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INDUSTRIAL ENGINEERING AND MANAGEMENT

MODELLING THE TRADE-OFF BETWEEN SERVICE LEVEL AND WASTE OF PERISHABLE GOODS AT FOOD RETAILERS

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Management Summary

Introduction

This research on the trade-off between service level and waste of perishable goods was done for Slimstock, an inventory optimization software and consultancy company. Food retailers find the trade-off between service level and waste challenging, as an excess of perishable goods leads to waste due to expiration. Since generic waste estimation models regarding a certain service level without specific assumptions are unknown for food retailers, developers of inventory management systems are unable to include these into the functionalities of the software. Therefore, the decisions on how to reduce waste, when ordering, are dependent on human judgement instead of an analytical method, resulting in an increased risk of food waste and not achieving the desired service levels. This leads to the following research objective:

"Develop an analytical method that estimates the probability of food waste of perishable goods based on a given service level before ordering, leading to minimizing food waste whilst achieving service levels in the future for food retailers."

Context analysis

Typical inventory characteristics for supermarket stores are the (R,s,nQ)-policy, the presentation stock (a manually set minimum stock on shelf), partial LIFO demand (customers pick goods with the longest remaining shelf life), the high number of order lines, fluctuating customer demand because of promotions or events, non-stationary demand throughout the year, and non-stationary demand throughout the week. For this research, the data of a supermarket client of Slimstock, called Supermarket, was used. A case study with data from Supermarket revealed that waste is mostly encountered in the agricultural and chilled assortment categories, most perishable goods are fast-movers, review and lead times are not always equal to one day, and waste occurs for many different shelf lives.

Literature

Although the number of papers on this matter is limited, models were found in the literature that estimate food waste regarding a certain target fill rate (the percentage of demand sold from shelf). In literature, waste is denoted by the relative outdating (the ratio between the expected daily outdating quantity and the expected daily demand). To model this, three approaches were found in the literature. The first is a simulation approach. The downside of simulation, however, is the long computation time. The second is an approximation approach. Van Donselaar & Broekmeulen (2012) derived two fast approximation methods, called z_A and z_B. The third is linear regression. These approximations are improved when adding variables to the regression that estimate waste. In all these models, the EWA-policy (Estimated Withdrawal and Aging) is assumed, which is a policy that predetermines the number of goods outdated in the upcoming cover period, which are added to the order level. However, to our knowledge, models concerning all characteristics from the previous paragraph are non-existent in literature.

Approach

We enhanced the models from literature to more accurately describe the food retail setting. First, we added the FIFO (first-in-first-out) fraction. Here, a fraction of 0.8 means that 80% of demand is met in FIFO order, and 20% of demand is met in LIFO order. Second, we modeled the yearly and weekly non-stationary demand for fast-movers by the Normal and Gamma distribution, depending on the coefficient of variation (the standard deviation of demand divided by the forecasted demand). Third, the presentation stock was added. We made some assumptions to simplify the model, such as immediate replenishment in the morning, and the exclusion of promotions and events for simplicity.

Second, we calculated the approximations z_A and z_B for each week separately, to account for non-stationary demand. Third, we improved the regression by adding seasonal effects to the variables and by adding a variable containing the FIFO fraction.

We experimented with the following SKU information: a 364-day forecast, historical sales without promotions or events, target service levels of 80%, 85%, 90%, 95%, 97%, and 99%, FIFO fractions of 1, 0.8, and 0.5, lead and review time, minimum and incremental order quantity, shelf life, and presentation stock. The experiments were performed for 898 representative SKUs from 20 different subsets of SKUs. These subsets consist of SKUs with the same shelf life, lead time, and review time.

Results

The final result of the approximation by regression of one representative SKU is shown in Figure 1. In the figure, the relative outdating is visualized regarding multiple target service levels and FIFO fractions.

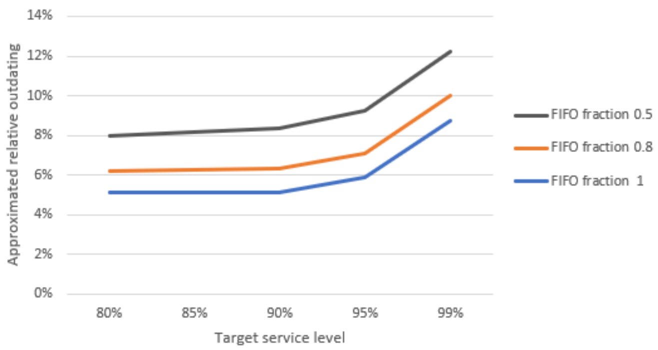

Figure 1: *The final Efficient Frontier of a representative SKU for three FIFO fractions and all target service levels.*

The performance of the models is measured by the approximation error. The simulation serves as a basis, and the performance of the approximations and regression are evaluated by the approximation error. This is defined as the relative outdating measured in our simulation, minus the approximated relative outdating. Important regression measurements are the adjusted R^2, which indicates the variability explained by the model, the RMSE, which indicates the variance of the residuals, and the p-value, which indicates the significance of the independent variables.

From the analysis of the simulation results, we can conclude that the incorporation of the FIFO fraction, non-stationary demand, and the presentation stock have a significant effect on the waste. Therefore, incorporation of these characteristics is necessary when estimating waste from a model. However, as the presentation stock seemed illogical and incorrect for some SKUs, the presentation stock was excluded from the approximations and regression. Furthermore, we concluded that regression was possible for subsets when the shelf life is two times the cover period at maximum. The average approximation error of the regression is -0.2%, with a standard deviation of 1.6%. For most subsets, the adjusted R^2 of the model was higher than 80%, the RMSE was 0.028 at most, and all variables had p-values below 5%. The performance of the approximations and regression are lacking for highly seasonal SKUs and especially for 99% target service levels, since waste grows exponentially rather than linearly in this case. Nevertheless, the outcome of the model gives a good indication for the expected waste for most SKUs.

Recommendations

The model can be used for estimating the waste percentage of an SKU. Next, supply planners can examine the effects of the change of the SKU's parameters. For future research, we suggest improving approximations for highly seasonal SKUs, incorporating promotions and events in the model, and researching to what extent the presentation stock is causing waste. We advise Slimstock to keep track of historical forecasts to better estimate the safety stock needed and to validate the model with actual waste percentages. Finally, we recommend Slimstock to apply the Implementation plan mentioned in Chapter 8 and execute a method that calculates the actual FIFO fraction of an SKU.

Acknowledgements

This research marks the end of my study Industrial Engineering and Management at the University of Twente. This thesis was written for Slimstock and is a result of more than half a year of hard work and perseverance. Writing this thesis was often challenging, but doing my internship at Slimstock made it worthwhile. First of all, although, considering the pandemic, I worked at the office in Deventer only a couple of times, I felt at home right away. I want to thank the Young Professionals of Slimstock for letting me in on all the YP calls and listening to my struggles each week. Secondly, I want to thank Nico for his help with programming. Thirdly, I would like to thank Bart for giving the answers to all my questions, so that I always left our meetings without any further concerns. And last but not least, I would like to thank Thijs, who has been the best supervisor. Thank you for all your support, time invested in my thesis, critical comments, the many discussions, and being a great friend.

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Lastly, I'm very grateful for the support of my family and friends since time spent aside from research is just as important as the time spent on research. First of all, I want to thank my parents, and especially my boyfriend, for their all-day support and encouragement. Furthermore, I would like to thank Matthijs and Robbert for their interest in the topic and the great joy during our productive and less productive study sessions. Finally, I thank my sorority for distracting me from my thesis on purpose.

During my life as a student, which took almost seven years, I learned a great deal about quantitative modeling in the production and logistics setting, working in groups, working efficiently, and communicating with others, all leading to me becoming an Industrial Engineer. I had a great time studying in Enschede and met amazing people, with whom I hope to spend more time after my graduation. I am proud of this research and looking forward to starting my career as part of the Young Professional program at Slimstock.

Eveline
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List of Abbreviations

CAO Computer Assisted Ordering

DC Distribution Center

EOQ Economic Order Quantity

EWA Estimated Withdrawal and Aging

FCC Fresh Case Cover

FIFO First In First Out

IOQ Incremented Order Quantity

LIFO Last In First Out

MOQ Minimum Order Quantity

RMSE Root Mean Squared Error

SKU Stock Keeping Unit

VIF Variance Inflation Factor

Nomenclature

α	Linear regression coefficient
β^*	Target service level
Δ	Change in inventory position
δ	Actual demand
Γ	Average daily remaining shelf life
γ	Average remaining shelf life
μ	Average daily demand
Φ	Normal cumulative density function
ϕ	Normal probability density function
ρ	EWA outdating moment
σ	Standard deviation
τ	Day in history
θ	Gamma scale parameter
B	Batch
b	Gamma shape parameter
D	Stochastic demand
d	Weekday
$E[X]$	Expected value of X
$EWAz$	Total expected EWA outdating
Ez	Expected EWA outdating
f	FIFO fraction
FC	Expected demand
G	Normal loss function
H	Historical demand
IP	Inventory Position
j	Age
k	Safety factor
L	Lead time
ls	Lost sales
M	Maximum shelf life
m	Month
MSF	Monthly seasonal factor
n	Number of cases
nQ	Order quantity
OO	Quantity on order
$P2$	Fill rate
PS	Presentation stock
Q	Case pack size
R	Review period
r	Remaining shelf life
R^2	Coefficient of determination
S	Order-up-to level
s	Reorder level
SL	Actual service level
SS	Safety stock
t	Day in future
TD	Total demand
TLS	Total lost sales
TO	Total outdating
TS	Total sales
u	Uniform distribution
W	Withdrawal from shelf
w	Week
WSF	Weekday seasonal factor
z	Expected relative outdating

z_A Expected approximated relative outdating type A

z_B Expected approximated relative outdating type B

z_{regr} Expected approximated relative outdating by regression

z_{sim} Expected simulated relative outdating

Introduction

The research for this thesis on the trade-off between service level and waste of perishable goods was done at Slimstock in Deventer. The research was conducted as part of the graduation assignment for the master's program Industrial Engineering and Management at the University of Twente.

This chapter is an introduction to the research and is constructed as follows. In Section 1.1 an introduction to Slimstock is given. The topic context is explained in Section 1.2 and serves as a background for this research and denotes the motivation and relevance of the topic. Afterwards, the problem investigated is explained in more detail in Section 1.3. Section 1.4 contains the research goal and objectives of the research. Lastly, Section 1.5 contains the research questions and reading guide.

1.1 Introduction to Slimstock

Slimstock was founded in 1993 in the Netherlands as an inventory optimization consultancy company. Slimstock's goal is to increase clients' efficiency, reduce inventory levels, and generate insight to inventory managers whilst increasing the service level. With over 1000 clients spread over 60 countries worldwide, Slimstock is the European market leader in the field of demand forecasting and inventory optimization. Nowadays, Slimstock helps companies to optimize their inventory in three ways. First of all, clients of Slimstock can use its software (called *Slim4*). The main functionalities of the software are demand forecasting, demand planning, and inventory management in order to get the right inventory at the right place at the right time. Secondly, consultancy is a big part of the activities of Slimstock. Advice is giving on, amongst others, assortment choice, promotions, and optimal production rates. Lastly, clients can follow training sessions, workshops, and seminars provided by the Slimstock Academy. Slimstock consists of several departments, one of which is the Development department. This department is responsible for improving the functional design of Slim4 and this research is conducted at this department.

1.2 Background

In 2017, the food waste per capita of the Dutch population was estimated between 106 and 147 kilograms (Soethoudt & Vollebregt, n.d.). This amounts to around 350 grams of food waste per person per day. Although the highest percentage of food waste comes from end-consumers, about 16% of all food is lost and wasted throughout the whole European supply chain from harvesting, hunting and foresting to consuming by households (Rutten, Nowicki, Bogaardt, & Aramyan, n.d.). In the same study, it was estimated that European food retailers and wholesalers account for 3.6% of the total food waste. For Dutch supermarkets specifically, it was estimated in a more recent study that around 1.7% of food does not end up with the end-consumer (Vollebregt, 2020). Amongst the foods, bread and pastries have the highest waste, with a proportion of 7.7% of the total not being sold. Fresh meats and fish have food waste of 2.9%, potatoes, vegetables and fruits have a proportion of 2.7% and dairy, eggs and ready-to-eat meals have a proportion of 1.4%.

In this research, the term food waste refers to food suitable for human consumption, but not consumed (Giuseppe, Mario, & Cinzia, 2014). For food retail environments such as supermarkets or food wholesalers, all food that is suitable for human consumption that is not sold counts as food waste. The role of inventory is to prevent getting out-of-stock, which can be defined as a product not present at the expected location (Aastrup & Kotzab, 2009). In food retail environments, on-shelf availability - the opposite of out-of-stock - is agreed to be an indicator for good customer service. Especially in case of food retailers, consumers want to buy from a wide variety of high quality and fresh products (Lebersorger & Schneider, 2014). In this thesis, on-shelf availability is made measurable by measuring the service level. What kind of service level, is discussed later on.

Kaipia et al. have concluded that expired best before dates is the most common reason for food waste in the food retail sector (Kaipia, Loikkanen, & Dukovska-Popovska, 2013). Especially for products such as milk, in case a shelf that is not empty is replenished with milk cartons that have a best before date of two days after the best before date of the first batch of milk cartons, in case of non-FIFO replenishing. In this case, if customers pick the cartons with the latest best before date, the first batch reaches its best before date in store and therefore cannot be sold anymore. Other causes for waste are, amongst others, damage during transportation, incorrect packaging, oversupply (Eriksson, Strid, & Hansson, 2014) or consumers' aversion against suboptimal foods (de Hooge et al., 2017).

The consequences of food waste for retailers are that they are faced with high costs and social blame of being one of the biggest causes of food waste (Broekmeulen & van Donselaar, 2019; Lebersorger & Schneider, 2014). People no longer accept that so much food is wasted along the supply chain. Therefore, retailers (and other supply chain links) look for ways to reduce waste by prolonging shelf life or reduce oversupply without compromising the service level.

Therefore, the question is whether food waste can be minimized whilst achieving the predetermined service level in the future. And since the Dutch Ministry of Agriculture, Nature and Food Quality wants to have food waste reduced by 50% in 2030 (Vollebregt, 2020), it becomes clear that researching the trade-off between service level and waste is important as well as relevant. In the next section, we dive into the problems that food retailers perceive concerning the estimation of food waste.

1.3 Problem statement

The previous section was about the food supply chain in general, but more specifically the food retailer clients of Slimstock too have difficulties with the trade-off between service level and waste. The problems that are encountered are presented visually in a problem cluster in Figure 1.1.

First of all, there is little applied knowledge of models that estimate the waste resulting from a given service level in the food retail market in practice. Both retailers and consultancy companies supplying inventory management systems are lacking knowledge on how to analytically estimate waste based on a given service level. Trivial problems, such as the Newsvendor Problem, are different from the current problem since some assumptions do not apply to the current problem. Examples are LIFO or random picking by consumers instead of FIFO picking and the fact that this model is applicable to products with a shelf life of one day.

As a consequence, developers of inventory management systems are unable to include the estimation of waste into the functionalities of the software. Consequently, food retailers lack knowledge about the probability of waste given a certain service level. This means that they are unknowing whether a small change in service level leads to a

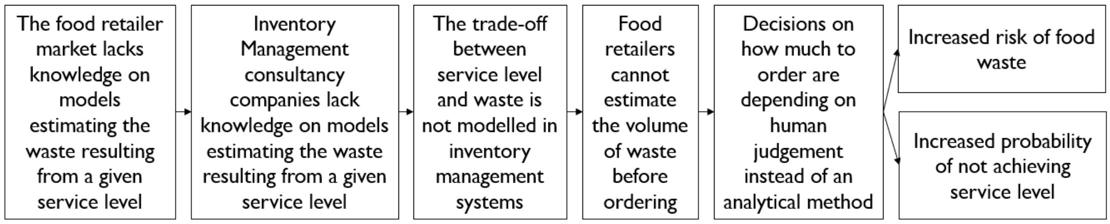


Figure 1.1: *Problem cluster of modelling the probability of food waste.*

substantially larger probability of food waste. The same holds for the reverse scenario. At this moment, food retailers cannot determine the expected service level for a given limit of food waste either.

Since the probability of food waste cannot be estimated before orders are placed, reviewing service level and waste is possible only after some time has passed. When a lot of expired products are left in-store, a manager might intuitively order fewer products next time. In the other situation, in which products were out-of-stock early in the day, the manager might decide to order more next time. This boils down to a situation in which the optimal order advice given by inventory management systems are ignored and the order size is manually adjusted.

Consequently, in situations in which fewer products are ordered than advised by an inventory management system, there is an increased probability of not achieving service levels. This entails lower customer satisfaction and a lower profit margin because of lost sales. Another perceived problem is the increased risk of food waste in case more products are ordered than advised by the inventory management system. This is a problem since the costs of expired products are very high. Not only costs for harvesting, assembling, producing, transporting, and/or staging products are incurred, but if not sold also costs are incurred for removing from the shelves and dispensing. Whereas no revenue is earned on these products. Furthermore, checking shelves on expired products is labour-intensive. Therefore, suboptimal situations like out-of-stock or food waste should be avoided. Taking this all into consideration, the problem statement is as follows:

"Food retailers lack pre-order knowledge on the probability of food waste given a certain service level, leading to a situation in which food waste is not minimized and/or the service level is not achieved."

The next section describes the research objective, belonging to this problem statement.

1.4 Research objective

The objective of this research is to develop an analytical method that estimates the probability of food waste of perishable goods. The research is restricted to perishable goods with a shelf life from 2 to 30 days (Broekmeulen & van Donselaar, 2009). Secondly, in this research, the forecasting of product sales is out-of-scope. We assume forecasts for products are adequate and given. Furthermore, the method can estimate the probability of food waste based on a given service level, denoted by the client of Slimstock in Slim4.

Furthermore, this research is restricted to the food retail environment, i.e. supermarkets. More details on this choice can be found in Section 2.2. Physical shops are taken into account, but distribution centres are not taken into account. This means that in physical shops the consumer can select the products he/she wants to buy in a LIFO, FIFO or random manner. Furthermore, the method is usable for an environment in which replenishment of products is done periodically and in small batches (Broekmeulen & van Donselaar, 2009). Lastly, the method is verified and validated such that it estimates the expected waste resulting from a target service level. The objective of this research is therefore defined as follows:

"Develop an analytical method that estimates the probability of food waste of perishable goods based on a given service level before ordering, leading to minimizing food waste whilst achieving service levels in the future for food retailers."

In order to achieve the research objective, we have made research questions. The questions are answered one by one to gradually come to the solution. These questions and plan of approach are discussed in the next section.

1.5 Research questions

Each question entails one chapter and consists of multiple sub-questions. First of all, the current way of working and opportunities are analysed. After researching the first research question, it should be clear how the - for this research relevant - components of Slim4 work, what are the business and order characteristics of food retailers, how are clients informed and how do they make decisions based on the data in Slim4, what products of Slimstock's client Supermarket contain the highest waste percentage and what the requirements of the new models entail. The information needed mostly comes from a Slim4 training, through meetings with employees of Slimstock, and corresponding literature afterwards. This leads to the following (sub)questions:

1. What does the ordering process look like for food retail clients?

    a. *How are replenishment orders currently generated for Slimstock's food retail clients*

    b. *What is the service level measure and in what way is the service level taken into account?*

    c. *What information and decision support on food waste do Slimstock's clients get when placing a replenishment order?*

    d. *What are the requirements of the new models?*

    e. *For what products in the product assortment of Supermarket are the new models most relevant?*

Through an extensive literature study, we find out the most important theories on modelling service level and waste, determine important parameters and variables, and calculate or approximate the expected waste. This leads to the following research questions:

2. What can we learn from literature about modelling service level and waste?

    a. *How can substitution, shelf life, non-stationary demand, presentation stock, and partial FIFO demand be modelled?*

    b. *What models concerning both the calculation of expected waste and the target service level are described in literature?*

    c. *How do we calculate the expected waste?*

The answer to the next question contains the developed models on the basis of the literature found that estimates the probability of waste given a certain service level. The notation and assumptions are explained, parameters and variables are given, and we denote the alterations made on the models in literature. This leads to the following research questions:

3. How are the models formulated that estimate the expected waste on the basis of the target service level for food retailers?
    a. *What is the design of the models?*
    b. *What assumptions are made?*
    c. *What equations, parameters and variables are used?*
    d. *What alterations are made on the models from literature?*

After answering the next research questions, it should be clear how the experimental settings are defined and how the models can be evaluated. We therefore answer the following research questions:

4. How can the models be evaluated?
    a. *How is the needed data obtained?*
    b. *How can we measure the performance of the model?*
    c. *How is the model validated and verified?*
    d. *What experimental design is relevant?*

In the next chapter, experiments are performed with empirical data from a client of Slimstock to see how the models perform. The results of the experiments reveal under which circumstances Slimstock's clients can expect what amount of waste given a certain service level. This research question is answered through experiments or simulation with the data of a client of Slimstock of which conclusions are derived. This leads to the following research questions:

5. What is the performance of the models?
    a. *What is the influence of non-stationary demand, mixed FIFO-LIFO withdrawal, and presentation stock on the expected waste?*
    b. *For what shelf lives is the model relevant?*
    c. *Which variables are the best predictors of expected waste?*
    d. *How does the Efficient Frontier look like when considering the model alterations?*

The thesis structure is as follows. Chapters 2 to 6 answer research questions 1 to 5, with each research question in one chapter. Chapter 7 entails the final conclusion. Lastly, Chapter 8 contains the discussion, and implementation plan for Slimstock, as well as recommendations and suggestions for future research.

Context analysis

For the first research question, we combine insights from a user training of Slim4, meetings with Slimstock's employees, data analysis, and literature. In the first section of this chapter, the general inventory model and its parameters used in Slim4 are explained. Section 2.2 describes characteristics of food retail clients, such as assortment, supply and inventory management, and customer demand. In the third section, the knowledge obtained from Section 2.1 and Section 2.2 are combined to give an overview of how food retail clients of Slimstock currently work with Slim4 and what decision making support on food waste they get. Next, Section 2.4 states the results on a preliminary data analysis on waste for Supermarket, a client of Slimstock. Finally, Section 2.5 describes what the requirements and conditions are for the implementation of the model or improvement in Slim4. This chapter ends with a conclusion that answers the following research questions:

1. What does the ordering process in Slim4 look like for food retail clients?
    a. *How are replenishment orders currently generated for Slimstock's food retail clients*
    b. *What is the service level measure and in what way is the service level taken into account?*
    c. *What information and decision support on food waste do Slimstock's clients get when placing a replenishment order?*
    d. *What are the requirements of the new model?*
    e. *For what products in the product assortment of Supermarket is the new model most relevant?*

2.1 Slim4 inventory model

In this section, the inventory management system of Slimstock, called Slim4, is explained. Slim4 is a Computer Assisted Ordering (CAO) system, which means that the system proposes an order quantity and a decision maker proceeds the ordering process (Haijema, 2011). In Slim4, inventory is managed in four steps, namely by demand classification, statistical forecasting and demand planning, calculations for order advice, and optimizing replenishment.

2.1.1 Demand classification

Before calculations are done on what to order when for each *Stock Keeping Unit* (SKU), Slim4 first classifies each SKU by its historical sales in a certain period. The classification of SKUs is important since forecasts of demand are calculated differently for products from different demand classes. All products are classified on historical demand and distinctions are, for example, made between fast-movers and slow-movers.

2.1.2 Statistical forecasting and demand planning

The next step is statistical forecasting and demand planning. The demand of all SKUs is forecasted based on the historical sales and demand classes. Subsequently, users of Slim4 can manually alter the forecasts when it is expected or known that demand will be in- or decreasing. For example, the sales are expected to be higher when a promotion in the form of discounts is coming or when an event, such as Christmas, is approaching. However, as explained in Section 2.5, promotions are not taken into account in this research.

2.1.3 Calculations of order advice and optimizing replenishment

Next, calculations are done on how much to order and when. The inventory management system in Slim4 for most clients is set up as an (R, s, nQ) inventory policy. In this policy, the *inventory position* (IP), defined as the stock on hand plus the pipeline inventory minus the backorders, is checked to see if it falls on or below the *reorder point* (s). If the inventory position is below the reorder point, an order advice is generated, with size $n \cdot Q$. Here, n is the number of case pack sizes and Q is the case pack size. It depends on the *review period* (R) whether an order is actually placed by the Slim4 user. The inventory model is continuous (R=1 day) or periodic (R≥2). In case the review period is not over yet, no order is automatically placed.

The IP after replenishment should cover at least the *expected demand* $E[D]$ during the *lead time* (L) and *review period* (R) of the replenishment $E[D_{L+R}]$ and the *safety stock* SS. Such that the current IP plus $n \cdot Q$ is equal to or larger than s. The safety stock for, for example, fast-movers is calculated from the *safety factor* (k), which results from the *target service level (fill rate)* β^* set by the client and the Normal Loss function (Silver, Pyke, & Thomas, 2017), times the standard deviation of the demand during lead time and the review period (σ_{L+R}). The service level in Slim4 is defined as the fill rate, which is the (expected) percentage of demand sold from the shelf. The exact calculations of the order level and other parameters are discussed in Section 3.4.1.

