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Increasing the rate of discharge conversations by pharmacy assistants

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Abstract

The main goal of this article is to reduce medication verification errors at Ziekenhuis Groep Twente (ZGT). This is tried by reducing the rate of not happening discharge conversations. The research question is: How to increase the number of discharge conversations at ZGT, in order to reduce medication verification errors? Scenarios are tested with help of a simulation verified by the queueing network analyzer (QNA). The effect of digital phone calls and another server distribution is tested. The combination of these scenarios lead to a 47 % reduction of the rate of not happening discharge conversation.

Keywords: queuing theory, network of queues, pharmacy assistant, medication verification, discharge conversations, QNA

1 Introduction

Despite the relatively high-level quality of the Dutch health care system, it is shown that still patients suffer from medical mistakes, which could eventually lead to death [4]. In order to reduce this number, many activities took place in order to increase attention to the topic and to improve the safety of patients [13]. One of these activities, which is of huge importance, includes the information exchange between patients and health care providers [8]. An important issue contributing to a better information exchange, are pharmacy assistants and the admission and discharge conversation they take [10]. In the hospitals of Ziekenhuis Groep Twente (ZGT) these problems are also observed and the medical staff would like to optimize the processes of medication verification. Although it seems that enough pharmacy assistants are available for admission and discharge conversations, the rate of occurred discharge conversations is still lower than hoped. Therefore, specifically, the goal of this article is to propose improvements for the medication verification process at ZGT, to increase the rate of discharge conversations. In a broader sense, this article proposes a network of multi-server queues, that is able to optimize processes, like the medication verification process. Therefore, the research question of this report is stated as: How to increase the number of discharge conversation at ZGT, in order to reduce medication verification errors?

In the second Chapter the underlying literature of this article will be presented in a short literature review. Thereafter, the process of medication verification is presented together with the model of this process represented by the queueing network with M|M|s queues, the Queueing Network Analyser (QNA) with GI|G|s queues and the Discrete Event Simulation

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(DES). After this, a step-wise verification of the DES will be presented, with help of the Queueing model and the QNA. In chapter 4, the verified DES is used to analyse the real process of medication verification at ZGT. Different scenarios are tested and compared to each other to answer the research question. Finally, the article is ending with a discussion and conclusion.

2 Literature Review

In this chapter, articles are presented related to this research. Furthermore, some underlying articles related to the theory are presented. Kelly [6] presents the first queueing network with different patient types. This is extensively used in the simulation. To verify this simulation the queueing network analyzer (QNA) of Whitt [15] is used. Morover, Whitt writes about the performance of the QNA and the performance of a G|G|m queue [14, 16]. This article is built upon these articles.

Furthermore, this article builds upon other scientific research related to queueing networks and health care systems. Bourne et al. [2] are presenting an article to reduce medications errors. The cause of this reduction is based upon changing the management style. This research could help in finding scenarios in the analysis of the process at ZGT. Nevertheless, this article is not presenting a queueing network. Cochran and Roche [3] are presenting a queueing network for the emergency department and Green and Savin [5] present an article to reduce delays with a queueing approach. Furthermore, Zonderland and Boucherie [18] are implementing the QNA for an outpatient clinic, which is even more useful for this research. For the optimization of the model, the article of Bahadori et al. [1] is really useful. In this article, Bahadori et al. [1] describe how a pharmacy system could be optimized with help of scenarios.

3 The Model

3.1 The Problem

As mentioned above, the process of medication verification at ZGT is considered. The medication verification occurs at one of the two hospitals of ZGT. These hospitals are located in Almelo and Hengelo. The medication of every arriving and leaving patient at the hospital needs to be verified by pharmacy assistants. Important elements of this verification are admission and discharge conversations. These conversations lead to the significant reduction of medication verification errors, which leads to better health of patients [10]. A patient may enter the hospital for two reasons. First, a patient could unplanned enter the hospital. For example, if the patient arrives as a consequence of an emergency. In such a case, the patient arrives at the first aid (spoedeisende hulp (SEH) in Dutch). Another type of an unplanned arrival is at the acute medical assessment unit (acute opname afdeling (AOA) in Dutch) [12]. Both on the SEH and AOA admission conversations take place. Secondly, entering could occur due to a planned appointment, for example, a knee surgery. For planned appointments two types of admission conversations take place. First, the patient could have an admission conversation at the outpatient clinic (pre-operatieve screening (POS) in dutch) a few days before the surgery. In the other case a patient is admitted immediately to the department (DEP). Dependent on the state of the patient, some patient do not have a admission conversation. In such a case, due to, for example, a bad mental illness or physical state of the patient, conversations are not possible. When a patient is transmitted from the SEH to a department and no admission conversation took place at the SEH, this conversation will still occur at the department. This is also applicable for patients arriving at a department without a visit to the POS. In some cases it might be that a admission conversation is taken twice, at the previous department, POS or SEH, and the new department. These may occur since it might be that the medication is not clear after the first admission conversation. Furthermore, the patients at the department in Hengelo always have a conversation by protocol.

Discharge conversations only take place at the departments. In this case the AOA is also seen as a department. So it might be that a patient leaves the AOA with a discharge conversation. Nevertheless, the discharge conversation at the department does not always take place, since nurses send patients home before the official discharge is planned or because the preparation of discharge takes too long. The main reason for a long preparation time is bad communication between pharmacy assistants and doctors. Again, patients in a bad state are not having discharge conversation. Although the conversations does not take place, medication verification occurs, just as in the case with admission conversation. The problem of not occurring discharge conversations, while these could happen is treated in this report.

3.2 Models

One of the ways to model the process of admission and discharge conversation is with help of a discrete event simulation (DES). To verify the DES, three steps will be performed. First, a queueing model will be proposed, in which it will be assumed that all stations/queues are $M|M|s$ queues. For these types of networks, characteristics of this model could be calculated in a simple way. Second, a queueing network analyser (QNA) will be introduced, an algorithm to calculate waiting times for networks with GI|G|s queues [15]. The outcome of the QNA will be compared to the analytically calculated results from step 1 to verify the QNA. Consequently, the QNA could be used to state characteristics of the queueing network with queues other than $M|M|s$ queues, like $GI|G|s$ queues. Finally, the results from the QNA will be used to verify a discrete event simulation of the network with data from ZGT. In the following subsections the queuing model, QNA and the DES will be described in detail with corresponding formulas.

3.2.1 Queuing Model

The queuing model used is represented in Figure 1. Pharmacy assistants are having conversations with patients during admission and discharge. Admission conversations take place at three departments, at the emergency room (SEH), department (DEP) and at the medical assessment unit (POS). After an admission conversation at the SEH or POS, it might happen that the patient has again a conversation at the DEP, see also Section 3.1. The AOA as represented in Section 3.1 will be part of department, as mentioned before. After admission a patient is treated at a department (Dep. Treatment) by nurses. It is assumed that there is always place to treat a patient, so there is no waiting time. That is why a infinite number of servers is assumed. After treatment at the department a discharge needs to be prepared (Dep. Preparation). After preparation, a discharge conversation takes place (Dep. Discharge) or not. This depends on the sojourn time at queue 5 if this sojourn time at queue 5 is too long the patient leaves before the discharge conversation occurred. Furthermore, there is a number of patients that are excluded for discharge conversations at all, for example because of the department they are lying on or the patient does not have any medication.

