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THE IBOR REFORM

A study on the basis spread between ARR and IBOR

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Abstract

Interbank Offered Rates (IBOR) have served as the go-to risk-free rate in the financial sector for decades. After the need emerged to transition to a more transparent rate, fully based on actual transactions, this has led to the introduction of Alternative Reference Rates (ARR) as a replacement for the IBOR. This research aims to analyze the structural differences between the IBOR and ARR for the Sterling, Dollar and Euro. The Sterling IBOR is the GBP Libor and will be replaced with the SONIA. For the Dollar zone, we have analyzed the IBOR FED Funds which is being replaced by the Dollar ARR; the SOFR. The commonly used Euro zone IBOR is the EONIA and is replaced for the €STR. We analyze the basis spread, which is defined as the difference between the overnight zero rates of the ARR and the IBOR, to determine which challenges are encountered by the structural differences. We analyze the rates in three different phases. First we analyze the general movements and statistics of the data. Next we use several regression models to better understand the behavior, auto-correlation and similarities between the rates. Finally, we forecast the IBORs, ARRs and basis spread and measure the accuracy. From the three phases we conclude that major transition challenges are caused by structural differences between the IBOR and ARR per currency zone. We have identified that the major challenges are the recalibration of models, the renegotiation of existing contracts, dispute resolution between parties due to a different interpretation of spreads and the need for new accounting guidance due to a difference in value, behavior and stability of the rates of the ARRs. These challenges will have to be addressed as soon as possible and more (global) guidance is needed to make sure the transition is completed before the possible discontinuation of the IBOR in the last quarter of 2021.

Keywords Interbank Offered Rates · Alternative Reference Rates · Basis spread · GBP Libor · SONIA · FED Funds · SOFR · EONIA · €STR

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Acronyms

BBA British Banking Association

BS Basis Swap

CCS Cross Currency Swap

CDS Credit Default Swap

EFFR Effective Federal Fund Rate

EONIA Euro Overnight Index Average

€STR Euro Short-Term Rate

EURIBOR Euro Interbank Offered Rate

FRN Forward Rate Note

FSOC Financial Stability Oversight Council

IBOR Interbank Offered Rate

IRS Interest Rate Swap

ISDA Interantional Swaps and Derivatives Association

LIBOR London Interbank Offered Rate

ON Overnight

OTC Over-The-Counter

pre-€STR Pre-Euro Short-Term Rate

SOFR Secured Overnight Financing Rate

SONIA Sterling Overnight Index Average

TS Tenor Swap

1. Introduction

The Interbank Offered Rates (IBOR) have served as a reference rate for variable-rate financial instruments for the past decades. These IBORs are collective terms for the London Interbank Offered Rate (LIBOR), Euro Interbank Offered Rate (EURIBOR) and Tokyo Interbank Offered Rate (TIBOR), Hong Kong Interbank Offered Rate (HIBOR), Singapore Interbank Offered Rate (SIBOR) and others. This rate is best explained as the rate for interbank lending on an unsecured basis, underpinning worldwide trade in financial products. In 2012, in the tail of the financial crisis, scandals arose in which several banks were accused of manipulating these London Interbank Offered Rates (LIBORs).

This scandal resulted in the head of the Financial Conduct Authority (FCA) and the head of the Commodity Futures Trading Commission (CFTC), to simultaneously announce that panel banks are no longer compelled to submit IBORs quotes post 2021. This has resulted in the need of transitioning from IBORs to Alternative Reference Rates (ARRs). The big difference is that the IBORs are based on average rates large banks reported, which are less based on actual transaction due to the low frequency of transactions for interbank lending. Since the new ARRs take into account more types of transactions compared to the IBORs, there are more actual transactions to determine the rate.

The Financial Stability Board (FSB) established the Official Sector Steering Group (OSSG) to lead the IBORs reform and focus on the advancement of ARRs. The Sterling Overnight Index Average (SONIA), Secured Overnight Financial Rate (SOFR), Tokyo Overnight Average Rate (TONA) and the Swiss Average Rate Overnight (SARON) have been selected as the ARRs for the four major LIBOR currencies. The Euro Short-Term Rate (€STR) will be the Euro equivalent and these rates have first been published on October 2nd 2019.

The new Alternative Reference Rates will be fully transaction based and not prone to subjective interpretation which is not the case for the Interbank Offered Rates. The differences between the IBORs and ARRs are described in Section 1.3.

1.1. History of Libor

LIBOR has been the industry leading rate for unsecured lending between large banks for the past forty years. It originated from a Greek banker that arranged a transfer of $80 million based on the funding costs of reference banks [1]. This was the start of the LIBOR method in 1969. In 1986, the British Bankers' Association (BBA) gathered this data to officially take control and formalize the rates. After the start of posting LIBOR in the British Pound, US Dollar and Japanese Yen, other currencies have followed such as the Euro and the Swiss Franc. Nowadays, the International Exchange (ICE) is the administrator.

Nowadays, the LIBOR is still available in the five currencies mentioned and in seven different tenors which are 'Overnight', '1 week', '1 month, '2 months, '3 months, '6 months', and '12 months'. In order to determine these rates, a panel of several banks is asked to answer the following question. "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11am?". The amount of banks in the panel differ depending on the quoted currency.

At the end of 2018, over $460 trillion in financial contracts were LIBOR-referenced contracts [2]. Since these rates depict the reported rates of these panel banks, and not fully transaction based rates, this has resulted in a possible manipulative tendency of the LIBOR. This is what came to light in 2012, when major banks reporting LIBOR rates where manipulating this rate for one of two reasons. The first is the fact that they manipulated these rates in order to improve their positions of outstanding derivatives. The second is to manipulate the LIBORs to give the impression that these banks were more creditworthy than they actually were.

The level of LIBOR reported also gives a good indication on the health of the financial markets and individual banks. A higher LIBOR rate suggests less stability and trust by banks and thus in the financial market. During periods of financial instability, for instance in the last recession, the spread between the USD LIBOR and OIS was high compared to periods of financial stability [3].

IBORs are calculated by taking the rates posted by the panel banks, trimming a few of the lowest and highest rates depending of the number on contributors (panel banks) and taking the average of the remaining rates. This way, the rate would quote a reliable level without the outliers and represent the overall interbank lending rates of the market [4].

1.2. The development of the ARRs

The Financial Stability Oversight Council (FSOC) and the Financial Stability Board (FSB) identified several risks regarding Libor referencing contracts. From this, the Alternative Reference Rate Committee was created (ARRC) to address these risks [5].

The ARRC designed four objectives in order to lead the transition away from IBOR to determine the best ARRs. The first two objectives were related to the best practices of the newly proposed rates namely identifying them for the ARRs and contract robustness. This first objective focuses on deciding which of the existing interest rates would potentially take over the IBORs. In order to make this decision, several factors were taken into account such as liquidity of the specific interest rate market, robustness of the market, etc. After identifying potential ARRs and determining contract robustness, it was time to look at the characteristics of the potential ARRs that would either disrupt or ease the implementation. This was summarized in an adoption plan. The last objective was related to the implementation success and planning. To determine how well suitable the potential ARRs are, the easy of implementation is an important factor for a fast adoption. Focusing on these four objectives has led to identifying the IBOR alternatives.

In this process, the ARRC looked at both secured as unsecured rates, OIS linked to a specific rates and several term rates instead of overnight rates. As these rates had similar downfalls as the IBORs, they were not suitable. In addition to the manipulative nature of the LIBOR as stated in 1.1, the LIBOR rates had other shortcomings as well. Some of these are the lack of liquidity in times of financial distress, with this even being the case for short-term wholesale transaction in steady financial times.

After assessing the potential ARRs using the four main objectives as criteria, the ARRC appointed three rates as the ARRs for the Dollar, Sterling and Euro respectively. The Secured Overnight Financing Rate (SOFR) was appointed as the ARR for the Dollar, the Sterling Overnight Financing Rate (SONIA) for the Sterling and the Euro Short-Term Rate (€STR) for the Euro. The SOFR solves the main issue of the LIBOR robustness since it reflects over $800 billion in actual daily market transactions [6]. The SONIA was picked as the ARR for the Sterling zone due to the near-risk-free level of the overnight rate and its robustness of transnational volumes [7]. Similar reasons lead the working group on euro risk-free rates to unanimously recommend €STR [8].

1.3. IBORs versus ARRs

There are structural differences between IBORs compared to an ARRs, especially regarding the forward-looking term rates vs overnight rates, so we address this topic in a more detailed way. The IBOR rates are forward-looking rates that are based on historical data. Both IBOR and ARR are based on historical data, but there is a difference in the importance of the historical data in the determination of the rate. The IBORs are calculated as the trimmed mean of rates submitted by the panel bank, which are an answer on the question at what rate funds can be borrowed. This process includes an interpretation of costs by the panel banks. ARRs are actually fully transaction-based, ruling out this subjective interpretation by banks. This results in a different relation with the historical data. In addition, the difference between the IBORs and ARRs is the fixing of the rate. IBORs are fixed in advance, which means that their value is based on historical data, but the rate is then fixed for the tenor period. This offers certainty of funding costs due to the known upcoming interest rate payments. Other structural differences are in the methodology, publication time and credit premium inherent in the rate [9].

Forward-looking rates versus backward looking rates

As stated before, the IBORs are forward-looking rates while the ARRss are backward-looking rates since they are calculated based on the transactions of the previous night. This means the rates can be calculated using historical data based on actual transactions. The difference with IBORs is that they are forward looking rates. Forward rates are rates that are known at the beginning of the interest period. An example is fixing the GBP Libor rate at the beginning of a period. For ARRs, this is done at the end of the interest period. This calls for the need of a term rate for the ARRs, a backward-looking term rate. A backward-looking term rate can be calculated using the proposed compounded setting in arrears methodology. This methodology compounds the daily overnight rates over the relevant IBOR period. This allows a tenor rate to be calculated using overnight rates. The disadvantage is that the rate will only be available at the end of the period. This is briefly explained is section 1.2

Difference in sensitivity for credit and liquidity risk

Credit risk is the risk of a counter party default resulting in a loss for a transaction. [10]. The definition of liquidity risk is two-fold. First of all, liquidity risk is the risk that a firm is not able to borrow liquidity in order to fund its assets. The second is the risk of not being able to sell a holding at its theoretical price [10]. There is a difference in sensitivity regarding credit and liquidity risk between IBORs and ARRs. Loans between financial institutions that reference LIBOR are prone to credit risk due to default risk of the counter party. If we compare this with the ARRs, these are nearly risk-free. With regard to liquidity risk, the liquidity premium will gradually change as the ARRs markets gain liquidity.

1.4. Relevance

Due to the different nature of IBORs and ARRs, the transition from the old to the new rates will face certain challenges. These challenges will have to be addressed before the IBORs are potentially discontinued in 2021.

1.4.1. Challenges

The challenges have been identified by EY and can be categorized in ten different topics. These ten challenges are visible in Table 1. Certain impact categories have been identified as well. A plus-sign in the table indicates that the challenge has a direct impact on the impact category [11].

Table 1: *Impact from IBOR reform*

	Impact category				
Challenges	Modeling	Transition speed	Data availability	Hedge accounting	Renegotiating contracts
Regulatory uncertainty	-	+	-	-	+
Operations and technology upgrades	+	-	-	+	-
Recalibration of models	+	-	-	-	-
Lagging liquidity	-	-	+	+	-
Renegotiation of existing contracts	+	+	-	-	+
Dispute resolution	+	+	-	-	+
Lack of global coordination	+	-	-	+	-
New accounting guidance	-	-	-	+	-
Lack of term rates	+	+	+	-	-
An unclear future	-	+	-	-	-

Table 1 shows some of the challenges that are being faced due to the IBOR transition to new ARRs. By analyzing the IBOR, ARR and basis spread (the difference of ARR minus IBOR) per currency zone, differences per currency zone will be identified, acknowledging possible challenges and determining what causes the challenge. We expect the need to renegotiate existing contracts to be identified as a challenge. The newly proposed methods are coordinated for the derivatives market, but this is not the case for cash products and some other contracts. In order to determine what the valuation is and whether this valuation is fair is the first major hurdle. When the new valuation method turns out to have a negative impact on one of the parties, reaching an agreement may well be very difficult [11]. Examples of such cash products are bonds, syndicate loans, floating rate notes (FRN) and securitised products. We expect to identify several differences between the current IBORs and new ARRs, resulting in the need to renegotiate.

By analyzing the developing basis spreads, we expect not all of the previously stated challenges to be identified. This thesis researches the behavior of the basis spread between the different currency zones. As defined, the basis spread is the difference between the ARR and IBOR. If we take another look at Table 1 with this goal in mind, we expect to identify not all of the challenges. Since parties determine the curves they use in financial modeling themselves instead of a central administrator, this will be impacted if the new ARRs behave differently. Therefore there is a difference in the curves that parties use, resulting in a different valuation and the need to recalibrate models. We expect the Alternative Reference Rates to behave different structurally, which will seen in the difference between the rate, the basis spread. For this reason, we expect the recalibration of models to be a challenge as well.

In Chapter 3, we use descriptive statistics to analyze the data and determine any differences in the behavior of the rate. Next, in Chapter 4 we use the regression models to explain the rates behavior in terms of structural components. The regression models and their found significant order are used to forecast the interest rates and basis and to check the accuracy. This leads to an insight in the current IBOR interest rate behavior and the behavior of their introduced ARR. Besides analyzing the current IBOR and the new ARR, we focus on analyzing the basis spread per currency zone. The analysis of the spread provides insights in what challenges may be encountered and why in this major transition. The goal of this thesis is:

"To identify the main challenges of the IBOR transition by analyzing the behavior of the basis spread."

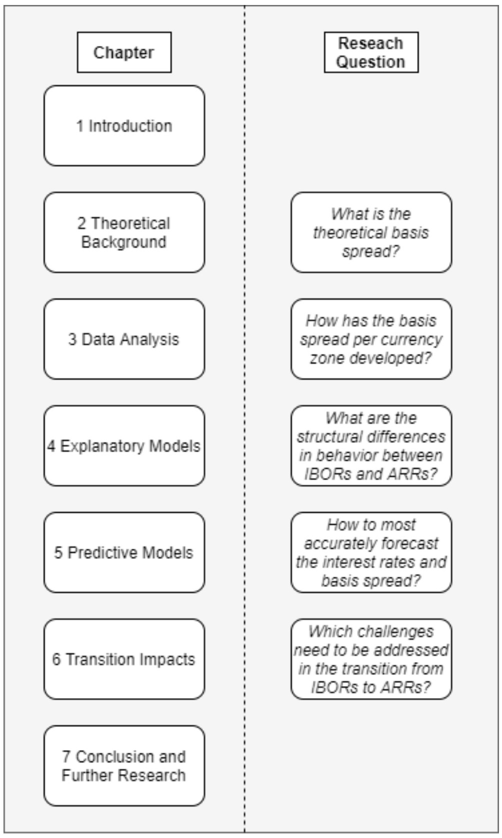
To reach this goal, the main research question is formulated as:

"What challenges are encountered in the transition from Interbank Offered Rates to Alternative Reference Rates by the structural differences between the rates?"

The designed model will have the purpose of forecasting the future spreads between the current IBORs and their proposed substituting ARRs per currency. A regression model is used for this purpose. In order to answer the main research question, several other question need to be addressed first. The following sub-questions have been formulated.

- *How has the basis spread per currency zone developed?*
- *What are the structural differences in behavior between the Interbank Offered Rates and Alternative Reference Rates?*
- *Are there structural differences in the forecastibility of the rates?*
- *Which challenges need to be addressed in the transition from Interbank Offered Rates to Alternative Reference Rates?*

Figure 1: *Thesis structure and chapter content*



2. Regression methods and methodology

In this chapter, we explain the differences between the old interest rates and the newly proposed risk-free rates. In addition, we explain the methods used for regressions.

2.1. Basis spread

Throughout this thesis, the basis spread for the Sterling, Dollar and Euro are analyzed. The basis spread for a currency zone at day t is calculated using the daily zero rates. The formulas for the Sterling, Dollar and Euro basis spread are seen in (1), (2) and (3) respectively.

$$SterlingBasis_t = SONIA_t - GBPLibor_t \tag{1}$$

$$DollarBasis_t = SOFR_t - FEDFunds_t \tag{2}$$

$$EuroBasis_t = €STR_t - EONIA_t \tag{3}$$

The basis spread for the pre-€STR Euro data is also needed. (4) shows this formula.

$$pre\text{–}EuroBasis_t = (pre\text{–}€STR_t) - EONIA_t \tag{4}$$

The ideal situation is not a situation where the basis spread is as small as possible, but as stable as possible. For that reason, it is important to analyze the basis spread using the regression models as well, instead of only focusing on the ARRs and IBORs. The €STR - EONIA spread could be fixed since the €STR is a newly introduced rate. Setting a fixed spread for an already existent IBOR and ARR causes far more problems compared to fixing an existent IBOR with a new ARR.

2.2. Ordinary Least Squares (OLS)

The Ordinary Least Squares (OLS) estimation optimizes the parameters of a linear equation such that the sum of the squared deviations of the independent variable is as small as possible. The formula of the statistical model is given in (5) [12].

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \tag{5}$$

The parameters β_0 and β_1 from (5) are estimated such that they are as close to Y_i, which means the smallest possible error term. These numerical estimates are $\widehat{Y_i}$ for Y_i, $\widehat{\beta_0}$ for β_0 and $\widehat{\beta_1}$ for β_0 resulting in (6).

$$\widehat{Y_i} = \widehat{\beta_0} + \widehat{\beta_1} X_i \tag{6}$$

The formula of the sum of squares of the residuals is given in (7). The parameters β_0 and β_1 are optimized such that the equation is minimized. In this equation $e_i = (Y_i - \widehat{Y_i})$ which is the observed residual.

$$SS(residuals) = \sum_{i=1}^{n} (Y_i - \widehat{Y_i})^2 \tag{7}$$

$$e_i = Y_i - \widehat{Y_i}$$

$$= \sum e_i^2$$

The vector for the OLS estimation is shown in (8).

$$\hat{\beta} = (X'X)^{-1} X'y \tag{8}$$

2.3. Auto Regressive (AR) regression

The first statistical test that is used is based on an Auto Regressive (AR) model. This means that the output variable is linearly dependent on its previous values. The formula of a general AR(p) function is shown in (9) [13].

$$X_t = \phi_1 X_{t-1} + ... + \phi_p X_{t-p} + \epsilon_t \tag{9}$$

Where ϵ_t is the error term that is independent and identically distributed random variable with a mean of 0 and a variance σ^2. The notation for the prediction of X_{n+1} is $\hat{X}_{n+1}$, which is based on the previous known values,

$$\hat{X}_{n+1} = \phi_1 X_n + ... + \phi_p X_{n+1-p} \tag{10}$$

2.4. Auto Regressive Moving Average (ARMA) regression

The Auto Regressive Moving Average model contains of two parts. The first is the auto-regression model shown in Section 2.3, and the second part suggests a smoothing if the values are greater than zero. If the values are negative, this increases the differences. The formula is shown in (11).

$$X_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_1 \epsilon_{p-q} \tag{11}$$

Adding the Auto Regressive part shown in (9) to (11), this gives us the ARMA(p,q) formula.

$$X_t = c + \epsilon_t + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \tag{12}$$

where

- *c is a constant*
- *ϵ_t is the error term also called white noise*
- *ϕ_i is constant-value for AR components*
- *θ_i is constant-value for MA components*

This can be rewritten such that

$$\phi(L)Y_t = c + \Theta(L)\epsilon_t \tag{13}$$

This process is stable when the conditions of the roots of the $\phi(L)Y$ polynomial are met. Disregarding the Moving Average component, this gives us the simplest first-order case,

$$\begin{aligned}(1 - \phi_1 L)y_t = \epsilon_t \Rightarrow \\ y_t = \phi_1 y_{t-1} + \epsilon_t\end{aligned}$$

2.5. Hurst exponent

The Hurst exponent measures the long-term memory of a timeseries. The Hurst exponent, developed by Harold Edwin Hurst, gives an indication of the behavior of the timeseries related to the autocorrelation. The results can roughly be divided into three brackets. These are a Hurst exponent value of 0.5, a value between 0 and 0.5, and a value between 0.5 and 1.

A Hurst exponent value of 0.5 indicates the timeseries follows a random walk. A random walk is best explained as a stochastic process of which the path take random one-step forward moves. This random walk is the sum of the white noise elements. A Hurst exponent value between 0.5 and 1 indicates a persistent behavior of the timeseries. This persistent behavior indicates a trend. The last division are the Hurst exponent values between 0 and 0.5. This indicates a mean-reverting nature of the timeseries. This mean-reversion effect is import for our analysis, since this indicates that overall the rates revert to their long-term mean.

The Hurst exponent will be used to analyze the individual timeseries data and to determine its behavior, whether that is a mean-revertion, random walk or trending.

3. Data Analysis

In this section, we describe and analyze the data of the overnight zero rates for the Sterling, Dollar and Euro. In Section 3.1 we describe our data collection process and the data range. In Section 3.2, the individual interest rate timeseries are analyzed per currency zone. Section 3.3 focuses on the basis spread for these zones.

