July 29, 2019

# MASTER THESIS

## ON THE OPTIMIZATION OF LOSSY ENERGY STORAGE DEVICES

Ties H. F. Westendorp
Student number: 1368362

Faculty of Electrical Engineering, Mathematics and Computer Science (EEMCS)
Chair of Discrete Mathematics and Mathematical Programming (DMMP)

Committee:
prof. dr. Johann L. Hurink
ir. Martijn H. H. Schoot Uiterkamp
dr. ir. Marco E. T. Gerards

UNIVERSITY OF TWENTE.

# Abstract

Emergent technologies such as electric vehicles (EVs) and renewable energy sources (RESs) are causing a shift towards an increased electrical energy use. The unpredictable production of RESs, such as solar panels and wind turbines, as well as the high peak consumption of EVs contribute to a discrepancy of production and consumption that can not be resolved in the traditional centralized energy supply chain without further investments in grid reinforcement. The use of schedulable smart appliances allows us to even out this discrepancy before the transformer. In this thesis we consider a controllable smart energy storage device where we account for energy conversion losses. We show that local reparation methods that account for losses applied to the lossless solution perform poorly, that the problem is $\mathcal{NP}$-complete, and propose polynomial-time exact methods for certain parameter choices, and a heuristic for the general case. A case study is conducted to evaluate the performance of the heuristic method.

# Contents

# 1. Introduction

*The increasing electrification of our daily energy consumption, as well as the unpredictability of low-level energy generation through solar panels and wind turbines, gives rise to a highly peaked electricity throughput at the transformer level. High production and consumption peaks at the transformer level are linked to system failures and local power outages, traditionally requiring reinforcement of in-place electrical systems to maintain integrity of the low voltage grid. The cost of reinforcement will rise dramatically if no measures are taken, given the higher penetration of solar panels and high-peak energy demand devices expected in the near future. Demand side management approaches, where the energy profile is evened out before the transformer, offer an alternative way to lower stress on the grid. Evening out the energy profile involves bridging the temporal divide between consumption and production peaks, requiring short-term storage of energy. In this thesis we consider mathematical optimization models for energy storage devices where we account for energy conversion losses. In this chapter we give some background information about demand side management and elaborate on the issues that necessitate it. Furthermore, the problem considered in this thesis is presented, and the research questions regarding it are stated. Finally, a short outline of the rest of this report is given.*

## 1.1 Energy transition

In recent decades there has been an increased interest all around the world in becoming independent of fossil fuels due to their contribution to global warming and climate change through the greenhouse effect. As a result, clean renewable energy sources (RESs) are nowadays more prevalent (see Figure 1.1), and the share of electrical energy in our day-to-day energy usage is increasing both in peak and overall demand – to the point where further increases may not be accodomodated by the current low-voltage (LV) grids.

This energy transition is accompanied by major technical issues for the current electricity grid. Foremost of these is that RESs are less flexible in when they generate electricity; rooftop photovoltaics (PV) only generate electricity when the sun is shining, and wind turbines only when the wind is blowing. Additionally, these systems occasionally generate more electricity than is required at that time. When this happens, an inverted power flow is sent upstream, potentially damaging electrical components [44]. The grid needs to be upgraded before this happens, when PV become more widespread and no further actions are taken. Related problems are reported by distribution grid operators around the world [4, 22, 36, 40, 41].

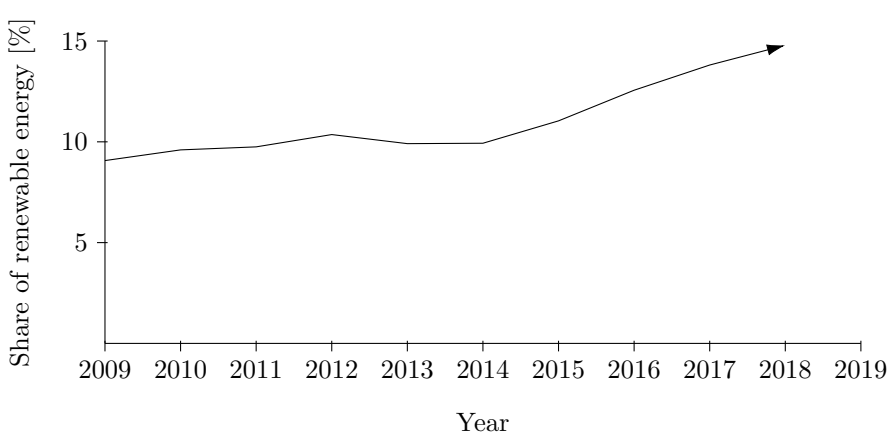



**Figure 1.1:** Share of electricity production from renewable sources as percentage of consumption in the Netherlands. Source: CBS [14].

Energy consumption, on the other hand, is usually directly triggered by human interaction, and so most appliances require energy only when humans are there to operate them. This causes consumption peaks to occur just before and after regular working hours. Due to the simultaneity of these peaks, a further increase in the share of high peak energy demand devices, e.g. electric vehicles (EVs), may overburden the LV grid [29]; fully charging an EV takes about a factor nine times the current average daily residential energy consumption per person (approximately 1500 kWh per person per year [12]), and fast charging at 150kW for three minutes already equals about twice a person's average daily electric energy usage. Further increases in EV penetration are likely to happen in the near future given the key role it plays in the climate goals of countries all around the world [6, 10, 31, 39, 48] (see Figure 1.2). Consequently, we will shortly require ways to accommodate for this increase.

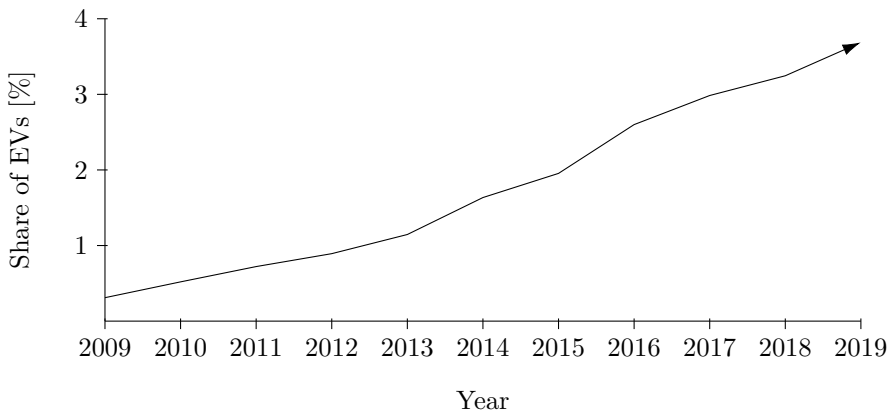



**Figure 1.2:** Share of EVs among registered personal cars in the Netherlands. Source: CBS [15].

## 1.2 Smart grids

For the reasons outlined above, there is an obvious interest in structurally lowering both consumption and production peaks in the energy profile, i.e., to do peak shaving. And because of the loss of flexibility at the supply side, in order to obtain the desired peak shaving capabilities, there should conversely be an increase of flexibility elsewhere in the grid: on the consumer side. Influencing consumer behaviour in order to match demand with the supply (rather than the other way around) is called demand side management (DSM). More information on DSM can be found in Section 2.1. A way to achieve it is through the use of smart, schedulable appliances incorporated in a communicating electricity grid, called a *smart grid*. Through communication and subsequent control mechanisms, smart devices can agree on and execute a common schedule that flattens their cumulative energy profile over a period. Any appliance that does not (always) need ad hoc activation, or whose activation can be automated, can be adopted into such a smart infrastructure. Typical examples are: dish washers and washing machines (referred to as time-shiftable devices), EVs, and energy storage devices.

Assuming we have an accurate prediction of all relevant parameters, we may attempt to employ mathematical optimization techniques to determine how to optimally schedule the activation of devices. Optimal in this case means the best possible schedule in the mathematical model with respect to the chosen objective. However, it turns out that the problem of simultaneously optimizing multiple devices is intractable, regardless of the objective of the optimization problem (as feasibility checking of multiple time-shiftable devices is already $\mathcal{NP}$-complete [24]). Furthermore, other requirements can make such a centralized approach undesirable in practice, such as not wanting to communicate expected energy consumption of individual households for privacy or security reasons. This leads us to consider optimizing devices in isolation, and aggregating their schedules at a higher level. If we subsequently choose as objective to minimize the Euclidean distance

between the energy profile and some target profile, we refer to this procedure as *profile steering* [24]. Profile steering is explained in more detail in Section 2.3.

## 1.3 Utilizing storage devices

Ideally, consumption and production peaks of an energy profile are simultaneously shaved by cancelling them out against each other. This can for instance be done by using energy storage devices to temporarily store excess energy from PV. Afterwards, i.e. whenever an excessive amount of energy is to be consumed, we can assist the grid by discharging the energy storage. When the storage is owned and used by the distribution grid operator, it can be used to support the grid by directly performing peak shaving. Figure 1.3 shows the effect of a Lithium-ion battery on the energy profile of a single household with PV: peaks are significantly reduced such that the highest consumption peak is halved in height, and the consumption peak is completely removed.

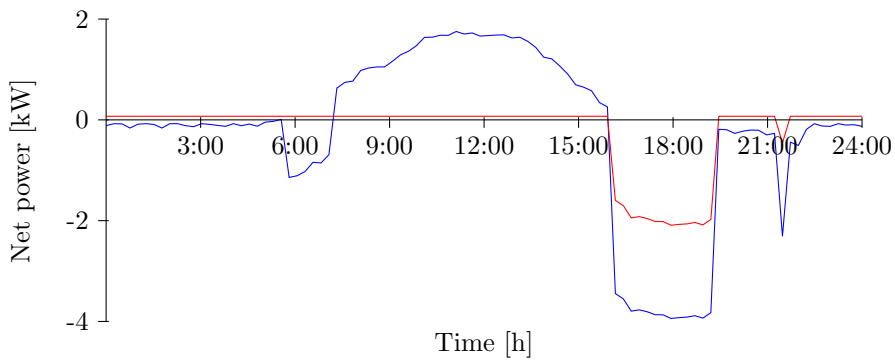



**Figure 1.3:** Energy profile of a single household on a typical day in summer, without (blue) and with (red) energy storage device.

Consumers require an economical incentive to participate in peak shaving with energy storage. A way to do so is through arbitrage created by varying energy prices over time. Consumers will be incentivized to exploit the arbitrage by charging the energy storage when energy is cheap, and discharging it when energy is more costly. A result is that consumers will be incentivized to use their energy storage to assist the grid if the prices are correlated with the neighborhood energy profile. Whenever the energy delivered to the grid yields less than taking energy from the grid costs, consumers are incentivized to use their own generated energy. When the generated energy is not immediately required, it may prove beneficial to temporarily store it.

In the Netherlands, home owners are currently not incentivized to limit the peaks of their energy profile, as the current net metering policy (salderingsregeling [47]) does not take into account peak production or consumption. Consuming the energy generated from one's own PV is not even incentivized under this policy, as the energy supplier is legally required to accept locally generated energy, and to only charge people for the difference in their annual consumption and production. Whenever a household generates more electricity than it consumes on an annual basis, the energy supplier is required to give a reasonable compensation for the surplus. Therefore, there is no financial gain in using locally generated energy instead of energy from the net under this policy. As such, under this policy it does not pay off to participate in peak shaving or to utilize storage as consumer [33]. This policy will be phased out starting in 2023 [49], after which it will gradually make way for a policy where only the feed-in subsidy remains. The goal is to incentivize the usage of locally generated energy over energy from the grid due to a lower compensation for delivering back energy to the grid. In this way, from an economical viewpoint people should prefer to even out their energy profile on a household level (possibly employing storage). Further details on the new policy are scarce, except that one of the aims is to keep the length of the payback period for PV approximately the same, at around seven years.

## 1.4 Research question

To effectively use an energy storage device, we require an adequate model of it. Optimization models for energy storage devices are typically very much simplified, ignoring e.g. energy conversion losses. In this thesis we extend an existing model by including exactly this feature, and we study the properties of the new, lossy model. It turns out that after the introduction of conversion losses, the problem becomes $\mathcal{NP}$-complete, implying that it is impossible to devise a general exact algorithm that runs in polynomial time (unless $\mathcal{P} = \mathcal{NP}$). This report addresses the following main question:

- *Can energy storage devices, where we account for energy conversion losses, effectively support the electricity grid over their lifetime?*

We split this question into the following subquestions:

- *How important are conversion losses in practice, and how relevant are they for optimization procedures?*
- *Are consumer-owned energy storage devices economically viable in the near future?*
- *Can we distinguish cases that are computationally tractable to optimally solve?*
- *Can an efficient approximate algorithm be devised?*

These questions will be answered in turn, before returning to the main question in Chapter 7.

## 1.5 Contributions

To summarize, in this thesis we extend existing models of van der Klauw [55] for the scheduling of energy storage devices by accounting for energy conversion losses. In general, unless $\mathcal{P} = \mathcal{NP}$, we can not find an efficient exact algorithm for this problem, though certain parameter choices lead to instances of the problem that can be solved to optimality in polynomial time. For the remaining cases we propose a novel constructive heuristic. It solves a restricted version of the problem to optimality, where some state variables are a priori restricted in sign (non-positive or non-negative) such that the optimal solution to the original problem is found when the appropriate signs are chosen. An implementation of the method in Python is incorporated in the optimization suite of the Decentralized Energy Management Toolkit (DEMKit) software package [27]. Finally, the performance of the method when applied to realistic artificial cases is investigated.

## 1.6 Outline

The structure of this thesis is as follows. In Chapter 2 we provide additional background information on smart grids, scheduling approaches for smart appliances, and battery modelling. The goal of this chapter is to place our research in context, to reflect on its applicability, and to give an overview of existing solution methods.

Chapter 3 introduces the model used in the rest of this thesis, and places it in a mathematical formalism. This model is solved in subsequent Chapters 4-5, in which we respectively investigate the importance of considering losses in the optimization process, and how to properly do so.

In Chapter 4 we consider two naive methods to account for the incurred energy conversion losses. We investigate the performance of these methods as compared to a linear programming approach. Several different objective functions are considered. We also investigate the economic feasibility of a residential energy storage for several efficiency values.

In Chapter 5 we show that the considered problem is $\mathcal{NP}$-complete in general, implying it may be inherently difficult to generally solve. We isolate parameter choices for which we can extend an existing solution approach to find optimal solutions, and we introduce a novel heuristic for the general problem.

In Chapter 6 we investigate the performance of the given heuristic in a realistic case study.

Chapter 7 concludes this thesis with a comparison of the proposed solution methods, discussion of the results and encountered problems, a projected future of energy storage in residential settings and future work.

# 2. Background

*The loss of flexibility at the supply side of the energy supply chain due to a higher penetration of uncontrollable renewable energy sources necessitates an increase of flexibility at the consumer side. We discuss several ways in which consumer behaviour can be influenced to better match the given energy supply. The emergence of RESs requires a paradigm shift in our current energy supply chain, as methods for utilizing the flexibility of multiple devices simultaneously are otherwise not scalable. The current centralized energy paradigm will have to make way for a decentralized paradigm, where part of the computational effort of scheduling can be delegated to the individual devices, making this approach more scalable with respect to the number of devices that can be considered during optimization. Several methods for device scheduling from the literature are listed. We reflect on the applicability of our model by looking at the workings of physical batteries, and finally consider some adverse effects control can have.*

## 2.1 Demand side management

A higher penetration of local energy production through RESs causes a loss of flexibility at the supply side of the energy supply chain, in turn necessitating an increase of flexibility at the consumer side to avoid excessive costs for grid reinforcement. Matching consumer demand to available supply, called demand side management or demand-response, is an effective way of flattening energy profiles. It can be achieved in multiple ways:

- **Better education of the public:** Government campaigns may make people more conscious of their energy footprint. They may incentivize them to use less energy altogether, or to prompt them to consider overproduction issues when deciding whether to purchase solar panels. Dealing with overproduction issues becomes especially important when the compensation for delivering back energy to the grid is low compared to the price for consuming energy. A meta-analysis of multiple experimental studies on energy conservation concludes that, on average, information strategies led to a reduction in energy consumption of 7.4% [16]. Energy audits and consultation were most effective, with an average decrease of 13.5%. Strategies focussing on financial aspects generally backfire, as they may highlight that the discomfort of energy conservation may not be worth it financially.

- **Financial incentives:** Day-night tariffs are a typical example that have been around for some time (at least 1926 in the Netherlands [53]). However, there are a few apparant problems with simple flat schemes: they do not incentivize energy usage whenever there is an abundance of locally generated energy, the obtained savings are often not worth the trouble [18], and the influence of these tariffs on consumer behaviour is hard to predict. Other pricing schemes such as time-of-use (ToU) pricing, off-peak savings, or quadratic prices perform better in these regards. Additionally, if local generation of energy is rewarded less than the purchase of external energy costs, consumers are incentivized to use their own energy first.

- **Home energy management systems (HEMS):** An integral part of future housing is, quite surely, an energy monitoring and management system. Through interfacing with a neighborhood-level controller, they may come to a consensus on proper activation times for certain smart appliances (such as washing machines, dish washers, heat pumps, EVs) in order to flatten the aggregated energy profile for an entire neighborhood. This sidesteps the problem of determining and distributing the prices in case of dynamic tariffs, and potentially gives much better results. Furthermore, by assuming an automated framework, there is no longer a reliance on active consumer participation to flatten energy profiles (besides initial registration of devices), ideally offering both improved comfort and a flattened energy

profile. Taking part in such a controlled system can be either made mandatory through legislation, or be economically incentivized through periodic rewards based on participation.

A future system may use a mixture of direct control for controllable loads, and financial incentives for uncontrollable base loads (appliances that are not schedulable, but for which we may still incentivize specific timed usage). More information about DSM can be found in [54].

## 2.2 Smart appliances

Through the use of schedulable smart appliances both production and consumption peaks can be cut-off. These appliances work within specified comfort bounds rather than ad hoc, allowing the energy profile to be evened out through automated activation. For this reason, such devices are also called distributed energy resources (DERs). A proper energy management approach should be able to handle all kind of DERs. It therefore makes sense to identify a number of device classes that generalize all possible kind of flexibility behaviours, in order to facilitate the interaction between an energy management system (such as a HEMS) and the DERs. One such classification is given in the Energy Flexibility Platform and interface (EF-Pi) [20], in which the following device types are distinguished:

- **Uncontrollables** are those devices that together constitute the base load. Their energy consumption or production is not schedulable and the devices should directly function when required by the consumer. Examples include computers, televisions, and coffee machines.

  A subset of the uncontrollable devices are *curtailable* devices. These devices can be shorted on their energy consumption/production (possibly leading to a loss of comfort). The typical example is rooftop PV with controllable inverter. Controllable inverters are already legally required in Germany and California [11, 21], due to a high penetration of rooftop PV systems. While potentially being beneficial for peak shaving, curtailment is generally not desirable from an economical or user comfort point of view, and should therefore only be used as a last resort;

- **Time-shiftables** are devices with a fixed energy profile after activation, where the activation time can be varied (within certain comfort bounds). Typical examples are pool pumps, washing machines, and dish washers;

- **Buffers** are devices that utilize some form of internal storage of energy, and can (dis)charge over several time periods in a flexible manner. The effective local use of a buffer may be able to simultaneously shave production and consumption peaks by cancelling them out against each other. Examples include EVs, batteries, and heat storage devices;

- **Unconstrained** devices encompass all other devices that may have a very customized modus operandi specifically designed for supporting the grid. On a residential level such devices generally do not occur. An example is a neighborhood-level electricity generator.

The problem of scheduling multiple devices simultaneously turns out to be intractable for a large number of devices, independent of the considered objective, as feasibility checking of multiple time-shiftable devices is already difficult (technically, it is $\mathcal{NP}$-complete [55]). Furthermore other requirements can make such a centralized optimization approach undesirable in practice, such as not wanting to communicate (across the grid) expected energy consumption of individual households for privacy and security reasons.

Overcoming these complications of the centralized energy management paradigm can be done by considering devices or households in isolation, and then potentially using some procedure to aggregate results. Energy management and device scheduling of this kind is appropriately called decentralized energy management (DEM).

## 2.3 Device scheduling

The optimal scheduling of devices involves using their flexibility to redistribute the cumulative energy consumption profile in such a way that certain measures are optimized. In a pricing setting this boils down to minimizing costs or exploiting arbitrage created by the varying energy tariffs, while in peak shaving this boils down to cancelling out consumption and production peaks against one another. Respectively these correspond to the consumer's point of view, and the distribution grid operator's point of view. We list several methods for approaching these problems below. Control approaches can roughly be divided into three classes: local control, central optimization, and market-based control.

### Profile steering

In the Profile Steering approach [24] (see Algorithm 1) scheduling decisions are made by a central controller, which aggregates proposed schedules of individual appliances. This optimization procedure is therefore a mixture of central optimization and local control. During each iteration, the best improving schedule is consolidated with the current schedule until no more (or too little) improvement is found. As the central controller repeatedly asks for a new schedule of each registered appliance, the controllers of the appliances should be able to compute a new schedule relatively fast and with limited memory. For this reason, standard generic solvers are inadequate in this context, and there is a need for specialized algorithms. In [55], the author derives such algorithms for a variety of device classes. One such device class is a lossless energy storage device, which we extend in this thesis by additionally considering conversion losses.

