only take place in the case of large market corrections. Our intuition is based on a life cycle analysis for just one person. It is also possible that there is some sort of balance between the transaction of different generations of participants. This could mean that the impact of the transaction cost is limited. But more research is needed to prove this.

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Appendix A

Formula sheet capital calculations

$$\mathrm{S_t} = \begin{cases} S_{t-1} \times (1 + I_t + IR) \times (1 + CP_t)\ , & t \leq 67 \\ 0\ , & t \geq 67 \end{cases}\ , \tag{A.1}$$

where S_t, I_t, and CP_t are the salary, inflation, and career path percentage at time *t* respectively and IR is the inflation raise.

$$\mathrm{F_t} = \begin{cases} F_{t-1} \times (1 + I_t + IR)\ , & t \leq 67 \\ 0\ , & t \geq 67 \end{cases}\ , \tag{A.2}$$

where F_t and I_t are the franchise and inflation at time *t* respectively and IR is the inflation raise.

$$\mathrm{PB_t} = \begin{cases} S_t - F_t\ , & t \leq 67 \\ 0\ , & t \geq 67 \end{cases}\ , \tag{A.3}$$

where PB_t, S_t, and F_t are the pension benefit, salary, and franchise at time *t* respectively.

$$\mathrm{P_t} = \begin{cases} PB_t \times AP_t\ , & t \leq 67 \\ 0\ , & t \geq 67 \end{cases}\ , \tag{A.4}$$

where P_t, PB_t, and AP_t are the premium, pension benefit, and accrual percentage at time *t* respectively.

$$\mathrm{PBF_t} = \begin{cases} \frac{CU_{t-1}}{A_t}\ , & t \leq 67 \\ 0\ , & t \geq 67 \end{cases}\ , \tag{A.5}$$

where PBF_t, CU_t, and A_t are the pension benefit, capital ultimo, and annuity factor at time t respectively.

$$REUR_t = (CU_{t-1} + P_t + PBF_t) \times ((LC_t \times ROE_t) + ((1 - LC_t) \times ROI_t)) \ , \tag{A.6}$$

where $REUR_t$, CU_t, P_t, PBF_t, LC_t, ROE_t, and ROI_t are the return EUR, capital ultimo, premium, pension benefit, life cycle, return on equity and return on interest rate at time *t* respectively.

$$CU_t = CU_{t-1} + P_t + PBF_t + REUR_t \ , \tag{A.7}$$

where CU_t, P_t, PBF_t, and $REUR_t$ are the capital ultimo, premium, pension benefit, and return EUR at time *t* respectively.

Appendix B

Inputs capital calculations

Age	Year	Inflation	Contribution	Career path	Age	Year	Inflation	Contribution	Career path
25	1	0,76%	16,5%	8,558%	**46**	22	1,86%	20,7%	0,000%
26	2	0,96%	16,7%	6,663%	**47**	23	1,87%	21,0%	0,000%
27	3	1,03%	16,9%	5,261%	**48**	24	1,89%	21,3%	0,000%
28	4	1,08%	17,1%	3,465%	**49**	25	1,90%	21,6%	0,000%
29	5	1,17%	17,2%	3,220%	**50**	26	1,94%	21,9%	0,000%
30	6	1,22%	17,4%	2,974%	**51**	27	1,98%	22,2%	0,000%
31	7	1,28%	17,6%	2,658%	**52**	28	1,98%	22,6%	0,000%
32	8	1,33%	17,8%	2,343%	**53**	29	1,95%	22,9%	0,000%
33	9	1,42%	17,9%	2,244%	**54**	30	1,88%	23,3%	0,000%
34	10	1,51%	18,1%	2,146%	**55**	31	1,83%	23,7%	0,000%
35	11	1,55%	18,3%	1,552%	**56**	32	1,83%	24,2%	0,000%
36	12	1,60%	18,5%	0,958%	**57**	33	1,82%	24,6%	0,000%
37	13	1,64%	18,7%	1,144%	**58**	34	1,82%	25,1%	0,000%
38	14	1,68%	18,9%	1,331%	**59**	35	1,82%	25,6%	0,000%
39	15	1,72%	19,1%	1,090%	**60**	36	1,81%	26,1%	0,000%
40	16	1,75%	19,3%	0,848%	**61**	37	1,81%	26,7%	0,000%
41	17	1,79%	19,5%	0,494%	**62**	38	1,80%	27,2%	0,000%
42	18	1,83%	19,7%	0,794%	**63**	39	1,80%	27,8%	0,000%
43	19	1,85%	19,9%	0,786%	**64**	40	1,80%	28,4%	0,000%
44	20	1,85%	20,2%	0,491%	**65**	41	1,79%	29,0%	0,000%
45	21	1,85%	20,4%	0,245%	**66**	42	1,79%	29,6%	0,000%

