Private Equity Valuation

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Management Summary

Private equity is a major asset class in alternative investments, renowned for its non-transparent characteristics. Being a large investor in private equity, NN Group is interested in predicting performance of this asset class. Factors driving private equity performance and relation of private equity to traditional investment markets are of key interest to NN Group.

This research investigates the possibility of predicting performance of private investments. We investigate the possibility of predicting performance at the fund level and at the portfolio company level. We are able to develop a framework for performance prediction at both levels. Due to the private nature of the asset class and unforeseen circumstances of a global pandemic outbreak we had limited access to resources. We base our results on a literature study and opinions of professionals at NN Group.

We investigate the utility of a management tool developed to model the life cycle of illiquid alternative assets. We find that the model fits our requirements and can be used to predict performance of private equity funds. The model can also be extended and used to assess the impact of new investments and make investment and management decisions accordingly.

Using the model effectively calls for an investigation in the growth factors of private equity. We identify potential growth drivers in private equity. Fund manager's alpha is a controversial factor when it comes to driving growth in private equity. Although we identify the fund manager characteristics as one of the determinants of fund performance, we recommend further investigation in the direction of quantifying alpha as well as other performance drivers.

For predicting performance at the portfolio company level, we investigate the company valuation methods. We choose Market Approach because of its simplicity and the nature of data accessible. We define the criteria for selection of listed peer companies of the private companies. We develop a framework for creating a dynamic index that represents the private equity portfolio in terms of a hypothetical public market portfolio. The private equity portfolio can be evaluated based on this dynamic index.

Due to lack of resources and data we are unable at this stage to develop and test a prototype of the models. This research and the framework developed can be used by any person or organisation that invests in private equity funds as Limited Partners to predict performance.

We have organised the report so that it is easy to navigate to the area of the reader's interest. The first two chapters are introductory. They set the expectations and give details regarding the problem statement, research formulation, and nuances of private equity as an investment class. Thereafter, each chapter is dedicated to discussing specific research goals. Chapters 3 and 4 focus on treating the portfolio at the fund-level for making valuation predictions. Chapters 5 and 6 focus on predicting performance at the portfolio company level. At the end of each chapter, we have provided a discussion or summary of the ideas discussed in that chapter for the convenience of a busy reader. The final chapter gives an overview of all the research questions addressed in this project and briefly describes and discusses the results from each chapter.

Preface

During the intense period of the last examinations of my graduation program I solicited at NN Group for a thesis project. This project concerned the valuation of private equity funds and predicting performance. I have a strong interest in corporate finance and equity valuation. This is my first experience with researching private equity. This thesis project appeared to me a wonderful opportunity to learn about private equity in combination with asset management so I gladly accepted the challenge.

The start of my thesis was eventful, just a week after I started at NN Group the world went under a bizarre lockdown owing to the rapid spreading of the deadly virus of Covid-19. This affected the scope of the project undertaken. After the initial phase of literature analysis I spent a long time determining an appropriate research method. This research lacked the data that is available to other academic researchers; this made it difficult to execute a straightforward analysis. The general challenge of this research is to find a research method that could cope with the available data and the research objectives.

This research could not be completed without the help of several people. Since this thesis project marks the end of my study I would like to thank my parents who supported me and never lost confidence in me actually completing this study. I would like to thank my brother, Ameya, who supported me at all times. I would like to thank my friend Samiksha for standing by me in these testing times.

At NN Group I would like to thank Ralph van Hien, my company supervisor, helped me learning the nuances of private equity, finding information sources and helped me whenever possible. Ralph and I held weekly sessions about the research developments, which were helpful to reflect on my progress.

Last but not least I would like to thank Reinoud Joosten, my professor from the University of Twente, who took the time to understand my framework, gave me valuable feedback during several meetings. I would like to thank Abhishta for his valuable feedback and guidance.

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1 Introduction

1.1 Nationale Nederlanden Group

NN Group N.V. is the biggest Dutch life insurer and third largest Dutch asset management firm. Headquartered in The Hague, NN provides insurance and financial services across 18 countries in Europe, Asia, and the Americas. The group provides retirement services, pensions, insurance, investments, and banking to approximately 18 million customers. NN Group includes Nationale-Nederlanden, NN Investment Partners, ABN AMRO Insurance, Movir, AZL, BeFrank, and OHRA

This financial giant has rather humble origins that can be traced back to the mid-19th century. Gerrit Jan Dercksen with his nephew Christiaan Marianus Henny founded Assurantie Maatschappij tegen Brandschade, in Zutphen on 12 April 1845. Until the end of 19th century, the firm was focused exclusively on the fire insurance business, the company grew and rebranded itself as Assurantie Maatschappij tegen Brandschade De Nederlanden, popularly called **'De Nederlanden'**. In the 1900s the company expanded its business to life insurance activities, business insurance, transport insurance and adopted the new slogan 'Alle Verzekeringen' (All Insurance). The company faced many troubles in the first half of the 20th century owing to an economic slowdown and the Second World War. The company survived through the effects of the Second World War and rebuilt its position in the market by 1960 as the largest non-life insurer and the second largest life insurer in the Netherlands.

In 1863, the Rotterdam underwriter Simon van der Held, along with the attorney William Siewertsz van Reesema, founded a modern life insurance company, The Nationale Levensverzekering-Bank, commonly known as **'De Nationale'**. De Nationale relied on actuarial approach unlike its contemporaries and was conservative in its use of mortality tables. This combination spelled great success for the firm even through epidemics like smallpox and cholera. By the end of the 1930s, Nationale was one of the country's biggest insurers. Following a rather conservative approach, Nationale did not expand its operations abroad. Instead, the company had a number of successful acquisitions of insurance companies within the Netherlands. The Nationale too faced extreme difficulties during the period of the Second World War and had to endure loss of quality employees and infrastructure.

By 1962, **'De Nederlanden'** and **'De Nationale'** decided to join hands as the two largest firms in the industry as it was better to cooperate than to compete. Today, the **Nationale Nederlanden Group** or the NN Group is involved in a wide range of financial businesses and provides services like insurance, asset management and banking across 20 countries. The Group maintains and increases its wealth by participating in traditional and alternative investment instruments.

1.2 Research Formulation

Private equity is a behemoth of the alternative investment sector with currently about 5000 billion dollars worth assets under management globally. Projections by leading information management firms indicate that the sector is set to grow up to 9000 billion dollars worth within the next five years. Despite the stressful times of the global pandemic, when the markets all around crashed, the investments pattern and return profile of private equity seemed unaffected. The question must be asked - what is happening? A quick scan of the field revealed that the private equity sector operated from behind a veil of trade secrecy. The data available is sparse and self-reported by managers bringing into question its veracity and reliability. With enough financial expertise, the cash flows, the returns and the residual value can be manipulated and the actual returns are known only after the investments are realized. The private equity fund is a black

box with many blind spots. The ambiguity surrounding the fair value of private equity affects the decision making of investors, potential buyers and sellers.

If a tree falls in a forest and no one is around to hear it, does it make a sound? We begin our quest of truth at the Financial Reporting department of Alternative Investments at NN Group.

1.2.1 Problem Identification

The Alternative Investments portfolio at NN includes private equity, private debt, and real estate. NN wants to gain more insights into the private equity portfolio of Alternative Investments. NN participates in private equity funds as a Limited Partner. The private equity funds are each managed by a General Partner, who on behalf of NN invests in portfolio companies with an objective of providing the investor (NN) a maximum risk-adjusted return and earning a performance incentive for self.

The General Partner manages the investments and provides quarterly management reports to the Limited Partners. Although the quarterly reports vary in format and depth of information, each report contains all the necessary information regarding the development in the portfolio companies, the fair value of the fund and its underlying companies, the accounts of cash flow, and the balance sheet of the portfolio. It may also contain additional information like descriptions of the portfolio companies and course of action to manage the portfolio company.

The value of the private equity portfolio is reported in the quarterly and annual financial statements of NN. Therefore, the private equity portfolio is reviewed on a quarterly basis and audited on an annual basis by both the internal and external auditor. On a quarterly basis, the Alternative Investment Reporting team updates the fair market value of the private equity portfolio. The update is based on quarterly management reports provided and prepared by the General Partner. The management report contains information regarding the fair value of the private equity fund and its underlying portfolio companies. The management report is released by the General Partner 45 days after the end of each quarter. So, the reported valuations are known 45 days after quarter-end and 90 days after the year-end. As a result of the lagged reporting by the General Partner, the private equity portfolio is most likely not reported at fair value at a specific reporting date in the NN financial statements. The value of the private equity portfolio reported in the financial statements of NN is stale and lags by a quarter.

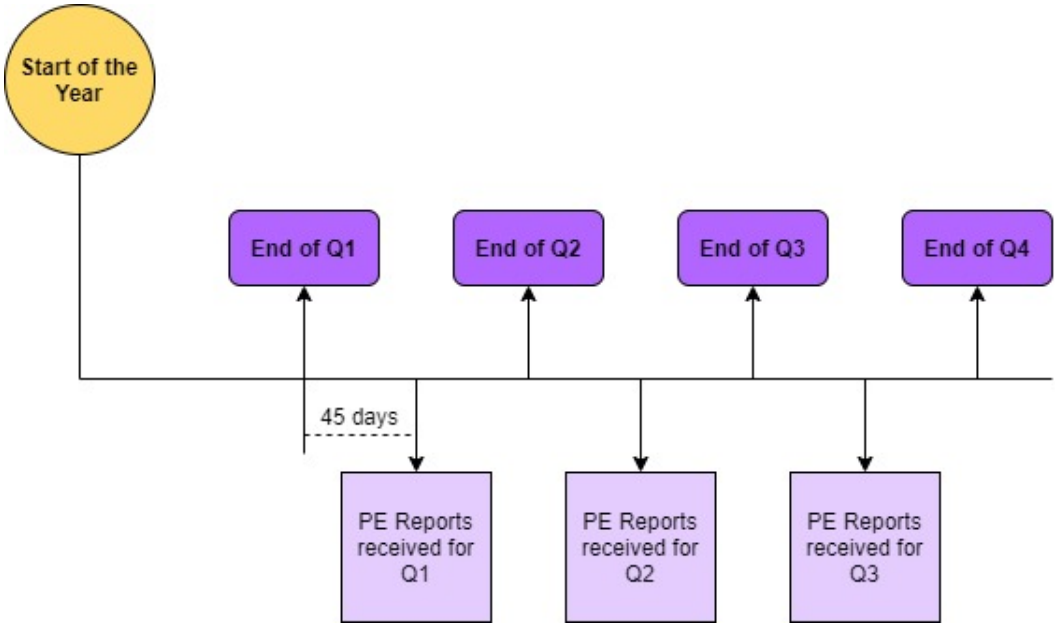

Figure 1.1: Timeline of the Company Quarterly Financial reports and Private Equity Fund reports.

Figure 1.1 shows the lag in the timelines for the company to release its financial reports for the general public and the reports received on private equity funds. The financial report at the end of the first quarter reports investments in private equity from the previously received report. For example, the financial report released at the end of Quarter 2 by NN to its shareholders will be reporting its private equity portfolio based on the stale report, which in this case, is the PE Reports for Q1.

In addition to complications due to the time-lagged nature of the valuations, there is ambiguity around the accuracy and authenticity of the valuations. Due to the non-disclosure agreement between the General Partner and the portfolio companies, Limited Partners like NN do not have access to the financial reports of the portfolio companies. NN relies solely on the valuation of portfolio companies provided by the General Partner. This calls for an improvement in the internal control on the fair value measurement of the private equity funds and more specifically the underlying portfolio companies.

Although in-line with the industry practices, the time-lagged evaluation of private equity investments results in a distorted view of the investment portfolio. Lack of sufficient information further calls in question the valuations provided by the General Partner. As a result, there is exposure to inherent valuation risk in the private equity portfolio of NN.

Our study focuses on the problem of valuation risk and ways to minimize it. With the problem identification at the center of this study, the research goal, defined at the highest level, is to *Develop a predictive model for the fair value of the private equity portfolio.*

The goal of this research is translated into the research question - *How can the fair value of the private equity portfolio be forecasted?*.

The intention of NN is to mitigate the effects of time-lagged valuations and develop a more robust internal control over the valuations. We focus on the progressive changes in the fair value of the portfolio companies which will then help to understand the fair value of the private equity fund.

We identify a series of scaffolding research questions, the answers to which would lead to the answer to the main research question.

- What are the characteristics of private equity investment?
- Can market movement in private equity be predicted?
- What drives performance of private equity?
- What data are available to understand and project fair value progression?
- How can the fair value of portfolio companies be predicted without access to company accounts?
- How can the fair value of private equity portfolio be derived from the drivers of private equity?

1.2.2 Research Approach

The research is structured around the objective to be able to predict the fair value of a private equity portfolio. Since the COVID-19 pandemic hit the world at the beginning of this project, we had limited access to resources. Due to this we shall be developing a framework for the main objective backed by academic research.

We have approached the process of value prediction in two ways. First, we adopt a fund-level approach and attempt to predict the fair value of a fund as a whole. The next approach goes a step deeper and attempts to predict fair values at the portfolio company levels. Generally

speaking, private equity funds are collections of investments in private companies which are then called the portfolio companies as they are now part of the investment portfolio. Theoretically, the sum total of the fair value of all the portfolio companies should be the fair value of the private equity fund. The research approach that we follow is explained briefly next.

- *Study the characteristics of the private equity universe.*
  This objective requires study of the private equity industry and acquiring the relevant academic knowledge.

  For a Fund-Level Model:
  We treat the private equity portfolio at the fund level. The characteristics of the models and the growth-factors are investigated from a fund-level perspective.

- *Find a relevant model that could be used to predict the fair values of the private equity funds.*
  Based on the understanding of the problem and knowledge of the private equity industry, scout the academic literature for a model that could be used to predict the fair values of private equity funds.

- *Find the relevant performance drivers for private equity.*
  Any model that projects the value curve for the PE fund is certain to have incorporated the factors of growth. Based on the extent of the detail of the model, we will probe into the performance drivers for the private equity industry.

  For a Portfolio Company-Level Model:
  In this approach, we want to map each portfolio company to its so-called “identical twin" from the public market. We are trying to theoretically create a collection or index of public market companies that resembles the private equity portfolio of the company (NN). Valuation and value prediction of the private equity portfolio is then based off of valuation of the index thus created.

- *Define the dimensions for determining a peer company for the portfolio company.*
  This objective requires analysis of the relevant literature and the research available on peer analysis of comparable companies.

- *Develop an algorithm for creating an index similar to the private equity portfolio.*
  Based on the results and conclusions from the previous objective, we shall be developing a blueprint for selecting peers for portfolio companies.

- *Define the valuation metric*
  Once we have a peer group for each portfolio company, we define the relation between the value of the portfolio company and the market indicator of its value. Through this objective we will determine the choice of the valuation metric.

2 Private Equity - An Overview

2.1 Introduction

As the cut-throat competition in the technologically advanced and information-dense traditional investment universe intensifies, investors are looking at other avenues to diversify risk and enhance their returns. Alternative investments being atypical when it comes to information transparency, regulation, tax considerations and correlation with the traditional investments have gained significant interest from investors of various types since the mid-1990s. Alternative investments are however not risk-free and may be correlated to traditional investments in certain economic conditions. Alternative investments include Hedge Funds, Private Equity Funds, Real Estate, Commodities, Infrastructure, Tangible Assets such as art, antiques, vintage items, and collectibles, and Intangible Assets such as intellectual property rights.

Private equity funds invest in privately-owned companies or in public companies with the intent to take them off the public market and own them privately. In this chapter, we shall briefly discuss the types of private equity strategies, the private equity market structure, the private equity fund characteristics and the valuation methods used in private equity market.

2.2 Private Equity Strategies

- **Venture Capital Funds**
  Venture Capital (VC) funds identify profitable ideas and invest in companies that are new and have a potential for fast growth. The fund manages the portfolio company (the venture) through its different stages of growth. VC funds are high-risk funds and potentially high-return investments. The three stages of a portfolio company when a VC can invest are discussed below.
  - **Seed stage** : At this stage, the company is not established and has no financial history. The VC funds the company to conduct research tests and develop a viable product.
  - **Start-up stage** : In this stage, the company needs funds to set up operations, begin product development, marketing, etc.
  - **Expansion stage** : In this stage, the company has somewhat stabilized. The company needs funds to grow by increasing production, expanding to new markets, or may need additional working capital. Investment of this type is also called Growth Equity.
- **Buyout Funds**
  Buyout funds, as the name suggests, buy the company from current shareholders. The fund manager usually sits on the board of the company, and drives changes and growth from a management perspective. The buyout is usually executed in conjunction with financial debt and is hence called leverage buyout.
- **Mezzanine Funds**
  Mezzanine funds invest in established companies that are unable to raise capital from traditional markets. These companies issue subordinated debts that have warrants or rights to convert to common stock. Mezzanine debt provides a relatively stable cash flow and generates lower returns than other types of PE funds.

