MASTER THESIS

Exploiting Information From Analyst Target Prices Using Portfolio Optimization

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Abstract

This thesis studies the investor portfolio selection problem. I examine whether a risk-averse investor can use information from consensus target prices, issued by financial analysts, in the construction of the portfolio weights. The weights of the portfolio are directly estimated as a function of the security's characteristics through optimizing the average utility that the investor would have obtained over the sample period. I find that the investor can in fact exploit analyst information through consensus target prices. The target price implied return and target price implied dispersion are informative to investors and provide new information not yet captured by traditional variables. I empirically show the investor can use this information to construct a portfolio that provides robust out-of-sample performance. The investor deviates from the market portfolio and optimally overweighs firms that have greater target price implied returns and underweights firms with a large dispersion in target prices. By doing so the investor obtains an economically and statistically significant alpha of 310bps (280 bps) over the value-weighted (equal-weighted) benchmark portfolio. Moreover, the portfolio obtains an economically and statistically significant alpha of 240bps a month, over the portfolio that only includes the traditional characteristics identified by literature such as size, book-to-market, momentum, investment, and profitability. The results remain robust even after controlling for the Fama-French-Carhart risk-factors. The results are not completely spanned by the Fama-french-Carhart factors indicating that returns are not fully explained by exposure to the well-known risk factors. The results are robust to short-sale restrictions, various utility functions, and across samples.

Keywords: *Optimal portfolio selection, Analysts; Target prices, Firm characteristics, Optimization, Equity factors, CRRA, Risk-aversion.*

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1. Introduction

The consensus with the finance literature leans towards the notion that expected returns are predictable, at least to some extent. The vast literature on expected returns identifies an ever-growing number of potential variables that allegedly carry predictive ability to explain the cross-section of expected return. For example, Fama and French (1993, 1998, 2006, and 2008) have repeatedly documented persuasive anomalies in stock returns driven by certain risk-factors that span the cross-sectional distribution of expected returns. Investors could construct a portfolio themselves by selecting assets that load high on the factor attributes (i.e. through portfolio sorts) to reap the factor premia associated with them (Lewellen, 2015). Alternatively, Daniel & Titman (1997) argue that firm-level characteristics drive these stock expected stock returns. On top of the cross-sectional return information previously mentioned, financial analysts seek to quantify firm-specific information into their forecasts to derive the fair value of a firm. Incorporating and combining cross-sectional and qualitative information such as analyst target price in a portfolio in an efficient and robust manner has proven to be a difficult endeavor. Modern portfolio theory formulated by Markowitz has since its establishment been a cornerstone for the construction of mean-variance efficient portfolios that allow an investor to optimize a portfolio conditional upon the first two moments. However, optimizing mean-variance efficient portfolios requires modeling the expected returns and estimating covariances for a large number of securities. This often leads to high estimation risk and yields unstable portfolios over time. Moreover, traditional portfolio optimization methods often find it difficult to take into account relevant information such as analyst-forecasts, risk-premia, prior views, or economic conditions. Finally, the non-normality of the return distribution may render traditional methods such as mean-variance optimization inappropriate as these fail to take into account fat-tails or skewness preferences that are potentially relevant for risk-averse investors (Barroso & Santa-Clara, 2015). Finally, the information conveyed through analyst target-prices is broadly available but varies substantially amongst analysts, therefore coming up with a decision rule to incorporate securities based on target prices is difficult. Consequently, we are faced with a series of questions, which characteristics do provide timely, independent information about average returns? Do these characteristics enable an investor to achieve outperformance? And if so, how does the investor efficiently incorporate these variables into a portfolio that is robust and persistent across time. These questions lead me to the following research question: Is a risk-averse investor able to increase the performance of her portfolio by exploiting information from analyst target-prices? The questions will be answered through the following series of sub-questions.

1. Is there predictive ability in the analyst-forecasts?
2. Do analyst forecasts add value for an investor after controlling for additional variables?
3. How does the portfolio performance change if we do not allow an investor to short assets?
4. How does the portfolio performance change in an investor becomes more risk-averse?
5. Is the portfolio performance robust across objective functions and samples?

The goal of this thesis is to study whether analyst forecasts convey useful information that an investor can use in the portfolio selection to improve the portfolio performance relative to the benchmark portfolios. I will examine the added value of analyst forecasts by directly incorporating the consensus target prices of financial analysts into the portfolio optimization. Lewellen (2015) shows that frm-level characteristics carry predictability for the cross-section of stock returns that can be included in portfolio selection throughout of sample regression. However, as Lamoureux & Zhang (2014, 2020) express focusing on expected returns might lead to wrong inferences especially in instances where time-sensitive variables are employed that have shown to be less persistent. In addition to this limitation, characteristic-based strategies that generate large alphas might entail large fat tails making them undesirable for risk-averse investors (Lamoureux & Zhang, 2020). Therefore to shift the focus from expected return to expected utility I apply the methodology of Brandt, Santa-Clara and Valkanov (2009), in which portfolio weights are parametrized as a function of the portfolio assets' cross-sectional characteristics that maximize the utility that the investor would have received over the sample period. The methodology has been applied by DeMiguel, Plyakha, Uppal and Vilkov (2013) who incorporate information from options markets into the selection problem and find significant outperformance. Fletcher (2017) examines the efficacy of several trading strategies using mean-variance preferences in the context of the U.K market and finds those parametric portfolios using size, book-to-market, and momentum outperform both passive benchmarks as well as alternative mean-variance trading strategies. Lamoureux & Zhang (2014, 2020) also provide comprehensive evidence in favor of several characteristics that provide significant out-of-sample returns that are not spanned by the Fama-french-Carthart risk-factors robust over various risk-aversion levels. In the light of this research, I use characteristics that are supported by these prior studies and complement current literature by examining two new variables based upon analyst target prices, which are (1) target price implied return and (2) target price implied dispersion. My study provides new evidence on the benefits of using analyst information in constructing the optimal portfolio weights. The beforementioned papers all allow for unlimited leverage or restrict leverage all together. I also follow Ammann, Coqueret and Schade (2016) and constrain the leverage (i.e. allow but limit) the investor is allowed to take on in order to provide feasible solutions to the selection problem. The methodology that I use enables us to account for non-linearities and departure from normality such that the final solution better reflects the preferences of a hypothetical risk-avererse investor. The study will be applied to a large sample of firms that are or have been listed across the NYSE, AMEX, and NASDAQ. This provides great insight into the various drivers of returns of stocks listed on these exchanges. It is one of the first studies that applies the traditional risk-based characteristics and combines these with analyst target-prices in a comprehensive manner such that robust portfolio weights can be constructed. In line with previous studies, I perform a range of robustness tests such as constraining the ability to use leverage i.e. shorting-assets, vary the risk-aversion of an individual investor, and enforce a restriction on the market-capitalization of firms included in the sample.

The results show that there exists predictive ability in analyst forecasts that enable an investor to improve the performance of her portfolio across a range of metrics. The informative component remains pervasive after the introduction of the traditional characteristics size, value, momentum, profitability, and investment and after controlling for the Fama-French-Carhart factors. First, in line with the literature, I find that if short-sales are allowed an investor, that includes all characteristics, optimally tilt his or her portfolio away from the value-weighted equilibrium portfolio towards small-cap firms, firms with a greater book-to-market ratio, firms that have earned high past 12-month returns, firms that invest less, and towards firms that are more profitable, on average. This results in significant positive style exposure towards the *market* factor, *size*, *value*, and *profitability* factor while it leads to a significant negative style exposure towards the *momentum* factor. By doing so the investor obtains a portfolio that outperforms the value-weighted (equal-weighted) benchmark by as much as 90bps (60bps), a month. I find that the optimized portfolios use the characteristics to construct a portfolio that reduces or even eliminates the negative skew of the value-weighted benchmark portfolio resulting in a symmetrical portfolio in line with findings from Lamoureux & Zhang (2020).

Both the characteristic target-price-implied-return and target-price-implied dispersion provide new information to the optimization in a portfolio where short sales are allowed. The investor overweighs firms that are characterized by positive target price implied returns and underweights towards firms that show a large dispersion of analyst target prices. By incorporating information from both characteristics, a constant relative risk-averse investor (CRRA) investor with risk-aversion of five (y=5) earns an alpha of 310 bps, a month (t=11.42) over the value-weighted benchmark portfolio and an alpha of 280bps (t=10.71) over the equal-weighted benchmark. The results show that if the characteristic analyst-target-price-implied-return is included in the optimization, the additional performance accounted for by the characteristic target-price-dispersion is as much as 93bps (t=5.38) a month relative to the portfolio including target-price-implied-return. This shows that both the target-price-implied characteristics improve portfolio performance and therefore including both characteristics improves the performance when the investor is allowed to assume short positions. The complete portfolio, which includes both analyst characteristics, obtains a monthly alpha of approximately 240bps (t=8.79) over the traditional portfolio. The optimal portfolio tops all prior portfolios in terms of risk-adjusted performance but also shows the largest monthly transaction cost and turnover. I show that a large part of the performance resides on the short-leg of the portfolio that hedges the market in downturns.

When short sales have been constrained the performance of the optimal portfolio reduces substantially. In this instance, the optimal portfolio, that includes both target-price characteristics earns a significant alpha of 50bps (t=3.77) a month over the value-weighted portfolio and a significant alpha of 20bps (t=5.732) over the equal-weighted benchmark portfolio. The outperformance remains significant after controlling for the Fama-French-Carhart factors. By restraining short-sales, the characteristic target-price implied dispersion no longer adds significant performance to the portfolio and has therefore become redundant in the optimization. The complete portfolio that including both analyst characteristics continues to obtain a monthly alpha of 23bps (t=6.34) over the portfolio that includes only the traditional characteristics. This shows that even though restricting short-sales reduces both the absolute and risk-adjusted-performance of the optimized portfolios the investors' outperformance remains robust.

The investor preferences play a central role in estimating the active portfolio tilts that define the optimal portfolio weights. By varying the risk-aversion parameter, I show that more risk-averse investors optimally make a trade-off between mean-variance efficiency on the one hand and positive skewness on the other hand. As a result, a more risk-averse investor seeks to hedge a greater portion of her portfolio which improves her Sharpe-ratio but reduces the probability of earning very large payoffs. This is comparable to a risk-averse investor that dislikes lottery-like payoffs with occasional large profits. At greater risk-aversion levels a constant relative risk-averse (CRRA) investor seeks to minimize the probability of very large negative returns over the course of the investment period such that drawdowns are minimized. These findings are in line with Brandt et al. (2009) and Lamoureux Zhang (2020). When short-sales are prohibited the investor is unable to hedge the market in the worst months through taking short positions and consequently, the risk-adjusted return becomes a direct tradeoff between mean returns and variance. As a result, both the absolute risk-adjusted performance as well as the relative outperformance over the benchmark portfolios decrease with increasing risk-aversion.

Furthermore, I find that the optimization remains robust across a range of objective functions. An investor with mean-variance preferences does not consider the third and fourth moment of the return distribution and as a result, optimally invests in a portfolio with larger mean-variance efficiency but characterized by larger occasional drawdowns relative to that of a CRRA investor. An investor that prefers to maximize her Sharpe ratio holds a portfolio with a large risk-adjusted-performance but does so by accepting a portfolio with a much larger probability mass in the tails of the distribution resulting in a larger probability of extreme returns. The results over the various objective functions carry over to the portfolios without short sales and only differ in terms of magnitude. Finally, I show that the results remain robust after considering a sample that restricts the inclusion of firms based on market capitalization. Reaffirming that the results are not driven by small-cap firms.

2. Literature review

The literature on portfolio management has been propelled by the introduction of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). The model's basic premise is that the expected return of an asset is a function of the prevailing risk-free rate and the correlation of the firm-returns with the market returns. According to the model, a firm with a higher correlation with the market carries a greater exposure towards market risk and consequently will earn higher returns. The model asserts that all risk-averse investors will invest their capital in the portfolio that offers the greatest return per unit of risk, that is the mean-variance efficient market-portfolio. The capital asset pricing model (CAPM) rests on the conjecture there exists a positive relationship between the beta and the expected return. However longstanding evidence documents the inadequacy of the capital asset pricing in explaining the cross-section of average returns. The relation between average returns and the market beta is flatter than predicted by the Sharpe (1964) and Lintner (1965) capital asset pricing model; see for example Fama & French (1992, 1996). They show that the relation between beta and the average returns is weak and the explanatory power is lacking. This led them to conclude that beta alone cannot account for the complete cross-section of average expected returns of equities and propose the use of additional factors. The ongoing debate even led to, albeit contended, introduction of strategies such as betting-against beta (see e.g. Frazzini & Pedersen, 2014), which take a short position in high-beta assets in order to exploit the consistent underperformance of such assets.

The search for variables that can predict the cross-sectional differences in average returns between firms has produced an ever-growing list of contributing variables often referred to as the 'factor zoo' (Feng, Giglio & Xiu, 2000). This reflects the fact that many of these variables are found to be redundant when used in combination with other variables and therefore remain subject to debate. However, some of these variables, beyond the aggregate market return, have been shown to consistently offer sources of return and therefore investors should incorporate stocks in their portfolio that fit or have exposure towards these sources of return. Whether these exceptional returns stem from underlying risk factors, firm characteristics or are natural outcomes of behavioral patterns is a topic of discussion that I will address in section 2.2.

The literature around portfolio management has mostly resolved around return-based factors and rather limited attention has been given to alternative factors. This thesis contributes by investigating the value of analyst target prices. There may be merit in incorporating analyst target prices into optimal portfolio selection as this might allow the investor to tap into, previously disregarded, new information that is potentially very valuable. The next paragraphs will highlight the literature on portfolio optimization and make way through the vast literature around cross-sectional asset-pricing and alternative characteristics. I will explain why these particular characteristics are valuable to investors, what drives their returns and why I incorporate them into the portfolio optimization later on. I will start with an assessment of the literature around the parametric portfolio methodology proposed by Brandt et al. (2009). Next, I will review the literature on analyst-forecasts as these are the main variables of interest for my thesis. The literature review will conclude with a review of the literature on the cross-sectional variables that are size, value, momentum, profitability, and firm-investment.

2.1. Portfolio optimization

2.2. Traditional approaches

One of the first approaches towards disciplined and quantitative portfolio construction is defined by Markowitz (1952). Markowitz laid the foundation for modern portfolio theory (MPT) and mean-variance optimization by illustrating the concept of diversification in which the investor does not only care about maximizing expected returns but also about the variance of the returns. Under mean-variance optimization, the investor maximizes her return given a certain level of risk that yields the mean-variance efficient portfolio. Despite the elegance and rationality of the method the practical appeal of the methodology is lacking. The problem with mean-variance optimization is that the output becomes very unstable due to the instability of the ill-conditioned covariance matrices that need to be estimated. This makes it difficult to form portfolios that offer robust performance over time without putting excessive constraints on the portfolio. Moreover, the method is cumbersome for a portfolio with a large number of assets as the dimensionality results in a large estimation risk of the variances and expected returns (Michaud, 1989). Finally, the approach exclusively accounts for the first two moments, return and variance disregarding the second and fourth, skewness and kurtosis.

Another prominent approach to asset allocation would be to manage the inputs into the model rather than the optimization process, these procedures are referred to as Bayesian methods. A Bayesian method seeks to incorporate both prior information as well as current or exogenous information (Basile & Ferrari, 2016). One of the applications is that of Black and Litterman (1992). The optimization methodology of Black and Litterman (1992) allows investors to construct portfolios and input absolute or relative subjective beliefs and add additional parameters or constraints. These (sometimes excessive) constraints reduce the instability of the portfolio but still require many views before the portfolio weights can be derived. Moreover, the approach still requires the variance-covariance matrix to be estimated and subsequently inverted. Various Bayesian extensions have tried to reduce the large estimation risk of the early Markowitz, sample-based, mean-variance optimization using various statistical approaches such as the shrinkage of estimators (see e.g. Jorion, 1985 & Ledoit and Wolf, 2004) or by imposing generalized moments constraints (see e.g. Jagannathan and Ma, 2003). Nevertheless, these fixes only achieve modest empirical success (DeMiguel et al., 2009). Other Bayesian approaches are that of Pastor and Stambaugh (2000) who refine the portfolio choice by incorporating a set of priors based upon asset-pricing models such as those of Fama and French (1992) & Daniel and Titman (1997). Hjalmarsson & Manchev (2012) argue that naïve regression-based modeling only considering conditional expected return performs worse relative to a standard equal-weighting scheme not only due to poor mean-variance estimates but also due to changing parameters and varying risk-premia over time. Hence regression-based models perform well in-sample but quickly deteriorate when applied out-of-sample.

Alternative optimization techniques in which the conditional moments have been ignored altogether have gained ground. Under such techniques the expected returns are completely disregarded, giving less room for estimation errors. Examples of such strategies are equal-weighting, minimum-variance, or risk-parity strategies. The minimum-variance portfolio is the portfolio with the smallest variance that also lies on the efficient frontier while the equal-weighted portfolio is most straightforward and reflects assigning equal weight to all assets. DeMiguel et al. (2009) find that the equal-weighting strategy dominates all previously discussed optimization techniques including strategies explicitly accounting for estimation risk further highlighting the inability to correctly define a portfolio allocation. Under a risk-parity optimization, the portfolio-weights are defined such that the assets contribute equal to the overall portfolio risk, in line with risk-budgeting literature. The weight allocated to each asset becomes higher as the asset's volatility and correlation with other assets becomes smaller (Braga, 2015). Under naïve risk-parity, which assumes equal pair-wise correlations, the optimal weights are calculated as the inverse of the standard deviation. Braga (2015) finds that the risk-parity approach provides real benefits of diversification, relatively lower turnover, and transaction costs but remains inferior in terms of financial efficiency relative to more naïve approaches such as equal-weighting. Also, these techniques do not allow for incorporating prior views concerning the expected returns of the underlying assets or for incorporating cross-sectional firm-level information.

2.3. Parametric portfolio policy

Brandt et al. (2009) (BSV) introduce a novel methodology that allows the portfolio weights to be constructed by drawing inferences from cross-sectional firm-level information. BSV refer to the methodology as the parametric portfolio policy as it requires estimating the parameters in order to construct the optimal portfolios. By combining the firm-level information with the market-equilibrium an investor is able to construct a portfolio with a large number of assets, that provides the highest utility, conditional upon the specified utility-function. Under the methodology of Brandt et al. (2009) the effective weight that a firm receives in the portfolio results from the benchmark weight and a deviation from that market-weight depending on the characteristics of the firm at that specific point in time. This methodology is able to exploit both information in the cross-section of stocks as well as information of characteristics across time, as the parameters to be estimated are allowed to vary over time. The parameters of interest are estimated such that the utility that the investor would have obtained over the sample period, depending on its objective function, is maximized. The parameters to be estimated hereafter referred to as coefficients, result in an over-or underweighting of the optimal portfolio from the market equilibrium towards the most optimal allocation. This means that the benchmark weightings are adjusted in order to tilt towards firms with certain characteristics to the extent that they provide excess returns. If the parameter that we estimate is positive, the characteristic is associated with above-average returns, and therefore the portfolio increases its exposure towards this characteristic. A negative coefficient implies the opposite and hence means the portfolio takes a negative allocation to the underlying characteristic relative to the benchmark portfolio. The objective-function that I use allows the optimization to take into account all higher moments of the return distribution. Usually only the first and second moment i.e. returns and variances are included while the third and fourth, i.e. skewness and kurtosis, are being ignored.

It is important to establish the relation between firm characteristics and risk-factors early on. Firm-specific characteristics are descriptors that are calculated using individual firm-level data. Factors are variables that reflect or proxy for common sources of underlying risk for which investors receive returns. The descriptors or characteristics relate to common risk factors as the former are used to construct the factor-portfolio that proxy for some underlying risk. However, these factor-risks are not tied to individual characteristics but reflect the fact that firms with these characteristics show similar patterns of returns. The descriptors will be referred to as stock characteristics and the risk-factors, popularized by Fama and French, used in the regression analysis will be referred to as style factors. For now, I take an agnostic stance on the source of these returns that I will review in section 2.4.

Hjalmarsson and Manchev (2012) present the benefits of parametrizing weights directly as a function of underlying characteristics rather than making them the outcome of complex estimation procedures. They derive a closed-form solution to the methodology of BSV and find that the methodology delivers superior results compared to the naïve trading strategy that inputs conditional means into the regression optimization. DeMiguel et al. (2009) compare the BSV optimization to a naïve diversification strategy such as equal weighting (1/N). For a portfolio with a small number of asset the increased performance of the BSV-portfolio does not cover the increased transaction costs relative to the equal-weight (1/N) strategy. However, for portfolios with a larger number of assets the strategy does earn a greater Sharpe ratio even after taking into account transaction costs. This shows that incorporating cross-sectional firm-level information such as firm characteristics rather than solely statistical estimator does lead to improvement of portfolio performance.

In a follow-up paper, DeMiguel et al. (2013) find that by incorporating size, value and momentum the portfolio annualized Sharpe ratio yields approximately 0.66 relative to 0.59 for the benchmark portfolio, with short-sale constraints. Moreover, they find that it is useful to incorporate option-implied information in the portfolio-construction beyond the usual characteristics as this significantly increases the Sharpe to 0.76. Their performance is robust across both daily and monthly rebalancing. Zhang (2013) finds that the methodology of BSV is effective in tilting the optimal portfolio towards characteristics that provide greater returns on average. Their portfolio actively tilts towards small-cap stocks, value-firms, and firms with higher past 12-month returns. The portfolio yields an annualized alpha of 540 bps, while allowing for short-sales, over the Fama-french-Carhart factors throughout 1974-2010. They find this outperformance to be persistent over bull- and bear markets and robust across different horizons. Ammann et al. (2016) find that the introduction of a short-sale constraint to the portfolio optimization limits the variation of the portfolio parameters and therewith also the deviation from the benchmark weights. Less variation of the active portfolio also leads to greater stability of the weights across time, greater diversification and fewer transaction costs. Medeiros et al. (2014) apply the portfolio method in the Brazilian market between 2001 and 2013. By optimizing upon book-to-market, size and momentum characteristics the parametric portfolio earns an out-of-sample average annualized return of 35.9% relative to 14.9% on a value-weighted portfolio. The optimized portfolio earns a Sharpe ratio of 1.59 relative to the 0.68 for the benchmark portfolio. Moreover, when short sales are restricted the annual return reduces to 22.5% which is still well above the return of standard benchmark portfolios. Fletcher (2017) examines the benefits of using stock-characteristics to model optimal weights using the model of BSV in the context of the United Kingdom. Specifically, they include size, value and momentum characteristics in the optimization and estimate portfolios over the period of 1981 till 2012. They find that the characteristics based portfolio yields a monthly excess return of 407 bps with a sharpe ratio of 0.293 while the benchmark value-weighted (equal-weighted) portfolio over the same timespan earned approximately 32 bps (57 bps) a month with a respective Sharpe ratio of 0.078 (0.118). Remarkably, both of the portfolios yield a negative CER as a result of the high volatility of the portfolios, penalizing the utility function. The characteristic based portfolio also outperforms the conditional mean-variance portfolios that earn monthly excess returns of around 176bps with a Sharpe ratio of 0.132 (a month) on average. After constraining leverage (no short sales) and accounting for transaction costs the mean excess return decreases to 104 bps and volatility decreases to 543 bps while the CER increases to 0.305. This shows that constraining the optimization leads to a more consistent and stable portfolio with more attractive risk-adjusted returns. Finally, they find that the superior performance is concentrated in the beginning years of their sample that could indicate that stock return predictability has weakened in recent years.

Lamoureux and Zhang (2020) apply a modified version of BSV in which they incorporate various characteristics such as momentum, book-to-market, size, beta, residual standard deviation and last-year same month return. Lamoureux and Zhang (2020) argue that the methodology of BSV performs an optimal joint sort, similar to the Fama-French portfolio sorts, on multiple characteristics simultaneously. Moreover, they find that the penalty imposed on the portfolio for including multiple characteristics increases with the risk-aversion level. Therefore, greater dimensionality will likely result in greater estimation risk. It is therefore key to identify relevant variables that do not add unnecessary volatility to the parameters. Performance-wise they document that, contingent on the loss function specified, the optimal portfolio consistently earns a substantial positive alpha’s relative to the Fama-French-Carhart (1997) model and Sharpe ratio double that of the benchmark. They note that when risk-aversion levels increase, the portfolio's exposure toward the risk factors increases, leaving a smaller portion of performance unexplained by traditional risk factors at the cost of a lower alpha.

DeMiguel et al. (2020) examine the effect of including transaction cost in the BSV optimization while using a mean-variance objective function. They argue that the optimal portfolio parameters are chosen such that the significant negative skew of the benchmark is removed resulting in more symmetric optimal portfolios in line with Lamoureux and Zhang (2020). In their most recent study they find that the optimal portfolio that incorporates size, value, investment, and profitability earns as much as 23.6% on average each year with a Sharpe ratio of 1.072 relative to a return of 8.5% a year for the value-weighted (equal-weighted) portfolio with a Sharpe ratio of 0.56 (0.48). Hence, adding investment and profitability benefits the optimal portfolio substantially and therefore significantly outperforming the standard benchmark portfolios. The Fama-French 5-factor alpha found is close to 102 bps a month on average (t=3.59). Finally, they document that it is beneficial to incorporate combine multiple characteristics in the optimization as it smoothens the parameters. Exploiting the cross-section using multiple characteristics is advantageous as it will substantially reduce the portfolio transaction due to trading diversification and hence increase the investors' utility. The intuition is that as the portfolio rebalances, the over-and underweighting over various characteristics balance each other and therefore reduce trading costs.

2.3.1. Optimal portfolios over various asset classes

In the original paper of BSV the portfolio parameters that are estimated through optimization remain constant across time and properties. This means that the deviations from the market portfolio are only due to the variation of the characteristics. However, the parametric portfolio optimization is flexible in that it is possible to incorporate time-varying conditioning variables into portfolio selection. Plazzi et al. (2011) apply the methodology to a portfolio of commercial real estate and let the optimal parameter vary with the realization of a proxy for business cycle risk allowing them to adjust the composition of their optimal portfolios accordingly. By optimal tilting towards characteristics, the portfolio return improves from 2.57% towards 13%, then by letting the parameter vary across time the performance is marginally improved to 13.4%. Barroso and Santa-Clara (2015) test the relevance and performance of currency tilting portfolios and show that the optimal portfolio consistently outperforms naïve benchmark portfolios both in in-sample as out-of-sample tests. Specifically, the strategy results in a significant improvement in Sharpe ratios after considering transaction costs and a large reduction in drawdowns. The optimal portfolio alpha is as much as 207 bps a month (t=3.74) after controlling for the Fama-French-Carhart factors, indicating that economic returns stem from sources other than the traditional risk factors. Ranganathan, Lohre and Nolte (2019) extent the research of currency factor tilting and confirm the earlier findings of Barroso and Santa-Clara (2015). Their evidence shows that the benefits from currency tilting are mainly derived from the cross-sectional variation of the characteristics rather than the time-series variation in state-variables. This means that most of the performance gain results from tilting rather than timing characteristics.

In summary, the evidence on the performance of the methodology of BSV is promising, most portfolios outperform their respective benchmark portfolios using a variety of characteristics proposed by academic literature. The consensus is that the optimal portfolio tilts towards characteristics that have been providing greater abnormal returns in the past. The question remains what factor or characteristics to include. The research concerning new factors and especially non-return-based factors has been thin and this is where this thesis will contribute. The next section will examine the literature with respect to return and non-return-based factors.

2.4. Cross-sectional asset pricing

2.4.1. Value & Size

There has been ample empirical research that has demonstrated the above-average cross-sectional returns of factors beyond those already explained by the market factor. Variables that are often associated with variation in the cross-section patterns of returns are book-to-market ratio, market capitalizations, past returns, profitability, and investment. In an efficient market with rational investors that are naturally risk-averse, assets are priced rationally, and stock risks are multidimensional. Under the risk-based model posited by Fama and French (1993, 1996) returns reflect compensation for bearing systematic risk demanded by their respective investors. Variables that are related to cross-sectional average returns must proxy for the sensitivity to common risk factors. One such dimension of risk is proxied by size, measured as market capitalization while the other, value, is proxied by the book-to-market ratio (Fama & French, 1992, 1993). This implies that firms with similar characteristics that co-vary with each other are exposed to a common underlying risk. I will address them both in the following paragraph.

The *size* effect was first documented by Banz (1981) who demonstrated that a firm's market equity appears to be inversely (negatively) related to cross-sectional average returns. Under the assumption of efficient markets, firm size serves as a proxy for a systematic risk factor. Small firms can suffer from prolonged earnings depressions relative to large firms and are therefore more subject to business cycle risk. Fama and French (1993) show that these particular stocks co-move together and as a result produce similar patterns of returns. This suggests that firm-size is associated with a common risk factor that can explain the negative relationship between firm size and expected return. The size effect has been shown to obtain most of its predictive ability from small stocks but also appears pervasive and persistent across varying size groups (Fama & French, 2008). The relationship between average returns and market beta seems almost non-exist whereas the relationship between size and average return appears statistically strong and positive (Fama & French, 1992, 1993). French and Fama (2015) show that the average spread of a small-minus-big portfolio is approximately 0.29% (t=2.31).

A vast amount of literature has documented that higher book-to-market (value-firms) firms have earned higher stock returns on average and outperform the market over a length of time. Rosenberg et al. (1984) identify a strategy that buys high book-to-market stocks and sells low book-to-market stocks obtains significant abnormal profits. The firms' book-to-market ratio provides a powerful characterization of cross-sectional average returns as the ratio appears positively related to average returns (Fama & French, 1992). The fact that higher book-to-market firms have provided greater average cross-sectional returns is generally accepted, yet, till today there is still little consensus whether these abnormal returns are due to mispricing, originate from investors demanding a risk-premium for bearing systematic risk, or simply stem from high book-to-market firms having greater returns (Daniel & Titman, 1997).

The first string of literature resorts to risk-based explanations for the value premium. Under the risk-conjecture investors demand a higher return for holding such firms as they load high on some unknown distress factor. Other risk-based theories relate the value-premium to the asset-duration of a firm. A large portion of the earnings of growth firms (low book-to-market firms) are weighted to the future hence these firms are subject to a greater interest rate sensitivity on their investments relative to value firms. However, counter to simple intuition, Dechow, Sloan and Soliman (2004) find that long-duration assets, i.e. growth firms, have produced lower average returns historically. This suggests that equity investors actually choose a long investment horizon and do not demand a premium for holding growth firms stocks that have greater sensitivity to expected return shocks. Another notion along similar lines is that price of risk expressed through the discount rate moves independently from cashflow-risk. In a world where investors are more averse to cashflow-risk, returns on short-horizon equity such as value-stocks, vary more with fluctuations in cashflows whereas returns on growth stocks, as long-horizon equity, vary with fluctuations in discount rates. Altogether, this results in a greater return on equity on value stocks for which investors demand greater cashflow premium relative to a lower return on growth stocks which are more sensitive to discount rates, that investors do not mind bearing (Lettau & Wachter, 2007). Binsbergen, Brandt and Koijen (2012) show via a synthetic portfolio of dividends strips that short-term dividends carry a higher risk-premium vis a vis the market portfolio, which represent a stream of all future dividend income, hereby showing that the term-structure of equity-returns is in fact downward-sloping. Complementary, Weber (2018) documents a downward sloping equity-curve which would imply that short duration stocks earn a premium, but find that low-duration stocks, that face no short-sale constraints, do not earn a premium relative to unconstrained high duration stocks. Therefore, they suggest that in fact, short-sale impediments might explain the difference in expected returns consistent with a theory of mispricing. The spread-differential is largest in most short-sale constrained stock, showing that only in these instances low-duration (value) stocks outperform high-duration (growth stocks).

The second string of literature rests on behavioral arguments that challenge the efficient market hypothesis. According to Lakonishok, Schleifer and Vishny (1994) the abnormal, value premium represents a contrarian investment strategy that opposes 'naïve' strategies followed by other investors. Following this intuition, low market-to-book firms are overpriced while firms with high market-to-book ratios are underpriced. A 'contrarian' strategy that involves buying out-of-favor value stocks and selling 'in-favor' growth stocks represents an effective strategy that takes advantage of mispricing in the cross-section of stock returns. Mispricing might result from investors extrapolating past-earnings-growth too far into the future, overreaction, or biased investment decisions (Lakonishok et al., 1994). When, eventually, the (wrongly) anticipated growth rate does not materialize the relative return of growth firms will deteriorate. Under the behavioral model, the value and size premium will continue to exist as long as investors will behave irrationally to new information or when limits to arbitrage prevent these anomalies from being exploited by arbitrageurs.

Third and final, Daniel and Titman (1997), take a somewhat different stance and argue that the cross-sectional return patterns associated with small and high book-to-market firms are not due to covariances of returns but result from similar firm-specific stock characteristics. They posit that the proposed risk-factors are not associated with a return-premium and therefore cannot be considered compensation for bearing systematic risks. The return on the stocks is therefore unrelated to the underlying covariance structure of the returns but rather the result of firm characteristics. Although the book-to-market stocks appear to co-vary with each other, this is not the result of similar underlying risks associated with them but is due to these firms simply having similar properties; e.g. similar industries, regions, etc. Daniel and Titman (1997). If this is the case then the abnormal returns associated with these characteristics might reflect behavioral biases such as the overextrapolation of past growth rates suggested by Lakonishok et al. (1994). Another possibility is that the returns stem from sustained mispricing due to priors that investors hold towards firms with certain characteristics.

Whatever the source of the returns may be, an investor should seek exposure to these assets as they have provided sustained and persistent returns in the past and may well do so in going forward. Moreover, the book-to-market ratio is negatively related to profitability and firm investment, and therefore firms with low book-to-market ratios are on average more profitable and tend to invest more (Fama & French, 2006). Fama and French (2008) show that the relationship between book-to-market is consistently positive across size groups. French and Fama (2015) demonstrate that the value effect remains strongly persistent over time and show a value sorted portfolio[1] earns as much as 0.37% on average a month (t=3.20). Moreover, they show that the cross-sectional average return for the book-to-market portfolio falls for firms that have larger market equity relative to small-cap stocks hence the value effect appears to be stronger amongst small stocks. More specifically, when double sorting on size and value, a small-value portfolio earns on average 0.53% a month (t=4.05) while a big-value portfolio earns 0.21% (t=1.69). Finally, Hou, Xue and Zhang (2020) find that the book-to-market ratio remains a persistent predictor for stock returns over the period of 1967 till 2014. The high-minus-low portfolio continues to earn as much as 0.59% a month (t=2.84).