**Algorithm 1** Profile steering algorithm

```
function ProfileSteering(f, M, T, X, X_m, ε)
    Request an initial schedule x_m of each device m
    repeat
        x ← Σ_{m∈M} x_m                                  ▷ Determine the current aggregated schedule
        for m ∈ M do
            x̂_m ← arg min_{x̃_m ∈ X_m} f(x − x_m + x̃_m)    ▷ Construct candidate schedules
            δ_m ← f(x) − f(x − x_m + x̂_m)                 ▷ Improvement made by m
        end for
        m̂ ← arg max_{m∈M} δ_m                            ▷ Find device with best improvement
        x_m̂ ← x̂_m                                        ▷ Update schedule of m̂
    until δ̂_m̂ < ε                                        ▷ Repeat as long as sufficient progress is made
    return x
end function
```

### PowerMatcher

The PowerMatcher [34] is an example of a market-based control DSM approach. Double-sided auctions are used to offer the consumer the possibility to sell their flexibility to interested parties. In this approach, each smart appliances comes equipped with a bidding function, indicating which price it is willing to pay for operation. This function may vary over time, and depend on internal mechanics of the appliance (such as state of charge of an EV or battery), e.g. making an EV more eager to charge closer to the deadline. This approach is optimal in the sense that the global optimum of the summed utilities coincides with the market equilibrium, and is Pareto optimal with respect to the market model. Disadvantages of this approach are the need for a bidding function for each appliance (they play a central role, but it is unclear exactly how they should be chosen), there are no guarantees for load balancing, and there is an implicit loss of flexibility due to postponement; early after going to auction devices may be reluctant to operate, causing them to generally postpone activation to a non-optimal point.

#### Game theoretic approach

Multi-agent decision processes can be modeled as (non-)cooperative games. In a smart grid, every consumer is considered a player of such a game. An overview of game theoretic methods applied in the smart grid can be found in [52]. In [46], a neighborhood with multiple batteries is considered. A best-response algorithm is used to find a pure Nash equilibrium with no guarantee on optimality. Players pay for their share of the total consumption at each time step, where the cost function is quadratic at each time step – implying load balancing to be beneficial for all players. Despite the lack of optimality guarantees, simulations indicated good performance for all simulations. Investigation of the influence of conversion losses indicated a significant impact on participation behaviour. The peak-to-average (PAR) ratio reduction was halved in the case of 91% efficiency as compared to the lossless case. Similarly, a lower reduction in energy bills for all participants occurred in the lossy case. The authors refer to possible improvement through a more advanced billing scheme: basing the prices on the respective success of each participant in reducing the PAR ratio may directly incentivize optimizing in that direction. The battery model used in this paper includes some non-linear mechanics, and battery actions are restricted to four modes of operation (idle, halfway charging, full charging, and demand fulfillment).

#### Alternating direction method of multipliers

In [50], the authors consider the problem of simultaneously scheduling a fleet of EVs in a pricing and peak shaving setting. The central problem is decomposed into $N+1$ coupled problems (one for each EV, and one for the aggregator) using the Alternating Direction Method of Multipliers (ADMM) method [23]. ADMM is an augmented Lagrangian method that iteratively improves the aggregated schedule using partial updates for the dual variables. ADMM has better asymptotic convergence properties compared to the profile steering approach, though it also introduces some complications: extensibility to multiple device types still has to be investigated, ADMM requires the specification of a penalty parameter but offers no method of determining it, and finally even for single devices ADMM requires multiple iterations. A further comparison of both methods is warranted, though we do not do so in this thesis.

## 2.4 Battery modelling

In order to match demand and supply of energy we can employ short-term energy storage devices (buffers in EF-Pi), such as electrical batteries. In order to make good use of these devices, we require a good model of their capabilities and input-response behaviour. This model can subsequently be optimized using mathematical programming techniques to obtain a sequence of (dis)charging decisions. It therefore makes sense to look different ways to model the inner workings of the most common residential energy storage device: an electrical battery.

Batteries are thermo-electro-chemical systems that are used for the temporary storage of energy. Inside the battery energy is stored in chemical bonds, while electrical energy can be charged to and discharged from it. The base components of a battery include: two electrodes, a separator, and an electrolyte. During discharge an oxidation reaction causes a reductant to donate electrons at the anode (one of the electrodes), and conversely a reduction reaction causes an oxidant to accept them at the cathode (the other electrode). The resulting flow of electrons can be used to power electrical appliances. During charging the reactions are reversed. Energy dissipates in the form of heat or radiation during the charge and discharge processes, and charge may be lost over time due to internal resistance. The yield relative to the input from a battery after a full charge-discharge cycle is called the round-trip efficiency.

The slow diffusion of reactants at either electrode can cause non-linear effects to arise: when the battery is discharged faster than diffusion can take place the reactants may accumulate at the poles, causing the battery's effective capacity to diminish until further diffusion has taken place. Hence, batteries may, for a short period, effectively have some amount of unavailable charge just after usage. This is called the recovery effect. Furthermore, the assumption that the battery voltage stays constant during discharge, instantly dropping to zero when empty, does not hold in practice; during discharge the voltage gradually lowers, and

the effective capacity is lower for high discharge currents. This is called the rate capacity effect. For the purpose of optimization, such effects are often ignored and simpler models are adopted.

In general, batteries can be modeled on different levels:

- **Electro-chemical models** describe the chemical processes inside the battery as a system of non-linear differential equations. Electro-chemical models can obtain very high accuracy, but require lots of parameters to properly work.
- **Electrical-circuit models** try to emulate the electrical behaviour of a battery by modelling it as a circuit with similar electrical properties. Electrical-circuit models require less configuration, but still require a lot of experimental data. While otherwise fine, these models are less accurate in predicting battery lifetime.
- **Analytic models** abstract away from the internal specifics of the battery and instead try to model the observed dynamics directly. An example is the Kinetic Battery Model (KiBaM) [38], which models the recovery effect by approximating the diffusion process as two bounded basins (one for the unavailable, and one for the available charge) with some fixed conductance between them.
- **Stochastic models** more or less assume the battery is a black box. Characteristics of the physical device are modelled by emergent properties of some randomized process.

The accuracy and complexity of these models are strongly related, causing the more accurate models to not be suitable for use in optimization procedures. A survey of battery models is given in [32].

## 2.5 Adverse effects of control

So far, we have mostly considered the positive effects of control and optimization in DEM. Aside from the overhead of introducing and running control mechanisms in the current energy supply chain, such mechanisms also introduce some other issues related to privacy and peak reinforcement in a pricing setting.

### Privacy and security

Energy consumption profiles may contain privacy sensitive information of the consumer, and should therefore be handled as such: details on energy consumption should be communicated as locally as possible, and should not be stored (for an extended period) without consent or legitimate reason according to GDPR. Information that can be deduced from energy consumption are things such as: number of residents, home occupancy, and/or ownership of an EV or rooftop PV. Failing to protect this data may therefore compromise the privacy and security of the consumer, as access to it may allow third parties to, for example, send targeted advertisements, or plan a burglary.

Despite best efforts to the contrary, electrical systems may sometimes fail. In particular, electrical batteries may catch fire [8] due to incorrect control, puncture damage, over-charging, or local short-circuiting from contact with a low resistance conductor. There are multiple reported cases of burn injuries [37] and even deaths caused by fires of faulty smartphone batteries. Naturally, these safety concerns have to be addressed before a widescale rollout of residential energy storage devices can happen. The sea-salt battery [2, 17] is an emergent battery technology which is much safer to use and easier to dispose of than Li-ion batteries.

### Peak reinforcement

Control mechanisms allow for the automated optimization of certain objectives. Consumers are generally assumed to want to minimize costs, while the grid operator may want to minimize peaks. These objectives may not align: there can be a significant increase in power passing the transformer by trading in energy using e.g. day ahead prices of the spot market [43] (for intraday prices the effect is similar but reduced). In general, traditional market mechanisms do not solve local grid problems.

The observed effect can be easily explained: as all consumers are simultaneously incentivized to shift their energy use to a period of low cost, peaks may not be reduced but merely shifted. Additionally, consumption peaks may occur at times where there would have been none before. Hence the automated scheduling of smart appliances in this setting potentially only worsens the situation.

One of the root causes of these problems is that people receive the same prices at the same time. Dynamic price tariffs, where different people receive different energy tariffs at the same time, may be a way to deal with this. The fairness of such schemes is still heavily debated [42], and it is unclear how exactly the prices should be determined and fairly divided among the populace.

We do not investigate the influence of losses on the adverse effects in a pricing setting in this report.

# 3. Model

*In order to make optimal charge and discharge decisions for the considered energy storage device, we require a mathematical model to optimize. In this chapter, we formulate the models used in the rest of the thesis. The considered model is an extension to the one presented in [55]. The novel contribution is the addition of conversion losses. At the end of this chapter we summarise the model assumptions.*

## 3.1 High-level description

We start by giving a high-level description of the properties of the derived model. In the next section we place this description in a mathematical formalism.

Our model captures charge and discharge decisions of an energy storage device for a finite amount of time steps $\mathcal{T} = \{1, \ldots, T\}$. We denote the state of charge at the end of time step $t$ by $SoC_t$, where initially the storage device has a state of charge equal to $SoC_0$. Furthermore, the amount of (dis)charging done at time step $t \in \mathcal{T}$ is denoted by $x_t$, where positive values indicates the storage device charges, and negative values indicates the device discharges. The charging decisions $\mathbf{x} = (x_1, \ldots, x_t)$ are bounded from below and above, and the goal is to minimize a convex and separable objective function $f(\mathbf{x}) = \sum_{t \in \mathcal{T}} f_t(x_t)$, where all $f_t$ are convex. Typical choices for $f_t(x_t)$ in the field of DEM are:

- linear pricing: $f_t(x_t) = c_t x_t$, where $c_t$ is the unit price of energy at time step $t$;
- feed-in subsidy: $f_t(x_t) = c_t (x_t - p_t)^+ + s_t (x_t - p_t)^-$, where $s_t \leq c_t$, $y^+ = \min\{0, y\}$ and $y^- = \max\{0, y\}$, $c_t$ as before, $s_t$ is the unit subsidy for fed-in energy, and $p_t$ is the energy profile at time $t$. Under a feed-in subsidy policy delivering back energy to the net incurs a monetary reward that is lower than the energy cost, thus promoting self-consumption;
- quadratic deviations: $f_t(x_t) = (x_t - p_t)^2$, where $p_t$ as before. By minimizing the quadratic deviations objective, we use the storage device to steer the energy profile towards zero.

For simplicity we assume there is no latent charge (as might happen inside an electrical battery due to slow diffusion of reactants), which means the entire state of charge is available for usage at all times. Losses are incurred during (dis)charging, such that a fraction of the in- and outflow is lost when the storage is used. Figure 3.1 is an abstract representation of our model. The resulting model is an extension to the one presented in [55].

Energy conversion losses cause energy to be lost during the charging and discharging processes. This is modeled by losing a fixed fraction of the in- or outflow: one unit of charge from the net leads to a charge

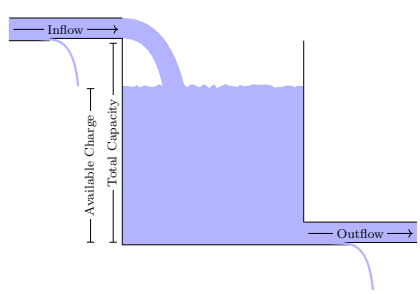



**Figure 3.1:** Simplified lossy storage model

| | RTE [%] | Decay [% energy/day] |
|---|---|---|
| PHES | 70-85 | $\approx 0$ |
| CAES | 57-85 | $\approx 0$ |
| FES | 70-95 | 1.3-100 |
| SCES | 90-98 | 20-40 |
| SMES | 90-98 | 10-15 |
| NaS | 70-90 | 0.05-20 |
| LA | 70-82 | 0.033-0.3 |
| NiCd | 60-70 | 0.067-0.6 |
| Li-Ion | 85-98 | 0.1-0.3 |
| ZnBr | 60-75 | 0.24 |
| PSB | 57-75 | $\approx 0$ |
| VR | 60-85 | 0.2 |
| Seasalt | 80-90 | ? |

**Table 3.1:** Efficiency parameter ranges for different energy storage devices, as seen in [51]. Abbreviations used: pumped hydroelectric energy storage (PHES), compressed air energy storage (CAES), flywheel energy storage (FES), supercapacitor energy storage (SCES), superconducting magnetic energy storage (SMES), sodium sulphur battery (NaS), lead acid battery (LA), nickel cadmium battery (NiCd), lithium-ion battery (Li-ion), zinc bromine flow battery (ZnBr), polysulphide bromide flow battery (PSB), and vanadium redox flow battery (VR). RTE value for seasalt battery from [3].

increase of $\eta_c \in (0, 1]$ in the energy storage, and conversely discharging $\eta_d \in [1, \infty)$ energy from the battery delivers one unit of energy to the net. Consequently, circulating one unit of energy from the net through the energy storage back to the net yields $\eta_c/\eta_d$, a quantity known as the *round-trip efficiency* (RTE) of the energy storage device. Conversion losses most notably occur due to internal resistance in the form of friction, spillage, heat emission, or radiation. When the state of charge is not directly measurable (which is the case for electrical batteries) it is usually assumed that charging and discharging contribute equally to the loss of energy, such that: $\eta_c = 1/\eta_d = \sqrt{\text{RTE}}$.

Another kind of loss is static discharge losses, which occurs over time either due to leakage or due to uncontrolled spontaneous discharge. This case has been studied in [55] within the context of heating, ventilation, and air conditioning (HVAC) systems. The problem we present here remains convex after introduction of this type of loss, and it can easily be accounted for. Therefore, for convenience and without loss of generality, we do not consider it in our model.

In Table 3.1 typical ranges of values for the RTE and decay rate per day for different types of energy storage technologies. This table gives a simplified view of the working of batteries, as in practice the RTE depends strongly on the mode of operation.

## 3.2 Mathematical formulation

In this section we place the description of the previous section in a mathematical formalism. More precisely, we formulate the optimization problems of scheduling the (dis)charging decisions of lossless and lossy energy storage devices. A first formulation of our energy storage problem, based on the previous section, is:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}), \\ \text{s.t.} \quad & SoC_t^{\min} \leq SoC_t \leq SoC_t^{\max} \quad \forall t \in \mathcal{T}, \\ & x_t^{\min} \leq \Delta SoC_t \leq x_t^{\max} \quad \forall t \in \mathcal{T}, \end{aligned} \tag{3.1}$$

where $SoC_t^{\min}$ and $SoC_t^{\max}$ are capacity bounds, and $x_t^{\min}$ and $x_t^{\max}$ are (dis)charging bounds. The constraints in (3.1) state that the (dis)charge rate of the battery and its state of charge should be within these

bounds. To make the model complete, we need to relate the state of charge to the charging decisions. When we disregard losses their relation is straightforward, as the state of charge at time $t$ is simply the accumulated effect of all (dis)charge decisions up to time $t$ starting from the initial state of charge:

$$SoC_t = SoC_0 + \sum_{t'=1}^{t} x_{t'}.$$

Thus, we obtain the following optimization problem for lossless storage devices:

**Problem 3.1.** ***Lossless Storage Problem:*** *Given an initial state of charge $SoC_0$, capacity bounds $SoC_t^{\min}$ and $SoC_t^{\max}$, and (dis)charge bounds $x_t^{min}$ and $x_t^{max}$, the optimization problem for the lossless energy storage device is:*

$$\begin{aligned} min_{\mathbf{x}} \quad & f(\mathbf{x}), \\ s.t. \quad & SoC_t^{\min} \leq SoC_0 + \sum_{t'=1}^{t} x_{t'} \leq SoC_t^{\max} \quad && \forall t \in \mathcal{T}, \\ & x_t^{\min} \leq x_t \leq x_t^{\max} && \forall t \in \mathcal{T}. \end{aligned}$$

When we take into account conversion losses, the state of charge at time $t$ depends on the charging decisions $x_t$, and the conversion rates $\eta_c$ and $\eta_d$ in the following manner:

$$SoC_t = SoC_0 + \sum_{t'=1}^{t} \left(\eta_c x_{t'}^+ + \eta_d x_{t'}^-\right). \tag{3.2}$$

where $x_t^+ = \max\{x_t, 0\}$ is the positive part and $x_t^- = \min\{x_t, 0\}$ is the negative part of $x_t$. Plugging (3.2) into (3.1) gives us the following formulation of the Lossy Storage Problem:

**Problem 3.2.** ***Lossy Storage Problem:*** *Given an initial state of charge $SoC_0$, conversion rates $0 < \eta_c \leq 1 \leq \eta_d$, capacity bounds $SoC_t^{\min}$ and $SoC_t^{\max}$, and (dis)charge bounds $x_t^{min}$ and $x_t^{max}$, the optimization problem for the lossy energy storage device is:*

$$\begin{aligned} min_{\mathbf{x}} \quad & f(\mathbf{x}), \\ s.t. \quad & SoC_t^{\min} \leq SoC_0 + \sum_{t'=1}^{t} \left(\eta_c x_{t'}^+ + \eta_d x_{t'}^-\right) \leq SoC_t^{\max} \quad && \forall t \in \mathcal{T}, \\ & x_t^{\min} \leq \eta_c x_t^+ + \eta_d x_t^- \leq x_t^{\max} && \forall t \in \mathcal{T}. \end{aligned}$$

Note that we can remove the losses in Problem 3.2 by setting $\eta_c = \eta_d = 1$, doing so reduces Problem 3.2 to Problem 3.1.

It is useful to make the distinction between the charging decision $x_t$, and the internal change in state of charge $\tilde{x}_t$ at time $t$. These quantities are related in the following manner:

$$\begin{aligned} \tilde{x}_t &= \eta_c x_t^+ + \eta_d x_t^-, \\ x_t &= \tfrac{1}{\eta_c}\tilde{x}_t^+ + \tfrac{1}{\eta_d}\tilde{x}_t^-. \end{aligned} \tag{3.3}$$

Since the second transform in (3.3) will occur frequently, we introduce the following notation for it: $T_\eta(x) = \frac{1}{\eta_c}x^+ + \frac{1}{\eta_d}x^-$.

In Problem 3.2 the charging decisions occurred inside the objective, while the internal change in state of charge occurred in the constraints. Using the above equations we can obtain an equivalent (but differently formulated) model, by expressing the state space in terms of $\tilde{x}_t$ rather than in $x_t$, thus moving the losses from the constraints to the objective:

**Problem 3.3.** ***Lossy Storage Problem (alternative form):*** *Given an initial state of charge $SoC_0$, conversion rates $0 < \eta_c \leq 1 \leq \eta_d$, capacity bounds $SoC_t^{\min}$ and $SoC_t^{\max}$, and (dis)charge bounds $x_t^{\min}$ and $x_t^{\max}$. The optimization problem for the lossy energy storage device is:*

$$\begin{aligned} min_{\tilde{\mathbf{x}}} \quad & \tilde{f}\left(\tilde{\mathbf{x}}\right) = f\left(\tfrac{1}{\eta_c}\tilde{\mathbf{x}}^+ + \tfrac{1}{\eta_d}\tilde{\mathbf{x}}^-\right), \\ s.t. \quad & SoC_t^{\min} \leq SoC_0 + \sum_{t'=1}^{t} \tilde{x}_{t'} \leq SoC_t^{\max} \quad \forall t \in \mathcal{T}, \\ & x_t^{\min} \leq \tilde{x}_t \leq x_t^{\max} \quad \forall t \in \mathcal{T}. \end{aligned}$$

Although it is perhaps more natural to visualize the lossy storage problem as formulated in Problem 3.2 we will instead work with the alternative formulation offered in Problem 3.3, unless stated otherwise. Note that the feasible sets of both problems are compact. Hence when the objectives we consider are continuous, the Weierstrass' extreme value theorem for general topological metric spaces asserts that the optimal solution exists – it only remains to find it.

## 3.3 Model assumptions

In formulating the models we have made some implicit and explicit assumptions. We summarise them below:

**Assumption 1:** *The initial state of charge $SoC_0$ is measurable (and given).*

**Assumption 2:** *The objective is convex and separable: $f\left(\mathbf{x}\right) = \sum_{t\in\mathcal{T}} f_t\left(x_t\right)$, where each $f_t$ is convex.*

**Assumption 3:** *Static discharge and energy conversion losses are independent. Furthermore, the corresponding parameters can be independently determined.*

**Assumption 4:** *The storage device behaves linearly; non-linear effects of the storage device are negligible.*

The accuracy of these assumptions differs with the considered storage device: in a pumped hydroelectric storage system the state of charge can be directly measured as the water level in the upper reservoir. This is not the case for an electric battery. As the state of charge is not directly measurable, it may also be harder to separate the effects of static and conversion losses in an electric battery.

# 4. Importance of losses

*In this chapter we quantify the importance of considering losses during optimization. In particular, we assess the deterioration of the objective value when we momentarily disregard the effect of losses. We propose two intuitive approaches for amending solutions of the lossless problem to account for losses, and show through simulations that they perform poorly as compared to the optimal solution of the lossy problem. These results serve as motivation for further exploration of the problem. Finally, we briefly investigate economic feasibility.*

## 4.1 Amending the solution

When the round-trip efficiency is relatively high (RTE $\approx 1$), we expect the optimal solutions of the lossless and lossy problems to be very similar: applying the lossless solution to a lossy storage with high efficiency should yield only a slight deterioration in objective value. Nevertheless, the solution may become infeasible due to there being a lower state of charge than anticipated, causing the given solution to attempt to further discharge the storage whilst it is already empty. Consider a 40 kWh storage device with 70% RTE with the quadratic deviations objective, where the profile values are obtained by aggregating the net energy consumption of ten households. The blue graphs in Figure 4.1a show the optimal charging decisions (left) and state of charge (right) when the storage is lossless, and the red graphs indicate the same quantities when the optimal lossless solution is applied to the lossy storage. The solution quickly becomes infeasible, getting as low as negative half of the maximum capacity at the end of a single day. Two simple ways in which we can amend the lossless solution to account for this are:

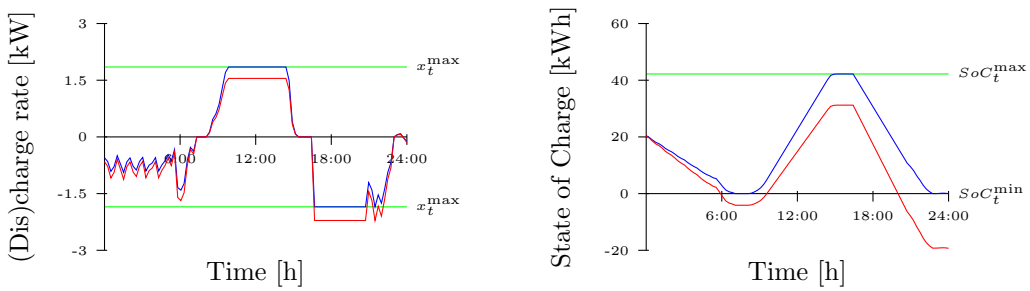
- **repair** the solution: whenever the state of charge is about to exceed its bounds, it is set equal to that bound instead. When the changes in state of charge bounds are not too erratic, this gives a feasible solution (otherwise, the problem can be preprocessed first). This method attempts to remain close to optimality by staying as close as possible to the original optimal solution, but due to feasibility problems will generally not be able to;

- **rescale** the solution: we transform the solution by considering the lossless solution to express the internal energy $\tilde{x}_t$ of the lossy storage. In other words, all positive valued $x_t$'s are scaled by $\frac{1}{\eta_c}$, and all negative valued $x_t$'s are scaled by $\frac{1}{\eta_d}$. Essentially we are plugging the solution to Problem 3.1 into Problem 3.3. This method attempts to remain feasible, but in doing so does not remain optimal as the objective toward which we optimize has changed.

