UNIVERSITY OF TWENTE.

Cost and risk-reduction benefit of hedging inflation risk for Dutch defined-benefit pension funds

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August 2020

Thesis submitted in partial fulfilment of the requirements for the degree of MSc. in Industrial Engineering and Management – Financial Engineering and Management

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Acknowledgements

This thesis report marks the end of my journey as a graduate student in the Netherlands, a faraway land that greeted me with open arms and offered me more than just a joyous stay. I want to thank the most important people who made this learning experience unforgettable.
I thank Dr. Berend Roorda, my thesis instructor who accompanied me in the redaction of this paper even during the Great Lockdown. From the first day I arrived at the University of Twente, he was kind to me and vividly encouraged me to work hard. Whenever I had questions regarding an exercise, he would help unhesitatingly.
I wholeheartedly thank NN Investment Partners for the immeasurable kindness and warmth its employees greeted me with since I joined the ICS department. My internship was a terrific learning experience. I learned a great deal from my thesis supervisor, Robert Berkhout, who patiently answered all my questions pedagogically. I thank him for rigorously proof-reading this report multiple times and providing constructive feedback to improve it. I also thank Berber de Backer for honing my quantitative modelling skills, Ingmar Minderhoud for introducing me the Dutch pension fund industry and Arjen Monster for lending me his books on inflation hedging.
I am appreciative to my friends who supported me during my hectic integration in this country.
Most importantly, I want to thank my beloved parents Khaled and Jocelyne Zorba for their relentless support even when financial hardships and tragedies befell them. My mother never doubted I will succeed, and this pushed me to work even harder to make her proud. My parents are both the bravest and most inspiring people I've ever met. They are my inexhaustible source of energy. I dedicate this thesis to them entirely.

Walid Zorba
Athens, August 18th 2020

Abstract

Inflation has recently garnered the attention of Liability-Driven Investment (LDI) strategists because of its unusually low level. Though it might seem desirable on the surface, it alarms asset management firms as they expect a sharp increase in the upcoming years due to its supposed ergodic nature. As such, the promised benefits could wane unless Dutch pension funds hedge inflation risk from their balance sheet. The problem is, if inflation still stays low, the hedge will damage their assets. The overarching goal of this technical report is to determine whether it is worthwhile hedging inflation risk from a Dutch pension fund's balance sheet or not. NN Investment Partners manages the portfolios of Dutch pension funds. Its role is to ensure their clients remain sufficiently well-funded to pay their *soft* obligations in full by providing them with sound recommendations based on future market expectations. To capture a wide-range of possible scenarios, we build an Economic Scenario Generator (ESG) in MATLAB that produces future asset returns, in a Time-Frequency Representation. Concretely, we simulate inflation-sensitive balance sheet items as stochastic variables by modelling inflation swap rates and interest swap rates as a *composed* Ornstein–Uhlenbeck process, and global equity as a composed random walk with an upward drift. Afterwards, we import our figures to the company's balance sheet simulation tool and observe what happens to the coverage ratio distribution at the end of our time horizon every time we gradually increase the inflation hedge ratio; the results are remarkable. The more we hedge, the higher the expected nominal and *real* coverage ratio; conversely, the less we hedge, the lower the expected nominal and real coverage ratio. Moreover, the risk profile of the pension fund improves substantially as the volatility of its real coverage ratio distribution decreases when we increase the hedge ratio. Therefore, if our simulation is correct, we advise Dutch pension funds to hedge their inflation exposure entirely to jointly a) maximize their funding position and b) minimize their balance sheet risk. However, we do recognize our models are based on a set of assumptions. For instance, they violate the risk-neutral framework; both models rely on *true* probabilities instead (i.e., historical probabilities). We identify other possible limitations and discuss their practical implications on the Dutch pension landscape towards the end of the paper.

Keywords: BEIR, Fisher hypothesis, ARIMA, Term structure of inflation rates, Machine Learning, Time series forecasting, Stochastic modelling, model validation, Risk Management, Economic Scenario Generator, Liability-Driven Investment

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1 Introduction

For the past three decades, inflation was not perceived as a serious threat to promptly address by the Dutch pension industry. What most participants did not realize, is that during this period, their purchasing power was cut by almost half. What's more, retirees are disproportionately exposed to this peril as their nominal benefits do not always adjust to the prevailing rate of inflation. Consequently, the real value of their assets is significantly deteriorating and poverty fears draw nearer to reality by the day. Luckily, inflation-hedging strategies exist to cope with this situation. We pick up the slack by explaining the most widely employed by investment management firms throughout this report.

The opening chapter introduces NNIP in conjunction with its role in managing inflation risk. Section 1.2 presents the current state of affairs of the Dutch pension system, and Section 1.3 diligently enumerates the risks encountered by pension funds with special attention devoted to the risk posed by inflation. Subsequently, Section 1.4 discloses the assignment's raison d'être and, Section 1.5 explains why inflation risk matters. Section 1.6 formulates the critical question of this technical report. In Section 1.7, we refine and define the research objective, i.e., the research questions which will drive the rest of the report forward. Finally, in Section 1.8, we present the outline of the graduate thesis.

1.1 Company description

NN Investment Partners (NNIP) is the asset management division of NN Group, the largest insurance provider in the Netherlands. It is headquartered in the Hague and one of its activities is fiduciairy management of Dutch pension funds. Stated differently, pension funds delegate some of their investment decisions (but not responsibilities) to NNIP which acts as their fiduciary agent. As of 2019, NNIP has 287 billion EUR in AuM and 58 billion EUR under advice.

Yet, such a paramount responsibility requires cutting-edge skills, otherwise the company's business activities will grind to a halt. And since the competition in the domestic market is fierce, risk management competencies are highly prized. I have been contracted in this environment to join the ICS (Integrated Client Solutions) department as a thesis intern.

One of the activities of ICS is to advise pension funds and insurers on how to manage risk successfully. Specifically, it designs hedging strategies to protect pension fund portfolios from inflation risk so that they can guarantee inflation-protected benefits to their retirees. Its core constituents are astronomers, applied mathematicians and economists.

NNIP's overarching goal is to become the best fiduciary manager in the Netherlands.

1.2 Pension fund sector in the Netherlands

Pension funds are financial institutions mandated to provide future retirement income to their members in exchange of premium payments received during the contributor's career. In a nutshell, they work as follows: they collect annuities from their active participants, invest the proceeds in financial assets and, once they retire, pay them the promised or realized benefits. An ancillary function they dispense, is risk-sharing amidst their members. In the Netherlands, three sources of pension benefits structured around "pillars" exist:

- Pillar 1: Public pension system as codified in the AOW (compulsory)
- Pillar 2: Private pension system (mostly compulsory)
- Pillar 3: Individual Retirement Accounts (optional)

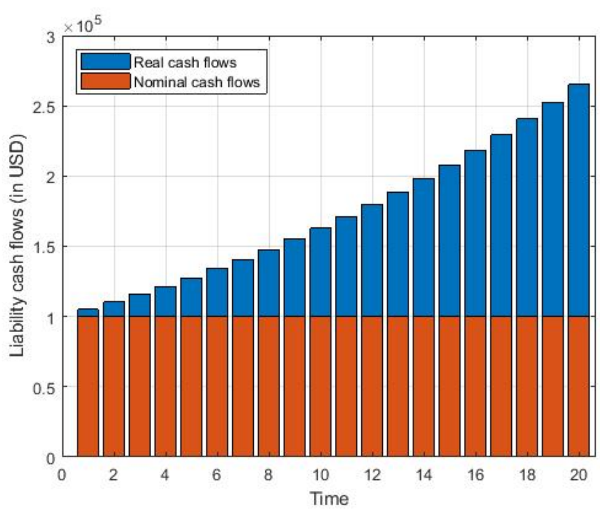
State pension consists of an indiscriminate flat-rate benefit that depends solely on household status and the minimum wage.
In contrast, private pension funds are run as stand-alone non-profit entities, and are typically classified in two groups: defined-benefit (DB) and defined-contribution (DC) schemes.
Whilst both of them invest their assets to earn real returns, a DB scheme is managed collectively and sometimes employer-sponsored whereas a DC scheme is configured as an matching agreement between the employer and the employee whereby both regularly credit the account. This implies DB plans are riskier than DC plans because in case of bankruptcy, the pensioner's account disappears. But unlike DC plans, they promise guaranteed lifetime payments. Secondly, the benefits are indexed to the prevailing rate of inflation. In addition, when markets are bear, the employer can disburse money from his/her own capital into the account to offset any losses sustained; however, this happens rarely. That explains why DB plans are far more coveted. Indeed, more than 90% of the Dutch labour force subscribes to Pillar 2, 99% of which are DB contributors. It is worth noting that collective defined-contribution (CDC) schemes exist as well; they are hybrid plans combining elements of DB and DC schemes.
Increasingly, employers are phasing out DB pension schemes in favour of DC and CDC schemes, thus shifting the risk to the employee. The relative size of DC pension funds is small compared to DB pension funds which has been historically accruing. Therefore, we focus on DB schemes. But the lessons learned from DB plans can also be used for DC plans because even DC plans make regular cash-flow payments.
Henceforth, we shall exclusively focus on DB pension funds.

1.3 Balance sheet risks

Pension funds are exposed to a host of different risks that might compromise their ability to meet their promised life-time payouts, i.e., obligations. These exist because the environment in which they operate is fraught with uncertainties induced in part by long-term payment commitments. This means, balance sheets might not be depicted faithfully since their items are recorded on a present value basis.
Such examples include longevity risk tied to the increasing life-expectancy of

pensioners; this results in higher payout ratios which reduces the pension fund's coverage ratio over time (because its liabilities increase).
Another looming threat is market risk; since pension funds invest heavily in the stock-market, an undesirable movement could lead to catastrophic losses in their position.
Also, they face interest rate risk; ideally, they want the rates to increase so that the present value of their liabilities decreases. On the other hand, if the rates fall, the value of their liabilities will increase, thus reducing the nominal coverage ratio.
Payment benefits contain a noteworthy embedded option: a Cost-of-Living Adjustment (COLA) provision to counteract the effect of inflation. Concretely, it means pension funds index their payments to the expected rate of inflation.
For instance, if the promised amount is \$100,000 p.a. and the expected inflation rate is 5% p.a., then the pension fund will pay out \$105,000 the first year provided the coverage ratio is above a certain level. This indexation is, by law, conditional on the level of the coverage ratio. Conversely, no indexation transpires in the absence of inflation. The rationale behind this conditional indexation is to preserve the retirees' purchasing power as nominal benefits ignore inflation's eroding effect. Thus, pension funds have an exposure to inflation risk. It is important to monitor it, because it is one of the major risks pension funds face; in fact, inflation rates are compounded to the liability cash-flows. This implies, periodic inflation payments can increase at an exponential fashion.
To illustrate our point, let's assume a retiree receives \$100,000 p.a. from his plan, and the expected rate of inflation is again 5% p.a. Fig. 1 shows that after 15 years, the inflation liabilities overtake the promised nominal benefits themselves.

Figure 1: Pension benefits with COLA provision



Though NNIP is not a pension fund itself, it must still deal with this issue as it advises pension funds on how to hedge balance sheet risks of which inflation is one. NNIP is responsible for sound advice and looking ahead for its client base. But If the recommendations are ineffective, its clients will abandon NNIP in favor of its competitors. Therefore, the company has an incentive to provide satisfactory solutions to pension funds and insurers to remain profitable, especially since inflation risk affects the vast majority of the contributors. Thus, NNIP must strengthen its capabilities on inflation risk advising because it is the problem owner.
Following the 2007 credit crisis, the House of Representatives passed the FTK (Financial Assessment Framework), a bill compelling pension funds to publicly disclose their earmarked solvency buffers as well as any indexation ambitions they might have for transparency purposes. In practice however, inflation does not form part of the solvency charge calculation; therefore, the solvency charge will not be part of the discussion.

1.4 Genesis of the assignment

We previously stated that pension funds in the Netherlands are run as non-profits wherein assets are invested to pay pension benefits. A noteworthy trait is that their balance sheets are deprived of equity. This signifies, their assets should be at least equal to the promised liabilities otherwise benefits will shrink since the liabilities overtake the assets. The coverage ratio is a KPI that measures the paying ability of pension funds. It is expressed as follows:

$$CR = \frac{PV_A}{PV_L}$$

with CR=nominal or real coverage ratio, PV_A = present value of assets and PV_L=present value of liabilities.
By law, the valuation is based on a mark-to-market approach. For liabilities, pension funds follow the FTK regulation and use the risk-free interest rate curve for liability valuation. The Dutch government and the DNB had many debates on whether this discount rate should change to a higher interest rate that takes into account higher returns. But presently, they are both unwilling to challenge the status quo unless these two conditions are met:

1. Stop treating pension benefits as risk-free cash-flows
2. Eliminate inter-generational capital flows within the pension fund

Unless these issues are settled, the risk-free rate prevails.
A coverage ratio of 100% means the fund is in a position to service all its future obligations whereas above or below 100% indicates the fund is either over-funded or under-funded.
If the inflation rate increases, then the real coverage ratio will decrease and vice-versa. This means, pension funds should purchase inflation-protection in the event of an upswing to guarantee future payment benefits. However, Dutch

pension funds turned a blind eye to inflation risk in part because they thought it did not pose a credible threat to the balance sheet. NNIP cites other reasons as well:

1. The nominal coverage ratios were historically low, so, pension funds were not allowed to grant inflation-adjusted benefits by law
2. Some pension funds expected inflation rates to remain low for a prolonged duration
3. Asset managers believed they could reduce the inflation risk from the balance sheet by simply lowering the interest rate hedge

In lieu, they focused solely on hedging interest rate risk because coverage ratios were too low to worry about indexation of pension benefits anyway. In contrast, British pension funds have always hedged both risks since conditional indexation is mandated by law.

But recently, there has been a renewed interest in hedging inflation in the Netherlands too because the rates are at an all-time low. Whilst it seems counter-intuitive, there is a logical explanation for that: if inflation rises abruptly, the real coverage ratio will go down. That is because, it is more likely to witness a sudden uptick in inflation when it is far below the 2% target [1] than when it is above it. That is why, when inflation exceeded the target, Dutch pension funds did not bother hedging it because the rates would theoretically fall, thus increasing the real coverage ratio in return. At that time, hedging inflation would have been akin to catching a falling knife. An additional reason why NNIP advised against inflation hedging, was that nominal coverage ratios were relatively low; and according to the Dutch central bank (DNB), the COLA should only be exercised if the nominal funding ratio hits the 120% threshold.

Still, some of NNIP's clients are well-funded (i.e., CR above 130%), and want to protect their principal. Verily, a growing number of pension funds are asking for advise on inflation-protection, and NNIP must look into this otherwise it will lose its Dutch customers.

But the firm does not have an inflation curve in its ESG yet. This is problematic for three main reasons:

1. NNIP cannot derive the real coverage ratio which is indispensable to make informed decisions on capital requirements that guarantee future benefits.
2. It cannot make meaningful investment decisions on behalf of its clients so long as inflation remains unaccounted for.
3. It cannot perform balance sheet simulations to gauge the fund's ability to meet its future liabilities under different economic stress scenarios.

[1] In the pursuit of price stability, the European Central Bank strives to maintain an inflation rate close to 2% p.a. in the Euro area. This entails, in case of an inflation drop, it will react by pushing it back to the 2% level.

So, NNIP must change the status quo by adopting an active role in managing inflation risk; to stay ahead of the curve, the company must integrate it in its ESG (that we build upon) alongside market risk and interest rate risk. Only then, the strategic advisors will truly understand the impact of inflation on the client's portfolio. That is why this assignment was commissioned.
To recapitulate, Dutch pension funds ask for inflation protection if and only if these two conditions are satisfied:

1. Possess an initially high nominal coverage ratio and expect high future inflation

2. Enact regulatory changes focusing on real pension benefits. But this is a big change and cannot happen overnight

1.5 Problem identification

Liability-Driven Investment (LDI) strategists routinely recourse to the OTC market to trade zero-coupon inflation swaps (ZCIS), inflation-linked swaptions and inflation-protected bonds amongst others to hedge inflation risk. Nevertheless, since these hedges are costly, it is crucial to anticipate from the get-go what the future rate of inflation will be to warrant such operations. For instance, if the projected rate of inflation is 0% p.a. for the next few decades, then it is not worth hedging it at all. Moreover, the average duration of nominal liabilities in the Netherlands hovers around 20 years. Adjusted for inflation, it climbs to 26 years [2]. This entails, a parallel shift of +1% in the inflation curve increases the fund's soft liabilities by 26%. Therefore, it would be unwise to sit on the fence. But woefully, we cannot accurately predict the rate of inflation for such long-term horizons. This poses a double problem for LDI strategists:

1. They cannot adequately optimize their portfolios to guarantee future cash inflows whilst preventing funding shortfalls (i.e., coverage ratio below 120%).

2. They cannot accurately estimate the cost of hedging inflation risk.

Notwithstanding, we can forecast long-term inflation rates by examining its mathematical properties. Based on a set of assumptions, we sketch the true distribution of possible expected inflation values.

1.6 Problem formulation

The European Central Bank (ECB) dictates the Euro-zone's monetary policy. Through open market operation (OMO), it reduces the short-term interest rate by conducting expansionary monetary policy (i.e., money printing) [1]. The intention is to spur investing, spending and ultimately growth by enticing individuals and businesses to borrow credit at lower yields. But such an operation

[2] It constitutes the average *real* liability calculated by the company's actuaries.

results in a rise in inflation. Oppositely, by contracting the credit supply, the short-term interest rate rises; as a result, the inflation rate declines. In this case, the goal is to jointly stop "overheating" the economy and rein in inflation.
Fortunately, the ECB follows an inflation-targeting (IT) regime to control short-term inflation; this implies, it should be easier to predict inflation than under a non-IT scenario. In fact, Hall and Jaaskela (2011) demonstrated that the adoption of an IT regime amongst central banks has improved significantly the forecasting accuracy of inflation whilst reducing its volatility in tandem [2]. Moreover, they noticed it exhibits mean-reverting properties, thus implying it's a stationary process (i.e., its characteristics are invariant under time shifts). But it's worth bearing in mind, the ECB's intervention is not overriding; there are exogenous variables beyond its control influencing inflation paths such as commodity price shocks or demographic shifts [3]. Moreover, OMO becomes ineffective at a certain point because of the liquidity trap (i.e., interest elasticity of demand drops). So, since randomness exists, we should use probabilistic models to generate its future possible outcomes.
Yet, a wide range of techniques exist; and each one has its own advantages and drawbacks. But virtually all of them require past data as input; and although past results are not always indicative of future results, it should be a good starting point for the model development phase. We can extract historical quotes from Bloomberg Terminal.
Following the European debt crisis, growth rates in the EU became sluggish. In fact, most countries have not recovered yet; the side effect of this prolonged stagnation is low inflation in the Euro-zone [4]. At some point, France even recorded a yearly inflation rate of 0%. That is advantageous for pension funds because they need not to grant inflation payments anymore, so, their inflation-adjusted liability cash-flows do not increase over time. Even so, can we guarantee this trend will persist indefinitely? This begs the question:
What is the cost and risk-reduction benefit of hedging inflation risk for Dutch DB pension funds?
To answer it, we will proceed in two phases: first, we build an Economic Scenario Generator (ESG) that includes an inflation tool in addition to other risk-sensitive assets. The goal of a long-term ESG is to generate realistic scenarios for possible future developments of the "main risk factors" (i.e., the relevant financial variables) for pension funds and insurers. Such risk variables include but are not limited to:

- Interest rate term structure (nominal and real)
- Inflation (realized and break-even inflation)
- Credit spreads
- Equity returns

An ESG can be used as part of portfolio construction and/or strategic risk monitoring. Our ESG is custom-made to suit NNIP's investment objectives. It will be less comprehensive than the company's current inflation-absent *nominal*

ESG, but still gives a very good approximation of the pension fund's overall portfolio dynamics. Since our ESG is modular, NNIP can always integrate functions of the code to the company's own in-house ESG. Our ESG will simulate three representative risky assets:

- Interest rate term structure (nominal and real)
- Inflation (realized and break-even inflation)
- Equity returns

In the second phase, we integrate our curves to NNIP's balance sheet projection tool to visualize how the pension fund's funding position evolves over our 15-year time horizon and calculate important statistics that cannot be derived analytically. Based on the results, we adjudicate on whether it is wise to hedge inflation risk from the balance sheet or not.

1.7 Research objective

Throughout the paper, we will follow Heerkens's MPSM methodology to tackle the business problems as outlined in his book [5].
We translate the problem formulation into a sequence of research questions we will address. So far, three important concerns have emerged in that logical order:

1. What is the best practice to hedge inflation risk? What are the shortcomings? In which direction can we make progress?
2. What are the different methods available to model inflation rates? Which ones are most suitable for Dutch define-benefit pension funds? How can we improve the model?
3. Is it still worth hedging inflation? In other words, does the benefit of hedging inflation in the long-term outweigh its short-term costs or not?

We are dealing with a knowledge problem because it can only be answered by acquiring additional knowledge. Therefore, our research will be spearheaded by academic papers, vocational articles and industry best practices. Inflation will be the principal variable of interest. But since want to visualize how it impacts the pension fund's entire portfolio, we must also simulate interest rates risk and global equity; this will give us a holistic picture of its balance sheet exposures. If we model inflation risk in isolation, it will not be helpful for the study as it would be impossible to measure its influence on the pension fund's total assets. So, modelling interest rates and equity returns is a corollary to investigate the purported risk-reduction benefits (or lack thereof) of hedging inflation risk.
Each section will be broken-down into sub-questions to avoid going off-the-mark. We shall build a picture of future inflation that reflects all the knowledge currently available.
It is worth noting this thesis fills an important gap in the literature in that it is the first of its kind to address this question.

1.8 Thesis outline

Before cutting through the chase, we first provide a run-through of the financial instruments employed by LDI strategists to hedge inflation; the reader will get acquainted to these in Chapter 2.
In Chapter 3, we present the Machine Learning techniques to forecast inflation. We probe their suitability and highlight their predictive shortcomings.
Next, in Chapter 4, we explore stochastic differential equations to describe market variables. We also process our raw data for the quantitative modelling phase.
In Chapter 5, we construct a *real* (i.e., inflation-adjusted) ESG on MATLAB to simulate our time-series.
Based on the simulation set, Chapter 6 reveals if it is worthwhile hedging inflation risk from the pension fund's portfolio or not. It is the most emblematic section of the paper.
Finally, Chapter 7 summarizes and concludes the technical report and ushers recommendations and best practices to industry practitioners on the best course of action to take for inflation risk management.

2 The mechanics of inflation-hedging

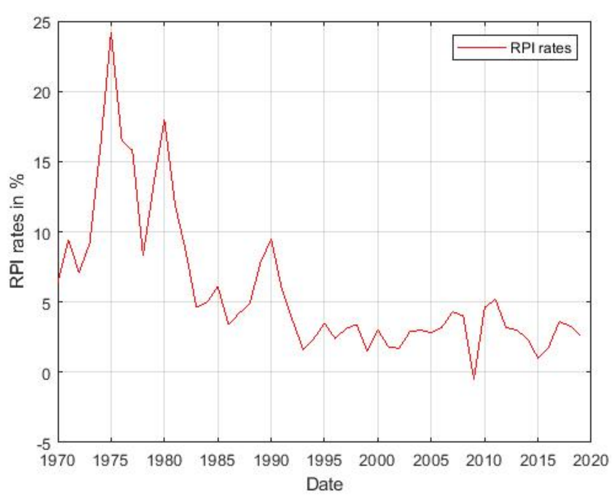
In this chapter, we jump feet-first to the heart of inflation hedging. LDI strategists possess a wide array of instruments to offset pension fund liabilities and maintain an acceptable coverage ratio level. But the most popular ones by far are inflation-linked bonds and swaps.
Sections 2.1 and 2.2 diligently breaks down ILBs and inflation swaps respectively. In Section 2.3, we explore the inflationary hedging properties of tangible assets and highlight their limitations. Finally, Section 2.4 narrates NNIP's state of play vis-à-vis inflation risk hedging.

2.1 Inflation-linked bonds

What are inflation-linked bonds? How can they offset pension fund liabilities? What are their limitations?

Inflation-linked bonds were once the juggernaut of liability hedging. In fact, the oldest ILB was issued by the Commonwealth of Massachusetts in 1780 to fund the American War of independence. The index tracked the price change of a basket of corn, wool and beef which had risen by 32 shortly before their issuance. However, they fell out of favour after the conflict ended. But they started garnering traction again once Great Britain's RPI peaked at 24.2% and 17.9% in 1975 and 1980 respectively. These "linkers" signaled the government had finally taken this issue seriously and would commit to lower inflation in the upcoming years. Surprisingly, Britain accomplished it triumphantly as Fig. 2 indicates:

Figure 2: UK RPI rates between 1970 and 2019



Fisher developed a theoretical framework to decompose nominal bond yields into three components: inflationary expectations, a real yield above the expected rate of inflation and a risk premium. The risk premium designates the compensation the investors require to hold the bond since it contains credit risk. Hence, the *simplified* Fisher equation is:

$$n = r + i^e + p$$

where n=yield on nominal bond; r= real yield; i^e=inflationary expectations and p=risk premium. Usually, government-issued securities are presumed to be risk-free, so p is negligible; withal, r is determined by time preference.
This implies, if inflation rates increase, the nominal yields should also increase as well. As a result, the PV of the liabilities will decrease which positively affect the pension fund's real funding position. If the COLA provision is exercised, its liability is valued as such:

$$PV_L = \frac{CF\,(1+i_1^e)}{(1+n)} + \frac{CF\,(1+i_2^e)(1+i_1^e)}{(1+n)^2} + ... + \frac{CF\,(1+i_T^e)(1+i_{T-1}^e)...(1+i_1^e)}{(1+n)^T}$$

where CF= periodic cash-flow benefits.
Let i_t be the realized annual inflation rate $\forall$t$= 1, 2, ..., T; so$,

$$PV_L = \frac{CF\,(1+i_t)}{(1+n)} + \frac{CF\,(1+i_t)^2}{(1+n)^2} + ... + \frac{CF\,(1+i_t)^T}{(1+n)^T}$$

Equivalently,

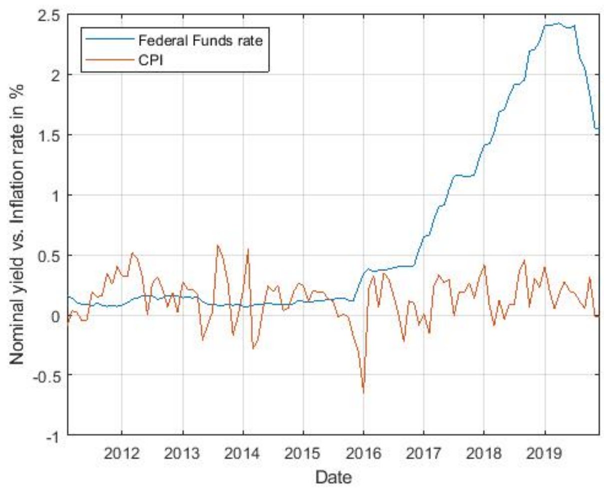
$$PV_L = \sum_{t=1}^{T} \frac{CF\,(1+i_t)^t}{(1+n)^t}$$

Let δ_t denote the compound indexation factor such that $\delta_t = (1+i_t)^t$. Therefore,

$$PV_L = \sum_{t=1}^{T} \frac{\delta_t\,CF}{(1+n)^t}$$

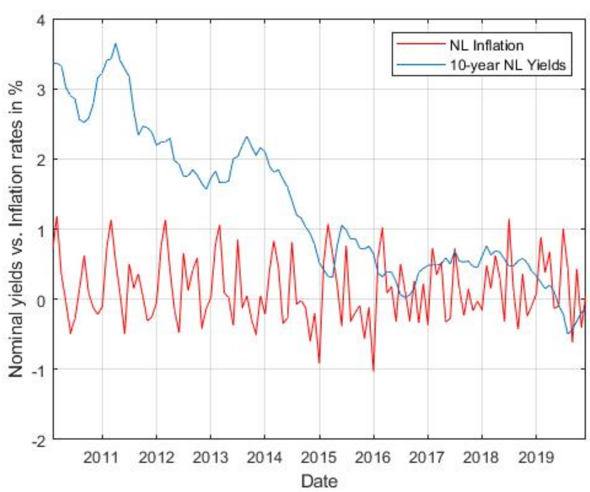
Ideally, pension funds want the nominal yields to rise, and expected inflation to decline to minimize PV_L. The Fisher equation suggests that when inflation rises, so does the nominal yield. But that is not what we observe in practice; Fig. 3 casts doubt on this stylized fact; indeed, we notice the US CPI outpacing nominal yields on several occasions meaning the real rates were negative up until 2015.

Figure 3: Federal Funds rate vs. US CPI between 2010 and 2019



Such a situation still persists in the Netherlands as Fig. 4 demonstrates:

Figure 4: 10-year NL yields vs. NL inflation between 2010 and 2019



Hence, if the realized inflation exceeds the discount rate i.e., $\delta_t > (1+n)^t$, it will negatively impact the pension fund's real coverage ratio thus becoming under-funded. Fortunately, they can resort to ILBs to hedge their liabilities.
Investments that target returns above the rate of inflation can protect and increase the pensioner's purchasing power. The principal on the TIPS adjusts automatically to the anticipated rate of inflation. This implies, the higher the

rate, the higher the principal redeemed. Moreover, it pays coupons; this represents the real rate (i.e., the return above the inflation rate). Another desirable feature is its embedded deflation floor which acts as a protective put in options trading with a strike price equal to the par value of the bond.
As a hypothetical example, let's consider a 10-year TIPS with a 2% coupon rate paid semi-annually and an expected inflation rate of 4%. At maturity, the bondholder receives nearly \$1486. Additionally, each coupon payment received will be paid on the inflation-adjusted principal value; the first payment is \$10.40, the second \$10.60 and so on.
Moreover, TIPS provide diversification benefits as they are uncorrelated to other assets classes; when inflation is on the upswing, TIPS gain value, but nominal bonds don't because the buying power of their cash-flow streams gets eroded. As for stocks, Nelson (1976) contradicts the Fisher Hypothesis by furnishing evidence that a negative relation exists between stock returns and anticipated levels of inflation [6]. A conjectural reason is that corporations under-perform against an inflationary backdrop which translates into lousy stock returns wherever markets are efficient. Still, the relation between equity performance and inflation rates is fuzzy.
Yet, ILBs suffer from many drawbacks thus, explaining why their desirability is waning. NNIP's strategic advisors list them herein:

1. They lack flexibility. The maturity of ILBs is generally standardized; this means if the pension liability duration jumps from 20 to 21 years, it won't be easy to find a bond with the same tenure [3] to hedge against inflation. This situation leads to mismatched cash-flows and convexity problems.

