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Use of Additional Transmitters for Increasing Detection Range in Harmonic Radar

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Summary

Since the mid-1970s, when the first papers about harmonic radars were published, the theory behind nonlinear radars has been extensively studied. New ways to measure the distance to targets of interest were proposed, new nonlinear elements for harmonic tags and different transmissions configurations were implemented. Despite this, one thing which still represents the bottleneck of the harmonic radars is their low range. The primary reason for the low range is the low power conversion efficiency of the passive nonlinear tag. There are many ways to improve the capacity of harmonic radars. In this paper, we focus on using additional transmitters.

Contents

Chapter 1

Introduction

1.1 Introduction

Harmonic radars are commonly used in situations where linear radar techniques fail due to the limited size of the object to be tracked, the necessity for high rejection of the environmental clutter, or where the weight plays an important role. There are multiple examples of these situations. For instance, in entomology, where tracking insects [1], [2] plays an essential role. Other examples are detecting RF equipment [3], sensing temperature remotely [4], monitoring vital human signs [5], alerting a vehicle driver of the presence of vulnerable road users [6]. In a typical harmonic radar configuration, a transceiver sends a modulated signal at some frequency f and some power P. And a nonlinear tag reflects a sum of of harmonics of frequency f of the modulated signal, which is then used to detect the location of the tag [3] [7] [8]. The bottleneck of such a system is its low range. The primary reasons for this are the low conversion efficiency of the nonlinear tag and high power free path loss due to high frequency resulting from a necessity for small tags. An easy solution for this is to increase the transmission power, which will increase the range according to range equation of the harmonic radar. However there are several problems with this approach. First of all high concentration of power in one location is dangerous. For high power, the transmitter gets complex [7], the equipment is expensive, and in extreme cases, the system can cause non-linearities. An extreme example of non-linearities due to a high concentration of power is the Luxembourg effect. Based on the nonlinear range equation of the harmonic radars [3] it can be observed that there are other methods to increase the range. By increasing the gain of antennas for instance. For this however, you need more antennas as it limits the area of detectability. Another method is to increase the power conversion efficiency of the nonlinear element. But this is usually out of control as it depends on the material and its properties.

1.2 Problem description

The previous section mentioned that although increasing the transmitter's power is a simple way to improve the range, it has some flaws. Instead of increasing the power directly, the power can be increased by adding additional transmitters [9]. These auxiliary transmitters will be approximately at the same distance as the main transmitter from the tag.It should be pointed out that in contrasts to main transmitter, which from now on will be called ranging transmitter, additional transmitters emit an unmodulated signal. The feasibility of this method can be tested using the power series model of the nonlinear elements. The power series model can be derived from the IV characteristic of the diode. The exponential function which relates voltage over diode to the current through the diode can be represented as a Taylor series [9]. For high-range application (which we are trying to research), only up to second power is needed, as the model very closely fits the actual data (if the input voltage is on the order of several thermal voltages of the Schottky diode, the second-order power model has a deviation of less than 10 percent [9]). It is essential to point out that the method used to measure the distance relies on the cross-correlation between the ranging signal used for transmission and the reflected signal from the tag after being demodulated by the local oscillator, tuned to the second harmonic of the ranging signal.

Influence of the auxiliary transmitters is described by the superposition of the original modulated signal from the ranging transceiver and a carrier signal characterized by an amplitude and a phase (this represents the influence of all auxiliary transmitters). Because the component of interest is the second harmonic, using the power series model the final expression for the tag output contains the signal from the ranging transceiver, intermodulation and carrier signals. So three different baseband signals can be used for cross-correlation. In general, only one of the baseband components is used, but all three components contain power which can potentially offer a better signal to noise ratio (SNR).

1.3 Research question

The problem stated above raises the question, is there something that can be done to increase the range? Most of the things mentioned in the introduction suggested that the biggest problem with harmonic radar comes from the design of the tag. Clearly, a more efficient tag design can improve system performance. Nonetheless the question still remains, is there something that can be done it terms of signal form and signal processing that can also result in an increase in efficiency? Is there any optimal solution in terms of ranging signal considering everything else is equal that can increase range/range resolution? Is this optimal solution feasible? What are the drawbacks of this solution?

Chapter 2

System Model

2.1 Principle of operation of harmonic radar

2.1.1 General description

The goal of this paper is to investigate the ranging capabilities of harmonic radar in the presence of auxiliary transmitters. In order to do this a model of the system has to be derived. Usually a power series model [10] is used to describe the behaviour of the harmonic radar. The power series model can be derived taking into account the behaviour of the nonlinear transponder. The transponder is commonly made of antenna used to intercept and send back the EM waves coming from the transceiver, a Schottky diode [3] which acts as the nonlinear element, and an inductor which helps with antenna matching. The general principle of operation is shown in figure 2.1 and the simplified model of the tag is shown in figure 2.2. Taking into account the model shown in figure 2.2 the input-output relationship of the tag can be derived. It should be pointed out that usually the tag is made so that it is resonant at the frequency of interest so that inductor impedance cancels the imaginary part of the antenna impedance. Without losing any analytical insight the model can be further simplified considering $i_d R_A \ll V_{in}$ which is justifiable for long distances, where i_d is the current through the diode and R_A is the real part of antenna impedance. As a result only the diode affects the current so

$$i_d = I_S(e^{\frac{V_D}{nV_T}} - 1) \tag{2.1}$$

where I_S is the saturation current of the diode, V_D is the voltage across the diode, n and V_T being respectively ideality factor and thermal voltage of the diode. Applying Taylor series approximation on the diode current in the form of Maclaurin series and making the change of variable $\alpha = \frac{1}{nV_T}$, the diode current becomes

$$i_d = I_S \sum_{n=1}^{+\infty} \frac{(\alpha * V_D)^n}{n!} \tag{2.2}$$

where $V_D \approx V_{in}$. From 2.2 at the maximum range when $i_d R_A << V_{in}$, the tag output current can be related to the input voltage via a power series.

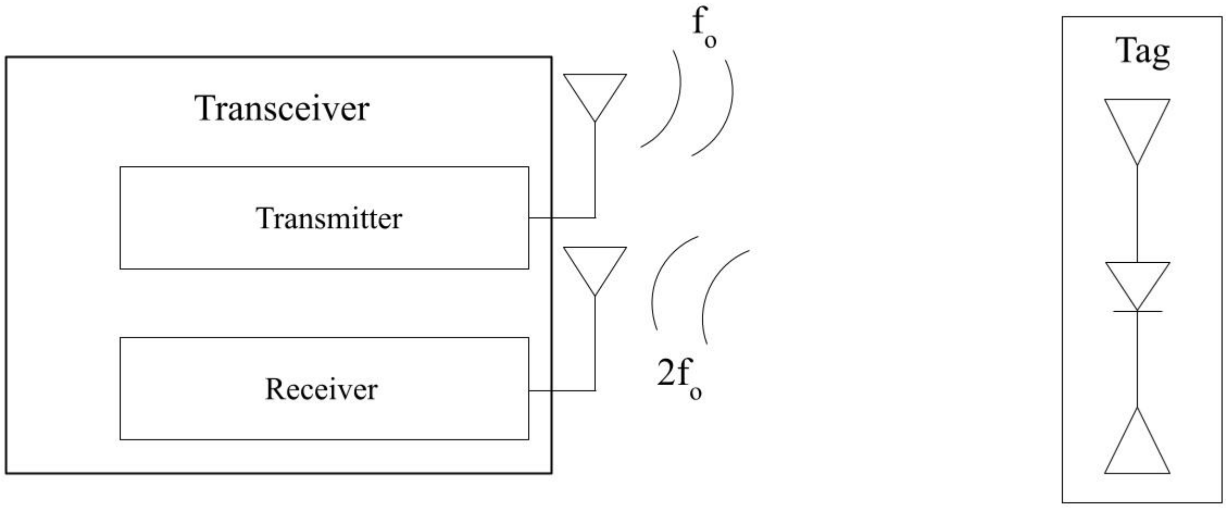

Figure 2.1: Operation principle of harmonic radar

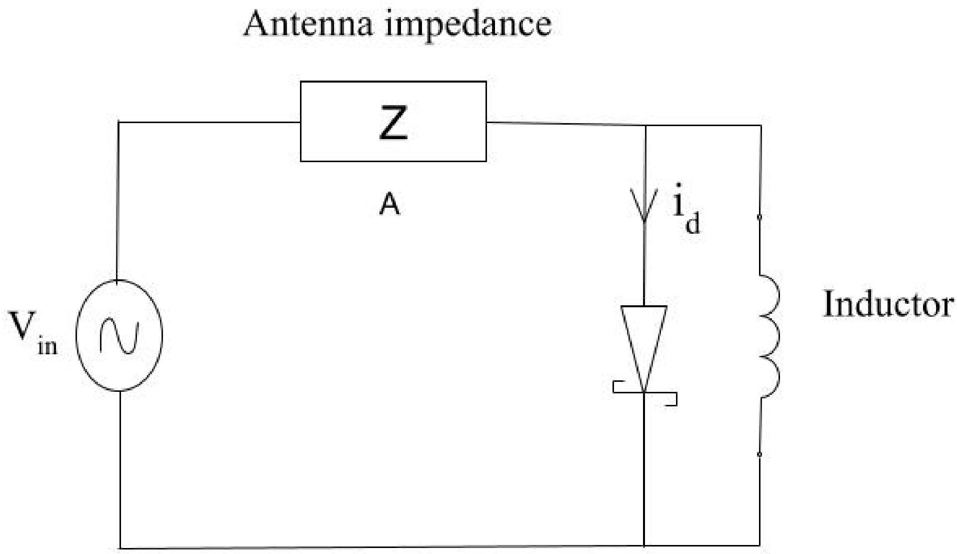

Figure 2.2: Simplified model of the tag

2.1.2 Ranging principle

General description

Consider $s_{tx} = A\Re(x(t)e^{j(\omega_0 t+\theta_0)})$ as the signal transmitted by the ranging transmitter, where A is the amplitude of the signal, $x(t)$ is a complex envelope used for ranging, θ_0 is the phase of the local oscillator, and $\Re$ denotes the real part of a complex number.