2.4.2. Momentum

Throughout literature cross-sectional momentum has been documented to yield above-average returns on assets for prolonged periods of time. Even though the effect is pervasive the source of the effect remains one of the prominent anomalies in the light of efficient markets. The factor was first examined by Jegadeesh and Titman (1993) who document that firms that have shown to provide above-average returns over the last 12 months will continue to do so for a certain period of time before they revert to their mean. These abnormal average returns they document are in line with a model of delayed overreaction to firm-specific information and cannot be attributed solely to exposure to a systematic or common underlying risk. Under the behavioral model investors buy past-winners and sell past-losers by which they temporarily move prices away from their long-run equilibrium values. This trading causes prices to overreact in the short term and revert to their long-term values over time. This short-term momentum and long-term reversal effect are also observed by Jegadeesh and Titman (2001) who attribute the effect to the serial correlation of firm-returns, not due to the cross-sectional variation in returns.

Under the model of under-reaction, investors might underreact to new information at first, such as earnings announcements, which are corrected over time giving rise to positive returns in the form of momentum profits. Conservatism bias leads investors to underweight new information resulting in a slow down diffusion of information in stock prices (Barberis et al., 1998). Momentum profits are consistent with both delayed overreaction and initial underreaction, however, under the underreaction hypothesis, post-formation returns are be assumed to be positive whereas in the delayed overreaction model the post-formation returns are expected to be negative.

The models described before attribute momentum to firm-specific returns or time-series patterns in firm-stock returns. Time-series momentum is related though not identical, to cross-sectional momentum. The former is a time-series phenomenon and results from positive autocorrelation between stock-returns whereas the latter results from covariances of the returns in the cross-section. Cross-sectional momentum takes into account a firm's relative return whereas time-series momentum is absolute in nature, taking into account only its own return history. Lewellen (2002) documents a systematic risk-component of momentum returns in the form of macro-economic risks. More specifically they show that the momentum effect is pervasive across size and book-to-market portfolios in addition to individual stocks and industries suggesting large cross-sectional variation. This suggests that covariance between returns, in contrast to underreaction, could better explain abnormal momentum returns. To reconcile both findings, Moskowitz et al. (2012) document the presence of both time-series momentum and cross-sectional variation simultaneously. They show that the persistent time-series momentum is driven almost entirely by positive auto-covariances of returns. The effect partially reverses after a year, consistent with the notion of initial underreaction and delayed overreaction. The (time-series) effect is robust across asset classes and persistent and therefore unlikely to be a compensation

[1] That are constructed on 2x3 factor sorts similar to (Fama & French, Common risk factors in the returns on stocks and bonds, 1993)

for crash risk or tail events. At the same time, they document significant cross-sectional correlation amongst assets and asset classes consistent with a risk-based story. These findings show that time-series and cross-sectional moment are two distinct phenomena but share much of the same drivers that are positive auto-covariances of returns.

Jegadeesh and Titman (1993) report that, between 1965 and 1985, the high-minus-low portfolios on prior six-months returns earn on average 1.1% (t=3.61) and 0.9% (t=3.54) at a horizon of 6 and 12-months, respectively. In a follow-up paper, Jegadeesh and Titman (2001) continue to find momentum returns of similar magnitude in the years thereafter. Rouwenhorst (1998) examines the profitability of momentum strategies in a European context and find similar results to Jegadeesh and Titman (1993); specifically, a portfolio on prior six month returns with a holding period of 6 or 12 months earns 1.28% (t=4.59) or 1.05% (t=3.48) a month on average, respectively. The momentum factor has been given further support and attention by Carhart (1997) who augmented the original Fama and French (1992) 3-factor model with the Jegadeesh and Titman (1993) momentum factor. The momentum premium carries strong marginal predictive explanatory power and is pervasive across all size sorted groups (Fama & French, 2008). Nevertheless, the effect appears more persistent amongst large size quantiles (Lewellen, 2002). Hurst et al. (2017) show that a time-series momentum strategy has earned an annualized excess return of 11% (after transaction costs) over the period of 1880 till 2016 using a global sample. The performance proves consistent across markets and asset classes indicating the robustness of the strategy. In a follow-up study, Hou et al. (2018) continue to find the momentum factor to be, albeit smaller, pervasive and persistent at 0.82% (t=3.49) and 0.55% (t=2.9) a month on average over a 6- and 12-month horizons respectively.

2.4.3. Profitability

The first versions of the three-factor model of Fama and French included the market, size and value as factors. Novy-Marx (2013) amongst others points out the incompleteness of the three-factor model proposed by Fama and French (1992) since the model is unable to explain a large portion of the variation in average returns related to profitability and investment. Therefore, by popular demand French and Fama (2015) add two additional factors, namely, profitability and investment. A vast number of academics have identified a positive relationship between firm profitability and average returns. There remains a debate whether these returns results from greater underlying risk or should be attributed to sustained mispricing. The introduction of a new variable, profitability is motivated by the dividend discount model. To illustrate the intuition behind the relationship between the profitability and average returns the dividend discount models are slightly reformulated. According to the dividend discount valuation model, the firm value is equal to the sum of all discounted future dividends. Under clean surplus accounting, the dividend discount model can be reformulated to a valuation equation[2]

According to valuation theory, when controlling for expected profitability and investment, firms with higher book-to-market ratios should have higher discount rates which implicitly mean greater returns. This also implies that holding everything all factors constant firms with higher (lower) valuations should have lower (higher) average returns. Moreover, for firms with a given book-to-market ratio and expected investment, higher profitability implies greater expected returns. Finally, holding all other factors constant, firms with higher expected growth in book equity due to greater re-investment generate lower expected returns (Fama & French, 2006). However, this does not mean that the theory can determine whether the expected returns derived from the valuation equation stem from irrational mispricing or from rational risk-pricing. Therefore, the ability of profitability and investment to predict

[2] $\frac{M_t}{B_t} = \frac{\sum_{t=1}^{\infty} E(Y_{t+1} - dB_{t+1})/(1+r)^1}{B_t}$, M_t is the firms market value at time t, B_t, represent the book equity at time t. Furthermore Y_{t+1}, are the firms' equity earnings, dB_{t+1}, represent the change in book equity per share from t-1 till t. Finally, r is the long-term average expected stock returns (for more information see: Fama & French, 2006).

future returns does not help us determine whether these returns are caused by risk and or to what extent these are the result of sustained mispricing (Fama & French, 2008).

Fama and French (2006) test the relationship and find mixed results. In cross-sectional regression analysis, they document a statistically significant positive strong relation between profitability and average returns with a slope of 1.1 (t=2.55). However, in portfolio tests, they note that the incremental average returns captured by the profitability factor are modest when controlling for Size and Book-to-market. Therefore although the predictive ability might be significant, the incremental economic benefit from the profitability forecast is marginal. Fama and French (2008) restrict their attention to firms with positive profitability. Accordingly, they find reliable evidence that higher positive profitability is associated with greater abnormal returns but there is no evidence that negative profitability is associated with abnormally lower returns. The effect is pervasive across size groups with a slope of 0.55, 1.19, and 0.75 for micro, small and big stocks respectively (t=1.50, 2.36, and 1.56, respectively). Profitability sorts (that include negative profitability stocks) produce weak economic results, where the average spread (value-weighted) for small and micro stocks is 0.36% and 0.79% a month (t=1.67, t=2.87) whereas big stocks earn -0.27% on average a month (t= -0.84).

Alternatively, firms with more productive assets should yield higher average returns versus a firm that own less productive assets as they should be priced similarly according to the valuation principle (Novy-Marx, 2013). This argument is consistent but not predicated upon risk-based asset pricing. Their results show that profitable firms generate significantly higher average returns relative to unprofitable firms even though the former might have greater valuation ratios and therefore would be more similar to a growth strategy. Cross-sectional regressions of Novy-Marx (2013) suggest that gross profitability has roughly the same power as the book-to-market ratio in predicting returns, with a significantly positive coefficient of 0.75 (t=5.49). Gross profitability subsumes the information from all other income-related variables and remains a powerful predictor of average returns even after controlling for book-to-market and size. Therefore, contrary to Fama and French, Novy-Marx (2013) find that profitability is in fact complementary to the book-to-market ratio of a firm in explaining the cross-section of average returns. The portfolio sorts show that profitable firms generate above-average excess returns that are similar to value firms, nevertheless, the firm resembles the characteristics and covariances of growth firms. This means that while profitable firms appear to be typical growth firms they are able to outperform the market despite their high valuations. Both value and growth portfolios are polluted with unprofitable and profitable firms, respectively. By excluding unprofitable firms from the value portfolio and profitable stocks from the growth portfolio performance is enhanced substantially relative to the unconditional portfolio.

With the beforementioned in mind, French and Fama (2015) propose a five-factor model that incorporates these state variables. Similar to the before Fama and French (2015) document that profitability is a strong predictor of cross-sectional average returns. Simultaneously they document that the profitability effect appears stronger amongst small firms, this is somewhat consistent with findings of Novy-Marx (2013) even though the latter concludes that the predictive power of profitability is left largely undiminished across size quartiles. Consistent with Novy-Marx (2013), Hou et al. (2018) find that gross profitability to lagged assets as measured by gross profits is a significant predictor of asset returns. The high-minus-low portfolio earns on average 0.38% per month (t=2.62) where Novy-Marx (2013) find that the portfolio earns 0.31% on average a month (t=2.49). Interestingly, Hou et al. (2018) find that the operating profitability factor (Operating profits-to-book equity), used in Fama and French (2015), earns on average 0.25% per month but the results appear insignificant (t=1.2).

2.4.4. Investment

Evidence shows that much of the variation in average returns stemming from investment and profitability is left unexplained by the three-factor model of Fama and French (1992). In response, various papers analyzed the relationship between firm average returns and firm investment. Firms that substantially increase their capital investments, subsequently, earn lower average returns relative to otherwise similar firms. Usually, increased investment expenditures are viewed as favorable as these are associated with greater investment opportunities while also to the ability to acquire investment capital at favorable terms. However, managers might oversell their investments and the resulting payoff might not be adequate to justify their expenditures. The intuition behind the negative relation stems from overinvestment or managerial hubris. Second, the hypothesized relationship might suffer from reverse causality, such that at times when stock prices are higher, firms may increase their capital expenditures. As a result, greater capital expenditures might not be indicative for higher stock returns (Titman, et al., 2004). Titman et al. (2004) document a negative relationship between abnormal capital investments and subsequent stock returns. A long-short strategy that goes long a portfolio with low investment and short a portfolio with high investment firms earns a significant return of 0.168% a month on average or the equivalent of 2.02% (t=2.91) per year over the period of 1973 till 1996, that remains persistent after controlling for size, book-to-market, and momentum effects. Furthermore, they document that the average excess return monotonically decreases particularly for firms with greater cashflows and fewer amount of debt with capital investments. This is in line with the agency problem of overinvestment stating in which excess funds are often funneled to suboptimal firm-investments.

Fama and French (2006) document a significant negative relationship between average returns and asset growth (t= -3.87). Fama and French (2008) document that asset growth with the equity component removed from the variable is weakly associated with pervasive abnormal returns. The observed negative relationship is present and statistically reliable in small and micro-cap stocks but is absent for large firms that account for over 90% of the total market cap. (Cooper, Gulen & Schill, 2008) also find firms' annual asset growth to be a strong predictor of cross-sectional average returns. Asset growth reflects the sum of individual sub-components of growth that appear either on the left or right-hand side of the balance sheet i.e. either the investing side or the financing side respectively.

A composite measure such as asset-growth captures information from the various components of growth and therefore able to predict the cross-section of average returns better than a single component of growth Cooper et al. (2008). They find that firms with large past investment, measured as the year-over-year asset growth rate, earn abnormal returns relative to low-growth firms. This is effect is strongly significant and remains persistent up to 5 years beyond the sorting year. In a similar vein to Titman et al. (2004), Cooper et al. (2008) show that large stock returns 'are likely to provide an impulse for large future increases in assets of high growth firms'. More specifically they show that the relationship between asset growth and returns performance is positive in the (5-years) pre-formation period but becomes negative after the post-formation period. The relationship between growth and returns appears to be perfectly monotonic across the board over the period from 1968 till 2003. More specifically a long-short asset growth (value-weighted) portfolio earns as much as -1.05% (t =-5.04) on average a month in the first year and remains persistent up to three years thereafter. Cross-sectional regression results show that the predictive ability of asset-growth is significant and economically strong across size groups and remains so after controlling for other predictors. Specifically, the regression analysis show a statistically negative relationship with a coefficient of -0.094, -0.079 and -0.059 (t= -5.18; t= -3.80; t= -3.60) for small, medium and large firms, respectively.

By double sorting on both investment and size, Fama and French (2015) find that the average returns of firms with high investments earn lower average returns relative to firms with fewer investments. A double portfolio sorted on size and investment show that small firms with high investment earn as much as 0.45% (t=5.49) on average a month while big firms with high investment portfolio earn 0.22% (t=2.0) on average a month, suggesting that the investment premium is greater for small stocks. Additionally, they document that in a five-factor model that includes both profitability and investment, the previously described value factor becomes redundant for describing average returns. This suggests that value is simply a proxy for investment and profitability. Hou et al. (2018) go even further and argue that value and momentum are noisy versions of the profitability and investment factor. The investment effect remains pervasive as Hou et al. (2018) report that a high-minus-low portfolio (investment-to-assets) earns on average -0.46% per month (t=-2.92) over the period of 1967 till 2014. The beforementioned findings are consistent with the behavioral hypothesis of Lakonishok et al. (1994) stating that the cross-sectional pattern of returns is due to mispricing and overextrapolation. It appears that the investors' expectations of future firm growth have been tied to past growth rates even though these future growth rates have been shown to follow a mean-reverting process. Furthermore, the evidence is consistent with the agency hypothesis stating that the asset-growth effect results from managerial over-investment and managerial empire building.

The rationale to include the subset of the beforementioned characteristics is in accordance with prior literature applying the BSV methodology. Most have only focused on the first three factors, size, book-to-market, and momentum. DeMiguel et al. (2020) take a more agnostic stance and consider a range of variables. They find only a subset of 6 variables to be robust and significantly improving performance amongst which investment (asset-growth), and gross profitability. Therefore to focus the efforts and reducing the risk of over-fitting the model I include the subset of the before mentioned variables.

2.5. Analyst forecasts

Most statistical optimization methods fail to incorporate information beyond standard return-based factors and resolve to optimize using past information. An alternative potential source of information are analyst forecasts in the form of recommendations or target prices. If analysts have the ability to add value by conveying new information to market participants or by identifying mispriced stocks this information could be valuable to include in the optimization. The ability of the analyst to forecast future expected stock returns is debated and at times also controversial due to various (some may say toxic) incentives that are involved with setting target prices. especially at sell-side firms. The efficient market hypothesis states that market-prices reflect all private and public information available. Under this model, there is no information uncertainty and the information onto which analysts base their forecasts has already been reflected in the prevailing market price. Under such assumption's analyst forecasts do not add value-relevant information and forecasts merely represent biased estimates of future stock returns (Jegadeesh & Wooij, 2006). However, when relaxing the efficient market hypothesis there is the possibility that analysts have the potential to reduce both estimation risk and information asymmetries. Despite the prevalence of analyst target prices and their widespread availability to investors, the role of analyst target prices on portfolio returns has been given little attention in the literature. That is compared to the extensive literature on earnings forecast and stock recommendations there are relatively few papers on the investment value of target price forecasts (Da et al., 2016). The next section discusses the literature both on target prices and recommendations and explains why I choose to investigate target prices.

2.5.1. Analyst forecasts are informative

The first string of literature asserts that analyst target price forecasts are informative and contribute to improved price efficiency. The early study of Womack (1996) is the first to document that it is beneficial for investors to take advantage of publicly available investment research in portfolio selection. In fact, they showed that firms recommended by analysts tend to outperform firms that are downgraded by analysts. Barber et al. (2001) document that positive consensus analyst recommendations are associated with positive abnormal returns. They show that portfolios formed based on positive recommendations result in a gross return of around 4.1%, on average, a year after controlling for risk exposures through size, book-to-market, and momentum factors. More specifically, a portfolio sorted on analyst recommendation that rebalances monthly earns as much as 79 basis points on average a month (t=3.197).

Jegadeesh et al. (2004) show that that level of consensus recommendation adds value only amongst stocks with favorable quantitative characteristics but is otherwise orthogonal to existing factors that predict stock returns. Their evidence suggests that the predictive power of the analyst forecasts is largely driven by exposure towards momentum stocks. Recommended stocks appear to be highly correlated with momentum indicators, whereas the consensus recommendation contradicts the expected return of the contrarian strategies i.e. contradicting the normative direction expected by literature. They find that the change or revision in analyst recommendation is more robust than the level of recommendations hence upgraded stocks outperform downgraded stocks. The reason for this being that the changes are less affected by the growth-bias compared to the level variable. Another reason is that analyst recommendation changes capture a qualitative component of stock-returns. Jegadeesh & Wooij (2006) document that characteristics-based-portfolio that takes into account analyst recommendation changes earns above-average returns beyond the inclusion of other predictive variables. This effect is persistent across a wide range of countries. Complementary to this Barber et al. (2010) find that average abnormal returns increase with both positive recommendation changes and the recommendation level.

More recent research argues that target-prices rather than recommendations are more informative to investors. Forecasts of price-target are different from analyst recommendations in that they are continuous, are more frequently revised, and have well-defined investment horizons, and therefore more informative (Bradshaw et al., 2013). Da & Schaumburg (2011) support this notion by arguing that target prices reflect a direct measure of fundamental value perceived by analysts rather than recommendations or earnings forecasts. Da & Schaumburg (2011) find evidence that a (within-industry) portfolio sorted on target price implied returns, yields an abnormal excess return of 212 bps a month (t=5.27) while controlling for size, value, and momentum and 210 bps (t=2.72) controlling for the characteristics of Daniel & Titman (1997). Cross-sectional regressions show that target prices have a predictive ability to forecast one-month-ahead returns with a coefficient of 31 bps (t=2.41) incremental to size, value, and profitability factors. These findings show that analyst price-target forecasts are distinct from the previously described analyst forecasts such as analyst recommendations. Bilinski et al. (2013) use an international set of data from over 16 countries from 2000 till 2009 and argue that analysts have the persistent and differential ability to issue accurate target prices. Specifically, they find that close to 59.1% of the target prices are met during the forecasting period while close to 44.7% of the target prices are met at the end of the forecasting horizon.

Chen, Da and Schaumburg (2015) incorporate target prices into their portfolio selection using the Black & Litterman (1992) methodology. Their findings show that investors can exploit information in analyst target prices and incorporate the information into their asset allocation decisions such that it yields a profitable portfolio that outperforms standard mean-variance optimized portfolios. Bradshaw et al. (2013) focus on analyst price targets over the period 2000 till 2009 and find weak evidence for the persistent differential abilities of analysts to accurately forecast target prices controlling for other variables such as lagged target price, momentum size, and book-to-market. They document that more optimistic target prices (i.e. a greater predicted return) exhibit greater forecast errors and are achieved less often during the forecasting period. Altogether it seems that analysts have somewhat of an ability to accurately and persistently forecast future stock prices by means of target prices. The reason that analyst target prices are continuous, provide valuable information according to recent literature, and are widely available is the reason I focus on target prices in the remainder of the thesis. The next section provides a perspective that opposes the idea of informative analyst forecasts

2.5.2. Analyst forecasts are uninformative

The second stream of literature disputes the ability of financial intermediaries of making accurate forecasts that are informative to investors. The biased analyst hypothesis suggests that analyst recommendations are a potential source of market frictions that contribute to sustained mispricing (Guo, Li & Wei, 2020). If this is the case, then analyst forecasts may contribute to the anomaly returns instead of resolving inefficiencies.

Bonini, Zanetti and Salvi (2010) examine the effectiveness of target prices to anticipate future market prices efficiently and find that forecasting errors are not distributed randomly and are significantly different from zero. Moreover, the prediction errors exhibit positive autocorrelation that suggests a systematic upward bias of forecasted prices. The forecasting error appears to increase with the target price implied return and the market capitalization of the firm. This suggests that analysts are biased mediators. Dechow & You (2019) examine both the bias and inaccuracy of target prices and identify various sources that contribute to (in)accuracy of such analyst target-prices. The results show that the bias related to the assessment of risk-factors contributes more to the overall forecasting error of target price implied returns than do job-related incentives or errors in the forecasting of firm-fundamentals. That is, analysts tend to overestimate returns from riskier stocks and herewith introduce noise to the target-price estimation. Specifically, using regression analyst they show that the long-term growth forecasts bias only explains for around 4.17% of the variation in target implied returns, job-related incentives account for 6.79% of the cross-sectional variation, and information regarding future stock returns for about 4%. Finally, risk-proxies account for around 17% of the variation in target-price returns and around 10.51% of target price forecast errors. Furthermore, they find that the relationship between target-price-implied-returns and future stock returns is weakly positive in predicting one-month ahead stock returns and weakly negative over 6- to 12-month horizons. This evidence suggests that information from target price implied returns is quickly reflected in stock price and thereafter conveys very little useful information in predicting long-term stock prices.

Engelberg et al. (2019) hypothesize that analyst forecast could uncover mispricing and offers a potentially powerful tool for investors in reaping returns associated with these inefficiencies. Therefore they suggest that analysts' target prices have predictive power for stock returns. However, Engelberg et al. (2019) and Guo et al. (2020) find that analysts largely ignore anomaly-related information in forming their forecasts which leads to contradicting return-forecasts predicted by the anomaly variables versus the forecasted target-price-implied-return. Similar to Jegadeesh et al. (2004) the analyst change in target price, in contrast to the absolute level, does appear to be informative and reflect information embedded in anomaly variables (Engelberg et al., 2019). Similar to Bonini et al. (2010) both studies find that the target-return forecasts are systematically biased upwards. Analysts tend to give more favorable recommendations to stocks classified as overvalued that will then earn below-average returns in the subsequent periods (Guo et al., 2020). Guo et al. (2020) conclude that in contrast to before, there is no return predictability for the level of analyst consensus recommendations. Altogether it appears the consensus on the informative value of target prices is mixed. Analyst target prices seem weakly associated with above-average abnormal returns, at least in the short term.

2.5.3. Analyst forecast dispersion

Forecast dispersion could reflect investor uncertainty concerning firm fundamentals. Therefore forecast dispersion may serve as a proxy for uncertainty around actual firm-value (Ramath & Shane, 2008). Diether et al. (2002) document a significantly negative relation between forecast uncertainty, as measured by dispersion in consensus estimates, and future stock returns while also finding a positive relation between analyst dispersion and past return variability and market beta. The negative sign of the former relationship stands in contrast with the notion that investors discount uncertainty and demand a premium for bearing greater risk. Diether et al. (2002) attribute the negative relationship between forecast dispersion and subsequent returns due to mispricing and differences in opinion between analysts rather than being a proxy for risk. Johnson (2004) reconciles the previous finding with the notion of efficient markets and suggests that the negative relation is a manifestation of the non-systematic risk that arises when asset values are unobservable, which could be the case when a firm carries excessive leverage. This information risk is unpriced and therefore expected returns will always decrease with greater idiosyncratic risk. Palley et al. (2019) find evidence that consensus target prices are useful in predicting future stock return if analysts are in agreement. Moreover, they find that future returns decrease with increasing consensus target price dispersion. They document a persistent four-factor alpha of 213bps (t=4.48) and a five factor-alpha of 139bps (t=2.89).

Da & Schaumburg (2011) assert that cross-sectional dispersion in target price serves as a proxy for liquidity-risk and hence implied dispersion could reflect differences in liquidity across stocks caused by information asymmetries. This leads to firms with greater forecast dispersion to earn greater returns due to the exposure to an aggregate liquidity risk factor for which investors demand a premium. Finally, consistent with the risk-based hypothesis Feng & Yan (2016) document a significantly positive relationship, throughout the period from 1999 till 2013, between target price dispersion and future stock returns for the next month while controlling for the size, book-to-market, momentum, and idiosyncratic volatility. The relationship is both economically significant as well as statistically significant and remains persistent up to a period of 24 months. They conclude that analyst target price dispersion can serve as a proxy for future idiosyncratic risk instead of being a measure of differences in opinion by analysts as argued by Diether et al. (2002). Concluding it seems that the literature surrounding analyst target prices is limited and has not yet reached a consensus whether consensus analyst target prices do in fact provide new valuable information. This thesis will fill a gap by providing new insights into the value of such forecasts to investors in the portfolio selection problem.

3. Methodology

3.1. Introduction into portfolio optimization

The methodology section consists of five sections. First I discuss the model that I use to optimize the portfolio. Second I discuss empirical design, third I discuss the performance measurement. Finally, I discuss the data and the construction of the variables. The next section presents the methodology that is used to construct the portfolio weights following Brandt et al. (2009). The methodology allows an investor to estimate the optimal weight in each asset as a function of both the stock characteristics (cross-sectionally standardized) and the equilibrium market-weighting. The optimal parameters of the model are estimated by maximizing the average utility that the investor would have obtained by implementing the policy over the sample period. By parameterizing the weights upon the stock characteristics, I avoid modeling the joint-distribution of returns and the oncoming statistical problems. The intuition behind the model is that an investor holds the efficient market portfolio and only deviates if the respective characteristics provide an attractive combination of risk and return across stocks and across time such that it increases the utility of the investor (Brandt, et al., 2009). By tilting towards these characteristics via either long or short exposure the investor optimally seeks to maximize her utility given his or her pre-specified risk tolerance. The risk tolerance is set arbitrary, this parameter allows us to test whether the performance of the optimal portfolio is persistent across investors with varying risk-aversion levels. The portfolio that I consider is fully invested in risky assets that implies that the sum of the benchmark weights and the active exposure always sum to one. That is, the total overweighting and underweighting to stocks is always equal to one hundred percent. I optimize the portfolio for an investor with CRRA-preferences to accommodate for the higher moments of returns, such as skewness and kurtosis.

3.2. The optimization model

Suppose a general portfolio is depicted by a vector of weights, $\omega_{i,t}$, with a return of $r_{p,t+1}$. Let N_t be the number of investable stocks at that particular point in time, *t*, that is allowed to vary over time. Then $r_{i,t+1}$ reflects the return of asset *i*, over the period *t* till *t+1*. Then the following equation (1) describes the respective portfolio return over the holding period:

$$r_{p,t+1} = \sum_{i=1}^{N_t} \omega_{i,t} * r_{i,t+1} \tag{1}$$

In the optimal portfolio the portfolio weights, $\omega_{i,t}$, are parameterized as a linear function of the characteristics. This is specified in equation (2).

$$\omega_{i,t} = \bar{\omega}_{i,t} + \frac{1}{N_t}\left(\theta^T \bar{\chi}_{i,k,t}\right) \tag{2}$$

Where, $\bar{\chi}_{i,k,t}$ is the vector of firm characteristics for firm *i*, characteristic *k*, observed at time *t*, standardized cross-sectionally to have a mean of zero and a unit standard deviation across all stocks at time *t*. Then, $\bar{\omega}_{i,t}$, reflects the weight in the benchmark portfolio (value-weighted) from which we choose to deviate or not. $\frac{1}{N_t}$, reflects a scaling term that allows for a time-varying number of assets in the portfolio and makes sure that the sum of the deviations always equal to one. The normalization by 1/N ensures that an increasing number of stocks still carries the same cross-sectional allocation to the characteristics. Without the normalization an increased number of stocks would result in larger average weights, meaning more aggressive tilts. Equation 2 reflects the notion of active portfolio management, by which the intercept reflects the weight of the asset in the benchmark. The right-hand side of equation (2), $\frac{1}{N_t}\left(\theta^T \bar{\chi}_{i,k,t}\right)$, illustrates the active tilt of the portfolio which is the extent to which the optimized (or optimal) portfolio deviates from the market portfolio. This tilt reflects an active over *or* under-weighting depending on the prevailing stock characteristics. The vector of characteristics, $\bar{\chi}_{i,k,t}$ is standardized as this ensures that the cross-sectional distribution of the characteristics is stationary across time. Second, the standardization ensures that the deviation from the benchmark portfolio always sums to 1. Then θ, *theta*, is a vector of coefficients that follows from the estimation procedure and reflects the weight that the characteristics receive in the parametric portfolio. In loose terms, the theta coefficient could be referred to as the loading or exposure of the portfolio towards certain characteristics, while this is true, it should *not* be confused with the loading or coefficients obtained from the regression analysis that reflects exposure to certain style factors. Combining the previous terms I can rewrite the return of the optimal portfolio in the subsequent period as equation (3:

$$r_{p,t+1} = \sum_{i=1}^{N} (\bar{\omega}_{i,t} + \frac{1}{N_t}(\theta^T \bar{\chi}_{i,k,t}) r_{i,t+1} \tag{3}$$

The investor faces the following problem (4) to choose the weights, $\omega_{i,t}$, that maximizes the conditional expected utility of the portfolio's return $r_{p,t+1}$ as follows:

$$\max_{\theta} E[\,u(r_{p,t+1})] = \max_{\theta}\ E\left[u\left(\sum_{i=1}^{N} \omega_{i,t} * r_{i,t+1}\right)\right] \tag{4}$$

Which can be rewritten as follows (5):

$$\max_{\theta} E[\,u(r_{p,t+1})] = \max_{\theta}\ E\left[U\left(\left(\bar{\omega}_{i,t} + \frac{1}{N_t}\theta^T \bar{\chi}_{i,k,t}\right) r_{i,t+1}\right)\right] \tag{5}$$

To estimate the portfolio weights in each period I will apply dynamic optimization meaning that I re-estimate the parameters every period. Note that in order to compute the portfolio in period *t+1*, the characteristics from the previous period, *t*, are used. The dynamic optimization applies a 'rolling horizon' which means that the estimation window rolls forward every period with one period equaling one month (DeMiguel, et al., 2013). For every subsequent optimization, the estimation window also rolls forward one period thus creating a different in-sample training set until the final period of the sample. The optimization is re-estimated each month which is the equivalent to an investment strategy that rebalances monthly. The estimation window (or sample window) that I choose has a length of τ=30 monthly observations which is close to 2.5 half years. That is approximately equal to the length that Fletcher (2017) uses. The intuition is that at the end of every period, I re-estimate the portfolio with all information covered by the estimation window. The total sample period, *T*, counts 240 periods. This means that every optimization uses the previous 2.5 years of monthly data which is rolled forward at every next optimization for a total of T$-\tau$ periods resulting in 209 rolling period returns.

The conditional expected utility from equation (5) translates into the following unconditional expected utility (6) for the utility function (U).

$$\max_{\theta} E[u(r_{p,t+1})] = \max_{\theta^T} \frac{1}{T} \sum_{T-\tau}^{T} U\left(\sum_{i=1}^{N_t} \left(\bar{\omega}_{i,t} + \frac{1}{N_t} \theta^T \bar{\chi}_{i,k,t}\right) r_{i,t+1}\right) \quad \textbf{(6)}$$

$$\bar{\omega}_{i,t} = \frac{ME_{i,t}}{\sum ME_{i,t}} \quad \textbf{(7)}$$

As described, $\bar{\omega}_{i,t}$ represents the starting point of our active strategy which is the initial benchmark portfolio from which we choose to deviate, or not, depending on the cross-sectional variation in characteristics and their relation to risk and return. Equation (7) shows how the value-weighted market portfolio, $\bar{\omega}_{i,t}$, is constructed in which, *ME*, represents the market-equity of firm *i*, at time *t*. Then the benchmark weights is simply the market-capitalization of a firm divided by the sum of all market capitalizations. This setup is chosen such that the number of firms in the portfolio may vary according to the available data. Choosing any other benchmark would result in unfeasible portfolios due to different constituents across times. Lohre and Hammerschmid (2017) state that the parametrization implicitly assumes that the chosen characteristics span the joint distribution of returns that are relevant for the portfolio optimization. This implies that the tilts that I find apply to the complete cross-section of assets over time, rather than being different for each individual assets.

The investor's utility function I use is one of Constant relative risk aversion (CRRA) as illustrated in (8). Where γ is the risk-aversion level specified. The advantage of using this particular utility function is that it also takes into account the higher moments of the return distribution such as kurtosis and skewness. The portfolio utility function is very flexible and can incorporate a variety of objective functions such as loss aversion, power utility, quadratic utility, while also objective functions such as maximizing the Sharpe ratio or maximizing the information ratio can be incorporated.

$$u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\gamma}}{1-\gamma} \quad \textbf{(8)}$$

3.2.1. No short-sale restriction

I employ a weight constraint that I impose to limit the leverage or amount of short sales that the portfolio is allowed to take on. This is the equivalent of a no-short sales restriction that some investors potentially have to adhere to. I limit the leverage by applying a penalty to the utility function. This procedure limits leverage up until a prespecified level, r*. $\propto$ is a positive number that enforces the restriction to be satisfied. The penalty is defined as shown below. Going forward I *always* restrict the portfolio leverage up to 100% (or 1) unless otherwise specified, such as in the case without short sales, in which the leverage is set to zero. By restricting the leverage to 100% the results of the portfolio optimization are more stable and reliably over time. Moreover, a 100% leverage is still feasible for parties that are allowed to assume short positions. I follow Brandt et al. (2009) and Fletcher (2017). Optimized portfolios that allow for short-sales (but still restricted to 1) will be denoted with *SS*, while portfolio that completely prohibits the use of short sales will be denoted with *NSS* (no-short-sales)

$$penalty = \propto \sum \left| \omega_{i,t}^{>r*} \right| \tag{9}$$

This translates into the following optimization problem depicted in (10). This simply incorporates the penalty from above into equation (6). As a result, a greater penalty due to greater leverage reduces the utility that an investor receives.

$$\max_{\theta} E[\,u(r_{p,t+1})] = \max_{\theta^T} \frac{1}{T} \sum_{T-\tau}^{T} u\left(\sum_{i=1}^{N_t} \left(\overline{\omega}_{i,t} + \frac{1}{N_t} \theta^T \bar{\chi}_{i,k,t} \right) r_{i,t+1} \right) - \propto \sum \left| \omega_{i,t}^{>r*} \right| \tag{10}$$

3.3. Portfolio construction

I will perform the optimization in several steps in which the number of characteristics included in the optimization differs per optimized portfolio. The structure of the portfolios is outlined in table 1. below. First I optimize two single characteristic portfolios individually using both the target-price-implied return and target-price-implied dispersion in a separate portfolio. Then I optimize the portfolio using the traditional characteristic from literature in order to obtain the baseline portfolio to which I can compare. To see the incremental value of the characteristics TIMR and TIMR I incrementally add these to the portfolio that includes the traditional, size, and book-to-market characteristics, the profitability and investment characteristic of French and Fama (2015) in combination with the Carhart momentum factor. Here I follow Lamoureux and Zhang (2014). This procedure will be followed for both short-sales as well as short-sale restricted portfolios. To conserve space in the text I have not reported the results from all incremental steps of adding the traditional characteristics to the optimal portfolio, these full tables can be found in the appendix and are referred to as the partial portfolios. These partial portfolio show the effect of incrementally adding a single characteristic. These partial portfolios are not reported unless there exist meaninfull differences.