Each station or queue is defined as i. The external arrival rate and total arrival rate

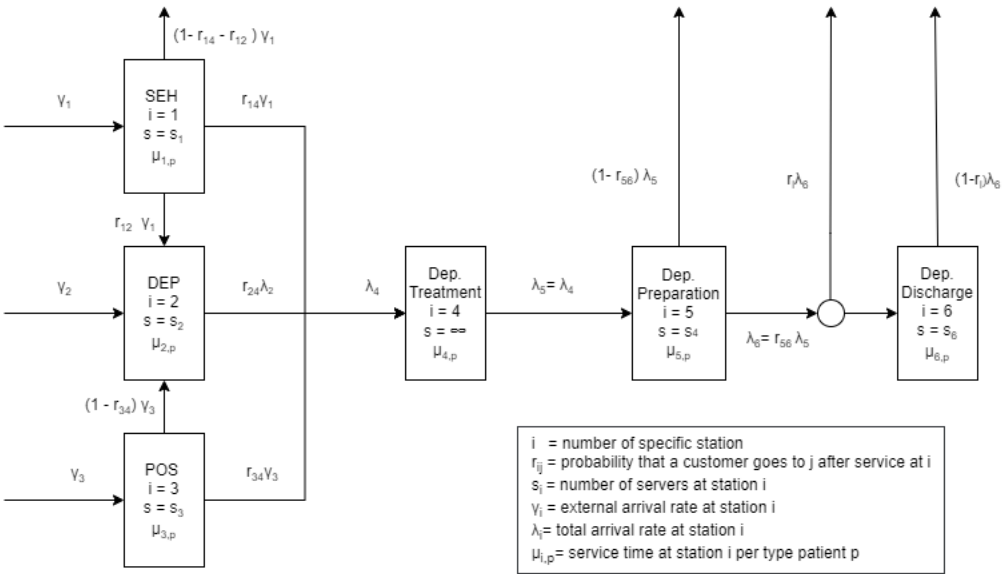

Figure 1: Model of process medication verification

at station i are defined as γ_i and λ_i, respectively. The transition probability from queue i to queue j is defined by r_{ij}. The number of servers at station i is s_i, and the service time at station i of patient type p is defined as $\mu_{i,p}$. The patient types p are based on age, since this states something about the total amount of medicines the patient uses and the ability of the patient to have a clear and concise conversation with the pharmacy assistant. There are in total 4 patient types, children (age 0-18), adults (age 18-50), aged adults (age 50-75), and the elderly (age 75+).

The queues are assumed to be M|M|s queues. Therefore, the arrival rates to the queues could be computed by [17]:

$$\lambda_j = \gamma_j + \sum_{i=1}^{J} r_{ij}\lambda_i \tag{1}$$

in which J is the number of queues, $J = 6$. For the model represented in figure 1 the solution to the system of traffic equations, expressed in the external arrival rates γ_i and the transition rates r_{ij}, is:

$$\begin{aligned}
\lambda_1 &= \gamma_1 \\
\lambda_2 &= \gamma_2 + r_{12}\gamma_1 + r_{32}\gamma_3 \\
\lambda_3 &= \gamma_3 \\
\lambda_4 &= \gamma_1(r_{12} + r_{24}r_{12}) + \gamma_2 r_{24} + \gamma_3(r_{34} + r_{24}r_{32}) \\
\lambda_5 &= \gamma_1(r_{12} + r_{24}r_{12}) + \gamma_2 r_{24} + \gamma_3(r_{34} + r_{24}r_{32}) \\
\lambda_6 &= r_{56}(\gamma_1(r_{12} + r_{24}r_{12}) + \gamma_2 r_{24} + \gamma_3(r_{34} + r_{24}r_{32})).
\end{aligned}$$

There are two options for a patient to leave queue 5. First, the transition probability r_{56} is defined as the the proportion of patient that should have a discharge conversation at queue 6. Second, the fraction $1 - r_{56}$ is the proportion of patients that do not have a discharge conversation at all, mentioned as the proportion of patient excluded by protocol before. Nevertheless, not all patients flowing from queue 5 to 6 are having a discharge conversation. There is a proportion r_l that flows out of the system before getting service at queue 6, see also Figure 1. This fraction r_l is based on the sojourn time at queue 5. If the sojourn time at queue 5 is larger than the accepted sojourn time, $EW_{acc,5}$, then the patient is leaving before service (r_l).

The average waiting time and average queue length at queue i are defined as $E[W_{q_i}]$ and $E[L_{q_i}]$, respectively. Using the traffic equations, the number of servers at station i, s_i, and the assumption that the arrival and services time are exponential, the following is applicable for $i \neq 4$ [17]:

$$E[L_{q_i}] = \frac{\rho_i^{s_i+1}}{(s_i - \rho_i)^2 (s_i - 1)!} \cdot P_{0_i} \tag{2}$$

$$E[W_{q_i}] = \frac{E[L_{q_i}]}{\lambda_i} \tag{3}$$

in which $\rho_i = \frac{\lambda_i}{\mu_i} < s_i$ and P_{0_i} is the steady state probability for 0 customers in queue i, defined as [17]:

$$P_{0_i} = \left(\sum_{n=0}^{s_i-1} \frac{\rho_i^n}{n!} + \frac{\rho_i^{s_i}}{(s_i - \rho_i)(s_i - 1)!} \right)^{-1}. \tag{4}$$

For $i = 4$, there is an infinite number of servers. Therefore, a patient does not wait at queue 4 and so $E[W_{q_4}] = 0$. The average sojourn time at queue i is defined as $E[W_i]$. The formula for the average sojourn time [17]:

$$E[W_i] = EW_{q_i} + \frac{1}{\mu_i}. \tag{5}$$

So knowing these formulas, r_l, could be calculated as

$$r_l = P(W_5 > EW_{acc,5}) \tag{6}$$

in which W_5 is the random variable representing the sojourn time in queue 5.

3.2.2 Queuing Network Analyzer

The queueing network analyser (QNA) is an algorithm, which calculates the average waiting time for networks with GI|G|s queues [15]. The QNA will be verified by the analytical calculations of the queueing model with M|M|s queues. The QNA will be used to verify the discrete event simulation for networks with GI|G|s queues. The verification is presented in Section 4. The algorithm for the queuing network analyzer (QNA) as described by Zonderland and Boucherie [18] is used. This QNA consists of five steps. Full description of this QNA could be found in Appendix B. Eventually, the waiting time at queue j, $E[W_j]$, is calculated as

$$E[W_j] \approx E[W_{M|M|s}] \frac{c_{A,j}^2 + c_{S,j}^2}{2}, \tag{7}$$

where $E[W_{M|M|s}]$ is the expected waiting time of queue j for a M|M|s queue [18]. Furthermore, $c^2_{A,j}$ and $c^2_{S,j}$ are the squared coefficients of variation (scv) of the inter-arrival and service times, respectively. These parameters are calculated in the first four steps of the QNA.

3.2.3 Discrete Event Simulation

A good way to replicate the original process of admission and discharge conversation is using a discrete event simulation (DES). As stated before, this DES will be verified by the QNA. The DES is implemented as illustrated in Figure 2. The general idea of the DES is adding and removing events from the event-list until the event StopSimulation is reached. There are five types of events which could be added and removed from the event-list: Arrival, Arrival2, ArrivalQ6 StartService and EndService. These events are illustrated as big squares in Figure 2. Each event needs a certain number of parameters to work. The arrival event needs a time, the type of the patient and the number of the queue. For StartService and EndService an extra input should be given, the number of the servers (see also Figure 2). This number ranges from 1 to s_k, the number of servers at queue k. ArrivalQ6 needs a time, type of a patient and the sojourn time of that patient in queue 5. Another important variable in the DES is the queue. The number of queues ranges from 1 to 6, since there are 6 queues in our queuing model. In the DES the queue is a dictionary with the numbers 1 to 6 as the keys. The values are lists of patients in the system, so the patient being served plus the waiting patients. The number of types are the same as defined in section 3.2.1. Within Figure 2, schedule 'Event' means the insertion of an event in the event-list. After implementation of an event, the event is deleted from the event list. After deletion, the next events is executed. This process continues until the event StopSimulation is reached. This event is based on the end time put in before starting the simulation.

To start the DES, some arrivals needs to be implemented by hand. Once one arrival is planned, for a specific type p, and at a specific q, i, the DES automatically plans new arrivals of that specific patient type at that specific queue. This means that for the earlier presented queuing model, an arrival should be planned at time 0, for type $1-4$, at queue 1, 2 and 4. Since there are four types and there are external arrivals at queue $1-3$. So in total, twelve events should be inserted, just to initiate the DES. Since after an arrival, automatically a new arrival is planned based on some arriving distribution, or as will be done later in this article, based on a list of data. Furthermore, within executing the event Arrival, after planning a new arrival, the number of servers at queue k will be compared with the queue length at queue key k (remember the queue is a dictionary). If this number of servers is bigger than the total length of the queue, it will be checked which server is available. After this available server i is found, out of a range 1 to s_i servers, StartService is planned for a certain patient at server i and the patient is put in the queue. If there is no server available, the patient will be placed in the queue with server $i = 0$. As probably noticed and mentioned above, putting a patient in a queue does not mean that he is already served, it is dependent on the number of server i if the patient is served. This character i may be confusing, since this character is also used for the queues. In the description of the DES the character k is used for queues.