2. Balance sheet inflation risk is so big, it requires a large upfront investment in ILBs to hedge that risk. And the bigger the investment, the lower the expected return of assets as less capital is disposable for higher returning assets.

3. ILBs suffer from liquidity issues; they are not widely available. Even in the countries they exist, their issuance is irregular.

Fortunately, swaps fix these problems.

[3] In theory, we should match the bond's Macaulay duration with the liability's nominal duration so that price risk and reinvestment risk offset each other. But we can ignore this rule since we presume pension funds will hold the bond till maturity.

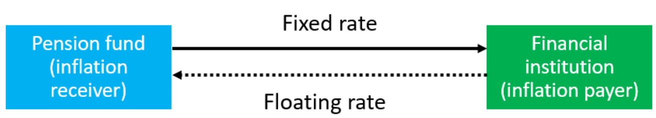
2.2 Inflation derivatives

What are swaps? How do they overcome the inherent limitations of inflation-sensitive bonds? Do they have any drawbacks themselves?

Inflation swaps, and especially zero-coupon inflation swaps (ZCIS) are used extensively by pension funds across the Euro area and Great Britain to transfer their risk exposure to a counter-party. Since their introduction, their growth has been unabated; they are the most actively traded inflation derivatives in the OTC market. This means, they are so liquid they can be customized to cover any duration.

An inflation swap is a bilateral agreement in which one counter-party, the payer, makes periodic payments to another party, the receiver, that depends on the realized inflation over a set period of time, and receives in exchange the fixed swap rate. In our case, the pension funds (or NNIP which acts on their behalf) are naturally at the receiving end, and the payer is either a derivatives broker or a hedge fund. The schematic diagram in Fig. 5 depicts how the deal unfolds:

Figure 5: Cash-flow exchange for ZCIS



The pension fund is always *long* inflation. At inception, to price the swap fairly and ensure the transaction is equitable for both participants, the current value of the fixed cash-flow must be equal to the floating cash-flow. Since no arbitrage exists at equilibrium, risk-neutral pricing theory dictates that swap rates should be unbiased predictors of future inflation rates. In fact, Ribero and Curto (2014) concluded that the 1-year ZCIS constitutes an accurate indicator of future inflation rates [7]. This resembles the expectations theory of the term structure of the interest rates stating that forward rates are determined purely by current spot rates as investors are risk-neutral. This claim holds if and only if the market is efficient. On balance, even if longer-term maturity swaps do not track the underlying index with surgical accuracy, they still carry valuable information that can be leveraged to forecast expected inflation. It is important to note that LDI strategists cannot directly use the observable inflation index itself to hedge inflation risk from a pension fund's portfolio; instead, they must resort to swaps because it is a *tradeable* market product. Equivalently, to manage interest rate risk, one must resort to interest rate derivatives.

The fixed swap rate corresponds to the break-even inflation rate (BEIR) which reflects the market's expectation on anticipated inflation rates. It is calculated thusly:

$$BEIR = n - r$$

with n=nominal yield on bond and r=real yield on ILB of similar maturity.

The receiver pays the fixed amount, known as fixed leg:

$$N * [(1 + BEIR)^T - 1]$$

On the other leg, the buyer pays the variable rate which corresponds to the realized compounded rate of inflation:

$$N * [\frac{I_T}{I_0} - 1]$$

At equilibrium:

$$N * [(1 + BEIR)^T - 1] = N * [\frac{I_T}{I_0} - 1]$$

where:
N= the notional amount
$BEIR$= the break-even inflation rate
I_0= the initial the index value
I_T= the index value at maturity

Payments are settled in arrears. In contrast to inflation-linked bonds, inflation swaps are not real-yield instruments; their payoff depends solely on the rate of inflation. Nonetheless, they require no upfront fees which explains why they are so attractive to asset managers; they can be used as much as the hedge requires while enough capital remains to be invested in higher-yielding assets. But some disadvantages exist constraining their usability.
Inflation-linked swaps are either traded directly with counter-parties or through a central clearing party. This requires accounts setups (i.e., margin accounts) and extensive legal documentation. Still, both markets are liquid and transaction costs are rather low. Secondly, portfolio managers must convince the clients that swaps are beneficial liability-hedging instruments; it can sometimes be a daunting task. Thirdly, pension funds must prove to the regulators that swaps are used solely for hedging purposes as it is illegal to speculate on them. Fourthly, the valuation analysis is hard to justify because the initial values are always forward-looking. It's worth noting ILBs are not bereft of this infirmity. Finally, there is always a probability that the opposing party will default on his contractual obligation: this is credit risk. However, to counter this risk, collateral is exchanged on a daily basis between NNIP and counter-parties. Still, Sovereign bonds are less likely to default because they are backed by governments.

2.3 Real Assets as Inflation-hedges

Which real assets possess inflationary hedging properties? What constrains their use in the asset management industry? Can we overcome these obstacles?

Unlike the previous inflation-hedging instruments, real assets are tangible products imbued with intrinsic value. Examples of such objects are Real Estate properties, Real Estate Investment Trusts (REIT) and even antique comic books [8]. These assets appreciate in value when inflation rates increase; this suggests, they have inflation-hedging abilities that can offset buying power depreciation caused by expected or unexpected inflation. But their inflation sensitivity is not on the same level as inflation swaps; inflation swaps are sensitive to expected inflation while real assets only to realized inflation.

Pension fund portfolios generally contain REITs. Nevertheless, these investment vehicles constitute only a partial hedge against expected inflation and perverse one against unexpected inflation. As such, they enjoy limited support amongst inflation-averse investors [9].

While residential Real Estate properties provide a nearly perfect hedge (the Real Estate CPI elasticity is equal 1.02 in the USA), they requires large upfront fees to acquire.

Although they can generate real returns, they are as impractical as ILBs since they are too expensive acquire and too illiquid to immediately convert to cash.

Gold is considered a decent inflationary hedge, but investors must incur storing costs if they stockpile too many bullion bars of it.

Consequently, despite the aforementioned products' salient feature, the best contender is the BEIR swap. Resultantly, we shall use historical BEIR quotes to simulate inflation in the scenario generation phase.

2.4 NN Investment Partners's Financial Engineering

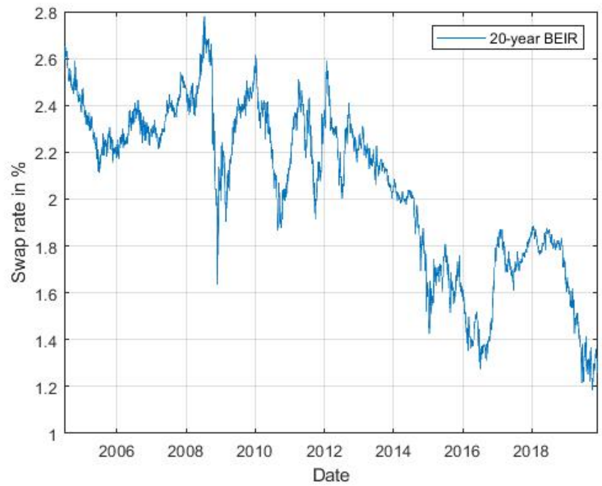
How does NNIP cope with inflation risk in the absence of an inflation curve? Is the method reliable? Can we fix this problem?

We previously stated that NNIP's ESG does not incorporate an inflation tool to derive the real coverage ratio distribution of Dutch pension funds.

Nevertheless, NNIP guides Irish clients with their investment objectives. Thus, the company came up with a temporary ad-hoc solution to forecast inflation rates: enter in a Euro swap agreement.

But according to NNIP, the BEIR curve contains a risk premium for unexpected inflation equal to +0.5% thus costing a lot of money to hedge. Indeed, we observe a mismatch between the quoted 20-year swap rate at different time points.

Figure 6: 20-year ZCIS rate from 2004 to 2019



By entering into this this agreement, NNIP would have clearly lost billions of EUR since it is a receiver. The cost of hedge in 2004 was around 2.6% p.a. for 20 years and dipped to 1.2% p.a. for 20 years in 2019; that's because the shape of curve changes constantly to reflect the market's expectations.
Secondly, NNIP argues the BEIR curve merely quotes a discrete prediction at any available ex-post date. But what the company needs is to estimate a prediction *interval* in which future inflation will fall, with a certain probability, given what has already been observed.
The panacea for such a woe is to construct a probabilistic forecast that captures a wide range of possible values inflation might take. This will help the company optimize their portfolios based on their sensitivity to inflation. The model would incorporate uncertainty, thus depicting a more realistic picture of inflation dynamics.
Hypothetically, if the model indicates that in 20 years, there will be a 95% chance that inflation lies between -0.001% and +0.001%, then NNIP might opt out of hedging altogether.

3 Forecasting inflation with Machine Learning

What is Machine Learning? Which Machine Learning techniques are used to forecast inflation? What are the main drawbacks? Can we fix them?

In this Chapter, we are concerned with the model selection phase; specifically, we probe a set of candidate models and pick the most appropriate one based on what we want to accomplish. Two classes of statistical models unsheathe from the literature: Machine Learning algorithms and stochastic models.
Section 3.1 opens with the definition and rational behind the Machine Learning (ML) as well as its limitations. Section 3.2 explores Neural Networks in the realm of time-series forecasting.
Section 3.3, ensues with Random Forest algorithm and its advantages over NN models.
Section 3.4 talks about Gradient Boosting models, an algorithm that is similar to Random Forest in its construction but characterized by the same predictive weakness.
Finally, Section 3.5 concludes this Chapter by comparing and contrasting Machine Learning algorithms and stochastic models.

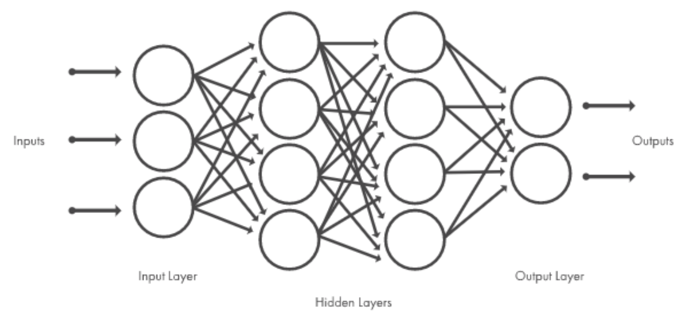
3.1 Machine Learning: motivation, relevance and limits

Machine Learning is the scientific study of algorithms and statistical models. It is a subset of Artificial Intelligence. The two major paradigms are supervised and unsupervised learning.
Under both settings, the algorithm performs specific tasks without the need of explicit human instructions. Instead, the execution is carried out based on statistical inferences. This implies, machine learning programs examine the patterns inside the sample data (i.e., the training data), generate an equation and make predictions on out-of-sample data accordingly. Thus, in the absence of split training data, Machine Learning is impotent. But, the difference lies in the presence of labeled data (i.e., known data) in the supervised learning paradigm. Specifically, the algorithm has a learning feedback mechanism allowing it to correct its answer (or label) because known data is used as input. In unsupervised learning, there are no pre-existing labels in the data set; as such, the algorithm must autonomously detect patterns and cluster them based on recognizable common properties [10].
Machine Learning is widely applied in many fields including engineering, medicine and finance. However, its efficacy is not identical across all disciplines; whilst it works nearly perfectly in radiology to spot early signs of lung cancer, it is fraught with false positives and inaccuracies in other disciplines.
But how does it fare in the realm of time series forecasting? We will showcase the findings of the state of the art literature on this subject.

3.2 Neural Networks

A Neural Network is computer system loosely based on the architecture of the human brain. More specifically, it consists of layers of nodes (i.e., artificial neurons) connected to one another through synapses [11]. These nodes are the core processing units of the network, and are represented by circles in Fig. 7:

Figure 7: Neural Network architecture



Three categories of layers exist: the input layer, the output layer and the hidden layers situated in-between. The signal travels from the input to the output, and each layer performs different data transformations. The output layer returns the final result which is usually a probability and its complement. The hidden layers do most of the computations required by the network.
Each node is attributed with a number between 0 and 1 and each synapse has a weight that reflects its linking strength. The inputs are multiplied to the corresponding weights and their weighted sum is propagated to the nodes in the next layer as input once the activation function is triggered.
If the predicted output is incorrect, the network will re-adjust its weights recursively until the right answer is obtained (i.e., usually, until the Residual Sum of Squares is minimized). This "learning" phase is called back-propagation.
Nakamura (2005) used a Neural Network to forecast short-term US inflation on quarterly basis using data from 1978 to 2003 [12]. The estimated model he calibrated is:

$$\hat{\pi}_{t+j} = L_1 tanh(I_1 x_{t-1} + b_1) + L_2 tanh(I_2 x_{t-1} + b_2) + b3 \tag{1}$$

where:

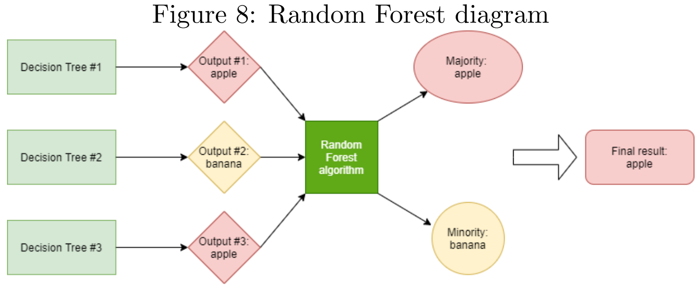
$\hat{\pi}_{t+j}$= NN inflation forecast j quarters in advance
x_{t-1}= One-period inflation lag
(L_1, L_2)= Layer weights
(I_1, I_2)= Input weights
(b_1, b_2, b_3) = biases

The author found that NN models tend to outperform univariate time-series models in the short-term, but suffer from over-fitting due to their multi-layered complexity. Over-fitting occurs when the algorithm is so closely associated to the training data, it becomes nigh impossible to generalize it to a wider data set [13]. As such, the model can no longer properly handle new data, and the forecast errors become too large to overlook. To overcome this hurdle, the model could skip data randomly to avoid data set memorization; still, the discarded data could contain valuable information, so it might be unwise to ignore it. Here, the model validator faces a dilemma over how much data suffices and what to sacrifice without compromising the model's reliability.
Furthermore, in contrast to stochastic modeling, NN is computationally greedy and incredibly data-driven; this means, it requires far more memory to store the data and train the model properly. Nevertheless, economic data is not always abundant, so the algorithm could be unusable in such instances. This is unsurprising as NN models were originally developed for image classification (in contrast to macroeconomic data, images of objects / living creatures are abundant); to teach the model how to recognize images, it must be first exposed to thousands of images at least during the training phase. Our data set for BEIR, interest rate swaps and global equity is made of 200 points; it *might* be insufficient to run a good NN model.
Finally, since the parameters generated are all deterministic, we deduce that NN models return single point predictions only. This means they could be good for hedge funds actively seeking to reap short-term gains (assuming the market is inefficient), but do not fit the bill for pension funds with long-term commitments.

3.3 Random forest algorithms

Random forests are also a supervised learning method that work by constructing multiple decision trees during the training phase. The final result depends on the output generated by the majority of the trees in the program. We schematically represent the elements of the system in Fig. 8:

Figure 8: Random Forest diagram



Concretely, a decision tree instantiates a learner (i.e., an array) that stores a variable based on a pre-specified criteria given the set of possibilities; for in-

stance, if Decision Tree #1 identifies the fruit as "red", it will hold an apple, otherwise it will be a banana. In this universe, a fruit is either an apple or a banana. Therefore, these binary trees are conditional operators. But sometimes, the tree wrongly classifies the variable if the data set possesses a high degree of entropy (i.e., randomness). To reduce this problem, the algorithm splits the data into smaller samples and runs the tests again until it prints the correct response. The optimal number of splits is reached when the information gain is maximized (or, equivalently, when the MSE is minimized).
If the majority of the trees store "apples", then the model predicts "apple".
Another interesting quirk of RF programs is, they can generate multivariate time-series models for forecasting purposes too.
In fact, Baybuza (2018) used an RF model to forecast short-term inflation in Russia [14]. He found that the accuracy of the monthly forecast is at least as good as the traditional forecasting methods. Specifically, he compared it with a Random Walk and an ARIMA model, and concluded that ML methods are slightly better in terms of accuracy from the second month onward. The reason is, RF models do not transform the raw data to make predictions unlike econometric models; basically, they ignore normality assumptions, structural breaks and function properly even in the presence of heteroskedasticity.
On the downside, RF models lack economic interpretability. On top of it, by design, they generate single point predictions (that might be wrong) for relatively short-term horizons just like NN models. To make matters worse, they suffer from the curse of dimentionality; this means, the further away in time RF models want to predict, the bigger the input space must become to accommodate such queries. The memory growth rate of such a process is exponential. Even though RF models are easier to train than their NN counterparts (i.e., NN models are still greedier), they are both unnecessarily cumbersome to use for time-series analysis. On the other hand, stochastic models are bereft of such data handling problems.
To summarize, RF algorithms are still inconvenient for NNIP.
Do boosting models cut the mustard?

3.4 Gradient Boosting models

Gradient Boosting (GB) resembles RF algorithms in that they constitute part of the ensemble learning methods of ML, and, uses the majority rule to predict the final outcome.
But they are more sophisticated because they ignore the independent learning assumption of regressors found in RF models.
It returns the best response function by computing the weighted sum of each predictor. If the answer is incorrect, it re-adjusts its weights sequentially during the training phase until the error is eliminated. This implies, GB is a supervised learning method as well. The trouble with GB programs is they might incorporate the noisy elements of the data into the parameters; as such, the fitted function can become unreliable.
A challenging task is to find the appropriate number of decision stumps; if there

are too many, there is a risk of over-fitting the function - even if the MSE is minimized.
To recapitulate, Gradient boosting is an ensemble method that uses weighted sums of regressors to produce better regressors. It starts by using a simple regression model, and the subsequent model is trained to predict the errors made by its previous version in a sequential fashion until no margin for improvement exists anymore. The overall prediction is returned based on the weighted sum of the collection.
Baybuza (2018), used a Gradient descent model to forecast Russian inflation. Similarly to RF, the model does not require prior data transformation to remove heteroskedasticity. The accuracy of the short-term monthly forecast is satisfactory since the MSE is relatively low. But auto-regressive models fared better in his experiment. Moreover, this ML model suffers from the same problem its aforementioned models do: it returns single point predictions only.

3.5 Summary

The literature reveals supervised Machine Learning methods for econometric forecasting has been scarcely applied, and the unsupervised learning paradigm has not been used in this domain since it serves different purposes.
The forecasting accuracy of Machine Learning models for short-term horizons is nearly as good as traditional models but, their predictive power deteriorates significantly in the long-term. Moreover, they generate in principle deterministic outputs whereas NNIP requires a bandwidth / distribution of possible values inflation might take. In contrast to stochastic models, they lack a stochastic component; without it, it is impossible to perform Monte Carlo simulations to adequately capture future uncertainties.
Yet, unlike stochastic models, they can handle non-numeric data which explains why they are so popular in signal processing and fuzzy control. Supervised Machine Learning algorithms can both forecast and classify data into categories based on inferences whereas stochastic models can only generate forecasts.
Although Machine Learning algorithms bear some resemblance with stochastic differential equations (i.e., they are both probabilistic models), they differ in the way they treat randomness; although it is possible to instruct the algorithm to return a range of possible values instead of a single number, the Machine Learning methods we reviewed were used to generate deterministic outputs only.
NNIP advisors argue it is unwise to predict 15 years ahead what the exact returns will be; therefore, it is recommended to avoid integrating such methods to the ESG. So, for our purposes, it is wise to model inflation, interest rates and equity returns as stochastic variables.

4 Stochastic methods to simulate inflation risk

What is the most suitable model to forecast long-term inflation for Dutch DB pension funds? What are the drawbacks? In which direction can we make progress?

Seemingly random changes in the financial markets have motivated the usage of *stochastic* models.
Stochastic models are either discrete-time or continuous-time processes.
In a discrete-time process, the random variable can take a countable number of values in a time interval, whereas the in the latter, it can take a continuous set of values.

4.1 Ornstein–Uhlenbeck process

The Ornstein-Uhlenbeck process is a time-homogeneous stochastic differential equation (SDE) that was initially developed by Dutch physicists to study the movements of dust particles under friction.
It was subsequently co-opted by Ferguson (2018) [15] to forecast inflation rates for the next 20 years. A variant of this model is employed at NNIP's ESG to model various asset classes as well. It bears some resemblance to the Brownian motion, but contains additional parameters. Its distinctiveness stems from the fact that the process exhibits mean-reversion; this means, the variable drifts towards its long-term mean over time. This is congruent with what Hall and Jaaskela (2011) have observed. As such, we will apply it because it adequately replicates one of the defining behavioral traits of the time series.
It is modelled as follows [16]:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t \tag{2}$$

where:

X_t=the random variable (i.e., the inflation rate)
κ=mean-reversion speed
θ=the drift (i.e, the long-term mean)
σ=the diffusion coefficient
W_t=Wiener process such that $W_t - W_{t-1} \sim N(0, \sigma^2)$

If κ is positive, then the equilibrium is attractive (i.e., inflation converges to its mean). Otherwise, it is repulsive. In the model validation phase, we expect a positive κ since inflation is presumed to exhibit mean-ergodicity.
We solve this SDE for X_t as demonstrated hereunder:

$$dX_t = \kappa\theta dt - \kappa X_t dt + \sigma dW_t$$

$$dX_t + \kappa X_t dt = \kappa\theta dt + \sigma dW_t$$

$$e^{\kappa t} dX_t + \kappa e^{\kappa t} X_t dt = \kappa\theta e^{\kappa t} dt + \sigma e^{\kappa t} dW_t$$

We apply Itô's product rule:

$$d(e^{\kappa t}X_t) = \kappa\theta e^{\kappa t}dt + \sigma e^{\kappa t}dW_t$$

We integrate from 0 to T:

$$\int_0^T d(e^{\kappa t}X_t) = \int_0^T \kappa\theta e^{\kappa t}dt + \int_0^T \sigma e^{\kappa t}dW_t$$

$$e^{\kappa T}X_T - e^{\kappa 0}X_0 = \kappa\theta\frac{e^{\kappa T} - e^0}{\kappa} + \sigma\int_0^T e^{\kappa t}dW_t$$

$$X_T - X_0e^{-\kappa T} = \theta(1 - e^{-\kappa T}) + \sigma e^{-\kappa T}\int_0^T e^{\kappa t}dW_t$$

$$X_T = X_0e^{-\kappa T} + \theta(1 - e^{-\kappa T}) + \sigma e^{-\kappa T}\int_0^T e^{\kappa t}dW_t \tag{3}$$

We see X_t is normally distributed because the integral of a deterministic function with respect to a Brownian motion is Gaussian. We can therefore affirm, the OU is a stationary Gauss-Markov process; this implies, it displays auto-correlation. This is consistent with what NNIP claims: past data contains useful information to forecast inflation rates. This model fits this criteria.
We distil the SDE's moments from its solution. We first derive the mean:

$$E[X_T] = E[X_0e^{-\kappa T} + \theta(1 - e^{-\kappa T}) + \int_0^T e^{-\kappa(T-t)}dW_t]$$

$$E[X_T] = X_0e^{-\kappa T} + \theta(1 - e^{-\kappa T}) \tag{4}$$

We now reproduce the variance formula to derive its analytical solution as well:

$$Var[X_t] = E[(X_T - E[X_T])^2]$$

$$Var[X_t] = E[(\sigma\int_0^T e^{-\kappa(T-t)}dW_t)^2]$$

By applying Itô's isometry, we obtain a deterministic integral:

$$Var[X_t] = \sigma^2\int_0^T e^{-2\kappa(T-t)}dt = \sigma^2\frac{1 - e^{-2\kappa T}}{2\kappa}$$

Finally, we get:

$$Var[X_t] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{5}$$

The OU process is a continuous-time stochastic process that exhibits mean-revertion. As such, we are interested in knowing what happens to the moments as time progresses. To learn more about its behaviour, we compute its limiting distributions; let us start with its expectation:

$$\lim_{T\to\infty} E[X_T] = \lim_{T\to\infty}[X_0e^{-\kappa T} + \theta(1 - e^{-\kappa T})]$$

We know:

$$\lim_{T\to\infty} e^{-T} = 0$$

Therefore, we have:

$$\lim_{T\to\infty} E[X_T] = X_0 \lim_{T\to\infty} e^{-\kappa T} + \theta(1 - \lim_{T\to\infty} e^{-\kappa T})$$

So,

$$\lim_{T\to\infty} E[X_T] = \theta \tag{6}$$

Eq. (6) algebraically demonstrates that in the long-term, the OU process converges to its own mean.
This is congruent with the ECB's current monetary policy which states that inflation should never exceed the 2% threshold. As such, we can assume the EU's inflation mean is just under 2%. Let's apply the limit to the variance:

$$\lim_{T\to\infty} Var[X_T] = \lim_{T\to\infty} [\frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T})]$$

So,

$$\lim_{T\to\infty} Var[X_T] = \frac{\sigma^2}{2\kappa} \tag{7}$$

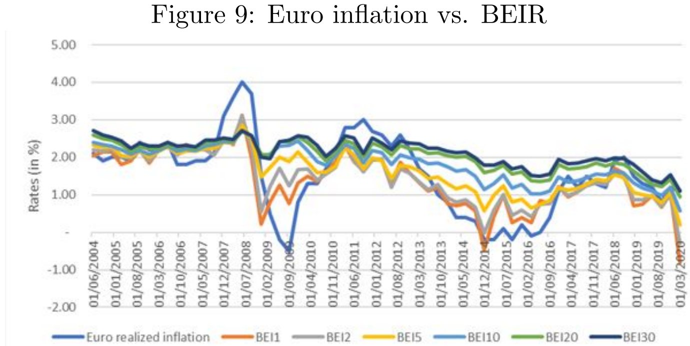
Eq. (7) shows the variance is inversely proportional to the mean-reversion rate. Concretely, this means the further inflation is from its long-term mean (i.e., θ), the faster the ECB will try to push it towards θ, thus increasing κ's value. That is logical; if the rate of inflation attained 30% in the Euro area, the ECB would try to bring it down to equilibrium far more aggressively than if it was at 3% to stave off hyper-inflation. This can be achieved either by selling bonds in the open market or increase the required reserve ratio. Conversely, the higher the mean-reversion rate, the lower the volatility; if inflation is close to its equilibrium, the ECB will not pursue any action to change its current position.
It is worth noting this model can handle negative rates as well; this was inconceivable in the past. To overcome this "problem", the Cox–Ingersoll–Ross model was put forth; its deterministic component is identical to the OU process, but the diffusion term is written as: $\sigma\sqrt{X_T}$ such that $X_T > 0$. Nowadays, this perception has ironically reversed, thus rendering the CIR model unusable; negative interest rates and deflation are sprawling all across Western Europe, thus becoming the norm.
Next, we start with the data processing part.

4.2 Data Preparation

We extract quarterly BEIR quotes with 1, 2, 5, 10 and 30-year tenures from Bloomberg Terminal. We use ZCIS instead of the Dutch CPI rates to simulate the curves because we can only hedge balance sheet risk by resorting to a market instrument. The BEIR is an ideal candidate since it is a tradable security that tracks inflation reasonably well as we explained in Sections 2.2 and 2.3. As a

result, indexation will depend on expected inflation, not inflation itself. The study period stretches back from 1970 to 2020; so, we have 200 observations in total. Although it seems extensive, LDI specialists insist that past information might provide valuable insights about the future. From a statistical perspective, increasing the sample size of the data should narrow the width of the estimated confidence interval the random variable falls into; so, the decision to examine such a large sample stands up to logical scrutiny.
The oldest traded ZCIS was issued in June 2004; before that date, inflation derivatives did not exist. But the nominal ESG is programmed to analyse data starting 1970. This means we have a void period between 1970 and 2004. Fortunately, we do not have this problem with interest rate swaps nor global equity. Nevertheless, if we do not fill this gap, the ESG will run into compiling errors, and our *real* ESG will not produce realistic results if we only use post-2004 data.
So, we back-fill the missing data by using the Dutch CPI as a proxy since Euro inflation rates are published on an annual basis only. It is a judicious decision because Dutch and Euro rates are highly correlated to each other.
In Fig. 9, we observe that realized Euro inflation is generally lower than its BEIR counterparts. The only exceptions transpire during the sub-prime mortgage crisis (2007) and the early phases of the European Debt crisis (2011 to 2013).

Figure 9: Euro inflation vs. BEIR



We deduce the spot rate is more volatile than BEIR swaps as it is more responsive to financial crisis. This is normal because swaps rates track spot rates not the other way around. Moreover, the reason longer maturity BEIRs are higher than shorter ones is because of counter-party risk; the probability of default increases with time. As such, the risk premium demanded by the fixed-leg investors rises. It is plausible to believe liquidity risk premium explains BEIR movements too since the spread between short and long-term maturities shrunk starting 2020. This implies investors are increasingly asking for longer BEIR maturities due to their grim economic outlook; the pandemic has wiped out any short-term growth prospects in the Euro zone.
We proceed this way: to back-fill BEI1 (i.e., 1-year BEIR), we use Dutch CPI rates since 1-year ZCIS are unbiased predictors of future inflation. This means,

from 1970 to 2004, the 1-year BEIR and the Dutch inflation rate are identical. To account for credit-risk, we must ensure this relation always holds: $BEI1 < BEI2 < BEI5 < BEI10 < BEI30$. Therefore, we compute the mean of each BEI time-series and back-fill them by the sum of the inflation rate plus the difference between the BEI mean and the inflation rate mean. For example: if $mean(inflation) = 2.5\%$ and $mean(BEI30) = 3.5\%$ during the same period, the difference between them is $\Delta_{BEI30-spot} = 1\%$. Hence, from 1970 to 2004, BEI30's missing data will be filled as follows:

$$BEI30_{1970-2004} = DutchCPIrate + \Delta_{BEI30-spot} \tag{8}$$

We implement this fix to all BEIR maturities. There is one drawback to this method: the correlations between the different maturities increase.