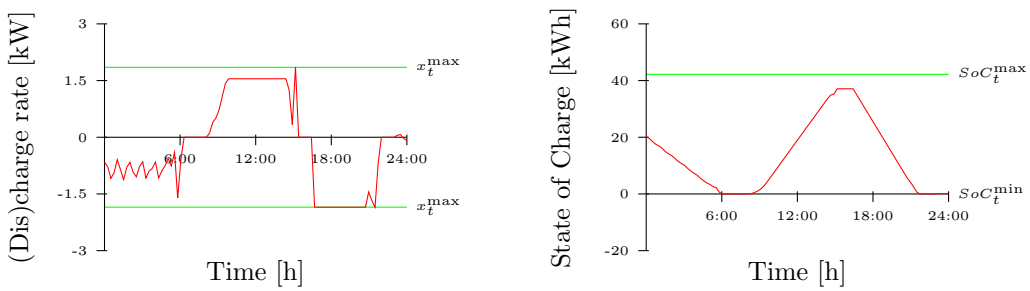
As can be seen in Figure 4.1a, simply ignoring the losses leads to an infeasible solution as the state of charge ends up lower than expected due to not having taken into account the incurred losses. The blue line indicates the optimal lossless schedule, and the red line the realized profile induced by using the lossless schedule in a lossy device with 70% RTE. The discrepancy between the lossless model and lossy reality is huge: around 20:00 the model thinks the storage is halfway full, while it is already empty! At the end of the day, the state of charge equals negative half of the capacity.

In Figure 4.1b losses are accounted for via the repair strategy. The (dis)charging profile is essentially the same as in the original solution, except for parts where it is not feasible to perform the prescribed action. The resulting schedule is feasible again, but doesn't ever use the full capacity of the battery. Furthermore, for larger time horizons, the effective capacity will continue to diminish further over time.

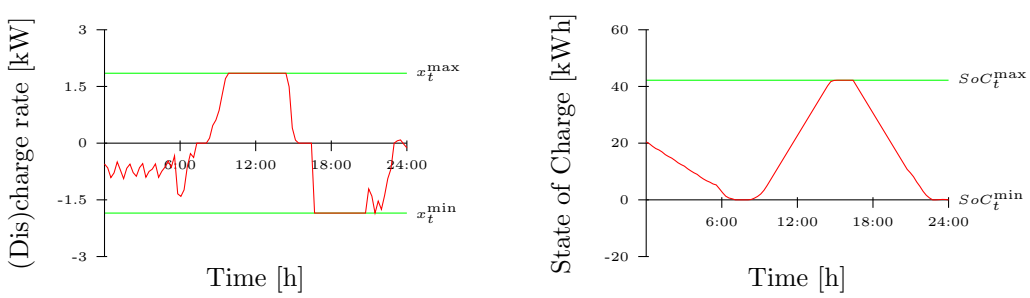
In Figure 4.1c, we directly translate the lossless schedule to the lossy case by rescaling the schedule. As such, it certainly is feasible, but may not be optimal. Both the repaired and rescaled solutions reduce to the original solution when the efficiency approaches 100%.




**(a)** Lossless solution applied to a lossy storage device with 70% RTE. The blue line indicates the lossless schedule, and the red line indicates the realized profile induced by using the lossless schedule in the lossy storage device.




**(b)** Repair method applied to the lossless schedule.




**(c)** Rescale method applied to the lossless schedule.

**Figure 4.1:** (Dis)charge rate and state of charge of storage devices under different schedules.

### 4.2 Bounding the deterioration

In this section, we use the rescale method to impose a bound on the change in objective from the lossless to lossy case when we consider the linear pricing objective. Although often returns diminish due to losses, we show that there are cases in which the objective improves due to losses. Let $v^*(\eta_c, \eta_d)$ denote the objective value of the optimal solution to an instance of Problem 3.3 (such that $v^*(1,1)$ is the optimal objective value of the lossless case). Lemma 4.2 gives a bound on their difference. Although the given bounds are not very tight, they at least indicate that the solution deteriorates proportional to $\frac{1}{\eta_c}$ (or $\frac{1}{\sqrt{\text{RTE}}}$) in the linear case.

**Lemma 4.1.** *When $f_t(x_t) = c_t x_t$, $\eta_c = 1/\eta_d$, and $x^{\min} = \min_t x_t^{\min} = -\max_t x_t^{\max} = -x^{\max}$:*

$$v^*(\eta_c, \eta_d) - v^*(1,1) \leq \sum_{t \in \mathcal{T}} c_t^+ \left(\frac{1}{\eta_c} - 1\right) x^{\max}.$$

*Proof.* We prove the bound by using the rescale method. Let $\mathbf{x}$ be the optimal solution to the instance of Problem 3.1. Applying the rescale method to $\mathbf{x}$ gives us a feasible solution to the instance of Problem 3.3: $\frac{1}{\eta_c}\mathbf{x}^+ + \frac{1}{\eta_d}\mathbf{x}^-$, with objective value $v^{\text{res}}(\eta_c, \eta_d)$, such that: $v^*(\eta_c, \eta_d) \leq v^{\text{res}}(\eta_c, \eta_d)$. In the following derivation we use the index sets $\mathcal{T}^+ := \{\, t \in \mathcal{T} \mid x_t \geq 0 \,\}$ and $\mathcal{T}^- := \{\, t \in \mathcal{T} \mid x_t < 0 \,\}$.

$$\begin{aligned}
v^*(\eta_c, \eta_d) - v^*(1,1) &\leq v^{\text{res}}(\eta_c, \eta_d) - v^*(1,1) \\
&= \left(\frac{1}{\eta_c} - 1\right) \sum_{t \in \mathcal{T}} c_t x_t^+ + \left(\frac{1}{\eta_d} - 1\right) \sum_{t \in \mathcal{T}} c_t x_t^-, \\
&\leq \left(\frac{1}{\eta_c} - 1\right) \sum_{t \in \mathcal{T}} c_t^+ x_t^+ + \left(\frac{1}{\eta_d} - 1\right) \sum_{t \in \mathcal{T}} c_t^+ x_t^- \\
&\leq \left(\frac{1}{\eta_c} - 1\right) x^{\max} \sum_{t \in \mathcal{T}^+} c_t^+ + \left(\frac{1}{\eta_d} - 1\right) x^{\min} \sum_{t \in \mathcal{T}^-} c_t^+ \\
&= \sum_{t \in \mathcal{T}} c_t^+ \cdot \max\left\{ \left(\frac{1}{\eta_c} - 1\right) x^{\max}, \left(\frac{1}{\eta_d} - 1\right) x^{\min} \right\} \\
&= \sum_{t \in \mathcal{T}} c_t^+ \left(\frac{1}{\eta_c} - 1\right) x^{\max}.
\end{aligned}$$

□

This result tells us that when all costs are non-positive (which *do* occur, although rarely, in the real world [4]), the lossy storage device will never be worse in objective value than the lossless storage device. This can be motivated by considering that non-positive costs indicate that energy consumption is rewarded. The lower the round-trip efficiency, the more the storage device is capable of consuming energy. We give two minimal examples below:

- The optimal solution to the following Lossy Storage Problem with negative linear cost:

$$\begin{aligned}
\min_x \quad & -x, \\
\text{s.t.} \quad & 0 \leq \eta_c x_t^+ + \eta_d x_t^- \leq 1,
\end{aligned}$$

  turns out to be strictly increasing in $\eta_c \in (0, 1]$: $-\frac{1}{\eta_c}$. It follows that negative prices allow us to find improvement when we introduce losses.

- And similarly for the quadratic deviations objective with positive profile:

$$\begin{aligned} \min_x \quad & (x-2)^2, \\ \text{s.t.} \quad & 0 \leq \eta_c x_t^+ + \eta_d x_t^- \leq 1, \end{aligned}$$

  where the optimal objective value equals 0 when $\eta_c \in \left(0, \frac{1}{2}\right]$ and $\left(\frac{1}{\eta_c} - 2\right)^2$ when $\eta_c \in \left[\frac{1}{2}, 1\right]$, which is an increasing function in $\eta_c \in (0, 1]$.

We show in Chapter 5 that exactly these cases, where we have negative prices or positive profiles, turn out to be difficult to solve in general.

## 4.3 Numerical study

In this section we determine the impact of the introduction of conversion losses for different objectives and several RTEs. In particular, we consider the deterioration of the optimal objective as a function of the round-trip efficiency. The schedules generated by the proposed repair and rescale methods are compared to the optimal solution for the linear pricing, feed-in subsidy, and quadratic deviations objectives. Optimal solutions are found by turning Problem 3.3 into an Integer Linear Programming problem [5]. In the case of the quadratic deviations objective, this methodology is not exact, but rather an approximation.

Throughout our simulations we consider single days in isolation with a time step every 15 minutes. We consider a 42.2 kWh Lithium-ion battery, having (dis)charge bounds of 7.4kW (1.85 kWh per 15 minutes) – comparable to normal charging of batteries found in average modern EVs (BMW i3: 19–42 kWh, Nissan Leaf II: 40–62 kWh, Tesla Model S: 50–100 kWh [**?**]) with a single phase AC charger at 230V and 32A. Our simulations thus allow for the interpretation of using the vehicle-to-grid (V2G) capabilities of a latent EV. An appropriately dimensioned residential energy storage device can be much smaller than this.

### Linear pricing

We first consider the linear pricing objective: $f_t(x_t) = c_t x_t$. For the unit energy prices $c_t$ we take day-ahead prices from the energy spot market of 2014. We simulate one week of each season, and a single day with a day-night tariff of €0.21/kWh by day and €0.18/kWh by night (the night tariff is active from 23:00h till 7:00h). An assessment of the efficacy of the repair and rescale approaches as compared to the optimal schedule can be found in Table 4.1. For many instances, the repair and rescale methods return a schedule that *costs* money (see Table 4.2). We do not trade on those days instead. When the RTE is lower than the ratio between the lowest and highest energy prices ($18/21 \approx 0.8571$ for the day-night tariff), there is no sense in trading as no profit can be made. The amount of lost revenue per day due to conversion losses is evaluated in Figure 4.2. In particular, the difference in objective between the optimal lossy solution and lossless solutions is plotted.

| | | Round-trip efficiency | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
| Spot prices | Optimum | €0.0005 | €0.0066 | €0.0544 | €0.1957 | €0.4490 | €0.8021 | €1.2724 |
| | Repair | €0 | €0 | €0 | €0 | €0.1004 | €0.5722 | ” |
| | Rescale | €0 | €0 | €0 | €0 | €0.1029 | €0.5845 | ” |
| Day-night | Optimum | €0 | €0 | €0 | €0 | €0.4003 | €0.8443 | €1.2660 |
| | Repair | €0 | €0 | €0 | €0 | €0.3132 | €0.7896 | ” |
| | Rescale | €0 | €0 | €0 | €0 | €0.3301 | €0.8101 | ” |

**Table 4.1:** Average daily revenue from energy trading using a lossy energy storage device, where no trading is done on unprofitable schedules.

| | | Round-trip efficiency | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
| Spot prices | Repair | -€2.9676 | -€2.2596 | -€1.5516 | -€0.8443 | -€0.1387 | €0.5669 | €1.2724 |
| | Rescale | -€3.5507 | -€2.6137 | -€1.7400 | -€0.9210 | -€0.1496 | €0.5799 | ” |
| Day-night | Repair | -€1.5924 | -€1.1160 | -€0.6396 | -€0.1632 | €0.3132 | €0.7896 | €1.2660 |
| | Rescale | -€1.9033 | -€1.2886 | -€0.7151 | -€0.1770 | €0.3301 | €0.8101 | ” |

**Table 4.2:** Average daily revenue of the repair and rescale methods from energy trading using a lossy energy storage device, where trading is still done on unprofitable schedules.

As long as a household is not a net producer annually, the linear pricing setting corresponds to the net metering policy. Under linear pricing, even for high round-trip efficiencies and strongly varying prices taken from the energy spot market, the arbitrage opportunities for energy traders do not warrant the investment in an energy storage device in the near future: even for the optimal schedule at 100% efficiency, and Lithium-ion battery prices being as low as $62/kWh in 2030 [25] (or €55.40/kWh), the expected payback time of the considered battery is about 5 years – approximately equal to its lifetime. For reference, the average global cost of Lithium-ion batteries was $176/kWh (or €157.26/kWh) in 2018 [25], giving an expected payback time of about 15 years. Appropriate dimensioning of the battery size does not help in this regard, as both revenue and costs scale with the battery dimension.

Assuming the expected battery prices of 2030, if the efficiency and lifetime of this battery technology can be further improved it may become economically feasible to purchase an electical battery dedicated to energy trading within the next 20 years. Specifically, if the average lifetime of Lithium-ion batteries can be extended to eight years a daily revenue above €0.80/day makes dedicated energy trading batteries economically viable, thus requiring an RTE of around 95% for economic viability (for the current five year lifetime a daily revenue above €1.28/day is required). If the lifetime can be further extended to ten years, a daily revenue above €0.64/day suffices, for which an RTE between 90% and 95% is required. Using the battery of a latent EV for energy trading may be worthwhile already – given that the battery is already available. A more thorough analysis of economic feasibility in a linear pricing setting can be found in [45], where the authors additionally show that rewarding EV owners for participation in grid support is required to counter the monetary losses incurred by device ageing.

Finally, accounting for losses after the fact using either the repair or rescale methods has poor performance for the linear pricing objective. At 90% RTE – a typical value for Lithium-ion batteries (see Table 3.1) – these

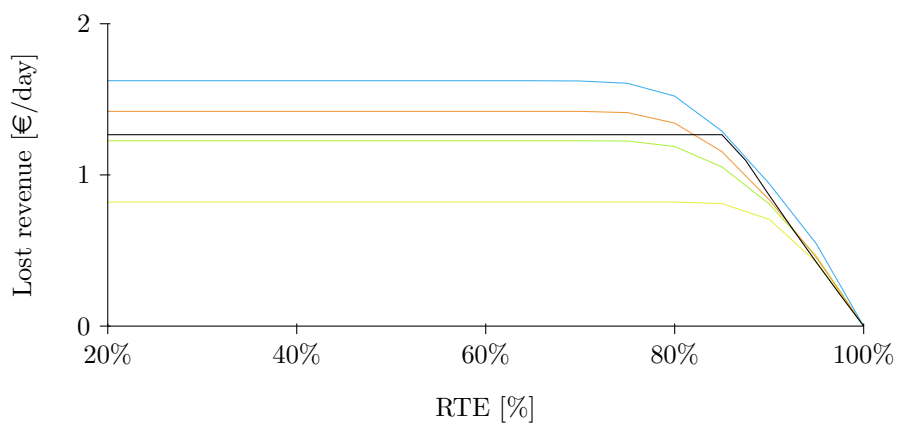


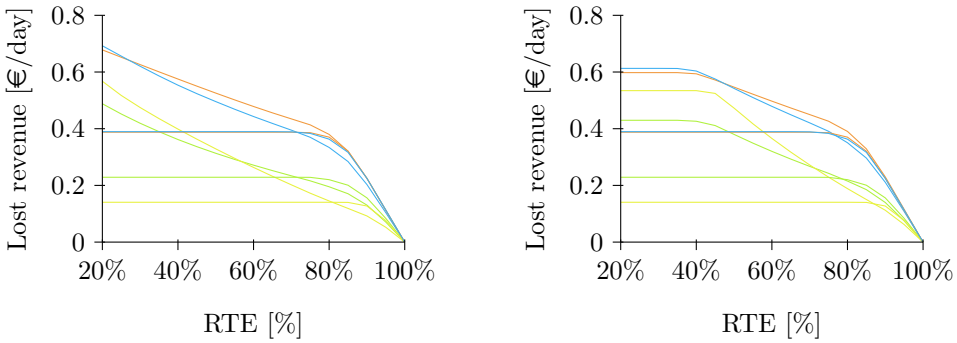

**Figure 4.2:** Lost revenue due to energy conversion losses as function of round-trip efficiency. Green, yellow, orange, and blue correspond to the averages of the simulations where the energy spot prices are taken from days in spring, summer, autumn, and winter respectively. The black line indicates the day-night tariff.

methods yield between 20-25% of the revenue that could have been obtained under the optimal schedule for the spot market prices, and about 80% for the day-night tariff.

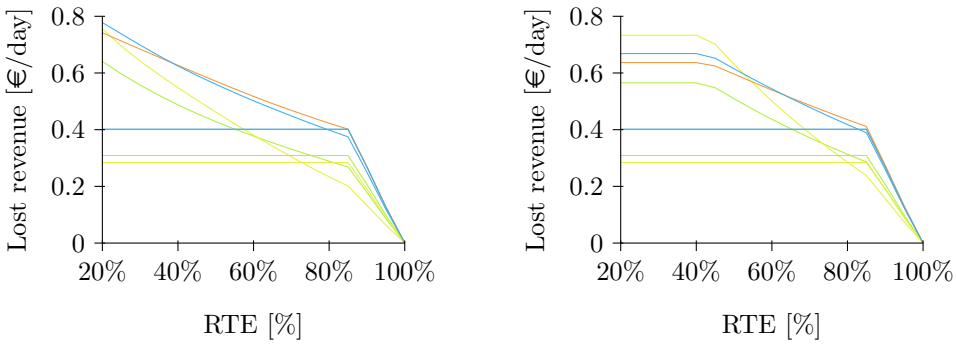
**Feed-in subsidy**

Next, we consider the feed-in subsidy objective: $f_t(x_t) = c_t(x_t - p_t)^+ + s_t(x_t - p_t)^-$, where $s_t \leq c_t$. Take the energy prices $c_t$ as before, the subsidy $s_t$ is taken constant for all time steps and is either equally zero everywhere or taken as 50% of the lowest energy price through the year, and the energy profile $p_t$ is generated by the Artificial Load Profile Generator (ALPG) [28]. We consider 105 isolated households, where approximately half of the houses have solar panels, and the solar irradiation data is taken from [35, Twenthe 2014]. One week per season is simulated, where all days and housholds are taken as isolated instances. The average lost revenue due to energy conversion losses are plotted in Figure 4.3a for spot market prices, and Figure 4.3b for the day-night tariff. Households with and without PV are averaged separately. Also see Table 4.3 and Table 4.4 for the daily revenue due to the battery for houses with and without PV respectively.

These results are revealing on multiple levels: the revenue due to the battery does not change at all for households without PV for different subsidy levels implying it is never discharged beyond the household's own energy demand, and compared to linear pricing the daily revenue due to the battery is much higher for households with PV (except for 95% and 100% efficiency). Using the same calculations from the previous section, we find, as expected, that households without PV should not invest in an energy storage device.




