BACHELOR THESIS

Stop-skipping pattern of bus line optimizing waiting, travel and in-vehicle times

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Preface

The contents of this report contribute to a better operating system of the Twents bus line. By applying the stop-skipping strategy a new pattern of the bus route appears which lowers bus travel time. The 'Twents' bus service is located in the area where I live so that makes it even more interesting. Furthermore, I used to travel with the bus, and I have experienced common problems that occur while taking the bus myself. This encourages the search for better solutions.

The data used in this research is provided by Keolis Nederland in cooperation with the University of Twente. The content is confidential and will therefore not share this with others and only results are evaluated in this report.

COVID-19 did not only change the use and service of the buses, but also the writing of this thesis report. This report is fully written from home which is different from the regular process. No travel time is needed to move to the company location and working alone lowers distraction weaknesses.

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Summary

This bachelor thesis is written in cooperation with Keolis Nederland and the University of Twente. It starts off with an introduction of the problem and states the research question that will be answered at the end of this report. This will be followed by a literature review, which contains similar works and preliminary research. The literature review is used to gain knowledge about the subject and gather information regarding the theory of stop-skipping. This results in the methodology for this research which is divided into two parts: First, the methodology for the data analysis will be described and second the model formulation of stop-skipping strategy. Section 6 presents the results of the data analysis, a substantiation for the bus line choice and the model results. In the conclusion the research question is answered, and a discussion is written to add all points of attention. At last, recommendations for further research are given.

Public transit company Keolis Nederland experiences revenue losses due to active COVID regulations in the Netherlands. These losses are caused by a lower demand and therefore new strategies are urgent in order to keep the losses minimal. This research covers the strategy of stop-skipping which skips 1 or more stops to reduce the travel time. This strategy is applied on a bus line in the region

'Twente' in the Netherlands. The choice for this bus line is reinforced by a data analysis on all bus lines in the area to see what bus lines suffer the most.

The stop-skipping strategy is displayed as a model that contains a mixed-integer non-linear mathematical program. Central is an objective consisting of three terms that is minimized based on constraints. These three terms are the passenger waiting time, bus travel time and passenger in-vehicle time, respectively. With branch and bound the model finds an optimal pattern where 1 or more bus stops are skipped. The values of the three objective terms are compared to the old situation where all bus stops are served.

The solution of the model shows a deviating stop-skipping pattern in the last trip. Therefore, an adapted solution is analysed by using the same stop-skipping pattern for trip 4 as found in trip 2. Comparing the time values of the adapted solution with the current situation showed that the bus travel time and in-vehicle time decreases with 2.6% and 7.1% respectively, but the passenger waiting times increases with 5,6%. A sensitivity analysis confirms the choice for the cost factors in this research and shows that other cost factors lead to either unrealistic stop-skipping pattern where too many stops are skipped, resulting in high waiting times.

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1 Introduction

The public transit sector in the Netherlands has suffered a lot from the COVID-19 regulations that became active in March 2020. Face masks in public transport are mandatory and people are requested to keep 1.5-meter distance from each other. Furthermore, the regulations included closure of schools and cultural activities which contributes to a decrease in trips. This decrease is experienced in public transport as well, which leads to huge revenue losses among public transit companies. These costs can largely be compensated by the Dutch government, but the remainder should still be paid by the transit companies themselves.

The bus service Twents, a sub-service of one of the bigger public transit companies Keolis Nederland, experienced similar trip decreases which can be deduced from the number of check-ins and check-outs. Currently, buses operate under normal circumstances although the demand has decreased.

Because the COVID pandemic is still dominating the way of living there is an urgent need of new developments of better operating services. These developments should contribute to a decrease of the operational costs. The costs for the staff and the costs for operating a bus are mostly determining the amount of the operational costs. A good way to decrease the operational costs is by lowering the travel time of the bus (Lee, Shariat, & Choi, 2014). Furthermore, the wishes of the stakeholders should be taken into account. The most important stakeholder, the user, should be satisfied because they altogether provide the revenue of the transport companies. This is done using the next two factors: the user waiting times and the user in-vehicle time. Generally, users have a higher appreciation for low waiting and in-vehicle times. This research therefore considers the travel time, user waiting time and user in-vehicle time in order to decrease the operational costs of Twents. This decrease is obtained by following a strategy called stop-skipping. This strategy skips certain stops in a bus line to decrease the travel time of the bus, resulting into the following research question:

"*What is the best stop-skipping pattern of a Twents bus line considering user waiting times, user in-vehicle time and bus travel times?*

The model is tested on a bus line within the system of Twents. To determine what bus line will be useful upon testing, a data analysis of the recent trips on all bus lines is done. A bus line is chosen based on passenger division, landscape type, and the demand division. The stop-skipping model is a mixed integer non-linear program (MINLP). This refers to a program with continuous and binary variables and non-linear equations in the objective and/or constraints (Guide, 2020). The model is solved with the branch and bound algorithm. This implies that it determines a solution, which will be further split into several sub solutions. This keeps going until an optimal solution is found. The solution consists of optimal values for the travel time, user waiting time and user in-vehicle time which will be compared to the current situation when all bus stops are served. The model determines the optimal solution by changing the value of the binary variables, which indicate if a bus stop is served or not. Therefore, the optimal solution will contain a stop-skipping pattern as well. To assess the robustness of the optimal solution a sensitivity analysis is conducted. This analysis will cover the values of the cost factors that determine the cost of the travel time, user waiting time and user in-vehicle time. At last, a discussion about this research weaknesses and possible improvements is raised and recommendations for further research are given.

2 Literature review

This section contains the literature written about the stop-skipping concept that will be used in order to optimize the bus service. There is already quite a lot of literature written about the stop-skipping before COVID because it was proven to lower the travel time of the bus. (Liu, Yan, Qu, & Zhang, 2013) defined the stop-skipping as a scheme to skip one or more stops to reduce the travel time of the bus. This can be applied if the bus is late and behind schedule to avoid dwell times and increase operation speed. (Gkiotsalitis & Cats, 2021) listed a number of literature works which use the stop-skipping strategy for optimization. A recurring feature which shows up in all stop-skipping literature is the use of an optimization model. This model consists of variables, constraints and an objective. This objective consists of the different costs that are made during a bus trip, and these costs will be optimized.

(Gkiotsalitis & Cats, 2021) also makes a distinction between multiple and single trips that are considered. The advantage of evaluating an objective for only one trip is the model simplicity. The model can be solved with relatively low computation time and a simple solving method. However, consecutive trips do play a role in the decision making of stop-skipping. It is often discouraged to skip two consecutive trips to avoid very long waiting times for passengers. This can only be prevented by considering multiple trips.

The study by (Liu, Yan, Qu, & Zhang, 2013) focuses next to the stop-skipping strategy on deadheading problems. This is an advanced strategy of the stop-skipping problem and focusses on bus lines where the deadheading bus departs (almost) empty to a designated stop. It skips stops based on the same grounds as the stop-skipping strategy. This study also criticises previous studies for aiming too much on reducing passenger waiting times instead of reducing passengers' in-vehicle travel time and operating costs. Therefore, the objective includes three terms that takes care of the passenger waiting times, in-vehicle travel time and operating costs due to bus travel time. A similar study is conducted by (Zhang, Huang, Liu, & Vu, 2020). Additionally, the capacity of the bus is taken into account as well as a few other factors which result in a more complex model. The agent-based modelling and simulation is used to simulating the bus system which has never directly been used on studying bus operating systems before.

Another study by (Wang, De Schutter, van den Boom, Ning, & Tang, 2014) develops a more efficient bilevel approach, which involves a mixed-integer nonlinear programming problem (MINLP). The study values passenger satisfaction which is influenced by waiting times, passenger in-vehicle times and the number of transfers. The model objective includes the same terms as (Liu, Yan, Qu, & Zhang, 2013) with two additional penalties for waiting time of passengers left by the last train and for the arrival time of the last train.

More recent studies are done using the stop-skipping strategy to solve COVID-19 related problems. The demand decreased sharply in public transport as stated in (Hörcher, Singh, & Graham, 2021), which causes imbalance between the costs and revenues of operating the service. Due to this, public transport companies were forced to make changes in their services and adjust timetables (Vickerman, 2021), (Wielechowski, Czech, & Lukasz Grzeda, 2020). A study by (Gkiotsalitis K. , 2020b) investigates the stop-skipping strategy on a rolling horizon. This indicates that real time is taken into account by solving the model repeatedly, by moving the considered time interval forward. Three types of methods were mentioned to solve the model:

- Brute force

This method does not contain any algorithm or heuristic and is trying all possible solutions until it finds a good solution corresponding to the given objective, variables and constraints. (Gkiotsalitis K., 2020b) refers to this method as a good alternative if the solution set is not too extensive.

- Sequential hill climbing (S-HC)

S-HC is a heuristic approach which evaluates the stop-skipping options in a sequential order. This results in a way smaller solution set than the brute force and branch & bound approach.

- Genetic algorithm

The genetic algorithm is a more complex algorithm that uses a population in order to get to the best solution. The population has members which are represented by a matrix which contains values of 0 and 1. This algorithm also contains a smaller computation time because its evaluation set is determined by the population size multiplied by the maximum number of generations.

Comparing the mentioned studies, there are a lot of possible methods to solve a stop-skipping problem. A study by (Gkiotsalitis K., 2020a) uses the stop-skipping strategy to develop a better bus operating system. The model developed is a MINLP and is tested on the bus line between Hengelo and Enschede. The model is solved with branch-and-bound approach. The objective of the MINLP consists of three terms, the costs of the waiting time, bus travel time and in-vehicle crowding. The study of (Gkiotsalitis K., 2020a) provides very useful information as it considers the same research area, and the model is not too complex. Although, there are a few differences:

1. The use of capacity restrictions

The study considers two types of capacity, soft capacity and hard capacity. The soft capacity is the capacity that is allowed under the social distancing measurements and the hard capacity is the regular capacity. Keolis stated not to refuse people if the soft capacity is reached. They mention it to be the full responsibility of the user considering exceeding the soft capacity. Therefore, no distinction between soft and hard capacity is considered and only the maximum capacity of a bus is taken into account.