3.1. Data collection description

To be able to answer are main research question, we first need to analyse the interest rate timeseries and the basis timeseries. We collect the zero rates of the overnight interest rates using Bloomberg. The data is collected between 23-04-2018 and 19-11-2019. The starting data has been chosen due to the reform of the SONIA. The calculation methodology for the SONIA resulting in the Adjusted SONIA with data available from 23-04-2018. The €STR rate was first published on 02-10-2019, resulting in a limited data availability for this ARR. The total number of data points per rate are shown in Table 2.

Table 2: *Number of observations zero rates*

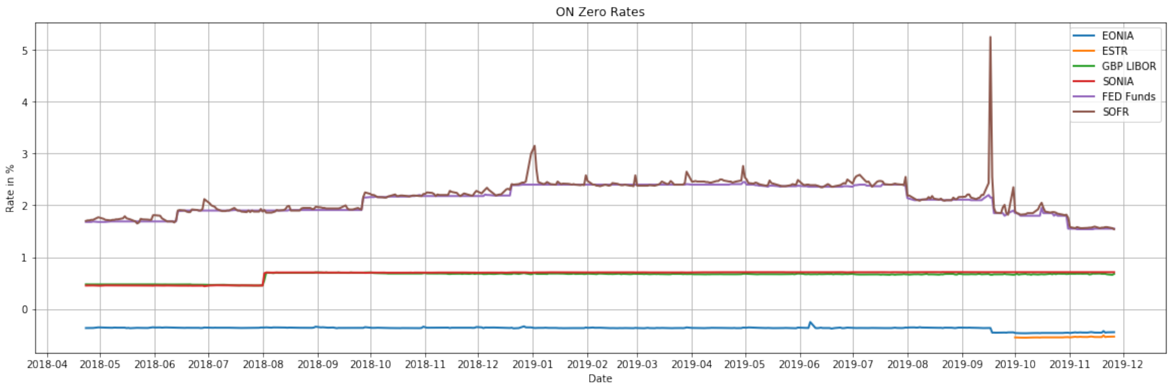
	EONIA	€STR	GBP Libor	SONIA	FED Funds	SOFR
No. Obs.	418	42	418	418	418	418

To clarify the rates in Table 2, the EONIA is the old rate for the Euro and the €STR is the new ARR. For the Sterling zone, the GBL Libor is analyzed as the OLD IBOR and the SONIA is the new ARR. In the Dollar zone, we have analyzed the FED Funds rate as the IBOR and the SOFR as the new ARR.

3.2. Interest rate timeseries of IBORs and ARRs

We analyze the interest rate timeseries of the daily quoted overnight zero rates from 23-04-2018 up to 26-11-2019 per currency. We start with the actual zero rates for each interest rate. Later, we will standardize the data set for regression purposes. When we look at Figure 2, we see the six different interest rates with the Dollar rates at the top, the Sterling rates in the middle and the Euro rates at the bottom.

Figure 2: *Overnight Daily Zero Rates*

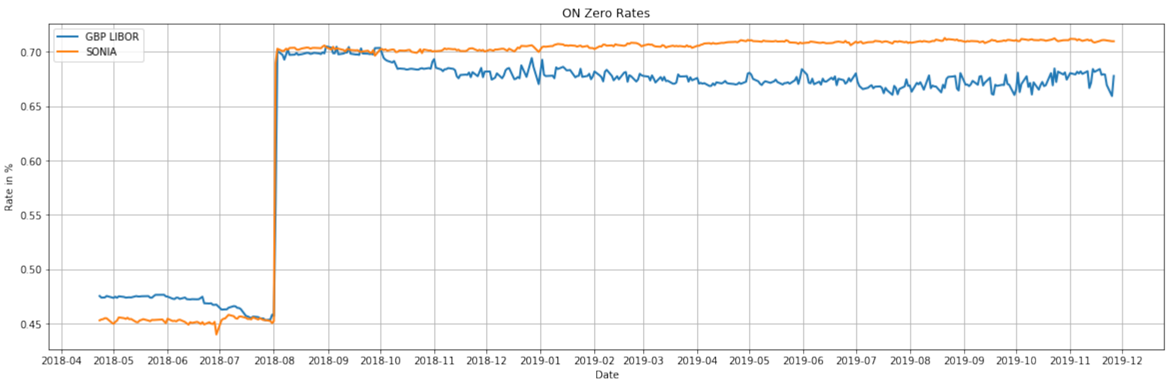


In Sections 3.2.1 to 3.2.3 we will analyze individual times series data of the IBOR and ARR per currency. Later on, in Section 3.3, we analyze the basis timeseries which is the difference between the overnight zero rates.

3.2.1. Sterling timeseries

As previously mentioned, the IBOR for the Sterling is the GBP Libor whereas the ARR is the SONIA. The IBOR and ARR on the first glimpse look to move similarly, although the GBP Libor seems more volatile. If we look at the starting point of the rates, we see that the GBP Libor is higher than the SONIA. This would seem logical, due to the additional risk components that are part of the GBP Libor. Since the SONIA is the volume-weighted mean rate of the central 50% of actual transactions and the GBP Libor is a forward-looking rate based on bank speculations, the GBP Libor rate contains more risk components compared to the SONIA. What stands out is that the GBP Libor becomes lower and stays below the SONIA throughout the data period of the rate movement.

Figure 3: *Overnight Sterling Zero Rates*



In Table 3, the descriptive statistics of the rates are shown. An important factor influencing the statistics is the steep increase at the beginning of August 2018. As a result of a government decision to be able to handle the market fluctuations as a result of the Brexit, The Bank Of England raised the interest rate [14]. The volatility of the GBP Libor, denoted by the standard deviation, is slightly lower compared to that of the SONIA, as expected looking at Figure 3. Both rates are highly skewed to the left and the kurtosis indicates the shape distribution of the data is flat-topped.

Table 3: *Descriptive statistics Sterling rates: GBP Libor and SONIA*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
GBP Libor	417	0.641479	0.67388	0.006461	0.080382	-1.656283	0.860396	0.517754
SONIA	417	0.661986	0.70550	0.009320	0.096540	-1.711950	0.944509	0.501226

The Hurst exponent is described in Section 2.5. In Table 3, we notice a Hurst exponents of approximately 0.5, indicating a random-walk. We expect this result to originate from the enormous increase in August 2018. To test this, we analyze the data starting in September. The results are shown in Table 4.

Table 4: *Descriptive statistics Sterling rates after*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
GBP Libor	322	0.677134	0.67619	0.000081	0.009003	0.925322	0.894180	0.14322509
SONIA	322	0.706642	0.70780	0.000013	0.003614	-0.499819	-0.982232	0.1755502

We now observe entirely different values. Although the mean is relatively similar, we observe the volatility of the rate to be very different. The volatility has decreased significantly, indicating a more stable rate. In addition, we observe that the GBP Libor has a higher volatility, indicating it is less stable compared to the SONIA. In Table 3, we concluded from the Hurst exponents that both rates followed a random walk. For the data set starting after the increase in August 2018, we observe both rates to have a mean-reverting nature. We observe entirely different statistics when the increase in August 2018 is included or excluded in the timeseries. This shows the impact of the sudden increase or decrease of an interest rate on the ability to understand the rate.

Next we look at the return of the rates. We find the return by calculating the difference compared to the previous day. Figure 4 shows the daily return of the GBP Libor.

Figure 4: *Overnight Daily Rates*

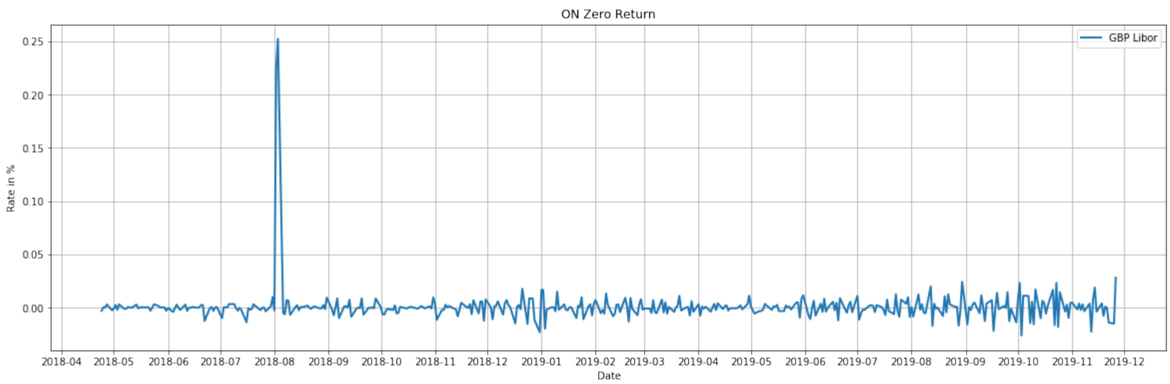


Figure 5 shows the daily returns of the SONIA. We observe the spike in August 2018 as the result of the steep increase shown in Figure 3. The descriptive statistics for both the GBP LIBOR and the SONIA are shown in Table 5.

Figure 5: *Overnight Daily Rates*

Looking at Table 5, we notice that the return of the SONIA is more than two times a volatile compared to the GBP Libor return. In addition, we notice a very high skewness and kurtosis in both scenarios. The Hurst exponent value of nearly zero indicates a high mean-reverting nature, which is what we expect for returns.

Table 5: *Descriptive statistics Sterling returns: GBP Libor and SONIA*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
GBP Libor return	416	0.000997	0	0.000324	0.017998	11.215716	148.384278	0.013355
SONIA return	416	0.001332	0	0.000671	0.025913	20.008080	405.402223	0.013445

3.2.2. Dollar timeseries

Looking at the USD rates in Figure 6, we see the rates closely follow each other but the Secured Overnight Financing Rate looks more volatile compared to the FED Funds. The rates have three jumps upwards in 2018 and three jumps downwards in 2019. These jumps are caused by governmental decisions of manipulating the interest rates to influence the current economic heath of the country. What stands out is, although the SOFR follows the FED Funds, the spikes occur mostly at months-end. This is the result of an effect called window-dressing. With window-dressing, large companies change their portfolio at the end of a month, quarter or year, by selling bad or average performing stocks/products and buying attractive products. The goal is, when showing their investors the portfolio when it is performing bad, to make their investors feel that the portfolio they currently own will be attractive in the future instead of showing bad performing stocks that no one knows [15]. This effect increases the volatility of the FED Funds rate.

Figure 6: *Overnight Dollar Zero Rates*

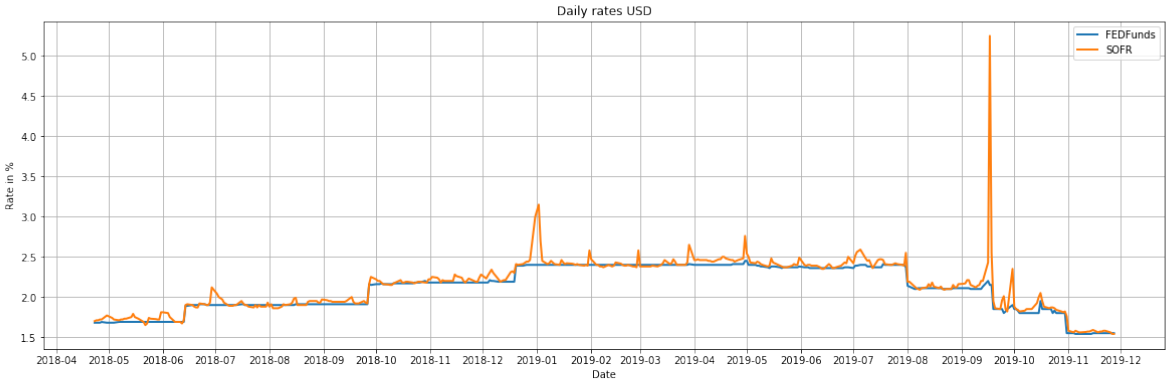


Table 6 shows the descriptive statistics of the Dollar rates, similar to the previous section. As expected, we notice that the SOFR is more volatile compared to the Effective Federal Funds Rate. Different from the Sterling rates, the shape of the distribution of the Dollar rates is very different with the FED Funds is slightly skewed to the left with a bell shaped form while the SOFR is rightly skewed and has a heavy tail. The FED Funds rate follows a Geometric Brownian Motion (GBP) while the SOFR is strongly mean reverting.

Table 6: *Descriptive statistics Dollar rates: FED Funds and SOFR*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
FED Funds	417	2.106717	2.17	0.075523	0.274814	-0.429453	-1.119729	0.496200
SOFR	417	2.155983	2.19	0.108150	0.328861	1.861730	17.638024	0.104938

Next we look at the returns of the Dollar rates. We immediately notice that the FED Funds returns are less volatile and even have several days in which the rate is stable.

Figure 7: *Overnight Daily Rates*

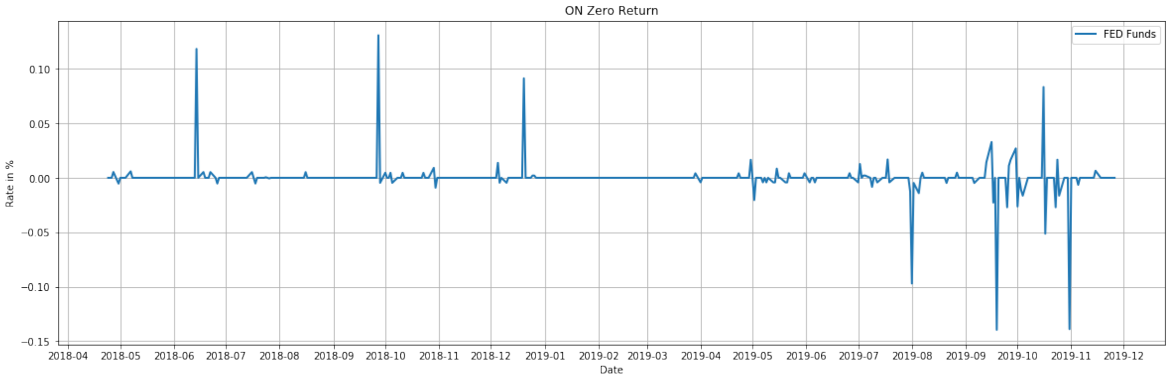
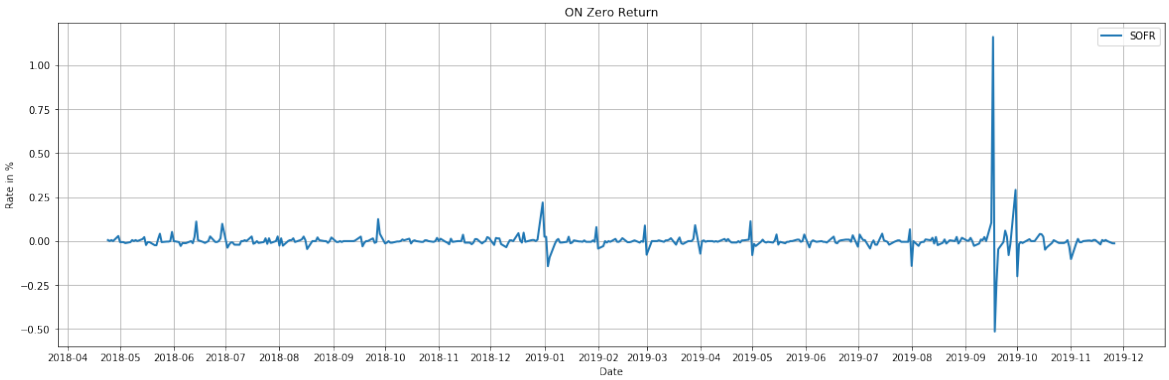


Figure 8: *Overnight Daily Rates*



Looking at Table 7, the SOFR is indeed more volatile compared to the FED Funds. Unlike the Sterling rates, we now notice both Dollar rates being strongly mean-reverting. Regarding the shape of the distributions, we observe that this time both rates are heavy tailed.

Table 7: *Descriptive statistics Dollar returns: GBP Libor and SONIA*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
FED Funds return	416	-0.000064	0	0.000256	0.015995	-0.706635	51.448409	-0.020373
SOFR return	416	0.001785	0	0.005056	0.071103	9.631702	177.731513	0.002339

3.2.3. Euro timeseries

Figure 9 shows the actual zero rates for the EURO zone. The spread between the EONIA and the €STR has been fixed at 8.5 basis points as determined by The Brattle Group. This is the result of the ECB where they decided that the EONIA will continue to exist, but based on a fixed spread between the €STR that was determined by the available data of the pre-€STR.

Figure 9: *Overnight Euro Zero Rates*

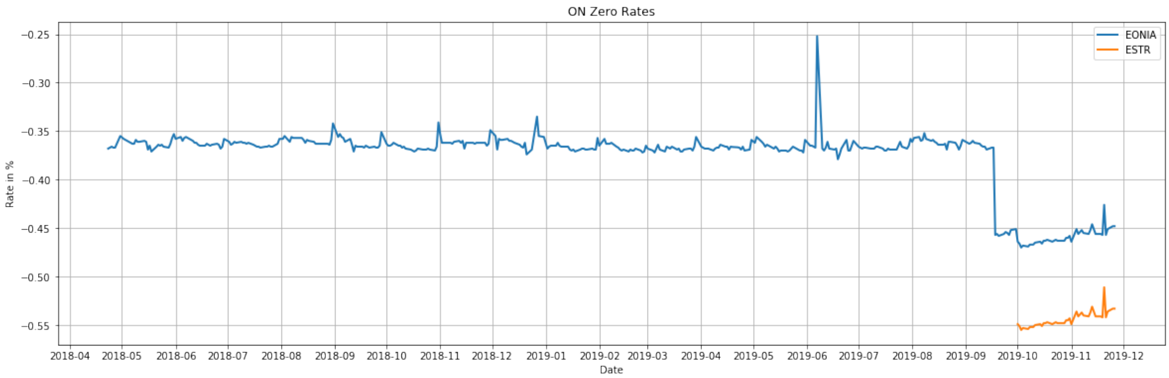


Figure 10 shows EONIA and pre-€STR rates from April 23rd 2018 up to the 2nd of October 2019 when the €STR was first published. One can immediately see that there is no fixed spread between the EONIA and the pre-€STR, with the rates behaving differently. To get a better understanding of the difference between the two rates, the descriptive statistics are seen in Table 8.

Figure 10: *Overnight Daily Rates*

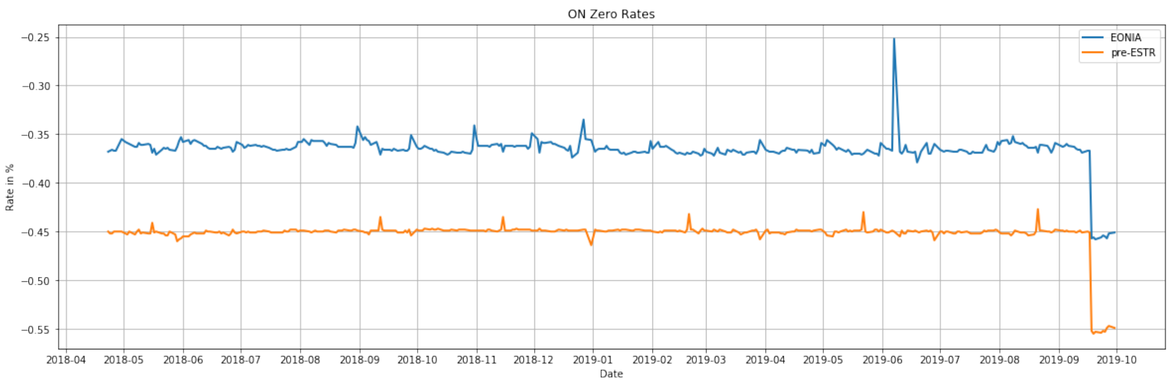


Table 8: *Descriptive statistics Euro rates: EONIA, pre-€STR and €STR*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
EONIA	376	-0.366053	-0.365	0.000254	0.015940	-3.154345	28.033205	0.142553
pre-€STR	376	-0.452203	-0.450	0.000251	0.015857	-5.909523	34.741149	0.200106
€STR	41	-0.543854	-0.547	0.000068	0.008245	1.674078	4.899443	0.101033

In order to compare the rates, we use a similar timeseries length which is up to the moment the €STR is first published, resulting in 376 observations for the EONIA and the pre-€STR. The volatility of both rates described as the variance is very similar. In addition, both rates are negatively skewed and have a heavy tail. The pre-€STR is more mean-reverting but both rates show this nature. For the €STR, there are only 41 data points available but in this period the rate shows very little volatility. The €STR shows a mean-reverting nature as well. The returns of all three rates are shown in Figure 11 with the descriptive statistics in Table 9.