Appendix C

Mortality table

Age	Mortality rate	Age	Mortality rate	Age	Mortality rate	Age	Mortality rate
25	0,05%	**56**	0,48%	**87**	12,74%	**118**	59,28%
26	0,05%	**57**	0,54%	**88**	14,23%	**119**	59,73%
27	0,04%	**58**	0,59%	**89**	15,82%	**120**	60,14%
28	0,05%	**59**	0,64%	**90**	17,32%	**121**	100,00%
29	0,05%	**60**	0,72%	**91**	18,98%		
30	0,05%	**61**	0,79%	**92**	20,78%		
31	0,05%	**62**	0,87%	**93**	22,65%		
32	0,06%	**63**	0,96%	**94**	24,60%		
33	0,06%	**64**	1,07%	**95**	26,61%		
34	0,06%	**65**	1,18%	**96**	28,66%		
35	0,07%	**66**	1,30%	**97**	30,74%		
36	0,07%	**67**	1,43%	**98**	32,83%		
37	0,07%	**68**	1,59%	**99**	34,91%		
38	0,08%	**69**	1,75%	**100**	36,96%		
39	0,09%	**70**	1,92%	**101**	38,97%		
40	0,09%	**71**	2,16%	**102**	40,91%		
41	0,10%	**72**	2,41%	**103**	42,78%		
42	0,11%	**73**	2,67%	**104**	44,57%		
43	0,13%	**74**	2,99%	**105**	46,26%		
44	0,14%	**75**	3,33%	**106**	47,85%		
45	0,15%	**76**	3,74%	**107**	49,33%		
46	0,17%	**77**	4,21%	**108**	50,71%		
47	0,20%	**78**	4,71%	**109**	51,98%		
48	0,21%	**79**	5,24%	**110**	53,14%		
49	0,24%	**80**	5,88%	**111**	54,21%		
50	0,26%	**81**	6,64%	**112**	55,18%		
51	0,30%	**82**	7,34%	**113**	56,05%		
52	0,32%	**83**	8,25%	**114**	56,84%		
53	0,36%	**84**	9,20%	**115**	57,56%		
54	0,39%	**85**	10,25%	**116**	58,19%		
55	0,44%	**86**	11,49%	**117**	58,77%		

Appendix D

Cholesky decomposition

Copula models have become very popular and well-studied among the scientific community. They are used to model the relationship between random variables. Cholesky decomposition is a copula under the assumption of a Gaussian distribution. We can use Cholesky decomposition due to the assumption of the random variables distribution given in Equation 3.8. It is widely used to transform uncorrelated random variables to correlated random variables for simulation purposes. We have also used this in our research because in the multivariate regression model the random variables are correlated. To transform the random variables to correlated random variables with the use of Cholesky decomposition, we have used the covariance matrices given in the tables below as input. The Cholesky decomposition is represented in the following formula:

$$A = L \times L^T \tag{D.1}$$

Where *A* is a symmetric positive definite matrix and *L* is the lower triangular matrix.

L is called the Cholesky factor of *A*. Once again we have used Matlab to perform the Cholesky decomposition and to transform the uncorrelated random variables into correlated random variables. In this research we have assumed a two-state Markov regime switching component and therefore the Cholesky decomposition is done twice, for both State 1 and State 2. First, we have converted the covariance matrix into the correlation matrix. Then by using the *Chol* Matlab command we have computed the Cholesky matrix based on the correlation matrix. The next step is to fill a matrix with random variables and multiply it by the Cholesky matrix. Finally, this is multiplied by the standard deviation, which is the square root of the diagonal values of the covariance matrix. This results in a matrix with correlated random variables. To test the correctness of the correlated random variables, the standard deviation of the correlated random variables should be approximately the same as the standard deviation extracted from the covariance matrix. We have done all these steps are done for both of the states, so as a result two matrices with correlated random variables have been constructed and used as inputs for the simulation.

	Level	**Slope**	**Curvature**	**Inflation**	**GDP**	**MSCI**
Level$_{t-1}$	3.62E-06	-3.52E-06	-2.68E-06	1.63E-07	2.91E-08	3.16E-06
Slope$_{t-1}$	-3.52E-06	5.11E-06	-1.26E-06	5.62E-08	-8.40E-09	-4.60E-06
Curvature$_{t-1}$	-2.68E-06	-1.26E-06	3.16E-05	-1.57E-08	-6.25E-08	4.79E-05
Inflation$_{t-1}$	1.63E-07	5.62E-08	-1.57E-08	2.11E-06	4.43E-09	-8.56E-08
GDP$_{t-1}$	2.91E-08	-8.40E-09	-6.25E-08	4.43E-09	6.09E-09	-2.59E-07
MSCI$_{t-1}$	3.16E-06	-4.60E-06	4.79E-05	-8.56E-08	-2.59E-07	1.29E-03

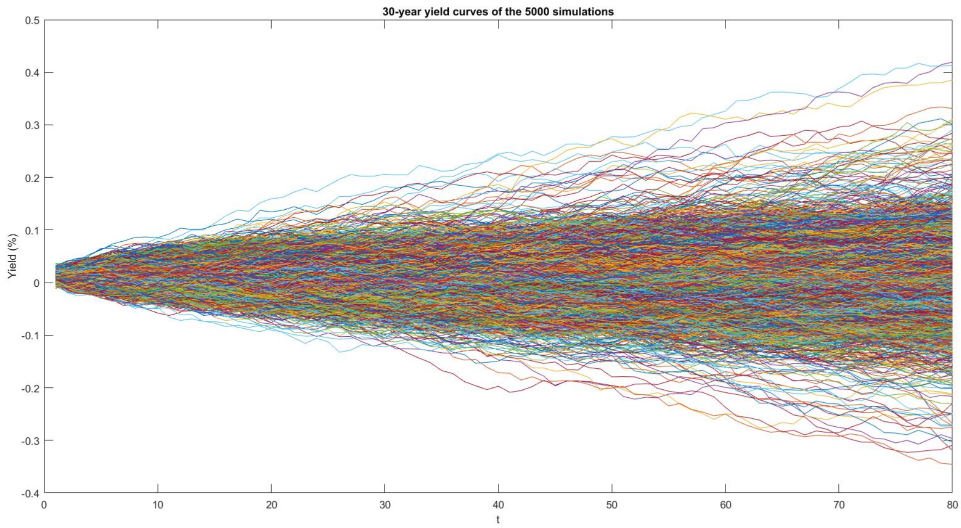
Table D.1: Covariance matrix State 1.