2. The lack of passenger in-vehicle travel time

Because this research values the stakeholders' requirements and wishes, the passenger in-vehicle travel time must be considered. This means that the capacity penalty in the objective as described in (Gkiotsalitis K., 2020a) will be replaced by a passenger in-vehicle cost.

This research will therefore provide an evaluation of a stop-skipping problem with a three terms objective considering costs for passenger waiting time, passenger in-vehicle time and bus travel time. The branch-and-bound method is used to solve the MINLP model and multiple trips will be considered, which means there is dealt with a rolling horizon. The model will be tested on a bus line in Twente, a region in the Netherlands, to see if it has actual benefits. These bus lines will further on be determined based on a data analysis of the passenger trips occurring in the bus service for the time span between March 2019 and July 2020. In the next section a detailed description on how the bus lines are determined is given. In table 2, a summary is given of the discussed literature works and their characteristics and table 1 provides an overview of different objective terms.

Table 1 Objective terms

Objective terms	
O1	Passenger waiting time
O2	Bus travel time

O3	Passenger in-vehicle time
O4	Energy consumption
O5	Bus capacity maximization
O6	Cost 'forced off' passengers
O7	Access time

Table 2 Summary of stop-skipping literature

Study by	*Strategy*	*Trips considered*	*Math program*	*Objective terms*	*Solution method*
(Gkiotsalitis K., 2020a)	Stop-skipping	Multiple	Mixed-integer nonlinear	O1 + O2 + O5	Branch and bound
(Gkiotsalitis K., 2020b)	Stop-skipping	Multiple	Integer nonlinear	O1 + O2 + O3	Brute force and heuristics
(Gkiotsalitis K., 2019)	Stop-skipping	Multiple	Integer nonlinear		
(Chen, Adida, & Lin, 2013)	Holding	Multiple	Linear	O1	Heuristics
(Liu, Yan, Qu, & Zhang, 2013)	Stop-skipping and deadheading	One	Integer nonlinear	O1 + O2 + O3	Genetic algorithm
(Lee, Shariat, & Choi, 2014)	Stop-skipping	Multiple	Mixed-integer nonlinear	O1 + O2 + O3 + O7	Genetic algorithm
(Sun & Hickman, 2007)	Stop-skipping	One	Integer nonlinear	O1 + O2 + O6	Brute force
(Zhang, Huang, Liu, & Vu, 2020)	Stop-skipping and holding	Multiple	Mixed-Integer linear	O1 + O3	Stochastics
(Wang, De Schutter, van den Boom, Ning, & Tang, 2014)	Stop-skipping	Multiple	Mixed-integer nonlinear	O1 + O2 + O3 + O4	Bi-level approach
This research	Stop-skipping	Multiple	Mixed-integer nonlinear	O1 + O2 + O3	Branch and bound

The solution found by the model contributes to the described problem that is caused by COVID-19. Because of the lower demand along the bus line, the stop-skipping model has more opportunity than with an actual demand. With the new solution the bus will save travel time, which will save costs and thus minimize the revenue losses cause by COVID-19.

3 Methodology

As a result of the decision by the Dutch government to take measures against COVID-19, the number of passengers making use of bus service ‘Twents’ dropped. For adaption of the stop-skipping strategy research must be done to the demand of the bus lines. A bus line with a high frequency would result in a much more effective solution than a bus line with a small frequency. Since COVID-19 is the major cause of the problem, a distinction is made between two scenarios: (1) A pre-COVID scenario, which is the period before the Dutch government took measurements against the virus and (2) a COVID scenario, which is the period after two months that the measurements were taken. By

comparing these two scenarios, the bus lines that change the most can be identified. These changes are evaluated based on three criteria: average trip demand per week, passenger type and total demand per bus stop. From the results, one bus line is chosen that will be used for stop-skipping. As mentioned before, the model will be solved with the branch-and-bound method. This approach is based on principle that a set of solutions can be divided into smaller subsets (-, 2001). For each of these subsets, an evaluation will be done in order to find the best solution among them. The results of the stop-skipping strategy will be returned as the total passenger waiting times, total passenger in-vehicle time and total bus travel time. This will be compared to the situation now which does not apply stop-skipping and a conclusion is drawn. The remainder of the methodology is split into two parts, the data analysis of the bus service demand and the model formulation of the stop-skipping part.

3.1 Data analysis

The data analysis consists of three parts that together form the substantiation for the decision of the chosen bus line. The average demand per week is determined to see which bus lines show the biggest differences of demand between the COVID- and pre-COVID scenario. Second, the changes in passenger division between the COVID- and pre-COVID scenario are analysed to exclude bus lines that show a deviation of the passenger division. Third, the demand per stop is determined to track down the bus lines that have the biggest count of low demand bus stops. The fourth section contains the landscape division of the bus lines. This is done because a bus line with a mixed landscape is favoured.

3.1.1 Average demand per week

The average demand per week for each bus line is determined by using a data set of trip frequencies. This dataset contains the number of trips for all dates and hours for each line between March 2019th and July 2020th. First, the frequencies of each column are stored in a separate table. Then, for each line the number of trips is grouped for each date. The result is analysed and the average demand for each week is calculated. From these averages, a new average is calculated which gives the average week average. This is done for both scenarios and the difference is determined by

$$\Delta \boldsymbol{Q} = \boldsymbol{Q}_{precovid}^{weekly} - \boldsymbol{Q}_{covid}^{weekly} \quad \textbf{Equation 1}$$

3.1.2 Passenger division

The passenger division is determined for all bus lines for the two scenarios. Since the dataset of trip frequencies contains only general information, the bigger set is used that contains data about number of passengers, date, stop_in, Stop_out, hour, line_in, line_out, audience and week number. The passenger division is determined by grouping the number of passengers for each audience type for each line. The two scenarios are compared by dividing the number of passengers for each passengers group by the total number of passengers. This is shown as:

$$\boldsymbol{N_{audience}} = \frac{\boldsymbol{Q_{audience}}}{\sum_{i=1}^{m} \boldsymbol{Q_{audience,i}}} \qquad \textbf{Equation 2}$$

Where i is the index $1 \leq i \leq m$, where m is the end index. This gives the percentages for each passenger group and gives a clear view of the differences between the two scenarios. The result is stored in a bar plot as well as in a table.

3.1.3 Demand per stop

The demand is determined in a same way as the passenger division but instead of grouping the passengers by the audience, they are grouped by the stops. The result is stored in the same way as the passenger division. The demand per stop is used to see if a bus line has a lot of bus stops that have potential to be skipped. When a line only has stops with high boardings, the eventual waiting times will be a lot higher than a line that has stops with low boardings. The low demand bus stops are detected by computing the relative boarding rates of each bus stop and counting the bus stop that satisfies

$$\sum_{i=0}^{|S|} \boldsymbol{Q_i}, \quad \boldsymbol{Q_i} \leq \mathbf{0.001} \qquad \textbf{Equation 3}$$

So, the bus stops that have a boarding rate of 0.1% of the total boardings are counted.

3.2 Model formulation

The model considers multiple trips where one trip is described as $N = \{1, 2, \ldots, |N|\}$ where N is the number of trips occurring within a certain time horizon. At the start of a new time horizon, there is already determined what the skipped stops of all trips within this time horizon will be. Two extreme cases can occur: First, the time horizon contains only 1 trip, which means that the stop-skipping strategy is adjusted to a single trip problem as described in (Liu, Yan, Qu, & Zhang, 2013). Second, the time horizon includes the all-daily trips of the bus line, which makes it tactical planning problem (Gkiotsalitis, 2019). This section will further elaborate on the assumptions made for simplification. Also, the nomenclature that is used will be listed and the variables and constraints are defined and at last, theproblem objective is formulated. These will form together the mathematical model of the stop-skipping strategy.

3.2.1 Assumptions and nomenclature

For almost every model that is tried to simulate reality, assumptions are made. The assumptions for this model are as follows:

1. The arrival rate of passenger at bus stops are random. This is because it cannot be determined how passenger coordinate their arrival times based on the bus service.(Welding, 1957); (Randall, Condry, Trompet, & Campus);
2. Buses that serve the same line do not overtake each other. This means that if a bus fails due to circumstances and cannot finish their line, the remaining of the trip will be unserved. This assumption is often taken while investigating bus service on a single line. (Chen, Adida, & Lin, 2013); (Gkiotsalitis K., 2020a); (Gkiotsalitis K., 2020b);

3. An origin-destination pair cannot be skipped by two consecutive bus trips of the same line (Sun & Hickman, 2007), (Fu, Liu, & Calamai, 2003). This, in order to prevent negative effects of increased passenger frustration and prevent higher waiting times because of unserved passengers at the skipped station.
4. For all stops served by the bus, the number of passengers boarding, and alighting is known. (Sun & Hickman, 2007)
5. The capacity of the buses is not considered explicitly. Looking at the concerned bus lines it is very unlikely that the maximum capacity will be reached (Sun & Hickman, 2007). Furthermore, it could be that the maximum capacity is reached in a very extreme situation, but for simplification and feasibility this will not be concerned.