Figure 11: *Overnight Daily Returns*

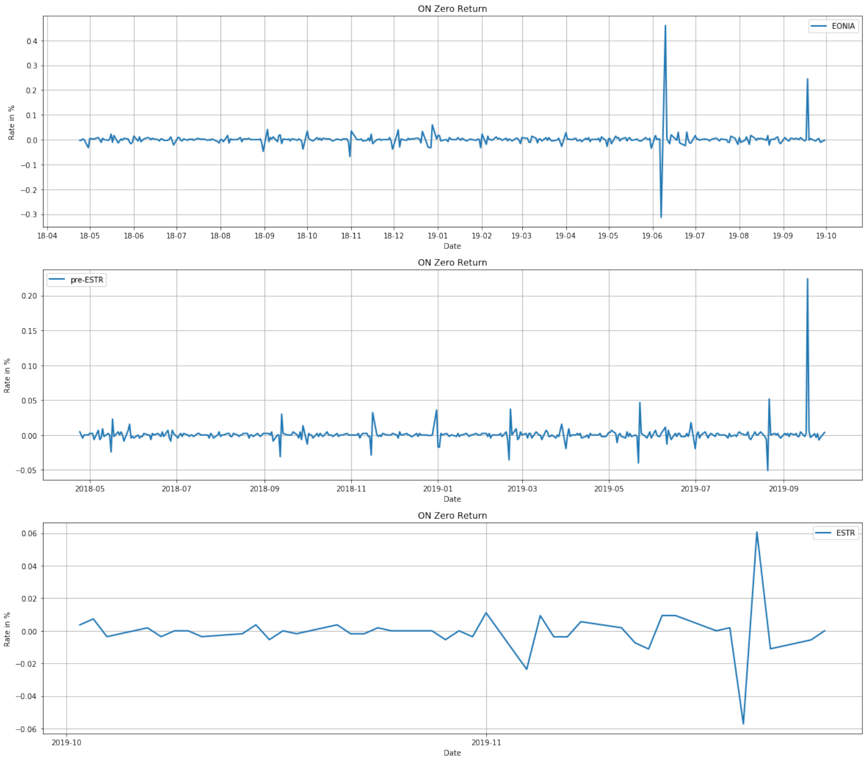


Table 9: *Descriptive statistics Euro returns: EONIA, pre-€STR and €STR*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
EONIA return	375	0.001066	0	0.001129	0.033602	5.661553	120.342809	-0.004498
pre-€STR return	375	0.000620	0	0.000198	0.014064	10.794846	171.904428	-0.098946
€STR return	40	-0.000633	0	0.000218	0.014767	0.331937	12.301858	0.144783

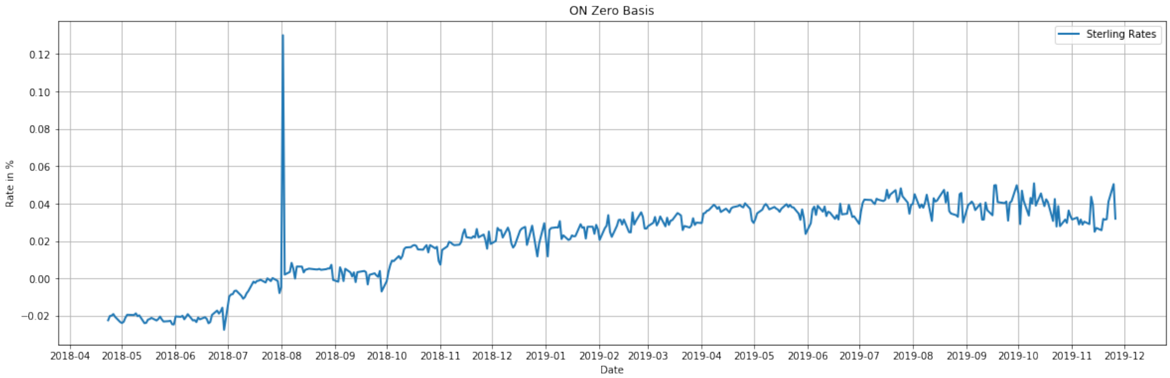
3.3. Historical basis timeseries

We want to analyze the basis spreads between the ARRs and IBORs. The basis spread is found by subtracting the overnight zero IBOR rate from the ARR for each currency as described in Section 2.1. This leaves us with the observed basis spread for the Sterling, Dollar and Euro.

3.3.1. Sterling basis timeseries

In Figure 12, the Sterling basis timeseries is shown. (1) in Section 2.1 shows this formula. We notice an increase in the basis spread level. This indicates that the difference between the SONIA and the GBP Libor is increasing over time. The peak in August 2018 was the result of a governmental decision as described in Section 3.2.1. The descriptive statistics are shown in Table 10.

Figure 12: *Sterling Basis Timeseries*



The first thing we notice in Table 9, is the low mean of the rate. The spread is approximately 0.02%, so 2 basis points. The Sterling basis increases over time but remains fairly small. From the variance we conclude that the volatility of the Sterling basis is relatively low. The Hurst exponent tells ut the rate is strongly mean-reverting.

Table 10: *Descriptive statistics Sterling basis*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
Sterling basis	417	0.020507	0.02742	0.000458	0.021384	-0.460842	0.942034	0.101159

3.3.2. Dollar basis timeseries

In Figure 13, the Dollar basis timeseries is shown. (2) in Section 2.1 shows this formula. At the first glimpse, the basis spread seems to move back to zero. The basis seems fairly volatile, but the basis shows no increase or decrease as seen for the Sterling basis. The spike in September 2019 is the result of an event called the 'SOFR Surge Event'. Due to a combination of events, namely $60 billion in treasury debt maturities that impacted available cash, in combination with $115 billion of investment grade debt and the lack of cash as a result of the upcoming corporate tax payments, the SOFR increased with 282 basis points [16].

Figure 13: *Dollar Basis Timeseries*

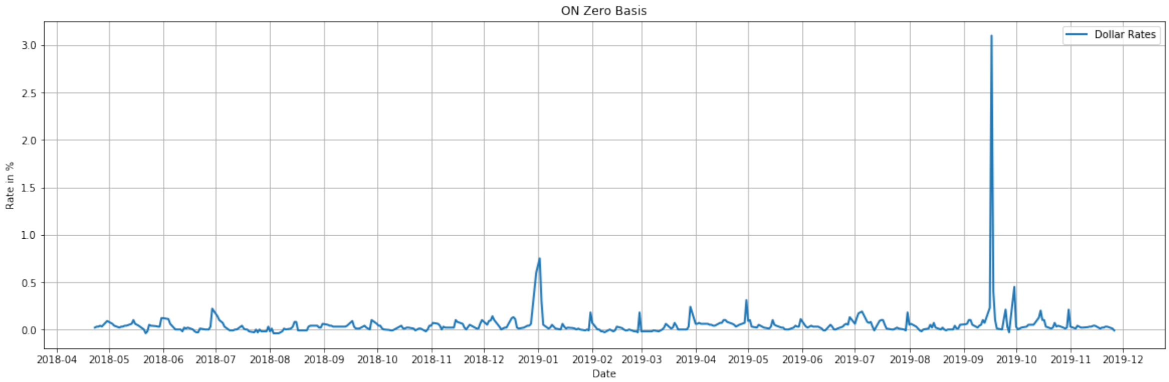


Table 11 shows the descriptive statistics for the Dollar basis. Looking at the results, we notice that the volatility is much higher compared to the Sterling basis. The data is rightly skewed and also heavily tailed to the right, as a result of the many spikes, mostly the result of window-dressing. With a value for the Hurst exponent close to zero, we conclude that the Dollar basis is strongly mean reverting.

Table 11: *Descriptive statistics Sterling basis*

	Count	Mean	Median	Variance	Stand. Dev.	Skewness	Kurtosis	Hurst
Dollar basis	417	0.049266	0.029999	0.028477	0.168752	14.733612	259.110637	0.028269

3.3.3. Euro basis timeseries

The Euro basis as shown in (3) of section 2.1 is a fixed spread of 8.5 basis points. For that reason, we analyze the basis spread for the pre-€STR and EONIA as shown in (21).

$$EuroBasis_t = pre - €STR_t - EONIA_t \quad (14)$$

Looking at Figure 14, although the spread is near the now fixed spread of 8.5 basis points, it stands out it is far from stable. The basis shows to be very volatile with both upward as downward spikes. The major spike in June 2019 is the result of in increase in the EONIA.

Figure 14: *pre-€STR Basis Timeseries*

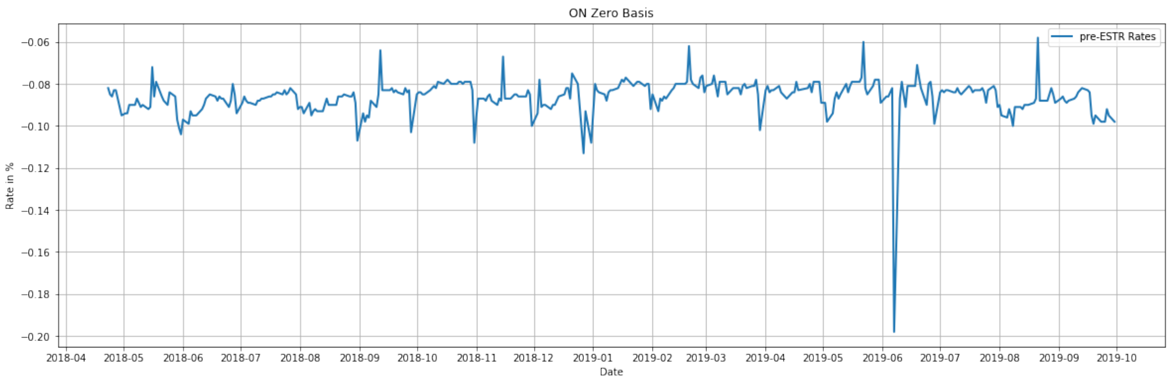


Table 12 shows the descriptive statistics of pre-€STR Euro basis. The spread has a mean of approximately 8.6 with a median of 8.5 basis points. The overall volatility is fairly low as compared to the Sterling and Dollar basis. The data is skewed to the right with a heavy tail as a results from the spikes. The Hurst exponent is nearly zero, indicating the basis is heavily mean-reverting.

Table 12: *Descriptive statistics pre-€STR Euro basis*

	Count	Mean	Median	Variance	Stand. Dev	Skewness	Kurtosis	Hurst
pre-€STR Euro basis	369	-0.086062	-0.085000	0.000077	0.008785	-5.684479	71.649399	0.026495

3.4. Key finding data analysis

After analyzing the data, the main differences in the behavior per rate can be summed up in four different categories. The first category is the average difference of the interest rate or the average of the basis spread. The difference in basis indicates the structural difference between the IBOR and the proposed ARR. If this basis spread is stable or even fixed such as in the Euro zone, this will ease the transition. If the basis spread is large and volatile, transitioning the IBOR exposure and referencing products to a new ARR will result in many challenges. In Section 3.3.1, we observed that the Sterling basis spread has been increasing due to a decrease of the GBP Libor. The Dollar basis spread is fairly stable, but experiences month-end volatility due to the movements in one of the underlying rate, the SOFR. A stable spread between the FED Funds and the SOFR will allow a more smooth transition.

The second category is the volatility. In general, a difference in the volatility will result in a different behavior of the rate itself, different risk and financial models, but also different valuations of financial products and derivatives referencing the interest rate. The SONIA overall is more volatile than the GBP Libor. Since value of options increase as volatility increases, the possible discontinuation of the Libor will also hugely affect the derivatives market. If the market does not prepare itself sufficiently for the transition and possible discontinuation of the IBOR rates, this could trigger volatility.

The third and fourth categories are asymmetry and mean-reversion. A rate is asymmetric if spikes go in just one direction instead of both directions. A rate is mean-reversing if after an increase or decrease, the rate eventually goes back to long-term mean. For the Sterling rates seen in Table 12, we observe that the rates are not asymmetric and only the Sterling basis is mean-reverting. The Dollar rates on the other hand show different results. The FED Funds is nor asymmetric, nor mean-reverting. The SOFR and Dollar spread are both asymmetric and mean-reverting. For the Euro zone, we observe that the EONIA, pre-€STR and the Euro spread are mean-reverting. For an interest rate to be mean-reverting on the long run can be important for exposure management. If the SOFR had no mean-reverting tendency, the SOFR and the Dollar spread would gradually increase at each spike or month-end, increasing difficulty for exposure management. The speed of the mean-reverting nature is important as well for financial instruments referencing the interest rate.

Table 13: *Difference in behavior from regression analysis*

	Mean	Volatility	Asymmetry	Mean-reverting
GBP Libor	0.641479	0.006461	No	No
SONIA	0.661986	0.009320	No	No
Sterling spread	0.020571	0.000458	No	Yes
FED Funds	2.106717	0.075523	No	No
SOFR	2.155893	0.108150	Yes	Yes
Dollar spread	0.049266	0.028477	Yes	Yes
EONIA	-0.366553	0.000254	Yes	Yes
pre-€STR	-0.452203	0.000251	No	Yes
Euro spread	-0.086062	0.000077	No	Yes

The identified structural differences between the Interbank Offered Rates and Alternative Reference Rates are in line with some of the pre-mentioned challenges in Section 1.4.1. This difference in behavior between the IBOR and the ARR is depicted by the basis spread. From Table 13, we can conclude that the proposed ARRs are structurally different compared to the IBORs. In addition, the basis spreads are different per currency zone, indicating each currency zone needs a unique regulation, guidance and fallback language. The structural differences will form a major challenge in the transition from IBORs to ARRs.

In Section 3.2.1 we measured the descriptive statistics of the Sterling rates after the steep increase in August 2018 seen in Figure 3. We observed a big difference in volatility and mean-reverting nature. This shows the impact of a governmental increase or decrease on the data and its behavior. These increases or decreases should be taken into account and addressed as outliers if needed in order to better understand the actual behavior of the rates.

4. Explanatory models

In this section, we start with analyzing the six interest rate timeseries and the three basis spread timeseries in Section 4.1 using the models described in Chapter 2. In Sections 4.1.1 and 4.1.2 the Dollar IBOR and ARR are analyzed using an AR and ARMA model respectively. In Section 4.1.3, a cross-sectional OLS is used to express the old rate in the new for the Dollar zone. Next we use the same models to analyze the basis timeseries in Section 4.2.

4.1. Interest rate timeseries

This section analyzes the Dollar interest rate timeseries using different regression models. This is also done for the Sterling and Euro interest rate timeseries in Appendix A.2. The findings will be discussed at the end of this chapter.

In this Section, we analyze both the actual overnight zero rates as the standardized data. Standardizing data is the process of subtracting the mean and dividing by the standard deviation. This results in the 'standard normal', which is a mean of 0 and a standard deviation of 1. This is,

$$Z = \frac{X - \mu}{\sigma}$$

4.1.1. FED Funds interest rate timeseries

First, we analyze the FED Funds interest rate timeseries using an auto regressive model and an auto regressive moving average model. The OLS method will be described in Section 4.1.2, since we define the new ARR as a function of the current IBOR.

Auto Regressive Model

Similar to what is seen for the Sterling rates, we start with an Auto Regressive model to analyze the Dollar IBOR rate, the FED Funds (Effective Federal Funds Rates). After testing the number of lags for the AR(p) model, we found that only the first lag, which represents $FEDFunds_{t-1}$, is significant. This gives us the AR(1) formula shown in (15).

$$FEDFunds_t = \alpha + \beta_1 FEDFunds_{t-1} + \epsilon_t \tag{15}$$

Table 14: *AR(1) results Dollar IBOR: FED Funds*

	α	β_1	α error	β_1 error	α p-value	β_1 p-value	Log like.	No. obs.
FED Funds	1.8528	0.9955	0.263	0.004	0.000	0.000	845.710	417
FED Funds stand.	-0.9251	0.9955	0.957	0.004	0.334	0.000	306.588	417

Table 14 shows the results of the AR(1) model for both data sets. We notice that the β_1 is very high, indicating high corrlation between the value of the FED Funds at *t-1* and the value at t. Since the values for β_2 and β_3 for *t-2* and *t-3* respectively were both insignificant, this can be interpreted as the rate to be fairly volatile.

Auto Regressive Moving Average Model

We now add the moving average components and analyze the data with the ARMA(p,q) model. After analyzing the data with the ARMA(3,3) model, we find that the inverse of the Hessian Matrix gives NA values for the FED Funds data set. Adjusting the p and q to the parameters that fit the data set results in finding that the moving average component is always insignificant. Dropping this brings us to the AR(1) model previously described.

4.1.2. SOFR interest rate timeseries

Auto Regressive Model

We analyze the data using the Auto Regressive model. We once again test the maximum value for p for the AR(p) model. An AR(3) model is the maximum model where all the lags are significant. The formula is shown in (16) with the results in Table 15.

$$SOFR_t = \alpha_t + \beta_1 * SOFR_{t-1} + \beta_2 * SOFR_{t-2} + \beta_3 * SOFR_{t-3} + \epsilon_t \qquad (16)$$

Table 15: *AR(3) results Dollar ARR: SOFR*

	SOFR	SOFR stand.
α	2.1268	-0.0889
β_1	0.5682	0.5682
β_2	0.1163	0.1163
β_3	0.2004	0.2004
α error	0.078	0.237
β_1 error	0.048	0.048
β_2 error	0.055	0.055
β_3 error	0.048	0.048
α p-value	0.000	0.708
β_1 p-value	0.000	0.000
β_2 p-value	0.036	0.036
β_3 p-value	0.000	0.000
Log like.	106.427	-357.827
No obs.	417	417

Looking at the results of the overnight zero rates, we notice that the value of the β's decreases as the lag increases. This is what we would expect. The most recent value, the SOFR at *t-1*, is a better estimator of the current value at time t compared to the value at *t-3*. The α value for both the original overnight zero rate as for the standardized data is insignificant. This component is therefore dropped.

Auto Regressive Moving Average Model

We once again test the optimal values for p and q in the ARMA(p,q) model and conclude to use ARMA(1,1). (12) shows the ARMA(p,q) formula in Section 2.4. The results of the ARMA(1,1) are shown in Table 16.

Table 16: *ARMA(1,1) results Dollar ARR: SOFR*

	AR(1)	MA(1)	AR(1) error	MA(1) error	AR(1) p-value	MA(1) p-value	Log like.	No. obs.
SOFR	0.9995	-0.6964	0.001	0.054	0.000	0.000	109.500	417
SOFR stand.	0.9798	-0.6504	0.014	0.069	0.000	0.000	-351.806	417

The AR(1) value is close to one, indicating a strict relation between the value of the SOFR at t and *t-1*. For both the overnight zero rates as the standardized rates, the values for the MA(1) are negative. This is similar to the formula notation by Box and Jenkins.

4.1.3. Sterling cross-sectional OLS

As stated before, the Dollar Alternative Reference Rate is the Secured Overnight Financing Rate (SOFR). We start with an OLS analyses, to analyze the relationship between the IBOR and the ARR. For the Dollar rates, the formula is shown in (17).

$$SOFR_t = \alpha_t + \beta_t * FEDFunds_t + \epsilon_t \tag{17}$$

For the overnight zero SOFR data, we see that the α_t is insignificant. This is the value for the intercept, the position where the line crosses the y-axis of the plot. The coefficient β_t is significant. The value for the Adjusted R-squared is 0.737 which is high, but not comparable with Sterling OLS results. This may be due to the effect of window-dressing.

Table 17: *OLS results Dollar ARR: SOFR*

	α_t	β_t	α_t error	β_t error	α_t p-value	β_t p-value	Adj. R^2	No. obs.
SOFR	-0.0086	1.0275	0.064	0.030	0.893	0.000	0.737	417
SOFR stand.	0	0.8586	0.025	0.025	1.000	0.000	0.737	417

The effects of window-dressing are visible in the Secured Overnight Financing Rate. In order to analyze this effect, we perform a regression in which we flatten the period from the last two days of the month until the first two days. This removes most of the impact of the month-end window-dressing effects. In Figure 15, we see the SOFR and the FED Funds rate with flattened month-end rates. In addition, we remove the outlier on the 17th of september 2019. This event is known as the 'SOFR Surge Event'. Due to a combination of events, namely $60 billion in treasury debt maturities that impacted available cash, in combination with $115 billion of investment grade debt and the lack of cash as a result of the upcoming corporate tax payments, the SOFR increased with 282 basis points. This events has a major impact on the regression results. If we filter these window-dressing events and run the OLS regressions again, we get very different results.