Having done these calculations, Slim4 sets up an order advice with a replenishment order quantity of at least the *minimum order quantity* (MOQ). In case a bigger replenishment order quantity than the MOQ is needed, the MOQ is incremented with the *incremented order quantity* (IOQ) until the replenishment order quantity is large enough. Lastly, the replenishment can be optimized, for example, by adding more products and therefore optimizing a full truck load.

Although the (R,s,nQ) policy is most common for clients, Slim4 can work with other policies and calculations of the order quantity. Examples are the *economic order quantity* (EOQ) and an (R, s, S) inventory policy. When the order quantity nQ is calculated with the EOQ, ordering costs, and holding costs are taken into account when determining the replenishment order quantity. Finally, Slim4 is capable of determining the variable order quantity based on an (R, s, S) policy, where after the review period a quantity of *order-up-to level* (S) minus the inventory position is ordered when the IP falls below the reorder point (s) such that the IP after replenishment is equal to the order-up-to level.

2.2 Food retail characteristics

The food clients of Slimstock, including food manufacturers, food wholesalers and club stores (large retailers specializing in bulk-sized products (Cai, Volpe, Schroeter, & Mancino, 2018)), supermarket chains, and superettes (small supermarkets with self-service features (Cai et al., 2018)), are quite diverse. At first, it was thought that supermarket chains, as well as food wholesalers, club stores, and superettes were eligible for this research, but many of the perishable foods and customer demand characteristics as described in the rest of this section do not apply to food manufacturers, food wholesalers, club stores, and superettes. Therefore, the decision was made to focus this research (and therefore the rest of this section) on supermarket chains only.

Although different supermarket chains differ in their strategies based on pricing, service or assortment with both food and non-food (Solgaard & Hansen, 2003), there are also many comparisons in terms of assortment, supply processes, inventory management, and customer demand.

2.2.1 Assortment

The supermarket store assortment can be divided into multiple types, namely:

1. non-food, such as magazines and sanitation products,
2. food, which can be divided into more categories, namely:
   a. non-perishables with a store shelf life of thirty days or longer, such as rice or sauces,
   b. perishable products with a store shelf life of eight to thirty days, such as milk and meat,
   c. ultra-fresh perishables, which have a store shelf life of two to seven days, such as ready-to-eat meals, vegetables, and fruits,
   d. bakery products, such as bread, which have a store shelf life of one day and are disposed of after one day (Slimstock-Consultant, 2020; van Donselaar, van Woensel, Broekmeulen, & Fransoo, 2006).

A characteristic of perishables is the significantly lower rate of deterioration in certain circumstances, such as a refrigerated environment for ready-to-eat meals, as opposed to regular supermarket store temperature. Furthermore, Van Donselaar et al. (2006) pointed out that perishables have more average weekly sales in cubic meters and a lower coefficient of variation of weekly sales compared to non-perishables. Only perishable products with a store shelf life of two to thirty days (items b. and c.) are examined in this research. This is because non-perishables do not fit into the scope, and bakery products are replenished multiple times a day depending on the customer demand.

2.2.2 Supply processes

Before explaining the supply processes of supermarket stores, the meaning of some words are defined. When *supermarket chain* is stated, a certain supermarket brand is meant. A supermarket chain usually consists of multiple supermarket locations and one or more distribution centers (DCs). With a *supermarket location*, we mean one location of the supermarket chain, that has a store where customers can shop. The supermarket location can be further divided into the *store warehouse*, where products are delivered from trucks, and the *store*, where shelves with products are replenished from the inventory in the store warehouse, and where customers shop for products.

The delivery time of replenishment is different for each product, supermarket chain and stores (Slimstock-Consultant, 2020). However, the process of supplying products is quite the same for most supermarket chains. The supply chain of supermarket chains consists of multiple food producers and wholesalers that supply non-food and food with a long store shelf life to a DC. There, the products are repacked and distributed to one or more supermarket locations. Food producers and wholesalers of perishable foods, however, usually supply supermarket locations directly, without interference of a DC (Slimstock-Consultant, 2020; van Donselaar et al., 2006) or use cross-docking. The goal of the direct delivery by producers is to reduce the lead time. Because of this reason, only supermarket stores and not DCs are taken into account in this research. Replenishment trucks come to the supermarket location every day at different times, though usually in very similar routes (Slimstock-Developer, 2020). Not every product is replenished every day, but if a product is replenished in the store warehouse by a truck, this is at most once a day. However, replenishment of the product's shelf from the store warehouse could happen more than once a day (Slimstock-Consultant, 2020). In general, the delivery frequency of perishable goods is higher than the delivery frequency of non-perishables (van Donselaar et al., 2006).

2.2.3 Inventory management

The inventory policy of supermarket stores is usually modelled with an (R,s,nQ)-policy. The order quantity is rounded up to n case pack sizes Q, i.e. it is not possible to replenish half a six-pack of soda cans. Furthermore, when ordering the lead time and review time of the product are taken into account. Typical for the food retail are short review and lead times. However, a review time of one day does not necessarily mean that a product can be ordered on each day. As a traditional week consists of five working days and two days weekend, a limited number of products can be ordered or replenished during the weekend, i.e. the review (and lead) times are not static for a product.

A typical inventory characteristic in supermarket stores is the *presentation stock*. Shelves filled with products are just as much part of overall product presentation as good-looking packaging. Supermarket store managers generally want shelves (visually) filled, even though they risk expiration of products when more products are on the shelf than necessary. Furthermore, when multiple batches of one product are on the shelf, and the batches have different shelf lives, managers strive to fill the shelf with products with a shorter store shelf life on the first row. Usually, a product is picked *first-in-first-out* (FIFO), but some consumers pick the product with the longest available store shelf life, and pick *last-in-first-out* (LIFO) (Li, Yu, & Wu, 2016). However, in the last couple of years supermarket chains have started to discount products that have a short remaining store shelf life, in order to promote buying FIFO and influence customer demand (Chen, Pang, & Pan, 2014).

2.2.4 Customer demand

Striving for high availability is a characteristic of food retail customer demand. For supermarket chains, the availability of products is most important (Slimstock-Consultant, 2020), while maintaining below a certain level of food waste. Food waste is a high cost item for supermarket chains (Slimstock-Consultant, 2020) and usually the managing board sets a maximum budget (Van Donselaar & Broekmeulen, 2012) or maximum percentage (Slimstock-Consultant, 2020) for waste disposed. Lastly, the likelihood of substitutes implies that customers buy another product or the retailer experiences lost sales, and that therefore backordering is not part of the inventory policy.

Another characteristic of supermarket store customer demand is the high number of *order lines*. Typical for supermarket stores compared to other businesses in retail are the many customers (or order lines), i.e. a customer of a supermarket store usually buys less than 5 cucumbers at once instead of 100 such as customers of a wholesaler. Besides, the number of customers per day varies throughout the week. Customer demand for supermarket stores is typically non-stationary within a week, meaning that more products are bought on Fridays and Saturdays than on other days of the week (Broekmeulen & van Donselaar, 2009). Furthermore, another characteristic of supermarket customer demand is that customers often buy substitutes or sales are lost, i.e., when the favourite salad of the customer is sold out, the customer chooses another salad or the customer does not buy a salad at all.

Lastly, food retail products are highly susceptible to demand variations around promotions and events. Demand forecasts and actual sales during and right after a promotion or event are highly influenced. Since promotions often take place, forecasting demand during promotions is not difficult when taking historical sales into account (Slimstock-Consultant, 2020). However, during promotions, the variability in demand increases, and so do the probabilities on stock-out or waste.

2.3 Ordering process

Now that the theoretical inventory model of Slim4 and some supermarket chain and store characteristics are explained, this section explains how in practice decisions on order quantity and waste are taken when supermarket chain clients use Slim4. But first, a more detailed description of the service level is given. The service level in Slim4 is defined as the *fill rate*, which is the (expected) percentage of demand sold from the shelf. The target fill rate is a tactical or strategical parameter of the product, and for most clients of Slimstock the target fill rate of a product is determined by estimating the importance of the product on-shelf (Slimstock-Developer, 2020). The target fill rate can be based on the ABC-classification and/or volatility of demand. In supermarket stores, a slow-moving perishable item gets a target fill rate of 70%, whereas a fast-moving item gets a target fill rate of more than 95% (Jiang, Shi, & Shen, 2019).

Currently, there is no insight on what the probability of waste of products on the shelves after replenishment is, when deciding on how much to order. In case the company wants to overrule the calculations, tailor-made logic is configured to manipulate the order advice to the client-specific needs. Furthermore, the inventory manager or planner is always able to adjust the actual order quantity.

2.4 Case study

In this section, we perform an analysis of the data of a Dutch supermarket chain and client of Slimstock. We further refer to this supermarket chain as Supermarket. Supermarket has integrated Slim4 in all its stores and one DC. All stores contain over 20.000 products and both non-food and food products are part of the assortment of Supermarket. For this analysis, Supermarket provided product information and transaction data of two consecutive months of all stores and DC, and two years of historical sales. Since the DC is out of scope, the data of the DC is taken into account. The studied period in this case study was the transaction and product information data of two consecutive months. The data provided are transactions of all shops and product information, such as the average store shelf life, MOQ/IOQ, and target service level.

New and end-of-life products were excluded from the analysis. SKUs were removed from the analysis when they satisfied at least one of the following criteria: (1) SKUs marked as end-of-life, (2) SKUs removed from the assortment in the studied period, (3) SKUs introduced into the assortment during the studied period, (4) SKUs without sales in the studied period.

The data cleaning process was short: A small percentage of the products had a set shelf life of 0, which is not possible in practice. Based on other products in the same assortment category with the same characteristics, the assumption was made that the shelf life was 7, 3 and 1 days for flowers & plants, agricultural and bakery products respectively. Perishables in the assortment are covered by the categories of bakery products, agricultural, cheese, meat, chilled, flowers & plants, and groceries.

The goal of the analysis was to find out what part of the assortment generates most waste and should, therefore, be the focus of this research. For this analysis, we selected all transactions with type 'waste' and used all SKUs that had at least one 'waste' transaction in the researched period. Within the products with waste, we differentiate between products with a low waste percentage and a high waste percentage, such that about 50% of the products have a high waste percentage. The threshold for this is 8%, i.e. around 50% of the products that encountered waste at least once in the research period have a waste percentage of 8.01% or higher and are defined as products with a high waste percentage. The products with a low waste percentage are denoted by the orange color in Figure 2.1. The results of the analysis are as follows.

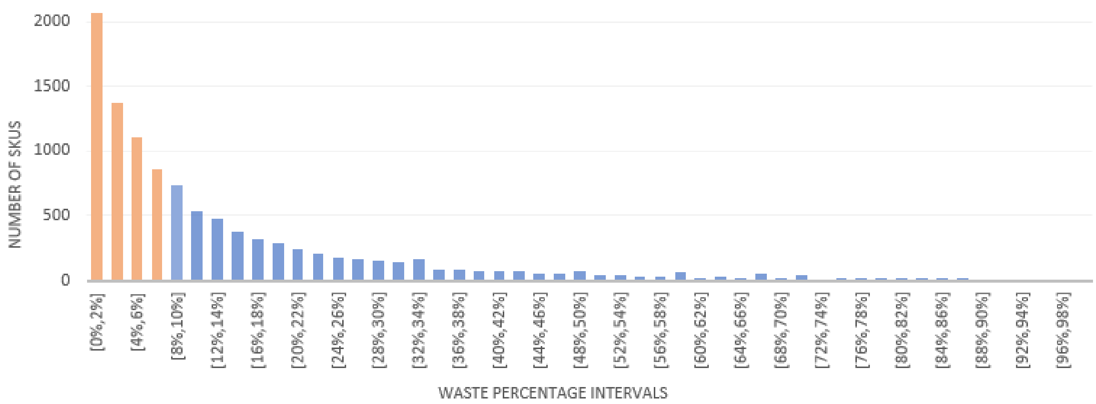


Figure 2.1: *Distributions of waste of perishable SKUs in the studied period*

- This research should focus on those assortment categories that experience most waste. When we take a look at the assortment categories of wasted products, about 55% of them are covered by the agricultural and chilled assortment categories. Thus, this research focuses on products from the agricultural and chilled assortment categories only.

- About 90% of the products with a high waste percentage are fast-movers, defined as products with at least 24 customer order lines per year (Gelders & Looy, 1978). This means that the research is focused on fast-movers.

- 77% of the products with a high waste percentage have a review time of 1 day, called continuous review. The other products have a review time of two days or more. This means that the to-be-developed model should not only assume continuous review.

- About 11% of products with a high waste percentage have an MOQ (batch size) resulting in an inventory that is equal to or larger than the demand during shelf life. We computed this by diving the MOQ by the demand during shelf life (as will be explained in Section 3.3). This means that for 11% of the products with a high waste percentage, one of the problems is an MOQ that is too large (or sales that are too low).

- One would expect that products with a short shelf life (seven days or shorter) are the products experiencing more waste compared to products with a longer shelf life. However, of the products with a high waste percentage in the chilled assortment category, 92% has a shelf life between 8 and 30 days. This means that all shelf lives up to 30 days should be included. Note that most product shelf lives are fixed because of an expiration date on the package. For some agricultural products, however, the shelf life is variable, e.g. a mango in a crate might last three or four days depending on environmental factors.

2.5 Conditions for implementation

In addition to the scope defined in Section 1.4, the analytic method should estimate the expected waste of a perishable product given a certain service level. The opportunities for this are high since currently waste is not explicitly taken into account into the standard (R,s,nQ) inventory model. The to-be-developed model should help inventory managers or planners with the decision making on order quantities based on service level and expected waste. A limitation is that the model should not decrease Slim4's performance on inventory management. Furthermore, the user-friendliness of Slim4 may not be pressurized and the new method should be well-structured and easy to understand, as logistic managers prefer this in practice (Haijema & Minner, 2019).

Thirdly, the model should be an add-on to the current inventory policy and parameters described in Section 2.1, otherwise, there is no practical relevance to this research. This does not mean that the model should be entirely based on the current parameters and variables in Slim4, but implementing a method that calls for many extra data points is impractical.

Finally, the model should serve as a decision making aid for tactical decisions. This means that the expected waste for a group of products or a single product is determined during a tactical service level analysis. Changes in the order quantity will, therefore, not influence the displayed expected waste. Furthermore, a tactical model implies that operational decisions such as promotions and discounting products with a store shelf life of 0 or 1 day, are out of scope for this research.

2.6 Conclusions

In conclusion, this chapter guides the literature review in the following direction. We will read literature considering an extension to the standard (R,s,nQ) ordering policy that estimates waste for fast-moving perishable products with a fixed or variable shelf life ranging from 2 to 30 days in the chilled and agricultural assortment categories. The model should be easy to understand and support tactical decisions. The inventory policy preferably takes a service level metric (such as the fill rate) into account. Other necessities are presentation stock, mixed FIFO and LIFO withdrawal, substitution, lost sales, non-stationary demand, a positive review time, and a positive lead time. Lastly, models should be found that determine waste and service level simultaneously.

Literature

In this chapter, we find out about the most important models and theories on modelling service level and waste through an extensive literature study. The process of this study is explained in Section 3.1. The conclusion of Chapter 2 gave direction to this literature search with assumptions and characteristics the to-be-developed model should adhere to. Section 3.2 explains how these assumptions and characteristics are modelled in literature. Models concerning service level and waste simultaneously are described in Section 3.3. Some calculations of expected waste are exact. These calculations can be found in Section 3.4. Other calculations are approximations of expected waste, that can be found in Section 3.5. This chapter ends with a conclusion that answers the following research questions:

2. What can we learn from literature about modelling service level and waste?
    a. *How can substitution, shelf life, non-stationary demand, presentation stock, and partial FIFO demand be modelled?*
    b. *What models concerning both the calculation of expected waste and the target service level are described in literature?*
    c. *How do we calculate the expected waste?*

3.1 Search process

This section contains the search process of the literature found and used in this chapter. The objective of this research is to develop a model that estimates the probability of food waste of perishable goods based on a given service level. Therefore, the goal of the literature search was to find literature considering tactical models that model the trade-off between service level and waste, and models that determine the expected waste. As described in Section 2.6, the literature should consider estimating waste for fast-moving perishable products, with a fixed or variable shelf life, a presentation stock, mixed FIFO and LIFO withdrawal, substitution or lost sales, non-stationary weekly demand, a positive review time, and/or a positive lead time. The literature described in this chapter was sourced from Google Scholar and the University of Twente Worldcat catalogue from September to November 2020. The main search terms used were a combination of perishable or food, waste or outdating or disposal, estimate or model, inventory management, service level, fill rate, and/or shelf life.

3.2 Assumptions and characteristics

Before we explain what models were found that estimate the expected waste resulting from a given target service level, we first research some assumptions and characteristics that are applicable to this research. Namely, how to model substitution, shelf life, non-stationary demand, presentation stock, and how to model partial FIFO withdrawal. First of all, The research on substitution was very limited, i.e. substitution was only considered for a two-product case, making it inapplicable to this research. Besides, no data is available on substitution. Furthermore, no scientific literature was found on overriding safety stocks by means of presentation stock. This makes sense since most research on safety stocks is about calculating safety stocks, and manually adjusting safety stocks is therefore not a logical subject for research, although this does occur in practice for various reasons. For shelf life, modelling partial FIFO withdrawal and stochastic non-stationary demand more results were found. These characteristics are discussed in the next two sections.

3.2.1 Shelf life

One literature review has mentioned that in perishable inventory literature, product shelf life and customer demand play the most important role (Chaudhary, Kulshrestha, & Routroy, 2018). We consider the shelf life first. The product shelf life or product lifetime can be fixed, or random. An example of random lifetimes is fresh fruits and vegetables (Chaudhary et al., 2018). Chen & Lin (2002) mention that the deterioration time can be modelled by a Normal distribution and this is the most used distribution for shelf life in real-world cases. Another paper mentions Exponential distribution for the shelf life (Duong, Wood, & Wang, 2015), suitable for products with a very short shelf life. However, most papers assume fixed shelf lives since this simplifies calculations.

3.2.2 Partial FIFO withdrawal

As explained in Section 3.4, partial FIFO withdrawal can be modelled by a FIFO fraction, which is the fraction of demand withdrawn in FIFO order. The research done on product's FIFO fractions is limited (Bastiaanssen, 2019). The papers found by Bastiaanssen mentioned different FIFO fractions, ranging from 0.25, to 0.6 or even 0.9. In his own research, Bastiaanssen found that FIFO fractions differ per store, but differences between products were most significant. Consequently, it should not be assumed that FIFO fractions are equal for different products in the same product category.

3.2.3 Non-stationary demand

Stochastic demand can be modelled by using multiple probability distributions, such as Normal, Lognormal, and Exponential for fast-moving products (Chaudhary et al., 2018). Models that include stochastic as well as time-varying (non-stationary) demand better serve inventory models than models that only consider stochastic demand, but contributions in research on more than two products are limited (Chaudhary et al., 2018). In the literature found, two calculations were often used to model non-stationary demand. Firstly, in Rossi (2010), the demand is modelled as the expected demand on day t (Rossi, 2013). Secondly, in the paper of Pauls-Worm et al. (2014), demand is modelled by a Normal distribution in a certain time period and differs per day. For all products in the research, the empirical standard deviation was replaced by a certain value, such that the coefficient of variation is 0.33 for each product (Pauls-Worm, Hendrix, Haijema, & van der Vorst, 2014). This ensures that the probability that the demand is less than 0 items is almost zero. According to Silver, Pyke and Thomas (2017), the Normal distribution can only be used if the ratio σ_L/μ_L is smaller than 0.5. Otherwise, it is more desirable to use the Gamma distribution for the demand (Silver et al., 2017), or another PDF positive on the x-axis, such as the Lognormal distribution since with these distributions only positive values for expected demand are realizable.

3.3 Modelling waste and service level simultaneously

One of the first papers written on perishable inventory management was by Van Zyl in 1964 about replenishment policies for a single echelon inventory system for perishable goods with a fixed lifetime and stochastic demand (Van Donselaar & Broekmeulen, 2012). It was argued at that time that different policies for non-perishable and perishable goods were needed since *"the assumption that an item can be stored indefinitely in warehouses does not hold for perishable goods"* (Balugani, Lolli, Gamberini, Rimini, & Babai, 2019). Today, perishable inventory models are a hot topic, which is demonstrated by a large increase in the number of papers published on the subject from 2012 compared to before 2010 (Janssen, Claus, & Sauer, 2016). Most papers research other ordering policies than the standard (R,s,nQ)-policy in the setting of food retail, food production, blood, and medicines, such as an (R,s,nQ,Qmax)-policy where the order quantity is nQ but at most Q_{max}. Adjusting the standard ordering policy in these papers is often done with minimizing costs as the objective. In some of these models, costs for disposing of outdated/expired products are taken into account. However, models with an objective to minimize waste are rare, which is also denoted by Jansen et al. (2016). Among these models that estimate expected waste, only some take the service level into account (Janssen et al., 2016; Bijvank & Vis, 2011). Furthermore, when models account for a service level, it is mostly assumed that the target level is already set instead of proposing a method that determines the best target service level. In all perishable literature inventory models, only two papers were found with the objective to model service level and waste simultaneously.

First of all, Van Donselaar & Broekmeulen (2012) derived two approximations for the expected waste, also denoted as the *relative outdating* z. The authors define the relative outdating as the ratio between the expected daily outdating quantity and the expected daily demand. More about the approximations is explained in Section 3.5. By calculating the approximations for every fill rate percentage, an Efficient Frontier is obtained as a result. The Efficient Frontier can be seen in Figure 3.1, where the expected outdating is set out to the fill rate. The lines in the figure represent all products with a certain shelf life M. The figure should be read as follows: if a product has a shelf life of 3 days and the maximum outdating target is at most 10%, the target fill rate is set to 77%, and vice versa. Clearly, the expected outdating grows exponentially when the fill rate is approaching 100%. The approximations were calculated by assuming a FIFO withdrawal policy and stationary demand. Since FIFO underestimates outdating when a part of the products on shelf is withdrawn in LIFO order, this means that the Efficient Frontier represents a lower bound for the outdating percentage for any given fill rate or an upper bound for the target service level as the diagram is symmetric.
Secondly, Broekmeulen & Van Donselaar (2019) derived another Efficient Frontier based on the assumptions of Van Donselaar & Broekmeulen (2012) and slightly reformed the approximations. In this paper, the Efficient Frontier was determined for each store, by analyzing specific item-store combinations, and a distinction was made between assortment categories. The authors argued that analyzing each store separately is fairer since each store experiences different sales for each item.

Finally, Broekmeulen & van Donselaar (2019) propose a first indicator of outdating, called the Fresh Case Cover (FCC). It is a simple formula, defined as the case pack size Q divided by the demand during shelf life, also denoted as:

$$FCC = \frac{Q}{M\mu} \tag{3.1}$$

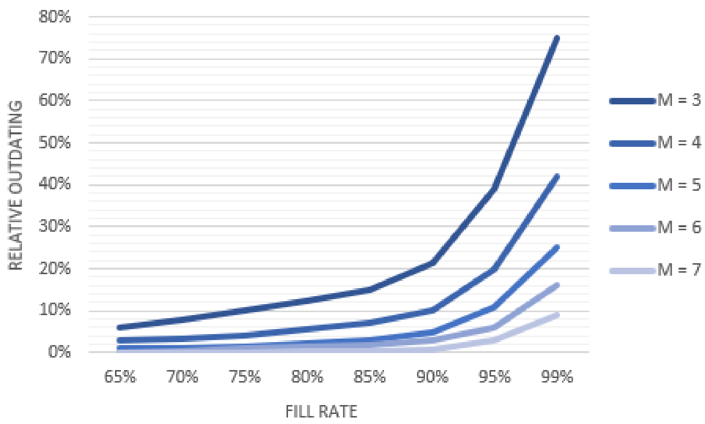


Figure 3.1: *Efficient Frontier per shelf life (Van Donselaar & Broekmeulen, 2012)*

Note that the *case pack size* is expressed as Q, and the *order quantity* is expressed as n_tQ. If a product's FCC > 1, this means that the case pack size is too large for its demand during shelf life. This is an indicator that either the demand should be enlarged, the shelf life should be elongated or the case pack size should be reduced. Furthermore, an FCC $>$ 1 indicates that the service level is achieved and has outdating as a result, independently of the set target service level or outdating target. For products with an FCC < 1, in theory, the lower the FCC, the higher the probability of achieving the service level and the lower the expected outdating.