Most of the parameters and variables are all defined based on a trip and its stops. The set $N = \{1, 2, ..., |N|\}$ is considered to be the set of bus trips. Each trip has a fixed number of stops which is formulated as $S = \{1,2, ..., |S|\}$. The following parameters are used:

Table 3 Parameters

Parameter	***Description***
$t_{i,j}$	The travel time needed for a bus trip i to get from stop $j-1$ to stop j
p_1	Alighting time for a passenger
p_2	Boarding time for a passenger
δ	Difference of bus acceleration and deceleration
λ_{jk}	The passenger arrival rate at stop j for passengers who travel to stop k for k > j
w_1	Cost factor for first term regarding waiting times
w_2	Cost factor for second term regarding travel times
$\tilde{d}_{i,1}$	Planned departure time from first stop to last stop
$\widetilde{w}_{1,jk}$	Number of passengers waiting for the first trip travelling from j to k where $k > j$

Where $i \in N$, $j \in S$ and $k \in S \mid k > j$.

3.2.2 Variables and constraints

The variables used to determine the constraints are:

Table 4 Equations

Variable	***Description***
$x_{i,j}$	A decision variable for trip i which is 1 if stop j is served and 0 otherwise
$d_{i,j}$	The departure time of trip i at stop j
$a_{i,j}$	The arrival time of trip i at stop j
$k_{i,j}$	The dwell times of trip i at stop j, i.e., the time of boarding and alighting of passengers
$w_{i,jk}$	The number of passengers waiting for trip i and travelling from stop j to stop k for $k > j$
$I_{i,jk}$	The number of passengers that are skipped by bus i and travel from stop j to stop k
$m_{i,j}$	The number of passengers that are skipped by bus i at stop j
$u_{i,j}$	The number of passengers boarding trip i at stop j
$v_{i,j}$	The number of passengers alighting trip i at stop j
$b_{i,jk}$	The number of passengers boarding trip i at stop j who travel to stop k for $k > j$
$\gamma_{i,j}$	The bus passenger load of trip i departing from stop j
$h_{i,j}$	The bus headway between trip i and $i-1$ at stop j

Where $i \in N, j \in S$ and $k \in S \mid k > j$

Additionally, a set of constraints are developed. The number of passengers waiting for trip i and travelling from stop j to stop k is determined by

$$w_{i,jk} = \begin{cases} \lambda_{j,k} * h_{i,j} \ \forall i \in N \setminus \{1\}, j \in S \setminus \{|S|\}, k \in S: k > j \\ \widetilde{w}_{j,k} \ for \ i = 1, j \in S \setminus \{|S|\}, k \in S: k > j \end{cases} \quad \textbf{Equation 4}$$

For the first trip there is a special case because $h_{i,j}$ cannot be defined yet. Therefore, the number of passengers is written as the parameter $\widetilde{w}_{j,k}$. Also, j cannot be the last stop of S because then stop k cannot be larger than j and $k > j$ always holds. The number of skipped passengers is determined by

$$I_{i,jk} = w_{i,jk} - w_{i,jk} * x_{i,j} * x_{i,k} \forall i \in N, j \in S \setminus \{|S|\}, k \in S: k > j \quad \textbf{Equation 5}$$

At the last stop passengers cannot board and thus not be skipped. This number is formulated into a new variable to determine the number of passengers at a certain stop j

$$m_{i,j} = \sum_{j=i+1}^{|S|} I_{i,jk} \forall i \in N, j \in S \setminus \{|S|\} \quad \textbf{Equation 6}$$

The expected number of passengers that is waiting for trip i and will board the bus at stop j is determined by the number of passengers travelling between stop j and k.

$$u_{i,j} = \begin{cases} x_{i,j} * \sum_{k=j+1}^{|S|} w_{i,jk} * x_{i,k} \forall i \in N, j \in S \setminus \{|S|\} \\ 0, \forall i \in N, j \in 1 \end{cases} \quad \textbf{Equation 7}$$

Again, at the last stop there is a special case which has no boardings, and $u_{i,|S|}$ is therefore zero. The number of expected alightings is a similar equation as the expected boardings but is dependent on the passengers travelling between j and k.

$$v_{i,j} = \begin{cases} x_{j,i} * \sum_{k=1}^{j-1} w_{i,kj} * x_{i,k,} \forall i \in N, j \in S \setminus \{1\}, k \in S \mid k > j \\ 0, \forall i \in N, j \in S \setminus \{|S|\}, k \in S, k \leq j \end{cases} \quad \textbf{Equation 8}$$

When $k \leq j$, $b_{i,jk} = 0$ because the bus can only serve stops for $k > j$. The dwell time $k_{i,j}$ of a trip i is the total time of boarding and alighting and is written as

$$b_{i,jk} = \begin{cases} x_{i,j} * w_{i,jk} * x_{i,k}, \forall i \in N, \forall j \in S \setminus \{|S|\}, k \in S \mid k > j \\ 0, \forall i \in N, j \in S \setminus \{|S|\}, k \in S, k \leq j \end{cases} \quad \textbf{Equation 9}$$

When $k \leq j$, $b_{i,jk} = 0$ because the bus can only serve stops for $k > j$. The dwell time $k_{i,j}$ of a trip i is the total time of boarding and alighting and is written as

$$k_{i,j} = \max(p_1 * u_{i,j}; p_2 * v_{i,j}), \forall i \in N, j \in S\backslash\{1\}$$ **Equation 10**

Because the bus has different openings for boarding and alighting, the highest time is decisive and the maximum of the two is taken. The in-vehicle passenger load is also dependent on the boarding and alighting rates. The load of a trip i travelling from stop j to $j+1$ is determined by

$$\gamma_{i,j} = \begin{cases} \gamma_{i,j-1} + u_{i,j} - v_{i,j}, \forall i \in N, j \in S\backslash\{1, |S|\} \\ u_{i,1} \ \forall i \in N, j \in 1 \\ v_{i,|S|} \forall i \in N, j \in |N| \end{cases}$$ **Equation 11**

The load at the first stop is equal to all boarding passengers and the load at the last stop is equal to all alighting passengers, expecting all passengers to leave the bus at the end. The arrival time of a bus trip i at stop j is dependent on the departure time of the previous stop, the bus travel time and the time needed to decelerate and accelerate. This is written as

$$a_{i,j} = \begin{cases} d_{i,j-1} + t_{i,j} + \frac{\delta}{2} * (x_{i,j-1} + x_{i,j}), \forall i \in N, j \in S\backslash\{1,2\} \\ \tilde{d}_{i,1} + t_{i,2} + \frac{\delta}{2} * (x_{i,1} + x_{i,2}), \forall i \in N \end{cases}$$ **Equation 12**

For the arrival time at stop two, the departure time at stop $j-1$ is replaced by the planned departure time at the first stop. The departure time for the other stops is equal to the arrival time and the dwell times:

$$d_{i,j} = a_{i,j} + k_{i,j}, \forall i \in N, j \in S\backslash\{1\}$$ **Equation 13**

Because there is assumed that buses do not overtake each other the headway between two buses is determined by the arrival time for bus trip i at stop j minus the departure time for bus trip $i-1$ at the same stop.

$$k_{i,j} = \begin{cases} a_{i,j} - d_{i-1,j}, \forall i \in N\backslash\{1\}, j \in S\backslash\{1\} \\ d_{i,1} - d_{i-1,1}, \forall i \in N\backslash\{1\}, j \in 1 \end{cases}$$ **Equation 14**

The first stop is a special case because buses cannot arrive at this stop thus not have an arrival rate. Therefore, the headway is determined based on the difference of the planned departure times.

At last, the binary variable $x_{i,j}$ determines if a stop is served or not. It returns one if the stop is served and 0 if not. This yields in the following constraints:

$$x_{i,j} \in \{0,1\}, \forall i \in N, j \in S$$ **Equation 15**

$$x_{i,1} = x_{i,|S|} = 1, \forall i \ \in N$$ **Equation 16**

$$\left(x_{i-1,j} * x_{i-1,k}\right) + \left(x_{i,j} * x_{i,k}\right) \geq 1, \forall i \in N \backslash \{1\}, j \in S, k \in S \mid k > j \qquad \textbf{Equation 17}$$

Equation 15 determines the possible values of x. Equation 16 is defined because the first and last stop should always be served and equation 17 makes sure that no two consecutive trips skip the same stop.

3.2.3 Objective

As mentioned before, this research aims to minimize the passenger waiting times, bus travel times and passenger in-vehicle travel times. Therefore, the objective consists of three terms each representing one of these objectives. Each term is multiplied with a cost factor $w1$,$w2$ and $w3$, respectively. The first term calculates cost due to the waiting time of the passengers who arrive after the bus has left from stop j, which is the boarding rate multiplied by half of the bus headway. The second term calculates the cost that is caused by the travel time of the buses and the third term computes the passenger in-vehicle bus time by multiplying the equation of the second term with the number of passengers boarding between two stops. The objective is shown in equation 18.

$$\begin{aligned} f(x) = w_1 * \sum_{i=2}^{|N|} \sum_{j=1}^{|S|-1} (u_{i,j} - m_{i,j}) * \frac{h_{i,j}}{2} + m_{i,j} * \left(\left(\frac{h_{i-1,j}}{2} \right) + k_{i-1,j} + h_{i,j} \right) + \ w_2 \\ * \sum_{i=2}^{|N|} \sum_{j=2}^{|S|} \left(t_{i,j} + \left(k_{i,j} + \delta\right) * x_{i,j}\right) + \ w_3 \\ * \sum_{i=2}^{|N|} \sum_{j=1}^{|S|-1} \sum_{k=j+1}^{|S|} b_{i,jk} * \sum_{l=j+1}^{k} \left(t_{i,l} + \left(k_{i,l} + \delta\right) * x_{i,l}\right) \end{aligned} \qquad \textbf{Equation 18}$$

The constraints and objective together form the mathematical program for the stop-skipping model. This is formulated as

$$(Q): \min Z = f(x) \mid x \in F(x) = \{x \mid x \ satisfies \ equation \ (4 - 17)\}$$

The program is a non-linear program because the objective contains multiplications in its terms and the constraints have both linear and non-linear aspects. Furthermore, the variables contain a decision variable that is bound to be the binary variable ($x_{i,j}$). This binary variable is set to 1 if a stop is served and 0 if it is skipped. Therefore, the set of possible solutions is equal to $2^{|N|*|S|}$. The branch and bound algorithm is used to find the optimal solution. From this set of solutions $2^{|N|*|S|}$, the model will compute a set of feasible solution, which is done by branching. This are the solutions that satisfy all the constraints. The feasible solutions are divided in sub solutions and are evaluated until the best solution is found (-, 2001). This process is called bounding. Figure 1 provides a simplified overview of the process of branch and bound.