2.3 Private Equity Market Structure

The market space is classified in two ways.

- Organised Market: Institutional investors such as banks, insurance companies, pension funds, high net-worth individuals, operate in the organised markets. Investments mostly take place through private equity (PE) funds.
- Informal Market: In this market, investments are made directly in private firms by angel capital, family, friends, and fools. Additionally, funding also comprises of the founder's savings and efforts.

2.3.1 Investing in Private Equity

Investment in private equity can be made in many ways. As shown in Figure 2.1, direct investment can be made in privately owned companies without going through the fund route. Investments are made indirectly by participating in PE funds. Indirect investments can also be made by participating in funds of fund. Investment in private equity can also be indirectly achieved by investing in publicly listed PE firms.

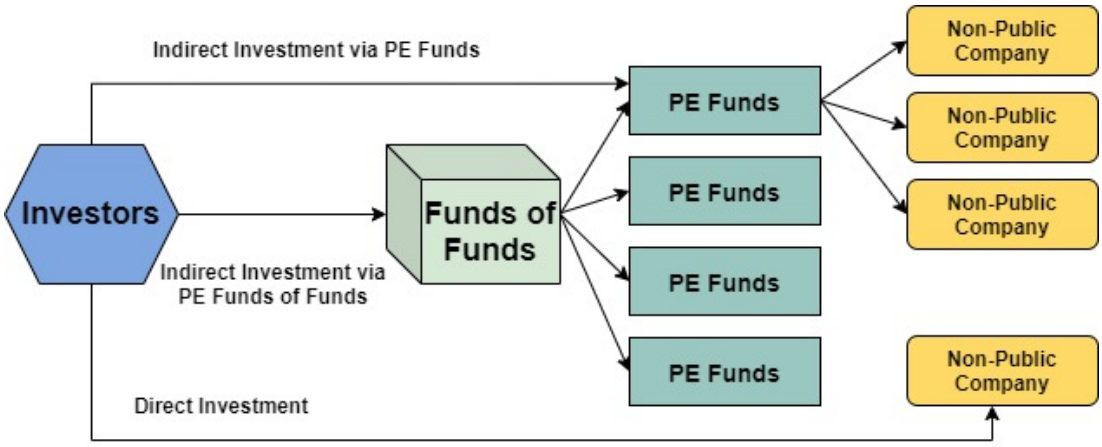

Figure 2.1: Styles of Investments in Private Equity.

2.3.2 Fund Structure

A private equity fund is an investment vehicle through which investments can be diversified among different private firms, which become portfolio companies. Fund investments are usually not offered to the general public and hence may be differently regulated.

Fund management companies (also called PE firms) set up PE funds that are managed by professionals referred to as fund managers or General Partners (GPs). LPs are usually institutional investors, high net-worth individuals, and other investors that are assumed to understand the risk associated with this asset class.

GP and the LPs have the quintessential principal-agent relationship. This exposes the LP to ethical risks. While the GP theoretically bears unlimited liability, the fund structure ensures limited liability and reduced taxation to the LPs at the cost of information transparency.

2.4 Private Equity Fund Lifecycle

The fund, once created, goes through four main phases. These phases generally overlap and there is no strict demarcation.

- **Fund Raising**
  A fund is created by the GP, and LPs are invited to participate in the fund. Interested LPs consent to participation by committing capital to the fund and funds are thus raised. The committed capital is the cash investment promised by an LP over the life of the fund. It is

also the LP's maximum liability. Once the necessary funds are raised, the fund is closed to new entrants. The LPs that have committed to the fund cannot leave without penalties.

- **Investing**
  During this time the GP is sourcing and evaluating potential investments, conducting business and valuation due diligence, negotiating term sheets. As and when the GP finds a suitable investment opportunity a capital call is raised. The LPs make the capital contribution. The vintage year is the year a fund commences its operations and raises the first capital call. During the pre-defined investment period, the GP invests in various portfolio companies.

- **Holding**
  After investing in a portfolio company the fund becomes a significant shareholder of the portfolio company. In the holding phase, the fund managers employ management strategies to increase or create shareholder value. Shareholder value is created by various strategies like cost reduction, operational improvement, company restructuring, talent upgrades, and expansion.

- **Divesting**
  The fund can exit the investment in three basic ways - IPO, trade sales or secondary buyout.

  Trade sale is the most common way of exiting the investment in private equity. In this, the portfolio company is sold to one of its competitors or a strategic buyer.

  IPO is the most expensive option and is considered only by experienced managers when the market conditions are favorable. An IPO can potentially lead to significantly larger benefits than trade sales or secondary buyout.

  Secondary buyout happens when the portfolio company can neither be sold to another buyer, nor be made public through an IPO. The portfolio company is sold to another private equity firm. This typically generates less value compared to IPO or trade sales.

  There also exists a rather unpleasant way of exiting the investment, that is by writing it off and accepting the loss of invested capital and efforts.

2.5 Valuations

A significant part of portfolio management is to evaluate the portfolio company. The valuation process is conducted many times over the portfolio-life of the company to monitor the growth of the investment. There are various ways to evaluate a company and the accounting reports like the balance sheet, the statement of cash flow are at the heart of all the valuation processes. Although the portfolio companies maintain accounting reports, they are not under any obligation to issue financial statements publicly. Another noteworthy point is that the confidentiality agreement protects sensitive information. According to the confidentiality agreement, the fund manager, that is the GP, has exclusive access to the financial statements of the portfolio companies and performs the valuations. The fund manager reports only the final results of financial analysis of these companies in the form of quarterly and yearly fund reports to the LPs.

We discuss the different approaches to the evaluation process below.

- **Asset approach**: The theory underlying the asset-based approach is that the value of a business is equal to the sum of the value of its assets. Models developed in this approach equate the value of a firm to the market value of its net assets.

- **Market approach**: This approach is based on the assumption that comparable firms in the same industry will have similar financial ratios also called valuation multiples. The

idea of efficient markets is implicit to market based valuation approach. The most common valuation measures used in comparable company analysis of public companies are enterprise value (EV) to sales (EV/S), price to earnings (P/E), price to book (P/B), and price to sales (P/S). The same idea when extended to private firms uses ratio of EV and Earning Before Income Tax, Depreciation and Amortization (EBITDA) EV/EBITDA and EV/Total Revenue.

- **Discounted Cash flow approach**: In this approach, various methods are employed to generate the present value of the company based on projected future cash flows. A popular model is the dividend discount model which is typically useful when a company has a history of issuing dividends at a regular interval. The model may be modified to represent better cash flows of a general company, which may not issue dividends. The model of this type discounts the free cash flow to equity, which represents the *dividend paying capacity* instead of actual dividends paid.

Each valuation approach is based on assumptions and involves many factors that depend on the judgment of the analyst. As a result, the value of private equity portfolio is subjective.

2.6 Performance Measurement

Measuring performance of the private equity portfolio tracks the progress in valuation of the portfolio. A simple and effective way to track progress of an investment in a private equity fund is the multiple method. The progress is measured in multiples of Paid-in-Capital. Cash outflow is the Paid-in-Capital, contributed by the LP. Cash inflow is the distribution from the fund and the Residual Value is the value of the fund which is yet to be divested and may be enhanced in the future. The ratio is called Total Value to Paid-in-Capital (TVPI) which can be subdivided into Distributed Capital to Paid-in-Capital (DPI) and Residual Value to Paid-in-Capital (RVPI). A ratio greater than 1 indicates that the investment has been successful in generating profits. Equation 2.1 shows the relation of the ratios and the cash flows.

$$TVPI = DPI + RVPI = \frac{\Sigma Distributions}{\Sigma Contributions} + \frac{\Sigma ResidualValue}{\Sigma Contributions} \tag{2.1}$$

This method, although practical, has a major drawback. It does not account for the time value of money. To compensate for this, the Internal Rate of Return (IRR) is also used to measure the performance of investments in private equity. It is the discount rate that makes the net present value (NPV) zero.

Both the multiple and IRR depend on the valuation of the portfolio, which as we previously stated is subjective. IRR is a forward looking metric as is NPV. Public Market Equivalent (PME) is a benchmarking performance measurement. It compares the performance of the private equity against public markets. For calculating a PME in its simplest form, we first select a public market index to be compared with the private equity portfolio as a reference. We then simulate trade on the reference index based on activity in the private equity fund. Each time the fund calls for contributions (distributions), we simulate a buy (sell) action of the reference index. We maintain a record of all the simulated trades thereby creating a synthetic portfolio. At the end of the final period, the IRR of the cash flow stream of this synthetic portfolio is referred as the PME. It is used not only to track performance of the private equity against the public markets but also as an indicator of fund manager's skills to generate profits.

3 Illiquid Alternative Asset Fund Modelling

Takahashi and Alexander [2002] developed a management tool to predict cash flows and valuation of illiquid alternative asset funds. The model is flexible and can be applied to a wide variety of investment vehicles.

Takahashi and Alexander [2002] studied the history of modeling illiquid assets and state that until the early 1990s, investments in this asset class were made based on certain simple rules-of-thumb, like splitting the capital and investing a fixed amount periodically in all of the venture capitalist funds in the market so as to average out the exposure. As the alternative asset market grew and became more sophisticated, such simple rules were becoming obsolete. In order to keep up with the growing investment market, Takahashi and Alexander [2002] developed the new model based on four criteria.

- Firstly, the model is intended to be simple yet sensible on a theoretical basis.
- Secondly, the model should be able to incorporate and respond to actual capital flows and asset value changes in real-time.
- Thirdly, the model should be sensitive to varying return scenarios and varying rates of investments and distributions.
- Finally, the model should be flexible such that it is applicable to a variety of asset types.

3.1 The Model

The model comprises of inputs as indicated in Figure 3.1, such that we are able to predict capital contributions (C), distributions (D), and Net Asset Value (NAV) of the fund at any given time *t*.

Input	Description	Output	Description
RC	Rate of contribution	*C*	Capital contributions
CC	Capital commitment ($)	*D*	Distributions ($)
L	Life of the fund (years)	*NAV*	Net asset value ($)
B	Factor describing changes in the rate of distribution over time		
G	Annual growth rate (%)		
Y	Yield (%)		

Figure 3.1: Model inputs and outputs, Takahashi and Alexander [2002].

3.1.1 Capital Contributions

The rate at which investments are made in a fund varies with time. Capital contributions are concentrated heavily in the early years of a fund's life. Capital contribution at any given time, *t*, is the fraction of the remaining capital commitment. Equation 3.1 indicates the capital contribution at time, *t*, is the product of the rate of contribution (RC) and the difference between the committed capital (CC) and the capital paid-in (PIC) until the time *t*. The paid-in-capital *PIC*, at time *t*, is the sum total of all the contributions made previously, as indicated in Equation 3.2.

$$C_t = RC_t * (CC - PIC_t) \tag{3.1}$$

$$PIC_t = \sum_{1}^{t-1} C_n \tag{3.2}$$

To see the model for contributions in action, Takahashi and Alexander [2002] have arbitrarily taken *RC* to be 25% in year 1 and 33.3% in year 2 followed by 50% in all the following years. Capital commitment *CC* is 100% by definition. The paid-in-capital (*PIC*) for year one is 0% of the capital commitment, as no previous payments have been made to the fund before it's commencement. Table 3.1 is a snapshot of the calculations worked out according to the *RC* defined as such for 4 years. Panels in Figure 3.2 show the graphical progression of rate contribution, outstanding commitment and contributions over the years.

Capital Commitment	100%

Year (t)	Rate of Contribution (RC_t)	Paid-in-Capital (PIC_t)	Outstanding Commitment ($CC\text{-}PIC_t$)	Capital Contribution (C_t)
1	25%	0.00%	100.00%	25.00%
2	33.3%	25.00%	75.00%	24.98%
3	50%	49.98%	50.03%	25.01%
4	50%	74.99%	25.01%	12.51%

Table 3.1: Yearly calculations for capital contributions.

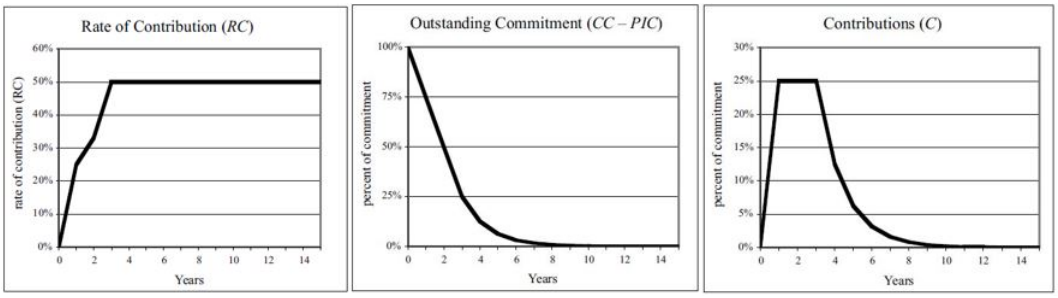

Figure 3.2: Capital contributions, Takahashi and Alexander [2002].

3.1.2 Distributions

Distributions from investment funds vary over the lifetime of the funds. They typically increase over time, are most concentrated in the mid-life phase of the fund and eventually decline as the fund matures. Distribution at time t is modelled according to Equation 3.3. We can see that the distribution at time t is directly proportional to the Net Asset Value *(NAV)* of the fund in the previous time period. In fact, the fund value *(NAV)* grows at the rate of G over unit time and a fraction of it, determined by the distribution rate *RD*, is given away as distributions. .

$$D_t = RD_t[NAV_{t-1} * (1+G)] \tag{3.3}$$

The rate of distribution, *RD* is modelled according to Equation 3.4. The yield, Y sets a minimum distribution level which is useful for income-generating assets like real estate. For other assets that are not income-generating, the yield can be set to zero. The life of the fund is represented as L. The rate of distribution is controlled by the Bow factor B. Figure 3.3 shows the effect of the Bow factor on rate of distribution. As the Bow factor increases, the distributions are delayed in the initial stage and accelerate at a higher rate at the later stages.

$$RD_t = Min[Max[Y,(t/L)^B],1] \tag{3.4}$$

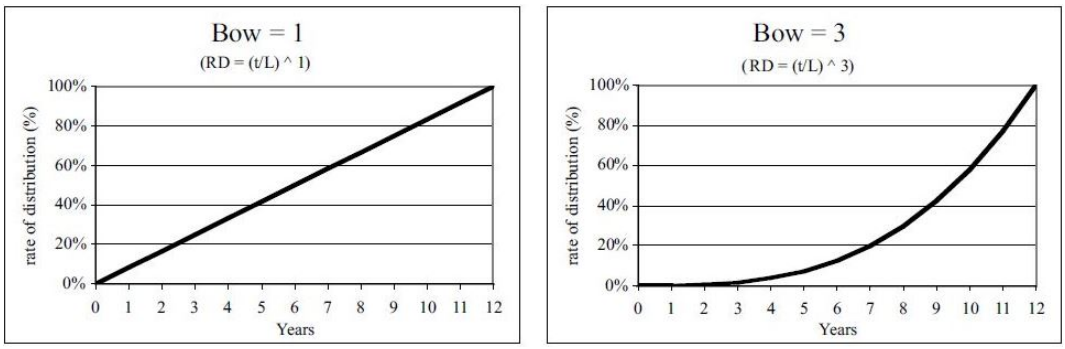

Figure 3.3: Rate of Distribution with different Bow factor values, Takahashi and Alexander [2002].

3.1.3 Net Asset Value

The NAV of the illiquid alternative asset fund is modelled recursively. Equation 3.5 shows that NAV of the investment fund at a time t is equal to the NAV at the end of previous time period *t-1*, grown at the rate G plus the contribution until time t minus the distributions until time t.

$$NAV_t = [NAV_{t-1} * (1+G)] + C_t - D_t \tag{3.5}$$

To see the model working, Takahashi and Alexander [2002] have arbitrarily assumed the growth rate (G) to be 13% and a Bow factor (B) of 2.5 for a fund of life (L) 12 years and that the fund does not generate a regular income, that is, yield (Y) is 0%. The assumptions previously stated for rate of contribution in the model for contributions are valid also for modelling the fund NAV. Panels in Figure 3.4 show projections thus made in this sample model for the NAV.

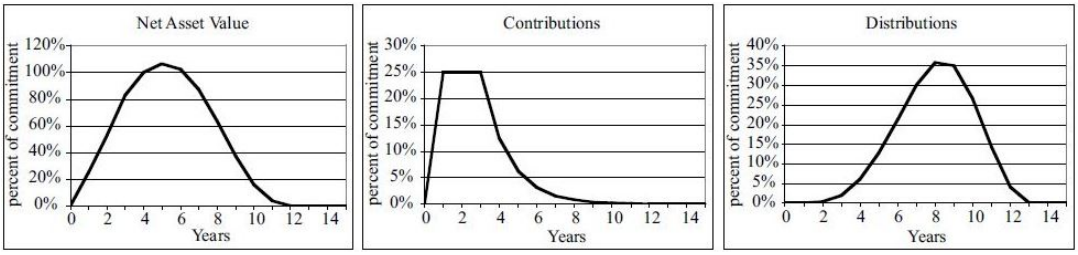

Figure 3.4: Sample model, Takahashi and Alexander [2002].