3.4 Exact calculations for the expected outdating

Early models that calculated the expected outdating were based on the calculations of Nahmias (1982). The author based the model on stochastic demand and unrealistic assumptions, such as a fixed shelf life of two days, zero lead time, continuous review, a FIFO issuing policy, and backlogging. Chiu (1995) extended this model by adding a positive lead time (Chiu, 1995). In both papers, the expected outdating was calculated based on the order quantity. In another paper, the number of outdates is calculated by taking the remaining shelf life of the batches in the inventory into account, rather than calculating the expected outdates based on the order quantity (Duan & Liao, 2013). Furthermore, this model is capable of estimating the number of outdates for a certain planning horizon longer than $L + R + M$ in the beforementioned models. The paper of Lowalekar & Ravichandran (2017) bases the expected outdates on the age of the inventory too (Lowalekar & Ravichandran, 2017). However, this paper concerns itself in the setting of a blood bank environment, with assumptions and model characteristics that do not fit the fast-moving retail setting of this thesis, such as a Poisson distribution, not more than one outstanding order per product per unit time, and the assumption that a batch starts aging only after the older batch is completely removed from inventory.

The most interesting papers found that calculate outdating with exact calculations are the papers of Broekmeulen & Van Donselaar (2009) and Haijema & Minner (2019). Both papers estimate the expected outdating with exact calculations based on the age of the inventory in a (deterministic) simulation. The two models are built under different assumptions leading to different model characteristics. Before we dive into the equations of these papers, first the equations are explained that are used in the standard (R,s,nQ)-policy by Silver, Pyke and Thomas (2017) and the EWA-policy in the next subsection.

3.4.1 (R,s,nQ)-policy equations

In the (R,s,nQ) policy, an order advice is generated when the inventory position IP_t on day t just before placing an order is below the dynamic reorder level s_t. Let L be the lead time, R the review time, and $L+R$ the cover period. The reorder level is the dynamic safety stock plus the dynamic expected demand during the cover period. We define this as follows.

$$s_t = SS_t + \sum_{i=t+1}^{t+L+R} E[D_i] \tag{3.2}$$

where the safety stock SS_t is calculated as

$$SS_t = k_t \cdot \sigma_{D_{L+R}t} \tag{3.3}$$

where k_t is the safety factor on day t, and $\sigma_{D_{L+R}t}$ is the dynamic standard deviation of the demand during the cover period (Silver et al., 2017). Equation 3.2 is true if the demand during the cover period is modelled by a Normal distribution. The standard deviation of the demand during the cover period is calculated as

$$\sigma_{D_{L+R}t} = \sigma_{Dt} \cdot \sqrt{L+R} \tag{3.4}$$

For the determination of the safety factor, the target fill rate β^* is incorporated. The first step to estimate the safety factor in a lost sales situation is to use Equation 3.5.

$$G(k_t) = \frac{n_t Q}{\sigma_{D_{L+R}t}} \cdot \frac{1-\beta^*}{\beta^*} \tag{3.5}$$

Let $G(k_t))$ be the normal loss function of k_t and n_tQ the order quantity. In the book of Silver, Pyke, and Thomas, n_tQ and β^* in Equation 3.5 are static values resulting in a static value for the safety factor k_t unless the value of $\sigma_{D_{L+R}t}$ changes. In this thesis, however, n_tQ is not a static value and could change per day. Therefore, the value of k_t could change every day. We define n_tQ based on a existing formula (van Donselaar & Broekmeulen, 2013) made applicable to non-stationary demand as follows:

$$n_t Q = \max\left(MOQ, \sum_{i=t+1}^{t+R} E[D_i]\right) \tag{3.6}$$

As for $G(k_t)$ in Equation 3.5, the normal loss function is given in Equation 3.7.

$$G(k_t) = \phi(k_t) - k_t \cdot (1 - \Phi(k_t)) \tag{3.7}$$

where $\phi(k_t)$ and $\Phi(k_t)$ are the probability density function and the cumulative density function of the standard Normal distribution respectively.

In case $IP_t < s_t$, a number of case packs n_t of size Q are ordered such that the inventory position after replenishment is exactly or just above the reorder level. The number of case packs n_t to order can be calculated as follows.

$$n_t = \left\lceil \frac{s_t - IP_t}{Q} \right\rceil \tag{3.8}$$

where $\lceil x \rceil$ rounds up x to the nearest integer.

3.4.2 EWA-policy

This subsection explains the EWA or *Estimated Withdrawal and Aging* policy that is used in the paper of Broekmeulen & Van Donselaar (2009) described in the next subsection. The EWA-policy differs from the standard (R,s,nQ)-policy by not only taking the inventory position of the current day into account, but also the expected outdating in $L+R-1$ days when determining the ordering quantity. This means that the EWA-policy orders the rounded difference between the order level and inventory position plus the expected outdating, whereas the standard inventory policy only orders the rounded difference between the order level and inventory position. The example below explains why it is important to correct the order quantity for the expected outdating.

Consider the example used in the former sections. The product has a lead time of 1 day, 1 day review period, a fixed safety stock of 15 units, a case pack size of 1, expected demand of 10 items, and a physical inventory of 20 items. The inventory consists of two batches, namely batch A of 15 items with shelf life $t+1$ day and batch B with 5 items with shelf life of $t+4$ days. For now, let's assume FIFO demand. On day $t+1$ the order level is equal to the safety stock plus expected demand in $R+L-1=1$ day, or $15+10=25$. Using Equation 3.8 and the standard (R,s,nQ)-policy, the order quantity of day $t+1$ is equal to $(25-20)/1=5$ items. However, the EWA policy uses a slightly different equation to calculate the order quantity, namely an extension of Equation 3.8. The number of case packs to order on day t is dependent on the expected outdating $EWAz_i$ in the next $R+L-1$ days and is defined as:

$$n_t = \left\lceil \frac{s_t - IP_t + \sum_{i=t+1}^{t+L+R-1} z_i}{Q} \right\rceil \tag{3.9}$$

The expected outdating can be determined by Equations 3.10, 3.12, and 3.13. Assuming FIFO withdrawal and an actual demand equal to the expected demand, the withdrawal of items on day $t+1$ is equal to 10. All items in batch A will expire after day $t+1$, meaning that all non-sold items of this batch will be outdated. Since the number of items in batch A is 15, and the number of items withdrawn from batch A is 10, 5 items will be outdated. Therefore, $EWAz_{t+1}$ is 5, and the order quantity on day $t+1$ is equal to $(25-20+5)/1=10$ units.

Compared to the standard (R,s,nQ)-policy, the EWA-policy orders 10 instead of 5 units. This is important, since on day $t=2$ the (R,s,nQ)-policy expects the inventory position to be $25+5=30$ units. However, the inventory position will be $25+5$ - $5=25$ units, since 5 units with expiration date $t+1$ are outdated on day $t+2$. Therefore, on day $t+2$, the inventory is 5 items short, which might results in lost sales. However, for the EWA-policy this is not the case. Since the EWA-policy resulted in an order quantity of 10 units, the inventory position on day $t+2$ is equal to $25+10$ - $5=30$ units, which is exactly the number of units needed.

3.4.3 Equations from Broekmeulen & Van Donselaar (2009)

This paper takes either strict FIFO or LIFO withdrawal, a fixed safety stock for each week, and non-stationary demand into account. These last assumptions are usually not applicable to food retail. However, other assumptions and characteristics of the model fit the retail setting in this thesis well. The calculations of this paper were used in many other papers, and are as follows.

The maximum shelf life M results from the best-before date stamped on the product by the manufacturer. The amount of outdating depends on how many items from each batch are chosen by the customers. In the FIFO case, customer demand is satisfied by the oldest batch. Let B_{tr} be the number of units with the same remaining shelf life r on day t, where r is between 1 and M days. Furthermore, let W_{tr} be the number of units withdrawn from a batch with remaining shelf life r on day t. The withdrawal W_{tr} is the minimum of the remaining batch size B_{tr} and the unsatisfied demand from older batches on the shelf, such that

$$W_{tr} = \min\left[B_{tr}, \delta_t - \sum_{i=1}^{r-1} W_{ti}\right] \tag{3.10}$$

Here, the customer demand δ_t is the actual demand on day t. For example, a shelf with one product contains two batches, one of size 15 with $r = 1$, and one of size 5, with $r = 4$. Furthermore, the actual demand is 10. Equation 3.10 then ensures that 10 items with $r = 1$ are withdrawn from the shelf, and zero items with $r = 4$ are withdrawn.

In the LIFO case, customer demand is satisfied by the newest batch. Therefore, the withdrawal of a batch on day t is the minimum of the remaining batch size and the unsatisfied demand from fresher batches on the shelf, with remaining shelf life r ranging from M to 1 days. The formula is given in Equation 3.11.

$$W_{tr} = \min\left[B_{tr}, \delta_t - \sum_{i=r+1}^{M} W_{ti}\right] \tag{3.11}$$

If we take the same example as before, Equation 3.11 ensures that five products with $r = 4$ and five products with $r = 1$ are withdrawn.

At the end of each review period, a replenishment decision takes place which determines the order quantity and thus the batch size for $L + 1$ days ahead, also denoted as $B_{t+L+1,m}$. After each day, batches with $r = 1$ are disposed of, and the other batches are updated to account for aging and withdrawal. This is denoted as

$$B_{t+1,r-1} = B_{tr} - W_{tr} \tag{3.12}$$

Let z_t be the estimated amount of outdating on day t. The batch with a remaining shelf life of one day will be disposed of, such that

$$z_t = B_{t,1} - W_{t,1} \tag{3.13}$$

Taking the same example as before, in a strict FIFO case, the batch with $r = 1$ day ($B_{t,1}$) contains 15 items. After withdrawal on day t ($W_{t,1}$) of 10, the amount of outdating at the end of day t (z_t) is equal to 15 - 10 = 5 items. In a strict LIFO case, the withdrawal on day t of the with batch $r = 1$ day ($W_{t,1}$) is 5. This means that the amount of outdating at the end of day t (z_t) is equal to 15 - 5 = 10.

3.4.4 Equations from Haijema & Minner (2019)

This paper is relevant since it assumes that a fraction of demand is met in FIFO order and the rest is met in LIFO order. As in the former paper, the authors assume each product has a maximum fixed shelf life M, a lead time L and review time R. Furthermore, n_tQ is the number of products ordered on day t and $B_{t,j}$ is the number of products in stock that are j periods old. When an order is placed in period t, the quantities of outstanding orders and the stocks levels are known. The inventory position IP_t is, therefore, the quantity ordered that arrives in the next 1 to L days plus the batches currently on the shelves, such that

$$IP_t = \sum_{j=1}^{L+M-1} IP_{t,j} = \sum_{j=1}^{L} n_{t-j}Q + \sum_{j=1}^{M-1} B_{t,j} \tag{3.14}$$

Demand occurs after ordering and replenishment, and is assumed to be stochastic and stationary with mean μ and standard deviation σ. A fraction f of demand is met in FIFO order, so fraction $(1-f)$ of demand is met in LIFO order. The fraction f and the outdating that results from it have a non-linear relationship. Therefore, $\sqrt{f}$ instead of f is used in formulas to denote the non-linear relationship. The result is that the FIFO demand distribution is fitted on a mean $f\mu$ with standard deviation $\sqrt{f}\cdot\sigma$ and the LIFO demand distribution is fitted on a mean $(1-f)\mu$ with standard deviation $\sqrt{(1-f)}\cdot\sigma$.

Haijema and Minner present a fast way to estimate outdating $R+L-1$ periods ahead. The estimated outdating is defined as

$$z_t = \sqrt{f}\cdot z_t^{FIFO} + (1-\sqrt{f})\cdot z_t^{LIFO} \tag{3.15}$$

where z_t^{FIFO} is the estimated outdating if the demand equals the mean demand over $R+L-1$ periods and all products are withdrawn in FIFO order. Taking the same example as in section 3.4.3, with a FIFO fraction of 0.8, a z_t^{FIFO} of 5, and a z_t^{LIFO} of 10, the estimated outdating is equal to $\sqrt{0.8}\cdot 5 + (1-\sqrt{0.8})\cdot 10 = 5.53$ items.

The outdating estimation of Haijema and Minner focuses only on products with an expiration date within the next $R+L-1$ periods. E.g. when the maximum fixed shelf life is 5 days, $R=1$, and $L=1$ we are interested in products that now have age $j=4$ days since these products will expire in $R+L-1=1$ day. Let $IP_{t,j}$ be the inventory position of a batch of a product with age j on day t. The outdating estimate for FIFO is then the difference between the old products on the shelf and the mean demand:

$$z_t^{FIFO} = \left(\sum_{j=m-R-L+1}^{m-1} IP_{t,j} - (R+L-1)\mu\right)^+ \tag{3.16}$$

where x^+ is equal to $max(0,x)$. The estimated outdating in case of LIFO demand z_t^{LIFO} is calculated by taking the withdrawal from younger products into account, such that

$$z_t^{LIFO} = \left(\sum_{j=m-R-L+1}^{m-1} IP_{t,j} - \left((R+L-1)\mu - \sum_{j=1}^{m-R-L} IP_{t,j}\right)^+\right)^+ \tag{3.17}$$

Although these exact outdating calculations are useful, in case of supermarkets this simulation needs to be performed for thousands of products. Haijema and Minner note that a faster way to determine the expected outdating is preferable. This is done by approximations, and these are described in the next section.

3.5 Approximations for the expected outdating

Since a simulation can take up a large amount of computation time, two researchers have tried to come up with approximations of the expected outdating. The approximations are explained below. Both approximations assume the *Estimated Withdrawal and Aging* policy and FIFO withdrawal.

3.5.1 Approximations

The following approximations stand on some assumptions that are not at all applicable to a perishable food case, but these were the only approximations found in literature. However, the approximations can be used in the perishable food case since the approximations are improved by multiple linear regression, explained in the next section.

The first approximation of the relative outdating is based on the assumptions of stationary demand and ample inventory on hand due to a large safety stock, and only the replenishment of the large safety stock is considered. At day 0, a large order is placed to replenish the safety stock. The replenishment will arrive L days later in the store and will outdate M days after. The EWA-policy always looks $L + R - 1$ days ahead to see how many products will be outdated. The earliest day the EWA-policy identifies this outdating is at the review moment on (or immediately following) day $L + M - (L + R - 1) = M - R + 1$. This is at day $\lceil (M - R + 1)/R \rceil \cdot R$, which is equal to $\rho = \lfloor M/R \rfloor \cdot R$, where $\lceil x \rceil$ resp. $\lfloor x \rfloor$ denote the nearest integer higher (resp. lower) than or equal to x. So, if today is day 0, we will order a batch with shelf life M and lead time L, on day ρ the EWA-policy will identify that this batch expires in $L + R - 1$ days.

On day ρ, the age of the batch ordered on day 0 has age $L + M - \rho$. All products not sold in $L + M - \rho$ days will be identified as outdated in $L + R - 1$ days. So, the modified inventory position at day ρ is equal to $||(L + m - \rho)\mu||$, where μ is the average daily demand and $||x||$ means the rounded number of x. Since the EWA-policy aims to order at least the order level minus the inventory position plus the estimated outdating, the number of case pack sizes to order on day ρ (n_ρ) is equal to

$$n_\rho = \left\lceil \frac{s - ||(L + M - \rho) \cdot \mu||}{Q} \right\rceil \tag{3.18}$$

where Q is the case pack size. Since we order every $\rho = \lfloor M/R \rfloor \cdot R$ days, the expected relative outdating can be approximated by Equation 3.19:

$$z_A = \frac{1}{\rho\mu} \cdot E\left[\left(\left\lceil \frac{s - ||(L + M - \rho] \cdot \mu)||}{Q} \right\rceil \cdot Q - D_\rho \right)^+ \right] \tag{3.19}$$

where D_ρ is a stochastic variable representing the demand during ρ days. Substituting Equation 3.18 as constant c and knowing X is a stochastic variable, the expected value in Equation 3.19 can be calculated as

$$E[(c - x)^+] = \sum_{x=0}^{c} (c - x) \cdot P(X = x) \tag{3.20}$$

where $P(X = x)$ is the probability that the stochastic variable X is equal to value x. Note that for this calculation, the demand should be modelled by a discrete distribution. In the paper, the Mixed binomial, Geometric, Negative binomial, and Poisson distributions are used. The demand of the items are fitted to the distributions and parameters are determined using the paper of (Adan, Eenige, & Resing, 1995). More explanations on this method are given in Section 4.4.

The second approximation is based on the assumptions that demand is stationary and the inventory position just after (potentially) ordering is uniformly distributed between $s-1$ and $s-1+Q$ when an order is placed if and only if the inventory position is equal to $s-1$. We can write the uniformly distributed change in the inventory position as $\Delta \sim u[0,Q]$. Note that this is a different setting than the setting described in (Hadley & Whitin, 1963), where an order is placed when the inventory position is equal to or below s and therefore the inventory position just after (potentially) ordering is uniformly distributed between $s+1$ and $s+Q$.

Another assumption is that no outdating takes place in the first $L+M-1$ days. Then, the expected outdating at the end of day $L+M$ is equal to $E[(s-1+\Delta-D_{L+M})^+]$, where D_{L+M} is again a stochastic variable. Therefore, the relative outdating can be approximated by:

$$\begin{aligned} z_B &= \frac{1}{\rho\mu} \cdot E\left[\left(s-1+\Delta-D_{L+M}\right)^+\right] \\ &= \frac{1}{\rho\mu \cdot Q} \cdot \sum_{i=1}^{Q} E\left[\left(s-1+i-D_{L+M}\right)^+\right] \end{aligned} \tag{3.21}$$

The authors tested both approximations z_A and z_B by comparing them to results of multiple simulations, for which calculations explained in Sections 3.3.1 and 3.3.2 were used to simulate the reorder points, order levels and order quantities for many weeks under different input parameters. It should be noted that the approximations were only used for products with up to 30% of outdating. The authors did subsequent research in 2019, where the approximations were generalised for outdating percentages above 30% of which more is explained later in this section. However, for products with outdating percentages of at most 30%, the approximations had low approximation errors. For products with a short shelf life, z_A had the lowest approximation error, but z_B had the lowest average approximation error overall. Next, the authors tried to improve the outdating approximation by regression. This is explained in the next subsection.

3.5.2 Regression

A multiple linear regression model is used to determine what are the most important factors for calculating the dependent variable. Furthermore, a regression equation can be used to predict other factors of interest that were not tested. The dependent variable in this case is the regression-based approximation for the relative outdating z_{regr}. By taking the estimated outdating from the simulation and calculating the independent variables, the coefficients of the linear regression model $\alpha_i (i = 0, ..., 7)$ can be determined. The regression formula is as follows:

$$\begin{aligned} z_{regr} = \alpha_0 + \alpha_1 \cdot \frac{\sigma}{\mu} + \alpha_2 \cdot \frac{SS+Q-1}{\mu} + \alpha_3 \cdot \left(\frac{Q}{\mu} - R\right)^+ + \alpha_4 \cdot \left\lceil \frac{s}{Q} \right\rceil \cdot \frac{Q}{\mu} + \\ \alpha_5 \cdot (1-P2^*) + \alpha_6 \cdot z_A + \alpha_7 \cdot z_B \end{aligned} \tag{3.22}$$

The reasoning behind the variables is as follows:

- Variable $\frac{\sigma}{\mu}$ stands for the coefficient of variation, which measures the variation with respect to the demand.
- According to Chazan & Gal (1977), the absolute outdating has a lower and upper bound, which are a function of the maximum inventory level divided by the shelf life, or $\frac{S}{m}$ in case of an (R,S) policy. As explained earlier, the (R, s, nQ)-policy has maximum inventory $s-1+Q$, therefore, the function is changed to $\frac{s-1+Q}{m}$, or can be written as $\frac{SS+(L+R)\cdot\mu-1+Q}{m}$. However, since we are interested in the relative outdating, the function should be corrected for the shelf life, hence $\frac{SS+(L+R)\cdot\mu-1+Q}{m\cdot\mu}$. However, the regression is performed per L, R, and M combination. Therefore, the values of L, R, and M are redundant. Hence, the function changes to $\frac{SS+Q-1}{\mu}$.
- The case pack size Q has a large effect on the outdating when the case pack size is larger than the expected demand during R. Hence variable $\left(\frac{Q}{\mu}-R\right)^{+}$. With $\frac{Q}{\mu}$ we denote the number of days it takes for one case pack size to be bought and R is the number of days in the review period. Such that, when $\frac{Q}{\mu}$ is larger than R, there's more relative outdating as a consequence.
- When analysing the simulation results, the authors noticed more relative outdating when Q was relatively large compared to s, hence the term $\lceil\frac{s}{Q}\rceil\cdot\frac{Q}{\mu}$.
- The variable $1-P2^{*}$ stands for the effect of the fill rate.
- Lastly, variables z_A and z_B stand for the approximations mentioned in the previous subsection.

By calculating the z_{regr} for every fill rate percentage and the safety stock that fits this fill rate, an Efficient Frontier (Section 3.3) is obtained as a result.

In their research of 2019, the authors removed variable $(1-P2^{*})$ from the regression, as this variable had little added or even a negative value on z_{regr}, probably since the target service level is also incorporated in the safety stock in variable two. Furthermore, the approximation was generalized for products with a outdating percentage of over 30%. A weighted combination of z_A and z_{regr} was used rather than z_{regr}, namely:

$$z_{regr}^{'}=\frac{z_{regr}+z_A^{x+1}}{1+z_A^{x}} \tag{3.23}$$

since z_A performed well for high outdating SKUs, and in fact much better than Equation 3.22. Therefore, Equation 3.23 ensures that $z_{regr}^{'}$ approaches z_A when x is high. In the authors' case, $x=3$ yielded the best results.

3.6 Conclusions

In conclusion, expected waste or outdating can be modelled with exact calculations or approximations, both of which involve the target service level. When the expected outdating is determined for many target service levels, an Efficient Frontier can be drawn. Research on certain assumptions and characteristics of the food retail environment showed that modelling substitution is difficult, shelf life is best modelled as a fixed value, both FIFO and LIFO withdrawal is assumed, and non-stationary demand can be modelled by a

Normal distribution, or Gamma distribution in case of high variability. Tables 3.1 and 3.2 denote the papers that are of most use in this thesis, namely (1) Broekmeulen & Van Donselaar (2009), (2) Broekmeulen & Van Donselaar (2019), (3) Haijema & Minner (2019), and (4) Van Donselaar & Broekmeulen (2012), and their characteristics and assumptions.

For estimating the expected outdating on the basis of a service level, papers (2) and (4) are most extensive, paper (1) is useful for order level calculations and the EWA policy, and paper (3) is useful for modelling mixed FIFO and LIFO withdrawal. Since presentation stock was not mentioned in literature, this subject is addressed in the next chapter.

Table 3.1: *Relevant papers, their characteristics and assumptions (1)*

	Safety stock	**Presentation stock**	**Policy**	**Lead and Review time**
(1)	Per weekday	No	Lost sales	Positive
(2)	Fixed	No	Lost sales	Positive
(3)	Fixed	No	Lost sales	Positive
(4)	Fixed	No	Lost sales	Positive

Table 3.2: *Relevant papers, their characteristics and assumptions (2)*

	Shelf life	**Service level**	**FIFO/LIFO**	**Demand**
(1)	Fixed	Fill rate	FIFO	Stochastic and non-stationary
(2)	Fixed	Fill rate	FIFO	Stochastic and stationary
(3)	Fixed	Fill rate	Mixed	Stochastic and stationary
(4)	Fixed	Fill rate	FIFO	Stochastic and stationary

Model design

In this chapter, we develop our model based on the equations mentioned in Chapter 3 and the standard inventory model described in Section 2.1. First of all, Section 4.1 denotes the approach of the model. Secondly, Section 4.2 provides the assumptions. Next, the model is explained with the simulation first in Section 4.3, the approximations second in Section 4.4, and the regression last in Section 4.5. Then, it is explained how the Efficient Frontier is constructed in Section 4.6. Finally, this chapter ends with a conclusion that answers the following research questions:

3. How is the model formulated that estimate the expected waste on the basis of the target service level for food retailers?

   a. *What is the design of the model?*

   b. *What assumptions are made?*

   c. *What equations, parameters and variables are used?*

   d. *What alterations are made on the models from literature?*

4.1 Approach

The design of the model is as follows. First of all, a simulation model is formulated for all perishable products in all stores of Supermarket to determine the expected outdating for each target fill rate. The paper of Broekmeulen & Van Donselaar (2009) is used as a basis and the model described in Van Donselaar & Broekmeulen (2012) and Broekmeulen & Van Donselaar (2019) is fully built as a start. However, the model is enhanced to more accurately describe reality. Since in the former model strict FIFO withdrawal is assumed, the calculations are enhanced by using the FIFO fraction f by Haijema & Minner (2019). Furthermore, demand is modelled with a Normal distribution when the coefficient of variation is smaller than or equal to 0.5 and a Gamma distribution else. Other alterations of the model are the presentation stock, and non-stationary weekly and yearly demand.

Secondly, the approximations of Van Donselaar & Broekmeulen (2012) are calculated. Considering the computational complexity of the approximations, it was chosen not to alter the approximations to the FIFO fraction, non-stationary demand, and presentation stock.

Thirdly, the regression formula is formulated to improve the outdating approximation as performed in the papers of Van Donselaar & Broekmeulen (2012) and Broekmeulen & Van Donselaar (2019). Furthermore, new variables are formulated in order to account for the modifications of non-stationary demand, presentation stock, and the FIFO fraction. Then, the coefficients of the regression equation are calculated for all shelf life, lead time, and review time combinations.

Lastly, an Efficient Frontier is constructed to see the effect of the target service levels on the outdating percentages. An overview of the approach is visible in Figure 4.1.