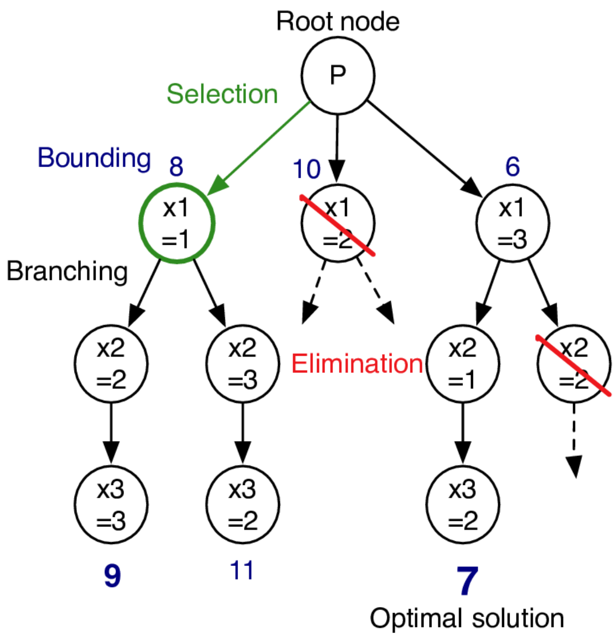

Figure 1 Branch and bound algorithm

4 Results

This section will first provide the result of the data analysis. The second part contains the model solutions tested on the bus line that is chosen from the data analysis.

4.1 Data analysis

The bus lines that exist in Twente are divided in six sections: Enschede city, Hengelo city, Almelo city, regional, local and students. In table 7 in Appendix A the characteristics of the bus lines are shown. Because there is not data provided after the COVID measures came for the local and student sections, these are not analysed any further. The other bus lines are used in the next three sections to investigate the three criteria.

4.1.1 Passenger Division

Looking at the passenger division for both scenarios show the following. The distribution of passengers has for all lines approximately the same structure. Most of the users are students, followed by adultsand anonymous users. The change after the COVID measures became active resulted in a relative drop of student users for all lines whereas the adult and anonymous users remain the same or increased. Especially line 26 and line 64 show a big shift of the distribution. This can be seen in figure 2, 3, 4 and 5.

Pre-COVID	COVID

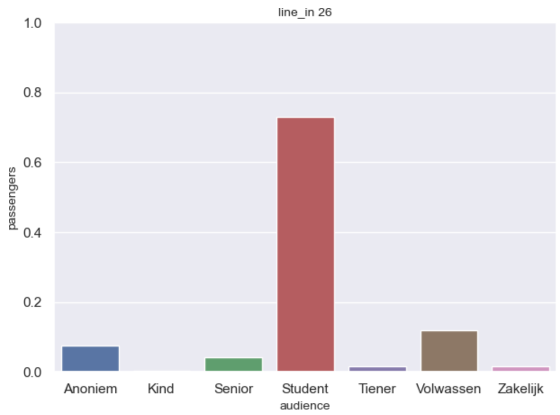

Figure 2 pre-COVID passenger distribution line 26

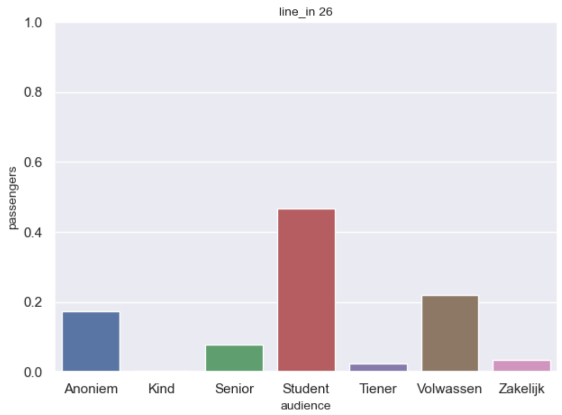

Figure 3 COVID passenger distribution line 26

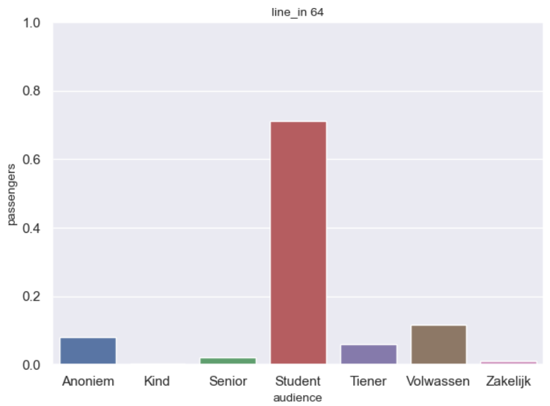

Figure 4 Pre-COVID passenger distribution line 64

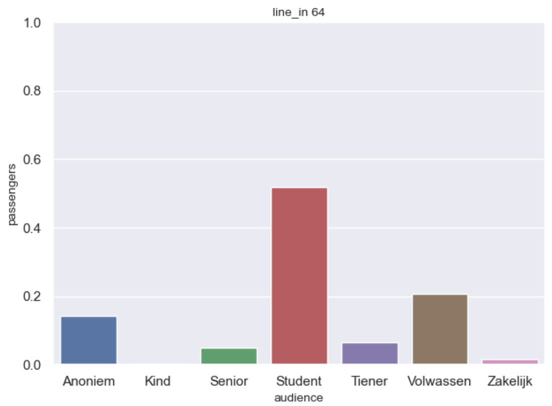

Figure 5 COVID passenger distribution line 64

The shift of passenger types can be explained by looking at the absolute numbers of the passenger division. This shows that the number of students decreased a lot while the anonymous and adult number hardly changed. The reason for the drop of students is the closure of schools which keeps them at home. Because all bus lines show a similar change in user type, no decision is made yet on selecting specific bus lines here.

4.1.2 Average demand per week

The difference of the demand averages for each line is shown in table 8 in appendix B. In figure 6, the average frequencies during the COVID scenario for each line are presented. Apparently, the first five bus lines are the highest frequency lines together with line 9 and 62.

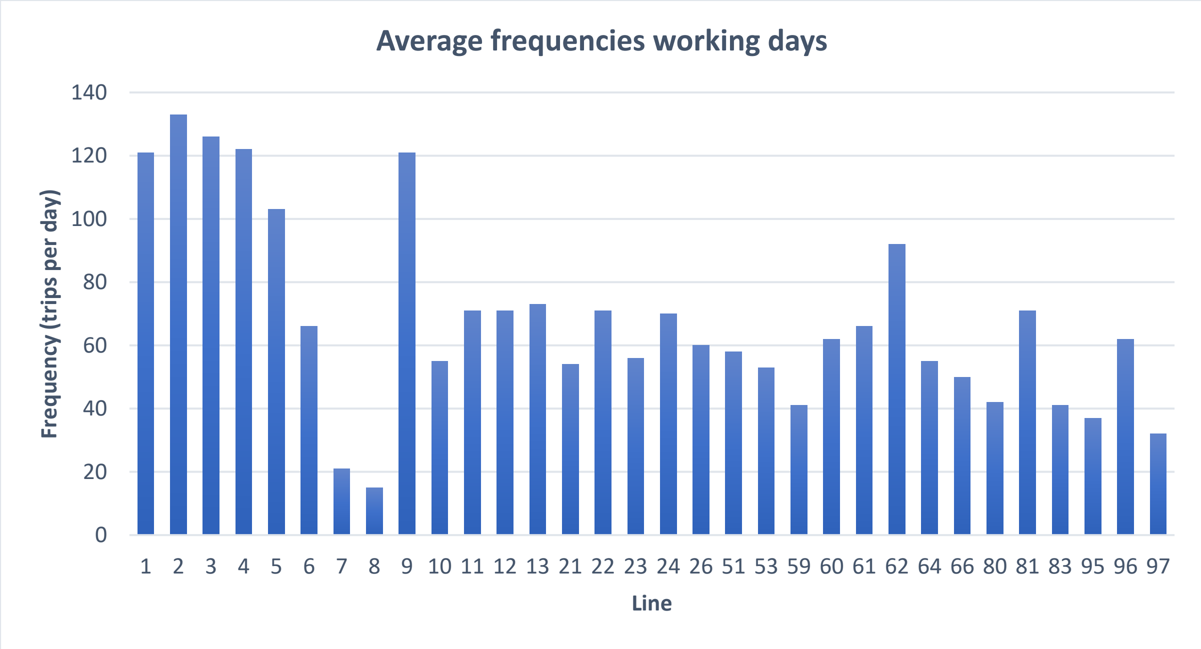

Figure 6 Weekly demand averages

For the model, it is important to investigate a bus line with a sufficient frequency in order to collect the data that is needed. Especially with the COVID scenario, the frequency dropped already and if a line with too few trips is investigated it is harder to determine a reliable solution with the stop-skipping strategy. The highest frequency lines also show the biggest demand drop. The differences are shown in figure 7.