3.2 Validation of the Model

For validating the model, Takahashi and Alexander [2002] tested it against a sample of 33 venture capital funds picked from Yale University's investment portfolio. They first charted the historical data of NAV, contributions and cumulative distributions from these funds. Then they checked whether the model thus developed could track the historical data when fed with appropriate inputs. They found that their model was successful in this endeavour as it successfully met all the four criteria defined previously.

Figure 3.5 shows the panels where NAV, contributions and cumulative distributions from the historical data as well as the modelled values are charted. The bars represent the historical data and the line represents the curve developed by using the model. The inputs for the model in this case are as follows: 20% growth rate, 20-year life, 29% rate of contribution in year one, 30% rate of contribution in subsequent years, a Bow of 1.2, and a yield of 0%.

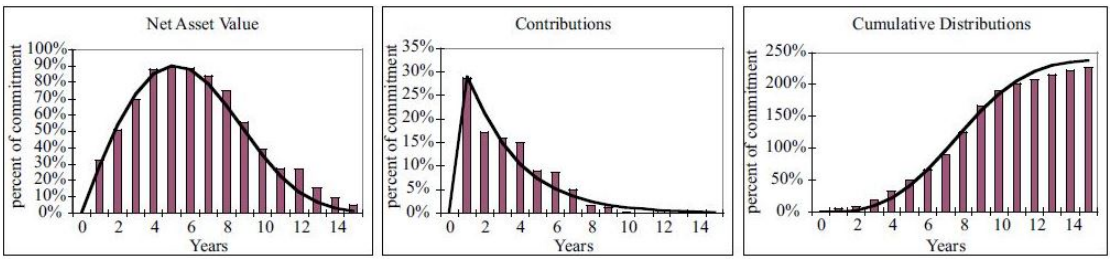

Figure 3.5: Model compared to historical data, Takahashi and Alexander [2002].

The generalisation of trends for contributions and distributions is consistent with the actual data. Contributions are concentrated in the initial years and taper off as the life of the fund progresses. Distributions, on the other hand are concentrated mostly in the mid-life period of the fund as can be seen by a sharply positive slope of the curve of cumulative distributions. This makes the model meet the first criterion, that is, to be simple yet sensible on a theoretical basis.

The second criterion is to be able to incorporate and respond to actual capital flows and asset value changes in real-time. To check for this, Takahashi and Alexander [2002] developed a base model for the data on the funds with vintage year 1993. The data was made known to the model until the year 2000 and the model made future data projections for subsequent years. Figure 3.6 shows panels for curves of NAV, contributions and distributions of the said data. The shaded bars represent the actual data and the unshaded bars are the values projected by the model. Although the fund outperformed the predictions, they state that the base model was able to use actual data and make reasonable future projections. The base model Yale uses for venture capital funds has the inputs as follows : 13% growth rate, 12-year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a Bow of 2.5, and a yield of 0%.

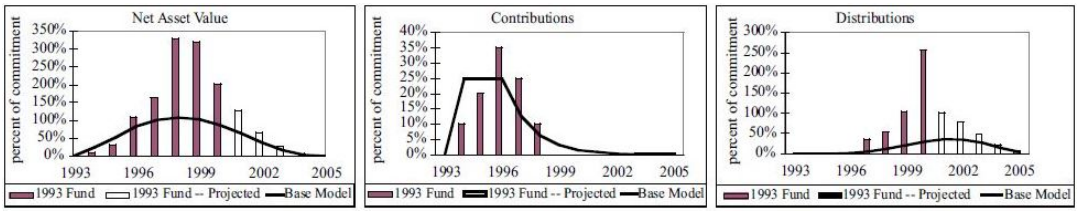

Figure 3.6: Model compared to 1993 vintage year venture capital data, Takahashi and Alexander [2002].

The third criterion was that the model should be sensitive to varying return scenarios and varying rates of investments and distributions. To test the model on this criterion, a similar exercise was carried out for the fund with vintage years between 1984 and 1986. This was a period of economic recession and social crisis after the second world war. With the inputs defined at 7% growth rate, 18-year life, 20% contribution rate in year one, 25% contribution rate in year two, 30% contribution rate in subsequent years, a Bow of 2.2, and a yield of 0%, the model fits the historical data as shown in Figure 3.7.

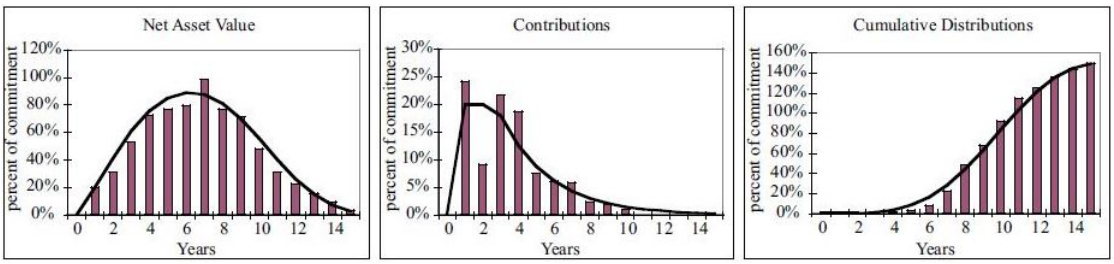

Figure 3.7: Model compared to 1984-1986 vintage year venture capital data, Takahashi and Alexander [2002].

The ability of the model to adapt to varying economic conditions is lucid when we compare the model inputs that were used for fitting the entire data set of 33 venture capital funds and the inputs used to fit the data from those venture capital funds that have vintage only in the most distressed years. Table 3.2 enlists the input factors used in each case.

Input / Data	Annual Growth rate	Bow Factor	Rate of Contribution in Year 1	Rate of Contribution in Year 2	Rate of Contribution for all future years
For all the sample funds	20%	1.2	29%	30%	30%
For funds with vintage 1984-1986	7%	2.2	20%	25%	30%

Table 3.2: Inputs for modelling funds for varying economic scenarios.

The annual growth rate for funds started investing in the distressed times is only 7% as compared to the 20% growth rate of aggregate of funds that have vintages spread over at least a decade. A slower growth during challenging economic times is reasonable. Similarly, during challenging economic times, the distributions are expected to be delayed. The Bow factor, which is key in determining the rate of distribution is comparatively higher for the funds with vintage in distressed times than for the funds in general. When the investment environment is unfavourable, investment opportunities are likely more difficult to come by. We expect the rate of contribution to be lower in distressed times than in times of normal or average economic stability . When the necessary adjustments are made to the model, it is able to fit the general data as well as data from the period unfavourable for investments. The model is thus sensitive to varying return scenarios and varying rates of investments and distributions and meets the third criteria.

By altering the inputs of the model, we can use it to represent other illiquid assets such as real estate, which generally has cash yields and more traditional private equity assets like the leverage buyout funds. The model satisfies the fourth criteria well as it is able to fit different asset types.

3.3 Discussion

The model developed by Takahashi and Alexander [2002] is useful in our project. The model is simple to use and understand. The model is sensitive to real-time cash-flow changes in the fund. The prediction for contributions depends on the outstanding capital commitment. This

means the predicted cash-flow value of contribution at any time will have accounted for the past values of actual contributions. Similarly, distributions and NAV predictions adapt to actual past values. The model is flexible. It is able to adjust to scenarios with lower than expected growth and also scenarios of unfavorable investment environment. When the environment is not favorable for investment, contributions are called at a slower pace. Likewise, in case of an unfavorable exit environment, the distributions may slow down or the life on the fund may be extended. The model fits the purpose of our project, that is to be able to predict the NAV of the private equity portfolio.

Additionally, the model can be extended for further analysis at the asset class level or the portfolio level. By varying the commitments to the funds we can analyse the cumulative impact on cash flow projections and risk exposure. Similarly, we can analyse the effects of new commitments.

A drawback of the model is that the success of the model depends on the quality of the inputs. Assumptions for developing a base model requires in-depth understanding of the asset class the model is intended to be used. For example, if we want to use it to project real estate funds, we cannot use the same base model assumptions that we use for venture capital funds. The market for the two funds are vastly different. An understanding of the industry, backed by research will be necessary to make educated guesses for the Bow factor, rate of contributions and growth rate. Although the model is simple, it is reliable only when defined by an expert.

The focus of this research is to be able to predict the NAV of the private equity fund on a quarterly basis. With an appropriate choice of inputs, the model can be used for this purpose. We restate Equation 3.5 here. When we take the unit of time to be a quarter of a year, that is 3 months or 90 days, the model is geared to the time-frequency of our interest.

$$NAV_t = [NAV_{t-1} * (1 + G)] + C_t - D_t \tag{3.5}$$

The private equity reports are released 45 days after the end of a quarter. So, we receive the value for NAV_t at time t+0.5. In order to know the value without delays, we wish to make predictions. We already know the NAV of the fund from the previous report. This means, the NAV_{t-1} value is known to us. At the end of a quarter, we are already aware of the contributions made during that quarter. This means, the C_t value is known and does not require to be predicted. Also, the distribution for the quarter has already been made. This means, the D_t value is also known and does not require to be predicted. Predicting the NAV value of the fund now can be narrowed down to essentially predicting the internal rate of return or the growth rate of the fund .

This bring us to the fundamental query - What does the growth in private equity depend on? We proceed in the next chapter to explore the determinants of growth in private equity.

4 Value Determinants in Private Markets

Our goal is to be able to predict the performance of private equity. In this chapter, we adopt an approach of treating the private equity fund as the investment unit. We explore the factors that affect movements in the valuation of private equity. An investigation of the returns-profile of private equity will be useful to be able to identify factors affecting the growth of investment in this sector and could enable us to model returns in private equity. Private equity performance is measured in terms of Internal Rate of Return (IRR) due to the irregularity of cash flows during its lifetime. A benchmarking metric that compares private equity performances with the public market is called Public Market Equivalent (PME).

4.1 Literature

Aigner et al. [2008] identify factors that contribute to the growth of PE fund value using multivariate regression analysis. Their study is based on 64 realised and 40 mature funds screened from the data-set acquired from a European fund-of-funds. About 55% of these 104 funds belong to the North American region and the rest to the European region.

They use a weighted least square model for this analysis instead of the ordinary least square (OLS) regression model because evidently, the assumption of constant variance of the error term is in violation when tested. It important that the explanatory variables are uncorrelated to successfully test the impact of each on the dependant variable while controlling others. Aigner et al. [2008] ensure this by using variance inflation factor, a measure of severity of multicollinearity, for every regression.

They measure fund growth in three ways - firstly as gross PME, secondly as gross IRR and lastly as the percentage of a fund's investments that generated losses. These are the dependant variables used in each of the multivariate regression analysis. The use of gross IRR and gross PME makes sense as it circumvents the problem of mixing the effects of management fees and carried interest with the fund's performance. The independent variables are the factors that affect the fund's growth. We explain the independent variables below.

- Buyout Ratio: It is the percentage of deals in an equity fund categorised as buyout deals. Buyout deals are typically reliable in generating returns compared to the extremely risky venture capital deals. This variable checks the risk undertaken by the fund.
- Experience of GP: Experience can be in terms of the number of years spent in the industry or in terms of funds managed. In this analysis Aigner et al. [2008] consider both.
    1. Years of experience : It is the time span between the vintage year of the GP's first fund and the most recent vintage of the GP. A logarithmic value is applied because it is assumed that each additional year of experience but with a diminishing marginal effect.
    2. Number of funds : It is the number of funds managed by the GP including the current fund.
- Level of interest rates: This is defined by two variables. Aigner et al. [2008] consider the interest rate at the vintage year of the fund and the average interest rate during the fund's life. They use 3-months U.S. Treasury rates for the North American funds and German "Driemonastgeld" for the European funds.
- Economic trend: Defined by two variables, economic trend comprises of nominal GDP at the vintage year of the fund and the average nominal GDP of during the lifetime of the

fund. U.S. GDP represents the funds from North America and Germany is the proxy for the European funds.

- Development of stock markets: This is defined by two variables, one represents the stock market development in the vintage year of the fund and the other represents the overall movement of the market during the lifetime of the fund. The return of MSCI World Performance Index performance represents the development in stock market in the vintage year of each fund. The average return, calculated as a geometric mean, of the annual returns of MSCI World Index during a fund's lifetime.

- Fund size: The commitment towards each fund is represented in logarithmic scale as the authors suppose the fund size influences the dependant variables with diminishing marginal effects.

- Commitments in vintage years: It represents the world private equity environment as it is the amount of money committed to private equity funds worldwide. Logarithmic scale accounts for the diminishing marginal effects.

- Diversification: Diversification in terms of size is measured in the number of portfolio companies each fund has. The Herfindahl-Hirschman Index is used to represent diversification across regions, industry sectors, stages of investment.

Figure 4.1 is a snapshot of the summarised results of the analysis. The buyout ratio has a positive influence on the performance of the fund and is an indicator of reduced loss. Both economic trend and development of stock market indicate that there may be a rule of thumb - funds that begin investing in times of prosperity and/or stability tend to perform poorly. The vintage year GDP and vintage year stock market condition have a negative influence on the fund performance. Another general rule that emerges from the result is that the average economic growth and stock market growth during the life of the fund influences its performance positively. In any case, investing at the beginning of a period of financial development will influence the fund positively. It is interesting to notice this in combination with the effects of worldwide commitments to private equity in the year of vintage. If the worldwide investments are on rise, this not only is expected to have a negative influence on the PME of a fund that starts investing in that year but also has a positive influence on the percentage of loss. Finally, The experience of the GP has a positive influence on the fund performance but also on the percentage of loss. It seems that with more experience a GP gets, the chances of loss also increase. Aigner et al. [2008] explain this bewildering result stating that more experienced GP not only have developed expertise in managing funds but also are willing to undertake more risky investments.

Independent Variable	Dependent Variable	Influence
Buyout Ratio	IRR/PME	+
	Percentage of Loss	−
Experience of GP	IRR/PME	+
(log (years) and number of funds)	Percentage of Loss	+
Level of Interest Rates		
(3-months U.S. Treasury Rate)	IRR/PME	−
Economic Trend		
• over Total Lifetime of Fund (GDP)	IRR/PME	+
• in Vintage Year (GDP)	IRR/PME	−
Development of Stock Markets		
• over Total Lifetime of Fund (MSCI)	IRR	+
• in Vintage Year (MSCI)	IRR	−
Log of Fund Size (optimal €24 million)	IRR/PME	−
Commitments in Vintage Year	PME	−
	Percentage of Loss	+
Diversification		
• Number of Portfolio Companies	PME	+
• Financing Stages	IRR/PME	+
• Industry Sectors	IRR/PME	?
• Regions	IRR/PME	?

Figure 4.1: Summary of regression analysis, Aigner et al. [2008].

Welch [2014] attributes the conflicting views regarding private and pubic market co-movements to the accounting practices of the industry. He states that private equity risk characters are based on accounting Net Asset Values (NAV), accounting being the key-word here. This underestimates the systematic risk (β). He identifies that there is a conflict of interest in manipulating reported NAV as it affects fundraising in the short term for the PE firm and in the long term smoothing of returns supports the claim of low risk and diversification characteristics touted by the managers. According to the reformed accounting principles, the valuation should be based on the fair value of the fund and not on NAV. In his study of co-movement of PE funds from Europe and US markets and global capital markets, he concludes that (β) nearly doubles from what one would typically expect and (α) disappears as a result of implementing updated accounting principles. The disappearance of α essentially implies that the risk-return dynamic of the private equity is no different from that of the public stock market.

Boyer et al. [2018] constructed indices of buyout firms using proprietary data of secondary market prices of private equity stakes to measure the risk and returns of private equity investments. They compare these transaction-based indices with the NAV-based indices. The NAV-based indices are developed from proprietary data obtained from information and financial data firms like *Prequin* and *Burgiss*. They are composed of securities of the publicly traded private equity firms. From a comparative study of transaction-based and NAV-based indices, they conclude that private equity is much more correlated to the general public market than one would ex-

pect from just a study of the NAV-based indices. They further state that the alpha (α) of this transaction-based private equity index is not statistically significant, i.e., does not differ from zero. Essentially, their study indicates that investing in private equity is no different from investing in a typical public index fund, with regards to risk-return characteristics.

Robinson and Sensoy [2016] study the cyclicality and performance measurement in private equity and find that there are significant co-movements in public and private markets. They state that the cyclicality of the co-movement is a result of exposure to the same business conditions and investment opportunities.