**(a)** Energy prices taken from spot market. Left: zero subsidy, right: 50% of lowest price as subsidy.




**(b)** Energy prices taken from day-night tariff. Left: zero subsidy, right: 50% of lowest price as subsidy.

**Figure 4.3:** Lost revenue due to energy conversion losses as function of round-trip efficiency. Green, yellow, orange, and blue correspond to simulations where the energy spot prices are taken from days in spring, summer, autumn, and winter respectively. Results are averaged separately for houses with and without PV. The houses with PV stand to lose more of their revenue due to inefficiencies in the energy storage device.

| | Subsidy | RTE | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|---|
| Spot prices | 0% | Optimum | €0.4867 | €0.5101 | €0.5396 | €0.5795 | €0.6405 | €0.7183 | €0.8031 |
| | | Repair | €0.3244 | €0.3778 | €0.4423 | €0.5183 | €0.6084 | €0.7057 | ” |
| | | Rescale | €0.1674 | €0.2404 | €0.3311 | €0.4377 | €0.5580 | €0.6829 | ” |
| | 50% | Optimum | €0.1912 | €0.2214 | €0.2574 | €0.3035 | €0.3704 | €0.4537 | €0.5437 |
| | | Repair | €0.0592 | €0.0933 | €0.1438 | €0.2198 | €0.3199 | €0.4306 | ” |
| | | Rescale | €0.0166 | €0.0468 | €0.0994 | €0.1821 | €0.2937 | €0.4181 | ” |
| Day-night | 0% | Optimum | €0.5713 | €0.5981 | €0.6235 | €0.6479 | €0.7520 | €0.8601 | €0.9594 |
| | | Repair | €0.3870 | €0.4483 | €0.5243 | €0.6127 | €0.7252 | €0.8441 | ” |
| | | Rescale | €0.1973 | €0.2826 | €0.3904 | €0.5162 | €0.6648 | €0.8158 | ” |
| | 50% | Optimum | €0.2200 | €0.2549 | €0.2880 | €0.3198 | €0.4308 | €0.5455 | €0.651 |
| | | Repair | €0.0701 | €0.1089 | €0.1664 | €0.2549 | €0.3810 | €0.5171 | ” |
| | | Rescale | €0.0190 | €0.0533 | €0.1123 | €0.2094 | €0.3492 | €0.5009 | ” |

**Table 4.3:** Averaged daily revenue from feed-in subsidy using a lossy battery for different round-trip efficiency values for households with PV.

| | Subsidy | RTE | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|---|
| Spot prices | 0% | Optimum | €0.0002 | €0.0021 | €0.0131 | €0.0422 | €0.1033 | €0.1906 | €0.2866 |
| | | Repair | €0 | €0 | €0.0012 | €0.0148 | €0.0784 | €0.1766 | ” |
| | | Rescale | €0 | €0 | €0.0014 | €0.0146 | €0.0776 | €0.1781 | ” |
| | 50% | Optimum | €0.0002 | €0.0021 | €0.0131 | €0.0422 | €0.1033 | €0.1906 | €0.2866 |
| | | Repair | €0 | €0 | €0.0012 | €0.0148 | €0.0784 | €0.1766 | ” |
| | | Rescale | €0 | €0 | €0.0014 | €0.0146 | €0.0776 | €0.1781 | ” |
| Day-night | 0% | Optimum | €0 | €0 | €0 | €0 | €0.1155 | €0.2380 | €0.3489 |
| | | Repair | €0 | €0 | €0 | €0 | €0.0844 | €0.2193 | ” |
| | | Rescale | €0 | €0 | €0 | €0 | €0.0840 | €0.2196 | ” |
| | 50% | Optimum | €0 | €0 | €0 | €0 | €0.1155 | €0.2380 | €0.3489 |
| | | Repair | €0 | €0 | €0 | €0 | €0.0844 | €0.2193 | ” |
| | | Rescale | €0 | €0 | €0 | €0 | €0.0840 | €0.2196 | ” |

**Table 4.4:** Averaged daily revenue from feed-in subsidy using a lossy battery for different round-trip efficiency values for households without PV.

For households with PV, only the zero subsidy case warrants the purchase of an energy storage device for both the spot market prices and day-night tariff. Given zero feed-in subsidy and a doubled battery lifetime, Lithium-ion batteries will be viable in 2030. Similarly, given one and a half times the lifetime and a 2% to 3% increase in efficiency Lithium-ion batteries will also be viable in 2030 (for the day-night tariff).

The repair and rescale methods perform relatively well in some scenarios for this objective: at 90% only a few cents are lost each day as compared to the optimal solution in the 90% efficiency, day-night tariff, household with PV setting. For other settings, around 12% of the revenue is lost.

The revenue due to the battery increases at the expense of the revenue due to PV. In reality, viability therefore depends strongly on the level of subsidization given for PV, and the feed-in subsidy should steadily be reduced to zero according to current battery prices. Furthermore, the revenue due to the battery will decrease for every additional schedulable appliance introduced in the household.

Correctly dimensioning the battery potentially increases the daily revenue, as the revenue does not scale linearly with the battery capacity. Consider the same battery as before, but scaled down by a factor ten: 4.22 kWh maximum capacity with 0.74 kW (dis)charge bounds (0.185 kWh per 15 minutes). Table 4.5 shows the average daily revenue due to the battery in the optimal schedule for households with PV. While costs go down by a factor ten, the average daily revenue due to the battery approximately only goes down by a factor two. The level curves indicate the €/kWh battery prices at which point the corresponding subsidy and

| Subsidy | Round-trip efficiency 70% | 75% | 80% | 85% | 90% | 95% | 100% | |
|---|---|---|---|---|---|---|---|---|
| 0% | €0.2949 | €0.3045 | €0.3147 | €0.3267 | €0.3422 | €0.3600 | €0.3789 | ≤150 €/kWh |
| 10% | €0.2589 | €0.2698 | €0.2812 | €0.2943 | €0.3108 | €0.3295 | €0.3494 | |
| 20% | €0.2228 | €0.2351 | €0.2477 | €0.2618 | €0.2794 | €0.2991 | €0.3198 | ≤ 125 €/kWh |
| 30% | €0.1868 | €0.2004 | €0.2141 | €0.2294 | €0.2480 | €0.2686 | €0.2902 | |
| 40% | €0.1508 | €0.1656 | €0.1806 | €0.1970 | €0.2166 | €0.2382 | €0.2607 | ≤ 100 €/kWh |
| 50% | €0.1147 | €0.1309 | €0.1471 | €0.1646 | €0.1852 | €0.2077 | €0.2311 | ≤ 75 €/kWh |
| 60% | €0.0787 | €0.0962 | €0.1136 | €0.1322 | €0.1538 | €0.1773 | €0.2016 | |
| 70% | €0.0427 | €0.0615 | €0.0800 | €0.0997 | €0.1224 | €0.1468 | €0.1720 | ≤ 50 €/kWh |
| 80% | €0.0123 | €0.0272 | €0.0465 | €0.0673 | €0.0910 | €0.1164 | €0.1424 | |
| 90% | €0.0015 | €0.0063 | €0.0170 | €0.0352 | €0.0596 | €0.0859 | €0.1129 | ≤ 25 €/kWh |
| 100% | €0.0000 | €0.0006 | €0.0043 | €0.0135 | €0.0312 | €0.0557 | €0.0833 | |

**Table 4.5:** Averaged daily revenue from feed-in subsidy using a lossy battery for different round-trip efficiency values for households without PV.

RTE pair gives enough incentive to invest in residential electrical batteries. We assume a five year lifetime with no battery degradation, and that the solar panels are already cost-neutral. Given the projected price for 2030, and no significant improvement in the battery technology itself, a subsidy of 50% or lower gives enough incentive to consumers to purchase energy storage. The expected revenue due to the battery will only increase over time as the subsidy level continues to drop.

#### Quadratic deviations

Finally, we consider the quadratic deviations objective $f_t(x_t) = (x_t - p_t)^2$, where $p_t$ as before. Once more, one week per season is simulated, where all days and houses are taken as isolated instances and the results are averaged for the houses with and without PV. We again consider the 42.2 kWh storage device. The change of the optimal objective value as compared to the lossless battery as function of the RTE is plotted in Figure 4.4. See Table 4.6 for a comparison of the objectives of the optimal solution and the repair and rescale methods. The averaged objective of the PV instances without battery is 19.1814, and 9.5188 for the instances without PV. Combining these statistics with Table 4.6 indicates that the use of a battery is roughly twice as effective at profile steering for households with PV.

On about 2% of the days, the repair and rescale methods gave a schedule that was worse than doing nothing at 90% RTE. Instead of executing that schedule, the battery did not operate on those days instead. Hence, roughly 7 days each year a household is not using it's storage device for load balancing. By itself, this is not a huge problem. However, the days on which this occurs across multiple households correlate. Simultaneous failure to perform load balancing may still cause overloading on those days. As the grid has to be dimensioned for such eventualities, this defeats the purpose of having batteries in the first place.

| | | Round-trip efficiency 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|
| PV | Optimum* | 14.4703 | 14.4060 | 14.3428 | 14.2807 | 14.2200 | 14.1605 | 14.1019 |
| | Repair | 15.0844 | 14.9091 | 14.7363 | 14.5683 | 14.4010 | 14.2436 | ” |
| | Rescale | 14.4829 | 14.4137 | 14.3471 | 14.2829 | 14.2208 | 14.1605 | ” |
| No PV | Optimum* | 7.70462 | 7.61521 | 7.52930 | 7.44685 | 7.36764 | 7.29144 | 7.2180 |
| | Repair | 7.97168 | 7.83508 | 7.70109 | 7.57274 | 7.44740 | 7.32739 | ” |
| | Rescale | 7.72209 | 7.62595 | 7.53552 | 7.45007 | 7.36898 | 7.29177 | ” |

**Table 4.6:** Averaged daily Euclidean norm using a lossy energy storage device.

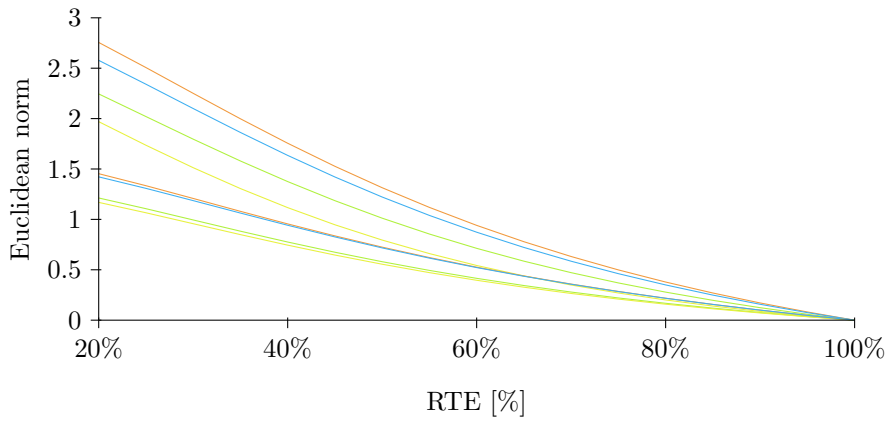



**Figure 4.4:** Changes in Euclidean norm of the net energy profile as function of round-trip efficiency. Green, yellow, orange, and blue correspond to simulations where the energy profiles are taken from days in spring, summer, autumn, and winter respectively. Results are averaged separately for houses with and without PV. The houses with PV suffer more due to inefficiencies in the energy storage device.

## 4.4 Conclusion

In this chapter we sought to answer the first two of our subquestions:

- *How important are losses in practice, and how relevant are they for optimization procedures?*
- *Are consumer-owned energy storage devices economically viable in the near future?*

We only considered Li-ion batteries in both cases.

From the obtained results we conclude that high energy conversion efficiencies (above 90%) are crucial for achieving desired arbitrage exploiting and peak shaving capabilities in order to make consumer-owned battery storage viable. The optimal revenue of the lossy case as compared to the lossless case quickly diminishes for both the linear pricing and the feed-in subsidy scenarios. Not properly accounting for these losses worsens the situation: using naive methods, such as the repair or rescale method, leads to significant financial losses of around 75% of the total revenue at 90% RTE and 30% at 95% RTE as compared to the optimal lossy solution in the linear pricing setting for spot market prices. For the day-night tariff, the financial losses are about 25% at 90% RTE and 5% at 95% RTE. The feed-in subsidy objective also deteriorates when using the repair or rescale method rather than the optimal solution, but slightly less so with around 15% financial losses at 90% RTE. For the quadratic deviations objective, the averaged objective deteriorates less strongly due to either losses or the repair/rescale method as compared to both pricing settings. However, the days on which the repair and rescale schedules (about 2% of the cases) are disadvantageous correlate across different households. The simultaneity of these events therefore still necessitates a method that incorporates the consideration of losses into the optimization process for this case.

In a profile steering approach, multiple instances of the storage problem have to be solved in a short amount of time, requiring us to be able to solve individual instances in the order of milliseconds. This makes the used linear programming models inadequate in practice. We conclude that a new method is called for that incorporates conversion losses into the optimization process.

To make residential electrical battery storage economically viable in the near future, a low feed-in subsidy is required. Low feed-in subsidies prompts owners of PV to use their own generated energy. The revenue due to the battery therefore mainly consists of saving costs by matching supply and demand over time. For lower feed-in subsidies the revenue due to the battery increases at the expense of the revenue due to PV. Therefore, unless PV prices drop at the same rate as the devaluation of PV due to the lowered feed-in subsidy, rooftop PV will have to be continued to be subsidized in the future. Table 4.5 indicates that for a feed-in subsidy below 50% of the energy price residential Li-ion batteries may be economically feasible from 2030.

# 5. Analysis

*In this chapter we derive methods for solving the problems presented in Section 3.2. We begin with stating the derivation of the solution method to the Lossless Storage Problem, and show that this derivation fails in deriving a general solution method for the Lossy Storage Problem, as it turns out to be $\mathcal{NP}$-hard in general. We show how optimal solutions can be computed for certain parameter choices in the objectives are derived using subdifferential calculus. For the general problem, we propose a novel heuristic approach, of which we assess the performance in a case study in Chapter 6.*

## 5.1 Lossless storage problem

In this section we discuss a polynomial time solution approach for Problem 3.1 (the lossless problem). The method we present was originally derived by van der Klauw [55]. In his thesis, the Lossless Storage Problem is referred to as the Battery Charging (BC) problem.

First note that we can always add a dummy time step to the end of any instance of the Lossless Storage Problem in which we can force the state of charge to a specific level $C$ without influencing the schedule in the rest of the time steps. To do so, we add a time step $T+1$ with the following properties:

$$\begin{aligned} f_t(x_{T+1}) &= 0, & & \\ SoC_{T+1}^{\min} &= C, & x_{T+1}^{\min} &= C - SoC_T^{\max}, \\ SoC_{T+1}^{\max} &= C, & x_{T+1}^{\max} &= C - SoC_T^{\min}. \end{aligned}$$

The optimal solution of the original problem can be obtained by discarding the charging decision of the dummy time step. In the rest of this section we only consider instances that have undergone this transformation, i.e. that have an equality constraint for the state of charge in the final time step. Equivalently this means we only consider instances where $SoC_T^{\min} = SoC_T^{\max} = C$.

Solving the Problem 3.1 relies on solving an intermediate problem, which corresponds to the Lossless Storage Problem where we have relaxed all but the last constraint (which is an equality). This intermediate problem is called the Electric Vehicle Charging (EVC) problem, because, when $x_t^{\min} \geq 0$, it models the problem of charging an EV battery to some capacity without allowing vehicle-to-grid (V2G) functionality. The EVC problem belongs to the class of resource allocation problems, of which an overview can be found in [30]. Formally, the EVC problem can be stated as follows:

**Problem 5.1.** ***Electric Vehicle Charging Problem:*** *Given an initial state of charge $SoC_0$, desired state of charge $C$, and (dis)charge bounds $x_t^{\min}$ and $x_t^{\max}$, the electric vehicle charging problem is:*

$$\begin{aligned} min_{\mathbf{x}} \quad & f(\mathbf{x}), \\ s.t. \quad & SoC_0 + \sum_{t=1}^{T} x_t = C \\ & x_t^{min} \leq x_t \leq x_t^{max} \qquad \forall t \in \mathcal{T}. \end{aligned}$$

There is a strong relation between the optimal solutions of instances of Problem 3.1 and Problem 5.1 if $SoC_T^{\min} = SoC_T^{\max} = C$ and their objective functions are equal, allowing us to solve Problem 3.1. Suppose we have such instances, and we have used some method to solve the EVC instance. If the EVC solution is a feasible solution to the BC instance, it must be an optimal solution to it. If it is not feasible, the convexity of the problem suggests that a local reparation may maintain optimality. Concretely: let $k$ be the time step where the maximal violation of state of charge bounds occurs, then there exists an optimal solution to the BC instance where the state of charge is exactly the bound at $k$. Lemma 5.1 proves this claim.

**Lemma 5.1.** *Consider an instance of Problem 3.1 with* $SoC_T^{\min} = SoC_T^{\max}$ *and let* $\mathbf{y}$ *be an optimal solution to the corresponding instance of Problem 5.1 obtained by ignoring the cumulative bounds for all indices except the last. Assume that* $\mathbf{y}$ *is not feasible for the considered instance of Problem 3.1 and let* $k$ *be the index at which the cumulative bound is maximally violated, i.e.* $k = \arg\max_t \left\{ \sum_{t'=1}^{t} y_{t'} - SoC_t^{\max}, SoC_t^{\min} - \sum_{t'=1}^{t} y_{t'} \right\}$. *Then, there exists an optimal solution* $\mathbf{x}$ *to the considered instance of Problem 3.1 such that, if* $\sum_{t=1}^{k} y_t > SoC_k^{\max}$ *then* $\sum_{t=1}^{k} x_t = SoC_k^{\max}$ *and, on the other hand, if* $\sum_{t=1}^{k} y_t < SoC_k^{\min}$ *then* $\sum_{t=1}^{k} x_t = SoC_k^{\min}$.

See Lemma 5.1 in [55] for the full proof. The proof uses the convexity of the objective.

Furthermore note that if we know the state of charge level at some time step, we can decouple the problem into two new EVC problems: one in which we constrain the last state of charge to be the given bound, and one in which we assume the initial state of charge is the given bound. These new problems can be solved in a similar manner.

This gives us a divide-and-conquer approach for Problem 3.1 relying on our ability to solve Problem 5.1. The full procedure is outlined in Algorithm 2. Its time complexity is $O(T(T + \mathcal{C}_{\text{EVC}}(T)))$, where $\mathcal{C}_{\text{EVC}}(T)$ is the time complexity of solving an instance of Problem 5.1 consisting of $T$ time steps using the procedure $optEVC$. By $\mathbf{f}_{t \to t'}$ we denote the vector $(f_t, \ldots, f_{t'})$ obtained by restricting $\mathbf{f}$ to the range of indices $t$ through $t'$. In the remainder of this section we describe a so-called water filling approach [9, p. 245] to solve Problem 5.1 with linear or quadratic objectives. For both types of objective, the approach runs in $O(T \log T)$ time.

**Algorithm 2** Lossless Storage Problem with $SoC_T^{\min} = SoC_T^{\max}$

```
function OPTBC(T, f, x^min, x^max, SoC^min, SoC^max, SoC_0)
    x = optEVC(T, f, x^min, x^max, C_T, SoC_0)
    if x is feasible then
        return x
    end if
    k ← arg max_k { Σ_{t=1}^k x_t − SoC_k^max, SoC_k^min − Σ_{t=1}^k x_t }
    if Σ_{t=1}^k x_t > SoC_k^max then
        v ← SoC_k^max
        SoC_k^min ← SoC_k^max
    else
        v ← SoC_k^min
        SoC_k^max ← SoC_k^min
    end if
    for t = k + 1, ..., T do
        SoC_t^min ← SoC_t^min − v
        SoC_t^max ← SoC_t^max − v
    end for
    x_{1→k} ← optBC(k, f_{1→k}, x^min_{1→k}, x^max_{1→k}, SoC^min_{1→k}, SoC^max_{1→k}, SoC_0)
    x_{k+1→T} ← optBC(T − k, f_{k+1→T}, x^min_{k+1→T}, x^max_{k+1→T}, SoC^min_{k+1→T}, SoC^max_{k+1→T}, v)
    return x
end function
```

**Waterfilling approach**

We use the Karush-Kuhn-Tucker (KKT) optimality conditions of Problem 5.1 to arrive at a solution procedure. The KKT conditions [9, p. 244] are necessary conditions for a solution of an optimization problem to be optimal. Consider the following general optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}), \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq 0, \quad \forall i = 1, \ldots, m, \\ & h_j(\mathbf{x}) = 0, \quad \forall j = 1, \ldots, l. \end{aligned} \tag{5.1}$$

The KKT conditions of (5.1) are as follows: if $f : \mathbb{R}^n \to \mathbb{R}$, $g_i : \mathbb{R}^n \to \mathbb{R}$, and $h_j : \mathbb{R}^n \to \mathbb{R}$ are continuously differentiable at a point $x^*$, and if $x^*$ is a local optimum of (5.1) and (5.1) satisfies some regularity conditions, then there exist KKT multipliers $\mu_i$, $i = 1, \ldots, m$, and $\lambda_j$, $j = 1, \ldots, l$, such that:

$$\begin{aligned} \nabla f(x^*) + \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^*) &= 0, \\ g_i(x^*) &\leq 0, \quad \forall i = 1, \ldots, m, \\ h_j(x^*) &= 0, \quad \forall j = 1, \ldots, l, \\ \mu_i &\geq 0, \quad \forall i = 1, \ldots, m, \\ \mu_i g_i(x^*) &= 0, \quad \forall i = 1, \ldots, m. \end{aligned}$$

Furthermore, when the Problem 5.1 is convex, i.e. when $f, g_1, \ldots, g_m$ are convex functions and $h_1, \ldots, h_l$ are affine functions, the KKT conditions are also sufficient for the solution to be optimal.

An example of the aforementioned regularity conditions is Slater's condition. It requires the existence of a strictly feasible point $\overline{\mathbf{x}}$ such that $g_i(\overline{\mathbf{x}}) < 0$ for all $i = 1, \ldots, m$, and $h_j(\overline{\mathbf{x}}) = 0$ for all $j = 1, \ldots, l$. This condition holds for every instance of Problem 5.1. Note that having $\sum_{t \in \mathcal{T}} x_t^{\min} = C$ or $\sum_{t \in \mathcal{T}} x_t^{\max} = C$ trivializes Problem 5.4, as there exists only a single feasible solution in either case, consisting of fully discharging or fully charging respectively. We can therefore assume that $\sum_{t \in \mathcal{T}} x_t^{\min} < C < \sum_{t \in \mathcal{T}} x_t^{\max}$ holds, implying the existence of a strictly feasible point $\overline{\mathbf{x}}$:

$$\overline{x}_t = x_t^{\min} + \frac{C - \sum_{t' \in \mathcal{T}} x_{t'}^{\min}}{\sum_{t' \in \mathcal{T}} \left( x_{t'}^{\max} - x_{t'}^{\min} \right)} \left( x_t^{\max} - x_t^{\min} \right), \quad \forall t \in \mathcal{T}. \tag{5.2}$$

It follows that Slater's condition holds. This regularity condition holds for every instance of Problem 5.1. Therefore, we can readily use the KKT conditions to determine solutions for Problem 5.1.