For line of sight (LOS) path, the power of the signal at the transmitter P_t and received power P_r are related through Friis equation [11]

$$P_r = \frac{P_t G_T(\omega_0) G_R(\omega_0) c^2}{(2d\omega_0)^2} \tag{2.3}$$

where $G_T(\omega_0)$ is the gain of the transmitting antenna at ω_0, $G_R(\omega_0)$ is the gain of the receiving antenna of the tag at ω_0, d is the distance between them and ω_0 is the frequency of operation. In addition to this, there is also a multiplicative factor k which represents the effectiveness of power delivery to the diode and it is mostly the result of matching the antenna impedance to that of the diode. Taking this into account the final formula is

$$P_r = \frac{k P_t G_T(\omega_0) G_R(\omega_0) c^2}{(2d\omega_0)^2} \tag{2.4}$$

In a noise-free environment, the signal at the tag is given by

$$s(t) = A_r \Re(x(t-t_d)e^{j(\omega_0(t-t_d)+\theta_0)}) \tag{2.5}$$

where A_r is the amplitude of the received signal that can be obtained from equation 2.4, $t_d = \frac{d}{c}$ is the time EM waves need to travel to the tag located at distance d from the transceiver, and c is the speed of light in vacuum.

Focusing only on the second power in equation 2.2 (second harmonic is the typical signal used in harmonic radars [10]), current through the diode at the second harmonic due to voltage $s(t)$ is

$$i_d(t|2\omega_0) = I_S \frac{\alpha^2}{4} A^2 \Re(x^2(t-t_d)e^{2j(\omega_0(t-t_d)})). \tag{2.6}$$

This current is then converted to EM waves through the load of the antenna which represents its power conversion factor. The power received back is again related to the one sent through equation 2.4 and the signal is delayed by the time $t_d = \frac{d}{c}$. From here, the signal that arrives at the ranging receiver is

$$s_{rx}(t) = \gamma i_d(t-t_d) \tag{2.7}$$

where γ takes into account the effect of power conversion of the tag plus the decay due to square law. Taking into account 2.6, the signal 2.7 can be written as

$$s_{rx}(t) = \gamma I_S \frac{\alpha^2}{4}(A^2 \Re(x^2(t-2t_d)e^{2j(\omega_0(t-2t_d))})). \quad (2.8)$$

If we substitute $\beta = \gamma I_S \frac{\alpha^2}{4}$, then the signal after quadrature down-conversion becomes

$$s_{rx}(t) = \beta A^2 \Re(x^2(t-2t_d)e^{2j(\omega_0(t-2t_d)+\theta_0)}). \quad (2.9)$$

The phase component $e^{-4j\omega_0 t_d + 2\theta_0}$ can be removed by using a Costas loop, and βA^2 can be written as A_r so the signal after quadrature down-conversion becomes

$$s_{rx}(t) = A_r x^2(t-2t_d). \quad (2.10)$$

Complex envelope $x(t)$ is deterministic so crosscorrelation can be used to find the time shift $2t_d$. From here, it results that the function used for crosscorrelation should have the propriety that $y(t) = x^2(t)$. Taking this into account the crosscorrelation can be written as

$$R_{sy}(\tau) = \int_0^T s_{rx}(t) y^*(t-\tau) dt$$

where T is the integration time. This can be further simplified to autocorrelation of $y(t)$

$$R_{sy}(\tau) = \int_0^T A_r y(t-2t_d) y^*(t-\tau) dt = A_r R_y(\tau - 2t_d) \quad (2.11)$$

where $R_y(t)$ is the autocorrelation of $y(t)$. Finally the peak value of the $R_y(t)$ is at $t=0$ so for $R_y(t-2t_d)$ the peak will be at $t = 2t_d$. Thus, the distance to the tag is equal to

$$d = 0.5c \, arg \, max R_{sy}(\tau). \quad (2.12)$$

Target detection

Equation 2.6 presents an idealized version of the ranging principle. It is very easy to see that in this configuration, as long as you received a signal, you can find the range to the target. In reality, like any physical system, the harmonic radar is prompted to noise. As $s_{rx}(t)$ represents a complex envelope, the noise is presented on both real and imaginary part of the complex signal and can be characterized by complex additive white Gaussian noise (AWGN) with power spectral density (PSD) equal to N_0.

Consider now a noisy crosscorrelation

$$R_{noisy}(\tau) = \int_0^T (s_{rx}(t) + n(t)) y^*(t-\tau) dt = R_{sy}(\tau) + \int_0^T n(t) y^*(t-\tau) dt \quad (2.13)$$

where $R_{sy}(\tau) = \int_0^T (s_{rx}(t) y^*(t-\tau) dt$ and $n(t)$ is complex AWGN with PSD N_0. It is easy to prove that the SNR at the peak cross-correlation is equal to

$$SNR = \frac{max|R_{sy}(\tau)|^2}{N_0 T \int_0^T y(t) y^*(t) dt}. \quad (2.14)$$

Taking equation 2.7 into account, it is easy to observe that for small value of the $s_{rx}(t)$, or its total absence, the peak of $R_{noisy}(\tau)$ can be the peak of the noise. So to solve this problem a threshold is usually set where the value of the peak is compared against this threshold. While the threshold might be set in different ways, potentially yielding different values for the minimum detectable SNR, it is generally true that a higher SNR will lead to a better estimation performance. This suggests that an increase in system SNR defined by 2.8, directly corresponds to an increase in range. Therefore, for simplicity, the optimality problem will be from now on defined in terms of the increase in SNR.

2.1.3 Range resolution

To be able to solve or even define the problem of optimality in terms of range resolution you need a definition for it. Throughout the paper two definitions will be used interchangeably. It should be stated that these two definitions are the same in nature, they are just defined in two different domains.

Time domain definition

Consider the time domain representation of the signal after applying the crosscorrelation and consider two situations presented in figures 2.3 and 2.4. It is clear from the figures that if T is defined as the time duration of the pulses presented in figures 2.3/2.4, then the pulses have to be at least T apart from each other so that they can be clearly distinguished. Define T_r as the time between two closes points around the peak of the signal with the amplitude at least $-20dB$[1] lower than the peak value. Then the range resolution is given by

$$\sigma R = \frac{cT_r}{2}. \tag{2.15}$$

Frequency domain definition

Consider a rectangular function in frequency domain with bandwidth BW. The time domain representation of the signal is a $sinc$ function with period $\frac{1}{BW}$. A $sinc$ function follows the definition of the T_r in time domain for $T_r = \frac{1}{BW}$. The range resolution, defined in terms of bandwidth, is then

$$\sigma R = \frac{c}{2BW} \tag{2.16}$$

where BW is the bandwidth of the rectangular signal in frequency domain equivalent to the frequency representation of the output of the matched filter.

[1]The $-20dB$ may be seen as an arbitrary value and in some sense it is. The reason this value was chosen is because of more general definition in frequency domain where bandwidth dictates the values of the range resolution.

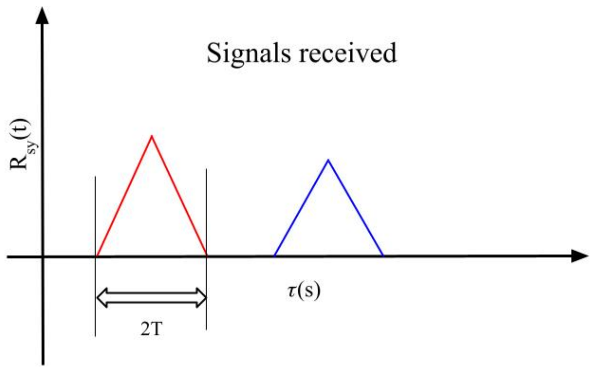

Figure 2.3: Targets easily separable after applying crosscorrelation

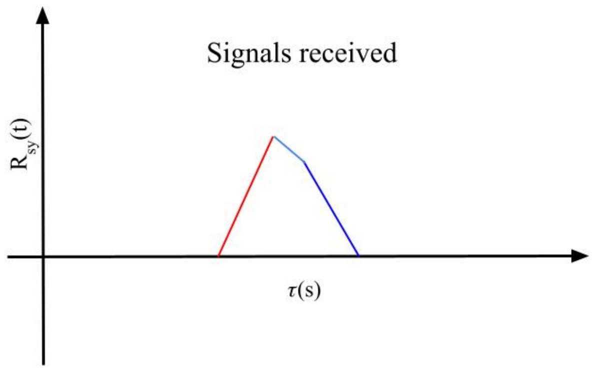

Figure 2.4: Targets inseparable after applying crosscorrelation

2.2 Operation with auxiliary (helper) transmitters

2.2.1 Model derivation

Now that the power series model was derived, equation (2.2) can be used to model the behaviour of the system with auxiliary transmitters which is illustrated in figure 2.5.

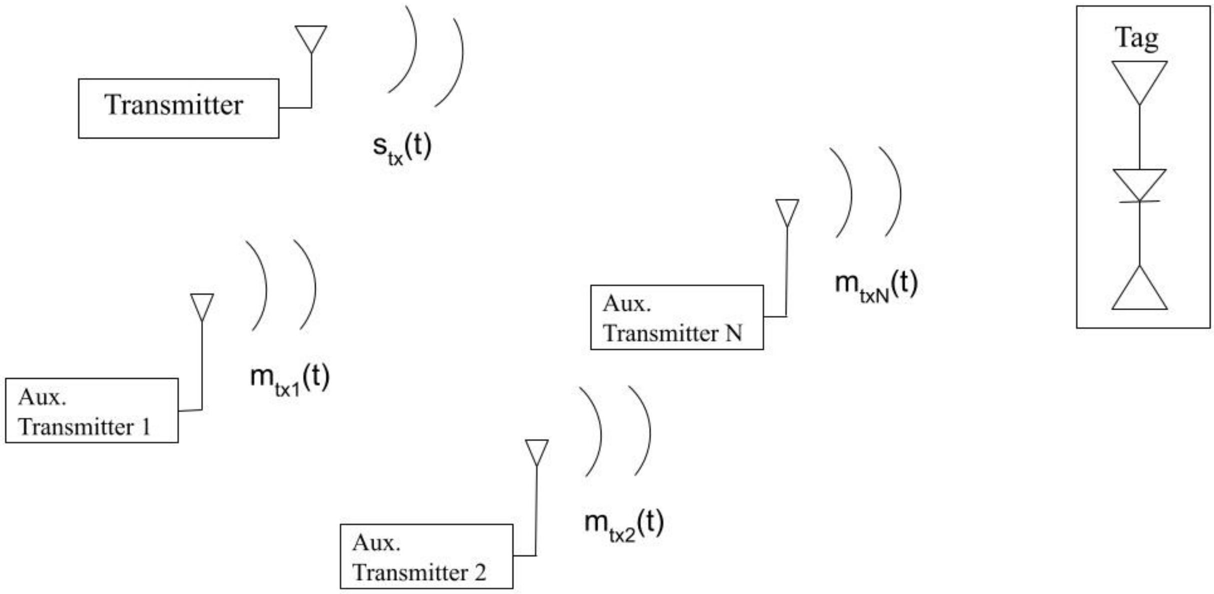

Figure 2.5: Operation with auxiliary transmitters

For auxiliary transmitters, the signal emitted by the antenna can be defined as $m_{txi} = A_i \Re(e^{j(\omega_0 t + \theta_i)})$, where i is the number of the auxiliary transmitter (e.g., for N auxiliary transmitters $i \in [1, N]$), A_i and θ_i being respectively the amplitude of the signal and the phase of the local oscillator of the auxiliary transmitter i.