While some research goes as far as claiming that private markets and public markets move in sync such that systematic risks do not contribute towards the returns, there is no lack of studies that claim just the opposite. Kaserer and Diller [2004] in their study of drivers of returns for European private equity use fund-level cash flow data and compare PME and market-excess IRR of the fund to public market return indicators. Their study is unable to find significant evidence for the returns of private equity to be related to public stock markets. They state that information about investment opportunities travels in the private equity universe at a much slower pace compared to the public market as the private market is not a continuous one. Consequently, the returns in private equity are much more volatile than in public market prices. As the knowledge and skills are differently distributed among different funds, Kaserer and Diller [2004] state and also demonstrate through regression analysis of IRRs of the subsequent funds that subsequent returns run by a management team are correlated. This furthers the claim that returns of private equity funds depend on the skills of the manager or the management team.

Kaplan and Schoar [2005] also find evidence for persistence of fund performance in their study of private equity. The fund managers who have outperformed the industry in one fund are most likely to repeat the success in the next fund. Their study is robust and keeps in check the possibility of induced persistence due to the continuation of investments from one fund to the next. To avoid the problem they have also compared the performance of a fund of a GP with the second previous fund, that is the fund previous to the previous fund.

Kreuter and Gottschalg [2006] conducted experiments on the data of 615 private equity funds from American and European markets to study the effect of different characteristics of a fund manager on the performance of the fund. They also had access to the professional history of all the fund managers involved in the fund operation. They find that there is persistence in the performance of funds that are managed by the same GP. That is the measure of the past performance of a GP's fund is correlated with the performance of the subsequent fund of the GP. There is a strong correlation of fund returns and the manner in which GP conducts the deals, which is measured by the variance in the number of investment deals made during the fund life. Although funds are affected by external factors, performance persistence is driven by GP's ability to generate returns. Their study shows a strong correlation between GP's prior experience measured in the number of funds handled and fund returns.

From a study of the literature, we can conclude that some value determinants are consistently significant and the importance of others is inconclusive. We believe we have enough evidence to consider macroeconomic factors, the economic environment during the vintage of the fund, and the movements of the public market as value drivers in private equity. We have conflicting literature on the role of fund managers in generating returns although the literature on the role of fund managers in generating consistently superior returns is consistent. Siding with the literature that claims no role of fund manager in generating returns is counter-intuitive. In addition to selecting investment opportunities and making timely investments, the fund manager of a private equity fund uses his management skills and his position in the portfolio companies to augment and create value. If we want to include a fund manager's characteristic skills in a model to determine fund performance, we need to study further what constitutes alpha and to what extent it affects the fund's success. The nature of data required for our investigation are

privately owned by data firms. We require sufficient fund-level data that include information on timelines, cash flows, fund manager's professional history, and much more. Due to the lack of funds associated with this project, we are unable to purchase such a database at the moment. Purchasing database will aid this study not only in clarifying the confusion surrounding the role of alpha (α), but also will be instrumental in demonstrating the effect and significance of other value determinants on the private equity funds similar to the one we want to focus on. The database can also be of help in checking the robustness of the model. As we are unable to access the necessary data, further research in the role of manager is out of the scope of our study.

4.2 Experiment

We conduct a broad comparison of private and public markets by running basic correlation tests. We compare the movement of index of publicly listed private equity firms with indices that represent public markets. We then explain why publicly available data falls short in conducting further research into understanding the role of alpha (α) in private equity returns.

4.2.1 Technical Understanding

Correlation is defined as the statistical association of any two random variables. It is an indicator of the degree to which a pair of variables is related. The method to calculate correlation depends on the nature of the data and the relation between the pair of variables.

Pearson's Correlation

The Pearson's correlation method checks for the strength of linear association of two random variables. It is obtained by taking the ratio of the covariance of the two variables in consideration, normalized to the square root of their variances.

Consider random variables X and Y. The correlation coefficient is given by ρ_{XY}. The Pearson's correlation is mathematically defined as per Equation 4.1.

$$PCorr(X,Y) = \rho_{XY} = \frac{cov(X,Y)}{\sigma_X * \sigma_Y} \tag{4.1}$$

Spearman's Rank Correlation

The Spearman's Rank correlation check for the the strength and direction of monotonic association between two variables. The Spearman's correlation between two variables is the same as the Pearson correlation between the rank values of those two variables. Spearman's correlation is typically used to determine correlation for ordinal data.

Consider data sets X and Y. The data are converted into rankings rgX and rgY. The Spearman's correlation is mathematically defined as per Equation 4.2.

$$SCorr(X,Y) = \rho_{rgXrgY} = \frac{cov(rg_X, rg_Y)}{\sigma_{rg_X} * \sigma_{rg_Y}} \tag{4.2}$$

According to Capital Asset Pricing Model (CAPM), the return of a security is a combination of systematic return and unsystematic return. The systematic return is proportionally related to the market return by a factor popularly called beta (β). The unsystematic return is independent of market movements and is represented by alpha (α). Equation 4.3 shows the formula relating returns of a security and the market returns.

$$Return_{security} = \alpha + \beta * Return_{market} \tag{4.3}$$

β is the sensitivity of the returns of a security to the changes in the market. Consider a security S and a market M, such that R_s are the returns on security and R_m are the market returns over

some time 't'. The β of the security S relative to the market M is mathematically defined as per Equation 4.4. The implied α is determined as shown in Equation 4.5.

$$\beta(S,M) = \frac{cov(R_s, R_m)}{var(R_m)} \tag{4.4}$$

$$\alpha = Return_{security} - \beta * Return_{market} \tag{4.5}$$

4.2.2 Data

The data on private equity firms in terms of cash flows, investments are not publicly available. Such data need to be purchased from data firms like *Prequin* which specialize in data and information of the alternative investments world. Due to shortage of funds, such data are unavailable to us at the moment. Instead, we turn to publicly listed private equity firms as a broad representation of the private equity investment class. The private market movements are represented by data of **S&P Listed Private Equity Index**. The index is designed to provide tradeable exposure to the leading publicly-listed private equity companies. This index has been chosen as it comprised of the leading listed private equity companies of the world. The geographical breakdown of composition of S&P Listed Private Equity Index is shown in Figure 4.2. From the figure, it is evident that the leading private equity firms are concentrated mainly in North America and Europe.

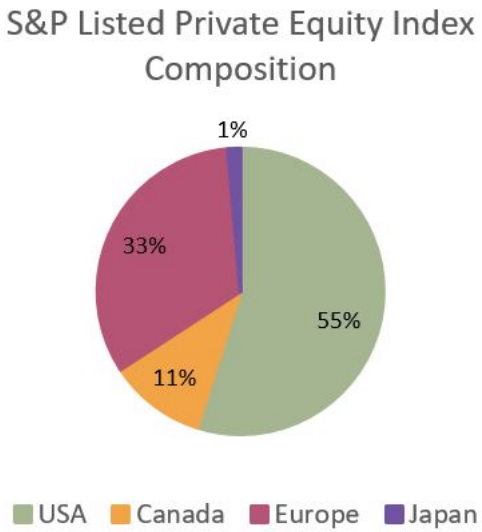

Figure 4.2: SP Listed Private Equity Index: Geographical Breakdown as of May 2020.

The public market movements are represented by data obtained of the different public market indices. In order to check for differences due to effect of different geographical location, we use a global stock market index and several region specific indices. Table 4.1 shown below, gives us an idea of the indices used and the markets they reflect.

Index	Markets
S&P Global 1200	Global public stock market.
S&P 500	Public stock market of the USA.
DJIA	Public stock market of the USA.
MSCI MidCap European Index	Public stock market of Europe.
DAX	Public stock market of Germany.
CAC40	Public stock market of France.
iShares MSCI World Small Cap	Small caps in developed markets worldwide.

Table 4.1: List of indices representing Public Markets.

All data are obtained from the internet as they are available from websites like Standard & Poor, MSCI, YahooFinance and Investing.com. We use data from the year 2010 to the year 2020. Figure 4.3 compares a breakdown of the geographical regions of S&P Global 1200 and S&P Listed Private Equity Indices. The two indices seem similar in terms of geographical composition. 66% of constituents of both are from North America. Europe has a healthy representation in both, although the global index, true to its name and motive is more diverse.

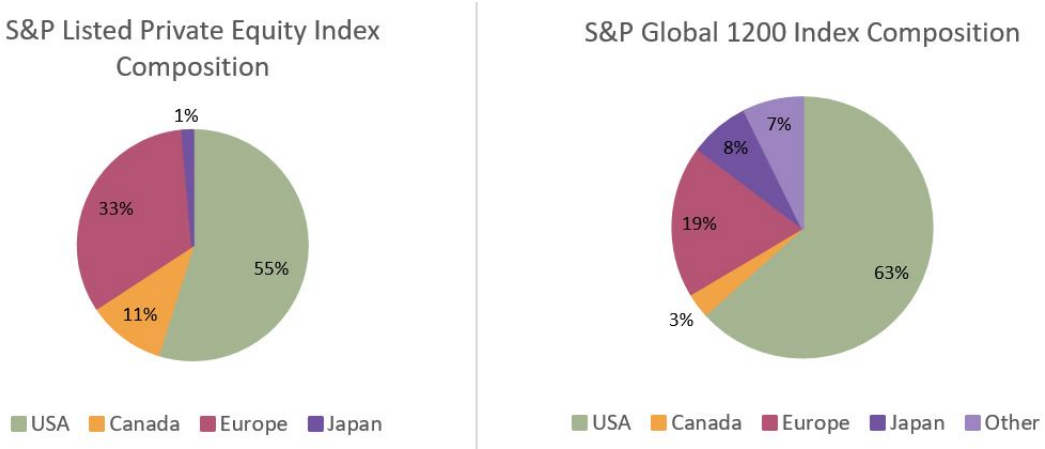

Figure 4.3: Geographical representation in SP Listed Private Equity Index and SP Global 1200 Index.

4.2.3 Statistic

Our objective here is to determine whether the movements in private equity market are associated with the public market movements. Movements in markets are monitored by the value of 'return' on a stock price. We have hence compared daily returns on public and private market data. Suppose P_t is the price of stock P at the end of day t. The return on stock P on day t is given by r_t as shown in Equation 4.6.

$$DailyReturn = r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{4.6}$$

4.3 Result

We find that the experiment we run generates results consistent with the results of Boyer et al. [2018]. The index of publicly listed private equity firm shows a high correlation with the global public index. The β of private equity index is extremely high and the alpha is not significantly different from 0 and is negative. Figure 4.4 shows the plot of daily returns of S&P Listed Private Equity Index vs the daily returns of S&P Global 1200 Index. A clear trend is visible that indicated positive relation. Additionally, the regression line indicates a β of 1.19 and an α of -0.01%.

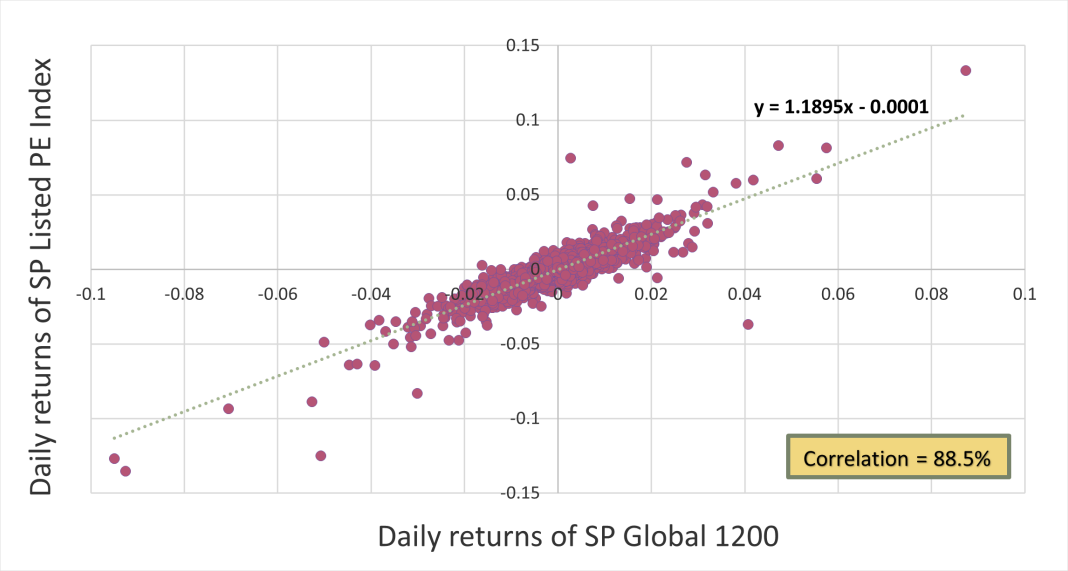

Figure 4.4: Correlation of Listed Private Equity Market and Public Market for data from years 2010 to 2020.

Index	Pearson Correlation	Spearman Rank Correlation	Beta	Alpha
SP Global 1200	88.48%	83.21%	119%	-0.01%
SP500	82.30%	75%	97%	-0.02%
DJIA	80.71%	72%	97%	-0.02%
MSCI European	74.86%	65%	83%	-0.0001%
DAX	71.46%	64%	72%	-0.01%
CAC40	74.14%	66%	73%	-0.0004%
Average	**79%**	**71%**	**90%**	**-0.01%**

Table 4.2: Correlations, β and α of Private Equity Market in relation with Public Market Indicators for data from years 2010 to 2020.

The correlation, β and the implied α of the private equity index with other public indices are summarized in Table 4.2. By comparing the listed private company index to the public market indices, we see a pattern. The correlation, Pearson and Spearman measures, are very high. On an average, the private market is nearly 80% correlated with the public market. This implies a very high average β of 90%. The alpha values as can be seen in the table are negative and practically zero. It is interesting to note here that the private market index shows a higher correlation with the public market indicators of the USA. We should be expecting this result as the S&P Listed Private Equity Index has 55% of its components from the USA market and only 33% from the European region.

4.4 Discussion

An average β of as high as 0.9 creates an illusion which diminishes role of α to the point of insignificance in earning return on investments. This is counterintuitive as it challenges the very fundamental idea of investment in private equity. Private equity returns are believed to be based on skills of the fund manager in selecting portfolio companies; management skills

employed to make investments profitable. Owing to illiquidity and information asymmetry, we have enough rationale to doubt the results of high correlation.

Correlation gives us a broad idea whether or not two random variable move simultaneously in the same direction. Since correlation and causation are not necessarily related, the correlation in public and private markets could be spurious. It is bold to assume the stock value of the listed private equity firms represents private markets. In valuation of private markets, we are interested particularly in the valuation of the underlying assets which are the portfolio companies. The stock price is a representative of the present value of future revenue stream. Using stock prices of listed private equity firms can be problematic and misleading for many reasons. Firstly, the main stream of revenue for private equity firms is the management fees. Profit from investment in underlying assets forms only a fraction of the revenues to be generated. So, the stock price reflects more the financial health of the private equity firm rather than that of its underlying assets. Firms that are funds of funds pose a double layer of ambiguity. Secondly, large private equity firms hold other investments like real estate, traditional stock market funds, hedge funds, etc. It is not realistic to learn what portion of stock price can be attributed to private market investments. There is no reason for listed private equity firms to be impacted any differently by market sentiments than other listed firms. The correlation between the listed private equity firms and the public market indices may simply be a result of general market sentiments.

Cointegration is generally cited as a more reliable regression tool for studying pair-wise movements. Two sets of variables are said to be cointegrated when a linear combination of the said variables has a lower order integration. Cointegration works for non-stationary time series. It is widely used in forex trading. While correlation gives a general idea about overall direction of movement, cointegration helps us identify to what extent variables are sensitive to each other's movements. Stock market prices (P_i) are non-stationary time series but our value of interest, the stock returns (r_i) are stationary time series . By taking the first difference of stock prices, we eliminate the non-stationary effect when we obtain stock returns. This is further confirmed by the Augmented Dickey Fuller Test. We therefore forego the option of using cointegration to study relative movements of private and public markets.

4.5 Summary

To be able to predict private equity performance, we studied the model developed by Takahashi and Alexander [2002] and concluded that we require to model the growth factor in order to achieve our goal. From a study of the literature, we identify a number of factors that drive the growth of private equity market. The five factors that could be used to model the growth rate are listed below.

- Fund Manager
- Interest rates
- Economic trends
- Stock market movements
- Private equity investment trends

Experience of the fund manager in terms of years and number of funds previously managed is an important indicator of the fund's potential. Studies show evidence for consistent returns when managed by a particular manager. Additionally, the probability of a fund of a manager transitioning to a higher performance quartile depends on that fund manager's previous performance. Obtaining authentic information on the fund manager's professional history is a challenge. Another difficulty is to be able identify and convert the return generating skills into numeric value that is the fund manager's α.

The public market seems to have an effect on private market as well. Funds that have vintage year in a period of stock market stability tend to perform poorly as compared to funds that begin investing in times of distress. Stock market movements have a positive influence on the growth of private equity funds during their lifetime. That being said, in order to incorporate this information for modelling growth in PE fund, we need to further investigate into the extent to which these effects are significant.

