Swap Spread Models and Their Implications for Dutch Sovereign Interest Rate Risk Hedging

Research Report

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Executive Summary

The Dutch State Treasury Agency (DSTA), which is responsible for the management of the Dutch state's debt, posed some questions regarding interest rate risk. With the usage of swaps, the DSTA can hedge against rising interest rates and change the effective duration of bonds ahead of time. These are tools to comply with its risk indicators, the weighted average time till refixing and the 12-month refixing amount. The goal of these indicators is to diminish interest risk by ensuring stable interest rate payments. Both swap hedge strategies require a combination of two short term receiver swaps and one long term payer swap. The alternative is to not use the swaps or issue a long-term bond. The usage of these swap strategies uncovers the exposure to the future swap spread. The swap spread is the swap rate minus the sovereign rate on the Dutch sovereign bonds. The goal of the research was to identify the swap spread and describe how it behaved in the past and might behave in the future.

The first step was to define the individual parts of the swap spread, being bonds, swaps, sovereign rates, and swap rates. The sovereign rate curve was constructed by zero-coupon rates, because of its availability and maturity correction by Bloomberg. The swap rates are determined by the Euribor 6m interest rate swaps. A 20-year period for 13 different maturities ranging from 1- to 30-year of the data was retrieved from Bloomberg and separated into three different periods (before, during, and after the crisis), where the middle period contains the credit and euro crisis. A temporal analysis was performed on all different time series. This resulted in the observation that the swap spread can be described by a random walk, where its changes follow an ARMA(1,1) model. This model is then extended into an ARMAX model which was able to reduce the variance of the residuals around 30% for the post-crisis period by introducing liquidity and credit premium proxies. Finally, the swap spread was decomposed using Nelson-Siegel's interest rate model, which led to the observation that swap spreads in periods of crisis primarily exist out of a downward slope, while it exists majorly out of shift and curvature in periods of non-crisis. The consequence is that while the sovereign rates generally are lower than the swap rates, in times of crisis the distance between them becomes less the higher the maturity becomes.

To hedge against increases in sovereign rates and reduce the eventual interest rate volatility, swaps could be used. The resulting swap spread has lower volatility than the zero-coupon rate, which means that the strategy is indeed volatility reducing. However, it is important to consider the maturity periods in question, because volatility and estimation errors increase over time. The costs have been quite high in the past since interest rates have mostly been decreasing the last 20 years, nonetheless, the strategy has become much more viable since interest rates have become negative. The decision to use swaps to extend the effective duration of bonds is dependent on the future swap spread and two present swap spreads. The analysis leads to its explanation by implied future rate arguments. What it says is that when the real/forecasted future swap spread is higher than the implied future swap spread, then it is better to issue the short-term bond combined with extending swaps.

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1 Introduction

1.1 Background

The Dutch State Treasury Agency (DSTA) is responsible for the management and financing of the state debt. This is mainly done by issuing Dutch State Loans (DSL) and Dutch Treasury Certificates (DTC). The former (DSL) are loans with maturities of more than a year, and the latter (DTC) less than a year. Next to financing the state debt the DSTA also has the responsibility to regulate the treasury balance, manage the treasury banking, and oversee the financial payments for the Dutch State (Rijksoverheid, n.d.).

The DSTA has several departments to fulfil its day-to-day obligations. One of them is the department Policy and Risk Management State Debt. This branch is responsible for establishing policy frameworks and guidelines for areas like the interest risk profile of the state debt, the controlling of credit risk, the policy regarding financing decisions, and the policy regarding treasury banking (Dutch State Treasury Agency, n.d.). In line with this task the department drafts the interest rate risk policy (Dutch: renterisicokader) that controls the interest risk created by the financing of the state debt. This policy describes two indicators of interest risk: average time to refixing and the 12-month refixing amount (Dutch: renterisicobedrag or RRB). The average time to refixing is essentially the mean time to refixing interest payments, including the swaps to transform interest rates. This indicator serves as a long-term measure of risk. The refixing amount is the percentage amount of the state debt for which the interest rate needs to be determined in the coming 12 months. Therefore, this indicator serves as a short-term measure of risk (Ministerie van Financiën, 2019).

To artificially change the average maturity of the debt portfolio the DSTA has in the past taken on swap contracts to exchange floating and fixed rates. This is done by converting fixed rates to floating rates and converting those floating rates to a fixed rate with a lower or higher maturity. In this manner, the average time to refixing of the portfolio can be shortened or extended. Another method used to alter the average time to refixing of the portfolio is to prematurely close out swap contracts (Zondag, 2016).

These swap contracts do not necessarily come without cost but depend on the development of the swap rates. Additionally, swap contracts are not necessarily a perfect instrument to alter the interest rate on the debt portfolio. Discrepancies between the government bond rates and the swap rates are not uncommon and are referred to as the swap spread. It is in essence the swap rate minus the interest paid on government bonds. This spread changes over time for different maturities can be influenced by which bonds get issued over time and has an erratic appearance, therefore it is hard to make out trends in its movement and predict its future state.

Every two years, the DSTA evaluates the interest rate goals such as the average time to refixing of the portfolio and the 12-month refixing amount (Hoekstra, 2019). The first time of this evaluation is at the end of 2021. The results of this report are used as input for this evaluation.

1.2 The assignment

The department Policy and Risk Management State Debt would like to have analyzed empirically how significant movements in the swap spread have been in the past, and how they could develop in the future. More precisely, what is the underlying model or structure that drives the movements in the swap spread over different maturities and how has it behaved in the past. In addition to understanding its causes and history, further questions arise about the spread's consequences, or more importantly what its consequences for the risk profile of the state debt are. This latter question and the former together form the focus of the proposed assignment. Zooming in on the latter question about the policy, the department mentions the central aim of maintaining an acceptable balance between risks and costs. What an acceptable balance exactly entails and what metrics should be used to measure it, is up for further investigation. Therefore, without making it the aim of the research, the subject of balancing risk and costs will mostly be present, if not be central, during the recommendations chapter in the research. However, the modelling and historical analysis of the swap spread remains the main goal of this research and the policy implications are a byproduct of this report.

1.3 Problem Context

Suppose that the DSTA has committed to issuing a 5-year bond after exactly two years. This might for example be announced in a funding plan and can therefore not be deviated from. However, the interest rates might rise during this period, which means that the DSTA committed to an issuance that will cost the taxpayer more overall interest rate costs. To lessen this impact, the DSTA proposes to use a swap structure that profits from a long term 7-year payer swap, covered by shorter-term receiver swaps that fit the foresight period and the bond period. This structure is visualized in Figure 1-1.

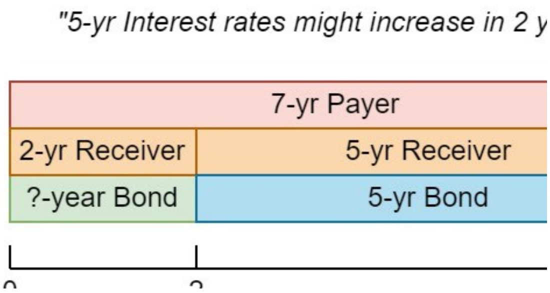

Figure 1-1 Rising interest rate hedge structure.

The issue of the bond cannot be changed which brings the risk that the yield might unfavourably change between the time of commitment and issuance. The first two years the swap construct pays and receives the swap rates $F(0,2) - F(0,7)$ and 2-years later for 5 years the payments are the swap rates $F(2,5) - F(0,7)$. The 5-year yield that needs to be paid after 2-years is $-\big(y(2,5)\big)_5$ and is added to both sides to complete the full comparison. This inequality illustrates when the usage of swaps is worth the effort

$$\big(F(0,2) - F(0,7)\big)_2 + \big(F(2,5) - F(0,7)\big)_5 - \big(y(2,5)\big)_5 > -\big(y(2,5)\big)_5.$$

The 5 subscript indicates a yearly payment span of 5 years after 2 years from now, and the 2 subscript a period of 2 years from now, and are added to distinguish their duration and discounting amount but keep the inequality simple. The yields present in the inequality could be omitted, however, keeping it is important to maintain the reality faced by the DSTA, which is the swap spread against the yield. The inequality can be rewritten as

$$\big(F(0,2) - F(0,7)\big)_2 + \big(SS(2,5) - F(0,7)\big)_5 > -\big(y(2,5)\big)_5.$$

This result shows why the movements of the swap spread are important since it is the presently unknown value on the left side. The swap spread will be more formally defined later on, but for now, it is considered to be the swap rate minus the sovereign rate, i.e. $SS(t,M) = F(t,M) - y(t,M)$. Most of the left-hand variables can be determined at the present time, which means that the decision is mostly dependent on the future swap spread against the future yield. Another important factor is the volatility or possible change of the swap spread against that of the yield. The DSTA prefers more stability and wonders if with the usage of swaps it is indeed more stable. Therefore, a question is whether the swap spread generally moves less than the yields.

Another possible scenario is the market preference of short issuance (5-year), while long issuance (7-year) is preferred by the DSTA. The DSTA's preference can come from present risk indicators or just a general market view from the DSTA. The market preference can be expressed by the DSTA's primary dealers. An option of the DSTA would be to issue short and swap the interest rates to the longer period or to ignore the primary dealer advice and issue the longer period. This scenario is visualized in Figure 1-2.

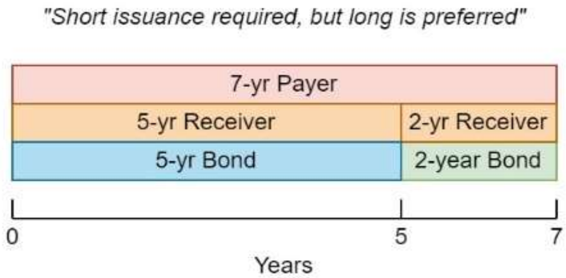

Figure 1-2 Maturity extension hedge structure.

The next inequality expresses the question of whether issuing short with swaps is better than issuing long, where $F(0,5) - F(0,7) - y(0,5)$ express the payments of the first 5 years and $F(5,2) - F(0,7) - y(5,2)$ express the payments in the last 2 years. The payments of a long term 7-year bond are expressed by $\big(-y(0,7)\big)_5 + \big(-y(0,7)\big)_2$. The inequality is

$$\big(F(0,5) - F(0,7) - y(0,5)\big)_5 + \big(F(5,2) - F(0,7) - y(5,2)\big)_2 > \big(-y(0,7)\big)_5 + \big(-y(0,7)\big)_2.$$

This can once again be transformed by taking all presently known values to one side like

$$\big(F(5,2) - y(5,2)\big)_2 > \big(F(0,7) - y(0,7)\big)_5 - \big(F(0,5) - y(0,5)\big)_5 + \big(F(0,7) - y(0,7)\big)_2.$$

It can be rewritten as

$$\big(SS(5,2)\big)_2 > \big(SS(0,7) - SS(0,5)\big)_5 + \big(SS(0,7)\big)_2.$$

Once again, the future swap spread makes an appearance on the left side. However, this time all terms can be expressed with swap spreads. Because the DSTA is faced with similar scenarios, it would like to have more information about the swap spread and its movement so that it can properly assess the risks it is facing by using swaps.

1.3.1 Further considerations

From the first conversations with the department, a few possible influences on the rates and curves already came to light. A notable one is the shift caused by the DSTA itself when it puts new bonds up for auction. This comes from the reasoning that when the DSTA issues new bonds the supply of this bond will increase. At the same time, the buyer of the bonds receives a fixed coupon rate, which they might want to exchange for a floating rate. This increase in the demand for payer swaps, which in turn influences the swap rates, results in a change in the value of the DSTA's swap portfolio. Therefore, the decision to issue new loans not only influences the debt portfolio, but also the swap portfolio.

Another point of attention is what sovereign rate should be used for comparison with the swap rates. By default, the yield curve with the bond yields for maturities comes to mind, however, this is not the only curve that can express states of the bonds. One could also translate the yield curve to an implied 6m-forward rate curve using non-arbitrage arguments (Yasuoka, 2018, p. 9). Another would be the zero-coupon yield curve, where the yields are translated to their zero-coupon equivalent. The spot curve, where the spot rates instead of yields are plotted over the maturities, would also be an alternative. (The spot curve is theoretically the same as the yield curve when only zero-coupon bonds are considered, however.) There could be even more different kinds of curves, and which one should and shall be used for modelling is addressed later in the research. A further question is if one should look at the nominal spread, or if it would be better to look at a zero-volatility spread (z-spread).

1.4 The Core Problem

The core problem can be described as a missing empirical and quantitative motivation for the interest risk policy about swap spread risk management. The problem is that empirical knowledge and insights are missing, and the DSTA would like to obtain them. This knowledge problem is built upon smaller problems, like the ones mentioned in the previous section. This means that the different possible curves and the auction effect need to be considered, among others. These smaller problems will be accounted for in the sub-questions of the three main research questions that will be defined later in section 1.7.

1.5 The Problem Approach

The process followed during the data analysis of the research is similar to the CRISP-DM (CRoss-Industry Standard Process for Data Mining) model by Shearer (2000) because the proposed analysis of the swap spread time series follows the same steps in practice. The model is guidance for what step to follow when data mining. It's a cyclic process existing out of:

- Business Understanding
- Data Understanding
- Data Preparation
- Modelling
- Evaluation
- Deployment.

Its presence won't have a big impact on the final report, however, it is a nice tool to help structure the workflow during the research, making the research's execution more streamlined.

During the execution, there will also be given updates to the stakeholders every one or two weeks. It is important to share the findings with stakeholders and others to see if the project is still on track, or to see if in the meantime requirements have or should be changed because of new insights. From the interactions, it should also become clear what kind of solutions are preferred in terms of scenarios and end results. This communicative process manifests continuously during the execution of all the steps, just like the documentation of the report. The most crucial assumptions made choices, and findings are documented. Writing these down at the right places and sowing them together into readable chapters, is the last step and will create the final thesis report.

In addition to data, literature is required to get a sense of what has been done in the past and what can be extended upon. This is obtained by literature study about previous research and general methods/theories in books. This research takes place at the end of this chapter and the whole of the second chapter. Further literature is used incidentally in further chapters.

1.6 The Research Problem

The swap spread is central in the research. Therefore, the main research question is:

How can swap spreads be defined, what underlying model drives swap spreads, and how can swap spread models serve decision-making?

This in turn can be related to the research goal with the SMART formulation:

The DSTA (**specific**: who) wants to model the swap spread (**specific**: what) for and together with the DSTA (**specific**: where/who) before the end of 2021 (**time-bound**), so that the results can be used as input comparing the effect of cost and risk of bond issuance vs interest rate swaps (**relevant**/**specific**: why). This goal is attained when all known modelling best practices have been applied and the relative best among the models is selected (**Measurable**, only when the selection criteria in the first phase of the research are worked out). If no such model arises, then another conclusion must be made about the possibilities of modelling the swap spread (**Attainable**, for example, if no such model exists, then non-existence is also a result).

1.7 The Research Questions

The three central research questions are made according to the structure described by Verschuren & Doorewaard (2016). This means that the first question focuses on the creation of assessment criteria and proposing possible approaches to creating a model for explaining the swap spread. The second question aims at creating and testing the models against the criteria, leading to the results that can be compared. Finally, the third question's goal is to make conclusions and recommendations, which in this case focuses on the model's implications for comparing costs and risks of interest risk management through either bond issuance vs interest rate swaps.

1.7.1 What are possible methods for modelling swap spreads and how is their performance measured?

The main challenge is to determine a model that explains the swap spread over time and for different maturities. Before considering the whole structure, one would first investigate its parts and see what is already known about them. Therefore, those individual subjects require a structured literature review to find out what is already known about them. The two main rates from which the swap spread exists out of are the government bond interest rates and the swap rates. For the interest rates, it is important to also look at the different curves that describe the interest rates for different maturities. Furthermore, a question is dedicated to looking at the relationship between government bond issuance and a change in swap rates. Thereafter, the swap spread itself is further investigated in the existing literature. At the same time, the available data is explored to see what information they reveal at first sight. The fitting sub-questions are:

- What are government bonds and their interest rates?
- What is a yield curve and what other curves describe the relations between bonds and their maturities that could potentially be relevant for calculating swap rates?
- What are swaps and swap rates and why are they relevant for a debt manager?
- How are swap spreads influenced by other variables?

After understanding the phenomena at hand, the possible method of describing their underlying structure is investigated. This would require taking up time series analysis literature and writing down the methods that lend themselves to the cases of interest rate time series. The sub-question to be answered is:

- What do time series analysis and interest rate modelling theory recommend to modelling the swap spread?