	Level	**Slope**	**Curvature**	**Inflation**	**GDP**	**MSCI**
Level$_{t-1}$	3.93E-06	-6.58E-06	-3.26E-06	3.93E-07	5.15E-08	3.02E-05
Slope$_{t-1}$	-6.58E-06	2.54E-05	4.20E-07	1.15E-07	2.32E-07	5.99E-05
Curvature$_{t-1}$	-3.26E-06	4.20E-07	6.52E-05	-1.30E-08	-1.72E-07	-2.07E-05
Inflation$_{t-1}$	3.93E-07	1.15E-07	-1.30E-08	3.48E-06	4.42E-07	7.59E-08
GDP$_{t-1}$	5.15E-08	2.32E-07	-1.72E-07	4.42E-07	1.86E-07	1.74E-06
MSCI$_{t-1}$	3.02E-05	5.99E-05	-2.07E-05	7.59E-08	1.74E-06	3.32E-03

Table D.2: Covariance matrix State 2.

Appendix E

Plot simulated yield curves



Appendix F

Matlab codes

F.1 Assessment framework

Capital calculations

```
%------------------------------------------------------------------------
%{
THE DATA BELOW MUST BE LOADED TO BE ABLE TO RUN THIS SCRIPT
use spreadsheet link:
KansOverleving(uit sheet 3), Inflatie, CarrierePad, Premie, ...
    RendementAandelen,
RendementRente, SpotRate, GegevenOverlevingskans, Leeftijd(uit ...
    sheet 3),LifeCyle, VasteAnnuiteit
%}
%------------------------------------------------------------------------
%Calculating annuity factor
Rate = SpotRate;
[Koopsom,SubKoopsom] = ...
    KoopsomFunctie(Leeftijd,KansOverleving,Rate);     %functie ...
    aanroepen
%------------------------------------------------------------------------
%CAPITAL DEVELOPMENT CALCULATIONS
%basic input data
beginsalaris = 27000;
pensioenleeftijd = 67;
franchise = 15304;
opslaginflatie = 0.005;

gamma = [2 5 10];
%------------------------------------------------------------------------
```

```
DC_leeftijd_idx = 1;
DC_dt_idx = 2;
DC_salaris_idx = 3;
DC_franchise_idx = 4;
DC_pg_idx = 5;
DC_premie_idx = 6;
DC_uitkering_idx = 7;
DC_rEUR_idx = 8;
DC_kvoorbio_idx = 9;       %kapitaalvoorbio
DC_rBio_idx = 10;
DC_knabio_idx = 11;        %kapitaalnabio

vullen = [];                                  %het cre eren van ...
    een array voor de kapitaalberekeningen
vullen(1,DC_salaris_idx) = beginsalaris;      %initialiseren ...
    beginsalaris
vullen(1,DC_franchise_idx) = franchise;       %initialiseren ...
    franchise
vullen(1,DC_knabio_idx) = 0;                  %initialiseren ...
    kapitaal ultimo

VT_Leeftijd_idx = 1;
VT_Periodes_idx = 2;
VT_Kans67_idx = 3;
VT_Uitkering_idx = 4;

AantalScenarios = 5000;

DG = zeros(1,AantalScenarios);
utility = zeros(AantalScenarios,length(gamma));
%------------------------------------------------------------
for scenario = 1:AantalScenarios
    LeeftijdStap = 25;
    for dt = 1:43
        vullen(dt,DC_leeftijd_idx) = LeeftijdStap;
        if dt == 1
            LeeftijdStap = LeeftijdStap - 1;
        end
        if LeeftijdStap >= min(Leeftijd)
            if LeeftijdStap < pensioenleeftijd
                vullen(dt,DC_dt_idx) = dt -1;
                vullen(dt,DC_salaris_idx) = ...
                    vullen(dt-1,DC_salaris_idx) * (1+Inflatie(dt-1) ...
                    + ...                       opslaginflatie) * ...
                    (1+CarrierePad(dt-1));
                vullen(dt,DC_uitkering_idx) = 0;
            else
                vullen(dt,DC_dt_idx) = pensioenleeftijd - ...
```

```
                    min(Leeftijd);
                vullen(dt,DC_salaris_idx) = 0;
                vullen(dt,DC_uitkering_idx) = - ...
                    vullen(dt-1,DC_knabio_idx) / Koopsom(dt-1);
            end

            vullen(dt,DC_franchise_idx) = ...
                vullen(dt-1,DC_franchise_idx) * (1+Inflatie(dt-1) + ...
                opslaginflatie);
            vullen(dt,DC_pg_idx) = ...
                max(0,(vullen(dt,DC_salaris_idx) - ...
                vullen(dt,DC_franchise_idx)));
            vullen(dt,DC_premie_idx) = vullen(dt,DC_pg_idx) * ...
                Premie(dt-1);

            vullen(dt,DC_rEUR_idx) = (vullen(dt-1,DC_knabio_idx) + ...
                vullen(dt,DC_premie_idx) + ...
                 vullen(dt,DC_uitkering_idx)) *(LifeCycle(dt-1) * ...
                    Aandelen(dt-1,scenario) + ...
                 (1-LifeCycle(dt-1))*ReturnRente(dt-1,scenario));

            vullen(dt,DC_kvoorbio_idx) = ...
                vullen(dt-1,DC_knabio_idx) + ...
                vullen(dt,DC_premie_idx) + ...
                 vullen(dt,DC_uitkering_idx) + vullen(dt,DC_rEUR_idx);
            vullen(dt,DC_rBio_idx) = 0;

            vullen(dt,DC_knabio_idx) = vullen(dt,DC_kvoorbio_idx) ...
                + vullen(dt,DC_rBio_idx);

        end
        LeeftijdStap = LeeftijdStap + 1;
    end

%------------------------------------------------------------
    %Eerst het berekenen van de DG en daarna invullen in de ...
        utility functie
    maturity=1:78;
    PV = sum(Uitkeringen(1,:)./(1+SpotRate(2:79)).^maturity);
    for i = 1:42
        PV = PV *(1+ReturnRente(i,scenario));
    end
    DG(scenario) = vullen(43,11) / PV;

    for riskaversion = 1:length(gamma)
            utility(scenario,riskaversion) = ...
                DG(scenario)^(1-gamma(riskaversion)) / ...
                (1-gamma(riskaversion));
```

```
    end
end

MeanUtilityGamma2 = mean(utility(:,1));
MeanUtilityGamma5 = mean(utility(:,2));
MeanUtilityGamma10 = mean(utility(:,3));
ExpectedArray = [MeanUtilityGamma2 MeanUtilityGamma5 ...
    MeanUtilityGamma10];

ExpectedUtility = array2table(ExpectedArray);
ExpectedUtility.Properties.VariableNames = ...
    {'Gamma_is_2','Gamma_is_5','Gamma_is_10'};
```