4.3 Parameter Estimation methods

Now that the data is complete, we ensue our study by estimating the optimal value of each parameter (i.e., κ, θ and σ). This step is crucial in that if the fit is satisfactory, we can expect the forecast to be reliable as well. Still, a good fit is a necessary but *insufficient* condition to judge the appropriateness of a model. We run robustness tests later in this paper.

Two routines exist to calculate the optimal parameters: Maximum Likelihood Estimation (MLE) and Ordinary Least Squares (OLS). If the underlying distribution is normal, they should both yield the same results. Regardless, we will explore both routines and chose the best one.

4.3.1 Ordinary Least Squares

Avellaneda uses the OLS method to estimate the OU's best parameters. He approximates it to a first order auto-regressive process with a constant term [17]. Then, he uses an OLS regression to minimize the Residual Sum of Squares and matches the parameters of both equations to solve for the unknowns.

The AR(p) model is defined as:

$$y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + ... + b_p y_{t-p} + \epsilon_t \tag{9}$$

Equivalently,

$$y_t = a + \sum_{i=1}^{p} b_i y_{t-i} + \epsilon_t \tag{10}$$

where:

$(y_{t-1}, y_{t-2}, \dots, y_{t-p})$ are the past values of y_t
$(a, b_1, b_2, \dots, b_p) \in \mathbb{R}^{p+1}$
ϵ_t is a white noise process such that $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

An AR(p) model is a stochastic process in which the variable's current value depends only on its own lagged values plus an error term. If the model is good, the

errors must be independent and identically distributed. It is a popular econometric model because it is not as prone to over-fitting problems as its ARIMA counterpart [4].
Following Eq. (9) and (10), AR(1) is written as:

$$y_{t|t-1} = a + by_{t-1} + \epsilon_t \tag{11}$$

In fact, Baciu (2015) used AR(p) and ARIMA(p,d,q) models to forecast monthly inflation in Romania [18], a country imbued with an IT regime. The author used as input realized inflation from 1993 to 2013.
Regarding the OU process, we assumed $T \in [0;+\infty[$. But we can generalize the expressions by re-writing the moments of the distribution thusly:

$$E[X_{t+\Delta t}] = X_t e^{-\kappa \Delta t} + \theta(1 - e^{-\kappa \Delta t})$$

$$Var[X_{t+\Delta t}] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa \Delta t})$$

$$X_{t+\Delta t} = X_t e^{-\kappa \Delta t} + \theta(1 - e^{-\kappa \Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \epsilon_t \tag{12}$$

Such that ϵ_t follows N[0;1]. Eq. (11) represents the **discretized** form of the OU's analytical solution.
From Eq. (11) and (12), we deduce that:

$$b = e^{-\kappa \Delta t} \Rightarrow \kappa = -\frac{ln(b)}{\Delta t} \tag{13}$$

$$a = \theta(1 - e^{-\kappa \Delta t}) \Rightarrow \theta = \frac{a}{1 - b} \tag{14}$$

$$SE = \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \Rightarrow \sigma = SE \sqrt{\frac{-2ln(b)}{(1 - b^2)\Delta t}} \tag{15}$$

This is the method employed by NNIP to estimate the correct parameters. Yet, since it requires many intermediate steps, the simulated model is not as satisfactory as the one generated by the MLE method which requires fewer steps. We thus explore the MLE optimization algorithm.

4.3.2 Maximum Likelihood Estimation

In an MLE setting, we answer the question: "given the model, which parameter values make the observed data most likely?". This method has been used by Vega[19]. We minimize sum of squares of residuals e_i, given by:

$$e_i = X_{t+\Delta t} - [X_t e^{-\kappa \Delta t} + \theta(1 - e^{-\kappa \Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \epsilon_t] | i = 2, 3...200 \tag{16}$$

[4] ARIMA(p,d,q) models are extensions of AR(p) processes

In MATLAB, we use the *normpdf* function to return a probability density function evaluated at e_i with zero mean and an arbitrarily chosen standard deviation. This is in-line with what we explained in Section 4.1; the OU process assume the underlying distribution is log-normal. We try to maximize the probability density function by selecting error values that are as close to zero as possible. To do so, our gradient descent routine must iteratively search the best (or optimal) values for our parameters. Next, we take the natural logarithm of our PDF values and sum them up. Finally, we maximize the final value by tinkering with the parameters once again.
Henceforth, we employ the MLE method to calculate the optimal parameters.

5 *Real* Economic Scenario Generator refinement

Which risk variables other than inflation should we include in the study? How to simulate them? Can we improve the models?

To recapitulate, we want to simulate inflation risk, interest rate risk and market risk as these are the main risk factors affecting the pension fund's balance sheet as we explained in Section 1.3; these three assets provide a good approximation of the overall portfolio risk. We can introduce bond yields to the study, but according to NNIP's LDI strategists bond yields are very well captured by the interest rate hedge, and credit spread returns are captured by market risk (so, they are included in equity returns).
Nevertheless, we first enhance the OU model by adopting NNIP best practices and run econometric tests to ensure the improved *composed* OU model is suitable for the risk variables.

5.1 Model Improvement

In Section 4.1, we stated one of the fundamental underlying assumptions of the OU is that errors are independent and identically distributed.
From a financial perspective, this argument is weak, because market variables are often correlated. For instance, we cannot dispute the fact that interest rates and inflation are somewhat related. To account for those interactions, NNIP came up with an improved version of the classical OU model: a composed OU model.
Concretely, NNIP's quantitative analysts introduced correlated error terms between financial variables; as such, the ϵ_t from Eq. (12) no longer follows a standard normal distribution. Henceforth, the error term is a correlated standard multivariate normally distributed function. In MATLAB, we use the command *mvnrnd*, and apply it to generate all 2,000 scenarios. The correlation matrix used as one of its parameters is the historical correlation between our time-series. The idea is that between 2021 and 2035, our variables will interact in a similar fashion they did between 1970 and 2020. After all, the goal quantitative modelling if to faithfully replicate historical trends.
The reason will call it a *composed* model is because it is an operation that takes two functions, an OU and the PDF of a joint normal distribution, to produce a third function: an OU with correlated error terms. We write it as: $OU(PDF(x))$ for all x in X.
NNIP argues it is a more judicious way to simulate an economy involving many variables. Since the pension fund's portfolio contains many risk assets, it is wise to employ this augmented model for our simulations.

5.2 Robustness tests

In this section, check the reliability of the composed OU SDE to model our risk variables. More specifically, we run a) serial correlation tests and b) statistical model validation.

5.2.1 Serial correlation

We generate the correlograms of BEIRs, IRs and global equity to determine whether the composed OU process is appropriate to model our time-series or not. Fig. 10 and 11 reveal that BEI1 and IR1 have lagged correlation; this indicates we can used past data to make predictions. So, BEI risk and interest rate risk are not completely random processes; the latter should be easier to forecast since it contains 11 significant lags in its auto-correlation plot whereas inflation has got only 5. The results can be generalized for all maturities as Appendix A.2 indicates. Thus, we can employ the same model to simulate both inflation risk and interest rate risk.

Figure 10: BEI1 Correlogram

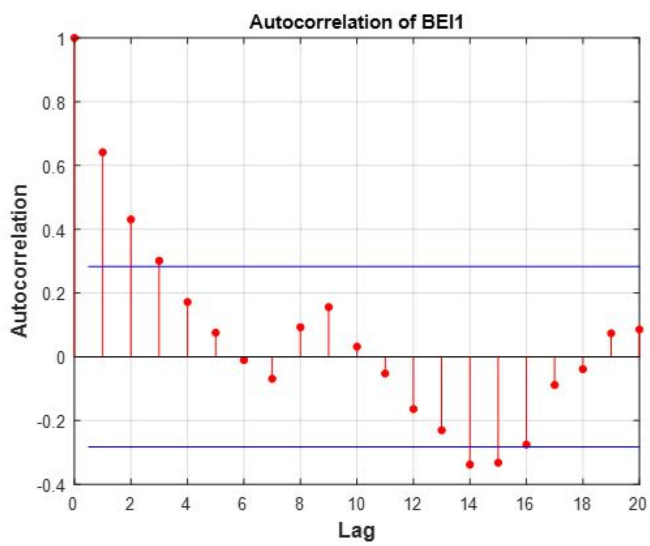
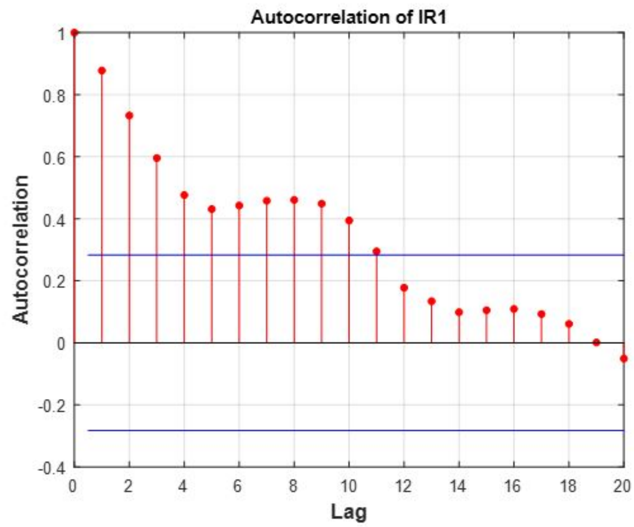
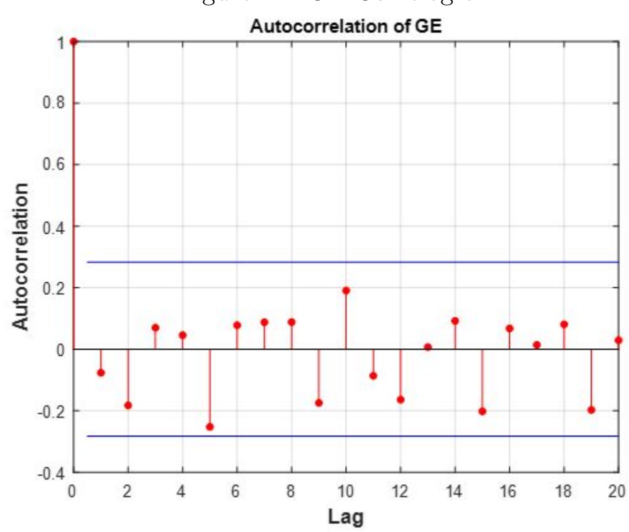


Figure 11: IR1 Correlogram



Unfortunately for NNIP, global equity bears no serial correlation. This is consistent with NNIP's claims; it implies we should not use the same equation to model equity returns.

Figure 12: GE Correlogram



The reason we can forecast interest rates and inflation is because governments dictate their levels to a large degree; on the other hand, stock returns are determined mostly by market forces. To ensure the composed OU is unsuitable for global equity, let us visualize the model-generated outputs to compare them with the historical data.

5.2.2 Model validation

We plot the 11 simulated time-series from 1970 to 2020 and compare them with the historical data. If the graphs are similar, then the interpolated output has sufficient fidelity. This is a necessary and sufficient condition to check the appropriateness of the model; as we said in Section 3.5, even Machine Learning models can replicate historical data with great accuracy, but their predictive performance deteriorates in out-of-sample spaces.
Fig. 13 and 14 show the simulated composed OU graphs for BEI1:

Figure 13: Historical BEI1 quotes

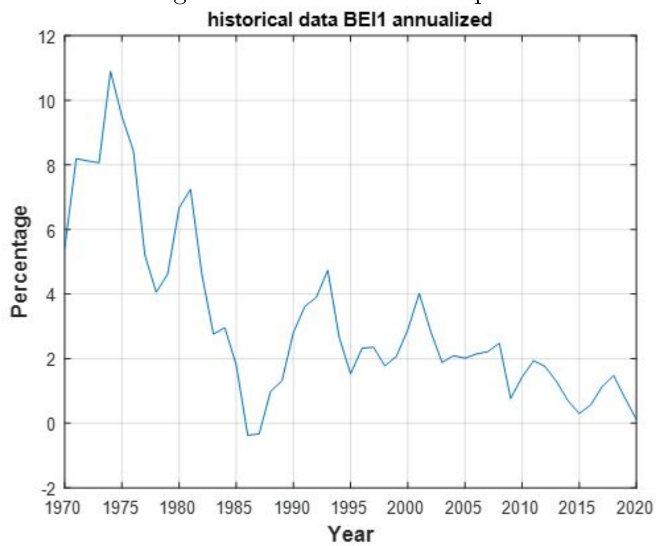
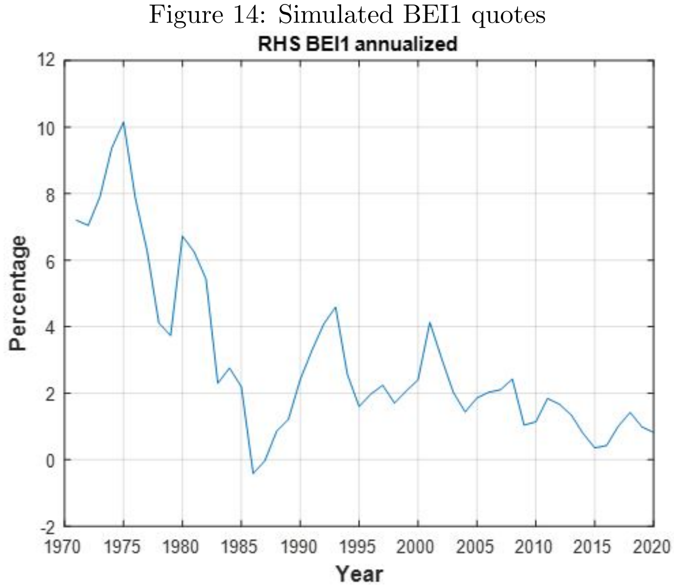


Figure 14: Simulated BEI1 quotes



Similarly, we plot IR1's historical and simulated composed OU model in Fig. 15 and 16 respectively:

Figure 15: Historical IR1 quotes

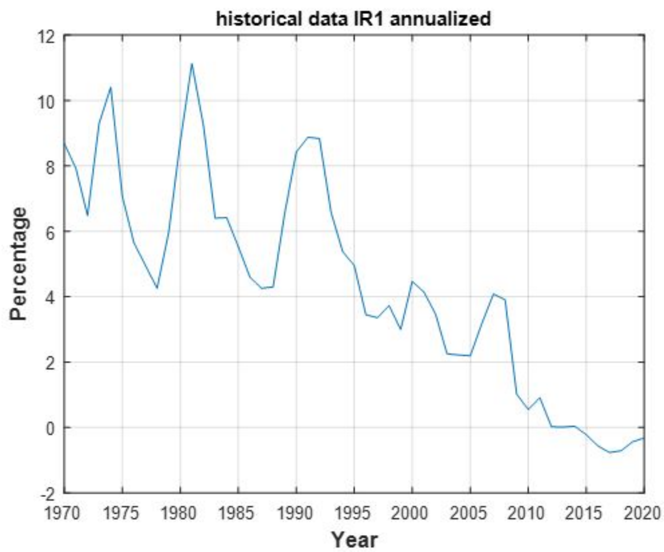
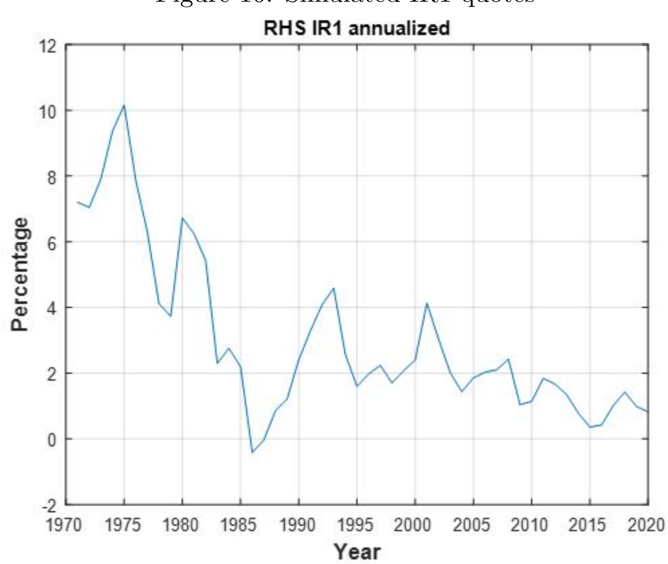
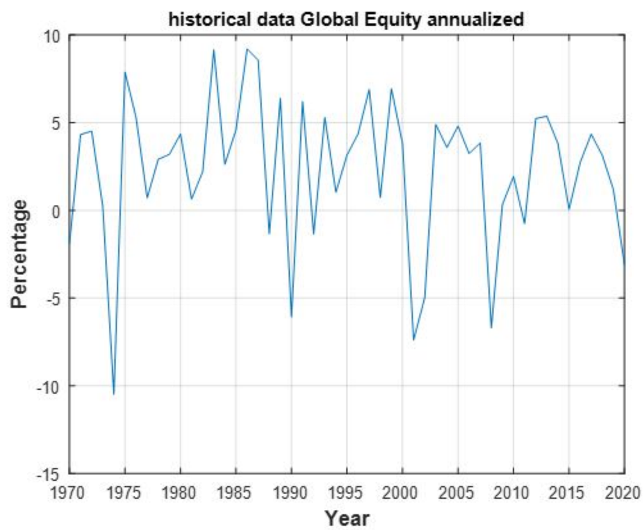


Figure 16: Simulated IR1 quotes



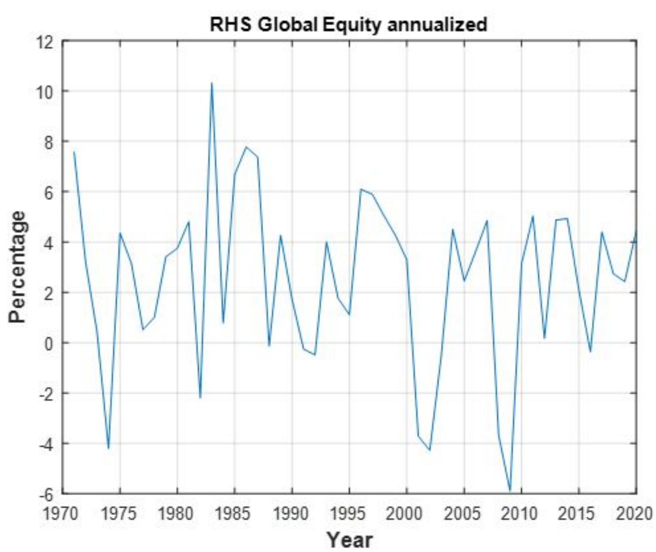
The simulated graphs do reassemble their historical counterparts. This is normal, since we previously proved inflation and interest rates exhibit serial correlation and the OU process is designed by construction to handle time-series with serial correlation. Even NNIP's LDI experts are aware of this fact. Furthermore, they know equity returns do not exhibit serial correlation, so, our composed OU model cannot describe global equity accurately.

Figure 17: Historical GE returns



As Fig. 17 and 18 proves, there is a mismatch between historical data and simulated output.

Figure 18: Simulated GE returns

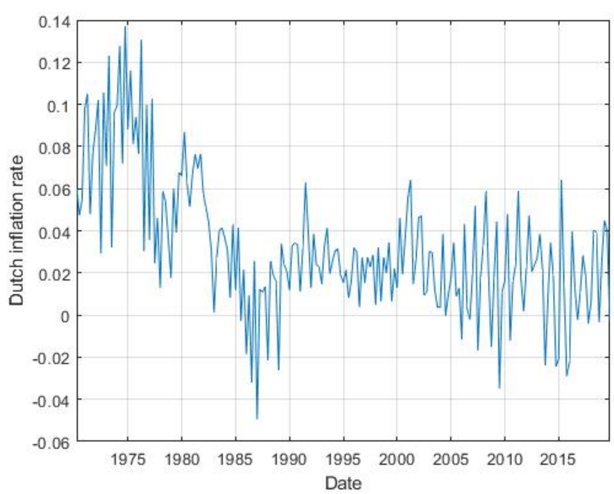


We observe equity returns skyrocket in the early 1970s; but in the simulated plot, it shows a huge dip at the same period. The same problem exists in 2020. This coincides with what Fig. 12 demonstrates; if the time-series is bereft of auto-correlation, then the composed OU is not suitable for modeling. As such, we must find another model.
We fix this problem by constructing a *composed* Random Walk model by slightly altering the composed OU process in the next Section.

5.3 Inflation rate risk

We previously stated the κ_{BEIR} vector of each of our BEIR time-series must contain positive numbers since inflation is a mean-reverting process; it is indeed the case. We also proved analytically in Eq. (6) their long-term mean should converge towards their respective historical mean; indeed, the MLE algorithm returned these results as Appendix A.1 shows. This means our MATLAB code functions correctly; however, the θ values are all far higher than 2%.

Figure 19: NL inflation from 1970 to 2020



The historical average of the Dutch inflation rate is 3.19%; and since all the BEIR curves are based on the Dutch CPI, they all have a θ above 3.19%. Fig. 19 points two events driving the mean upward:

1. The 1973 Oil embargo
2. The 1979 Iranian Revolution

These crises triggered a massive increase in Dutch inflation peaking at 14% and 9% respectively; these unexpected upswings were caused by oil shortages. Moreover, the Netherlands did not follow an IT regime at that time. It officially

adopted one in the early 2000s; that explains why, there are much fewer volatility clusters starting this decade.
Additionally, NNIP's strategists assume the ECB will succeed in its mission to reign in inflation; as such, expected inflation cannot exceed 2% p.a. Therefore, we hard-code the real ESG thusly:

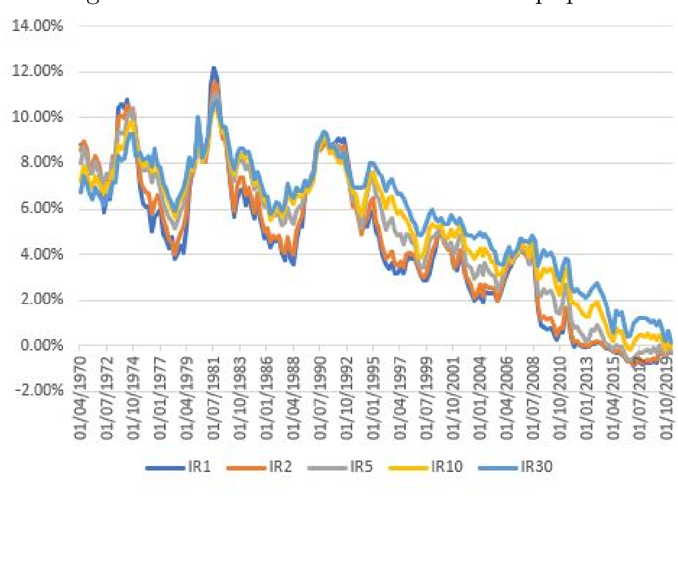
$$[BEI1, BEI2, BEI5, BEI10, BEI30] = [2.0\%, 2.1\%, 2.2\%, 2.3\%, 2.4\%] \quad (17)$$

The right-hand side of the expression is the θ_{BEIR} vector. It is the only parameter of the time-series that is user-generated at the requisition of NNIP; the rest of the parameters (i.e, κ and σ) are determined by the MLE optimization algorithm.
Ostensibly, by conserving σ and κ, we want to faithfully model the stochastic behaviour of our BEIR curves; even though we modified the levels for political considerations, we did not alter the remaining unique characteristics of the curves (i.e., volatility and mean-reversion speed).
In the next phase, we integrate interest rate risk and equity returns in the real ESG.

5.4 Interest rate risk

We apply the same reasoning for interest rates; as such, we also want to simulate interest rate curves using interest rate swaps with 1, 2, 5, 10 and 30 year maturities since the pension fund's average real liability duration is 26 years. Unfortunately, we encounter the same problem as with BEI rates; Fig. 20 shows the interest rate quotes were high for all key maturities;

Figure 20: Evolution of interest rate swap quotes



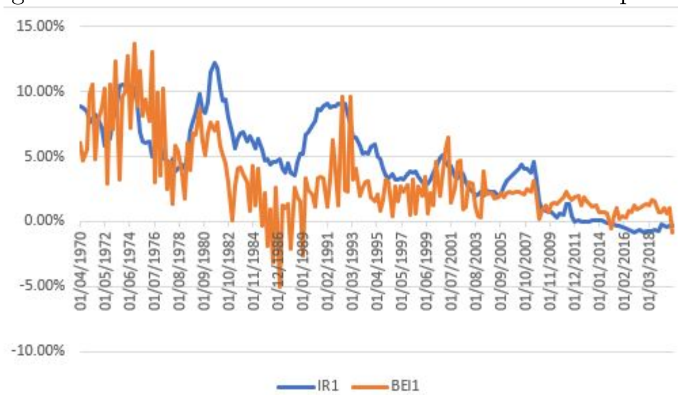
Historically, the interest rate averages were much higher than the user-generated θ_{BEIR} vector in Eq. (17).

Figure 21: Interest rate swaps: averages

Maturities:	historical average:
IR1	4.42%
IR2	4.55%
IR5	5.05%
IR10	5.37%
IR30	5.75%

So, we resort to the same fix we did for BEI rates. NNIP believes that interest rates and inflation are positively correlated because of the Fisher equation (cf. Section 2.1); Indeed, in Fig. 22 we observe this relation was mostly valid until the real Euro swap rates started becoming negative in 2008.

Figure 22: Relation between interest and inflation swap rates



Since interest rates should be slightly higher than inflation according to Fisher, we hard-code a IR's θ_{IR} vector as follows:

$$[IR1, IR2, IR5, IR10, IR30] = [2.1\%, 2.2\%, 2.3\%, 2.4\%, 2.5\%] \tag{18}$$

Similar to Section 5.1, the other coefficients are determined purely by the MLE routine.
Hitherto, BEIR and IR models are based on historical data for the volatility and mean-reversion, and expert opinion for the means. But how to model equity returns?

5.5 Market risk

We simulate equity returns (proxied by the MSCI index) because it is one of the main risks weighing in the pension fund's portfolio; specifically, it represents market risk.

To transform the composed OU into a composed RW, we set the mean-reversion speed κ close to zero. It is reasonable because if the time-series is not attracted to its long-term mean, it should theoretically behave like a composed Random Walk with a positive drift. As a result, we model equity returns by:

$$X_{t+\Delta t} = X_t e^{-\kappa \Delta t} + \theta(1 - e^{-\kappa \Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \epsilon_t \tag{19}$$

With kappa very small such that $\kappa = 0.0001$.

The rest of the parameters (i.e, σ and θ) are determined purely by the MLE algorithm.

Now that our models are built, we proceed with the scenario generation phase.

5.6 Simulation

We run 2,000 Monte Carlo simulations for each key variable from year 2021 to 2035; this means, we should have 30,000 yearly curves in total. We compute the summary statistics of the simulated trials and compare them with the historical data of each asset.

Figure 23: Descriptive Statistics for historical BEI rates

	BEI1	BEI2	BEI5	BEI10	BEI30
min:	[-4.9615	-4.9615	-3.7265	-3.5002	-3.1165]
max:	[9.9434	9.9434	11.1784	11.4047	11.7884]
mean:	[2.6313	2.6578	3.5655	3.7988	4.1582]
median:	[1.6814	1.6938	2.4351	2.6614	3.0451]
mode:	[-4.9615	-4.9615	-3.7265	-3.5002	-3.1165]
std:	[3.2930	3.273	3.4269	3.4184	3.4285]
range:	[14.9049	14.9049	14.9049	14.9049	14.9049]
prctile 2.5%:	[-3.2049	-3.2049	-1.9699	-1.7436	-1.3599]
prctile 97.5%:	[9.8395	9.8395	11.0745	11.3008	11.6845]

Figure 24: Descriptive Statistics for predicted BEI rates

	BEI1	BEI2	BEI5	BEI10	BEI30
min:	[1.2758	1.4053	1.4736	1.5817	1.6872]
max:	[2.7187	2.8243	2.9246	3.0124	3.1422]
mean:	[2	2.1	2.2	2.3	2.4]
median:	[1.9985	2.0966	2.2019	2.3033	2.3992]
mode:	[1.2758	1.4053	1.4736	1.5817	1.6872]
std:	[0.3498	0.3464	0.3522	0.3469	0.3503]
range:	[1.4429	1.419	1.451	1.4307	1.4550]
prctile 2.5%:	[1.3141	1.4388	1.5108	1.62	1.7232]
prctile 97.5%:	[2.6809	2.7854	2.8863	2.9759	3.1003]

We observe the range of the historical BEI rates is much larger than that of the simulated ones. As explained in Section 5.3, this difference is due to the crises in the 1970s and abscence of IT regime. NNIP's LDI strategists believe the ECB can control inflation rates in the Euro area, thus explaining why the simulated variables are less volatile and more clustered around the input-generate means. Consequently, no oil shock scenarios should take place in the future.

Figure 25: Descriptive Statistics for historical IR

	IR1	IR2	IR5	IR10	IR30
min:	[-0.8400	-0.798	-0.569	0.1084	0.6211]
max:	[10.4139	9.9342	9.8896	9.745	9.9950]
mean:	[4.3252	4.4463	4.9385	5.278	5.6623]
median:	[4.0628	4.3445	5.1296	5.8605	6.3400]
mode:	[-0.8400	-0.798	-0.569	0.1084	0.6211]
std:	[3.2195	3.1421	3.0457	2.7291	2.5085]
range:	[11.2539	10.7322	10.4586	9.6366	9.3739]
prctile2pt5:	[-0.7492	-0.6923	-0.371	0.1857	0.8130]
prctile97pt5:	[10.3154	9.9014	9.3992	9.2035	9.4535]

Figure 26: Descriptive Statistics for predicted IR

	IR1	IR2	IR5	IR10	IR30
min:	[-0.3842	-0.2986	-0.3633	-0.3357	-0.3160]
max:	[6.2360	6.2268	6.2598	6.2292	6.2652]
mean:	[2.1	2.2	2.3	2.4	2.5]
median:	[3.0801	3.1372	3.1156	3.109	3.1367]
mode:	[-0.3842	-0.2986	-0.3633	-0.3357	-0.3160]
std:	[1.7638	1.7518	1.7724	1.7641	1.7613]
range:	[6.6201	6.5253	6.6231	6.5649	6.5812]
prctile2pt5:	[-0.2524	-0.174	-0.2371	-0.209	-0.1924]
prctile97pt5:	[6.1182	6.1219	6.1448	6.1224	6.1529]

We notice our simulated interest swap rates also differ from the historical data. Similarly to BEI data, NNIP assumes interest rates exhibit mean-reversion; as a result, the simulated data is less volatile, and more centered around the means.