1.7.2 How are the proposed models created and tested, and what is their performance?

Before the models can be created, the data needs to be transformed in such a way that it can be used by the models. After the data preparations, the proposed models are created and tested according to the predefined performance criteria. Their overall performance is then compared to each other. With the comparison results, it is possible to conclude which model captures trends present in the swap spread the best according to the criteria. The sub-questions are:

- Where does the data come from and how is it prepared for modelling?
- How are the models created from the data?
- What are the resulting models, and which one is considered best?

1.7.3 *What can be concluded about the swap spread's structure, and what does this mean for the DSTA's interest risk policy?*

From here on, the implications of the findings are considered for the DSTA. It can be considered as the conclusions and recommendation section of the report. The sub-question is:

- How do the findings relate to the DSTA?

1.7.4 *Overview of research questions*

Later, in the report when there is a reference to a research question by only its number, the number corresponds to the question in the overview below.

1. **What are possible methods for modelling swap spreads and how is their performance measured?**
   - 1.1. What are government bonds and their interest rates?
   - 1.2. What is a yield curve and what other curves describe the relations between bonds and their maturities that could potentially be relevant for calculating swap rates?
   - 1.3. What are swaps and swap rates and why are they relevant for a debt manager?
   - 1.4. How are swap spreads influenced by other variables?
   - 1.5. What do time series analysis and interest rate modelling theory recommend to modelling the swap spread?
2. **How are the proposed models created and tested, and what is their performance?**
   - 2.1. Where does the data come from and how is it prepared for modelling?
   - 2.2. How are the models created from the data?
   - 2.3. What are the resulting models, and which one is considered best?
3. **What can be inferred from the created models and what does this mean for the DSTA's interest risk policy?**
   - 3.1. How do the findings relate to the DSTA?

1.8 The research design, methods, and data collection techniques

The three research questions divide the research process into:

- Literature study and criteria establishment
- Model creation and testing
- Conclusions and policy recommendations

Henceforth, the data/information requirement can be divided into the same kind of categories. The first central research question aims at creating a theoretical foundation for understanding the subject at hand and establishing model performance criteria accordingly. Therefore, this central question mostly requires literature and some data for premature data exploration. The interest rate and swap rate data will be collected using a Bloomberg terminal. The literature will be collected through Elsevier's ScienceDirect, Google Scholar, and internal documents of the DSTA. ScienceDirect will offer a register of previous research that has been done in the field. Google Scholar and Google itself will offer a rawer source of literature, which is not necessarily researching reports or articles, but for example books with the theory about interest modelling or time series analysis.

The second central question shouldn't require more literature and if it should, then the first central question is expanded. Therefore, the second questions mostly aim at the data from the Bloomberg terminal and use it in the creation of the models. The practical data analysis and modelling are done in Python where the used packages are documented, and the resulting script is added to the appendix.

The third central question mainly uses the results generated in the previous central question as input. The structures and predictions (if possible) of the models need to be related to the portfolio and funding strategy of the DSTA to see what its impact might be. It requires data about the current portfolio of the DSTA to see what the impact of changes in the swap spread is.

Table 1-1 below summarizes the research strategy. Internal refers to internal documents of the DSTA.

Table 1-1 Research strategy overview.

Sub Question	Data	Method	Sources
1.1.	Literature/ interest rate data	Structured Literature Review/ Data Visualization	ScienceDirect/ Google Scholar/ Bloomberg/ Internal
1.2.	Literature/ interest rate data	Structured Literature Review/ Data Visualization	ScienceDirect/ Google Scholar/ Bloomberg/ Internal
1.3.	Literature/ swap rate data	Structured Literature Review/ Data Visualization	ScienceDirect/ Google Scholar/ Bloomberg/ Internal
1.4.	swap rate data/ Auction dates	Time Series Analysis	Bloomberg/ Internal
1.5.	Literature/ swap rate data/ interest rate data	Structured Literature Review/ Data Visualization	ScienceDirect/ Google Scholar/ Bloomberg/ Internal
2.1	swap rate data/ interest rate data	Time Series Analysis	Bloomberg
2.2.	swap rate data/ interest rate data	Time Series Analysis	Bloomberg
2.3.	swap rate data/ interest rate data	Time Series Analysis	Bloomberg
3.1.	Modelling results	Observation	Experiments

1.9 Literature Study

To get a quick overview of what research into interest rate swap spreads has been done in the past, a query in the ScienceDirect register from Elsevier is performed. Since the subject of research is the interest rate swap spread, it makes sense to include the whole term in the query. This gives 43 results. The final query is, therefore: "interest rate swap spread". Based on their titles and summaries these eleven articles are deemed to be relevant:

- A Recursive Algorithm for Default Risk Adjustment in Interest Rate Swaps (Fehle, 1998)
- Fiscal policy events and interest rate swap spreads: Evidence from the EU (Afonso & Strauch, 2007)
- The effect of Fed monetary policy regimes on the US interest rate swap spreads (Huang & Chen, 2007)
- Analyzing the Volatility Transmission on the Eoniaswap Market (Făt & Mutu, 2012)
- What drives the Libor–OIS spread? Evidence from five major currency Libor–OIS spreads (Cui et al., 2016)
- Modelling Australian interest rate swap spreads by mixture autoregressive conditional heteroscedastic processes (Chan et al., 2009)
- Determinants of interest rate swap spreads (Lang et al., 1998)
- What determines the yen swap spread? (Azad et al., 2015)
- An empirical analysis of the Australian dollar swap spreads (Fang & Muljono, 2003)
- Modeling volatility and changes in the swap spread (In et al., 2003)
- Asymmetric dynamics in correlations of treasury and swap markets: Evidence from the US market (Toyoshima et al., 2012)

1.9.1 Summarizing Chronological Overview

Lang et al. (1998) argue that interest rate swaps create a surplus that is shared among the counterparties to accommodate the risks involved, and that this, in turn, affects swap spreads. They argue that swap spreads indicate how counterparties divide swap surplus among each other. The authors conclude that if bond spreads (corp. bond yield minus gov. bond yield) of single-A firms increase then so do the swap spreads. If the spreads of agencies increase, then so does the swap spread, however, the degree depends on the competitiveness of the market. If single-A firms undertake higher risk, then they are required to offer higher spreads to attract agencies into a swap contract. Finally, they conclude that swap spreads contain a procyclical element and that the rise of single-A firm spreads in a bad economy, will make it more profitable for other single-A firms to separate themselves from lower credit quality firms.

Fehle. (1998) continued the premise that with arbitrage arguments and the absence of default risk and other costs, the fixed payments on swaps should equal the yield of a government bond with equal maturities. The author, therefore, proposes a model that explains existing swap spreads using default risk variables. With this, the author was able to derive implied swap rates from default rates. In the conclusion, the author states, however, that there should be doubt in the statement that default risk mainly influences the pricing of swaps. He, therefore, proposes that future research is required to refine the framework of swap pricing.

Fang & Muljono. (2003) investigated the relationship between the Australian dollar interest rate swap spread and the term structure of the interest rate. They also investigated determinants of interest rate swap spreads. Their data had a daily sample period from 6 December 1996 to 31 December 1999. To model the changes in the swap spread they use a 6-term regression model with the explaining variables being the 3-month treasury note rates, differences between m-year and 1-year maturity treasury note rates (the slope), curvature factors implied by the Nelson-Siegel model, the spread between 3-month Bank Bill Swap Rates (BBSR) and 3-month treasury note rates, the spread between AA corporate bond yields and Treasury yields, and the bid-ask spread in the m-year Australian dollar swaps. The authors found that the Australian swap spreads are positively affected by the level of curvature, but negatively affected by the slope of the Treasury note rates. They also concluded that liquidity does not add much towards explaining the swap spread. The findings suggest that the Australian dollar swap spreads mostly exist out of a credit risk premium.

In et al. (2003) used a model that allows relationships between volatilities of different maturities to capture determinants that influence changes in the U.S. interest rate swap spreads. They used a regression model where the change in the swap spread is explained by the change in the level of the 90-day Treasury bill interest rates, the change in the slope of the Treasury bill yield curve, the squared change (volatility proxy) in the 90-day Treasury bill rate, the change in the spread between AAA- and BAA-rated bonds, and the change in the spread between 3-month LIBOR and the 90-day Treasury bill rate. The authors further extended this by incorporating a multivariate EGARCH model to determine the volatilities of the swap spreads. They conclude that the multivariate EGARCH model captures the volatilities of the swap spreads quite well. From the regression, they observed that changes in interest rate volatility and changes in the spread between AAA- and BAA-bonds have a positive relationship with changes in the swap spread. Negative relations with changes in the level of interest rates and changes in the slope of the interest term structure were also observed.

Afonso & Strauch (2007) set out to investigate the impact of fiscal policy events on interest rate swap spreads. They looked at fiscal policy events of 2002 and laid them next to weekly swap spread data. The authors used a regression model with auto-regression of degree 1, the US spread of this and the previous week, the bid-ask spread for 10-year government bonds, the average implied volatility of call and puts, the slope of the US yield curve, and dummy variables that represent the fiscal policy events. The authors found significance for the autoregressive and US swap spread terms, unfortunately, the fiscal policy events did not show to be of much influence on the swap spread.

Huang & Chen. (2007) investigated the structure of swap spreads concerning economic shocks among different federal monetary policy regimes. The authors partition the data according to the different Fed monetary policies. To these partitions, they applied a vector autoregressive model with the variables being: 2-year maturity swap spreads, 10-year maturity swap spreads, the slope of the treasury yields term structure, the differential between Baa and Aaa rated bonds with similar maturity, volatility generated by an EGARCH model, and a liquidity premium by subtracting 3-month treasury by Eurodollar rates. From these variables, the authors conclude most notably using WALD tests that the yield slope strongly influences the swap spread for 2-year maturities. However, for 10-year maturities, this is less present. Using variance decomposition, they further conclude that the variance of the swap spread is mostly explained by itself, meaning that in the vector auto regression model the swap spread variable is influenced the most by its previous value in time. Among the many findings the authors conclude that in the partitions, the treasury slope, liquidity premium, interest rate volatility, and default premium all influence the swap spread variance. They further noted the different explanatory roles of these variables during different monetary policy regimes.

Chan et al. (2009) looked at Australian dollar interest rate swap spreads using MARCH (mixture autoregressive conditional heteroscedastic) models. They used the daily differenced 3, 5, and 10-year maturity swap spread time series (DSS 3, 5, and 10) for their modelling. The time frame of the data was from 3 January 2000 to 29 December 2006. For the three DSS time series, they consistently identified a MARCH(2; 3, 0; 1, 0) model. They interpreted this model as an AR(3)-ARCH(1) model with small independent shocks/breaks. The authors argue that the fitted model can therefore accommodate outliers/shocks in the data. For further research, they propose to investigate models that can catch the dependence between the DSS time series, since they are not independent.

Toyoshima et al. (2012) used an asymmetric dynamic conditional correlation (A-DDC) model to capture correlations between treasury and swap markets. The period of the data that was used ranged from February 9, 2006, to May 31, 2011. The authors estimated the conditional variances using an AR-EGARCH model. Instead of using a normal distribution for the error term, they used generalized error distribution, which in addition to the normal distribution incorporates the thickness of the tails (kurtosis). The A-DCC model they used to have the form of a degree-1 AR process with a crisis term added. This crisis term is a Boolean variable that is positive (1) when the time falls within a 2007 to 2011 financial crisis period. The authors conclude that only the conditional correlation of the 2-year maturity differs from the others, which is possibly explained by their influence of market forecast of monetary policy changes by the Federal Reserve Board. They also observed that the coefficients for the financial crisis proxies are all negative, implying that arbitrage transactions between swaps and treasuries had decreased in scale during the crisis.

Făt & Mutu. (2012) used cointegration tests to discover equilibrium relationships between Eonia and Eoniaswap rates. Furthermore, they fitted an ARFIMA-FIGARCH model to capture volatility relationships between Eoniaswap rates at different maturities. The authors used Zivot Andrews' structural break test to identify structural breaks in the series. They used the Hurst exponent and GPH test to identify long term memory in the series, which they were able to detect. They fitted ARFIMA(1,dm,1) – FIGARCH(1,d_v,1) was able to remove the temporal relations and heteroscedasticity from the residuals. The authors conclude that the long-term memory of the Eoniaswap rates makes the derivatives less profitable for banks, but more suitable for efficient market risk management.

Azad et al. (2015) investigated Japanese yen interest rate swap spreads price risks. They developed an AR model including terms for business cycle risk, skewness risk, correlation risk, default risk, and liquidity risk. The business cycle and skewness risk variables are obtained using a FS-GARCH model. Their empirical findings show that business cycle risk has a countercyclical impact on the yen swap spread and is positively correlated with skewness risk. When the correlation between underlying interest rates is high, the yen swap spread is negatively correlated with correlation risk and is positively correlated if their correlations are low.

Cui et al. (2016) investigated determinants of the USD, GBP, EUR, JPY, and CHF, LIBOR-IOS spreads. The authors use a general unrestricted model together with weekly data. They applied an algorithm to optimize the variables for the model. They ended up taking the differenced logarithm of the spread and explaining it by structure break dummy variables combined with n-explaining variables with 12-week lags. The explaining variables were chosen to be dummies for systemic credit risk, counterparty risk, market volatility, market liquidity, bank leverage, SMI, IPG, and real GDP. The authors conclude that systemic credit and default risk, market volatility, and counterparty risk are the three main drivers of the spread during the crisis and in the long run.

1.9.2 *Overview of explaining variables from the articles*

Not all the articles offer an insight in possible explaining variables, however, a lot do. The ones that do are the variables mentioned below:

- Procyclical elements
    - Lang et al. (1998) concluded that procyclical elements are present in the swap spread, which means that it moves together with GDP. Azad et al. (2015) found a similar relation between business cycle risk and its countercyclical impact on the yen swap spread.
- Spot rate curvature
    - Fang & Muljono (2003) observed that the Nelson-Siegel curvature positively affect the swap spread, which could mean that lessening demand and supply discrepancies increase the swap spread. This is of course on the basis that curvature represents a demand and supply correction curve.
- Treasury note rates
    - Fang & Muljono (2003) also observed that the treasury notes slopes had a negative impact on the swap spread. This slope is the difference between m-year and 1-year Treasury note rates. In et al. (2003) also discovered this negative relationship between the slopes and swap spreads. Huang & Chen (2007) discovered a similar relationship, however, they note that this relationship is stronger for 2-year maturities than 10-year maturities. Which is a result of the fact that a slope decrease is more noticeable for shorter periods than longer periods. They also give it as a variable that explains the variances present in the swap spread

    - In et al. (2003) further, conclude that the 90-day Treasury bill rate has a negative impact on the changes in the swap spread.
    - Huang & Chen (2007) discovered that interest rate volatility impacts the swap spread volatility. Cui et al. (2016) add market volatility to this list, meaning that a volatile market creates a wider swap spread.
    - Azad et al. (2015) found that when the correlation between underlying interest rates is high, then the swap spread is negatively correlated with regard to correlation risk. This relation is positive otherwise.
- Different credit bond spreads
    - In et al. (2003) concluded that the spread between AAA- and BAA-bonds has a positive relationship with changes in the swap spread.
- Liquidity
    - Huang & Chen (2007) note that liquidity premiums influence the variance of the swap spread. However, Fang & Muljono. (2003) note that liquidity does not add much towards explaining the swap spread.
- Default/credit risk premium
    - Fang & Muljono. (2003) conclude that the Australian dollar swap spreads mainly exist out of a credit risk premium. Huang & Chen (2007) support this by concluding that default premiums influence the swap spread variance. Cui et al. (2016) also support this, by concluding that it is one of the main drivers in the spreads. They also mention counterparty risk to be a role, however, this can also be credit risk.

A handful of possible influences have come to light. Credit risk premium and Treasury note slopes seem to be favourites among the researchers and Liquidity and credit risk are the main interpretations of the swap spread. The Treasury notes are in the context of this research represented by the Dutch sovereign rates. With the usage of Nelson-Siegel decompositions, the slopes are going to be extracted into a single parameter as proposed and executed in section 3.2.5. The credit measures are going to be covered by a comparison between the Dutch and German bonds, however, this could also be considered a proxy for credit premiums. The procyclical elements will get a role represented by the Dutch GDP, the AEX, and Dutch inflation rates. The credit and liquidity premiums will be further represented by the bid and ask spreads for both bonds and swaps, and the difference between those spreads.

2 What are possible methods for modelling swap spreads and how is their performance measured?

This chapter covers the definitions of bonds, yield curves, swaps, and ultimately the swap spread. It then continues on possible influences on the yield curves by other factors and how the swap spread time series are analyzed and used for modelling.