Because EM waves obey the superposition principle, V_D in the equation 2.2 can be thought as the superposition of all EM waves incident on the tag. Taking this into account, the signal arriving at the transponder will have the following form

$$s(t) = A\Re(x(t - t_d)e^{j(\omega_0(t-t_d)+\theta_0)}) + \sum_{i=1}^{N} A_i\Re(e^{j(\omega_0(t-t_{di})+\theta_i)}). \tag{2.17}$$

Note that for simplicity, the notation for signal amplitudes remains the same here but their values now follow from the received signal power given in equation 2.4. The term $\sum_{i=1}^{N} A_i\Re(e^{j(\omega_0(t-t_{di})+\theta_i)})$ can be further simplified to $A_v\Re(e^{j(\omega_0 t+\theta_h)})$, where

$$A_v = |\sum_{i=1}^{N} A_i e^{j(\theta_i-\omega_0 t_{di})}| \tag{2.18}$$

$$\theta_h = \arccos\left(\frac{\sum_{i=1}^{N} A_i\Re(e^{j(\theta_i-\omega_0 t_{di})})}{A_v}\right). \tag{2.19}$$

With 2.18/2.19, equation 2.17 can be rewritten as

$$s(t) = A\Re(x(t - t_d)e^{j(\omega_0(t-t_d)+\theta_0)}) + A_v\Re(e^{j(\omega_0 t+\theta_h)}). \tag{2.20}$$

This equation can be now substituted in equation 2.2.

One thing to notice is that coefficients of the powers in the power series decay exponentially in form of α^n. In addition to this, from Friis equation it is easy to see that for an increase in frequency the received power decreases. This suggests that a better option is to focus only on the second harmonic whose amplitude is given by all the components from power of 2 and up. For $s(t) << 1$ only the contribution from power two can be taken into account, so for $2\omega_0$ we obtain

$$\begin{aligned} i_d(t|2\omega_0) = I_S\frac{\alpha^2}{4}\Big(A^2\Re(x^2(t - t_d)e^{2j(\omega_0(t-t_d)+\theta_0)}) + 2AA_V\Re(x(t - t_d)e^{j(\omega_0(t-t_d)+\theta_0+\theta_h)}) \\ + A_V^2\Re(e^{2j(\omega_0 t+\theta_h)})\Big). \end{aligned} \tag{2.21}$$

For diode current defined by 2.21, the signal that arrives at the receiver is given according to equation 2.9 by

$$\begin{aligned} s_{rx}(t) = \beta\Big(A^2\Re(x^2(t - 2t_d)e^{2j(\omega_0(t-2t_d)+\theta_0)}) + 2AA_V\Re(x(t - 2t_d)e^{j(\omega_0(t-2t_d)+\theta_0+\theta_h)}) \\ + A_v^2\Re(e^{2j(\omega_0 t+\theta_h)})\Big). \end{aligned} \tag{2.22}$$

2.2.2 Overview of constellation of helper transmitters

One thing that comes to mind when using additional transmitters is if their relative position to each other matters. There are three reasons why their relative position would matter.

First is the difference in the phase of the local oscillator which influences the superposition of the waves. Second, they have different distance to the tag, so that adds a new random phase difference. Finally, because of different distances to the tag their power near the tag is also different, although they may start with the same power. The reason for the last one being LoS path loss due to the inverse square law. For small enough relative distances $\delta d << d$, where δd is the distance to the ranging transmitter from auxiliary transmitter, the influence is insignificant. Nonetheless, for big enough transmit power $A_V > A$ the power loss due to δd is equal to $10log(\frac{1}{1+\frac{\delta d}{d}})$.

The intuitive answer based on the analysis above is to put the auxiliary transmitters as close to the ranging transmitter as possible. However an analytical proof based on a mathematical model would be more suggestive.

As pointed out earlier there are three reasons why the relative distance would matter. And although the phase of the local oscillator does not depend on the position of the transmitter, the maximum achievable power depends (it should be made clear that there is also a phase component depending on the relative position of the transmitter to the tag but that one is considered random). There are two main factors associated with ideal relative position: uniform power distribution around an imaginary circle with the center at the receiver and maximization of power on this circle. To solve the problem we can define a Cartesian coordinate system with the center at the ranging transmitter/receiver as shown in figure 2.4. One thing to take into account is that it is considered that $\delta h << d$ where δh is the relative height between the transmitters. Now, we assign vectors $\vec{r_i}$ to the auxiliary transmitters in Cartesian system defined in figure 2.4. In addition to this, $\vec{R}$ corresponds to the position of the tag. In such a system, the distance between the helper transmitter and transponder is defined as $|\vec{R} - \vec{r_i}|$. Then the problem mentioned above can be stated mathematically as

$$\begin{cases} \frac{\partial L}{\partial \theta} = 0, R = |R|e^{j\theta}, \theta \in [0, 2\pi] \wedge |R| = const \\ |\nabla_r L = 0, |R| = const \wedge \forall \theta \in [0, 2\pi] \wedge |r_i| < |R| \end{cases} \tag{2.23}$$

where $L = \sum_{i=1}^{N} \frac{A_i}{||\vec{R} - \vec{r_i}||}$ and A_i is the amplitude of the unmodulated signal coming from the auxiliary transmitters. It has to be pointed out that this presents a simplified model since not all the waves will add constructively. Moreover, the signal strength will also depend on the path taken, frequency, temperature, etc. Nonetheless, this is the maximal achievable signal strength.

To find the solution one does not actually need to solve 2.23. For constant amplitude, there is a trivial solution which in fact provides most of the insights. Based on the form of the first equation in 2.23 a trivial solution will be a geometrical form which has a symmetry around rotation with some random angle θ. This geometrical form is a circle. For such a form the second equation in 2.23 is true for $|r_i| \to 0$. So the best arrangement of auxiliary transmitters are on a circle around the ranging transmitter/receiver with as small radius of the circle as possible.

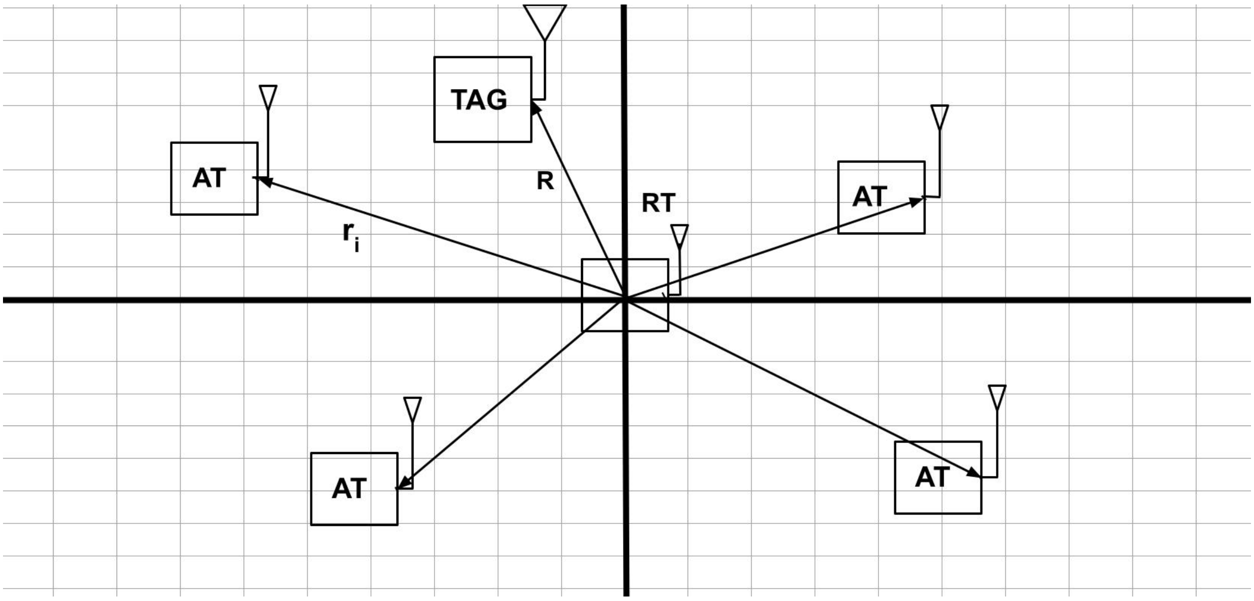

Figure 2.6: Coordinate space with center at ranging transmitter (RT) and auxiliary transmitters (AT) distributed around

Chapter 3

Proposed Solution

3.1 Statement of problem to be solved

Chapter 2 provided a full description of the system model to be used. Taking this model into account, we can start our description by focusing on equation 2.22

$$s_{rx}(t) = \beta(A^2x^2(t-2t_d)e^{2j(-2\omega_0 t_d+\theta_0)}) + 2AA_v x(t-2t_d)e^{j(-2\omega_0 t_d+\theta_0+\theta_h)}) + A_v^2 e^{2j(-2\omega_0 t_d+\theta_h)}) \tag{rep. 2.22}$$

Investigating this equation we can observe that there are four components $(A_V, t_d, \theta_0, \theta_h)$ which for most of the purposes can be considered random. One thing to notice is that for slow enough targets (slow as defined by $Tv < \sigma R$, where T is the pulse duration, v average velocity of the target and σR range resolution) these components are wide sense stationary random processes for the duration of measurements. So two situations can be considered: $A_v, t_d, \theta_0, \theta_h$ are treated as some deterministic constants and their random behaviour is studied after a full solution is derived, or their random nature is taken into account in designing optimal solution. For $A_V, t_d, \theta_0, \theta_h$ being deterministic, this corresponds to the situation of an estimator which moves the randomness from $A_V, t_d, \theta_0, \theta_h$ to that of the estimators of these values. For simplicity, in the following analysis, the solution will be presented considering an ideal situation when $A_V^2 e^{2j(-2\omega_0 t_d+\theta_h)}$,$e^{2j(-2\omega_0 t_d+\theta_0)})$, $e^{j(-2\omega_0 t_d+\theta_0+\theta_h)})$ can be perfectly removed and A_V is deterministic. The solution where the environment is stochastic is presented in the appendix.

As pointed out in the introduction, the goal is to find a complex envelope and a corresponding match filter which offer an optimality in terms of range/range resolution for a configuration described by 2.22. Consider following conditions for optimality:

1. The amplitude of the complex envelope $x(t)$ can be maximum 1. Increasing the amplitude of the complex envelope will increase the SNR, which will give unfair advantage to some signals.