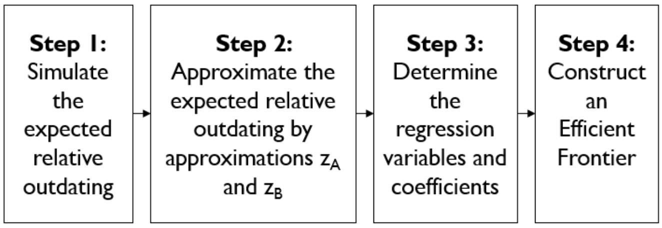


Figure 4.1: *Flow diagram of the research approach.*

4.2 Assumptions

We start building the model by formulating the following model assumptions.

Deterministic lead time: Products are assumed to have a positive and deterministic lead time, i.e. when ordering it is exactly known on what day the product is replenished.

Periodic review: Products are assumed to have a positive and deterministic review period, i.e. if R = 1, products are reviewed every day. If R > 1, products are reviewed less frequently. Thus, products can only be ordered on their review day.

Presentation stock: Some products have a safety stock predetermined by the retailer. The type of presentation stock used is a minimum safety stock, i.e. when the calculated safety stock is below the presentation stock, the presentation stock is used instead. We assume the presentation stock is fixed during the year.

Immediate replenishment: Products are received at the start of the day before the demand takes place and the products are put on the shelf directly. This means that no products are replenished from the store warehouse during the day.

Perfect supplier reliability: It is assumed that all products that are ordered are delivered exactly after a certain lead time and in the right quantities.

Quantities and shelf lives of batches are known: The assumption is made that retailers know exactly how many products are on shelf, what the quantities of the batches are, and what the remaining shelf lives are. This is usually not the case in practice, but this assumption is necessary to model the EWA-policy.

No substitution: Substitution in case of a stock-out is true in a retail setting, but no data is available on whether the product bought was a substitution of another product. Furthermore, limited research is performed on substitution and its effects on more than two products. This has the following two implications:

- **Unsatisfied demand is lost:** It is assumed that excess demand is lost when the number of items on shelf is insufficient to satisfy demand.
- **Products' demand distributions are independent:** It is assumed that there are no dependencies between the demands of products and each item is handled on its own.

Partial FIFO & LIFO withdrawal: Products with multiple batches on the shelf are susceptible to LIFO withdrawal. It is assumed that a certain fraction of demand is withdrawn in LIFO order. Bastiaanssen (2019) has conducted a method to determine the

FIFO fraction of a product. Since the determination of FIFO fractions is out-of-scope for this thesis, assumptions are made. These are explained in Section 5.4.

Non-stationary demand: Demand is non-stationary during the week, i.e. the average number of products sold on one day of the week is different from the average demand of another day of the week. Furthermore, the demand for some SKUs is susceptible to seasonality during the year.

Stochastic demand: Demand is unknown before ordering and follows a Normal distribution when the coefficient of variation is below or equal to 0.5, and follows a Gamma distribution else.

Fixed maximum shelf life: In the retail setting, a product could be spoiled even before the expiration date or average shelf life. However, no data exists on the probability of a product expiring before the expiration date or average shelf life. Thus, for all products, the shelf life is assumed to be fixed. Therefore, the probability of a product expiring before the expiration date is zero. Furthermore, it is assumed that aging of items starts no earlier than when the items are brought into the store.

Items that are not sold during their shelf life are discarded: When a product is not withdrawn from the shelf on the day of expiration, the product is disposed of by a store employee at the end of the day.

The price of a product is constant over time: Although in the retail setting prices change for many reasons, such as a discount on products with a remaining shelf life of one day, for simplicity it is assumed that the price remains constant throughout time. This implies that promotions are not taken into account in the model.

4.3 Simulation

This section describes the entire simulation model. First, the notation and calculations are explained in subsection 4.3.1. Next, the logic of the simulation model is explained in subsection 4.3.2.

4.3.1 Notation

In this subsection, we introduce the sets and indices, parameters, and variables used in the model. Furthermore, we explain how the data was obtained when no data was available. Note that all notation applies to all SKUs, i.e. for all products in all stores.

Sets

First of all, sets and corresponding indices are introduced. For simplicity, a year consists of 52 weeks and 364 days. The indices are necessary to keep track of the day of the simulation, the remaining days left until the SKU expires, and the week of the year.

- τ for day in history ($\tau \in 1, ..., 730$)
- t for day of the simulation ($t \in 1, ..., T$)
- r for the remaining shelf life ($r \in 1, ..., M$)
- w for the week of the year ($w \in 1, ..., 52$)

Parameters

This subsection contains all the parameters that serve as an input for the model. First, we express the experimental input parameters of the model, which are explained in more detail in Chapter 5. The experimental input parameters are:

- f for FIFO fraction
- β^* for target service level (fill rate)

Next, we express the parameters that are denoted in Slim4 by the clients of Slimstock, and in this case Supermarket. The data used in Slim4 is stored in a database, which consists of several large tables, of which a transaction table with detailed information on product transactions (date, quantity, number of order lines), and an article table with characteristics of the article (lead & review time, MOQ, IOQ, two years of historical demand) are most important. Other tables contain the store shelf life, the amount of outdating in two consecutive months, seasonal demand patterns, and weekly demand patterns. The input parameters retrieved from Slim4 are as follows:

- L for Lead time in days
- R for Review period in days
- M for maximum fixed shelf life in days
- Q for the case pack size in items
- PS for Presentation Stock in items
- H_τ for historical demand of day τ in items
- FC_t for forecast of demand (expected demand) on day t in items

Some parameters in the previous list need further explanation. First, the list contains parameters H_τ for the historical demand on day τ and FC_t for the expected demand on day t. Starting the simulation on Monday August 31^{st}, 2020 (the first day of week 36 in 2020), FC_1 represents August 31^{st}, 2020. However, H_{730} represents Monday September 3^{rd}, 2018, since this day is the first day of week 36 in the year 2018. FC_0 and H_0 do not exist. The case pack size Q of an SKU is set equal to the MOQ of the SKU.

Next, the model uses parameters which are not retrieved from Slim4, but calculated by an equation in Table 4.1, namely the average and sample standard deviation of daily demand in a certain week.

For safety stock calculations in the simulation, the standard deviation of demand is needed. This standard deviation is normally calculated per day. For simplicity, the decision was made to calculate the standard deviation per week, such that the standard deviation of daily demand σ_w is similar for each weekday in week w. In order to compute σ_w, the average of daily demand μ_w in week w was calculated first, but is only used in this calculation. Both parameters were calculated using two years of historical data when available, such that, for example, the average and sample standard deviation of the daily demand of week 1 were calculated by data from week 1 in 2019 and week 1 in 2020. However, σ_w is not only used in the safety stock calculations, but also in the calculations of actual demand δ_t, mentioned in Section 4.3.2, Process Sales.

Table 4.1: *Model parameters, notation, and equations*

Parameter	Symbol	Equation
Average of daily demand in week w	μ_w	$\frac{\sum_{t=7w-6}^{7w} H_\tau}{7} + \frac{\sum_{t=7(52+w)-6}^{7(52+w)} H_\tau}{7}$
Standard deviation of daily demand in week w	σ_w	$\sqrt{\frac{\sum_{t=7w-6}^{7w}(H_\tau - \mu_w)^2 + \sum_{t=7(52+w)-6}^{7(52+w)}(H_\tau - \mu_w)^2}{14-1}}$

Table 4.2: *Model variables, notation, and equations*

Variable	Symbol	Equation
Actual demand on day t	δ_t	$\delta_t^{FIFO} + \delta_t^{LIFO}$
Withdrawal on day t of batch with remaining shelf life r	W_{tr}^{FIFO}	$\min\left[B_{tr}, \delta_t^{FIFO} - \sum_{i=1}^{r-1} W_{ti}\right]$
Withdrawal on day t of batch with remaining shelf life r	W_{tr}^{LIFO}	$\min\left[B_{tr}, \delta_t^{LIFO} - \sum_{i=r+1}^{M} W_{ti}\right]$
Stock on-hand on day t of batch with remaining shelf life r	B_{tr}	$\max\left[B_{t-1,r+1} - W_{t-1,r+1}^{FIFO} - W_{t-1,r+1}^{LIFO}, 0\right]$
Expected outdating EWA on day t	Ez_t	$\max\left[B_{t,1} - W_{t,1}^{FIFO} - W_{t,1}^{LIFO}, 0\right]$
Total expected outdating EWA on day t	$EWAz_t$	$\sum_{i=t+1}^{t+l+R-1} Ez_i$
Inventory position on day t	IP_t	$B_{tr} + OO_t$
Order quantity on day t	n_tQ	$\max\left[0, \left\lceil \frac{s_t - IP_t}{Q} \right\rceil \cdot Q\right]$
Quantity on order before ordering on day t	OO_t	$\sum_{i=t-L-R+1}^{t-1} n_iQ$
Safety factor on day t	k_t	See Equations 3.5 to 3.7
Safety stock on day t	SS_t	$\max\left[PS, k_t \cdot \sigma_i \cdot \sqrt{L+R}\right],\ i = \left\lceil \frac{t}{7} \right\rceil$
Order level on day t	s_t	$SS_t + \sum_{i=t+1}^{t+L+R} FC_i + EWAz_t$

Variables

Table 4.2 contains the variables used in the model, along with the notation and equations. A distinction was made between output variables (see Table 4.3) and all other variables (see Table 4.2). As for variables W_{tr}^{FIFO}, W_{tr}^{LIFO}, B_{tr}, IP_t, n_tQ, k_t and s_t, explanations are given in Sections 3.4 and 3.5.1. As for δ_t, SS_t, Ez_t, and $EWAz_t$, more explanation is given below.

Actual demand δ_t is used to denote the actual demand on day t in the simulation. It is a stochastic variable. More explanation is given in Section 4.3.2, subsection Process Sales.

As for variable Expected outdating EWA on day t, Ez_t, is the expected outdating determined by the EWA policy. As the values of the FIFO withdrawal W_{tr}^{FIFO} and LIFO withdrawal W_{tr}^{LIFO} are calculated independently of each other, the sum of W_{tr}^{FIFO} and W_{tr}^{LIFO} can be higher than $B_{t,1}$. Therefore, the Ez_t is a max-function such that the expected outdating is always a positive integer value.

Furthermore, the Total expected outdating EWA on day t, $EWAz_t$, comprises the total expected outdating using the EWA policy in the next $L + R - 1$ days. The value of $EWAz_t$ is used to correct the Inventory Position IP_t before ordering.

The safety stock SS_t on day t is slightly different from the safety stock equation in Section 3.4. First of all, in case a presentation stock PS is installed for a product, the calculations for the safety stock are overruled when the presentation stock is higher than the calculated safety stock. Furthermore, when calculating the safety stock, the standard deviation of the forecast error is usually used. As the data is not sufficient to obtain the forecast error, the variation in demand is used instead. Since the standard deviations are calculated per week, the calculation of i ensures that the right standard deviation of the current week is used. Lastly, the safety factor is at least 0, since a negative safety stock is not possible.

Lastly, Table 4.3 contains other variables, their symbols, and their equations that serve as output of the simulation model. At the end of each simulation day, the outdating on that day z_t, the lost sales ls_t, and the average remaining shelf life γ_t are registered. Where the outdating is the number of products with a shelf life of one day, the lost sales is the maximum of the demand not met from shelf and 0, and γ is the average remaining shelf life of the products on shelf.

Subsequently, at the end of the simulation with length T, the totals are counted, namely the total outdating TO, the total demand TD, the total lost sales TLS, the total sales TS, and the average remaining shelf life Γ. Lastly, the actual service level SL is calculated as 1 minus the lost sales percentage, and the relative outdating z_{sim} is calculated by dividing the total outdating by the total demand.

Table 4.3: *Simulation output variables, notation, and equations*

Variable	**Symbol**	**Equation**
Outdating on day t	z_t	$B_{t,1} - W_{t,1}$
Lost sales on day t	ls_t	$\max\left[\delta_t - \sum_{r=1}^{M} B_{t,r}, 0\right]$
Average remaining shelf life on day t	γ_t	$\frac{\sum_{r=1}^{M} r \cdot B_{t,r}}{\sum_{r=1}^{M} B_{t,r}}$
Total outdating	TO	$\sum_{t=1}^{T} z_t$
Total demand	TD	$\sum_{t=1}^{T} \delta_t$
Total lost sales	TLS	$\sum_{t=1}^{T} ls_t$
Total sales	TS	$TD - TLS$
Average remaining shelf life	Γ	$\frac{\sum_{t=1}^{T} \gamma_t}{T}$
Actual service level	SL	$1 - \frac{TLS}{TD}$
Relative simulated outdating	z_{sim}	$\frac{TO}{TD}$

4.3.2 Simulation logic

This section describes the logic of a day of the simulation model. Every simulation day is the same and follows the following steps: receive orders, place new orders, draw demand, process sales, age the inventory, dispose of expired items, save daily results, and start a new day. This process is visualized by a flowchart in Figure 4.2. The process steps are explained below the flowchart.

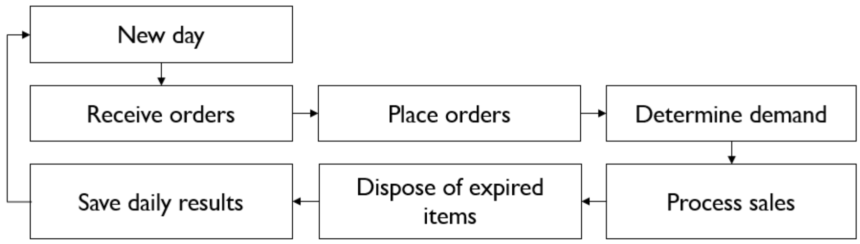


Figure 4.2: *Flowchart of the simulation model*

Receive orders

At the start of the first day of the simulation, the order level for the current day s_t is calculated by using the equations in Table 4.2. This order is received immediately, such that sales take place on that day. At the start of any other new day, the orders that are due on this day are received. Because of lead and review times, not all products receive orders each day. As a new order is received, the current inventory is updated with the number of items from the new batch.

Place orders

The next event on a day is to place orders. In accordance with the assumptions in Section 4.2, all batch quantities and their shelf lives are known. The number of products to order n_tQ is calculated by subtracting the inventory position IP_t from the order level s_t, which are calculated with equations in Table 4.2. Where IP_t is the on-hand inventory $B_{t,r}$ plus the quantity on order OO_t minus the expected outdating in $R + L - 1$ days $EWAz_t$ determined by the EWA policy. The logic of determining the expected outdating in $R + L - 1$ days is as follows:

Starting with $i = t + 1$ and the expected outdating $EWAz_i$ on day $i = t + L + R - 1$ is equal to 0:

While $i \leq t + L + R - 1$:

1. Determine the estimated FIFO withdrawal W_{ir}^{FIFO} and estimated LIFO withdrawal W_{ir}^{LIFO} of batch with remaining shelf life r on day i by using the equations in Table 4.2. We assume the actual demand δ_i in period i is equal to the expected demand in period i: FC_i. Assuming a FIFO fraction exactly equal to the assumed FIFO fraction f, the FIFO demand FC_i^{FIFO} on day i is $f \cdot FC_i$ and FC_i^{LIFO} is $FC_i - FC_i^{FIFO}$.

2. For all remaining shelf lives r, determine the remaining batch sizes B_{ir} of each batch at the end of period i by the equation in Table 4.2.

3. The expected outdating Ez_i on day i is equal to the remaining items left in batch $B_{i,1}$ since these items expire at the end of day i. Remove these items from inventory, save the quantity as expected outdating Ez_i on day i and age the remaining inventory.

4. Add the number of outdated items Ez_i on day i to the total expected outdating $EWAz_i$ on day i. Continue with step 1, while $i < t + L + R - 1$ and simulate the next day using $i = i + 1$. Stop otherwise.

The total expected outdating on day i is equal to $EWAz_i$ and is added to the order level.

Determine demand

After orders are placed, actual sales take place. But before the sales take place, it must be determined how many items are bought on this day and from which batch. In order to process the actual store demand for a product, a random number representing the total demand is drawn from a probability distribution. The Normal distribution is used in case the standard deviation σ_w of the current week w divided by the forecast FC_t on day t is smaller than or equal to 0.5. Otherwise, the Gamma distribution is used. The Normal distribution uses two parameters, namely the mean, which is equal to the forecast FC_t on day t. The second parameter is the standard deviation σ_w of week w. For the Gamma distribution, the parameters are b and θ. Using FC_t and σ_w from the Normal distribution, shape parameter b and scale parameter θ are calculated as follows (based on (Silver et al., 2017)):

$$b = \frac{(FC_t)^2}{\sigma_w^2} \tag{4.1}$$

$$\theta = \frac{\sigma_w^2}{FC_t} \tag{4.2}$$

Process sales

The random number drawn from the probability distribution in the previous step represents the total demand of that day. Next, the LIFO and FIFO demand is determined. As input parameter, there is the FIFO fraction f. However, the LIFO demand is not exactly $1 - f$ times the total demand on each day. Therefore, LIFO demand is determined by drawing a random number from the Binomial distribution with parameters number of trials δ_t, the actual demand determined in the previous step, and success factor $1 - f$, the LIFO fraction. Consequently, all non-LIFO demand is FIFO demand. Starting with the LIFO demand, products are retrieved from the store shelf and the products with the longest shelf life are picked first. Then, products with the shortest shelf life are picked to fulfill FIFO demand.

Dispose of expired items

At the end of each day, products with a remaining shelf life of 1 day are removed from inventory and the number of items is noted as outdated. Then, the total number of items in inventory is counted.

Save daily results

In the last step of the day, daily results are saved. This entails the number of items delivered, the sales, the lost sales, the demand, the number of items outdated, the starting inventory, the ending inventory and the average remaining shelf life of the inventory at the end of the day.

New day

Then, a new day is started by increasing the day number t with 1, increasing the weekday number d with 1, reducing the remaining shelf lives r of all batches in inventory with one day, initializing all variables, and counting the number of items in inventory at the start of the day.

4.4 Approximations

This section describes the calculations of the stationary approximations z_A and z_B from Section 3.5.1. First, additional notation is needed. For simplicity, all notation used in both the approximations as well as the regression are denoted in this section.

4.4.1 Notation

Sets

The approximations and regression equation use the following sets and indices.

- d for the day of the week ($d \in 1, ..., 7$)
- m for the month of the year ($m \in 1, ..., 12$)

Parameters

The following parameters are used in the approximations and regression equation:

- WSF_d for weekday seasonal factor on weekday d
- MSF_m for monthly seasonal factor of month m

Next, three parameters are calculated by a formula in Table 4.4.

Table 4.4: *Model parameters, notation, and equations*

Parameter	Symbol	Equation
Expected daily demand	μ	$\frac{\sum_{t=1}^{364} FC_t}{364}$
Standard deviation of expected daily demand	σ	$\sqrt{\frac{\sum_{t=1}^{364}(FC_t-\mu)^2}{364-1}}$
EWA outdate period	ρ	$\left\lfloor \frac{M}{R} \right\rfloor \cdot R$

First of all, the expected daily demand μ and the sample standard deviation of expected daily demand σ were calculated. Both parameters were calculated with forecasts of 364 days, including all days of a regular year, except August 30^{th}. Note that σ is also used when calculating the safety stock or order level in the approximations, as the forecast error is unknown. Since one of the assumptions of the developed model entails that promotions are not taken into account, the daily demand of all SKUs was corrected for promotions by subtracting the promotional demand from the daily demand.

Lastly, as described in Section 3.5.2, on day ρ the EWA-policy identifies that a batch ordered on day 0 expires in $L+R-1$ days.

Variables

The approximations use one stochastic variable D_x which represents the demand of x days. The probability density function of D_x is retrieved by fitting a probability distribution on the summation of x days for all days of the year. We explain the fitting of the distributions in the next subsection.

4.4.2 Approximation equations

Recall approximation z_A mentioned in Section 3.5.2:

$$z_A = \frac{1}{\rho\mu} \cdot E\left[\left(\left\lceil \frac{s - ||(L+M-\rho)\cdot\mu||}{Q} \right\rceil \cdot Q - D_\rho\right)^+\right] \tag{4.3}$$

Note that both approximations assume FIFO withdrawal and non-stationary demand. It seems obvious to alter both approximations such that they assume partial FIFO withdrawal and non-stationary demand. However, due to the computational complexity of the approximations, it was chosen not to alter the approximations, but the regression formula instead. This regression formula can be found in the next Section. Recall from Section 3.5.1 approximation z_B:

$$\begin{aligned} z_B &= \frac{1}{\rho\mu} \cdot E\left[\left(s - 1 + \Delta - D_{L+M}\right)^+\right] \\ &= \frac{1}{\rho\mu \cdot Q} \cdot \sum_{i=1}^{Q} E\left[\left(s - 1 + i - D_{L+M}\right)^+\right] \end{aligned} \tag{4.4}$$

where the change in the inventory position is uniformly distributed between $s-1$ and $s-1+Q)$, such that $\Delta \sim u[0,Q]$. However, in Slimstock's case, the inventory position is between s and $s-1+Q$. Namely, $n \cdot Q$ is ordered when the inventory position is equal to $s-1$, yielding a new inventory position of at least s in case the old inventory position was below $s-1$ and a new inventory position of $s-1+Q$ when the old inventory position was exactly $s-1$. Therefore, the change in inventory position Δ is uniformly distributed over $[1,Q]$ instead of $[0,Q]$. However, this does not change approximation z_B in Equation 4.4 as the summation over i is already between 1 to Q.

In order to determine the expected value in the approximations, the following formula is used:

$$E[(c-x)^+] = \sum_{x=0}^{c} (c-x) \cdot P(X=x) \tag{4.5}$$

where $P(X = x)$ is the probability that the stochastic variable X is equal to value x. Note that for this calculation, the demand should be modelled as stationary demand by a discrete distribution instead of the continuous distributions in the simulation. In the paper, the Mixed Binomial, Geometric, Negative Binomial, and Poisson distributions are used. The method of determining the right distribution and parameters is explained in the paper of Adan, et al (1995). In this method, the demand of the items is fitted to the distributions by considering the mean and variance of the demand. Based on the variance to mean ratio, it is determined whether Mixed Binomial, Geometric, Negative Binomial, or Poisson distribution best fits the variance to mean ratio and then the needed parameters are calculated.

4.5 Regression

This section describes the alterations to the multiple linear regression formula described in Section 3.5.3. With the same logic as in the paper, we remove the $(1 - P2^*)$ variable. The regression formula is as follows:

$$z_{regr} = \alpha_0 + \alpha_1 \cdot \frac{\sigma}{\mu} + \alpha_2 \cdot \frac{SS + Q - 1}{\mu} + \alpha_3 \cdot \left(\frac{Q}{\mu} - R\right)^+ + \alpha_4 \cdot \left\lceil \frac{s}{Q} \right\rceil \cdot \frac{Q}{\mu} + \alpha_5 \cdot z_A + \alpha_6 \cdot z_B \tag{4.6}$$

To incorporate the FIFO fraction, presentation stock, yearly seasonality and weekly seasonality into the regression, new variables need to be formulated. The variables and their reasoning are as follows:

- A variable concerning the FIFO demand, is the FIFO fraction f. But, another variable is possible. As explained in Section 3.4.4, the FIFO fraction and the outdating resulting from it, have a non-linear relationship, namely the relationship of $\sqrt{f}$. Therefore, the effect of both variable f as well as variable $\sqrt{f}$ need to be tested.

- A variable is needed concerning the yearly seasonality. A measure for seasonality in demand forecasting is the monthly seasonal factor MSF_m of month m. The monthly seasonal factor is calculated by calculating the mean demand for each month, considering multiple years of data, and calculating the total average yearly demand by summing up the average demands of 12 months. Then, we normalize the yearly and monthly demands, such that the average yearly demand is 12. The monthly demands are normalized accordingly, which has a monthly seasonal factor as a result for each month. A measure for seasonality during the year, therefore, could be the variance of the monthly seasonal factors. We denote this as σ^2_{MSF}. Note that an SKU with significant yearly seasonality has a σ^2_{MSF} higher than 0, whereas an SKU without significant seasonality during the year has a variance equal to 0.

- Lastly, a variable is needed for the weekly seasonality. Using the same logic as mentioned in the previous point, weekly seasonality could be measured by the variance of the weekday seasonal factors WSF_d on weekday d. We denote this as σ^2_{WSF}. We calculate the $WSFs$ by normalizing the total average weekly demand to 7. Note that an SKU with significant weekly seasonality has a σ^2_{WSF} higher than

0, whereas an SKU without significant seasonality during the week has a variance equal to 0.