Assume the $f_t$ functions to be strictly convex, which implies that their derivatives $f_t'$ are continuous, strictly increasing, and invertible. The KKT conditions of Problem 5.1 allow us to explicitly derive the optimal $x_t$. A feasible solution $\mathbf{x}$ to Problem 5.1 is optimal if and only if there exist multipliers $\lambda, \mu_1, \ldots, \mu_T$ such that:

$$\begin{aligned} f_t'(x_t) + \mu_t &= \lambda \quad \forall t \in \mathcal{T}, \\ \mu_t^+ (x_t - x_t^{\max}) &= 0 \quad \forall t \in \mathcal{T}, \\ \mu_t^- (x_t^{\min} - x_t) &= 0 \quad \forall t \in \mathcal{T}, \end{aligned} \tag{5.3}$$

Here, we have combined the multipliers of the box constraints of $x_t$ into one multiplier. From (5.3) it directly follows that:

$$\begin{aligned} \lambda < f_t'(x_t^{\min}) &\Rightarrow \mu_t < 0 \Rightarrow x_t = x_t^{\min}, \\ \lambda > f_t'(x_t^{\max}) &\Rightarrow \mu_t > 0 \Rightarrow x_t = x_t^{\max}, \\ \lambda = f_t'(x_t) &\Rightarrow \mu_t = 0 \Rightarrow x_t = (f_t')^{-1}(\lambda). \end{aligned} \tag{5.4}$$

Hence, we can write $x_t$ as a function $\lambda$: $x_t = x_t(\lambda)$. To ensure feasibility, we only still require that $\sum_{t=1}^{T} x_t(\lambda) = C$ holds. Because we assumed $f_t'$ to be strictly increasing, $x_t(\lambda)$ will be non-decreasing in

$\lambda$. The result is that we can gradually increase $\lambda$ until the constraint is satisfied, at which point we have obtained the optimal solution according to the KKT conditions.

Consider the quadratic objective $f_t(x_t) = \frac{1}{2}a_t x_t^2 + b_t x_t + c_t$ where $a_t > 0$, then (5.4) prescribes the following expression for $x_t(\lambda)$:

$$x_t(\lambda) = \begin{cases} x_t^{\min} & \text{if } \lambda < a_t x_t^{\min} + b_t, \\ \frac{\lambda - b_t}{a_t} & \text{if } a_t x_t^{\min} + b_t \leq \lambda \leq a_t x_t^{\max} + b_t, \\ x_t^{\max} & \text{if } \lambda > a_t x_t^{\max} + b_t. \end{cases} \tag{5.5}$$

Essentially, this equation states that every $\lambda$ is in some sense a "fill-level" of a series of $T$ basins, where every basin $t$ relates to the gradient of the corresponding $f_t$, and the bounds $x_t^{\min}$ and $x_t^{\max}$. This interpretation explains why this method is often referred to as water filling or valley filling. See Figure 5.1.

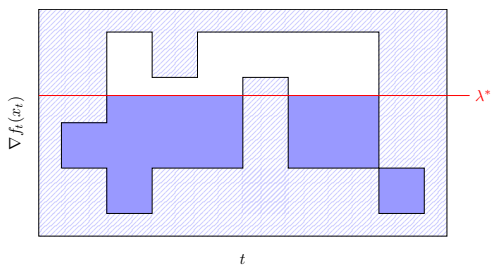



**Figure 5.1:** Water filling approach for the case of quadratic objective functions.

The case where $f_t(x_t) = a_t x_t + b_t$ can be solved in a similar fashion, except that each basin shrinks down to a point. This implies that the $x_t$ should be "filled" in order of increasing $a_t$. Specifically, for any optimal $\mathbf{x}^*$ if $a_t < a_{t'}$, then $x_t^* < x_t^{\max}$ implies $x_{t'}^* = x_{t'}^{\min}$. Adding a dummy node, where $f_t(x_t) = 0$, can be seen as adding a time step with linear objective where $a_t = b_t = 0$. As the dummy node is always capable of meeting the desired state of charge $C$ the fill level should never exceed zero in the presence of a dummy node.

For both types of objectives, the water filling approach relies on iterating through the endpoints of the basins in order. Thus, the running time is dominated by sorting these points, which can be done in $O(T \log T)$ time. Algorithm 3 outlines the procedure when considering quadratic objectives. It appears as Algorithm 4.1 in [55]. It is a primal-dual method [19], as both primal and dual variables are simultaneously estimated.

An $O(T)$ time algorithm exists based on median search [26]. However, this approach is inefficient for the relatively small values of $T$ typically encountered in DEM.

#### Relaxation of the lossy problem

In this section we show that the previous approach generally no longer works for the lossy problem. We relax the storage charging problem to a corresponding EV charging problem (with only an equality constraint at the end), and show that the relaxed solution does not necessarily correspond to the optimal solution in the original problem for both the linear pricing and quadratic deviations objectives. Consider a lossy energy storage charging problem with $T = 2$, and the following choice of parameters (where $\lambda \in [0, 1]$):

$$\begin{aligned} \mathbf{x}^{\min} &= [-1, -1], & \mathbf{x}^{\max} &= [\ \ 1, \ \ 1], \\ \mathbf{SoC}^{\min} &= [-1, \ \ 0], & \mathbf{SoC}^{\max} &= [\ \ \lambda, \ \ 0], \\ \eta_c &= \tfrac{1}{2}, & \eta_d &= 2. \end{aligned}$$

First, we consider the linear pricing objective $f_t(x_t) = c_t x_t$, where $c_1 = -2$ and $c_2 = -1$. The optimal relaxed solution charges maximally on the first time step and discharges maximally on the second time step. Therefore, if Lemma 5.1 would extend to the lossy problem, we would be led to believe that $x_1 = \lambda$ and

**Algorithm 3** Water filling Approach for Problem 5.1 with $f_t(x_t) = \frac{1}{2}a_t x_t^2 + b_t x_t + c_t$

```
function OPTEVC(T, f, x^min, x^max, C, SoC_0)
    A ← {a_1 x_1^min + b_1, ... a_T x_T^min + b_T}
    B ← {a_1 x_1^max + b_1, ... a_T x_T^max + b_T}
    Sort A and B in non-decreasing order
    Take α as the first value from A with associated interval t
    A ← A \ {α}, λ̃ ← α, Ĉ ← Σ_{t∈T} x_t^min, η ← 1/a_t
    while Ĉ < C do
        Take α as the first element from A with associated time interval t
        Take β as the first element from B with associated time interval t'
        γ ← min {α, β}, Δ ← min {γ − λ̃, (C − Ĉ)/η}
        λ̃ ← λ̃ + Δ, Ĉ ← Ĉ + Δη
        if Ĉ < C and α = γ then
            A ← A \ {α}
            η ← η + 1/a_t
        else if Ĉ < C and β = γ then
            B ← B \ {β}
            η ← η − 1/a_t'
        end if
    end while
    return x(λ̃) using (5.5)
end function
```

$x_2 = -\lambda$ is optimal. This solution gives an objective value of $-\frac{7}{2}\lambda$. However, the solution $x_1 = -1$ and $x_2 = 1$ has an objective value of $-1$, which is smaller when $\lambda < \frac{2}{7}$.

Next, we consider the quadratic deviations objective $f_t(x_t) = (x_t - p_t)^2$, where $p_1 = 6$ and $p_2 = 5$. We assume the formulation of Problem 3.2 where the losses occur inside the constraints. We can find the optimal solution geometrically, as the level curves of the given objective are concentric circles centered around $(6, 5)$, see Figure 5.2. Again, if Lemma 5.1 would extend to the lossy problem, we would be led to believe that $x_1 = \lambda$ and $x_2 = -\frac{\lambda}{4}$ is optimal. However, the solution $x_1 = -\frac{1}{4}$ and $x_2 = 1$ outperforms it when $\lambda < \frac{1}{34}\left(76 - \sqrt{4161}\right)$.

Hence, Lemma 5.1 fails in general for the lossy storage problem. This is due to the fact that the objectives $(f_t \circ T_\eta)$ are generally no longer convex. In Section 5.3.1 we isolate cases that can still be solved in a similar manner. In the next section, we show that the problem is $\mathcal{NP}$-hard in general.

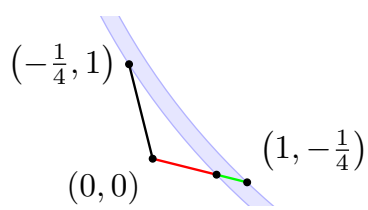



**Figure 5.2:** Hitting level curves over the feasible region of the relaxed problem. For some values of $\lambda$ the solution $[\lambda, -\lambda/4]$ obtained from the relaxation performs worse than $[-\frac{1}{4}, 1]$ (red), while in others it performs better (green).

## 5.2 $\mathcal{NP}$-completeness proof

In this section we consider the decision version of Problem 3.3, Problem 5.2, which involves deciding whether we can find a feasible solution to Problem 3.3 which has an objective value no greater than some threshold, and we show that it is $\mathcal{NP}$-complete. We consider the linear pricing objective with constant negative prices. Evidently, Problem 5.2 is in $\mathcal{NP}$: checking whether some certificate $\overline{\mathbf{x}}$ solves an instance can be done in linear time. We show by a reduction from the Subset Sum Problem (Problem 5.3) that the problem is $\mathcal{NP}$-hard.

**Problem 5.2.** ***Decision version of Lossy Storage Problem:*** *Given a time horizon $\mathcal{T} = \{1, \ldots, T\}$, efficiency rates $0 < \eta_c < 1 < \eta_d$, a threshold $\tau$, box constraints $x_t^{\min}$, $x_t^{\max}$, initial state of charge $SoC_0$, and cumulative bounds $SoC_t^{\min}$ and $SoC_t^{\max}$ for $t \in \mathcal{T}$, can we find $x_t$ for $t \in \mathcal{T}$ such that it is feasible with respect to the given bounds:*

$$SoC_t^{\min} \leq SoC_0 + \sum_{t'=1}^{t} x_{t'} \leq SoC_t^{\max} \qquad \forall t \in \mathcal{T},$$
$$x_t^{min} \leq x_t \leq x_t^{max} \qquad \forall t \in \mathcal{T},$$

*and the total objective does not exceed the threshold $\tau$ (where $f_t(x_t) = c_t x_t$ and $T_\eta(x_t) = \frac{1}{\eta_c} x_t^+ + \frac{1}{\eta_d} x_t^-$): $\sum_{t \in \mathcal{T}} (f_t \circ T_\eta)(x_t) \leq \tau$?*

**Problem 5.3.** ***Subset Sum Problem:*** *Given a set of integers $S = \{\alpha_1, \ldots, \alpha_n\}$ with $0 < \alpha_1 \leq \ldots \leq \alpha_n$ and a goal $G$, can we find a subset $\mathcal{I} \subseteq \{1, \ldots, n\}$ such that $\sum_{t \in \mathcal{I}} \alpha_t = G$?*

Let an instance `Subset` of Problem 5.3 be given with integers $S = \{\alpha_1, \ldots, \alpha_n\}$ and goal $G$, then we can transform it to an instance `Storage` of Problem 5.2 with time horizon $T = n$ and the following parameters:

$$x_t^{\min} = -\alpha_t, \qquad x_t^{\max} = \alpha_t \qquad t = 1, \ldots, n,$$
$$SoC_t^{\min} = -\sum_{t=1}^{n} \alpha_t, \qquad SoC_t^{\max} = \sum_{t=1}^{n} \alpha_t \qquad t = 1, \ldots, n-1,$$
$$SoC_0 = 0, \qquad SoC_n^{\min} = SoC_n^{\max} = \sum_{t=1}^{n} \alpha_t - 2G,$$
$$\tau = \sum_{t=1}^{n} \frac{c_t}{\eta_c} \alpha_t + G \qquad c_t = -\frac{\eta_c \eta_d}{\eta_c + \eta_d} \qquad t = 1, \ldots, n.$$

Note that this transformation is possible in polynomial-time: all parameters (of which there are a linear amount) can be determined in linear time. It only remains to prove that it is a reduction, i.e. that each YES or NO-instance of Problem 5.3 respectively transforms into a YES or NO-instance of Problem 5.2. We will prove this in the rest of this section, but first we reflect on the connection between these two problems.

For instances with constant negative prices it turns out to be optimal to maximally charge and discharge on each time step. Assume we have some optimal solution $\mathbf{x}$ where we *do not* maximally charge and discharge, such that there are time steps $t$ and $t'$ where: $x_t \in (0, x_t^{\max})$ and $x_{t'} \in (x_{t'}^{\min}, 0)$. Then, we can construct an improving solution $\mathbf{y}$, equal to $\mathbf{x}$ on all time steps except $t$ and $t'$, that has lower objective than $\mathbf{x}$. Take $\delta = \min\left\{x_t^{\max} - x_t, x_{t'} - x_{t'}^{\min}\right\}$, then if we take $y_t = x_t + \delta$, and $y_{t'} = x_{t'} - \delta$ we have found a feasible solution that improves upon the objective of $\mathbf{x}$ by:

$$\begin{aligned}\frac{1}{\eta_c} c_t (y_t - x_t) - \frac{1}{\eta_d} c_{t'} (y_{t'} - x_{t'}) &= \frac{1}{\eta_c} c_t \delta - \frac{1}{\eta_d} c_{t'} \delta \\ &= \left(\frac{1}{\eta_c} - \frac{1}{\eta_d}\right) c_t \delta < 0,\end{aligned}$$

contradicting the optimality of $\mathbf{x}$. Therefore, it is always optimal to maximally charge and discharge on each time step. The 'hardness' of the problem comes from deciding on which interval to charge and discharge to

begin with. As will shortly become clear, the reduction was chosen such that for a certificate $\mathcal{I}$ of `Subset` the corresponding certificate $\mathbf{x}$ of `Storage` will maximally discharge for every interval $t \in \mathcal{I}$ and otherwise maximally charge.

We will first prove that if `Subset` is a YES-instance for Problem 5.3, then the corresponding `Storage` instance is a YES-instance for Problem 5.2.

**Lemma 5.2.** *If `Subset` is a YES-instance for Problem 5.3, then `Storage` is a YES-instance for Problem 5.2.*

*Proof.* Suppose that `Subset` is a YES-instance, then there exists some $\mathcal{I} \subseteq \{1, \dots, n\}$ such that $\sum_{t \in \mathcal{I}} \alpha_t = G$. We will show that the following solution to `Storage` is feasible with respect to the box and state of charge constraints, and that the threshold $\tau$ is met:

$$\begin{aligned} x_t &= x_t^{\min} = -\alpha_t && t \in \mathcal{I}, \\ x_t &= x_t^{\max} = \alpha_t && t \notin \mathcal{I}. \end{aligned}$$

By construction, $x_t$ satisfies its box constraints for all $t \in \mathcal{T}$. They also satisfy their state of charge constraints for all $t \in \mathcal{T}$:

$$\begin{aligned} SoC_t &\geq -\sum_{k=1}^{t} \alpha_k \geq -\sum_{k=1}^{n} \alpha_k = SoC_k^{\min} && t = 1, \dots n-1, \\ SoC_t &\leq \sum_{k=1}^{t} \alpha_k \leq \sum_{k=1}^{n} \alpha_k = SoC_k^{\max} && t = 1, \dots, n-1, \\ SoC_n &= \sum_{t \notin I} \alpha_t - \sum_{t \in I} \alpha_t = \sum_{t=1}^{n} \alpha_t - 2 \sum_{t \in I} \alpha_t = SoC_n^{\min} = SoC_n^{\max}. \end{aligned}$$

It only remains to show that the threshold $\tau$ is met:

$$\begin{aligned} \sum_{t=1}^{n} \left(f_t \circ T_\eta\right)(x_t) &= \sum_{t \notin \mathcal{I}} \frac{c_t}{\eta_c} \alpha_t - \sum_{t \in \mathcal{I}} \frac{c_t}{\eta_d} \alpha_t \\ &= \sum_{t=1}^{n} \frac{c_t}{\eta_c} \alpha_t - \sum_{t \in \mathcal{I}} \left( \frac{1}{\eta_c} + \frac{1}{\eta_d} \right) c_t \alpha_t \\ &= \sum_{t=1}^{n} \frac{c_t}{\eta_c} \alpha_t + \sum_{t \in \mathcal{I}} \alpha_t = \sum_{t=1}^{n} \frac{c_t}{\eta_c} \alpha_t + G = \tau. \end{aligned}$$

Hence, `Storage` is a YES-instance.

□

To be able to prove the converse we require Lemma 5.3, which gives a linear lower bound on the objective at each time step. The bound is obtained from the linear approximation of $\left(f_t \circ T_\eta\right)(x_t)$ through $x_t = -\alpha_t$ and $x_t = \alpha_t$. We will later show that this linear approximation summed over $t \in \mathcal{T}$ equals $\tau$, implying infeasibility of the solution if the inequality is strict on any time interval. Therefore, any certificate of `Storage` being a YES-instance must have $x_t = -s_t$ or $x_t = s_t$ for each $t \in \mathcal{T}$.

**Lemma 5.3.** *For $t = 1, \ldots, n$ we have that: $(f_t \circ T_\eta)(x_t) \geq -\frac{1}{2}x_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t$. This inequality is strict when $x_t \in (-\alpha_t, \alpha_t)$, and is met with equality when $x_t = -\alpha_t$ or $x_t = \alpha_t$.*

*Proof.* We verify the lemma for the points $x_t = -\alpha_t, 0, \alpha_t$, and use (piecewise) linearity of both sides of the inequality to prove the remaining cases when $x_t \in (-\alpha_t, 0)$ or $x_t \in (0, \alpha_t)$.

- First, we consider the case where $x_t = -\alpha_t$:

$$\begin{aligned}
(f_t \circ T_\eta)(x_t) &= -\frac{c_t}{\eta_d}\alpha_t \\
&= \frac{\eta_c}{\eta_c + \eta_d}\alpha_t \\
&= -\frac{1}{2}\left(\frac{\eta_d - \eta_c}{\eta_c + \eta_d} - 1\right)\alpha_t \\
&= \frac{1}{2}\alpha_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t \\
&= -\frac{1}{2}x_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t
\end{aligned}$$

- Next, we consider the case where $x_t = \alpha_t$:

$$\begin{aligned}
(f_t \circ T_\eta)(x_t) &= \frac{c_t}{\eta_c}\alpha_t \\
&= -\frac{\eta_d}{\eta_c + \eta_d}\alpha_t \\
&= -\frac{1}{2}\left(\frac{\eta_d - \eta_c}{\eta_c + \eta_d} + 1\right)\alpha_t \\
&= -\frac{1}{2}\alpha_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t \\
&= -\frac{1}{2}x_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t
\end{aligned}$$

- Finally, we consider the case where $x_t = 0$:

$$(f_t \circ T_\eta)(x_t) = 0 > -\frac{1}{2}x_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t$$

Since $(f_t \circ T_\eta)(x_t)$ is linear with slope $\frac{c_t}{\eta_c}$ for $x_t \geq 0$, it follows that any point from $(0, \alpha_t)$, i.e. $x_t = \lambda s_t$ for $0 < \lambda < 1$, will be met with strict inequality:

$$\begin{aligned}
(f_t \circ T_\eta)(x_t) &= (f_t \circ T_\eta)((1-\lambda)0 + \lambda\alpha_t) \\
&= (1-\lambda)(f_t \circ T_\eta)(0) + \lambda(f_t \circ T_\eta)(\alpha_t) \\
&> -\frac{1}{2}\lambda\alpha_t - \frac{1}{2}\frac{\eta_d - \eta_c}{\eta_c + \eta_d}\alpha_t.
\end{aligned}$$

The case where $x_t \in (-\alpha_t, 0)$ follows analogously by linearity of $(f_t \circ T_\eta)$ for $x_t \leq 0$.

□

**Lemma 5.4.** *If* `Subset` *is a NO-instance for Problem 5.3, then* `Storage` *is a NO-instance for Problem 5.2.*

*Proof.* Given that `Subset` is a NO-instance, there does not exist an $\mathcal{I} \subseteq \{1, \ldots, n\}$ such that $\sum_{t \in \mathcal{I}} \alpha_t = G$. Assume `Storage` is a YES-instance with certificate $\mathbf{x}$. By summing over $t = 1, \ldots, n$ on both sides of Lemma 5.3, we obtain the following:

$$\begin{aligned}
\sum_{t=1}^{n} (f_t \circ T_\eta)(x_t) &\geq -\frac{1}{2} \sum_{t=1}^{n} x_t - \frac{1}{2} \frac{\eta_d - \eta_c}{\eta_c + \eta_d} \sum_{t=1}^{n} \alpha_t \\
&= -\frac{1}{2} \left( \sum_{t=1}^{n} \alpha_t - 2G \right) - \frac{1}{2} \frac{\eta_d - \eta_c}{\eta_c + \eta_d} \sum_{t=1}^{n} \alpha_t \\
&= G - \frac{1}{2} \left( \frac{\eta_d - \eta_c}{\eta_c + \eta_d} + 1 \right) \sum_{t=1}^{n} \alpha_t \\
&= G + \sum_{t=1}^{n} \frac{c_t}{\eta_c} \alpha_t = \tau,
\end{aligned} \tag{5.6}$$

where the first equality follows from the $n$th state of charge constraint and feasibility of $\mathbf{x}$. It follows from (5.6) that if any of the summed inequalities for $t = 1, \ldots, n$ is strict, the threshold is not met, implying that $\mathbf{x}$ is not a solution to `Storage`. Therefore, again by Lemma 5.3, either $x_t = -\alpha_t$ or $x_t = \alpha_t$ for every $t = 1, \ldots, n$. Define $\overline{\mathcal{I}} = \{ t \in \mathcal{T} \mid x_t = -\alpha_t \}$, then we get from the $n$th state of charge constraint that:

$$\begin{aligned}
\sum_{t=1}^{n} \alpha_t - 2G = \sum_{t=1}^{n} x_t &= \sum_{t \notin \overline{\mathcal{I}}} \alpha_t - \sum_{t \in \overline{\mathcal{I}}} \alpha_t \\
&= \sum_{t=1}^{n} \alpha_t - 2 \sum_{t \in \overline{\mathcal{I}}} \alpha_t.
\end{aligned}$$

We have found an $\overline{\mathcal{I}}$ such that $\sum_{t \in \overline{\mathcal{I}}} \alpha_t = G$, contradicting the assumption that `Subset` is a NO-instance. The assumption that `Storage` is a YES-instance was wrong: `Storage` is a NO-instance. □

Theorem 5.1 combines the preceding lemmas to state that Problem 5.2 is $\mathcal{NP}$-complete. By noting that the state of charge bounds on the first $n-1$ time steps can never be exceeded, and the last bound is an equality, it also follows that the "Lossy Electric Vehicle Charging Problem" (see Problem 5.4 from the next section) is $\mathcal{NP}$-complete as well by the same reduction.