2. The energy of the complex envelope can be maximum the energy of the rectangular

signal with amplitude 1 with the same pulse duration as the complex envelope $x(t)$. Providing more energy will also result in higher SNR.

3. The pulse duration of the complex envelope should be the same for all candidate signals. Increasing the pulse duration of the complex envelope will increase the energy of the envelope, which will again result in higher SNR.

4. There are some signals which for the same pulse duration have higher bandwidth. An example of this is a linear frequency modulated signal .While their bandwidth is a fair advantage, in general the whole system is bandwidth-limited. So in order to make the comparison fair, it is considered that the bandwidth of the complex envelope can not be higher than that of the Barker code of length 11 of the same time duration. The Barker code was chosen arbitrarily, the bandwidth can be limited to any reasonable value.

These conditions can be translated mathematically to

$$|x(t)| \leq 1 \tag{3.1}$$

$$\int_0^T |x(t)|^2 dt \leq T \tag{3.2}$$

$$T = const \tag{3.3}$$

$$BW(x(t)) \leq BW(Br(t, 11)) \tag{3.4}$$

where $Br(t, 11)$ is the time domain representation of a Barker code of length 11. Given the constraints 3.1 – 3.4 the problem of finding an optimal solution can be formulated as

$$|\nabla SNR| = 0 \tag{3.5}$$

$$|\nabla BW(R_{s\mu}(\tau))| = 0 \tag{3.6}$$

where

$$SNR\left(x(t), \mu(t)\right) = \frac{max|R_{s\mu}(\tau)|^2}{N_0 T \int_0^T \mu(t)\mu^*(t)dt} \tag{3.7}$$

$$s_{rx}(t) = \beta(A^2 x^2(t - 2t_d) + 2AA_V x(t - 2t_d)) \tag{3.8}$$

$$R_{s\mu}(\tau) = \int_0^T s_{rx}(t)\mu^*(t - \tau)dt \tag{3.9}$$

Here, $x(t)$ is the complex envelope used for ranging, T is the pulse duration of the ranging signal,$BW(*)$ is the bandwidth of the signal (the definition used does not matter as long as it is the same for every signal used).

Before moving to the actual solution, several things have to be pointed out. First of all, it is easy to see that the conditions mentioned above limit the form of the complex envelope to $|x(t)| = 1$. While having a unit magnitude complex envelope restricts the space of possible solutions, in general it is assumed as fact that the solution should be on a unit circle. Second, it is very unlikely that a solution to this system of equations exits. The goal in this case is finding a solution as close to optimality as possible.

3.2 Problem analysis

The optimal problem stated above represents a system of equations of two variables $(x(t), \mu(t))$. This suggests dividing the problem in steps and building the solution by considering one of the functions as known and solving for the other one.

3.2.1 Optimal filter

Lets examine the problem of finding the optimal function $\mu(t)$, which gives the highest SNR when SNR is defined by equation 3.7 and $x(t)$ is known. Because the noise is modeled as complex AWGN while $x(t)$ represents an analytic form of a real physical signal, the solution for this problem is well known and it is given by the match filter [11]. As a result, the maximum SNR as a function of $\mu(t)$ for a given complex envelope $x(t)$ is equal to

$$\mu(t) = gs_{rx}(t), c \in \mathbf{C} \tag{3.10}$$

where g is some complex constant.

Using the proprieties of the matched filter, equation 3.7 can by simplified to

$$SNR(x(t)) = \frac{max(\int_0^T s_{rx}(t)(s_{rx}(t))^* dt)}{N_0 T}. \tag{3.11}$$

Recognizing that T and N_0 do not depend on the complex envelope $x(t)$ and $s_{rx}(t)$ can be shifted by $2t_d$ without affecting the result, equation 3.11 can be written as

$$max_{x(t)}[\beta^2 \int_0^T (A^2x^2(t) + 2AA_V x(t))(A^2x^2(t) + 2AA_V x(t))^* dt]. \tag{3.12}$$

Because $|x(t)| = 1$ and β, A, A_V are considered constants, moreover $x(t)$ can be written as $e^{j\phi_x(t)}$, equation 3.12 translates to

$$max_{x(t)}[\int_0^T x^2(t)x^*(t)dt + \int_0^T ((x^2(t))^* x(t)dt] = max_{x(t)}[2\int_0^T cos(\phi_x(t))dt] \tag{3.13}$$

where

$$max_{x(t)}[\int_0^T cos(\phi_x(t))dt] = T, \phi_x(t) = 2n\pi rect(\frac{t-\frac{T}{2}}{T}) \quad n \in \mathbf{N}. \tag{3.14}$$

Equation 3.14 suggests that the maximum SNR for $s_{rx}(t)$ is given by $x(t) = rect(\frac{t-\frac{T}{2}}{T})$. One thing worth mentioning here is that for $x(t) = rect(\frac{t-\frac{T}{2}}{T})$ the DC term in equation 2.22 can not be removed anymore, so it contributes to the SNR. In this case, SNR approaches that of the system without auxiliary transmitters where the amplitude is equal to $A + A_V$. For the general case of using match filter

$$SNR(x(t)) = \frac{A^4T + 4A^2A_v^2T + 4A^3A_v\int_0^T cos(\phi_x(t))dt}{N_0T}. \tag{3.15}$$

Inspecting equation 3.15 it is easy to notice that the maximum SNR is obtained for $x^2(t) = x(t)$. In more rigorous terms, if $< x^2(t), x(t) >$ is defined as the inner product between $x^2(t)$ and $x(t)$, then SNR is directly proportional to the value of the real part of the inner product. The reason for this is the fact that $\Re(< x^2(t), x(t) >) = \int_0^T cos(\phi_x t)dt$.

3.2.2 Bandwidth

The optimal solution for SNR as derived above is obtained when $x(t) = rect(\frac{t-\frac{T}{2}}{T})$). The bandwidth of $s_{rx}(t)$ for $x(t) = rect(\frac{t-\frac{T}{2}}{T})$) is equal to $\frac{1}{T}$. For matched filter, the crosscorrelation preserves the bandwidth so $BW(R_{s\mu}(\tau)) = BW(s_{rx}(t))$, which results in a range resolution $\sigma R = \frac{c}{2BW(R_{s\mu}(\tau))} = \frac{cT}{2}$. This is the worse bandwidth you can obtain for a signal with pulse duration T. A better solution has to be obtained which can give a better range resolution while still having a high SNR. The solution can be derived by starting with the form of the crosscorrelation in 3.9

$$R_{s\mu}(\tau) = \int_0^T \beta^2(A^2x^2(t-2t_d) + 2AA_vx(t-2t_d))(A^2x^2(t-\tau) + 2AA_vx(t-\tau))^*dt$$

It is easy to see that this equation can be simplified to

$$R_{s\mu}(\tau) = \beta^2 R_\mu(\tau - 2t_d). \tag{3.16}$$

Taking this into account, the range resolution of $R_{s\mu}(\tau)$ is exactly the same as of $R_\mu(\tau)$. It is also important to point out that because after quadrature down-conversion the complex envelope is obtained directly, the cross correlation is also going to be complex. So for ranging, the magnitude of the cross correlation is used:

$$\begin{aligned}|R_\mu(\tau)| =&|A^4\int_0^T e^{j(2\phi_x(t)-2\phi_x(t-\tau))}dt + 4A^3A_v\int_0^T cos(2\phi_x(t) - \phi_x(t-\tau))dt \\ &+ 4A^2A_v^2\int_0^T e^{j(\phi_x(t)-\phi_x(t-\tau))}dt|.\end{aligned} \tag{3.17}$$

Section 2.1.3 proposed two definitions for the range resolution of the harmonic radar. Focusing on the time domain definition, it is not difficult to see that high range resolution translates to $\frac{|R_\mu(\tau)|}{|R_\mu(0)|} << 1$ for τ close to 0. Applying this to equation 3.17, results in

$$\frac{d^2\phi_x(t)}{dt^2} >> \frac{2\pi}{T}. \tag{3.18}$$

In general, focusing on equation 3.17, the highest range resolution, as defined by the time domain definition, is given by a signal which has a high value for the second derivative of the instantaneous phase of $x(t)$. In more rigorous terms

$$\frac{d^2\phi_x(t)}{dt^2} = \frac{2\pi}{T\tau} \tag{3.19}$$

where τ is the width of the main lob. A thing to notice is that for such a signal $x^2(t)$ and $x(t)$ are orthogonal or have a very small inner product. As a general rule, investigating equation 3.17 it can be declared that to increase the bandwidth of the system defined by $s_{rx}(t)$, the inner product between $x^2(t)$ and $x(t)$ has to be as small as possible(for small inner product $x^2(t)$ and $x(t)$ do not overlap in frequency domain,because $x(t)$ has to have bandwidth of atleast Barker code for the case of $A \approx A_V$ and $x^2(t)$ and $x(t)$ not overlapping in frequency domain —$R_\mu(\tau)$— should have higher bandwidth). The opposite statement is generally not true, a small SNR system does not guarantee a high range resolution. The reason for this is while range depends on the absolute value of the inner product, SNR is proportional to the real part of the inner product. An example of such a signal is $x(t) = e^{j\frac{\pi}{2}} rect(\frac{(t-\frac{T}{2})}{T})$, for which the real part of inner product between $x^2(t)$ and $x(t)$ equal to 0 but the absolute value is maximum. The relationship between inner product and range resolution can be proven as follows. According to power theorem $< x^2(t), x(t) >=< Y(\omega), X(\omega) >$, where $< *, * >$ is the inner product and $Y(\omega)$, $X(\omega)$ are the Fourier transforms of $x^2(t)$ and $x(t)$. For $A_V >> A$, cross-correlation in 3.17 approaches $|R_\mu(\tau)| = |4A^2A_V^2 \int_0^T e^{j(\phi_x(t)-\phi_x(t-\tau))}dt|$, that means that $x(t)$ has to have the bandwidth of the Barker code to fulfil the requirement for range resolution. Moreover for $A >> A_V$, the opposite happens and $x^2(t)$ has to have the bandwidth at least of the Barker code. Because for $x(t) = e^{j\phi_x(t)}$ both $x(t)$ and $x^2(t)$ have the same energy ,results that $< Y(\omega), Y(\omega) >=< X(\omega), X(\omega) >$. The highest bandwidth for A and A_V close in value to each other is given for $Y(\omega)$ and $X(\omega)$ not overlapping in frequency domain. For this situation $< Y(\omega), X(\omega) >= 0$. Because of $< Y(\omega), Y(\omega) >=< X(\omega), X(\omega) >$,in case $X(\omega)$ and $Y(\omega)$ overlap in frequency domain, increasing the bandwidth of $Y(\omega)$ results in lowering the energy in the frequency band of $X(\omega)$,which results in lowering the value of $< Y(\omega), X(\omega) >$. Finally for $X(\omega)$ and $Y(\omega)$ occupying the same frequency band, because of $< Y(\omega), Y(\omega) >=< X(\omega), X(\omega) >$ results $x^2(t) = x(t)$. While the highest bandwidth is given by equation 3.18 there are multiple signals which follows the equation. Because the goal is to find an optimal both in SNR and range resolution,results that a maximum SNR

for equation 2.18 has to be found

$$e^{j\phi_x(t)} = e^{j(At^2+Bt+C)} \tag{3.20}$$

SNR is proportional the the real part of the inner product between $x(t)$ and $x^2(t)$,so maximum SNR is given by

$$max_{A,B,C}\Re(\int_0^T e^{j(At^2+Bt+C)}dt) \tag{3.21}$$

A is dictated by the range resolution so it is considered a constant, because C is the phase component which has a value between $[-1, 1]$ and B is a frequency which lowers the average of $e^{j(At^2+Bt+C)}$. Results that for maximum real part of inner product $B, C = 0$.