This results in a regression formula incorporating non-stationary demand and a FIFO fraction, which is necessary since the approximations assume stationary demand and FIFO withdrawal only. In conclusion, the following regression equation is tested in Chapter 6:

$$z_{regr} = \alpha_0 + \alpha_1 \cdot \frac{\sigma}{\mu} + \alpha_2 \cdot \frac{SS + Q - 1}{\mu} + \alpha_3 \cdot \left(\frac{Q}{\mu} - R\right)^+ + \alpha_4 \cdot \left\lceil \frac{s}{Q} \right\rceil \cdot \frac{Q}{\mu} + \alpha_5 \cdot z_A + \alpha_6 \cdot z_B + \alpha_7 \cdot f + \alpha_8 \cdot \sigma^2_{MSF} + \alpha_9 \cdot \sigma^2_{WSF} \tag{4.7}$$

As in the paper of Van Donselaar & Broekmeulen (2012), we improve the performance of the regression by removing the outdating when it is considered negligible. I.e.: we set the relative outdating of the simulation to 0 when z_{sim} is smaller than or equal to 0.1%. Furthermore, we only accept relative outdating as a positive number, as the relative outdating cannot be negative. Therefore, we set $z'_{regr} = (z'_{regr})^+$. Lastly, we correct the relative outdating z_{regr} for high outdating with the following formula:

$$z'_{regr} = \frac{z_{regr} + z_A^4}{1 + z_A^3} \tag{4.8}$$

4.6 Constructing the Efficient Frontier

Once the coefficients of the regression formula are calculated, an Efficient Frontier can be obtained. As the coefficients are general for each shelf life, lead time, and review time combination, many target service levels can be tested. To derive an Efficient Frontier, z_{regr} is calculated for all integer target service levels within the range and including 80% to 99%. With all these datapoints, a graph is constructed displaying the Efficient Frontier.

4.7 Conclusions

In this chapter, a model was formulated that estimates the expected outdating on the basis of the target service level. This model concerns three parts, namely a simulation, approximations, and multiple linear regression. All three parts include characteristics not taken into account in a single model in literature, namely a FIFO fraction, presentation stock, and non-stationary yearly and weekly demand. Before testing this model, providing an Efficient Frontier, and describing the results in Chapter 6, Chapter 5 first describes the experimental design used to test the model.

Experimental design

In this chapter, we define the methods to evaluate the models described in the previous chapter and define the experimental setup. First, in Section 5.1, we discuss the inclusion and exclusion of SKUs for evaluation of the model. Next, Section 5.2 discusses how to measure the performance of the model. Furthermore, Section 5.3 describes how the model is validated and verified and Section 5.4 defines the experimental setup. Finally, this chapter ends with a conclusion that answers the following research questions:

4. How can the models be evaluated?
   a. *How is the needed data obtained?*
   b. *How can we measure the performance of the model?*
   c. *How is the model validated and verified?*
   d. *What experimental design is relevant?*

5.1 Data selection

5.1.1 Inclusion and exclusion of SKUs

In this section, we describe the process of selecting SKUs for the model. The product information and transaction data provided by Supermarket used for the case study in Section 2.4 are used for the evaluation of the model as well. We restrict ourselves to SKUs that fit the scope and that have sufficient data available. The inclusion and exclusion criteria of SKUs can be found in Table 5.1 and are explained below. At the end, we picked only the SKUs that matched all inclusion criteria.

Table 5.1: *Data inclusion and exclusion criteria*

Inclusion criteria	**Exclusion criteria**
Shelf life $\leq$ 30 days	Shelf life $>$ 30 days
Store SKUs	DC SKUs
Enough data available	Not enough data available
Sales during studied period	No sales during studied period
Mature SKUs during the studied period	SKUs with an introduction or end-of-life during studied period
Stocked SKUs	Non-stocked SKUs
Chilled and Agricultural assortment categories	Other assortment categories
At least 24 orderlines per year	Less than 24 orderlines per year
Expected demand of at least 10 during the cover period	Expected demand of less than 10 during the cover period
Non-fixed safety stock	Fixed safety stock
Non-fixed order level	Fixed order level

All SKUs of tens of stores of Supermarket were considered. In terms of the scope, SKUs in the DC were not selected, as well as SKUs with a store shelf life of more than 30 days. Furthermore, SKUs were excluded when they satisfied at least one of the following criteria: (1) SKUs marked as end-of-life, (2) SKUs removed from the assortment in the studied period, (3) SKUs introduced into the assortment during the studied period, (4) SKUs without sales in the studied period, and (5) SKUs that are non-stocked items.

As for the analysis in Section 2.4, SKUs with the highest outdating percentages were mostly part of the chilled and agricultural assortment categories. Furthermore, these SKUs were most likely to be fast-movers. Therefore, only SKUs from these assortment categories and SKUs that have at least 24 order lines per year were included. About 58,000 SKUs remained. Furthermore, to adequately model the SKUs with the continuous Normal and Gamma distributions, SKUs must have an expected demand of at least 10 units in the cover period (Silver et al., 2017). Therefore, SKUs with a lower expected demand were excluded. Then, about 6,900 SKUs remained.

Lastly, SKUs with a fixed safety stock (presentation stock) or fixed order level were removed from the dataset. The reason for this is that a change in the target fill rate is not affecting the safety stock or order level of the SKU. Therefore, the Efficient Frontier of the SKU is a straight line. Since this is not the interest of this research, these SKUs were removed from the dataset.

About 6,700 SKUs remained that belonged to all stores of Supermarket. The average expected daily demand range between 0.8 and 161 items a day. The lead times of the SKUs are 1, 2, or 6 days and the review times of the SKUs range between 1, 2, 5, and 7 days. The MOQs, IOQs and presentation stock all range between 1 and 90 items. Furthermore, about 10% of these SKUs have a coefficient of variation above 0.5. Lastly, 86% of the SKUs registered outdating in the studied period.

5.1.2 Selection of sample set

When searching for the right sample size, two aspects are important. First of all, the sample size should be small enough such that the total run time of the simulation is manageable. Secondly, the sample size should be large enough such that the effect of the regression variables can be measured with a certain precision. A rule of thumb in literature is to have at least 10 observations per regression variable (Harrell Jr, Lee, & Mark, 1996). As the regression equation in the model has at least 6 and at most 9 variables, the minimum number of observations should be at least 90. Since each SKU is simulated with 6 different target service levels (see Section 5.4), at least $90/6 = 15$ SKUs are necessary per single lead time L, review time R and shelf life M. Table 5.2 states all lead time, review time, and shelf life combinations up to shelf life 16, as well the number of SKUs belonging to that combination. For simplicity, combinations with less than 15 SKUs were left out of the table.

Table 5.2: *SKU combinations per lead time, review time, and shelf life*

M	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
L=1, R=1	2349	459	304	168	274	138	218		58		108	331		
L=2, R=1		102												
L=2, R=2		22	29	32	15	163	475	94	358	165	83			
L=2, R=5												141		
L=5, R=7														15

As we want to compare coefficients of combinations, we choose combinations with similar characteristics. Therefore, it was chosen to evaluate the model with R = 1 or 2 and L = 1 or 2 only. Furthermore, to reduce the run time and total number of SKUs, only SKUs with a shelf life of at most 13 days were chosen. Then, 50 SKUs were picked per

R, L, M combination when possible. This resulted in 898 SKUs. When selecting the SKUs for the final sample, it was taken into account that the selection of 898 SKUs is a representative selection of the 6,700 SKUs, by selecting SKUs with different expected demands, IOQs, MOQs, presentation stock, and coefficient of variation. 898 SKUs are a good representation of 6700 SKUs when determined for a 95% confidence interval and an allowed error of 5% and doing a t-test as explained in Appendix A.

5.2 Performance measurement

To determine whether the model achieves its purpose, it should be tested whether the model adequately estimates the expected outdating. A key measure for the approximations and regression is the approximation error. Van Donselaar & Broekmeulen (2012) define the approximation error as *"the relative outdating measured via simulation minus the approximated relative outdating* [of the approximations and regression]*"*. This comes to $z_{sim} - z_A$ for approximation z_A, $z_{sim} - z_B$ for approximation z_B and $z_{sim} - z_{regr}$ for the regression. This shows for what SKUs the approximations and regression are most suitable. When calculated for each SKU, the average approximation error and standard deviation of the approximation error can be obtained. This shows what approximation performs best in general.

Furthermore, in case of the multiple linear regression, the adjusted R^2 (coefficient of determination) is an important measurement since it indicates to what extent the independent variables explain the dependent variable. The closer R^2 to 100%, the better. The RMSE is a second measurement for the regression. The RMSE (Root Mean Square Error) is the square root of the variance of the residuals. A residual is the difference between the data point and the regression line. The lower the RMSE, the better. Lastly, an important measurement for linear regression is the p-value. The p-value denotes the significance of an independent variable on the outcome of the dependent variable.

5.3 Model verification and validation

Even though many assumptions are made that simplify the programmed model, each part of the model should reflect a realistic situation and therefore verification and validation of the model are important. This section therefore discusses the verification and validation of the model.

5.3.1 Verification

With the process of verification, we check whether the model meets its specifications, reflects a realistic situation, consists of proper programming, and performs as described in Chapter 4. A well-programmed but simplified simulation model programmed in Python and used by employees of Slimstock was used as a start. After verifying and adequately understanding this model, small functionalities were added each time. And after adding a new functionality, the output of the model was tested. Programming was done under the supervision of a developer of Slimstock. The data extraction was done using SQL and analyses were performed using Excel.

In order to get realistic results, a warm-up period was brought in. The reason behind this is that on day 1 of the simulation, one big order is placed that consists of only one batch for each product. As this is not a realistic situation for some products, time will pass until a situation exists with realistic inventories. The warm-up period was calculated by the Welch graphical procedure as described in Appendix A and had 30 days as a result. This is a somewhat logical number as the longest shelf life in the sample set is 13 days and 30 days is therefore roughly two times the product shelf life. The consequence of a warm-up period is that output is only saved for the simulation period after the warm-up period.

Furthermore, the required number of replications and run time are determined such that the output of the model is no coincidence. These were calculated by performing a t-test with a 95% confidence interval and an allowed error of 5%. As described in Appendix A, the total run length is three years and the number of replications is set to nine.

Next, we tried to verify the simulation model by comparing the results of the simulated expected outdating percentage to the actual outdating percentage in reality. We define the outdating percentage as the total outdating divided by the total sales. To compare the outdating percentages, the simulation results of all SKUs with their target service level, their presentation stock, and a FIFO fraction of 0.8 were compared to the actual outdating percentage of the SKU. A FIFO fraction of 0.8 was chosen since this resulted in an overall lowest approximation error. The actual outdating percentages stem from the data from two months in 2020. The actual and simulated outdating percentages of each M, L, and R combination can be found in Tables 5.3 and 5.4 respectively. From these tables, we observe that when the simulation results of a subset is higher than 5%, the outdating is usually overestimated in the simulation. Especially for subset M7L2R2. Whereas for subsets with a lower average outdating percentage, the outdating is usually underestimated. However, we must note that the actual and simulated outdating are difficult to compare. An explanation for this is the fact that only two months of outdating data were available, whereas a full year is simulated. And the actual outdating percentage includes outdating from promotions, which are not taken into account in the simulation. Furthermore, actual outdating from the past is compared to estimated outdating from the future. Taking this all into consideration, there is not enough data to verify the model on actual outdating.

Nevertheless, there are other ways to show that the simulation model calculates the outdating in a correct manner, namely by showing that the model provides results that are expected. In this case, the simulated expected outdating should be higher when an SKU has characteristics that cause outdating. To test this, the simulated outdating was set out to all six regression variables in the paper of Van Donselaar & Broekmeulen (2019) in Figure 5.1. Overall, all variables show a positive trend, which is also implied in the paper. Furthermore, all regression variables in the figure have values that seem to be logical, such as the values of z_A and z_B that always fall in the interval [0,1]. Therefore, we can conclude that the calculations behind z_{sim} and the regression variables seem to be right.

The approximations and regression were first tested with relatively stationary SKUs, such that the approximations and simulation result in a somewhat equal value and we can verify that the approximations calculate the outdating correctly. Furthermore, the model in the paper of Van Donselaar & Broekmeulen (2012) was exactly rebuild for verification. In their paper, the average approximation errors of z_A and z_B were 2.79% and 1.51% respectively. When rebuilding the model ourselves, the approximation errors were 0.13% and 0.09% respectively. A reason for the high performance compared to the paper could be that this research uses quite a narrow sample set of SKUs.

Lastly, the calculation of the regression coefficients should be verified. We used the same settings as the paper of Van Donselaar & Broekmeulen (2012), namely stationary

Table 5.3: *Actual outdating percentages per M, L, and R combination.*

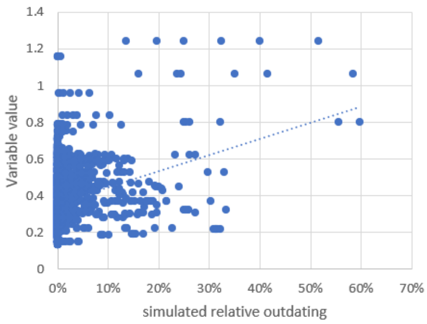
Shelf life M	**L = 1, R = 1**	**L = 2, R = 2**	**L = 2, R = 2**
3	9.9%		
4	5.4%	6.2%	10.3%
5	9.0%		12.6%
6	4.5%		6.5%
7	6.2%		5.3%
8	3.8%		5.8%
9	3.0%		7.6%
10			7.1%
11	4.7%		6.1%
12			4.5%
13	4.1%		4.9%

Table 5.4: *Simulated outdating percentages per M, L, and R combination.*

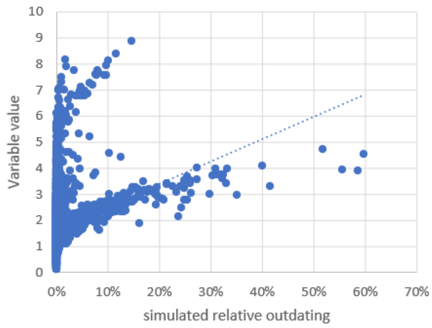
Shelf life M	**L = 1, R = 1**	**L = 2, R = 2**	**L = 2, R = 2**
3	10.3%		
4	0.5%	2.1%	19.3%
5	0.1%		4.0%
6	0.0%		4.1%
7	0.0%		5.9%
8	0.0%		2.3%
9	0.0%		3.7%
10			0.3%
11	0.0%		0.2%
12			0.1%
13	0.0%		0.0%

demand, no presentation stock, and FIFO demand. Furthermore, variable five from their paper $1 - P2$ was removed from the regression formula as suggested by the authors. Only the dataset is different. The results of the coefficient calculation can be found in Table 5.5, where coefficients α_1 to α_6 belong to the variables in Figure 5.1. The results in our research were the same from the results in their paper in some cases. At a first glance, we observe that in most subsets either z_A or z_B has a high value representing a good predictor. This was also the case in the paper. Secondly, we observe a negative value for the coefficient of variable α_0, which was the same in the paper. Thirdly, we see sporadically negative values of coefficients. This is also the case in the paper. However, some results in our research are different from the paper. Firstly, we can see an adjusted R^2 ($R^2 - adj$) of 100% for a shelf life of 13 days. In this case, the simulated outdating is equal to 0, such that z_{regr} is equal to 0, all coefficients are equal to 0, and the adjusted R^2 is equal to 100%. Lastly, for some combinations have an adjusted R^2 close to 0%. This is often caused by only one SKU, which becomes clear in the following residual plot in Figure 5.2. This is an SKU with a 99% target service level, which is an outlier when comparing its simulated relative outdating to that of other SKUs.

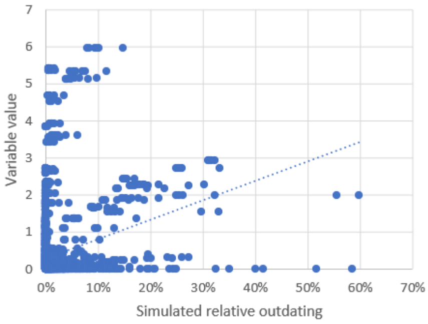
In Figure 5.2 we can see that one SKU is causing the low adjusted R^2. Meanwhile, the residual is only 0.2%, a number which is almost negligible in practice. Lastly, the



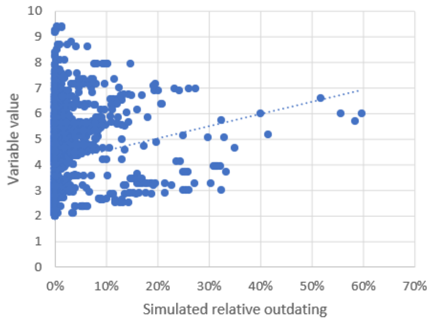
(a) $\frac{\sigma}{\mu}$



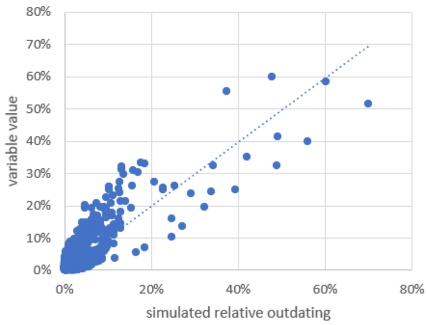
(b) $\frac{SS+Q-1}{\mu}$



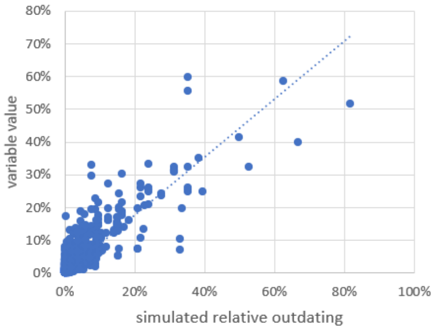
(c) $\frac{Q}{\mu} - R$



(d) $\frac{s}{Q} \cdot \frac{Q}{\mu}$



(e) z_A



(f) z_B

Figure 5.1: *Positive relations between simulated output and regression variables.*

regression only has a high adjusted R^2 for combinations with shelf lives 3, 4, and 7. In all cases, either z_A or z_B is a good predictor for the relative outdating. In these combinations, many SKUs have a relative outdating unequal to 0, therefore, the regression works well. Thus, it seems the regression and the calculation behind it are correct, while concluding that it is necessary to have enough SKUs with simulated outdating. In conclusion, all calculations of the model seem right although we cannot validate them exactly. Therefore, the verification is justified. In Chapter 6, we determine the right minimum relative outdating, such that the regression works for all combinations and we can determine for which product groups this method is relevant.

Table 5.5: *Validation of the regression coefficients.*

M	3	4	5	6	7	8	9	10	11	12	13
L = 1 & R = 1											
R^2-adj	93.7%	81.4%	36.2%	5.1%	85.2%	4.6%	66.7%		44.9%		100%
α_0	0.009	-0.004	-0.001	-0.000	1.1E-5	-0.000	-0.000		-2.6E-5		0
α_1	-0.148	0.019	0.001	0.000	-0.000	0.000	0.000		-6.1E-5		0
α_2	-0.018	0.001	0.000	7.1E-5	2.6E-5	9.4E-5	0.000		-4.6E-6		0
α_3	0.044	0.001	-8.3E-5	-4.0E-5	-2.4E-5	-7.4E-5	-0.000		7.5E-6		0
α_4	0.022	-7.6E-5	0.000	2.0E-5	4.2E-6	-9.9E-6	-3.8E-5		6.0E-7		0
α_5	0.030	0.338	1.385	0.016	0.694	73.57	1301.8		-0.054		0
α_6	1.029	1.351	-0.37	-0.027	0.121	-9.812	-193.9		0.067		0
L = 2 & R = 1											
R^2-adj		90.8%									
α_0		-0.063									
α_1		0.126									
α_2		-0.029									
α_3		-0.003									
α_4		0.002									
α_5		0.048									
α_6		1.310									
L = 2 & R = 2											
R^2-adj		90.6%	63.8%	66.0%	91.9%	42.4%	82.0%	62.5%	64.9%	11.9%	100%
α_0		-0.182	-0.002	-0.018	-0.021	-0.010	-0.007	-0.000	-0.005	-0.002	0
α_1		0.132	-0.015	0.014	0.023	0.007	-0.002	0.000	0.004	0.001	0
α_2		0.041	0.012	0.007	0.011	0.004	0.004	0.000	-2.3E-5	0.000	0
α_3		0.012	0.005	6.8E-5	-0.012	-0.002	-0.001	-6.5E-5	-0.000	-0.000	0
α_4		0.012	0.005	0.000	-0.012	-0.002	-0.001	0.000	-0.000	-0.000	0
α_5		0.134	1.746	-0.006	1.718	0.177	0.699	0.264	-0.251	-39.21	0
α_6		0.240	0.579	1.155	0.577	0.022	0.354	-0.057	1.096	-12.53	0

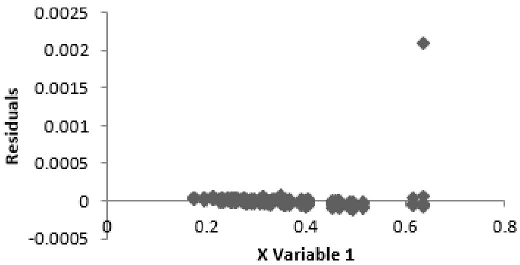


Figure 5.2: *Residual plot of variable 1 of combination M8L1R1.*

5.3.2 Validation

Validation is about whether the model fulfills its intended purpose and meets its requirements. We had to validate whether the designed and programmed model yields the completion of the research goal: model the expected outdating on the basis of the target service level. The model design was regularly validated by two Slimstock developers. Furthermore, when questions arose, these were answered by consultants working for and an employee of Supermarket.

5.4 Experimental setup

This section explains the experimental setup of the evaluation of the model. The experimental factors are the FIFO fraction and target service levels.

The fraction of demand met in FIFO order f for each SKU is not known for Supermarket, which is in general representative for the entire industry (Slimstock-Developer, 2020). Since the literature study in Chapter 3 yielded no clear results on a FIFO fraction, it was discussed with Supermarket that a FIFO fraction of 0.8 could be used for the simulation. In order to see a clear effect of the FIFO fraction, it was chosen to model with a FIFO fraction of 1 and 0.5 as well.

Since Supermarket currently has target service levels between 80% and 97% for all its products, this setting serves as a basis for the experimental target service levels. In order to simulate SKUs in a wide setting to see the effect of target service levels on the expected outdating whilst limiting the total run time of the simulation, the experimental settings chosen for the target service levels are 80%, 85%, 90%, 95%, 97% and 99%. This implies 3 (FIFO fractions) times 6 (target service levels) = 18 experiments per SKU, where the first experiment is done for a FIFO fraction of 1 and target service level of 80%.

5.5 Conclusions

In conclusion, in this chapter the experimental setup was formulated to evaluate the model described in Chapter 4. 898 SKUs were selected, with fast-moving, chilled or agricultural, minimum presentation stock, lead times of 1 or 2 days, review times of 1 or 2 days, varying MOQs and IOQs, and shelf lives between 3 and 13 days as characteristics. The warm-up period was determined for 30 days, the number of runs is set to 9 and the run time of the simulation is 3 years. The experimental FIFO fractions are 1, 0.8, and 0.5. Besides, the experimental target service levels are 80%, 85%, 90%, 95%, 97%, and 99%. Lastly, the performance of the approximations and regression is measured by the approximation error, p-value, RMSE, and R^2. In conclusion, the experimental parameters are as follows:

Table 5.6: *Input parameters of the simulation*

SKUs	**Replications**	**FIFO fractions**	**Target service levels**
898	9	1, 0.8, 0.5	80%, 85%, 90%, 95%, 97%, 99%

Results

In this chapter, experiments are performed with empirical data from a client of Slimstock to see how the model performs. Section 6.1 describes how the alterations to the model (mixed FIFO-LIFO demand, non-stationary demand, and presentation stock) influence the simulated expected outdating. The validation of the model in Section 5.3 showed that not all shelf life, lead time, and review period combinations are relevant for regression. Therefore, Section 6.2 discusses which combinations are eligible for regression. Thereafter, Section 6.3 shows the regression formula with new variables and the performance of this regression formula. Finally, this chapter ends with a conclusion that answers the following research questions:

5. What is the performance of the models?
    a. *What is the influence of non-stationary demand, mixed FIFO-LIFO withdrawal, and presentation stock on the expected waste?*
    b. *For what shelf lives is the model relevant?*
    c. *Which variables are the best predictors of expected waste?*
    d. *How does the Efficient Frontier look like when considering the model alterations?*

6.1 The influence of the model alterations

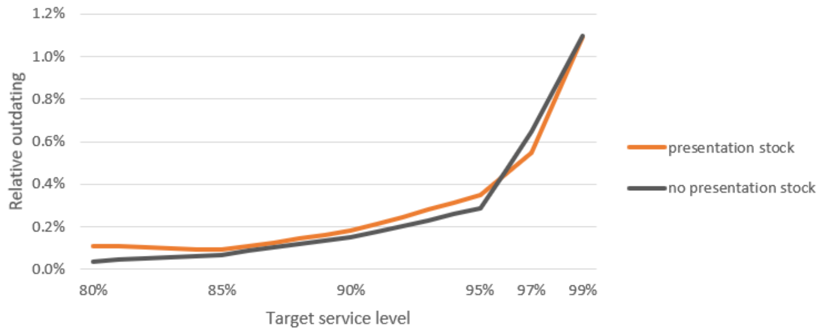
This section describes the impact of non-stationary demand, the FIFO fraction, and presentation stock on the expected outdating. Therefore, we compare the simulation results and approximation results with these model alterations to the basic model with strict FIFO withdrawal, stationary demand, and no presentation stock.