The economic trends, represented by the GDP of the fund's domicile, have a similar effect on the fund's growth as the stock market movements. Funds that have vintage year in a periods of economic prosperity and higher GDP tend to perform poorly than those that begin investing in a period of economic crisis. During the lifetime of the fund, the trends in GDP positively influence the growth in private equity. This gives us a general idea and to learn specific effect on growth rate, further study is imperative.

The interest rates of the fund's domicile also affect the growth of private equity. Periods of higher interest rates tend to slow down the growth and vice versa.

Finally, as the dynamics of demand and supply would dictate, the trends in investment in private equity is also an indicator of a fund's success. If a fund begins investing in a period when the investment and investment opportunities are scarce, it is more likely to be successful than in period where the market is flooded with investor's money. This is a comparative factor. To be able to model growth based on this factor defining a continuous relation between private equity market investments and growth indicators of the funds is a start.

Fund's growth rates could be modelled by refining each of the five factors. Modelling growth rate of a private equity fund seems out of reach for now. The data required to study the details of fund manager's α are classified. Studying the effect of GDP, interest rates and stock market on private equity calls for access to data on private equity funds which are proprietary and so are the data on private equity investment trends. Due to the major constraint of access to the data, we pause our work on modelling growth rate of funds here. We proceed from a fund-level approach to a portfolio-company level approach to model fund growth in the next chapter.

5 Peer Selection & Valuation

With the central focus on predicting the fair value of the PE investment portfolio, this chapter aims at developing a framework for selecting comparable listed companies for portfolio companies. We assume that we can relate private market companies to public companies as they differ only in ownership status and are similar in their businesses and factors affecting their businesses. We are unable to test the framework developed at this stage because of limited access to resources which is a unique situation due to outbreak of a global pandemic.

5.1 Valuation method

The PE portfolio consists of PE funds which are essentially collections of unique portfolio companies. Figure 5.1 shows the structure of the private equity portfolio. Theoretically, predicting the fair value of each of these portfolio companies and consolidating the result will give the fair value of the whole PE investment portfolio.

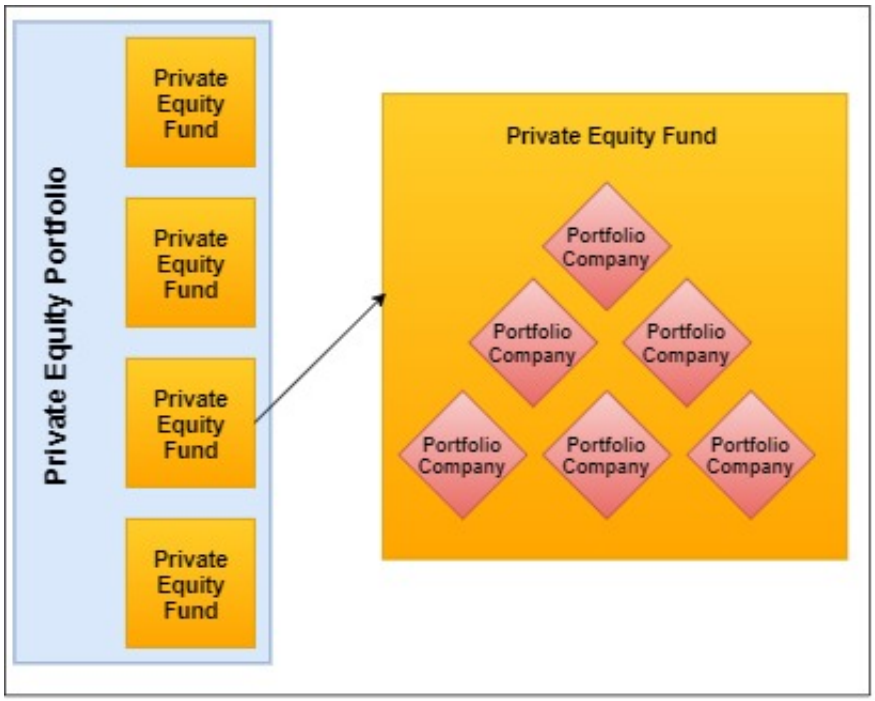

Figure 5.1: Structure of Private Equity Portfolio.

The models developed based on the approaches described in Section 2.5 may each lead to different results and imprecise valuations. This inconsistency is justified by the variety of assumptions that are inherent to each approach and judgement calls made while developing the models.

The market approach of valuation is based on the law of one price, which states that identical assets should sell for same price. The market approach to equity valuation assumes that comparable firms of an industry have comparable valuation multiples. It is a relative method of valuation. It is not uncommon to use different models simultaneously for company valuation and then compare the results. Although the meaning of the term 'comparable' remains undetermined the simplicity of the Market Approach has made it the popular choice in the industry, and it is used by most practitioners as their primary method of stock valuation. Asquith et al. [2005] have conducted a study on the content of equity analyst reports and the effect their

release has on markets. They indicate that more than 99% of reports refer to the valuation multiples where as only about 12.8% of the analysts use present value method. Demirakos et al. [2004] discovered that analysts prefer multiple based models over the present value models for equity valuation.

The main factors justifying the recommendation of the Market Approach are its industry-wide acceptance and the nature of information available. In this study, we deal with private equity from a Limited Partner position. The financial statements are thus inaccessible. This automatically renders the Discounted Cash Flow Method and Asset Based Approach redundant. The market approach requires much less information compared to the other approaches and the information it requires is accessible. We proceed the research by selecting the Market Approach for valuation of portfolio companies.

5.2 Approach for Selection of Comparable Firms

The accuracy of relative valuation models depends on the relevance of the peer groups. Identifying companies with the most identical cash flow projections as the target firm would lead to the ideal peer group of the target firm. This is not only computationally expensive but also defeats the purpose of using the multiples valuation method.

As there can be different opinions on what considers a company comparable to the target company, there is no one-size-fits-all solution to this objective. The method for selecting comparable firms (or peers) of the target firm depends on the degree of accuracy intended and the nature of information available. Broadly speaking, there exist three approaches to determining a peer group of the target firm.

- Industrial Classification Approach : This approach advocates the importance of using Industrial Classification Codes like SIC, GICS, and NACE that segment the industry based on common characteristics shared in the products, services, production and delivery system of a business.
- Valuation Fundamentals Approach : In this approach the peers are selected from the cross-section of the industry based on the closeness of their valuation fundamentals like profitability, growth and risk.
- Co-searched Approach: In this more recently developed approach, the peer groups are based on search traffic patterns on the internet or dedicated websites. It is based on the idea that firms that are co-searched are economically-related.

In one of the early studies to address the confusion surrounding peer selection methods, Alford [1992] analyzed the effect of the choice of the set of the comparable firms on valuation accuracy and found that the valuations are most accurate when the comparable firms are selected on the basis of industrial classification. The choice of comparable firms in his study was based on combination of factors like industrial classification, risk and growth prospects. His study demonstrates that further combining risk and earning growth with industrial classification does not improve valuation accuracy but is just as effective. In a more recent study, Cheng and McNamara [2000] confirmed that industrial classification is most instrumental in defining the group of comparable firms. They found that combining the industrial code with Return on Equity data gives the most accurate valuation for the target firm whose value is unknown.

Bhojraj and Lee [2002] developed a systematic technique to develop a peer group independent of the choice of multiple used for valuation. Each valuation multiple can be replaced by a “warranted multiple" for each firm. The warranted multiples are developed such that they reflect the large sample relation between a firm’s valuation multiple and variables that explain cross-sectional variations in the ratio. The estimated warranted multiple becomes the basis of our comparable firm analysis. They found that this method of peer selection was superior to

the industrial classification methods used in the industry for inferring the value of the target firm.

The studies on developing efficient peer groups fundamentally depend on the industrial classification. Moreover, these studies are not able to capture the effect of cross-industrial economic relations. Lee et al. [2014] challenged the fundamental definition of industry and developed a novel method of identifying economically related firms based on search traffic patterns on the internet. They applied a co-search algorithm to Electronic Data-Gathering, Analysis, and Retrieval (EDGAR) website (provided by the U.S. Securities and Exchange Commission). They found that the firms that are chronologically adjacent in searches by the same individual are fundamentally or economically related. Search based peer groups outperform the groups formed on the basis of industrial classification.

The Valuation Fundamental Approach and the newly developed Co-search Approach, although promising, require access to data that are classified and/or can be purchased from data firms. Given the constraint of limited access to data, these approaches cannot be employed for this project as of yet. Industrial classification is indispensable from the process of peer group selection. The literature supports the idea that a combination of industry classification and fundamental characteristics of the firm is effective for the selection of comparable firms. This project shall proceed to develop a method based on a combination of Industrial Classification and Fundamental Characteristics of the firm to determine group of comparable firms.

5.3 Developing Framework for Peer Selection

In this section, we discuss the development of a framework for selecting comparable firms. To recapitulate, this project aims at predicting the valuation of private equity funds of which privately owned firms are typical the basic unit. Predicting valuation of all units would theoretically lead to predicting valuation of the fund. The investment is in the form of a Limited Partner and restricted access to accounting information does not allow for direct value predictions. The framework being developed here aims to map the privately owned firms to their peers in public market and derive valuations and predict performance by comparable company analysis.

As concluded in the previous section, we shall develop a peer group for the target firm based on Industrial Classification and Fundamental Characteristics of the firm. In his book, Damodaran [2012], Damodaran states that from valuation standpoint, if assets are comparable, it implies that they have similar cash flows, risk and growth potential in addition to being in the same business. We define four dimensions for peer group selection:

1. Industrial Classification Code: The markets around the world are organized by assigning standard industrial codes to different business activities. Matching the industrial code will help us spot companies with identical business activities.

2. Geographical Location: Businesses are affected by the law of the land. Matching the location will equalize the companies being compared on fronts like tax structure, labor market factors, restrictions accounting standards and macroeconomic factors.

3. Risk: The risk associated with a business determines the potential future profit and growth.

4. Growth Prospects: Activities like investment in R&D, expansion to new territories, restructuring of business model affect the potential future profit and growth.

While Industrial Classification and Geographical Location are independent in nature, Risk and Growth Prospects work in combination to identify the most similar peer. The dimensions for peer group selection are discussed below.

5.3.1 Industrial Classification Code

There is a variety of industrial classification codes developed by different governments and organizations based on different criteria like line of business, revenue source, product and market. The industrial classifications under consideration for this project are described below.

1. SIC Code
   The Standard Industrial Classification (SIC) is the earliest industrial classification system. It was commissioned by the government of the USA and developed by an Interdepartmental Committee on Industrial Classification operating under the jurisdiction of the Central Statistical Board with the objective “to develop a plan of classification of various types of statistical data by industries and to promote the general adoption of such classification as the standard classification of the Federal Government.” Most of the the studies on industrial classification analysis use SIC as their basis of research.

   Although still popular in research, SIC is somewhat obsolete and is in the process of being replaced by North American Industry Classification System (NAICS), developed in 1997 by conjoined efforts of the governments of Canada, USA and Mexico, which is an improvement over SIC as it allows for flexibility of definition of industry and is sensitive to changes in emerging markets.

   Both SIC and NAICs are production based and are developed to organize industries and to be able to measure, analyze and communicate results of industry analysis in standardized way. They are not particularly designed for financial and market research.

2. GICS Code
   MSCI Inc.and Standard & Poor’s (S&P) developed the Global Industry Classification Standard (GICS) in 1999. As the GICS methodology describes its aim as “to improve transparency and efficiency in the investment process" it can be said that GICS is developed keeping in mind the needs of finance research. GICS is designed to be market demand oriented in its analysis and classification of companies globally. The philosophy of market based classification is that the company performance and earning patterns depend on the dynamics of the market a company serves as opposed to production based classification which implies that companies producing identical products are economically related. The GICS code is jointly assigned by S&P and MSCI to individual companies on the basis of their annual reports. A team of specialists and finance professionals is responsible for the process of assigning the GICS codes. The GICS System and assignment of GICS code to the companies is revised annually.

SIC employs a 4-digit coding system where as GICS uses 8-digit coding system. Both classification systems have a hierarchical, top-down structure that begins with general characteristics and narrows down to the specifics. Figures 5.2 and 5.3 show examples of classification under the structures of SIC and GICS classification systems.

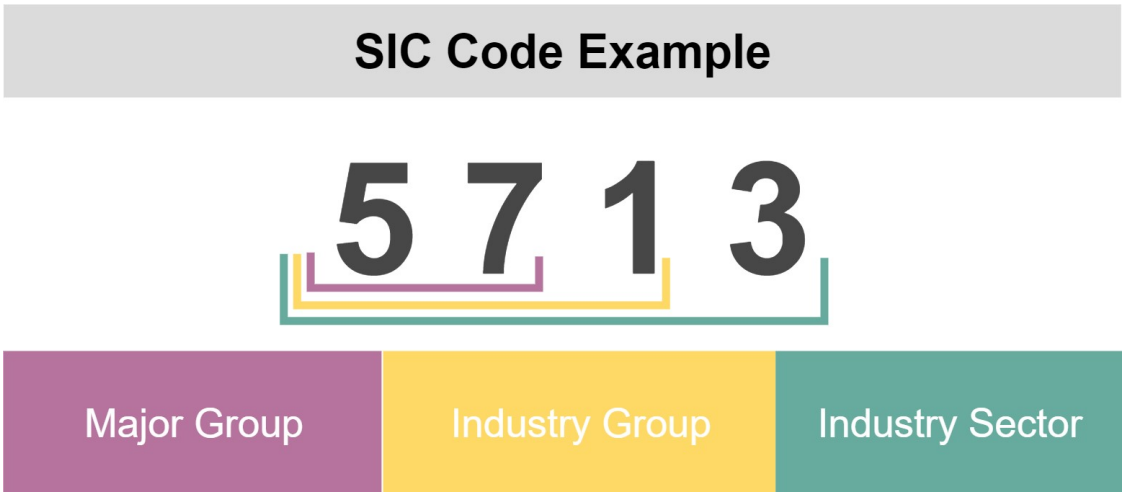

Figure 5.2: Example of classification code according to the SIC Classification Systems.

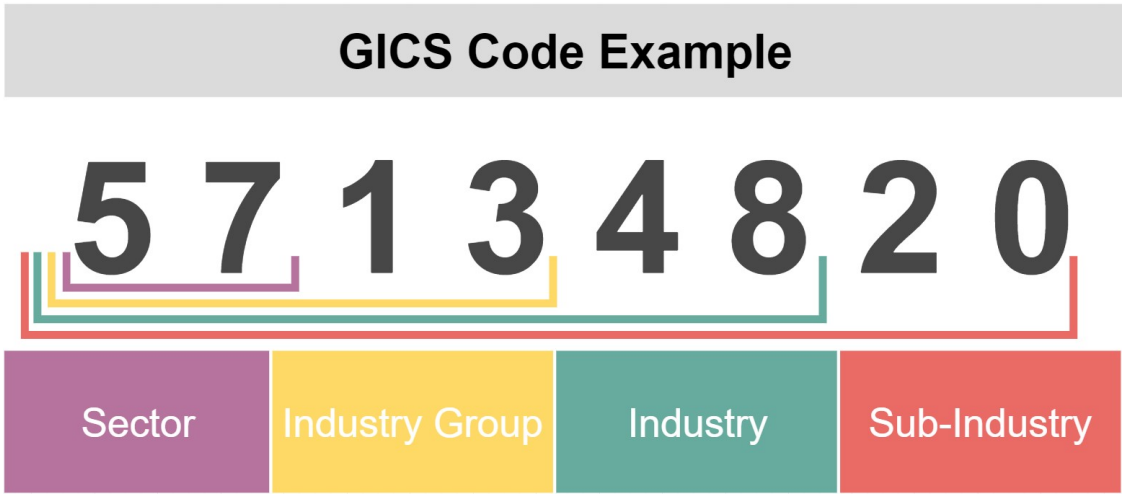

Figure 5.3: Example of classification code according to the GICS Classification Systems.

Choice of Classification System

As GICS is a market based classification, it becomes an obvious choice for developing an index to predict performance of economically related private firms. Moreover, GICS is designed with a global perspective and is more responsive to changes in the market as it is updated annually as opposed to SIC which is updated every five years. Bhojraj et al. [2003] compared four different industrial classification systems for capital market research and found that industry cluster based off of GICS system is significantly better at explaining stock return co-movements and cross-sectional firm-level variation in valuation multiples and forecast growth rates as compared to SIC.

Other market-based industrial classification systems such as the Industry Classification Benchmark (ICB) developed by FTSE International Limited and The Refinitiv Business Classification (TRBC) developed by Refinitive could be considered in place of GICS. It would be required to study the comparison of valuation accuracy when using each of these classification schemes to determine best suited classification system. The industry codes developed by finance data firms are not open source and need to be purchased. Due to monetary limitations, this research is unable to conduct experiments among popular market-based industrial classification sys-

tems to evaluate the efficiency of each on different fronts like valuation multiples, sales growth etc.