Annuity factor

```
%Calculate annuity factors (Dutch: Koopsom)
function [Koopsom,SubKoopsom] = ...
    KoopsomFunctie(Leeftijd,KansOverleving,SpotRate)

InputTable = [Leeftijd KansOverleving];

pensioenleeftijd = 67;
SubKoopsom = zeros(80,111);
Koopsom = zeros(81,1);

for LeeftijdStap = min(Leeftijd):104
    for dt = 0:(length(SpotRate)-1)

        if LeeftijdStap + dt > max(Leeftijd)
            SubKoopsom(LeeftijdStap-24,dt+1) = 0;
        else
            if LeeftijdStap + dt < pensioenleeftijd
                SubKoopsom(LeeftijdStap-24,dt+1) = 0;
            else
                elementToFind1 = LeeftijdStap + dt;
                elementToFind2 = LeeftijdStap;
                colToReturn = 2;

               SubKoopsom(LeeftijdStap-24,dt+1) = ...
                   InputTable(elementToFind1 == InputTable(:,1), ...
                   colToReturn) / ...
                    InputTable(elementToFind2 == InputTable(:,1), ...
                        colToReturn);

            end
        end
```

```
            Koopsom(LeeftijdStap-24) = Koopsom(LeeftijdStap-24) + ...
                (SubKoopsom(LeeftijdStap-24,dt+1) / (1+SpotRate(dt+1))^dt);
        end

end

end
```

F.2 Model building

Dynamic Nelson-Siegel

```
% two steps optimization, optimize constant lambda in the DNS
% Level, slope and curvature1
% excluding the macro factors

mytenors=maturity';

% select appropriate period
swapsdata=swapsdata(startval:endval,:);

%$ optimization
options=optimset('Display','iter','TolFun',10^(-25),...
'TolX',10^-8,'MaxFunEvals',1000000,'MaxIter',500);
lambda=0.8;
[lambda, fval]=fminsearch(@(lambda) ...
    optim_lambdas(lambda,mytenors,swapsdata),lambda,options);

%$ Calculate loading and coefficients using optimised lambda
myloadings=LSCfunctie(lambda,mytenors);
mybetas=OLS(swapsdata',myloadings);                   % ...
    Level,Slope,Curvature

%$ Calculate estimates yield
myyields=NS_curves(myloadings,mybetas);

%LSC function

function factors = LSCfunctie(lambda,maturity )
% this function calculates the factor loadings
% lambda=vector (2*1)
% maturity= Nx1 vector of tenors