Within StartService, only EndService will be scheduled. The time of EndService is based on the distribution of the service times. These distribution might, for example, be exponential, uniform or any other distribution of the service rates. During the execution of the event EndService, a patient of a certain type at an certain queue, will be deleted from the queue. After deletion, the number of servers will be compared with the length of

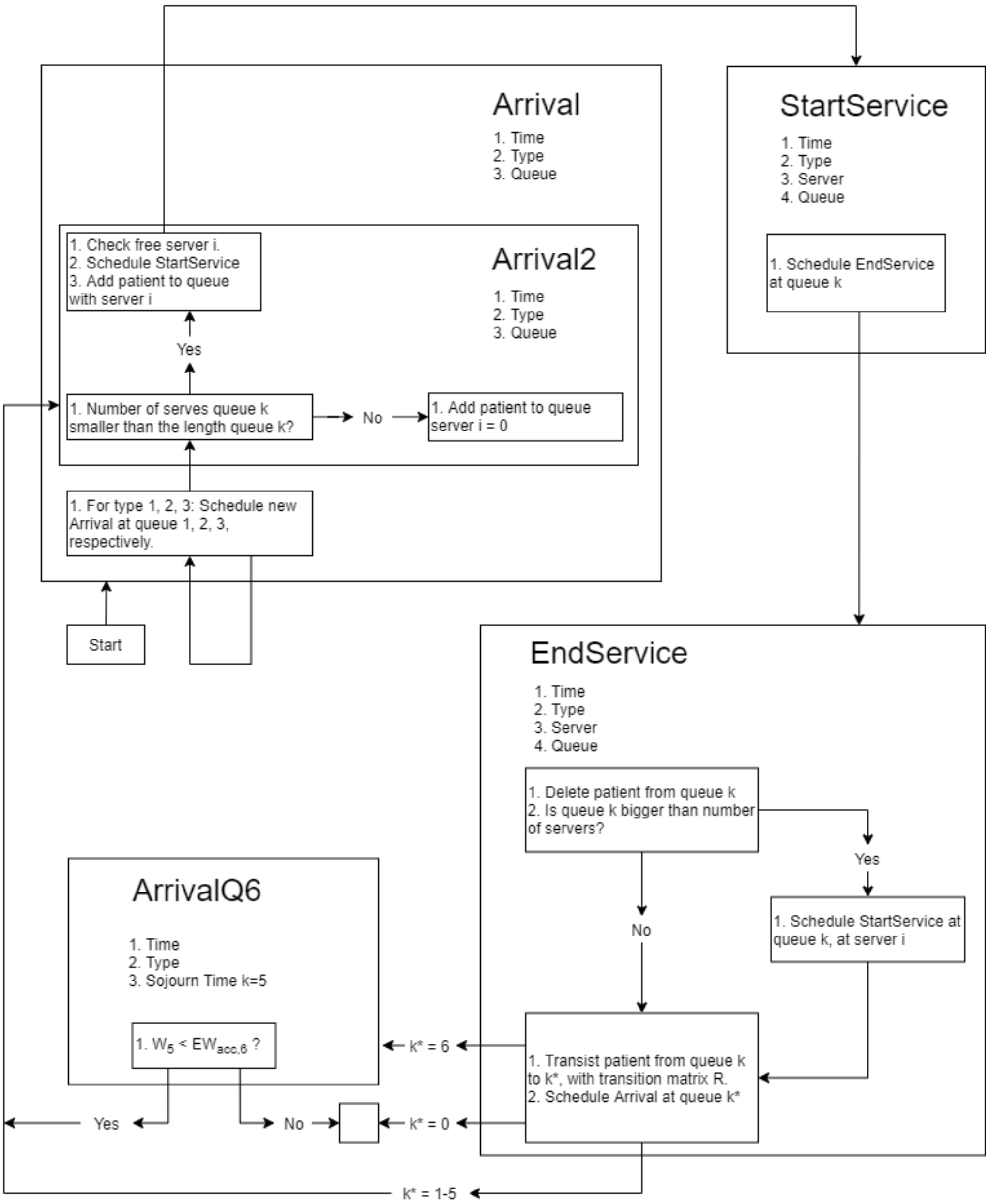

FIGURE 2: Flowchart Discrete Event Simulation (DES)

the queue. If this queue is bigger than the number of servers than the event StartService will be inserted. After this or if the queue is smaller than the number of service the patient will be transmitted to another queue. This transition is based on a transition matrix R . This matrix have to be put in before the simulation is started. With help of a random number generator, the new queue will be chosen, with help of R. If a patient is transmitted to another queue k^*, an Arrival2 event at queue k^* is inserted. Arrival2 is part of Arrival and is almost the same as Arrival. The difference between Arrival and Arrival2 is that at Arrival2 a new arrival is not rescheduled. The transition matrix has also the possibility to transmit a patient to queue 0, which is meaning that the patient goes out of the system. The transition from queue 5 to queue 6 is special, because this transition depends on the sojourn time at queue 5. If k^* is equal to 6, the event ArrivalQ6 is inserted in the eventlist. Within the ArrivalQ6 event it is checked if the sojourn time of the patient coming from queue 5 is smaller than the accepted waiting time, $EW_{acc,5}$. If this is the case the patient is following the same procedure as in Arrival2. If this is not true, the patient will leave the system, so $k^* = 6$ will be changed into $k^* = 0$. Since the QNA does not satisfy this possibility, for verification a accepted waiting time is chosen, such that every patient arriving at queue 6 will also be served.

4 Results Verification

4.1 Queueing model with M|M|s queues

In this subsection the waiting time is calculated for a network of M|M|s queues. For this purpose, the formulas as presented in section 3.2.1 are used, formulas 5 and 4. The exact numbers of the verification are stated in appendix A. The resulting waiting times are displayed in table 1.

	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
$E[W_j]$	0.0723	0.0001	0.5333	NaN	0.2164	3.5072

Table 1: Calculated waiting times for M|M|s queues at station i.

These numbers are calculated with the programming language Matlab and are four-decimal precise. For $i = 4$ Matlab gives NaN, not a number. Since there are an infinite number of servers at queue 4, which does not satisfy the formula 5. The average waiting time at queue 4 is equal to 0, as stated earlier.

4.2 Queueing Network Analyser with M|M|s queues

In this section the correctness of the QNA is verified. After the verification, the QNA will be used to compute the waiting times for networks of other types of queues, like $G|GI|s$ queues. The steps of the QNA as stated in [18] are performed, see also appendix B. The QNA is implemented in the programming language Matlab. The inputs for the QNA are:

γ: external arrival rates at each queue i, γ_i.

R: transition matrix, with entries r_{ij}, transitions from queue i to queue j.

s: number of servers at each queue i, s_i.

μ: services rates at each station i, mu_i

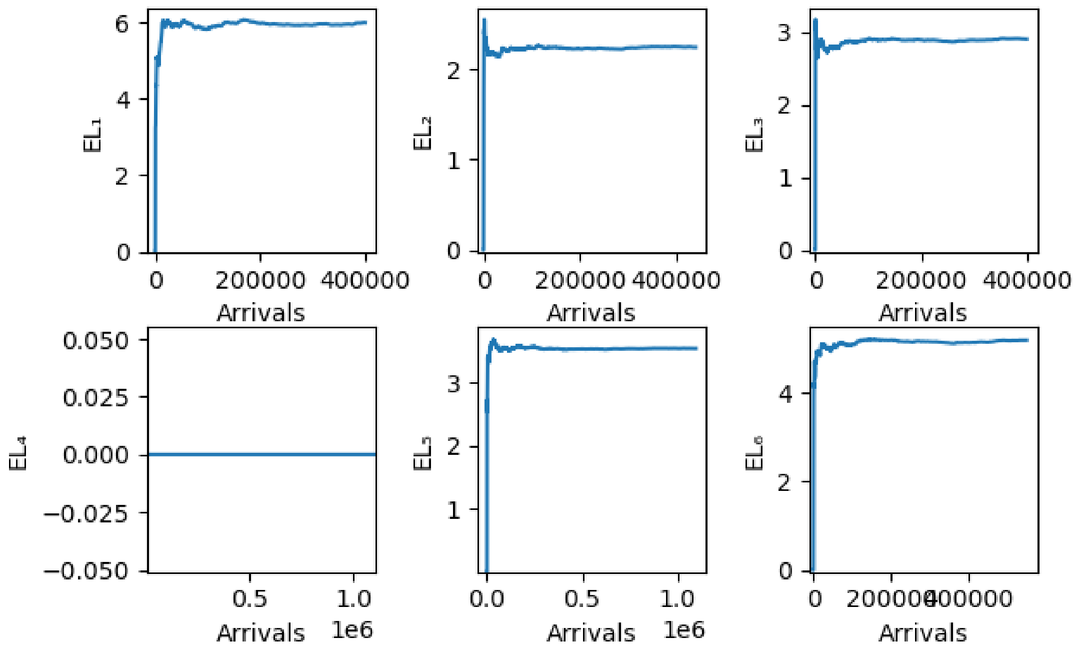

FIGURE 3: Average waiting queue length over the number of arriving patients.