2.1 What are government bonds and their interest rates?

2.1.1 What is a bond?

A bond is a type of loan, where instead of periodic instalments, the full amount is paid of instantly after a certain period. This full amount is called the principal and the final payoff date is called the maturity of the contract. Bonds can have interest-like payments embedded in their contracts, which are called coupons. These coupons are percentages based on the principal and are paid to the bond holder periodically until the contract matures. A bond with no coupons is also referred to as a zero-coupon bond.

Bonds are issued with the purpose of cash management in mind. When an organization needs more liquidities for its operations or projects it can issue bonds to obtain them. The DSTA uses bonds for the same purposes. The DSTA has a central role when it comes to getting liquidities for government and government-related organizations. Therefore, it has daily and longer-term quotas of cash acquisition that are operationally met by the dealers in their dealing room. These government-issued bonds are non-surprisingly called government, treasury, or sovereign bonds. Regularly, the yields on these bonds are used as a proxy for a risk-free rate with which cash flows can be discounted in a risk-neutral world. There is a clear distinction between short term and long-term bonds issued by the DSTA, where the former is called Dutch Treasury Certificates (DTC) and the latter Dutch State Loans (DSL). DTCs are short-term DSLs, however, they are always zero coupon-bearing.

Yields on amounts that still need to be (re-)financed in the future are the major interest rate risk faced by the DSTA. They are the periodic cost of the outstanding loans. When the interest payments are known for a longer period, the DSTA deems them to be stable and less risky. However, too long is also unfavourable, because a general market interest rate can decrease further, which makes the fixed interest payments more expensive in hindsight. This dynamic of long and short interest rate payments is one of the challenges faced by the DSTA's risk management and is a major part of what is called interest rate risk. The DSTA uses a measure that they call average time to refixing (Ministerie van Financiën, 2019) to monitor this specific interest rate risk. It is simply the weighted average maturity of outstanding interest payments. Fixed interest payments on a swap would also count towards this measure.

2.1.2 The present value of a bond

Suppose a bond pays coupons at dates $T_1, T_2, T_3, \dots, T_N$, which together form the set T. The value of a bond at time $t < T_N$ is given by

$$B_{c,P}(t,T) = \sum_{\tau \in T:\, \tau > t} \frac{cP}{(1+y(T_N))^{\tau - t}} + \frac{P}{(1+y(T_N))^{T_N - t}}.$$

Here c is the coupon percentage, P the principal, and $y(T_N)$ the yield to maturity. Because the principal is a constant and can be taken outside of the equation, a more compact notation is

$$B_c(t,T) = \sum_{\tau \in T:\, \tau > t} \frac{c}{(1+y(T_N))^{\tau - t}} + \frac{1}{(1+y(T_N))^{T_N - t}}.$$

Which can be multiplied by the principal if it needs to be incorporated. Because of its simplicity, the later equation is used. The value of a zero-coupon 1-euro principal bond is given by

$$B_0(t,T) = \frac{1}{(1+y(T_N))^{T_N - t}}.$$

Yields are assumed to be constant for every period in the previous equations, however, this need not be the case. Therefore, an alternative to the bond price equation is

$$B_c(t,T) = \sum_{\tau \in T:\, \tau > t} \frac{c}{(1+r(\tau - t))^{\tau - t}} + \frac{1}{(1+r(T_N - t))^{T_N - t}},$$

where the yield from before is now considered a function of time. This means that every period has a different interest rate. To avoid confusion between the yield to maturity curve, this curve is called the spot rate curve, and it will be denoted by $r(t)$. With the use of zero-coupon bonds, a spot rate curve can be obtained by

$$r_0(T_N) = \left(\frac{1}{B_0(0,T)}\right)^{\frac{1}{T_N}} - 1.$$

The curve produced by this method is called the zero-coupon curve. Ideally, the interest rate curve is the graph expressed by the zero-coupon yield function because it is quite straightforward to compute. However, zero-coupon bonds are not always available on the market, which means that coupon-bearing bonds need to be used to create a mix between a spot rate and zero-coupon curve.

2.2 What is a yield curve and what other curves describe the relations between bonds and their maturities that could potentially be relevant for calculating swap rates?

2.2.1 The basic interest rate curve

One of the intuitions behind a regular interest curve is that lending out money for a longer time means a higher credit risk, because the probability that a counterparty goes bankrupt is higher, the longer the period. Therefore a higher yield is required for longer periods. Furthermore, benchmark interest rates might decrease over the period, which can have positive or negative effects on the outstanding loans. The same goes for borrowing money, but then the risks are reversed. The rate for borrowing is generally higher than lending, otherwise, banks would not deem the practice worth their effort. The relation implied by the liquidity/credit premium of increasing interest is expressed by

$$\frac{dr(t)}{dt} > 0.$$

However, it is observed that the difference decreases over time (diminishing returns), giving it the shape of a curve and making the function concave. This means that the second derivative is negative

$$\frac{d^2 r(t)}{dt^2} < 0.$$

However, reality shows that these expected properties of the yield curve do not need to hold and that there are more factors other than credit risk that play a role in its shape. Figure 2-1 is an example of this discrepancy, where a 1-year rate is higher than the 2-year rate.

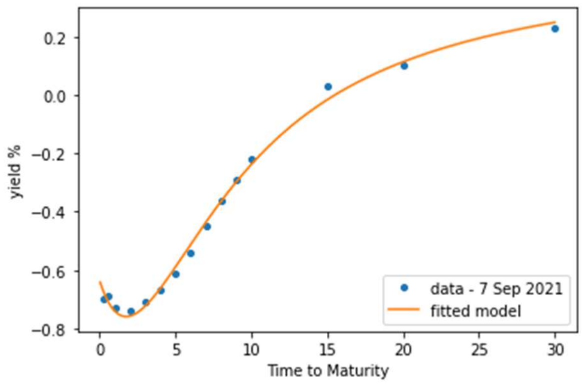

Figure 2-1 The zero curve of September 7, 2021 (data retrieved from Bloomberg).

The next section argues why this might be.

2.2.2 *Money Market vs Capital Market*

The discrepancy introduces the concepts of money market and capital market. By an explanation of Investopedia (Majaski, 2021): the money market is the market of short-term financial products. The capital market is its long-term counterpart and therefore regards the market of long-term financial instruments. Simply said, they divide the total market into short-term and long-term. Looking back at Figure 2-1, the assumption violation happens in the short-term period, before the 1-to-2-year maturity period. This could be due to liquidity, trade volume, and/or supply and demand. If the demand for cash products decreases and the supply does not, then the equilibrium price of the cash products is relatively low, compared to its present value. If this is disproportionally the case for cash products against capital products, then the yield implied from these market prices also becomes disproportionately high, so much that shorter rates are higher than the longer rates. It proposes the question: what has the biggest impact on the price? Is the bond price decided by present value calculations and liquidity/credit premium arguments, or, as the money market suggests, is the bond price fully decided by demand and supply? Even so, the answer might be a mix of both, where in the cash market demand and supply has the greatest influence, while in the capital market it is mostly influenced by liquidity/credit premiums. Section 2.4.1 will continue on this subject, while the next section is dedicated to interest rate movement over time.

2.2.3 *Zero curve evolutions over time*

The area of interest is not merely the zero curve at a single moment, but also how the zero curve evolves over time. To support this field of interest, the notation of the interest rate function is extended to

$$r(t,\tau).$$

Which can be read as the $(\tau - t)$-year interest rate from time t to maturity τ, if time is expressed in years. The bond price equation then is extended into

$$B_c(t,T) = \sum_{\tau\in T:\tau>t} \frac{c}{(1+r(t,\tau-t))^{\tau-t}} + \frac{1}{(1+r(t,T_N-t))^{T_N-t}}.$$

This notation also reveals one of the challenges encountered when measuring a fixed rate over time. Suppose one would like to determine a n-year yield over time. At initial time zero a m-year, where $n < m$, the bond's n-year cash flow is paid exactly n-years from now. However, as one day passes this same cash flow is now paid $n - 1$ days from now. This decrease in period makes it impossible to directly determine the n-year without approximation. This approximation becomes more prevalent, the more days pass. This stresses the importance of the estimation techniques discussed in the next section and appendix B.

2.2.4 *Deciding what curve to use*

Yield to maturity, spot rate, and zero-coupon curves have been mentioned in the previous sections. However, choosing what curve to use and how to establish it is no easy task. The first idea is to create one from observable bond prices. This is easier said than done, and the text in appendix B explores this path. In summary, the text explores multiple methods of curve creation. It starts with the bootstrapping method (Hull, 2018), which requires the gaps of missing maturities to be filled by other means. This introduces interpolation methods from linear to cubic splines (Cox, 1995). The Nelson-Siegel (Nelson & Siegel, 1987) model is introduced shortly thereafter. Some further investigation in review papers (Schmidt, 2011) also shines a light on no-arbitrage and market models, like the LIBOR market model. It is argued which method or model is best to use, which results in the choice of the Nelson-Siegel model. It is because of its simplicity, it gives generally better results than splines (Lorenčič, 2016), and it is widely used in the past by central banks (Bank of International Settlements, 2005). However, the central banks used an extension on the model made by Svensson (1995).

Unfortunately, these findings are not used further during the research, which is the reason that the full text is added to the appendix and summarized instead. The reason for not being useful is that in hindsight these calculations are too time-consuming to do within a reasonable amount of time. The optimization methods used in fitting Nelson-Siegel curves to cash flows take too long to compute and the results are not of much quality. Therefore, it is decided to take this curve for granted and use the zero-coupon data provided by Bloomberg. This saves a lot of time, which can be invested in better explaining the swap spread. More details about the Bloomberg curves are covered in section 3.1.3.

The Nelson-Siegel model is still used, however, but in another context. It is used later on to decompose the zero-coupon and swap spread in terms of shift, slope, and curvature. Therefore, the next section will explain what the model exactly entails and later in section 3.2.5 the decomposition will be explained.

2.2.5 *Nelson, Siegel, and Svensson*

The Nelson-Siegel model mentioned in the previous section proposes the interest rate for maturity M as

$$r(M) = \beta_1 + \beta_2 \frac{\lambda}{M}\left(1 - e^{-\frac{M}{\lambda}}\right) + \beta_3\left(\frac{\lambda}{M}\left(1 - e^{-\frac{M}{\lambda}}\right) - e^{-\frac{M}{\lambda}}\right),$$

and Svensson extends this to

$$r(M) = \beta_1 + \beta_2 \frac{\lambda_1}{M}\left(1 - e^{-\frac{M}{\lambda_1}}\right) + \beta_3\left(\frac{\lambda_1}{M}\left(1 - e^{-\frac{M}{\lambda_1}}\right) - e^{-\frac{M}{\lambda_1}}\right)$$

$$+\beta_4\left(\frac{\lambda_2}{M}\left(1-e^{-\frac{M}{\lambda_2}}\right)-e^{-\frac{M}{\lambda_2}}\right).$$

When this model is taken to structure the zero curve, the changes in interest rate over time can instead be captured by changing the parameters over time like

$$r(t,M)=\beta_1(t)+\beta_2(t)\frac{\lambda(t)}{M}\left(1-e^{-\frac{M}{\lambda(t)}}\right)+\beta_3(t)\left(\frac{\lambda(t)}{M}\left(1-e^{-\frac{M}{\lambda(t)}}\right)-e^{-\frac{M}{\lambda(t)}}\right).$$

The parameters have become functions of time; hence they have become stochastic processes on their own.

All the parameters have a contribution to the structure of the zero curve. They have similar roles as the three principal components from the principal component analysis (PCA) done on swap rates (Hull, 2015, pp. 193–196). The first one (β_1) is a parallel shift as seen in Figure 2-2. As the name suggests it simply shifts the curve up or down.

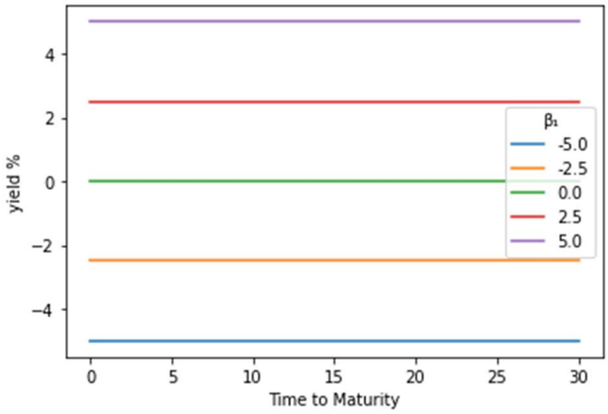

Figure 2-2 The shift component's shapes.

The second parameter (β_2) is for the slope component. It signifies the amount of difference between lower and higher rates. Some of the possible shapes can be seen in Figure 2-3.

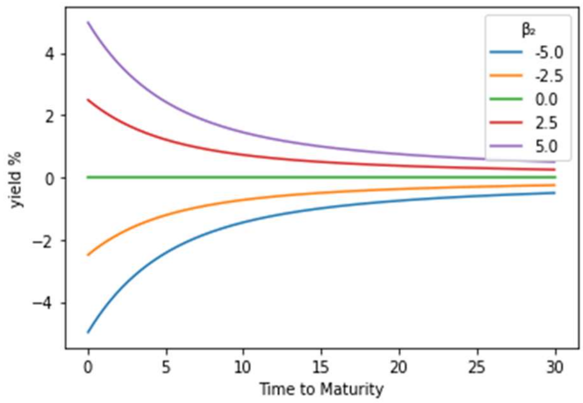

Figure 2-3 The slope component's shapes.

The third (β_3) and fourth (β_4) component both represent a bend in the curve as illustrated in Figure 2-4. This bend, if rightly adjusted, can represent the valley between the money and capital market. Svensson (1995) added the second bend (β_4)

component to "increase flexibility and improve the fit." However, the addition is not supported by the PCA (Hull, 2015, pp. 193–196) results and might seem more like overfitting.

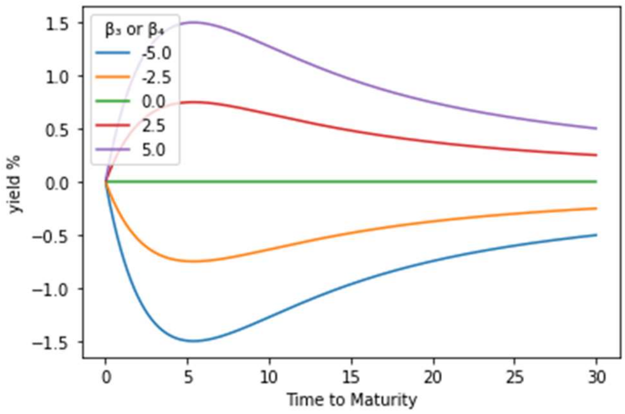

Figure 2-4 The bending component's shapes.

The two lambda parameters (λ_1 and λ_2) signify the rate of convergence to the graphs asymptote. This asymptote is given by

$$\lim_{M\to\infty} r(M) = \beta_1,$$

which means that the parameter $\beta_1$can also be interpreted as the yield on long-term investments. What is considered long, is up for debate. Another thing that is worth investigating, is what happens when time goes to 0. Intuitively, the yield should be zero, since a 100 euros now, should be worth a 100 euros now. However, the model gives (by using l'Hôpital's rule)

$$\lim_{M\to 0^+} r(M) = \beta_1 + \beta_2.$$

If this is assumed to be strictly zero, then $\beta_1 = -\beta_2$ and the NS-model has one parameter less to optimize, which makes it take the form of

$$r(M) = \beta_1\left(1 - \frac{\lambda}{M}\left(1 - e^{-\frac{M}{\lambda}}\right)\right) + \beta_3\left(\frac{\lambda}{M}\left(1 - e^{-\frac{M}{\lambda}}\right) - e^{-\frac{M}{\lambda}}\right).$$

Another way to weakly incorporate the assumption is by adding a point to the data which corresponds to having equal cash flows at time 0. When fitting the curve, it will then go approximately through that point, but it leaves open some flexibility.

To demonstrate how such parameters are chosen and what a fitted curve might look like, a quick calculation is performed on a zero curve from September 7, 2021. The curve is retrieved from Bloomberg, which itself uses different and mostly unknown means of creating it. Nonetheless, a fit can be made using OLS and non-linear least squares. The convenient thing about estimating the parameters is that when the residuals of differences in yields are minimized, the Betas can be estimated using OLS, which means that only the Lambdas need to be estimated by non-linear means. This is not the case when bond prices are used for estimation, because bonds prices are dependent on yields in a non-linear manner, which would make the total equation non-linear. The fit resulted in the curve depicted in Figure 2-5. The blue dots in this graph are the data points from Bloomberg.