Figure 27: Descriptive Statistics for historical GE rates

	GE
min:	-38.303
max:	38.0195
mean:	8.2184
median:	15.7463
mode:	-38.303
std:	16.9031
range:	76.3225
prctile2pt5:	-28.5733
prctile97pt5:	32.4469

Figure 28: Descriptive Statistics for predicted GE rates

	GE
min:	-25.0969
max:	31.5964
mean:	8.1926
median:	13.6721
mode:	-25.0969
std:	16.8062
range:	56.6933
prctile2pt5:	-23.8891
prctile97pt5:	31.4196

Our ESG simulated equity returns satisfactorily compared to BEI rates and Interest rates. Indeed, the expected drift is nearly identical to the historical mean, and so is the volatility. It important to simulate this asset correctly, because Dutch pension funds allocate most of their capital in stocks. We proceed to the hedging simulation phase.

6 Inflation-hedging: a cost-benefit analysis

We ensue the study by exporting the previously generated 2,000 simulations of each asset class to the company's balance sheet tool.
Given the scenario set, we can simulate a client's balance sheet over time. This simulation is used as part of the portfolio construction process and can also be employed to review dynamic strategies. By employing such an approach, we can study the whole distribution of the balance sheet / funding ratio over time. This enables us to answer questions that cannot be answered using an analytical approach either because the analytical solution is too tedious to derive or it does not exist. Such questions include but are not limited to:

- What is the shortfall probability?
- What is the 97.5% VaR of the portfolio?
- What is the impact of using derivative strategies?

Our objective is to observe how the pension fund's real and nominal coverage ratio evolves over time given different hedge scenarios. Stated differently, how does inflation-hedging affect the risk-return profile of the pension fund? If our simulation set is reliable, what will be its coverage ratio 15 years from now? Will NNIP be able to fulfill its fiduciary duty towards its client or not? What is the optimal inflation hedge ratio based on our projections? We answer these burning questions in this Chapter.
Section 4.1 gives an overview of the balance sheet model. Section 4.2 presents the empirical results in case NNIP refrains from hedging inflation from its client's portfolio. Section 4.3 compares the outcomes when we gradually increase the hedge ratio at different points. Finally, Section 4.4 issues recommendations and caveats based on our investigation.

6.1 Model description

At its core, the balance sheet tool is an Excel file programmed in VBA language. It is composed of four main sheets:

1. Scenario input (i.e., the simulation set imported from MATLAB)
2. Balance sheet allocations / constraints
3. Balance sheet scenario generation
4. Output scenarios

In the first sheet, we just paste the raw values of the 11 time-series randomly generated 2,000 times each through the MATLAB code. The entire process is diligently explained in Chapters 4 and 5. These values do **not** change for the rest of the experiment.

The second sheet is a user input balance sheet template; specifically, the portfolio manager dictates the asset allocation of the pension fund's portfolio. In our case, we allocate 50% of the AuM in global equity, and the remainder goes to cash and bonds. Although we simulate the portfolio of a fictitious pension fund, these allocations are realistic according to NNIP.

We are aware that more assets exist in a pension fund's balance sheet (e.g. high-yield bonds, treasury securities, etc.) but they all are captured by our modeled risk factors; so, NNIP argues our simulation set should give a realistic representation of real-life risks. For example, high-yield bonds are adequately captured by market risk instantiated by global equity; that is because high-yield bonds are almost insensitive to interest rate risk and very sensitive to movements in the stock-market. We also ignore credit risk because it does not weigh heavily on treasury notes.

Moreover, in the same sheet, the LDI strategist decides what the interest rate hedge ratio and the inflation hedge ratio should be. We set the interest rate hedge at 50%; it corresponds to an industry standard. Manifestly, if interest rates increase by +1%, the pension fund's liabilities will decrease by half this amount. On the other hand, if interest rate drops by 1%, the liabilities increase by only half the amount. NNIP chiefly resorts to swaps for its hedges because bonds are costly. We will vary the inflation hedge at the following levels: 0%, 20%, 25%, 40%, 50%, 60%, 75% and 100% in the upcoming Sections. Naturally, a 0% inflation hedge means the LDI strategist decided not to swap the principal at all.

The third sheet is just a Macro to lunch the stochastic simulation. But in the back-end, it performs an important operation: it multiplies the returns of the input sheet by the allocations on the balance sheet. The interest rate hedge is calculated based on our change in interest rates. The same applies for the inflation hedge.

Finally, the fourth sheet furnishes the results of our long-lasting study; once the simulation is over, it returns two distributions with their respective moments: a) the nominal coverage ratio distribution and b) the real coverage ratio distribution. We can also see two tables with 2,000 rows (i.e., simulations) for each distribution illustrating the evolution of the coverage ratio from Year 1 to Year 15. We analyze the information further down.

6.2 Hedge-free scenario

Let us observe what happens if we refrain from hedging inflation risk from the balance sheet. Fig. 29 and 30 showcase the outcomes of the simulation:

Figure 29: Nominal Coverage ratio at 0% hedge ratio

Scenario	1
No of scenari	2000

	Overall
Average	117.4%
volatility	64.8%
Skewnes	0.82
Kurtosis	0.72
Minimun	-13.6%
Maximur	414.6%

Percentile	Value
1.0%	12.24%
2.5%	20.84%
10.0%	44.14%
25.0%	67.99%
50.0%	109.96%
75.0%	152.43%
90.0%	210.67%
97.5%	261.61%
99.0%	308.22%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	100.00%	142.41%	139.29%	148.00%	153.28%	144.87%	149.23%	165.03%	168.14%	172.51%	186.36%	193.51%	194.35%	193.32%	202.91%	100.00%	100.00%	Impact
1	100.00%	142.41%	139.29%	148.00%	153.28%	144.87%	149.23%	165.03%	168.14%	172.51%	186.36%	193.51%	194.35%	193.32%	202.91%	100.00%	100.00%	1.02910798
2	100.00%	137.37%	143.30%	142.81%	146.80%	151.29%	151.99%	145.01%	139.96%	139.90%	154.76%	170.54%	187.94%	199.33%	209.93%	100.00%	100.00%	1.09933442
3	100.00%	135.68%	149.14%	153.60%	154.22%	157.29%	160.96%	159.50%	175.09%	177.96%	189.11%	197.47%	210.13%	244.37%	251.74%	100.00%	100.00%	1.51742978
4	100.00%	139.59%	148.37%	151.08%	156.10%	145.09%	148.65%	152.81%	152.04%	159.77%	150.79%	156.78%	170.49%	170.22%	170.17%	100.00%	100.00%	0.70168718
5	100.00%	140.15%	143.45%	147.88%	154.28%	167.91%	172.39%	170.85%	198.19%	204.58%	203.73%	203.74%	211.88%	222.55%	223.49%	100.00%	100.00%	1.23492048
6	100.00%	143.98%	143.96%	148.79%	156.43%	169.48%	178.61%	198.13%	198.75%	204.66%	220.27%	238.45%	247.55%	257.63%	262.40%	100.00%	100.00%	1.62399328
7	100.00%	141.51%	147.88%	148.79%	156.79%	162.86%	175.18%	179.76%	184.13%	191.70%	212.35%	224.66%	229.39%	252.01%	260.38%	100.00%	100.00%	1.60382758
8	100.00%	139.46%	139.11%	139.63%	138.12%	141.68%	139.58%	138.75%	138.93%	135.51%	137.29%	132.02%	133.85%	142.35%	131.31%	100.00%	100.00%	0.31312011
9	100.00%	140.00%	148.92%	156.23%	168.86%	173.95%	185.15%	190.26%	201.63%	224.80%	243.38%	247.64%	280.79%	302.56%	309.50%	100.00%	100.00%	2.09501283
10	100.00%	138.55%	136.80%	147.33%	144.85%	156.67%	155.94%	154.40%	168.13%	179.37%	189.08%	195.46%	203.34%	196.98%	212.09%	100.00%	100.00%	1.12087692
11	100.00%	138.59%	148.52%	155.43%	166.55%	162.74%	170.89%	172.51%	166.98%	167.17%	178.69%	185.77%	177.33%	182.81%	182.89%	100.00%	100.00%	0.82890528
12	100.00%	135.77%	142.19%	141.78%	129.31%	136.62%	141.21%	144.37%	142.92%	143.03%	144.42%	146.93%	146.56%	158.09%	166.39%	100.00%	100.00%	0.66390856
13	100.00%	142.44%	155.51%	165.66%	175.64%	181.36%	203.98%	212.25%	212.01%	212.03%	226.23%	239.64%	265.67%	282.37%	296.07%	100.00%	100.00%	1.96070915
14	100.00%	144.86%	154.10%	164.02%	167.62%	155.46%	157.93%	150.74%	153.67%	144.84%	144.98%	141.90%	140.10%	143.98%	157.45%	100.00%	100.00%	0.57447161
15	100.00%	143.21%	140.89%	137.62%	138.88%	138.19%	145.11%	149.16%	152.86%	158.49%	159.35%	154.53%	157.14%	158.07%	155.78%	100.01%	100.01%	0.55775866
16	100.00%	141.12%	145.44%	150.34%	154.32%	162.83%	160.67%	157.63%	172.28%	169.24%	170.92%	171.69%	164.92%	159.78%	152.03%	100.00%	100.00%	0.5203013
17	100.00%	142.88%	156.42%	168.56%	166.12%	173.32%	188.19%	209.44%	198.96%	208.09%	231.52%	261.52%	284.44%	304.33%	310.67%	100.00%	100.00%	2.10673129
18	100.00%	140.53%	144.29%	137.37%	140.61%	141.60%	140.60%	137.92%	133.92%	135.58%	135.12%	138.15%	135.31%	133.79%	144.74%	100.00%	100.00%	0.44736965
19	100.00%	143.39%	154.08%	163.73%	172.06%	175.06%	178.88%	189.06%	204.76%	221.15%	233.70%	248.07%	271.12%	283.76%	293.28%	100.00%	100.00%	1.93282111
20	100.00%	140.22%	137.46%	153.23%	164.45%	170.10%	186.28%	187.58%	194.07%	228.15%	229.20%	243.62%	256.98%	262.79%	279.49%	100.00%	100.00%	1.79493674
21	100.00%	142.57%	134.94%	132.18%	139.81%	138.56%	142.58%	143.82%	165.50%	161.31%	172.29%	184.47%	201.72%	209.35%	210.13%	100.00%	100.00%	1.10131009
22	100.00%	136.01%	143.76%	148.26%	163.77%	168.61%	185.55%	200.66%	215.36%	217.12%	227.66%	237.52%	237.89%	232.48%	261.82%	100.00%	100.00%	1.61818907
23	100.00%	140.95%	147.76%	162.31%	158.74%	167.78%	178.48%	172.83%	178.40%	181.88%	194.67%	217.69%	237.85%	245.01%	260.68%	100.00%	100.00%	1.60676331

Figure 30: Real Coverage ratio at 0% hedge ratio

	Overall
Average	68.3%
volatility	44.4%
Skewnes	0.75
Kurtosis	1.07
Minimur	-25.2%
Maximur	272.7%

Percenti	Value
1.0%	-9.83%
2.5%	-4.19%
10.0%	16.74%
25.0%	35.23%
50.0%	63.27%
75.0%	94.24%
90.0%	125.66%
97.5%	167.56%
99.0%	198.13%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	82.72%	88.17%	96.21%	100.80%	107.63%	103.95%	110.77%	118.96%	114.29%	113.70%	121.99%	125.07%	134.48%	151.04%	100.00%	100.00%	Impact
1	77.51%	82.72%	88.17%	96.21%	100.80%	107.63%	103.95%	110.77%	118.96%	114.29%	113.70%	121.99%	125.07%	134.48%	151.04%	100.00%	100.00%	0.9487
2	77.51%	87.80%	95.00%	105.74%	107.89%	110.38%	117.96%	119.69%	129.17%	132.17%	141.80%	149.10%	160.73%	162.28%	171.89%	100.00%	100.00%	1.2177
3	77.51%	83.20%	92.90%	92.29%	94.98%	102.37%	100.82%	104.44%	105.46%	110.02%	112.68%	115.62%	126.02%	138.75%	142.11%	100.00%	100.00%	0.8335
4	77.51%	78.71%	77.14%	79.29%	79.16%	74.53%	73.06%	73.36%	77.37%	75.16%	71.26%	71.10%	78.01%	78.44%	76.40%	100.00%	100.00%	-0.0143
5	77.51%	88.46%	94.06%	94.99%	98.22%	96.34%	99.00%	107.63%	115.95%	119.78%	117.15%	124.91%	132.99%	144.55%	153.47%	100.00%	100.00%	0.98
6	77.51%	81.66%	89.99%	96.87%	103.28%	101.39%	103.61%	108.18%	116.60%	115.85%	116.17%	121.85%	123.41%	133.52%	140.27%	100.00%	100.00%	0.8098
7	77.51%	83.75%	81.95%	84.14%	87.55%	91.94%	92.66%	95.49%	95.25%	102.32%	109.67%	117.24%	115.35%	123.58%	132.06%	100.00%	100.00%	0.7038
8	77.51%	81.93%	85.60%	86.58%	83.51%	80.25%	78.42%	86.00%	82.89%	83.33%	87.56%	89.99%	88.47%	90.17%	91.15%	100.00%	100.00%	0.176
9	77.51%	85.03%	92.40%	97.36%	92.65%	100.08%	108.17%	107.49%	116.31%	117.43%	122.93%	127.05%	138.38%	145.83%	164.41%	100.00%	100.00%	1.1211
10	77.51%	81.55%	84.26%	85.63%	84.93%	91.96%	94.73%	95.48%	102.93%	108.16%	112.17%	113.39%	123.31%	127.93%	133.29%	100.00%	100.00%	0.7196
11	77.51%	81.45%	83.28%	79.99%	87.43%	84.21%	90.39%	99.01%	100.41%	98.67%	104.94%	99.30%	97.90%	98.90%	99.84%	100.00%	100.00%	0.2882
12	77.51%	86.21%	86.84%	92.03%	90.82%	97.41%	100.16%	107.26%	112.79%	117.53%	128.36%	143.05%	137.08%	136.45%	145.02%	100.00%	100.00%	0.871
13	77.51%	83.34%	86.40%	92.85%	103.93%	105.50%	114.61%	121.70%	132.58%	126.90%	124.42%	135.33%	148.59%	157.76%	157.48%	100.00%	100.00%	1.0318
14	77.51%	85.77%	89.41%	88.41%	99.17%	94.16%	91.78%	85.70%	83.48%	77.40%	70.56%	65.62%	68.01%	72.87%	73.18%	100.00%	100.00%	-0.0559
15	77.51%	85.67%	91.10%	95.99%	89.97%	96.62%	100.60%	104.81%	109.48%	111.26%	114.40%	124.55%	133.58%	131.63%	133.38%	100.01%	100.01%	0.7208
16	77.51%	86.71%	89.72%	97.22%	105.10%	110.43%	109.33%	112.93%	114.51%	118.07%	118.51%	118.95%	121.70%	116.43%	119.82%	100.00%	100.00%	0.5459
17	77.51%	81.80%	84.32%	87.94%	89.97%	93.65%	100.53%	95.47%	97.31%	103.54%	109.43%	127.01%	128.82%	134.81%	134.14%	100.00%	100.00%	0.7307
18	77.51%	84.27%	82.26%	81.29%	89.94%	87.19%	87.99%	84.67%	83.64%	80.48%	77.14%	77.20%	74.11%	69.62%	70.77%	100.00%	100.00%	-0.0869
19	77.51%	84.88%	87.03%	91.72%	104.35%	115.90%	125.25%	124.01%	140.09%	156.55%	165.86%	182.02%	192.79%	190.92%	179.61%	100.00%	100.00%	1.3173
20	77.51%	81.49%	83.89%	94.17%	106.92%	114.96%	125.62%	128.44%	133.81%	151.75%	157.60%	161.80%	173.66%	175.28%	201.06%	100.00%	100.00%	1.594
21	77.51%	84.82%	89.46%	84.23%	83.66%	84.12%	88.68%	87.19%	88.52%	89.36%	94.58%	93.39%	96.58%	94.26%	99.47%	100.00%	100.00%	0.2833
22	77.51%	82.82%	88.30%	89.57%	98.10%	97.20%	97.91%	103.67%	104.46%	103.97%	112.98%	127.33%	134.09%	141.37%	149.52%	100.00%	100.00%	0.929
23	77.51%	82.89%	92.86%	103.90%	105.23%	106.65%	117.61%	117.25%	111.34%	112.05%	123.85%	132.71%	138.41%	144.08%	149.66%	100.00%	100.00%	0.9309

In our universe, the hypothetical pension fund was founded in Year 1. Once created, its assets and liabilities are equal, thus explaining why its nominal coverage ratio is equal to 100%. As such, Year 1 corresponds to 2021 in the real world and Year 15 corresponds to 2035.

It is worth keeping in mind we annualized the data generated from the real ESG beforehand.

We notice the real coverage ratio's figures are lower than the nominal coverage ratio's figures; it is because to obtain Table 30's numbers, we divide the nominal coverage ratio by the *inflation*-adjusted liabilities. In other words, the compound indexation factor δ_t, is missing from the nominal coverage ratio's denominator. As a result, the nominal coverage ratio is always larger or equal than the real coverage ratio. LDI advisors closely monitor the *real* coverage ratio because it reflects the pensioner's true buying power. On the other hand, nominal figures are important for two reasons:

1. Determine if the COLA provision can be exercised (i.e., $CR > 120\%$)

2. Derive the real coverage ratio

We observe a sharp jump in the nominal coverage ratio of nearly 40% at Year 2 across all scenarios; this translate to a modest increase of 5% in real terms. From Year 2 to Year 10, the nominal coverage ratio increases, albeit at a slower rate. And from Year 10 to Year 15, its rise accelerates until it reaches 220% on average. We observe the same pattern for the zero-hedge real coverage ratio; indeed, after 15 years, the pension fund's expected real coverage ratio winds up at 131% after indexation. The "impact" column calculates the coverage ratio's rate of return between Year 15 and Year 1 in both cases. We compute it like this:

$$E[CR] = \frac{\sum_{i=1}^{N}[\frac{CR_{Year15}-CR_{Year1}}{CR_{Year1}}]_i}{N} \tag{20}$$

So, as the summary statistics above the simulation tables indicate, by forgoing the inflation hedge and investing 50% of its capital on global equity, the pension fund's expected nominal coverage ratio return is 117.4% in nominal terms and 68.4% in real terms. That is hardly a surprise; the latest MSCI quarterly return was recorded on 31/03/2020; it was -15.48%. The stock-market had slumped following the stay-at-home orders issued by governments to curtail the spread of COVID-19. This translated into low equity yields. This date corresponds to the latest input used by the real ESG to forecast global equity. The composed random walk's role is to faithfully replicate global equity's historical trend; since global equity's drift was historically [much] higher than -15.48%, the shape of our curves in the next immediate period will be upward sloping most of the time. Since the composed random walk's drift is positive, the overall trend will also be positive; with hindsight, our model is partially vindicated because right after the crash, investors started buying the dips thus raising the stock prices (and returns) in a V-shaped fashion. So, for a pension fund created right in the midst of a pandemic, its funding position will drastically improve if it invests its capital in stocks -even if the inflationary hedge is nonexistent.

As a caveat, we demand investment professionals to take our results with a small grain of salt as our ESG is not risk-neutral; we follow empirical probabilities instead. As such, our gains might be slightly overestimated since the model's drift is higher than the risk-free rate which has been dipping into negative territory since the corona outbreak. Indeed, the historical average of equity returns is around 8% p.a.

Risk-neutral models are used to price derivative instruments, not for scenario generation.

Regarding the second moment, we see that nominal coverage ratio's return distribution is more volatile than the real coverage ratio's return distribution. Fig. 31 and 32 are histograms of the simulated distributions.

Figure 31: Nominal coverage ratio distribution

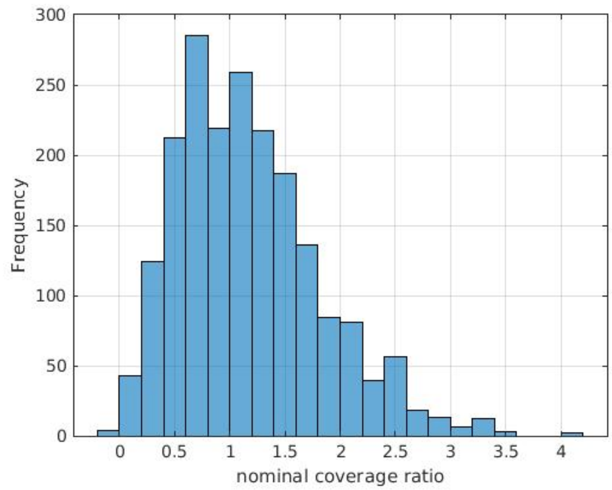
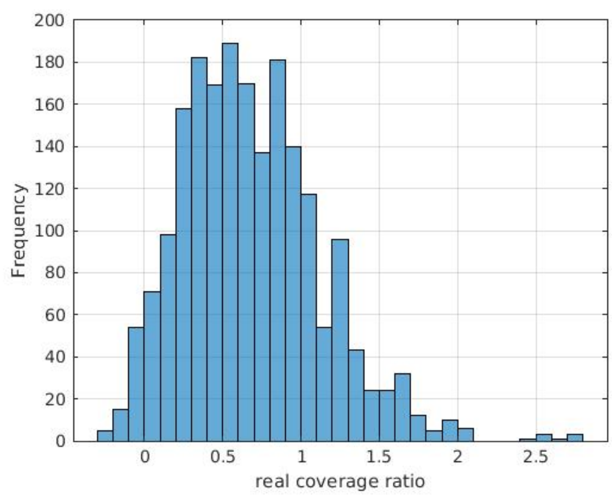


Figure 32: Real coverage ratio distribution



They are both log-normally distributed with positive skewness; this means the data is clustered in the left side and exhibits a fat tail on the right. So, the pension fund should expect to incur frequent small losses (i.e., coverage ratio dips) and rare but massive gains to cover the losses. This is a desirable feature. Kurtosis is a scalar denoting a type of risk. The kurtosis risk is positive for both probability density functions (PDF); this means they have fat tails, implying they can take extreme values (positive or negative) quite frequently. They are

leptokurtic. And although the real PDF has less volatility, it has slightly fatter tails.
On balance, we demonstrated the coverage ratio gains were considerable in a zero-hedge environment. But how do the central moments of the PDFs change when we introduce an inflation hedge?

6.3 Hedging scenario

We ensue the study by setting the inflation hedge ratio at 20%. Fig. 33 and 34 show the results of the simulation:

Figure 33: Nominal coverage ratio at 20% hedge ratio

Scenario	1
No of scenario	2000

	Overall
Average	132.0%
volatility	75.6%
Skewnes	0.82
Kurtosis	0.66
Minimum	-14.7%
Maximun	467.6%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkracht	Koopkracht vs ind	
	100.00%	152.24%	145.92%	154.69%	159.93%	147.08%	153.27%	171.03%	172.79%	179.04%	196.25%	202.91%	203.03%	199.56%	207.97%	100.00%	100.00%	Impact
1	100.00%	152.24%	145.92%	154.69%	159.93%	147.08%	153.27%	171.03%	172.79%	179.04%	196.25%	202.91%	203.03%	199.56%	207.97%	100.00%	100.00%	1.07968
2	100.00%	144.09%	149.59%	146.02%	150.33%	155.17%	154.16%	145.16%	137.34%	136.68%	151.77%	168.29%	186.06%	198.59%	209.03%	100.00%	100.00%	1.09031
3	100.00%	143.63%	158.36%	164.18%	163.82%	165.49%	170.41%	167.26%	186.70%	188.72%	201.97%	211.60%	225.05%	264.91%	272.91%	100.00%	100.00%	1.72905
4	100.00%	150.34%	162.76%	165.19%	171.93%	157.41%	162.75%	168.07%	165.05%	176.50%	164.98%	173.45%	189.86%	188.89%	189.41%	100.00%	100.00%	0.89408
5	100.00%	147.30%	149.62%	154.68%	161.57%	179.38%	183.92%	179.00%	210.86%	217.52%	216.55%	214.15%	222.03%	232.34%	231.50%	100.00%	100.00%	1.31503
6	100.00%	154.65%	151.33%	155.52%	163.35%	180.26%	191.10%	214.89%	213.03%	220.46%	240.16%	261.90%	272.52%	282.49%	286.56%	100.00%	100.00%	1.86564
7	100.00%	150.70%	159.54%	159.52%	168.77%	175.01%	190.61%	195.07%	200.19%	207.51%	231.68%	244.86%	250.60%	276.89%	284.55%	100.00%	100.00%	1.84553
8	100.00%	148.85%	146.82%	146.93%	145.70%	151.50%	149.17%	145.42%	146.65%	141.80%	142.80%	135.13%	137.95%	148.37%	133.50%	100.00%	100.00%	0.33498
9	100.00%	148.35%	157.56%	165.33%	182.92%	187.04%	198.94%	205.36%	217.32%	246.38%	268.65%	272.56%	311.92%	337.25%	341.31%	100.00%	100.00%	2.41306
10	100.00%	147.90%	144.43%	157.91%	154.56%	168.16%	165.99%	163.28%	178.86%	191.72%	202.94%	210.56%	218.02%	208.53%	225.85%	100.00%	100.00%	1.25853
11	100.00%	147.93%	160.63%	171.06%	183.73%	179.06%	187.98%	187.23%	179.32%	179.78%	193.06%	203.72%	192.56%	199.43%	199.00%	100.00%	100.00%	0.99005
12	100.00%	142.57%	150.60%	148.31%	131.84%	139.18%	144.06%	145.99%	142.84%	141.80%	141.27%	141.74%	142.28%	155.22%	163.12%	100.00%	100.00%	0.63119
13	100.00%	152.03%	168.43%	179.73%	189.44%	195.93%	222.89%	231.33%	227.83%	228.65%	247.00%	261.26%	290.94%	309.59%	326.12%	100.00%	100.00%	2.26116
14	100.00%	154.13%	164.90%	178.05%	179.04%	163.84%	167.51%	159.51%	164.25%	154.23%	157.26%	155.02%	151.37%	154.87%	173.85%	100.00%	100.00%	0.73848
15	100.00%	152.15%	146.94%	140.99%	144.22%	141.16%	148.65%	152.57%	155.82%	162.15%	162.30%	154.05%	155.19%	156.41%	153.20%	100.01%	100.01%	0.53201
16	100.00%	149.15%	153.57%	157.40%	160.07%	168.99%	166.07%	161.11%	178.38%	173.50%	175.20%	175.84%	166.73%	161.47%	151.24%	100.00%	100.00%	0.51239

Figure 34: Real coverage ratio at 20% hedge ratio

	Overall
Average	78.4%
volatility	48.3%
Skewnes	0.80
Kurtosis	1.13
Minimun	-16.7%
Maximur	294.1%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkracht	Koopkracht vs indexatie ambiti	
	77.51%	88.43%	92.36%	100.56%	105.18%	109.28%	106.76%	114.79%	122.26%	118.61%	119.73%	127.92%	130.65%	138.82%	154.80%	100.00%	100.00%	Impact
1	77.51%	88.43%	92.36%	100.56%	105.18%	109.28%	106.76%	114.79%	122.26%	118.61%	119.73%	127.92%	130.65%	138.82%	154.80%	100.00%	100.00%	0.99725
2	77.51%	92.09%	99.17%	108.12%	110.48%	113.21%	119.64%	119.81%	126.74%	129.13%	139.06%	147.13%	159.12%	161.68%	171.15%	100.00%	100.00%	1.20821
3	77.51%	88.08%	98.64%	98.64%	100.90%	107.70%	106.74%	109.52%	112.46%	116.67%	120.34%	123.90%	134.97%	150.41%	154.06%	100.00%	100.00%	0.98763
4	77.51%	84.77%	84.62%	86.69%	87.18%	80.86%	79.99%	80.69%	83.99%	83.03%	77.97%	78.66%	86.87%	87.05%	85.04%	100.00%	100.00%	0.09713
5	77.51%	92.97%	98.11%	99.36%	102.87%	102.92%	105.62%	112.76%	123.37%	127.36%	124.52%	131.29%	139.35%	150.91%	158.97%	100.00%	100.00%	1.05097
6	77.51%	87.71%	94.60%	101.25%	107.85%	107.84%	110.86%	117.33%	124.98%	124.79%	126.66%	133.83%	135.85%	146.40%	153.19%	100.00%	100.00%	0.97646
7	77.51%	89.19%	88.42%	90.21%	94.24%	98.80%	100.82%	103.61%	103.55%	110.75%	119.65%	127.79%	126.02%	135.78%	144.31%	100.00%	100.00%	0.86191
8	77.51%	87.45%	90.35%	91.10%	88.08%	85.81%	83.81%	90.13%	87.50%	87.20%	91.07%	92.11%	91.17%	93.98%	92.67%	100.00%	100.00%	0.19557
9	77.51%	90.10%	97.77%	103.03%	100.37%	107.61%	116.23%	116.02%	125.36%	128.70%	135.70%	139.83%	153.72%	162.54%	181.30%	100.00%	100.00%	1.33911
10	77.51%	87.05%	88.96%	91.78%	90.62%	98.71%	100.83%	100.97%	109.50%	115.60%	120.39%	122.15%	132.21%	135.44%	141.94%	100.00%	100.00%	0.83125
11	77.51%	86.94%	90.07%	88.04%	96.45%	92.65%	99.43%	107.46%	107.83%	106.12%	113.38%	108.89%	106.30%	107.89%	108.64%	100.00%	100.00%	0.40165
12	77.51%	90.53%	91.98%	96.27%	92.60%	99.24%	102.18%	108.46%	112.73%	116.52%	125.57%	138.00%	133.07%	133.97%	142.17%	100.00%	100.00%	0.8342
13	77.51%	88.94%	93.58%	100.74%	112.10%	113.97%	125.23%	132.63%	142.47%	136.84%	135.84%	147.54%	162.72%	172.97%	173.46%	100.00%	100.00%	1.23794
14	77.51%	91.26%	95.67%	95.97%	105.92%	99.23%	97.34%	90.68%	89.23%	82.41%	76.53%	71.68%	73.49%	78.38%	80.80%	100.00%	100.00%	0.04245
15	77.51%	91.02%	95.02%	98.34%	93.43%	98.69%	103.05%	107.20%	111.60%	113.83%	116.52%	124.16%	131.92%	130.25%	131.17%	100.01%	100.01%	0.69236
16	77.51%	91.64%	94.73%	101.79%	109.02%	114.61%	113.01%	115.43%	118.56%	121.04%	121.48%	121.84%	123.04%	117.67%	119.19%	100.00%	100.00%	0.53783

At first sight, we notice an increase in the expected nominal coverage ratio of nearly +15% compared to the zero-hedge scenario. If the COLA provision is activated, it goes down from 132% to 78.4%.
Moreover, the volatility of the nominal funding ratio in the 20% hedge scenario has increased compared to the previous zero-hedge scenario and understandably so; the BEI swaps are risky derivative products imbued with variable financial performance. When we add them to the asset side of the pension fund's balance sheet, there are strings attached: they come with additional risk which translates

into increased volatility. If the underlying rate moves in the unwanted direction, our assets could become impaired; as such, swaps are double-edged swords. Similarly to the zero-hedge case, the real coverage ratio's volatility has decreased compared to its nominal counterpart; it fell to 48.3% from 75.6%. This happened because the swaps removed the ex-ante risk from the balance sheet; we therefore deduce that break-even inflation increased since the hedge was effective. This is logical as the preponderance of BEI curves generated by the ESG are upward sloping since the latest historical BEIRs (that were used as input in the first simulation) were far below the 1% - 2% bandwidth. This ongoing anomaly is exacerbated by the corona-virus pandemic. Furthermore, we stated earlier the composed Ornstein–Uhlenbeck process is stationary, mean-ergodic and, most importantly, mean-reverting; we manually set the θ_{BEIR} vector to [2.0%, 2.1%, 2.2%, 2.3%, 2.4%] so that the time-series could eventually converge to these values. By construction, the model ensures the BEI trends are upward sloping most of the time hence explaining why the inflation hedge was beneficial to the pension fund. Had the latest historical BEI quotes with their respective maturities been above the θ_{BEIR} vector, the ESG would have generated mostly downward sloping curves thus, reducing the coverage ratio in case an inflation hedge was in place.