VarExt: variance of external inter-arrival rates.

VarS: variance of service times.

The exact filled in numbers could be found in Appendix A, as stated before. The QNA model should give the same results as the analytical calculations, when the same inputs are used. The results of the QNA are indeed the same as the analytic calculations, exposed in table (1).

4.3 Discrete Event Simulation with M|M|s and G|GI|s queues

When verifying the discrete event simulation, first a good warm-up period needs to be defined. This warm-up period is defined by looking at the average waiting queue length over time. An example of a picture in which this is represented, is in Figure 3

After this warm-up period steady state is reached. Therefore, within a picture this warm-up period could approximately be seen. From Welch's graphical approach the following formula could be deviated [7]:

$$\left| \frac{\frac{1}{2d}\sum_{k=1}^{2d} D_k}{\frac{1}{d}\sum_{k=1}^{d} D_k} - 1 \right| < \alpha \tag{8}$$

with

$$D_k = \frac{1}{N}\sum_{n=1}^{N} D_{nk} \tag{9}$$

in which N is the number of runs, D_{nk} is the observation of the k-th observation in the n-th run, and d is the index of observation k. The parameter alpha is determined by the graphical representation. In the following simulations the value for α are ranging between 0.025 and 0.0001. For the waiting times after the warm-up period 95% confidence intervals are set up, by the following equation [9]:

$$95\% - CI = \left(\bar{X} - c * \frac{S}{\sqrt{N}}, \bar{X} + c * \frac{S}{\sqrt{N}} \right). \tag{10}$$

In this equation $\bar{X}$ is the sample mean, c is the critical value for $\alpha/2$, two tailed t-test, and S is the sample standard deviation. The confidence intervals are based on 20 runs, $n = 20$. Dependent on the distribution of the inter-arrival and service times the DES takes about 3 to 7 minutes to run for t = 100,000. It is assumed during verification that one time unit is equal to a quarter. When simulating the real situation one time unit is equal to a hour. The t-distribution is used to determine the confidence intervals with 19 degrees of freedom, namely $20 - 1$, so $c = 2.093$ [9]. The sample mean and sample variance are calculated over data after the warm-up period. The discrete event simulation (DES) should give a confidence interval for the average waiting time at station i. First it will be checked if the DES verifies the M|M|s queues. For all the queues the waiting time for de QNA and DES are given. Moreover, a 95%-CI is given in table 2, for the results of the DES. The data as described in Appendix A is used as input.

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
i = 1	0.0723	0.0720	(0.0713, 0.0728)
i = 2	0.0001	$1.0983 * 10^{-4}$	$(0.9929 * 10^{-4}, 1.2037 * 10^{-4})$
i = 3	0.5333	0.5359	(0.5318, 0.5400)
i = 4	NaN	0.0000	(0.0000,0.0000)
i = 5	0.2164	0.2154	(0.2123, 0.2185)
i = 6	3.5072	3.4808	(3.3808, 3.5809)

TABLE 2: Waiting times for QNA and DES for M|M|s queues.

The confidence intervals as presented in Table 2 are all falling around the real mean, meaning the means calculated with the QNA. So the DES is verified for M|M|s queues. Actually this part should be about verifying general inter arrival and service times. Therefore, the QNA and the DES are compared for uniform service times and uniform inter-arrival times. It should be noticed that the QNA is also a approximation for $GI|G|s$ queues, which becomes better when the load, $\rho = \frac{\lambda}{s\mu}$, converges to one [15]. Therefore other service rates are used then the previous cases. These new input numbers could again be found in Appendix A. Nevertheless, the load at queue 4 will never converge to 1, since it is a infinite server queue, so this may lead to a wrong approximation. Another changing variable in Appendix A is the variance since the variance of a uniform distribution differs from the variance of a exponential distribution [11]. In Table 3 the results for GI|M|s queues are given with an uniform arrival process. For the queues 1 to 3 the results for the QNA and the DES are almost the same. The results for queue 5 and 6 differ quite a lot between the DES and the QNA. This may be a result of the fact that queue 4 has an infinite number of servers [15]. Furthermore although the loads for the queues are changed these are still about 0.9, and could converge further to 1.

The big errors for queue 5 and 6 coming also back for a network of M|G|s queues, see Table 4. There is also a big error at queue 5 for a network of GI|G|s queues, see Table 5.

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
k = 1	6.0758	6.0408	(5.8489, 6.2327)
k = 2	1.9992	1.8529	(1.7885, 1.9173)
k = 3	2.8860	2.8875	(2.8065, 2.9686)
k = 4	NaN	0.0000	(0.0000,0.0000)
k = 5	1.5914	1.3686	(1.3470, 1.3903)
k = 6	3.5057	3.2774	(3.1925, 3.36236)

TABLE 3: Calculated waiting times for uniform inter arrival times (GI|M|s queues).

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
k = 1	6.0758	5.9585	(5.6450, 6.2720)
k = 2	1.9035	1.9479	(1.9002, 1.9956)
k = 3	2.8860	2.9365	(2.8715, 3.0015)
k = 4	NaN	0.0000	(0.0000,0.0000)
k = 5	1.0416	0.7774	(0.7652, 0.7899)
k = 6	2.1141	2.0414	(1.9295, 2.1533)

TABLE 4: Calculated waiting times QNA and DES for uniform service times (M|G|s queues).

If the hypothesis is true that the big errors in Tables 3, 4, and 5 are because of the infinite server queue 4, this error should disappear when the infinite number of servers is replaced by a finite number or if queue 4 is deleted from the network. Both options are tested for uniform inter-arrival and service times. Furthermore, within these new test the loads are increased to about 0.95 for each queue. The new input values could again be found in Appendix A. The results of these two test could be found in Table 6 and Table 7.

The two new tests also result in big errors for the last two/three queues. This might still be a result of the load, which could still converge more to 1. On the other hand the waiting times for queue 1 to 3 in Tables 6 and 7, did not become significantly more precise than the waiting times in Table 3, 4 and 5. It is hard to state if this is applicable for queue 4,5, and 6, since in contrast to queue 1, 2, and 3, the queues, 4,5, and 6 do not have external arrivals and the effect of external arrivals are not clear. Although the QNA is able to handle networks with G|GI|s queues [15], the performance of these networks are not tested [14]. Whitt [14] is testing the performance of networks of G|GI|1 queues. In addition, Whitt [16] is exposing approximations for one single GI|G|s queue. So the performance of the QNA for a network G|GI|s queues is not exactly known. More about this issue is in the discussion. Although the simulation does not equal the QNA approximations for general inter-arrival and service times, the results in 2 and the fact that the QNA is almost true for the queues 1, 2 and 3 are convincing enough to state that the DES is verified well enough. Summarized, the following two reasons are also stating why the verification is well enough. First, if there is a mistake in the simulation, it should have been noticeable in Table 2. Second, if by some reason the simulation is still not correct due to some other reason, this is also not needed. Since scenarios are compared in the analysis, the focus is on the proportion of differences between scenarios, not on precise waiting times. Furthermore, it is not the case that the error is ten times bigger or ten times smaller, the error is less than 60%.