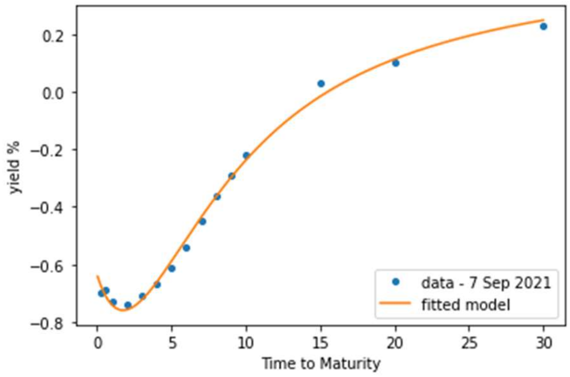

Figure 2-5 The data points and the fitted curve.

Table 2-1 contains the resulting parameters of the fit in Figure 2-5. What is interesting to see is that both Lambdas are approximately equivalent. This means that the extra addition of Svensson was unnecessary in this case and that third and fourth Betas can be added up to form the main bending parameter.

Table 2-1 Estimated parameters for the Nelson-Siegel-Svensson model

β_1	β_2	β_3	β_4	λ_1	λ_2
0.521	-1.159	-0.680	-1.320	2.581	2.581

Figure 2-6 illustrates how the Betas contribute to the curve. It is interesting to see how the right combination of the slope and bending components can create a valley that separated the money and capital market.

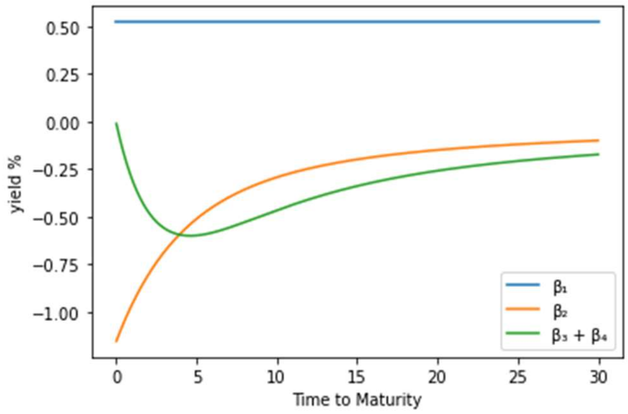

Figure 2-6 Decomposition of the fitted model

2.3 What are swaps and swap rates and why are they relevant for a debt manager?

2.3.1 *What is a swap?*

A swap is an agreement where two parties agree to exchange cash flows at predetermined dates in the future. In the case of interest rate swaps, these cash flows are based on interest rates. One party agrees to pay a fixed amount on future dates, based on a fixed rate called the swap rate. The other party pays a floating amount, based on a floating interest rate, like Euribor. The floating rate used for a period is "fixed" one period prior. Noteworthy is that the DSTA takes on the opposite role of the investor, meaning that the equations need to be negated from their perspective. However, for the sake of consistency, the equations described in this report are from the perspective of the investor.

2.3.2 *What is the value of a swap?*

Suppose a swap exchanges cash flows at dates $T_1, T_2, T_3, \dots, T_N$, which together form the set T. The present value of a swap at time $t < T_N$ is given by

$$S_K(t,T) = K \sum_{\tau \in T:\tau > t} \frac{V(\tau-\Delta\tau) - F(0,T_N)}{(1+y(t,\tau-t))^{\tau-t}},$$

where K is some fixed amount of money, $\Delta\tau$ distance between two dates, and $V(\tau - \Delta\tau)$ is the floating (variable) interest rate function, which could represent accumulated floating rates like Euribor. F is the fixed interest rate (swap rate) at some time with some given maturity, y is the interest rate used to discount the cash flows, T_N is the maturity of the swap, and T is the set of payment dates. This equation assumes that swaps can be discounted by the same interest rate as bonds. The floating rate is fixed the period before its usage and $\Delta\tau$ signifies this period between the current and previous data in the sum. If the periods between all cash flows dates are equal, then $\Delta\tau$ is some predetermined constant. In this situation, the 'owner' of the swap pays fixed and receives floating. The fixed exchange position of the owner determines the name of the swap, which in this case makes it a payer swap. Just like with the principal of bonds, the constant K can be taken out of the equation for simplicity making it

$$S(t,T) = \sum_{\tau \in T:\tau > t} \frac{V(\tau-\Delta\tau) - F(0,T_N)}{(1+y(t,\tau-t))^{\tau-t}}.$$

A receiver swap is the reversed cash flow direction of a payer swap, which is the same as negating the equation

$$-S(t,T) = -\sum_{\tau \in T:\tau > t} \frac{V(\tau-\Delta\tau) - F(0,T_N)}{(1+y(t,\tau-t))^{\tau-t}} = \sum_{\tau \in T:\tau > t} \frac{F(0,T_N) - V(\tau-\Delta\tau)}{(1+y(t,\tau-t))^{\tau-t}}.$$

From now on $S(t,T)$ is referred to as a payer swap and $-S(t,T)$ a receiver swap.

2.3.3 *How do swaps relate to bonds?*

Combining a payer swap with a bond can make the coupons become floating instead of fixed, creating

$$B_c(t,T) + S(t,T) = \sum_{\tau \in T:\tau > t} \frac{c}{(1+y(t,T_N-t))^{\tau-t}} + \frac{1}{(1+y(t,T_N-t))^{T_N-t}} + \sum_{\tau \in T:\tau > t} \frac{V(\tau-\Delta\tau) - F(0,T_N)}{(1+y(t,T_N-t))^{\tau-t}}.$$

To make the fixed payments overlap, the amount of $\frac{c}{F(0,T_N)}$ swaps need to be bought.

Hence

$$B_c(t,T) + \frac{c}{F(0,T_N)} S(t,T) =$$
$$\frac{c}{F(T_N)} \sum_{\tau \in T:\tau > t} \frac{V(\tau-\Delta\tau)}{\left(1+y(t,T_N-t)\right)^{\tau-t}} + \frac{1}{\left(1+y(t,T_N-t)\right)^{T_N-t}},$$

making the bond receive floating coupons $\frac{c}{F(0,T_N)} V(\tau - \Delta\tau)$. Instead of selling the swap contract at a predetermined price, most swap contracts are sold for "free" ($S_P = 0$), however, the costs are embedded in the swap rate $F(T)$. Thus, instead of quoting the price of the swap, the swap rates are quoted by market makers.

A final remark should be made about the $\frac{c}{F(0,T_N)}$ term that represents the swaps required to negate the fixed payments. One could have another goal in mind for swaps that requires not the fixed payments to match, but the volumes. This results in a fixed exchange that is non-zero.

2.4 How are swap spreads influenced by other variables?

2.4.1 Demand and supply

It has been argued before in section 2.2.2 that demand and supply play a certain role in the pricing of financial products. Multiple payments at different times in the future demonstrate the importance of understanding the time value of money. However, an extra dimension is important to consider, which stems from the fact that the payments together have become a financial product. Furthermore, it is a scarce product, meaning that there is not an infinite amount available. This gives it the influences of demand and supply.

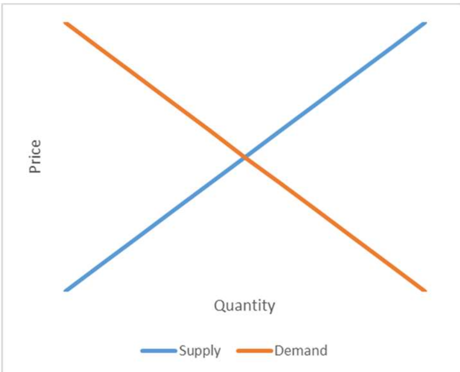

Figure 2-7 Fictitious demand and supply curves example.

In the theoretical pricing equations of the swaps and bonds given in the previous sections, the time value of money is represented by the sovereign rate curve functions. However, there is no such variable that considers the demand and supply of the instrument. In the theory of an efficient market, a product's price is determined by the equilibrium where demand meets supply on the curves. In section 2.2.2 a handful of possible bond price and yield determinants were discussed. In addition, it might be that the theoretical price based on liquidity and credit premiums, is the demand-supply equilibrium in a general market. However, for shorter maturities, the market might have a higher general supply or lower general demand, which moves the equilibrium to a lower price (a lower bond price, means a higher yield). The result is the elevated cash market rates observed in Figure 2-1. This would mean that there is a theoretical yield curve that purely follows the credit/liquidity risk reasoning and an additive curve that solely exist out of supply and demand. The question that remains is how these two can be decomposed from the observable curve in the market. The Nelson-Siegel decomposition might have the answer since the slope component agrees with the expected shape of the theoretical yield curve imposed by credit premiums, which means that the curvature component could represent the influences of supply and demand. This leaves the shift component to be divided among the two. It is hard to prove this however, that is why this will remain just a possibility for now. The next section will investigate the possible impact of bond issuances on swap rates.

2.4.2 Swap demand and sovereign bond issuances

There is a possibility that swap rates can be influenced by the announcement and issuance of old or new old bonds. To investigate this possibility, the auctions so far in 2021 are inspected and there were six auctions where the maturity of the bond was still to be determined. These bonds are most interesting because they are irregular, and their announcement was completely new information for the market. These auctions were on the 23rd of March, 28th of April, 25th of May, 22nd of June, 13th of July, and 31st of August. They respectively had a volume of 1.95, 2.35, 1.95, 2.25, 2.78, and 2 billion. Table 2-2 contains the means and standard deviations of the swap rate changes. The swap rates data used for computing these ranges from 28-9-2020 to 27-9-2021 and is retrieved from Bloomberg.

Table 2-2 Means and standard deviations of swap rates (1-year data from Bloomberg).

	4-year	5-year	12-year	15-year	20-year	25-year
Mean	-0,0663%	-0,0845%	-0,1443%	-0,1568%	-0,1693%	-0,1741%
SD	1,1679%	1,4655%	2,2240%	2,3085%	2,3935%	2,4535%

Figure 2-8 illustrates the impact these auctions had on the change in swap rates by plotting how many standard deviations the change was from the mean.

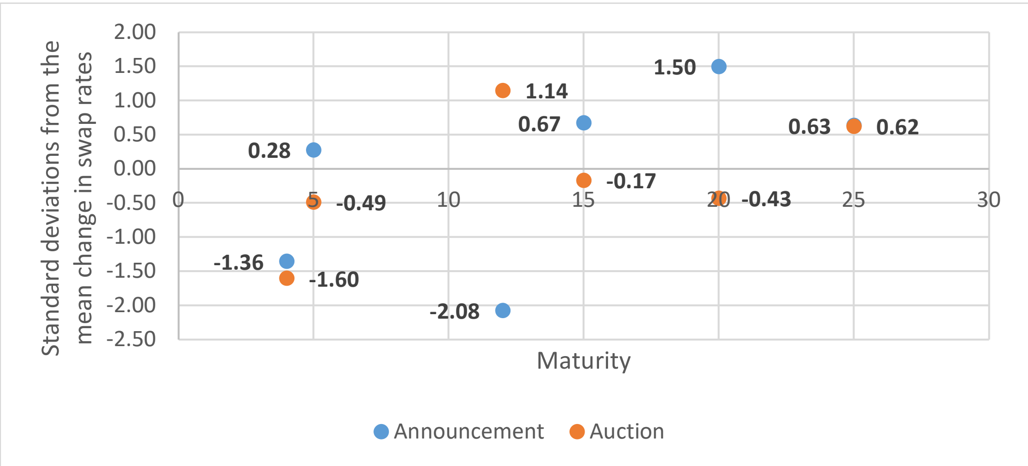

Figure 2-8 Swap rate change deviations and auctions.

From this graph, it seems that most auctions had an impact for which the change in swap rates is within two standard deviations from the mean, except for the 12-year swap rate, which slightly went past it. The observation that they mostly lie within two standard deviations from the mean does not make a promising prospect for the impact of auctions on swap rates, since one would like to reject equal means. Six samples do of course not make a rigorous conclusion, however, for what it is worth, Dutch auctions themselves and their announcements seem not to have a significant impact on swap rates. More detailed and extensive research might shine a better light on this matter, however, this surpasses the scope of this research.

2.5 What do time series analysis and interest rate modelling theory recommend to modelling the swap spread?

This section starts with the pretext that the data is already prepared for usage because the focus here lies on the modelling process. The data preparation process is explained later in section 3.1. The book used for the modelling process is *Elements of Time Series Econometrics: An Applied Approach* (Kočenda & Černý, 2015). The text hereafter is based on the methods explained in that literature, if not, then the alternative source is specified.

2.5.1 The memory of the swap spread

The first goal is to filter out temporal relatedness in the time series by fitting the swap spread to Auto-Regressive Moving Average (ARMA) models. The auto regression refers to how much the next value is dependent on the previous, the moving average by how much previous residual values influence the next value in time. The aim is to reduce the time series to residuals using ARMA models, such that the residuals can be further explained by external variables. After selecting the external variables through linear fitting and significance testing with the residuals, they can be incorporated into the ARMA model by refitting an ARMAX model, which is an ARMA model with external variables.

The modelling starts with uncovering if the swap spread contains a memory of its previous values. Before that is possible, however, the stationarity of the time series needs to be determined. Both the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test lend themselves to this purpose. The null hypothesis of the ADF test states that the time series has a unit root, which means that it is not stationary. A failure to reject the null hypothesis (H0) might therefore indicate non-stationarity. The KPSS test switches the hypotheses. Here the null hypothesis states that the time series is stationary. This means that the stochastic process has mean-reversion, which is a requirement for ARMA processes. Conclusively, if the ADF test leads to the rejection of H0 and the KPSS test does not lead to the rejection of H0, then the time series is assumed stationary. The ADF is also known as a unit root test and the KPSS as a stationarity test, because of their zero hypotheses.

Either way, next to stationarity statistics, graphing the autocorrelation functions (ACF) is also a way to uncover the structure of the time series. The ACF and Partial ACF (PACF) graphs can uncover auto regression or moving averages present in the time series. Furthermore, if stationarity cannot be assumed, then these graphs might possible still show the next step necessary to make the series stationarity by, for example, differencing.

Stationarity and the ACF visuals lead to the next step of ARMA model fitting. The freedoms here are the degrees of the moving average and auto regression. To optimize the model, one would initially try to minimize the squared mean error, however, because economists like to have parsimonious models and overfitting is a common mistake, it is better to look at information criteria, like the Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). Both criteria have different intentions in mind, where AIC tries to favour models that are close to an unknown reality, the BIC assumes that the set of models created contains the true model. Both measures have their pros and cons, and it is wise to consider both. On a closing note, it is good to consider that the fit of an ARMA-like model does not mean that the real process behind the swap spread follows these rules. Reality is more complex and is driven by the individual behaviour of many investors and traders. What the ARMA model does is describe the macro results of individual behaviour and shape it into the form of temporal dependence. Therefore, the ARMA model should not be interpreted as the real swap spread producing model, but as a way to catch the (coincidental) memory present in the swap spread.

After fitting and selecting the preferred ARMA model, what remains should be residuals. To see if these residuals have dependence among samples, statistical tests like the Ljung-Box can be performed. To add a second perspective a second residual dependence test, the Breusch-Godfrey test (Breusch, 1978) (Godfrey, 1978), is added. The tests look at the autocorrelations and infer if the data is independently distributed. The zero hypotheses state that the data is independently distributed. Therefore, a rejection of H0 is not the desired result for a good ARMA fit. A non-rejection of both tests would give the most confidence independence of residual samples.

2.5.2 *Explaining the variation of the residuals*

Once the residuals are considered to be independent, the aim is to explain them by external variables. Where the ARMA model explains temporal relations, a regression model for the residuals might explain the reason behind its movements. The aim is then to reduce the variance of the residuals as much as possible. These explaining variables ought to be the ones proposed in section 2.4 and further laid out in section 2.1. The variables are individually regressed with the swap spread for all maturities. After the regression, one would like to know if the variable significantly helped explain the swap spread. This question can be answered with an F-test, where the zero hypothesis takes that the new regression model does not capture the variance better than the old regular residual model. A rejection of H0 would therefore mean that the added variable has some explanatory power.

After the significant explaining variables have been recognized, they can be incorporated into the ARMA model. This incorporation is done by refitting the ARMA parameters together with the external variables, hence everything is fitted from scratch and the external variables are not fitted on top of the existing ARMA model. The resulting ARMAX model's performance is then measured by how much the variance is reduced by adding the external variables.