Figure 15: *SOFR timeseries reduced window-dressing effect*

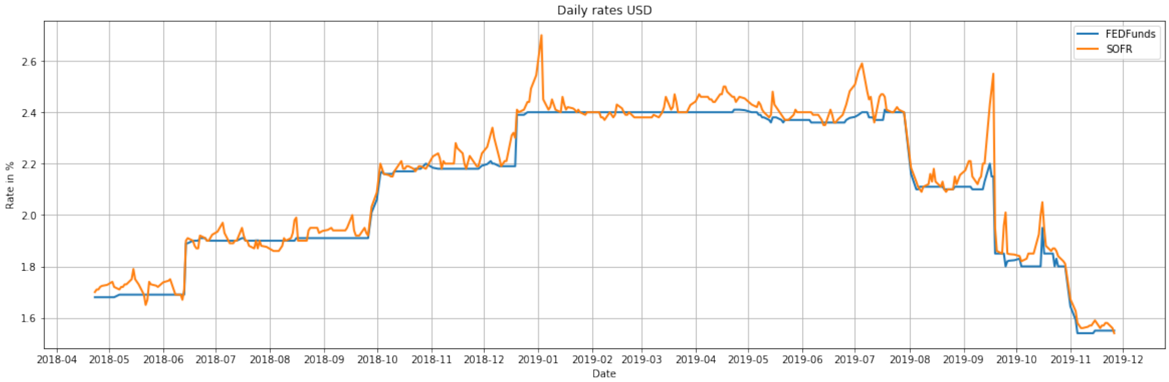


Table 18 shows the result of the OLS regression for the USD interest rates where the month-end effects are flattened. We comapre the adjusted R^2 of both regression tests. Table 14 showed a value of 0.737 while the adjusted R^2 is currently 0.969 which means that the FED Funds almost identically reflects the SOFR. This means that the SOFR Surge Event and the window-dressing effects are responsible for most of the deviation between the rates.

Table 18: *OLS result reduced window-dressing*

	α	β	α error	β error	α p-value	β p-value	Adj. R^2	No. Obs.
$SOFR = \alpha + \beta * FEDFunds + \epsilon$	0.0085	1.0114	0.019	0.009	0.649	0.000	0.969	417

Since the window-dressing effect is a recurring month-end effect, we continue our regressions with the original overnight zero rates and with the standardized data without smoothing the month-end rates. We add lags for the FED Funds rate, to analyze the effect of the lagged variables. Since the GBP Libor lag for *t-3* is insignificant, the formula for the Dollar rates is shown in (17).

$$SOFR_t = \alpha_t + \beta_1 * FEDFunds_t + \beta_2 * FEDFunds_{t-1} + \beta_3 * FEDFunds_{t-2} + \epsilon_t \quad (18)$$

The results are shown in Table 19. We notice that, apart from the α term, all terms are significant. It stands out that the value for β_2 is much higher than β_1. This indicates that the value of the FED Funds at *t-1* is a better estimation of $SOFR_t$ than the value of the FED Funds rate at t. The R^2 is exactly the same as for the AR(1).

Table 19: *OLS results Dollar ARR: SOFR*

	SOFR lag	SOFR lag stand.
α	-0.0065	0.0034
β_1	0.5581	0.4678
β_2	1.0214	0.8543
β_3	-0.5530	-0.4615
α error	0.065	0.025
β_1 error	0.260	0.218
β_2 error	0.362	0.302
β_3 error	0.261	0.218
α p-value	0.920	0.892
β_1 p-value	0.033	0.033
β_2 p-value	0.005	0.005
β_3 p-value	0.035	0.035
Adj. R^2	0.737	0.737
No. obs.	417	417

4.2. Basis timeseries

We now analyze the basis spreads per currency zone. The basis spread formulas are described in Section 2.1.

4.2.1. Sterling basis timeseries

Auto Regressive Model

We start with the Sterling basis spread. From (1) we find the Sterling basis spread is the GBP Libor subtracted from the SONIA. The maximum number lags to add that are significant is 6. The formula for the AR(6) model are shown in (19).

$$SterlingBasis_t = \alpha_1 + \sum_{i=1}^{6} \beta_i * SterlingBasis_{t-i} + \epsilon_t \tag{19}$$

The results are shown in Table 20. Although the third lag is increasing compared to the second, overall the β's are decreasing as the i increases. The constant term is insignificant. The result indicates that the spread between the rates stays fairly stable for longer periods of time. The standardized rates show different results. The added fifth and sixth lag show insignificance and therefore the an AR(4) model is used for the standardized data. β's one to four are decreasing as i increases, similar to original zero rates.

Table 20: *AR(6) and stand. AR(4) results Sterling spread*

	Sterling spread	Sterling spread stand.
α	0.0150	0.0022
β_1	0.2273	0.3106
β_2	0.1400	0.1853
β_3	0.2032	0.2365
β_4	0.1633	0.1684
β_5	0.1071	
β_6	0.1413	
α error	0.017	0.044
β_1 error	0.048	0.048
β_2 error	0.050	0.049
β_3 error	0.049	0.049
β_4 error	0.049	0.048
β_5 error	0.050	
β_6 error	0.049	
α p-value	0.367	0.961
β_1 p-value	0.000	0.000
β_2 p-value	0.005	0.000
β_3 p-value	0.000	0.000
β_4 p-value	0.001	0.001
β_5 p-value	0.031	
β_6 p-value	0.004	
Log like.	1414.612	393.484
No. obs.	417	417

Auto Regressive Moving Average Model

Testing the p and q value in the ARMA(p,q) results in a maximum ARMA(1,1) model for the overnight zero rates and the standardized data. The results are shown in Table 20.

Table 21: *ARMA(1,1) results Sterling basis*

	AR(1)	MA(1)	AR(1) error	MA(1) error	AR(1) p-value	MA(1) p-value	Log like.	No. obs
Sterling basis	0.9987	-0.8165	0.002	0.026	0.000	0.000	1421.032	417
Sterling basis stand.	0.9784	-0.7014	0.011	0.038	0.000	0.000	396.207	417

For the ARMA(1,1) model, both coefficients are significant. We notice a value of AR(1) that is almost one. This indicates the value of the Sterling spread *t-1* at *t-1* is a good estimator of the value at *t*. The negative moving average value. This indicates a auto correlation in the data.

4.2.2. Dollar basis timeseries

Auto Regressive Model

We now focus on the Dollar basis spread. (2) shows the Dollar basis spread as the FED Funds subtracted from the SOFR. Only the first lag in the AR(P) model is significant, therefore the formula for the AR(1) model are shown in (20),

$$DollarBasis_t = \alpha_1 + \beta_1 * DollarBasis_{t-i} + \epsilon_t \tag{20}$$

The results are shown in Table 22. Different from the Sterling basis results, we notice the β_1 to be fairly small. We conclude that the value of the Dollar basis spread at *t-1* is a weak estimator of the value at *t*. The standardized data shows similar results, although the β_1 is higher. The constant value α is insignificant due to the standardization.

Table 22: *AR(1) results Dollar basis*

	α	β_1	α error	β_1 error	α p-value	β_1 p-value	Log like.	No. obs.
Dollar basis	0.0492	0.2610	0.011	0.047	0.000	0.000	165.525	417
Dollar basis stand.	0.0003	0.3102	0.036	0.046	0.992	0.000	-307.203	417

Auto Regressive Moving Average Model

Testing the values for p and q with the coefficients being significant results in an AR(1,0) model for overnight zero rate Dollar basis spread. The results are shown in Table 22. The standardized data does shows significance for the moving average coefficient. The results for the ARMA(1,1) are shown in Table 23.

Table 23: *ARMA(1,1) results Dollar basis*

	AR(1)	MA(1)	AR(1) error	MA(1) error	AR(1) p-value	MA(1) p-value	Log like.	No. obs.
Dollar spread stand.	0.6444	-0.3824	0.153	0.191	0.000	0.046	-305.117	417

The value for the auto regressive coefficient has increased significantly compared to the AR(1) value. We notice a moving average value of 0.3824, indicating a moving mean of the residual errors.

4.2.3. Euro basis timeseries

Auto Regressive Model

The last basis spread we analyze is the Euro basis spread between the pre-€STR and the EONIA. The formula for this basis spread is shown in (4). The results for the AR(2) model are shown in Table 24.

Table 24: *AR(2) results Euro basis*

	α	β_1	β_2	α error	β_1 error	β_2 error	α p-value	β_1 p-value	β_2 p-value	Log like.	No. obs.
Euro basis	-0.0862	0.2681	0.1028	0.001	0.052	0.052	0.000	0.000	0.047	1255.873	373
Euro basis stand.	-0.0011	0.2699	0.1039	0.043	0.052	0.052	0.980	0.000	0.045	-287.600	373

From the original Euro basis spread timeseries results we conclude that the first and second lag are limited estimators for the value of the $EuroBasis_t$. Similar results are found for the standardized data.

Auto Regressive Moving Average Model

The ARMA formula is shown in Section 2.4. Analyzing the Euro basis spread using this model, gives us the results in Table 25. The results are from an ARMA(1,1) model.

Table 25: *ARMA(1,1) results Euro basis*

	AR(1)	MA(1)	AR(1) error	MA(1) error	AR(1) p-value	MA(1) p-value	Log Like.	No. obs.
Euro basis	0.7633	-0.5531	0.084	0.106	0.000	0.000	1242.348	369
Euro basis stand.	0.7640	-0.5531	0.084	0.106	0.000	0.000	-283.397	369

For the original data, we observe a high value for the first regressive leg, indicating the rate is strongly dependent on the value of the previous day. The standardized data shows similar results.

4.3. Interest rate timeseries findings

Dollar interest rate observations

We start with the observations for the Dollar timeseries. From Table 5 we concluded that the SOFR has a mean that is 5 basis points higher compared to the FED Funds. In addition, the rate is more volatile with month-end window dressing effects as well. Comparing Figure 7 and 8 indicates the FED Funds rate has periods in which the rate is stable, while the SOFR fluctuates daily. This difference in volatility has an impact on the valuation of new products after the transition. An increase in volatility may result in a beneficial situation for certain parties while others are disadvantaged. In addition, financial products referencing the interest rate will have a different value if volatility changes. The Hurst exponent indicates the FED Funds rate shown no sign of a reversion to its mean while the SOFR is strongly mean-reverting. Figure 6 shows us some asymmetry, where it looks like the SOFR is almost constantly higher and only peaks upward before returning to the mean. The are no downward peaks.

Looking at the regression results, we find that the FED Funds rate at time t can only significantly be estimated by the value at *t-1* using an auto regressive model. The β is very close to 1. This shows the residuals are serially correlated at the first lag. A value of 1 would indicate the process to be a random walk.

The regression results for the SOFR are very different compared to the FED Funds results. First of all, the Auto Regressive results show three significant lags, with each β decreasing as the lag increases. The ARMA(1,1) model is significant indicating, besides the value of SOFR at *t-1*, the past error can be used as to estimate the error at t.

The Dollar basis timeseries is analyzed in Section 3.3.2 and in Section 4.2.2. Table 10 shows us the mean of the Dollar basis is approximately 0.049. From Figure 13 we notice that the basis rarely drops below the mean with most movements and spikes going upward. This originates from the SOFR spikes and the way the spread is defined in (2). Testing the mean-reverting nature of the rate using the Hurst exponent shows us a value of 0.028, indicating a very strong mean-reverting nature for the Dollar basis. Looking at the Auto Regressive results, what stands out is the low value for the β_1 indicating relatively low serial correlation. The residual error of the moving average in the ARMA model does show significance.

Sterling interest rate observations

Appendix 3.2.1 and Section 3.3.1 describe the Sterling interest rates timeseries and basis timeseries. The mean of the SONIA is higher than the GBP Libor, opposite to what one would expect due to the term premium and credit risk components inherent in the GBP Libor. This decrease in GBP Libor indicates some asymmetry between the rates. Both rates show no sign of mean-reverting tendency, as a result of the increase in August 2018. Testing the Hurst exponents starting at September shows both rates to have a mean-reverting tendency, but not much as seen for the Dollar timeseries. The volatility of both rates is fairly similar, although the SONIA is less volatile in more recent events.

Looking at the regression results in Appendix **??**, we conclude from the Auto Regressive model results that the GBP Libor shows strong positive correlation at its first lag, with weaker negative correlation at the second lag and a decreasing positive correlation at the third lag. This negative value indicates a reversion toward an equilibrium value.

From Table 40, we conclude that the SONIA shows strong correlation for its first lag. Since only the first lag is significant, this means only the first lag can be used for estimating the value if SONIA at t. There is significant moving average component. Reducing the model gives us the AR results that have already been discussed.

For the Sterling zone, we lastly descirbe the findings for the Sterling basis results from Section 4.2.1. From Table 19 we conclude that the overnight zero Sterling rates basis at time t can be estimated with significant six legs. For the standardized data, we observe four significant legs. Different from the Dollar basis, this suggest the basis to be more stable. This can be concluded when comparing Table 9 and 10. It stands out that the Sterling basis in Figure 12 is increasing, and becoming more volatile. The ARMA(1,1) results in Table 20 indicate a high correlation with the first AR(1) coefficient.

Euro interest rate observations

In Section 3.2.3, we described the Euro timeseries data. Since the spread between the €STR and the EONIA is fixed, we analyze the period prior to the €STR. Therefore, we compare the pre-€STR and the EONIA. In Figure 10 we observe that, although the spread between the rates is fairly stable, it does show volatility. The EONIA has more spikes resulting in an increased volatility. From the Hurst exponent in Table 7, we conclude that both rates are mean-reverting. The EONIA Hurst exponent value is closer to zero, which indicates a stronger mean-reverting nature compared to the pre-€STR. The returns in Table 8 show difference between the two rates. The EONIA has a higher mean and a much higher volatility, as a result of the spikes seen in Figure 10.

We analyzed the Euro interest rate timeseries in Section **??**. We start with the Euro IBOR EONIA results in Section A.2.3. From Table 41, we conclude that estimating the EONIA at time t can be done with three significant lags. We notice the β to decrease as the i increases, which is what we would expect. The standardized data shows similar results. The Auto Regressive Moving Average results are shown in Table 42. Since the overnight data is not stationary, only the standardized data is analyzed. We conclude the ARMA(1,1) model coefficients are significant, indicating the error at *t-1* can be used to estimate the value of the EONIA at t.

The pre-€STR rate is the ARR for the Euro rates. From Table 43 we conclude that only the first lag is significant. The value of the β_1 is very close to one, indicating strong correlation with the first lag. Since only the first lag is significant, this indicates a more volatile process. The ARMA(1,1) model shows is not significant for MA(1). Reducing the model gives the AR(1) model as discussed.

Lastly, we summarize the findings for the Euro basis spread analyzes from Section 3.2.3 and 4.2.3. Looking at Figure 14 and Table 11, we observe the volatility of the basis to be very low. In addition, the median of the Euro basis is the exact fixed spread that is used between the €STR and the EONIA. With a Hurst exponent value of almost 0, we observe the rate is strongly mean-reverting. Looking at the analyzes of the rate using regressions, we observe an AR(2) model in Table 23. The values of the β's are relatively low compared to the Sterling and Dollar basis values. The Euro basis at time t can be estimated with an ARMA(1,1) model.

Conclusion

The explanatory models used give us an indication of the behavior, auto-correlation of the rate and provide us with the significant coefficients to forecast the values in Chapter 5. We observe that in each currency zone, there is no single model with a specific parameter that outpeforms the other models for the IBORs and ARRs. In Chapter 3, we identified several differences in the basis statistics of the rate, indicating a different behavior. The regression models in this chapter verify that the IBOR and ARR for each currency zone are structurally different. We therefore conclude that, based on our findings in this chapter, the difference with regard to behavior and auto-correlation will cause additional challenges and therefore impact the ease and speed of the transition.

5. Predictive models

In this chapter, the regression models of Chapter 4 are used to predict the rate. We use in-sample forecasting and validate the predicted results with the actual results. The results of the prediction fit are used to describe what regression model can best be used to predict the rates. These findings, in combination with the observation discussed in Section 4.3, are used to discuss the transition impact in Chapter 6.

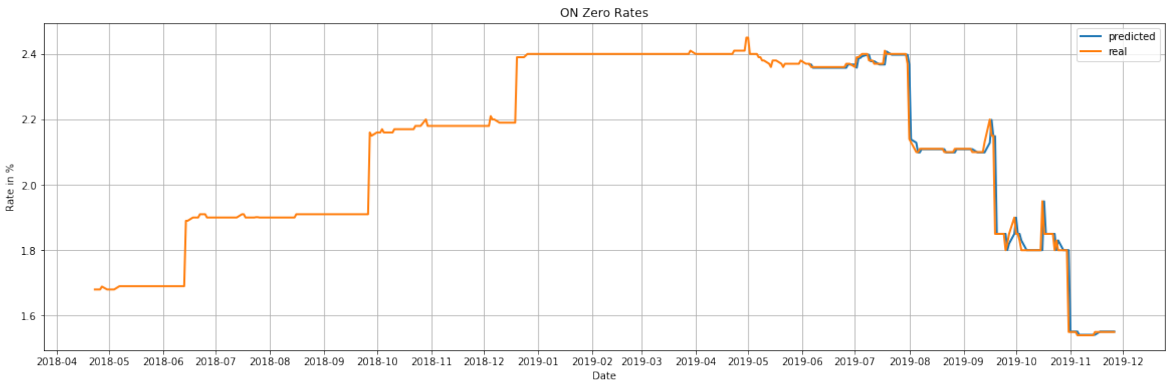
5.1. Interest rate timeseries

5.1.1. FED Funds interest rate

Auto regressive

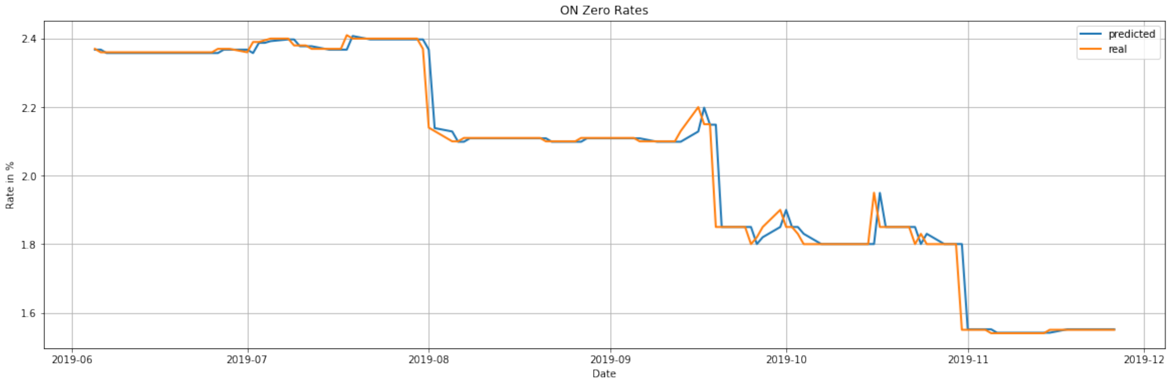
Section 4.1.1 discussed the auto regressive model for the FED Funds data, resulting in the use of an AR(1) model. For this reason, we predict the FED Funds rate using an AR(1) model as well. Figure 16 shows the AR(1) one-step ahead predictor for the FED Funds rate.

Figure 16: *FED Funds AR(1) prediction 30%*



We observe that the predicted values closely follow the actual rates. Figure 17 shows only the predicted observations compared to the actual values. We notice that, due to the auto regressive lag, the predicted values consistently change after the actual overnight zero rates change.

Figure 17: *FED Funds AR(1) prediction sample 30%*



The delay in volatility change results in a small misfit between the actual and predicted rates. The size of the misfit is tested using the Mean Square Error (MSE) and the Mean Absolute Percentage Error (MAPE), with the results shown in Table 26. The MSE is the square value of the sums of the errors between the predicted and actual value for each data point. Since this causes positive errors to cancel out the negative errors, we also use the MAPE. MAPE is the average of the sum of the absolute difference between actual value at t and the predicted value at t, divided by the actual value at t.

Table 26: *AR(1) prediction results Dollar IBOR: FED Funds*

Sample prediction size	MSE	MAPE
90%	0.0025188547	1.06314
80%	0.0025119061	0.96175
70%	0.0021190860	0.80674

We observe that the best predictions with the lowest MSE is for the smallest training data set used. This is contradictory to what literature suggest. We expect this difference to originate from an increase of volatility in the final months of the data set. This is also seen for the MAPE results. In Figure 17, we notice that the increased volatility causes the predicted values to deviate more compared to the stable periods in August. The model performs better compared to the one-step naive forecast for the 70% and 80% sample, but this is due to the lacking volatility in these periods.

Auto regressive moving average

In Section 4.1.1 we used the Auto regressive moving average model to analyze the overnight zero rates. We concluded that an ARMA(1,1) model was not suitable for the data set.

5.1.2. SOFR interest rates

Auto regressive

After testing the maximum number of lags of the AR(p) model for the SOFR overnight zero rates in Section 4.1.2, an AR(3) model was used to analyze the data. This model is now used to predict the SOFR rate and to analyze the predicted values compared to the actual values.