3.3 Proposed solution

The solution analysis can be summarized as follows.

1. To increase the SNR of the system as defined by $s_{rx}(t)$, the inner product between $x^2(t)$ and $x(t)$ has to also maximized. The highest SNR is obtained when $x^2(t)$ and $x(t)$ are collinear.

2. To increase the range resolution of the system defined by $s_{rx}(t)$, the inner product of the $x^2(t)$ and $x(t)$ has to minimized. The highest bandwidth is obtained when $x^2(t)$ and $x(t)$ are orthogonal and instantaneous derivative of the phase of $x(t)$ is as high as possible.

3. It is not possible to come up with a complex envelope $x(t)$ for $s_{rx}(t)$ that has both a peak in SNR and in range resolution. Although such a signal exists when no auxiliary transmitters are used.

4. The best solution when $\frac{A}{A_V} >> 1$ or $\frac{A_V}{A} >> 1$ is to focus only on the terms $x^2(t)$ or $x(t)$, which one has the highest power contribution.

Taking into account everything mentioned above, plus the strict condition of a range resolution higher that of that of the Barker code of length 11, two possible solutions are proposed. It should be pointed out that the main region of operation of these two solutions are for $\frac{A_V}{A} \in [0.1, 10]$. As stated above when $A_V >> A$ or $A >> A_V$, it is more optimal to focus only on one of the terms.

Linear frequency modulation

Consider a complex envelope of form

$$x(t) = e^{j(F_0+f_n)t} rect(\frac{t-\frac{T}{2}}{T}) \tag{3.22}$$

where T is the duration of the pulse, F_0 is a constant frequency used as an offset and $f_n = nf$ for $(n-1)T_f \le t \le nT_f, n \in [0, N_f - 1]$ and $T_f = \frac{T}{N_f}$. It is easy to see that for a frequency modulation scheme, $x^2(t)$ will have double the bandwidth of $x(t)$. You can observe this in figure 3.1 where this increase is shown.

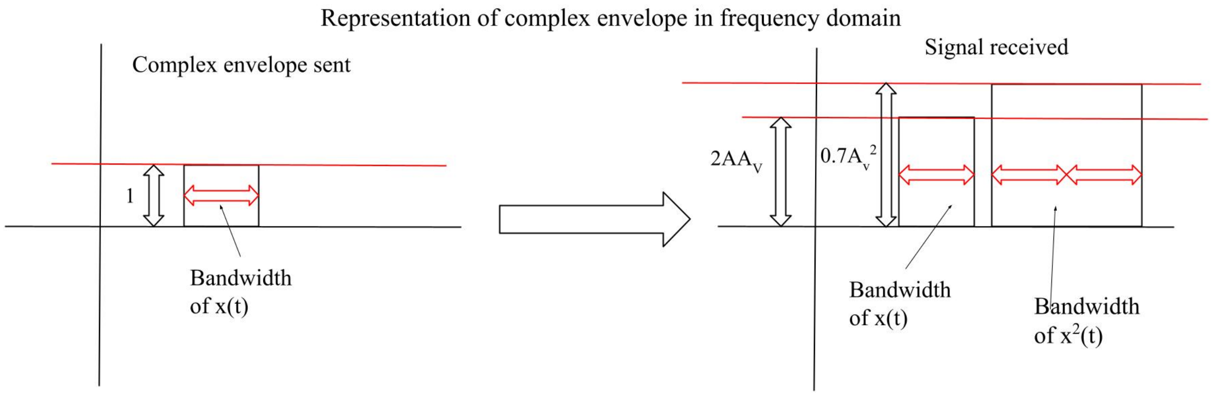

Figure 3.1: Frequency domain representation of the complex envelop sent and signal received

It has to be pointed out that figure 3.1 shows only one of the many situations that can happen. In reality, $x^2(t)$ and $x(t)$ can overlap in frequency domain or be far apart; they can have different amplitude or can be a the same level. Nonetheless, this suggests that for a frequency modulation technique the output bandwidth of the system has a bandwidth in the range[1] (BW, 3BW) where BW is the bandwidth of $x(t)$. The highest SNR for $x(t) = e^{j(F_0+f_n)t} rect(\frac{t-\frac{T}{2}}{T})$ is obtained when

$$\mu(t) = x^2(t) + \frac{2A_V}{A} x(t). \tag{3.23}$$

The SNR for equation 3.23 is equal to

$$SNR = \frac{(A^4T + 4A^2A_v^2T)}{N_0T}. \tag{3.24}$$

Equation 3.23 suggests that the optimal filter requires knowledge of A and A_V, which generally are not known in advance. Moreover, the signal at the receiver also contains phase components which were disregarded earlier in the analysis. Note that in contrast to 3.8, the full received signal is defined by 2.22. Their effect on the SNR and range

[1] You can actually ensure that the bandwidth is always 3BW. Analysing figure 3.1 it can be seen that you can insure three times bandwidth for $(F_0 + (N_f - 1)f) \le 2F_0$ when $\mu(t) = 4\frac{A_v}{A}x^2(t) + x(t)$ while for digital domain we have that $2(F_0 + (N_f - 1)f) < \frac{F_s}{2}$ where F_s is the sampling frequency.

resolution should be studied. The starting point for this is the $s_r x(t)$ as received in a realistic scenario.

$$s_{rx}(t) = \beta(A^2x^2(t-2t_d)e^{2j(-2\omega_0 t_d+\theta_0)}) + 2AA_v x(t-2t_d)e^{j(-2\omega_0 t_d+\theta_0+\theta_h)}) + A_v^2 e^{2j(-2\omega_0 t_d+\theta_h)}). \quad \text{(rep. 2.22)}$$

Here, $A_v^2 e^{2j(-2\omega_0 t_d+\theta_h)}$ can be easily removed for $x(t)$ being a frequency modulated signal because it is a DC term. Since $x(t)$ and $x^2(t)$ are orthogonal, $e^{2j(-2\omega_0 t_d+\theta_0)})$ and $e^{j(-2\omega_0 t_d+\theta_0+\theta_h)}$ cab be removed by considering the sum of magnitudes of cross-correlation of $s_{rx}(t)$ and the individual terms $x(t)$ and $x^2(t)$. Therefore, the only term that has to be estimated is $\frac{2A_V}{A}$. Then, $\mu(t)$ takes the form of

$$\mu(t) = x^2(t) + \alpha x(t). \tag{3.25}$$

The SNR for $\mu(t)$ described by 3.25 and $x(t)$ described by 3.22 is given by

$$SNR = \frac{(A^2T + 2AA_v\alpha T)^2}{N_0T(T+\alpha^2T)}. \tag{3.26}$$

Figure 3.2 shows 3.26 as a function of α for different values of $\frac{2A_v}{A}$. From the figure, it can be concluded that for an α in the region of $[\frac{0.2A_v}{A}, \frac{20A_v}{A}]$ the maximum deviation from the optimal value is $-3dB$; for $A_v \approx A$ the maximum deviation from the optimal is also $-3dB$, while the optimal solution for $A_v > A$ is to increase the value of α and the best solution for $A < A_v$ is to decrease α.

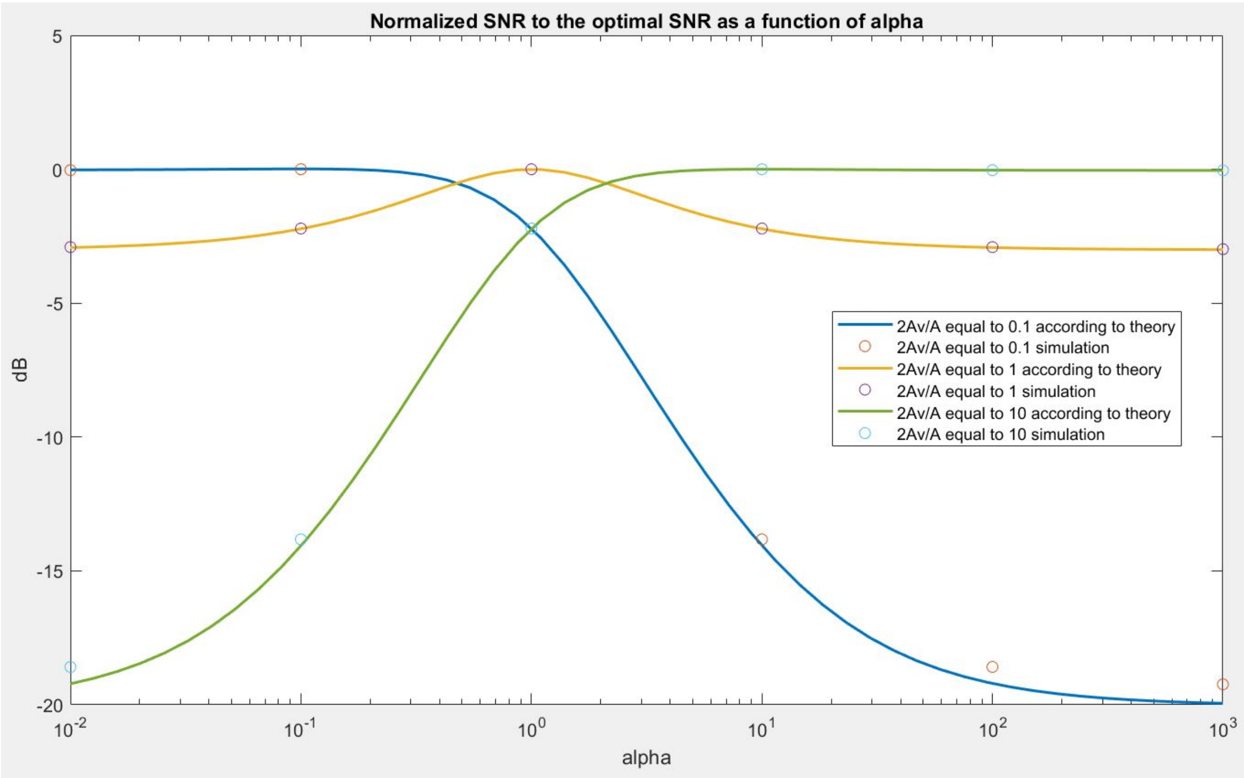

Figure 3.2: Normalized SNR to the optimal SNR for frequency modulation as a function of α.