In conclusion, in this project we use GICS classification system.

5.3.2 Geographical Location

Geographical location is an important dimension for peer selection as it can account for the different operating procedures, demographics, consumer behaviour, business models, pricing, tax and legal structures companies face based on their location. Moreover the macroeconomic factors of the region like economic growth rates, interest rates and inflation also have an impact on valuation of businesses as we discussed in Section 4, based on results of Aigner et al. [2008]. We select the country of registration as the indicator of geographical location. In the case where there is a lack of sufficient peers from the same country, a peer should be chosen from the same continent.

5.3.3 Risk

Business risk is often associated with the size of the firm. Larger firm tend to be more stable typically because of their history, stable earning patterns, diversification of projects, stable and often proven management team and techniques. Smaller firms on the other hand tend to lack management expertise, internal system controls and typically have a narrow spectrum of projects. Fama and French [1993] conclude in their landmark study of risk factors in stock returns that size and book value-to-market value ratio are sufficient to explain the returns of a stock excess of the market returns.

In finance literature, the size of the company is used to determine the risk profile of the business. Alford [1992] and Cheng and McNamara [2000] have indicated that valuation accuracy is affected by firm size when relative valuation techniques are used. Valuations are more accurate for the larger firms than smaller firms. Bhojraj et al. [2003] indicate the importance of considering size as one of the influencing factors in the valuation process in their study that compares effectiveness of various industrial classification standards in accuracy of multiple based valuation.

Size has been proven to be a reliable proxy for business risk. We use size to incorporate effect of risk while selecting peers for firm valuation. Firm size can be defined in many ways depending on the purpose. Market capitalization, number of employees, enterprise value, total sales, total assets and more can serve as firm size. This calls forth the question - which measure of firm size would be most suit the objective of firm valuation?

In a recent study Dang et al. [2018] identify the issue of lack of research on measurements of size and their effect, and address it by doing the necessary research. They conducted a regression analysis to test the explaining power of different measures of size with regards to representative specifications of executive compensation, board of directors, investment policy, corporate control, financial policy, payout policy and firm diversification and performance. The three measures of size studied are total assets, total sales and market value of equity. Their motivation behind this choice is the popularity of these measures in literature as they learned from a survey of 100 research papers. They found that Total Assets has high explanatory power for majority of the representative specifications. It is more relevant for firm diversification, capital structure and investment policy. Total Sales is an important proxy when one is concerned about dividend policy and cash holdings. Market value of equity performs well on firm risk, capital structure and investment policy.

Total Assets can be a good choice. Total Assets is an indicator for a firm's resources and growth potential, it is a forward-looking indicator of size. Market capitalization, although as successful as total asset, cannot be used to link public and private firms as the comparison would be highly

subjective. Also, the lack of information on private firms will make the use of both - total sales and market capitalization impossible.

'Total Assets' being an item from accounting statements is inaccessible for private firms. So, we suggest 'Net Assets' be used in place of Total Assets as a proxy for firm size which represents business risk. Additionally, Net Asset accounts for leverage and debts, theoretically making it more robust.

5.3.4 Growth Prospects

The valuation of a firm depends on its growth prospects which represent present value of future income. A firm with potential of generating higher future income can be an attractive asset. It is important to base the valuation progression of a target firm on the peers that show similar growth prospects and growth trends. The commonly used indicators of future income include projected values of items from the income statement of the company such as sales, operating income, Earnings Before Interest, Taxes, Depreciation, and Amortization (EBITDA). Our interest lies particularly with the performance of EBITDA as value driver as this is the only value driver that is accessible from the portfolio companies.

Liu et al. [2002] challenge the idea that certain value drivers are more relevant for certain industries. Their study compares and ranks six value drivers in their capacity to provide accurate valuation across industries. We are interested in the performance of EBITDA as compared to other value drivers to check it's reliability. The experiments are conducted on public companies and we assume the result would be valid for comparing firms irrespective of their ownership status. The six value drivers that they compare are enlisted in Table 5.1. In the table we also see the variables that they use as proxy for each of the value drivers. They conducted their experiments on the yearly data collected from the database of COMPUSAT and IBES from the years 1982 to 1999.

Accruals allow managers to reflect their judgement about future prospects. Book value of equity (BV) is linked to the firm value and is of interest while monitoring growth of a firm. Similarly, the trend in growth of sales or revenue over time is an indicator of a firm's growth. Accrual flows encompass for actual earnings data obtained from COMPUSAT and IBES labelled as 'Sales' and 'IACT' respectively. Sum of forward earnings aggregates the separate forward earning forecasts. It represented as ES1, which is the sum of earnings per share (EPS) forecasts for each of the next five years, and ES2 which is the sum of the present value of those forecasts.

No.	1	2	3	5	4	6
Value drivers	Accrual Stocks	Accrual Flows	Sum of forward earnings	Intrinsic value measures	Cash Flow	Forward looking Information
Variables representing the value drivers	BV	Sales	ES1	P1	CFO	EPS1
		IACT	ES2	P2	FCF	EPS2
				P3	MCF	EG1
					EBITDA	EG2

Table 5.1: Value drivers and variables used in the work of Liu et al. [2002].

The intrinsic value measures are based on residual income approach, by which the value of a firm is the sum of present book value of equity and present value of future residual incomes. The variables P1 and P2 represent the intrinsic value when the projections of residual incomes are considered for up to five years out in the future. P1 assumes a constant residual income after year 5 and P2 assumes it to be 0. P3 considers forecasts for 12 years ahead and assumes a linear

growth in profitability from year 3 onward. Cash flows are difficult to manipulate and hence may provide a more accurate representation of growth of a firm. The cash flows measures used are cash flow from operation (CFO), free cash flow to debt and equity holders (FCF), maintenance cash flow (MCF) and EBITDA. Forward looking information plays out in two approaches. One approach considers P/E ratio as the forward looking indicator of value creation. Accordingly, EPS1 and EPS2 are the mean earning forecasts over the next 1 and 2 years respectively. The other approach considers the PEG ratio as the forward looking indicator of value creation. The earning-growth combinations EG1 and EG2 combine EPS2 and a long term mean earnings growth rate *g*, such that EG1 = EPS2(1+*g*) and EG2 = EPS2**g*.

In their analysis, Liu et al. [2002] follow the traditional ratio representation where price of a firm i in the year t (p_{it}) is directly proportional to the value driver of the firm in the same year (x_{it}) as indicated in Equation 5.1 where ϵ_{it} is the pricing error. They estimate the value for β_t by applying the condition that expected value of pricing error to be zero. This is a harmonic mean estimate of β_t according to Equation 5.2. The pricing error is as shown in Equation 5.3. To evaluate performance of the value drivers, they examine measures of dispersion of the distribution of the pricing error.

$$p_{it} = \beta_t x_{it} + \epsilon_{it} \tag{5.1}$$

$$\beta_t = \frac{1}{\mathbf{E}[\frac{x_{it}}{p_{it}}]} \tag{5.2}$$

$$\frac{\epsilon_{it}}{p_{it}} = \frac{p_{it} - \hat{\beta}_t x_{it}}{p_{it}} \tag{5.3}$$

Their results are summarized in Figures 5.4, 5.5 and 5.6. In Figure 5.4 we can see the chart derived from a histogram for pricing errors for selected value drivers. The column width for the histogram is 0.1 (or 1% of price). For example, for EPS2, the fraction of the sample with pricing error between 0 and -0.1 is about 18%.The graph of EPS2 has thin tails and a tapered and pinched top. This means, the probability of a very small error is high but a probability of error being large is very small when EPS2 is the value driver. EBITDA is able to predict the valuations within 20% of observed prices for almost 40% of firm years.

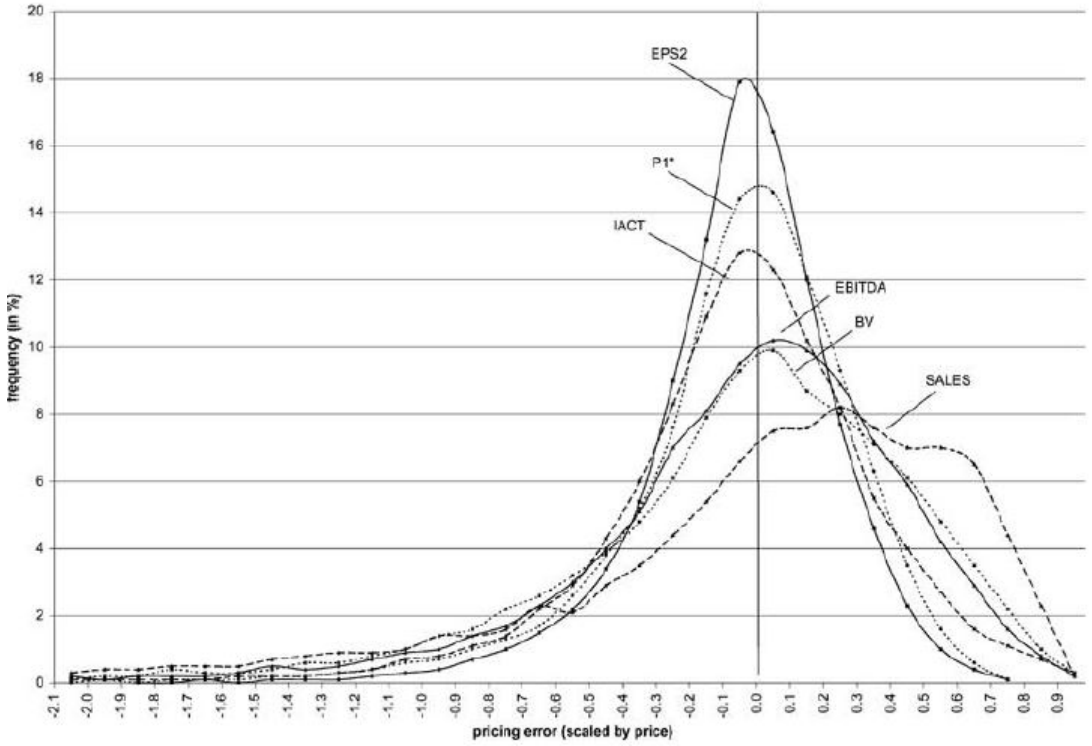

Figure 5.4: Pooled distribution of pricing error, Liu et al. [2002].

For comparing the value drivers across industries, Liu et al. [2002] pooled the valuation results for each industry and ranked the value drivers based on interquartile range for pricing errors. Figure 5.5 is a record of number of times each a value driver was ranked first, second and so on while testing across 81 industries defined by IBES data. For example, EPS2 ranked first in results of 66 industries, second in 11 industries, third in 2 industries and fourth in 2 industries out of a total of 81 industries evaluated. The value driver of our interest, EBITDA, ranks fourth best for 44 industries and fifth best for 21 industries out of 81 industries studied.

	rank					
	1	2	3	4	5	6
SALES	0	0	2	5	14	60
BV	1	3	5	17	41	14
EBITDA	1	1	10	44	21	4
IACT	0	12	53	11	3	2
P1*	13	54	9	2	2	1
EPS2	66	11	2	2	0	0

Figure 5.5: Performance across industries, Liu et al. [2002].

We see next that there is evidence for consistency of the rankings of value drivers over time. Figure 5.6 charts the interquartile ranges for distribution of pricing errors for each value driver over a period of 17 years from 1982 to 1999. EBITDA has fourth least interquartile range for pricing errors and is consistent over the years.

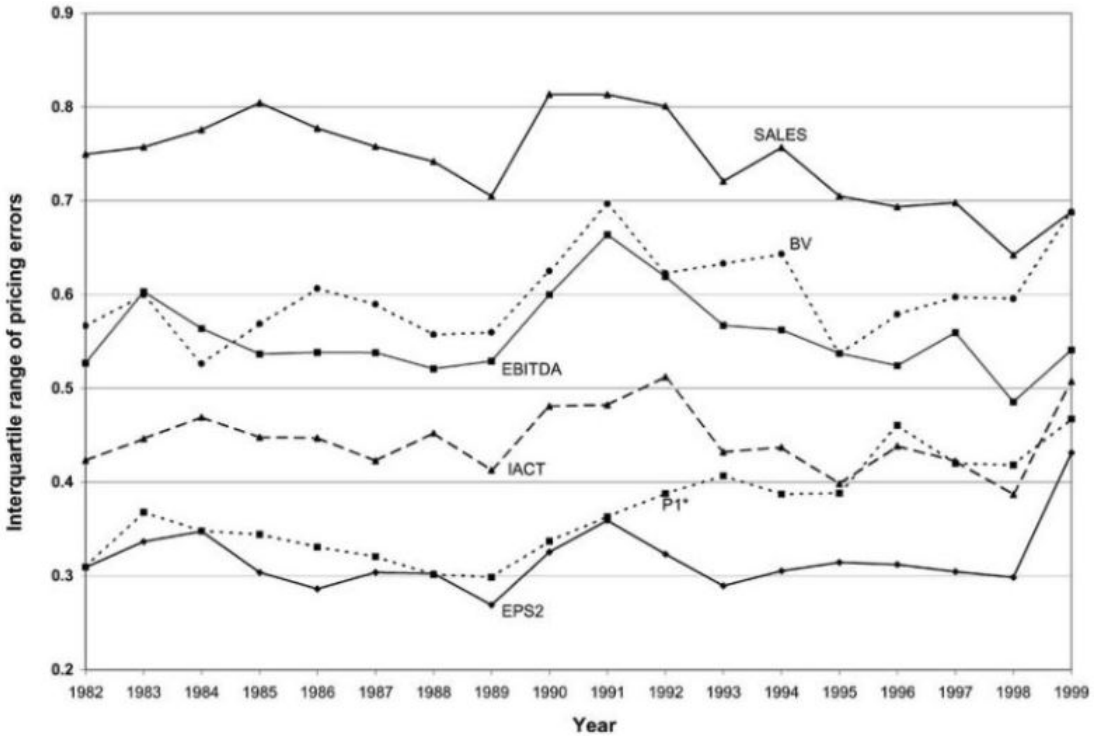

Figure 5.6: Performance across years, Liu et al. [2002].

Furthermore, Liu et al. [2002] confirm that forward earning estimations provide superior results as compared to historical measures and that the accuracy improves as the forecast horizon lengthens. Lie and Lie [2002] independently conducted similar experiments where they grouped comparable firms and checked the actual valuation of the firms against the valuation derived from the median of value drivers for the group. They also indicate that forward measures of earnings are able to give more accurate valuations. The result that forward looking measures are more useful in forecasting firm valuation comes as no surprise since the present value of an asset is the present value of its future cash flows and forward looking measures involve analytical projections of future cash flows.

Combining the two studies, we consider the trend in forward looking EBITDA for comparing growth prospects of the target firms with their peer. The data available on target firms are stale in time, so we must be careful to compare EBITDA values of target firms and database from the same time frame. Strength of linear correlation of EBITDA trends adjusted for time shall determine the suitability of a public company for being the peer of a target firm. Since the index developed is dynamic and can be updated as frequently as each quarter, it is unnecessary to assume that the trends and correlations of EBITDA values will remain the same.

5.4 Measurement of Valuation Multiple

As discussed in Section 5.3.4, the value driver we follow is EBITDA. Valuation multiple of the portfolio company will be based on the average NAV-EBITDA ratio of the market determined by the peer group. We could use simple arithmetic mean, geometric mean, harmonic mean, median or even weighted mean as our measure of average for the valuation multiple. Naturally, the use of each will have a different impact on equity valuation as each process treats data differently. To illustrate the effect of choice of measurement, we look at the panel in Figure 5.7. PE ratio of five random chemical firms is used to determine the average PE valuation multiple for the peer group. Consider a firm that has earnings of 10 units. If simple arithmetic mean is the measure of the industry average valuation multiple, the valuation of our firm would be

$23.6 * 10 = 236$. If harmonic mean is used instead, our firm will be valued at 136 instead. In this section we explore the most suitable measure of the average for valuation.

	ALB	WLK	HUN	PX	FF	Total
Historical PE (2015)	39.5	8.1	43.6	20.0	6.9	
Size (total assets in $M, 2014)	5,161	5,214	10,567	19,704	579	41,225
% Size	12.5%	12.6%	25.6%	47.8%	1.5%	100.00%

Panel B: Multiple Averages Calculation

Simple Arithmetic Mean	$AM = \frac{\sum_{i=1}^{n} Multiple_i}{n}$	$AM = \frac{39.5 + 8.1 + 43.6 + 20.0 + 6.9}{5} = 23.6$
Harmonic Mean	$HM = \frac{n}{\sum_{i=1}^{n} \frac{1}{Multiple_i}}$	$HM = \frac{5}{\frac{1}{39.5} + \frac{1}{8.1} + \frac{1}{43.6} + \frac{1}{20.0} + \frac{1}{6.9}} = 13.6$
Geometric Mean	$GM = \sqrt[n]{\prod_{i=1}^{n} Multiple_i}$	$GM = \sqrt[5]{39.5 \times 8.1 \times 43.6 \times 20.0 \times 6.9} = 18.1$
Weighted Average (by size)	$WA = \sum_{i=1}^{n} w_i (Multiple_i)$	$WA = (0.125 \times 39.5) + (0.126 \times 8.1) + (0.256 \times 43.6) + (0.478 \times 20.0) + (.014 \times 6.9) = 26.8$
Median	Central Position	20.0

Figure 5.7: Averages of valuation multiple using different methods, Plenborg and Pimentel [2016].