```

```
J=size(maturity,1);
% factors (L,S,C)
factors=[ones(J,1),(1-exp(-lambda*maturity))./(lambda*maturity),...
(1-exp(-lambda*maturity))./(lambda*maturity)-exp(-lambda*maturity)];

end
```

```
% NS Curves function

function NS_rates = NS_curves(factors,beta )
% computes DNS rates given the betas and factor loadings
% factor= aantal tenors x aantal factoren
% beta=aantal factor x aantal maanden/observaties

NS_rates=factors*beta;
NS_rates=NS_rates';
end
```

```
% OLS function

function beta = OLS(my_y,my_x )
% fits an OLS through data
% y=Nxd vector of data--> N=mat, d=maanden
% x= Nxfactor--> yieldcurve params at time=t

% Calculate the beta estimates.
vc=inv(my_x'*my_x);
beta=vc*(my_x'*my_y);

end
```

```
% optim_lambdas function

function my_error = optim_lambdas(lambda,tenors,swapsdata)
% This function is used to optimize lambda using a two steps approach
% this is the first step

% Compute loadings in DNS-matrix: aantal tenors x aantal factoren
myloadings=LSCfunctie(lambda,tenors);

% Compute coefficients- OLS
mybetas=OLS(swapsdata',myloadings);
%
```

```
% Estime DNS curves
mycurves=NS_curves(myloadings,mybetas);

% Compute error
% penalty
p1=0;
if lambda<0.0 || lambda>0.99
    p1=1000000;
end

my_error=sum(sum(((swapsdata-mycurves).^2).*100000,2))+p1;

end
```

Markov regime switching

```
%Markov regime switching script

%load dependent variabeles
%load independent lagged variabeles

k=2;                                % Number of States
S=[1 1 1 1 1 1 1];                  % Defining which parts of the ...
    equation will switch states

advOpt.distrib='Normal';            % The Distribution assumption ...
    ('Normal', 't' or 'GED')
advOpt.std_method=1;                % Defining the method for ...
    calculation of standard errors. See pdf file for more details
advOpt.useMex=1;                    % Defining use of mex version ...
    of likelihood function
advOpt.diagCovMat=0;

[Spec_Out]=MS_Regress_Fit(dep,indep,k,S,advOpt);
```

Cholesky decomposition

```
function [FinalCorrRVState1,FinalCorrRVState2] = ...
    Cholesky(Horizon,NrVariables,Chol1,Chol2,SigmasState1,SigmasState2)

UncorrRVState1 = normrnd(0,1,Horizon,NrVariables);
UncorrRVState2 = normrnd(0,1,Horizon,NrVariables);

CorrRVState1 = UncorrRVState1 * Chol1;
CorrRVState2 = UncorrRVState2 * Chol2;
```

```

FinalCorrRVState1 = bsxfun(@times,CorrRVState1,SigmasState1);
FinalCorrRVState2 = bsxfun(@times,CorrRVState2,SigmasState2);

end
```

Simulation

```
%Simulation script

NrSimulations = 5000;
NrYears = 80;
Maturity=50;

Horizon = NrYears * 12;
State = zeros(Horizon,1);
Deltas = zeros(Horizon,6);
Absolute = zeros(Horizon,6);
Yield = zeros(NrYears,Maturity,NrSimulations);
AandelenMaandelijks = zeros(Horizon,NrSimulations);

% pre allocation and cholesky decomposition
NrVariables = 6;
FinalCorrRVState1 = zeros(Horizon,NrVariables);
FinalCorrRVState2 = zeros(Horizon,NrVariables);

CovState1 = Spec_Out.Coeff.covMat{1, 1};
CovState2 = Spec_Out.Coeff.covMat{1, 2};
CorrState1 = corrcov(CovState1);
CorrState2 = corrcov(CovState2);
Chol1 = chol(CorrState1);
Chol2 = chol(CorrState2);
SigmasState1 = cov2corr(CovState1);
SigmasState2 = cov2corr(CovState2);

coefmat1=zeros(NrVariables,NrVariables);
coefmat2=coefmat1;
for i=1:NrVariables
    coefmat1(:,i)=Spec_Out.Coeff.S_Param{1,i}(:,1);
    coefmat2(:,i)=Spec_Out.Coeff.S_Param{1,i}(:,2);
end

coefmat=zeros(NrVariables,NrVariables,2);
coefmat(:,:,1)=coefmat1;
coefmat(:,:,2)=coefmat2;