Figure 18: *SOFR AR(3) prediction sample 30%*

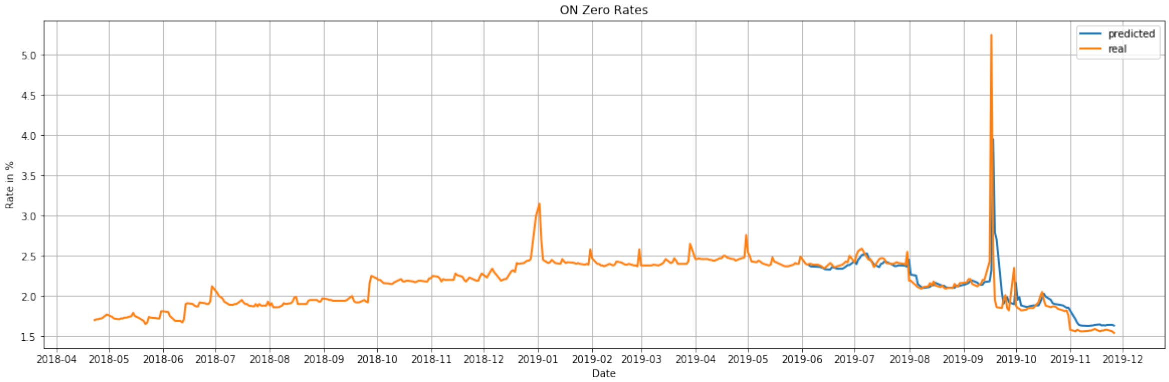


Figure 19 shows the results for the entire actual rate in orange and the last 30% predicted values in blue. We notice that, although the SOFR is very volatile, the AR(3) predictions are fairly accurate. Table 28 shows the MSE for the AR(3) predictions. The MAPE values are extremely high. Similar to the OLS results, the error in percentages become very high since the actual SOFR rates have extreme spikes that are smoothed out by the model, therefore the errors increase.

Table 27: *AR(3) prediction results Dollar IBOR: SOFR*

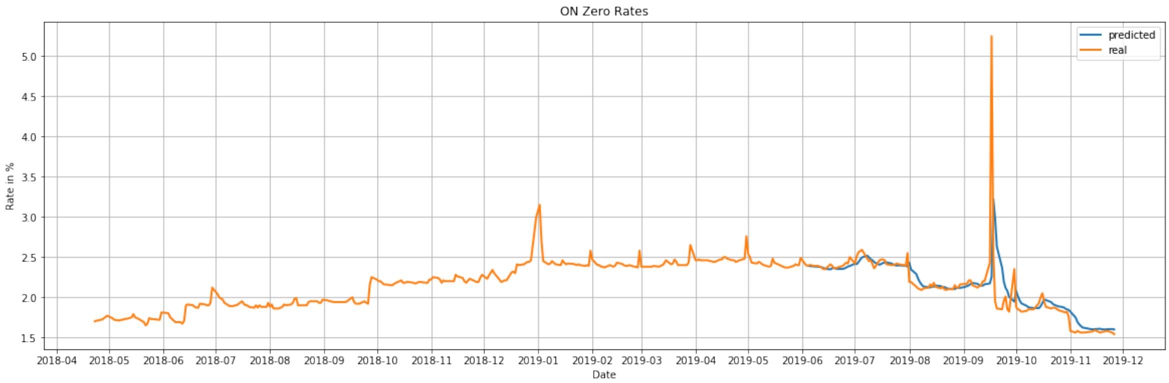
Sample prediction size	MSE	MAPE
90%	0.0082741595	4.39263
80%	0.1527322753	5.79525
70%	0.1031037220	4.53734

We observe that the best prediction is the one where the training data contains 90% of the total data set, the prediction size is therefore 10%. This is the predicted outcome. This sample prediction size significantly outperforms the other two tests. This is caused by the absence of the previously described SOFR Surge event in the data set. Both the 20% and the 30% test do contain this event, which causes the misfit between the actual and the predicted values.

Auto regressive moving average

From Section 4.1.2 we conclude an ARMA(1,1) model is used for the prediction of the SOFR rates. In Figure 20, we see the actual SOFR rates and the predicted values for the last 30% of the data.

Figure 19: *SOFR ARMA(1,1) prediction sample 30%*



We notice that, because of the moving average in the model, the prediction is more smooth compared to the AR(3) model. Since the SOFR is a fairly volatile rate, the smoothing nature of the ARMA model predictions results in slightly better predictions. The best prediction is once again based on the largest training data set. The SOFR Surge event occurs at around 85% of the data sample. This causes the second test, with a training sample size of 80%, to perform the worst. The MAPE values show similar results. Although the ARMA(1,1) model predictions look close to the actual values, the percentage errors are high due to the extreme values at month-end.

Table 28: *AR(3) prediction results Dollar ARR: SOFR*

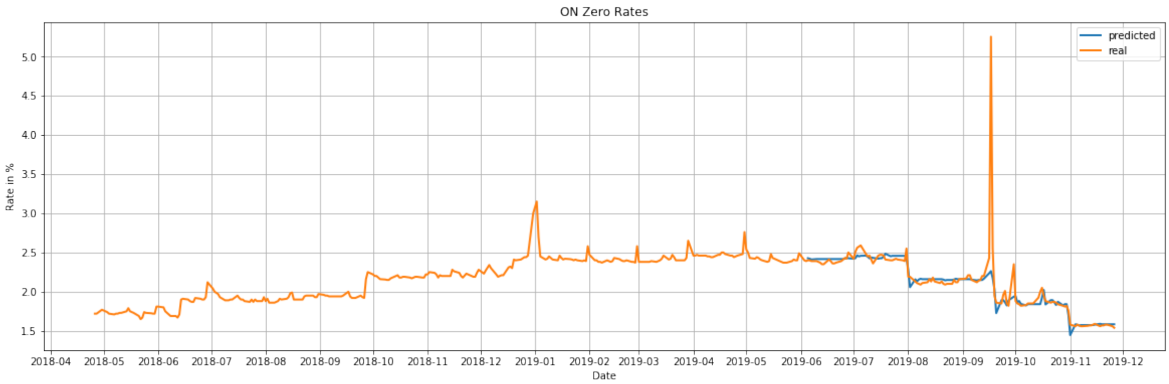
Sample prediction size	MSE	MAPE
90%	0.0073926788	3.90216
80%	0.1463217522	5.93769
70%	0.0986050860	4.54201

5.1.3. Sterling cross-sectional OLS

Ordinary Least Squares

Similar to the Ordinary Least Squares regression for the Sterling rates, three lags are significant as shown in 4.1.2. Predicting the last 30% of the SOFR values using three lags of the FED Funds rate as described in (18) is shown in Figure 16. We notice that the predicted SOFR is far less volatile, since it is based on the FED Funds.

Figure 20: *SOFR OLS prediction sample 30%*



Testing the goodness-of-fit of the predicted values is done by calculating the MSE. The values of the MSE for a training data set consisting of 70%, 80% and 90% of the original data set in shown in Table 27. The best prediction results are achieved for the largest training data set. This corresponds to what is expected. We observe that MAPE for all three tests is very high, even larger than two. The OLS model predictions for the SOFR are based on the FED Funds values. Due to the higher volatility of the SOFR and the window-dressing events effecting the rate, the error values become very high.

Table 29: *OLS prediction results Dollar IBOR: SOFR*

Sample prediction size	MSE	MAPE
90%	0.0062808866	2.09373
80%	0.1142952340	2.99382
70%	0.0769918736	2.64416

5.2. Basis spread timeseries

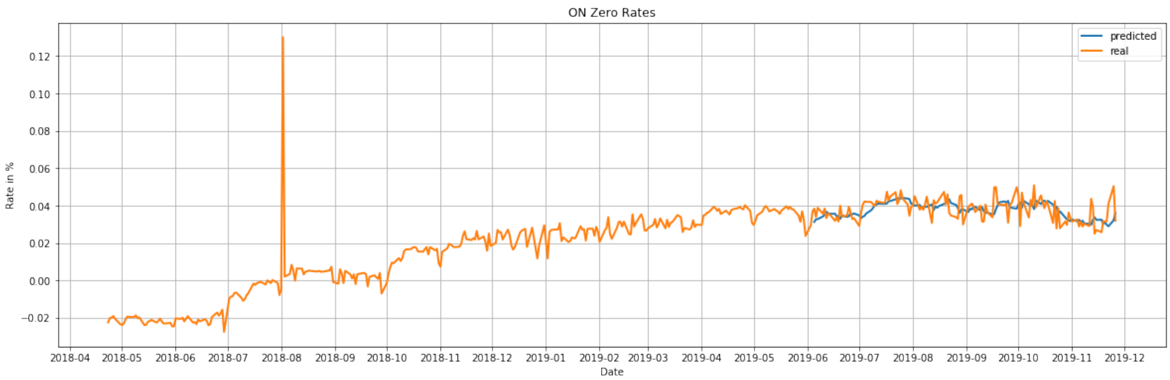
We now forecast the basis spreads per currency zone. In Section 4.2 the model parameters were determined. The values found are used to forecast the basis spread data and to test the accuracy of the different regression models.

5.2.1. Sterling basis

Auto Regressive

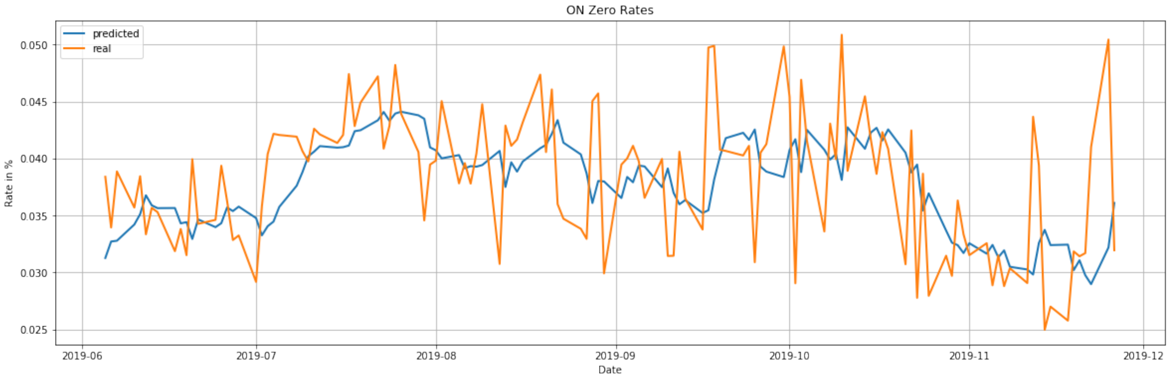
In Section 4.2.1 an AR(6) model is used to analyze the Sterling basis spread. Similar to Section A.3.1, the forecast values is tested using the MSE with 70%, 80% and 90% of the original data set used as training data for the forecast model. Figure 21 shows the Sterling basis spread data and the predicted results for the last 30% of the data.

Figure 21: *Sterling basis spread AR(6) prediction sample 30%*



The Sterling basis spread is far more volatile compared to the predicted values using the AR(6) model. The predicted values accurately follow the direction of the actual values, but in a smoother way. Figure 22 shows only the data for the last 30% of the data set. It becomes clear that the actual rates are more volatile with spikes both upwards and downwards while the predicted values follow the overall trend.

Figure 22: *Sterling basis spread AR(6) last 30% data set*



To test the accuracy of the predicted values, we test both the MSE as the R-squared of the predicted values. The results are shown in Table 30. What stands out is that, although the MSE are very low for each of the sample size tests, the R-squared values is very low. The reason for the R-squared to be so low, is a result of volatility of the Sterling basis spread. The best prediction results are for the smallest training data set, contradictory to what we would expect. The MAPE values are so extreme due to the nature of the model. Since an AR(6) model is used with low weights for each coefficient as seen in Table 29, the predicted rate is smoothed out and is not as volatile as the actual Sterling basis spread. This results in a high level of errors for the predicted rates.

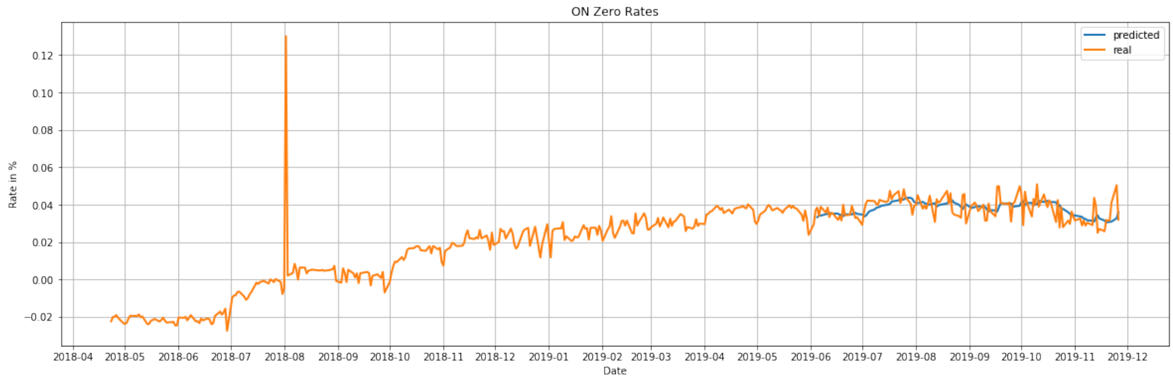
Table 30: *AR(6) prediction results Sterling basis spread*

Sample prediction size	MSE	R^2	MAPE
90%	0.0000442731	0.0812278955	11.11492
80%	0.0000389877	0.0609273629	12.84640
70%	0.0000307997	0.1120519982	11.11492

Auto regressive moving average

The auto regressive moving average model is used next to predict the Sterling basis spread and test the accuracy of the model. An ARMA(1,1) model is used for the Sterling basis. Figure 23 shows both the actual Sterling basis spread and the predicted values using the ARMA(1,1) model.

Figure 23: *Sterling basis spread ARMA(1,1) last 30% data set*



Similar to the AR(6) prediction result in Figure 21, we notice that the predicted values are not as volatile compared to the actual Sterling basis spread. The performance of the ARMA(1,1) model predictions is determined by calculating the MSE and the R-squared of the predicted values. These are shown in Table 31.

Table 31: *ARMA(1,1) prediction results Sterling basis spread*

Sample prediction size	MSE	R^2	MAPE
90%	0.0000418851	0.1307859064	14.15560
80%	0.0000360530	0.1316142971	12.60135
70%	0.0000285724	0.1762645521	10.93366

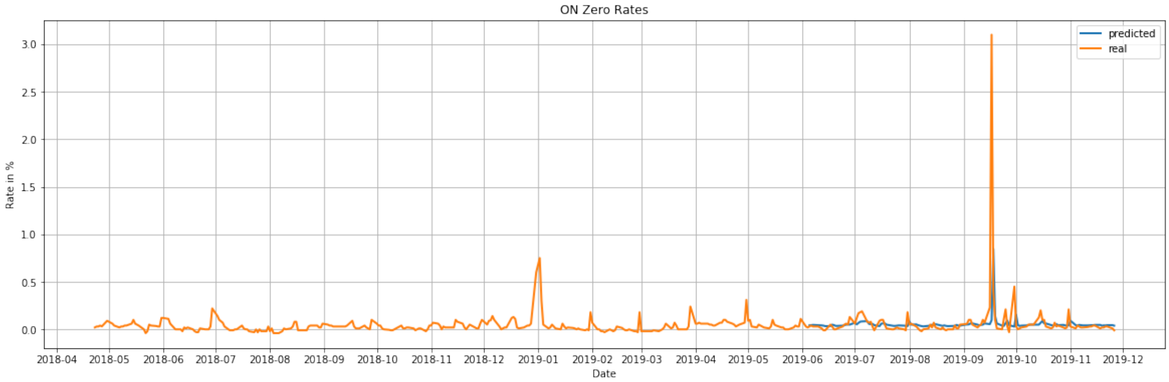
It stands out that once again the best performing test is the one with a training data set that is 70% of the total data set. Since the model performance is increasing as the training data decreases, we conclude that model performance is mostly reliant on the volatility of the actual Sterling basis spread. The more volatile the actual data, the worse the prediction. This is also the case for the MAPE results of the ARMA(1,1) predictions as previously explained for the AR(6) results in Table 30.

5.2.2. Dollar basis

Auto regressive

The Dollar basis spread is shown in Figure 13 of Section 3.3.2. In Section 4.2.2 an AR(1) model was used to analyze the Dollar basis spread. The predicted values for the last 30% of the data and the actual Dollar basis spread timeseries are shown in Figure 24.

Figure 24: *Dollar basis spread AR(1) last 30% data set*



Next, the model performance is tested by calculating the Mean Square Error and the R-squared of the predicted data compared to the actual Dollar basis spread rates. The results are shown in Table 32. No MAPE values are available for the Dollar basis spread since the spread regularly is zero. This results in infinite values when dividing with 0.

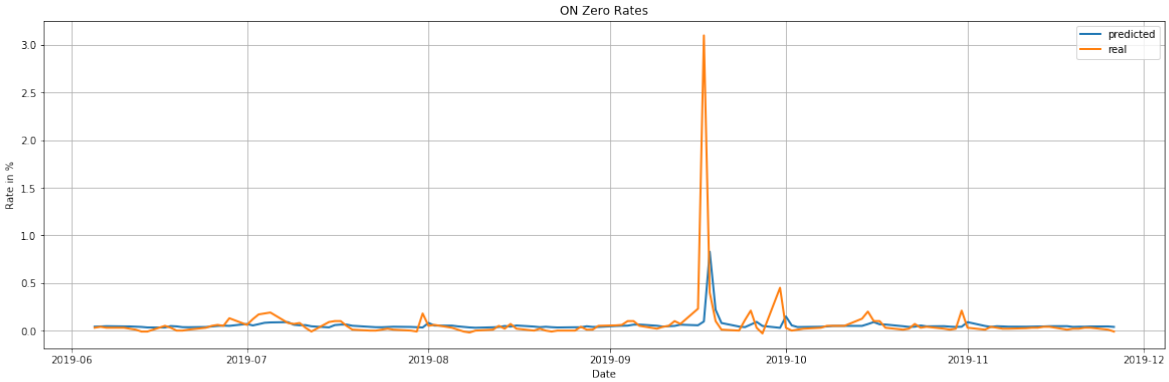
Table 32: *AR(1) prediction results Dollar basis spread*

Sample prediction size	MSE	R^2
90%	0.0022050479	-0.0754770567
80%	0.1155470428	0.0082399468
70%	0.0775053905	0.0150702308

Auto regressive moving average

An ARMA(1,1) is used to predict the Sterling basis spread timeseries as this is the significant model found in Section 4.2.2. The predicted values and the actual Dollar basis spread values for the last 30% are shown in Figure 25.

Figure 25: *Dollar basis spread ARMA(1,1) last 30% data set*



Looking at Table 33, we notice that the best performance of the MSE is for the 90% training data set while the highest R-squared is achieved for the 70% training data set. In Figure 25, we observe that the predicted values follow the movements of the actual values, but are not as volatile, which becomes especially clear for the upward peaks in August. The MSE results are very low, indicating small errors deviations between the predicted and the actual values, nevertheless the R-squared values are low as well, indicating the predictions are not that accurate.

Table 33: *ARMA(1,1) prediction results Dollar basis spread*

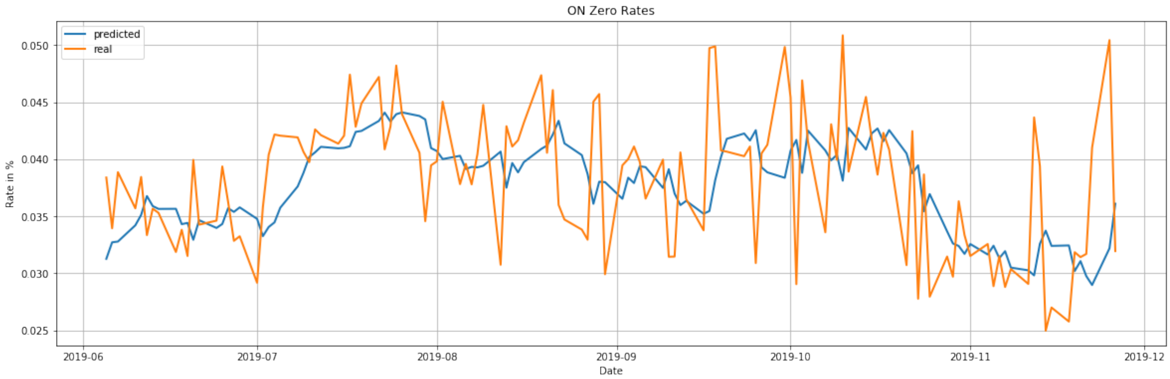
Sample prediction size	MSE	R^2
90%	0.0022035970	-0.0747694464
80%	0.1155904636	0.0078672584
70%	0.0775247580	0.0148241110

5.2.3. Euro basis

Auto regressive

The Euro basis spread is the spread between the pre-€STR and the EONIA. In Section 4.2.3, we concluded an AR(2) model could be used to forecast the values of the basis spread. The actual and predicted Euro basis spread are shown in Figure 26.

Figure 26: *Euro basis spread AR(2) last 30% data set*



We notice that the actual overnight Euro basis spread is far more volatile compared to the predicted values for the AR(2) model. Table 34 shows the accuracy tests for the predicted values. We notice that the best performing model test is for the 80% data set, since this generates the lowest MSE and MAPE, and the highest R-squared. Still we observe high errors for each test and MAPE values far greater than one. This indicates the predicted values are still not accurate. This is due to the difference in spikes, resulting in large squared or percentage error between the actual and predicted values.