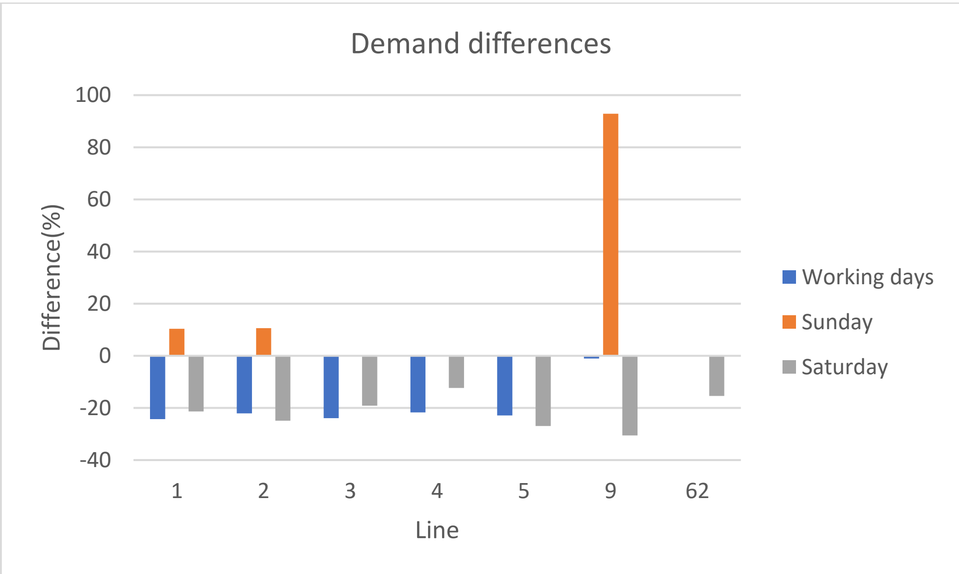

Figure 7 Demand difference after COVID

It shows that line 1-5 show a drop of approximately 20% on the working days, whereas the demand for line 9 and 62 stays the same.

4.1.3 Demand per stop

The previous section showed the number of lines that experience the highest demand difference. These lines will therefore be used for the analysis of the demand per stop. The demand per stop is determined for the COVID-period only. The counted bus stop for each line that satisfy the constraint of having less boardings than 0.1% of the total is shown in figure 8.

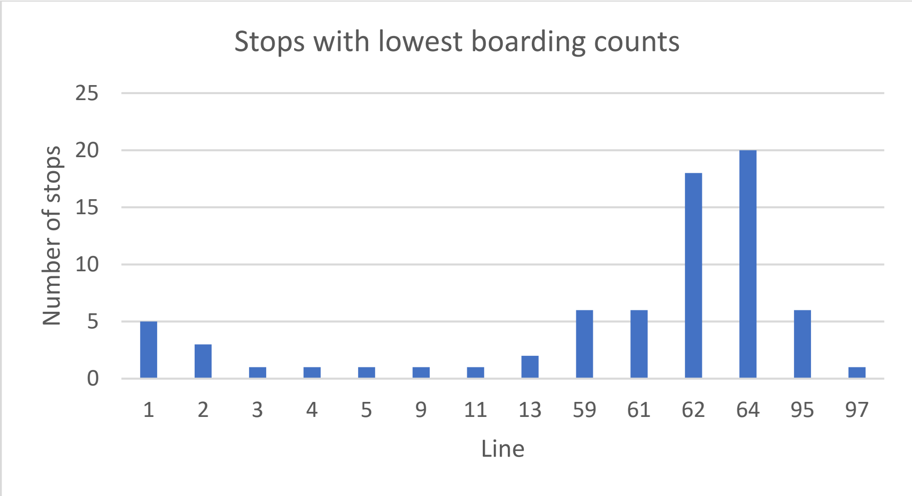

Figure 8 Stops with lowest boarding counts

So, especially the regional lines show a higher number of stops that have low boardings. Therefore, these lines have a lot of potential for the stop-skipping strategy. However, the frequencies are lower than the urban lines, which should be taken into account when making the choice.

4.1.4 Landscape

The type of landscape plays a role in the travel behaviour of the user. This type can be either urban or rural. Urban landscapes are defined as stops within built-up area. These are areas consisting of all cities with more than 25000 citizens. The other bus stops are defined as rural. For the city lines the landscapes are considered to be fully urban, whereas the regional lines show a mix of urban and rural landscape. The bus line that are still in the selection and are placed in a mixed landscape are shown in table 5.

Table 5 Landscape

Line	Stops	Urban	Rural
59	34	17	17
61	27	7	20
62	57	19	38
64	80	22	58
95	48	18	30
97	31	0	31

4.2 Comparison and decision

Comparing the result of the previous four sub-section shows a number of lines that have a good potential for testing. The average demand per week shows that line 1 till 5 and 9 have the highest frequencies and together with line 62 the highest difference between the two scenarios for the working days. Furthermore, from the demand per stop analysis is found that line 1 -5 and 9 have very little stops with low boarding rates. When comparing line 1 to 62, line 62 has much more stops with low boarding rates which makes this one a lot more interesting. Line 62 also has a mixed landscape which might explain this big difference. In the literature, line 62 has already been investigated one by a study of (de Weert & Gkiotsalitis, 2021). The aim of this study was to minimize the revenue losses by using a short-turning strategy. This allows buses to finish their trip earlier in order to decrease travel time and thus operational costs. With this case study existing, the decision is made to minimize the revenue losses with the stop-skipping strategy. Although the terms are different compared to (de Weert & Gkiotsalitis, 2021) the objective is similar.

The cons of line 62 is the recent change of this route because of the new N18 road. The changes involve the removal of bus stops in Usselo, which is located between Hengelo and Enschede, and several bus stops in Enschede. Because the used data still contains the data for these stops, it is much harder to determine the demand over this line. Therefore, there is decided to not look further on this line.

While comparing line 1-5 with each other they are all high frequency lines (see figure 6). Line 2 has the highest frequency of 133 and also the biggest difference compared with the COVID-scenario which is 44. The demand per stop analysis showed that 6 of its stops have a boarding rate of below 0.1% of the total boardings. Line 1 also has 6 stops which meets this constraint but has 14 more stops than line 2 which makes it less useful. Although, the landscape of line 2 is urban, it is chosen as a case study for the stop-skipping model.

4.3 Case study

4.3.1 Description

As mentioned in previous section, the concerned bus line is line 2. The bus stops depend on the direction of the bus, because the route is different between the two start and end stops, Enschede; Disselhoek and Enschede; Buizerdstraat, respectively. The route with 26 stops starts in Enschede; Disselhoek and ends in Enschede; Buizerdstraat which is shown in figure 9. Enschede has almost 160.000 inhabitants which causes line 2 to be a high frequency bus line.

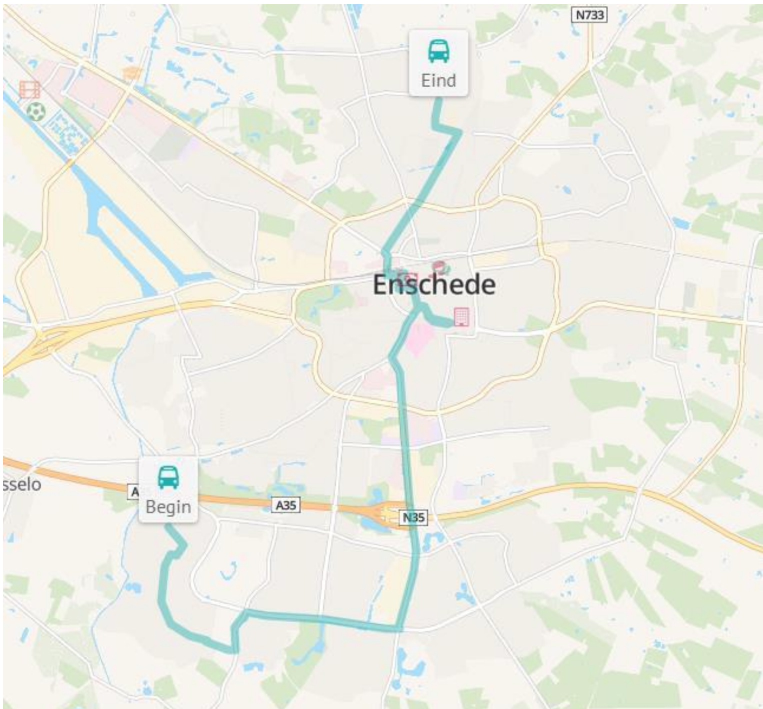

Figure 9 Bus line 2

The bus line is active from 5:47 till 23:15 in the weekdays but has different travel times within this period. Between 7:09 till 18:27 the bus travel time is the lowest and expected to be 30 minutes.

4.3.2 Data

The passenger demand is shown in the OD-matrix of bus line 2 shown in table 9 in Appendix C. This table has dimension $|S|$ = 26 and because only one direction is considered, the lower left side is containing zeros. The demand of this table are average values based on 8 weeks and a normal distribution is used to have a more realistic simulation. This also prevents the presence of zeros in the table due to the lacking amount of data. The demand is determined for each trip separately, which result in a different OD-matrix for each trip. The frequency is expected to be at its highest between 8:00 and 9:00. The buses have a headway of 15 minutes which makes it 4 buses operating per hour. Thus $|N| = 4$.

The additional travel time due to acceleration and deceleration, δ, is approximately 20 seconds and the time for boarding and alighting is $p_1 = 4$ and $p_2 = 2$, respectively (Fu, Liu & Calamai, 2003). The unit costs c_1, c_2 and c_3 are $\frac{20\$}{h}, \frac{50\$}{h}$ and $\frac{10\$}{h}$, respectively (Fu, Liu & Calamai, 2003). An overview of the parameters is shown in table 6.