3 How are the proposed models created and tested, and what is their performance?

This chapter covers the used data, the modelling process, and its results. The first section covers where the data comes from and how it is adjusted for modelling. It also covers how the dataset is split into periods of crisis and why the zero-coupon curve can be used instead of yield to maturity. The second section covers the execution of the modelling process. It starts with how the data was made stationary, followed by how the right ARMA model is chosen. The section ends with the linear regression, and how the swap spread is decomposed in shift, slope, and curvature. The last section covers the results from modelling, what the parameters of the ARMA and linear regression are, and what they might mean.

3.1 Where does the data come from and how is it prepared for modelling?

Plenty of data is used to create and explain the swap spread. Not only are there quite a few variables, but the time series also spans a period of 20 years. This period is from the 21st of September 2001 to the 21st of September 2021. This period exists out of 5218 business days. Furthermore, the rate-like variables also have an extra dimension, the different maturities. The maturities used are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, and 30 years. These 13 maturities and 5218 workdays mean that the rate-like data already contains 67834 points of observations. The next sections explain the thoughts behind the data and how these amounts of data were dealt with.

3.1.1 The what's and why's of data

All of the data is retrieved from the Bloomberg terminal using Bloomberg's excel API. The excel data is then imported into python for preparation and modelling purposes. The next paragraphs will cover what data is exactly imported, why it is used, and how it is handled.

Zero-coupon data

Both Dutch and German zero-coupon data is used to create variables. The Dutch zero-coupon data is used as a smoother proxy of yield to maturity and is used together with the swap spread data to create the swap spread time series. It is also used together with the German zero-coupon data to create a comparative spread variable for both Dutch and German bonds. The spread was computed by subtracting the German rates from the Dutch rates. All data points are readily available for both countries, which means that no missing value handling was required for this set.

Swap rate data

The mid swap rates and bid-ask swap rates are both retrieved from Bloomberg. The mid rates are used to create the swap spread time series and the bid-ask rates are used to compute the bid-ask spread. Like the zero-coupon data, all data points were available, which makes it convenient to use without adjustments. The swap spread data is used for ARMA modelling and the swap bid-ask spread data is used as an explanatory variable together with the Dutch bond YTM bid-ask spread data for the ARMA residuals.

Yield to Maturity data

Like the swap rates, the mid, ask, and bid data was imported. However, YTM is a bit trickier when it comes to its values and missing data. The n-year YTM is determined by the bond which is closest to the year in question. The 0.5-year (6-month) data is only observable from 13 July 2006, which means that it exists out of 3964 data points. The 6-month bonds are named here because the generic 1-year YTM is not provided by Bloomberg. Therefore, the 1-year YTM is linearly approximated using the 6-month rate and the 2-year rate. The bid-ask YTM data is used together with swap rate bid-ask spread data to compute the swap spread ask-bid spread

Inflation data

This dataset is available monthly and is retrieved for the period of 1 January 2001 until 31 August 2021. The daily values are linearly approximated from the closest monthly values. The values after 31 August 2021 are denoted as no answer (N/A). This data is used as an explaining variable in the linear regression of the residuals from the ARMA model.

Gross Domestic Product data

The data is provided by Bloomberg on a yearly basis starting from 1 January 2001 until 31 December 2019. Like the inflation data, the unknown values are linearly approximated between the yearly known values. All values after 31 December 2019 are noted as N/A. Furthermore, the values are indexed/normalized using the first known observation, which is the approximated value for 21 September 2001. This data is used as an explanatory variable in the residuals.

AEX data

The AEX data is daily observable for the period of 20 years. The data needs no further preparation and is used in the linear regression as an explaining variable.

3.1.2 The separation of the time series into pre-, mid-, and post-crisis

By request and interest of the DSTA, the datasets are divided into three different periods. These periods are pre-, mid-, and post-crisis. The crisis refers to the credit and euro crisis. These crises do not have exact dates on when they started and ended, but the internet’s most famous encyclopedia has some suggestions. The credit crisis (Wikipedia, n.d.-b) seems to have started around the summer of 2007 and seems to have dwindled down halfway through 2011. However, the euro crisis (Wikipedia, n.d.-a) started before the credit crisis ended around early 2010. This crisis then went on till mid-2015 when things started to look better. To reflect this period the data is split at the first of August 2007 and at the first of June 2014. The dataset is separated into three series existing out of 1527, 1768, and 1907 workdays and together with the 13 maturities creates 39 different time series to individually analyze. Figure 3-1 illustrates what this division looks like.

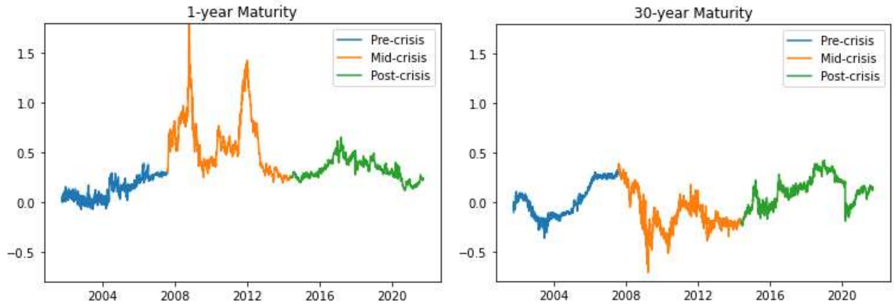

Figure 3-1 The 1-year and the 30-year swap spread divided into the crisis periods.

The peaks of crisis are visible in the mid-crisis part. Therefore, it seems like the crisis periods have been caught correctly.

3.1.3 *YTM vs Zero-coupon*

Yield to maturity and zero-coupon rates have been a subject before in earlier chapters. However, the decision needs to be made which one is used to create the swap spread time series. To get a grasp of their similarity, consider the graph from Bloomberg in Figure 3-2.

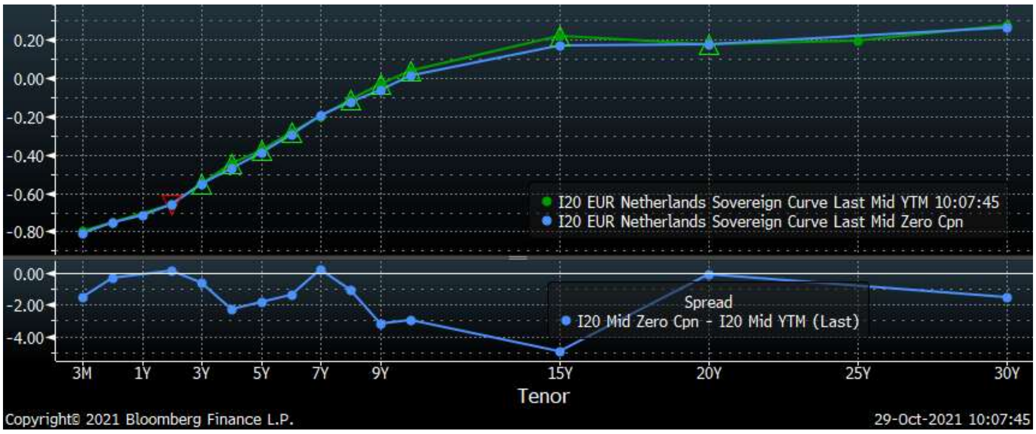

Figure 3-2 The zero-coupon curve, YTM curve and their spread on 29 October 2021.

The spread present in the graph shows that differences are not uncommon, especially for the 15-year maturity. The question raised is if this is always the case and to what degree. Figure 3-3 on the left shows the mean difference and standard deviation of 5218 workdays (20-years), except for the 1-year maturity for which the series exists out of 3964 workdays.

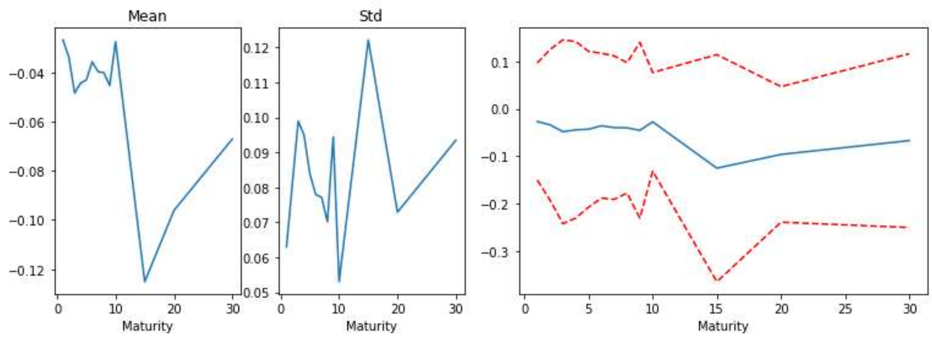

Figure 3-3 Mean, standard deviation and 99% confidence interval of YTM/0-cpn spread.

All maturities seem to have a negative mean YTM/0-cpn spread and are most extreme for the 15-year maturity, which shows a spread around 12 basis points (bp). The same extremity seems to be present in the standard deviations, where the 15-year maturity takes the lead. The negative mean could be explained by the existence of coupons which generally raises the YTM rates higher than the zero-coupon rates. Figure 3-3 on the right combines both graphs into a 95% confidence interval under the ungrounded but convenient assumption that the observations are normally distributed. The graph shows that the upper bound is more stable around the 10bp, while the lower bound emphasizes the errors introduced at the 15-year maturity. The lower bound seems to fluctuate more around the -25bp. Are the means significantly different from zero, however? The t-test test statistics are given by $t = \frac{m-\mu_0}{s/\sqrt{n}}$, where m is the sample mean, s the sample standard deviation, n the sample size, and μ_0 the zero hypothesis mean, which in this case is zero. With a two-sided confidence interval with a significance level of 1%, the required sample mean to reject the zero mean hypothesis is $|m| > 2.576 * s/\sqrt{n}$. Figure 3-4 shows the result of this where the blue line represents the minimum required sample mean and the orange line is the observed sample mean.

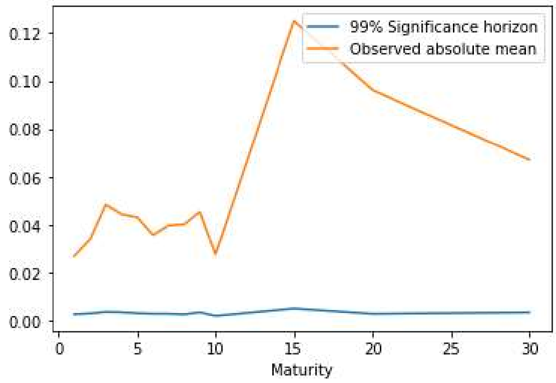

Figure 3-4 Graph of the significance levels and observed absolute means.

All the hypotheses of zero means are rejected with 99% confidence and the alternative hypothesis that the average spread between the YTM and zero-coupon curves are non-zero for all maturities is accepted.

Does this mean that the zero-coupon values cannot substitute YTM? Not necessarily, because the extremities shown by the 15-year maturity could even mean that the zero-coupon curve is a better representation of YTM than the roughly approximated YTM curve itself. Remember that the YTM value is determined by the bond that is closest to its maturity, however, this is without a correction for this discrepancy. The short term YTMs are never that far from their benchmark bonds, however, the long term YTMs are much more affected by this design. The 15-year YTM can be determined by a bond with a maturity between 10 to 20 years. If the respective closest bond has 12-year left, then this 3-year gap towards 15 can be quite erroneous. The same does not go for shorter-term maturities where the jumps are much shorter between the closest maturity benchmarks. However, why is this error the biggest for 15-year maturities, but less for the even longer-term maturities? Even for those maturities, the error is still larger than the shorter maturities, however, the lesser impact can be explained by a decreasing rate of change for farther maturities. Interest rates generally increase at a decreasing rate over the maturities, which also means that the error decreases for using neighbouring bonds further up the maturity line. Therefore, the observed error can be a case against using the YTM curve for the swap spread, instead of the zero-coupon curve. Furthermore, the zero-coupon curve also has more observations for the 1-year maturity, where the YTM curve is lacking till mid-2006. These are the arguments that favour the use of the zero-coupon curve as a substitute for YTM in the swap spread calculation, resulting in its use as such from now on. With that decision, the swap spread values were calculated using the zero-coupon data. The statistics of this dataset can be found in Appendix D.

3.2 How are the models created from the data?

This section covers the modelling process and methods introduced in section 2.5 and what decisions were made during the process. The first part covers how the dataset was determined to be stationary. The second part covers the ARMA modelling, followed by the third part about the linear regressions. The fourth part covers the ARMA extension to ARMAX using explaining variables. The fifth and final part covers how the swap spread was decomposed in shift, slope, and curvature.

3.2.1 Making the data Stationary

The preliminary graphs of the autocorrelation (ACF) and partial autocorrelation (PACF) graphs in Figure 3-5 reveal some of the temporal relations present in the swap spread. The slope present in the ACF indicates a very strong memory or that differencing is required to make the time series stationary. Differencing means that the new time series becomes the daily changes in the swap spread.

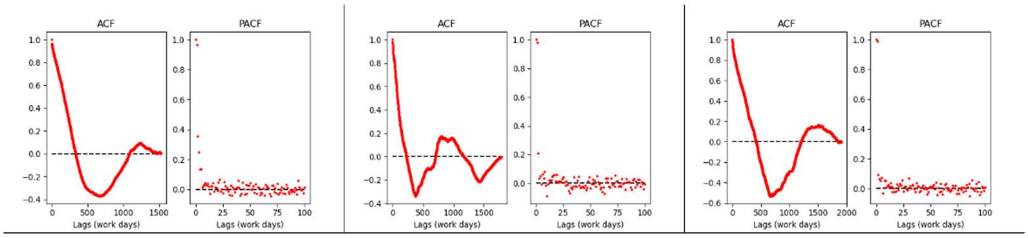

Figure 3-5 ACF & PACF of swap spread data for 5-year maturity and pre-, mid-, and post-crisis.

The stationarity is determined using the KPSS-test and ADF-test. The H0 of the ADF assumes non-stationarity and the H0 of KPSS assumes stationarity. These options are encoded and explained in Table 3-1.

Table 3-1 ADF & KPSS table code interpretations.

ADF	KPSS	Code
H0 not rejected	H0 not rejected	0
H0 not rejected	H0 rejected	1
H0 rejected	H0 not rejected	2
H0 rejected	H0 rejected	3

The tests have been performed for all crisis periods and all maturities (39 time series in total) for a significance level of 1%. For the regular swap spread, no test was able to assume stationarity as can be concluded from Table 3-2.

Table 3-2 ADF & KPSS test results before differencing.

Maturities:	1	2	3	4	5	6	7	8	9	10	15	20	30
Pre-crisis	1	1	1	1	1	1	1	1	1	1	1	1	1
Mid-crisis	1	1	1	1	1	1	1	1	1	1	1	1	1
Post-crisis	1	1	1	1	1	1	1	1	1	1	1	1	1

However, after differencing the data, i.e., looking at the changes of the swap spread, the tests give mostly promising results. Table 3-3 illustrates those results.

Table 3-3 ADF & KPSS test results after differencing.

Maturities:	1	2	3	4	5	6	7	8	9	10	15	20	30
Pre-crisis	2	2	2	2	2	3	3	3	3	3	2	2	2
Mid-crisis	2	2	2	2	2	2	2	2	2	2	2	2	2
Post-crisis	2	2	2	2	2	2	2	2	2	2	2	2	2

The codes of 2 indicate that both tests point towards stationarity. Unfortunately, this is not the case for the five pre-crisis maturities from 6 till 10. Here the tests disagree with each other, where the ADF rejected the unit root hypothesis and KPSS rejected stationarity. Because both tests reject the zero hypothesis it's hard to conclude which one is more correct, since both reject it with confidence. Figure 3-6 shows the ACF and PACF of the respective 8-year maturity before (left) and after (right) the data is differenced.

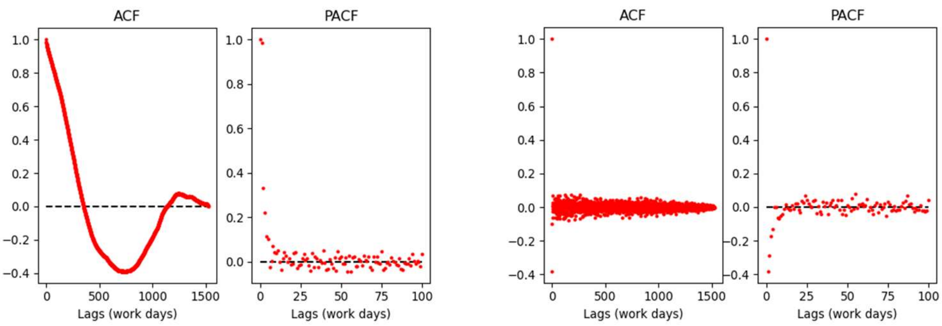

Figure 3-6 Before and after differencing the pre-crisis 8-year maturity swap spread.