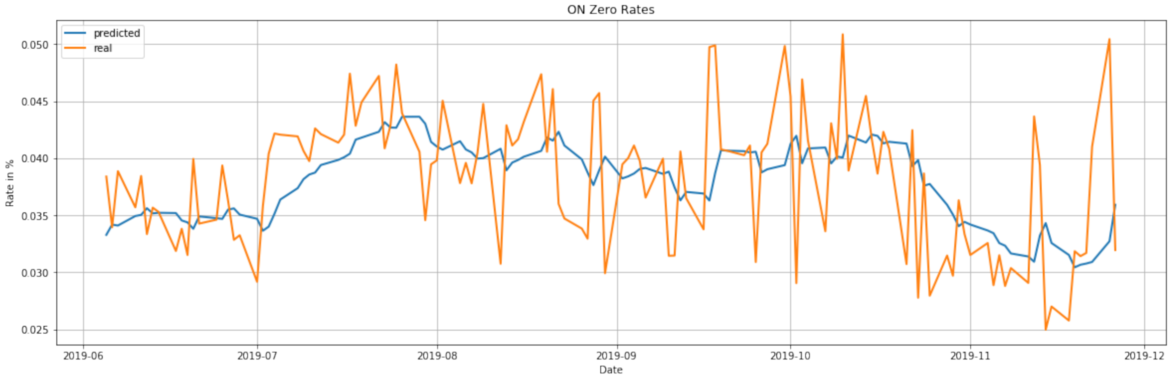
Table 34: *AR(2) prediction results Euro basis spread*

Sample prediction size	MSE	R^2	MAPE
90%	0.0000429276	0.1190278853	4.97184
80%	0.0000348246	0.2232819543	4.84692
70%	0.0001591373	-0.0032022806	6.03968

Auto regressive moving average

Next we forecast the pre-€STR Euro basis spread using an ARMA(1,1) model. We observe that the moving average causes the predicted values to be more smooth compared to the AR(2) results. We once again observe the actual Euro basis spread to be far more volatile compared to the predicted values.

Figure 27: *Euro basis spread ARMA(1,1) last 30% data set*



The results for the ARMA(1,1) predictive values in Table 35 are similar to the AR(2) results from Table 34. Since the longest training data set does not lead to the best prediction, we conclude that the predictive model performance is reliant on the temporary performance of the rate, especially with regard to volatility. The extremely high MAPE values indicate the relative errors are very high for the predicted ARMA(1,1) values.

Table 35: *ARMA(1,1) prediction results Euro basis spread*

Sample prediction size	MSE	R^2	MAPE
90%	0.0000416055	0.1461609152	4.69788
80%	0.0000339609	0.2425459925	4.64315
70%	0.0000416055	0.0069454193	5.91201

5.3. Key Findings predictive models

Sterling rates

The Sterling model prediction results are shown in Section A.3.1. The results of the GBP Libor, the Sterling IBOR rate, show the predictions are not very accurate. The Mean Square Error results are very low since the interest rates small number, and taking the square of the residuals results in an even smaller number. In addition, the downside of using the MSE is the fact that positive and negative errors cancel one another. The GBP Libor predictions show most accurate results for the 70% sample size used. Contradictory to one expectations, this indicates that the model performance is mostly reliant on the volatility of the rate during the prediction period. The volatility of the GBP Libor increases in the last months of the data set, resulting is a larger Mean Absolute Error Percentage. The best prediction accuracy results are for the AR(3) model.

The SONIA model predictions perform significantly better compared to the GBP Libor results. The only models that is not accurate is the OLS model. The OLS model consists of the lags of the GBP Libor. In Chapter 3, we observed that the Sterling spread was increasing as a result of a decrease in the GBP Libor. This results in a volatile prediction of data, while the actual SONIA values are very stable in that period. For that reason, the AR(1) model performs significantly better, with a MAPE value for all three scenarios very close to zero. This indicates the model predictions are very accurate.

The Sterling basis spread is gradually increasing as the GBP Libor decreases. In addition, the basis is very volatile. Testing the performance of the AR(6) model predictions and the ARMA(1,1) model predictions indicate very little accuracy of the models used. Due to the six legs in the AR(6) model, the prediction results are smoothed resulting in a less volatile behavior compared to the actual Sterling spread. This difference in volatility causes the lack in accuracy of the predicted values.

Dollar rates

For the FED Funds rate, we observe that only an AR(1) model is used and no ARMA(1,1) model shows significant coefficient results. The volatility of the FED Funds rate is mostly caused by governmental regulations causing the rate to increase or decrease. We observe that the model prediction are not very accurate, with MAPE values close to or greater than one, indicating the one-step naive forward performs better for the 90% test scenario.

The prediction results of the SOFR rate show very little accuracy. As previously described, the SOFR rate experiences increased volatility at month-end. In addition, the 'SOFR surge event' is part of the data set, increasing the misfit of the predicted values. We find that, similar to other rates, the model prediction accuracy is very much dependent on the daily volatility of the interest rate.

The Dollar basis spread values are predicted using an AR(1) model and an ARMA(1,1) model. Compared to the Sterling and Euro basis spread, the Dollar basis spread is fairly volatile. The MSE and R-squared indicate the predicted values do not accurately represent the actual rates. The main reason for the high MSE result is the SOFR surge event. The AR(1) model predictions are slightly more accurate compared to the ARMA(1,1) result although the difference is negligible.

Euro rates

The Euro forecast result are shown in Section A.3.4. The SONIA prediction models results for the AR(3) model and the ARMA(1,1) model show MAPE values higher than one. This indicates that the model performs worse than taking the values at *t-1* as the predicted values for *t*. Although Figures 35 and 36 look accurate, the test values indicate differently. The pre-€STR rate OLS prediction in Figure 37 indicate that the model predictions are not accurate. The EONIA and pre-€STR rate do not move parallel, resulting in a misfit of the predicted data. The AR(1) model performs significantly better than the OLS model. Although there is still a misfit, the 70% and 80% scenario MAPE values are approximately 65% and 68% respectively. Analyzing the errors indicate that most of the error occurs due to the drop in September 2019.

The Euro basis spread is described in Section 3.3.3, where we observed that the basis is mean-reverting but has a relatively low volatility. The AR(2) and ARMA(1,1) prediction results are shown in Table 23 and 24. The ARMA(1,1) model predictions are slightly more accurate, although both rates still show significant error.

Summarized

We observe that, except for the SONIA, the predicted values for all interest rates and basis spreads show significant inaccuracy of the predicted values. As stated, the accuracy of the predicted values is extremely dependent on the daily volatility of the rates. The SONIA model predictions perform best since the overnight zero rates of the SONIA are very stable, resulting in an accurate prediction. This difference in behavior and especially in volatility between the current IBORs and their new replacement ARRS will heavily affect the transition and acceptance rate of the ARRs as described in Chapter 6.

6. Transition impact

The transition from Interbank Offered Rates that have been used for decades to Alternative Reference Rates is one of the major challenges for the financial sector at this point in time. The Alternative Reference Rate Committee appointed groups for each currency zone to find the best alternative for the Interbank Offered Rates. Not only should the rates be a good alternative in the form of offering a relatively smooth transition, it should especially fix the shortcomings of the IBORs.

In Section 1.4.1, ten challenges of this transition are described. Financial Institutions referencing IBOR rates face these challenges due to the transition, and should prepare themselves before the transition deadline at the end of 2021. These ten issues have to be addressed in order to ensure a smooth transition and quick market acceptance of the new Alternative Reference Rates. In this thesis, we aimed to identify several of these challenges by analyzing the overnight zero rates of the current IBORs, the proposed ARRs and the basis spreads between these rates per currency zone. If the new Alternative Reference Rate is structurally different from the current Interbank Offered Rate, the identified challenges will become even harder and the transition from IBORs to ARRs will be delayed.

The first challenge identified by analyzing the overnight zero rates is the 'Recalibration of models'. IBORs are widely accepted and used in interest rate risk models, financial modeling, risk modeling and used as a discount factor in valuation techniques [11]. Any change in value, behavior or stability will result in the need to recalibrate the existing models since the structural difference do not allow similar models to be used. In Chapter 5, auto regressive models were used to forecast the rate and determine the behavior and predictability which can be used for valuation techniques. In addition, the stationarity of the rates are important. The Dollar basis spread is stationary, but highly volatile, especially at month-end. This is caused by the volatility spikes of the SOFR, the replacement rate for the FED Funds. From the AR(1) and ARMA(1,1) model, we conclude that the stationary process is captured by the auto regressive terms, but the volatility spikes can not. This indicates that, although the Dollar basis spread tends to return to zero, the spread is not stable. Due to the difference between the FED Funds and the SOFR, this will result in the need to recalibrate and redesign the existing model in order to reference the SOFR.

For the Sterling basis spread, we conclude that the basis spread is not stationary since on average it is increasing. The auto regressive models predictions show little accuracy, due to the volatility difference between the rates as described in Section 3.4. The auto regressive models for the GBP Libor show similar results. Due to the increased volatility of the GBP Libor, the residual errors increase, resulting in higher MAPE results. The only accurate auto regressive prediction is the AR(1) model for the SONIA rate. In Figure 34, we observe that the SONIA is very stable, resulting in accurate prediction of the auto regressive model. The difference between the current IBOR and the new ARR, depicted in the Sterling basis spread, indicate there is a value change between the rates. In addition to the value change, the behavior of the rates is structuraly different, resulting in the need to recalibrate existing models for the Sterling rate.

The Euro basis spread auto regressive models are used analyze the behavior, predictability and the stationarity of the rate. We conclude that the auto regressive models do not accurately predict the Euro basis spread, with the misfit occurring due to the volatile nature of the spread. From Section A.3.4, we conclude that the auto regressive models show little accuracy for the EONIA predictions while the pre-€STR are slightly more accurate. For the Euro zone, a fixed spread between the EONIA and the €STR has been set a 8.5 basis points. This fixed spread allows the recalibration of the models to become simpler and accelerates the transition from EONIA to the Euro Short-Term rate.

The second challenge is the 'Renegotiation of existing contracts'. The ISDA is coordinating a standard protocol for derivatives to transition from IBOR to ARR, but such coordination of a standard language is not available for cash products. This results in the need to renegotiate existing cash products that reference IBOR post 2021. This renegotiation process can result in large legal and reputation risk. If the ARRs behave very similar to the current IBORs and can be accepted as a similar rate, this will quicken the renegotiation process and reduce the possible legal and reputational risk. If the value of the ARR is significantly different, this will result in a value change of the existing contracts.

For the Dollar zone, we observe that the average spread is approximately five basis points. What stands out is the volatility of the spread, especially at month-end. The volatile behavior of the rate results the timing of the value renegotiation to be very important. If timing plays a role in the determination of a fair renegotiated value, this will be important in the acceptance of the Alternative Reference Rates.

The Sterling basis spread has an average of approximately two basis points, but the spread is increasing, starting at -2 basis points and increasing to 4 basis points at the end of 2019. Similar to the Dollar basis spread observations, we observe that the data is not stationary, and therefore the timing becomes important. This will negatively influence the transition from GBP Libor to SONIA. The Euro spread has been fixed at 8.5 basis points. This eliminates the difference in behavior and in addition

The cross-sectional regression results indicate to what extend the ARR can be predicted as a function of the IBOR. This is important for clients to understand the revaluation and to reach a valuation both parties believe is fair. When the cross-sectional regression model is not able to accurately forecast the ARRs, this will result in the additional legal risk and reputational risk. For the Dollar zone, we observe in Figure 18 that the SOFR can not accurately be expressed as a function of the FED Funds. The general movements are captured but the rates behave differently, resulting in a low accuracy of the cross-sectional model. Expressing the SONIA as a function of GBP Libor results in a high level of inaccuracy, due to the difference is value, behavior and stability. The Euro zone cross-sectional model shows similar inaccurate result. The fixed spread between the €STR and the EONIA result in a completely accurate cross-sectional model, reducing the renegotiation need and eliminating the importance of timing.

The third challenge is 'Dispute resolution'. As stated in Chapter 1, the spread between the ARR and the IBOR is influenced by a difference in premiums, both credit, liquidity and term. Since the Alternative Reference Rates are new and financial institutes determine their own way of adjusting the premiums, this will result in a different valuation between financial institutes. In this thesis, we have shown that IBORs and ARRs a structurally different in value change, behavior and stability, most likely resulting in a different valuation between parties. The lack of historical data will contribute to this difference, since there is no data to back-test new valuations, resulting is more extreme differences and therefore more disputes.

The last main challenge identified is the 'New accounting guidance'. Financial instruments, contracts and derivatives need to be recognized and accepted as eligible hedging items in specified accounting documents. The documents provide guidance on what is allowed. We have identified that the proposed ARRs are different in value, behavior and stability from the IBORs. This will result in the need for new accounting guidance, which will need more time to be developed. In addition, the lack of historical-data will delay the maturity of the ARR contracts and derivatives markets, also slowing down this process. Global guidance and coordination is needed to overcome this challenge as soon as possible. If this is not done quickly, this might result in the need for more time to transition from IBORs to ARRs.

7. Conclusion

In 2012 it was decided that the heavily used Interbank Offered Rates had be replaced. This decision was taken due to concerns on the sustainability of the IBOR rates and manipulative tendency. The Alternative Reference Rate Committee formed working groups for each currency, and set the goal to find the replacement rate for the IBOR rate in that particular currency, the Alternative Reference Rate.

For the Sterling, Dollar and Euro zone, we analyzed the existing IBOR, the replacement ARR and the basis spread. This basis spread is the difference between the IBOR and ARR. IBORs, and especially the LIBOR, are the most commonly used reference rate in the financial sector. Transitioning away from these rates brings implications. EY identified several challenges that they expect will be encountered during the transition. This research aims to answer the following main research question:

"What challenges are encountered in the transition from Interbank Offered Rates to Alternative Reference Rates by the structural difference between the rates?"

To answer this research question, we designed three phases in which different aspects of the IBORs, ARRs and the basis spreads are analyzed. The first phase is the standard data analysis by determining descriptive statistics, behavior and stability of the rate. We observed differences in the mean, volatility, asymmetry and the mean-reverting nature of the rates. The second phase was using explanatory regression models to understand the nature of the rates with regard to their auto-regressive components, moving averages and cross-sectional test between the IBORs and ARRs. In the last phase, we used the regression models to forecast the values and analyze the accuracy of the predicted values. This resulted in insights with regard to the behavior of the rates. The three phases have led to the identification of four major challenges in the IBOR transitions due to structural differences between the rates. These four challenges are:

Recalibration of models

We concluded that, due to the different behavior of the rates, especially with regard to volatility, currently used models will have to be recalibrated. Interest rate risk models, financial modeling, risk modeling will have to be adjusted and in addition a new discount factor rate will have to be implemented since Libor is currently the go-to discount rate. The month-end volatility spikes of the SOFR and the increasing spread and difference in volatility for the Sterling basis cause this need for recalibration. The pre-€STR analyses shows similar results, although for this currency, a fixed spread between the €STR and the EONIA has been set at 8.5 basis points, reducing the difficulty of the recalibration.

Renegotiation of existing contracts

For derivatives, a standard language has been developed to transition products that are IBOR referencing to ARR linked-products. As this is not available for cash products, the cash contracts that reference IBOR and especially long-term contracts maturing post 2021 will have to be renegotiated to determine a new 'fair' value. We have identified several differences between the IBORs and ARR, complicating this renegotiation. In addition we have used a cross-sectional regression to forecast the ARR as an function of the IBOR. This resulted in a high level of inaccuracy. Since the ARR can not be accurately expressed as a function of the IBOR, clients may find it hard to interpret the new rate and agree to a renegotiated value of the existing contracts referencing IBOR post 2021. We conclude that this will increase the difficulty of the challenge since we have shown that the new ARRs are different in value, and stability, resulting in disputes when determining the fair revaluation and therefore complicating the transition.

Dispute resolution

Using the three analyses phases, we have identified that the proposed ARRs are different in valuation, behavior and stability. Since financial institutes such as banks are mostly themselves responsible for the risk and financial models used, the difference between the rates depicted as the basis spread per currency zone will most likely result in a different valuation between two parties. Similar to the renegotiation of existing contract challenge, if a party finds itself negatively revalued do to a difference in modeling the spread, this will result in disputes and therefore risk and reputational risk.

New accounting guidance

The last challenge identified is the challenge in new accounting guidance. For accounting standards, financial products need to be recognized as eligible for hedging purposes. If this is not the case yet, one is not allowed to use these instruments for hedging purposes such as hedge accounting. The new accounting guidance and standards are not available yet for all ARRs. The identified differences indicate that there are structural differences between the rates, resulting in the need for new accounting guidance which will take time to be developed and accepted. This forms a major challenge for the smooth acceptance of the ARRs.

Summarized

The Alternative Reference Rates replace Interbank Offered Rates and eliminate the subjectivity of the rate and in return offer transparency due to the rate being fully transactional based. This is a major benefit. Although the Alternative Reference Rates solve the downsides of the Interbank Offered Rates, the transition itself brings new challenges and complications. We have identified structural differences between the currently used IBOR and the new ARR with regard to a value change, the behavior of the rates and the stability, causing these four major challenges identified. The challenges will have to be addressed in time, in order to allow the transition to be accepted in time and for financial institutes to be ready before possible discontinuation of the IBOR rates at the end of 2021.

8. Discussion and further research

Discussion

In this thesis, we analyzed the overnight zero rates for three different currency zones to identify the main challenges encountered in the transition from Interbank Offered Rates to Alternative Reference Rates due to the structural difference in value, behavior and stability.

The main reason for transitioning from IBORs to ARRs is to eliminate the subjectivity of the IBORs by the transparent, fully transactional based ARRs. As stated in Chapter 6, two out of the four challenges exist due to a subjectivity in the (re)valuation of contracts and re-calibration of models. This indicates that, although the rates themselves are fully transaction based and therefore eliminate subjectivity in the determination of the daily quoted rate, subjectivity will still play a major role for financial institutes in their modeling and curve construction. In addition, the lack of standardization between currency zones still results in major differences.

We have analyzed the daily overnight zero rates as of the introduction of the new methodology of the SONIA. This results in approximately 1,5 years of data available, and for the Euro analysis even less. The reliability of these data sets and the results are impacted by the limited amount of historical data. In order to make sure the findings in this research hold, the data should continuously be analyzed during the coming year to provide further insights for the IBOR transition.

The analysis of the data has been used to identify the main challenges encountered by the transition. To identify these challenges, we built on an existing paper of EY. In addition to the list in this paper, many different challenges and unique challenges per currency zone are encountered, but this has been left out of this thesis due to the scope. By looking beyond this list of challenges, one could identify additional challenges due to a difference in value, behavior and stability of the IBORs and ARRs.

Also, the data used originate from a relatively stable period of financial health. This could indicate that the results we find now are not representative for other periods of economic health, such as recession. In addition, for each currency zone there has been a steep increase or decrease of the rate as a result of a governmental decision to change the level of the interest rate to influence the health of the financial sector. This has influenced the volatility of the rates and therefore the interpretation of our findings.

Several questions arise after reading the conclusions of this research thesis. The first question that comes to mind is to what extent the Euro zone transition is already finished due to the introduction of a fixed spread between the Interbank Offered Rate (EONIA) and the Alternative Reference Rate (€STR). The answer is; it is not. To clarify, the existence of a fixed spread between the rates does allow several of the identified challenges to become less relevant, but this does not mean the transition is already finished. Although fixed, both rates coexist next to each other, with different liquidities of the market and the knowledge that the EONIA will be discontinued in the near future. This results in a different bid-ask spread for both rates, although the spread between the IBOR and ARR is fixed. This brings several new challenges, and therefore we conclude that, although the transition becomes easier by introducing a new ARR with a fixed spread, the transition from EONIA to €STR is still a major challenge.

The second question is to what extent the standard language protocol coordinated by the ISDA already solves many of the challenges identified in this thesis. The answer is; it does not. Although the standard protocol offers a coordinated approach, it does not offer a solution for the transition. The standard language is a consultation that is continuously updated throughout the process. It therefore offers not a solution, but a guidance for the financial institutes. In addition, the standard language is designed by the ISDA. This means that the consultation only applies to derivatives in the ISDA region, but it is not forced upon the financial institutions. Summarized, the standard protocol does offer a support since it slowly provides more clarity, but it does not offer a solution for the transition from IBOR to ARR.

The last question is to what extent do the new ARRs eliminate the manipulative possibilities and the subjectivity of the IBORs. During the Great Recession, there was little to no market for interbank lending, resulting in a subjective rate that could easily be manipulated, which happened. One could ask whether this could also be possible for the new ARRs and what would stop major financial institutions to agree to a trade to manipulate the rate. The big difference with the Alternative Reference Rates is the pool of transactions it is based upon. Instead of just the interbank lending market, the ARRs are also based on the wholesale lending market. Since smaller banks and corporates can not access secured lending by an institution such as the FED or ECB, they have to access the wholesale lending market. Even in times of financial distress resulting in the lack of interbank lending, the wholesale market can provide the necessary transactions to base the rate upon.