Table 6 Input variables

Parameter	Unit	Value
$\boldsymbol{N}$	$-$	$\{1, 2, \ldots |N|\}$
$\boldsymbol{S}$	$-$	$\{1,2, \ldots, |S|\}$
$\boldsymbol{\delta}$	s	20
$\boldsymbol{\lambda}$	$-$	$|S| * |S|$ $matrix\ with\ arrival\ rates$
$\boldsymbol{p_1}$	s	4
$\boldsymbol{p_2}$	s	2
$\boldsymbol{c_1}$	\$	20
$\boldsymbol{c_2}$	\$	50
$\boldsymbol{c_3}$	\$	10

4.3.3 Application

The data mentioned in previous sections are the input for the mathematical program described in section 5.2.3. The solution for the program is found using the model solver Gurobi in Python. The general model structure in Gurobi consists of parameters, variables, constraints and an objective which makes it very easy to use. This model consists of 14979 continuous, 3024 integer and 3024 binary variables. Next to that, it contains 15431 quadratic and 100 general constraints.

The model is first relaxed with a dual simplex. This method relaxes the model in order to avoid infeasibility. The model is called 'infeasible' if there is no solution such that all constraints are met. After this root relaxation the first objective solution is found which is 1.193954e+02. This solution is determined in 6.02 seconds. However, this solution is not valid because it does not meet all the constraints of the MINLP. To find a feasible solution, the branch and bound algorithm is used. The initial objective solution is used to determine further solutions, which continues until one optimal solution is determined. The total number of solutions is determined by $2^{|S|*|N|}$. Thus, the solution set will contain $2^{26*4} = 2.028241e31$ possible solutions. Therefore, the model computation time is limited to 1800 seconds. This enables the model to have enough time to find a feasible solution and not have the computation time too long.

Because the demand input is based on a normal arrival scale the solution output of each run differs. Therefore 10 consecutive runs are executed, and the average value of each term is determined. The average arrival rates are used to see what the skipped bus stops are.

4.3.3.1 Model process

After approximately 1000 seconds the model finds the first feasible solution which is approximately 2e+02. This is shown in figure 10. The incumbent is defined as the possible solutions found over time. The goal of the modal is to make the gap between the incumbent and the lower bound zero. In the figure can be seen that the gap is narrowed down after ±1000 seconds but does not reach zero, because the computation is terminated after 1800 seconds. The gap is then 13.4%, which means that the value of the solution found could be 13.4% lower since it can take lower values until the lower bound is reached.

Figure 10 Model process towards optimal solution

4.3.3.2 Optimal solution

The optimal solution found by the model resulted in the following pattern of serving and skipped stops shown in figure 11. For some reason, the average demand number in the OD-matrix were not able to find an optimal solution. Therefore, an approximate average is used determined from ten simulations that uses demands that are normal distributed.

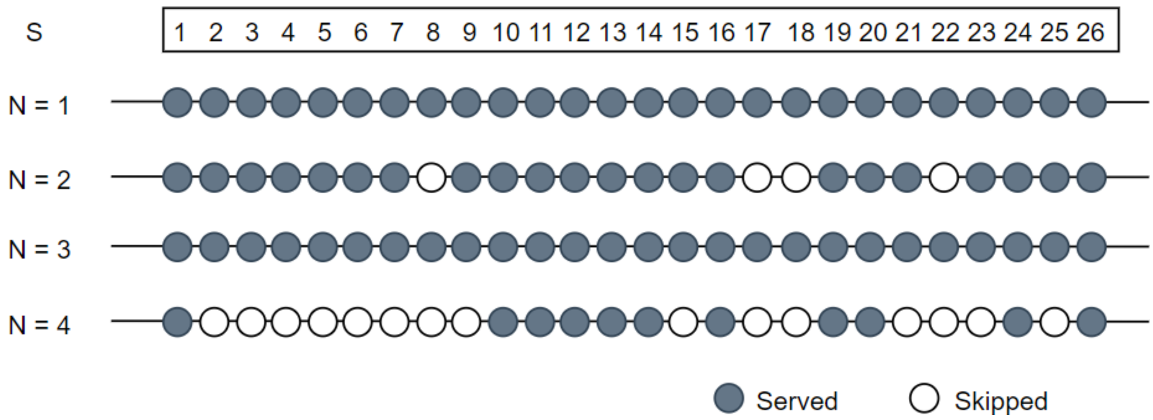

Figure 11 Stop-skipping pattern bus line 2

The model determined to skip 3 stops on $N = 2$ and 15 stops on $N = 4$. $N = 1$ and $N = 3$ stay the same and this is caused by the constraint that two consecutive trips cannot skip the same stop. This is determined to be most effective by skipping stops every even trip number, no matter how many trips per hour there are. The difference in number of skipped stops between trip 2 and 4 is rather unusual but shows up in every simulation. The only input variables that change with each trip are the arrival rates, but this should only be able to create small differences.

4.3.3.3 Cost comparison

The new stop-skipping pattern is compared to the current situation with all stops served. For both situations the total passenger waiting time, total bus travel time and total passenger in-vehicles between 8:00 and 9:00 a.m. are compared and shown in figure 12.

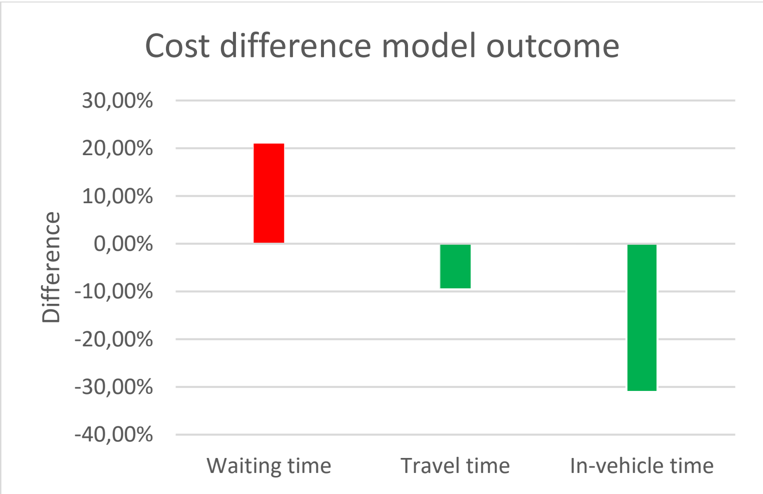

Figure 12 Cost and time comparison original model outcome

From the figure can be seen that with the stop-skipping strategy the total waiting time increases with 21.049%. The total travel and passenger in-vehicle time decrease with 9.4196% and 31.046%, respectively. The total costs decreased with 6,19488%. The huge changes of the total waiting and in-vehicle time are caused by the big number of skipped bus stops in trip 4 as seen in figure 11. With all the skipped stops, passengers are not able to board which results in an increase of the waiting times and a decrease of the in-vehicle time. For this reason, there is chosen to change the pattern of trip 4 to the same pattern as trip 2 presented in figure 13.

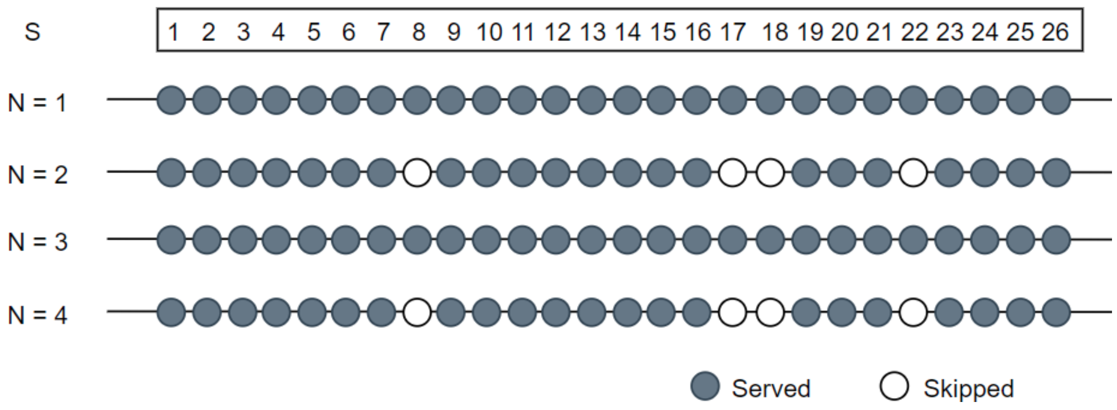

Figure 13 Stop-skipping pattern adapted situation

For this adapted solution, the total waiting, travel and in-vehicle time are compared to the current situation with all stops served. This is shown in figure 14.

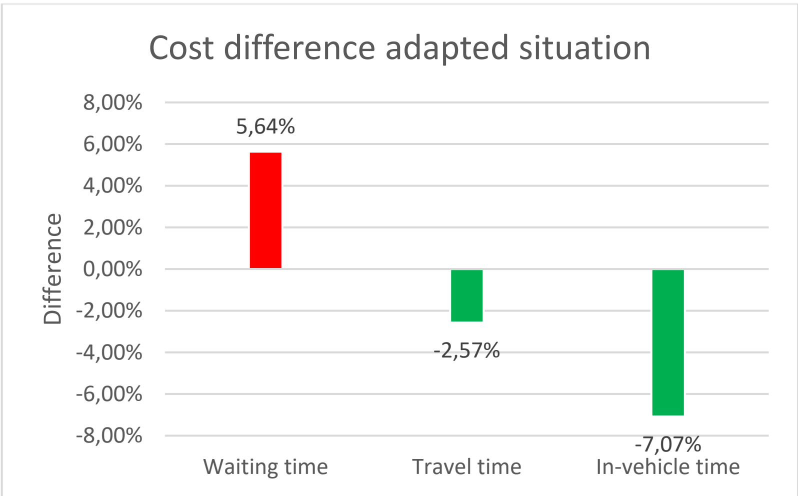

Figure 14 Cost and time comparison adapted pattern

The adapted situation still shows in increased waiting time of 5,635264%. The total travel and in-vehicle time decreased with 2.5653% and 7.0747% respectively and the total time decreased with 1.3731%. So, with the adapted situation, the total time decrease is smaller, but the total waiting time increase is acceptable.