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
k = 1	3.0379	3.0057	(2.8446, 3.1669)
k = 2	1.0439	0.9611	(0.9430, 0.9792)
k = 3	1.4430	1.3751	(1.3527, 1.3975)
k = 4	NaN	0.0000	(0.0000,0.0000)
k = 5	1.0328	0.6113	(0.6058, 0.6168)
k = 6	2.1125	1.8027	(1.7460, 1.8593)

Table 5: Calculated waiting times for QNA and DES with uniform inter arrival and service times (GI|G|s queues).

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
k = 1	3.0379	2.9414	(2.8595, 3.0233)
k = 2	2.8431	2.7264	(2.6430, 2.8098)
k = 3	3.0972	3.0768	(2.98104, 3.1726)
k = 4	1.7695	1.3223	(1.2837, 1.3608)
k = 5	1.8428	1.2254	(1.2013, 1.2494)
k = 6	4.2482	3.5901	(3.4412, 3.7389)

Table 6: Calculated waiting times for network of 6 finite server G|GI|s queues.

5 Results Analysis

Since sufficient reasons are given why the DES could be used to simulate the real situation, the DES could be used to test the real system. By the idea of Bahadori et al. [1], scenarios are tested and compared to each other. The scenarios that are going to be tested are listed below. These scenarios are chosen since, it is indicated as a solution for the problem by pharmacy assistants and/or it is thought of as a solution myself during the internship at ZGT or during the process of this research.

S_0: The original, real scenario.

S_1: Conversations are replaced by digital phone calls.

S_2: The number of servers per station is changed.

S_3: The scenarios S_1 and S_2 together.

First, scenario S_0 is discussed, together with the relating parameters. These parameters include the number of patient types, the arrival and service rates for each patient type, the transition matrix for each patient type, the distribution of pharmacy assistants over the different queues, the working times and the accepted sojourn time at queue 5, $EW_{acc,5}$. The exact numbers of some of these parameters could be found in Appendix C. Two types of data-files are delivered to estimate these values. First, data of the admissions and discharges are delivered by the application managers at ZGT. The most important parts of this data includes the arrival time, the leaving time and the age of a patient. The age is used to categorize patients in four patient types. The arrival time and the leaving time is used to determine the mean service time and the variance of the service time at queue 4. Furthermore, the arrival time is used in order to create a list of arrivals to the system

	$E[W_j]$		
Queue	QNA	DES	95%-CI DES
k = 1	3.0379	2.9363	(2.8449, 3.0277))
k = 2	2.8431	2.7406	(2.6343, 2.8470)
k = 3	3.0972	3.0397	(2.9244, 3.1551)
k = 4	1.7695	1.2890	(1.2636, 1.3144)
k = 5	4.2420	3.5445	(3.4488, 3.6403)

Table 7: Calculated waiting times for network of 5 G|GI|s queues.

according to the working times. The pharmacy assistants are working each day a week from 8:00-17:30 with breaks from 10:00-10:20, 12:00-12:30, and 15:00-15:20. These breaks are also included in the arrival times. Even though some departments are having breaks in turns. Within the transitions between queues also extra time is included when the end time of a service is not during working times. This extra time is included in the end-service time. The transition rates are also deviated from the first Data-file. Some transitions could not be extracted from the data-file like, r_{10} and r_{12}. These transitions are set to 0. The external arrivals are also treated a bit different. There are only probabilities that a patient came via the SEH, DEP or POS. Therefore, a random generator generates a number from 1 to 3 with these probabilities, whenever an Arrival Event needs to schedule a new arrival. Instead of always reschedule an Arrival at a certain queue, k = 1,2,3. A new arrival is scheduled from a list, therefore, the warm-up periods are almost not included for the real case simulations.

Furthermore, there is a second data-file that delivers information about the service times. This second file is delivered by pharmacy assistants. Since the response of the pharmacy assistants was low in a very short time frame, the variances of the services are adapted by hand, to prevent extreme variances. These assistants had to fill in the service times for each patient type for the POS, SEH, department and AOA, for admission and discharge. The service times consisted out of three parts, preparation, walk to/of the patient and the conversation itself. Later the service times for the AOA and department are added for the DEP. This is done with help of data-file 1, from which the proportion of patients at the AOA could be extracted. In the DES the service times are uniform distributed. Therefore, it was also needed to reduce the variance of the service times since, otherwise, the service times could have become negative. There is not a uniform distribution at queue 4. The service time for queue 4 is log-normal distributed. A well-known distribution for estimating the stay at the hospital. From the second data-file together with knowing how the number of pharmacy assistants in the weekend differ, a distribution of servers over the different queues could be made. Since pharmacy assistants are handling both admission and charge conversations in some cases the number of servers is not an integer. In such cases, the number of pharmacy assistants is rounded to an integer. Again, all the exact parameters could be found in C.

The only parameter that is not defined yet is the accepted sojourn time at queue 5, $EW_{acc,5}$. This variable is defined with help of the DES. From the first data-file the r_l could be determined for each patient type p. This the proportion of patients that leave the hospital, before a necessary conversation took place. The results are presented in Table 8 and are rounded to 4 decimals. These fractions are combined to one value for r_l, since in this research this combined fraction is tried to reduce. There is no priority in patient types. The total fraction r_l is computed with help of the fractions of patient types arriving at queue 6. This is resulting in $r_l = 0.3691$. To simulate this rate with the simulation, the

Type	0-17	18-49	50-74	75+
r_l	0.6494	0.4305	0.3297	0.3700

TABLE 8: Proportion of leaving patients before discharge conversation.

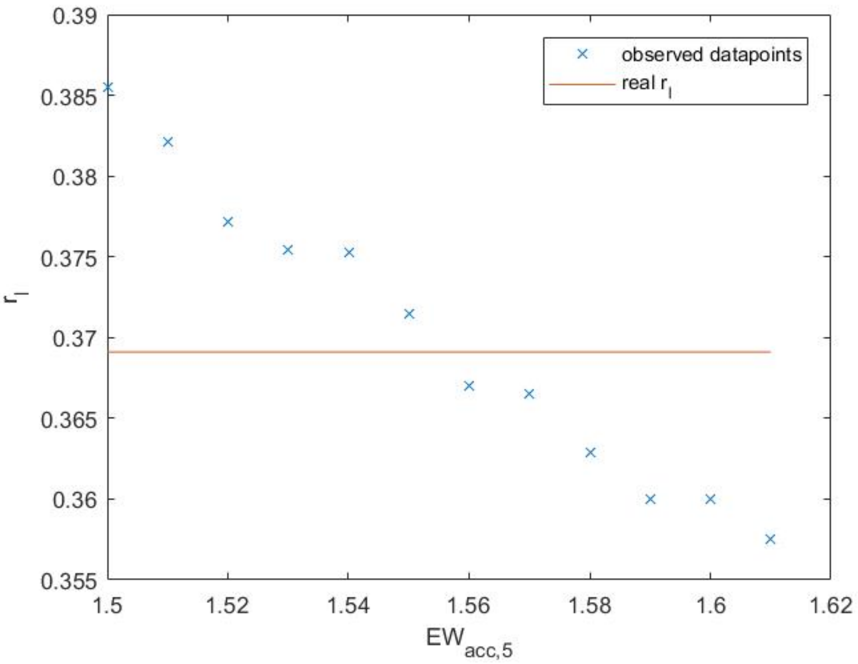

FIGURE 4: The probability r_l for different accepted sojourn times at queue 5.

simulation is run with a range of different accepted sojourn times. During this simulation the r_l is also estimated. The accepted sojourn time that leads to the least error with the real $r_l = 0.3691$, is taken as real value for the accepted sojourn time. The values for the accepted sojourn times ranges between 1.5 and 1.61. For these values, the r_l could be simulated, see Figure 4. From this test, the accepted sojourn time at queue 5 is chosen to be on $EW_{acc,5} = 1.555$. This sojourn time is used to test the other scenarios. The estimated r_l is in this case estimated by 0.3715, see also the results for scenario S_0 in Table 9.

Scenario S_0 is described above with all related parameters. In scenarios S_1, S_2 and S_3 these parameters are changed, and the outcomes for r_l are compared with S_0. In the first scenario, S_1, the effect of replacing a part of the conversations by digital phone calls is tested. When a digital phone call takes place the mean service time decreases. The walking time is about 1 to 2 minutes. The average time a patient picks up the phone is estimated about 30 seconds, 0.5 minutes. So the service time changes at the queues 1, 2, 3 and 5. In scenario S_1 three cases are tested.