Also, the IBOR panel banks were only asked to quote the level of the rate which allowed the possibility to manipulate, especially in times of financial distress. The ARRs are based on actual daily transactions, meaning that in order to manipulate the rate, the transactions will actually have to be executed. This requires skin-in-the-game, very different from 'just' quoting a rate. We conclude that there is a difference in the subjectivity and the possibility to manipulate the ARRs compared to the IBORs. This does not mean that the subjectivity and manipulative possibilities are eliminated by transitioning from an IBOR to ARR. We still identify several challenges resulting from subjectivity in the ARR and are aware that the ARR market can still be manipulated, although this might become more difficult.

Further research

We distinguish two areas for further research. Firstly, more complex models can be used to forecast the interest rate values and the basis spread. In this thesis, we have analyzed the rates using several auto-regressive models and a cross-sectional model. In order recommended to extent the types of models used to verify our findings. A recommendation would be to add specific models for modeling the short-rate. If the forecast values of such a models are very accurate, this can be used by financial institutions to try and predict the rate and use this to overcome some of the challenges identified in this thesis. This will accelerate the total transition from IBOR to ARR.

Secondly, we have analyzed the overnight zero rates for the Interbank Offered Rate, the Alternative Reference Rate and the basis spread for three currency zones, that is the Sterling, Dollar and Euro. It is recommended to analyze the zero rates at different maturities as well, to see whether the concluded findings still hold. Due to a different volume per maturity, this might result in the identification of additional or different challenges. In addition, research for different currency zones can be added such as for the Japanese Yen and the Swiss Frank. We have also only looked at one IBOR and the replacement ARR for three currency zones. Research could be expended by adding other rates per currency zone as well, such as the USD Libor and the Euribor. This might provide additional findings.

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A. Appendix

A.1. Data description

Pearson correlation

To measure the correlation between the interest rates per currency, we use the Pearson correlation coefficient. This coefficient measures the linear correlation between two variables, in our case between the old IBOR and newly proposed ARR per currency. Since we have time series data, we plot the Pearson coefficient as a time variable as well. The values range between 1 and -1, with 1 meaning positively correlated, -1 meaning negatively correlated and 0 indicating no correlation. We set a rolling window of 4. We only do this for the USD rates and the GBP rates, since the EURO rates have a fixed spread which leads to a correlation of 1. Before determining the Pearson coefficient, we standardize the data. Standardizing the data is a technique in which the data is rescaled such that the mean is 0 and the standard deviation is 1.

We first calculate the correlation between the different interest rates based on the entire time series period. These results are shown in Table 35. Next we determine the rolling window correlations.

Table 36: *Correlation between interest rates*

Rates	Overall Pearson r
USD interest rates	0.859
GBP interest rates	0.987
EURO interest rates	1.000

The results for the daily FED Funds and SOFR correlation are shown in figure 10. Looking at the rolling window correlation of the USD rates, we see the daily rates are mostly positively correlated. Different from what is seen in figure 28 for the GBP interest rates, the USD interest rates show longer periods of a rolling-window Pearson correlation value of 0.

Figure 28: *Overnight Daily Rates*

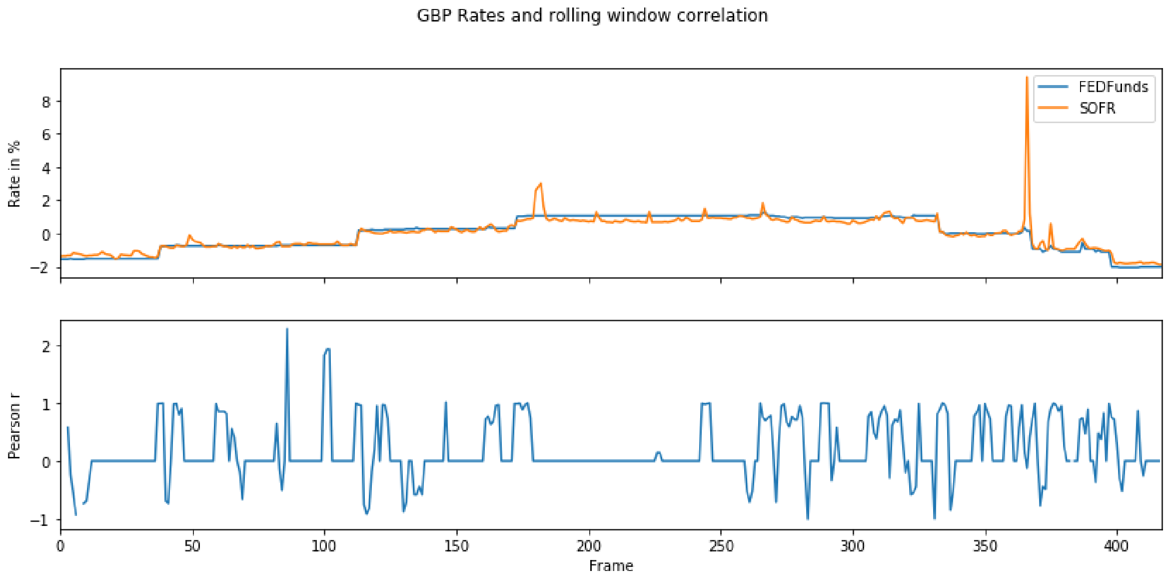
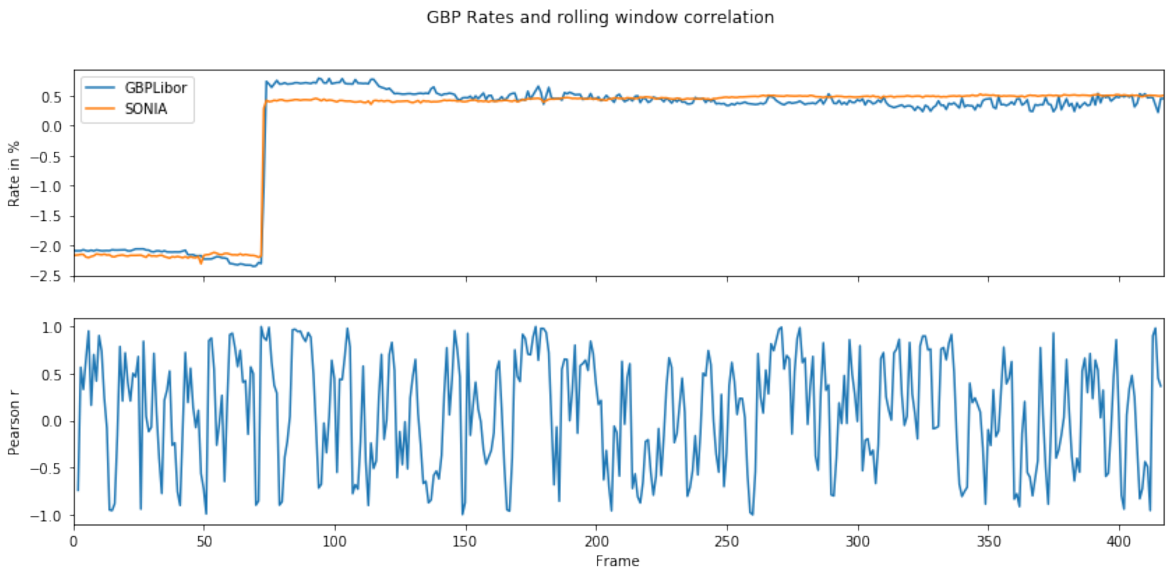


Figure 29 depicts the rolling window Pearson correlation results for the GBP interest rate. What stands out is the fact the the correlation values are extremely volatile and swinging between -1 and 1. Although the rolling correlation is very volatile, the overall Pearson correlation as shown in Table 35 is still almost 1.

Figure 29: *Overnight Daily Rates*



A.2. Explanatory models

A.2.1. GBP Libor interest rate timeseries

We first look at the results of analyzing the Sterling IBOR rate, the GBP Libor.

Auto Regressive Model

The Auto Regressive model is explained in Section 2.3, with the general formula shown in (10). We use an AR(3) model, which means a lag of three days. The specified formula for the GBP Libor is given in (21).

$$GBPLibor_t = \alpha + \beta_1 GBPLibor_{t-1} + \beta_2 GBPLibor_{t-2} + \beta_3 GBPLibor_{t-3} + \epsilon_t \quad (21)$$

Table 37: *AR(3) results Sterling IBOR: GBP Libor*

	α	β_1	β_2	β_3	α p-value	β_1 p-value	β_2 p-value	β_3 p-value	Log like.	No. obs.
GBP Libor	0.6182	1.2946	-0.4231	0.1220	0.000	0.000	0.000	0.013	1355.028	417
GBP Libor stand.	-0.2901	1.2946	-0.4231	0.1220	0.682	0.000	0.000	0.013	303.287	417

Table 36 shows the results using both the normal as the standardized data. For the actual zero rates, both the constant as the three lags are significant. We notice that the first lag, $GBPLibor_{t-1}$ contributes the most to the value of the GBP Libor at $t-1$. After testing with AR(4), the fourth lag is insignificant. For the standardized data, we see similar results, except for the constant term α that is insignificant, which is logical given the standardization.

Auto Regressive Moving Average Model
Next, we analyze the actual zero rates and the standardized data using an ARMA(1,1) model. This means both the auto regressive component as the moving average has two lags. Since the initial AR coefficients for the ARMA(p,q) model with $p > 1$ and $q > 1$ is not stationary, the ARMA(1,1) model is evaluated. The results are shown in Table 37.

Table 38: *ARMA(1,1) results Sterling IBOR: GBP Libor*

	AR(1)	MA(1)	AR(1) p-value	MA(1) p-value	Log like.	No. obs.
GBP Libor stand.	0.9904	0.3364	0.000	0.000	303.912	417

A.2.2. SONIA interest rate timeseries

Ordinary Least Squares model
Analyzing the Sterling ARR, the SONIA, and the ARRs in the coming sections, we start with a linear OLS. The formula for the Sterling zone is shown in (22).

$$SONIA_t = \alpha_t + \beta_t * GBPLibor_t + \epsilon_t \tag{22}$$

The result for the overnight zero rates and the standardized data is shown in Table 38. For the overnight zero rates, the α_t and β_t are significant, while this is not the case for standardized data. Here, the α_t is insignificant, which is a logical outcome given the data is standardized. The Adjusted R-squared is 0.975 which is very high. Since the adjusted R-squared gives an indication of model performance, this shows that the GBP Libor at t strongly predicts the SONIA at t, indicating the move similarly.

Table 39: *OLS results Sterling ARR: SONIA*

	α_t	β_t	α_t error	β_t error	α_t p-value	β_t p-value	Adj. R^2	No. obs.
SONIA	-0.0987	1.1858	0.006	0.009	0.000	0.000	0.975	417
SONIA stand.	8.513e-16	0.9874	0.008	0.008	1.000	0.000	0.975	417

To get a better understanding of this correlation between the SONIA and GBP Libor, we add lags of the GBP Libor. We test the number of lags by checking whether the results are significant or insignificant. For the Sterling rates, the third GBP Libor lag is insignificant, therefore the function is shown in (23).

$$SONIA_t = \alpha_t + \beta_1 * GBPLibor_t + \beta_2 * GBPLibor_{t-1} + \beta_3 * GBPLibor_{t-2} + \epsilon_t \quad (23)$$

The results are shown in Table 39. Since the values for the third lag of the GBP Libor are insignificant, we rerun the model using two lagged variables for the GBP Libor. For the overnight zero rates, all components are significant. The β_1 has the highest impact with a value of 1.3437, which is for the GBP Libor at t. The impact decreases as the lags increase. The adjusted R-squared has a value of 0.975 which is the same as for (22).

Table 40: *OLS results Sterling ARR: SONIA*

	SONIA lag	SONIA lag stand.
α	-0.0980	-0.0006
β_1	1.3437	1.1211
β_2	-0.4070	-0.3414
β_3	0.2483	0.2094
α error	0.006	0.008
β_1 error	0.080	0.066
β_2 error	0.126	0.106
β_3 error	0.079	0.067
α p-value	0.000	0.938
β_1 p-value	0.000	0.000
β_2 p-value	0.001	0.001
β_3 p-value	0.002	0.002
Adj. R^2	0.975	0.975
No. obs.	414	414

Auto Regressive model

We now focus on the Auto Regressive model for the SONIA, similar the analysis for the GBP Libor. After tested, all added lags are insignificant resulting in the basic AR(1) formula shown in (24).

$$SONIA_t = \alpha + \beta_1 SONIA_{t-1} + \epsilon_t \tag{24}$$

Table 41: *AR(3) results Sterling IBOR: GBP Libor*

	α	β_1	α error	β_1 error	α p-value	β_1 p-value	Log like.	No. obs.
SONIA	0.6237	0.9947	0.081	0.005	0.000	0.000	1259.435	417
SONIA stand.	-0.3974	0.9947	0.839	0.005	0.636	0.000	284.073	417

Table 40 shows the results using both the normal as the standardized data. In both cases, the SONIA at *t-1* almost exactly explains the rate for *t*. As explained, added lags are insignificant. The α for the standardized data is insignificant, which is expected due to the standardization.

Auto Regressive Moving Average model

Testing the data using the ARMA model, we find that no moving average coefficient is significant. Therefore we reduce the model to an AR(1) model of which the results have been discussed in Table 40.

A.2.3. EONIA interest rate timeseries

The last rates that we analyze individually are the Euro rates. We start with the Euro IBOR rate, the EONIA. For the EONIA rate, we start with an AR(p) model, followed by an ARMA(p,q) model.

Auto Regressive Model

We start with analyzing the EONIA with the AR(p) model. The maximum value for p is tested such that the coefficients are still significant. From this test, it follows that we use an AR(3) model for the EONIA data. The results are shown in Table 41.

Table 42: *AR(3) results Euro ARR: EONIA*

	EONIA	EONIA lag
α	-0.3697	-0.2202
β_1	0.5564	0.5564
β_2	0.2090	0.2090
β_3	0.1514	0.1514
α error	0.006	0.370
β_1 error	0.051	0.051
β_2 error	0.058	0.058
β_3 error	0.054	0.054
α p-value	0.000	0.552
β_1 p-value	0.000	0.000
β_2 p-value	0.000	0.000
β_3 p-value	0.005	0.005
Log like.	1212.895	-341.501
No. obs	376	376

For the actual EONIA zero rates, we notice that the value for β_1 is the largest, hence the value of EONIA at *t-1* is the best estimator for the value at *t*. The value of β_i decreases as i increases. This indicates that more recent values have a higher explanatory power compared to older values. This effect is what we notice for the standardized data as well. The difference is that the α is insignificant, as a result of standardizing the data.

Auto Regressive Moving Average Model

For the ARMA(p,q) model, we test the values for p and q such that the model best fits the data. Since the EONIA data is not stationary, the ARMA model fails to provide data. The standardized data is stationary. An ARMA(1,1) model is used with the results shown in Table 42.

Table 43: *ARMA(1,1) results Euro ARR: EONIA*

	AR(1)	MA(1)	AR(1) error	MA(1) error	AR(1) p-value	MA(1) p-value	Log like.	No. obs.
EONIA stand.	0.9637	-0.4236	0.024	0.057	0.000	0.000	-341.127	376

Looking at the results, we notice the first lag of the standardized EONIA data is very high. The moving average coefficient is significant with a value of -0.4236.

A.2.4. pre–€STR interest rate timeseries

Ordinary Least Squares model

The Euro Alternative Reference Rate is the €STR. Since the spread between the EONIA and the €STR is fixed at 8.5 basis points, we analyze the pre-€STR rates instead of the €STR rates. The formula for the relation between the pre-€STR and the EONIA is shown in (25).

$$pre\text{–}€STR_t = \alpha_t + \beta_t * EONIA_t + \epsilon_t \tag{25}$$

Table 43 shows the results of this simple OLS regression. The constant value α_t is insignificant. The β_t on the other hand is significant with a value of 0.8455. This shows that the value of the EONIA at day t is a fairly accurate estimator for the value of the pre-€STR at t. Looking at the adjusted R-squared with a value of 0.718, we notice this is lower compared to the Sterling and Dollar rates.

Table 44: *OLS results Euro ARR: pre-€STR*

	α_t	β_t	α_t error	β_t error	α_t p-value	β_t p-value	Adj. R^2	No. obs.
pre-€STR	-0.1435	0.8435	0.010	0.027	0.000	0.000	0.718	376
pre-€STR stand.	-0.0043	0.8455	0.027	0.027	0.875	0.000	0.718	376

We now add additional terms to (25), by adding lagged variables of the EONIA. From testing the number of lags that are significant, we conclude to add two EONIA lags. The formula is shown in (26).

$$pre\text{–}€STR_t = \alpha_1 + \beta_1 * EONIA_t + \beta_2 * EONIA_{t-1} + \beta_3 * EONIA_{t-2} + \epsilon_t \tag{26}$$

The results are shown in Table 44. We notice that the value of the β's decreases as the value of i in *t-1* increases. This indicates the impact of the past rates decline as time goes on, which is a logical result. The adjusted R-squared returns a value of 0.774, which is an increase compared to the results in Table 43, where no lagged variables of the EONIA are added. Due to standardizing the data, the pre-€STR standardized α is insignificant.

Table 45: *Lagged OLS results Euro ARR: pre-€STR*

	pre-€STR lag	pre-€STR lag stand.
α	-0.0956	-2.625e-15
β_1	0.5366	0.5394
β_2	0.2526	0.2441
β_3	0.1854	0.1714
α error	0.010	0.025
β_1 error	0.040	0.040
β_2 error	0.046	0.044
β_3 error	0.042	0.039
α p-value	0.000	1.000
β_1 p-value	0.000	0.000
β_2 p-value	0.000	0.000
β_3 p-value	0.000	0.000
Adj. R^2	0.774	0.774
No. obs.	373	373

Auto Regressive Model

Next, we analyze the pre-€STR data with an Auto Regressive model. Testing the maximum parameters for the AR(P) model, indicates to use the an AR(1). This is shown in (27).

$$pre\text{–}€STR_t = \alpha_1 + \beta_1 * pre - €STR_{t-1} + \epsilon_t \tag{27}$$

The results are shown in Table 45. For the original zero rates, we conclude that the value of the pre-€STR at *t-1* is a very good estimator of the value at time *t*. The intercept term α is significant. The standardized data shows very similar results, with the intercept being insignificant as a result of the standardization.

Table 46: *OLS results Euro ARR: pre-€STR*

	α	β_1	α error	β_1 error	α p-value	β_1 p-value	Log like.	No. obs.
pre-€STR	-0.4578	0.9613	0.008	0.020	0.000	0.000	1359.360	373
pre-€STR stand.	-0.3494	0.9613	0.525	0.020	0.506	0.000	-185.407	373

Auto Regressive Moving Average Model

Adding the moving average components to the AR model gives us an ARMA(p,q) model as described in Section 2.4. For larger values of p and q, the data is not stationary. Reducing the values shows us the only model without insignificance is an ARMA(1,0) model or AR(1) model. These results are already shown in 45.

A.3. Predictive Models

A.3.1. GBP Libor interest rates

Auto regressive model

To validate the regression results from Section A.2.1, we predict the last 10%, 20% and 30% of the GBP Libor and compare those with the actual rates. In Section A.2.1, we concluded that the first three legs of the auto regressive model were significant. For that reason, we now predict the GBP Libor using the same AR(3) model.

Figure 30: *GBP Libor AR(3) prediction 30% sample*

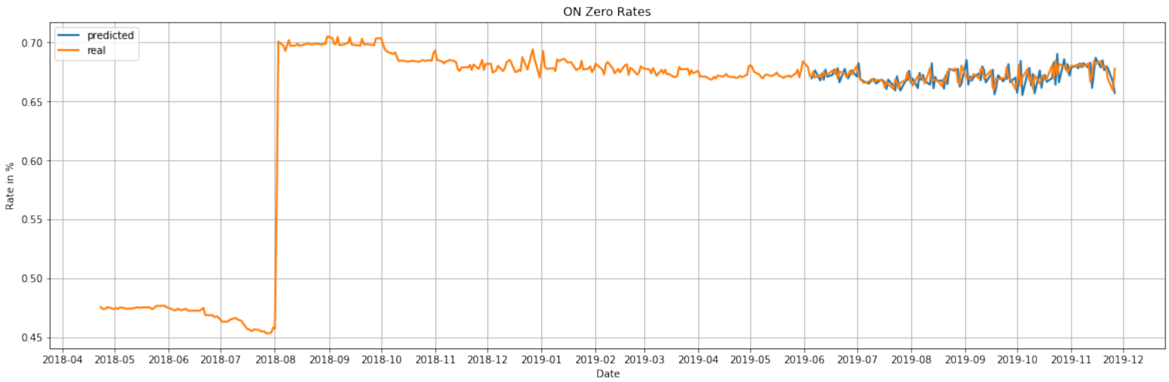


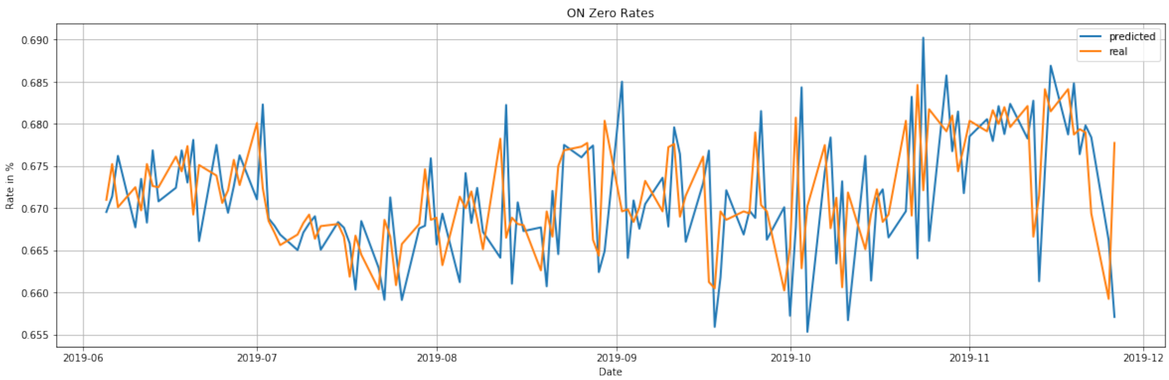
Figure 30 shows shows the result of the predicted 30% compared to the actual rate. We observe that the predicted rates (blue line) are very close to the actual rates (orange line). To test the misfit of the predicted values, we calculate the MSE for the 10%, 20% and 30% predictions. The results are shown in Table 46.