Baker and Ruback [1999] conducted the first dedicated study of comparative analysis of the performance of different measures on S&P500 data to challenge the then existing notion of the median being the best valuation statistic as it eliminates the effects of outliers. Their study compares simple arithmetic mean, harmonic mean, median and weighted average measures of valuation statistic. They strongly advice in favour of using harmonic mean and further emphasise that use of harmonic mean will eliminate the risk of overestimation which is inherent to the usage of simple arithmetic mean. Liu et al. [2002] conducted a comprehensive research on equity valuation using value drivers. They mention superiority of results when the harmonic mean is used as opposed to simple arithmetic mean or median as one of their important findings. Herrmann and Richter [2003] indicate in their study, the arithmetic mean overestimates the market price where as the harmonic mean underestimates it. The median measure performs better than harmonic mean in estimating the market price. This results contradicts the findings of Liu et al. [2002]. Further investigation reveals that Liu et al. [2002] had eliminated outliers from their sample. When Herrmann and Richter [2003] trim the 1% extreme values in valuation multiples and relevant fundamentals at both ends of the distribution in a similar attempt to delete observations with extreme values, both studies are consistent in favouring the use of harmonic measure for valuation multiple as well.

As the leading studies agree on the use of harmonic mean for accurate valuation, we choose harmonic mean as our measure of the valuation multiple.

5.5 Summary

In this section we dealt with the private equity portfolio at the portfolio company level for valuation and value prediction. We developed a framework for creating an index that would represent the private equity portfolio in the public market with the aim of equity valuation. We choose the market approach of valuation which has the inherent assumption that the market is correct on average. The market approach values a company by comparing the fundamental

ratios to the market average. We then proceed to select an approach for determining comparable companies or peers. We came across the most widely used approach that is the industrial classification method, where in the companies are organised according to their business models. Another approach is to group firms based on their valuation fundamentals. These two approaches are most commonly used in the industry depending on the problem statement. We also learn about a novel approach being developed which groups firms according to a co-search algorithm in which the companies that are searched consequently on the websites for the database are believed to be similar. Although it is interesting to consider the new approach, we develop a conservative framework and choose to combine industrial classification and valuation fundamentals to define a peer group. We identified four dimensions for peer group selection as Industrial Classification, Geographical Location, Risk and Growth Prospects. We then chose the indicators of each dimension based on our analysis of existing literature. Figure 5.8 enlists the dimensions and their indicators.

Dimension	Indicator
Industry Code	GICS
Location	Country
Risk	Net Assets
Growth Prospects	Trend in EBITDA

Figure 5.8: Dimensions for the development of index and their indicators.

Lastly, we discussed the measurement of our valuation multiple. EBITDA is our value driver and NAV-EBITDA ratio of the portfolio company will be compared to the average ratio of its peers. We addressed the issue of effect of the choice of averaging method on valuation. We chose harmonic mean as it is a conservative measure and eliminates the risk of overestimation.

When each portfolio company is represented by a group of public peers, we shall have an index that can be used for valuation of our private equity portfolio. In the next section we focus on creating a heuristic for defining an index representing the PE investment portfolio in the public market. The index developed is intended to predict the value progression of the portfolio companies. A successfully developed index that is able to link the PE investment portfolio with the public markets will be instrumental in predicting the valuation of the investments in the PE investment portfolio.

6 Index Development

We have discussed the dimensions for peer selection and the valuation process of a company once the peer group is determined. In this section, we develop an heuristic for automation of peer group selection for each portfolio company and hence the index representing private equity portfolio in the public market. The dimensions for our peer group selection process are Industrial Classification Code, Geographical location, Risk and Growth Prospects. Information for the public companies on these dimensions is available on databases like Bloomberg and Compusat. For the portfolio companies, all the information is derived from the quarterly reports, except the Industrial Code. The current practice is to manually assign an Industrial Classification Code to the portfolio companies to track investments made in different industries. Assigning of the codes when done manually is prone to errors and will negatively affect the peer group selection and eventually the valuation. We suggest a heuristic to develop a program that is able to assign the Industrial Code minimising human dependency and judgemental errors.

6.1 Algorithm for Peer Selection

In this section we discuss the heuristic by which the peer group for each target firm can be selected based on the dimensions previously discussed. The four dimensions - Industrial Classification, Geographical Location, Risk and Growth Prospects, are not of the same kind. Industrial Classification and Geographical location are independent and categorical. Risk and Growth Prospect, on the other hand, function in combination and there can be a spectrum of degree of similarity. Figure 6.1 shows the flow of steps in formulating the peer group.

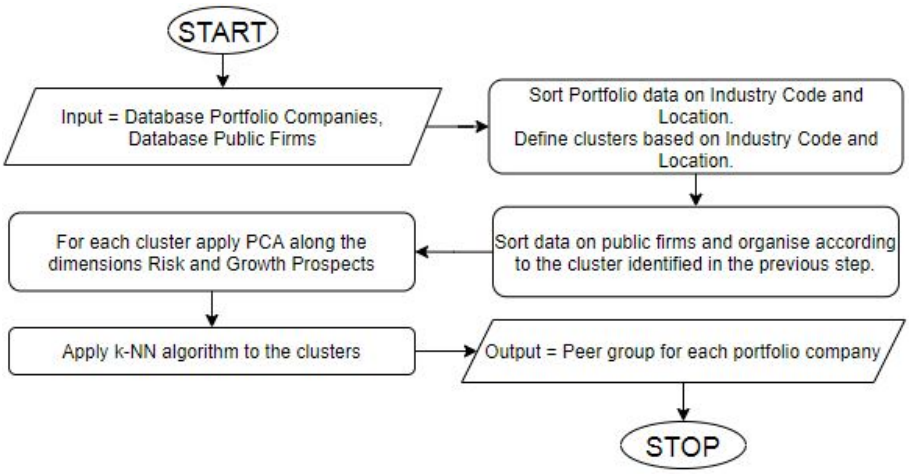

Figure 6.1: Algorithm for generating peer group for portfolio companies.

When we prepare our data, we need to organise the information on our private equity portfolio. We assign an Industrial Code to each Portfolio Company. We also assign the tags for geographical location, that is country and continent to each portfolio company. We sort the data and create clusters of Industry and Location combination. Figure 6.2 gives a visual representation of the groups in which portfolio companies are sorted before we proceed to the next steps. The 15 portfolio companies in Figure 6.2, have an industry code, which is either blue or yellow and are are from countries red or green. Consequently, they can be organised into 4 clusters that can be identified by Industry Code and Location.

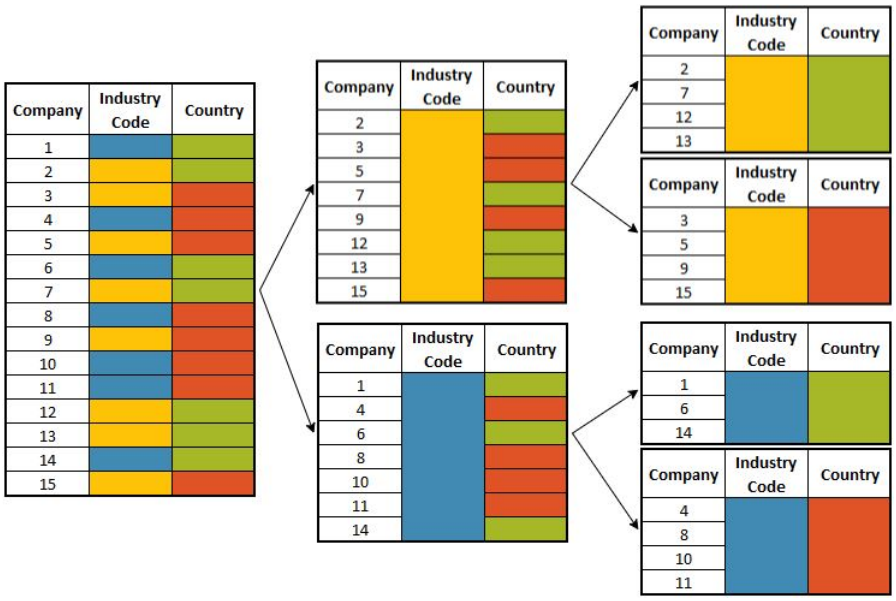

Figure 6.2: Portfolio Companies sorted and organised according to Industrial Classification Code and Geographical Location.

Similarly, we organise the data on public firms to correspond to the clusters formed by portfolio companies. We filter the public firms first on the basis of Industrial Code, and then on the basis of Geographical Location to create intermediate peer group for each portfolio firm. As mentioned in Section 5.3.2, we want to match location by country of the company. In case there are no public peers with the same country, we match continent of the companies. Figure 6.3 shows an example of an intermediate peer cluster for a cluster of portfolio companies.

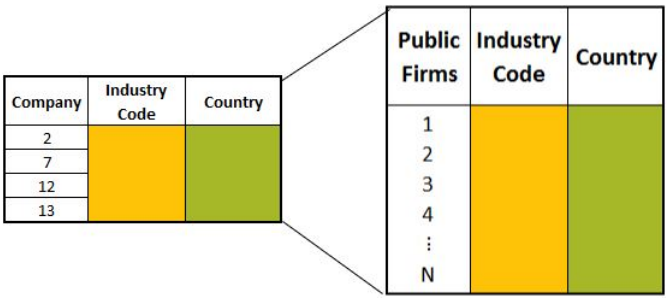

Figure 6.3: Intermediate peer cluster for further refining process.

The intermediate clusters of peers and portfolio companies shall now be refined on the basis of risk and growth prospects by employing Principal Component Analysis (PCA). Finally a k-nearest neighbour (k-NN) algorithm will generate the list of peer group for each portfolio company. Figure 6.4 is a visual representation of the intended way of applying the k-NN method. Each circle represents a company that has identical Industrial Classification Code and Geographical location. The yellow circles represent the public companies and the purple circles represent the portfolio companies. The data point for each company is plotted along the axis of Growth Prospects and Risk.

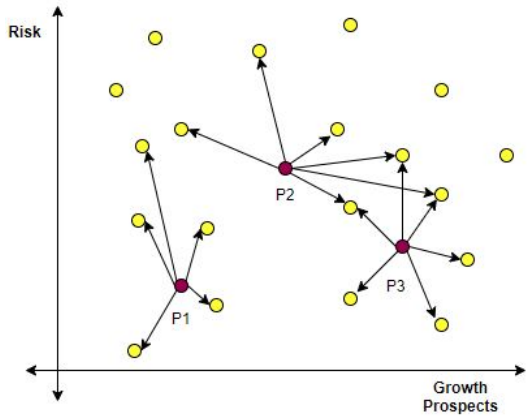

Figure 6.4: Finding peers for portfolio companies by k-NN method.

Cooper and Cordeiro [2008] conducted experiments in optimal equity valuation using multiples especially to determine the best size of the set of comparable firms. In their research, they selected peers based on industry and long-term growth forecasts from the IBES database. They find that the number of comparable firms should be at least 5 and not exceed 10 to ensure a minimum error in measurement and bias. We therefore recommend to limit 'k' in the k-NN algorithm to 5 at the least and 10 at the most.

6.2 Assigning Industrial Classification Code

The industrial classification system of this project, GICS, is a market based classification system. Market based industrial classification maps company performance and earning patterns of a company to the dynamics of the market they serve. Assignment of industrial code according to the GICS methodology is a complex process that requires professional expertise and access to the company's revenue patterns according to the methodology explained in the methodology section on the official website of GICS Methodology, the firm that has developed the GICS system. Lack of information on portfolio company, misunderstanding of the description of the business, inadequate knowledge of GICS methodology and lack of professional expertise may lead to incorrect assignment of GICS code to the portfolio company. Incorrect assignment of industrial code to the portfolio company will have far-reaching consequences and will fail to develop a group of 'peers' for the portfolio company. The industrial classification codes for public companies are available on data platforms but we need to assign the industrial classification code for the portfolio companies on our own. As of now, the assignment of industrial codes to the portfolio companies is done manually by the Limited Partner. We run the risk of incorrect assignment of industrial classification code to the portfolio company. To mitigate this risk we suggest the use of machine learning for assigning the industrial code to the portfolio companies. Since we are following GICS system, the portfolio company should be assigned a GICS code based on the GICS methodology. Although we do not have access to relevant information for this, like the revenue patterns of portfolio companies, we could make use of descriptive information available on the portfolio firm to make an educated guess about its industrial classification code.

Shown in Figure 6.5 are a few examples of typical textual descriptions of portfolio companies. The examples of description are created based on the information in the quarterly reports of the private equity portfolio studied. The General Partner (GP) provides us, the Limited Partners (LP) with the quarterly reports. Since there is no standard format for creating reports, the descriptive text may not be uniformly rich in information.

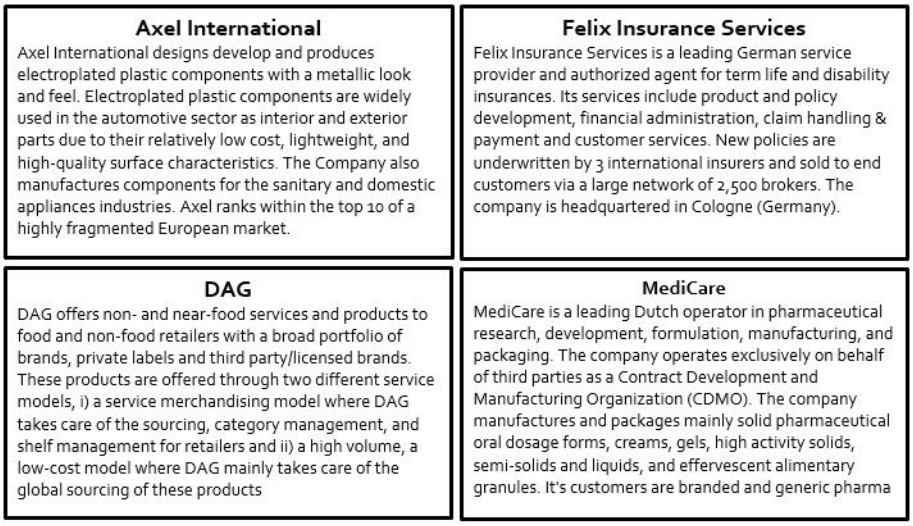

Figure 6.5: Sample of descriptive texts for Portfolio Companies.

In Figure 6.6, we can see the description of a few of the GICS code and Figure 6.7 is an actual of description of a public company on Bloomberg database.

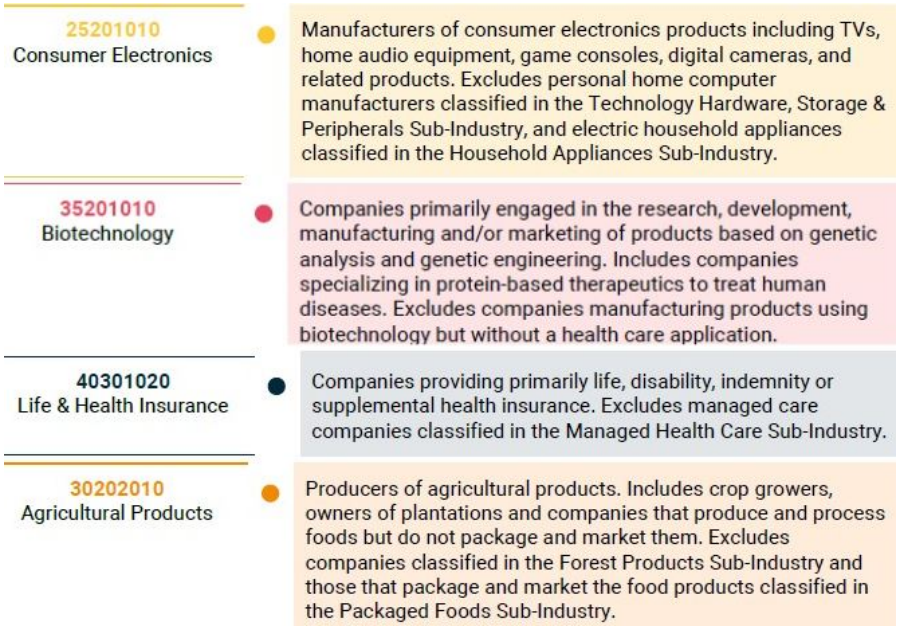

Figure 6.6: Sample of descriptive texts for GICS Code, GICS Methodology, (www.msci.com/gics).