C_1: Testing when 50% of the conversations are replaced by digital phone calls.

C_2: Testing when 75% of the conversations are replaced by digital phone calls.

C_3: Testing when 100% of the conversations are replaced by digital phone calls.

The results are exposed in Table 9. The second scenario S_2 is based upon another distribution of servers at the queues. When a pharmacy assistant is shifted from queue k

to queue k^*, the waiting time at queue k is increasing, but the waiting time at queue k^* is decreasing. The waiting time of queue 5 needs to decrease in order to reduce r_l. The waiting time at the SEH should not increase, since emergencies are important to cover quick. So two cases are possible. The first case: remove a pharmacy assistant from the POS and add that pharmacy assistant to the Dep. Preparation. The second case: remove a pharmacy assistant from the DEP and add this assistant to Dep. Preparation. Another possibility would be to check the first case with the condition that every patient leaving from the POS is not checked again at the DEP, so this means that $r_{32} = 0$. This results in a lower waiting time at queue 2. For case 2 this needs not to be checked. Since this extra rule, $r_{32} = 0$, does not lead to changes in the waiting time of the POS. In total there are three cases to be checked:

C_1: At DEP, 3 pharmacy assistant and at Dep. Preparation 4. ($r_{32} \neq 0$)

C_2: At POS 2 pharmacy assistant and at Dep. Preparation 4. ($r_{32} \neq 0$)

C_3: At DEP, 3 pharmacy assistant and at Dep. Preparation 4. ($r_{32} = 0$)

In the original process there are 4 pharmacy assistants at DEP, 3 pharmacy assistants at the POS and 3 pharmacy assistants at the Dep. Preparation. The results are again exposed in Table 9. In scenario S_3 the scenarios S_1 and S_2 are combined. Since there are three cases for S_1 and three cases for S_2, there are in total nine cases for S_3. These cases are defined as $C_{m,n}$, where m is the corresponding case in S_1 and n the corresponding case in S_2. The results of this last scenario are also exposed in Table 9. The overall reduction of r_l in percentages is presented in Table 10. As expected the leaving probability, r_l, reduces when scenario S_1, S_2, and S_3 are applied. The combination of scenario S_1 and S_2 in scenario S_3 seems the most effective, especially case $C_{3,2}$.

Scenario	Case	r_l	95%-CI	Queue	EW_j	95%-CI
S_0	C_1	0.3715	(0.3697, 0.3734)	k = 1	0.0114	(0.0094, 0.0135)
				k = 2	0.9533	(0.9500, 0.9566)
				k = 3	0.0960	(0.0879, 0.1042)
				k = 5	1.0644	(1.0604, 1.0685)
				k = 6	0.0215	(0.0205, 0.0224)
S_1	C_1	0.3514	(0.3496, 0.3532)	k = 1	0.0099	(0.0084, 0.0114)
				k = 2	0.9109	(0.9078, 0.9140)
				k = 3	0.0919	(0.0879, 0.0959)
				k = 5	1.0117	(1.0076, 1.0157)
				k = 6	0.0258	(0.0247, 0.0269)
	C_2	0.3368	(0.3352,0.3385)	k = 1	0.0120	(0.0104, 0.0136)
				k = 2	0.8877	(0.8848, 0.8907)
				k = 3	0.0960	(0.0915, 0.1005)
				k = 5	0.9784	(0.9746, 0.9821)
				k = 6	0.0270	(0.0262, 0.0278)
	C_3	0.3283	(0.3266, 0.3302)	k = 1	0.0091	(0.0078, 0.0104)
				k = 2	0.8690	(0.8662, 0.8718)
				k = 3	0.0995	(0.0946, 0.1045)
				k = 5	0.9580	(0.9542, 0.9618)
				k = 6	0.0291	(0.0283, 0.0300)

Scenario	Case	r_l	95%-CI	Queue	EW_j	95%-CI
S_2	C_1	0.2327	(0.2310, 0.2343)	k = 1	0.0099	(0.0084, 0.0114)
				k = 2	1.6230	(1.6148, 1.6313)
				k = 3	0.1031	(0.0971, 0.1090)
				k = 5	0.7335	(0.7307, 0.7362)
				k = 6	0.0662	(0.0648, 0.0676)
	C_2	0.2283	(0.2265,0.2300)	k = 1	0.0101	(0.0087, 0.0114)
				k = 2	0.9514	(0.9484, 0.9544)
				k = 3	0.2144	(0.2078, 0.2209)
				k = 5	0.7254	(0.7226, 0.7283)
				k = 6	0.0677	(0.0660, 0.0695)
	C_3	0.2306	(0.2289, 0.2322)	k = 1	0.0096	(0.0082, 0.0110)
				k = 2	1.6020	(1.5936, 1.6104)
				k = 3	0.0897	(0.0852, 0.0942)
				k = 5	0.7306	(0.7276, 0.7336)
				k = 6	0.0675	(0.0658, 0.0691)
S_3	$C_{1,1}$	0.2106	(0.2091, 0.2121)	k = 1	0.0109	(0.0095, 0.0122)
				k = 2	1.5384	(1.5317, 1.5451)
				k = 3	0.0927	(0.0875, 0.0979)
				k = 5	0.6994	(0.6964, 0.7024)
				k = 6	0.0789	(0.0767, 0.0811)
	$C_{1,2}$	0.1978	(0.1961, 0.1994)	k = 1	0.0092	(0.0079, 0.0106)
				k = 2	1.4887	(1.4834, 1.4939)
				k = 3	0.0878	(0.0831, 0.0925)
				k = 5	0.6783	(0.6755, 0.6811)
				k = 6	0.0884	(0.0865, 0.0904)
	$C_{1,3}$	0.2117	(0.2100, 0.2134)	k = 1	0.0101	(0.0086, 0.0115)
				k = 2	1.5124	(1.5076, 1.5171)
				k = 3	0.0877	(0.0830, 0.0924)
				k = 5	0.6994	(0.6966, 0.7022)
				k = 6	0.0775	(0.0761, 0.0790)
	$C_{2,1}$	0.1978	(0.1963, 0.1993)	k = 1	0.0078	(0.0063, 0.0092)
				k = 2	1.4892	(1.4840, 1.4944)
				k = 3	0.0897	(0.0856, 0.0937)
				k = 5	0.6788	(0.6763, 0.6814)
				k = 6	0.0888	(0.0868, 0.0909)
	$C_{2,2}$	0.1945	(0.1929, 0.1960)	k = 1	0.0099	(0.0085, 0.0114)
				k = 2	0.8872	(0.8844, 0.8899)
				k = 3	0.2184	(0.2108, 0.2259)
				k = 5	0.6718	(0.6691, 0.6745)
				k = 6	0.0877	(0.0855, 0.0900)
	$C_{2,3}$	0.1968	(0.1953, 0.1984)	k = 1	0.0095	(0.0079, 0.0111)
				k = 2	1.4646	(1.4605, 1.4687)
				k = 3	0.1023	(0.0960, 0.1085)
				k = 5	0.6760	(0.6735, 0.6785)
				k = 6	0.0874	(0.0852, 0.0896)
	$C_{3,1}$	0.1861	(0.1845, 0.1878)	k = 1	0.0092	(0.0077, 0.0103)

Scenario	Case	r_l	95%-CI	Queue	EW_j	95%-CI
				k = 2	1.4584	(1.4534, 1.4634)
				k = 3	0.0880	(0.0841, 0.0918)
				k = 5	0.6617	(0.6588, 0.6645)
				k = 6	0.0948	(0.0926, 0.0970)
	$C_{3,2}$	0.1851	(0.1835, 0.1868)	k = 1	0.0098	(0.0081, 0.0114)
				k = 2	0.8665	(0.8635, 0.8694)
				k = 3	0.2139	(0.2072, 0.2206)
				k = 5	0.6563	(0.6535, 0.6591)
				k = 6	0.0955	(0.0932, 0.0977)
	$C_{3,3}$	0.1878	(0.1862, 0.1894)	k = 1	0.0109	(0.0089, 0.0129)
				k = 2	1.4330	(1.4282, 1.4378)
				k = 3	0.0923	(0.0876, 0.0969)
				k = 5	0.6619	(0.6594, 0.6645)
				k = 6	0.0938	(0.0916, 0.0959)