Table 1. Empirical design

Table 1. presents the various portfolio strategies that will be optimized and describes what characteristics will be included in the optimization. The first two singleton portfolios include one single analyst characteristic. The remainder of the portfolios includes multiple characteristics starting with the (1) traditional portfolio followed by (2) the portfolio including target-price-implied return (3) and the portfolio including target price implied return *and* target price implied dispersion

Portfolio No.		Characteristics included in the optimization	Abbreviation
Singleton characteristics			
	(1)	Target price implied return	TIMR
	(2)	Target price implied dispersion	TIMD
Multiple characteristics			
Traditional	(1)	Size, Book-to-market, Momentum, Profitability, Investment	ME, BTM, MOM, PROF, INV
Extended	(2)	Size, Book-to-market, Momentum, Profitability, Investment, TIMR	ME, BTM, MOM, PROF, INV, TIMR
Extended	(3)	Size, Book-to-market, momentum, profitability, Investment, TIMR, TIMD	ME, BTM, MOM, PROF, INV, TIMR, TIMD

Table 2. Performance metrics

The table presents the various performance metrics used. i, is used to denote a firm and t is used to denote time. $\bar{\omega}_{i,t}$, hence reflects the benchmark weight of firm, i at time t. $\omega_{i,t}$, reflects the weight of firm, i at time t. *N*, denotes the total number of firms in the sample. R_p reflects the portfolio time-series annualized return. $\bar{R}p$, reflect the portfolio time-series average return. r_f, is the prevailing risk-free rate. σ_p^2 reflect the portfolio time-series standard deviation. y, is the risk-aversion parameter set at 5 unless otherwise stated. R_b denotes the benchmark portfolio. The scaling term is equal to the number of months in a year which is 12. F, represents the cumulative distribution over the optimization horizon. The return-threshold, rt, is set to be the return of the value-weighted benchmark portfolio. MAR reflects the minimal accepter return which is set equal to 0. δ_{MAR}, is the portfolio downside deviation over the period. A_t, is a vector of weights.

Measurement	**Abbreviation**		**Measurement of variable**
Value-weight benchmark	*VW bmk*	Brandt et al. (2009)	$R_{VW,t} = \sum_{i=1}^{N} \bar{\omega}_{i,t} * r_{i,t}$
Equal-weight benchmark	*EW bmk*	DeMiguel et al., (2009), Fletcher, (2017)	$R_{EW,t} = \sum_{i=1}^{N} \frac{1}{N_{i,t}} * r_{i,t}$
Sharpe Ratio	*SR*	Sharpe (1994)	$\frac{Annual\ excess\ return}{Annual\ St.dev.} = \frac{\sqrt[t]{\left(1+(R_p-r_f)\right)^{12}}-1}{\sqrt{12} * \sqrt{\frac{\Sigma(R_p-\bar{R}p)^2}{T-1}}}$
Information Ratio	*IR*	Goodwin (1998)	$\frac{Annual\ active\ return}{Annual\ tracking\ error} = \frac{\sqrt[t]{\left(1+(R_p-R_b)\right)^{12}}-1}{\sqrt{12} * \sqrt{\frac{(\Sigma R_p-R_b)^2}{T-1}}}$
Sortino Ratio	*ST*	Sortino & Price, (1994)	$\frac{(R_p - MAR)}{\delta_{MAR}} = \frac{(R_p - MAR)}{\sqrt{\sum_{t=1}^{n} \frac{min[(R_p - MAR, 0)]^2}{t}}}$
Omega Ratio	*Omega*	Shadwick & Keating (2002)	$Omega = \frac{\int_r^{\infty}(1-F(x))dx}{\int_{-\infty}^{rt} F(x)dx}$
Certainty equivalent return	*CER*	Goto & Xu, (2015), Fletcher, (2017),Lamoureux & Zhang, (2014)	$CER = R_p - \left(\frac{y}{2}\right)\sigma_p^2$
Hefindahl-Hirshman-index	*HHI*	Goetzmann & Kumar, (2008), Goto & Xu, (2015)	$HHI = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} \omega' \omega_{i,t}$
Turnover	*TN**	Ammann et al. (2016), DeMiguel et al. (2009) Fletcher, 2017	$TN = \sum_{i=1}^{N} \lvert \omega_{i,t+1} - \omega_{i,t} \rvert$
Proportional transaction costs.		Brandt et al. (2009), DeMiguel et al. (2020)	$z_{i,t} = A_t(0.006 - 0.0025 * ME_{i,T}),$
Transaction Costs	*TC*	See *TN**	$TC = \lvert \omega_{i,t+1} - \omega_{i,t} \rvert * z_{i,T}$
Total monthly Turnover	*TTN*	See *TN**	$TTN = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} \lvert \omega_{i,t+1} - \omega_{i,t} \rvert$
Total monthly Transaction Costs	*TTC*	See *TN**	$TTC = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} TC_{i,t}$

3.4. Performance measurement

After the portfolio optimization, I will measure the performance of the optimal portfolio in a comprehensive manner leaving very few stones unturned. The performance metrics that I include are described in table 2. I include the annualized return, the standard deviation of returns, Sharpe ratio, the information ratio, Sortino ratio, Omega ratio, the Certainty equivalent return, the Herfindahl-Hirshman-index, skewness (third moment) and the kurtosis (fourth moment) of the returns. All metrics are annualized unless stated otherwise. First using the time-series of monthly out-of-sample returns generated by the optimized portfolio, I calculate the portfolio annualized returns, standard deviations, skewness, kurtosis for the respective period. The benchmark portfolio weights, $\overline{\omega}_{i,t}$, as presented in Table 2 should be chosen carefully as these represent the market portfolio weights from which the optimized portfolio will deviate. These weights also define the value-weighted benchmark portfolio. In the value-weighted portfolio, a firm carries a weight proportional to the firms' respective market capitalization. In the equal-weighted portfolio, each firm carries an equal-weight depending on the number of firms in the portfolio for that specific point in time. Both portfolios are attractive portfolios to invest in as they represent a fairly diversified portfolio by construction. The weights in the benchmark portfolios change when the respective market capitalizations change or when firms enter or exit the sample. Fama & French (2008) stress that microcaps are influential drivers of returns in equal-weighted, as they represent on average 3% of the market value but account for about 60% of the total number of stocks. This might lead to an unrepresentative picture of the average cross-sectional return and the factor exposure. DeMiguel et al. (2009) conclude that in order to evaluate the performance of a particular strategy for optimal asset allocation the equal-weight portfolio should always be used as a benchmark portfolio. Therefore to mitigate the size bias and provide a complete comparison, I choose to include both portfolios as a benchmark. Both portfolios are rebalanced monthly.

The Sharpe ratio (*SR*) is calculated by dividing the annualized mean monthly excess return by the annualized monthly standard deviation of the excess returns (Sharpe, 1994). The metric is scaled towards an annual measure. The Sharpe ratio shows the return that the portfolio has earned per unit of risk in excess of the risk-free rate. This metric provides insight into the return in comparison to the total risk that the investor faces. The information ratio (*IR*) is calculated by dividing the active return by the tracking error. The active returns is calculated as the annualized excess returns that the optimal portfolio earns over the benchmark portfolio. The benchmark portfolio in this case is the value-weighted portfolio. The tracking error is calculated by calculating the average squared deviation of returns between the optimized portfolio and the benchmark portfolio. This metric is then annualized by multiplying it with the square root of 12 (Goodwin, 1998). The information ratio indicates with what consistency the optimal portfolio has over or underperformed the benchmark over the estimation period. The difference between the Sharpe ratio and Information ratio is that the Sharpe ratio considers the total risk while the information ratio only considers the excess risk relative to the benchmark. The next metric is the Sortino ratio, *ST*, in which *MAR*, reflects the minimum acceptable return over the optimization horizon, which is set, 0. This reflects the fact that risk is only associated with bad outcomes, specified as returns below the *MAR*. This separates the good variance that lies above the MAR from the unwanted variance, specified as variance from returns that lie below the MAR (Sortino & Price, 1994). The Omega function takes into account the complete set of information by considering all moments of the return-distribution. The metrics take into account the whole return distribution including the information in the tails and therefore provides a powerful characterization of the risk-reward trade-off of the distribution. The function allows to rank and compare returns of different portfolios, in which a higher ratio is better. The function is defined following Shadwick & Keating (2002), where *F,* represents the cumulative distribution over the optimization horizon. The return-threshold, *rt*, is set to be the return of the value-weighted benchmark portfolio. The certainty equivalent return (*CER*) can be interpreted as the risk-free rate at which the investor is indifferent between holding investing in the risk-free rate or holding the risky portfolio. A higher CER indicates that the portfolio has more desirable risk-return characteristics from a mean-variance perspective (Goto & Xu, 2015). This measure is defined following Fletcher (2017).

Turnover (*TN*) is calculated as in Fletcher (2017) and DeMiguel et al. (2013). The average turnover per holding period (*TTN*) is calculated as the absolute average percentage of wealth traded at every portfolio rebalancing period. The absolute wealth traded in each period is defined as the absolute difference in portfolio weights between the rebalancing dates. A high average monthly turnover reflects a large portion of the portfolio being sold and rebought in the underlying asset at the rebalancing date which also translates into greater transaction costs. Following Brandt et al. (2009), Ammann et al. (2016), and DeMiguel et al. (2020) I let the transaction costs be proportional to a firms' market capitalization. First I define the proportional transaction costs parameter as: $z_{i,t} = A_t(0.006 - 0.0025 * ME_{i,T})$, where $ME_{i,T}$ is the market capitalization of a firm, i, at time T, normalized by the maximum market capitalization across all firms at that period. A_t is a vector that reflects the decreasing transaction costs over time in a linear manner, ranging from 5 in January 1980 to 1.0 in January 2002, after which it remains equal to 1.0. The transaction costs are then calculated as the proportional transaction costs multiplied by the absolute average wealth traded every period. Finally, the total monthly transaction costs associated (TTC) with implementing the portfolio on a monthly basis are calculated as the average of the sum of the monthly transaction costs.

The portfolio diversification is measured using the Herfindahl-Hirschman index (HHI). The metric is calculated as the sum of the squared market-capitalization (Goetzmann & Kumar, 2008; Goto & Xu, 2015). The index takes the lowest value when all assets receive equal weightings such as in an equal-weight portfolio (1/N) alternatively the index takes larger values when the variability of weights across the assets increases and the concentration risk increases as a consequence.

Moreover, I monitor the third and fourth moments of the return-distribution. These provide an insightful characterization of the return distribution that is contingent upon the investor utility function that specifies the investor preferences for specific moments. Monitoring these moments is especially important if we relax the assumption that investors have CRRA preferences. Brandt & Aït-sahali (2001) Show that preferences for different moments between CRRA investors and equally risk-averse mean-variance investors do explain differences in stock-holdings between the two. Skewness is the third moment that describes the (a)symmetry of the return distribution. Investors generally prefer positively skewed return distributions vis a vis negatively skewed distributions indicating they prefer to hold assets that carry a greater probability of providing above-average returns at the cost of lower mean-variance efficiency. Skewness can serve as a proxy for jump-risk in which there is a larger probability of earnings either very large positive or negative returns. That means holding a portfolio with large positive skewness resembles a portfolio providing a lottery-like type payoff, with returns being either close to zero or being extremely positive. Equivalently a negatively skewed return distribution yields a higher probability of earnings large negative returns relative to positive returns holding all other factors constant. Investors tend to accept a greater variability of returns in exchange for greater positive skewness (Arditti, 1970; Kraus and Litzenberger, 1976). Mitton & Vorkink (2007) show that investors trade-off diversification in order to hold a portfolio with greater average positive skewness. The fourth moment, kurtosis measures the probability mass in the tails of the distribution relative to its overall distribution. A higher probability mass in the tails of the distribution reflects a greater probability of obtaining extreme returns. In general, investors are assumed to be kurtosis averse hence avoid investing in assets that provide a large uncertainty of extreme events that is investors dislike investing in assets with greater probability mass in the tails. Xiong and Idzorek (2011) show that investors with mean-variance preferences in general prefer a return-distribution with smaller kurtosis.

Moreover, I report two metrics related to the downside risk which are the VaR (different from the var, variance) and the CVaR using a confidence level of 97.5%. The former is the value-at-risk that reflects the potential loss for a given probability of 97.5%. The latter refers to the Conditional value-at-risk or the expected shortfall (ES) that reflects the conditional expectation of loss given that the loss is beyond the VaR given a certain probability. The optimization takes into account higher moment preferences a result the mean-variance-based metrics are no longer adequate in reflecting the empirical risk-return trade-off. When the returns deviate from normality the gaussian VaR and CVaR (ES) become less reliable as these metrics assume normality (Boudt et al., 2008; Xiong and Idzorek, 2011).

Therefore, in order to account for the potential non-normality of the return distribution, I use the modified VaR and CVaR risk-measures (See Boudt et al. (2008 for more information). For moderate values of skewness and kurtosis, the modified versions are good approximations of the gaussian metrics. When the return distribution becomes more negatively skewed the modified VaR and CVaR tend to be too pessimistic and Gaussian VaR and CVaR are too optimistic (Boudt, et al., 2008). The reverse also holds true as the data is positively skewed. In that case, the gaussian VaR and CVaR become to pessimistic whereas the modified ones become to optimistic. To ensure consistency, I check both the confidence level and the lower bound of the skewness. The lower bound of the confidence interval is limited by investor preferences for kurtosis while the upper bound is constraint by the value of skewness[3] (Cavenaile & Lejeune, 2012). The reporting level of 97.5% that I choose falls within the appropriate range and is, therefore, an admissible level for providing consistent results across the portfolios.

Table 3. Regression design

The table presents the regression design used to evaluate the optimized portfolios. $R_{p,t}$, reflects the return of the optimized portfolio. $R_{EW,t}$ and $R_{VW\,,t}$, reflect the equal and value-weighted benchmark respectively. *MKT* denotes the market factor, *HML* is the value factor, *SMB* is the Size factor, *RMW* is the profitability factor, *CMA* is the Investment factor finally, *UMD* is the momentum factor. r_{ft} is the prevailing risk-free rate.

	Regression equation	Literature
(a)	$R_{p,t} = \alpha_p + \beta_p(R_{EW,t}) + \varepsilon_{p,t}$	(Jensen, 1968)
(b)	$R_{p,t} = \alpha_p + \beta_p(R_{VW\,,t}) + \varepsilon_{p,t}$	(Jensen, 1968)
(c)	$R_{p,t} - r_{ft} = \alpha_p + \beta_1(MKT_t - r_{ft}) + \beta_2(HML_t) + \beta_3(SMB_t) + \beta_4(RMW) + \beta_5(CMA) + \beta_6(UMD) + \varepsilon_{p,t}$	Fama & French (1992, 1993,1998, 2008, 2015) ; Carhart (1997), Lamoureux & Zhang (2014), Lewellen (2015)

In order to gauge the performance of the optimal portfolios, I perform a regression analysis against the benchmark portfolios and the risk factors identified by the literature. This regression design is discussed in table 3. The first regression **(a)** regresses the returns of the optimal portfolio against the returns of the equal-weighted benchmark: where R_p are the time-series returns of the optimal portfolio. The second **(b)** regression regresses the returns of the optimal portfolio against the time-series returns of the value-weighted benchmark. This follows the procedure of Brandt, et al. (2009). Thereafter I perform a regression analysis in order to examine the style (factor) exposure of the portfolio and in order to see whether the returns of the portfolio are fully captured by the traditional risk factors. This regression is depicted in equation **(c).** The dependent variable in this regression is the optimal portfolio return in excess of the risk-free rate. Next $MKT - r_f$ refers returns of a value-weighted market portfolio in excess of the risk-free rate, *HML* (High-Minus-Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. *SMB* (Small Minus Big) is the average return on the

[3] The minimum skewness at which the mVaR is reliable is around -7.6 at the 95% confidence level while it is equal to -0.98 at the 99% confidence level.

three small portfolios minus the average return on the three big portfolios. *RMW* (Robust minus Weak) is the average return on the two portfolios with strong operating profitability minus the average return on the two portfolios with lower operating profitability. *CMA* (Conservative minus Aggressive) is the average return on the two portfolios with a conservative investment policy minus the average return on the two portfolios with an aggressive investment policy. Finally, *UMD* (momentum) is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios (Fama & French, 1992). R_{EW} and R_{VW} refer to the equal-weighted and value-weighted benchmark respectively. Both are constructed from the firm-available observations in the dataset. The null hypothesis states that the performance of the optimized portfolio is equal to the performance of the benchmark portfolio hence $H_0\colon \alpha_{portfolio} = 0$. The t-value is for the evaluating the hypothesis that the optimal portfolio performance is no better than the benchmark portfolio performance. A significant t-statistic will lead me to reject the null hypothesis that the performance is no better than the benchmark (DeMiguel et al., 2013).

3.5. Data & variables

3.5.1. Overview

The data for the optimization comes from three different sources: (1) the fundamental accounting information is derived through Compustat, (2) End-of-the-day stock price information and simple returns are sourced from the Center for Research in Security Prices (CRSP) database accordingly (3) the analysts' target prices are obtained from IBES. Both CRSP, Compustat, and IBES are accessible via the WRDS database. The sample period is from ranges from January 2000 till December 2019. The starting date is restricted by the availability of IBES data through WRDS. The Fama-French five factors (MKT, SMB, HML, CMA, and RMW) and the additional momentum factor (UMD) to control for risk-factors are obtained through the online data library of K.R. French. The presence of microcaps can result in an unrepresentative picture of the cross-sectional characteristics in the sample (Fama & French, 2008). As a hurdle, I restrict the dataset to only include firms above the NYSE 20th percentile of market-capitalization measured (Ammann et al., 2016; Lewellen, 2015). This ensures microcaps are excluded from the sample. The final sample consists of 347.145 firm-month observations. The next sections will address the different databases finally section 3.6 describes the construction of the individual variables. The merging process of the three databases is described in the appendix.

3.5.2. Compustat

Fundamental accounting data to calculate the stock-characteristics are obtained from Compustat (North America). The following variables are obtained through WRDS Compustat with their respective item codes in between brackets. These accounting metrics are obtained quarterly, any observations in the intermediate months between the quarters are obtained by carrying forward the most recent observation. (1) Total assets (atq), (2) Interest and related expense (xintq), (3) Total liabilities (ltq), (4) Selling, general and administrative expenses (xsgaq), (5) Stockholders' equity (seqq), (6) Income before extraordinary items (ibq), (7) Preferred stock (redeemable) (pstkrq), (8) Income taxes (txdiq), (9) Preferred stock total (pstkq), (10) Deferred taxes and investment tax Credit (txditcq), (11) Cash and short term investments (cheq), (12) Deferred taxes (txdbq), (13) Current liabilities (lctq), (14) Total revenue (revtq), (15) Debt in current liabilities (dlcq), (16) Costs of goods sold (cogs), (17) Income tax payable (txpq), (18) Depreciation and amortization (dpq), (19) Earnings per shares excluding ext. (epspxq), (20) Property plant and Equipment (ppegtq) and finally (21) Inventories (invtq) and finally (22) the company

3.5.3. CRSP

The CRSP Monthly stock files provide five different kinds of information. The first includes the complete name and identifying information that is the historical CUSIP, share class, and SIC codes amongst others. Second, the file contains the historical price and trading volumes of the respective securities traded on the NYSE, AMEX, and Nasdaq stock exchanges. Third, the file contains delisting information used to calculate the appropriate delisting returns. Fourth the file contains information concerning the distributions in the form of dividends, share buybacks, stock splits or other special distributions. Finally, the file contains the share outstanding values. The following variables are obtained: (1) date, (2) permanent security number assigned to all CRSP securities (Permno), (3) the permanent company identification number assigned to all CRSP companies with share issues (Permco), (4) the firms' CUSIP that is subject to name or changes in the capital structure (CUSIP), (5) Cumulative adjustment factor (cfacshr) (6), Price per share (prc), (7) Holding Period Return (ret), (8) Holding Period Return excluding dividends (retx) and finally the (9) the two-digit share code describing the types of shares that are traded (shrcd). I filter the shares to include only ordinary common shares with share codes 10 & 11. This ensures that only ordinary common shares incorporated in the US enter the sample, closed-end funds, REITS, and Trusts are excluded. Moreover, I restrict the sample to include firms listed to the NYSE, AMEX and NASDAQ with the exchange codes 1, 2, and 3 respectively (Lamoureux & Zhang, 2014; Fama & French, 2015; Da & Schaumburg, 2011) Figure 1. presents the distribution of firms across the beforementioned exchanges.

For the next step, I obtain the firm and security delisting file from WRDS. I construct the security's holding period adjusted return (retadj) using the firm holding period return (ret, adjusted for dividends and stock splits) and the firm's delisting return when the firm is or has been delisted in the past. CRSP updates the delisting returns on an ongoing basis up to ten years after the delisting, if needed (Beaver, McNichols & Price, 2006). The holding period return (ret) is the firm's last available return revised for cash or price adjustments such as dividends and splits. Both are simple returns. I use the firms' PERMNO identifier of the firms to match the delisting returns from the delisting file with the holding period returns from CRSP. The securities adjusted return has been calculated by comparing the value after delisting with the price at the latest trading date. The value that is observed after delisting is not necessarily equal to the last trading date price as it can include a price from another exchange or the total value of final distributions to the shareholders. The treatment of these returns is necessary as failing to do so leads to biased estimates of stock-returns and incorrect inferences (Beaver, et al., 2006). This de-listing adjustment ensures that the return on the final date of the firms listing is properly accounted for and minimized missing values on the last trading day.

3.5.4. IBES

From IBES the following items are obtained with their IBES in between brackets. (1) the firm 8-character CUSIP identifier (CUSIP), (2) the statistical period at which the estimate becomes effective (STATPERS), (3) the mean analyst price target forecast (MEANPTG). The mean estimate is calculated as the arithmetic average of all outstanding target price estimates for a particular statistical period. To study the effect of analyst dispersion I also obtain (4) the standard deviation of the target price (STDEV). IBES updates its estimates at every mid-month that is the Thursday preceding the third Friday of the month and hence new data becomes available every month. I treat the estimates as if they are end-of-the-month data and therefore by construction introduce a lag. The measure of target price that I use is the consensus target price provided by IBES. The consensus information obtained from IBES is likely to be the most representative information that is readily available to investors (Palley et al., 2019). The construction of the variables is further explained in section 4.5 and the accompanying table 4, to be found in the appendix.

3.6. Construction of the characteristics

Table 4, in the appendix, presents the construction of the variables in detail. First I derive the book-equity as depicted below. When one of the variables is not available I derive the book-value of equity by using the first item that is fully available for the firm. If the stockholder's equity is not available, I use the common equity (ceq) *or* total assets (atq) minus total liabilities (tlq). If deferred taxes and the investment tax credit is not available (txditcq), I use the deferred taxes (txdbq) plus investment tax credit (itccy) from the cash flow statement. If the total redeemable preferred stock is not available I use the Total preferred stock (pstkq). This approach follows the procedure of Fama and French (1992) by which I extract alternative items when the full line item is unavailable or missing. All accounting variables going forward are derived by using data from the last fiscal quarter ending at least four months ago following Hou et al. (2015, 2020). For example, if the second quarter fiscal data becomes available on 5 September (or 25 September) of year *t*, then the data from the first fiscal quarter data is used to construct the optimal portfolio at the beginning of the next period. For intermediate months I impute the book-equity forward based on book equity from the previous quarter. For example, the book value of equity is lagged one quarter to the most recent observation, this means that the lag of the book-equity is a least 4 months at the end of every quarter. Finally, I exclude all observations for which I observe a negative book value of equity for the respective year.

The size (*ME*) characteristic is constructed in line with Fama and French (1993) as the price (*PRC*) times the shares outstanding (SHARES). To calculate the market capitalization of the firm, I merge the securities with different PERMNO's but identical PERMCO's. i.e. if a firm has multiple listings I merge the market-equities into one and retain the largest PERMNO. The value characteristic (*btm*) is constructed in line Clifford and Frazzini (2013), who suggest that one period lagged prices provide more information than a 6-moth or annual lag in price in constructing valuation ratios. They show that this small adjustment can lead to significantly better portfolio performance. The variable is constructed by scaling the quarterly lagged book-equity by the one-period lagged market equity of the firm. The 12-month momentum return (M12) is calculated in line with Jegadeesh and Titman (1993). The cumulative past performance is calculated over the previous 12-months and excludes the front-month return

Gross profitability (GRPROF) is constructed in line with Novy-Marx (2013) and is defined as total revenue (revtq) minus Costs of goods sold (cogsq) scaled by total assets (atq). Again all variables are obtained every quarter and the intermediate months are filled forward. Gross profitability is lagged four periods to ensure that all information has been adequately reflected in the market. Novy-Marx (2013) suggest that gross profitability is a better proxy than current earnings employed by Fama and French (2006) as it represents a firms' true economic profitability. The items are a more informative measure of profitability as it includes fewer noise and is less affected by accounting decisions relative to items further down the income statement. Cashflow profitability (CFPROF) is constructed in line with Lakonishok et al. (1994) as total income before extraordinary items (ibq) plus income taxes (txdiq) plus depreciation and amortization (dpq) scaled by the one-period lagged market value of equity. Operating profitability or return of equity (OPPROF) is defined as revenues (revtq) minus cost of goods sold (cogsq) minus selling, general and administrative expenses (xsgaq) minus interests expenses (xintq) scaled by book equity from the previous quarter (Fama and French, 2015). Investment (INV) is constructed as the annual change in Property, plant and equipment (ppegtq) plus the annual change in inventories (invtq) scaled by the firms total assets (atq) (Chen et al., 2011; French & Fama, 2015). Increases (decreases) in property plant and equipment capture the capital investments (des-investment) in long-term assets employed for company operations. The change in inventories captures the investments in working capital employed that are used during the normal operating cycle such as materials, supplies, and other short-lived assets. Asset growth (ASTCHG) is constructed in line with

Cooper et al. (2008) as the year-on-year change in the change in total assets. Cooper et al. (2008) suggest that this variable subsumes various other characteristics proposed before. In fact, the measure appears to be the strongest variable documented in predicting the cross-section of equity returns. Asset growth retains its ability to predict returns also for large-cap firms, which other predictors lose much of their predictive ability.

The target-price-implied return that analysts provide is an ex-ante prediction of the performance of the stock over the coming twelve months. The metric is calculated as the mean analyst target price scaled by the previous period stock-price (Da & Schaumburg, 2011; Engelberg et al., 2019; Palley et al. 2019). The measure reflects a direct measure of analyst expectations of the future stock returns. As Feng and Yan (2016) note scaling by current stock prices leads to stock-return predictability by construction and therefore suggest using one period lagged stock returns. Consequently, I use a lagged price similar to the construction of the size characteristic. Implied analyst dispersion is constructed as the standard deviation of the consensus analyst forecast scaled by the lagged stock price (Diether et al. 2002; Da & Schaumburg, 2011; Palley et al. 2019). The full metrics are described in table 4. in the appendix.

4. Results

4.1. Descriptive statistics

Table 5. presents a summary of the descriptive statistics of characteristics that are included in the portfolio optimization. Figure 1. below provide a time-series overview of the number of firms in included in the sample. The number of firms in the sample varies depending on data that is available at that time, the maximum number of firms in the sample is 1814 firms, reached at September 2018. The minimum number of firms is equal to 724, at the start of the sample and grows steadily in the years thereafter except for the period surrounding the global financial crisis (end '07 '08). In the sample the average number of firms listed is at the NYSE is close to 53%, the average number of firms listed at the Nasdaq is close to 46% and finally, the number of firms listed at the AMEX is close to 1%. The figures are comparable to those of Da & Schaumburg (2011) that find that on average 54%, 43%, and 3% of their sample are listed on the NYSE, Nasdaq, and AMEX, respectively.

Figure 1. Number of firms in the sample across time

The figure below present a time-series overview of the number of firms in the sample across the various exchanges. The blue dotted line represent the total number of firms listed across all exchanges. The black line represent firms noted at the NYSE (exchange code 1), the red dotted line represents firms noted at the AMEX (exchange code 2), the green dotted line represents firms noted at the NASDAQ (exchange code 3). The sample period range from January 2000 till December 2019.

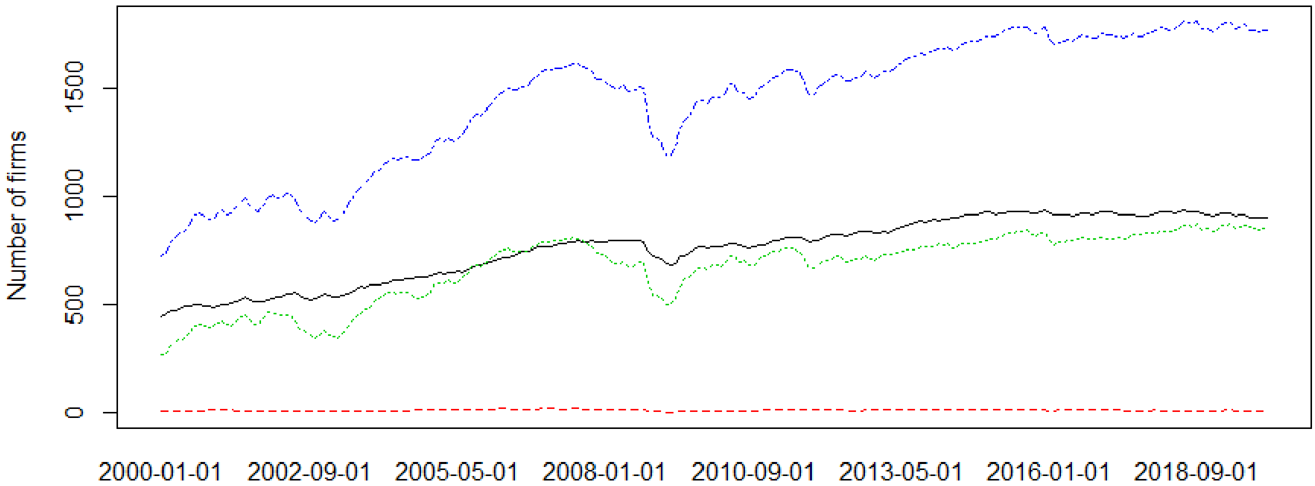


Table 5. shows that the average monthly time-series return of the firms in the sample is equal to 1.1% with a standard deviation of 12.9%. This is somewhat similar to Lewellen (2015) who find an average monthly return of 1.27% with a standard deviation of 14.79%. Fletcher (2017) find a monthly average return of approximately 1.02% with a standard deviation of 9.82% over the period of 1991 till end 2012. All firm-characteristics have been winsorized at their 1% and 99% percentiles to provide meaningful results (Palley et al., 2019). The minimum market capitalization across all firm observation is at least 194 mln. consequently, the risk the cross-sectional dispersion across characteristics is driven by micro-cap firms is mitigated (Fama & French, 2008). Nevertheless, the average firm market capitalization is close to 4.92bln with a median firm size of approximately 1.4bln indicating that a sample includes a substantial portion of relatively smaller firms. This is similar to Lewellen (2015) that find an average market-capitalization of 4.63bln. The correlation between market-equity and the book-to-market ratio is significantly negative similar to Novy-Marx (2013). The average book-to-market ratio is close to 0.529 indicating that on average the firm's market capitalization is rough twice the amount of net assets that the firm carries on its balance sheets. The standard deviation of the metric is close to 0.40. Fletcher

(2017) find a slightly lower book-to-market value of 0.436 with a standard deviation of 0.273. The difference is due to the fact that their sample includes relatively many large firms characterized by lower book-to-market values, on average. The average past twelve-month returns is close to 15.11% percent with a standard deviation of almost 50%, highlighting a wide dispersion in returns of the underlying firms. This is also reflected in the minimum (maximum) past 12-month return of -70% (225%). Similarly, Fletcher (2017) finds a twelve-month momentum return of 15.14% with a standard deviation of approximately 42%. The momentum characteristic appears negatively related to the book-to-market ratio, in line with Novy-Marx (2013). This shows that growth firms tend to have generate higher momentum returns. The average annual gross profitability is approximately 8.07% with a standard deviation close to 6.65%. The average annual cash flow profitability or cash flow yield is substantially lower (as expected) at 1.93% and also shows a greater variation with a standard deviation of 3%.

Table 5. Descriptive statistics

The table below provides the summary statistics of all characteristics included in the portfolio optimization. *ME,* reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return excluding the front month, *INV,* represent the investment characteristics. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristics target-price-implied return and *TIMD* reflects the characteristics target-price-implied-dispersion. *ASTCHG,* represent the annual change in total assets. *CFPROF,* reflects the cashflow profitability and finally *OPPROF*, reflects operating profitability.