The after differencing figure seems to be fully relieved of the strong autoregressive memory. The same goes for all other maturities and periods. Therefore, from here on all differenced series are assumed stationary, even though the KPSS and ADF disagree for a few of them. The reason to take this conflict among tests as a given is because the goal is to explain the swap spread in general. The underlying assumption here is that the structure should be the same and that only the parameters themselves can be different. It also prevents an overfitting view for the periods and maturities.

3.2.2 *ARMA, BIC, AIC, and Residuals*

The next step after assuming stationarity is deciding the degrees of temporal relatedness present in the time series. The ACF and PACF of the 8-year maturity after differencing (rightmost two) in Figure 3-6 seems to indicate some negative autocorrelation around 4 workdays back in time. The graphs of the other maturities seem to indicate a similar relation around 4 workdays or less. For this reason, all possible ARMA(p, q) combinations are considered where both degrees can range from 0 to 6. Thus, 49 different models are fitted to the 39 different time series, which results in a total of 1911 different models. The models are fitted using state-space techniques provided by the python Statsmodels package. These techniques transform the ARMA equations to the state space format and use the Kalman filter method to determine likelihoods. For all 39 different time series, the models with the lowest BIC or AIC are saved. This results in the scores in Table 3-4. A BIC score of 17 for (0, 1) means that 17 out of the 39 time series, the minimal BIC value was achieved by the (0, 1) model. The same goes for the AIC scores. The models with zero scores for both AIC and BIC were excluded from the table.

Table 3-4 Minimized BIC and AIC configuration scores for each period and maturity.

ARMA(p, q)	*BIC*	*AIC*	*% of Total*
(0, 1)	17	2	26,76%
(0, 2)	7	3	14,08%
(1, 1)	6	2	11,27%
(1, 2)	3	3	8,45%
(2, 1)	2	3	7,04%
(0, 3)	1	3	5,63%
(0, 6)	0	4	5,63%
(3, 0)	1	1	2,82%
(2, 6)	0	2	2,82%
(3, 5)	0	2	2,82%
(3, 6)	0	2	2,82%
(4, 6)	0	2	2,82%
(5, 6)	0	2	2,82%
(0, 0)	1	0	1,41%
(1, 0)	1	0	1,41%
(1, 3)	0	1	1,41%
(1, 4)	0	1	1,41%
(2, 2)	0	1	1,41%
(2, 5)	0	1	1,41%
(3, 4)	0	1	1,41%
(4, 5)	0	1	1,41%
(5, 4)	0	1	1,41%
(6, 0)	0	1	1,41%

The BIC measure seems to be mostly pointing towards the (0, 1) model, however, the AIC model leans more towards the (0, 6) and is very indifferent among the (0, 2), (1, 2), (2, 1), and (0, 3) models. Given the slight differences in scores, the AIC measure does not give a conclusive direction. To combat this indifference and others, the residuals are tested for the presence of independence using the Breusch-Godfrey and Ljung-Box tests. Like the stationarity testing, the hypotheses rejection combinations have been assigned codes as described in Table 3-5, where the 3 would be the most desired result.

Table 3-5 Breusch-Godfrey & Ljung-Box table code interpretations.

Breusch-Godfrey	Ljung-Box	Code
H0 not rejected	H0 not rejected	0
H0 not rejected	H0 rejected	1
H0 rejected	H0 not rejected	2
H0 rejected	H0 rejected	3

For both tests, the number of workday lags used was up to 253 to capture the yearly recurrences, and the confidence level was 1%. Table 3-6 to Table 3-9 contain the results of the testing on the residuals of the (0, 1), (0, 2), (1, 1), and (1, 2) models. These models were picked to see how the rejections change for the top models.

Table 3-6 Residual dependence rejection codes for ARMA(0,1) residuals.

Maturities:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*	*Total*
Pre-crisis	3	1	1	1	1	0	0	1	0	0	1	1	0	10
Mid-crisis	1	1	3	3	3	3	1	1	1	1	0	1	3	22
Post-crisis	1	1	1	0	0	0	0	1	1	0	0	0	0	5
Total	5	3	5	4	4	3	1	3	2	1	1	2	3	37

Table 3-7 Residual dependence rejection codes for ARMA(0,2) residuals.

Maturities:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*	*Total*
Pre-crisis	1	1	1	0	0	0	0	0	0	0	1	1	0	5
Mid-crisis	1	1	3	3	3	3	1	1	1	1	0	1	1	20
Post-crisis	1	1	1	0	0	0	0	1	1	0	0	0	0	5
Total	3	3	5	3	3	3	1	2	2	1	1	2	1	30

Table 3-8 Residual dependence rejection codes for ARMA(1,1) residuals.

Maturities:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*	*Total*
Pre-crisis	1	1	1	0	0	0	0	0	0	0	1	1	0	5
Mid-crisis	1	1	3	1	1	3	1	1	1	1	0	1	1	16
Post-crisis	0	0	1	0	0	0	0	1	1	0	0	0	0	3
Total	2	2	5	1	1	3	1	2	2	1	1	2	1	24

Table 3-9 Residual dependence rejection codes for ARMA(1,2) residuals.

Maturities:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*	*Total*
Pre-crisis	1	1	1	0	0	0	0	0	0	0	1	1	3	8
Mid-crisis	1	1	3	1	1	3	1	1	1	1	0	1	3	18
Post-crisis	0	1	1	0	0	0	0	1	1	0	0	0	0	4
Total	2	3	5	1	1	3	1	2	2	1	1	2	6	30

From the tables, it is apparent that both the Ljung-Box and Breusch-Godfrey test seem to prefer (1, 1) over the rest. Furthermore, the dependence is almost always rejected for the mid-crisis period by the Breusch-Godfrey test. Table 3-10 is filled with the AIC and BIC minimizing model parameters for the maturities in the mid-crisis period. Table 3-10 AIC and BIC preferences for the mid-crisis period

Table 3-10 AIC and BIC preferences for the mid-crisis period

Maturities:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
AIC	(1,4)	(3,5)	(5,4)	(3,4)	(2,1)	(4,5)	(2,1)	(2,1)	(2,5)	(3,0)	(3,5)	(0,3)	(5,6)
BIC	(1,0)	(3,0)	(1,1)	(2,1)	(2,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)

For the AIC it seems that the mid-crisis period is informationally wise better explained by higher degree models. Perhaps, in periods of crisis, the changes in swap spreads contain more temporal dependence which would have been trivially small otherwise. In other words, crisis periods might cause a structural break in the model of temporal dependence. However, AIC tries to pick a model closest to an unknown reality. The result is that AIC generally over fits, while BIC under fits. Therefore, to get a general explanatory model for the swap spread, it is best to find some middle ground between the two. Therefore, the conclusion is to choose the ARMA(1, 1) model as the model that generally fits the temporal relations present in the changes of the swap spread. What's left now are the residuals of the model and their explanation by external variables.

3.2.3 *Residual Variance, Linear Regression, and F-tests*

Using ordinary least squares, every variable gets separately fitted to all the swap spreads per period and maturity. The regression generates an F-test statistic, which describes if the model with the explaining variable explains the swap spread better than the model without it. These tests are conducted with a significance level of 1% and a rejection of the zero hypothesis means that the variable has some explaining power. The amount of H0 rejections per period per explaining variable is summarized in Table 3-11 to Table 3-15. The numbers can range from 0 to 13. A 7, for example, would mean that the zero hypotheses were rejected for 7 out of the 13 maturities. The variables are differenced, which means that the changes in variables are fitted to the changes in the swap spread. The first, Table 3-11, summarizes the rejection for the Dutch Gross Domestic Product, AEX, and inflation.

Table 3-11 F-test rejections of GDP, AEX, and Inflation.

	GDP	*AEX*	*Inflation*
Pre-crisis	0	4	0
Mid-crisis	0	11	0
Post-crisis	6	8	0

The numbers present are quite low, except for the AEX. What this means is that in general the movements in GDP and inflation do not significantly explain the movements in the swap spread. The AEX is more promising, but mostly for the mid-crisis period. Table 3-12 shows the explanatory significance of the YTM bid-ask spread for all its maturities. The columns here and in the next tables do not represent the maturities of the swap spread, but the maturities of the explanatory variable. The maturities of the swap spread have been summed up to create the values.

Table 3-12 F-test rejections of swap rate bid-ask spreads.

Maturities Swap BA spread:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
Pre-crisis	13	13	13	13	13	13	13	13	13	13	13	13	13
Mid-crisis	9	11	11	12	12	12	13	12	12	12	12	11	11
Post-crisis	9	13	13	13	13	13	13	13	13	13	13	13	13

The numbers here are much higher. It seems that the 7-year swap rate bid-ask spread has the most explaining power since it is significant for all the maturities and crisis periods. If this variable is regarded as a measure of liquidity, then it is possible to conclude that the change in liquidity on long term Dutch bonds has a relation with the change swap spread. The next table, Table 2-1, contains the F-test rejection amounts for the Dutch bond YTM bid-ask spread.

Table 3-13 F-test rejections of bond YTM bid-ask spreads.

Maturities YTM BA spread:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
Pre-crisis	0	0	0	0	13	0	0	0	0	0	0	0	0
Mid-crisis	0	1	0	3	13	0	1	0	5	4	0	0	0
Post-crisis	0	1	1	0	13	0	0	0	0	0	0	0	0

The interesting result in this table is that only the 5-year YTM bid-ask spread shows significance. More so is the fact that it shows significance for all periods and maturities. Table 3-14 shows the results of the spread between the German and Dutch zero-coupon curves.

Table 3-14 F-test rejections of Netherlands and German zero-coupon spreads.

Maturities 0-cpn spread:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
Pre-crisis	1	2	10	1	7	3	6	4	11	12	11	11	12
Mid-crisis	9	3	10	13	12	11	9	9	9	10	10	10	12
Post-crisis	2	1	6	9	9	10	10	10	9	9	9	8	8

This variable seems to show a wide variety of results for the different maturities. The 3 and 9- to 30-year zero-coupon spread seems to be promising for explaining the swap spread. If the difference between German and Dutch rates are assumed to be purely credit-based (while it probably also contains liquidity), then it is possible to conclude that the change in credit spread between the mentioned maturity has some explaining power for the swap spread. Furthermore, this variable is the same as the negated difference in the Dutch and German swap spread, since $SS_N - SS_G = (F - y_N) - (F - y_G) = y_G - y_N$. Therefore, the variable can also be interpreted as the difference in liquidity/credit premium for Germany and the Netherlands. If, of course, the swap spread can be defined as such. Table 3-15 contains the rejection values for the swap spread ask-bid spread.

Table 3-15 F-test rejections of swap spread bid-ask spreads.

Maturities swap spread AB spread:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
Pre-crisis	0	13	13	13	13	13	13	13	13	13	9	13	13
Mid-crisis	0	10	11	12	10	12	12	12	12	12	12	12	12
Post-crisis	4	9	13	13	13	13	13	13	13	13	13	13	12

Once again, the high numbers are mainly present for all maturities, except the shorter ones. This variable was defined as $(F_A - y_A) - (F_B - y_B)$. The right-hand side of the equation shows why it can be called the ask-bid spread of the swap spread. It seems to have quite a bit of explaining power. The variance reduction needs to indicate which maturity is best used for the final ARMAX model.

3.2.4 *From ARMA to ARMAX*

The selection of the maturities of the swap rate bid-ask spread and the YTM bid-ask spread is made by the highest rejection values, which were for the 7-year and 5-year respectively. The variance reduction percentages in Table 3-16 also agree with this choice.

Table 3-16 Variance reduction percentages of the external variables.

% Var. reductions	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*15*	*20*	*30*
Swap rate	6,9	7,0	6,0	8,0	8,7	8,7	8,9	7,4	8,6	8,4	7,7	7,4	7,1
Bid-ask	0,8	1,2	1,2	2,6	2,7	2,4	2,4	2,9	2,8	2,7	2,6	2,3	2,4
spread	0,5	1,3	2,0	2,2	2,3	2,7	3,0	2,6	2,4	2,5	2,3	2,1	1,8
NL-DE	0,4	1,6	5,8	0,5	0,5	0,3	0,5	0,4	0,9	1,2	1,6	1,2	1,6
zero-cpn.	0,9	0,7	2,8	5,1	6,6	6,4	5,4	5,5	5,0	4,9	4,1	3,7	2,8
spread	0,3	2,0	2,3	0,9	1,9	3,4	3,9	3,4	3,2	3,2	3,0	2,5	0,7
Sw.Sprd.	26,3	7,3	7,3	8,4	8,1	8,8	8,6	7,0	8,4	8,1	29,7	30,7	6,8
Bid-ask	0,0	1,1	1,3	3,1	0,7	3,1	3,1	3,8	3,8	3,5	3,2	3,4	3,6
spread	0,2	0,6	2,5	3,1	13,1	2,9	3,2	2,8	2,3	2,5	2,1	2,0	1,6
YTM	26,7	0,0	0,1	0,1	4,0	0,0	0,0	0,0	0,0	0,1	27,5	27,2	0,0
Bid-ask	0,0	0,2	0,0	0,2	8,9	0,1	0,1	0,2	0,3	0,2	0,0	0,1	0,1
spread	0,0	0,0	0,1	0,1	24,5	0,0	0,0	0,0	0,0	0,1	0,1	0,0	0,0

The choice for the Netherlands-Germany zero-coupon spread and swap spread bid-ask spread is less trivial. The most variance reduction by the linear regression is observed for the 3-year and 20-year respectively. Therefore, these two maturities are used, even though for the zero-coupon spread the amount of F-test rejections is decreased. In addition to these variables, the Dutch zero-coupon slope parameter is also added, however, its computation is explained in the next section. After incorporating these variables into a new ARMAX model using the same package used for the ARMA model, the variance reduction can once again be determined. Note that the ARMAX model is created from the ground up, which means that is not built on top of the ARMA model, but individually created. The resulting variance reductions can be found in Table 3-17.

Table 3-17 Residual variance reduction parameters from the ARMAX model.

% Residual Variance reduction	1	2	3	4	5	6	7	8	9	10	15	20	30
Pre-crisis	79,2	3,4	2,5	5,8	7,1	6,4	7,8	7,6	7,5	6,6	9,6	7,0	5,5
Mid-crisis	32,3	22,0	29,4	37,0	38,2	35,6	35,9	30,8	27,6	25,6	12,8	11,5	11,6
Post-crisis	44,2	30,9	41,4	41,4	35,0	35,6	35,0	31,7	29,2	25,1	19,1	18,9	20,0

The residual variance reduction can be quite high for certain maturities. An interesting pattern to notice is that the reductions seem to decrease when maturities increase for the post-crisis period. However, there are a few exceptions like the 2-year maturity. The pre-crisis period does not seem to have as much variance reduction as the other, but there is an extremely high number for the 1-year maturity. The mid-crisis period seems to show similar results as the post-crisis period. In section 3.3.2 the parameters of the model are further laid out.

3.2.5 *Nelson-Siegel and the slopes of crisis*

In section 2.2.5 the Nelson-Siegel model was introduced. This model parsimoniously describes interest rate curves. If swap rates are assumed to have the same kind of interest rate structure as spot rates, then an interesting case can be made when the two curves are compared. The Nelson-Siegel model

$$r(T) = \beta_1 + \beta_2 \frac{\lambda}{T}\left(1 - e^{-\frac{T}{\lambda}}\right) + \beta_3 \left(\frac{\lambda}{T}\left(1 - e^{-\frac{T}{\lambda}}\right) - e^{-\frac{T}{\lambda}}\right)$$

seems exponential at first sight, however, it is a linear combination of exponential functions. If the long-term parameter λ is assumed to be equal for both the swap rate and spot rate curves, then their difference can be described by the difference of the beta parameters. This perspective opens the door to decomposing the swap spread into parts of shift, slope, and curvature. It takes the form of

$$S_S(T) = F(T) - r(T) = (\beta_1 - \beta_1^*) + (\beta_2 - \beta_2^*)\frac{\lambda}{T}\left(1 - e^{-\frac{T}{\lambda}}\right) + (\beta_3 - \beta_3^*)\left(\frac{\lambda}{T}\left(1 - e^{-\frac{T}{\lambda}}\right) - e^{-\frac{T}{\lambda}}\right)$$

where the swap spread becomes the spread between the parameters. The goal is to fit a model to three different curves at the same time: the swap rate curve, the sovereign rate curve, and the swap spread. Therefore some adjustments need to be made to the error to minimize. The result is an equally weighted error function among the three curves. This error takes the form of:

$$Error = \frac{1}{3}|y - y_{NS}| + \frac{1}{3}|S - S_{NS}| + \frac{1}{3}|S - y - (S_{NS} - y_{NS})| = \frac{1}{3}\left(|e_y| + |e_S| + |e_S - e_y|\right)$$

This means that the optimization also considers that the error difference among the swap and zero-coupon curves should not be too high. This causes the fit to be close to both curves, but also the swap spread. For optimization, the division by three can be omitted. The least-squares function of the Scipy package in python was used to minimize the squared error. It used the Trust Region Reflective Algorithm together with the smooth approximation of the l1 loss function. A result of this fitting to the zero-coupon and swap curves can be observed in Figure 3-7.