TABLE 9: Results for different scenarios

Scenario	Case	Reduction r_l	95%-CI
S_1	C_1	5.41 %	(0.0493, 0.0590)
	C_2	9.34 %	(0.0888, 0.0977)
	C_3	11.63 %	(0.1112, 0.1209)
S_2	C_1	37.36 %	(0.3693, 0.3782)
	C_2	38.55 %	(0.3809, 0.3903)
	C_3	37.93 %	(0.3750, 0.3838)
S_3	$C_{1,1}$	43.31 %	(0.4291, 0.4371)
	$C_{1,2}$	46.76 %	(0.4633, 0.4721)
	$C_{1,3}$	43.01 %	(0.4256, 0.4347)
	$C_{2,1}$	46.76 %	(0.4635, 0.4716)
	$C_{2,2}$	47.64 %	(0.4724, 0.5192)
	$C_{2,3}$	47.03 %	(0.4659, 0.4743)
	$C_{3,1}$	49.91 %	(0.4945, 0.5034
	$C_{3,2}$	50.17 %	(0.4972, 0.5061)
	$C_{3,3}$	49.45 %	(0.4902, 0.4988)

TABLE 10: Reduction of r_l in percentages.

6 Discussion

This discussion consists out of theoretical and practical implications. The discussion is started with theoretical implications. In the theoretical implications, the limitations of this report are discussed. These limitations are opportunities for future research. The limitations are split into three parts. The first part is about future research in ZGT. The second part is about how to improve this specific research. The final part is about what future research that could take place in mathematical sense.

In order to reduce medication verification errors at ZGT, in future research the focus could be shifted from increasing the discharge rate to, for example, increasing the admission rate or on how to improve bad communication between doctors, nurses and pharmacy assistants, which could also lead to medication verification errors. It is strongly advised that in such a case, first the notation of admission and discharge become more consequent. It is hard to extract data from the available data, since some same cases having different notations. Furthermore, future research would be easier if the information about patients is organized in timelines. So per timeline the admission and discharge of a patient, including his visit to the SEH, POS or department. This is also one of the points, which could lead to a better results in this specific research. Another point of improvement would be to estimate the service times better, The estimation of the service times are now based on three pharmacy assistants, and these values are much deviating. Furthermore, the results of this research could be improved by a better distribution of servers or with the option to let servers work for a proportion of the day. Next to a better distribution of servers, the research results could also be improved by including more patient types. For example, types per age per department. A prioritizing of the patients would also improve this research. Mathematically, an important step needs to be done in analyzing the performance of the QNA. Which factors leads to better estimations of the QNA. In such a way simulation could be verified better. The verification as described in chapter 4 was not optimal because of this point. In future research, the verification may be extended as well.

In the practical implications, it is discussed what the results of this research could mean in practice. First, it is important to mention that the results in this report are not exact numbers for the real problem, since some assumptions are made, which may be incorrect, see theoretical implications. This report is about the relationship between scenarios. The results for scenario S_3 are the best, especially the cases $C_{3,1}$, $C_{3,2}$ and $C_{3,3}$. Nevertheless these cases are based on the case in scenario S_1 that all conversation become digital. This way of taken conversations may also lead to medication verification errors. Since elderly people may not understand the process of digital phone calls. Moreover, the expression of a patient is a good indicator for the pharmacy assistant to trust a patient during medication verification. This recognizing of expressions may be lost when applying digital phone calls. Furthermore, the assumption is made that the average time to pick up a phone is 30 seconds, which may also deviate much, for example, when a patient does not pick up the phone at all. Summarized, by the previous mentioned reasons, the cases $C_{3,n}$ are not advised. The advise would be to start in ZGT with implementing the first two cases of S_1, to see the effect of digital phone calls on the leaving rate r_l. After this testing period other scenarios/cases could be added. Especially, case $C_{1,2}$ and $C_{2,2}$ are advised, since the waiting time at the POS is relatively low. Furthermore, with the simulation program other scenarios could be tested. For example, it could be tested what the effects are of changing working times. Any scenario that is based on changing the arrival rates, service rates, server distribution, or working times, could be tested.

7 Conclusion

In this article, the process of medication verification at ZGT is tested, with help of a simulation. The simulation is verified by the queueing network analyzer. It is advised to implement the cases $C_{1,2}$ and $C_{2,2}$. These cases include testing the process when 50 / 75 % of the conversations are digital and when a pharmacy assistant is shifted from the POS to discharge preparation. Other scenarios could also be tested on advise of the simulation presented in this report. Shifting the working times may lead, for example, to a decrease of the waiting times, see Appendix D. The scenario that working times are split in two parts may also lead to an increasing waiting time.

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A Filled in numbers verification

In Table 11, the "1", "2" and "3" after rate, variance and after servers, are representing three situations. The first situation is verification with a infinite server queue at position 4. The second situation represents verification with queue 4 as a finite server 4. In situation three, the infinite queue is left out. So there are only 5 queues.

	Queues		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
External	M|M|s	Rate 1	2.0	2.0	2.0	0	0	0
rates		Var 1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0
	GI|M|s	Rate 1	2.0	2.0	2.0	0	0	0
		Var 1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0	0	0
	M|G|s	Rate 1	2.0	2.0	2.0	0	0	0
		Var 1	$\frac{1}{4}$	$\frac{1}{4}$	1/4.0	0	0	0
	GI|G|s	Rate 1/2/3	2.0	2.0	2.0	0	0	0
		Var 1/2/3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0	0	0
Service	M|M|s	Rate 1	1.5	1.5	1.5	1.5	1.5	1.5
rates		Var 1	$\frac{1}{2.25}$	$\frac{1}{2.25}$	$\frac{1}{2.25}$	$\frac{1}{2.25}$	$\frac{1}{2.25}$	$\frac{1}{2.25}$
	GI|M|s	Rate 1	0.7	0.35	1.1	1.5	1.2	1.5
		Var 1	$\frac{1}{0.7^2}$	$\frac{1}{0.35^2}$	$\frac{1}{1.1^2}$	$\frac{1}{1.5^2}$	$\frac{1}{1.2^2}$	$\frac{1}{1.5^2}$
	M|G|s	Rate 1	0.7	0.35	1.1	1.5	1.2	1.5
		Var 1	$(\frac{2}{0.7})^2\frac{1}{12}$	$(\frac{2}{0.35})^2\frac{1}{12}$	$(\frac{2}{1.1})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$	$(\frac{2}{1.2})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$
	GI|G|s	Rate 1	0.7	0.35	1.1	1.5	1.2	1.5
		Var 1	$(\frac{2}{0.7})^2\frac{1}{12}$	$(\frac{2}{0.35})^2\frac{1}{12}$	$(\frac{2}{1.1})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$	$(\frac{2}{1.2})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$
	GI|G|s	Rate 2	0.7	0.33	1.05	1.2	1.2	1.5
		Var 2	$(\frac{2}{0.7})^2\frac{1}{12}$	$(\frac{2}{0.33})^2\frac{1}{12}$	$(\frac{2}{1.05})^2\frac{1}{12}$	$(\frac{2}{1.2})^2\frac{1}{12}$	$(\frac{2}{1.2})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$
	GI|G|s	Rate 3	0.7	0.33	1.05	1.2	1.5	
		Var 3	$(\frac{2}{0.7})^2\frac{1}{12}$	$(\frac{2}{0.33})^2\frac{1}{12}$	$(\frac{2}{1.05})^2\frac{1}{12}$	$(\frac{2}{1.2})^2\frac{1}{12}$	$(\frac{2}{1.5})^2\frac{1}{12}$	
Servers		1	3	7	2	100	5	2
Servers		2	3	7	2	5	5	2
Servers		3	3	7	2	5	2	

Table 11: Rates and server distributions

The transition matrix in Table 12 is suitable for the situations 1 and 2. For situation three this matrix is a 5x5 matrix, with the same probabilities but without queue 4.

r_{ij}	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6
i = 1	0.0	0.05	0.0	0.7	0.0	0.0
i = 2	0.0	0.0	0.0	1.0	0.0	0.0
i = 3	0.0	0.05	0.0	0.95	0.0	0.0
i = 4	0.0	0.0	0.0	0.0	1.0	0.0
i = 5	0.0	0.0	0.0	0.0	0.0	0.5
i = 6	0.0	0.0	0.0	0.0	0.0	0.0

Table 12: Transition matrix

B Queuing Network Analyser

Step 1. Calculate the aggregate arrival at queue j, λ_j:

$$\lambda_j = \gamma_j + \sum_{i=1}^{J} \lambda_i r_{ij} \tag{11}$$

In this equation γ_j is the external arrival rate at queue j, J is the number of queues, and r_{ij} is the transition rate from queue i to queue j.