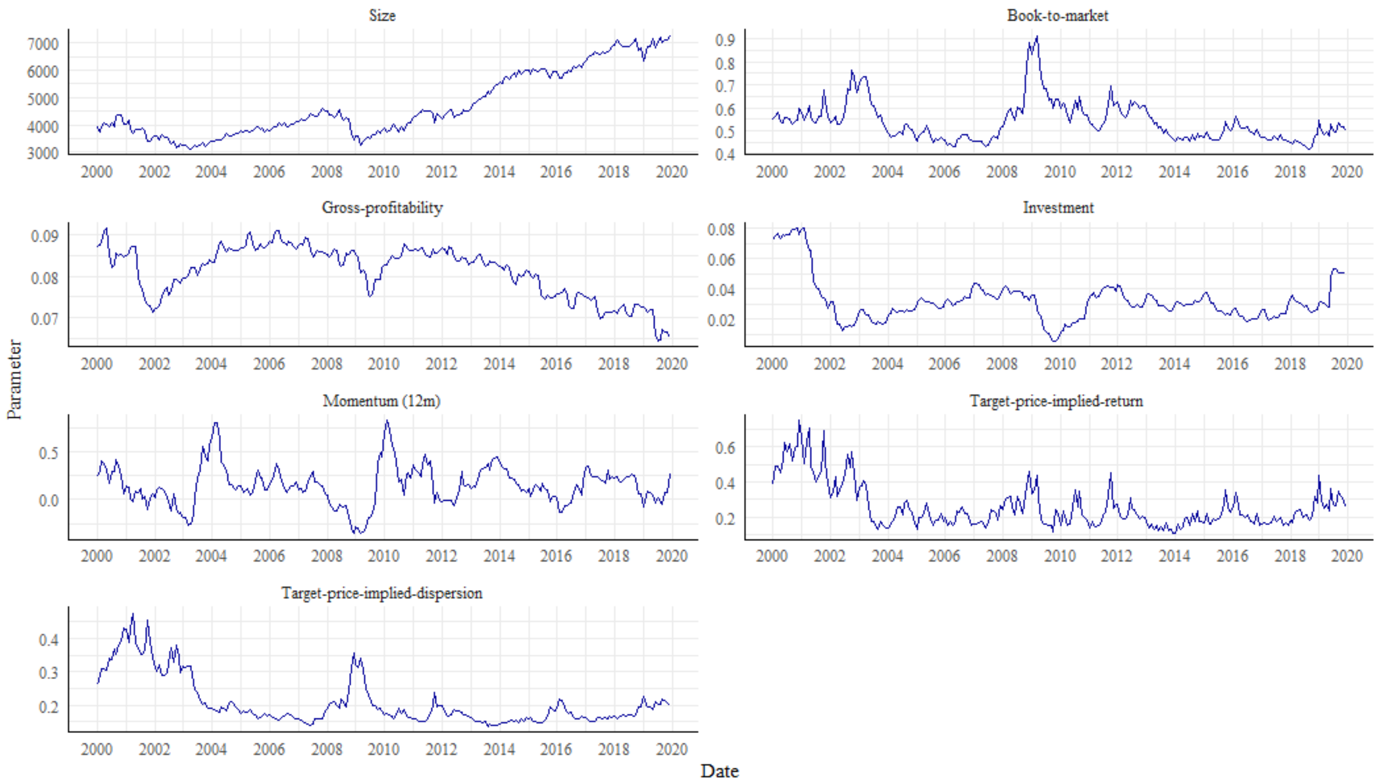
	ME	BM	MOM	INV	GRPROF.	TIMR	TIMD	ASTCHG	CFPROF	OPPROF
min	194	0.03	-69.6%	-14.11%	-11.9%	-20.6%	0.0%	-30.9%	-8.6%	-30.6%
max	35594	2.31	225.9%	45.74%	29.5%	212.2%	140.7%	187.2%	14.0%	59.2%
median	1441	0.44	8.8%	0.81%	7.3%	16.2%	13.9%	4.8%	1.8%	5.5%
mean	4916	0.53	15.1%	3.13%	8.1%	24.2%	19.6%	12.0%	1.9%	6.0%
std.dev	8509	0.40	47.4%	7.81%	6.6%	32.8%	20.3%	30.4%	3.0%	10.3%

The average annual operating profitability is approximately 6.03% with a standard deviation of close to 10.26%. The variance across all profitability metrics is large indicating there is a large heterogeneity of firms within the sample with different levels of profitability. The average profitability within our sample is slightly below the 10% operating profitability that Novy-Marx (2013) finds. This could be due to the fact that the sample includes a vast number of small firms that are less likely to benefit from economies of scale. Table 6, in the appendix, shows that all profitability related variables are significantly positively correlated, as expected. The correlation plot shows that gross profitability, cash flow and operating profitability are all significantly positively correlated to firm size in line with findings of Novy-Marx (2013). The average yearly cross-sectional investment represents close to 3.13% of the total assets, while the average asset change is close to 11.96%. The correlation between firm investment and year-over-year asset-change is as large and significant, as expected. The average target-price-implied-return is close to 24.16% somewhat higher compared to Da and Schaumburg (2011) find an average target price implied return of 40% with a median of 24% over the period of 1999 to 2010, but they comment this is substantially higher than one would expect. Palley et al., (2019) find a target-price-implied-return of 21.7% with a median of 14.4% over the period. Finally, Da et al. (2016) find an average (median) target-price-implied returns of 25.95% (18.27%) over the period of 1999 to 2011.

Finally, I find an average mean (median) target-price-implied dispersion of close to 19.6% (13.9%) This is similar to Palley et al. (2019) that find a mean (median) of 18% (13.3%) on average over the period of 1999 to 2018. Figure 2. below, plots the average cross-sectional characteristics over time. The figure shows there exists a large is time-series variability of the characteristics. The period surrounding the financial crisis ('07'08) is characterized by a large drop in market-equity accompanied by a large increase in the average book-to-market value and a large decrease in profitability. Moreover, the average firm appear to delay investments that only traced back to previous levels years later. The average target-price-implied returns peaked during the period before the dot.com crash (late 2002) after which is retraced to lower levels. Finally, the target-price-implied dispersion also seems to increase during times of market turmoil.

Figure 2. Average characteristics across time

Figure 2. displays the cross-sectional means of the firm characteristics plotted over the period as of 2000 until end 2019. The values are obtained by calculating the cross-sectional averages and plotting these over time. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristics, *MOM* reflects the 12-month momentum return excluding the front month, *INV*, represent the investment characteristics. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristics Target-price-implied return and *TIMD* reflect the characteristics, Target-price-implied-dispersion. *ASTCHG,* represent the annual change in total assets. *CFPROF*. reflect the cashflow profitability and finally, *OPPROF,* reflects operating profitability.



4.2. Benchmark portfolios

In order to provide a starting point for my analysis, I gauge the performance of the two benchmark portfolios using both simple statistics as well as regression analysis. The results have been presented in table 6. First I will discuss the portfolio performance, thereafter I will discuss the portfolio characteristics in terms of weights. Finally, I discuss the regression statistics that provide insight into the style-exposures. The table should be read from left to right as every subsequent column presents a different portfolio. This format will be the same going forward.

Table 6. Benchmark portfolios

Table 6 presents the properties and performance of the benchmark portfolios. Panel A, presents the characteristics of the portfolios. Panel B presents the regression analysis. All metrics are calculated using monthly returns. The first set of rows present the average minimum and maximum weight in the underlying assets: Min and Max w_i%, The average absolute portfolio weight, Av. $|w_i|$ %, The average sum of negative weights: Av. sum w_i<0, The average fraction of negative weights Av. % w_i<0, The annualized return $\bar{r}$, the standard deviation: $\sigma(\bar{r})$, The Sharpe ratio: SR, the skewness, kurtosis, VaR (97.5%), CVaR (97.5%), The Sortino ratio (MaR=0%), the Herfindahl index: *HHI*, the average transaction costs: *TTC*, the average portfolio turnover: *TTN* and the Certainty equivalent return: *CER*. Panel B present the time-series regression results of the risk-factor upon the portfolio returns. The market, size, value, profitability, investment and momentum factor are denoted with *eMKT*, *SMB*, *HML*, *RMW*, *CMA* and *UMD*, respectively. The Adj. R^2 represent the adjusted R-squared. Note: *p<0.1 , **p<0.05, ***p<0.01. t-statistics are presented in bold.

Panel A.	Benchmark portfolios	
	Value-weighted (VW)	Equal-weighted (EW)
Min w_i%	0.00%	0.02%
Max w_i%	0.61%	0.02%
Av. $|w_i|$ %	0.02%	0.02%
Av. sum w_i<0	0.00%	0.00%
Av. % w_i<0	0.00%	0.00%
$\bar{r}$	9.62%	13.96%
$\sigma(\bar{r})$	15.11%	18.78%
Sharpe ratio	0.55	0.67
Skewness	-0.73	-0.33
Kurtosis	2.07	1.59
VaR (97.5%)	-9.45%	-10.70%
CVaR (97.5%)	-15.42%	-16.63%
Sortino (Mar= 0%)	0.294	0.362
HHI	0.758	0.141
TTC	0.04%	0.03%
TTN	7.54%	5.50%
CER	3.91%	5.14%
	Panel B.	
Constant	0.0004	0.003***
	1.05	**8.454**
eMKT	1.015***	1.035***
	96.053	**92.384**
SMB	0.152***	0.735***
	9.063	**41.386**
HML	-0.119***	-0.114***
	-7.022	**-6.385**
RMW	-0.032*	-0.099***
	-1.357	**-3.988**
CMA	0.051*	0.002
	1.865	**0.071**
UMD	-0.028***	-0.112***
	-3.169	**-11.974**
Adj. R^2	0.987	0.99

Table 6. shows that the annualized return of the value-weighted (equal-weighted) benchmark is close to 9.6% (14%). Moreover, the standard deviation of the portfolio is close to 15.1% (18.8%). As a result the Sharpe ratios of the value-weighted (equal-weighted) portfolio in question yield 0.55 (0.67). Plyakha et al. (2014) find the value-weighted (equal-weighted) portfolio earns an annualized return of 13.9% (11.5%) over the period of 1994 till 2010. Moreover, DeMiguel et al., 2009 find that both benchmarks earn an annualized Sharpe ratio close to 0.60. Medeiros, et al. (2014) find a Sharpe ratio of 0.679 (0.701) for the value-weighted (equal-weighted) benchmark over the period of 2001 till 2013 in the Brazilian market.

The figures show that the equal-weighted portfolio earns a greater risk-adjusted return in terms of Sharpe ratio relative to the value-weight portfolio. The Sortino ratio, which only considers the standard deviation of the (negative) downside risk, is somewhat larger for the equal-weighted portfolio relative to the value-weighted portfolio. The value-weighted benchmark returns are slightly more negatively skewed with also greater kurtosis relative to the equal-weighted portfolio resulting in a larger probability of earning negative returns. The return distribution of both benchmark portfolios is visualized in the cumulative density plot in figure 3, below. The plot shows that the value-weighted portfolio carries a slightly larger probability mass centered in the tails of the distribution. An investor with a risk aversion parameter of y=5 earns a greater certainty equivalent return on the equal-weighted portfolio compared to a value-weighted portfolio due to the higher return and only slightly higher standard deviation. The downside risk of the portfolios is summarized using the VAR and the expected shortfall (CVAR) at the 97.5th percentile. The equal-weighted portfolio demonstrates somewhat greater downside risk with a VAR (CVAR) of 10.70% (16.63%) at a confidence level of 97.5% relative to the VaR (CVaR) of 9.45% (15.42%) for the value-weighted portfolio. Figure 4. presented in the appendix shows the sensitivity of the VaR across a range of confidence intervals for both benchmark portfolios. Both benchmark portfolios have a return-distribution that departs from normality and this shows that at greater confidence levels the Modified and Gaussian VaR diverge, stressing the importance of using the modified VaR and CVaR. The plots show that both benchmark portfolio showing similar sensitivity to a changing CI but differ in terms of the absolute level of VaR

The portfolio weights presented in the first rows in panel A. give insight into the construction of both benchmark portfolios. By construction, the value-weighted portfolio assigns a greater weight to firms with a larger market capitalization and equivalently a lower weighting to smaller firms. The minimum (maximum) weight assigned to the assets in the value-weighted portfolio is 0% (0.61%) which results in an average absolute weight of approximately 0.02%. The equal-weighted portfolio gives an equal weight across all securities in the universe which also results in an average absolute weight of approximately 0.02%. The minimum and maximum weight change over time as the number of stocks in the portfolio varies. Both portfolios only assign positive weights to securities and therefore the minimum weight in a particular security is always positive. The HH-index of the equal-weighted (value-weighted benchmark is close to 0.142 (0.759) demonstrating that the equal-weighted benchmark portfolio is more diversified compared to the value-weighted. In fact, the equal-weighted benchmark carries the highest feasible diversification in the portfolio universe (of this sample). Therefore the equal-weighted portfolio sets a lower-bound for the HHI index that serves as a yardstick for the other portfolios.

Given the different composition of the benchmarks, the returns of the respective portfolios will also differ. Panel B presents the results of the regression analysis. Both portfolios have been regressed upon the Fama-French-Carhart factors to determine the style exposures of both benchmarks and to examine the drivers of the underlying performance. The regression also allows us to examine the underlying characteristics of the benchmark constituents of both benchmark portfolios. Both portfolios have significant exposure to the market (*MKT*) as appears from the significant market coefficient. The market betas for the value and equal-weighted portfolios are 1.015 and 1.035 respectively (t= 96.053

and t= 92.384). These are fairly close to 1 as you would expect from benchmark portfolios. This shows that the benchmark portfolio do approximate the market-portfolio specified by Fama and French. What seems counter-intuitive is the significant alpha of 30bps a (t= 8.454) of the equal-weighted portfolio relative to the Fama-french-Carhart factors. In contrast, the value-weighted portfolio earns a negligible and non-significant alpha of only 4 bps a month. The positive and significant alpha of the equal-weight portfolio is the result of the methodology by which both the Fama-French-Carhart factors and the value-weighted benchmark portfolio have been constructed. Both the Fama and French factor portfolios and the value-weighted benchmark have been constructed using a value-weighted procedure and therefore are good substitutes for one another[4]. This finding is in line with the notion that these benchmark portfolios should represent well-diversified passive indices and therefore should have abnormal returns or alphas that are close to zero. The non-zero alpha of the equal-weighted portfolio results from the fact that the portfolio overweight small and growing firms. If these firms outperformed in the respective period, a portfolio that overweighs these factors, such as the equal-weighted benchmark will yield positive excess returns (Cremers et al., 2012). Plyakha, et al. (2014) also find a positive four-factor *annualized* alpha of 185 bps of the equal-weight portfolio albeit non-significant, their sample, however, covers larger firms listed at the SP500.

Both benchmark portfolios have a significant tilt towards small-cap firms as observed by the positive coefficient on the size (*SMB)* factor. Both the value and equal-weighted carry a significant positive loading of 0.152 and 0.735 (t= 9.063, 41.386) respectively. By construction, the equal-weighted portfolio allocates greater weights towards small-cap firms and therefore inherently receives a greater exposure towards the small-cap factor vis a vis the value-weight portfolio. Both portfolios have an equally large negative exposure towards the value factor (*HML*) with coefficients of -0.12 and -0.11 (t= -7.022, -6.385) respectively. This demonstrates that both benchmark portfolios have significant exposure towards growth firms rather than value firms. These findings can be reconciled with the fact that the benchmark portfolios incorporate firms from both the NYSE, AMEX, and NASDAQ of which the latter includes a lot of small growth firms. Therefore the benchmarks are expected to have a greater exposure towards relatively smaller, fast-growing firms, vis a vis a diversified benchmark portfolio such as the S&P500, that largely consist of blue-chip firms with a proven track record. Both the value and equal-weighted benchmark yield a small but significantly negative loading towards the profitability factors (RMW) of -0.03 and -0.1, respectively (t= -1.357, -3.988). Both portfolios carry a negative exposure toward the profitability factor as appears from the statistics. The size of the coefficients indicates that the value-weighted benchmark slightly overweight's profitable firms relative to the equal-weighted benchmark. Only the value-weighted portfolio has a significant positive coefficient of 0.05 (t= 1.865) towards the investment factor (*CMA*) indicating that the value-weighted portfolio has exposure towards firms that tend to make greater investments in PPE. In contrast, the equal-weight portfolio has a negligible non-significant coefficient close to zero (t=0.07).

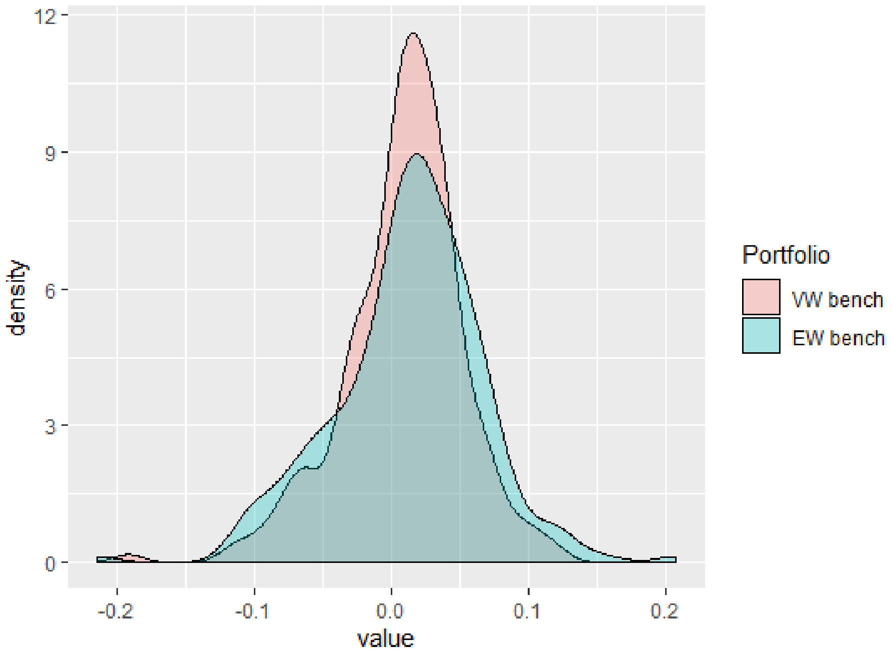
Finally, the regression statistics show that both benchmark portfolios carry a negative exposure towards the momentum factor. The value and equal-weighted portfolio carry a negative coefficient of -0.028 and -0.112 (t= -3.169,-11.974) respectively towards the momentum factor. The fact that the equal-weighted portfolio has a substantially lower coefficient relative to the value-weighted benchmark is due to the re-balancing of the value-weighted portfolio that results in overweight in momentums stocks relative to the equal-weighted portfolio that rebalances to the mean. At times when a firm quickly appreciates in value, the value-weighted portfolio places more weight on these firms that have gained value in the previous period. Holding all else equal one would expect the value-weighted benchmark to carry a positive momentum coefficient due to this effect, but due to the large overweight to small firms, the coefficient remains low. Our findings are confirmed by Plyakha, et al. (2014) that also document the

[4] See (Fama & French, The cross-section of expected stock returns, 1992) for details

equal-weight benchmark carries a substantially large exposure towards the size (*SMB*) factor. Moreover, they find that while both the equal and value-weighted benchmark portfolios carry a significant negative exposure toward the momentum factor the exposure of the equal-weight benchmark is significantly more negative. The adjusted R-square of both the value and the equal-weight portfolio are 0.987 and 0.990 respectively. Both figures are close to 1 which indicates that the pattern of returns of both respective portfolios is well captured by the Fama-French-Carhart factors included in the regression.

Figure 3. Cumulative density plot of the benchmark portfolios

The plot shows the cumulative density plot of the return distribution for both benchmark portfolios. The pink surface reflects the value-weighted benchmark portfolio (VW) and turquoise surface reflects the equal-weighted benchmark (EW). Both plots correspond to the information presented in table 6.



4.3. Individual portfolios

Table 7. Individual portfolios

Table 7. presents the results for the optimal portfolios optimized using a single characteristic. Panel A. presents the results for the portfolio optimized using the characteristic target-price-implied return (TIMR), SS represent the portfolio in which short-sales are allowed. NSS represent the portfolio without short sales (no-short-sales). Panel B present the results for the portfolio optimized using the characteristics target-price-implied-dispersion (TIMD). The second row of the table presents the theta coefficient. Thereafter the first set of rows present the average minimum and maximum weight in the underlying assets: Min and Max w_i%, The average absolute portfolio weight: Av. $|w_i|$ %, The average sum of negative weights: Av. sum w_i<0, The average fraction of negative weights: Av. % w_i<0, The annualized return $\bar{r}$, the standard deviation: $\sigma(\bar{r})$, The Sharpe ratio, the skewness, kurtosis, VaR (97.5%), CVaR (97.5%), The Sortino ratio (MaR=0%), the Omega ratio, the average monthly transaction costs: *TTC*, the average monthly portfolio turnover: *TTN* and the Certainty equivalent return: *CER*. The risk-aversion is set at, *y*=5. All metrics use simple returns and are annualized unless otherwise stated.

	Benchmark portfolio		Panel A: TIMR		Panel B: TIMD	
	VW	EW	SS	NSS	SS	NSS
Theta coefficient			1.758	0.027	-0.576	-0.005
Min w_i%	0.00%	0.02%	-0.18%	0.00%	-0.04%	0.00%
Max w_i%	0.61%	0.02%	1.06%	0.55%	0.58%	0.55%
Av. $|w_i|$ %	0.02%	0.02%	0.03%	0.02%	0.02%	0.02%
Av. sum w_i<0	0.00%	0.00%	-29.94%	0.00%	-5.25%	0.00%
Av. % w_i<0	0.00%	0.00%	16.47%	0.00%	4.41%	0.00%
$\bar{r}$	9.62%	13.96%	37.47%	10.00%	13.10%	9.59%
$\sigma(\bar{r})$	15.11%	18.78%	22.52%	15.17%	18.72%	15.08%
Sharpe ratio	0.545	0.666	1.582	0.568	0.624	0.544
Skewness	-0.734	-0.325	-0.132	-0.728	-0.064	-0.737
Kurtosis	2.067	1.588	0.951	2.047	3.269	2.071
VaR (97.5%)	-9.45%	-10.70%	-10.63%	-9.45%	-9.39%	-9.44%
CVaR (97.5%)	-15.42%	-16.63%	-15.39%	-15.41%	-11.42%	-15.41%
Sortino (Mar= 0%)	0.294	0.362	0.870	0.304	0.350	0.293
Omega ratio			7.155	7.334	2.329	0.424
TTC	0.04%	0.03%	0.11%	0.03%	0.10%	0.03%
TTN	7.54%	5.50%	19.49%	6.17%	16.98%	6.47%
CER	3.91%	5.14%	24.78%	4.25%	4.34%	3.90%

4.3.1. Target-price-implied-return

Table 7. presents the results from the portfolio optimized using the individual characteristics target price-implied-return and target price implied dispersion. First, these characteristics are considered in isolation in order to gauge the strength and direction of the effects. Ammann, et al. (2016) show that constraining leverage reduces the sensitivity of the portfolio performance to the selection of firm-characteristics and risk-aversion level as a result the risk of model misspecification is significantly reduced. Therefore I report the results with short sales (SS) and without short sales (NSS). In the instance with short sales, the leverage is restricted to 1 in order to provide meaningful results.

I first address the results in Panel A, which concerns the characteristic TIMR, and later continue to examine the characteristic TIMD in panel B. The table shows the cross-sectional tilt towards TIMR is consistently positive for both the short-sale and short-sale prohibited portfolio. The short-sale portfolio optimized upon the characteristic TIMR yields a theta coefficient of 1.758. This shows that the optimized portfolio optimally deviates from the benchmark portfolio by overweighting firms characterized by positive target price implied returns. The coefficient of TIMR returns reduces to 0.027 when short sales are prohibited, indicating that the investor still overweighs these firms but substantially less so. The portfolio now approximates the benchmark.

In the first instance (with SS) the investor that incorporate the TIMR-characteristics in the portfolio selection, earns a large annual return of 37.47% with a standard deviation of 22.52%. This results in a large Sharpe ratio close to 1.582. The return distribution is negatively skewed and has a kurtosis of 0.951, which is substantially lower relative to the kurtosis of both benchmark portfolios. Moreover, the optimized portfolio yields a Sortino and Omega ratio of 0.87 and 7.16. After restricting short sales the performance drops to 10%, on average, a year with a standard deviation of 15.17% resulting in a Sharpe, Sortino, and Omega ratio of 0.568, 0.304 and 7.33 respectively. This shows that while constraining leverage results in a substantially lower Sharpe and Sortino ratio, the omega ratio slightly increases as the higher moments are taken into account. These results also show that a large part of the performance stems from the short-leg of the portfolio that underweights firms relative to the value-weight portfolio and enables the investor to hedge the market in downturns. A large divergence from the moments of the benchmark is penalized and results in a decreasing Omega ratio. It also shows that the returns of the constrained optimal portfolio carries a lower excess risk relative to the value-weighted benchmark portfolio imposing the short-sale restriction. With short sales, the certainty equivalent returns is close to 25% which is well above the 18.2% that Medeiros et al. (2014) find using a set of characteristics that include size, book-to-market, and momentum. The Certainty equivalent return decreases to 4.2% as short sales are prohibited, indicating that the investor would accept a lower risk-free rate in compensation of exposure towards the risky portfolio. The CER of 4.2% after short sales is close to the value of 6.7% that Medeiros et al. (2014) find conditional upon size, book-to-market and momentum. These results are interesting because Lamoureux and Zhang (2020) find that no characteristics that are used as a singleton (i.e. one a standalone basis) are able to construct a portfolio whose out-of-sample certainty equivalent return is greater than the market portfolio.

In the case with short sales, the average minimum (maximum) weight in the underlying assets is -0.18% (1.06%). The minimum (maximum) weight in the case without short sales is 0% (0.55%) respectively. The large active bets contribute to the observed shape of the return distribution and attribute to the performance of the portfolio. Large concentrated bets in firms that generate large positive returns result in a return distribution with less negative skewness and a smaller positive kurtosis. The investor that is able to go short owns a large short position of around (-) 29.94% in the underlying assets of the portfolio. The average fraction of shorted-stocks is close to 16.5%. One can observe that the portfolio has a large portfolio turnover of almost 20% of the total portfolio. This indicates that at every rebalancing period close to 20% of the total portfolio is bought and sold, resulting in average monthly transaction costs of close to 0.11% (i.e. approximately 1.3% annually). In contrast, the short-sale constrained investor does no carry short positions and as a result, carries less concentrated positions. This leads to a smaller portfolio turnover close to 6.2% with monthly transaction costs yielding approximately 0.03%, a fraction of the short-sale portfolio. These findings are in line with Ammann et al. (2016) showing that constraining the leverage leads to optimal portfolios that are easier and cheaper to implement.

4.3.2. Target-price-implied-dispersion

Panel B presents the results for the variable target price implied dispersion. The table follows the same structure as before. The optimal portfolio with short-sales carries a negative tilt of -0.576. The portfolio deviates from the benchmark portfolio by underweighting firms with a large dispersion of target-prices. The optimal portfolio achieves an annualized return of close to 13.10% with a standard deviation of 18.72% resulting in a Sharpe ratio of around 0.625. The minimum (maximum) weight assigned to the underlying assets is close to -0.04% (0.58%). Moreover, the average sum of negative weights is approximately 5.25% of the total portfolio. The faction of shorted stocks is now close to 4.4%. The average absolute weight remains close to 0.2%, similar to both benchmark portfolios. The portfolio VaR (CVaR) is close to 13.10% (11.42%), which is a slight improvement (deterioration) relative to the equal-weight (value-weight) benchmark portfolio.

After prohibiting short sales the average theta coefficient approximates zero, this naturally lead the portfolio weights to converge to the benchmark weights. The minimum weight is zero by construction while the maximum weight is close to 0.55%. The negligible tilt shows that the optimal portfolio for which short sales are restricted does not actively deviate from the value-weighted benchmark portfolio. Consequently also the performance metrics are close to identical to the value-weighted benchmark portfolio. The annualized performance decreases to 9.59% with a standard deviation of 15.08% yielding a Sharpe ratio of 0.544. Moreover, the portfolio earns a Sortino ratio of 0.293 and yields a certainty equivalent of 3.9% (y=5) identical to the benchmark.

After examining both singleton (individual) portfolios that include the characteristics, target price implied return, or target price implied dispersion one can conclude that a CRRA-investor seeks to positively tilt her optimal portfolio towards firms that offer positive target-price-implied-returns. The active tilt towards firms with greater target-price-implied-returns remains persistent after restricting short sales but does decrease in magnitude. As a result, the performance of the former decreases substantially. I also show that the investor actively deviates from the market portfolio by underweighting firms characterized by large target price dispersion. The effect disappears completely as short sales restrictions are imposed, resulting in an optimized portfolio that converges to the benchmark portfolio in terms of asset-allocation and risk-adjusted return. The question that arises is whether these active deviations are robust to the inclusion of additional characteristics that have been shown to predict the cross-section of expected returns. The next paragraphs will elaborate.

Table 8. Incremental Portfolio: With short-sales

The table presents the results for the incremental portfolios with short-sales. The characteristics are depicted in the first column, the corresponding time-series average coefficients can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristics, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristics. *GRPOF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristics: target-price-implied return and *TIMD* reflects the characteristic: target-price-implied-dispersion. Panel B. Presents the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01, t-statistics are presented in bold

	Benchmark portfolio		Traditional portfolio	Extended portfolios	
Theta	VW	EW	(1)	(2)	(3)
θ *ME*			-1.74	-1.13	-1.22
θ *BTM*			2.72	1.97	1.89
θ *MOM*			0.58	0.54	0.69
θ *INV*			-2.49	-1.67	-1.75
θ *GRPROF*			1.00	0.71	0.76
θ *TIMR*				2.17	2.55
θ *TIMD*					-1.38
Min w_i%	0.00%	0.02%	-1.14%	-0.83%	-0.96%
Max w_i%	0.61%	0.02%	1.15%	1.56%	1.44%
Av. $\lvert w_i \rvert$ %	0.02%	0.02%	0.06%	0.05%	0.05%
Av. sum w_i<0	0.00%	0.00%	-79.95%	-71.40%	-75.34%
Av. % w_i<0	0.00%	0.00%	30.74%	29.30%	29.911%
$\bar{r}$	9.62%	13.96%	23.62%	54.88%	58.54%
$\sigma(\bar{r})$	15.11%	18.78%	23.39%	28.75%	24.16%
Sharpe ratio	0.545	0.666	0.943	1.838	2.336
Skewness	-0.734	-0.325	0.632	0.631	0.230
Kurtosis	2.067	1.588	3.697	3.290	1.603
VaR (97.5%)	-6.97%	-7.97%	-10.50%	-11.10%	-9.44%
CVaR (97.5%)	-11.22%	-12.34%	-12.62%	-12.55%	-13.68%
Sortino (Mar= 0%)	0.294	0.362	0.560	1.102	1.399
Omega ratio			2.218	5.901	8.435
Info. ratio			0.994	2.409	3.179
HHI	0.758	0.141	1.959	1.819	1.767
TTC	0.04%	0.03%	0.46%	0.46%	0.85%
TTN	7.54%	5.50%	78.64%	78.64%	145.30%
CER	3.91%	5.14%	9.94%	34.22%	43.95%

Table 8. Panel B Regression analysis

	VW	EW	(1)	(2)	(3)
Constant			0.009***	0.027***	0.031***
			3.296	**7.829**	**10.069**
VW bmk			1.264***	1.537***	1.259***
			20.315	**19.715**	**18.392**
$Adj. R^2$			0.664	0.651	0.619
Constant			0.006***	0.024***	0.028***
			2.83	**8.498**	**10.705**
EW bmk			1.102***	1.351***	1.095***
			27.351	**27.006**	**23.331**
$Adj. R^2$			0.782	0.778	0.723
Constant	0.0004	0.003***	0.009***	0.029***	0.03***
	1.05	**8.454**	**5.652**	**10.669**	**11.421**
eMKT	1.015***	1.035***	0.968***	1.108***	1.062***
	96.053	**92.384**	**19.953**	**14.169**	**13.86**
SMB	0.152***	0.735***	0.959***	1.347***	1.099***
	9.063	**41.386**	**12.478**	**10.871**	**9.056**
HML	-0.119***	-0.114***	0.322***	0.118	0.144
	-7.022	**-6.385**	**4.152**	**0.946**	**1.177**
RMW	-0.032*	-0.099***	0.244**	-0.02	0.243
	-1.357	**-3.988**	**2.271**	**-0.114**	**1.429**
CMA	0.051*	0.002	0.62***	-0.111	-0.081
	1.865	**0.071**	**4.933**	**-0.548**	**-0.41**
UMD	-0.028***	-0.112***	-0.371***	-0.428***	-0.155**
	-3.169	**-11.974**	**-9.164**	**-6.567**	**-2.423**
$Adj. R^2$	0.987	0.99	0.885	0.803	0.732

4.4. Incremental portfolios: with short-sales

Table 8. presents the results of the optimization using the traditional characteristics. Three portfolios are presented, the first optimal portfolio (1) includes all traditional characteristics that are size, book-to-market, momentum, investment, and gross-profitability. The following portfolios (2) and (3) represent the portfolios that include the characteristics: target price implied return (TIMR) and target price implied dispersion (TIMD), respectively. The characteristics are included in addition to the traditional characteristics in order to examine the incremental value of adding multiple characteristics and test whether the tilts remain persistent. The full table that incrementally (step-wise) adds the traditional characteristics can be found in the appendix as table 9. The table has been divided into two panels, Panel A and Panel B which are then subdivided into sections 2 and 3 sections respectively. Panel A, presents the portfolio tilts expressed by the theta coefficient, presented in the top section of Panel A. The bottom section of Panel A. presents the performance of the various portfolios across a range of metrics that shall be discussed later. The first two sections of Panel B, present the regression results of the various portfolios upon both the equal and value-weighted benchmark portfolios. The third section of table 4. Panel B. presents the results of the regression of the optimal portfolios upon the Fama-French-Carhart factors. The table should be read from left to right as every next column to the right includes an additional characteristic in order to gauge the relative incremental change. First, I will examine the optimized portfolio coefficients presented in the first rows. This will be followed up by an examination of the style tilts as expressed by the factor regressions in Panel B, to gain a more in-depth perspective of the optimal portfolios and offer insights in style tilts of the optimal portfolio relative to the value and equal-weighted benchmarks. Third and final I will provide an in-depth examination of the performance of the portfolios.

4.4.1. Characteristics

The active tilts of the optimal portfolios are discussed in terms of their theta coefficient or simply coefficient, in short. The coefficient that is reported is a time series average tilt of the optimal portfolios and represents the average tilt of the optimal portfolio relative to the benchmark portfolio over the investment period. The first section of Panel A table 4 presents these coefficients.

The first portfolio includes all traditional characteristics, size, book-to-market, momentum, investment and profitability and is therefore call the traditional portfolio. The table shows that the coefficient of the size characteristic, presented in the first row, is consistently large and negative in line with the findings of Ammann et al. (2016) and Brandt et al. (2009). The coefficient appears large and negative across all optimized portfolios. Note that the characteristics size, denoted by *ME,* is inversely related to the s*ize (SMB)* factor of Fama and French. The negative average tilt shows that the portfolio consistently seeks to deviate from the market-portfolio portfolio by actively underweighting firms with a *larger* market capitalization. This finding is in line with the notion that firms with a smaller market capitalization offer above-average cross-sectional returns relative to larger firms (Fama & French, 1992). The average coefficient becomes smaller as multiple characteristics are introduced due to overlapping exposures, this effect is clearly visible in table 9 to be found in the appendix. The active portfolio tilts are contingent upon the underlying correlation and covariance structure of the characteristics. Multiple characteristics show large underlying positive correlations, as a result, part of the exposure to the size characteristics is captured by other coefficients. As these characteristics contain overlapping information it is no longer optimal to hold a substantially large tilt across the respective characteristics and the average coefficient decreases.

The second row presents the book-to-market characteristic. The book-to-market coefficient is consistently large and positive across all portfolios but also decreasing with the inclusion of additional characteristics. This effect is most evident in table 9 in the appendix. The large theta coefficients across the board, albeit smaller than of the size characteristic, show that the optimal portfolios seek to deviate from the value-weighted benchmark portfolio by overweighting firms characterized by a greater book-to-market ratio. This is in line with the theory of Fama and French (1992, 1993) stating that firms with greater book-to-market ratios offer a larger cross-sectional average return. Brandt et al. (2009) find a coefficient for the book-to-market characteristic of similar magnitude.