It is worth mentioning our in-house Ornstein–Uhlenbeck process is not risk-neutral; if it was, we would have had an equal number of upward sloping and downward sloping BEI curves to eliminate the possibility of arbitrage gains. In this probability space, each BEIR should act like a Martingale with respect to their latest respective historical market quote. In the risk-neutral world, NNIP should have been indifferent between hedging or not since economic profits are nonexistent. If they do exist however, there was a mispricing in Fig. 5: either the fixed leg is under-priced, or, realized inflation has an intrinsically random component. At any cost, mean-reverting models are hardly ever risk-neutral, and asset managers believe markets are inefficient.

To better visualize how the other moments have been impacted, we plot both PDFs hereunder:

Figure 35: Nominal coverage ratio distribution at 20% hedge ratio

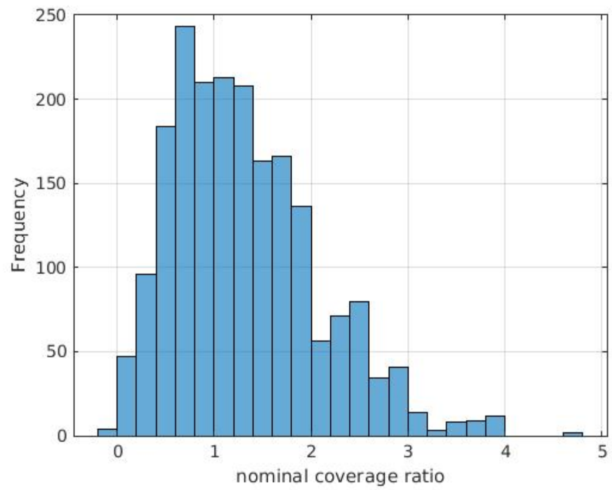
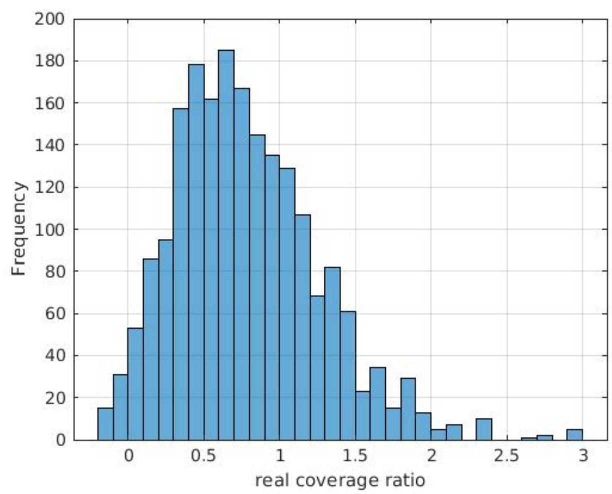


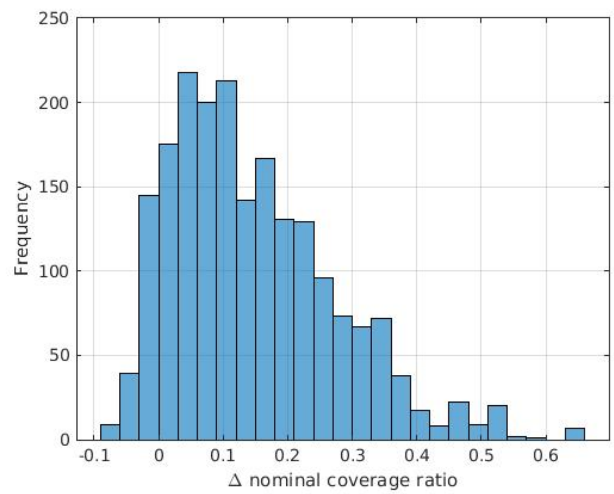
Figure 36: Real coverage ratio distribution at 20% hedge ratio



The distributions still exhibit positive skewness and kurtosis risk. What's more, the real PDF is always shifted to the left by a scalar compared to the normal PDF; this difference represents the nominal coverage ratio portion that was destroyed by inflation.

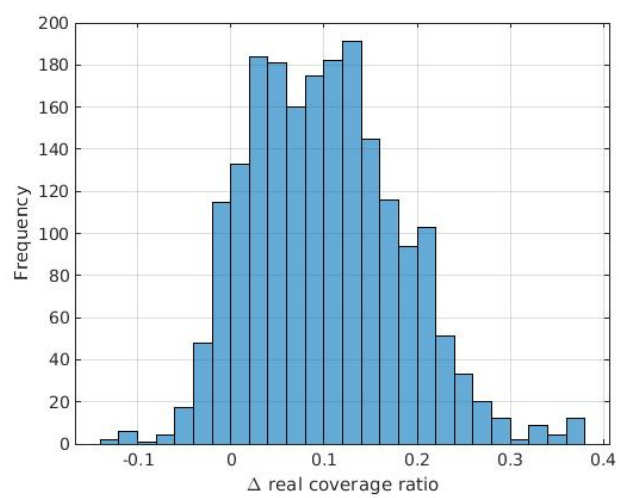
To quantify the benefit, we plot two PDFs illustrating the difference between the 20% hedge and zero-hedge case.

Figure 37: $\Delta_{20\%-0\%}$ nominal coverage ratio distribution



More precisely, Fig. 37 was created by subtracting the nominal coverage ratio distribution at 0% hedge (cf. Fig. 31) from the nominal coverage ratio distribution at 20% hedge (cf. Fig 35).

Figure 38: $\Delta_{20\%-0\%}$ real coverage ratio distribution

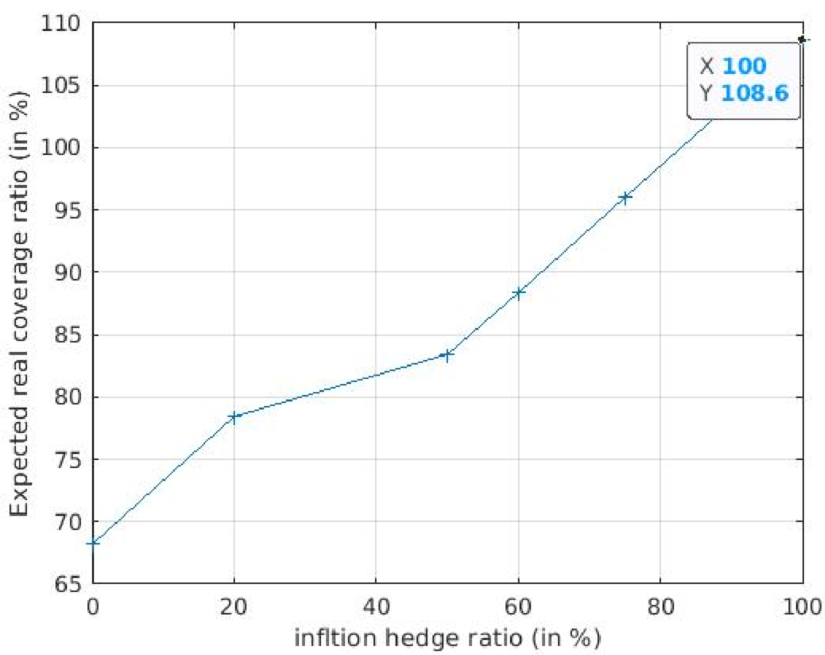


Likewise, Fig. 38 was created by subtracting the real coverage ratio distribution at 0% hedge (cf. Fig. 32) from the real coverage ratio distribution at 20% hedge (cf. Fig 36).

The mean of $\Delta_{20\%-0\%}$ real PDF is 10.07%; this means, by hedging inflation

rate risk at 20%, the retirees can expect a real buying power appreciation of 10.07% in 15 years. It is a bonanza for the pension fund. However, the benefit gained from the hedge is smaller than equity appreciation; Fig. 30 indicates the real gain in buying power was 68.3% without the inflation hedge between Year 1 and Year 15. Therefore, the incremental benefit of hedging inflation risk is small, but not negligible.
Is there an optimal hedge ratio? At which point it becomes undesirable to hedge inflation rate risk? We answer these questions by monitoring the expected change in the real coverage ratio's return when we gradually increase the level. By repeating the simulation, we derive the following results:

Figure 39: Expected real coverage ratio with respect to the inflation hedge ratio



The outputs of each simulation are in Appendix A.4. The expected inflation-adjusted coverage ratio increases when the hedge ratio increases albeit at a decreasing rate. When the pension fund swaps 100% of its principle, its funding ratio increases to 108.6%; it corresponds to a 40% increase compared to the zero-hedge scenario. As a result, if the simulation is correct, we advise the pension fund to hedge inflation rate risk from its balance sheet entirely to maximize its funding position. The industry standard is to hedge approximately 70% of the principle amount, but we infer, based on our study, the optimal hedge ratio for our hypothetical pension fund is 100%.

7 Recommendations and Conclusion

Our simulation results suggest that by hedging inflation risk from the pension fund's balance sheet, the retirees' real buying power increases considerably; not only they keep up with expected inflation, they expected a real coverage ratio increase of 40% over a 15-year time horizon. In truth, by hedging their entire inflation exposure, pension funds can expect to maximize their funding position, and reduce their risk profile thus responding dutifully to their fiduciary obligations. Therefore, based on our simulation runs, we recommend NNIP to set the inflation hedge ratio at 100% since we expect an increase in realized inflation in the upcoming years. Moreover, since global equity is the driving force behind the coverage ratio increase, we suggest allocating more capital on stocks. However, we urge some caution since our scientific experiment is based on a series of assumptions that are not always unassailable.

First, our simulation is just one possible version of the future based on historical probability distributions. We cannot guarantee it will necessarily take place.

The use of an Ornstein-Uhlenbeck process is well-documented in the corpus of inflation forecasting, and NNIP's modified version enhances it considerably since it now account for correlated exogenous market variables like global equity and interest rates. Still, this composed model is not risk-neutral; consequently, our results are too optimistic. In a risk-neutral world, the pension fund would not have been able to earn more than the BEIR quote itself i.e., between 0.5% p.a. and 3% p.a. of the swaped principle. In our study, we earned 40% from the inflation hedge alone. Even if inflation is intrinsically random, it is unlikely to earn such a return even over a 15 year period.

Furthermore, our composed Random Walk is not risk-neutral either; maybe if we used a Geometric Brownian motion to simulate global equity instead, we would have gotten lower gains for the coverage ratio. Since the Random Walk's drift is based on historical data, the expected return will be much higher than the risk-free rate which explains why even in the absence of an inflation hedge, the expected coverage ratio is that high.

Nevertheless, our model incorporates correlated error terms meaning it constitutes a noteworthy improvement over the literature's traditional Markov models assuming independent error terms.

We did not use true probabilities to price the swap rates; we used them to sketch the probability distribution of several hedging positions NNIP might take. As such, even though the ESG is not risk-neutral, it has some theoretical validity.

Our composed stochastic models are more suitable for NNIP's mandate than supervised Machine Learning algorithms and traditional time-series models for three reasons:

1. They possess a stochastic term to create Monte Carlo simulations
2. They account for and reproduce the interactions between market variables
3. The probability of capturing the correct values for break-even inflation, interest rate swaps and equity returns in the long-term is higher since

composed stochastic models generate a range of values, not a single point prediction.

Theoretically, ML programs could be set up to return confidence intervals, but Nakamura (2005) and Baybuza (2018) used them to generate discrete predictions only. This is not helpful for NNIP at this form, but we advise the company's quantitative analysts to investigate the possibility of creating ML models that return bandwidths instead of single points and compare their predictive power to that of the currently employed composed stochastic models. Such a research has not been undertaken yet by the company.

Another issue is the constant volatility assumption of our models. In real life, there are volatility clusters in the markets meaning we should perhaps enhance our models further by assigning time-varying volatility instead. But it will complicate the calculations and risk over-fitting our models which are already sophisticated enough.

Although the back-filling method is cogent, the pre-2004 BEI curves are parallel to each other because they are separated by a fixed scalar; as a result, there might not be enough curve risk in our simulated inflation curves thus rendering the quotes cheaper. On the other hand, NNIP believes it has been partially offset in part by the user-generated θ_{BEIR} vector; they think the BEIs are a bit overprice since realized inflation has been below the 2% mark for the past 5 years approximately.

Most importantly, our real ESG is just a minimum viable product; we simulated three essential variables present in the pension fund's portfolio hoping to capture all the market risks. But in reality, there are more asset classes.

Finally, NNIP's ESG incorporates a Markov-switching regime to account for business cycle changes; our real ESG does not possess one. Maybe it renders our simulation less reliable.

Despite these technical limitations, the model improvement, the scenario generation routine and the methodology to derive expected coverage ratios remain very relevant and practitioners can learn from this report and update their technical framework to improve the Dutch pension fund industry.

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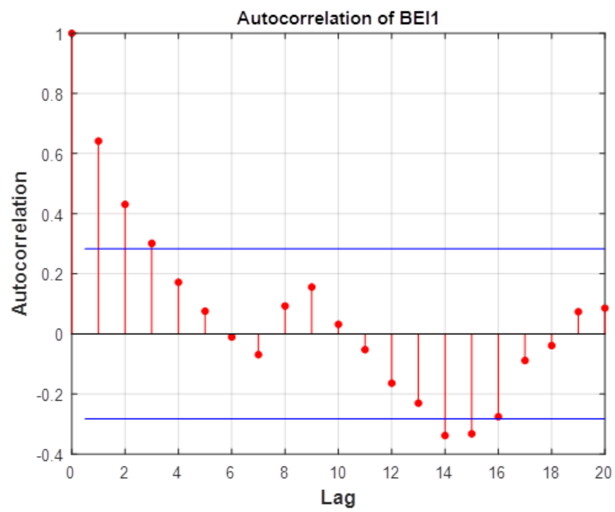
A Appendix

A.1 MATLAB code

The author is bound by a confidentiality agreement not to share the code with third parties.

A.2 Autocorrelation plots

Figure 40:



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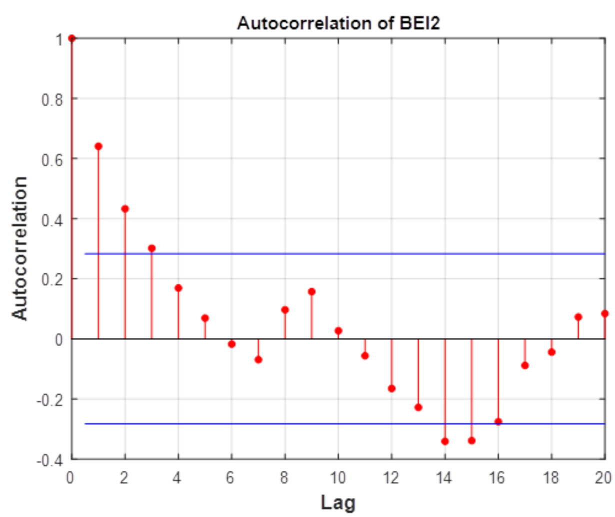

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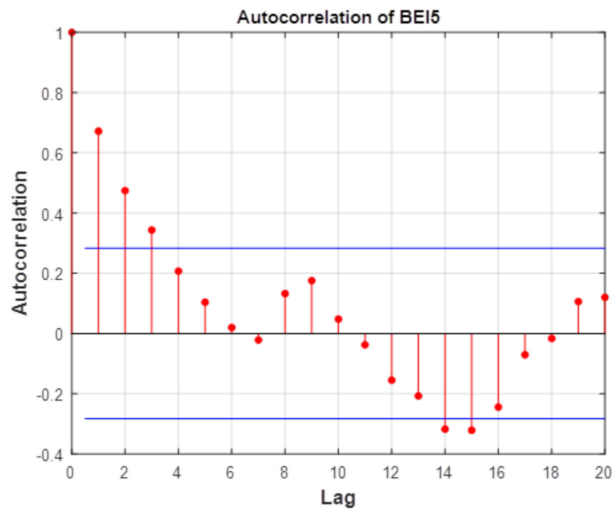

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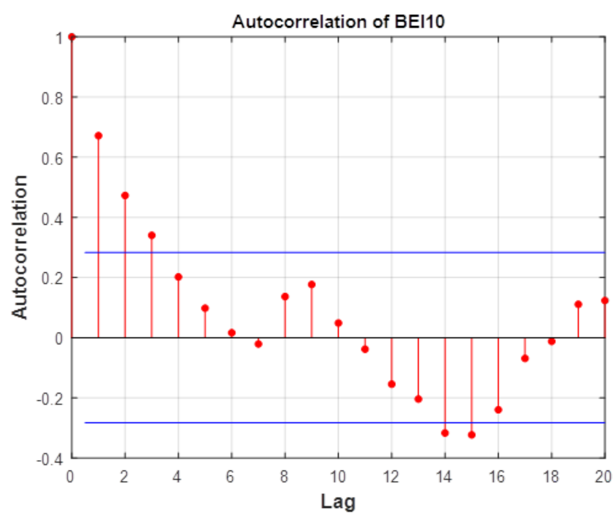

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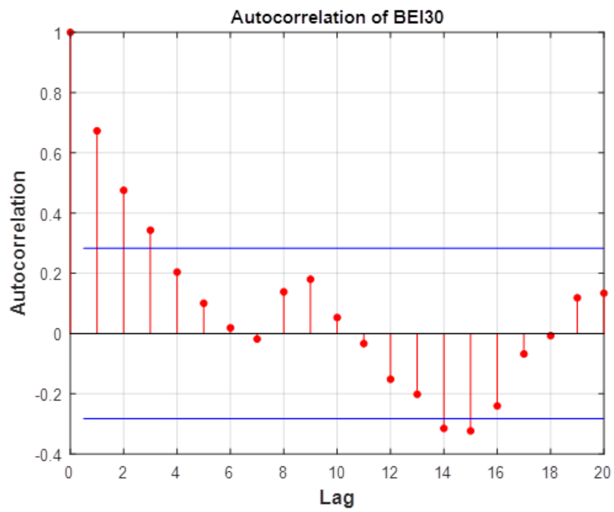

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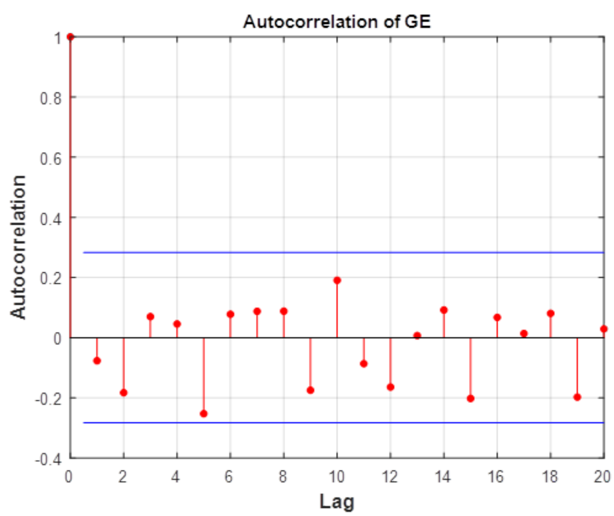

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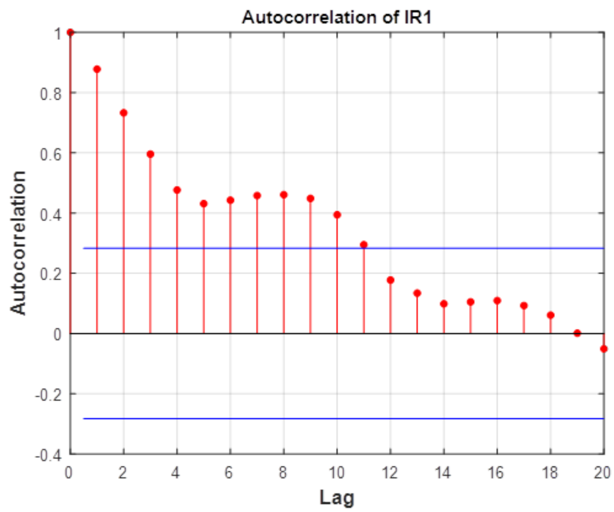

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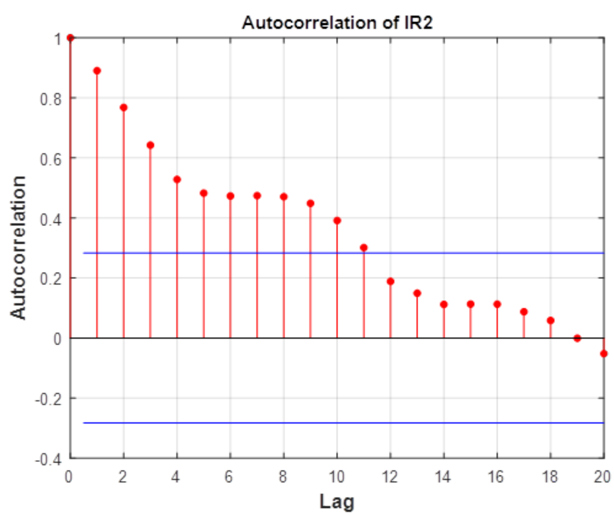

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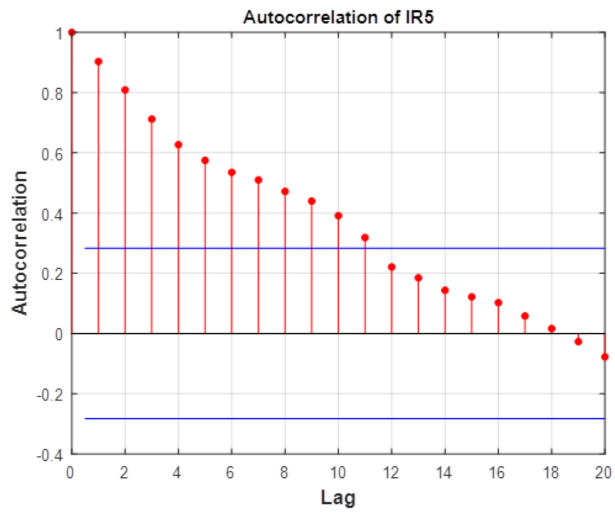

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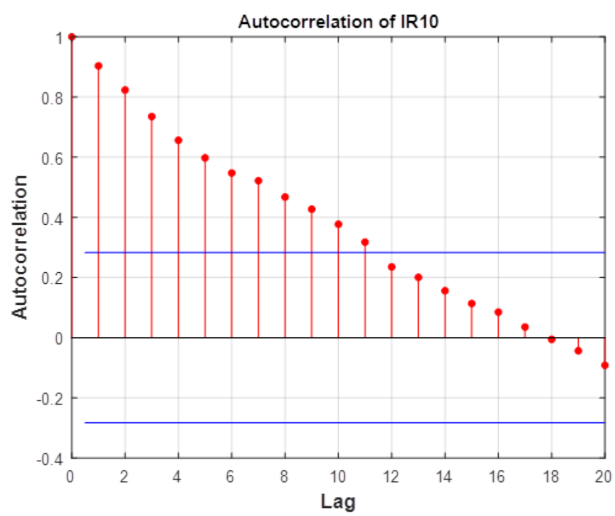

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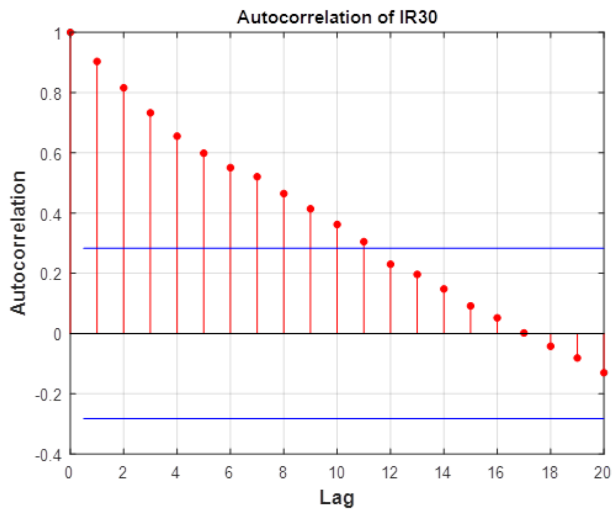
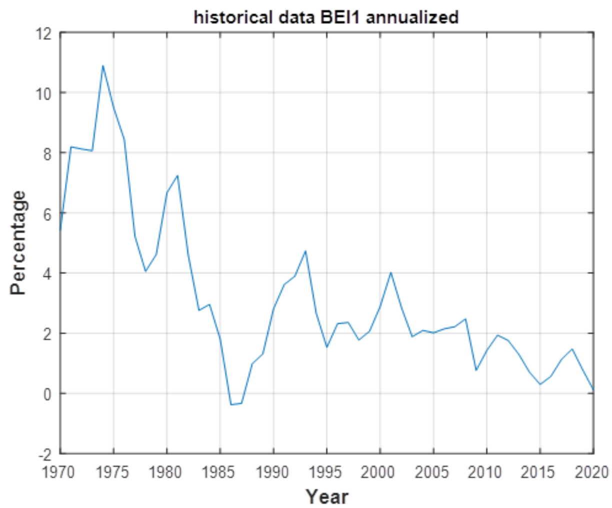


Autocorrelation of IR30
Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
12
14
16
18
20

A.3 Time-series: historical vs. simulated

Figure 51:



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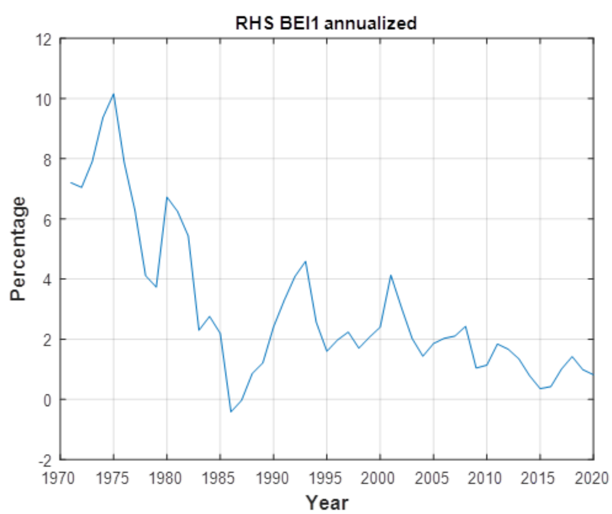

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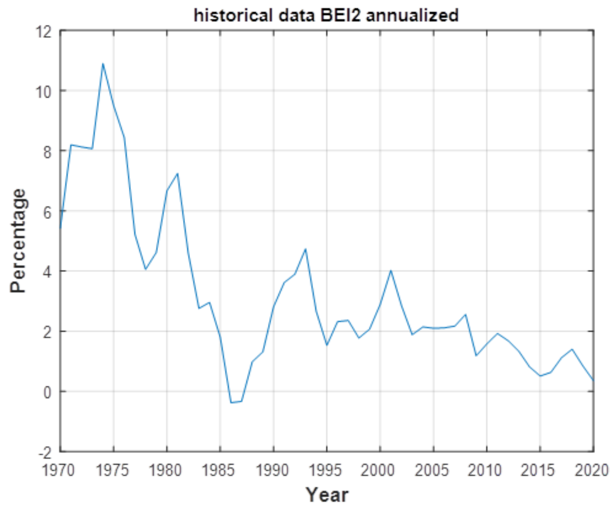

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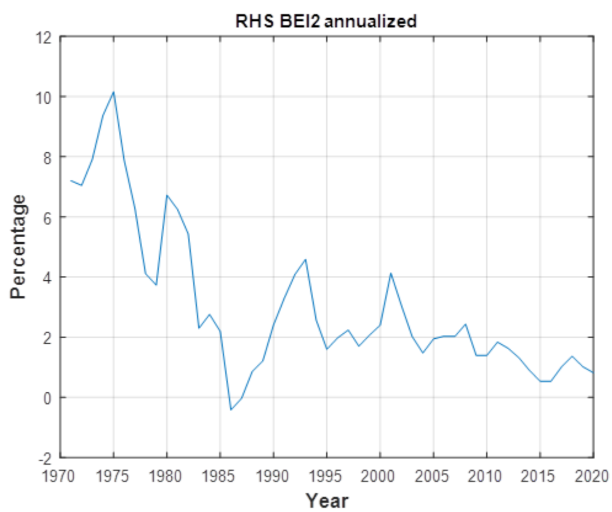