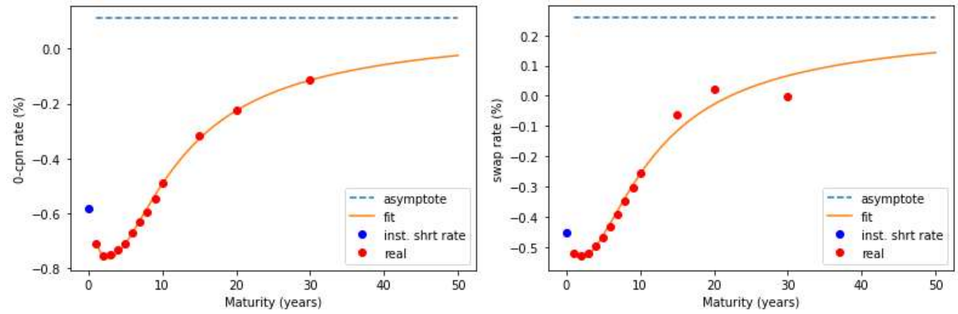

Figure 3-7 Zero-coupon and swap rate curve Nelson-Siegel fit (20 November 2020).

The zero-coupon seems to align well, but the swap rate is a bit less at the later maturities. The respective swap spread fit is plotted in Figure 3-8.

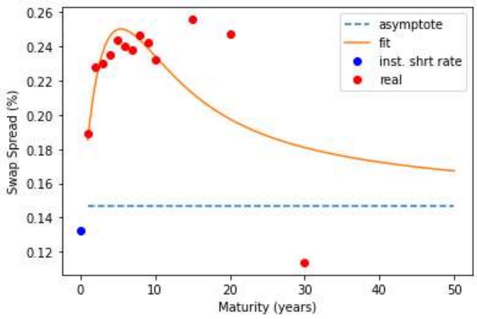

Figure 3-8 Swaps spread Nelson-Siegel fit (20 November 2020).

This swap spread fit seems to be a bit more hectic compared to its real values, especially for the longer maturities. This is not generally the case for the fits, however, as can be seen in Figure 3-9.

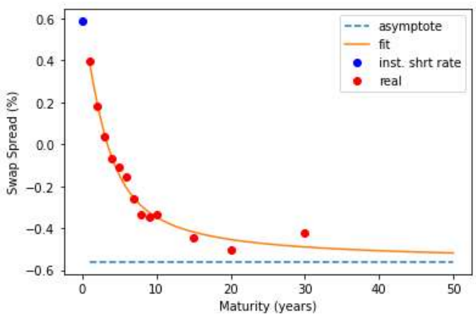

Figure 3-9 Swap spread Nelson-Siegel fit (22 May 2009)

The reason for these errors could lie in the assumption of equal convergence parameters (lambda) for both the zero-coupon and swap curves. Furthermore, in the first case, the 30-year swap rate is lower than the 20-year swap rate. This inconsistency in the curve is then stronger reflected in the swap spread, resulting in a worse fit. Figure 3-10 shows the average parameter values for each crisis period.

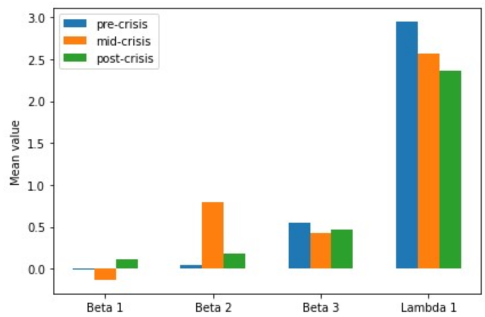

Figure 3-10 Average Nelson-Siegel parameter values per period.

The shift (Beta 1) and curvature (Beta 3) do not look that interesting across crisis periods, but slope (Beta 2) does. The slope parameter is much higher for the crisis period than for non-crisis periods. This can also be observed in the mean NS curves per period in Figure 3-11.

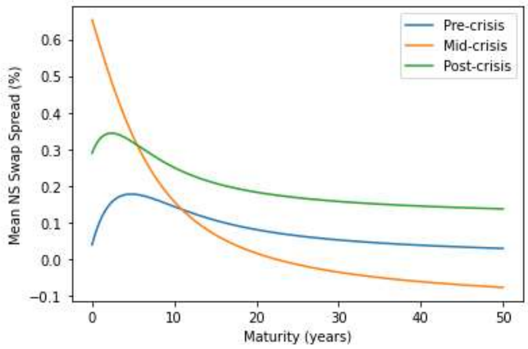

Figure 3-11 Mean NS swap spread per maturity and period.

The crisis period majorly shows a downward sloping curve, while the other periods are more shaped by curvature and shift. The same shapes can be observed in the regular mean swap spread curves in Figure 3-12. Note that the y-axis limits differ in both graphs.

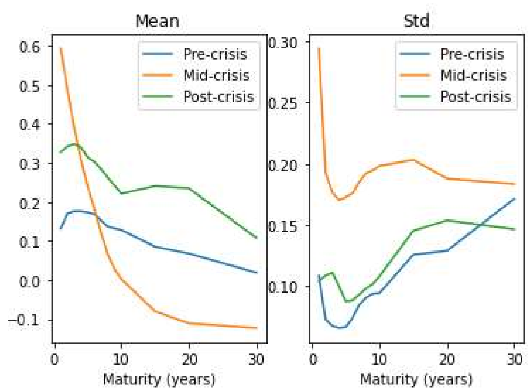

Figure 3-12 Mean swap spread per maturity and period.

What all of this means is that in periods of non-crisis, the swap spread is mostly determined by curvature, while in periods of crisis this gets overruled by a difference in slope. Also, note that the swap spread becomes negative at some point for the longer-term rates. This means that those swap rates have become lower than the zero-coupon rates. The shapes of the curves for the three crisis periods have been further illustrated in Figure 3-13 to Figure 3-15. These graphs contain on the left side the mean Nelson-Siegel fitted curve with standard deviation bounds, and on the right the same but then for the unfitted data, i.e. the normal swap spread observed in the market. Note that the y-axis in the graphs might differ.

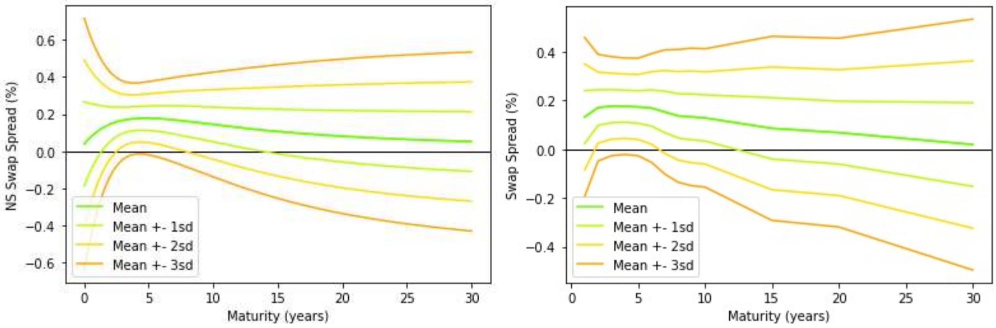

Figure 3-13 Mean pre-crisis NS swap spread (left) and normal swap spread (right) with standard deviation bounds.

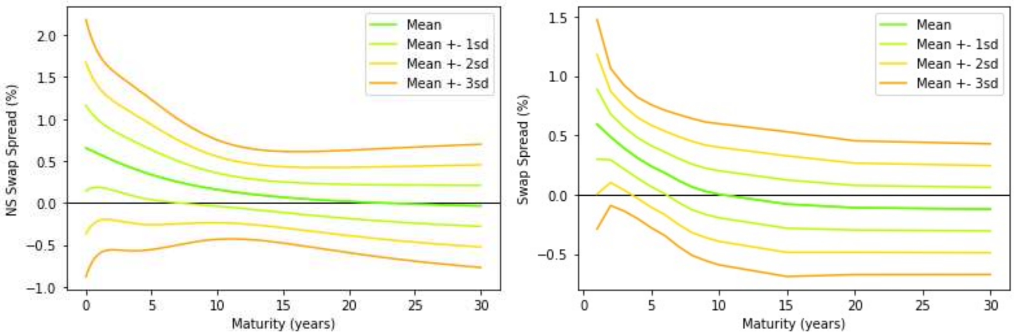

Figure 3-14 Mean mid-crisis NS swap spread (left) and normal swap spread (right) with standard deviation bounds.

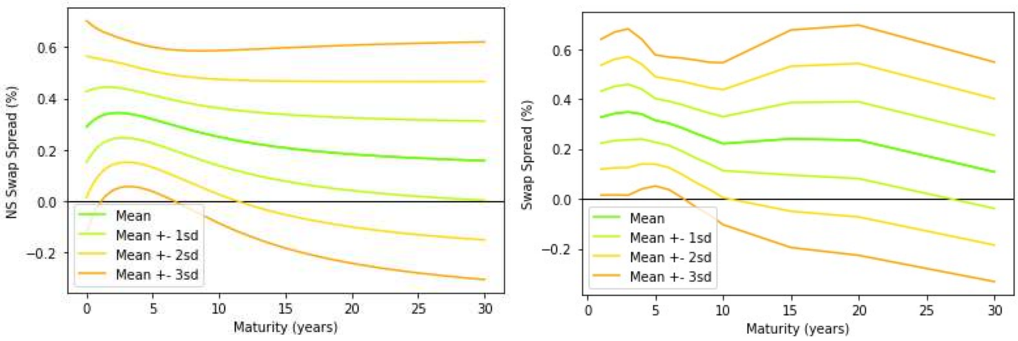

Figure 3-15 Mean post-crisis NS swap spread (left) and normal swap spread (right) with standard deviation bounds.

The first thing to remark is that the mid-crisis period graphs are much larger in magnitude, which is a result of the overall increased volatility in times of crisis. The important thing to take from these graphs is to which degree the swap spread can be negative. The first thing to observe is that the swap spread is more likely to be negative the higher the maturity is. This is primarily because of the decreasing mean and increasing standard deviation of the swap spread over time. This negativity seems to be the strongest in the mid-crisis period where the mean itself is negative for the longer maturities, more so in the normal swap spread graph on the right side. The post-crisis period around and before the 5-year maturity appears to have the lowest degree of possible negativity since even the 3 standard deviations bounds are non-negative for the normal swap spread. The NS swap spread differs here the more it goes to the 0 maturity. The least likely negative maturity appears to be the 4-year maturity. The importance of non-negative swap spread comes from the decision inequalities in section 1.3 and they will be further used in the conclusions in section 4.1.1.

3.3 What are the resulting models, and which one is considered best?

This section covers the final results of the models in more detail. The first part focuses on the ARMA model and its parameters, and the second part on the linear regression and its parameters.

3.3.1 What is the final ARMA model and its parameters?

The final product in the previous section was not a single ARMA model, but 39 to be precise. They all have different parameters which complicate matters further. Table 3-18 contains the average parameters per period.

Table 3-18 Average ARMA parameters per period.

Period	*Constant*	*AR 1*	*MA 1*	*Res. Var.*
Pre-crisis	0,000153	0,110134	-0,678976692	0,000232
Mid-crisis	-0,00021	0,091586	-0,337609692	0,000967
Post-crisis	0,000122	0,403019	-0,509104877	0,000178
Average	0,000021	0,20158	-0,508563754	0,000459

These parameters are the averages of the different maturities combined, but it gives some insights into the details of the models for the different periods. A complete overview of the individual values can be found in Appendix E and Figure 3-16 to Figure 3-18.

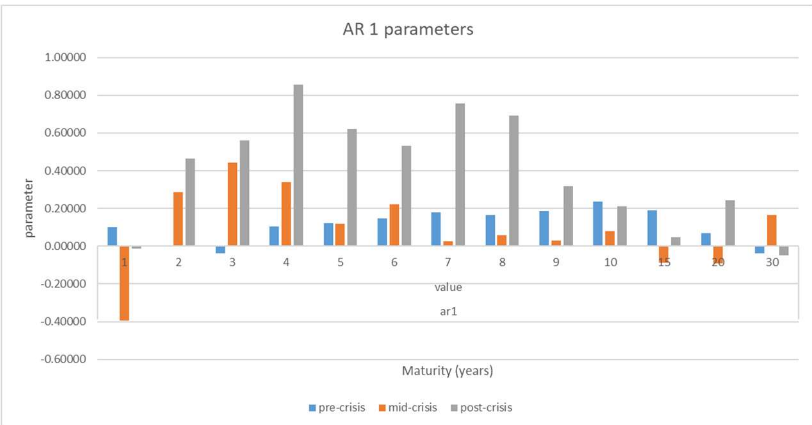

Figure 3-16 Autoregressive parameters per period per maturity.

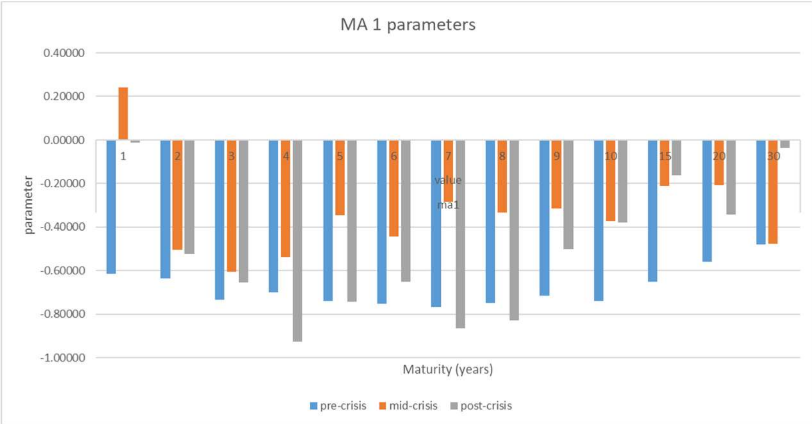

Figure 3-17 Moving Average parameters per period per maturity.

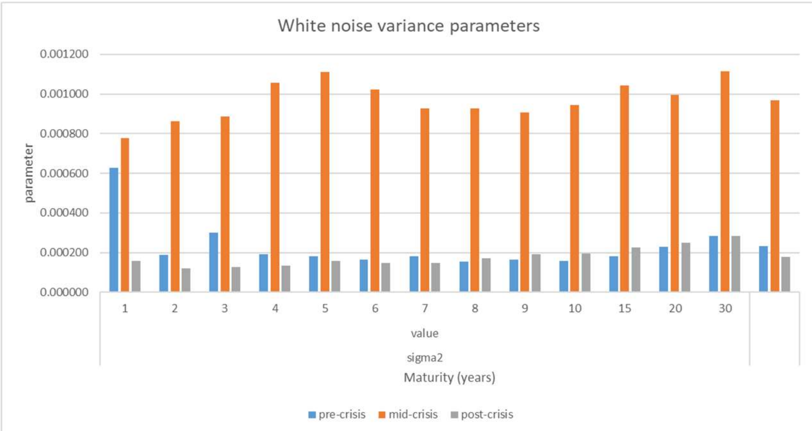

Figure 3-18 Residual variance parameters per period per maturity.

The figures and tables in the appendix show that the parameters can differ quite a lot over the maturities. The most interesting of these are the 1-year maturity parameters for the mid-crisis period. The parameters here are around $-\frac{2}{5}$ for the autoregression and $\frac{1}{4}$ for the moving average. This indicates that the 1-year maturities can follow quite a different structure than the other maturities during periods of crisis. As expected, the residual variance is structurally higher in crisis periods than in non-crisis periods, but quite stable over the maturities, except for the 1-year maturity pre-crisis variance.

The individual parameters have standard errors with which the z-values and p-values of the parameters are computed. The zero hypothesis for the z-test is assuming that the parameter is zero. Thus, a rejection of this is required to be confident that the parameter is non-zero. The confidence level applied here is 1%. Table 3-19 shows the number of zero hypotheses that were rejected per period.