The third portfolio includes the momentum characteristic defined as the last twelve-month cumulative return minus the front month. The average theta coefficient is as large as 0.58, 0.54, and 0.69 for the traditional, and extended portfolios, respectively. The coefficient is positive across all optimal portfolios showing that the portfolios consistently deviate from the benchmark portfolio towards firms that show greater return in the previous 12-months. Brandt et al. (2009) also find a positive momentum coefficient albeit somewhat larger in magnitude. The overweight to momentum comes at the expense of a smaller allocation to the size characteristic but simultaneously increases the coefficient to the book-to-market characteristic. The correlation matrix shows that the correlation between the momentum characteristic and the book-to-market characteristic is large and negative. Hence by overweighting momentum firms the portfolio implicitly tilts towards growth firms and smaller firms. As a consequence, on top of the *explicit* expose already in place (due to the active size and book-to-market-tilt) the portfolio takes a larger, *implicit*, tilt towards book-to-market and a smaller tilt towards small-cap firms through the momentum characteristic. The positive tilt provides evidence in favor of the momentum factor of Jegadeesh an Titman, (1993) and the notion of time-series momentum (Moskowitz et al., 2012). So far the results have been in line with Brandt, et al. (2009) and Zhang (2013) that also find that the deviations of the optimal portfolio from the benchmark portfolio decrease with a firm's market-capitalization, increase with both the book-to-market ratio, and the past 12-month return.

The table also shows that the theta coefficient for investment characteristic is as large as -2.49, on average, for the first portfolio and decreases to -1.67 and -1.75 with the introduction of TIMR and TIMD respectively. This shows that the optimized portfolios strongly underweight firms, relative to the benchmark portfolio, that have carried out substantial investments over the last year. This supports the assertion that firms that make higher investments earn lower average returns in the subsequent period. This effect appears consistently negative regardless of the investment characteristic that has been defined[5], providing evidence in favor of the investment narrative. This finding is in line with Ammann, et al. (2016) who also find optimal portfolios to benefit from including the characteristics asset-growth in the selection problem. Row five presented the loading on the profitability characteristic (GRPROF). The theta coefficient is large and positive across all three portfolios showing that the optimal portfolio seeks to deviate from the value-weighted benchmark portfolio by overweighting firms characterized by greater profitability. This effect is persistent and strong regardless of the profitability definition used in the optimization[6]. The introduction of the investment and profitability characteristics comes at the expense of the book-to-market coefficient. These findings are in line with DeMiguel et al. (2020) and French & Fama (2015) that show that due to the underlying covariance structure both variables capture overlapping information and introducing the respective characteristics at the same time goes at the expense of the book-to-market characteristic.

The final two rows, corresponding the extended portfolios 2 and 3 in the last columns, present the coefficients for the analyst-characteristics, target price implied returns and the target price implied dispersion, respectively. The first analyst characteristic, target price implied return is consistently positive. The large positive loading shows that the portfolio optimally seeks to deviate from the

[5,5] These results are omitted in Table 4 but can be found in the appendix table 9. All investment & profitability metrics result in roughly the same results.

benchmark portfolio by overweighting firms that receive a positive target price implied return in line with the previous results from the portfolio that only included the characteristic as a singleton. Note that it does not yet tell us whether the characteristic improves portfolio performance and is valuable to the investor. Interestingly, the coefficient is larger relative to the coefficient as presented in the singleton portfolios due to the interaction with the other characteristics. The final row shows that the theta coefficient of target price implied dispersion of the optimal portfolio is as great as -1.38 this shows that the portfolio seeks to underweight firms that show a greater dispersion in target prices, in line with the results from the previous exercise of the singleton characteristics. The remainder of the characteristics only modestly change, indicating that the characteristics provide robust and independent information that only overlaps to some extent. These results show that the characteristics do still lead the investor to deviate her portfolio from the benchmark portfolio through exploiting the information from analyst consensus target-prices. This effect is persistent beyond the inclusion of the traditional characteristics, indicating the characteristics TIMR and TIMD provide new information to the optimization. The next section will examine the style exposure and investigate whether this information is valuable and leads to improved performance.

4.4.2. Style exposure

The bottom section of table 8. Panel B presents the time-series regression output against the benchmark factors. All optimal portfolios carry a strong and statistically significant exposure towards the market factor showing that the portfolios hold a significant portion of the market portfolio and deviate only if the characteristics offer an attractive risk-return tradeoff that benefits the investors' average utility.

The exposure towards the size (*SMB*) factor varies substantially across the portfolios. The factor regression shows that the optimal portfolios uniformly carry a substantially larger tilt towards the size (*SMB*) factor relative to both the equal and value-weighted benchmark, in line with the large negative theta coefficient presented earlier. The regression coefficient for size is significant and large as 0.959 (t=12.47) for the traditional portfolio showing a substantial exposure towards the size factor both in absolute and relative terms. The style exposure of the portfolios including TIMR and TIMD is significant and as large as 1.347 (t=10.87) and 1.099 (t=9.06), respectively. The exposure to the size factor of all optimized portfolios approaches that of the equal-weighted portfolio, showing similar factors coefficients albeit somewhat larger in magnitude.

The exposure towards the value factor (*HML*) becomes larger across all optimal portfolios[7] relative to both benchmark portfolios showing that incorporating the value characteristic indeed causes the optimal portfolio to tilt away from the equilibrium-weighting towards value-firms. The traditional portfolio carries a significant exposure towards the value-factor with a coefficient of 0.322 (t=4.15). After the introduction of the analyst characteristics, the value-factor coefficients remain positive but no longer significant. This shows that all optimized portfolios hold less absolute exposure toward growth firms and equivalently a greater exposure towards value firms compared to the market-portfolio. Interestingly, the exposure towards value firms does decrease as I include the analyst characteristics.

The exposure towards the profitability factor (*RMW*) varies substantially across the optimized portfolios due to the implicit exposure towards the factor resulting from other characteristics. Table 9 shows that when profitability is introduced as an additional characteristic the exposure toward the factor (*RMW*) increases towards 0.244 (t=2.271) and becomes significant at a 1% level. This shows the traditional portfolio overweighs profitable firms confirming earlier conclusions. The exposure to the profitability factor becomes non-significant after the introduction of TIMR or TIMD.

[7] After the introduction of the book-to-market-characteristic. Table 9 in the appendix shows that the optimal portfolio that is optimized using only the size characteristic carries a smaller exposure to the value-factor.

The traditional portfolio carries a significant exposure towards the investment factors (CMA) with a coefficient of 0.62 (t=4.93) but this becomes non-significant and even negative after the inclusion of the target-price-implied return and target-price-implied dispersion. The exposure towards the investment factor (*CMA*) increases steadily with the inclusion of additional characteristics showing positive implicit bets towards the factor. When the investment characteristics is introduced the coefficient becomes significantly positive with a coefficient of 0.62 (t=5.071). The significant exposure decreases with the introduction of TIMR and TIMD

Next, the exposure towards the momentum factors (*MOM*) is significantly negative across the board. This shows that the allocation taken towards the other factors results in a consistent and significant underweighting of momentum firms. This findings is confirmed by Ammann et al. (2016) who find that past returns (i.e. momentum) do not add any further value to the optimization due to its instability over time. The positive characteristic for momentum and negative exposure can be reconciled by the fact that even though the characteristic is robust when included in the conditioning set, the characteristics suffers from severe market downturns, especially during the period from March 2009 through December 2010 (Lamoureux & Zhang, 2014).

The previous findings show that the optimal portfolios, with short sales, seek to incorporate relevant the information from the various characteristics and this leads to significant factor exposures across the board. Overall one can see that the R-squared of the factor regressions decreases with the number of characteristics that have been included in the optimization. The traditional portfolio note an R-squared of 0.885 which decreases to 0.732 after including both TIMR and TIMD, This is substantially lower compared to the benchmark portfolios showing that the characteristics add information uncaptured by the traditional risk factors. These result show that the returns produced by the optimized portfolio largely lie outside the span of the Fama-French-Carhart Factors. Moreover, it shows that the information from target prices adds new information to the optimization beyond what is already reflected by the traditional characteristics, size, value, momentum, investment, and profitability together. This new information results in an overweighting to firms that show greater target price implied returns and an underweighting towards firms that show a greater dispersion in analyst target prices. The inclusion of the analyst-characteristics uniformly increases the exposure towards small-cap firms and momentum firms relative to the *traditional portfolio*. Moreover, the exposure to value firms decreases substantially. Finally, the exposures toward the investment and profitability factors decrease relative to the traditional portfolio. Note, however, that the initial conclusions still hold stating that the portfolio underweights small-cap firms and firms that pursue large investments while overweighting value firms, profitable firms, and firms with greater past 12-month returns, relative to the *benchmark portfolio*.

4.4.3. Performance

The second set of rows of Panel A, Table 8, presents the performance details of all respective portfolios. I will first address the traditional portfolio and proceeds by examining the performance of the optimized portfolios that extend the traditional portfolio by including analyst information.

The traditional portfolio that includes all characteristics, size, book-to-market, momentum, investment, and profitability is presented in the fourth column of the table. Earlier we observed that the portfolio actively deviates from the benchmark by tilting towards smaller firms, firms with greater book-to-market ratios, earning greater past returns, pursuing fewer investment and finally firms that show greater profitability, on average. These deviations from the benchmark leads the optimal portfolio to earn an annualized return of 23.62% with a standard deviation of 23.39%. The optimal portfolio yields a Sharpe ratio, information ratio, Sortino ratio, and Omega ratio of 0.943, 0.994, 0.560, and 2.218, respectively. All respective metrics are substantially larger compared to both the equal-weighted and value-weighted benchmark figures. The certainty equivalent of the traditional portfolio is approximately 9.9% vis a vis a CER of 3.91% (5.14%) for the equal-weighted (value-weighted) benchmark. This is in line with findings from Lamoureux and Zhang (2020) that find that combining multiple characteristics rather than singletons enable the optimized portfolio to outperform the market in terms of certainty-equivalent-returns. This leads us to conclude that the traditional portfolio outperforms the benchmark already by a margin both in annualized absolute returns as well as in terms of the performance ratios. The regression-analysis in panel B shows that the traditional portfolio outperforms the value-weighted benchmark portfolio by as much as 90bps (t=3.23) on average, a month statistically significant at the 1% level. Moreover, the optimal portfolio earns significant alpha of 60bps (t=2.83) a month over the equal-weighted benchmark portfolio. Regressing the portfolio returns upon the Fama-French-Carhart factors shows that the optimal portfolio outperforms the traditional factors with a monthly alpha of 90bps (t=5.65) statistically significant at the 1% level. These findings support the earlier conclusion that the traditional portfolio obtains an outperformance. The significant alpha of the final regression illustrates that the outperformance is not completely explained by the traditional factors which provide evidence in favor of the notion of Daniel and Titman (1997) and Lewellen (2015) asserting that the cross-sectional return patterns associated with the characteristics do not stem from covariances of returns but reflect firm-level characteristics.

The return-distribution of the traditional portfolio has become more positively skewed while also the kurtosis has increased substantially noting 0.943 and 3.692 respectively. This is in line with investors preferring more positive skewness but counter to the expectation of investors disliking greater kurtosis. The changing density of the return distribution illustrates the fact that the preferences of a CRRA-investor are difficult to capture by traditional performance measurements. The higher (more negative) modified VaR and CVaR, reflect that the CRRA investor trades-off greater returns for larger a downside risk. Overall one can conclude that the higher kurtosis is rewarded with greater returns, in line with the notion that investors require a premium for bearing higher kurtosis and consequently receiving a greater probability of tail risks. The portfolio statistics show that the minimum weight is negative, which is expected since short sales are allowed. The maximum weight allocated has increased to 1.15% relative to the benchmark portfolio that carries a maximum weight of 0.61%. The average absolute weight has increased to 0.06% indicating that the portfolio takes larger active bets. This is also reflected in the greater HHI of 1.95 relative to the benchmark portfolio. This shows that the traditional portfolio is less diversified and takes greater active bets in the underlying assets. The average sum of negative weights is close to -79% that is below the specified threshold of 100% leverage. The large hedge-portfolio of 79% also implies that the total long-leg of the portfolio sums to 179%. One trend that is visible across all portfolios is that the total monthly average portfolio turnover and transaction costs increase consistently with the introduction of new characteristics. This indicates that the increased portfolio performance comes at a cost of increased trading activity. This high-turnover due to large concentrated bets is also observed by Fletcher (2017) and DeMiguel et al. (2013).

Thus far I have shown that including the multitude of traditional characteristics increases the performance of the optimized portfolio substantially across the various performance metrics. I show that the investor can in fact use the traditional characteristics to obtain better performance compared to the benchmark portfolio. That is both the simple performance metrics well as the regression output has improved by incrementally including the traditional, size, book-to-market, momentum, investment, and profitability characteristics in the information set. The next step is to include the two characteristics, target price implied dispersion and target price implied return to examine the incremental value.

The first portfolio that includes target-price-implied-return (TIMR) earns a substantial annualized return of 54.88% with a standard deviation 28.75%. The portfolio yields a Sharpe ratio, information ratio, Sortino ratio and Omega ratio of 1.838, 2.409, 1.102 and 5.901 respectively. This reflects a substantial increase in absolute performance relative to all the previous portfolio as well as both benchmark portfolios. The certainty equivalent has increased to approximately 35% which is more than three times as much. Moreover, the regression output shows that the optimal portfolio earns a statistically significant positive alpha of 270bps (t=7.83) a month, on average over the value-weighted benchmark. Moreover, the optimal portfolio shows an strong positive outperformance of 240bps (t=8.45) over the equal-weighted benchmark portfolio statistically significant at the 1% level. Finally, the portfolio factor regression in the final rows shows that the optimal portfolio that includes the target price implied return yields a monthly alpha of 290bps (t=10.67) over the Fama-French-Carhart factor. also statistically significant at the 1% level. This examination clearly shows that including the target price implied return characteristics uniformly increases the performance of the portfolio across various performance metrics. The large outperformance of this portfolio highlights the fact that much of the returns of this optimal portfolio are left unexplained by the traditional risk factors. Therefore adding the characteristic target price implied return introduces new information to the optimization that is useful in construction the optimal weights. The statistics in the finals rows show that the increased performance does come at a cost. The portfolio is less diversified with an HHI of 1.81, indicating that the portfolio takes even larger concentrated bets compared to the traditional portfolio. Both the average monthly turnover and average monthly transaction costs have increased substantially to 131.91% and 0.77%. This shows that on average more than the total portfolio has been bought and resold at the monthly rebalancing date. The portfolio has average monthly transaction cost of 0.77% (approximately 9% annually), on average, which would greatly reduce the attractiveness of the portfolio in practice.

The last portfolio includes the second analyst characteristic, target-price-implied-dispersion (TIMD). The portfolio underweights firms, relative to the value-weighted benchmark portfolio, that show large target implied dispersion. By including this characteristic the portfolio returns further increases to 58.54% with a standard deviation of 24.16%. The incremental increase is smaller compared to before but still substantial. The Sharpe ratio, information ratio, Sortino ratio, and Omega ratio further increase to 2.336, 3.179, 1.399, 8.435, respectively. Moreover, the certainty equivalent increases to 43%. The increased performance is also reflected in the benchmark regressions that shows that the optimal portfolio yields a monthly alpha of 310bps (t=10.07), on average, over the value-weighted portfolio. Furthermore, the portfolio earns a statistically significant alpha of 280bps (t=10.71) over the equal-weighted portfolio. Finally, the portfolio outperforms the Fama-French-Carhart factor model with as much as 300bps (t=11.42) statistically significant at the 1% level. The positive and significant intercept of the times-series regression shows that the return of the characteristics based portfolio is not fully captured by the traditional risk-factors. The portfolio diversification has slightly improved relative to the previous portfolio. The HHI notes 1.76 which is in line with previous portfolios but substantially higher relative to the value and equal-weighted benchmark indices of 0.76 and 0.64. In line with previous findings, the table shows that's both the average transaction costs and average portfolio turnover increase further to 0.85% and 145.30%. Additional regression analysis[8] shows that the monthly alpha of the extended portfolio (3) over portfolio (2) is significant and a large as 93bps (t=5.376), a month. This shows that the characteristic TIMD does in fact add new information to the optimization if short sales are allowed. Moreover, the regression shows that the outperformance of the portfolio that includes both characteristics is as much as 241bps (t=8.79)[9], a month, over the traditional portfolio presented in column four.

Figure 6. presented an illustration of the 12-month (TTM) rolling performance of the portfolios over the investment period. The results indicate that the performance is not merely driven during one specific period in time. This shows that the optimized portfolio perform consistently over time. What is remarkable is that the optimized portfolio do also outperform in market downturns. This is line with the

[8] The regression regresses the returns of the extended portfolio (3) upon the portfolio that includes extended portfolio (2). The regression follows the same structure described in Table 3. These OLS-regression are not reported. Available upon request.

[9] The regression regresses the returns of the extended portfolio (3) upon the traditional portfolio. The regression follows the same structure described in Table 3. These OLS-regression are not reported. Available upon request.

notion that the short-leg of the portfolio provides a hedge during market-drawdowns. Figure 7. presented in de appendix plots the cumulative performance of the portfolios in table 8. with an initial investment of 1$. The plots reaffirm the observations made earlier. All optimized portfolios outperform the respective benchmark portfolio and the ability to short stocks provides a hedge in market-downturns. The portfolios that include both target-price-analyst (TIMR and TIMD) characteristics obtain the largest outperformance herewith confirming that investors can use the information from target-pricing in their portfolio to maximize the average utility she obtains. The results also show that the characteristics remain informative beyond the traditional characteristics. Similar to Brandt et al. (2009) the levered portfolios report large average sums of negative weights, sometimes almost as large as the whole portfolio due to the fact that the investor is allowed to short assets. The incremental introduction of multiple characteristics leads to an increase of both the average monthly transaction costs as well as the average monthly portfolio turnover. This shows that while new information is introduced that improves the performance also the trading activity increases substantially and therefore reduces the attractiveness of the portfolio for an investor. Such trading activity is often unfeasible and undesirable for investors as these are paired with increasing transaction costs. In combination with the large concentrated bets the portfolio may not be a feasible option for the investor to invest in therefore it would be interesting to examine the portfolio where leverage is constrained and investigate whether the outperformance remains persistent. It also allows us to gauge the impact of short-sales upon the portfolio concentration and trading activity.

Table 10. Incremental portfolios: No short-sales

The table presents the results for the incremental portfolios with ***no*** short-sales. The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristics, *MOM* reflects the 12-month momentum return, *INV* represents the investment characteristics. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristics Target-price-implied return and *TIMD* reflect the characteristic: Target-price-implied-dispersion. Panel B. Presents the results from the regression analysis described in table 3. The *VW* and *EW- bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01, t-statistics are presented in bold.

	Benchmark portfolio		Optimal portfolios		
Theta	VW	EW	(1)	(2)	(2)
θ *ME*			-1.78	-1.79	-1.79
θ *BTM*			0.59	0.34	0.34
θ *MOM*			0.02	0.02	0.02
θ *INV*			-0.02	-0.03	-0.03
θ *GRPROF*			0.01	0.02	0.02
θ *TIMR*				0.22	0.22
θ *TIMD*					-0.01
Min w_i%	0.00%	0.02%	0.00%	0.00%	0.00%
Max w_i%	0.61%	0.02%	0.26%	0.27%	0.27%
Av. $\lvert w_i \rvert$ %	0.02%	0.02%	0.02%	0.02%	0.02%
Av. sum w_i<0	0.00%	0.00%	0.00%	0.00%	0.00%
Av. % w_i<0	0.00%	0.00%	0.00%	0.00%	0.002%
$\bar{r}$	9.62%	13.96%	15.00%	18.27%	18.10%
$\sigma(\bar{r})$	15.11%	18.78%	20.02%	20.28%	20.15%
Sharpe ratio	0.545	0.666	0.677	0.827	0.824
Skewness	-0.734	-0.325	-0.111	-0.197	-0.232
Kurtosis	2.067	1.588	2.056	1.741	1.639
VaR (97.5%)	-6.97%	-7.97%	-11.07%	-11.08%	-11.05%
CVaR (97.5%)	-11.22%	-12.34%	-18.05%	-17.59%	-17.35%
Sortino (Mar= 0%)	0.294	0.362	0.375	0.446	0.443
Omega ratio			1.855	2.454	2.432
Info. ratio			0.734	1.159	1.161
HHI	0.758	0.141	0.193	0.180	0.179
TTC	0.04%	0.03%	0.06%	0.09%	0.09%
TTN	7.5%	5.5%	10.81%	15.00%	14.85%
CER	3.91%	5.14%	4.98%	7.99%	7.95%

Table 10. Panel B Regression analysis

	VW	EW	(1)	(2)	(3)
Constant			0.002**	0.005***	0.005***
			1.964	**3.762**	**3.770**
VW bmk			1.261***	1.279***	1.272***
			44.339	**45.08**	**45.782**
$Adj.R^2$			0.904	0.907	0.91
Constant			0.0002	0.002***	0.002***
			0.471	**5.637**	**5.732**
EW bmk			1.058***	1.074***	1.067***
			119.759	**136.871**	**141.856**
$Adj.R^2$			0.986	0.989	0.99
Constant	0.0004	0.003***	0.003***	0.005***	0.005***
	1.050	**8.454**	**5.55**	**9.326**	**9.379**
eMKT	1.015***	1.035***	1.043***	1.056***	1.057***
	96.053	**92.384**	**67.2**	**62.769**	**64.793**
SMB	0.152***	0.735***	0.78***	0.81***	0.803***
	9.063	**41.386**	**31.738**	**30.401**	**31.08**
HML	-0.119***	-0.114***	-0.03	-0.086***	-0.083***
	-7.022	**-6.385**	**-1.207**	**-3.209**	**-3.189**
RMW	-0.032*	-0.099***	-0.044	-0.137***	-0.135***
	-1.357	**-3.988**	**-1.273**	**-3.672**	**-3.74**
CMA	0.051**	0.002	0.066	-0.039	-0.033
	1.865	**0.071**	**1.638**	**-0.901**	**-0.771**
UMD	-0.028***	-0.112***	-0.222***	-0.191***	-0.179***
	-3.169	**-11.974**	**-17.125**	**-13.61**	**-13.201**
$Adj.R^2$	0.987	0.99	0.984	0.982	0.983

4.5. Incremental portfolio: without short-sales

4.5.1. Characteristics

The next section examines the optimized portfolios which short-sale restrictions imposed. This ensures that the optimal weights are never lower than zero. The results are presented in *Table 10*. The first portfolio shows that it is still optimal to deviate from the benchmark by underweighting large-cap firms. Similar to the portfolios with short sales, the size effect is present across the various portfolios. The average theta coefficient for the size characteristic is persistent in magnitude across all the portfolios. This is emphasized by the coefficients in the factor regressions which are also consistently close to 0.8 and statistically significant. The *SMB* coefficient for the traditional portfolio is 0.780 and statically significant at a 1% level (t=31.74), finally in the last portfolio, the factor coefficient has increased to 0.803 (t=31.08).

The *book-to-market* theta coefficients also remain consistently positive across the various portfolios. Similar to the portfolio in which cases were not allowed, the theta coefficient is consistently positive indicating a positive tilt towards value firms relative to the value-weighted benchmark portfolios. Similarly, the theta coefficient varies depending on what other characteristics have been included in the portfolio[10]. However, the factor regressions show a more nuanced picture. The exposure to value firms is negative across all portfolios. Nevertheless, the value exposure is larger compared to the benchmark portfolios justifying the positive theta coefficient. This shows that even though the factor exposure to the value factor remains negative on average, the negative exposure has decreased relative to the benchmark due to the active tilt towards value-firms.

The theta coefficient for *momentum* is consistently positive across all portfolios similar to when short sales were not prohibited. The magnitude of the theta coefficient has decreased substantially leading to a smaller deviation from the benchmark portfolio. The factor regression shows that exposure to the momentum factor is consistently negative and significant across all portfolios. The traditional The exposure to the momentum factor decreases further to -0.179 (t=13.201) as soon as all characteristics are included in the optimization, including TIMR and TIMD. This reveals that all optimal portfolios consistently have a lower exposure toward the momentum factor compared to both benchmark portfolios. Similar the size coefficient, the *investment* coefficient has also decreased substantially in magnitude. The sign remains consistently negative across all portfolios similar to the cases in which short sales were not prohibited. The regression shows the portfolio exposure towards the investment factor (CMA) is no longer significant and becomes negative for the portfolios including TIMR and TIMD.

The theta coefficient of the *profitability* characteristic is consistently positive consistent with the prior portfolios allowing for short sales. However, the magnitude of the coefficient has substantially decreased in line with the other characteristics. The factor regressions in panel B show that all portfolios carry a significantly negative exposure towards the profitability factor (*RMW*), most likely due to the implicit exposures towards other characteristics.

The theta coefficient for the *target price implied return* (TIMR) remains positive on average similar to the singleton portfolio and the portfolio in which short sales were allowed. Similar to before, the portfolios that include TIMR or TIMD carry different factor exposures vis a vis the traditional portfolio. The exposure to the size factor remains significantly large and positive with a regression coefficient on size of 0.81 (t=30.40) showing that the portfolio remains to have a substantial tilt towards small-cap firms. Moreover, the portfolio carries a regression coefficient of -0.086 (t=-3.209) towards the value factor indicating that the portfolio carries a negative exposure towards value firms and equivalently a positive exposure towards growth firms. The portfolio carries a significantly negative

[10] For a more in depth view on the incremental change of the characteristics, see table 11 in the appendix

exposure towards the profitability factor with a coefficient of -0.137 (t=-3.672). Finally, the portfolio carries a significantly negative exposure towards the momentum factor with a coefficient of -0.191 (t=-13.61). The final portfolio includes the characteristic *target-price-implied-dispersion* (TIMD). The theta coefficient has remained negative on average but decreased substantially in magnitude indicating that it no longer results in a substantial deviation from the equilibrium benchmark weights. As a result, the portfolio factor exposures are very similar to the portfolio including target price implied returns, showing only minor differences in the factor coefficients.

Overall the regression shows the optimized portfolios uniformly overweight to small firms at the expense of the exposure toward the remaining factors The underlying correlation structure plays a big role in explaining these findings. The explicit tilt towards the size characteristics results in an implicit underweighting of the remaining characteristics. All the theta coefficients are conform the expectations and mostly in line with the findings from the previous case in which short sales were not prohibited. Nevertheless, the average theta coefficients and the factor coefficients are closer to those of the benchmark portfolio demonstrating a smaller deviation from the benchmark positions. Except for the size factor, restricting short sales largely eliminates the positive exposures towards style factors that we saw in Table 8. Finally, the overall smaller deviations from the value-weighted benchmark portfolio result in optimal portfolios that tend to approximate the equal-weighted benchmark portfolio in terms of style exposures.

4.5.2. Performance

The portfolio statistics are presented in the second set of rows in Panel A of Table 10. I first discuss the traditional portfolio and follow up with the extended portfolios. The short-sale constraint enforces the minimum weight to be zero across all portfolios. Therefore by construction, the average sum of negative weights and the average fraction of shorted stocks, in the fourth and fifth column respectively, are equal to zero across all portfolios. The maximum weight in the portfolio is not constrained and therefore varies across the board but is never greater than 0.27%. The maximum weights are never greater than the value-weighted benchmark portfolio and are more comparable to that of the equal-weighted benchmark. This shows that the optimized portfolios are less concentrated, on average, and as result are more diversified. This is confirmed by the Hirschman-Herfindahl index that is now closer to that of the equal-weight benchmark. The average weights in the underlying assets, as presented in row three, is constant across all portfolios since the sum of over and under-weightings is always the same. The average weight in the underlying assets is equal to 0.02%, across all portfolios, which is identical to the average absolute weight of both benchmark portfolios. This shows that overweighting and underweighting firms affect the individual weights in the underlying assets but does not affect the average weights distributed across all assets.

The first thing to notice is that the annualized return across all portfolios has decreased substantially compared to the previous portfolios (with short-sales). The traditional portfolio now earns an annualized return of 15% with a standard deviation of 20.02%. This results in a Sharpe ratio of approximately 0.677. The skewness of this portfolio is slightly less negative while the kurtosis is also slightly smaller relative to the benchmark portfolios. This results in a Sortino, Omega ratio and, Information ratio of 0.375, 1.855, and 0.734, respectively. Finally, the traditional portfolio yields a certainty-equivalent of 4.98% which a gain (loss) of 1.06% (0.16%) compared to the value-weight (equal-weighted) benchmark. This is slightly lower than the 3.3% gain relative to the market-portfolio that Brandt et al. (2009) find. Medeiros et al. (2014) find a CER of 6.7% conditional upon the size, book-to-market, and momentum. Based on presented metrics I conclude that under short-sale constraints the traditional portfolio still outperforms the value-weighted benchmark but fails to outperform the equal-weighted benchmark. The regression statistics confirm this. The traditional portfolio yields an average monthly alpha of approximately 20bps over the value-weighted benchmark, statistically significant at the 5% level. The regression alpha of the portfolio over the equal-weighted portfolio is close to zero and statistically insignificant. Therefore I fail to reject the null hypothesis that the traditional portfolio offers an alpha that is not statistically different from zero. The traditional portfolio does however obtain a significant alpha of 30bps (t=5.55) over the Fama-French-Carhart factors, reaffirming the observation that the returns from the optimized portfolio are not completely explained by the traditional risk factors.

I proceed to examine the two optimized portfolios including TIMR and /or TIMD. The table shows that these two portfolios still achieve the greatest absolute performance relative to both the benchmark portfolios as well as the traditional portfolio. The optimized portfolio including the characteristic TIMR (TIMR and TIMD) yields an annualized return of 18.27% (18.10%) with a standard deviation of 20.28% (20.15%). This suggests that most of the performance improvement comes from the characteristic target price implied return. The portfolio including TIMR yields a Sharpe, Sortino, information and omega ratio of 0.827, 0.446 and 2.454, 1.159 respectively while the final portfolio including both analyst characteristics earns 0.824, 0.443, 2.432 and 1.161 respectively. These are close to identical. These results are reaffirmed in the regression results. The portfolio that includes the TIMR characteristic yields a monthly alpha of 50bps (t=3.76) relative to the value-weighted benchmark, statistically significant at a 1% level. Moreover, the portfolio earns a statistically significant alpha of 20bps (t=5.64) relative to the equal-weight portfolio. The optimized portfolios including both TIMR and TIMD yield almost identical results. Moreover, both the portfolios earn a significant of alpha of 50 bps each (t=9.33) and t=9.38), over the Fama-french-Carhart factors. The performance statistics and the regression results (non-reported[11]) show that including the characteristic TIMD does not further improve the performance of the portfolio relative to the portfolio with TIMR-characteristic as the regression alpha is economically negligible and insignificant. The regression[12] analysis shows that including both analyst characteristics does significantly improve the performance of the portfolio above that of the traditional portfolio. Specifically, by including both analyst characteristics the portfolio earns an alpha of 23bps (t=6.34), a month over the traditional portfolio. This results confirms the notion that analyst forecasts add value even when short sales are prohibited. It also shows that the characteristic target-price-implied-return provides valuable information that allows the investor to build a portfolio that is more profitable compared to all benchmarks.

The first takeaway from these results is that we can conclude that a traditional portfolio continues to outperform the value-weighted benchmark portfolio when short sales are prohibited. Only the final two portfolios that include information from analyst target prices outperform *both* the equal and value-weighted benchmark on a risk-adjusted basis. Including target-price-implied-dispersion (TIMD) does not improve the performance of the optimized portfolio if target price implied return is already included. This shows that all marginal improvements stem from the characteristics target price implied return. If we assume that including transactions are constant over time and add the average monthly transaction costs (TTC) as an overlay to the optimal portfolio alpha (50bps – 9 bps) the annualized alpha would still be as large as 41bps over the value-weighted benchmark. The alpha over the equal-weighted benchmark would marginalize to only 11bps a month. This leads me to conclude that including characteristics based upon target prices also leads to an improvement of performance in the optimal portfolio for which short-sales are prohibited. *Figure 8,* in the appendix, presents the 12-month rolling performance of the optimized portfolio. The performance is more in line with the benchmark portfolios. The

[11] The regression regresses the returns of the complete portfolio (Traditional + TIMR+ TIMD) upon the portfolio that includes TIMD
[10] The regression regresses the returns of the complete portfolio (Traditional + TIMR+ TIMD) upon the traditional portfolio
These OLS-regression are not reported. Available upon request.

outperformance remains pervasive but appears more gradual over time. *Figure 9*, in the appendix, presents the cumulative performance of the optimal portfolios with an initial investment of 1$. The cumulative outperformance continues to exists however, one should also note that the respective portfolios are also subject to greater downside risk. Overall the results show that analyst target prices in fact contain new information beyond that already spanned by traditional return-characteristics both for an investor that can short assets as well as investor who are prohibited from taking on short positions. Restricting short-sales has a large negative effect on the risk-adjusted return metrics but simultaneously improves the diversification of the portfolio. The HHI shows that the optimized portfolios are almost as diversified as the equal-weighted portfolio while the portfolio turnover and transaction costs are only slightly higher relative to the benchmark portfolios. This shows that the optimized portfolios that I present including analyst characteristics provide a CRRA investor with a feasible diversified investment opportunity, that offers persistent performance across time while only marginally increasing trading costs. 5

5. Robustness tests

5.1. Varying risk-aversion: With short-sales

The following paragraph will examine the characteristics and performance of the optimal portfolios across various hypothetical risk-aversion levels. The optimized portfolio will ultimately depend on the investors' preferences. Previously the risk-aversion level has been (arbitrary) set at y=5 for a CRRA investor. By varying the risk-aversion parameter it becomes clear how a CRRA investor would optimally invest his or her portfolio across the spectrum of risk-aversion levels.

5.1.1. Characteristics

Table 12. presents the results for the complete optimal portfolio, that includes all characteristics, for which short sales are allowed but restricted to 1 in line with prior exercises. I choose the portfolio with the most complete information set that includes all traditional and both analyst characteristics. When scanning the theta coefficients one can see that both the magnitude of the theta coefficient changes with the level of the risk-aversion level while also the sign of the theta coefficient changes for some characteristics, if y is increased sufficiently.

The sign of size coefficient is negative across all risk-aversion levels in line with previous findings, indicating a consistent underweighting of large-firms. The coefficient becomes more negative up to y=4 and then decreases again with increasing risk-aversion. The regression coefficient for the size factor decreases from 1.283 (t=10.52) at risk-aversion level y=1 to 0.41 (t=4.89) at a risk-aversion level of y=10 showing a decreasing exposure. This seemingly contradictory result is due to the underlying covariance structure of the characteristics. The other remaining characteristics simultaneously take an implicit bet, which in this case leads to a decrease of the actual exposure towards the size factor[13]. Theta coefficients that are associated with both risk and mean returns will decrease at higher risk-aversion levels (Brandt, et al., 2009). A risk-averse investor attributes a larger part of the returns to risk and chooses to reduce the exposure toward the risk factor. Following this logic, the decreasing size exposure shows that the size characteristic is associated with both mean returns and risk.