6.1.1 Non-stationary demand

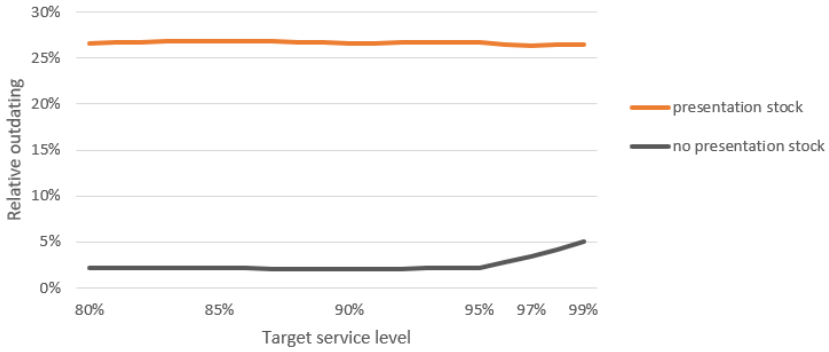
First of all, the effect of non-stationary demand is examined by changing the settings with stationary demand, no presentation stock, and a FIFO fraction of 1 to the case with non-stationary demand. In general, we conclude that the expected outdating decreases when the average demand increases. One explanation for this is that in these situations, the case pack size is relatively small when the demand is high. Therefore, the case pack size is never too large for its demand during shelf life. Furthermore, we see that the relative outdating increases when the demand is modelled as non-stationary. Besides, this is confirmed by the statistics since the average and standard deviation of the relative outdating are equal to 1.7% resp. 4.9% for the non-stationary case, whereas these are only 0.8% resp. 3.2% for the stationary case.

6.1.2 Presentation stock

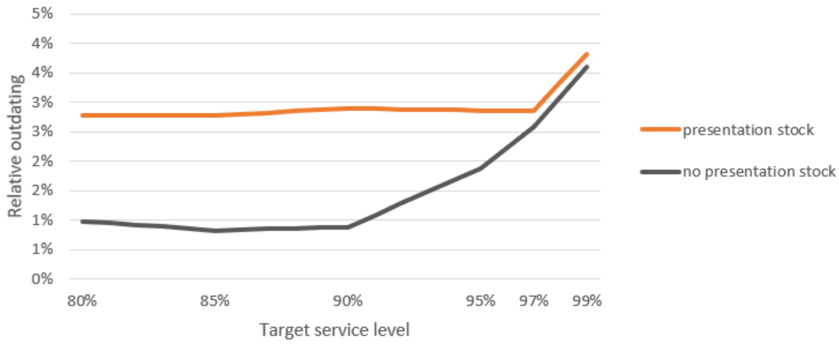
Next, we study the effect of the presentation stock on the expected outdating in a case with non-stationary demand and 100% FIFO withdrawal. The setting of a presentation stock resulted in three different types of results, as depicted in Figure 6.1. First of all, we observe SKUs for which the outdating is hardly affected by the presentation stock, such as the SKU in Figure (a). About 75% of SKUs fall into this category. This includes many SKUs that have (approximately) 0% outdating with or without presentation stock,



(a) No difference between outdating with or without presentation stock



(b) High increase when applying presentation stock



(c) Presentation stock representing a 95% service level

Figure 6.1: *The effects of presentation stock on the relative outdating.*

or SKUs that have a relatively high increase from 0.000001 to 0.0002, but which is negligible since it boils down to about 0% outdating in the absolute sense. The fact that the presentation stock has hardly any effect on the estimated outdating is due to a presentation stock that is lower than or approximately equal to the calculated safety stock. In Figure (a) we observe that for a target service level of 97%, the outdating when applying presentation stock is even lower than the situation without presentation stock. This is explained by the fact that in some cases the presentation stock leads to ordering more often leading to fewer peaks in order quantities, therefore reducing outdating, which is also observed when employing a $Qmax$ ordering policy (Haijema & Minner, 2019).

Secondly, SKUs exist for which the presentation stock yields a huge outdating increase for all target service levels, such as the SKU in Figure (b). This is the case for around 10% of the SKUs. The most extreme case consists of an SKU with absolutely 0% outdating without presentation stock and approximately 40% outdating with presentation stock.

The question arises whether the set presentation stock fits the SKU. The presentation stock of these 10% of SKUs is such that the outdating is no longer caused by a certain target service level. Therefore, approximating the expected outdating on the basis of a target service level makes no sense. In the remainder of this chapter, we therefore exclude the presentation stock from our analysis. Hence, the presentation stock is not taken into account in the regression equation or approximations.

Thirdly, some SKUs have presentation stocks representing a certain service level, such as the SKU in Figure (c). This includes the remaining 15% of SKUs. It becomes clear that installing this presentation stock for the SKU in Figure (c) to a target service level of 97% or lower actually serves as a 97% target service level.

Overall, the average and standard deviation of the relative outdating are equal to 1.7% resp. 4.9% for the case of non-stationary demand and no presentation stock, which increased to 3.1% and 9.0% with presentation stock. We conclude that presentation stock has a big effect on the outdating, at least for 25% of all 898 SKUs.

6.1.3 FIFO fraction

Furthermore, we examine the effect of the FIFO fraction on the expected outdating in a setting with non-stationary demand and no presentation stock for all SKUs in Figure 6.2.

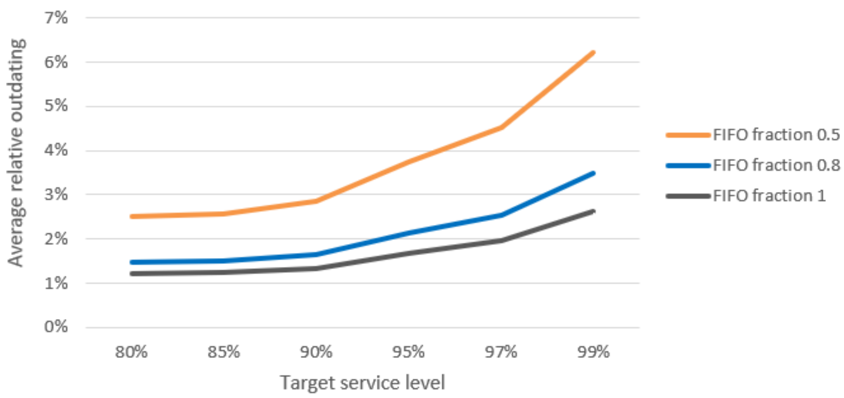


Figure 6.2: *The effects of the FIFO fraction on the relative outdating.*

First of all, significantly more outdating is generated when the FIFO fraction is lower than 1. Furthermore, the difference in outdating between fractions for an 80% service level is smaller than the difference in outdating between fractions for a 90% service level. However, in the figure we see the increase in outdating between FIFO fractions 0.8 and 0.5 is much higher than the increase between FIFO fractions 1 and 0.8. This means a linear decrease of the FIFO fraction does not cause a linear increase of the relative outdating. This is depicted in Figure 6.3. In this figure, the relative outdating concerning a certain FIFO fraction was predicted by the relative outdating from a FIFO fraction of 1, plus the relative outdating from a FIFO fraction of 1 times $(1 - \sqrt{f})$. For example, the relative outdating from FIFO fraction 1 is equal to 12%. The relative outdating from a 0.5 FIFO fraction is equal to $12\% + 12\% \cdot (1 - \sqrt{0.5}) = 15.5\%$. The figure was made for one representative SKU for an 80% target service level.

Taking this all into consideration, this section shows that the three model alterations (non-stationary demand, presentation stock, and mixed FIFO-LIFO demand) highly influence the outdating estimate, and cannot be left out of a model that estimates outdating

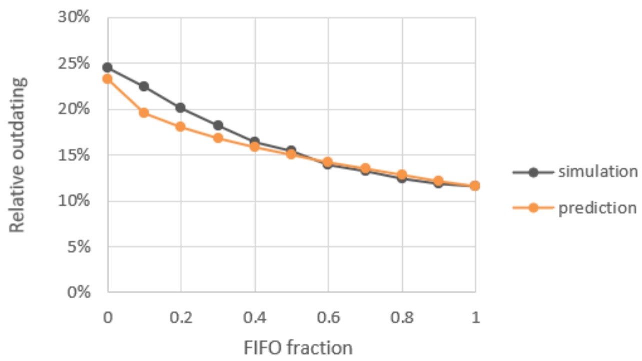


Figure 6.3: *Relative outdating prediction per FIFO fraction.*

when these alterations apply to the SKUs. However, as stated before, the presentation stock is not taken into consideration in the remainder of this chapter.

6.2 Determination of suitable subsets

When validating our model with the settings of Van Donselaar & Broekmeulen in Section 5.3, it became clear that regression does not work for all M, L, and R subsets. This was the case when either the simulated outdating was approximately zero or one SKU decreased the adjusted R^2 of the subset by being the only extreme outlier. This is supported by Figure 6.4, in which we see somewhat of a pattern between the average relative outdating of a subset and the adjusted R^2 when determining the regression coefficients. At least, this is the case for all subsets with an average relative outdating less than 10%. Note that subset M4L2R2 was left out of the figure since the relative outdating of the subset is an outlier. Figure 6.4 shows that when the average relative outdating of a subset is close to 1% or under, the adjusted R^2 decreases rapidly. Furthermore, in Figure 6.5 we observe a pattern between subsets with a high percentage of SKUs with estimated relative outdating above 0.1% and the adjusted R^2 of those subsets. Furthermore, the average relative outdating of each subset is denoted in Table 6.1, in which we see a clear pattern between the average relative outdating and the shelf life relative to the cover period. The average relative outdating is calculated by averaging the relative outdating for all target service levels and FIFO fractions. From the table, we observe the relative outdating is especially large when M is close to $R + L$. Besides, Table 6.1 shows that the relative outdating increases rapidly when M is close to $L + R$.

For subset M4L1R1, with an average relative outdating of 1.2%, the adjusted R^2 is relatively high, namely 69%. For subset M10L2R2, with an average relative outdating of 0.9%, the adjusted R^2 is 36%. Both subsets have a high number of SKUs with estimated outdating, namely 63% of the M4L1R1 subset, and 59% of the M10L2R2 subset. Therefore, we conclude that the low adjusted R^2 for subset M10L2R2 is caused by bad predictors, and not a lack of SKUs with outdating. Furthermore, the next subsets (M5L1R1 and M11L2R2) have much fewer SKUs with outdating and a much lower adjusted R^2. We therefore include all subsets up to M4L1R1 and M10L2R2 in the regression. These subsets are bold in Table 6.1. Consequently, the included subsets are shelf lives 3 and 4, for L=1 and R=1, shelf life 4 for L=2 and R=1, and shelf lives 4 to 10 for L=2 and R=2. We conclude that we can draw a line when shelf life M is close to $2 \cdot (L + R)$.

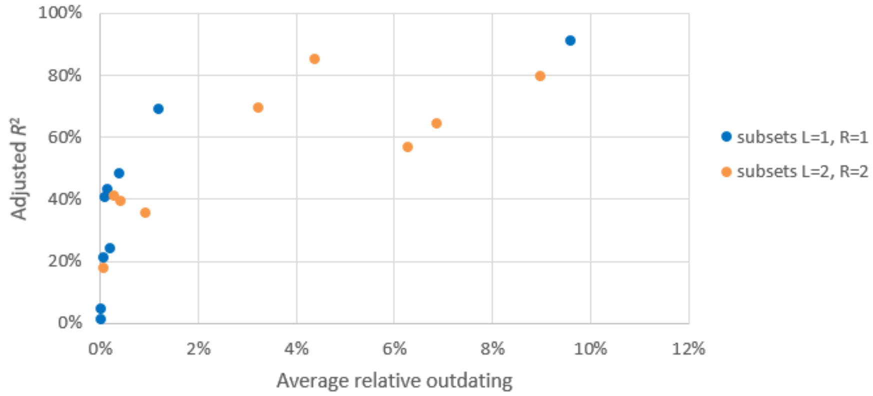


Figure 6.4: *The relation between the average relative outdating and the adjusted R^2 for all M, L, and R combinations.*

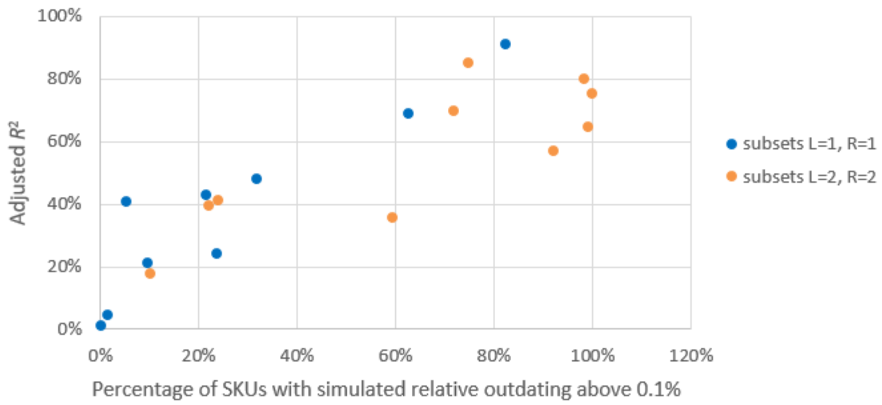


Figure 6.5: *The relation between the estimated outdating percentage and the adjusted R^2 for all M, L, and R combinations*

Table 6.1: *Average relative outdating per M, L, and R combination.*

Shelf life M	**L = 1, R = 1**	**L = 2, R = 1**	**L = 2, R = 2**
3	**9.6%**		
4	**1.2%**	**3.6%**	**22.8%**
5	0.5%		**6.3%**
6	0.2%		**6.8%**
7	0.1%		**9.0%**
8	0.1%		**3.2%**
9	0.1%		**4.4%**
10			**0.9%**
11	0.0%		0.4%
12			0.3%
13	0.0%		0.1%

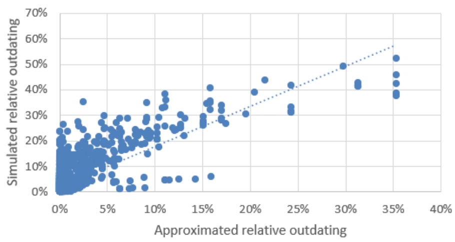
6.3 Performance of the new regression equation

This section describes the newly developed regression variables and their performance. First, the new regression variables are explained and their effects on the relative outdating are shown. Later in this section, these variables are used in the regression formula and its performance is tested. Lastly, the approximation errors are analyzed to conclude for which type of SKUs the model fits well. Apart from the new model, the original approximations and regression formula were tested. This is described in Appendix B.

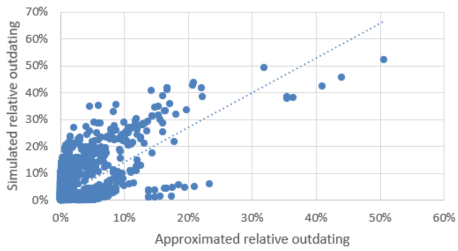
6.3.1 New regression variables

First of all, z_A and z_B were improved to account for seasonality throughout the year. This was done by calculating z_A and z_B for every week of the year, by making the order level s and average daily demand μ dependent on week w. In this matter, the parameters are changed to s_w and μ_w in Equations 4.3 and 4.4. It seems logical to also let stochastic variable D_ρ be dependent on the week. However, this implies that a probability distribution is be fitted to D_ρ with only 7 days as data points. Since this is not enough to correctly fit a probability distribution, D_ρ was not changed.

$z_A^{'}$ and $z_B^{'}$ perform slightly better than the original z_A and z_B denoted in Appendix B, with an average and standard deviation of the approximation error of 2.7% and resp. 4.7% for $z_A^{'}$ and 2.0% resp. 5.2% for $z_B^{'}$. The results are shown in Figure 6.6. However, for some SKUs, $z_A^{'}$ and $z_B^{'}$ still do not predict well, namely the few highly seasonal SKUs close to the x-axis in Figure 6.6. These are SKUs with a standard deviation of yearly demand higher than the average demand. For these SKUs, $z_A^{'}$ is a better predictor than $z_B^{'}$.



(a) z_A, improved



(b) z_B, improved

Figure 6.6: *Approximations z_A and z_B including seasonality.*

Next, almost all regression variables in Equation 4.7 were changed to account for non-stationary demand, partial LIFO demand, or other causes of relative outdating not taken into account in the original regression equation. The final equation is denoted in Equation 6.1. Explanations of the variables are given below.

$$\begin{aligned} z_{regr} = \alpha_0 + \alpha_1 \cdot \frac{\sigma_W}{\mu} + \alpha_2 \cdot \left\lceil \frac{s}{Q} \right\rceil \cdot \frac{Q}{\mu} + \alpha_3 \cdot z_A^{'} + \alpha_4 \cdot z_B^{'} + \\ \alpha_5 \cdot \left(\frac{SS}{\mu} \right)^+ + \alpha_6 \cdot \beta^* \cdot (1 - \sqrt{f}) + \alpha_7 \cdot \frac{Q}{M \cdot \mu} \end{aligned} \tag{6.1}$$

Variable $\frac{\sigma_W}{\mu}$: First of all, as described in Section 6.1, the variance in weekly demand has a strong effect on the relative outdating, whereas the variance of monthly demand is not very noticeable. However, in variable $\frac{\sigma}{\mu}$, both the weekly and monthly variance are incorporated in σ, resulting in a high coefficient of variation when both the weekly and monthly variance is high. Since the monthly variance is redundant, only the weekly variance is taken into account. However, the variance of the weekday seasonal factors σ^2_{WSF} did not predict the relative outdating well. Instead of the variance of the weekday seasonal factors, the variance of the demand in an average week was calculated. This was done by multiplying the average daily demand μ with the weekday seasonal factor WSF_w corresponding to weekday w for all seven days in the week. Afterward, the variance of these seven values is taken, such that we obtain the variance of the weekly demand σ^2_W. This standard deviation is used instead of σ_{WSF} and σ. Therefore, the variable representing the coefficient of variation is changed to $\frac{\sigma_W}{\mu}$.

Variable $\lceil \frac{s}{Q} \rceil \cdot \frac{Q}{\mu}$: This variable was slightly altered. With the same logic as described in the first paragraph, the safety stock in case of high monthly variance was set high, whereas outdating was mostly caused by the weekly variance in demand. Therefore, the standard deviation used in this variable was changed to σ_W.

Variables $z^{'}_A$ and $z^{'}_B$: Although the performance of $z^{'}_A$ and $z^{'}_B$ are not good for every SKU, the performance is good for most SKUs, as described in the previous subsection. Furthermore, these approximations performed best compared to the original z_A and z_B and a version of z_A and z_B with the σ_W incorporated in the calculation. Therefore it was chosen to incorporate approximations $z^{'}_A$ and $z^{'}_B$ in the regression equation.

Variable $(\frac{SS}{\mu})^+$: The VIF (variance inflation factor) of a variable should be at most 10, since a value of 10 indicates correlation with other variables in the formula, which is not preferred. In order to meet the restriction of all variance inflation factors below 10, variable $\frac{SS+Q-1}{\mu}$ was removed from the equation, since the correlation with either $z^{'}_A$ or $z^{'}_B$ was high. The second variable $\frac{SS+Q-1}{\mu}$ was replaced by variable $(\frac{SS}{\mu})^+$, since with the removal of the second variable, the target service level was barely incorporated in the regression equation, therefore the differences between the target service levels were too small. The target service level is incorporated in the safety stock SS, such that the differences between target service levels are clear.

Variable $\beta^* \cdot (1 - \sqrt{f})$: Next, variable f was changed. As described in Section 6.1, demand with a certain FIFO fraction follows a 1 - $\sqrt{f}$ relation. Therefore, the variable was changed to 1 - $\sqrt{f}$. However, it was also clear from that section that the effect of partial LIFO withdrawal becomes larger as the target service level becomes larger, i.e. the relationship of outdating with partial LIFO withdrawal is dependent on the target service level. Therefore, the variable was changed to $\beta^* \cdot (1 - \sqrt{f})$.

Variable $\frac{Q}{M \cdot \mu}$: On its own, variable σ^2_{WSF} was not a good predictor. Therefore, the standard deviation of weekly demand was taken instead, and this was applied in the first variable mentioned in the first paragraph. Furthermore, σ^2_{MSF} did not have any significant effect on the regression. More relative outdating only took place when the case pack size during a season was too large for its demand. Therefore, a measurement was needed to measure the case pack size relative to the demand during shelf life. Variable $(\frac{Q}{\mu} - R)^+$ was a variable in the regression equation, but it had to be removed because of a high VIF value. This brings us to the FCC metric, described in Section 3.3. The FCC is defined as $\frac{Q}{M \cdot \mu}$. This variable served the removal of variable $(\frac{Q}{\mu} - R)^+$ well, since these variables almost serve the same purpose of measuring the case pack size as opposed to the daily demand.

6.3.2 Performance of the regression

The regression equation of the former subsection was applied to the selected subsets. First, we applied the regression equation without the FIFO fraction variable to the cases with full FIFO withdrawal, non-stationary demand, and no presentation stock. The results are denoted in Appendix C. Comparing the FIFO to partial FIFO case, we conclude that it is feasible to use a regression equation for multiple FIFO fractions, since the model performs well. The adjusted R^2, RMSE, and coefficient values of each subset are found in the table below.

Table 6.2: *Performance of the developed regression formula.*

	intercept	$\frac{\sigma_W}{\mu}$	$\lceil \frac{s}{Q} \rceil \cdot \frac{Q}{\mu}$	$z_A^{'}$	$z_B^{'}$	$(\frac{SS}{\mu})^+$	$\beta^* \cdot (1-\sqrt{f})$	$\frac{Q}{M \cdot \mu}$
M3L1R1	-0.0882	0.1142	-0.0007	0.2075	0.1416	0.0932	0.1211	0.2396
M4L1R1	-0.0347	0.0937	0.0003	6.2214	-1.5023	0.0317	0.0638	0.0432
M4L2R1	-0.0971	0.0954	-0.0003	1.2430	1.7430	-0.0743	0.1662	0.0630
M4L2R2	-0.2380	0.2672	-0.0041	0.0669	0.0278	0.1418	0.4301	0.5761
M5L2R2	-0.1201	0.1644	0.0009	5.8659	-1.0738	0.0669	0.3194	0.3358
M6L2R2	-0.1241	0.1177	-0.0026	-0.9224	2.0901	0.0625	0.2477	0.3696
M7L2R2	-0.1229	0.1810	-0.0056	3.9413	-0.2236	0.0690	0.2528	0.2912
M8L2R2	-0.0653	0.1096	-0.0024	2.3707	0.3218	0.0358	0.1369	0.1568
M9L2R2	-0.0488	0.0589	-0.0033	1.7379	-0.2705	0.0357	0.1366	0.1504
M10L2R2	-0.0325	0.0226	-0.0003	-0.7846	0.4687	0.0135	0.0797	0.0999

	RMSE	$R^2 - adj$	average approximation error	st. dev. of approximation error
M3L1R1	0.028	95%	-0.5%	2.4%
M4L1R1	0.012	82%	-0.2%	1.1%
M4L2R1	0.014	89%	-0.2%	2.0%
M4L2R2	0.020	97%	0.0%	2.0%
M5L2R2	0.020	90%	-0.3%	1.7%
M6L2R2	0.015	92%	-0.1%	1.3%
M7L2R2	0.017	95%	-0.1%	1.6%
M8L2R2	0.020	83%	-0.4%	1.7%
M9L2R2	0.018	92%	-0.3%	1.6%
M10L2R2	0.010	64%	-0.2%	0.9%

From Table 6.2 it becomes clear that the formula performs well for all subsets (except M10L2R2) since the adjusted R^2 of these subsets is high and the RMSE is low. Although from the next table we see that all variables have a significant effect on the prediction of the relative outdating, from the second variable in Table 6.2 ($\lceil \frac{s}{Q} \rceil \cdot \frac{Q}{\mu}$) we see that the coefficient values are very small for each subset and mostly negative, which is not expected. Furthermore, we observe high and sometimes even negative coefficient values for $z_A^{'}$ and $z_B^{'}$, again indicating that these approximations do not fit all SKUs well. Lastly, the average and standard deviation of the approximation error were computed, which are -0.2% resp. 1.6%. This means the regression is slightly overestimating the simulated relative outdating. However, the values are still very low, and quite an improvement compared to $z_A^{'}$ and $z_B^{'}$.

Furthermore, the p-values and VIFs of the coefficients were calculated. Besides, the regression equation was tested for subsets not taken into account in Table 6.2. This is all denoted in Appendix D. We should note that this model was tested with the training set and not with a test set. One of the reasons is that some of the used subsets did not have more SKUs available to put in a test set. This means that it cannot be tested whether the model is overfitted.

Lastly, Van Donselaar & Broekmeulen (2012) used Equation 4.8 to correct the outdating for SKUs with more than 30% relative outdating, such that the outdating approximated by regression more closely resembled the simulated relative outdating. However, since z_A' and z_B' have worse approximations than z_{regr}, this formula remained unused in our research.