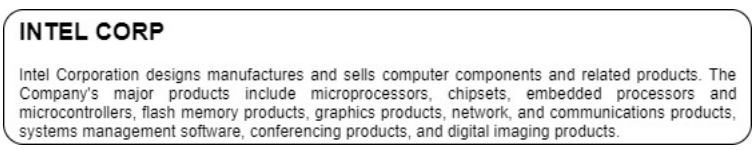

Figure 6.7: Sample of descriptive texts for public firm on Bloomberg Terminal, Guide to using Bloomberg, Damodaran, (http://pages.stern.nyu.edu/ adamodar/).

To automate and standardize assigning industrial codes, we suggest an algorithm. This step can be done by developing appropriate text comparing programs. We intend to compare the descriptive text of a portfolio company with the description of each GICS code. Based on the

degree of similarity, we assign a ranking to each GICS code for that company. We finally update the database for portfolio companies by assigning the code for which the descriptions have highest degree of similarity. Figure 6.8 shows a basic outline of desired steps and results. The accuracy and reliability of assigning industrial codes by this method will depend on the program developer's understanding of the GICS methodology.

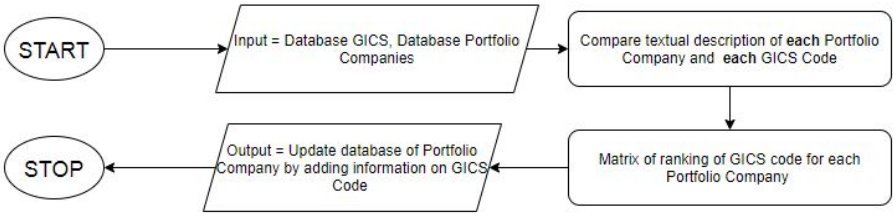

Figure 6.8: Algorithm for assigning a GICS code to Portfolio Companies.

6.3 Summary

In this section we developed a heuristic for selection of peer groups for each of our portfolio companies. We had previously discussed the parameters to consider while selecting peers and now we have also developed a framework to apply the parameters. The parameters of Industrial Code and Geographical location are used in combination to filter out data and sort the data to formulate clusters. Once the clusters are formed, we organise the data within each cluster along the dimensions of risk and growth prospects. We then intend to apply the k-NN algorithm to each cluster to find the nearest neighbours for each of the portfolio companies. We want to rely on the research of Cooper and Cordeiro [2008] which suggests that there is no improvement in valuation accuracy by considering less than 5 or more than 10 peers. We want calibrate the k-NN algorithm accordingly such that the for each portfolio company we have at least 5 and at the most 10 peers.

Currently our model of the index has only two continuous dimensions - Risk and Growth Prospects. With just two dimensions, the use of PCA is not essential. We still want to include it in our plan for implementation as a good practice. As the project progresses, we could expect more dimensions to be added to the index and hence the peer group formulation. PCA would be of help for dimensionality reduction.

Further, we address the issue of assigning Industrial Code to the portfolio companies. Presently, the industrial codes are assigned to the portfolio companies manually by the Limited Partner. Assigning industrial code requires in-depth understanding of the methodology of the classification system and of the revenue streams and target market of the portfolio company. Limited Partners have limited information about the portfolio companies and are blindsided. Assigning industrial codes manually is prone to errors due to lack of information and lack of expertise. Assigning incorrect industrial codes pose serious valuation risk. We suggest a method to overcome this risk by using text comparing programs to assign industrial codes to the portfolio companies.

7 Conclusion

In this project, we dealt with the issue of blind spots in private equity. The sector operates from behind a veil of secrecy and is vulnerable to manipulation of returns. We explore the issues faced by a Limited Partner, the NN Group as an investor in private equity. The company invests in private markets through various private equity funds. Being a limited partner, NN receives the quarterly reports from the fund managers with a lag of 45 days. This leads to inefficiencies in financial reporting as stale values are featured in the statements for private equity portfolio. Moreover, the valuation is subjective to the opinions and biases of the fund manager. NN is thus exposed to valuation risk. We have developed a framework for valuation of a private equity portfolio backed by research. A private equity portfolio is a collection of investments in private equity funds, which in turn are each collections of investments in private firms. We thus considered valuation at two levels - the fund level and the portfolio company level. Our results can be extended to track valuations of investments made not only in private equity but in all asset classes. The framework rests on foundation of sound research work and can be used to meet all kinds of valuation requirements.

7.1 Valuation at Fund Level

At the fund level we suggest using the 'Illiquid Alternative Asset Fund Model' developed by Takahashi and Alexander [2002]. The model when tested, fits the historical data from Yale University's investment portfolio. It is simple and based on a strong theoretical basis which is widely accepted in the profession. It is able to incorporate and respond to real time changes in asset values and capital flows. It is sensitive to changing economic environments and it is applicable to a variety of asset types. The simplicity and flexibility of the model make it an ideal choice for our project. Although our focus is predicting the NAV of private equity portfolio, this model is a robust tool that could additionally be used for a variety of financial analyses. It can be used to model cash flows, scenario analysis and analyse the effect of new commitments. The model allows us to predict the NAV of the private equity fund according to Equation 3.5, which is repeated below.

$$NAV_t = [NAV_{t-1} * (1+G)] + C_t - D_t \tag{3.5}$$

As we predict the NAV value by the end of the quarter, all the variables of the equation are known, except the growth rate 'G'. So, modelling fund valuation is theoretically narrowed down to modelling the growth rate of the fund. We then research the factors that determine growth in private markets.

The leading research on growth in private equity identify interest rates, economic trends, stock market movements and private equity investment trends to influence the performance of PE funds. A major disagreement exists when considering the effect of the fund manager on the fund's performance. Although some researchers have opposing views, sophisticated investigations using proprietary data have been able to provide strong evidence for the persistence in performance of a fund manager and correlation between fund returns and management style. We do not have access to proprietary data so we compared the Listed Private Equity Index with other public market indices to be able to generalise an overall α. We find absurdly strong correlation between the markets. This goes against common sense as private equity has much more information asymmetry and the involvement of the fund manager in its conception and growth cannot be overlooked.

From the study of multivariate analysis of the PE fund performances we identify five factors that should be used to model growth of PE funds. The factors are listed below.

- Fund Manager (α)
- Interest rates
- Economic trends
- Stock market movements
- Private equity investment trends

We require further analysis into each of these factors. We need to study the extent of effect each has on the PE fund's growth and develop a model for the growth. Defining each of the five factors mentioned above calls for further research. Access to the necessary data would allow for experimentation and trials to test different methods and learn from those. As we could not access the necessary data, we pause the development of model at the fund level here and move on to the next approach. We suggest this as a starting point for future researchers in this area.

7.2 Valuation at Portfolio Company Level

In this approach, we evaluate the portfolio companies. These are the private firms in which we have invested via PE funds. As we are the Limited Partners, we do not have access to the financial statements of the portfolio companies and must rely on the information provided by the General Partner, that is the fund manager. As a result, the Discounted Cash Flow method and more sophisticated analysis such as Option Pricing is not possible from a Limited Partner standpoint. Given the conditions, we are able to use only a Market Multiple valuation approach for the portfolio companies. Comparable company analysis is the most popular valuation method used in the industry and has its advantages of being simple, fast and reliable. The inherent assumption in this method is that the markets may be wrong on pricing individual companies but are correct on average. Success of this method depends on how accurately similar firms are selected for valuation. We developed a framework for peer group selection. We use a combination of industrial classification and valuation fundamentals for developing the selection approach. More recently developed approaches include monitoring internet traffic on dedicated websites and connecting companies that are searched together. We are skeptical about this approach as the assumptions are rather lofty. This approach assumes the internet traffic is rational, moreover the success of such an approach depends highly on the details and quality of the algorithm developed. Combining industrial classification and valuation fundamentals, we recognise from the literature the 4 factors to match for comparing assets for valuation to be industrial classification, cash flows, risk and growth potential. As we do not have access to the cash flows of portfolio firms, we do not pursue it. Instead we have added another dimension, that is the geographical location. We assume that cash flows of a firm are affected most by demand, law and tax regulations and other external macroeconomic factors which can be equalised by matching the geographical location. After a careful analysis of the literature and the information on portfolio companies available for comparison, we define 4 dimensions as shown in the Figure 5.8 based on which comparable peers should be selected for valuation of a portfolio company.

Dimension	Indicator
Industry Code	GICS
Location	Country
Risk	Net Assets
Growth Prospects	Trend in EBITDA

Figure 5.8: Dimensions for the development of index and their indicators.

Once a portfolio company's peer group has been selected based on the four dimensions, we evaluate the portfolio company by equating its the valuation fundamental ratio to the average of the peer group. The valuation fundamental of our choice is the EBITDA-NAV ratio. It is the only ratio available to compare due to information asymmetry between LP and GP. Nevertheless, literature supports the use of EBITDA to be effective for valuation. Finally, we analyse the literature on suitable measures of average for the valuation ratio from the peer group. Using different averages like simple mean, median and harmonic mean give different valuations. From literature we learn that the use of harmonic means and median generate more accurate valuation predictions as opposed to simple mean. We recommend using the harmonic mean.

7.3 Index Development

After defining the dimensions for peer selection we created an algorithm to be used. Automating the process will make it flexible and reliable to an extent. The portfolio companies and the public firms are first sorted into clusters by matching their industry codes and locations. The data in each cluster are plot along the dimensions of risk and growth prospects. The k-NN algorithm helps us filter at least 5 (at most 10) comparable companies for each portfolio company. Figure 6.1 shows the algorithm for peer group selection.

Once we have peer group for each portfolio company, we can calculate peer average of the comparable valuation multiple, that is EBITDA-NAV ratio.

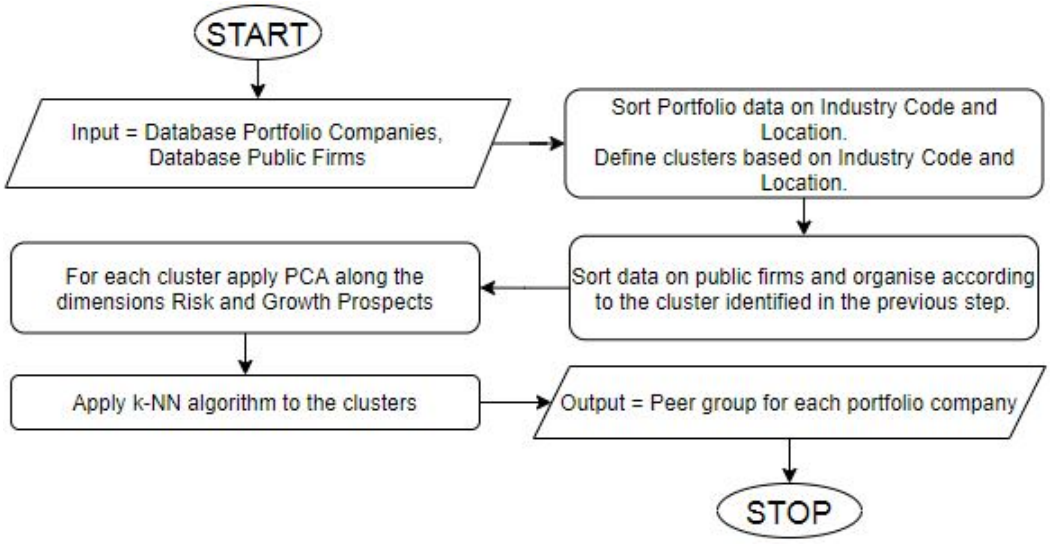

Figure 6.1: Algorithm for generating peer group for portfolio companies.

We noticed that assigning industrial classification codes to the portfolio company is currently done manually by the Limited Partner. Inadequate knowledge of the methodology of assigning codes will lead to erroneous valuations. Having a correct industrial code is crucial for the success of the valuation index. This can be ensured either by being attentive to this step or by automating it. For automation, we suggest an algorithm for assigning industrial codes to the portfolio companies. The success of the algorithm shall depend on the program developer's understanding of the methodology of the classification system. It is worth exploring the option of automating the assigning of industry codes as it will definitely reduce dependency on personnel, and will not be affect by the change in personnel. Figure 6.8 shows the algorithm for assigning the industrial classification codes to the portfolio companies. This is an outline of the process that we recommend. Text comparison is out of our purview and we must consult the relevant people to check the feasibility of this suggestion.

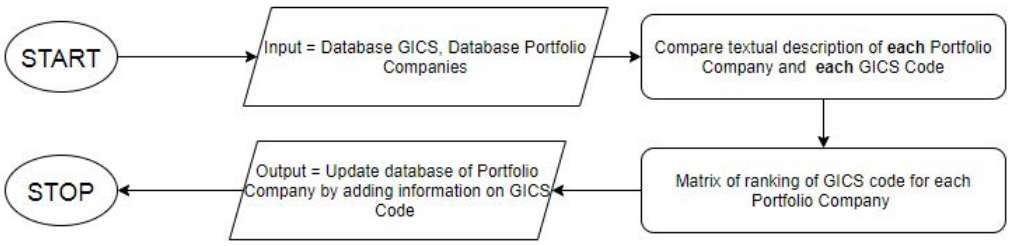

Figure 6.8: Algorithm for assigning a GICS code to Portfolio Companies.

In conclusion, we have developed framework for private equity portfolio valuation. We have taken two approaches to accomplish this. In the first approach we try to valuate the portfolio at the fund level. Takahashi and Alexander [2002] have developed the Illiquid Asset Fund Model, a version of which applies to private equity funds and can be used not only for valuation but also for managing alternative investments. In order to use the model for predictions, we must model the growth rate. We identify five factors that affect growth of private equity. We require further analysis into each of the factors in order to model growth rate. Access to proprietary data will be helpful in conducting experiments and analysis. Presently we do not have access to such data and recommend pursuing it in future.

The second approach to private equity valuation is by valuing portfolio companies. We suggest using market approach whereby we use comparable company analysis for valuation. Success of this depends on the selection of peer companies for comparison, which is subjective. We developed a framework for selection of peer group for companies considering the data constraints due to LP position on the investments. We also have finalised a valuation method after peer selection for the portfolio companies. We are unable to test our framework due to inaccessibility of relevant data. Our framework is based on leading researches of the industry which makes us confident of its success. As the company, NN, aims to develop a dynamic value tracking and value predicting system to manage its alternative investments, we also developed algorithms which would automate the peer selection based on machine learning. Finally, we suggest to reduce dependency on personnel for defining data points of portfolio companies as it requires insight and necessary expertise. We offer a possible solution to be explored to solve this problem in the way of a text comparing algorithm for assigning industrial codes to portfolio companies.

This research helps various stakeholders of investment industry. The framework developed can be used for valuation not only by investors in private equity but investors in the traditional equity funds as well. In addition to helping the blindsided private equity investor, this research can be helpful to venture capitalist firms to map and monitor the growth in value of their investments. Merger and Acquisition firms that drive inorganic growth can benefit from the insights provided in the research on the factors to be considered for an ideal peer group for valuation of firms. The research will help them identify the fair value of a target firm and also enables a projection of the value of a firm post-merger or post-acquisition.

This research can be made robust by addressing some of the shortcomings. We are currently suggesting one model for analysis of portfolio at the fund level. Although the use of the model is justified well, we would have liked to compare it with a few other such models. The research is based on literature study and will be more convincing if we are able to validate it with experiments on actual data. The data available to us on private equity investments was sparse and the data set was significant in terms of its size. This limited our ability to make any sensible comparisons with the public market. We have defined the factors to be considered for peer selection. After initial sorting and filtering, the clustering of firms is supposed to happen along risk and growth prospects. We tried to venture into the discussion of weighing risk and growth

prospect differently. At this point in the research, we believe it would be beneficial to learn from experimental validation the nature of weights, if any, that need to be applied to each of the two said factors. Lastly, we have not ventured into the details of the text comparison technique as one of the factors for determining a comparable firm. It is a unique suggestion and requires to be analysed by someone with expertise on the subject matter.

There is a huge scope for future researchers in this study. We have suggested the factors to be incorporated in the model for valuation at the fund level. Future researchers can further refine the definition of each factor and how they would be integrated in the model. The fund manager's alpha requires a special mention here as it is one of the controversial factors that defines our model. The industry is divided in its opinion on the significance of alpha, or rather the fund manager, in generating returns. A study of the characteristics of fund manager and the return that can be associated with a manager will further throw light on the issue. Another place for future researchers to start is by validating the framework. With access to the right kind of data, future researchers can validate different aspects of the framework and help improving it.Be it in validating the significance of the factors for fund-level model, the factors for peer selection, or the weights for the clustering of peers. Once the framework is validated and perfected, the next step would be to devise methods by which this could be integrated in the area of use such as the reporting system at NN Group. Our study is just the first step in the quest of truth in the highly secretive world of private equity.

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