The theta coefficient for the book-to-market characteristic remains large and positive in line with previous findings. The regression coefficient shows that exposure to the value characteristic increases with greater risk-aversion, albeit modestly. A more risk-tolerant investor holds a negative exposure toward the value characteristics whereas a more risk-averse investor holds a positive exposure towards the value exposure. The regression coefficient changes from -0.129 (t=-1.049) at risk-aversion level y=1 to a coefficient of 0.099 (1.166), at a risk-aversion level of y=10. The increase in exposure to the value factor indicates that the characteristic is more associated with expected returns rather than with risk. This is in contrast to Fama and French (1992) who specifically define value as a risk-factor that would imply a stationary effect.

Interestingly, the theta coefficient of the momentum coefficient increases monotonically and becomes positive at a higher risk-aversion level. The regression coefficient of the momentum factor is significantly negative -0.84 (t= -13.087) at risk-aversion level y=1 and becomes significantly positive at 0.36 (t=8.173) at y=10. Both findings show that the exposure towards momentum increases in a linear fashion for a higher risk-aversion. The theta coefficient of the profitability characteristic also increases in a constant linear manner as risk-aversion increases. However, the factor regression shows that even though the theta increases constantly, the underlying factor coefficient to the profitability factor varies substantially across risk risk-aversion levels. The coefficient of the profitability factor increases from a negative -0.488 (t=2.863), at y=1, to a positive coefficient of 0.113 (t=0.961), at y=10. The exposure to profitability and value increases at higher risk-aversion levels indicating that these are mostly associated

[13] Note that the size characteristic and the size factor (*SMB*) move in opposite direction by construction

with returns. The theta coefficient towards the investment characteristics becomes more negative with increasing risk-aversion levels. In accordance, the regression coefficient towards the investment factor increases from a negative non-significant -0.165 (t=0.827) towards a positive and statistically significant coefficient of 0.506 (t=3.692). This shows that the investment characteristic is both associated with risks and mean returns that risk-averse investor seeks exposure to.

The theta coefficient of the characteristics target-implied-return (TIMR) is stationary and large in magnitude across all risk aversion levels. This shows that the characteristic is persistent across various risk-aversion levels. The consistent large coefficient shows that the characteristic is not directly related to a risk factor. In contrast, the theta coefficient of the target implied dispersion (TIMD) decreases substantially at greater risk-aversion levels indicating that the characteristic is clearly associated with risk. An investor with risk-aversion level y=10 substantially underweights firms with greater implied dispersion as these are associated with greater risk for a CRRA investor.

Finally, one can also observe that as risk-aversion increases a CRRA investor optimally reduces is exposure towards the market factor. A risk-averse investor (y=1) carries a statistically significant market exposure of 1.145 (t=14.861) whereas a risk-averse investor (y=10) carries a substantially lower, albeit still significant, market exposure of 0.953 (t=18.015). This shows that the market in fact represents a systematic risk-factor. Consequently, a more risk-averse investor seeks less exposure to such risk. The performance statistics show that for moderate levels of risk aversion the skewness of the portfolio decreases while the risk-adjusted returns increase. This shows the investor trades positive skewness for greater mean-variance efficiency. This finding is in line Lamoureux and Zhang (2014) that find unconstrained investors are unwilling to pay for beta, as a measure of non-diversifiable risk, and therefore hold fewer high-beta stocks at greater risk-aversion levels.

5.1.2. Performance

The second set of rows in Panel A. present the portfolio statistics. When risk-aversion increases from y=1 to y=10, the average sum of shorted stocks increase from 63.1% to 83.3%. Meanwhile, the average fraction of shorted stocks only increases from 27.8% to 31.2%, respectively. This implies that a more risk-averse investor takes similar bets in the underlying assets but does so with greater leverage. The active short positions help by hedging the worst-performing stocks in the months where the market portfolio has large drawdowns (Brandt, et al., 2009). This is reflected in the minimum and maximum weights in the underlying assets. As the investor becomes more risk-averse she takes larger negative bets in the underlying assets while taking smaller positive bets in the underlying. This is reflected in the increasing Hirschman-Herfindahl index that starts at 1.68 for a risk-averse investor (y=1) and gradually increases to a value of 1.80 for risk-aversion level y=10. These findings correspond to those of Brandt, et al. (2009) that also document that a more-risk averse investor uses more leverage.

By changing the risk-aversion level the optimized portfolio re-allocates capital into different securities. Not surprisingly, the return-distribution also changes. With increasing risk-aversion the annual return decreases substantially from 61.08% (y=1) to 40.23% (y=10) while the standard deviation decreases from 33.58% to 16.31%, respectively. Remarkably, the Sharpe ratio increases from 1.757 to 2.611 when the risk-aversion increases from y=1 to y=6. Similarly, the Information, Sortino, and the Omega ratios all increase in a linear fashion when risk-aversion is increased to moderate levels. The risk-adjusted performance seems to reach a turning point when risk-aversion is increased beyond y=6, after which all performance metrics seem to decrease somewhat. At greater risk-aversion levels, the curvature of the the utility function is dominated by the utility that is obtained in the worst month (Brandt, et al., 2009). The regression statistics confirm our earlier findings. The optimized portfolios (across all levels of risk-aversion) uniformly obtain a statistically significant alpha of at least 230bps (210bps) over the value-weighted (equal-weighted) benchmark portfolio.

The VaR (CVaR) at y=1 notes 12.32% (18.12%) that decreases to 6.49% (8.60%) for y=10. Non-reported results show that the portfolio's maximum drawdown decreases from 37.22% for y=1 to 15.25% for y=10. This emphasizes the fact that at higher risk-aversion levels the downside-risk dominates the utility of a CRRA investor. Lamoureux and Zhang (2020) find that as risk aversion increase the optimal portfolio shift mass from the flanks to its tails resulting in a portfolio with a similar standard deviation to the benchmark. Figure 10, to be found in the appendix presents the cumulative density plot of the returns across various risk aversion levels (y=1, 5, 10). The plot shows that the optimal portfolio with y=1 is characterized by a large positive kurtosis and greater positive skewness relative to the optimal portfolio with risk aversion levels y=5 and y=10. The large kurtosis leads to a large probability mass in (both) tails of the return-distribution, something that an investor dislikes especially at higher risk-aversion levels. Our results are in line with Brandt et al. (2009) who document that at greater risk-aversion levels, the (CRRA) investor preferences correspond to a max-min criterion that results in greater downside (loss) aversion. Our results also confirm the notion of Lamoureux and Zhang (2020) that more risk-averse investors carry portfolios that are more leptokurtic and have a significantly lower inter-quartile range.

5.2. Varying risk-aversion: Without Short-sales (NSS)

5.2.1. Characteristics

The next section shows how the optimal portfolios respond to varying risk-aversion levels when short sales are prohibited. Table 13. follows the same structure as before in which risk-aversion levels increase when reading from left to right. The results are mostly in line with prior findings and confirm the earlier conclusions that the investor is able to exploit information from analyst target-prices in her portfolio selection. According to Brandt et al. (2009) all characteristics that are associated with risk show a decrease in the absolute value of their theta coefficient at higher risk-aversion levels. The theta coefficient of size decreases at greater risk-aversion levels, this is also reflected in the regression coefficient of the size factor (*SMB*) that decreases from 0.845 (t=28.345) to 0.602 (t=27.521). Therefore exposure to large-cap firms decreases as risk aversion increases, similar to the short-sale portfolio.

The coefficient of the book-to-market characteristic remains stationary and increases somewhat with increasing risk-aversion. The sign of regression coefficient remains negative but increases with at for higher risk-aversion. The portfolio optimally holds a smaller negative exposure towards the value factor relative to both the equal and value-weighted benchmark portfolio, in line with the short-sale portfolio.

Similar to the case with short sales, the momentum characteristic increases when the investor becomes more risk-averse. The regression statistics show that the investor remains underweighted momentum firms compared to both benchmark portfolios. In the short-sale portfolio, this underweight became overweight. The underweighting does decrease from -0.196 (t=-12.489) at y=1 to and -0.088 (t=-7.620) at y=10. The theta coefficient of the investment characteristic becomes increasingly negative at higher risk-aversion levels, similar to the short-sale portfolio. In accordance, the regression shows that the factor coefficient increases from a negative -0.057 (t=-1.167) for y=1 to a 0.055 (t=1.524). This shows that as risk-aversion increases, the investor seeks to increase her exposure towards the investment factor. The theta coefficient of the profitability characteristic has become very small and stays stationary across the board. The regression coefficient at y=1, is a large as -0.162 (t=-3.895) and slightly increases towards -0.081 (t=-2.647) when risk-aversion increase to y=10. Hence, a more risk-averse investor tends to decrease the underweight in the profitability factor relative to the benchmark portfolio. This confirms the findings of the short-sale portfolio.

The patterns produced by the two analyst characteristics are similar as in the portfolio with short-sales. While both characteristics tend to be associated with risk, target price implied dispersion decreases more with increasing risk-aversion. A more risk-averse investor decreases the exposure to firms that show a greater dispersion in its analyst forecasts while only moderately decreasing her exposure towards firms that show higher target-price implied returns.

5.2.2. Performance

The second set of rows in Panel A of table 13. present the distribution of the optimal portfolio weights'. By construction, the minimum weights are at least zero. Naturally, the average sum of absolute weight below zero is 0% as short-sales are prohibited. The portfolio diversification decreases somewhat as risk-aversion increases showing that the investor takes on more concentrated bets in order to reduce the probability of large drawdowns. At risk-aversion level y=7, the portfolio is most diversified, as observed by the HHI of 0.175 and thereafter increases to 0.192 for y=10. This similar phenomenon is observed in the portfolio with short-sales. The portfolio transaction costs and turnover show a similar pattern, both decrease up to y=7 after which they increase again. At greater risk-aversion levels the investor utility function corresponds to a max-min criterion in which the investor maximizes the minimal return it would have obtained over the investment horizon. Risk-aversion y=7 seems to be the optimal point after which the investor trades off a lower drawdown for greater transaction costs and lower diversification. Restricting short-sales clearly reduces the extreme bets of the portfolios which leads to the absolute risk in terms of VaR, CVaR, and variance of returns to decrease.

The change in portfolio allocation naturally translates into a different performance. The portfolio yields an annualized return of 19.76% (13.86%), for y=1 (y=10), with a standard deviation of 20.55% (18.01%). The Sharpe ratio decreases from 0.887 to 0.690, when the risk-aversion parameter increases from y=1 to y=10. Similarly, the Sortino ratio, Omega, and Information ratio decrease with monotonically increasing risk-aversion. Finally, an CRRA investor trades off a lower kurtosis for a more negatively skewed return distribution. This is illustrated in the cumulative density plot in figure 9. This figure shows that investor dislikes extreme returns and seeks to maximize the minimum return over the investment period, especially at greater risk-aversion levels. All portfolios yield a significant alpha of at least 40bps (20bps) over the value-weighted (equal-weighed) benchmark portfolios. Remarkable is that the very risk-averse investor (y=10) receives a negative certainty equivalent return. This is consistent with Lamoureux and Zhang (2014) that also find negative CER at risk-aversion levels of y=10 and larger. A negative CER would imply that the investor is unwilling to hold risky stocks. At y=10, the annualized alpha over the value-weighted benchmark portfolio continues to be significant and is as large as 20bps (t=2.62) a month, on average. At the same time, the alpha of the optimized portfolio with y=10 the equal-weighted benchmark has diminished to an insignificant 0bps (t=1.12).

5.3. Different objective functions

The foremost reason to take the utility function of a CRRA investor as a starting point for optimizing the investment portfolios is the fact that it takes higher moments of the return-distribution into account. However, Ammann, et al., 2016 argue that using the power-utility function of a CRRA investors might also lead to problems in the optimization as the utility function of a CRRA investor might fail to identify a global optimum. When the optimal solution results in it a local maximum the optimization presents a sub-optimal solution and the investor might not choose the best portfolio that reflects her preferences. In order to test the impact of the objective function, and test the versatility of other investor preferences, I conduct the same optimization as before while testing for different objective functions. I test the optimization with mean-variance preferences, log utility (which in the ideal situation, would correspond to y=1) and finally, I optimize the portfolio by maximizing the Sharpe ratio. I perform this exercise for both the portfolio with short sales as well as for the constrained portfolio without short-sales. Table 14, in de appendix, present the results for the beforementioned portfolios.

5.3.1. Log-utility

By scanning the results in table 14. One can already note that the theta coefficients are consistent across the various objective functions except for the log utility. The preferences under a log-utility function[14] are very similar to prior results, as log utility represents a special case of CRRA utility where the risk-aversion level is set at y=1. The results are indeed similar to the results obtained in the previous exercise where I explicitly set risk-aversion to 1. When short-sales are *allowed*, both the optimal portfolio with logarithmic-preferences and the power utility (CRRA with y=1) carry a positive theta towards the characteristic target-price-implied-dispersion and a negative theta towards the momentum characteristic, whereas the portfolios with other objective-portfolios carry contrasting signs. The magnitude and sign of the remaining coefficients is very similar to the other portfolios in table 14. The optimized portfolio carries a negative coefficient towards the size and investment characteristics and a positive sign towards the book-to-market, profitability, and target-price-implied return characteristics. For the portfolio in which short sales are *not allowed* the theta coefficients are in line with previous findings from the power-utility (CRRA) portfolios with risk-aversion y=1. The coefficients are also in line with results from the other objective functions except for target-price-implied-dispersion that carries a contrasting sign.

The distribution of weights is approximately the same across all objective functions with few slight nuances showing that the portfolio takes consistent bets. The portfolio optimized upon log-utility carries slightly lower minimum weights, greater positive weights, and carries a lower sum of negative weights. This results in a lower Sharpe, information, omega, and Sortino ratio compared to other portfolios. The logarithmic portfolio with short sales (no short sales) manages to achieve the second-highest absolute annual return of 61.11% (20.4%) a standard deviation of 34.88% (20.64%). The factor exposures of the log-utility portfolio (both SS and NSS) closely resemble the results of the CRRA portfolio (with y=1). In both cases, the portfolios carry a significant positive exposure towards the market and the size factor while also having a significant negative exposure towards the value, profitability, and momentum factor. The optimized portfolio obtains a monthly alpha of 290bps (t=7.01) and 250bps (t=7.44) over the value-weighted and equal-weighted benchmark portfolio, respectively. Both results are economically large and statistically significant at the 1% level. When short sales are prohibited the monthly alpha reduces to 60bps (t=4.67) and 40bps (t=8.54) over the value-weighted and equal-weighted benchmark portfolio, respectively. I conclude that the logarithmic-portfolio closely

[14] $Max\ E(log(1 + r_{pt+1}))$

resembles the results from the CRRA-portfolios, both in terms of exposures and performance and therefore the conclusions remain unchanged.

5.3.2. Mean-variance preferences

The theta coefficients of the portfolios optimized for a mean-variance utility function are very much similar to the CRRA-portfolio and the portfolio optimized upon the Sharpe ratio. The theta coefficients only differ in terms of magnitude. The mean-variance[15] objective function specifies that an investor would like to generate the highest possible return associated with a certain level of risk, given her risk tolerance. The portfolio with short-sales (without short-sales) yields an annualized return of 45.03% (14.41%) with a standard deviation of 17.06% (18.43%). The mean-variance investor optimizes the portfolio only upon the first two moments which has obvious implications on the shape of the return-distribution. If short sales are allowed the returns are negatively skewed. The portfolio yields a Sharpe ratio of 2.53 with an omega ratio of 7.88. Comparing these to the returns of the CRRA-portfolio, the results show that the investor makes an active tradeoff between a greater mean-variance efficient return at the cost of greater negative skewness in the portfolio. The results for the portfolio with no-short sales are very similar and the conclusions remain identical. A mean-variance investor has different style preferences vis a vis a CRRA investor. The mean-variance investor takes significant exposure towards both the investment factor and the momentum factor. It shows that the investment and momentum factors are more attractive from a mean-variance perspective but less so when taking into account higher moments. In contrast, both the size and the value factor seem less attractive. The optimized portfolio obtains a monthly alpha of 250bps (t=11.51) and 240 bps (t=11.17) over the value-weighted and equal-weighted benchmark portfolio, respectively. Both results are economically large and statistically significant at the 1% level. When short sales are prohibited the monthly alpha reduces to 20bps (t=2.66) and 10bps (t=1.89) over the value-weighted and equal-weighted benchmark portfolio, respectively. These results reaffirm the prior conclusion that the optimized portfolio outperforms both benchmark portfolios and the investor is better-off by incorporating information from target prices into the selection problem.

5.3.3. Sharpe-ratio

The final portfolio is set to maximize the Sharpe ratio[16]. The portfolio remains overweight, on average, towards the book-to-market, momentum, profitability, and implied return characteristics relative to the value-weighted benchmark portfolio. Moreover, the portfolio remains underweights towards the size, investment, and implied dispersion characteristics relative to the value-weighted portfolio. The fact that the portfolio still overweighs the characteristic target-price-implied return (TIMR) and underweights target-price implied dispersion (TIMD) supports prior conclusions and shows the active tilts are robust and in line with the CRRA portfolio. The distribution of weights is more or less similar to a CRRA investor as shown by the minimum (maximum) weights, the average absolute weights, and the Hirschman-Herfindahl index. The optimal portfolio with short-sales (no short sales) yields an annualized return of 67.93% (21.47%) with a standard deviation of 25.31% (20.83%) resulting in a Sharpe ratio of 2.59 (0.956). This is the highest Sharpe ratios across the optimal portfolios. The Sharpe ratio is almost 5 times as large as the value-weighted benchmark portfolio in the short-sale case and almost twice as large in the constrained case. The skewness and kurtosis of the portfolio with short-sales (without short sales) are 0.72 and 3.36 (-0.13 and 1.66), respectively, substantially larger than the benchmark portfolios. The greater absolute performance comes at the costs of a greater turnover and larger monthly

[15] $Max\ E(r_{pt+1}) - \left(\frac{y}{2}\right) * VAR(r_{pt+1})$,

[16] $Max\ E\left(\frac{r_{pt+1}-rf}{SD_{pt+1}}\right)$

transaction costs. An investor that optimizes her Sharpe ratio and is allowed to enter short position prefers style exposure towards the size, investment, and momentum factor while it holds negative exposure towards the value and profitability factor. If short-sales are restricted she loses her exposure towards the investment and momentum factor. This could be due to the inability to hedge bad performing assets in the markets' worst performing months (Brandt, et al., 2009). The optimized portfolio obtains a monthly alpha of 350bps (t=10.99) and 330bps (t=11.40) over the value-weighted and equal-weighted benchmark portfolio, respectively. Moreover, when short sales are prohibited the monthly alpha reduces to 70bps (t=4.966) and 40bps (t=8.449) over the value-weighted and equal-weighted benchmark portfolio, respectively. These results are economically large and statistically significant at the 1% level. Overall the results of the optimal portfolio that maximize the Sharpe ratio is similar to the portfolio of the CRRA investor and the conclusion that investor benefit from incorporating information from analyst-target prices still holds.

5.4. Sub-sample

An often-heard argument in portfolio optimization is that characteristics-portfolio are driven by the overrepresentation of small or micro-cap firms in the sample. To control for such sample-bias I re-optimize the complete traditional portfolios and the portfolios that include all analyst characteristics while restricting the sample to include only firms that fall above the 90th size percentile (NYSE cutoff). This way I arbitrarily restrict the sample to exclude only firms with a market capitalization of at least 2.671 bln. I do so for both short-sales and no-short sale portfolios. Similar tests have been performed by Fama & French (2008) and Lewellen (2015) as a simple check to test whether predictability is driven by smaller-caps stocks. The results are presented in table 15. The results for the portfolio with short sales are very much in line with the results observed before. The signs of the coefficients are identical to the full sample and have only changed in terms of magnitude. The coefficient towards the size characteristic has become more negative for both the short sale as well as short sales constrained portfolio as a natural result from the restriction on market-capitalization. The same goes for the book-to-market characteristic and profitability. The coefficients towards the momentum and investment characteristics have increased across both Short-sale and short-sale constrained portfolios. This shows that albeit the underlying firms change, the cross-sectional exposure remains more or less the same. The portfolio performance has changed in absolute terms but stayed the same in relative terms. This effect is underlined by Fletcher (2017) that find a positive relationship between the number of securities in the optimal portfolios and the performance. Also Fama & French (2008) and Lewellen (2015) find that return-predictability is weaker amongst large stocks. The traditional portfolio, which includes the size, book-to-market, momentum, investment, and profitability characteristic outperforms the benchmark portfolios both in the case with and without short sales. The resulting alpha is still large and significant and of similar magnitude as was the case in the full sample. This shows that the performance of the traditional portfolio is robust to changes in the sample.

The portfolios that include TIMR and TIMD characteristics also show strong persistent performance but change in terms of magnitude. The optimal portfolio that allows for short-sales yields an average monthly alpha of 170bps (t=8.17) and 160bps (t=7.66) over the value and equal-weighted-benchmark portfolio respectively. In the full sample, this was 310bps (t=10.07) and 280bps (=10.705). Nevertheless, regression results[17] show that the portfolio including the analyst target price characteristics still obtains a monthly alpha as large as 90bps (t=5.49), on average, over the traditional portfolio that includes the traditional characteristics (In the full sample this was 93 bps). Therefore I still conclude

[17] OLS regression is available upon request

that the additional characteristics target-price-implied-dispersion and target-price-implied-return provide new independent information to an investor. The same conclusion holds for the optimal portfolio for which short sales are constrained. The average monthly alpha of the portfolio with analyst-characteristics is as large as 10bps (t=5.06) (In the full sample this was 23bps) over the traditional portfolio.

6. Conclusion

This thesis studies the benefits of using analyst target price forecast into the portfolio selection problem. I parameterize the optimal portfolio upon different characteristics that have empirically been shown to span the cross-section of expected returns. The optimal parameters are then found by maximizing the average utility that the investor would have obtained over the sample period. The portfolios are constructed using firm-level information that is widely available to investors. In addition to size, value, book-to-market, investment, size and profitability, I incorporate consensus analyst target prices into the portfolio selection problem and examine whether this improves the utility that the investor receives by investing in the optimized portfolio.

The results from the individual portfolios show that the optimized portfolio deviates from the benchmark portfolio by actively overweighting firms that receive higher target price implied returns or firms that are characterized by a smaller dispersion of implied target price returns. The investor deviates from the market portfolio by overweighting firms with positive target price implied return while underweighting firms with large dispersion in target prices. By incorporating target price implied return or target price implied dispersion as a singleton an investor can attain an absolute outperformance relative to the market portfolio. The risk-adjusted return ratios and certainty equivalent return are all greater compared to the market-portfolio portfolio. The question then arises is whether the target-price characteristics remain persistent after the inclusion of traditional characteristics previously identified by literature.

Next, I show that a CRRA investor can improve her utility by investing in a portfolio that deviates from the benchmark portfolio by underweighting large-cap firms, and firms that have pursued large investments of the past year. Moreover, I show that the investor optimally overweighs firms with greater book-to-market values, that have shown greater past 12-month stock-returns and finally firms that have greater profitability (relative to the benchmark). By doing so the investor significantly outperforms the value-weighted (equal-weighted) benchmark by as much as 90bps (60bps), on average, a month. This result is both economically as well as statistically significant. The utility of the investor is further improved by including the characteristic target price-implied return in the optimization problem. The investor obtains a significant monthly alpha of 270bps (240bps), on average, over the value-weighted (equal-weighted) benchmark portfolio. Next, I show that the utility is maximized when *both* the characteristics, target price implied return and target price implied dispersion, are included in addition to the traditional characteristics. This particular portfolio yields a statistically significant monthly alpha of 310bps (280bps) over the value-weighted (equal-weighted) benchmark portfolio. Moreover, the regression alpha over the Fama-French-Carhart factors is as much as 300bps, a month, on average. The portfolio that includes both analyst characteristics obtain a significant monthly alpha of 241bps over the portfolio that include all the traditional characteristics. These results reveal that the returns of the optimized portfolio are not fully accounted for by exposure to the common risk factors or exposure to traditional characteristics such as size, book-to-market, momentum profitability or investment. These results lead me to conclude that a risk-averse investor can indeed use information from consensus analyst target price into the portfolio selection problem to obtain a greater out-of-sample utility both in an absolute sense as well as relative to the benchmark portfolios. An investor should therefore expand his information set beyond the traditional characteristics to include information conveyed by analyst target-prices.

In order to isolate the effect of short-sales on the portfolio, I re-optimize all portfolios but restrain the ability to assume short positions in the underlying securities. Restricting leverage leads to a notably lower annualized return, variance, and certainty-equivalent-return relative to the portfolios with short sales. This is due to the fact that unconstrained portfolios can exploit both positive and negative return-forecasts. The leverage also allows the portfolio to increase its exposure in the long-positions and therefore take more concentrated bets (Brandt et al., 2009). Nevertheless, I find that the traditional portfolio continues to outperform the value-weighted benchmark with a statistically significant alpha of 20bps a month. The alpha over the equal-weighted benchmark is no longer significantly different from zero showing that the returns of the traditional portfolio are mostly driven by exposure to the market factors. This confirms the notion of DeMiguel et al. (2009) that the equal-weighted benchmark delivers consistent and robust performance unmatched by most optimized models. Nevertheless, I show that by including the characteristic target price implied return in addition to the traditional characteristics the investor is in fact able to improve the performance of her portfolio. By doing so the portfolio yields a significant monthly alpha of 50bps (20bps) over the value-weighted (equal-weighted) benchmark portfolio. The characteristic target price implied dispersion does not improve the performance of the portfolio when short-sales are disallowed. The additional alpha that the investor earns by including analyst information into the portfolio optimization is close to 23bps, a month over the traditional portfolio. This result is both economically as well as statistically significant. This result is confirmed by the Fama-French-Carhart regression that shows that the particular portfolio earns a significant alpha of 50bps, a month, over the Fama-French-Carhart five-factor model. In general, if find that as a result of the short-sale restriction the active divergence from the equilibrium benchmark portfolio decreases substantially leading to a portfolio that approximates the benchmark. Moreover, the large concentrated active bets in the underlying assets disappear leading to a portfolio that is better diversified. Finally, restraining short-sales normalize the optimized portfolio which leads to a substantially lower average portfolio turnover and transaction costs compared to the short-sale portfolios. Therefore the constrained portfolio reflects a more feasible investment solution to the investor. Altogether I conclude that also constrained investors with short-sales prohibitions can exploit information from consensus target-prices to improve the performance of the portfolio relative to the benchmark portfolios and beyond the traditional characteristics.

In order to study the robustness of the results, I perform various tests. First I vary the risk-aversion level across the optimized portfolios that includes both analyst-characteristics. The results show that the investor modifies her portfolio by tilting towards characteristics that provide the greatest return given the level of risk that the investor wants to bear. The coefficient towards the characteristic target price implied return is consistently positive showing that the characteristic is only moderately associated with risk. The coefficient towards the target price implied dispersion becomes more negative at greater risk-aversion levels, indicating that the characteristic is associated with risk. By optimizing upon the firm-characteristics and taking into account the higher moments of the return distribution through the (CRRA) utility function the investor obtains a portfolio that is more positively skewed and displays smaller kurtosis. Up to moderate levels of risk-aversion the investor trades-off positive skewness for greater mean-variance efficiency. At very high risk-aversion levels the (CRRA) investor becomes increasingly averse to downside risk. The investor reduces the probability of experiencing large drawdowns resulting in a decrease in mean-variance efficiency. Altogether the optimized portfolios continue to significantly outperform the benchmark portfolios and herewith reaffirm the conclusion that the investor can exploit the information from target prices into her portfolio selection problem. In order to examine whether particular investor preferences drive the results, I re-optimize the portfolios using different objective functions. By doing so, I reaffirm the previous results and show that the results of the optimized performance are not ultimately dependent on specific investor preferences and remain robust across different objective functions. Finally, I control for the risk that the results are driven by small-cap firms. I restrict the sample to include only firms above the 90th size percentile. Counter to Fletcher (2017) I find the performance benefits to be persistent amongst the subset of firms characterized by larger market capitalization.

Overall my thesis provides new insights into the portfolio selection problem with respect to the value of consensus analyst target prices. According to my findings, investors should consider including firm characteristics in the optimization of their portfolio and augment the information set by including information disclosed by analysts in the form of target prices. I specifically show that target price implied returns and target price implied dispersion add value to an investor and provide new information beyond that already disclosed by the traditional firm-level characteristics.

Further research is required to investigate the underlying drivers of returns of the optimized portfolio that include firm-level information. More research needs to be done to examine what exactly drives the outperformance of the portfolio that includes analyst information in particular. By portioning firms into various industries or sectors, sub-portfolios one could examine what sectors or sub-portfolios outperform or underperform. It would prove interesting to investigate how this performance varies over multiple macro-economic cycles and examine if analysts can continue to offer valuable information across such cycles. Moreover, further research should investigate what resides in the short-leg of the portfolio and how the performance of the short-leg impact the total portfolio. An-in depth analysis could uncover what specific type of firms an investor would optimally over (under) weights in times of market distress. these relate to analyst target prices. Finally, it would be interesting to explore additional constraints upon the optimization such as active risk-budgets. One could introduce tracking-constraints to make the portfolio a feasible solution for large institutional investors that are often bounded by such risk-constraints.

7. Limitations

There are several limitations to the thesis that I conduct. First, the sample is constrained by the availability of analyst-target prices that only become available at the end of 1999. This means that the optimization is only performed over a relatively short period of time, not including multiple economic-cycles. Moreover, the optimization has been performed by including return-based factors that have been proven to predict the cross-section of expected returns. Albeit all results are obtained out-of-sample, the traditional characteristics included in the optimization are known to have explanatory value, hence the optimization is subject to a 'data snooping' bias. This bias is mitigated through the inclusion of non-return based factors such as analyst characteristics that have not yet been applied in such context. A third limitation is that the implementation of the optimal portfolios might be subject to liquidity constraints. I do not explicitly account for the probability of the underlying assets not being liquid enough or untradable. This limitation however is somewhat mitigated through restricting the sample by only including large-cap stocks. Finally transaction costs should be explicitly included in the optimization function in order to obtain feasible portfolios results that could be implementable in a cost-efficient manner. I leave this for further research.

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9. Appendices

Merging protocol

In order to properly merge the three aforementioned databases I make use of two linking tables. The first is the **CRSP/Compustat Merged Database – Linking Table**. From the linking table the following variables are retrieved. (1) The Compustat identifier (Gvkey), (2) the historical CRSP PERMNO ~ Compustat link (lpermno), (3) the link date at which the link became effective (linkdt), (4) the link end-date that is the last date at which the link was active (linkenddt), (5) the link-type (linktype) providing detailed information of the link data. I filter the data to include only *linktypes* "LU", "LC" and "LS" and herewith exclude firms that have inaccurate (or soft) links in. And finally (6) the primary issue indicator (linkprim) that indicates whether the link relates to a primary security or not. The reason for including this filter is to produce a constant primary security throughout the firms' history. I include "P", that is the primary market identified by Compustat in monthly security data. I include the "C", marker assigned by CRSP to overcome overlapping or missing ranges of primary markers from the Compustat database. And finally "J" marks joiner secondary issues of a firms identified by Compustat in monthly security data (WRDS, n.d.). Then the linktable is merged to the Compustat database based upon the Gvkey with the restrictions on *linktype* and *linkprim* described before. Next the firm-month observations (based upon the date) fall beyond of the effective link period are excluded. Subsequently, the CRSP dataset and the linked Compustat dataset are merged by Permno and Date. The second linking table that I use is the **IBES CRSP Link** table. The linking procedure follows the procedure specified by WRDS through their linking databases. From the linking table the I retrieve the following variables: (1) Permanent company identified as before (permno), (2) the 8-character historical CUSIP (corresponding to CRSP) (Ncusip), (3) the effective state time of the link (SDATE), (4) the effective end date of the link (EDATE). Subsequently the link table is merged to the full IBES data file by date and CUSIP. I filter out observations that occur before the effective link-date (SDATE) and that occur beyond the link-period has ended (EDATE) i.e. that lie outside the linked period. Finally, I merge the IBES dataset with the CRSP/COMPUSTAT dataset by date and the PERMNO identifier.

Table. 4. Construction of the variables

The table presents the construction of the characteristics. *i*, denotes a specific firm, at month *t*. ($\tau = 1$). *Sqq* is shareholders equity, *Ceq* is common equity, *Atq* is total assets, *Tlq* is total liabilities, *Txditcq* is deferred taxes and Investment tax credit, *Txdbq* is deferred taxes, *Itccy* is investment tax credit, *Pstkrq* is Total redeemable preferred stock and *Pstkq* is Preferred stock. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic Target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. ASTCHG reflect the firm year-over-year change in assets, CFPROF reflects cash flow profitability, OPPROF reflects the characteristics operating profitability.