Table 47: *AR(3) prediction results Sterling IBOR: GBP Libor*

Sample prediction size	MSE	MAPE
90%	0.0001055126	1.26146
80%	0.0000821901	1.09378
70%	0.0000623154	0.92578

In Table 46, we observe a contradictory aspect. We notice that the MSE decreases as the prediction sample size decreases as well. We would expect that the standard error would decrease as sample size increases. This is contradictory to our expectations. Figure 31 shows the predicted and real GBP Libor values for the last 30% of the data. Here we see why the MSE increases as sample size decreases. The GBP Libor is becoming more volatile at the end of the data set. This increase volatility causes the predicted values to deviate more compared to a more stable period, therefore the MSE increases as sample size increases. The Mean Absolute Percentage Error is a common method to measure the accuracy of the error measure for timeseries predictions. It measures the absolute percentage difference of the error, sums these values and divides by the number of observations. The reason why this is better than the MSE is since the positive and negative errors do not cancel each other out. Looking at the results, we notice that for the 90% and 80% sample has a MAPE error higher than 1. This indicates that the in-sample one-step forecast from the naive method performs better than the forecast values. The 70% test results are very close to one. This indicates that the predicted values deviate a lot from the actual overnight zero rates.

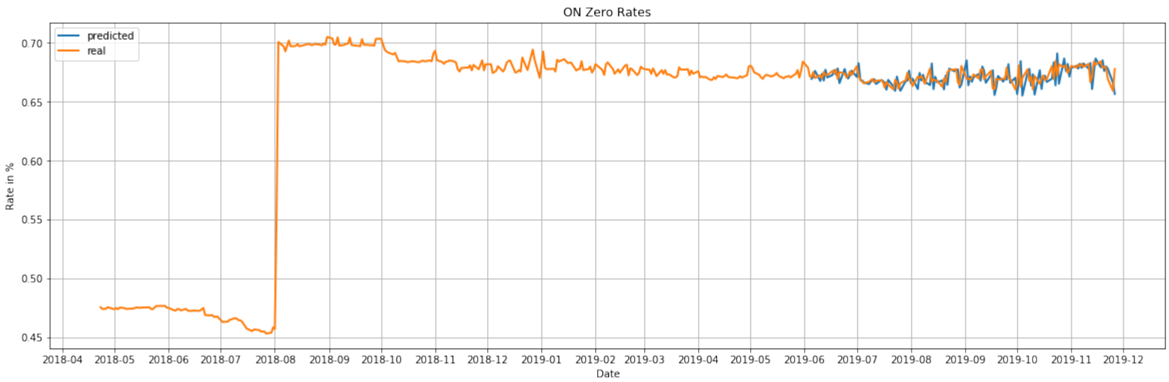
Figure 31: *GBP Libor AR(3) prediction 30%*



Auto regressive moving average

The Auto Regressive moving average model tested in Section A.2.1 resulted in the use of an ARMA(1,1) model. We test the ARMA(1,1) model using the same method as the AR(3) model.

Figure 32: *GBP Libor ARMA(1,1) prediction 30%*



In Figure 32 the GBP Libor prediction of the last 30% is shown. This means that the in-sample prediction size is 70% of the total data set, or 334 data points. We observe the ARMA prediction to follow the actual rates closely, although the rate seems to have a worse fit during the volatile period between 09-2019 and 10-2019. The MSE results are shown in Table 47.

Table 48: *ARMA(1,1) prediction results Sterling IBOR: GBP Libor*

Sample prediction size	MSE	MAPE
90%	0.0001097510	1.28283
80%	0.0000852523	1.11699
70%	0.0000646250	0.94359

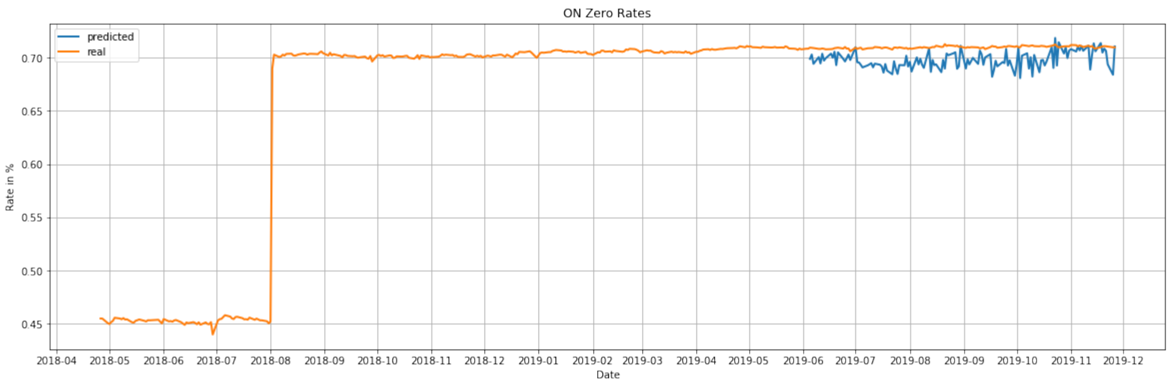
Similar to the AR(3) results, we observe that the MSE increases as the number of predictions decrease. We once again expect this effect to result from the increased volatility in the last period of the data set. If we compare the MSE values with the AR(3) values we notice that the AR(3) model predictions perform slightly better. Looking at the MAPE values, we once again observe two values greater than one. The individual MSE results cancel each other out when there are positive and negative values. We conclude that the added percentage error for all three test is very high, indicating a high level of deviation from the actual values.

A.3.2. SONIA interest rates

Ordinary Least Squares

As described in Section 2, an Ordinary Least Squares method is a linear regression method where the unknown parameters of the dependent variables are estimated such that the sum of the squared deviations of the independent variable in minimized. In Section A.2.2 we found that the first three legs of the ARR OLS regression shown in (23) are significant. In Figure 33, we notice that the predicted values are not very close to the SONIA values. The reason is the difference in volatility between the SONIA and the GBP Libor at the end of the data set. Comparing Figure 33 with Figure 3, we do notice that the predicted OLS values are closer to the SONIA values compared to the actual GBP Libor overnight zero rates.

Figure 33: *GBP Libor OLS prediction 30%*



To test the accuracy of the predicted values, we calculate the mean square error of the predicted values. Table 48 shows the MSE results for the OLS prediction. We notice that the most accurate prediction is for the largest training data set. The difference between the 70% and 80% test is very small. For the MAPE values, the misfit of the predicted OLS values becomes clear. All values are greater than one, indicating the model performs worse than a one-step forward naive forecast method. This means taking the values at *t-1* as the predicted values for *t* provides better results than the OLS regression model.

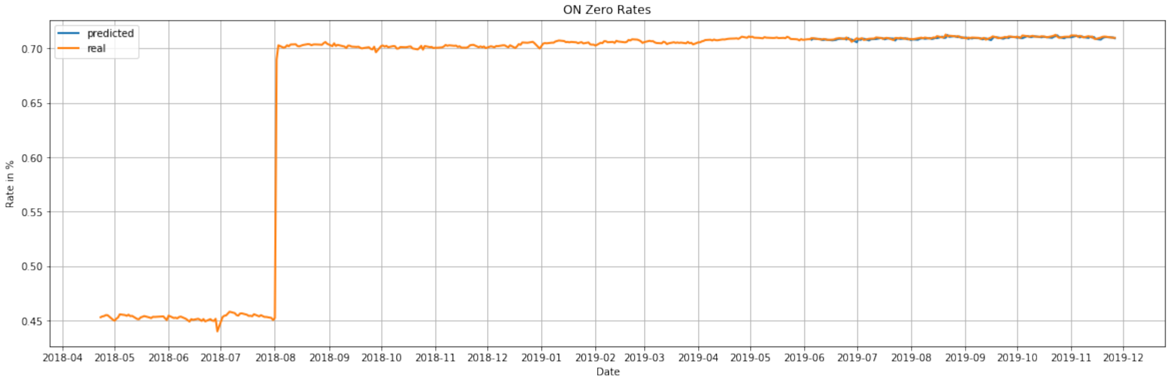
Table 49: *OLS prediction results Sterling ARR: SONIA*

Sample prediction size	MSE	MAPE
90%	0.0001824848	1.44687
80%	0.0002071095	1.68316
70%	0.0002042294	1.72924

Auto regressive

In Section A.2.2, we observed that only the first leg of the SONIA AR(p) model is significant. Therefore, the SONIA is predicted using an AR(1) model. Figure 34 shows the actual SONIA overnight zero rates in combination with the predicted last 30% using the AR(1) model.

Figure 34: *SONIA AR(1) prediction 30%*



In Figure 34, we notice that the predicted rate almost matches the real SONIA overnight zero rates. The blue lines are barely visible. In Figure 3 we observed that the SONIA is far less volatile with no increase of decreasing trend while we did notice this for the GBP Libor. Due to the absence of high volatility, the AR(1) model performs very well. The results are shown in Table 49. We notice that the lowest MSE is achieved when the predicted sample is the smallest. This is the expected result since the most observations are used to train the model, leading to a more accurate prediction. Different from the previous MAPE results for the GBP Libor OLS and the SONIA models, the MAPE values for the GBP Libor AR(1) model are low and therefore the model is accurate. For all three tests, the percentage error of the predicted values is approximately 11%, indicating an accurate forecast. An explanation for this result is the very low volatility of the SONIA during the sample period, resulting in an accurate prediction.

Table 50: *AR(1) prediction results Sterling ARR: SONIA*

Sample prediction size	MSE	MAPE
90%	0.0000009000	0.10716
80%	0.0000009712	0.11115
70%	0.0000010685	0.11547

Auto regressive moving average

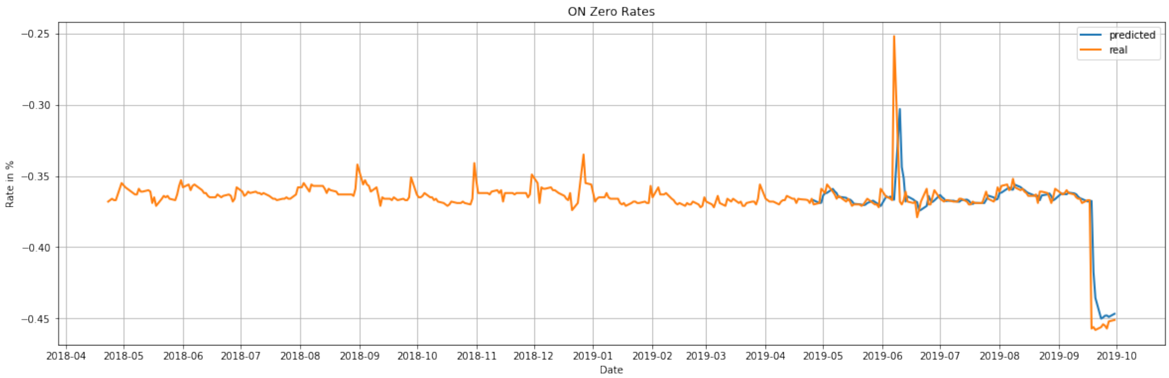
Reducing the ARMA(p,q) model in Section A.2.1 such that the coefficients are significant resulted in an ARMA(1,0) model, or an AR(1) model. The results of the predicted versus the actual values are already described in this Section.

A.3.3. EONIA interest rates

Auto regressive

To forecast the Euro IBOR rate, the EONIA, we use an AR(3) model as explained in Section A.2.3. Figure 35 shows the entire real data set and the last 30% of the data set with predicted values. Although the predicted values follow the actual rates closely, they tend to have less extreme spikes compared to the actual rates.

Figure 35: *EONIA AR(3) prediction sample 30%*



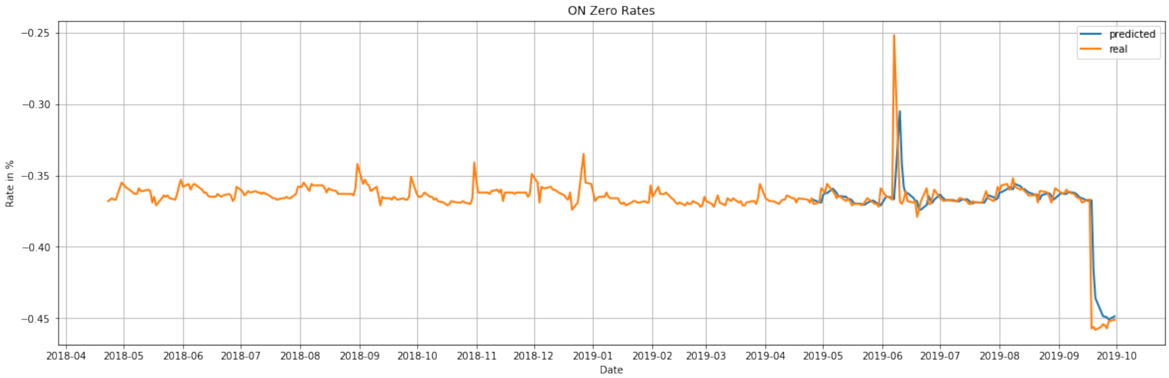
To test the prediction model performance, we once again test the prediction results with the Mean Square Error. The results are shown in Table 50. The best result is achieved for the 80% train data set. The reason why the models performs best at 80% is due to the events in the sample data. The 30% test covers the spike in July which increase the MSE, while the 10% data set is heavily effected by the drop of the rate in August. Since the 20% test starts after the spike in July in a relatively stable period, the AR(3) model performs best for this situation. The MAPE results are once again larger than one, indicating the model errors are still high. This suggest that the AR(3) forecast results are still not very accurate.

Table 51: *AR(3) prediction results Euro IBOR: EONIA*

Sample prediction size	MSE	MAPE
90%	0.0002817920	1.50764
80%	0.0001478210	1.14937
70%	0.0002662663	1.63506

Auto regressive moving average
From the ARMA(p,q) estimation in Section A.2.3, we conclude that an ARMA(1,1) model is used to predict the EONIA rates and test the result using a MSE estimation. Figure 36 shows the total actual EONIA rates and the predicted last 30% of the data.

Figure 36: *EONIA ARMA(1,1) prediction sample 30%*



The moving average aspect in the ARMA model slightly reduces the volatile movements of the rates, visible for the spike in July and the drop in August. The MSE results are shown in Table 51.

Table 52: *ARMA(1,1) prediction results Euro IBOR: EONIA*

Sample prediction size	MSE	MAPE
90%	0.0002862772	1.52112
80%	0.0001500028	1.15364
70%	0.0002650707	1.61830

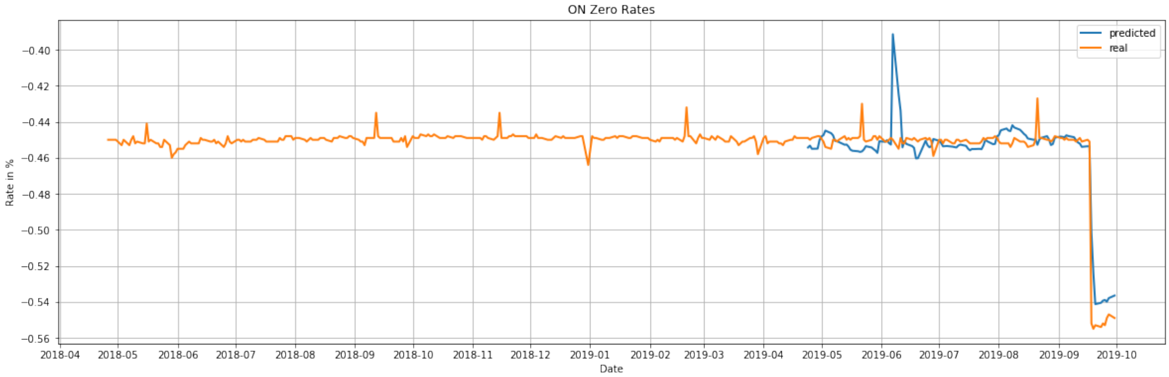
We observe similar results for the ARMA(1,1) model compared to the AR(3) model. We do notice that both the 90% and 80% ARMA(1,1) prediction are slightly less accurate compared to the AR(3) results. The 70% result is slightly more accurate for the ARMA(1,1). With regard to the MAPE results, we observe that all values are still greater than one, indicating the one-step naive forecast values are more accurate compared to the ARMA(1,1) results.

A.3.4. pre–€STR interest rates

Ordinary Least Squares

In Section A.2.4 we concluded that three independent variables were significant for the OLS model of the pre-€STR rate. The predicted values and the actual values are depicted in Figure 37. We notice that the predicted values that are based on the SONIA seem less accurate.

Figure 37: *pre-€STR OLS prediction sample 30%*



Testing the accuracy of the predicted values using by calculating the MSE gives us the results in Table 52. We observe that, although the rate does not seem to be very accurate, the MSE are still very small. We observe that the lowest MSE is achieved for the 80% training data set, with the highest MSE value for the 90% test. The highest MAPE results are also for the 90% data set, although the 80% and 70% test MAPE values are both greater than one. This indicated a lack of accuracy in the model.

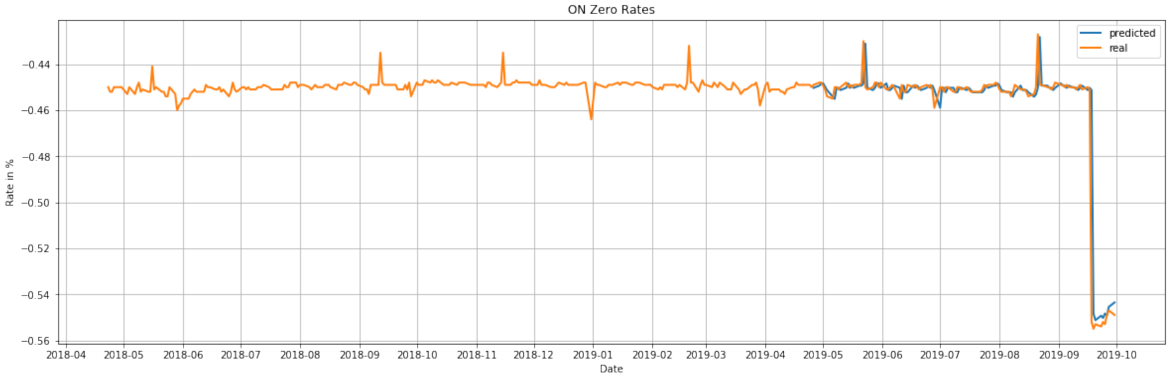
Table 53: *OLS prediction results Euro ARR: pre-€STR*

Sample prediction size	MSE	MAPE
90%	0.0001452867	1.37817
80%	0.0000854660	1.15041
70%	0.0001099855	1.29765

Auto regressive

An AR(1) model is used to predict the values for the pre-€STR rates as concluded in Section A.2.4. Similar to previous tests, different prediction sample sizes are used. In Figure 38 we notice that the pre-€STR rate is not volatile, resulting in a predicted rate that accurately follows the actual rate.

Figure 38: *pre-€STR AR(1) prediction sample 30%*



In Table 53 we notice that the best performance for both the MSE and the MAPE result is for the 70% data set. The Mean Absolute Percentage Error for the 90% data set is larger than one. As previously described, this indicates the one-step naive forecast is more accurate compared to the AR(1) prediction results. This is mainly due to the drop of the pre-€STR in September 2019 since the regressive aspect in the rate causes the prediction to react late. The MAPE values for the 70% and 80% training data test are lower than one, indicating the AR(1) model performs better.

Table 54: *AR(1) prediction results Euro ARR: pre-€STR*

Sample prediction size	MSE	MAPE
90%	0.0003053728	1.06858
80%	0.0001532136	0.68385
70%	0.0001102387	0.64743

Auto regressive moving average

In Section A.2.4 we concluded that no reduction of the p and q in the ARMA(p,q) model was significant, therefore we do not predict the pre-€STR rates using the Auto regressive moving average model.