As can be observed in figure 3.2, for optimal SNR, the value of the $2\frac{A_V}{A}$ has to be known. For the case of $A \approx A_V$ the value of the SNR does not fall below $-3dB$. For $A > 10A_V$ or $A < 10A_V$ the best option is to increase the value of the component $x^2(t)$ or $x(t)$, whichever holds most of the power. This will insure that the SNR decrease is minimized. Besides this, figure 3.2 strengths the idea that for $A >> A_V$ or $A_V << A$ it is more optimal to focus only on one of the components rather than a combination of them. In worse case scenario as suggested by the data in figure 3.2 $\mu(t) = x^2(t) + 2x(t)$ can be used. This will assure SNR as close to maximum as possible for situation when $A \approx A_v$,which is the main region of operation. For the case of $A_V > A$ the decrease in SNR is minimized in detriment to the situation when $A > A_V$. But because the reason why auxiliary transmitter are used in the first is that the can provide more power without

Quasi-Barker code

Consider a complex envelope of form

$$x(t) = e^{j\frac{\pi}{2}l(t)} rect(\frac{t-\frac{T}{2}}{T}) \tag{3.27}$$

where $l(t) = Qb(i)$, $Qb(i)$ is a quasi-Barker bit vector of length N, while $(i-1)T_r \leq t \leq iT_r$, $i \in [1, N]$ and $T_r = \frac{T}{N}$. It should be mentioned here that the goal of this paper is not in presenting the numerical values of these signals. They are easily derivable, for example by using least square method. The aim is to provide a general form of the signal which has the ability to have range resolution close to Barker for $s_{rx}(t) = A^2x^2(t) + 2AA_vx(t)$. An example of such a signal is presented in figure 3.3 and it has a numerical value of $[0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0]$. As can be seen from the figure, quasi-Barker code has a $9.8dB$ increase in peak value compared to the regular Barker code. It should be mentioned that the result was obtained for the situation when $A = 2A_V$. To get the highest SNR for quasi-Barker code as for the linear frequency modulated signal, a filter of form $\mu(t) = x^2(t) + \frac{2A_V}{A}x(t)$ has to be used. Again this represents the optimal case ,because the value of $\frac{2A_V}{A}$ is usually unknown a filter of form $\mu(t) = x^2(t) + \alpha x(t)$ is the practical approach. For $\alpha = \frac{2A_V}{A}$ the filter will approach optimal value ,in worse case scenario the filter can be changed to $\mu(t) = x^2(t) + 2x(t)$. The results obtained with this filter will be shown in numerical simulation section.

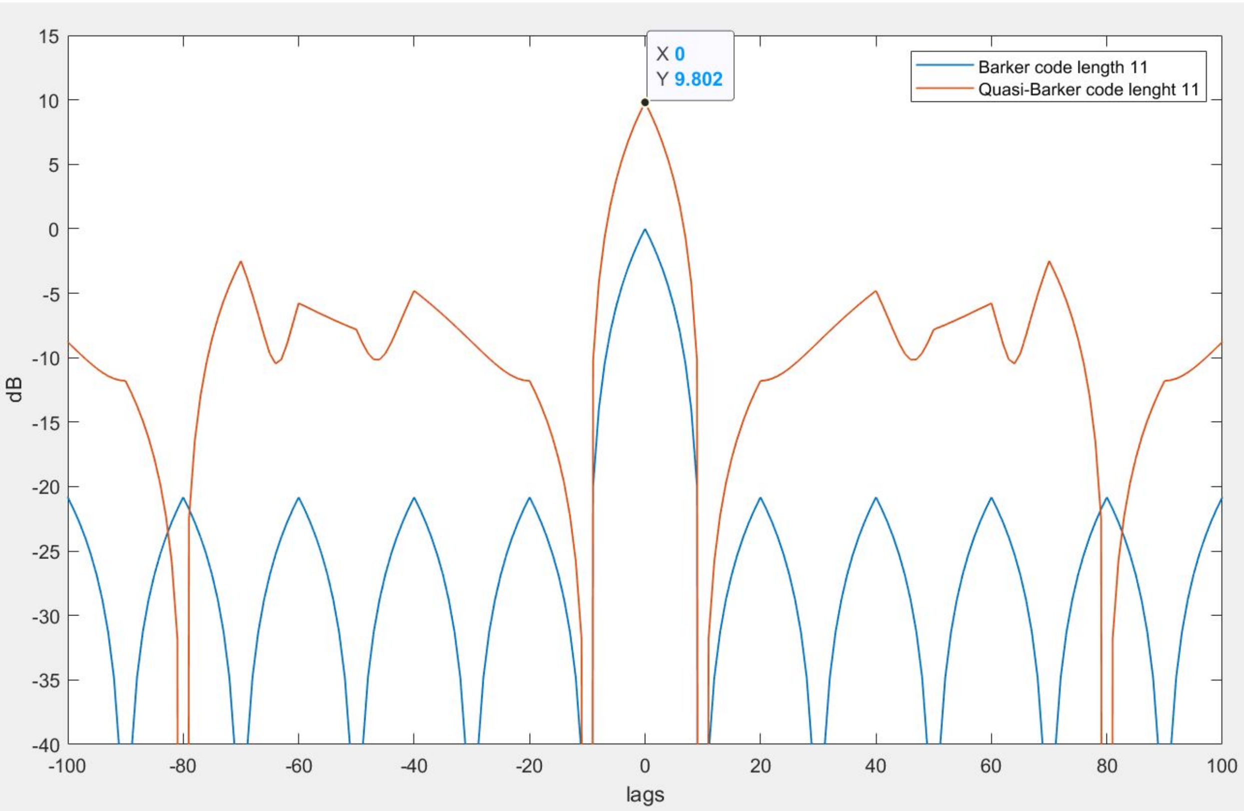

Figure 3.3: Normalized autocorrelation of Barker and Quasi-Barker code of length 11.Normalized to the peak of Barker autocorrelation

3.4 Numerical simulation

For numerical simulation the following assumptions are used. For range/range resolution comparison a Barker code of length 11 will be used as the state of art. The primary reason for this is that while rectangular function has the highest SNR it has a bad range resolution. For the complex envelope of the same energy/power and time duration as a rectangular function, Barker code will have a higher range resolution. For digital domain representation we set $F_s = \frac{1000}{T_{pulse}}$, $F_0 = \frac{F_s}{10}$, $N = 11$, $(N_f - 2)f = BW(barker)$, $\theta_h = U(0, 2\pi)$, and $\theta_0 = U(0, 2\pi)$. For the diode, a zero-bias Schottky diode SMS7630-040 is used, so that $I_s = 5mV$, $n_i = 1.05$ and $V_T = 25mV$. Finally, $R_A = 73\Omega$ which corresponds to the impedance of a half wavelength resonant frequency of a dipole antenna.

In order to prove the points mentioned in the analysis, figures 3.4-3.9 are presented. One of these points was that both linear frequency modulated signal and Quasi-Barker can provide a higher SNR than a regular Barker code, while having the same bandwidth or higher in comparison to Barker code of length 11. It should be stated that for regular Barker $s_{rx}(t) = 2AA_Vx(t)$,the reason for this is that for the case of $s_{rx}(t) = 2AA_Vx(t) + A^2x^2(t)$ Barker code will lose in resolution and the comparison will be unfair. The proof of potential SNR increase can be seen in figures 3.4 and 3.7. So for optimal filter as described by equation 3.10, both linear frequency modulated signal and Quasi-Barker code have a strictly higher SNR that a similar Barker code. For non-optimal case when $\mu(t) = x^2(t) + 2x(t)$, the SNR falls to $-0.6dB$ for $A_V >> A$ in case of linear frequency modulated signal and to $-0.1dB$ for Qausi-Barker. These values can be made smaller by increasing the value of α. It should also be pointed out that the graphs show a significant increase for $A >> A_V$. This happens because both linear frequency modulated signal and quasi-barker code rely on both $x^2(t)$ and $x(t)$ for SNR improvement. In addition to SNR increase, figures 3.5 and 3.6 show that there is a possibility that linear frequency modulated signal provides 3 times increase in bandwidth which translated to 3 times increase in range resolution. It has to be mentioned that this only happens for $A = 4A_V$ or $\mu(t) = 4\frac{A_V}{A}x^2(t) + x(t)$, in general bandwidth is between $[BW(x(t)), 3BW(x(t))]$. For the case of $\mu(t) = 4\frac{A_V}{A}x^2(t) + x(t)$,$x(t)$ and $x^2(t)$ not overlapping in frequency domain bandwidth of linear frequency modulated signal is independent of the value of A and A_V and it is always equal to $3BW(x(t))$. The drawback of such a configuration is that you lose in SNR, to estimate this lost compared to the optimal filter equation 3.26 can be used. One of the results during problem analysis was that orthogonality between $x^2(t)$ and $x(t)$ represents the major factor which dictates the trade off between SNR and range resolution. While indirectly this can be observed in figures 3.4,3.7,3.5 and 3.3 where the SNR and bandwidth of these signals are presented. A comparison between figures 3.4 and 3.7 puts forward the idea that quasi-Barker has the potential to provide higher SNR. The reason for this increase in SNR is a smaller bandwidth of the qusi-barker code. This can also be noticed by investigation equation 3.27 which idicates that for different vectors $l(t)$ both bandwidth and orthogonality between $x^2(t)$ and $x(t)$ changes. This as a direct result affects the SNR, explaining the higher SNR for quasi-Barker code. Finally, while both linear frequency modulated signal and quasi-Barker code show an increase in SNR for $A > A_V$(again this happens because for regular Barker code only component $x(t)$ was used, you can easily choose the component $x^2(t)$ and you will get the graphs mirrored on around $A = A_V$) for the case of $A_V < A$ they approach the one of only using $x(t)$. This suggest the idea that for big difference in the value of A and A_V it is more optimal to focus only on one of the components. The primary reason for this is while for a single component the value of α is not needed this is still the case for the situation of using both components. For a non optimal value of α the value of SNR will decrease.