Variable name	**Literature**	**Measurement of variable**
Book value of equity	Fama and French (1992, 1993)	$BE_{i,t=} \big(Stockholders\ equity_{i,t}\ (seqq)\ \boldsymbol{or}\ Common\ equity\ (ceq)\ \boldsymbol{or}\ (Total\ assets - Total\ liabilities)\big) + Deferred\ Taxes\ and\ Investment\ Tax\ Credit\ (txditcq)\ \boldsymbol{or}\ (Deferred\ taxes(txdbq) + investment\ tax\ credit\ (itccy)) + Total\ Redeemable\ Preferred\ stock\ (pstkrq)\ \boldsymbol{or}\ Total\ Preferred\ stock\ (pstkq)$
Market value of equity	Fama and French (1992, 1993)	$ME_{i,t} = SHARES_{\ i,t} * PRC_{i,t}$
Book-to-market	Fama and French (1992, 1993), Clifford and Frazzini (2013)	$BTM_{i,t} = \frac{BE_{i,t-4}}{ME_{i,t-1}}$
Momentum	Jegadeesh and Titman (1993) Lewellen (2015)	$MOM_{i,t} = \sum_{\tau=2}^{12} r_{i,t-\tau}$
Gross-profitability	Novy-Marx (2013)	$GRPROF_{i,t} = \frac{revtq_{i,t-4} - cogsq_{i,t-4}}{atq_{i,t-4}}$
Cashflow-profitability	Lakonishok et al. (1994)	$CFPROF_{i,t} = \frac{ibq_{i,t-4} + txdiq_{i,t-4} + dpq_{i,t-4}}{ME_{i,t-1}}$
Operating-profitability	Fama and French (2015)	$OPPROF_{i,t} = \frac{revtq_{i,t-4} - cogsq_{i,t-4} - xsgaq_{i,t-4} - xintq_{i,t-4}}{BE_{i,t-4}}$
Investment	Chen et al. (2011), French & Fama, (2006, 2008, 2015)	$INV_{i,t} = \frac{(ppegtq_{i,t-4} - ppegtq_{i,t-16}) + (invtq_{i,t-4} - invtq_{i,t-16})}{atq_{i,t-16}}$
Asset-change	Cooper et al. (2008)	$ASTCHG_{i,t} = \frac{atq_{i,t-4} - atq_{i,t-16}}{atq_{i,t-16}}$
Target-price-implied return	Da & Schaumburg, (2011), Engelberg et al. (2019), Palley et al. (2019)	$TIMR = \frac{TP_{i,t}}{Prc_{i,t-1}} - 1$
Target-price-implied dispersion	Diether et al., (2002), Da & Schaumburg, (2011), Palley et al. (2019)	$TIMD = \frac{STDEV\ TP_{i,t}}{Prc_{i,t-1}} - 1$

Table 6. Average time-series correlation plot

ME, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic Target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. ASTCHR reflect the firm year-over-year change in assets, CFPROF reflects cash flow profitability, OPPROF reflects the characteristics operating profitability. Note: *p<0.1 , **p<0.05, ***p<0.01. p-values are presented between parentheses

	RET	ME	BTM	MOM	INV	GRPROF	TIMR	TIMD	ASTCHG	CFPROF	OPPROF
RET	1.000	-0.027	0.152	-0.059	-0.077	-0.045	0.045	-0.007	-0.125*	0.025	-0.035
		(0.682)	(0.018)	(0.363)	(0.235)	(0.485)	(0.490)	(0.912)	(0.052)	(0.703)	(0.587)
ME		1.000	-0.518***	0.035	0.002	-0.677***	-0.280***	-0.434***	0.008	-0.501***	-0.337***
			(0.000)	(0.588)	(0.981)	(0.000)	(0.000)	(0.000)	(0.903)	(0.000)	(0.000)
BTM			1.000	-0.506***	-0.056	0.103	0.462***	0.577***	-0.244***	0.667***	0.332***
				(0.000)	(0.385)	(0.113)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
MOM				1.000	-0.030	0.183***	-0.339***	-0.370***	0.134**	-0.169**	-0.050***
					(0.641)	(0.004)	(0.000)	(0.000)	(0.039)	(0.009)	(0.000)
INV					1.000	0.231***	0.566***	0.434***	0.889***	0.392***	0.293
						(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.440)
GRPROF						1.000	-0.016	-0.032	0.244***	0.576***	0.729***
							(0.802)	(0.623)	(0.000)	(0.000)	(0.000)
TIMR							1.000	0.909***	0.572***	0.443***	0.142***
								(0.000)	(0.000)	(0.000)	(0.000)
TIMD								1.000	0.394***	0.385***	0.0396**
									(0.000)	(0.000)	(0.028)
ASTCHG									1.000	0.260***	0.201
										(0.000)	(0.542)
CFPROF										1.000	0.759**
											(0.002)
OPPROF											1.000

Figure 4a. Risk confidence sensitivity VaR (value-weighted benchmark)

The first plot depicts the Modified and Gaussian VaR over various confidence levels for the value-weighted benchmark portfolio. The second figure plot the same Modified and Gaussian VaR over various confidence levels for the equal-weight portfolio.

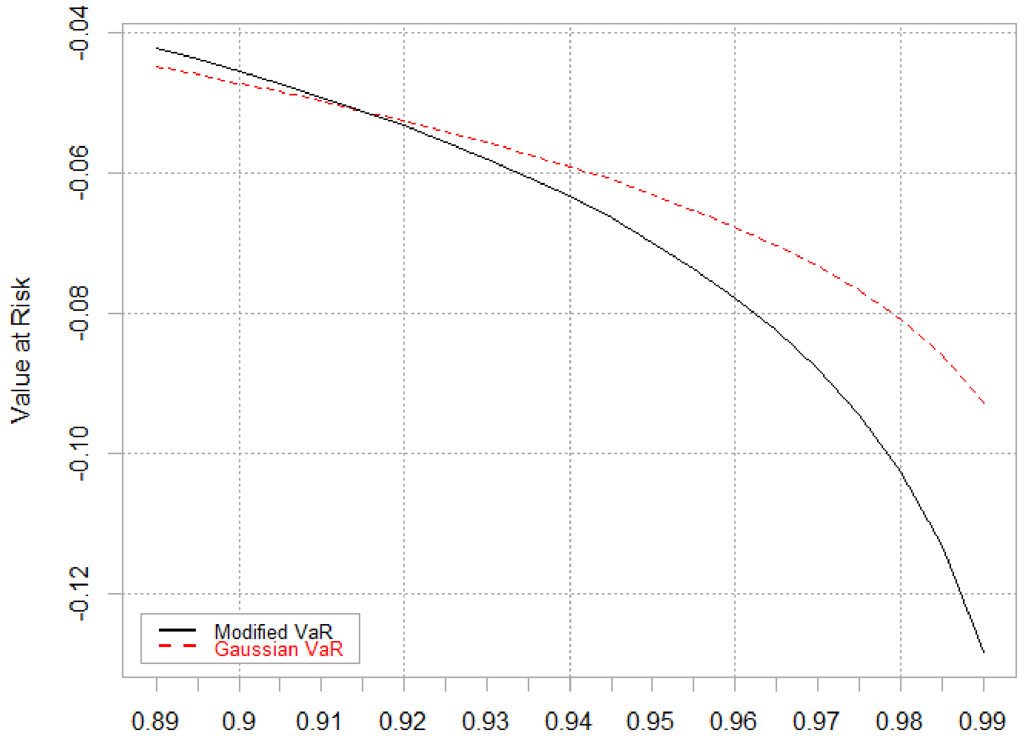

Figure 4b. Risk confidence sensitivity VaR (equal-weight benchmark)

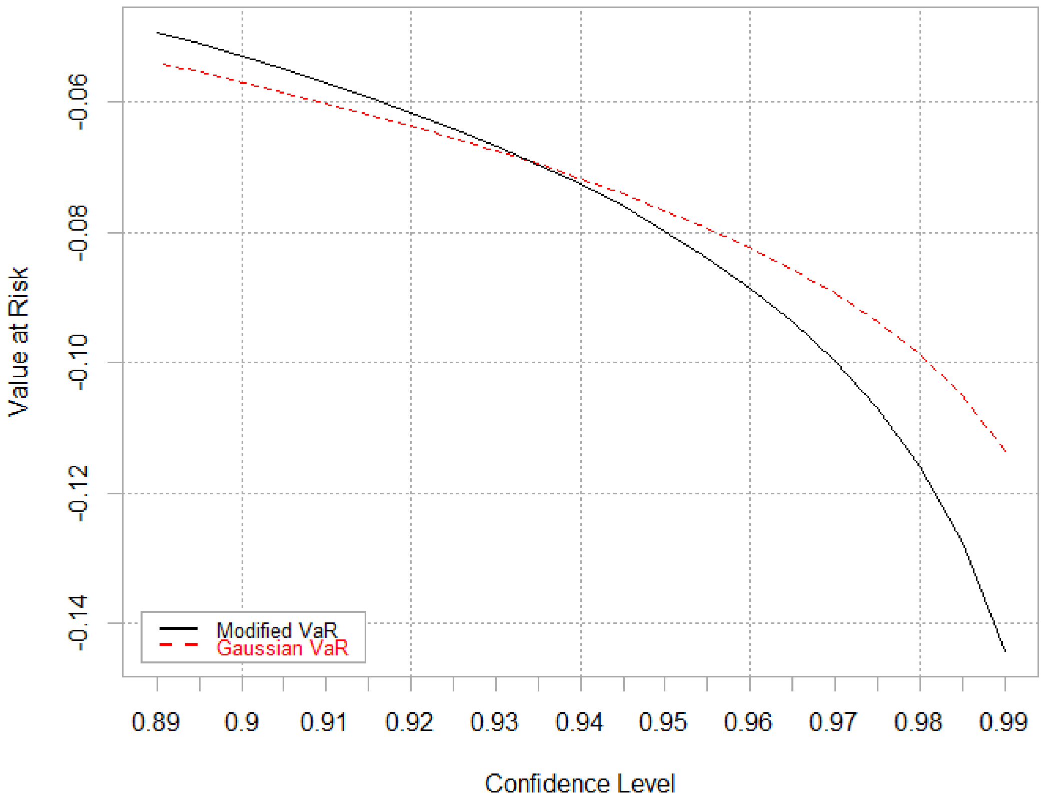

Table 9. Incremental Portfolio: With short-sales

Table 9. presents the results for the incremental portfolios with short-sales. The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. Panel B. Present the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	Benchmark portfolio								Partial portfolios	Traditional	Extended	Extended
Theta	VW	EW										
θ *ME*			-4.40	-3.72	-2.54	-1.71	-1.41	-1.78	-1.81	-1.74	-1.13	-1.22
θ *BTM*				2.10	3.54	2.68	1.75	2.37	1.84	2.72	1.97	1.89
θ *MOM*					1.58	0.58	0.67	0.80	1.15	0.58	0.54	0.69
θ *INV*						-2.46		-2.35	-2.18	-2.49	-1.67	-1.75
θ *ATCHG*							-2.85					
θ *OPPROF*								0.93				
ϑ *CFPROF*									1.00			
θ *GRPROF*										1.00	0.71	0.76
θ *TIMR*	-	-	-								2.17	2.55
θ *TIMD*	-	-	-									-1.38
Min w_i%	0.00%	0.02%	-0.68%	-0.66%	-0.46%	-1.08%	-1.32%	-1.07%	-0.98%	-1.14%	-0.83%	-0.96%
Max w_i%	0.61%	0.02%	0.17%	0.81%	1.30%	1.15%	0.81%	1.05%	1.08%	1.15%	1.56%	1.44%
Av. $\|w_i\|$ %	0.02%	0.02%	0.05%	0.05%	0.06%	0.05%	0.05%	0.05%	0.05%	0.06%	0.05%	0.05%
Av. sum w_i<0	0.00%	0.00%	-63.79%	-65.59%	-84.92%	-78.17%	-76.04%	-72.07%	-69.70%	-79.95%	-71.40%	-75.34%
Av. % w_i<0	0.00%	0.00%	27.53%	27.69%	31.44%	30.47%	30.12%	29.50%	29.08%	30.74%	29.297%	29.911%
$\bar{r}$	9.62%	13.96%	18.25%	21.25%	18.35%	22.72%	19.90%	19.05%	16.48%	23.62%	54.88%	58.54%
$\sigma(\bar{r})$	15.11%	18.78%	23.88%	28.91%	25.44%	24.66%	21.95%	20.87%	19.14%	23.39%	28.75%	24.16%
Sharpe ratio	0.545	0.666	0.702	0.682	0.662	0.858	0.837	0.840	0.784	0.943	1.838	2.336
Skewness	-0.734	-0.325	0.113	0.548	0.181	0.469	0.540	-0.101	-0.369	0.632	0.631	0.230
Kurtosis	2.067	1.588	0.730	3.820	2.641	3.482	4.072	0.981	0.619	3.697	3.290	1.603
VaR (97.5%)	-6.97%	-7.97%	-11.80%	-14.02%	-11.10%	-11.85%	-10.56%	-10.82%	-10.46%	-10.50%	-11.10%	-9.44%
CVaR (97.5%)	-11.22%	-12.34%	-15.15%	-19.80%	-17.99%	-17.58%	-15.93%	-15.19%	-14.16%	-12.62%	-12.55%	-13.68%
Sortino (Mar= 0%)	0.294	0.362	0.408	0.412	0.380	0.499	0.489	0.460	0.419	0.560	1.102	1.399
Omega ratio			1.704	1.818	1.665	2.116	1.929	1.919	1.673	2.218	5.901	8.435
Info. ratio			0.671	0.647	0.580	0.885	0.809	0.859	0.663	0.994	2.409	3.179
HHI	0.758	0.141	1.241	1.594	1.989	1.946	1.822	1.684	1.587	1.959	1.819	1.767
TTC	0.04%	0.03%	0.10%	0.22%	0.37%	0.40%	0.42%	0.44%	0.49%	0.46%	0.77%	0.85%
TTN	7.54%	5.50%	18.27%	38.77%	64.01%	68.81%	72.17%	75.33%	84.14%	78.64%	131.91%	145.30%
CER	3.91%	5.14%	4.43%	4.94%	7.51%	7.52%	7.85%	8.16%	7.33%	9.94%	34.22%	43.95%

Table 9. Panel B Regression analysis

	VW	EW	Partial portfolios							Traditional	Extended	Extended
Constant			0.004*	0.006*	0.005*	0.008***	0.007***	0.006***	0.005**	0.009***	0.027***	0.031***
			1.887	**1.765**	**1.616**	**2.827**	**2.697**	**2.842**	**2.422**	**3.296**	**7.829**	**10.069**
VW bmk			1.387***	1.624***	1.421***	1.352***	1.202***	1.19***	1.067***	1.264***	1.537***	1.259***
			26.301	**23.064**	**22.623**	**21.249**	**21.153**	**24.418**	**22.493**	**20.315**	**19.715**	**18.392**
Adj. R^2			0.769	0.719	0.711	0.684	0.682	0.741	0.708	0.664	0.651	0.619
Constant			0.001	0.002	0.001	0.005**	0.005**	0.004**	0.003*	0.006***	0.024***	0.028***
			0.911	**0.795**	**0.655**	**2.235**	**2.051**	**2.165**	**1.707**	**2.83**	**8.498**	**10.705**
EW bmk			1.232***	1.439***	1.242***	1.171***	1.02***	1.006***	0.899***	1.102***	1.351***	1.095***
			56.375	**37.969**	**33.055**	**28.392**	**25.716**	**30.67**	**26.993**	**27.351**	**27.006**	**23.331**
$Adj.R^2$			0.939	0.874	0.84	0.795	0.76	0.819	0.778	0.782	0.778	0.723
Constant	0.0004	0.003***	0.005***	0.008***	0.005***	0.009***	0.007***	0.005***	0.004***	0.009***	0.029***	0.03***
	1.05	**8.454**	**7.155**	**5.727**	**3.057**	**5.683**	**4.526**	**4.158**	**2.737**	**5.652**	**10.669**	**11.421**
eMKT	1.015***	1.035***	0.994***	1.082***	1.093***	1.005***	0.991***	1.051***	1.006***	0.968***	1.108***	1.062***
	96.053	**92.384**	**44.543**	**27.036**	**23.187**	**21.318**	**22.961**	**27.392**	**25.51**	**19.953**	**14.169**	**13.86**
SMB	0.152***	0.735***	1.537***	1.46***	1.225***	0.898***	0.644***	0.755***	0.789***	0.959***	1.347***	1.099***
	9.063	**41.386**	**43.48**	**23.029**	**16.406**	**12.021**	**9.418**	**12.433**	**12.629**	**12.478**	**10.871**	**9.056**
HML	-0.119***	-0.114***	-0.157***	0.194***	0.438***	0.351***	0.418***	0.469***	0.505***	0.322***	0.118	0.144
	-7.022	**-6.385**	**-4.393**	**3.037**	**5.811**	**4.656**	**6.059**	**7.651**	**8.006**	**4.152**	**0.946**	**1.177**
RMW	-0.032*	-0.099***	-0.218***	-0.053	0.273***	0.051	0.372***	0.345***	0.351***	0.244**	-0.02	0.243
	-1.357	**-3.988**	**-4.403**	**-0.601**	**2.612**	**0.489**	**3.897**	**4.065**	**4.019**	**2.271**	**-0.114**	**1.429**
CMA	0.051**	0.002	0.043	0.211**	0.232*	0.62***	0.614***	0.395***	0.319***	0.62***	-0.111	-0.081
	1.865	**0.071**	**0.735**	**2.034**	**1.899**	**5.071**	**5.486**	**3.979**	**3.117**	**4.933**	**-0.548**	**-0.41**
UMD	-0.028***	-0.112***	-0.165***	-0.565***	-0.298***	-0.424***	-0.377***	-0.142***	0.079***	-0.371***	-0.428***	-0.155***
	-3.169	**-11.974**	**-8.853**	**-16.935**	**-7.581**	**-10.781**	**-10.475**	**-4.443**	**2.412**	**-9.164**	**-6.567**	**-2.423**
$Adj.R^2$	0.987	0.99	0.977	0.949	0.908	0.902	0.897	0.91	0.887	0.885	0.803	0.732

Table 11. Incremental portfolios: NSS

Table 11. presents the results for the incremental portfolios with short-sales The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. Panel B. Present the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	Benchmark portfolio							Partial portfolios		Traditional	Extended	Extended
Theta	VW	EW										
θ *ME*			-1.98	-1.77	-1.78	-1.78	-1.73	-1.78	-1.78	-1.78	-1.79	-1.79
θ *BTM*				0.62	0.62	0.60	0.59	0.59	0.58	0.59	0.34	0.34
θ *MOM*					0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
θ *INV*						-0.01		-0.02	-0.02	-0.02	-0.03	-0.03
θ *ATCHG*							-0.01					
θ *OPPROF*								0.01				
ϑ *CFPROF*									0.02			
θ *GRPROF*										0.01	0.02	0.02
θ *TIMR*	-	-	-								0.22	0.22
θ *TIMD*	-	-	-									-0.01
Min w_i%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Max w_i%	0.61%	0.02%	0.08%	0.27%	0.27%	0.26%	0.26%	0.26%	0.26%	0.26%	0.27%	0.27%
Av. $\|w_i\|$ %	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Av. sum w_i<0	0.00%	0.00%	-0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Av. % w_i<0	0.00%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.001%	0.002%
$\bar{r}$	9.62%	13.96%	14.14%	15.12%	15.00%	14.96%	14.78%	15.08%	14.96%	15.00%	18.27%	18.10%
$\sigma(\bar{r})$	15.11%	18.78%	19.21%	20.17%	20.06%	20.01%	19.97%	19.99%	19.98%	20.02%	20.28%	20.15%
Sharpe ratio	0.545	0.666	0.661	0.677	0.675	0.675	0.668	0.682	0.676	0.677	0.827	0.824
Skewness	-0.734	-0.325	-0.332	-0.086	-0.120	-0.120	-0.125	-0.126	-0.120	-0.111	-0.197	-0.232
Kurtosis	2.067	1.588	1.445	2.128	2.008	2.015	2.040	2.010	2.049	2.056	1.741	1.639
VaR (97.5%)	-6.97%	-7.97%	-10.91%	-11.12%	-11.10%	-11.08%	-11.09%	-11.07%	-11.07%	-11.07%	-11.08%	-11.05%
CVaR (97.5%)	-11.22%	-12.34%	-16.62%	-18.23%	-17.99%	-17.98%	-18.06%	-17.97%	-18.04%	-18.05%	-17.59%	-17.35%
Sortino (Mar= 0%)	0.294	0.362	0.360	0.377	0.374	0.374	0.370	0.377	0.375	0.375	0.446	0.443
Omega ratio			1.750	1.863	1.847	1.845	1.840	1.870	1.848	1.855	2.454	2.432
Info. ratio			0.716	0.732	0.731	0.730	0.712	0.751	0.733	0.734	1.159	1.161
HHI	0.758	0.141	0.149	0.198	0.197	0.195	0.200	0.194	0.192	0.193	0.180	0.179
TTC	0.04%	0.03%	0.03%	0.06%	0.06%	0.06%	0.09%	0.06%	0.06%	0.06%	0.09%	0.09%
TTN	7.54%	5.50%	5.97%	10.95%	10.77%	10.81%	16.66%	11.04%	10.81%	10.81%	15.00%	14.85%
CER	3.91%	5.14%	4.91%	4.95%	4.94%	4.95%	4.81%	5.09%	4.98%	4.98%	7.99%	7.95%

Table 11. Panel B Regression analysis

	VW	EW							Partial portfolios	Traditional	Extended	Extended
Constant			0.002*	0.003*	0.002*	0.002*	0.002*	0.003**	0.002**	0.002**	0.005***	0.005***
			1.887	**1.955**	**1.947**	**1.946**	**1.867**	**2.032**	**1.96**	**1.964**	**3.762**	**3.77**
VW bmk			1.222***	1.268***	1.262***	1.26***	1.259***	1.259***	1.258***	1.261***	1.279***	1.272***
			49.638	**43.518**	**44.177**	**44.298**	**44.768**	**44.604**	**44.44**	**44.339**	**45.08**	**45.782**
Adj. R^2			0.922	0.901	0.904	0.904	0.906	0.905	0.905	0.904	0.907	0.91
Constant			-0.0001	0.0003	0.0002	0.0002	0.0001	0.0003	0.0002	0.0002	0.002***	0.002***
			-0.615	**0.486**	**0.436**	**0.428**	**0.212**	**0.633**	**0.457**	**0.471**	**5.637**	**5.732**
EW bmk			1.022***	1.065***	1.06***	1.058***	1.055***	1.057***	1.056***	1.058***	1.074***	1.067***
			452.413	**108.867**	**115.21**	**118.577**	**112.358**	**120.507**	**120.267**	**119.759**	**136.871**	**141.856**
$Adj.R^2$			0.999	0.983	0.985	0.985	0.984	0.986	0.986	0.986	0.989	0.99
Constant	0.0004	0.003***	0.002***	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	0.005***	0.005***
	1.05	**8.454**	**5.958**	**5.49**	**5.426**	**5.509**	**4.889**	**5.651**	**5.517**	**5.55**	**9.326**	**9.379**
$eMKT$	1.015***	1.035***	1.04***	1.044***	1.045***	1.042***	1.051***	1.043***	1.043***	1.043***	1.056***	1.057***
	96.053	**92.384**	**94.138**	**64.636**	**66.270**	**67.094**	**65.311**	**66.933**	**67.884**	**67.2**	**62.769**	**64.793**
SMB	0.152***	0.735***	0.811***	0.779***	0.779***	0.78***	0.768***	0.776***	0.781***	0.78***	0.81***	0.803***
	9.063	**41.386**	**46.322**	**30.456**	**31.203**	**31.694**	**30.146**	**31.422**	**32.073**	**31.738**	**30.401**	**31.08**
HML	-0.119***	-0.114***	-0.117***	-0.026	-0.023	-0.026	-0.02	-0.029	-0.021	-0.03	-0.086***	-0.083
	-7.022	**-6.385**	**-6.598**	**-0.991**	**-0.927**	**-1.049**	**-0.796**	**-1.152**	**-0.836**	**-1.207**	**-3.209**	**-0.083**
RMW	-0.032*	-0.099***	-0.099***	-0.036	-0.035	-0.042	-0.013	-0.047	-0.033	-0.044	-0.137***	-0.135***
	-1.357	**-3.988**	**-4.05**	**-1.013**	**-1.008**	**-1.232**	**-0.354**	**-1.363**	**-0.972**	**-1.273**	**-3.672**	**-3.74**
CMA	0.051**	0.002	-0.006	0.071*	0.072*	0.065	0.078*	0.065	0.055	0.066	-0.039	-0.033
	1.865	**0.071**	**-0.225**	**1.694**	**1.771**	**1.621**	**1.875**	**1.608**	**1.38**	**1.638**	**-0.901**	**-0.771**
UMD	-0.028***	-0.112***	-0.114***	-0.238***	-0.224***	-0.222***	-0.219***	-0.218***	-0.219***	-0.222***	-0.191***	-0.179***
	-3.169	**-11.974**	**-12.350**	**-17.651**	**-17.046**	**-17.109**	**-16.362**	**-16.77**	**-17.084**	**-17.125**	**-13.61**	**-13.201**
$Adj.R^2$	0.987	0.99	0.991	0.983	0.983	0.984	0.983	0.984	0.984	0.984	0.982	0.983

Figure 6. Rolling 12-month performance (With short-sales)

The figure displays the rolling 12-months performance (trailing twelve months) of the various portfolios corresponding to Table 8. The benchmark portfolio is the value-weighted benchmark portfolio (VW. Bench) and the equal-weight portfolio (EW. Bench). The traditional portfolio (1) refers to the portfolio that includes the characteristics: size, book-to-market, momentum, gross-profitability and investment. The extended portfolio (2) includes the traditional characteristic: TIMR The extended portfolio (3) includes the traditional characteristics + TIMR and TIMD. The optimized portfolios allow short-sales. The Returns, Standard deviation (1 corresponds to a 100%) and Sharpe ratio are depicted on the y-axis, respectively. The returns are simple returns.

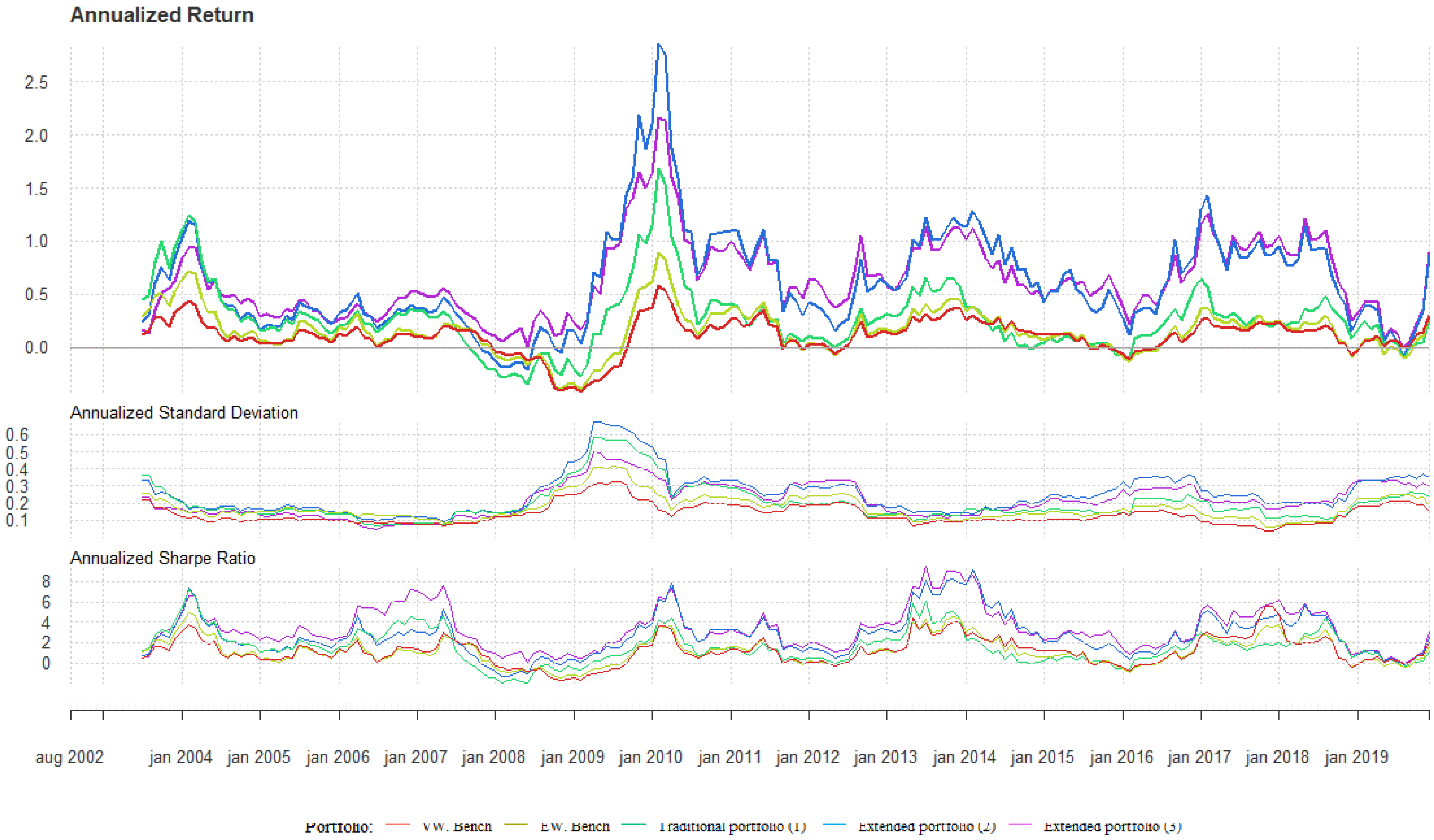

Figure 8. Rolling 12-month performance (Without short-sales)

The figure displays the rolling 12-months performance (trailing twelve months) of the various portfolios corresponding to Table 10. The benchmark portfolio is the value-weighted benchmark portfolio (VW. Bench) and the equal-weight portfolio (EW. Bench). The traditional portfolio (1) refers to the portfolio that includes the characteristics: size, book-to-market, momentum, gross-profitability and investment. The extended portfolio (2) includes the traditional characteristic: TIMR The extended portfolio (3) includes the traditional characteristics + TIMR and TIMD. The optimized portfolios **do not** allow for short-sales. The Returns, Standard deviation (1 corresponds to a 100%) and Sharpe ratio are depicted on the y-axis, respectively. The returns are simple returns.

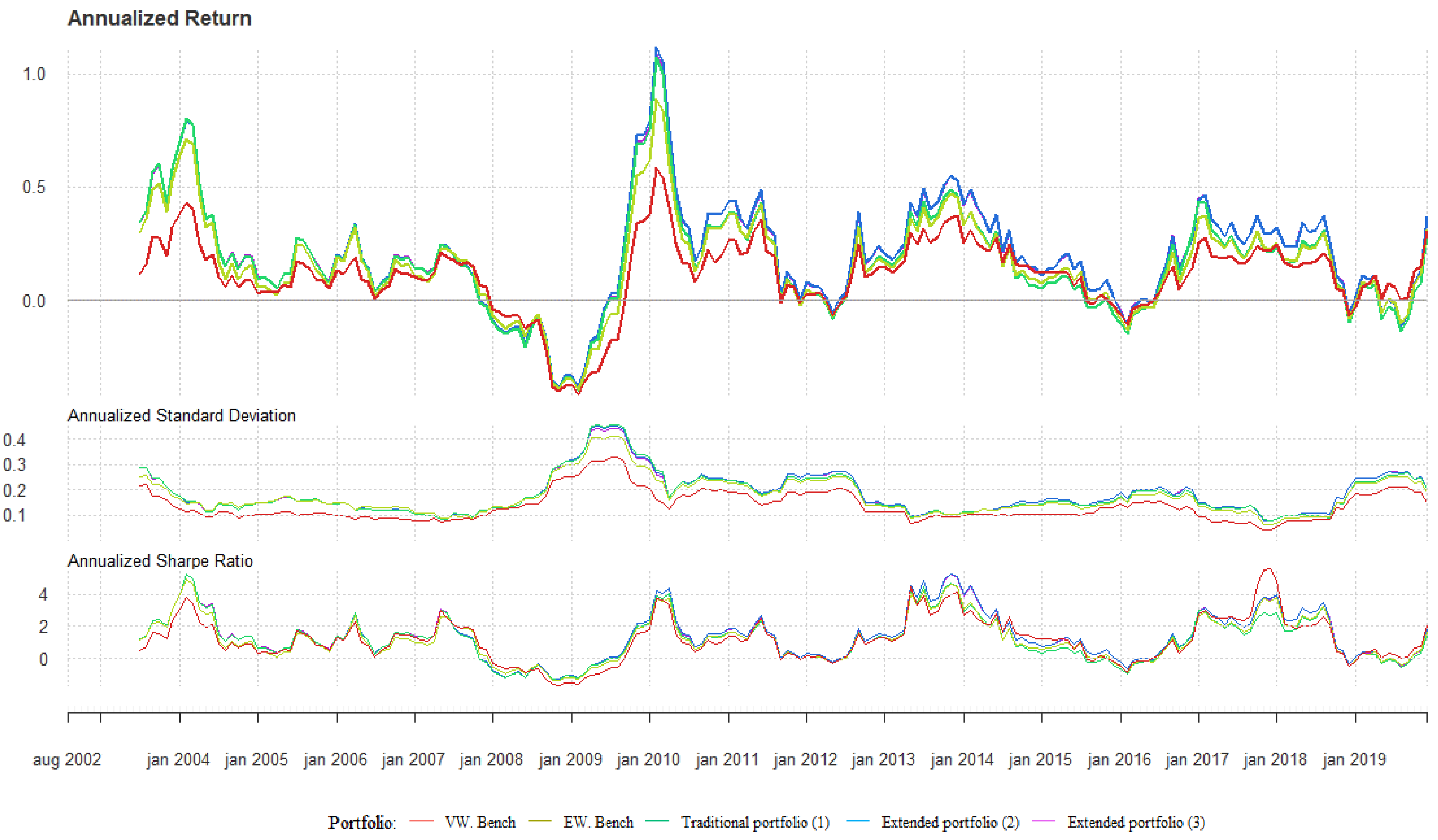

Figure 7. Cumulative performance of the portfolios (With short-sales)

Figure 7. displays the cumulative performance of the optimized portfolios and the benchmark portfolios with an initial investment of 1$ depicted by the horizonal blue line. Short sales are allowed and constrained to 1. The plotted portfolios correspond to the portfolios presented in **table 8.** The benchmark portfolio are the value-weighted benchmark portfolio (VW. Bench) and the equal-weight portfolio (EW. Bench). The traditional portfolio refers to the portfolio that includes the characteristics: Size, Book-to-market, Momentum, Profitability and Investment. The extended portfolio (2) includes the characteristics: TIMR The extended portfolio (3) includes both TIMR and TIMD.