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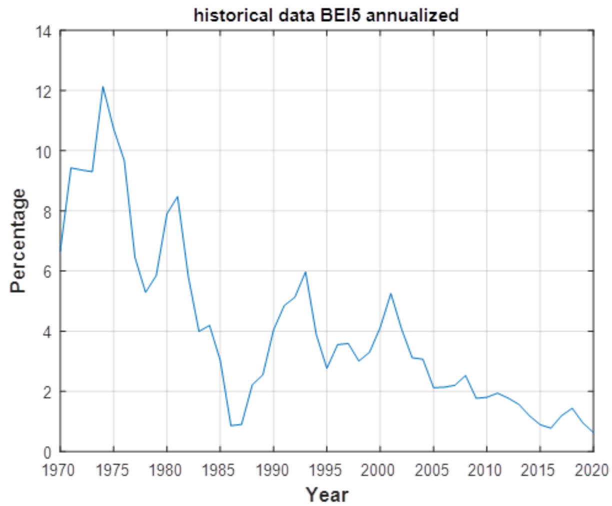

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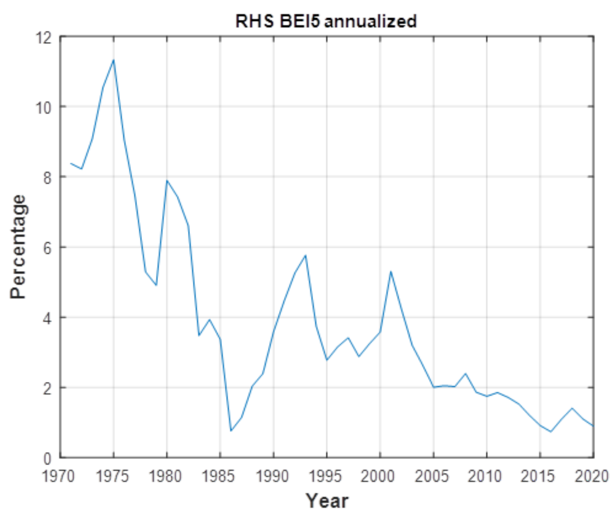

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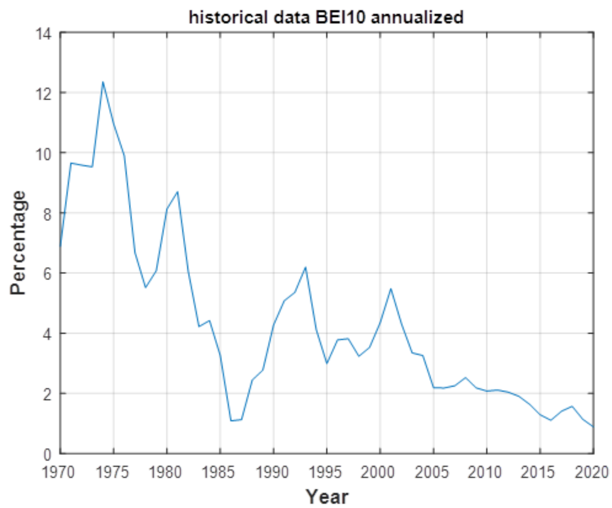

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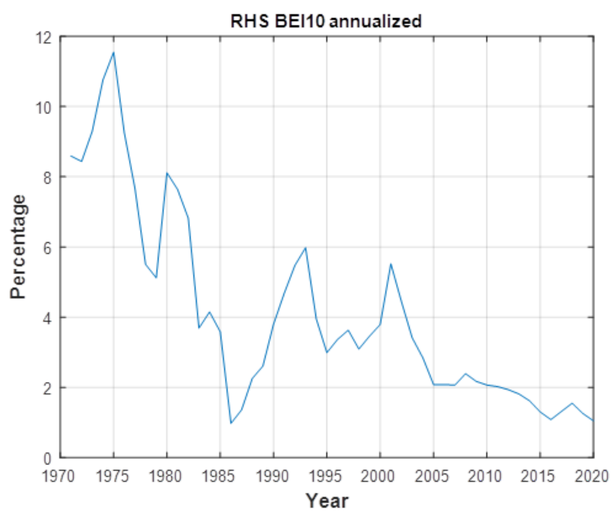

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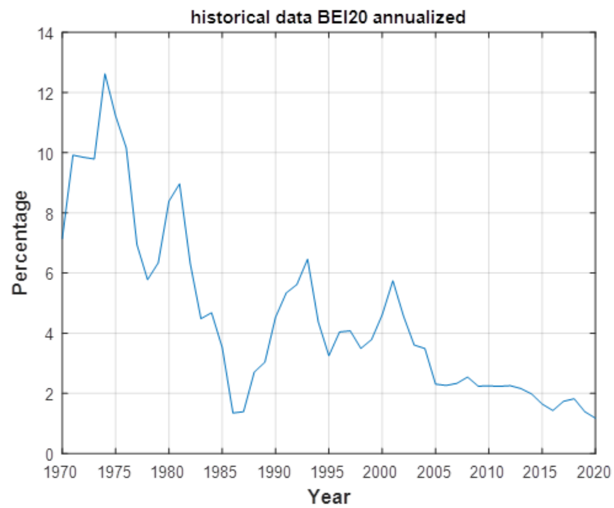

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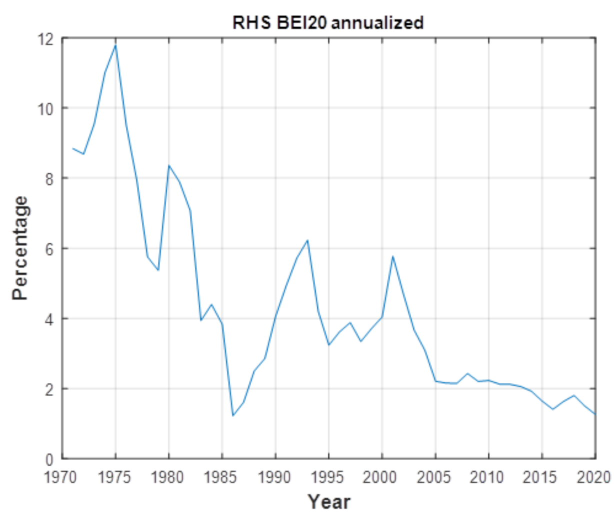

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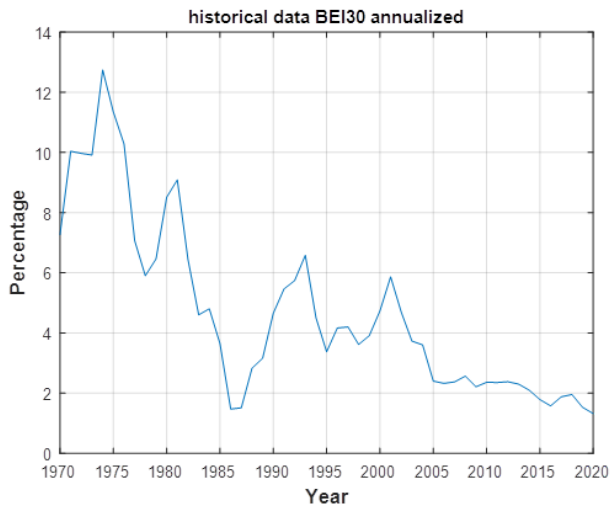

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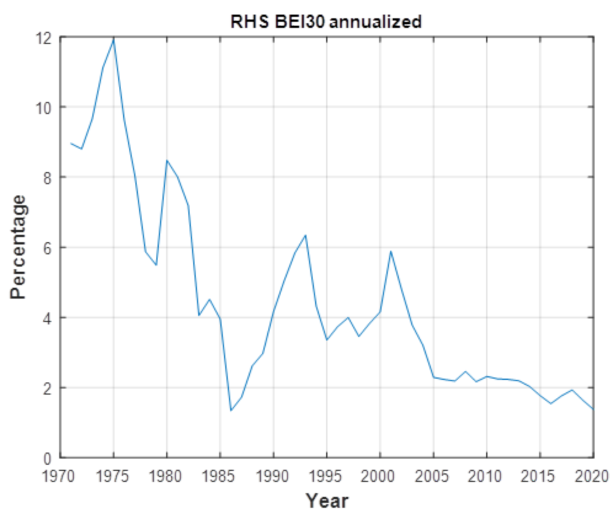

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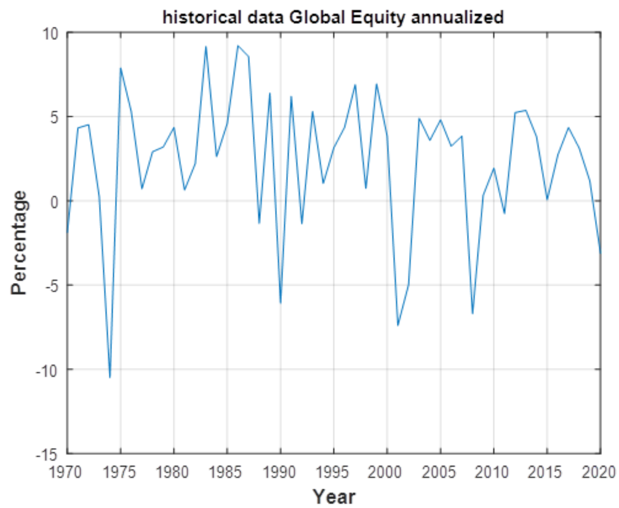

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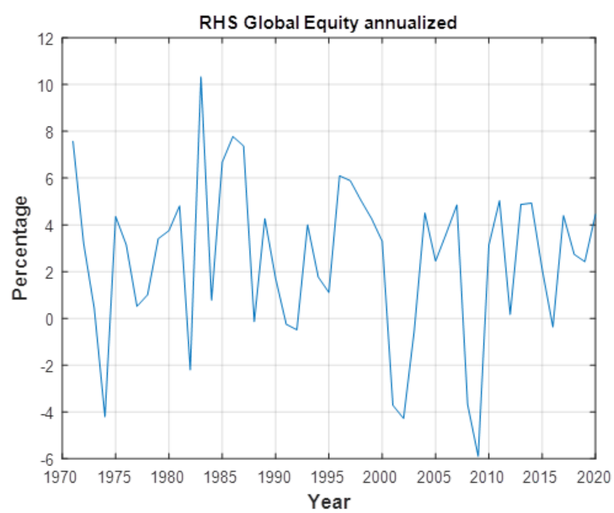

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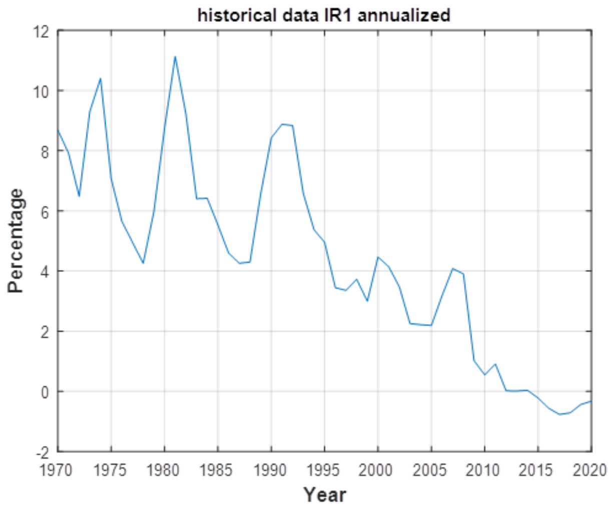

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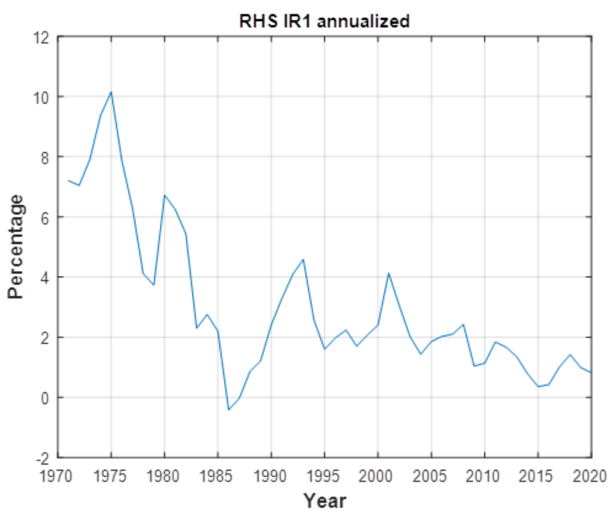

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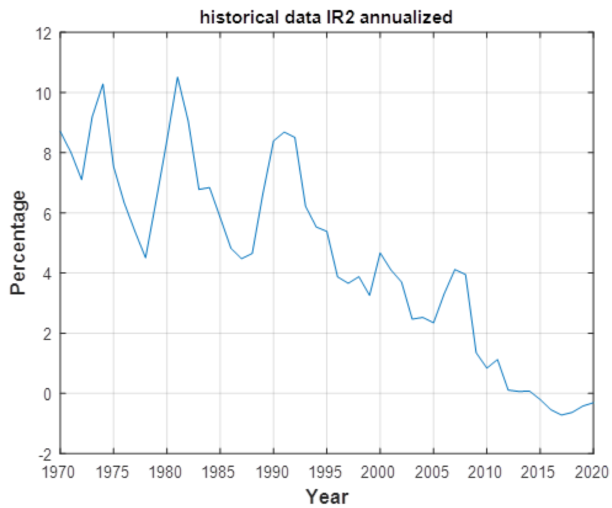

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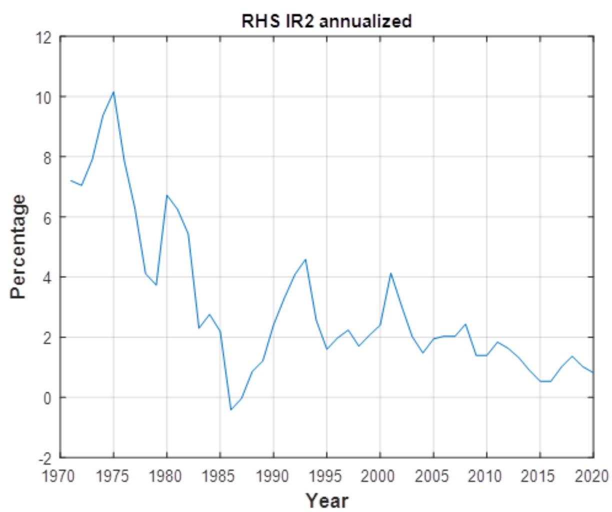

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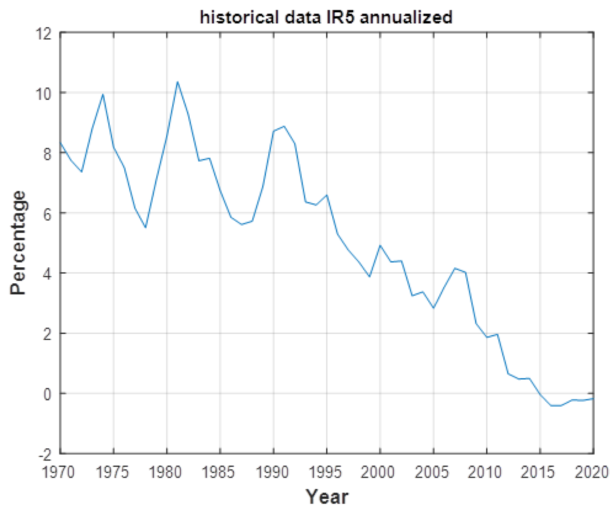

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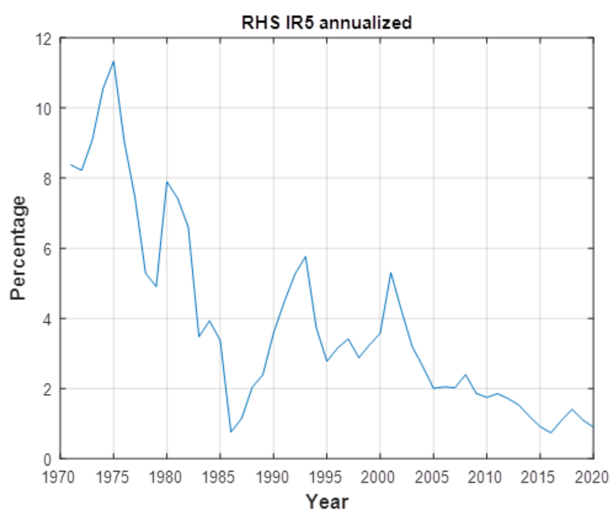


Figure 71:

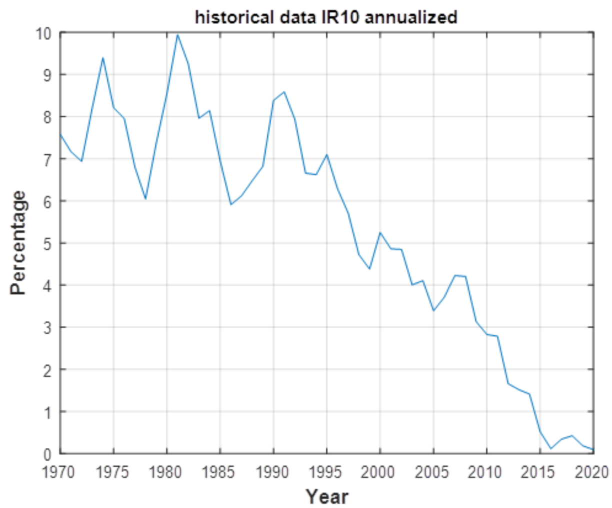


Figure 72:

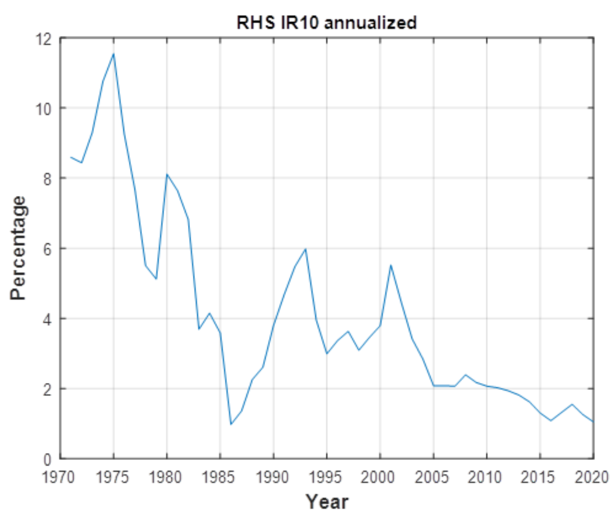


Figure 73:

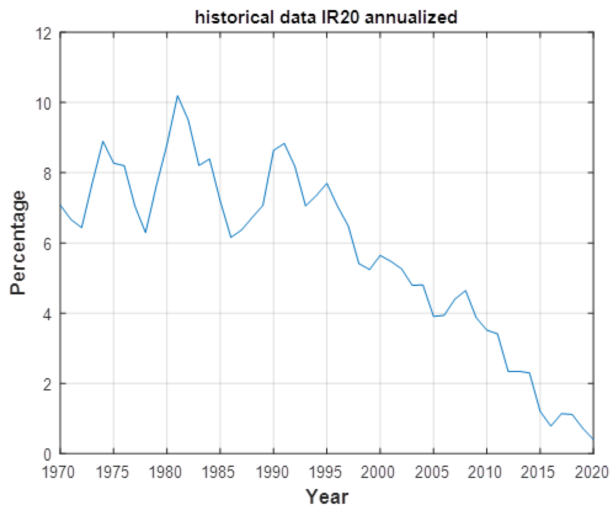
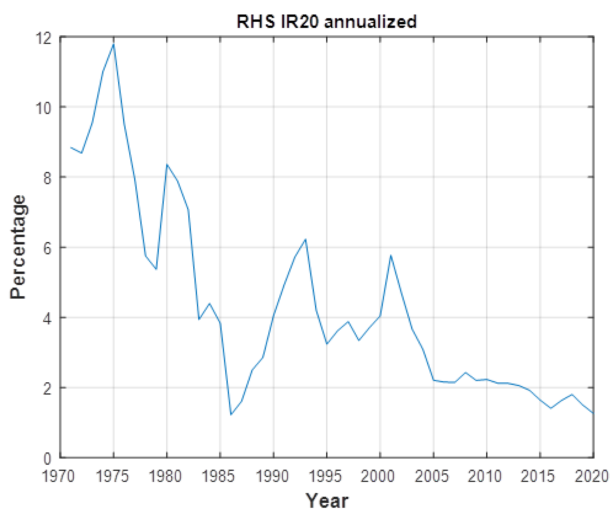


Figure 74:



A.4 Balance sheet simulations: outputs

Figure 75: Nominal coverage ratio at 0% hedge ratio

Percentile	Value
1.0%	12.24%
2.5%	20.84%
10.0%	44.14%
25.0%	67.99%
50.0%	109.96%
75.0%	152.43%
90.0%	210.67%
97.5%	261.61%
99.0%	308.22%

Scenario	1
No of scenario	2000

	Overall
Average	117.4%
volatility	64.8%
Skewnes	0.82
Kurtosis	0.72
Minimun	-13.6%
Maximur	414.6%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	100.00%	142.41%	139.29%	148.00%	153.28%	144.87%	149.23%	165.03%	168.14%	172.51%	186.36%	193.51%	194.35%	193.32%	202.91%	100.00%	100.00%	Impact
1	100.00%	142.41%	139.29%	148.00%	153.28%	144.87%	149.23%	165.03%	168.14%	172.51%	186.36%	193.51%	194.35%	193.32%	202.91%	100.00%	100.00%	1.02910798
2	100.00%	137.37%	143.30%	142.81%	146.80%	151.29%	151.99%	145.01%	139.96%	139.90%	154.76%	170.54%	187.94%	199.33%	209.93%	100.00%	100.00%	1.09933442
3	100.00%	135.68%	149.14%	153.60%	154.22%	157.29%	160.96%	159.50%	175.09%	177.96%	189.11%	197.47%	210.13%	244.37%	251.74%	100.00%	100.00%	1.51742978
4	100.00%	139.59%	148.37%	151.08%	156.10%	145.09%	148.65%	152.81%	152.04%	159.77%	150.79%	156.78%	170.49%	170.22%	170.17%	100.00%	100.00%	0.70168718
5	100.00%	140.15%	143.45%	147.88%	154.28%	167.91%	172.39%	170.85%	198.19%	204.58%	203.73%	203.74%	211.88%	222.55%	223.49%	100.00%	100.00%	1.23492048
6	100.00%	143.98%	143.96%	148.79%	156.43%	169.48%	178.61%	198.13%	198.75%	204.66%	220.27%	238.45%	247.55%	257.63%	262.40%	100.00%	100.00%	1.62399328
7	100.00%	141.51%	147.88%	148.79%	156.79%	162.86%	175.18%	179.76%	184.13%	191.70%	212.35%	224.66%	229.39%	252.01%	260.38%	100.00%	100.00%	1.60382758
8	100.00%	139.46%	139.11%	139.63%	138.12%	141.68%	139.58%	138.75%	138.93%	135.51%	137.29%	132.02%	133.85%	142.35%	131.31%	100.00%	100.00%	0.31312011
9	100.00%	140.00%	148.92%	156.23%	168.86%	173.95%	185.15%	190.26%	201.63%	224.80%	243.38%	247.64%	280.79%	302.56%	309.50%	100.00%	100.00%	2.09501283
10	100.00%	138.55%	136.80%	147.33%	144.85%	156.67%	155.94%	154.40%	168.13%	179.37%	189.08%	195.46%	203.34%	196.98%	212.09%	100.00%	100.00%	1.12087692
11	100.00%	138.59%	148.52%	155.43%	166.55%	162.74%	170.89%	172.51%	166.98%	167.17%	178.69%	185.77%	177.33%	182.81%	182.89%	100.00%	100.00%	0.82890528
12	100.00%	135.77%	142.19%	141.78%	129.31%	136.62%	141.21%	144.37%	142.92%	143.03%	144.42%	146.93%	146.56%	158.09%	166.39%	100.00%	100.00%	0.66390856
13	100.00%	142.44%	155.51%	165.66%	175.64%	181.36%	203.98%	212.25%	212.01%	212.03%	226.23%	239.64%	265.67%	282.37%	296.07%	100.00%	100.00%	1.96070915
14	100.00%	144.86%	154.10%	164.02%	167.62%	155.46%	157.93%	150.74%	153.67%	144.84%	144.98%	141.90%	140.10%	143.98%	157.45%	100.00%	100.00%	0.57447161
15	100.00%	143.21%	140.89%	137.62%	138.88%	138.19%	145.11%	149.16%	152.86%	158.49%	159.35%	154.53%	157.14%	158.07%	155.78%	100.01%	100.01%	0.55775866
16	100.00%	141.12%	145.44%	150.34%	154.32%	162.83%	160.67%	157.63%	172.28%	169.24%	170.92%	171.69%	164.92%	159.78%	152.03%	100.00%	100.00%	0.5203013
17	100.00%	142.88%	156.42%	168.56%	166.12%	173.32%	188.19%	209.44%	198.96%	208.09%	231.52%	261.52%	284.44%	304.33%	310.67%	100.00%	100.00%	2.10673129
18	100.00%	140.53%	144.29%	137.37%	140.61%	141.60%	140.60%	137.92%	133.92%	135.58%	135.12%	138.15%	135.31%	133.79%	144.74%	100.00%	100.00%	0.44736965
19	100.00%	143.39%	154.08%	163.73%	172.06%	175.06%	178.88%	189.06%	204.76%	221.15%	233.70%	248.07%	271.12%	283.76%	293.28%	100.00%	100.00%	1.93282111
20	100.00%	140.22%	137.46%	153.23%	164.45%	170.10%	186.28%	187.58%	194.07%	228.15%	229.20%	243.62%	256.98%	262.79%	279.49%	100.00%	100.00%	1.79493674
21	100.00%	142.57%	134.94%	132.18%	139.81%	138.56%	142.58%	143.82%	165.50%	161.31%	172.29%	184.47%	201.72%	209.35%	210.13%	100.00%	100.00%	1.10131009
22	100.00%	136.01%	143.76%	148.26%	163.77%	168.61%	185.55%	200.66%	215.36%	217.12%	227.66%	237.52%	237.89%	232.48%	261.82%	100.00%	100.00%	1.61818907
23	100.00%	140.95%	147.76%	162.31%	158.74%	167.78%	178.48%	172.83%	178.40%	181.88%	194.67%	217.69%	237.85%	245.01%	260.68%	100.00%	100.00%	1.60676331

Figure 76: Real coverage ratio at 0% hedge ratio

Percenti	Value
1.0%	-9.83%
2.5%	-4.19%
10.0%	16.74%
25.0%	35.23%
50.0%	63.27%
75.0%	94.24%
90.0%	125.66%
97.5%	167.56%
99.0%	198.13%

	Overall
Average	68.3%
volatility	44.4%
Skewnes	0.75
Kurtosis	1.07
Minimur	-25.2%
Maximur	272.7%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	82.72%	88.17%	96.21%	100.80%	107.63%	103.95%	110.77%	118.96%	114.29%	113.70%	121.99%	125.07%	134.48%	151.04%	100.00%	100.00%	Impact
1	77.51%	82.72%	88.17%	96.21%	100.80%	107.63%	103.95%	110.77%	118.96%	114.29%	113.70%	121.99%	125.07%	134.48%	151.04%	100.00%	100.00%	0.9487
2	77.51%	87.80%	95.00%	105.74%	107.89%	110.38%	117.96%	119.69%	129.17%	132.17%	141.80%	149.10%	160.73%	162.28%	171.89%	100.00%	100.00%	1.2177
3	77.51%	83.20%	92.90%	92.29%	94.98%	102.37%	100.82%	104.44%	105.46%	110.02%	112.68%	115.62%	126.02%	138.75%	142.11%	100.00%	100.00%	0.8335
4	77.51%	78.71%	77.14%	79.29%	79.16%	74.53%	73.06%	73.36%	77.37%	75.16%	71.26%	71.10%	78.01%	78.44%	76.40%	100.00%	100.00%	-0.0143
5	77.51%	88.46%	94.06%	94.99%	98.22%	96.34%	99.00%	107.63%	115.95%	119.78%	117.15%	124.91%	132.99%	144.55%	153.47%	100.00%	100.00%	0.98
6	77.51%	81.66%	89.99%	96.87%	103.28%	101.39%	103.61%	108.18%	116.60%	115.85%	116.17%	121.85%	123.41%	133.52%	140.27%	100.00%	100.00%	0.8098
7	77.51%	83.75%	81.95%	84.14%	87.55%	91.94%	92.66%	95.49%	95.25%	102.32%	109.67%	117.24%	115.35%	123.58%	132.06%	100.00%	100.00%	0.7038
8	77.51%	81.93%	85.60%	86.58%	83.51%	80.25%	78.42%	86.00%	82.89%	83.33%	87.56%	89.99%	88.47%	90.17%	91.15%	100.00%	100.00%	0.176
9	77.51%	85.03%	92.40%	97.36%	92.65%	100.08%	108.17%	107.49%	116.31%	117.43%	122.93%	127.05%	138.38%	145.83%	164.41%	100.00%	100.00%	1.1211
10	77.51%	81.55%	84.26%	85.63%	84.93%	91.96%	94.73%	95.48%	102.93%	108.16%	112.17%	113.39%	123.31%	127.93%	133.29%	100.00%	100.00%	0.7196
11	77.51%	81.45%	83.28%	79.99%	87.43%	84.21%	90.39%	99.01%	100.41%	98.67%	104.94%	99.30%	97.90%	98.90%	99.84%	100.00%	100.00%	0.2882
12	77.51%	86.21%	86.84%	92.03%	90.82%	97.41%	100.16%	107.26%	112.79%	117.53%	128.36%	143.05%	137.08%	136.45%	145.02%	100.00%	100.00%	0.871
13	77.51%	83.34%	86.40%	92.85%	103.93%	105.50%	114.61%	121.70%	132.58%	126.90%	124.42%	135.33%	148.59%	157.76%	157.48%	100.00%	100.00%	1.0318
14	77.51%	85.77%	89.41%	88.41%	99.17%	94.16%	91.78%	85.70%	83.48%	77.40%	70.56%	65.62%	68.01%	72.87%	73.18%	100.00%	100.00%	-0.0559
15	77.51%	85.67%	91.10%	95.99%	89.97%	96.62%	100.60%	104.81%	109.48%	111.26%	114.40%	124.55%	133.58%	131.63%	133.38%	100.01%	100.01%	0.7208
16	77.51%	86.71%	89.72%	97.22%	105.10%	110.43%	109.33%	112.93%	114.51%	118.07%	118.51%	118.95%	121.70%	116.43%	119.82%	100.00%	100.00%	0.5459
17	77.51%	81.80%	84.32%	87.94%	89.97%	93.65%	100.53%	95.47%	97.31%	103.54%	109.43%	127.01%	128.82%	134.81%	134.14%	100.00%	100.00%	0.7307
18	77.51%	84.27%	82.26%	81.29%	89.94%	87.19%	87.99%	84.67%	83.64%	80.48%	77.14%	77.20%	74.11%	69.62%	70.77%	100.00%	100.00%	-0.0869
19	77.51%	84.88%	87.03%	91.72%	104.35%	115.90%	125.25%	124.01%	140.09%	156.55%	165.86%	182.02%	192.79%	190.92%	179.61%	100.00%	100.00%	1.3173
20	77.51%	81.49%	83.89%	94.17%	106.92%	114.96%	125.62%	128.44%	133.81%	151.75%	157.60%	161.80%	173.66%	175.28%	201.06%	100.00%	100.00%	1.594
21	77.51%	84.82%	89.46%	84.23%	83.66%	84.12%	88.68%	87.19%	88.52%	89.36%	94.58%	93.39%	96.58%	94.26%	99.47%	100.00%	100.00%	0.2833
22	77.51%	82.82%	88.30%	89.57%	98.10%	97.20%	97.91%	103.67%	104.46%	103.97%	112.98%	127.33%	134.09%	141.37%	149.52%	100.00%	100.00%	0.929
23	77.51%	82.89%	92.86%	103.90%	105.23%	106.65%	117.61%	117.25%	111.34%	112.05%	123.85%	132.71%	138.41%	144.08%	149.66%	100.00%	100.00%	0.9309