```

```
Tenors=(1:Maturity)';
J=size(Tenors,1);
factors=[ones(J,1),(1-exp(-lambda*Tenors))./(lambda*Tenors),...
    (1-exp(-lambda*Tenors))./(lambda*Tenors)-exp(-lambda*Tenors)];

filter=(12:12:Horizon);

%Calculate simulated Δs
OnthoudenDeltas = zeros(960,6);
for Simulations = 1:NrSimulations
    [FinalCorrRVState1,FinalCorrRVState2] = ...
       Cholesky(Horizon,NrVariables,Chol1,Chol2,...
    SigmasState1,SigmasState2);
    FinalCorrRV(:,:,1)=FinalCorrRVState1;
    FinalCorrRV(:,:,2)=FinalCorrRVState2;
    for Runs = 1:Horizon

        TestState = rand;
        if Runs == 1
            if TestState < ...
               Spec_Out.filtProb(length(Spec_Out.filtProb),1)
                State(Runs) = 1;
            else
                State(Runs) = 2;
            end
        else
            if State(Runs-1) == 1
                if TestState < Spec_Out.Coeff.p(1,1)
                    State(Runs) = 1;
                else
                    State(Runs) = 2;
                end
            else
                if TestState < Spec_Out.Coeff.p(1,2)
                    State(Runs) = 1;
                else
                    State(Runs) = 2;
                end
            end
        end

        if Runs == 1
            Deltas(Runs,:) = StartDelta * coefmat(:,:,State(Runs)) ...
               + FinalCorrRV(Runs,:,State(Runs));
            Absolute(Runs,:) = StartAbsolute(Runs,:) + Deltas(Runs,:);
        else
            Deltas(Runs,:) = Deltas(Runs-1,:) * ...
               coefmat(:,:,State(Runs)) + ...
```

```
                    FinalCorrRV(Runs,:,State(Runs));
                Absolute(Runs,:) = Absolute(Runs-1,:) + Deltas(Runs,:);
            end
        end

    Yield(:,:,Simulations) = Absolute(filter,1:3)*factors' + ...
        Verschil'; % verschil is a matrix determined using excel to ...
        calibrate the yield curves
    AandelenMaandelijks(:,Simulations) = Deltas(:,6);

end

% Calibrate equity returns
Aandelen=zeros(5000,42);

for simulatie = 1:5000
    for jaren = 1:42
        Aandelen(simulatie,jaren) = ...
            prod(1+AandelenMaandelijks(1+12*(jaren-1):12*...
        jaren,simulatie))-1;
    end
end

Aandelen = Aandelen + DeltaAandelen; % DeltaAandelen is a matrix ...
    determined using Excel to calibrate the equity returns
```

Calibrating interest rate

```
%Calculate the average yield curves. This is used as an input for ...
    the Excel file to compute the  V e r s c h i l  matrix
Sommatie = zeros(80,50);

for simulations = 1:5000
    Sommatie = Sommatie + Yield(:,:,simulations);

end

Gemiddeld = transpose(Sommatie / 5000);
```

Interest rate returns

```
%Script to calculate the interest rate returns
NrSimulations = 5000;
clear Discounted
Discounted = transpose(25:67);
```

```
Discounted(:,2:NrSimulations+1) = 0;
ReturnRente = zeros(42,NrSimulations);

Maturity = 1:78;

Yield(:,51:78,:) = 0;
for i = 1:28
    Yield(:,50+i,:) = Yield(:,50,:);
end

for simulation = 1:NrSimulations
    for leeftijd = 1:43
        if leeftijd ==1
            Discounted(leeftijd,simulation+1) = ...
                sum(Uitkeringen(leeftijd,:) ./ ...
                (1+SpotRate(2:79)).^(Maturity));
        elseif leeftijd == 43
            Discounted(leeftijd,simulation+1) = VasteUitkering + ...
                sum(Uitkeringen(leeftijd,:) ./ ...
                (1+Yield(leeftijd-1,:,simulation)).^(Maturity));
        else
            Discounted(leeftijd,simulation+1) = ...
                sum(Uitkeringen(leeftijd,:) ./ ...
                (1+Yield(leeftijd-1,:,simulation)).^(Maturity));
        end

        if leeftijd > 1
            ReturnRente(leeftijd-1,simulation) = ...
                (Discounted(leeftijd,simulation+1) / ...
                Discounted(leeftijd-1,simulation+1)) - 1;
        end
    end
end
```