Step 2. Calculate the load of a server at queue j, ϕ_j:

$$\phi_j = \frac{\lambda_j E[S_j]}{s_j}. \tag{12}$$

In this equation $E[S_j]$ is the mean service time at queue j.

Step 3. Calculate the flow from queue i to queue j, λ_{ij}:

$$\lambda_{ij} = \lambda_i r_{ij} \tag{13}$$

and calculate the fraction of arrivals from queue i to queue j, q_{ij}:

$$q_{0j} = \frac{\gamma_j}{\lambda_j}, \qquad q_{ij} = \frac{\lambda_{ij}}{\lambda_j}. \tag{14}$$

In this equation q_{0j} denotes the fraction of external arrivals.

Step 4. Calculate the squared coefficient of variation (scv) at queue j for the inter arrival times ($c^2_{A,j}$) with help of the scv for the external inter arrival times (c^2_{0j}), and the scv of the service times ($c^2_{S,j}$).

$$c^2_{A,j} = a_j + \sum_{i=1}^{J} c^2_{A,i} b_{i,j}, \text{ with}$$

$$a_j = 1 + w_j \left((q_{0j} c^2_{0j} - 1) + \sum_{i=1}^{J} q_{ij}((1 - r_{ij}) + r_{ij}\phi_i^2 x_i) \right) \tag{15}$$

Furthermore,

$$x_i = 1 + \frac{1}{\sqrt{m_i}}(max(c^2_{S,i}, \frac{1}{5}) - 1), \tag{16}$$

in which m_i is the number of servers at queue i. In the continuation of this article, the letter s will be used. We also have,

$$b_{ij} = w_j q_{ij} r_{ij} (1 - \phi_i^2), \tag{17}$$

$$w_j = \left((1 + 4(1 - \phi_j)^2 (\eta_j - 1) \right)^{-1}, \tag{18}$$

and

$$\eta_j = \left(\sum_{i=0}^{J} q_{ij}^2 \right)^{-1} \tag{19}$$

Step 5. Calculate the mean waiting time at queue j, $E[W_j]$:

$$E[W_j] = E[W_{M|M|s}]\frac{c_{A,j}^2 + c_{S,j}^2}{2} \tag{20}$$

In this equation, $E[W_{M|M|s}]$ is the mean waiting time of an M|M|s queue.

C Parameters real system

The arrival rates in Table 13 are the probabilities that a customer comes via the SEH, DEP or POS. The service time 1 and 2 in the same Table are the parameters for either the uniform or the log-normal distribution. The transition rates in Table 14 are 0 when the cell is empty.

Type	Name	Queues					
		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0-17	Ext. Arrival Rates	0.0021	0.9895	0.0084			
	Service time 1	0.0083	0.0167	0.0333	1.7454*24	0.0167	0.0333
	Service time 2	0.3667	0.3667	0.3083	4.0701*576	0.2333	0.1750
18-49	Ext. Arrival Rates	0.0193	0.8909	0.0898			
	Service time 1	0.1000	0.1000	0.2250	2.0027*24	0.0333	0.0833
	Service time 2	0.4000	0.4000	0.2917	9.4412*576	0.2417	0.1917
50-74	Ext. Arrival Rates	0.0278	0.8019	0.1703			
	Service time 1	0.2167	0.2167	0.2167	3.3721*24	0.1333	0.1000
	Service time 2	0.3917	0.4083	0.5000	23.0095*576	0.2500	0.2167
75+	Ext. Arrival Rates	0.0366	0.8537	0.1097			
	Service time 1	0.2167	0.2167	0.2500	4.8597*24	0.1167	0.1167
	Service time 2	0.3917	0.4000	0.5000	28.8601*576	0.2667	0.2500
All	Distribution Servers	2	4	3	1000	3	2

Table 13: Service rates and distribution of servers of queues for four patient types

Type		Queues					
0-17		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
	i = 1				1		
	i = 2				1		
	i = 3		0.0021		0.9979		
	i = 4					1	
	i = 5						0.3235
	i = 6						
18-49		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
	i = 1				1		
	i = 2				1		
	i = 3		0.0214		0.9786		
	i = 4					1	
	i = 5						0.5337
	i = 6						
50-74		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
	i = 1				1		
	i = 2				1		
	i = 3		0.0665		0.9335		
	i = 4					1	
	i = 5						0.6336
	i = 6						
75+		i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
	i = 1				1		
	i = 2				1		
	i = 3		0.0354		0.9646		
	i = 4					1	
	i = 5						0.5097
	i = 6						

Table 14: Transition rates for four patient types

D Results extra scenarios

In this appendix, extra results are exposed for new scenarios. These are added, since these might also be useful as an advise. These are not included in the main text, since the time was insufficient to document it in a precise way.

S_4: Changing working times for admission.

S_5: Combining cases S_2 and S_4

The cases for S_4:

C_1: Working times admission 1 hour later.

C_2: Working times admission 1 hour earlier.

C_3: Working times admission 2 hours earlier.

The cases of S_5 are combining the cases 1 and 2 of S_2 and the cases 2 and 3 of S_4.

Scenario	Case	r_l	95%-CI	Queue	EW_j	95%-CI
S_4	C_1	0.3690	(0.3673, 0.3707)	k = 1	0.0459	(0.0406, 0.0511)
				k = 2	2.9983	(2.9939, 3.0027)
				k = 3	0.2462	(0.2359, 0.2564)
				k = 5	1.0622	(1.0584, 1.0660)
				k = 6	0.0217	(0.0209, 0.0225)
	C_2	0.3633	(0.3616, 0.3650)	k = 1	0.0075	(0.0062, 0.0088)
				k = 2	0.5371	(0.5342, 0.5400)
				k = 3	0.0557	(0.0540, 0.0574)
				k = 5	1.0419	(1.0379, 1.0459)
				k = 6	0.0213	(0.0205, 0.0221)
	C_3	0.3039	(0.3018, 0.3060)	k = 1	0.0207	(0.0109, 0.0305)
				k = 2	1.5831	(1.5712, 1.5949)
				k = 3	0.1961	(0.1740, 0.2182)
				k = 5	0.8814	(0.8776, 0.8852)
				k = 6	0.0212	(0.0203, 0.0222)
S_5	$C_{1,2}$	0.2260	(0.2245, 0.2275)	k = 1	0.0070	(0.0057, 0.0082)
				k = 2	1.1056	(1.0977, 1.1135)
				k = 3	0.0527	(0.0498, 0.0556)
				k = 5	0.7199	(0.7173, 0.7226)
				k = 6	0.0669	(0.0655, 0.0683)
	$C_{1,3}$	0.1629	(0.1611, 0.1647)	k = 1	0.0259	(0.0174, 0.0345)
				k = 2	2.4705	(2.4580 2.4829)
				k = 3	0.1910	(0.1719, 0.2102)
				k = 5	0.6075	(0.6049, 0.6101)
				k = 6	0.0690	(0.0672, 0.0709)
	$C_{2,2}$	0.2237	(0.2222, 0.2252)	k = 1	0.0083	(0.0070, 0.0097)
				k = 2	0.5382	(0.5356, 0.5408)
				k = 3	0.1382	(0.1335, 0.1429)
				k = 5	0.7130	(0.7104, 0.7156)
				k = 6	0.0658	(0.0644, 0.0673)
	$C_{2,3}$	0.1600	(0.1583, 0.1617)	k = 1	0.0177	(0.0077, 0.0276)
				k = 2	1.5816	(1.5714, 1.5918)
				k = 3	0.6878	(0.6595, 0.7162)
				k = 5	0.6013	(0.5988, 0.6039)
				k = 6	0.0672	(0.0649, 0.0694)

TABLE 15: Results for extra scenarios