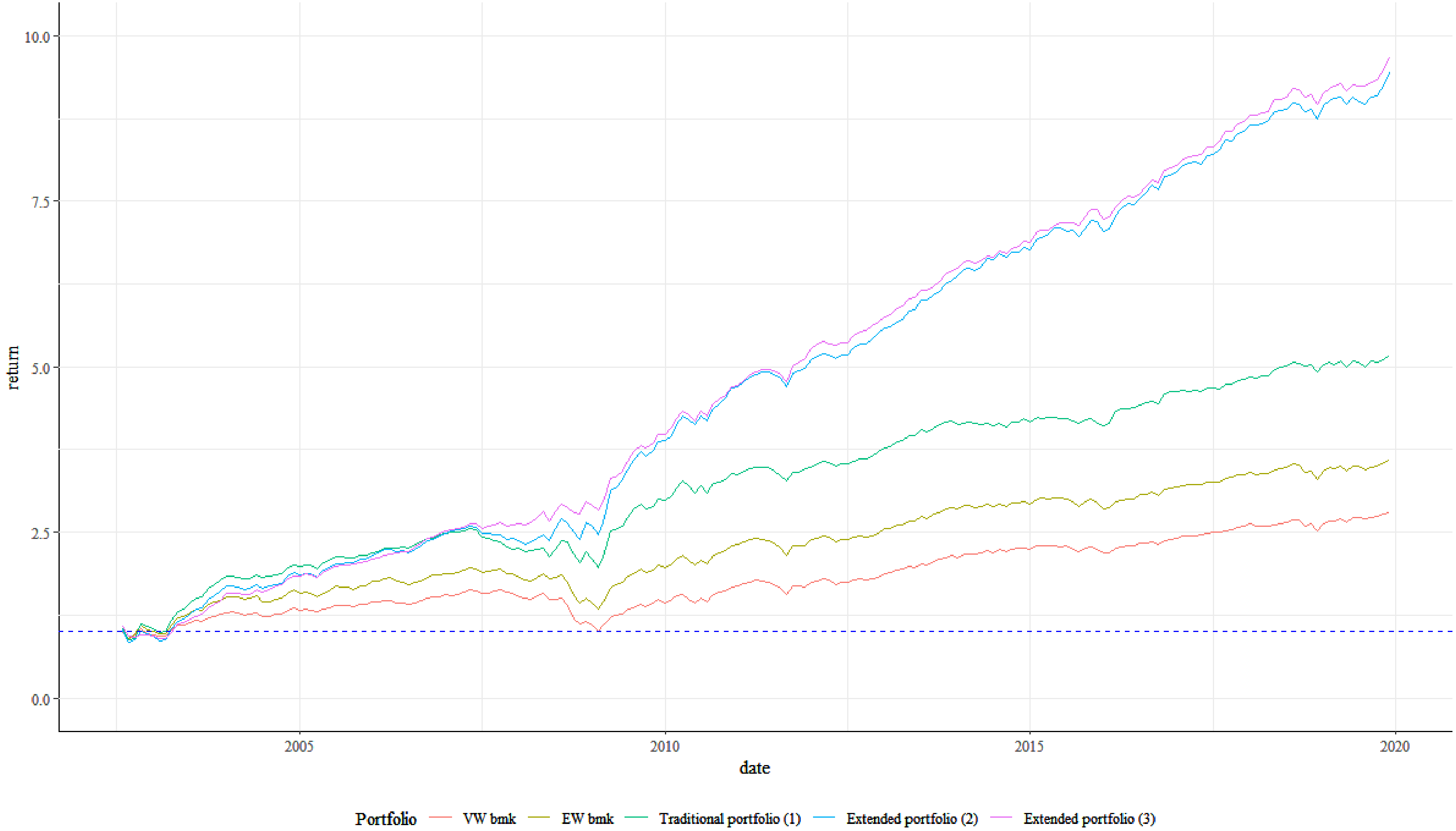

Figure 9. Cumulative performance of the portfolios (Without short-sales)

Figure 9. displays the cumulative performance of the optimal portfolio and the benchmark portfolios with an initial investment of 1$ depicted by the horizonal blue line. All portfolios are prohibited from entering short positions (NSS). The plotted portfolios correspond to the optimized portfolios presented in **table 10**. The benchmark portfolio are the value-weighted benchmark portfolio (VW. Bench) and the equal-weight portfolio (EW. Bench). The traditional portfolio refers to the portfolio that includes the characteristics: Size, Book-to-market, Momentum, Profitability and Investment. The extended portfolio (2) includes the characteristics: TIMR The extended portfolio (3) includes both TIMR and TIMD.

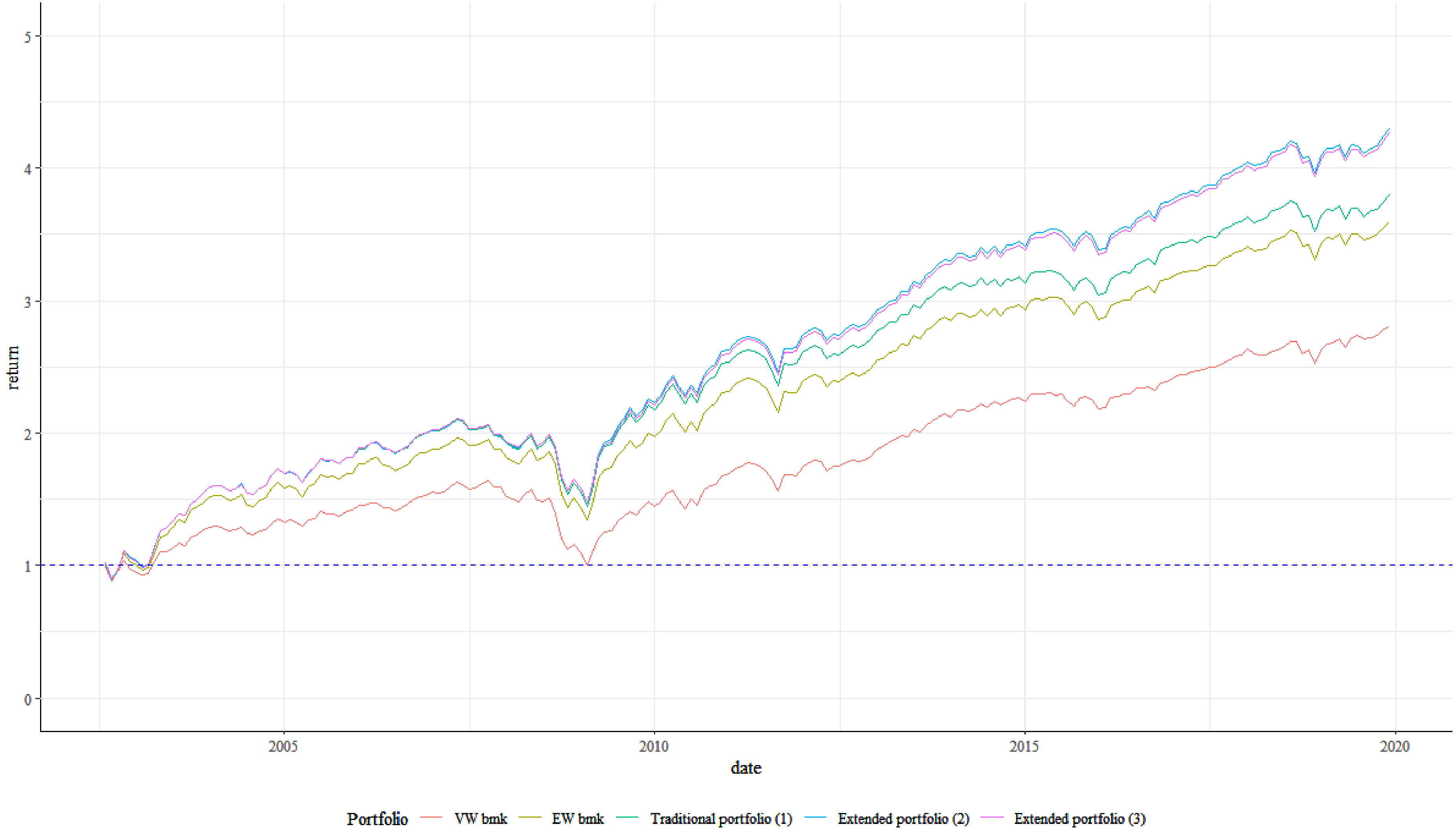

Table 12. Varying risk-aversion levels: With SS

Table 12. presents the results for the incremental portfolios with short-sales The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. Panel B. Present the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	Benchmark portfolio		Risk-aversion level							
Theta	VW	EW	y=1	y=2	y=3	y=4	y=5	y=6	y=7	y=10
θ *ME*			-0.93	-1.06	-1.18	-1.23	-1.22	-1.20	-1.18	-1.00
θ *BTM*			1.35	1.54	1.73	1.83	1.89	1.91	1.93	1.73
θ *MOM*			-0.53	-0.33	0.03	0.38	0.69	0.75	0.87	1.17
θ *INV*			-0.78	-1.00	-1.31	-1.57	-1.75	-1.83	-1.95	-1.90
θ *GRPROF*			0.18	0.30	0.46	0.62	0.76	0.83	0.93	1.12
θ *TIMR*			2.37	2.44	2.55	2.58	2.55	2.50	2.50	2.12
θ *TIMD*			0.40	0.09	-0.39	-0.90	-1.38	-1.58	-1.84	-2.35
Min w_i%	0.00%	0.02%	-0.55%	-0.62%	-0.72%	-0.83%	-0.96%	-1.02%	-1.09%	-1.22%
Max w_i%	0.61%	0.02%	1.65%	1.60%	1.56%	1.51%	1.44%	1.39%	1.32%	1.14%
Av. $\lvert w_i \rvert$ %	0.02%	0.02%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.06%
Av. sum w_i<0	0.00%	0.00%	-63.10%	-64.78%	-68.28%	-71.54%	-75.34%	-76.32%	-78.36%	-83.30%
Av. % w_i<0	0.00%	0.00%	27.82%	28.14%	28.75%	29.29%	29.91%	30.07%	30.43%	31.21%
$\bar{r}$	9.62%	13.96%	61.08%	61.57%	62.60%	60.05%	58.54%	59.07%	52.76%	40.23%
$\sigma(\bar{r})$	15.11%	18.78%	33.58%	31.85%	29.65%	27.62%	24.16%	21.79%	20.23%	16.32%
Sharpe ratio	0.545	0.666	1.757	1.867	2.039	2.097	2.336	2.611	2.507	2.356
Skewness	-0.734	-0.325	0.886	0.753	0.719	0.418	0.230	-0.112	-0.197	-0.089
Kurtosis	2.067	1.588	4.546	4.167	3.745	2.707	1.603	0.023	0.246	0.170
VaR (97.5%)	-9.45%	-10.70%	-12.32%	-12.10%	-10.91%	-10.99%	-9.44%	-8.50%	-8.27%	-6.49%
CVaR (97.5%)	-15.42%	-16.63%	-18.12%	-16.97%	-15.49%	-15.93%	-13.68%	-11.08%	-11.18%	-8.60%
Sortino (Mar= 0%)	0.294	0.362	1.058	1.109	1.234	1.224	1.399	1.592	1.512	1.473
Omega ratio			5.536	5.873	6.548	7.781	8.435	11.184	10.204	5.869
Info. ratio			2.240	2.414	2.678	2.872	3.179	3.817	3.644	2.808
HHI	0.758	0.141	1.677	1.685	1.721	1.733	1.767	1.760	1.767	1.805
TTC	0.04%	0.03%	0.70%	0.75%	0.82%	0.86%	0.85%	0.81%	0.76%	0.69%
TTN	7.54%	5.50%	119.96%	128.05%	141.46%	147.67%	145.30%	138.59%	130.88%	118.73%
CER	0.039	0.051	0.554	0.514	0.494	0.448	0.440	0.448	0.384	0.269

Table 12. Panel B Regression analysis

	VW	EW	y=1	y=2	y=3	y=4	y=5	y=6	y=7	y=10
Constant			0.029***	0.03***	0.031***	0.03***	0.031***	0.031***	0.028***	0.023***
			7.372	**7.863**	**8.578**	**9.27**	**10.069**	**12.05**	**11.74**	**10.448**
VW bmk			1.814***	1.708***	1.569***	1.496***	1.259***	1.172***	1.09***	0.823***
			20.329	**19.887**	**19.139**	**20.466**	**18.392**	**20.08**	**20.137**	**16.922**
Adj. R^2			0.665	0.655	0.637	0.668	0.619	0.659	0.66	0.578
Constant			0.025***	0.026***	0.028***	0.027***	0.028***	0.029***	0.026***	0.021***
			7.946	**8.579**	**9.316**	**10.186**	**10.705**	**12.557**	**12.282**	**9.827**
EW bmk			1.589***	1.502***	1.379***	1.303***	1.095***	0.994***	0.926***	0.659***
			27.866	**27.412**	**25.798**	**27.474**	**23.331**	**23.871**	**24.167**	**16.735**
$Adj.R^2$			0.789	0.783	0.762	0.784	0.723	0.732	0.737	0.573
Constant	0.0004	0.003***	0.034***	0.033***	0.033***	0.032***	0.03***	0.031***	0.027***	0.02***
	1.05	**8.454**	**12.718**	**12.069**	**11.892**	**12.117**	**11.421**	**13.423**	**13.26**	**11.094**
eMKT	1.015***	1.035***	1.145***	1.131***	1.095***	1.111***	1.062***	1.026***	0.998***	0.953***
	96.053	**92.384**	**14.861**	**14.154**	**13.415**	**14.631**	**13.86**	**15.148**	**16.706**	**18.015**
SMB	0.152***	0.735***	1.283***	1.343***	1.282***	1.153***	1.099***	0.805***	0.895***	0.41***
	9.063	**41.386**	**10.52**	**10.611**	**9.92**	**9.594**	**9.056**	**7.506**	**9.457**	**4.890**
HML	-0.119***	-0.114***	-0.129	-0.019	0.085	0.098	0.144	0.15	0.064	0.099
	-7.022	**-6.385**	**-1.049**	**-0.147**	**0.655**	**0.811**	**1.177**	**1.383**	**0.671**	**1.166**
RMW	-0.032*	-0.099***	-0.488***	-0.222	-0.053	-0.052	0.243	-0.141	0.023	0.113
	-1.357	**-3.988**	**-2.863**	**-1.253**	**-0.291**	**-0.308**	**1.429**	**-0.941**	**0.175**	**0.961**
CMA	0.051**	0.002	-0.165	-0.200	-0.287	-0.117	-0.081	0.238	0.298*	0.506***
	1.865	**0.071**	**-0.827**	**-0.964**	**-1.355**	**-0.595**	**-0.410**	**1.356**	**1.925**	**3.692**
UMD	-0.028***	-0.112***	-0.84***	-0.703***	-0.567***	-0.421***	-0.155***	0.026	0.114**	0.36***
	-3.169	**-11.974**	**-13.087**	**-10.559**	**-8.337**	**-6.661**	**-2.423**	**0.465**	**2.290**	**8.173**
$Adj.R^2$	0.987	0.99	0.859	0.832	0.798	0.799	0.732	0.743	0.768	0.719

Figure 10. Cumulative density function with short-sales

The plot below present the cumulative density function across a range of risk-aversion levels (y=10, y=5 and y=1). All portfolios allow for short sales (SS). The plot corresponds to the portfolios presented in Table 12.

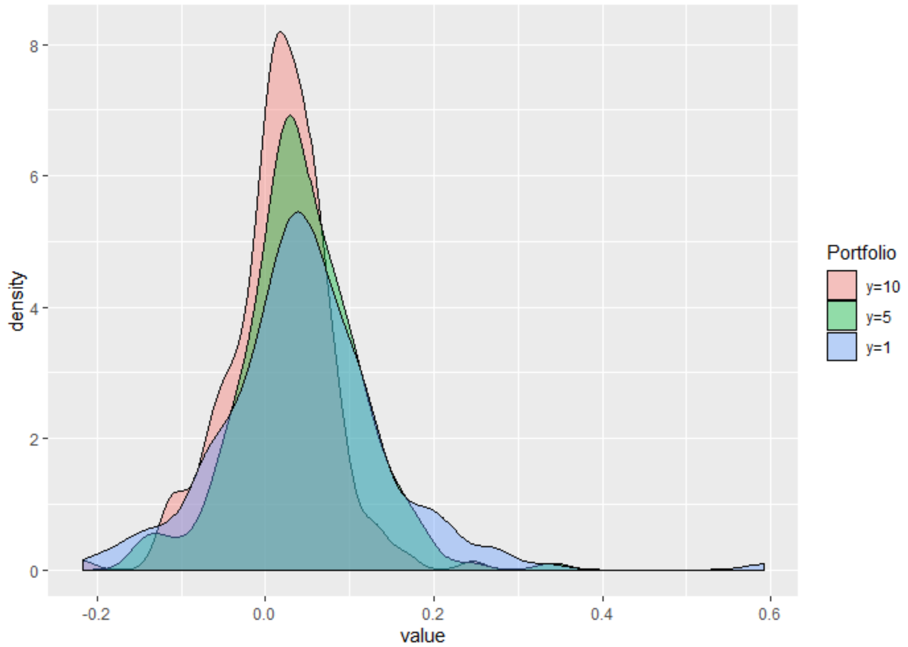

Table 13. Varying risk-aversion levels: NSS

Table 13. presents the results for the incremental portfolios with short-sales The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. Panel B. Present the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	Benchmark portfolio									
Theta	VW	EW	y=1	y=2	y=3	y=4	y=5	y=6	y=7	y=10
θ *ME*			-1.78	-1.79	-1.79	-1.79	-1.79	-1.79	-1.79	-1.35
θ *BTM*			0.25	0.28	0.31	0.33	0.34	0.35	0.36	0.30
θ *MOM*			0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.09
θ *INV*			-0.02	-0.02	-0.02	-0.03	-0.03	-0.04	-0.04	-0.05
θ *GRPROF*			0.01	0.02	0.02	0.02	0.02	0.01	0.01	-0.01
θ *TIMR*			0.33	0.29	0.27	0.24	0.22	0.19	0.17	0.06
θ *TIMD*			0.03	0.02	0.01	0.00	-0.01	-0.03	-0.03	-0.04
Min w_i%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Max w_i%	0.61%	0.02%	0.30%	0.29%	0.29%	0.28%	0.27%	0.25%	0.24%	0.23%
Av. $\|w_i\|$ %	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Av. sum w_i<0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Av. % w_i<0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$\bar{r}$	9.62%	13.96%	19.76%	19.53%	19.17%	18.67%	18.10%	17.60%	16.89%	13.86%
$\sigma(\bar{r})$	15.11%	18.78%	20.55%	20.46%	20.47%	20.35%	20.15%	19.95%	19.79%	18.01%
Sharpe ratio	0.545	0.666	0.887	0.880	0.862	0.843	0.824	0.808	0.778	0.690
Skewness	-0.734	-0.325	-0.174	-0.163	-0.145	-0.184	-0.232	-0.244	-0.273	-0.473
Kurtosis	2.067	1.588	1.746	1.735	1.829	1.713	1.639	1.632	1.605	1.683
VaR (97.5%)	-9.45%	-10.70%	-11.08%	-11.01%	-11.03%	-11.05%	-11.05%	-10.99%	-11.00%	-10.53%
CVaR (97.5%)	-15.42%	-16.63%	-17.62%	-17.45%	-17.67%	-17.47%	-17.35%	-17.24%	-17.19%	-16.62%
Sortino (Mar= 0%)	0.294	0.362	0.477	0.474	0.466	0.455	0.443	0.434	0.418	0.367
Omega ratio			2.720	2.702	2.629	2.530	2.432	2.363	2.243	2.018
Info. ratio			1.299	1.291	1.240	1.200	1.161	1.125	1.052	0.888
HHI	0.758	0.141	0.180	0.179	0.180	0.180	0.179	0.177	0.175	0.192
TTC	0.04%	0.03%	0.11%	0.10%	0.10%	0.09%	0.09%	0.08%	0.08%	0.08%
TTN	7.54%	5.50%	18.90%	17.08%	16.38%	15.64%	14.85%	14.02%	13.22%	15.15%
CER	0.039	0.051	0.176	0.153	0.129	0.104	0.080	0.057	0.032	-0.024

Table 13. Panel B Regression analysis

	VW	EW	y=1	y=2	y=3	y=4	y=5	y=6	y=7	y=10
Constant			0.006***	0.006***	0.005***	0.005***	0.005***	0.004***	0.004***	0.002***
			4.349	**4.318**	**4.100**	**3.935**	**3.77**	**3.613**	**3.307**	**2.622**
VW bmk			1.292***	1.288***	1.288***	1.282***	1.272***	1.261***	1.253***	1.16***
			43.602	**44.227**	**44.025**	**44.781**	**45.782**	**46.453**	**47.351**	**61.375**
Adj. R^2			0.901	0.904	0.903	0.906	0.91	0.912	0.915	0.948
Constant			0.003***	0.003***	0.003***	0.003***	0.002***	0.002***	0.002***	0.0004
			7.282	**7.247**	**6.504**	**6.062**	**5.732**	**5.452**	**4.722**	**1.12**
EW bmk			1.088***	1.083***	1.083***	1.077***	1.067***	1.057***	1.05***	0.955***
			130.086	**133.559**	**130.259**	**133.865**	**141.856**	**150.449**	**163.289**	**158.315**
$Adj.R^2$			0.988	0.988	0.988	0.989	0.99	0.991	0.992	0.992
Constant	0.0004	0.003***	0.006***	0.006***	0.006***	0.006***	0.005***	0.005***	0.004***	0.002***
	1.05	**8.454**	**10.002**	**10.092**	**9.735**	**9.526**	**9.379**	**9.126**	**8.529**	**4.601**
eMKT	1.015***	1.035***	1.056***	1.058***	1.058***	1.058***	1.057***	1.051***	1.051***	1.037***
	96.053	**92.384**	**56.110**	**58.725**	**59.137**	**61.212**	**64.793**	**67.033**	**70.688**	**75.104**
SMB	0.152***	0.735***	0.845***	0.833***	0.83***	0.816***	0.803***	0.793***	0.786***	0.602***
	9.063	**41.386**	**28.345**	**29.191**	**29.299**	**29.816**	**31.08**	**31.941**	**33.36**	**27.521**
HML	-0.119***	-0.114***	-0.125***	-0.111***	-0.101***	-0.09***	-0.083***	-0.081***	-0.071***	-0.075***
	-7.022	**-6.385**	**-4.164**	**-3.84**	**-3.516**	**-3.241**	**-3.189**	**-3.245**	**-2.969**	**-3.377**
RMW	-0.032*	-0.099***	-0.162***	-0.136***	-0.127***	-0.134***	-0.135***	-0.132***	-0.112***	-0.081***
	-1.357	**-3.988**	**-3.895**	**-3.412**	**-3.210**	**-3.51**	**-3.74**	**-3.8**	**-3.403**	**-2.647**
CMA	0.051**	0.002	-0.057	-0.044	-0.048	-0.044	-0.033	-0.035	-0.026	0.055
	1.865	**0.071**	**-1.167**	**-0.936**	**-1.033**	**-0.984**	**-0.771**	**-0.858**	**-0.686**	**1.524**
UMD	-0.028***	-0.112***	-0.196***	-0.198***	-0.203***	-0.193***	-0.179***	-0.172***	-0.164***	-0.088***
	-3.169	**-11.974**	**-12.489**	**-13.194**	**-13.639**	**-13.423**	**-13.201**	**-13.128**	**-13.197**	**-7.620**
$Adj.R^2$	0.987	0.99	0.978	0.979	0.98	0.981	0.983	0.983	0.985	0.984

Figure 11. Cumulative density function without short-sales

The plot below present the cumulative density function across a range of risk-aversion levels (y=10, y=5 and y=1). Short-sales are prohibited for all portfolios. The plot corresponds to the portfolios presented in table 13.

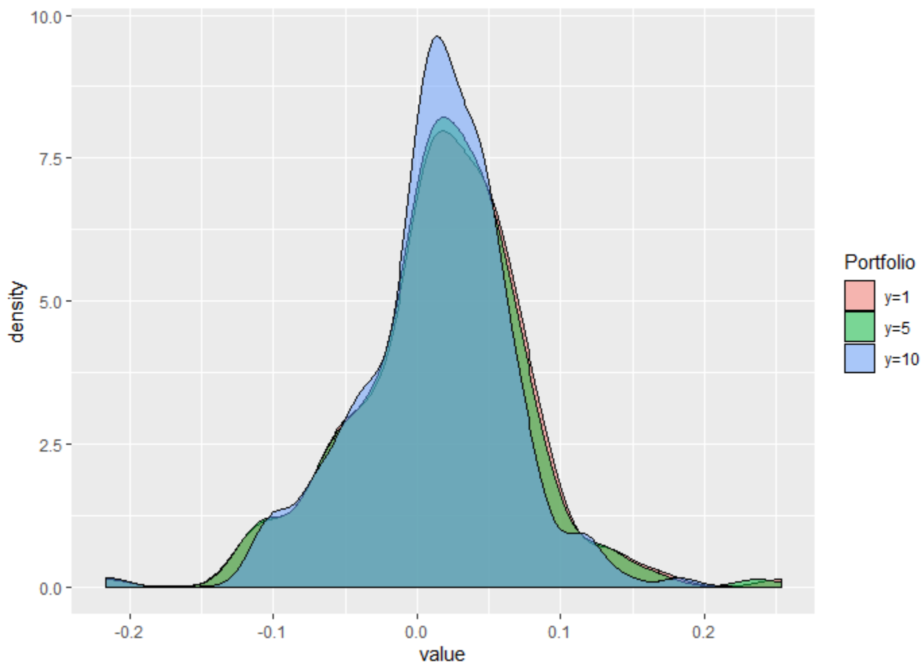

Table 14. Different objective functions

Table 13. presents the results for the incremental portfolios with short-sales The characteristics are depicted in the first column, the corresponding time-series average coefficient can be found in corresponding columns to the right. *ME*, reflects the size characteristic in mln, *BTM* is the book-to-market characteristic, *MOM* reflects the 12-month momentum return, *INV* represent the investment characteristic. *GRPROF* reflects the gross-profitability characteristic, *TIMR* reflects the characteristic target-price-implied return and *TIMD* reflect the characteristic Target-price-implied-dispersion. Panel B. Present the results from the regression analysis. The *VW* and *EW bmk* reflect the value and equal-weighted benchmark respectively. MV is the mean-variance portfolio. SR is the portfolio optimized upon the Sharpe ratio. Log represent the portfolio optimized upon log preferences. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	With SS				NSS			
Theta	MV	CRRA	SR	log	MV	CRRA	SR	log
θ *ME*	-1.09	-1.22	-1.33	-0.91	-1.44	-1.79	-1.85	-1.80
θ *BTM*	1.92	1.89	1.73	1.21	0.28	0.34	0.32	0.20
θ *MOM*	1.09	0.69	0.87	-0.61	0.11	0.02	0.07	0.02
θ *INV*	-1.87	-1.75	-1.64	-0.68	-0.05	-0.03	-0.06	-0.02
θ *GRPROF*	1.05	0.76	0.80	0.12	-0.01	0.02	0.02	0.01
θ *TIMR*	2.42	2.55	2.98	2.35	0.07	0.22	0.42	0.37
θ *TIMD*	-2.30	-1.38	-1.43	0.59	-0.04	-0.01	-0.04	0.04
Min w_i%	-1.17%	-0.96%	-0.98%	-0.53%	0.00%	0.00%	-0.02%	0.00%
Max w_i%	1.22%	1.44%	1.55%	1.70%	0.21%	0.27%	0.34%	0.30%
Av. $\lvert w_i \rvert$ %	0.06%	0.05%	0.05%	0.05%	0.02%	0.02%	0.02%	0.02%
Av. sum w_i<0	-81.77%	-75.34%	-75.80%	-63.05%	0.00%	0.00%	-0.07%	0.00%
Av. % w_i<0	30.95%	29.911%	29.98%	27.80%	0.00%	0.00%	0.07%	0.00%
$\bar{r}$	45.03%	58.54%	67.93%	61.12%	14.41%	18.10%	21.47%	20.43%
$\sigma(\bar{r})$	17.06%	24.16%	25.31%	34.88%	18.43%	20.15%	20.83%	20.64%
Sharpe ratio	2.532	2.336	2.597	1.693	0.704	0.824	0.957	0.916
Skewness	-0.318	0.230	0.721	0.999	-0.343	-0.232	-0.128	-0.155
Kurtosis	0.458	1.603	3.365	4.999	1.880	1.639	1.660	1.622
VaR (97.5%)	-5.20%	-9.44%	-5.26%	-7.95%	-7.77%	-8.17%	-8.07%	-8.11%
CVaR (97.5%)	-8.04%	-13.68%	-6.44%	-7.95%	-12.37%	-12.68%	-12.43%	-12.48%
Sortino (Mar= 0%)	1.502	1.399	1.718	1.029	0.378	0.443	0.515	0.493
Omega ratio	7.823	8.435	11.730	5.311	2.083	2.432	3.008	2.853
Info. ratio	3.280	3.179	3.549	2.126	0.900	1.161	1.450	1.375
TTC	0.674%	0.845%	0.911%	0.710%	0.081%	0.087%	0.118%	0.109%
TTN	115.59%	145.30%	156.27%	121.80%	14.64%	14.85%	20.21%	18.65%
CER	37.76%	43.95%	51.92%	30.70%	5.92%	7.95%	10.63%	9.78%

Table 14. Panel B Regression analysis

	MV	CRRA	SR	log	MV	CRRA	SR	log
Constant	0.025***	0.031***	0.035***	0.029***	0.002***	0.005***	0.007***	0.006***
	11.517	**10.069**	**10.985**	**7.006**	**2.664**	**3.77**	**4.966**	**4.673**
VW bmk	0.882***	1.259***	1.312***	1.88***	1.182***	1.272***	1.304***	1.298***
	18.006	**18.392**	**18.109**	**20.188**	**56.29**	**45.782**	**41.866**	**43.698**
Adj. R^2	0.608	0.619	0.611	0.662	0.938	0.91	0.894	0.902
	0.024***	0.028***	0.033***	0.025***	0.001**	0.002***	0.004***	0.004***
EW bmk	**11.168**	**10.705**	**11.400**	**7.436**	**1.868**	**5.732**	**8.449**	**8.544**
	0.723***	1.095***	1.124***	1.644***	0.979***	1.067***	1.101***	1.093***
	18.93	**23.331**	**21.769**	**27.37**	**184.092**	**141.856**	**115.53**	**136.276**
Adj. R^2	0.632	0.723	0.695	0.782	0.994	0.99	0.985	0.989
Constant	0.023***	0.03***	0.037***	0.034***	0.003***	0.005***	0.008***	0.007***
	12.123	**11.421**	**12.645**	**12.799**	**5.168**	**9.379**	**11.071**	**10.659**
eMKT	0.954***	1.062***	1.05***	1.153***	1.038***	1.057***	1.063***	1.062***
	17.335	**13.86**	**12.479**	**14.831**	**72.597**	**64.793**	**53.199**	**56.511**
SMB	0.566***	1.099***	0.991***	1.258***	0.654***	0.803***	0.877***	0.858***
	6.491	**9.056**	**7.439**	**10.216**	**28.889**	**31.08**	**27.725**	**28.82**
HML	0.079	0.144	-0.158	-0.229*	-0.075***	-0.083***	-0.117***	-0.141***
	0.894	**1.177**	**-1.177**	**-1.846**	**-3.293**	**-3.189**	**-3.67**	**-4.683**
RMW	0.133	0.243	-0.565***	-0.662***	-0.062**	-0.135***	-0.152***	-0.164***
	1.092	**1.429**	**-3.033**	**-3.848**	**-1.972**	**-3.74**	**-3.433**	**-3.951**
CMA	0.352**	-0.081	0.45**	-0.093	0.026	-0.033	-0.043	-0.028
	2.467	**-0.41**	**2.063**	**-0.462**	**0.704**	**-0.771**	**-0.84**	**-0.582**
UMD	0.288***	-0.155**	0.03	-0.919***	-0.118***	-0.179***	-0.198***	-0.191***
	6.27	**-2.423**	**0.424**	**-14.189**	**-9.904**	**-13.201**	**-11.903**	**-12.18**
$Adj. R^2$	0.722	0.732	0.705	0.867	0.984	0.983	0.975	0.978

Table 15. Restricted sample

The table includes all firms from the restricted sample above the 90th percentile of market-cap of the NYSE. The traditional portfolio includes the characteristics; Size, book-to-market, momentum, investment and profitability. The extended portfolio additionally includes TIMR + TIMD. Both short-sale (SS) and no-short-sale (NSS) portfolios are presented. Note: *p<0.1 , **p<0.05, ***p<0.01. t-tats are presented in bold.

	Benchmark portfolio		Traditional		Extended	
Theta	VW	EW	SS	NSS	SS	NSS
θ ME			-2.70	-1.02	-2.50	-1.00
θ BTM			1.95	0.46	1.71	0.37
θ MOM			0.95	0.09	0.82	0.11
θ INV			-1.26	-0.02	-1.09	-0.02
θ GRPROF			0.53	0.09	0.52	0.08
θ TIMR					1.61	0.12
θ TIMD					-1.53	-0.06
Min w %	0.00%	0.02%	-1.98%	0.00%	-2.30%	0.00%
Max w %	0.61%	0.02%	2.25%	0.60%	2.46%	0.56%
Av.absolute wi	0.02%	0.02%	0.14%	0.05%	0.14%	0.05%
Average sum w<0	0.00%	0.00%	-75.27%	0.00%	-80.54%	0.00%
Average % w<0	0.00%	0.00%	30.019%	0.001%	30.807%	0.002%
Ann. Return	9.62%	13.96%	20.64%	13.97%	31.52%	14.92%
St. dev.	15.11%	18.78%	16.04%	15.15%	16.29%	15.14%
Sharpe ratio	0.545	0.666	1.190	0.827	1.829	0.889
Skewness	-0.734	-0.325	-0.415	-0.627	0.129	-0.639
Kurtosis	2.067	1.588	0.967	2.002	1.418	1.879
VAR 97.5%	-6.97%	-7.97%	-0.085	-0.090	-0.069	-0.089
CVAR 97.5%	-11.22%	-12.34%	-0.123	-0.149	-0.102	-0.145
Sortino	0.294	0.362	0.636	0.431	1.103	0.461
Omega ratio			2.850	4.481	4.087	5.680
Info. ratio			1.569	2.217	2.110	2.636
HHI	0.758	0.141	4.583	0.565	5.004	0.539
TTC	0.04%	0.03%	0.31%	0.07%	0.62%	0.08%
TTN	7.54%	5.50%	56.22%	12.81%	113.43%	14.32%
CER	3.91%	5.14%	14.21%	8.24%	24.89%	9.19%

Table 15 Panel B Regression analysis

	VW	EW				
Constant			0.009***	0.003***	0.017***	0.004***
			6.001	**8.302**	**8.169**	**9.827**
VW bmk			0.956***	0.994***	0.845***	0.993***
			29.711	**110.065**	**18.158**	**107.407**
Ad. R2			0.809	0.983	0.614	0.941
Constant			0.007***	0.002***	0.016***	0.003***
			5.562	**2.986**	**7.659**	**3.856**
EW bmk			0.789***	0.784***	0.687***	0.782***
			34.629	**59.356**	**18.668**	**57.588**
Ad. R2			0.852	0.944	0.627	0.941
Constant	0.0004	0.003***	0.008***	0.003***	0.015***	0.003***
	1.05	**8.454**	**6.125**	**4.500**	**7.561**	**5.585**
eMKT	1.015***	1.035***	0.903***	0.986***	0.853***	0.99***
	96.053	**92.384**	**24.842**	**59.464**	**14.525**	**58.593**
SMB	0.152***	0.735***	0.607***	0.261***	0.571***	0.265***
	9.063	**41.386**	**10.539**	**9.936**	**6.145**	**9.919**
HML	-0.119***	-0.114***	0.089	-0.093***	-0.027	-0.096***
	-7.022	**-6.385**	**1.531**	**-3.501**	**-0.286**	**-3.562**
RMW	-0.032*	-0.099***	0.060	0.046	0.151	0.038
	-1.357	**-3.988**	**0.741**	**1.244**	**1.166**	**1.026**
CMA	0.051**	0.002	0.115	0.05	0.066	0.014
	1.865	**0.071**	**1.222**	**1.158**	**0.431**	**0.319**
UMD	-0.028***	-0.112***	0.084***	-0.051***	0.14***	-0.035**
	-3.169	**-11.974**	**2.778**	**-3.720**	**2.858**	**-2.518**
Adj. R2	0.987	0.99	0.863	0.968	0.654	0.967