F.3 Optimisation

Function capital calculations linear life cycle optimisation

```
function [MeanUtilityGamma2] = ...
    CapitalCalculationsOptimalisation(LCvar,SpotRate,Inflatie,...
CarrierePad,Premie,GegevenOverlevingskans,Leeftijd,...
KansOverleving,ReturnRente,Aandelen,Uitkeringen)
%--------------------------------------------------------------------------
```

```
x=(1:42)';
LifeCycle=LCvar(1)*(x-1)/41+LCvar(2); %lineaire functie
%-------------------------------------------------------------
%basic input data
beginsalaris = 27000;
pensioenleeftijd = 67;
franchise = 15304;
opslaginflatie = 0.005;

gamma = [2 5 10];
onthouden = 0;

AantalScenarios = 5000;
%-------------------------------------------------------------
%Initialisation
DC_leeftijd_idx = 1;
DC_dt_idx = 2;
DC_salaris_idx = 3;
DC_franchise_idx = 4;
DC_pg_idx = 5;
DC_premie_idx = 6;
DC_uitkering_idx = 7;
DC_rEUR_idx = 8;
DC_kvoorbio_idx = 9;
DC_rBio_idx = 10;
DC_knabio_idx = 11;

vullen = [];
vullen(1,DC_salaris_idx) = beginsalaris;
vullen(1,DC_franchise_idx) = franchise;
vullen(1,DC_knabio_idx) = 0;
DG = zeros(1,AantalScenarios);
utility = zeros(AantalScenarios,length(gamma));

%-------------------------------------------------------------
%Calculating annuity factor
Rate = SpotRate;
[Koopsom,SubKoopsom] = KoopsomFunctie(Leeftijd,KansOverleving,Rate);
%-------------------------------------------------------------
%CAPITAL DEVELOPMENT CALCULATIONS
for scenario = 1:AantalScenarios
    LeeftijdStap = 25;
    for dt = 1:43
        vullen(dt,DC_leeftijd_idx) = LeeftijdStap;

        if dt == 1
            LeeftijdStap = LeeftijdStap - 1;
        end
```

```

        if LeeftijdStap ≥ min(Leeftijd)
            if LeeftijdStap < pensioenleeftijd
                vullen(dt,DC_dt_idx) = dt -1;
                vullen(dt,DC_salaris_idx) = ...
                    vullen(dt-1,DC_salaris_idx) * (1+Inflatie(dt-1) ...
                    + ...                   opslaginflatie) * ...
                    (1+CarrierePad(dt-1));
                vullen(dt,DC_uitkering_idx) = 0;
            else
                vullen(dt,DC_dt_idx) = pensioenleeftijd - ...
                    min(Leeftijd);
                vullen(dt,DC_salaris_idx) = 0;
                vullen(dt,DC_uitkering_idx) = - ...
                    vullen(dt-1,DC_knabio_idx) / Koopsom(dt-1); ...
                                                        end

            vullen(dt,DC_franchise_idx) = ...
                vullen(dt-1,DC_franchise_idx) * (1+Inflatie(dt-1) + ...
                opslaginflatie);
            vullen(dt,DC_pg_idx) = ...
                max(0,(vullen(dt,DC_salaris_idx) - ...
                vullen(dt,DC_franchise_idx)));
            vullen(dt,DC_premie_idx) = vullen(dt,DC_pg_idx) * ...
                Premie(dt-1);

            vullen(dt,DC_rEUR_idx) = (vullen(dt-1,DC_knabio_idx) + ...
                vullen(dt,DC_premie_idx) + ...                  ...
                vullen(dt,DC_uitkering_idx)) *(LifeCycle(dt-1) * ...
                Aandelen(dt-1,scenario) + ...
                 (1-LifeCycle(dt-1))*ReturnRente(dt-1,scenario));

            vullen(dt,DC_kvoorbio_idx) = ...
                vullen(dt-1,DC_knabio_idx) + ...
                vullen(dt,DC_premie_idx) + ...
                 vullen(dt,DC_uitkering_idx) + vullen(dt,DC_rEUR_idx);
            vullen(dt,DC_rBio_idx) = vullen(dt,DC_kvoorbio_idx) / ...
                GegevenOverlevingskans(dt-1) - ...
                 vullen(dt,DC_kvoorbio_idx);
            vullen(dt,DC_knabio_idx) = vullen(dt,DC_kvoorbio_idx) ...
                + vullen(dt,DC_rBio_idx);

        end
        LeeftijdStap = LeeftijdStap + 1;
    end
%--------------------------------------------------------------------------
    maturity=1:78;
    PV = sum(Uitkeringen(1,:)./(1+SpotRate(2:79)).^maturity);
```

```
    for i = 1:42
        PV = PV *(1+ReturnRente(i,scenario));
    end
    DG(scenario) = vullen(43,11) / PV;

    for riskaversion = 1:length(gamma)
        utility(scenario,riskaversion) = ...
            DG(scenario)^(1-gamma(riskaversion)) / ...
            (1-gamma(riskaversion));
    end
end
MeanUtilityGamma2 = mean(utility(:,1))*-1;
MeanUtilityGamma5 = mean(utility(:,2))*-1;
MeanUtilityGamma10 = mean(utility(:,3))*-1;
%--------------------------------------------------------------------------

end
```

Linear optimisation code

```
%Optimisation linear life cycle
mynonln=@(LCvar) myres(Lcvar);
options=optimset( Display , iter );
aantal=100;
mysol=zeros(aantal,3);

for i=1:aantal
    Lcvar0=rand(1,2);
    [params_opt, F] = fmincon(@(Lcvar) ...
       CapitalCalculationsOptimalisation(Lcvar,SpotRate,Inflatie,...
    CarrierePad,Premie,GegevenOverlevingskans,Leeftijd,...
    KansOverleving,ReturnRente,Aandelen,Uitkeringen)...
    ,Lcvar0,[],[],[],[],[],[],mynonln,options);
    mysol(i,:)=[params_opt,F];
end
```

Inequality constraints linear life cycle optimisation

```
function [c,ceq] = myres(Lcvar)

x=(1:42);

c1=-(Lcvar(1)*(x-1)/41+Lcvar(2));
c2=(Lcvar(1)*(x-1)/41+Lcvar(2))-1;