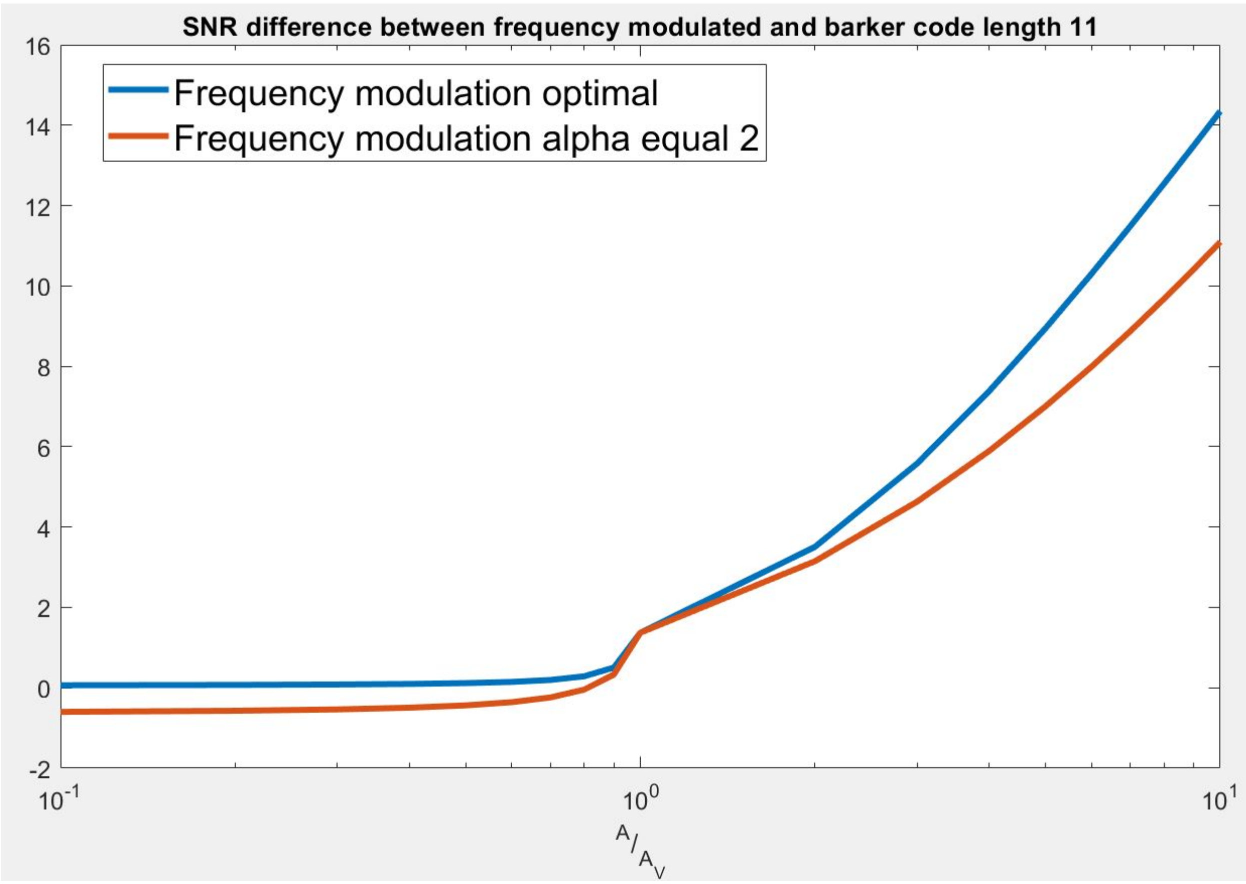

Figure 3.4: SNR difference between Barker code of length 11 and linear frequency modulated signal, both having the same bandwidth

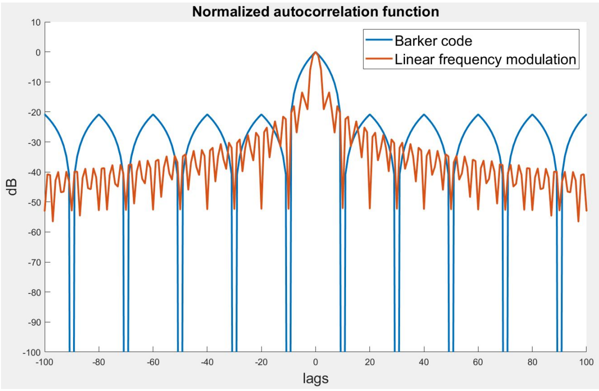

Figure 3.5: Autocorrealtion of barker code of length 11 and linear frequency modulated signal of the same bandwidth for $T = 1\mu s$

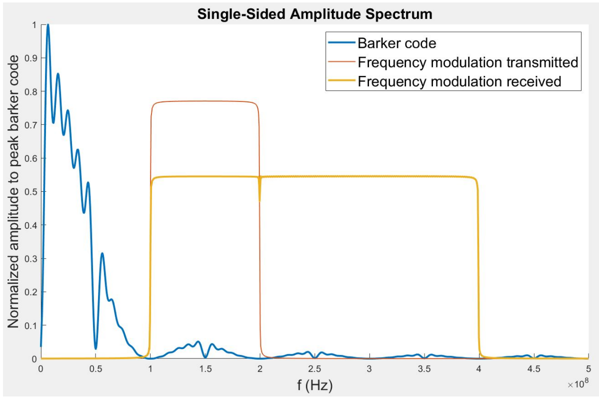

Figure 3.6: Frequency domain representation of the barker code of length 11 and the 3 times bandwidth of the signal $x^2(t)$ and $x(t)$

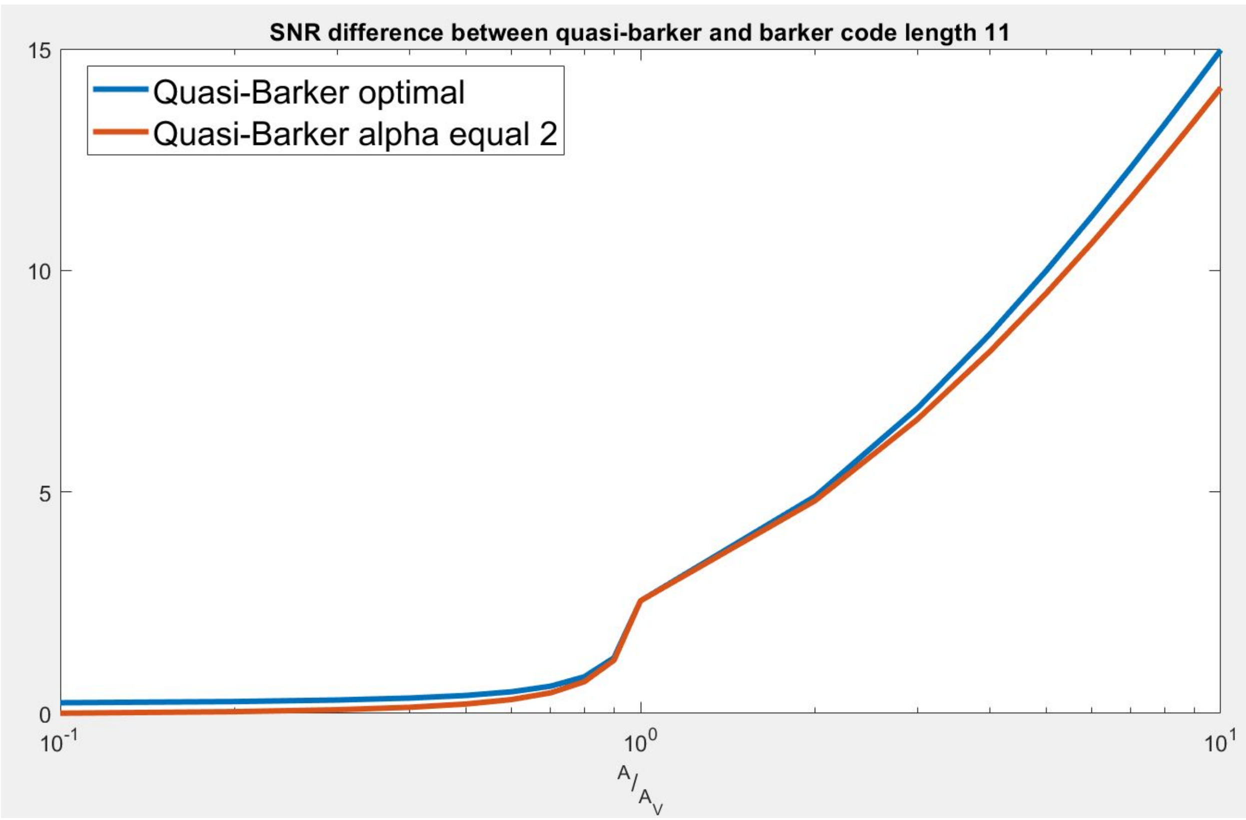

Figure 3.7: SNR difference between Quasi-Barker code of length 11 and Barker code of length 11

To evaluate the range improvement due to increase in SNR, a formula has to be derived which relates the increase in SNR to the increase in range. Consider some amplitudes A_{rang} and A_{aux} which are the amplitudes of the signals near ranging transmitter and auxiliary transmitter. The amplitudes arriving at the receiver are proportional to $\frac{A_{rang}}{d}$ and $\frac{A_{aux}}{d}$ (assuming that the distance between ranging and auxiliary transmitter is way smaller than the distance to the tag). Both $x^2(t)$ and $x(t)$ amplitudes are proportional to $\frac{1}{d^2}$, because of A^2 and $2AA_v$. Finally the signal arriving at the receiver is decreased by $\frac{1}{d}$ due to inverse square law. This translates to a total $\frac{1}{d^3}$ decay in amplitude.Taking this into account suppose we have two methods to estimate the range, method 1 and method 2. Then the ration between their maximum measurable distance for these two methods can be related to the difference in their SNR as follows

$$\frac{d_{max1}}{d_{max2}} = 10^{\frac{\Delta SNR}{60}} \tag{3.28}$$

where $\Delta SNR = SNR_1 - SNR_2$ is the difference of SNR in dB between methods 1 and 2, while d_{max1} and d_{max2} are estimate of the maximum measurable range for method 1 and respectively 2.

Using equation 3.28 on the graphs 3.4 and 3.7 estimate of the potential range increase can be determined. This potential increase is shown in figures 3.8 and 3.9. In general they just reinforce the points mentioned above, specifically that both linear frequency modulated signal and quasi-Barker code have the potential to increase the detectable range of the harmonic radar.