4.4 Sensitivity analysis

A sensitivity analysis is conducted to assess the validity of the cost factors. The values of the cost factors have a big impact on the outcome of the stop-skipping pattern.

4.4.1 Cost passenger waiting time

By changing the values of c_1, the effects on the total time and number of skipped stops are calculated. The result is shown in figure 15.

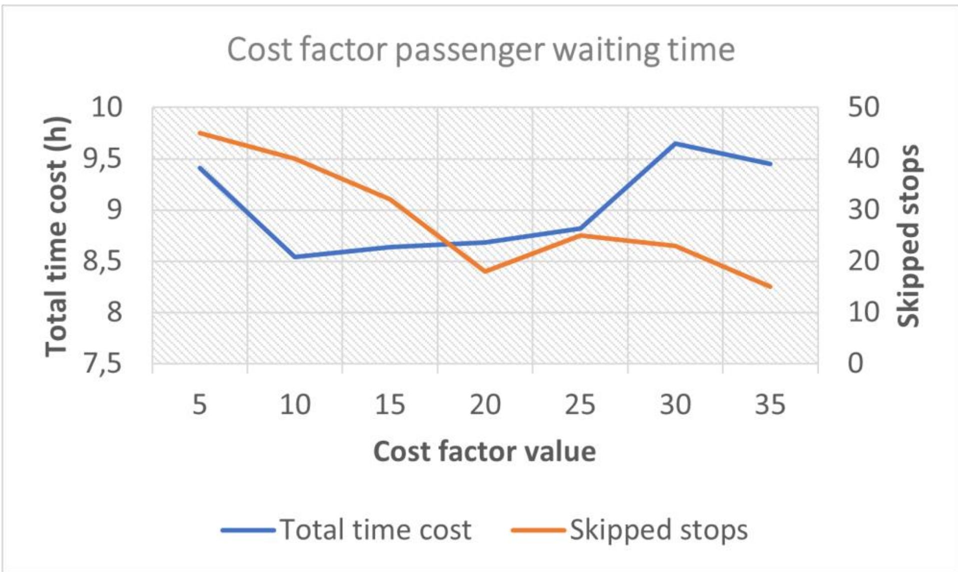

Figure 15 Sensitivity analysis waiting time cost

The analysis shows that a value of 20 has the lowest number of skipped stops compared to the other cost factors. Obviously, when c_1 is increased above 40 the waiting time decreases but this results in a higher total time compared to the total time belonging to a cost factor of 20.

4.4.2 Cost travel time

The changes of the objective terms as a result of changing the travel time cost factor $c2$ are shown in figure 16.

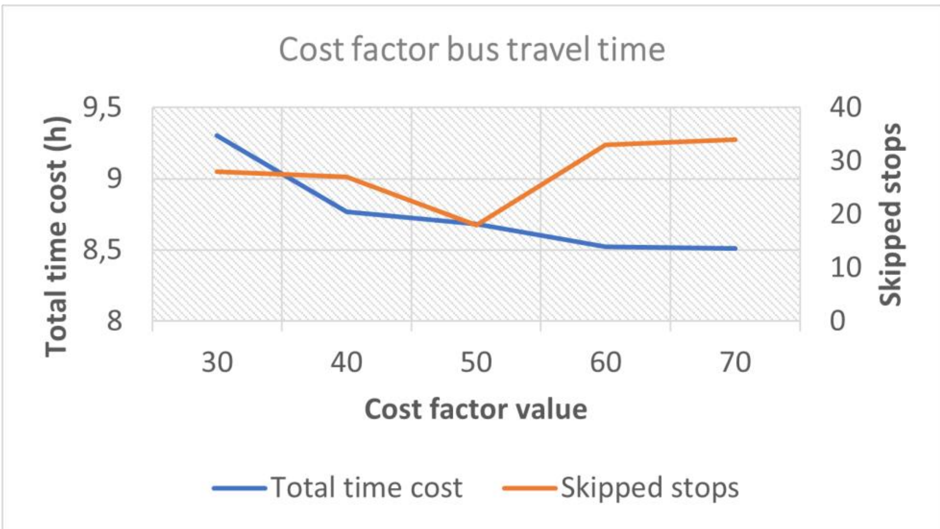

Figure 16 Sensitivity analysis travel time cost

There can be concluded that with a higher travel time cost, more stops are skipped. As a logical result of this, the in-vehicle time and the travel time decrease when the travel time cost factor increases. However, the waiting time is decreasing which is remarkable. Because 50 is the only value that gives a long computation time, the time cost factor can be stated very sensitive and is probably close to the optimal value.

4.4.3 Cost passenger in-vehicle time

The changes to the passenger in-vehicle time cost factor, c_3, are shown in figure 17.

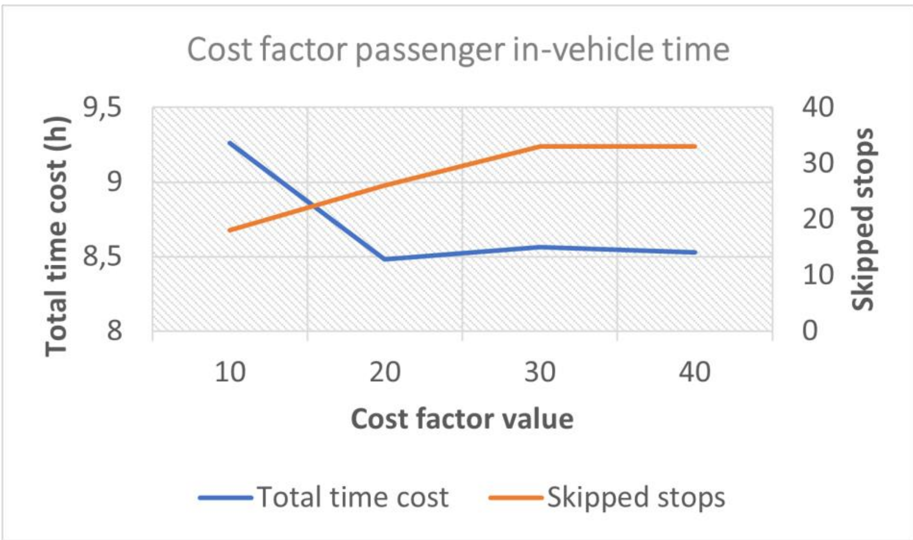

Figure 17 Sensitivity analysis passenger in-vehicle time

Figure 17 shows that a higher in-vehicle cost factor results in more skipped stops. On top of that, the skipped stops reach a maximum after the value of 33. This also substantiates the choice for a in-vehicle cost of 10.

5 Conclusion

In this thesis the stop-skipping strategy is used as main goal to reduce the travel time of buses operating in Twente commissioned by ‘Twents’, a sub service of Keolis Nederland. Through a data analysis including passenger division, demand division and landscape types, bus line 2 turned out to be the most useful option for optimalization. By establishing a model containing parameters, variables, constraints and objective an optimal solution is found by minimizing the value of this objective. The model is ranked as a MINLP (Mixed Integer Non-Linear Program) because the objective consists of multiplications and a binary variable is used to indicate if a stop is served or not. The model is solved with branch and bound and needs a computation time of at least 1000 seconds to find a solution. The outcome where several stops are skipped shows a decrease of 6,2% of the total time compared to the current situation where all stops are served. However, this solution shows a total waiting time increase of 21%, caused by an extensive amount of bus stops in trip 4. An adapted situation where trip 4 skips the same stops as trip 2 shows a total time decrease of 1.4% and a waiting time increase of 5,6%. This adapted situation shows that with stop-skipping an improved scenario can be reached but it cannot cogently be confirmed as an optimal solution.

Nevertheless, the adapted situation answers the research question as it is the best solution using the stop-skipping model. A sensitivity analysis showed that the optimal values for the cost factors are determined based on the computation time. The waiting time, travel time and in-vehicle time cost factors must be around 20, 50 and 10 respectively. This means that the factors chosen for this research are acceptable.

6 Discussion

The results of the stop-skipping model showed expectations but also inconsistencies. The first abnormality is the high number of skipped stops of trip 4. This is not realistic because a part of these bus stops has an average or even high boarding and/or alighting rate. Furthermore, this unrealistic pattern leads to incomplete time values. An explanation for this phenomenon could be that the cost for the waiting time is valued too low. As a result, the model values travel time more than waiting time and thus decide to skip the stop with the lowest waiting times, which in this case 15 stops. Another explanation could be that the objective is incomplete and is too focused on certain variables. Right now, the waiting time is determined by a consideration between the boarding rate $u_{i,j}$ and the skipped rate $m_{i,j}$. The travel time is determined by $t_{i,j}$ and the in-vehicle time includes the $b_{i,jk}$. This means that the alighting of passenger is not directly considered in the objective, and this might be the cause for the skipped stops in trip 4. A solution for this could be an additional term to the objective that takes the alighting passenger into account.

A second point of discussion is the correctness of the input data for the stop-skipping model. The arrival rates are deduced from the data between June 2nd and July 31st. This is a relatively short time period and resulted in a large amount of zero trips in the OD-matrix. It is not realistic as it would assume that there will always be zero trips between these stops in the future. This could be avoided when the data is sampled for a longer time period. Next to this, the COVID regulations did decrease the trips as well, which resulted in less data than there would be with a regular scenario. Another parameter that could be more precise are the inter station times. They are estimated based on the length between the bus stops and the planned arrival times. However, the inter station times are used in the constraints and objective so a better approximation should contribute to more reliable outcomes.

Another error that shows up during the computation of the solution is the value of the binary variables. Due to multiplications with the binary variables, some of their values slightly decreases under 1 or 0. Because of this, some constraints cannot behave properly, and the outcome is biased. Third, a verification of the model is done with a test case that contains a bus line with 9 stops. Although, there might still be errors in the formulation of constraints and/or objective due to limited knowledge of their implementation in the Gurobi model solver.