**Theorem 5.1.** *Problem 5.2 is $\mathcal{NP}$-complete (for constant negative costs).*

*Proof.* Lemmas 5.2 and 5.4 confirm that the given transformation is a polynomial-time reduction. The theorem now follows directly from the $\mathcal{NP}$-completeness of Problem 5.3, and because Problem 5.2 is in $\mathcal{NP}$. □

Due to a similar reduction by van der Klauw [56], instances of Problem 5.3 can also be reduced to instances of Problem 5.2 where $x_t^{\min}$, $SoC_t^{\min}$, and $SoC_t^{\max}$ are taken constant for all $t \in \mathcal{T}$, but where costs $c_t$ and

$x_t^{\max}$ may vary. For a given instance `Subset` of Problem 5.3 with integers $S = \{\alpha_1, \ldots, \alpha_n\}$ and goal $G$, he constructs an instance `Storage'` of Problem 5.2 with $T = n + 1$ in polynomial time with the following parameters:

$$
\begin{aligned}
x_t^{\min} &= -\alpha, & x_t^{\max} &= \alpha_t - \alpha, & t &= 1, \ldots, n, \\
x_{n+1}^{\min} &= -\alpha & x_{n+1}^{\max} &= G, & & \\
SoC_t^{\min} &= 0, & SoC_t^{\max} &= \sum_{t=1}^{n} \alpha_t, & t &= 1, \ldots, n+1, \\
SoC_0 &= n\alpha, & c_t &= -\frac{\eta_c \eta_d \alpha_t}{\eta_c \alpha + \eta_d(\alpha_t - \alpha)}, & t &= 1, \ldots, n, \\
c_{n+1} &< -\eta_c, & \tau &= \sum_{t=1}^{n} \frac{c_t}{\eta_c}(\alpha_t - \alpha) + \left(1 + \frac{c_{n+1}}{\eta_c}\right) G, & &
\end{aligned}
$$

where $0 < \alpha < \alpha_1$.

## 5.3 Tractable lossy instances

In the previous section we presented an $\mathcal{NP}$-completeness proof for the decision version of Problem 3.3. Despite the problem being hard in general, there are certain parameter choices that induce optimally solvable instances. In this section we extend the approach of Section 5.1 to instances of Problem 3.3 where the parameters ensure that convexity is maintained. We will take an analogous approach by considering the following intermediate problem:

**Problem 5.4.** ***Lossy Electric Vehicle Charging Problem:*** *Given an initial state of charge $SoC_0$, conversion rates $0 < \eta_c \leq 1 \leq \eta_d$, desired state of charge $C$, and (dis)charge bounds $x_t^{\min}$ and $x_t^{\max}$, the lossy electric vehicle charging problem is:*

$$
\begin{aligned}
min_{\tilde{\mathbf{x}}} \quad & \tilde{f}(\tilde{\mathbf{x}}) = f\left(\tfrac{1}{\eta_c}\tilde{\mathbf{x}}^+ + \tfrac{1}{\eta_d}\tilde{\mathbf{x}}^-\right), \\
s.t. \quad & SoC_0 + \sum_{t=1}^{T} \tilde{x}_t = C \\
& x_t^{min} \leq \tilde{x}_t \leq x_t^{max} \qquad \forall t \in \mathcal{T}.
\end{aligned}
$$

### 5.3.1 Conditions for convexity

As long as $\tilde{f}$ is convex, Lemma 5.1 holds, relating the optimal solutions of instances of Problem 3.3 and Problem 5.4 where $SoC_T^{\min} = SoC_T^{\max} = C$. Recall that from the separability of $\tilde{f}(\tilde{x}) = \sum_{t \in \mathcal{T}} (f_t \circ T_\eta)(x_t)$ where $T_\eta(x_t) = \frac{1}{\eta_c} x_t^+ + \frac{1}{\eta_d} x_t^-$, such that the convexity of the composed objectives $(f_t \circ T_\eta)$ for all $t \in \mathcal{T}$ implies that $\tilde{f}$ is convex. However, although $f_t$ and $T_\eta$ are both convex (by assumption and because $\eta_c < \eta_d$ respectively), their composition is not necessarily convex. The following result from convex analysis imposes a condition on $f_t$ that ensures convexity of $(f_t \circ T_\eta)$:

**Lemma 5.5.** *For any two functions $f$ and $g$, where $f$ is convex and non-decreasing, and $g$ is convex, their composition $(f \circ g)$ is convex.*

*Proof.* We prove convexity of $(f \circ g)$ directly through its definition. For any $\lambda$ with $0 \leq \lambda \leq 1$ it holds that:

$$\begin{aligned}(f \circ g)(\lambda x + (1-\lambda) y) &= f(g(\lambda x + (1-\lambda) y)) \\ &\leq f(\lambda g(x) + (1-\lambda) g(y)) \\ &\leq \lambda f(g(x)) + (1-\lambda) f(g(y)) \\ &= \lambda (f \circ g)(x) + (1-\lambda)(f \circ g)(y),\end{aligned}$$

where we have used that $g$ is convex and $f$ is non-decreasing in the first step:

$$\begin{aligned} g(\lambda x + (1-\lambda) y) &\leq \lambda g(x) + (1-\lambda) g(y) \\ f(g(\lambda x + (1-\lambda) y)) &\leq f(\lambda g(x) + (1-\lambda) g(y)),\end{aligned}$$

and the convexity of $f$ in the second step. □

The $f_t$ functions are generally not non-decreasing, thus $(f_t \circ T_\eta)$ (and in particular also $\tilde{f}$) is not guaranteed to be convex. Indeed, they turn out to be non-convex for certain relevant parameter choices. Hence, we now isolate conditions for convexity of three typical objectives that arise in DEM. These objectives are:

- linear pricing: $f_t(x_t) = c_t x_t$;
- feed-in subsidy: $f_t(x_t) = c_t (x_t - p_t)^+ + s_t (x_t - p_t)^-$, where $s_t \leq c_t$;
- quadratic deviations: $f_t(x_t) = (x_t - p_t)^2$.

**Linear pricing**

We first consider the linear objective $f_t(x_t) = c_t x_t$, where $c_t$ is the unit cost of energy at time $t$. Lemma 5.5 applied to this linear objective implies that $(f_t \circ T_\eta)$ is convex when the prices are non-negative. Conversely, we show that $(f_t \circ T_\eta)$ becomes non-convex when $c_t$ is negative. Negative prices – although rare – do occur in the real world to incentivize energy consumption when the excess of energy can be potentially harmful to the grid. This has happened on intraday markets across several countries: Germany, U.S., Australia, Switzerland, Belgium, France [4, 7]. The composed objective $(f_t \circ T_\eta)$ is a continuous piecewise linear function, where the two linear pieces meet at the origin:

$$(f_t \circ T_\eta)(x_t) = \begin{cases} \frac{1}{\eta_d} a_t x_t & \text{if } x_t < 0, \\ \frac{1}{\eta_c} a_t x_t & \text{if } x_t \geq 0. \end{cases}$$

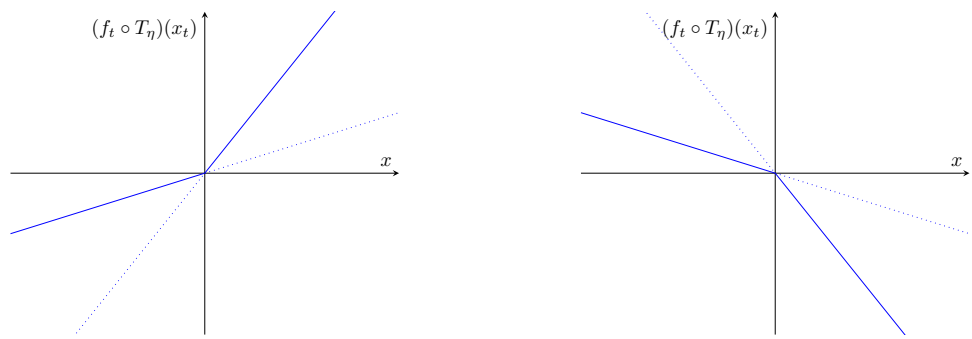
Individually the pieces are convex, so we are left to consider pairs of points that do not lie in the same halfspace. In particular, consider the convex combination of some $x_t > 0$ and $-x_t$. The objective function is non-convex when the averaged objective value of $x_t$ and $-x_t$ is less than the objective value at the origin. Respectively, the values at $x_t$ and $-x_t$ are: $\frac{1}{2}\frac{1}{\eta_c} c_t x_t$ and $-\frac{1}{2}\frac{1}{\eta_d} c_t x_t$. Therefore, the objective becomes non-convex whenever:

$$\frac{1}{2} \underbrace{\left(\frac{1}{\eta_c} - \frac{1}{\eta_d}\right)}_{>0} c_t x_t < 0 \Rightarrow c_t < 0.$$

Thus, $(f_t \circ T_\eta)$ is non-convex for negative prices $c_t$. See also Figures 5.3a, and 5.3b.

**Feed-in subsidy**

Next, we consider the feed-in subsidy objective $f_t(x_t) = c_t (x_t - p_t)^+ + s_t (x_t - p_t)^-$, where $c_t$ is the unit cost of energy at time $t$, $s_t$ is the subsidy at time $t$, and $p_t$ is the net energy profile of the considered household(s)




**(a)** Linear pricing with positive price **(b)** Linear pricing with negative price

**Figure 5.3:** The linear pricing objective remains convex for positive prices, and becomes non-convex for negative prices.

at time $t$. By expanding all branches of $f_t$ it follows that the composed objective $(f_t \circ T_\eta)$ can be written as a piecewise linear function. Consider the case where $x_t \geq 0$, and $x_t \geq p_t\eta_c$, then:

$$\begin{aligned}(f_t \circ T_\eta)(x_t) &= c_t\left(\tfrac{1}{\eta_c}x_t^+ + \tfrac{1}{\eta_d}x_t^- - p_t\right)^+ + s_t\left(\tfrac{1}{\eta_c}x_t^+ + \tfrac{1}{\eta_d}x_t^- - p_t\right)^- \qquad (x_t \geq 0, x_t \geq p_t\eta_c)\\ &= c_t\left(\tfrac{1}{\eta_c}x_t - p_t\right)^+ + s_t\left(\tfrac{1}{\eta_c}x_t - p_t\right)^-\\ &= \tfrac{c_t}{\eta_c}\left(x_t - p_t\eta_c\right).\end{aligned}$$

Similarly, expanding the other branches, we get that:

$$(f_t \circ T_\eta)(x_t) = \begin{cases} \frac{c_t}{\eta_c}\left(x_t - p_t\eta_c\right) & \text{when } x_t \geq 0, x_t \geq p_t\eta_c,\\ \frac{s_t}{\eta_c}\left(x_t - p_t\eta_c\right) & \text{when } x_t \geq 0, x_t < p_t\eta_c,\\ \frac{c_t}{\eta_d}\left(x_t - p_t\eta_d\right) & \text{when } x_t < 0, x_t \geq p_t\eta_c,\\ \frac{s_t}{\eta_d}\left(x_t - p_t\eta_d\right) & \text{when } x_t < 0, x_t < p_t\eta_c.\end{cases}$$

The above expression contains empty ranges: if $p_t$ is non-negative the third case can not happen. Similarly, if $p_t$ is negative the second case can not happen. This case distinction allows us to rewrite the expression as:

$$(f_t \circ T_\eta)(x_t) = \begin{cases} \frac{s_t}{\eta_d}\left(x_t - p_t\eta_d\right) & \text{when } x_t < 0,\\ \frac{s_t}{\eta_c}\left(x_t - p_t\eta_c\right) & \text{when } 0 \leq x_t < p_t\eta_c,\\ \frac{c_t}{\eta_c}\left(x_t - p_t\eta_c\right) & \text{when } p_t\eta_c \leq x_t,\end{cases} \qquad (p_t \geq 0)$$

$$(f_t \circ T_\eta)(x_t) = \begin{cases} \frac{s_t}{\eta_d}\left(x_t - p_t\eta_d\right) & \text{when } x_t < p_t\eta_d,\\ \frac{c_t}{\eta_d}\left(x_t - p_t\eta_d\right) & \text{when } p_t\eta_d \leq x_t < 0,\\ \frac{c_t}{\eta_c}\left(x_t - p_t\eta_c\right) & \text{when } 0 \leq x_t.\end{cases} \qquad (p_t < 0)$$

As these expressions are both continuous piecewise linear functions, they are convex if and only if the slopes in consecutive ranges form non-decreasing sequences, i.e. when:

$$\begin{aligned} p_t > 0 &\Rightarrow \tfrac{s_t}{\eta_d} \leq \tfrac{s_t}{\eta_c} \leq \tfrac{c_t}{\eta_c} \Leftrightarrow s_t \leq c_t \text{ and } s_t \geq 0,\\ p_t < 0 &\Rightarrow \tfrac{s_t}{\eta_d} \leq \tfrac{c_t}{\eta_d} \leq \tfrac{c_t}{\eta_c} \Leftrightarrow s_t \leq c_t \text{ and } c_t \geq 0,\\ p_t = 0 &\Rightarrow \tfrac{s_t}{\eta_d} \leq \tfrac{c_t}{\eta_c} \qquad\quad \Leftrightarrow s_t\eta_c \leq c_t\eta_d.\end{aligned}$$

A sufficient condition for the above is: $0 \leq s_t \leq c_t$ for all $t \in \mathcal{T}$, i.e. that both the subsidies and prices on each time step are positive. The objective functions $(f_t \circ T_\eta)(x_t)$ look similar to those of the linear objective, but are shifted and have additional point where the slope changes.

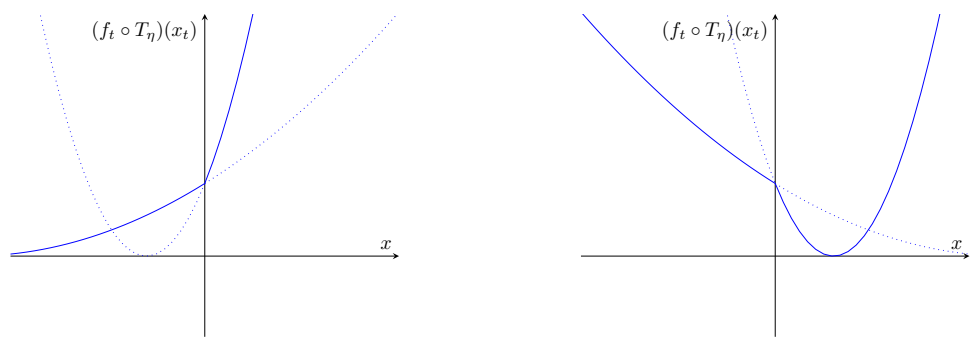
#### Quadratic Deviations

For the quadratic deviations objective function $f_t(x_t) = (x_t - p_t)^2$, where $p_t$ is the net energy profile at time $t$, Lemma 5.5 implies that the composed objective $(f_t \circ T_\eta)$ is convex when $p_t \leq \frac{1}{\eta_d} x_t^{\min}$. However, we will show it is convex for a larger range of parameter values.

Consider the more general quadratic function: $f_t(x_t) = \frac{1}{2} a_t x_t^2 + b_t x_t + c_t$ with $a_t > 0$. The composed objective $(f_t \circ T_\eta)$ consists of parts of two parabola that both pass through $(0, c_t)$, and have their minimum at different positions. The left parabola, whose value is assumed for negative $x_t$, has its minimum at $-\frac{\eta_d b_t}{a_t}$, and the right one, occurring for positive $x_t$, at $-\frac{\eta_c b_t}{a_t}$. The objective is convex precisely when the minimum of the left parabola is to the left of or on the same position as the minimum of the right parabola. This is the case when:

$$-\frac{\eta_d b_t}{a_t} \leq -\frac{\eta_c b_t}{a_t} \Rightarrow b_t \geq 0.$$

As a consequence, when considering the quadratic deviations objective the composed objective $(f_t \circ T_\eta)$ is convex whenever $p_t \leq 0$, and non-convex otherwise. See also Figure 5.4.




**(a)** Peak shaving with negative profile **(b)** Peak shaving with positive profile

**Figure 5.4:** The peak shaving objective remains convex for negative profile values, and becomes non-convex for positive profile values.

### 5.3.2 Subdifferential Karush-Kuhn-Tucker conditions

Analogously to the lossless case, we require a way to solve convex instances of Problem 5.4 in order to solve Problem 3.3. Due to the loss of differentiability of the objective function after the introduction of conversion losses, we require alternative KKT conditions based on subdifferentiation – a generalization of the derivative to non-differentiable convex functions.

Where the derivative of a differentiable function at a point is defined as the slope of the unique tangent line at that point, the subderivative of a convex function at a point is defined as the slope of any of the lines that is touching at that point and are everywhere touching or below the graph on the rest of the domain of the function. When there are non-differentiable 'kinks' in the function, multiple values exist. For differentiable convex functions, the derivative is the unique subderivative. The set of all subderivatives of $f$ at $x$ is called the subdifferential of $f$ at $x$:

**Definition 5.1.** ***Subdifferential:*** *The subdifferential $\partial f(x)$ of $f : I \mapsto \mathbb{R}^n$ at $x \in I$ (where $I \subseteq \mathbb{R}^n$ is an open convex set) is defined as the collection of all subgradients of $f$ at $x$:*

$$\partial f(x) = \{\, c \in \mathbb{R}^n \mid \forall y \in I : f(y) - f(x) \geq c \cdot (y - x) \,\}$$

Subdifferential KKT conditions [19] exist for convex optimization problems (5.1), where the continuous differentiability requirement is dropped:

**Theorem 5.2.** ***Subdifferential Karush-Kuhn-Tucker conditions:*** *If $f : \mathbb{R}^n \to \mathbb{R}$, and $g_i : \mathbb{R}^n \to \mathbb{R}$ are convex, and $h_j : \mathbb{R}^n \to \mathbb{R}$ are affine functions and Slater's condition holds, then $\mathbf{x}^*$ is the primal optimum and $\lambda^*$, $\mu^*$ is dual optimum of (5.1) if and only if:*

$$\begin{aligned} \partial f(x^*) + \sum_{i=1}^{m} \mu_i^* \partial g_i(x^*) + \sum_{j=1}^{l} \lambda_j^* \partial h_j(x^*) &\ni 0, \\ g_i(x^*) &\leq 0, \quad \forall i = 1, \ldots, m, \\ h_j(x^*) &= 0, \quad \forall j = 1, \ldots, l, \\ \mu_i^* &\geq 0, \quad \forall i = 1, \ldots, m, \\ \mu_i^* g_i(x^*) &= 0, \quad \forall i = 1, \ldots, m. \end{aligned} \tag{5.7}$$

Where, for sets $A, B \subseteq \mathbb{R}^n$, $A + B = \{\, a + b \mid a \in A, b \in B \,\}$ is the Minkowski sum. Note that in (5.7) $\partial f(x^*)$, $\partial g_i(x^*)$, $\partial h_j(x^*)$ are sets instead of scalars.

Slater's condition holds by (5.2), and the fact that the feasible sets of the lossless and lossy problem are the same. Hence, when $(f_t \circ T_\eta)$ is convex for all $t \in \mathcal{T}$, we can apply (5.7) to Problem 5.4 to obtain that $\mathbf{x}^*$ is primal optimal and $\lambda^*, \mu^*$ are dual optimal if and only if $\mathbf{x}^*$ is primal feasible and:

$$\begin{aligned} \partial(f_t \circ T_\eta)(x_t^*) + \mu_t^* - \lambda^* &\ni 0, \quad \forall t \in \mathcal{T}, \\ \mu_t^{*+} (x_t^* - x_t^{\max}) &= 0, \quad \forall t \in \mathcal{T}, \\ \mu_t^{*-} \left(x_t^* - x_t^{\min}\right) &= 0, \quad \forall t \in \mathcal{T}. \end{aligned} \tag{5.8}$$

In the remainder of this section we use (5.8) to solve Problem 5.4 for the linear pricing, feed-in subsidy, and peak shaving objectives. To simplify our analysis, we assume that $x_t^{\min} < 0 < x_t^{\max}$ for all $t \in \mathcal{T}$. Though the results can be easily adapted for the cases where $0 \leq x_t^{\min} \leq x_t^{\max}$ or $x_t^{\min} \leq x_t^{\max} \leq 0$.

#### Linear pricing

Let us again consider the linear objective $f_t(x_t) = c_t x_t$. Recall from Section 5.3.1 that the composed objectives $(f_t \circ T_\eta)$ are convex if and only if $c_t \geq 0$. Under this condition, the subdifferential of $(f_t \circ T_\eta)$ at $x_t$ can be explicitly determined as the following set-valued function:

$$\partial(f_t \circ T_\eta)(x_t) = \begin{cases} \left\{ \frac{c_t}{\eta_d} \right\} & \text{when } x_t < 0, \\ \left[ \frac{c_t}{\eta_d}, \frac{c_t}{\eta_c} \right] & \text{when } x_t = 0, \\ \left\{ \frac{c_t}{\eta_c} \right\} & \text{when } x_t > 0. \end{cases}$$

Using (5.8) this yields the following optimal solution:

$$\begin{aligned} &x_t = x_t^{\min}, && \mu_t < 0, && \lambda = \tfrac{c_t}{\eta_d} + \mu_t, \\ &x_t \in \left[x_t^{\min}, 0\right], && \mu_t = 0, && \lambda = \tfrac{c_t}{\eta_d}, \\ &x_t = 0, && \mu_t = 0, && \lambda \in \left(\tfrac{c_t}{\eta_d}, \tfrac{c_t}{\eta_c}\right), \\ &x_t \in [0, x_t^{\max}], && \mu_t = 0, && \lambda = \tfrac{c_t}{\eta_c}, \\ &x_t = x_t^{\max}, && \mu_t > 0, && \lambda = \tfrac{c_t}{\eta_c} + \mu_t. \end{aligned}$$

Imposing primal feasibility by requiring $\sum_{t\in\mathcal{T}} x_t = C$ to hold therefore again gives rise to a water filling approach. When the same breakpoint occurs in multiple time steps, the charge can be divided arbitrarily. A possible choice is to spread the charge equally on all tied time steps.