Figure 77: Nominal coverage ratio at 20% hedge ratio

Scenario	1
No of scenario	2000

	Overall
Average	132.0%
volatility	75.6%
Skewnes	0.82
Kurtosis	0.66
Minimun	-14.7%
Maximur	467.6%

Percentile	Value
1.0%	10.13%
2.5%	19.92%
10.0%	46.47%
25.0%	73.76%
50.0%	121.20%
75.0%	174.24%
90.0%	241.31%
97.5%	299.34%
99.0%	364.58%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkr	Koopkracht vs indexatie ambitie	
	100.00%	152.24%	145.92%	154.69%	159.93%	147.08%	153.27%	171.03%	172.79%	179.04%	196.25%	202.91%	203.03%	199.56%	207.97%	100.00%	100.00%	**Impact**
1	100.00%	152.24%	145.92%	154.69%	159.93%	147.08%	153.27%	171.03%	172.79%	179.04%	196.25%	202.91%	203.03%	199.56%	207.97%	100.00%	100.00%	1.0797
2	100.00%	144.09%	149.59%	146.02%	150.33%	155.17%	154.16%	145.16%	137.34%	136.68%	151.77%	168.29%	186.06%	198.59%	209.03%	100.00%	100.00%	1.0903
3	100.00%	143.63%	158.36%	164.18%	163.82%	165.49%	170.41%	167.26%	186.70%	188.72%	201.97%	211.60%	225.05%	264.91%	272.91%	100.00%	100.00%	1.7291
4	100.00%	150.34%	162.76%	165.19%	171.93%	157.41%	162.75%	168.07%	165.05%	176.50%	164.98%	173.45%	189.86%	188.89%	189.41%	100.00%	100.00%	0.8941
5	100.00%	147.30%	149.62%	154.68%	161.57%	179.38%	183.92%	179.00%	210.86%	217.52%	216.55%	214.15%	222.03%	232.34%	231.50%	100.00%	100.00%	1.315
6	100.00%	154.65%	151.33%	155.52%	163.35%	180.26%	191.10%	214.89%	213.03%	220.46%	240.16%	261.90%	272.52%	282.49%	286.56%	100.00%	100.00%	1.8656
7	100.00%	150.70%	159.54%	159.52%	168.77%	175.01%	190.61%	195.07%	200.19%	207.51%	231.68%	244.86%	250.60%	276.89%	284.55%	100.00%	100.00%	1.8455
8	100.00%	148.85%	146.82%	146.93%	145.70%	151.50%	149.17%	145.42%	146.65%	141.80%	142.80%	135.13%	137.95%	148.37%	133.50%	100.00%	100.00%	0.335
9	100.00%	148.35%	157.56%	165.33%	182.92%	187.04%	198.94%	205.36%	217.32%	246.38%	268.65%	272.56%	311.92%	337.25%	341.31%	100.00%	100.00%	2.4131
10	100.00%	147.90%	144.43%	157.91%	154.56%	168.16%	165.99%	163.28%	178.86%	191.72%	202.94%	210.56%	218.02%	208.53%	225.85%	100.00%	100.00%	1.2585
11	100.00%	147.93%	160.63%	171.06%	183.73%	179.06%	187.98%	187.23%	179.32%	179.78%	193.06%	203.72%	192.56%	199.43%	199.00%	100.00%	100.00%	0.99
12	100.00%	142.57%	150.60%	148.31%	131.84%	139.18%	144.06%	145.99%	142.84%	141.80%	141.27%	141.74%	142.28%	155.22%	163.12%	100.00%	100.00%	0.6312
13	100.00%	152.03%	168.43%	179.73%	189.44%	195.93%	222.89%	231.33%	227.83%	228.65%	247.00%	261.26%	290.94%	309.59%	326.12%	100.00%	100.00%	2.2612
14	100.00%	154.13%	164.90%	178.05%	179.04%	163.84%	167.51%	159.51%	164.25%	154.23%	157.26%	155.02%	151.37%	154.87%	173.85%	100.00%	100.00%	0.7385
15	100.00%	152.15%	146.94%	140.99%	144.22%	141.16%	148.65%	152.57%	155.82%	162.15%	162.30%	154.05%	155.19%	156.41%	153.20%	100.01%	100.01%	0.532
16	100.00%	149.15%	153.57%	157.40%	160.07%	168.99%	166.07%	161.11%	178.38%	173.50%	175.20%	175.84%	166.73%	161.47%	151.24%	100.00%	100.00%	0.5124
17	100.00%	153.16%	170.50%	185.48%	180.68%	188.54%	205.69%	235.03%	218.26%	227.70%	256.19%	290.03%	318.27%	341.47%	348.61%	100.00%	100.00%	2.4861
18	100.00%	149.34%	154.66%	145.09%	146.43%	148.32%	146.47%	143.81%	138.62%	141.78%	142.43%	146.58%	143.89%	143.76%	159.07%	100.00%	100.00%	0.5907
19	100.00%	152.62%	165.83%	176.79%	183.61%	184.20%	186.44%	198.88%	214.64%	231.05%	244.14%	258.34%	283.43%	297.94%	310.43%	100.00%	100.00%	2.1043
20	100.00%	149.95%	144.92%	162.25%	173.01%	177.80%	195.29%	195.90%	202.31%	239.80%	239.46%	255.59%	269.11%	275.31%	290.91%	100.00%	100.00%	1.9091
21	100.00%	151.68%	139.53%	137.17%	147.26%	145.16%	148.81%	150.52%	178.34%	171.89%	184.31%	200.27%	221.74%	232.07%	230.36%	100.00%	100.00%	1.3036
22	100.00%	144.13%	152.47%	157.65%	175.35%	181.39%	202.89%	220.44%	239.11%	240.62%	251.07%	259.38%	257.44%	248.27%	282.41%	100.00%	100.00%	1.8241
23	100.00%	150.34%	156.08%	171.65%	166.05%	176.83%	187.37%	180.00%	188.10%	192.12%	205.22%	231.66%	255.09%	262.49%	280.40%	100.00%	100.00%	1.804

Figure 78: Real coverage ratio at 20% hedge ratio

	Overall
Average	78.4%
volatility	48.3%
Skewnes	0.80
Kurtosis	1.13
Minimur	-16.7%
Maximu	294.1%

Percenti	Value
1.0%	-5.42%
2.5%	0.90%
10.0%	21.36%
25.0%	41.76%
50.0%	72.02%
75.0%	107.98%
90.0%	141.35%
97.5%	185.24%
99.0%	217.42%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambiti	
	77.51%	88.43%	92.36%	100.56%	105.18%	109.28%	106.76%	114.79%	122.26%	118.61%	119.73%	127.92%	130.65%	138.82%	154.80%	100.00%	100.00%	**Impact**
1	77.51%	88.43%	92.36%	100.56%	105.18%	109.28%	106.76%	114.79%	122.26%	118.61%	119.73%	127.92%	130.65%	138.82%	154.80%	100.00%	100.00%	0.997249916
2	77.51%	92.09%	99.17%	108.12%	110.48%	113.21%	119.64%	119.81%	126.74%	129.13%	139.06%	147.13%	159.12%	161.68%	171.15%	100.00%	100.00%	1.208209205
3	77.51%	88.08%	98.64%	98.64%	100.90%	107.70%	106.74%	109.52%	112.46%	116.67%	120.34%	123.90%	134.97%	150.41%	154.06%	100.00%	100.00%	0.987629319
4	77.51%	84.77%	84.62%	86.69%	87.18%	80.86%	79.99%	80.69%	83.99%	83.03%	77.97%	78.66%	86.87%	87.05%	85.04%	100.00%	100.00%	0.097134647
5	77.51%	92.97%	98.11%	99.36%	102.87%	102.92%	105.62%	112.76%	123.37%	127.36%	124.52%	131.29%	139.35%	150.91%	158.97%	100.00%	100.00%	1.050972502
6	77.51%	87.71%	94.60%	101.25%	107.85%	107.84%	110.86%	117.33%	124.98%	124.79%	126.66%	133.83%	135.85%	146.40%	153.19%	100.00%	100.00%	0.976456825
7	77.51%	89.19%	88.42%	90.21%	94.24%	98.80%	100.82%	103.61%	103.55%	110.75%	119.65%	127.79%	126.02%	135.78%	144.31%	100.00%	100.00%	0.861913211
8	77.51%	87.45%	90.35%	91.10%	88.08%	85.81%	83.81%	90.13%	87.50%	87.20%	91.07%	92.11%	91.17%	93.98%	92.67%	100.00%	100.00%	0.195574134
9	77.51%	90.10%	97.77%	103.03%	100.37%	107.61%	116.23%	116.02%	125.36%	128.70%	135.70%	139.83%	153.72%	162.54%	181.30%	100.00%	100.00%	1.339110306
10	77.51%	87.05%	88.96%	91.78%	90.62%	98.71%	100.83%	100.97%	109.50%	115.60%	120.39%	122.15%	132.21%	135.44%	141.94%	100.00%	100.00%	0.831252983
11	77.51%	86.94%	90.07%	88.04%	96.45%	92.65%	99.43%	107.46%	107.83%	106.12%	113.38%	108.89%	106.30%	107.89%	108.64%	100.00%	100.00%	0.401648864
12	77.51%	90.53%	91.98%	96.27%	92.60%	99.24%	102.18%	108.46%	112.73%	116.52%	125.57%	138.00%	133.07%	133.97%	142.17%	100.00%	100.00%	0.834198846
13	77.51%	88.94%	93.58%	100.74%	112.10%	113.97%	125.23%	132.63%	142.47%	136.84%	135.84%	147.54%	162.72%	172.97%	173.46%	100.00%	100.00%	1.237941427
14	77.51%	91.26%	95.67%	95.97%	105.92%	99.23%	97.34%	90.68%	89.23%	82.41%	76.53%	71.68%	73.49%	78.38%	80.80%	100.00%	100.00%	0.042454647
15	77.51%	91.02%	95.02%	98.34%	93.43%	98.69%	103.05%	107.20%	111.60%	113.83%	116.52%	124.16%	131.92%	130.25%	131.17%	100.01%	100.01%	0.692361203
16	77.51%	91.64%	94.73%	101.79%	109.02%	114.61%	113.01%	115.43%	118.56%	121.04%	121.48%	121.84%	123.04%	117.67%	119.19%	100.00%	100.00%	0.537826089
17	77.51%	87.69%	91.91%	96.77%	97.85%	101.88%	109.88%	107.13%	106.75%	113.30%	121.09%	140.86%	144.14%	151.26%	150.53%	100.00%	100.00%	0.942070804
18	77.51%	89.55%	88.18%	85.86%	93.66%	91.32%	91.65%	88.28%	86.58%	84.16%	81.32%	81.91%	78.80%	74.81%	77.78%	100.00%	100.00%	0.003461507
19	77.51%	90.34%	93.67%	99.04%	111.36%	121.95%	130.54%	130.45%	146.85%	163.55%	173.27%	189.55%	201.55%	200.46%	190.11%	100.00%	100.00%	1.45282556
20	77.51%	87.15%	88.45%	99.72%	112.49%	120.17%	131.69%	134.13%	139.50%	159.50%	164.65%	169.75%	181.86%	183.63%	209.27%	100.00%	100.00%	1.699963776
21	77.51%	90.24%	92.51%	87.41%	88.12%	88.12%	92.55%	91.25%	95.38%	95.23%	101.18%	101.39%	106.16%	104.50%	109.04%	100.00%	100.00%	0.406877537
22	77.51%	87.77%	93.64%	95.25%	105.04%	104.57%	107.06%	113.88%	115.98%	115.22%	124.60%	139.05%	145.10%	150.97%	161.28%	100.00%	100.00%	1.080765006
23	77.51%	88.42%	98.10%	109.88%	110.07%	112.40%	123.47%	122.12%	117.39%	118.36%	130.56%	141.22%	148.44%	154.36%	160.98%	100.00%	100.00%	1.076986273

Figure 79: Nominal coverage ratio at 50% hedge ratio

Scenario	1
No of scenari	2000

	Overall
Average	139.3%
volatility	81.2%
Skewnes	0.82
Kurtosis	0.63
Minimun	-15.3%
Maximur	494.1%

Percentile	Value
1.0%	8.33%
2.5%	19.91%
10.0%	48.28%
25.0%	76.64%
50.0%	129.05%
75.0%	185.54%
90.0%	255.01%
97.5%	323.20%
99.0%	388.23%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkr	Koopkracht vs indexatie ambitie	
	100.00%	157.16%	149.23%	158.03%	163.26%	148.19%	155.29%	174.03%	175.12%	182.30%	201.19%	207.61%	207.37%	202.68%	210.50%	100.00%	100.00%	**Impact**
1	100.00%	157.16%	149.23%	158.03%	163.26%	148.19%	155.29%	174.03%	175.12%	182.30%	201.19%	207.61%	207.37%	202.68%	210.50%	100.00%	100.00%	1.105
2	100.00%	147.44%	152.74%	147.62%	152.09%	157.11%	155.24%	145.23%	136.03%	135.07%	150.27%	167.17%	185.13%	198.21%	208.58%	100.00%	100.00%	1.0858
3	100.00%	147.61%	162.97%	169.47%	168.63%	169.59%	175.13%	171.14%	192.51%	194.10%	208.40%	218.67%	232.51%	275.19%	283.49%	100.00%	100.00%	1.8349
4	100.00%	155.71%	169.95%	172.24%	179.84%	163.57%	169.79%	175.71%	171.56%	184.87%	172.07%	181.78%	199.54%	198.23%	199.03%	100.00%	100.00%	0.9903
5	100.00%	150.88%	152.71%	158.08%	165.22%	185.11%	189.68%	183.07%	217.20%	223.99%	222.96%	219.36%	227.10%	237.24%	235.51%	100.00%	100.00%	1.3551
6	100.00%	159.99%	155.01%	158.88%	166.81%	185.65%	197.35%	223.27%	220.17%	228.35%	250.10%	273.63%	285.00%	294.93%	298.65%	100.00%	100.00%	1.9865
7	100.00%	155.30%	165.37%	164.89%	174.75%	181.08%	198.33%	202.72%	208.21%	215.41%	241.34%	254.97%	261.21%	289.34%	296.64%	100.00%	100.00%	1.9664
8	100.00%	153.55%	150.68%	150.58%	149.49%	156.41%	153.97%	148.75%	150.51%	144.94%	145.55%	136.68%	139.99%	151.39%	134.59%	100.00%	100.00%	0.3459
9	100.00%	152.52%	161.89%	169.88%	189.95%	193.59%	205.84%	212.91%	225.17%	257.17%	281.29%	285.01%	327.49%	354.59%	357.21%	100.00%	100.00%	2.5721
10	100.00%	152.58%	148.24%	163.21%	159.42%	173.91%	171.01%	167.71%	184.23%	197.90%	209.87%	218.11%	225.35%	214.31%	232.74%	100.00%	100.00%	1.3274
11	100.00%	152.61%	166.68%	178.88%	192.33%	187.22%	196.53%	194.59%	185.49%	186.09%	200.24%	212.69%	200.17%	207.74%	207.06%	100.00%	100.00%	1.0706
12	100.00%	145.97%	154.81%	151.58%	133.11%	140.47%	145.49%	146.80%	142.80%	141.18%	139.70%	139.15%	140.13%	153.79%	161.48%	100.00%	100.00%	0.6148
13	100.00%	156.82%	174.89%	186.77%	196.34%	203.21%	232.35%	240.87%	235.74%	236.95%	257.38%	272.07%	303.58%	323.20%	341.14%	100.00%	100.00%	2.4114
14	100.00%	158.76%	170.30%	185.06%	184.74%	168.03%	172.30%	163.89%	169.54%	158.92%	163.40%	161.57%	157.00%	160.32%	182.05%	100.00%	100.00%	0.8205
15	100.00%	156.62%	149.96%	142.67%	146.89%	142.65%	150.42%	154.27%	157.29%	163.97%	163.77%	153.80%	154.21%	155.58%	151.91%	100.01%	100.01%	0.5191
16	100.00%	153.17%	157.63%	160.93%	162.94%	172.07%	168.78%	162.86%	181.42%	175.63%	177.34%	177.92%	167.64%	162.31%	150.84%	100.00%	100.00%	0.5084
17	100.00%	158.30%	177.54%	193.93%	187.96%	196.15%	214.44%	247.83%	227.91%	237.51%	268.53%	304.28%	335.18%	360.05%	367.59%	100.00%	100.00%	2.6759
18	100.00%	153.75%	159.85%	148.94%	149.34%	151.68%	149.40%	146.76%	140.97%	144.88%	146.09%	150.79%	148.18%	148.74%	166.23%	100.00%	100.00%	0.6623
19	100.00%	157.24%	171.71%	183.32%	189.38%	188.77%	190.22%	203.79%	219.58%	236.00%	249.37%	263.47%	289.58%	305.03%	319.01%	100.00%	100.00%	2.1901
20	100.00%	154.82%	148.65%	166.76%	177.30%	181.65%	199.79%	200.06%	206.43%	245.63%	244.59%	261.57%	275.17%	281.56%	296.62%	100.00%	100.00%	1.9662
21	100.00%	156.24%	141.83%	139.67%	150.98%	148.46%	151.92%	153.86%	184.76%	177.18%	190.31%	208.17%	231.74%	243.43%	240.47%	100.00%	100.00%	1.4047
22	100.00%	148.19%	156.82%	162.34%	181.14%	187.78%	211.56%	230.33%	250.98%	252.37%	262.78%	270.31%	267.21%	256.16%	292.71%	100.00%	100.00%	1.9271
23	100.00%	155.03%	160.24%	176.32%	169.71%	181.36%	191.82%	183.59%	192.95%	197.24%	210.49%	238.64%	263.71%	271.23%	290.27%	100.00%	100.00%	1.9027

Figure 80: Real coverage ratio at 50% hedge ratio

	Overall
Average	83.4%
volatility	50.6%
Skewnes	0.82
Kurtosis	1.13
Minimur	-17.2%
Maximui	311.5%

Percenti	Value
1.0%	-4.38%
2.5%	2.95%
10.0%	24.01%
25.0%	45.79%
50.0%	75.79%
75.0%	114.27%
90.0%	149.07%
97.5%	197.31%
99.0%	221.62%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	91.29%	94.46%	102.74%	107.37%	110.10%	108.17%	116.81%	123.90%	120.77%	122.75%	130.88%	133.44%	140.99%	156.69%	100.00%	100.00%	**Impact**
1	77.51%	91.29%	94.46%	102.74%	107.37%	110.10%	108.17%	116.81%	123.90%	120.77%	122.75%	130.88%	133.44%	140.99%	156.69%	100.00%	100.00%	1.0215
2	77.51%	94.23%	101.25%	109.30%	111.78%	114.63%	120.48%	119.88%	125.53%	127.61%	137.69%	146.14%	158.32%	161.38%	170.79%	100.00%	100.00%	1.2034
3	77.51%	90.52%	101.51%	101.82%	103.86%	110.37%	109.70%	112.07%	115.95%	119.99%	124.17%	128.04%	139.45%	156.25%	160.03%	100.00%	100.00%	1.0647
4	77.51%	87.81%	88.36%	90.39%	91.20%	84.03%	83.45%	84.35%	87.30%	86.97%	81.32%	82.44%	91.31%	91.35%	89.36%	100.00%	100.00%	0.1529
5	77.51%	95.23%	100.13%	101.54%	105.19%	106.21%	108.93%	115.33%	127.08%	131.14%	128.21%	134.48%	142.54%	154.09%	161.72%	100.00%	100.00%	1.0865
6	77.51%	90.74%	96.90%	103.43%	110.13%	111.06%	114.48%	121.90%	129.17%	129.26%	131.91%	139.82%	142.07%	152.84%	159.65%	100.00%	100.00%	1.0598
7	77.51%	91.91%	91.65%	93.25%	97.58%	102.23%	104.90%	107.68%	107.71%	114.97%	124.64%	133.06%	131.35%	141.89%	150.44%	100.00%	100.00%	0.941
8	77.51%	90.21%	92.72%	93.37%	90.37%	88.59%	86.50%	92.20%	89.80%	89.13%	92.83%	93.17%	92.53%	95.89%	93.43%	100.00%	100.00%	0.2054
9	77.51%	92.63%	100.45%	105.86%	104.23%	111.38%	120.26%	120.29%	129.89%	134.34%	142.08%	146.22%	161.39%	170.90%	189.75%	100.00%	100.00%	1.4481
10	77.51%	89.81%	91.31%	94.85%	93.47%	102.08%	103.88%	103.71%	112.78%	119.33%	124.50%	126.52%	136.66%	139.19%	146.26%	100.00%	100.00%	0.8871
11	77.51%	89.69%	93.46%	92.06%	100.96%	96.87%	103.95%	111.68%	111.54%	109.84%	117.60%	113.69%	110.51%	112.38%	113.04%	100.00%	100.00%	0.4584
12	77.51%	92.69%	94.55%	98.39%	93.49%	100.15%	103.19%	109.06%	112.69%	116.01%	124.17%	135.47%	131.07%	132.73%	140.74%	100.00%	100.00%	0.8158
13	77.51%	91.75%	97.17%	104.68%	116.18%	118.21%	130.55%	138.10%	147.41%	141.81%	141.55%	153.65%	169.79%	180.57%	181.45%	100.00%	100.00%	1.341
14	77.51%	94.00%	98.81%	99.75%	109.30%	101.77%	100.13%	93.18%	92.11%	84.92%	79.52%	74.72%	76.22%	81.14%	84.61%	100.00%	100.00%	0.0916
15	77.51%	93.69%	96.97%	99.51%	95.16%	99.73%	104.28%	108.40%	112.66%	115.11%	117.58%	123.96%	131.09%	129.56%	130.07%	100.01%	100.01%	0.6781
16	77.51%	94.11%	97.24%	104.07%	110.98%	116.69%	114.85%	116.68%	120.58%	122.53%	122.96%	123.28%	123.70%	118.28%	118.88%	100.00%	100.00%	0.5338
17	77.51%	90.63%	95.71%	101.18%	101.80%	105.99%	114.55%	112.96%	111.47%	118.18%	126.92%	147.78%	151.80%	159.49%	158.72%	100.00%	100.00%	1.0478
18	77.51%	92.19%	91.13%	88.14%	95.52%	93.39%	93.49%	90.09%	88.05%	86.00%	83.41%	84.27%	81.15%	77.40%	81.28%	100.00%	100.00%	0.0487
19	77.51%	93.07%	96.99%	102.69%	114.86%	124.98%	133.19%	133.67%	150.23%	167.06%	176.98%	193.32%	205.92%	205.23%	195.37%	100.00%	100.00%	1.5206
20	77.51%	89.98%	90.73%	102.49%	115.27%	122.77%	134.73%	136.98%	142.34%	163.38%	168.18%	173.73%	185.96%	187.80%	213.38%	100.00%	100.00%	1.753
21	77.51%	92.96%	94.03%	89.00%	90.35%	90.13%	94.48%	93.28%	98.82%	98.16%	104.47%	105.39%	110.95%	109.61%	113.83%	100.00%	100.00%	0.4686
22	77.51%	90.24%	96.32%	98.08%	108.50%	108.26%	111.63%	118.99%	121.74%	120.85%	130.41%	144.91%	150.61%	155.76%	167.16%	100.00%	100.00%	1.1566
23	77.51%	91.18%	100.71%	112.87%	112.50%	115.27%	126.40%	124.55%	120.41%	121.52%	133.92%	145.48%	153.46%	159.50%	166.65%	100.00%	100.00%	1.1501

Figure 81: Nominal coverage ratio at 60% hedge ratio

Scenario	1
No of scenari	2000

	Overall
Average	146.7%
volatility	86.9%
Skewness	0.83
Kurtosis	0.62
Minimum	-15.9%
Maximum	520.7%

Percentile	Value
1.0%	5.62%
2.5%	19.89%
10.0%	49.66%
25.0%	79.77%
50.0%	135.70%
75.0%	196.54%
90.0%	270.36%
97.5%	339.79%
99.0%	409.92%

Dekkingsgraad Sw

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkr	Koopkracht vs indexatie ambitie	
	100.00%	162.07%	152.54%	161.38%	166.58%	149.30%	157.31%	177.03%	177.45%	185.56%	206.13%	212.31%	211.71%	205.80%	213.03%	100.00%	100.00%	Impact
1	100.00%	162.07%	152.54%	161.38%	166.58%	149.30%	157.31%	177.03%	177.45%	185.56%	206.13%	212.31%	211.71%	205.80%	213.03%	100.00%	100.00%	1.130256009
2	100.00%	150.80%	155.88%	149.23%	153.85%	159.06%	156.33%	145.31%	134.71%	133.46%	148.78%	166.04%	184.19%	197.84%	208.13%	100.00%	100.00%	1.08128841
3	100.00%	151.59%	167.57%	174.75%	173.43%	173.69%	179.85%	175.03%	198.31%	199.47%	214.82%	225.74%	239.97%	285.46%	294.07%	100.00%	100.00%	1.940675429
4	100.00%	161.09%	177.14%	179.29%	187.76%	169.73%	176.84%	183.34%	178.07%	193.23%	179.17%	190.12%	209.22%	207.56%	208.65%	100.00%	100.00%	1.086482653
5	100.00%	154.46%	155.80%	161.48%	168.87%	190.85%	195.45%	187.14%	223.54%	230.45%	229.37%	224.56%	232.17%	242.14%	239.51%	100.00%	100.00%	1.395138066
6	100.00%	165.33%	158.69%	162.24%	170.27%	191.04%	203.59%	231.66%	227.31%	236.25%	260.05%	285.35%	297.48%	307.36%	310.73%	100.00%	100.00%	2.10729008
7	100.00%	159.90%	171.21%	170.25%	180.74%	187.16%	206.05%	210.37%	216.24%	223.31%	251.01%	265.07%	271.81%	301.78%	308.72%	100.00%	100.00%	2.087232
8	100.00%	158.25%	154.54%	154.23%	153.27%	161.33%	158.76%	152.09%	154.37%	148.08%	148.31%	138.24%	142.04%	154.40%	135.68%	100.00%	100.00%	0.356840028
9	100.00%	156.70%	166.21%	174.43%	196.98%	200.14%	212.73%	220.46%	233.02%	267.96%	293.92%	297.47%	343.05%	371.93%	373.11%	100.00%	100.00%	2.731115656
10	100.00%	157.25%	152.06%	168.50%	164.27%	179.65%	176.03%	172.15%	189.59%	204.07%	216.80%	225.66%	232.69%	220.08%	239.62%	100.00%	100.00%	1.396189179
11	100.00%	157.28%	172.74%	186.69%	200.92%	195.38%	205.08%	201.95%	191.66%	192.39%	207.42%	221.67%	207.78%	216.05%	215.12%	100.00%	100.00%	1.15118847
12	100.00%	149.37%	159.02%	154.85%	134.38%	141.75%	146.91%	147.61%	142.76%	140.57%	138.13%	136.55%	137.99%	152.35%	159.85%	100.00%	100.00%	0.598477825
13	100.00%	161.61%	181.35%	193.80%	203.25%	210.50%	241.80%	250.40%	243.65%	245.26%	267.76%	282.88%	316.21%	336.81%	356.16%	100.00%	100.00%	2.561607408
14	100.00%	163.39%	175.70%	192.07%	190.45%	172.22%	177.08%	168.28%	174.83%	163.62%	169.54%	168.13%	162.64%	165.76%	190.25%	100.00%	100.00%	0.90247956
15	100.00%	161.09%	152.99%	144.35%	149.56%	144.13%	152.19%	155.98%	158.77%	165.80%	165.25%	153.56%	153.24%	154.75%	150.63%	100.01%	100.01%	0.506267454
16	100.00%	157.19%	161.69%	164.47%	165.82%	175.15%	171.48%	164.60%	184.47%	177.76%	179.47%	180.00%	168.55%	163.16%	150.45%	100.00%	100.00%	0.504474281
17	100.00%	163.44%	184.58%	202.39%	195.25%	203.76%	223.19%	260.63%	237.56%	247.32%	280.86%	318.54%	352.09%	378.62%	386.56%	100.00%	100.00%	2.865565651
18	100.00%	158.15%	165.04%	152.80%	152.25%	155.04%	152.33%	149.71%	143.32%	147.98%	149.75%	155.01%	152.47%	153.73%	173.40%	100.00%	100.00%	0.733979973
19	100.00%	161.86%	177.58%	189.85%	195.16%	193.34%	194.00%	208.69%	224.52%	240.95%	254.59%	268.61%	295.74%	312.12%	327.58%	100.00%	100.00%	2.275838941
20	100.00%	159.68%	152.38%	171.27%	181.58%	185.50%	204.30%	204.22%	210.56%	251.46%	249.72%	267.56%	281.24%	287.82%	302.33%	100.00%	100.00%	2.02330835
21	100.00%	160.80%	144.13%	142.16%	154.71%	151.77%	155.03%	157.21%	191.18%	182.47%	196.32%	216.07%	241.75%	254.79%	250.58%	100.00%	100.00%	1.50583209
22	100.00%	152.25%	161.17%	167.04%	186.93%	194.17%	220.24%	240.21%	262.86%	264.13%	274.49%	281.24%	276.99%	264.05%	303.00%	100.00%	100.00%	2.030026339
23	100.00%	159.73%	164.41%	180.99%	173.37%	185.88%	196.26%	187.18%	197.80%	202.36%	215.76%	245.63%	272.33%	279.97%	300.13%	100.00%	100.00%	2.001329868