At last, the question raises how this research will help in future problems. Although the outcome is not totally correct, it shows that the bus travel time and passenger in-vehicle time can be decreased. Furthermore, when stop-skipping will be applied at a bus line, the pattern will be announced in the travel planner applications. Most of the passengers will check the planner before they start their trip. When they see that their chosen bus stop of alighting is not served by the bus, a logical decision would be to go to a bus stop nearby that will be served. In this way, the waiting time of passengers will not be as high as the model outcome states. Next to that, the skop-skipping method is not really known in the Netherlands, whereas the public transport system is very extensive. It will be worth to test the principle of stop-skipping on especially longer bus lines, which include a lot of low boarding rate bus stops.

7 Recommendations

For further research the following highlights are proposed. First, the high number of bus stops causes the program to have a large computation time. When investigating a bus line with a lot of stops, the use of a well-developed computer is a must. 8 GB RAMS and at least 4 processors are recommended. Second, the arrival rates are recommended to be Poisson distributed. This distribution is often used for arrival rates and counts certain occurrences within a time period. The Poisson distribution was not suited for this research as the frequencies were too low. Therefore, it is also recommended to gather enough travel data. At last, other studies should consider other objective terms and no more than two terms per objective are needed. The travel time and passenger in-vehicle time behave the same when optimizing the model and therefore one of them is not actually necessary.

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9 Appendix

9.1 Appendix A: Bus lines

Table 7 Overview bus lines

Area	Line	Origin	Destination	Stops	Remarks
Enschede City	1	Universiteit	Wesselerbrink	40	
	2	Helmerhoek	Deppenbroek	26	
	3	Enschede	Glanerbrug	16	
	4	Stroinkslanden	Station (Enschede)	9	
	5	Zwering	Station (Enschede)	12	
	6	Stokhorst	Station (Enschede)	8	
	7	Station (Enschede)	Station (Enschede)	16	To Marssteden
	8	Hengelo Noord	Enschede Zuid	40	
	9	Hengelo	Enschede	13	
	802	P+R Zuiderval	Van Heekplein	2	
Hengelo City	10	Veldwijk	Station (Hengelo)	12	
	11	Hasseler Es	Station (Hengelo)	13	
	12	Gezondheidspark	Groot Diene	18	
	13	Hasseler Es	Station (Hengelo)	17	
Almelo City	21	Station (Almelo)	Windmolenbroek	12	
	22	Station (Almelo)	Windmolenbroek	8	
	23	Station (Almelo)	Schelfhorst	8	
	24	Station (Almelo)	Schelfhorst	10	
	26	Station (Almelo)	Twenteborg Ziekenhuis	11	
Regional	51	Almelo	Hengelo	31	
	53	Hengelo	Eibergen	26	
	59	Tubbergen	Haaksbergen	34	
	60	Oldenzaal	Enschede	37	
	61	Overdinkel	Enschede	27	
	62	Borculo	Denekamp	40	
	64	Overdinkel	Almelo	40	
	66	Neede	Oldenzaal	40	
	80	Hardenberg	Westerhaar	21	
	81	Ommen	Westerhaar	40	
	83	Vriezenveen	Almelo	18	
	95	Almelo	Borculo	40	
	96	Rijssen	Nijverdal	18	
	97	Holten	Haaksbergen	31	
	264	Denekamp	Almelo	35	
Local	505	Station (Enschede)	Hogeland	9	
	506	Enschede	Boekelo	13	
	508	Stadsdeelkantoor Zuid	Wkc Stroinkslanden	10	
	513	Nijverdal	Raalte	14	
	525	Station (Almelo)	Aalderinkshoek	9	
	530	Borne	Borne	18	via Bornschematen
	531	Borne	Stoom Esch	13	

	532	Borne	Letterveld	11
	591	Tubbergen	Bruinehaar	8
	592	Weerselo	Borne	14
	593	Oldenzaal	De Lutte	11
	594	Den Ham	Nijverdal	15
	595	Haaksbergen	Buurse	9
	596	Ootmarsum	Denekamp	16
	597	Nijverdal	Hellendoorn	14
	599	Rossum	Ootmarsum	15
Students	601	Vriezenveen	Almelo	3
	602	Sibculo	Almelo	6
	604	Daarlerveen	Almelo	8
	605	Vroomshoop	Almelo	5
	681	Den Ham	Almelo	29

9.2 Appendix B: Demand differences

Table 8 Average demand per week

	Difference		
	Weekend		Working days
Line	Saturday	Sunday	
1	33	-6	33
2	34	-9	44
3	33	0	30
4	28	0	17
5	30	0	38
6	1	0	3
7	nvt	nvt	1
8	nvt	nvt	3
9	1	-26	53
802	nvt	nvt	nvt
10	0	nvt	0
11	0	0	27
12	0	0	0
13	-1	0	26
21	1	nvt	0
22	0	0	0
23	1	nvt	0
24	0	0	0
26	0	nvt	1
51	2	1	-2
53	0	0	0
59	0	nvt	11
60	-2	nvt	3
61	0	0	26
62	1	0	42
64	0	0	10
66	1	nvt	4
80	nvt	nvt	2
81	-2	3	1
83	0	nvt	0
95	1	nvt	20
96	0	nvt	-3
97	1	nvt	23

9.3 Appendix C: OD-matrix

Table 9 OD-matrix bus line 2

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Enschede, Disselhoek	1	0	0	0	0,02069	0,0069	0	0	0,0069	0	0	0,01379	0,01379	0	0	0	0,15862	0,0069	0,07586	0,48276	2,15862	0,0069	0	0,04828	0,0069	0	0
Enschede, Dubbelinckhoek	2	0	0	0	0	0	0	0	0	0	0	0,0069	0,06897	0	0,0069	0,08276	0,29655	0,0069	0,01379	0,13103	2,17241	0,01379	0,02069	0,01379	0	0	0
Enschede, Graeshoek	3	0	0	0	0,01379	0	0	0	0	0	0	0,04138	0,07586	0	0	0,0069	0,15172	0,01379	0,09655	0,25517	2,11724	0	0	0,04138	0	0	0,03448
Enschede, Wkc Helmerhoek	4	0	0	0	0	0,01379	0,0069	0,0069	0	0,01379	0	0,06207	0,09655	0,0069	0,01379	0,02069	0,61379	0,05517	0,26207	0,66207	4,68276	0,01379	0,02069	0,13793	0,02069	0	0,03448
Enschede, Honinglohoek	5	0	0	0	0	0	0	0,0069	0	0,0069	0,0069	0,04828	0,16552	0,0069	0,21379	0,08966	1,4	0,11724	0,1931	1,22069	6,13793	0	0,02759	0,25517	0,03448	0	0,02759
Enschede, Beekwoudehoek	6	0	0	0	0	0	0	0	0	0	0,0069	0,11724	0,06897	0	0,0069	0,0069	0,62759	0,03448	0,6069	0,65517	4,74483	0,0069	0,0069	0,03448	0	0,0069	0,08276
Enschede, Boskamphoek	7	0	0	0	0	0	0	0	0	0,02069	0	0,0069	0,0069	0	0,02069	0	0,08966	0,04828	0,28276	0,12414	1,75862	0,01379	0	0,0069	0	0	0,14483
Enschede, Herikebrink	8	0	0	0	0	0	0	0	0	0	0	0	0,02069	0,05517	0	0	1,22069	0,11034	0,12414	0,91724	4,45517	0,01379	0,0069	0,04828	0	0,02069	0,05517
Enschede, Hengeveldebrink	9	0	0	0	0	0	0	0	0	0	0	0,0069	0,02759	0,01379	0,0069	0,0069	1,01379	0,03448	0,2069	0,16552	1,86207	0,0069	0	0,06207	0	0	0,18621
Enschede, Het Oosterveld	10	0	0	0	0	0	0	0	0	0	0	0,02069	0,15862	0,01379	0,04138	0,03448	1,32414	0,09655	0,31724	0,68966	8,02759	0,02759	0,01379	0,09655	0,13793	0,16552	0,71034
Enschede, Hertmebrink	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,01379	0,70345	0,04138	0,67586	0,71724	6,11724	0,04828	0,02069	0,31034	0,0069	0	0,08276
Enschede, Wkc Zuid	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,37241	0,01379	0,10345	0,58621	3,27586	0,02069	0	0,10345	0	0,03448	0,11034
Enschede, Buurserstraat	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0,0069	0,0069	0,44828	0,24828	0,13103	0,65517	7,75172	0,06207	0,04828	0,14483	0,02069	0	0,33103
Enschede, De Reulver	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,53793	0,0069	0,08966	0,14483	2,26897	0,0069	0	0,05517	0	0	0,05517
Enschede, Vlierstraat	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,06207	0,02069	0,27586	0,26207	3,8069	0,0069	0,11724	0,10345	0,02759	0	0,17931
Enschede, P+R Zuiderval	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,07586	0,30345	1,47586	0,02069	0,03448	0,3931	0	0,0069	0,02069
Enschede, Rabobank	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,01379	0,01379	0,65517	0,26897	0,03448	0,18621	0,02069	0	0,0069
Enschede, MST Haaksbergerstraa	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,25517	0	0	0,0069	0	0	0,25517
Enschede, Van Heekplein	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,6	0,03448	0,02069	0,48276	0,06207	0,05517	0,28966
Enschede, Centraal Station	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,92414	0,55862	18,8414	3,28966	3,28966	7,55172
Enschede, Boddenkamp	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,01379	0,0069	0	0,01379
Enschede, Deurningerstraat	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,0069	0	0,02759
Enschede, Roombeek	23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,06207
Enschede, Hulsmaatstraat	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,01379	0,14483
Enschede, Stroinksbleekweg	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,06207
Enschede, Buizerdstraat	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0