In this case each time step has not one, but two breakpoints: one at $\frac{c_t}{\eta_d}$, and one at $\frac{c_t}{\eta_c}$. At these breakpoints, if we can meet the equality bound within the given range for $x_t$, we set it to the required value, otherwise we increase $\lambda$ to the next breakpoint. The waterfilling method to solve Problem 5.4 with linear objective is shown in Algorithm 4.

**Algorithm 4** Waterfilling approach for Problem 5.4 with $f_t(x_t) = c_t x_t$.

```
function OPTLEVC
    A ← {c_1/η_d, ..., c_T/η_d}
    B ← {c_1/η_c, ..., c_T/η_c}
    Sort A and B in non-decreasing order
    C̃ ← SoC_0 + Σ_{t∈T} x_t^min, x ← x^min
    while C̃ < C do
        Take α as the first element from A with associated time interval t
        Take β as the first element from B with associated time interval t'
        if α < β then
            Δ ← min{−x_t, C − C̃}
            C̃ ← C̃ + Δ, x_t ← x_t + Δ
        else
            Δ ← min{x_t'^max − x_t', C − C̃}
            C̃ ← C̃ + Δ, x_t' ← x_t' + Δ
        end if
    end while
    return x
end function
```

**Feed-in subsidy**

Recall the feed-in subsidy objective $f_t(x_t) = c_t(x_t - p_t)^+ + s_t(x_t - p_t)^-$, and that this function is convex if $0 \le s_t \le c_t$ for all $t \in \mathcal{T}$. Note that, when $p_t\eta_d < x_t^{\min}$ or $p_t\eta_c > x_t^{\max}$, the objective reduces to a linear objective with unit cost of energy $c_t$ and $s_t$ respectively. We therefore can assume that w.l.o.g. $\frac{x_t^{\min}}{\eta_d} \le p_t \le \frac{x_t^{\max}}{\eta_c}$. When $p_t \ge 0$, the subdifferential of $(f_t \circ T_\eta)$ is given by:

$$\partial(f_t \circ T_\eta)(x_t) = \begin{cases} \left\{\frac{s_t}{\eta_d}\right\} & \text{when } x_t < 0, \\ \left[\frac{s_t}{\eta_d}, \frac{s_t}{\eta_c}\right] & \text{when } x_t = 0, \\ \left\{\frac{s_t}{\eta_c}\right\}, & \text{when } 0 < x_t < p_t\eta_c, \\ \left[\frac{s_t}{\eta_c}, \frac{c_t}{\eta_c}\right] & \text{when } x_t = p_t, \\ \left\{\frac{c_t}{\eta_c}\right\} & \text{when } x_t > 0. \end{cases} \qquad (p_t \ge 0)$$

When $p_t < 0$, the subdifferential looks similar. We obtain the following solution to (5.8) when $p_t \geq 0$:

$$\begin{aligned}
&x_t = x_t^{\min}, && \mu_t < 0, && \lambda = \tfrac{s_t}{\eta_d} + \mu_t,\\
&x_t \in \left[x_t^{\min}, 0\right], && \mu_t = 0, && \lambda = \tfrac{s_t}{\eta_d},\\
&x_t = 0, && \mu_t = 0, && \lambda \in \left(\tfrac{s_t}{\eta_d}, \tfrac{s_t}{\eta_c}\right),\\
&x_t \in [0, p_t\eta_c], && \mu_t = 0, && \lambda = \tfrac{s_t}{\eta_c},\\
&x_t = p_t\eta_c, && \mu_t = 0, && \lambda \in \left(\tfrac{s_t}{\eta_c}, \tfrac{c_t}{\eta_c}\right),\\
&x_t \in [p_t\eta_c, x_t^{\max}], && \mu_t = 0, && \lambda = \tfrac{c_t}{\eta_c},\\
&x_t = x_t^{\max}, && \mu_t > 0, && \lambda = \tfrac{c_t}{\eta_c} + \mu_t.
\end{aligned}$$

And for $p_t < 0$:

$$\begin{aligned}
&x_t = x_t^{\min}, && \mu_t < 0, && \lambda = \tfrac{s_t}{\eta_d} + \mu_t,\\
&x_t \in \left[x_t^{\min}, p_t\eta_d\right], && \mu_t = 0, && \lambda = \tfrac{s_t}{\eta_d},\\
&x_t = p_t\eta_d, && \mu_t = 0, && \lambda \in \left(\tfrac{s_t}{\eta_d}, \tfrac{c_t}{\eta_d}\right),\\
&x_t \in [p_t\eta_d, 0], && \mu_t = 0, && \lambda = \tfrac{c_t}{\eta_d},\\
&x_t = 0, && \mu_t = 0, && \lambda \in \left(\tfrac{c_t}{\eta_d}, \tfrac{c_t}{\eta_c}\right),\\
&x_t \in [0, x_t^{\max}], && \mu_t = 0, && \lambda = \tfrac{c_t}{\eta_c},\\
&x_t = x_t^{\max}, && \mu_t > 0, && \lambda = \tfrac{c_t}{\eta_c} + \mu_t.
\end{aligned}$$

Such that, after imposing the constraint $\sum_{t\in\mathcal{T}} x_t = C$, we obtain a water filling approach with three breakpoints for each time step.

**Quadratic deviations**

Finally, instead of the quadratic deviations objective function $f_t(x_t) = (x_t - p_t)^2$, we consider the more general quadratic objective $f_t(x_t) = \frac{1}{2}a_t x_t^2 + b_t x_t + c_t$, with $a_t > 0$, where $b_t \geq 0$ to ensure convexity. The subdifferential of $(f_t \circ T_\eta)$ is given by:

$$\partial(f_t \circ T_\eta)(x_t) = \begin{cases} \left\{\frac{a_t}{\eta_d^2}x_t + \frac{b_t}{\eta_d}\right\} & \text{when } x_t < 0,\\ \left[\frac{a_t}{\eta_d^2}x_t + \frac{b_t}{\eta_d}, \frac{a_t}{\eta_c^2}x_t + \frac{b_t}{\eta_c}\right] & \text{when } x_t = 0,\\ \left\{\frac{a_t}{\eta_c^2}x_t + \frac{b_t}{\eta_c}\right\} & \text{when } x_t > 0. \end{cases}$$

Giving us the following solution to (5.8):

$$\begin{aligned}
&x_t = x_t^{\min}, && \mu_t < 0, && \lambda = \tfrac{a_t}{\eta_d^2}x_t^{\min} + \tfrac{b_t}{\eta_d} + \mu_t,\\
&x_t = \tfrac{\lambda\eta_d^2 - b_t\eta_d}{a_t}, && \mu_t = 0, && \lambda \in \left[\tfrac{a_t}{\eta_d^2}x_t^{\min} + \tfrac{b_t}{\eta_d}, \tfrac{b_t}{\eta_d}\right],\\
&x_t = 0, && \mu_t = 0, && \lambda \in \left(\tfrac{b_t}{\eta_d}, \tfrac{b_t}{\eta_c}\right),\\
&x_t = \tfrac{\lambda\eta_c^2 - b_t\eta_c}{a_t}, && \mu_t = 0, && \lambda \in \left[\tfrac{b_t}{\eta_c}, \tfrac{a_t}{\eta_c^2}x_t^{\max} + \tfrac{b_t}{\eta_c}\right],\\
&x_t = x_t^{\max}, && \mu_t > 0, && \lambda = \tfrac{a_t}{\eta_c^2}x_t^{\max} + \tfrac{b_t}{\eta_c} + \mu_t.
\end{aligned}$$

Giving rise to a water filling approach with two basins per time step, as seen in Figure 5.5.

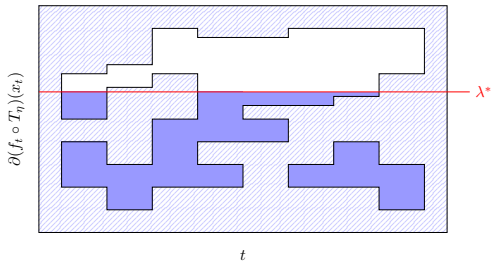



**Figure 5.5:** Water filling approach for the case of quadratic objective functions.

### 5.3.3 Maximal charging

In the previous section, we derived methods for solving convex instances of Problem 3.3. In this section, we consider a method to solve instances of Problem 3.3 where $f_t(x_t) = c_t x_t$ with $c_t < 0$ for some time steps. As we have seen, this problem is $\mathcal{NP}$-complete in general, hence it is unlikely that an exact efficient algorithm exists. Nevertheless, whenever negative prices occur we intuitively want to charge as much as possible. Theorem 5.3 gives us a condition under which this intuition turns out to hold true. As it turns out, if it is feasible to fully charge at all time steps where the prices are negative, then it is optimal to do so.

**Theorem 5.3.** *If there exists a feasible solution* $\mathbf{x}$ *to an instance of Problem 3.3 with linear objectives where* $x_t = x_t^{\max}$ *for all* $t \in \mathcal{T}$ *where* $a_t < 0$*, then each optimal solution* $\mathbf{y}$ *to the instance has* $y_t = x_t^{\max}$ *for all* $t \in \mathcal{T}$ *where* $a_t < 0$*.*

*Proof.* Define $\mathcal{I}$ as the set of time intervals where negative prices occur: $\mathcal{I} = \{t \mid a_t < 0\}$. Let $\mathbf{x}$ be a feasible solution with $x_t = x_t^{\max}$ for all $t \in \mathcal{I}$, and assume there exists some optimal solution $\mathbf{y}$ where for some time interval $t' \in \mathcal{I}$: $y_{t'} < x_{t'}^{\max}$. We show that $t'$ must be contained in an interval $[t_A, t_B]$ where: $\sum_{t=1}^{t_A} y_t = SoC_{t_A}^{\min}$ and $\sum_{t=1}^{t_B} y_t = SoC_{t_B}^{\max}$, where on the entire interval $y_t \leq x_t$ with strict inequality for $t'$, causing $\mathbf{x}$ to not be feasible. In all other cases, we can improve $\mathbf{y}$.

There must be some time step $t_B \geq t'$ where $\sum_{t=1}^{t_B} y_t = SoC_{t_B}^{\max}$, otherwise we could improve $\mathbf{y}$, contradicting its optimality, by increasing $y_{t'}$ by an amount up to:

$$\min\left(\underbrace{\min_{t=t',\dots,T}\left\{SoC_t^{\max} - \sum_{s=1}^{t} y_s\right\}}_{>0}, \underbrace{x_{t'}^{\max} - y_{t'}}_{>0}\right).$$

Let $t_B$ be the smallest such time step. There must also exist a time step $\bar{t} \notin \mathcal{I}$, $\bar{t} \leq t_B$ where $y_{\bar{t}} > x_{\bar{t}}$. If there were no such $\bar{t}$, then:

$$\begin{aligned} x_t &\geq y_t, & t &\leq t_B, t \notin \mathcal{I}, \\ x_t &= x_t^{\max} \geq y_t, & t &\leq t_B, t \in \mathcal{I} \setminus \{t'\}, \\ x_{t'} &= x_{t'}^{\max} > y_{t'}, \end{aligned}$$

such that, since $t_B \geq t'$, $\sum_{t=1}^{t_B} x_t > \sum_{t=1}^{t_B} y_t = SoC_{t_B}^{\max}$. This implies that $\mathbf{x}$ is infeasible, a contradiction. Let $\bar{t}$ be the last such time step, such that: $\downarrow$

$$y_t \leq x_t, \qquad t \in (\bar{t}, t_B].$$

Since $y_{\bar{t}} > x_{\bar{t}}$ and $y_{t'} < x_{t'}$, it follows that $\bar{t} \neq t'$. Therefore, either $\bar{t} > t'$ or $\bar{t} < t'$:

- Assume $\bar{t} > t'$, then we can improve $\mathbf{y}$ by increasing $y_{t'}$ and decreasing $y_{\bar{t}}$ by:

$$\min\left(\underbrace{\min_{t=t',\ldots,\bar{t}-1}\left\{SoC_t^{\max} - \sum_{s=1}^{t} y_s\right\}}_{>0}, \underbrace{x_{t'}^{\max} - y_{t'}}_{>0}, \underbrace{y_{\bar{t}} - x_{\bar{t}}^{\min}}_{>0}\right).$$

  The first argument is positive since, by definition, $t_B \geq \bar{t}$ is the first time step after $t'$ where $SoC_{t_B}^{\max} = \sum_{t=1}^{t_B} y_t$. The second and third arguments are positive by definition as well. It follows that $\mathbf{y}$ was not optimal, a contradiction.

- Next, consider $\bar{t} < t'$. Either there exists a time step $t_A \in [\bar{t}, t')$ where $\sum_{t=1}^{t_A} y_t = SoC_{t_A}^{\min}$, or there is not. If there is no such time, then we can improve $\mathbf{y}$ by increasing $y_{t'}$ and decreasing $y_{\bar{t}}$ by an amount up to:

$$\min\left(\underbrace{\min_{t=\bar{t},\ldots,t'-1}\left\{\sum_{s=1}^{t} y_s - SoC_t^{\min}\right\}}_{>0}, \underbrace{x_{t'}^{\max} - y_{t'}}_{>0}, \underbrace{y_{\bar{t}} - x_{\bar{t}}^{\min}}_{>0}\right).$$

  If there is such a time step $t_A$, then since $\bar{t}$ is the last time step before $t_B$ where $\mathbf{y}$ exceeds $\mathbf{x}$: $y_t \leq x_t$ for all $t \in (t_A, t_B]$, with strict inequality for $t'$. Therefore, $\sum_{t=1}^{t_B} x_t \geq \sum_{t=t_A+1}^{t_B} x_t + SoC_{t_A}^{\min} > \sum_{t=t_A+1}^{t_B} y_t + SoC_{t_A}^{\min} = \sum_{t=1}^{t_B} y_t = SoC_{t_B}^{\max}$, contradicting feasibility of $\mathbf{x}$.

Hence, there does not exist a time step $t' \in \mathcal{I}$ where for an optimal solution $\mathbf{y}$: $y_{t'} < x_{t'}^{\max}$.

□

Hence, if indeed there exists a feasible solution where we maximally charge on each time step with negative prices, we can then eliminate these time steps to yield a convex instance.

## 5.4 Sign restriction method

In the previous section we derived methods for solving the lossy EVC problem for certain parameter values. However, the imposed constraints on the parameters may not hold up in practice: too many negative prices may occur on a single day, there may be a negative feed-in subsidy (such that it becomes a penalty to feed energy into the net), or positive profile values make these methods inapplicable. In this section we propose a heuristic to deal with these scenarios. We show that restricting the sign of $x_t$ makes $(f_t \circ T_\eta)$ convex.

Although searching the entire space of the Lossy Storage Problem is hard, we will show that searching a single of its orthants can be done efficiently. For the moment consider that though we might not know the exact values of $x_t$ for any $t$, we might know their signs, i.e. know for each time step whether we are charging or discharging at that time. This corresponds to having an oracle for the Subset Sum problem in the given $\mathcal{NP}$-hardness proof. The choice of signs can be expressed as a partition $(I^+, I^-)$ of $\mathcal{T}$, such that:

$$t \in I^+ \Rightarrow \tilde{x}_t^{\min} = (x_t^{\min})^+, \tag{5.9}$$

$$t \in I^- \Rightarrow \tilde{x}_t^{\max} = (x_t^{\max})^-. \tag{5.10}$$

We can use the imposed sign restriction to absorb the efficiency coefficients into the parameters of the

objective. Assuming a quadratic objective function $f_t(x_t) = \frac{1}{2}a_t x_t^2 + b_t x_t + c_t$ we get that:

$$\begin{aligned}(f_t \circ T_\eta)(x_t) &= \begin{cases} \frac{1}{2}\frac{a_t}{\eta_c^2}x_t^2 + \frac{b_t}{\eta_c}x_t + c_t & t \in I^+ \\ \frac{1}{2}\frac{a_t}{\eta_d^2}x_t^2 + \frac{b_t}{\eta_d}x_t + c_t & t \in I^- \end{cases} \\ &= \tfrac{1}{2}\tilde{a}_t x_t^2 + \tilde{b}_t x_t + c_t,\end{aligned}$$

where

$$\tilde{a}_t = \begin{cases} \frac{a_t}{\eta_c^2} & t \in I^+ \\ \frac{a_t}{\eta_d^2} & t \in I^- \end{cases}, \text{and } \tilde{b}_t = \begin{cases} \frac{b_t}{\eta_c} & t \in I^+ \\ \frac{b_t}{\eta_d} & t \in I^- \end{cases}.$$

The Lossy Storage Problem restricted to a single orthant can thus be reduced to an instance of Problem 3.1. For other types of objective functions similar transformations exist. This procedure works as long as the set of objectives functions is complete under positive valued input-scaling, i.e. if the objectives we can solve are sufficiently parameterized to allow the absorption of positive factors from $x_t$.

Alternatively, observe that when we restrict $T_\eta$ to just positive or negative inputs, it reduces to a linear transform. From convex analysis, we know that $(f_t \circ T_\eta)$ is convex when $f_t$ convex and $T_\eta$ linear. As such, we only need a sign restriction on the time intervals that give rise to a non-convex objective. And so, given a solution method for the convex case, we obtain an exact procedure for the lossy case when we enumerate all possible sign restriction combinations for the non-convex $x_t$'s. Denote by $\mathcal{T}^{nc}$ the set of time steps for which the unrestricted objective is non-convex. If we denote by $g(I^+, I^-)$ the value obtained by imposing the sign restriction $(I^+, I^-)$ on $\mathcal{T}^{nc}$, we can restate Problem 3.3 as:

$$\min_{\substack{I^+ \subseteq \mathcal{T}^{nc} \\ I^- \subseteq \mathcal{T}^{nc} \\ I^+ \cup I^- = \mathcal{T}^{nc} \\ I^+ \cap I^- = \varnothing}} \quad g\left(I^+, I^-\right). \tag{5.11}$$

Taking the minimum over all restricted problems indeed gives the optimal solution of the original problem. However, it requires us to solve a number of convex instances that is exponential in the number of non-convex objectives, limiting its practical applicability only to instances of small size or where the number of non-convexities is small.

The notion of sign restriction also induces heuristic methods in which we do not search the entire state space, but only consider sign restrictions that are in some sense logical, or those which we expect the optimal solution to adhere. Some examples are:

- learning from historical data, for example using machine learning techniques on previously solved instances;
- parameter-based selection rules, for example for the profile steering objective $(x_t - p_t)^2$: whenever $p_t$ exceeds some threshold $\tau$ charge at time $t$, otherwise discharge;
- solving the corresponding Lossless Storage Problem, and copy the signs to the case where we *do* account for conversion losses;
- local search approaches with the neighborhood of sign flips.

We will compare these choices in a case study in the next chapter.

# 6. Case Study

*In the previous chapter, we developed methods to solve instances of the Lossy Storage Problem. Certain parameter choices allowed for exact and efficient solving, while for the remaining cases we introduced a heuristic approach based on sign restrictions. In this chapter, we evaluate the performance of the sign restriction methods outlined in Section 5.4, when applied to instances with the quadratic deviations objective* $f_t(x_t) = (x_t - p_t)^2$.

The sign restriction method from Section 5.4 is a heuristic, and offers no performance guarantees. In this chapter we therefore evaluate its performance on a set of non-convex instances of Problem 3.3 with quadratic deviations objective $f_t(x_t) = (x_t - p_t)^2$. We do not consider a pricing setting, since Theorem 5.3 offers us a means to solve all practical occurrences of negative prices.

We consider the case where each individual household has a residential-level Lithium-ion battery to flatten their energy profile. The energy profiles are the same as from Section 4.3: single days of individual households are considered in isolation, with time steps of 15 minutes, where a week per season is simulated. We only consider the households with PV, giving us a set of 52 households. The battery has 4.22 kWh maximum capacity with 0.74 kW (dis)charge bounds (0.185 kWh per 15 minutes) and starts out empty.

We require a sign restriction for each charging decision where in the corresponding time step production exceeds consumption. The choices we simulate are:

- **equal division:** an equal division of positive and negative signs, where the first quarter of each day is used for discharging, the next half for charging, and the final quarter for discharging;
- **profile sign:** whenever the profile is positive, we charge. Otherwise we discharge;
- **lossless sign:** whenever optimal solution the corresponding lossless instance charges, we charge. Otherwise we discharge.

Note that though we specify a sign restriction for each time step, it is only imposed for time steps with non-convex objective.

Next, we consider how we can further improve the obtained solutions. Whenever $x_t = 0$ for some $t$ where $(f_t \circ T_\eta)$ is non-convex, we can not worsen the solution by flipping the corresponding sign and solving for this new sign restriction. Indeed, in the worst case $x_t$ still equals 0 afterwards. This yields an iterated improvement method that halts when $x_t$ is non-zero for each time step with non-convex objective, or when no more improvement is found for any sign flip.

Another way by which we can improve the objective is through a more advanced local search method. We consider multi-start simulated annealing [1] with a static cooling strategy: $c_n = 0.999^n$. Each search consists of five restarts in which 50 sign flips are considered.