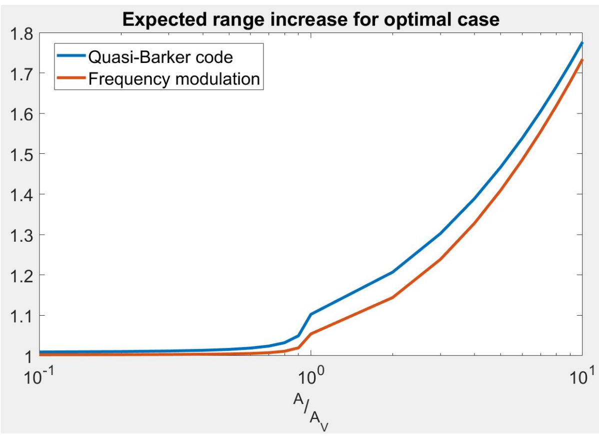

Figure 3.8: Expected range increase for optimal matched filter

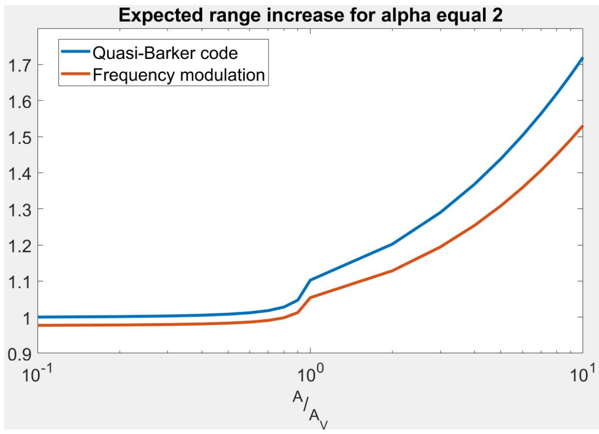

Figure 3.9: Expected range increased for alpha equal 2

Chapter 4

Conclusion and Recommendation

4.1 Conclusion

The problem of increasing the range/range resolution in harmonic radar system was considered. The starting point was a harmonic radar working in an environment with helper transmitters that emit unmodulated carrier signals. The goal was to find a complex envelope used for ranging which provides an optimality in terms of range and range resolution. The analysis showed that in the case of using helper transmitter there is no general complex envelope which has a peak in both SNR and range resolution. Nonetheless, the paper presents two signals which have the potential to have higher SNR than a Barker code while having the same bandwidth. It should be pointed out that for the same bandwidth, average power, energy and time duration of the ranging signal, it is possible to come up with signals that provide higher SNR than the two signals shown in this paper. The problem then becomes how much you want to lose in range resolution. The real benefit of these signals is that they rely on both $x^2(t)$ and $x(t)$ for range increase, while at the same time having a range resolution not lower than that of the single term $x(t)$. The latter point is proven by the analysis and numerical simulation that show that these signals have the potential to provide a higher SNR than only using $x(t)$ or $x^2(t)$ while having the same bandwidth as autocorrelation of $x(t)$.

4.2 Recommendations

Before mentioning any recommendations it should be stated that all the recommendations are for the region where $\frac{A}{A_V} \in [0.1, 10]$. For the case when $\frac{A}{A_V}$ is out of that region, the best solution is to focus on the components which provides most of the power. For the two solutions proposed one major drawback is that the value of the $\frac{2A_V}{A}$ has to be known. First of all while value of A_V is random ,the value of A is deterministic. Moreover for small number of auxiliary transmitter the value of A_V is easy to estimate. As the number of auxiliary transmitter increases the value of A_V decreases, because adding a high number of amplitude with different phase decrease the average value of the total amplitude. So then the designer is imposed to put auxiliary transmitter in such configuration that maximizes the average amplitude. This in exchange makes the estimation of A_V easier. Another approach is to apply and adaptive system to the problem. The value of α can be made adjustable so it maximises the SNR. Because the environment is noisy, there is a possibility that the system will maximize the noise or maybe the wrong component. To deal with this, a threshold can be designed that limits the probability of this situation. This threshold can be easily derived by arguing about the statistic of the ranging signal arriving at the receiver. Finally, the conditions imposed during the analysis of the problem are strict and can be actually loosen. Equation 3.15 shows the possibility to generate signals which can potentially offer higher SNR, this then translates into how much range resolution are you willing to lose.

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Appendix A

Appendix chapter

A.1 Solution analysis for random variables

The optimal problem stated above represents a system of equations of two variables $(x(t), \mu(t))$.This suggest dividing the problem in steps and building the solution by considering one the functions as known and solving for the other one.It was mentioned before that the nature of SNR is random.One thing to notice is that for a nonmoving target(or at least a one that moves really slow) the SNR is random in spacial space not temporal(temporal space is considered the time of measurement).As these spaces can be considered independent,$A_v, t_d, \theta_0, \theta_h$ can be treated as constants without affecting the optimality given by the expectation value.

So lets examine the problem of finding the optimal function $\mu(t)$,which gives the highest expected SNR,when SNR is described by equation 3.6 and $x(t)$ known.Because the noise is modeled as complex AWGN.Moreover $x(t)$ represents the analytic form of the real physical signal ,the solution for this problem is well known and it is given by the match filter.From here results that the maximum SNR as a function of $\mu(t)$,for a given complex envelope $x(t)$ is equal to

$$\mu(t) = cs(t), c \in \mathbf{C} \tag{A.1}$$

where c is a complex constant.

It should be clearly stated at this point that by analysing the form of the equation 3.9 and equation 2.4 it is very easy to notice that unless $x^2(t) = x(t)$ the solution clearly requires knowledge of $A_v, t_d, \theta_0, \theta_h$.And while they are treated as constant during the measurements,they are still random as in if their value is unknown.Because there is a possibility that the maximum is of form $x^2(t) = x(t)$,we will consider a hypothetical scenario where these values are known.In case the optimal solution is $x^2(t) = x(t)$ these values are not needed,as if $A_v, t_d, \theta_0, \theta_h$ are known.In case the solution is different from $x^2(t) = x(t)$,$\mu(t)$ is deterministic by nature so hypothetical scenario is clearly not realistic.Nonetheless in such a situation typically estimator will be used,this will move the random nature of $A_v, t_d, \theta_0, \theta_h$ to that of the estimators.So hypothetical scenario corresponds with situation of ideal estimators.This in principle makes the problem the one of designing a good estimator.

Using the proprieties of the matched filter for the hypothetical scenario,equation 3.6 can by simplified to

$$SNR(x(t)) = \frac{max(\int_0^T s(t)s*(t)dt)}{N_0T} \tag{A.2}$$

Recognizing that $A_v^2 e^{2j(-2\omega_0 t_d+\theta_h)}$ in $s(t)$ is a DC term so it can be easily removed,T and N_0 does not depend on complex envelope $x(t)$ and $s(t)$ can be shifted by $2t_d$ without affecting the result.Equation 3.10 can be written as

$$max_{x(t)}[\beta^2 \int_0^T (A^2x^2(t)a_q + 2AA_Vx(t)a_l)(A^2x^2(t)a_q + 2AA_Vx(t)a_l)^*dt] \tag{A.3}$$

where $\alpha_q = e^{2j(-2\omega_0 t_d+\theta_0)}$ and $a_l = e^{j(-2\omega_0 t_d+\theta_0+\theta_h)}$.Because $|x(t)| = 1$ and β,A,A_v(this is considered only spatially random) are considered constant,moreover $x(t)$ can be written as $e^{j\phi_x(t)}$,the maximum problem translates to

$$max_{x(t)}[\int_0^T x^2(t)x^*(t)a_qa_l^*dt+\int_0^T ((x^2(t))^*x(t)a_q^*a_ldt] = max_{x(t)}[2\int_0^T cos(\phi_x(t)+\theta_0-\theta_h-2\omega_0t_d)dt] \tag{A.4}$$

$\theta = \theta_0-\theta_h-2\omega_0t_d$ is a uniform random variable on interval $[0, 2\pi]$ so two possible situations are considered:

1.For $x^2(t)$ and $x(t)$ being orthogonal (all of this considering that DC component can be efficiently removed) $\int_0^T cos(\phi_x(t)+\theta_0-\theta_h-2\omega_0t_d)dt = 0$ by the definition of the orthogonality.Hence for $x^2(t)$ and $x(t)$ being orthogonal and hypothetical situation being true the value of the $x(t)$ does not influence SNR,which on average is given by

$$SNR_{average} = \frac{\beta^2(A^4T + 4*A^2A_aT)}{N_0T} \tag{A.5}$$

where A_a is the mean of the random variable A_v^2.It should be stated that this solution corresponds with the hypothetical scenario where the values of the $A_v, t_d, \theta_0, \theta_h$ can be estimated exactly.Because of the orthogonality for a non hypothetical scenario only the value of A_v has to be estimated.So for a realistic scenario SNR is given by

$$SNR = \frac{\beta^2(A^4T + 4*A^2A_v^2\alpha T)(A^4T + 4*A^2A_v^2\alpha T))}{N_0T(A^4T + 4*A^2A_v^2\alpha^2T)} \tag{A.6}$$

where α is dependant on the estimator of A_v.

2.For $x^2(t)$ and $x(t)$ not being orthogonal there is no possible way to get rid of the random variable θ unless $x^2(t) = x(t)$.Even if one component is eliminated,because $x^2(t)$ and $x(t)$ are not orthogonal removing one signal will decrease the power of the signal remaining,which will decrease the SNR.In the case matching filter is applied directly for θ uniformly distributed on $[0, 2\pi]$ on average $2\int_0^T cos(\phi_x(t)+\theta_0-\theta_h-2\omega_0 t_d)dt$ is equal to 0.On thing worth mentioning is that for $x^2(t)$ and $x(t)$ not being orthogonal to DC term,removing the DC becomes inefficient and decreases the SNR.In case DC term is kept and the matching filter is applied directly on the $s(t)$ as defined by equation 2.4,moreover $\phi_x(t) = 2n\pi$ where n is a natural number,the SNR is maximum and on average is given by

$$SNR_{average} = \frac{\beta^2(A^4T + 4A^2A_aT + A_bT)}{N_0T} \tag{A.7}$$

where A_a is the mean of the random variable A_v^2 and A_b the mean of the random variable A_v^4.When DC is perfectly removable A_b becomes 0.

As for the case of orthogonality this scenario corresponds with the hypothetical situation where the estimator is ideal.For the realistic scenario and considering only the A_v is estimated the SNR is given by

$$SNR = \frac{\beta^2 l(t)l^*(t)}{N_0T\int_0^T(A^2x^2(t) + 2AA_v\alpha x(t))(A^2x^2(t) + 2AA_v\alpha x(t))^*dt} \tag{A.8}$$

where $l(t) = (A^4T\alpha_q + 4A^2\alpha^2A_v^2\alpha_lT + \int_0^T(2A^3A_vx^2(t)x^*(t)a_l^* + 2A^3A_v(x^2(t))^*x(t)a_q^*)dt)$ and α depends on the estimator of A_v.