6.3.3 Analysis of the approximation errors

The result of the regression is an efficient frontier, which is shown in Figure 6.7. The SKU depicted in this figure is representative of many SKUs since for many SKUs we observe small or no differences in the relative outdating between the 80% and 90% target service level, and a high increase in the relative outdating between the 95% and 99% target service level. For all SKUs, an efficient frontier is made by using the regression equation for different input parameters, such as FIFO fractions and target service levels. In this subsection, we describe for which SKUs the regression equation performs well, and which type of SKUs need future research. A more elaborate analysis of the approximation errors can be found in Appendix E.

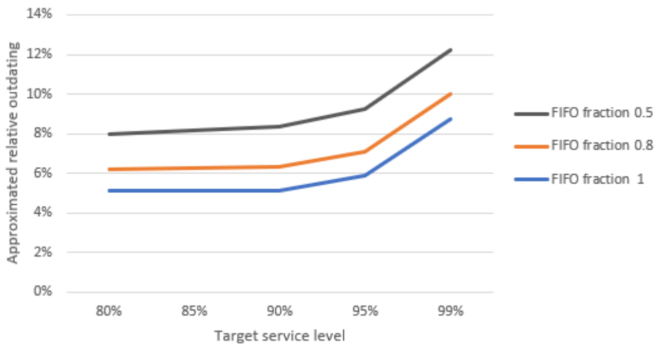
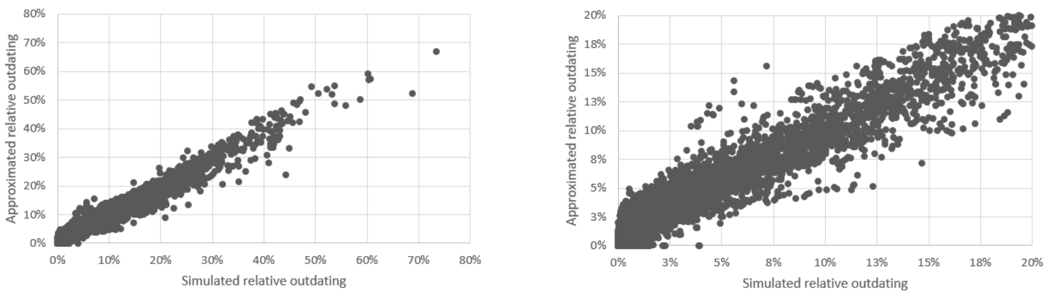


Figure 6.7: *The efficient frontier of a representative SKU for three FIFO fractions and all target service levels.*

First of all, we compare the approximated relative outdating by regression and the simulated relative outdating in Figure 6.8. We observe a clear relation between the approximated and simulated relative outdating, i.e. SKUs with low simulated relative outdating have a low approximated relative outdating, and the same is observed for SKUs with very high relative outdating.



(a) All SKUs.

(b) All SKUs within the 0% to 20% interval.

Figure 6.8: *The relation between the approximated relative outdating by regression and the simulated relative outdating.*

Aside from the large approximation errors, the relative outdating of many SKUs is estimated well by regression. Although the percentages of the approximated relative outdating by regression and the simulated relative outdating are rarely equal, the regression clearly shows the effects of target service levels seen in the simulation. In this case, the outcome of the regression mimics the outcome of the simulation, where no matter the choice of target service level between 80% to 90%, the expected relative outdating is the same. This means that the regression output is not valid for determining the exact expected relative outdating percentage, but rather for the change in relative outdating between two target service levels.

Diving further into the SKUs with high approximation errors, we observe three situation for which the regression does not approximate the outdating well. First of all, this is the case for highly seasonal items. Generally, the outdating for seasonal items (items with a high variance of monthly seasonal factors MSF) is overestimated by the regression. Secondly, we generally observe underestimation of the outdating for SKUs with a high variance of the WSF. Explanations on the high approximation errors can be found in Section 6.3.1. Unfortunately, a high MSF and WSF are not the only cause of high approximation errors since it partially depends on combinations with other regression variables. Therefore, it is not possible to clearly state from what MSF or WSF value the approximation error is high. Thirdly, we observe higher approximation errors for the 99% target service level, as the regression is linear rather than exponential.

After analysis, it became clear that the high approximation errors for SKUs with high weekly or monthly variance are due to the high approximation errors of the approximations z'_A and z'_B. This relation is shown in Figure 6.9. From this Figure we also see that high approximation errors of z'_B are slightly reduced when performing the regression, i.e. regression lowers the approximation error of z'_B. However, large approximation errors with z'_B cause large approximation errors of the regression, indicating that a good approximation of z'_B is key for each SKU.

Figure 6.9: *The relation between the approximation errors.*

6.4 Conclusions

In conclusion, this chapter shows that non-stationary demand, partial FIFO withdrawal, and presentation stock highly influence the expected outdating. Secondly, we perceived that the regression is only effective when the subset used for the determination of the regression coefficients contains sufficient SKUs with simulated outdating. The newly developed regression equation yielded high adjusted R^2 for all subsets. The approximation errors are highest for SKUs with a high variance in the monthly seasonal factors, a high variance in the weekly seasonal factors, or 99% target service levels.

Conclusion

In Section 1.5, we defined research questions to reach the research objective:

"Develop an analytical method that estimates the probability of food waste of perishable goods based on a given service level before ordering, leading to minimizing food waste whilst achieving service levels in the future for food retailers."

In the subsequent chapters, we answered these research questions. From the context analysis, it was clear that high waste percentages were mostly encountered at fast-moving perishable products in the chilled and agricultural assortment categories. Furthermore, presentation stock is a retail setting that had to be taken into account in the model. The literature study unveiled methods to model the expected relative outdating and the target service level (in the form of the fill rate) simultaneously whilst taking, amongst others, lost sales, non-stationary demand, lead and review times, and FIFO withdrawal into account. These methods were a simulation and approximations improved by regression. We developed models that estimate the expected relative outdating given a certain target service level, whilst incorporating non-stationary demand, partial FIFO withdrawal, and the presentation stock, for several target service levels. The model consists of four parts: the simulation, the calculation of the approximations $z_A^{'}$ and $z_B^{'}$, the determination of the regression coefficients, and the approximations of the relative outdating by regression.

The results showed that non-stationary demand, partial FIFO withdrawal, and presentation stock highly influence the expected food waste. Hence, considering these factors is essential when estimating the expected waste for the food retail setting and should not be left out of waste estimation models. The regression showed good results for most SKUs, since most approximation errors were (relatively) low. The effects on the expected increase of waste are well modeled by the regression equation. This will help supply chain planners to determine whether the increase in waste between, for example, a 90% and 95% target service level is worth the higher availability of products for customers.

However, the regression does not perform well for all SKUs in all situations. We perceived that the regression is only effective when the subset used for the determination of the regression coefficients contains sufficient SKUs with simulated waste. Otherwise, all SKUs with significant waste are seen as outliers. Secondly, the approximation errors are high for highly seasonal items, and especially for 99% target service levels of highly seasonal items.

Nevertheless, the method is promising, although more research is needed in different directions. The insights from the expected waste from the model can be beneficial for the inventory management of perishable items.

Discussion, recommendations, and implementation plan

This chapter contains the approach after the finalization of this research and the interpretation of the research. In Section 8.1, we discuss the scientific contribution and limitations of this research. Next, we provide suggestions for future research in Section 8.2. Furthermore, we elaborate on the practical implications of this research and provide an implementation plan in Section 8.3. Finally, in Section 8.4, we provide our recommendations to Slimstock.

8.1 Discussion

This research is an extension of the papers of Van Donselaar & Broekmeulen (2012) and Broekmeulen & Van Donselaar (2019). The scientific contribution is threefold. First of all, a scientific contribution is the addition of non-stationary demand and the FIFO fractions. Secondly, the regression equation is new. Thirdly, the demarcation of subsets suitable for regression is the last contribution. Furthermore, the addition of the presentation stock is a practical contribution of this research which, to our knowledge, was not found in the literature. The model is not only applicable for Slimstock and Supermarket but can be applied to other inventory management systems and other food retail clients.

One of the limitations of this research is the assumption that an SKU can be ordered every R days and the SKU is delivered exactly L days later. For example, orders are placed on Monday, Wednesday, Friday, and Sunday. In practice, this assumption does not hold, as an SKU with $R = 2$ might be ordered on Monday, Wednesday, Friday, and the order of Sunday is already ordered on Friday, since on Sunday no orders take place. It is expected that implementing this structure will cause more waste since SKUs are ordered for longer periods. Likewise, this holds for other assumptions of the model, namely an addition of closing days, immediate replenishment in the morning, perfect supplier reliability, and a fixed shelf life. However, for most of these assumptions it holds that the data needed to model the removal of this assumption is unavailable or unknown, such that modeling is no option. We expect that the relaxation of these assumptions do not greatly affect the waste. However, these assumptions should be taken into account when interpreting the model outcomes.

The second limitation of this research is the implemented EWA policy in the simulation. The EWA policy assumes that the number of batches, batch quantities, and the remaining shelf lives of the batches are known at the start of each day. However, this is often not the case in practice. Implementing EWA in the inventory management system requires guessing the batch information. This might lead to wrong guesses and, subsequently, too high or too low order quantities. Therefore, when interpreting the model outcomes, the supply chain planner should bear higher actual waste and lower actual service levels in mind.

Third, the estimation of the expected waste is made without considering price reductions (e.g. product promotions) and events (such as Christmas), for which different orders are placed and customer demand changes. Hence, the current model estimates waste for "regular" periods, and not the additional waste for events or promotions. This implies that the addition of promotions and events lead to more variation in demand, leading to more waste.

A fourth limitation is the set of SKUs used in this research. The set of SKUs contains only fast-moving SKUs from two assortment categories: agricultural and chilled. This means the research is not representative for slow-moving SKUs and SKUs from other assortment categories. We elaborate on this in Section 8.3.

Lastly, linear regression has some limitations, such as that the coefficients are determined by taking all SKUs in a subset into account, even when these SKUs are outliers. This means that outliers (e.g. 99% target service levels, or high-waste SKUs compared to others) are likely to be underestimated by the regression. However, as long as the subset of SKUs is large enough, it is unlikely that the estimation of waste for the SKUs apart from the outliers is significantly influenced by these outliers. Secondly, the question is whether linear regression was appropriate, knowing that there's a non-linear relationship between the target service level and relative outdating.

8.2 Future research

This section provides suggestions for future research. The first suggestion concerns the SKUs for which the regression does not perform properly: the highly seasonal SKUs. The large differences in demand cause the approximations to highly overestimate the expected waste. Therefore, the approximations should not be calculated with the demand of a year, but it should be researched whether the approximations per month or season estimate the waste well. Besides, more attention could be given to the calculation of the other regression variables per month or season. However, caution is needed such that the computation time is still fast. Secondly, the new approximation should not have the requirement to run it too often.

Secondly, SKUs with a maximum shelf life of one day were excluded from this research since the expected waste of these SKUs can be determined by the Newsboy Problem. Research should be done on whether the regression model also fits products with a maximum shelf life of one day and whether this outperforms the Newsboy Problem model or not.

Although promotions and events often take place in the food retail setting, this model does not take them into account. However, since promotions and events can cause waste, this influences the expected waste. Research should be conducted on the effects of promotions and events on waste. Furthermore, research should be done on how to incorporate these effects into the model such that the model outcome is not only useful for the weeks of the year without promotions or events.

Lastly, in this research, SKUs from many shops were combined in one subset. There is no knowledge on whether there are implicit differences between shops, so research should be conducted on that matter.

8.3 Implementation plan

In this section, we elaborate on how the presented model can lead to a concrete project and what are the first steps that Slimstock can take after this research is finalized. First of all, we need the following information of an SKU: a full year of daily demand forecast, historical sales without promotions or events, desired target service level(s), the actual or expected FIFO fraction, lead and review time, minimum and incremental order quantity, fixed shelf life, and presentation stock (if applicable). Furthermore, we advise Slimstock to keep track of the forecasts of products for each day in history. In this manner, the forecast error can be calculated, which can then be used to determine the needed safety stock. Hence, one requirement for the model to perform well is to only apply the model for clients according to the availability of this data and to ensure that the data is not mistaken.

The model consists of four parts: the simulation, the calculation of the approximations z'_A and z'_B, the determination of the regression coefficients, and the approximations of the relative outdating by regression. the simulation computation time is the longest, the approximations take longer since these are calculated for 52 weeks, and the computation times of the coefficients and regression are very fast. The models are an extension of a pre-programmed simulation model in Python. The reason for using the pre-programmed model is the simplicity of implementation after this research since Python is used by Slimstock employees for research purposes such as experimental setups. Therefore, experimentation with the model is possible immediately.

The implementation of the model in an inventory management system will lead to a calculated waste percentage per SKU. From these waste percentages, supply chain planners can determine what SKUs need more attention to reduce the waste. Experiments can be performed by altering certain parameters and calculating the expected results of these experiments. This leads to useful insights and might help in decisions such as whether to change the SKU parameters or phase out an SKU. Furthermore, in-depth simulations can be performed to investigate the influence of certain parameters on the given data-set. Besides, sensitivity analyses can be useful here to get the optimal setting for having the lowest expected waste.

The results have shown that the presentation stock has a significant effect on waste. However, the presentation stock is not yet incorporated in the approximations and regression formula. This can be done by changing the safety stock calculations. Then, the presentation stock too will be a parameter that can be changed and of which the effects can be simulated.

Although these insights are valuable, the implementation of the model might come with some risks and challenges. First of all, the dataset used for this model consists of fast-movers, and agricultural and chilled assortments groups only. This means that the model has yet to be tested for slow-movers and other assortment categories. Next, the method is not viable for subsets without simulated waste. Furthermore, the model is (currently) not suitable for highly seasonal SKUs, and the waste of outliers is not well predicted. This should all be taken into consideration.

Lastly, the model is not validated by the actual waste percentage of SKUs since only two months of data on actual waste were available. By simulating the past and comparing the simulated waste percentage to the actual waste percentage, the last step of the validation is complete.

8.4 Recommendations

This section contains several recommendations for Slimstock. The first recommendation is to examine all steps and questions mentioned in the Implementation plan in Section 8.3 and evaluate whether the model performs properly both for all SKU types as well as in the food retail setting including events and promotions. This is important since the model is promising, but has not yet proven itself in all conditions.

Secondly, determine whether the calculation of the FCC for each product for a certain period when ordering in Slim4 is insightful for clients. Then for each product, it is clear whether the order quantity is too large for the demand during shelf life.

One of the inputs of the regression model is the FIFO fraction. In this research, assumptions were made on the value of the FIFO fraction. However, as mentioned in Section 4.2, a method exists that calculates the actual FIFO fraction. The recommendation for Slimstock is to keep track of starting inventories for each day, such that this method can be executed and the actual FIFO fraction can be determined for each SKU.

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Appendices

A Warm-up, replications and run time calculations

This appendix explains the calculations between the warm-up period, number of replications and run time calculations of the simulation model. For determining the warm-up period, the average daily shelf life of the inventory of SKUs was tested for which it was expected to have the highest warm-up period of all SKUs. That is the case for SKUs with the longest shelf life, longest cover period and highest coefficient of variation in the sample set. Therefore, SKUs with a shelf life of 13 days and L and R of 2 days were tested. An 80% target service level was chosen, since the probability on an empty shelf is the highest for this service level and therefore the SKUs with the most volatile inventory are picked.

For the procedure, $n = 5$ replications were made and the run time was set to 364, as the anticipated warm-up period was determined at around two times the shelf life of the SKU, or 2 * 13 = 26. The Welch procedure to determine the warm-up period is as follows: Let Y_{ij} be the ith day of run j, for all $i = 1$ to m and $j = 1$ to n. For all independent days, the average daily inventory $\bar{Y}_i$ on day i was calculated as $\bar{Y}_i = \sum_{j=1}^{5} Y_{ij}$. Next, we choose the window w for the smallest value of w for which the graph looks "reasonably smooth" (Law, 2015). This window is used as moving average value. By using a moving average, oscillations are flattened. Based on past experience, we define a window of 30 days, such that the graph looks reasonably smooth in Figure A.1:

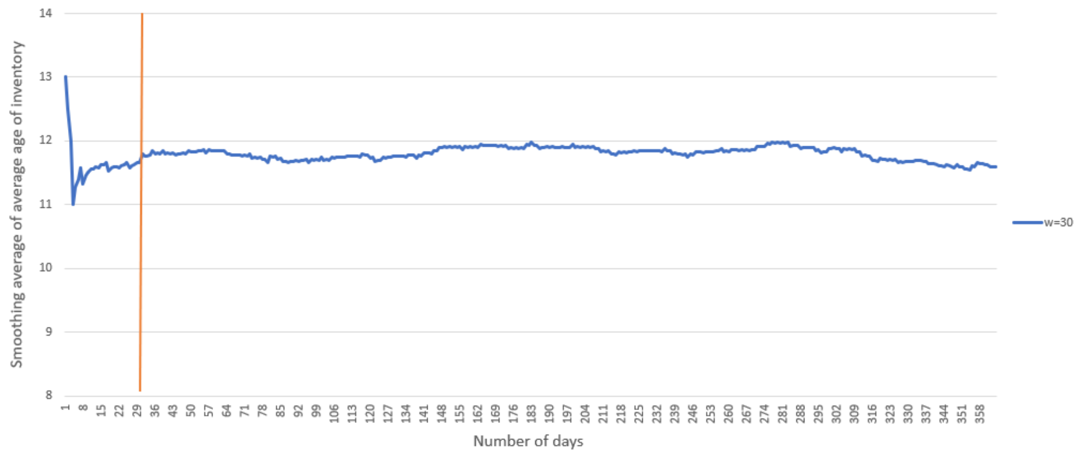


Figure A.1: *Simulation warm-up determination*

As can be seen in Figure A.1, the plot reaches stability at around day 29. Therefore, the warm-up period of the simulation was set to 30 days. Other SKUs were tested with the same procedure, such as an SKU with a shelf life of 10 and an SKU with a shelf life of 13 days and a lower coefficient of variation. As expected, the warm-up periods of these SKUs were lower than the warm-up period of 30 days.

Next, the number of replications was determined together with the run time of the model.

The right amount of replications and sufficient run time make sure that the output of the model lies within a reasonal small bandwidth. The determination of the replications and run time was done with a different SKU, namely an SKU with the shortest possible shelf life. Furthermore, the SKU had a very high coefficient of variation, such that the expected waste percentage per day varies a lot.

As a start, since the graph in Figure A.1 showed a gradual result of the age of inventory, a sufficient run length whilst doing 10 replications was anticipated to be only one year. Therefore, the SKU was tested with $n = 10$ replications and a run length of 364 days. The settings were a 95% confidence interval and a predefined error of 5%. For replication n the mean μ and variance σ^2 of the waste percentages were calculated for replications 1 to n. For each set of subsequent replications, the t-statistic was calculated with parameters 0.975 and n-1 degrees of freedom. Then, the error was calculated using the following equation:

$$Error = \frac{t \cdot \sqrt{\frac{\sigma^2}{n}}}{\mu} \tag{A.1}$$

When the error is smaller than or equal to the allowed predefined error of 5%, the number of replications is considered as enough. For a run length of 364 days, the error was always higher than the allowed error. Therefore, 10 replications are not sufficient for a run time length of 364 days. It was chosen to increase the run length by one year until at most 10 replications were sufficient. This was the case for a run length of three years, or 1028 days. Table A.1 shows that 9 replications are considered enough for the simulation, as the error of 9 replications is lower than the predefined error of 5%.

Table A.1: *Simulation warm-up determination for a three year run length*

Run	**Waste percentage**	**Mean**	**Variance**	**T-value**	**Error**
1	0.1360				
2	0.1511	0.1435	0.0001	12.706	0.6690
3	0.1338	0.1403	8.86E-05	4.3026	0.1666
4	0.1284	0.1374	9.5E-05	3.1824	0.1129
5	0.1317	0.1362	7.76E-05	2.7764	0.0803
6	0.1522	0.1389	0.0001	2.5706	0.0772
7	0.1293	0.1375	0.0001	2.4469	0.0673
8	0.1326	0.1369	8.9E-05	2.3646	0.0576
9	0.1381	0.1371	7.81E-05	2.3060	0.0496

B Performance of the model from literature

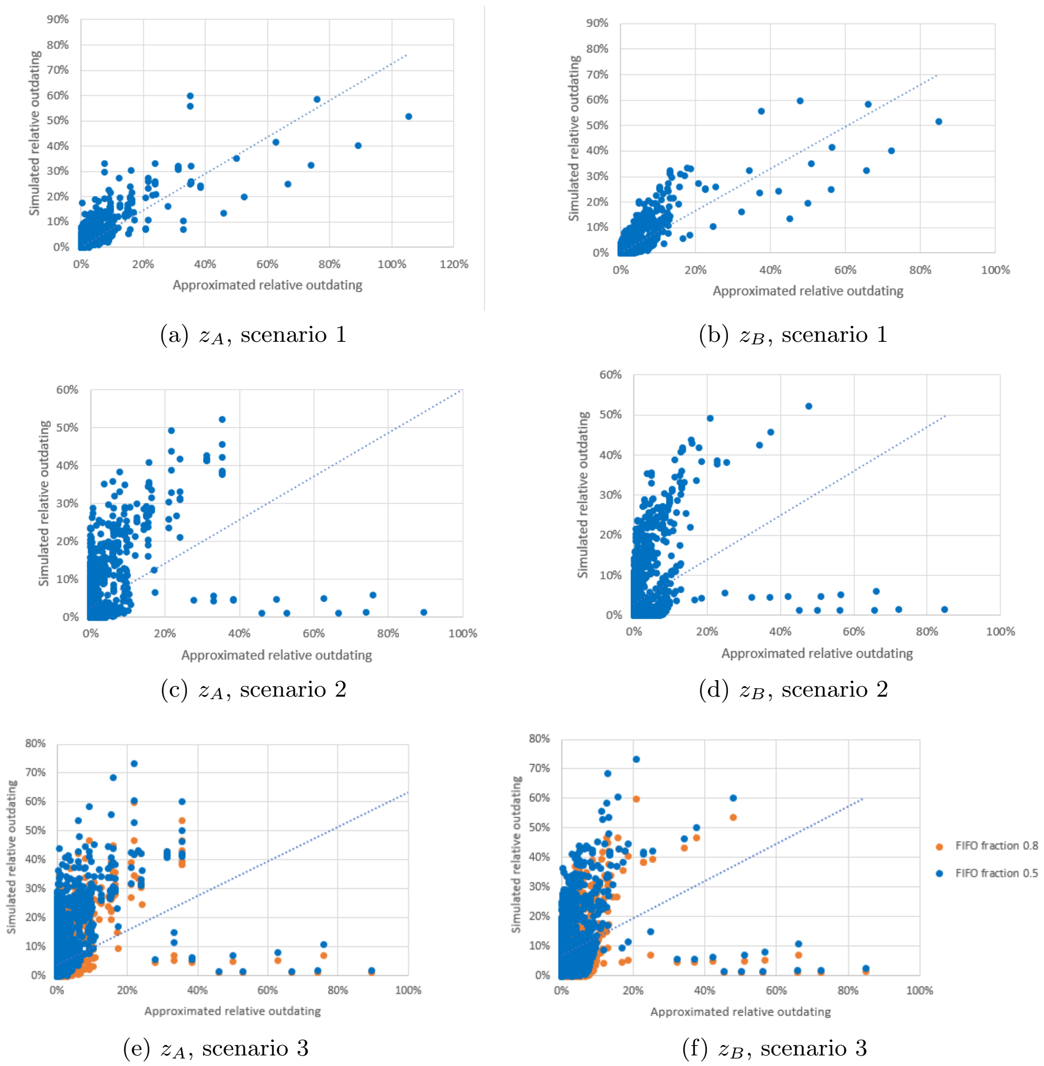
This appendix shows the performance of the model of Van Donselaar & Broekmeulen (2012) compared to the simulation results including non-stationary demand and the FIFO fractions. We first measure the performance of the approximations, then the performance of the regression formula.

B.1 Literature approximations performance

First, we test the performance of approximations z_A and z_B by the approximation error. The approximations are tested for three different scenarios:

1. stationary demand, and a FIFO fraction of 1
2. non-stationary demand, and a FIFO fraction of 1
3. non-stationary demand, and FIFO fractions of 0.8, and 0.5

Scenario 1 is already described in Section 5.3, where, just as in the paper, stationary demand and FIFO withdrawal are assumed. By comparing scenarios 1 and 2, the effect of non-stationary demand is determined. By comparing scenarios 2 and 3, we determine the effect of partial LIFO withdrawal. The results are shown in Figure B.1, where the simulated relative outdating is set out to the approximated relative outdating.



(a) z_A, scenario 1 (b) z_B, scenario 1

(c) z_A, scenario 2 (d) z_B, scenario 2

(e) z_A, scenario 3 (f) z_B, scenario 3

Figure B.1: *Approximations z_A and z_B under different scenarios.*

From figures B.1 (a) and (b), we conclude that z_A and z_B predict the expected outdating well, as expected. The same holds for the paper. And just as in the paper, z_B has a lower approximation error than z_A with an average and standard deviation of the approximation error of 0.2% resp. 3.4%, whereas z_B has 0.1% resp. 3.3%.