#### Results

The lossless sign choice outperforms the equal division and profile sign choices in almost all instances. Only in about 0.85% of the cases at 70% RTE and 0.5% of the cases at 85% RTE does the profile sign choice outperform it, and it is never outperformed by the equal division choice. Table 6.1 shows the average objective values attained by the different sign restriction choices for several round-trip efficiencies. The reported optimum is found by taking the signs of the solution to the ILP that approximates the given instance. The average objective of the lossless sign choice is lower than that of the optimum!

We consider an instance to be solved whenever its objective is lower or equal than the objective of the ILP solution of that instance. Table 6.2 shows the percentage of solved instances for several RTEs, for: the

lossless sign choice, lossless sign with iterated improvement, and lossless sign with simulated annealing. The simulated annealing method outperforms the rest, but also requires a lot more computation time. The lossless sign choice only requires us to solve two instances: one to determine the signs, and the other to determine the solution. The iterated improvement adds less than one pass on average to this, with a maximum of 12 additional passes encountered during simulations. The simulated annealing approach requires 250 additional passes. Given that the performance increase of simulated annealing is very small with respect iterated improvement, it may not be worth the computational overhead.

| | Round-trip efficiency | | | | | |
|---|---|---|---|---|---|---|
| | 70% | 75% | 80% | 85% | 90% | 95% |
| Optimum | 14.4703 | 14.4060 | 14.3428 | 14.2807 | 14.2200 | 14.1605 |
| Equal division | 14.4753 | 14.4105 | 14.3469 | 14.2845 | 14.2234 | 14.1636 |
| Profile sign | 14.4743 | 14.4096 | 14.3460 | 14.2837 | 14.2226 | 14.1628 |
| Lossless sign | 14.4698 | 14.4056 | 14.3425 | 14.2805 | 14.2198 | 14.1603 |

**Table 6.1:** Average objective value for several sign restriction methods and round-trip efficiencies.

| | Round-trip efficiency | | | | | |
|---|---|---|---|---|---|---|
| | 70% | 75% | 80% | 85% | 90% | 95% |
| Lossless sign | 82.1429% | 82.4274% | 83.6976% | 84.4286% | 86.5713% | 87.1154% |
| + iterated improvement | 86.1338% | 86.2857% | 86.3624% | 86.5698% | 87.0120% | 87.1428% |
| + simulated annealing | 86.4291% | 86.5721% | 86.7143% | 86.9378% | 87.1761% | 87.4286% |

**Table 6.2:** Percentage of optimally solved instances. An instance is considered solved if it performs just as well, or outperforms, the LP solution.

# 7. Conclusion and discussion

## 7.1 Research questions

In this thesis we considered the optimization problem where a lossy energy storage device is employed for either arbitrage seeking in a linear pricing or feed-in subsidy setting, or matching energy supply and demand. The research question we addressed is:

- *Can energy storage devices, where we account for energy conversion losses, effectively support the electricity grid over their lifetime?*

We splitted this question into the following subquestions:

- *How important are conversion losses in practice, and how relevant are they for optimization procedures?*
- *Are consumer-owned energy storage devices economically viable in the near future?*
- *Can we distinguish cases that are computationally tractable to optimally solve?*
- *Can an efficient approximate algorithm be devised?*

Correctly accounting for conversion losses turns out to be crucial, as naive approaches, such as the repair and rescale methods from Chapter 4, display poor performance for the linear pricing and feed-in subsidy settings. Even for high round-trip efficiencies the revenue due to the battery decreases sharply under these methods, as compared to the optimal solution. For the quadratic deviations objective the performance drop is less severe, yet still notable since the days on which this occurs across households correlate.

Simulations of a 4.22 kWh Li-ion storage device for several levels of feed-in subsidy and round-trip efficiencies were performed to investigate economic feasibility of residential energy storage devices. The simulations indicate that, given the current trend in Li-ion battery prices, it may from 2030 onward be economically feasible for consumers to invest in residential energy storage, given that the feed-in subsidy is 50% or less of the average energy price. This assumes that the consumer already owns rooftop PV that are cost-neutral.

In order to feasibly incorporate the new model into a decentralized energy management paradigm, instances of the problem have to be solvable in the order of milliseconds, with limited memory. This makes general approach such as ILP models inadequate, and requires us to consider tailored algorithms for this problem. For many parameter choices, an extension of existing approaches can be used to efficiently solve instances of the problem to optimality. This does not hold for, e.g., positive profile values or negative prices or subsidies. We proved that the lossy storage problem with negative linear prices is already $\mathcal{NP}$-complete when considered in isolation. Nevertheless, we derive a method to solve practical instances with negative prices, and show that the heuristic method performs well on instances with the quadratic deviations objective – being able to solve roughly 85% of the considered cases to optimality.

In practice, (dis)charge bounds and state of charge bounds will likely not change (much) over time. As the problem loses a lot of its structure under these assumptions, we suspect that such instances should be easy to solve. In particular, the given $\mathcal{NP}$-completeness proof does not hold under this constraint. However, we were unable to find a solution method for such instances.

The main contributions of this thesis are the implementation of the aforementioned solution methods in Python to be used inside DEMKit, an investigation in economic feasibility under the feed-in subsidy policy, and a $\mathcal{NP}$-completeness proof for the lossy storage problem with linear prices that are negative and fixed.

We answer our research question in the affirmative: the battery's capabilities, economic feasibility in the near future, and existence of adequate optimization techniques all contribute to a device that has great

potential to effectively support the grid. Lower battery costs, policy change from net metering to feed-in subsidy, and the availability of energy profile predictions and control mechanisms for energy storage are the last hurdles in the possible deployment of residential energy storage devices.

## 7.2 Data

The solar irradiation and price data were taken from 2014, as this was the only year I could get both datasets from, due to availability issues. Due to technological advances and inflation, we expect prices to strongly vary through the years. For an energy trader, the resulting effect of scaling the prices are minimal, as a flat increase or decrease in price does not affect arbitrage opportunities. Arbitrage opportunities will likely only increase in amount and magnitude in the near future: due to a higher penetration of local generation we expect prices across the day to vary stronger than in previous years, creating more opportunity for arbitrage. Therefore, in this regard results from Chapter 4 were slightly pessimistic.

Additionally, the choice of time horizon also influences the results: carry-over of energy to the next day is not rewarded as the simulation stops at 24:00, implying that, e.g., the last hour of the day in the day-night tariff does not contribute to the revenue. Considering either a rolling horizon, longer periods than single days, or somehow changing the problem to a 'steady-state' problem, where $SoC_T = SoC_0$, may help in this regard. Because low prices that are not followed by higher prices do not constitute an arbitrage opportunity, we essentially lose some revenue – reinforcing the pessimism of the results of Chapter 4. The effect of this is minimal though, as there is ample time to charge the storage from 0:00 till 7:00. Specifically, the maximum obtainable profit under linear pricing for the day-night tariff is $(c_{\text{day}} - c_{\text{night}})\, SoC^{\max} =$ €0.03/kWh $\cdot$ 42.2kWh = €1.266, corresponding to the results seen in Table 4.1. This indicates that considering a single day is sufficient in this case. We suspect that the same holds for spot market prices. Nevertheless, for the feed-in subsidy and quadratic deviations objectives it may still be useful to consider more time steps.

## 7.3 Consumer behaviour

Given rooftop PV, the revenue due to energy storage under the feed-in subsidy policy mainly consists of avoiding the purchase of energy by load shifting. Given a lower subsidy level consumers in households with PV are automatically incentivized to shift their load towards noon, regardless of whether they have an energy storage device. This is not accounted for in our simulations. When this happens, the revenue due to the battery will decrease, as consumer behaviour essentially takes over the task of load shifting via the energy storage in this case. A further study needs to be done to investigate the influence on revenue due to the battery; essentially the numerical study needs to be repeated, but some or all shiftable devices should be moved to a time around noon. Similarly, additional smart appliances may reduce the revenue due to the battery.

Given a household where the washing machine ($\tilde{5}$00 kWh per load) and dish washer ($\tilde{1}$500 kWh per load) are used twice a week, and their usage is shifted towards noon, already $\tilde{1}$0% of a households energy consumption can be shifted [13]. Given that manual shifting is more energy efficient than employing a lossy battery, we expect to gain about $(1 - RTE) \cdot 10\%$ of the energy costs on days where we shift this load at the cost of $RTE \cdot 10\%$ of revenue due to the battery.

# Bibliography


[1] E. Aarts and J. K. Lenstra, eds., *Local search in combinatorial optimization*, Princeton University Press, 2003. (Cited on page 51.)

[2] Aquion Energy, *Saltwater Battery — Safe Sustainable Batteries*. [Online] Retrieved from http://aquionenergy.com/ on July 22, 2019. (Cited on page 15.)

[3] ——, *S30-0080 battery stack product specification sheet*, (2016). [Online] Retrieved from http://www.blitzelectricalconsulting.com/wp-content/uploads/2016/11/Aquion_Energy_S30-0080_Product_Specification_Sheet.pdf on July 22, 2019. (Cited on page 18.)

[4] J. Berke, *Germany paid people to use electricity over the holidays because its grid is so clean*, Business Insider, (2017). [Online] Retrieved from https://www.businessinsider.com/renewable-power-germany-negative-electricity-cost-2017-12 on December 9, 2018. (Cited on pages 7, 23, and 41.)

[5] D. Bertsimas and J. N. Tsitsiklis, *Introduction to linear optimization*, Athena Scientific, 1997. (Cited on page 24.)

[6] S. Bitonto and T. Rico, *Electromobility in Germany: Vision 2020 and Beyond*, Germany Trade & Invest, Berlin, (2015). (Cited on page 8.)

[7] Bloomberg, *Power worth less than zero spreads as green energy floods the grid*, (2018). [Online] Retrieved from https://www.bloomberg.com/news/articles/2018-08-06/negative-prices-in-power-market-as-wind-solar-cut-electricity on July 15, 2019. (Cited on page 41.)

[8] N. Bogart, *Why lithium-ion smartphone batteries keep exploding*, Global News, (2014). [Online] Retrieved from https://globalnews.ca/news/1714748/why-lithium-ion-smartphone-batteries-keep-exploding/ on July 15, 2019. (Cited on page 15.)

[9] S. Boyd and L. Vanderberghe, *Convex optimization*, Cambridge University Press, 2004. (Cited on pages 32 and 33.)

[10] T. Bunsen, P. Cazzola, M. Gorner, L. Paoli, S. Scheffer, R. Schuitmaker, J. Tattini, and J. Teter, *Global EV Outlook 2018: Towards cross-modal electrification*, (2018). (Cited on page 8.)

[11] California Public Utilities Commission, *Rule 21 interconnection*. [Online] Retrieved from https://www.cpuc.ca.gov/Rule21/ on February 19, 2019. (Cited on page 12.)

[12] Centraal Bureau voor de Statistiek, *Elektriciteitslevering vanuit het openbare net; woningkenmerken, bewoners*, (2019). [Dataset, Dutch: Electricity delivery from the public net; residence features, residents] Retrieved from https://opendata.cbs.nl/#/CBS/nl/dataset/83882NED/table?ts=1556893815206 on February 19, 2019. (Cited on page 8.)

[13] ——, *Energieverbruik particuliere woningen; woningtype en regio's*, (2019). [Dataset, Dutch: Energy usage private residences; residence type and regions] Retrieved from https://opendata.cbs.nl/#/CBS/nl/dataset/71405NED/table?ts=1556893728591 on February 18, 2019. (Cited on page 54.)

[14] ——, *Hernieuwbare elektriciteit; productie en vermogen*, (2019). [Dataset, Dutch: Renewable electricity; production and capacity] Retrieved from https://opendata.cbs.nl/#/CBS/nl/dataset/82610NED/table?ts=1556893838303 on May 1, 2019. (Cited on page 7.)

[15] ——, *Personenauto's; voertuigkenmerken, regio's, 1 januari*, (2019). [Dataset, Dutch: Personal cars; vehicle features, regions] Retrieved from https://opendata.cbs.nl/#/CBS/nl/dataset/71405NED/table?ts=1556893728591 on May 6, 2019. (Cited on page 8.)

[16] M. A. Delmas, M. Fischlein, and O. I. Asensio, *Information strategies and energy conservation behavior: A meta-analysis of experimental studies from 1975 to 2012*, Energy Policy, 61 (2013), pp. 729–739. (Cited on page 11.)

[17] Dr Ten BV, *Sea Salt Battery*. [Online] Retrieved from https://www.drten.nl/portfolio_item/zeezout-batterij/?lang=en on July 22, 2019. (Cited on page 15.)

[18] Energieleveranciers.nl, *Het ingeslopen gevaar van dag en nachtstroom*, (2013). [Blog, Dutch: The danger of day and night tariffs] Retrieved from https://www.energieleveranciers.nl/blog/het-ingeslopen-gevaar-van-dag-en-nachtstroom on December 9, 2018. (Cited on page 11.)

[19] U. Faigle, W. Kern, and G. Still, *Algorithmic principles of mathematical programming*, vol. 24, Springer Science & Business Media, 2013. (Cited on pages 34 and 44.)

[20] Flexiblepower Alliance Network, *The energy flexibility platform & interface*, (2017). [Online] Retrieved from http://flexiblepower.github.io/technology/efpi on December 3, 2018. (Cited on page 12.)

[21] FNN, *VDE-AR-N 4105: Power Generation Systems Connected to the Low-voltage Distribution Network*, 2011. (Cited on page 12.)

[22] J. Franz, *California's electrical grid can't handle all the solar energy the state is producing*, Public Radio International, (2017). [Online] Retrieved from https://www.pri.org/stories/2017-07-20/california-s-electrical-grid-can-t-handle-all-solar-energy-state-producing on December 9, 2018. (Cited on page 7.)

[23] D. Gabay and B. Mercier, *A dual algorithm for the solution of non linear variational problems via finite element approximation*, Computers & Mathematics with Applications, 2 (1976), pp. 17–40. (Cited on page 14.)

[24] M. E. Gerards, H. A. Toersche, G. Hoogsteen, T. van der Klauw, J. L. Hurink, and G. J. Smit, *Demand side management using profile steering*, in 2015 IEEE Eindhoven PowerTech, IEEE, 2015, pp. 1–6. (Cited on pages 8, 9, and 13.)

[25] L. Goldie-Scot, *A behind the scenes take on lithium-ion battery prices*, BloombergNEF, (2019). [Online] Retrieved from https://about.bnef.com/blog/behind-scenes-take-lithium-ion-battery-prices on May 2, 2019. (Cited on page 25.)

[26] D. S. Hochbaum and S.-P. Hong, *About strongly polynomial time algorithms for quadratic optimization over submodular constraints*, Mathematical programming, 69 (1995), pp. 269–309. (Cited on page 34.)

[27] G. Hoogsteen, *A cyber-physical systems perspective on decentralized energy management*, PhD thesis, University of Twente, December 2017. (Cited on page 10.)

[28] G. Hoogsteen, A. Molderink, J. L. Hurink, and G. J. Smit, *Generation of flexible domestic load profiles to evaluate demand side management approaches*, in 2016 IEEE International Energy Conference (ENERGYCON), IEEE, 2016, pp. 1–6. (Cited on page 26.)

[29] G. Hoogsteen, A. Molderink, G. Smit, J. Hurink, B. Kootstra, and F. Schuring, *Charging electric vehicles, baking pizzas, and melting a fuse in Lochem*, CIRED: Open Access Proceedings Journal, (2017), pp. 1629–1633. (Cited on page 8.)

[30] T. Ibaraki and N. Katoh, *Resource allocation problems: algorithmic approaches*, MIT press, 1988. (Cited on page 31.)

[31] International Energy Agency, *New CEM campaign aims for goal of 30% new electric vehicle sales by 2030*, (2017). [Online] Retrieved from https://www.iea.org/newsroom/news/2017/june/new-cem-campaign-aims-for-goal-of-30-new-electric-vehicle-sales-by-2030.html on January 14, 2018. (Cited on page 8.)

[32] M. R. Jongerden and B. R. Haverkort, *Which battery model to use?*, IET software, 3 (2009), pp. 445–457. (Cited on page 15.)

[33] H. G. J. Kamp, *Evaluatie salderingsregeling (kamerbrief)*, Rijksoverheid Nederland, (2017). [Letter, Dutch: Evaluation net metering policy]. (Cited on page 9.)

[34] K. Kok, *The PowerMatcher: Smart Coordination for the Smart Electricity Grid*, PhD thesis, Vrije Universiteit Amsterdam, July 2013. (Cited on page 13.)

[35] Koninklijk Nederlands Meteorologisch Instituut, *Uurgegevens van het weer in Nederland*. [Dataset, Dutch: Hourly data of the weather in the Netherlands] Retrieved from http://projects.knmi.nl/klimatologie/uurgegevens on January 14, 2019. (Cited on page 26.)

[36] C. Latimer, *Too much of a good thing: solar power surge is flooding the grid*, The Sydney Morning Herald, (2018). [Online] Retrieved from https://www.smh.com.au/business/the-economy/too-much-of-a-good-thing-solar-power-surge-is-flooding-the-grid-20180606-p4zjs7.html on December 9, 2018. (Cited on page 7.)

[37] P. J. Mankowski, J. Kanevsky, P. Bakirtzian, and S. Cugno, *Cellular phone collateral damage: A review of burns associated with lithium battery powered mobile devices*, Burns, 42 (2016), pp. 61–64. (Cited on page 15.)

[38] J. F. Manwell and J. G. McGowan, *Lead acid battery storage model for hybrid energy systems*, Solar Energy, 50 (1993), pp. 399–405. (Cited on page 15.)

[39] J. F. Miller and D. Howell, *The EV everywhere grand challenge*, World Electric Vehicle Journal, 6 (2013), pp. 1008–1013. (Cited on page 8.)

[40] Nederlandse Omroep Stichting, *Te veel zonnepanelen in Groningen*, (2016). [Online, Dutch: Too many solar panels in Groningen] Retrieved from https://nos.nl/artikel/2118256-te-veel-zonnepanelen-in-groningen.html on December 9, 2018. (Cited on page 7.)

[41] ——, *Geen plek voor nieuwe zonneparken op stroomnetwerk*, (2019). [Online, Dutch: No more room for new solar pars on electricity network] Retrieved from https://nos.nl/artikel/2266953-geen-plek-voor-nieuwe-zonneparken-op-stroomnetwerk.html on January 11, 2019. (Cited on page 7.)

[42] S. Neuteleers, M. Mulder, and F. Hindriks, *Assessing fairness of dynamic grid tariffs*, Energy Policy, 108 (2017), pp. 111–120. (Cited on page 16.)

[43] S. Nykamp, M. G. Bosman, A. Molderink, J. L. Hurink, and G. J. Smit, *Value of storage in distribution grids – competition or cooperation of stakeholders?*, IEEE transactions on smart grid, 4 (2013), pp. 1361–1370. (Cited on page 15.)

[44] S. Nykamp, A. Molderink, J. L. Hurink, and G. J. Smit, *Statistics for PV, wind and biomass generators and their impact on distribution grid planning*, Energy, 45 (2012), pp. 924–932. (Cited on page 7.)

[45] S. B. Peterson, J. Whitacre, and J. Apt, *The economics of using plug-in hybrid electric vehicle battery packs for grid storage*, Journal of Power Sources, 195 (2010), pp. 2377–2384. (Cited on page 25.)

[46] M. Pilz, L. Al-Fagih, and E. Pfluegel, *Energy storage scheduling with an advanced battery model: a game–theoretic approach*, Inventions, 2 (2017), p. 30. (Cited on page 14.)

[47] PwC, *De historische impact van salderen*, (2016). [Online, Dutch: The historic impact of net metering] Retrieved from https://www.rijksoverheid.nl/documenten/rapporten/2016/12/15/de-historische-impact-van-salderen. (Cited on page 9.)

[48] Rijksdienst voor Ondernemend Nederland, *Elektrisch vervoer in Nederland – Highlights 2017*, (2017). [Dutch: Electric transport in the Netherlands – Highlights 2017]. (Cited on page 8.)

[49] Rijksoverheid Nederland, *Salderingsregeling verlengd tot 2023*, (2019). [Online, Dutch: Net metering policy extended till 2023] Retrieved from https://www.rijksoverheid.nl/actueel/nieuws/2019/04/26/salderingsregeling-verlengd-tot-2023 on April 26, 2019. (Cited on page 9.)

[50] J. Rivera, P. Wolfrum, S. Hirche, C. Goebel, and H.-A. Jacobsen, *Alternating direction method of multipliers for decentralized electric vehicle charging control*, in 52nd IEEE Conference on Decision and Control, IEEE, 2013, pp. 6960–6965. (Cited on page 14.)

[51] A. Rosin, *Energy storages*, (2012). [Online] Retrieved from http://www.ene.ttu.ee/elektriajamid/oppeinfo/materjal/AAV0170/EnergyStorages_ArgoRosin_13sept12.pdf on November 28, 2018. (Cited on page 18.)

[52] W. Saad, Z. Han, H. V. Poor, and T. Başar, *Game theoretic methods for the smart grid*, Signal Processing Magazine, 29 (2012), pp. 86–105. (Cited on page 14.)

[53] J. W. Schot, H. W. Lintsen, A. Rip, and A. A. A. de la Bruhèze, *Techniek in Nederland in de twintigste eeuw. Deel 2: Delfstoffen, energie, chemie*, Eindhoven: Stichting Historie der Techniek, (2000). [Dutch: Engineering in the Netherlands in the twentieth century. Part 2: Mineral resources, energy, chemistry]. (Cited on page 11.)

[54] P. Siano, *Demand response and smart grids – a survey*, Renewable and sustainable energy reviews, 30 (2014), pp. 461–478. (Cited on page 12.)

[55] T. van der Klauw, *Decentralized energy management with profile steering – resource allocation problems in energy management*, PhD thesis, University of Twente, May 2017. (Cited on pages 10, 12, 13, 17, 18, 31, 32, and 34.)

[56] ——, *NP-hardness of battery with round-trip efficiency.* [Unpublished], 2018. (Cited on page 39.)