Figure 82: Real coverage ratio at 60% hedge ratio

	Overall
Average	88.4%
volatility	53.1%
Skewnes	0.83
Kurtosis	1.13
Minimur	-17.8%
Maximui	329.8%

Percenti	Value
1.0%	-3.70%
2.5%	6.29%
10.0%	25.92%
25.0%	50.00%
50.0%	80.15%
75.0%	119.35%
90.0%	158.70%
97.5%	209.40%
99.0%	232.40%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	94.14%	96.55%	104.91%	109.56%	110.93%	109.57%	118.82%	125.55%	122.93%	125.76%	133.84%	136.23%	143.16%	158.57%	100.00%	100.00%	Impact
1	77.51%	94.14%	96.55%	104.91%	109.56%	110.93%	109.57%	118.82%	125.55%	122.93%	125.76%	133.84%	136.23%	143.16%	158.57%	100.00%	100.00%	1.0458
2	77.51%	96.38%	103.34%	110.49%	113.07%	116.05%	121.32%	119.94%	124.32%	126.09%	136.32%	145.16%	157.52%	161.08%	170.42%	100.00%	100.00%	1.1987
3	77.51%	92.96%	104.38%	105.00%	106.82%	113.04%	112.65%	114.61%	119.45%	123.32%	128.00%	132.18%	143.92%	162.08%	166.00%	100.00%	100.00%	1.1418
4	77.51%	90.84%	92.10%	94.09%	95.21%	87.19%	86.91%	88.02%	90.61%	90.90%	84.67%	86.22%	95.74%	95.65%	93.68%	100.00%	100.00%	0.2086
5	77.51%	97.49%	102.15%	103.73%	107.51%	109.50%	112.24%	117.90%	130.79%	134.93%	131.89%	137.67%	145.72%	157.27%	164.47%	100.00%	100.00%	1.1219
6	77.51%	93.77%	99.20%	105.62%	112.42%	114.29%	118.10%	126.48%	133.36%	133.73%	137.15%	145.81%	148.30%	159.28%	166.11%	100.00%	100.00%	1.1431
7	77.51%	94.63%	94.88%	96.28%	100.92%	105.65%	108.99%	111.74%	111.86%	119.19%	129.63%	138.33%	136.68%	147.99%	156.57%	100.00%	100.00%	1.0201
8	77.51%	92.97%	95.10%	95.63%	92.66%	91.38%	89.20%	94.26%	92.10%	91.06%	94.59%	94.23%	93.88%	97.80%	94.18%	100.00%	100.00%	0.2152
9	77.51%	95.17%	103.13%	108.70%	108.09%	115.15%	124.28%	124.55%	134.42%	139.98%	148.46%	152.61%	169.06%	179.26%	198.20%	100.00%	100.00%	1.5571
10	77.51%	92.56%	93.66%	97.93%	96.32%	105.45%	106.94%	106.46%	116.07%	123.05%	128.61%	130.90%	141.11%	142.94%	150.59%	100.00%	100.00%	0.9429
11	77.51%	92.43%	96.86%	96.08%	105.47%	101.10%	108.48%	115.90%	115.25%	113.56%	121.81%	118.49%	114.71%	116.88%	117.44%	100.00%	100.00%	0.5151
12	77.51%	94.85%	97.12%	100.51%	94.38%	101.06%	104.20%	109.66%	112.66%	115.50%	122.77%	132.94%	129.07%	131.50%	139.31%	100.00%	100.00%	0.7974
13	77.51%	94.55%	100.76%	108.62%	120.26%	122.45%	135.86%	143.57%	152.36%	146.78%	147.26%	159.75%	176.86%	188.18%	189.44%	100.00%	100.00%	1.4441
14	77.51%	96.74%	101.94%	103.53%	112.67%	104.31%	102.91%	95.67%	94.98%	87.43%	82.51%	77.75%	78.96%	83.89%	88.42%	100.00%	100.00%	0.1408
15	77.51%	96.37%	98.93%	100.68%	96.89%	100.77%	105.51%	109.60%	113.71%	116.39%	118.64%	123.77%	130.26%	128.87%	128.97%	100.01%	100.01%	0.6639
16	77.51%	96.58%	99.75%	106.36%	112.93%	118.78%	116.68%	117.92%	122.61%	124.02%	124.44%	124.72%	124.37%	118.90%	118.57%	100.00%	100.00%	0.5298
17	77.51%	93.57%	99.50%	105.59%	105.74%	110.10%	119.23%	118.80%	116.19%	123.06%	132.76%	154.70%	159.45%	167.72%	166.91%	100.00%	100.00%	1.1534
18	77.51%	94.83%	94.09%	90.42%	97.38%	95.46%	95.32%	91.90%	89.52%	87.84%	85.50%	86.62%	83.50%	80.00%	84.78%	100.00%	100.00%	0.0939
19	77.51%	95.81%	100.31%	106.35%	118.36%	128.01%	135.84%	136.89%	153.61%	170.56%	180.68%	197.08%	210.30%	210.00%	200.62%	100.00%	100.00%	1.5883
20	77.51%	92.81%	93.00%	105.26%	118.06%	125.37%	137.77%	139.83%	145.18%	167.25%	171.71%	177.70%	190.06%	191.98%	217.48%	100.00%	100.00%	1.8059
21	77.51%	95.67%	95.56%	90.59%	92.58%	92.13%	96.42%	95.31%	102.25%	101.09%	107.77%	109.39%	115.74%	114.73%	118.62%	100.00%	100.00%	0.5304
22	77.51%	92.71%	98.99%	100.92%	111.97%	111.94%	116.21%	124.10%	127.50%	126.48%	136.21%	150.77%	156.12%	160.56%	173.04%	100.00%	100.00%	1.2325
23	77.51%	93.94%	103.33%	115.85%	114.92%	118.15%	129.33%	126.99%	123.44%	124.68%	137.27%	149.74%	158.48%	164.64%	172.31%	100.00%	100.00%	1.2231

Figure 83: Nominal coverage ratio at 75% hedge ratio

Scenario	1
No of scenari	2000

	Overall
Average	157.6%
volatility	95.7%
Skewnes	0.83
Kurtosis	0.60
Minimun	-16.8%
Maximur	560.4%

Percentile	Value
1.0%	5.70%
2.5%	19.82%
10.0%	50.26%
25.0%	84.43%
50.0%	141.23%
75.0%	211.83%
90.0%	296.97%
97.5%	359.81%
99.0%	448.06%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkr	Koopkracht vs indexatie ambitie	
	100.00%	169.45%	157.50%	166.40%	171.58%	150.96%	160.33%	181.53%	180.94%	190.45%	213.54%	219.36%	218.21%	210.48%	216.82%	100.00%	100.00%	Impact
1	100.00%	169.45%	157.50%	166.40%	171.58%	150.96%	160.33%	181.53%	180.94%	190.45%	213.54%	219.36%	218.21%	210.48%	216.82%	100.00%	100.00%	1.16818652
2	100.00%	155.83%	160.60%	151.64%	156.50%	161.97%	157.95%	145.42%	132.74%	131.05%	146.53%	164.35%	182.78%	197.29%	207.45%	100.00%	100.00%	1.074521157
3	100.00%	157.55%	174.49%	182.69%	180.64%	179.84%	186.94%	180.85%	207.02%	207.54%	224.47%	236.34%	251.17%	300.87%	309.94%	100.00%	100.00%	2.099392547
4	100.00%	169.15%	187.92%	189.87%	199.63%	178.97%	187.41%	194.79%	187.83%	205.78%	189.81%	202.62%	223.75%	221.56%	223.08%	100.00%	100.00%	1.230780954
5	100.00%	159.82%	160.43%	166.58%	174.34%	199.45%	204.09%	193.25%	233.04%	240.16%	238.99%	232.37%	239.77%	249.48%	245.52%	100.00%	100.00%	1.455219662
6	100.00%	173.34%	164.22%	167.28%	175.46%	199.13%	212.96%	244.23%	238.02%	248.09%	274.96%	302.94%	316.20%	326.00%	328.85%	100.00%	100.00%	2.288526381
7	100.00%	166.79%	179.95%	178.30%	189.72%	196.27%	217.63%	221.85%	228.28%	235.16%	265.50%	280.22%	287.72%	320.45%	326.85%	100.00%	100.00%	2.268508659
8	100.00%	165.29%	160.32%	159.71%	158.95%	168.69%	165.95%	157.09%	160.16%	152.80%	152.44%	140.57%	145.11%	158.92%	137.32%	100.00%	100.00%	0.373234998
9	100.00%	162.96%	172.69%	181.25%	207.53%	209.97%	223.08%	231.78%	244.79%	284.14%	312.87%	316.16%	366.40%	397.94%	396.97%	100.00%	100.00%	2.969654218
10	100.00%	164.26%	157.78%	176.44%	171.56%	188.27%	183.57%	178.81%	197.64%	213.34%	227.19%	236.98%	243.69%	228.75%	249.94%	100.00%	100.00%	1.499431276
11	100.00%	164.28%	181.82%	198.42%	213.81%	207.61%	217.90%	212.99%	200.91%	201.85%	218.19%	235.13%	219.20%	228.52%	227.20%	100.00%	100.00%	1.272044667
12	100.00%	154.47%	165.33%	159.75%	136.28%	143.67%	149.05%	148.82%	142.70%	139.64%	135.77%	132.66%	134.78%	150.20%	157.39%	100.00%	100.00%	0.5739413
13	100.00%	168.80%	191.04%	204.36%	213.60%	221.42%	255.99%	264.71%	255.51%	257.72%	283.34%	299.09%	335.16%	357.23%	378.69%	100.00%	100.00%	2.786944255
14	100.00%	170.34%	183.80%	202.59%	199.01%	178.50%	184.27%	174.85%	182.76%	170.66%	178.75%	177.97%	171.09%	173.92%	202.55%	100.00%	100.00%	1.025482542
15	100.00%	167.79%	157.52%	146.87%	153.56%	146.36%	154.85%	158.53%	160.99%	168.54%	167.46%	153.20%	151.78%	153.50%	148.70%	100.01%	100.01%	0.48695825
16	100.00%	163.22%	167.79%	169.76%	170.13%	179.77%	175.53%	167.21%	189.04%	180.96%	182.68%	183.12%	169.91%	164.43%	149.85%	100.00%	100.00%	0.498539148
17	100.00%	171.15%	195.15%	215.08%	206.17%	215.17%	236.32%	279.82%	252.03%	262.03%	299.37%	339.92%	377.45%	406.48%	415.01%	100.00%	100.00%	3.150128535
18	100.00%	164.76%	172.82%	158.59%	156.61%	160.07%	156.73%	154.13%	146.85%	152.63%	155.24%	161.33%	158.90%	161.20%	184.15%	100.00%	100.00%	0.841458846
19	100.00%	168.78%	186.39%	199.65%	203.82%	200.20%	199.67%	216.05%	231.93%	248.38%	262.42%	276.31%	304.97%	322.75%	340.45%	100.00%	100.00%	2.404470629
20	100.00%	166.98%	157.98%	178.03%	188.00%	191.28%	211.05%	210.46%	216.74%	260.20%	257.41%	276.53%	290.33%	297.21%	310.89%	100.00%	100.00%	2.108947702
21	100.00%	167.64%	147.58%	145.90%	160.30%	156.72%	159.70%	162.23%	200.81%	190.41%	205.33%	227.92%	256.76%	271.84%	265.75%	100.00%	100.00%	1.65752784
22	100.00%	158.33%	167.70%	174.08%	195.61%	203.76%	233.24%	255.05%	280.67%	281.75%	292.05%	297.63%	291.65%	275.89%	318.45%	100.00%	100.00%	2.184465314
23	100.00%	166.77%	170.65%	188.00%	178.85%	192.67%	202.93%	192.56%	205.07%	210.04%	223.67%	256.10%	285.26%	293.08%	314.93%	100.00%	100.00%	2.149292329

Figure 84: Real coverage ratio at 75% hedge ratio

	Overall
Average	96.0%
volatility	57.2%
Skewness	0.84
Kurtosis	1.11
Minimum	-18.6%
Maximum	357.4%

Percenti	Value
1.0%	-2.78%
2.5%	7.42%
10.0%	29.83%
25.0%	54.91%
50.0%	87.71%
75.0%	130.93%
90.0%	172.06%
97.5%	227.52%
99.0%	253.11%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	98.43%	99.69%	108.18%	112.84%	112.16%	111.68%	121.84%	128.02%	126.17%	130.29%	138.28%	140.42%	146.42%	161.39%	100.00%	100.00%	Impact
1	77.51%	98.43%	99.69%	108.18%	112.84%	112.16%	111.68%	121.84%	128.02%	126.17%	130.29%	138.28%	140.42%	146.42%	161.39%	100.00%	100.00%	1.0822
2	77.51%	99.59%	106.47%	112.28%	115.02%	118.17%	122.58%	120.03%	122.50%	123.81%	134.27%	143.69%	156.31%	160.62%	169.86%	100.00%	100.00%	1.1915
3	77.51%	96.61%	108.69%	109.76%	111.25%	117.04%	117.09%	118.42%	124.69%	128.31%	133.75%	138.38%	150.63%	170.83%	174.96%	100.00%	100.00%	1.2574
4	77.51%	95.38%	97.71%	99.64%	101.23%	91.94%	92.11%	93.51%	95.58%	96.81%	89.70%	91.89%	102.38%	102.10%	100.15%	100.00%	100.00%	0.2922
5	77.51%	100.88%	105.19%	107.01%	110.99%	114.43%	117.20%	121.74%	136.35%	140.61%	137.42%	142.46%	150.49%	162.04%	168.59%	100.00%	100.00%	1.1752
6	77.51%	98.31%	102.66%	108.91%	115.84%	119.12%	123.54%	133.34%	139.65%	140.43%	145.02%	154.80%	157.63%	168.95%	175.80%	100.00%	100.00%	1.2681
7	77.51%	98.71%	99.73%	100.83%	105.94%	110.80%	115.11%	117.84%	118.09%	125.51%	137.12%	146.24%	144.68%	157.14%	165.77%	100.00%	100.00%	1.1387
8	77.51%	97.11%	98.66%	99.03%	96.10%	95.55%	93.24%	97.36%	95.56%	93.96%	97.22%	95.82%	95.91%	100.66%	95.32%	100.00%	100.00%	0.2298
9	77.51%	98.97%	107.15%	112.95%	113.87%	120.80%	130.33%	130.95%	141.21%	148.43%	158.03%	162.19%	180.57%	191.80%	210.87%	100.00%	100.00%	1.7206
10	77.51%	96.68%	97.18%	102.55%	100.59%	110.51%	111.51%	110.58%	121.00%	128.64%	134.77%	137.47%	147.78%	148.57%	157.08%	100.00%	100.00%	1.0266
11	77.51%	96.55%	101.95%	102.12%	112.24%	107.43%	115.26%	122.24%	120.82%	119.14%	128.14%	125.68%	121.01%	123.62%	124.03%	100.00%	100.00%	0.6003
12	77.51%	98.09%	100.97%	103.69%	95.72%	102.44%	105.72%	110.57%	112.61%	114.74%	120.68%	129.15%	126.06%	129.64%	137.18%	100.00%	100.00%	0.7698
13	77.51%	98.76%	106.15%	114.54%	126.39%	128.80%	143.83%	151.77%	159.78%	154.24%	155.83%	168.91%	187.46%	199.58%	201.43%	100.00%	100.00%	1.5988
14	77.51%	100.86%	106.64%	109.20%	117.74%	108.11%	107.08%	99.41%	99.29%	91.19%	86.99%	82.30%	83.06%	88.03%	94.14%	100.00%	100.00%	0.2146
15	77.51%	100.38%	101.86%	102.44%	99.49%	102.32%	107.35%	111.39%	115.30%	118.32%	120.22%	123.47%	129.02%	127.83%	127.31%	100.01%	100.01%	0.6426
16	77.51%	100.29%	103.51%	109.78%	115.87%	121.91%	119.44%	119.80%	125.64%	126.25%	126.67%	126.88%	125.38%	119.83%	118.10%	100.00%	100.00%	0.5237
17	77.51%	97.99%	105.20%	112.21%	111.66%	116.27%	126.24%	127.55%	123.27%	130.38%	141.50%	165.09%	170.94%	180.06%	179.20%	100.00%	100.00%	1.312
18	77.51%	98.79%	98.53%	93.85%	100.17%	98.56%	98.07%	94.62%	91.72%	90.60%	88.63%	90.15%	87.03%	83.89%	90.04%	100.00%	100.00%	0.1617
19	77.51%	99.91%	105.28%	111.84%	123.61%	132.55%	139.81%	141.72%	158.68%	175.82%	186.24%	202.73%	216.86%	217.16%	208.50%	100.00%	100.00%	1.69
20	77.51%	97.05%	96.42%	109.42%	122.23%	129.27%	142.32%	144.10%	149.45%	173.07%	177.00%	183.66%	196.20%	198.24%	223.64%	100.00%	100.00%	1.8854
21	77.51%	99.73%	97.84%	92.98%	95.92%	95.14%	99.32%	98.35%	107.41%	105.48%	112.72%	115.39%	122.93%	122.40%	125.80%	100.00%	100.00%	0.6231
22	77.51%	96.42%	103.00%	105.18%	117.17%	117.47%	123.07%	131.77%	136.14%	134.92%	144.93%	159.56%	164.39%	167.76%	181.86%	100.00%	100.00%	1.3463
23	77.51%	98.08%	107.25%	120.34%	118.55%	122.46%	133.73%	130.64%	127.98%	129.41%	142.31%	156.12%	166.00%	172.35%	180.80%	100.00%	100.00%	1.3327

Figure 85: Nominal coverage ratio at 100% hedge ratio

Scenario	1
No of scenari	2000

	Overall
Average	175.9%
volatility	110.6%
Skewnes	0.84
Kurtosis	0.59
Minimur	-18.2%
Maximur	626.7%

Percentile	Value
1.0%	5.84%
2.5%	17.41%
10.0%	51.02%
25.0%	90.71%
50.0%	156.90%
75.0%	243.01%
90.0%	336.54%
97.5%	410.23%
99.0%	511.62%

Dekkingsgraad Swap

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	100.00%	181.74%	165.78%	174.76%	179.89%	153.73%	165.38%	189.03%	186.76%	198.60%	225.90%	231.10%	229.06%	218.28%	223.14%	100.00%	100.00%	Impact
1	100.00%	181.74%	165.78%	174.76%	179.89%	153.73%	165.38%	189.03%	186.76%	198.60%	225.90%	231.10%	229.06%	218.28%	223.14%	100.00%	100.00%	123.14%
2	100.00%	164.23%	168.47%	155.65%	160.91%	166.82%	160.66%	145.61%	129.46%	127.03%	142.80%	161.54%	180.43%	196.36%	206.32%	100.00%	100.00%	1.0632424
3	100.00%	167.49%	186.01%	195.91%	192.65%	190.09%	198.75%	190.56%	221.53%	220.99%	240.54%	254.01%	269.82%	326.55%	336.39%	100.00%	100.00%	2.3639211
4	100.00%	182.59%	205.90%	207.50%	219.41%	194.37%	205.03%	213.87%	204.10%	226.69%	207.54%	223.45%	247.96%	244.90%	247.13%	100.00%	100.00%	1.4712781
5	100.00%	168.76%	168.14%	175.09%	183.45%	213.78%	218.51%	203.44%	248.89%	256.32%	255.02%	245.38%	252.45%	261.72%	255.54%	100.00%	100.00%	1.5553557
6	100.00%	186.68%	173.43%	175.69%	184.12%	212.60%	228.57%	265.18%	255.88%	267.83%	299.83%	332.26%	347.40%	357.08%	359.06%	100.00%	100.00%	2.5905869
7	100.00%	178.29%	194.53%	191.71%	204.68%	211.45%	236.92%	240.97%	248.35%	254.92%	289.66%	305.46%	314.23%	351.56%	357.06%	100.00%	100.00%	2.5706364
8	100.00%	177.03%	169.97%	168.83%	168.42%	180.97%	177.93%	165.43%	169.82%	160.66%	159.33%	144.47%	150.23%	166.45%	140.06%	100.00%	100.00%	0.4005599
9	100.00%	173.39%	183.50%	192.62%	225.10%	226.34%	240.32%	250.65%	264.41%	311.12%	344.46%	347.30%	405.31%	441.29%	436.72%	100.00%	100.00%	3.3672185
10	100.00%	175.95%	167.31%	189.67%	183.70%	202.63%	196.13%	189.90%	211.06%	228.78%	244.51%	255.85%	262.03%	243.19%	267.15%	100.00%	100.00%	1.6715014
11	100.00%	175.96%	196.95%	217.96%	235.30%	228.01%	239.28%	231.39%	216.34%	217.61%	236.15%	257.57%	238.23%	249.29%	247.35%	100.00%	100.00%	1.4734717
12	100.00%	162.97%	175.85%	167.92%	139.45%	146.88%	152.61%	150.85%	142.60%	138.10%	131.84%	126.18%	129.42%	146.61%	153.30%	100.00%	100.00%	0.5330471
13	100.00%	180.77%	207.19%	221.95%	230.85%	239.63%	279.62%	288.55%	275.28%	278.48%	309.30%	326.11%	366.75%	391.25%	416.25%	100.00%	100.00%	3.1625057
14	100.00%	181.92%	197.31%	220.13%	213.28%	188.97%	196.24%	185.81%	195.98%	182.39%	194.10%	194.36%	185.18%	187.53%	223.05%	100.00%	100.00%	1.2304875
15	100.00%	178.97%	165.08%	151.08%	160.24%	150.07%	159.28%	162.79%	164.68%	173.11%	171.14%	152.59%	149.34%	151.42%	145.48%	100.01%	100.01%	0.4547762
16	100.00%	173.27%	177.95%	178.59%	177.31%	187.47%	182.28%	171.57%	196.66%	186.29%	188.03%	188.32%	172.17%	166.54%	148.86%	100.00%	100.00%	0.4886473
17	100.00%	184.00%	212.75%	236.23%	224.38%	234.19%	258.19%	311.81%	276.16%	286.56%	330.21%	375.56%	419.73%	452.91%	462.44%	100.00%	100.00%	3.6244
18	100.00%	175.77%	185.79%	168.23%	163.89%	168.47%	164.05%	161.49%	152.73%	160.37%	164.39%	171.86%	169.62%	173.67%	202.06%	100.00%	100.00%	1.0205903
19	100.00%	180.32%	201.08%	215.98%	218.25%	211.63%	209.12%	228.32%	244.29%	260.76%	275.47%	289.14%	320.35%	340.48%	361.89%	100.00%	100.00%	2.6188568
20	100.00%	179.14%	167.31%	189.31%	198.71%	200.90%	222.31%	220.86%	227.05%	274.76%	270.23%	291.50%	305.49%	312.85%	325.17%	100.00%	100.00%	2.25168
21	100.00%	179.03%	153.32%	152.14%	169.61%	164.97%	167.48%	170.59%	216.86%	203.63%	220.35%	247.68%	281.78%	300.24%	291.04%	100.00%	100.00%	1.9103541
22	100.00%	168.48%	178.59%	185.82%	210.08%	219.73%	254.92%	279.77%	310.36%	311.13%	321.32%	324.96%	316.09%	295.61%	344.19%	100.00%	100.00%	2.4418636
23	100.00%	178.51%	181.06%	199.67%	187.99%	203.98%	214.05%	201.53%	217.19%	222.85%	236.86%	273.56%	306.81%	314.94%	339.59%	100.00%	100.00%	2.3958964

Figure 86: Real coverage ratio at 100% hedge ratio

	Overall
Average	108.6%
volatility	64.8%
Skewnes	0.84
Kurtosis	1.04
Minimur	-20.1%
Maximur	403.3%

Percenti	Value
1.0%	-1.82%
2.5%	11.26%
10.0%	34.04%
25.0%	60.54%
50.0%	100.89%
75.0%	146.14%
90.0%	196.69%
97.5%	249.70%
99.0%	294.11%

Dekkingsgraad Swap Reeel

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Koopkrach	Koopkracht vs indexatie ambitie	
	77.51%	105.57%	104.93%	113.61%	118.31%	114.22%	115.19%	126.88%	132.14%	131.57%	137.82%	145.69%	147.40%	151.85%	166.10%	100.00%	100.00%	Impact
1	77.51%	105.57%	104.93%	113.61%	118.31%	114.22%	115.19%	126.88%	132.14%	131.57%	137.82%	145.69%	147.40%	151.85%	166.10%	100.00%	100.00%	1.143
2	77.51%	104.96%	111.68%	115.25%	118.26%	121.72%	124.68%	120.19%	119.48%	120.01%	130.84%	141.23%	154.31%	159.87%	168.94%	100.00%	100.00%	1.1796
3	77.51%	102.71%	115.86%	117.71%	118.65%	123.71%	124.49%	124.78%	133.43%	136.62%	143.33%	148.73%	161.82%	185.41%	189.90%	100.00%	100.00%	1.45
4	77.51%	102.96%	107.05%	108.89%	111.26%	99.85%	100.77%	102.68%	103.86%	106.64%	98.08%	101.34%	113.46%	112.86%	110.95%	100.00%	100.00%	0.4315
5	77.51%	106.52%	110.25%	112.47%	116.79%	122.66%	125.48%	128.16%	145.62%	150.08%	146.64%	150.44%	158.45%	169.99%	175.47%	100.00%	100.00%	1.2639
6	77.51%	105.88%	108.41%	114.38%	121.56%	127.19%	132.59%	144.78%	150.12%	151.61%	158.13%	169.78%	173.18%	185.05%	191.95%	100.00%	100.00%	1.4765
7	77.51%	105.52%	107.81%	108.41%	114.29%	119.37%	125.32%	128.00%	128.47%	136.06%	149.60%	159.42%	158.01%	172.40%	181.09%	100.00%	100.00%	1.3364
8	77.51%	104.00%	104.59%	104.68%	101.82%	102.50%	99.97%	102.53%	101.32%	98.80%	101.62%	98.47%	99.29%	105.44%	97.22%	100.00%	100.00%	0.2543
9	77.51%	105.31%	113.86%	120.03%	123.52%	130.22%	140.40%	141.61%	152.53%	162.52%	173.99%	178.17%	199.75%	212.69%	231.99%	100.00%	100.00%	1.993
10	77.51%	103.56%	103.05%	110.24%	107.71%	118.94%	119.14%	117.44%	129.21%	137.95%	145.05%	148.42%	158.90%	157.94%	167.89%	100.00%	100.00%	1.1661
11	77.51%	103.41%	110.44%	112.18%	123.52%	117.98%	126.56%	132.80%	130.09%	128.45%	138.68%	137.68%	131.52%	134.86%	135.03%	100.00%	100.00%	0.7421
12	77.51%	103.49%	107.39%	108.99%	97.94%	104.72%	108.24%	112.07%	112.53%	113.48%	117.19%	122.84%	121.05%	126.54%	133.61%	100.00%	100.00%	0.7238
13	77.51%	105.76%	115.12%	124.40%	136.60%	139.40%	157.11%	165.44%	172.14%	166.67%	170.11%	184.17%	205.12%	218.59%	221.40%	100.00%	100.00%	1.8565
14	77.51%	107.71%	114.47%	118.65%	126.18%	114.45%	114.04%	105.64%	106.47%	97.46%	94.46%	89.88%	89.90%	94.91%	103.67%	100.00%	100.00%	0.3375
15	77.51%	107.07%	106.75%	105.38%	103.81%	104.92%	110.42%	114.39%	117.95%	121.52%	122.87%	122.99%	126.95%	126.10%	124.56%	100.01%	100.01%	0.607
16	77.51%	106.46%	109.78%	115.49%	120.76%	127.14%	124.04%	122.92%	130.71%	129.97%	130.37%	130.48%	127.05%	121.37%	117.32%	100.00%	100.00%	0.5137
17	77.51%	105.34%	114.69%	123.25%	121.52%	126.55%	137.93%	142.13%	135.07%	142.59%	156.08%	182.39%	190.09%	200.63%	199.67%	100.00%	100.00%	1.5762
18	77.51%	105.39%	105.92%	99.56%	104.83%	103.73%	102.66%	99.14%	95.39%	95.20%	93.85%	96.04%	92.90%	90.37%	98.80%	100.00%	100.00%	0.2747
19	77.51%	106.74%	113.58%	120.99%	132.37%	140.12%	146.43%	149.77%	167.13%	184.58%	195.51%	212.15%	227.81%	229.09%	221.63%	100.00%	100.00%	1.8594
20	77.51%	104.12%	102.11%	116.35%	129.20%	135.78%	149.92%	151.22%	156.55%	182.76%	185.81%	193.60%	206.45%	208.67%	233.91%	100.00%	100.00%	2.0179
21	77.51%	106.51%	101.65%	96.96%	101.49%	100.15%	104.16%	103.42%	115.99%	112.81%	120.96%	125.39%	134.91%	135.19%	137.77%	100.00%	100.00%	0.7775
22	77.51%	102.60%	109.68%	112.27%	125.84%	126.68%	134.51%	144.54%	150.54%	148.99%	159.45%	174.20%	178.16%	179.76%	196.56%	100.00%	100.00%	1.5359
23	77.51%	104.99%	113.79%	127.81%	124.61%	129.65%	141.05%	136.72%	135.54%	137.30%	150.70%	166.77%	178.54%	185.21%	194.96%	100.00%	100.00%	1.5154