```

```
c=[c1 c2];
ceq = [];

end
```

Function capital calculations dynamic life cycle optimisation

```
function [MeanUtilityGamma2] = ...
    CapCalcOptimDYN2(LifeCycle,SpotRate,Inflatie,CarrierePad,Premie,...
     Leeftijd,KansOverleving,ReturnRente,Aandelen,Uitkeringen,State)
%--------------------------------------------------------------------------

%basic input data
beginsalaris = 27000;
pensioenleeftijd = 67;
franchise = 15304;
opslaginflatie = 0.005;

gamma = [2 5 10];
onthouden = 0;
AantalScenarios = 5000;
%--------------------------------------------------------------------------
%Initialisation
DC_leeftijd_idx = 1;
DC_dt_idx = 2;
DC_salaris_idx = 3;
DC_franchise_idx = 4;
DC_pg_idx = 5;
DC_premie_idx = 6;
DC_uitkering_idx = 7;
DC_rEUR_idx = 8;
DC_kvoorbio_idx = 9;
DC_rBio_idx = 10;
DC_knabio_idx = 11;

vullen = [];
vullen(1,DC_salaris_idx) = beginsalaris;
vullen(1,DC_franchise_idx) = franchise;
vullen(1,DC_knabio_idx) = 0;

DG = zeros(1,AantalScenarios);
utility = zeros(AantalScenarios,length(gamma));

%--------------------------------------------------------------------------
%Calculating annuity factor
Rate = SpotRate;
```

```
[Koopsom,SubKoopsom] = KoopsomFunctie(Leeftijd,KansOverleving,Rate);
%------------------------------------------------------------------
%CAPITAL DEVELOPMENT CALCULATIONS
for scenario = 1:AantalScenarios
    LeeftijdStap = 25;
    for dt = 1:43
        vullen(dt,DC_leeftijd_idx) = LeeftijdStap;

        if dt == 1
            LeeftijdStap = LeeftijdStap - 1;
        end

        if LeeftijdStap ≥ min(Leeftijd)
            if LeeftijdStap < pensioenleeftijd
                vullen(dt,DC_dt_idx) = dt -1;
                vullen(dt,DC_salaris_idx) = ...
                    vullen(dt-1,DC_salaris_idx) * (1+Inflatie(dt-1) ...
                    + ...                opslaginflatie) * ...
                    (1+CarrierePad(dt-1));
                vullen(dt,DC_uitkering_idx) = 0;
            else
                vullen(dt,DC_dt_idx) = pensioenleeftijd - ...
                    min(Leeftijd);
                vullen(dt,DC_salaris_idx) = 0;
                vullen(dt,DC_uitkering_idx) = - ...
                    vullen(dt-1,DC_knabio_idx) / Koopsom(dt-1);
            end

            vullen(dt,DC_franchise_idx) = ...
                vullen(dt-1,DC_franchise_idx) * (1+Inflatie(dt-1) + ...
                opslaginflatie);
            vullen(dt,DC_pg_idx) = ...
                max(0,(vullen(dt,DC_salaris_idx) - ...
                vullen(dt,DC_franchise_idx)));
            vullen(dt,DC_premie_idx) = vullen(dt,DC_pg_idx) * ...
                Premie(dt-1);

            if State(scenario,dt-1) == 1
                vullen(dt,DC_rEUR_idx) = ...
                    (vullen(dt-1,DC_knabio_idx) + ...
                    vullen(dt,DC_premie_idx) + ...
                     vullen(dt,DC_uitkering_idx)) ...
                         *(LifeCycle(dt-1,1) * ...
                         Aandelen(dt-1,scenario) + ...
                     (1-LifeCycle(dt-1))*ReturnRente(dt-1,scenario));
            else
                vullen(dt,DC_rEUR_idx) = ...
                    (vullen(dt-1,DC_knabio_idx) + ...
```

```
                        vullen(dt,DC_premie_idx) + ...
                         vullen(dt,DC_uitkering_idx)) ...
                             *(LifeCycle(dt-1,2) * ...
                             Aandelen(dt-1,scenario) + ...
                         (1-LifeCycle(dt-1))*ReturnRente(dt-1,scenario));
            end

            vullen(dt,DC_kvoorbio_idx) = ...
                vullen(dt-1,DC_knabio_idx) + ...
                vullen(dt,DC_premie_idx) + ...
                 vullen(dt,DC_uitkering_idx) + vullen(dt,DC_rEUR_idx);
            vullen(dt,DC_rBio_idx) = 0;
            vullen(dt,DC_knabio_idx) = vullen(dt,DC_kvoorbio_idx) ...
                + vullen(dt,DC_rBio_idx);

        end
        LeeftijdStap = LeeftijdStap + 1;
    end

%----------------------------------------------------------------------
    %Coverage ratio and utility calculations
    maturity=1:78;
    PV = sum(Uitkeringen(1,:)./(1+SpotRate(2:79)).^maturity);

    for i = 1:42
        PV = PV *(1+ReturnRente(i,scenario));
    end

    DG(scenario) = vullen(43,11) / PV;

    for riskaversion = 1:length(gamma)
        utility(scenario,riskaversion) = ...
            DG(scenario)^(1-gamma(riskaversion)) / ...
            (1-gamma(riskaversion));
    end
end

MeanUtilityGamma2 = mean(utility(:,1))*-1000;
MeanUtilityGamma5 = mean(utility(:,2))*-1;
MeanUtilityGamma10 = mean(utility(:,3))*-100;
%----------------------------------------------------------------------
end
```

Dynamic optimisation code

```
options=optimset('Display','iter','MaxFunEval',4000);

aantal=4;
myF=zeros(aantal,1);
mypar=zeros(aantal,84);

for i=1:aantal
    LifeCycle0=rand(42,2);
    [params_opt, F] = fmincon(@(LifeCycle) ...
       CapCalcOptimDYN2(LifeCycle,SpotRate,Inflatie,CarrierePad,...
    Premie,Leeftijd,KansOverleving,ReturnRente,Aandelen,...
    Uitkeringen,State),LifeCycle0,[],[],[],[],lb,ub,[],options);
    myF(i,1)=F;
    mypar(i,1:42) = params_opt(:,1);
    mypar(i,43:84) = params_opt(:,2);
end
```