Next, we compare the approximation errors of z_A and z_B to the simulation results with non-stationary demand, and FIFO withdrawal. The average approximation errors and standard deviations are 2.14% resp. 6.72% for z_A and 2.08% resp. 6.92% for z_B. The results are shown in Figures B.1 (c) and (d). It becomes clear that adding non-stationary demand to the simulation means that the approximations perform worse. This is especially the case for a couple of SKUs close to the x-axis in the figures. These represent SKUs with high yearly seasonality. This causes a high coefficient of variation in the approximations, which means that the approximations expect high relative outdating. However, in the simulation we see that the order level fluctuates with the seasonality, such that the yearly seasonality is not necessarily causing more outdating. Therefore, the approximations highly overestimate the relative outdating of highly seasonal SKUs. Scatters close to the y-axis represent SKUs that have high weekly seasonality, resulting in high outdating in the simulation. However, the standard deviation in the approximations is not large, thus the approximations result in low relative outdating for these SKUs. Therefore, the approximations underestimate the outdating of many SKUs with high weekly seasonality.

Adding FIFO fractions results in even higher approximation errors and standard deviations with 4.75% resp. 8.32% for z_A and 4.70% resp. 8.49% for z_B in the case of FIFO fractions of 0.5 and 0.8. The plots are even more scattered in these cases, as shown in Figures B.1 (e) and (f). This is logical, as approximations z_A and z_B assume a FIFO fraction of 1. We observe that the relative outdating is higher when the FIFO fraction is lower, and that the approximated value is usually lower than the simulated value. We conclude that when non-stationary demand and FIFO fractions are part of the simulation, approximations z_A and z_B, in general, underestimate the outdating.

B.2 Literature regression performance

Next, we test the regression equation for three different scenarios:

1. stationary demand, and a FIFO fraction of 1
2. non-stationary demand, and a FIFO fraction of 1
3. non-stationary demand, and FIFO fractions of 1, 0.8, and 0.5

We measure the effect of non-stationary demand by comparing scenario 1 to scenario 2, and we measure the effect between FIFO demand and mixed FIFO-LIFO demand by comparing scenario 2 to scenario 3. Note that scenario 1 corresponds to the subset used for validation in Section 5.3. However, different results are shown in both tables since in this chapter relative outdating below 0.1% is considered as 0% relative outdating (see Section 4.5 for explanation), whereas in the former chapter this was not taken into account. The results are shown in Table B.1.

From the table, we see the RMSE, adjusted R^2, intercept, and coefficients of the variables of the regression formula for all chosen subsets for all scenarios. We first take a look at the RMSE and adjusted R^2. For most subsets, the regression performs reasonably well, with the adjusted R^2 above 80% in the scenario with stationary demand and FIFO withdrawal. As we apply the regression formula to the scenarios with non-stationary demand and even partial LIFO withdrawal, the regression performs worse and the RMSE, therefore, becomes larger. We provided Table B.2 with the p-values of the coefficients.

Table B.1: *Baseline performance and coefficients of the original regression formula for three scenarios.*

	RMSE	R^2-adj	intercept	$\frac{\sigma}{\mu}$	$\frac{SS+Q-1}{\mu}$	$\frac{Q}{\mu}-R$	$\lceil\frac{s}{Q}\rceil\cdot\frac{Q}{\mu}$	z_A	z_B
Scenario 1									
M3L1R1	0.024	89.1%	0.0108	-0.0678	0.0071	0.0347	0.0049	0.1746	0.4190
M4L1R1	0.003	79.3%	-0.0019	0.0085	0.0001	0.0017	0.0001	0.4577	1.1801
M4L2R1	0.008	90.1%	-0.0570	0.1164	-0.0255	0.0220	0.0002	0.0363	1.2696
M4L2R2	0.017	90.5%	-0.1837	0.1341	0.0415	0.0123	0.0187	0.1394	0.2132
M5L2R2	0.007	58.2%	-0.0112	-0.0117	0.0119	0.0060	0.0020	-0.1357	1.1402
M6L2R2	0.004	65.7%	-0.0172	0.0141	0.0067	0.0003	0.0005	0.2705	0.9818
M7L2R2	0.007	89.0%	-0.0231	0.0210	0.0126	-0.0121	-0.0020	1.5407	0.4313
M8L2R2	0.004	43.0%	-0.0092	0.0058	0.0039	-0.0022	-0.0001	0.1157	0.1160
M9L2R2	0.009	82.0%	-0.0065	-0.0015	0.0042	-0.0015	-0.0004	0.6961	0.3554
M10L2R2	0.001	51.3%	-0.0002	0.0002	0.0002	0.0000	-0.0001	0.1928	-0.0351
Scenario 2									
M3L1R1	0.044	86%	0.0054	-0.0133	-0.0184	0.1046	0.0203	0.0833	0.0938
M4L1R1	0.010	74%	-0.0191	0.0729	0.0116	0.0004	-0.0025	1.2944	1.9529
M4L2R1	0.010	83%	-0.0669	0.2030	-0.0325	0.0625	-0.0031	0.0531	1.1175
M4L2R2	0.018	95%	-0.1961	0.3498	0.1392	0.0561	-0.0046	0.0325	-0.5120
M5L2R2	0.014	84%	-0.0678	0.1633	0.0447	0.0032	0.0005	1.1488	-0.5710
M6L2R2	0.014	85%	-0.0622	0.1158	0.0425	0.0167	-0.0030	2.9119	1.6314
M7L2R2	0.018	91%	-0.1345	0.1948	0.0578	-0.0235	-0.0062	3.3540	-0.9699
M8L2R2	0.018	73%	-0.0571	0.0948	0.0249	-0.0092	-0.0032	0.8595	-0.1125
M9L2R2	0.012	95%	-0.0254	0.0257	0.0210	-0.0073	-0.0040	2.2164	-0.5525
M10L2R2	0.002	55%	-0.0063	0.0056	0.0032	-0.0004	-0.0003	0.0163	-0.0004
Scenario 3									
M3L1R1	0.035	92%	0.0473	-0.1362	-0.0061	0.0607	0.0126	0.1767	1.0169
M4L1R1	0.016	71%	-0.0361	0.0931	0.0259	-0.0025	-0.0005	0.5340	5.1159
M4L2R1	0.024	66%	-0.0846	0.1609	-0.0027	0.0420	0.0003	0.1068	1.9186
M4L2R2	0.054	75%	-0.1521	0.2613	0.1485	0.0378	-0.0042	0.0474	-0.1489
M5L2R2	0.041	57%	-0.0566	0.1591	0.0746	0.0165	0.0006	3.4937	-1.5375
M6L2R2	0.032	65%	-0.0721	0.0950	0.0641	0.0087	-0.0025	2.2703	2.0169
M7L2R2	0.034	80%	-0.1467	0.1842	0.0741	-0.0195	-0.0052	3.3326	-1.1505
M8L2R2	0.027	70%	-0.0785	0.1129	0.0375	-0.0143	-0.0030	0.9312	0.0294
M9L2R2	0.024	85%	-0.0595	0.0570	0.0357	-0.0189	-0.0037	2.0887	-0.8698
M10L2R2	0.014	36%	-0.0255	0.0231	0.0136	-0.0013	-0.0007	1.1401	-0.7197

Non-significant p-values (values above 5%) are highlighted. A cause of a low adjusted R^2 is insignificant variables. However, from the table, we see that this is not the case. When analyzing the residuals of each subset, it became clear that the regression performs with a low adjusted R^2 because of a few outliers with a high relative outdating. This is shown in Figure B.2 for subset M5L2R2 for scenario 3. Lastly, it became clear that often these outliers occur at a 99% target service level. This is logical, since a linear regression expects outdating to be linear according to the variables, but this is not the case for a 99% service level, since stock and outdating increase exponentially.

Next, we take a look at the intercept and coefficients. In almost all cases, the intercept is a negative value. This is not unexpected since this was also the case in the paper. However, as the shelf life becomes longer in scenario 1, variables 3 and 4 have a negative relationship with the relative outdating. In scenarios 2 and 3, this effect is increased, and even variable 6 has a negative relationship with outdating in some cases. Furthermore, the absolute values of the coefficients in the last two columns in scenario 2 and 3 become very large, underlining the fact that z_A and z_B no longer approximate the outdating well, as shown before in Figures B.1 (c) - (f). Lastly, we see that coefficients are very different between subsets. This is logical, because all subsets contain certain assortment categories with different characteristics, such that one subset cannot be compared to another subset.

Table B.2: *Baseline performance and p-values of the regression coefficients for three scenarios under the original regression formula.*

	$R^2 - adj$	**p1**	**p2**	**p3**	**p4**	**p5**	**p6**
Scenario 1							
M3L1R1	89.1%	0.000	0.000	0.000	0.000	0.000	0.000
M4L1R1	79.3%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R1	90.1%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R2	90.5%	0.000	0.000	0.000	0.000	0.000	0.000
M5L2R2	58.2%	0.000	0.000	0.000	0.000	0.000	0.000
M6L2R2	65.7%	0.000	0.000	0.000	0.000	0.000	0.000
M7L2R2	89.0%	0.000	0.000	0.001	0.000	0.000	0.000
M8L2R2	43.0%	0.000	0.000	0.000	0.000	0.000	0.000
M9L2R2	82.0%	0.000	0.000	0.000	0.000	0.000	0.000
M10L2R2	51.3%	0.000	0.000	**0.770**	0.000	0.000	0.000
Scenario 2							
M3L1R1	86%	0.043	0.000	0.000	0.000	0.000	0.000
M4L1R1	74%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R1	83%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R2	95%	0.000	0.000	0.000	0.000	0.000	0.000
M5L2R2	84%	0.000	0.000	0.000	0.000	0.000	0.000
M6L2R2	85%	0.000	0.000	0.000	0.000	0.000	0.000
M7L2R2	91%	0.000	0.000	**0.162**	0.000	0.000	0.000
M8L2R2	73%	0.000	0.000	0.000	0.000	0.000	0.000
M9L2R2	95%	0.000	0.000	0.000	0.000	0.000	0.000
M10L2R2	55%	0.000	0.000	0.000	0.000	0.000	0.000
Scenario 3							
M3L1R1	92%	0.000	0.000	0.000	0.000	0.000	0.000
M4L1R1	71%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R1	66%	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R2	75%	0.000	0.000	0.000	0.000	0.000	0.000
M5L2R2	57%	0.000	0.000	0.000	0.000	0.000	0.000
M6L2R2	65%	0.000	0.000	0.000	0.000	0.000	0.000
M7L2R2	80%	0.000	0.000	**0.189**	0.000	0.000	0.000
M8L2R2	70%	0.000	0.000	0.000	0.000	0.000	0.000
M9L2R2	85%	0.000	0.000	0.000	0.000	0.000	0.000
M10L2R2	36%	0.000	0.000	0.000	0.000	0.000	0.018

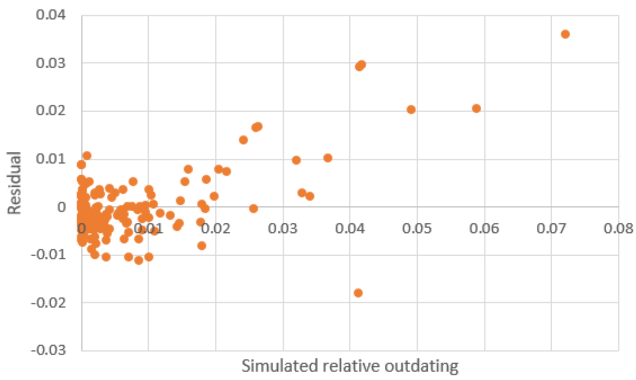


Figure B.2: *M5L2R2 regression residuals with a few outliers from scenario 3.*

C Regression results for FIFO withdrawal

This appendix shows the regression results in the situation of FIFO withdrawal. In Table C.1, the RMSE, $R^2 - adj$, and the coefficient values can be found. Furthermore, we compare the performance of the FIFO withdrawal case, with the mixed FIFO-LIFO withdrawal case from Section 6.4.2. The comparison is shown in Table C.2. Between the two cases, the $R^2 - adj$ is often higher in the FIFO case compared to the partial LIFO case, and the other way around. Overall, the regression equation performs well for both FIFO and partial LIFO cases, apart from subset M10L2R2.

Table C.1: *Performance of the developed regression formula with FIFO withdrawal.*

	$RMSE$	$R^2 - adj$	$intercept$	$\frac{\sigma_W}{\mu}$	$\frac{s}{Q} \cdot \frac{Q}{\mu}$	z_A'	z_B'	$\frac{SS}{\mu}$	$\frac{Q}{M \cdot \mu}$
M3L1R1	0.025	96%	-0.0327	0.1182	-0.0191	0.6566	-0.0243	0.0849	0.2142
M4L1R1	0.007	87%	-0.0172	0.0662	0.0007	6.2834	-1.5739	0.0105	0.0073
M4L2R1	0.007	91%	-0.0534	0.0761	-0.0006	1.2966	1.1121	-0.0632	0.0291
M4L2R2	0.012	98%	-0.2316	0.3081	-0.0059	0.1849	-0.1121	0.1163	0.5850
M5L2R2	0.010	91%	-0.0637	0.1550	0.0014	6.8274	-1.6336	0.0381	0.1457
M6L2R2	0.013	87%	-0.0963	0.1262	-0.0031	-2.2516	2.6919	0.0436	0.3050
M7L2R2	0.016	93%	-0.0713	0.1884	-0.0068	4.7277	-0.4375	0.0567	0.1681
M8L2R2	0.016	79%	-0.0374	0.0900	-0.0024	2.6448	0.1968	0.0224	0.1019
M9L2R2	0.012	95%	-0.0218	0.0283	-0.0031	1.9548	-0.2993	0.0211	0.1245
M10L2R2	0.002	62%	-0.0054	0.0048	-0.0001	0.1162	0.1320	0.0034	0.0204

Table C.2: *Comparison of the regression performance between FIFO and partial LIFO withdrawal.*

	$RMSE$ **FIFO**	$R^2 - adj$ **FIFO**	$RMSE$ **LIFO**	$R^2 - adj$ **FIFO**	**improvement**
M3L1R1	0.025	96%	0.028	95%	-1%
M4L1R1	0.007	87%	0.012	82%	-5%
M4L2R1	0.007	91%	0.014	89%	-2%
M4L2R2	0.012	98%	0.020	97%	-1%
M5L2R2	0.010	91%	0.020	90%	-1%
M6L2R2	0.013	87%	0.015	92%	5%
M7L2R2	0.016	93%	0.017	95%	2%
M8L2R2	0.016	79%	0.020	83%	4%
M9L2R2	0.012	95%	0.018	92%	-3%
M10L2R2	0.002	62%	0.010	64%	2%

D P-values, VIFs and other subsets in the regression

From Table D.1, we observe that all variables have p-values that are below 0.05, thus significant, except for variable $\frac{Q}{M \cdot \mu}$ for combination M7L2R2, which is 0.189. However, this does not compromise the performance of the regression equation with subset M7L2R2, because the adjusted R^2 is still 95%. Next, we present the VIFs of all variables.

Table D.1: *P-values of the regression variables.*

	$\frac{\sigma_W}{\mu}$	$\lceil \frac{s}{Q} \rceil \cdot \frac{Q}{\mu}$	$z_A^{'}$	$z_B^{'}$	$(\frac{SS}{\mu})^+$	$\beta^* \cdot (1-\sqrt{f})$	$\frac{Q}{M \cdot \mu}$
M3L1R1	0.000	0.000	0.000	0.000	0.005	0.000	0.000
M4L1R1	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R1	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M4L2R2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M5L2R2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M6L2R2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M7L2R2	0.000	0.000	0.000	0.000	0.000	0.000	**0.189**
M8L2R2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M9L2R2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M10L2R2	0.000	0.000	0.000	0.003	0.000	0.000	0.000

Table D.2: *Variance inflation factors of the regression variables.*

	$\frac{\sigma_W}{\mu}$	$\lceil \frac{s}{Q} \rceil \cdot \frac{Q}{\mu}$	$z_A^{'}$	$z_B^{'}$	$(\frac{SS}{\mu})^+$	$\beta^* \cdot (1-\sqrt{f})$	$\frac{Q}{M \cdot \mu}$
M3L1R1	1	2	5	6	2	1	3
M4L1R1	2	3	**16**	**17**	3	1	1
M4L2R1	2	3	**16**	**97**	**56**	1	5
M4L2R2	1	2	6	5	5	1	1
M5L2R2	1	2	4	4	3	1	1
M6L2R2	1	2	3	4	2	1	1
M7L2R2	2	3	4	5	6	1	1
M8L2R2	2	2	6	5	2	1	1
M9L2R2	1	3	**12**	9	3	1	4
M10L2R2	1	2	**11**	**11**	3	1	1

Most VIFs are below 10. However, in some cases, there is a too high correlation between $z_A^{'}$ and $z_B^{'}$. For these subsets, $z_A^{'}$ was removed from the regression equation and coefficients were determined again. This resulted in all VIFs below 10. The adjusted R^2 of subset M4L2R1 was then reduced by 2%, but the adjusted R^2 of subsets M4L2R2 and M10L2R2 were barely influenced.

The regression equation was also used to calculate all other subsets mentioned in Section 6.2 that were not tested above. Since the percentage of SKUs with outdating is low for each of those subsets, the regression was done for all subsets together. Since the shelf life, lead time, and review time were no longer fixed, another independent variable was added to the regression equation. From the analysis and Table 6.1 it is clear that the relative outdating reduces when the shelf life is large compared to the lead and review time. Therefore, the variable $\frac{M}{L+R}$ was added. However, the adjusted R^2 of the model is only 41%, meaning most SKUs with relative outdating are seen as outliers and are not well predicted. This, again, shows that a subset should have a large enough outdating percentage for regression to have good performance.

E Analysis of the regression approximation errors

This appendix contains an elaborate analysis of the regression approximation errors. First of all, we compare the approximated relative outdating by regression and the simulated relative outdating in Figure E.1. We observe a clear relation between the approximated and simulated relative outdating, i.e. SKUs with low simulated relative outdating have a low approximated relative outdating, and the same is observed for SKUs with very high relative outdating.

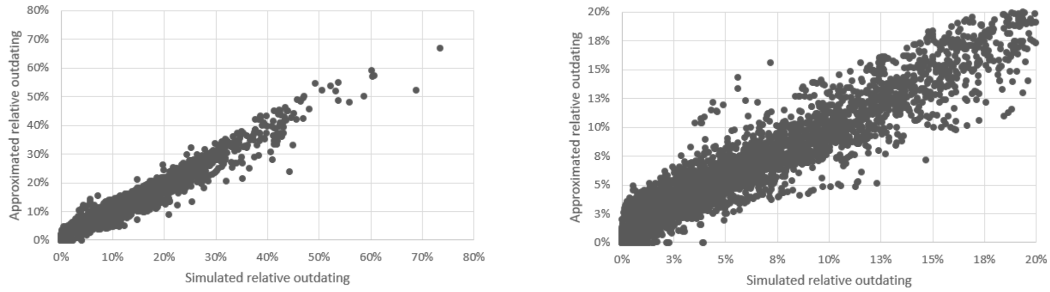
Next, we set out the simulated relative outdating to the approximation error in Figure E.2. We observe large outliers from relative outdating percentages larger than 20%. The highest approximation error is almost 21% for an SKU with the simulated outdating of 44%, meaning the regression approximates the relative outdating at $44\% - 21\% = 23\%$ instead of 44%.

The same holds for subset M10L2R2, for which the adjusted R^2 is just 64%. From Figure E.3 we observe a clear relation between the approximation error and simulated relative outdating.

Aside from the large approximation errors, the relative outdating of many SKUs is estimated well by regression. Although the percentages of the approximated relative outdating by regression and the simulated relative outdating are rarely equal, the regression clearly shows the effects of target service levels seen in the simulation. This is clearly seen in Figure E.4. In this case, the outcome of the regression mimics the outcome of the simulation, where no matter the choice of target service level between 80% to 90%, the expected relative outdating is the same. This means that the regression output is not valid for determining the exact expected relative outdating percentage, but rather for the change in relative outdating between two target service levels.

Diving further into the SKUs with high approximation errors, we observe three situation for which the regression does not approximate the outdating well. First of all, this is the case for highly seasonal items. Generally, the outdating for seasonal items (items with a high variance of monthly seasonal factors MSF) is overestimated by the regression. An example is shown in Figure E.5. Likewise, we generally observe underestimation of the outdating for SKUs with a high variance of the WSF. An example is shown in Figure E.6. Explanations on the high approximation errors can be found in Section 6.3.1. Unfortunately, a high MSF and WSF are not the only cause of high approximation errors since it partially depends on combinations with other regression variables. Therefore, it is not possible to clearly state from what MSF or WSF value the approximation error is high. Lastly, we observe higher approximation errors for the 99% target service level, as the regression is linear rather than exponential. The underestimation of the 99% service level is shown in Figure E.7.

After analysis, it became clear that the high approximation errors for SKUs with high weekly or monthly variance are due to the high approximation errors of the approximations z'_A and z'_B. This relation is shown in Figure E.8. From this Figure we also see that high approximation errors of z'_B are slightly reduced when performing the regression, i.e. regression lowers the approximation error of z'_B. However, large approximation errors with z'_B cause large approximation errors of the regression, indicating that a good approximation of z'_B is key for each SKU.



(a) All SKUs.

(b) All SKUs within the 0% to 20% interval.

Figure E.1: *The relation between the approximated relative outdating by regression and the simulated relative outdating.*

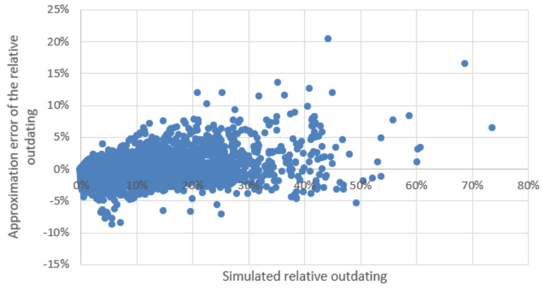


Figure E.2: *The relation between the approximated relative outdating by regression and the approximation error.*

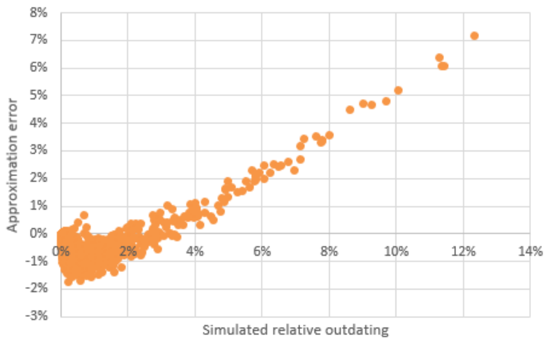


Figure E.3: *The relation between the approximated relative outdating by regression and the approximation error for subset M10L2R2.*

Figure E.4: *The regression relative outdating mimicking the simulated relative outdating.*

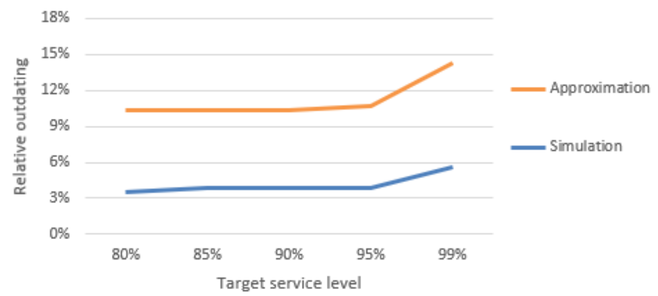


Figure E.5: *The overestimated outdating for highly seasonal SKUs.*

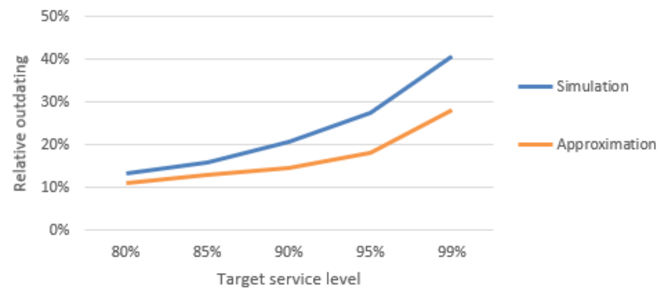


Figure E.6: *The underestimated outdating for SKUs with high weekly variance in demand.*

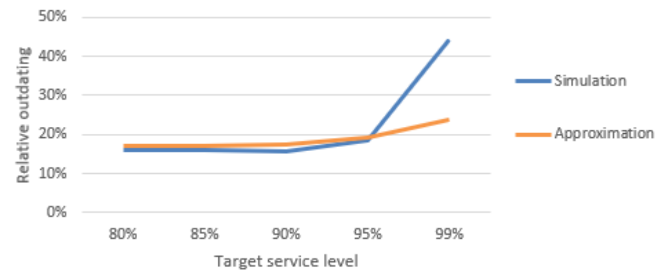


Figure E.7: *An example of an underestimation of the 99% target service level.*

Figure E.8: *The relation between the approximation errors.*