Table 3-19 Total H0 rejections per period.

Period	*Constant*	*AR 1*	*MA 1*	*Res. Var.*
Pre-crisis	0	9	13	13
Mid-crisis	0	6	13	13
Post-crisis	0	9	9	13
Total	0	24	35	39

For the constant, it is safe to assume that its value should be zero since no H0's have been rejected. The other two parameters are less decisive, however. The stronger of the two is the moving average parameter with 35 of the 39 hypotheses rejected. The non-rejection is present in the 1, 15, 20, and 30-year maturities in the post-crisis period. This means that for the short term and the longer-term periods indecisive if there is a moving average present. The residual dependence results in the earlier sections might support this for the later periods but not the early period since dependence was rejected for the models without this moving average. The longer periods showed indifference among those models, as none of them rejected H0. For the autoregressive parameter, the results are less obvious, especially for the mid-crisis period. The non-rejections for that period are the 5, 7, 8, 9, 10, 15, and 20-year. Because of this, it is hard to conclude if the mid-crisis period contains autoregressive properties. The preference of the BIC also pointed to a less complex model, while the residual dependence tests for the other two periods disagree. This possibly means that the pre- and post-crisis periods contains auto regression, while the mid-crisis period does not.

3.3.2 *How do the external variables contribute to the ARMA model?*

In the previous section, the values of the ARMA model parameters were explained and determined. This section expands on the model by considering the external variables and the resulting ARMAX model. The resulting average parameters of the ARMAX model can be found in Table 3-20.

Table 3-20 Average parameters of the ARMAX model per period.

ARMAX model parameters	*AR*	*MA*	*SBA7*	*YTMBA5*	*SSBA20*	*NGZCS3*	*NS Slope*	*Res. Var.*
Pre-crisis	0,0814	-0,6367	0,0433	0,0217	-0,0070	0,0433	-0,0014	0,0002
Mid-crisis	0,1310	-0,3248	0,1463	0,3039	-0,1081	0,1463	0,0001	0,0007
Post-crisis	0,5211	-0,6339	0,0472	0,3017	-0,0169	0,0472	0,0290	0,0001
Average	0,2445	-0,5318	0,0789	0,2091	-0,0440	0,0976	0,0092	0,0003

The first thing to notice is that the autoregressive and moving average parameters have slightly changed compared to the previous ones. This difference could be because of differences in estimation methods or the addition of external variables. For a quick comparison, the differenced swap spread and ARMA(X) residual variances are displayed in Table 3-21.

Table 3-21 Average residual variances after ARMAX fit.

	Diff. SS Var.	*ARMA Res. Var.*	*ARMAX Res. Var.*
Pre-crisis	0,000297	0,000232	0,000182
Mid-crisis	0,001029	0,000967	0,000708
Post-crisis	0,000178	0,000178	0,000125
Average	0,000501	0,000459	0,000339

The main takeaway from these results is that the swap spread can be explained by a few external variables. These are the YTM bid-ask spread, swap rate bid-ask spread, swap spread bid-ask spread, the Netherlands-Germany zero-coupon spread, and the slope of the zero-coupon curve. The first three can be considered as liquidity measures and differences in liquidity, while the latter two are more like credit measures. This is evidence for concluding that the swap spread exists out of liquidity and credit risk.

A drawback of the ARMAX model is the dependence on external time series observed at the same moment. What this means is that the model is not well suited for prediction, because this requires making predictions about the explaining variables as well. This is not impossible to do, but it would cause the model's complexity to increase significantly. Therefore, the ARMAX model is more useful to explain the swap spread, but predictions are better done with the ARMA model. The next section will use the results from this whole chapter and relate them to the hedge strategies introduced in the first chapter.

4 What can be inferred from the created models and what does this mean for the DSTA's interest risk policy?

This chapter's conclusions are drawn from the results in the previous chapter. These are directly related to the hedging decisions faced by the DSTA which were introduced in chapter 1. The chapter ends with final remarks about the explaining variables and what aspects/subject could use further research in the future.

4.1 How do the findings relate to the DSTA?

4.1.1 *Would the swap hedges have worked in the past?*

This section continues on the hedge decision inequalities from section 1.3. Using the historical data it is possible to see what the inequalities would have resulted in in the past. The first hedge was against rising interest rates and had the general inequality

$$\big(F(0,M_1) - F(0,M_1+M_2)\big)_{M_1} + \big(SS(M_1,M_2) - F(0,M_1+M_2)\big)_{M_2} > -\big(y(M_1,M_2)\big)_{M_2}.$$

If this inequality is true, then hedging is better than not hedging. Remember that the subscripts are there to substitute extensive discounting formulas and keep the equation simple. In the following example $M_1 = 2$ and $M_2 = 5$. The next figures illustrate the constant and variable parts present in the inequality. Figure 4-1 visualizes the three different components of the inequality.

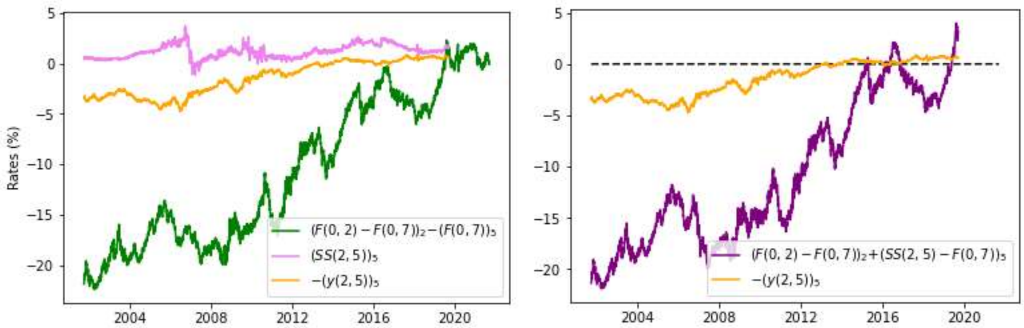

Figure 4-1 Historical inequality components (left) and inequality results (right).

The graphs illustrate the movements of the different parts in the past. Note that the future rates have been shifted back in time, which means that in this case, the last 2 years are missing since they have not happened yet. The term with present values seems to have been increasing steadily over time while the future negative 0-cpn rate and swap spread seem quite stable in comparison. However, the negative 0-cpn rate has been steadily increasing too, but at a lesser rate. The aim of the DSTA is not to save costs or make profits with the swap hedge, but to lessen the volatility of the future value by trading the negative yield for the swap spread. The graphs seem to depict a picture where the volatility is not less, at least not compared to the hedge costs incurred. To illustrate when the hedge might have worked in the past, the future 0-cpn part is moved to the left side of the inequality in Figure 4-2 in the left graph.

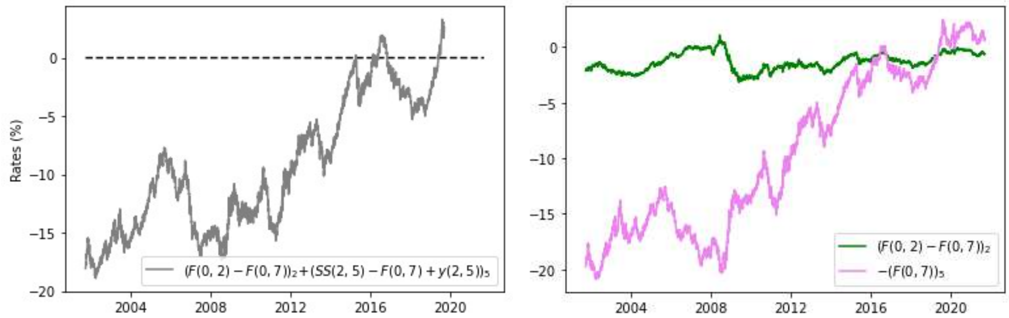

Figure 4-2 Historical inequality results (left) and present value terms decomposition (right).

What the graph shows is that the hedge would have been quite an expensive choice in the past. However, as the years go on the hedge starts to seem more profitable. The volatilities for the whole period were 0.040% $(\Delta 0cpn)$ and 0.023% (ΔSS). This means that the variance has been reduced by 31.93%. The question that remains is if such a reduction of volatility is enough for the costs induced to make the hedge a viable option. This cannot be answered here because it depends on the risk appetite of the DSTA. However, what can be agreed upon is that the hedge has become more attractive over the years and that it should not be put on the shelves just yet. Decomposing the constant term in 5-7 year swap rate spread and negative 7-year swap rate illustrates that the 7-year swap rate is the main driver of cost as can be seen on the right in Figure 4-2. For one, the swap rate is paid for a total of 7 years, where only the first 2 years are covered by a 2-year receiver swap. This means that the last 5 years are uncovered in the constant term. The 7-year swap rate has been steadily decreasing over time, however, which has resulted in its lessening impact on the hedge costs. The current environment of negative interest rates is therefore a good pretext to consider the usage of swaps to hedge against rising interest costs on committed bond issuances.

The second inequality mentioned was to extend the maturity of a short term bond with swaps and looked like

$$SS(M_1, M_2)_{M_2} > \big(SS(0, M_1 + M_2) - SS(0, M_1)\big)_{M_1} + \big(SS(0, M_1 + M_2)\big)_{M_2}.$$

If this inequality is true, then it is better to issue a short bond and extend it with swaps, than it is to issue the full long-term bond. In this example $M_1 = 5$ and $M_2 = 2$. The key results can be seen in Figure 4-3 and Figure 4-4.

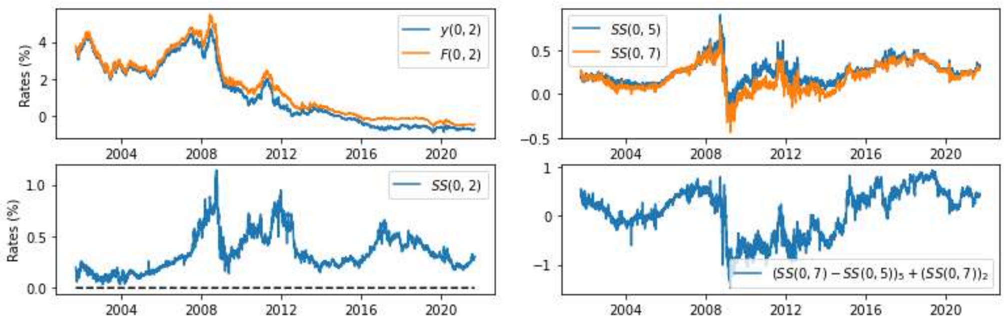

Figure 4-3 2-year zero-coupon rates, swap rates, and swap spread (left) and 5 & 7-year swap spread and their spread (right).

The first graphs on the left are the 2-year zero-coupon and swap rates and their (swap) spread below it. The figure on the right illustrates the two swap spreads and the total cost representing the right side of the inequality. These costs (abbreviated to $SS_I(5,2)$ because of the arguments later on) are then compared to the future 2-year swap spread in the top part of Figure 4-4 and their spread below it.

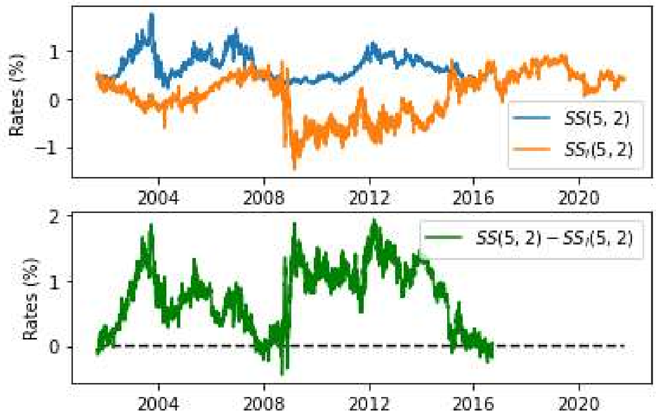

Figure 4-4 Presently known costs vs. 2-year future swap spread.

The bottom graph in the figure is mostly positive, which means that short issuance with swaps would have turned out better in the past. To get a better grasp on what the inequality and this result means, the following implied future yield equality can be considered

$$\left(y(0,M_1)\right)_{M_1} + \left(y_I(M_1,M_2)\right)_{M_2} = \left(y(0,M_1+M_2)\right)_{(M_1+M_2)} = \left(y(0,M_1+M_2)\right)_{M_1} + \left(y(0,M_1+M_2)\right)_{M_2}.$$

The same argument holds for the swap rates, which means that these equalities can be subtracted from one another to create an implied future swap spread equality. By rearranging the terms the following equality is created

$$\left(SS_I(M_1,M_2)\right)_{M_2} = \left(SS(0,M_1+M_2) - SS(0,M_1)\right)_{M_1} + \left(SS(0,M_1+M_2)\right)_{M_2}.$$

This equality has the same terms as the inequality, which can therefore be simplified to $SS(M_1,M_2) > SS_I(M_1,M_2)$. The inequality, therefore, says that short issuance with swaps is preferred, when the real/forecasted future swap spread is higher than the implied future swap spread. The bottom graph in Figure 4-4 illustrates that the real future rates were mostly higher than what has been implied in the past. Therefore to decide on what to choose both the implied swap spread and the present swap spread need to be considered. The present swap spread should then be used to forecast its future value. The next section will cover this forecasting with the conditional expectancy of the swap spread and its estimation error.

4.1.2 *What can be expected from the swap spread?*

Suppose that the swap spread can be described with an ARMA model (which might be far from reasonable), the total change in the swap spread k workdays in the future can be estimated by the equations found in Appendix G. The estimate given there for the swap spread and its limiting case is

$$SS_k^* = E[SS_k|SS_0,\Delta SS_0,\varepsilon_0] = SS_0 + (\Delta SS_0 + \varepsilon_0)\sum_{i=1}^{k} a^i - \varepsilon_0 a^k$$

and

$$\lim_{k\to\infty} SS_k^* = SS_0 + \frac{\Delta SS_0 + \varepsilon_0}{1-a}.$$

The expected error of this estimate is zero, however, the variance of this error is described by

$$Var[SS_k - SS_k^* | \Delta SS_0, \varepsilon_0] = \sigma_\varepsilon^2 \left(1 + \sum_{i=1}^{k-1}\left(1 + (a+b)\sum_{j=0}^{i-1} a^j\right)^2\right).$$

This variance does not converge to some value, however, if k is high enough the change in variance converges to

$$\lim_{i\to\infty}\left(1 + (a+b)\sum_{j=0}^{i-1} a^j\right)^2 = \left(1 + \frac{a+b}{1-a}\right)^2 = \left(\frac{1+b}{1-a}\right)^2.$$

This means that every time step added to k increases the variance by approximately $\sigma_\varepsilon^2 \left(\frac{1+b}{1-a}\right)^2$. However, this also depends on how fast the term converges. If the initial swap spread, swap spread change, and residual are taken to be $SS_0 = 0,\ \Delta SS_0 = 1$ and $\varepsilon_0 = 0$, then these limiting values for the crisis periods are like the ones in Table 4-1 when they are computed using the ARMA parameters from section 3.3.1.

Table 4-1 Limiting estimation values for the crisis periods.

Limiting values	*Pre-crisis*	*Mid-crisis*	*Post-crisis*
Estimate	1,1237647	1,1008197	1,6750952
Error Variance	0,0000302	0,0005141	0,0001204

The main takeaway from these values is that the estimate always pushes the swap spread up higher when its last change was positive and lower when it was negative. The error variance keeps on growing over time, however, it does so 5 times faster in the period of crisis, than the period after it.

4.1.3 *Final remarks regarding the swap spread and explaining variables*

The previous section ended with the estimates and their errors for the future swap spread. These were based on the ARMA model from section 3.3.1 however, not the ARMAX model with explaining variables from the section after that. The reason for this was that the ARMAX model requires more knowledge about the structure of the explaining variables. This would require further analysis of these variables and would make the formulae in the previous section much more complex. Furthermore, the explaining variables are observed at the same as the dependent variable. This would mean that there is no time causality in the model because they need to be observed at the same time. This would make future estimates rather hard to make since it would also require estimates of the explaining variables.

What can be deduced from the explaining variables and their variance reducing properties is that swap spread partially or fully exists out of liquidity and credit risk. The bid-ask spread variables mostly represent liquidity risk, while the German benchmark spread mostly represents credit risk. Of course, it can be argued that all these variables are mixes of credit and liquidity risk, but one more so than the other. The variance is not the only evidence for the swap spreads interpretation as credit & liquidity risk, because the graphs of the 1-year swap spread and the Dutch credit default swaps prices imply similar results. Figure 4-5 shows the former and Figure 4-6 the latter.

Figure 4-5 The 1-year swap spread

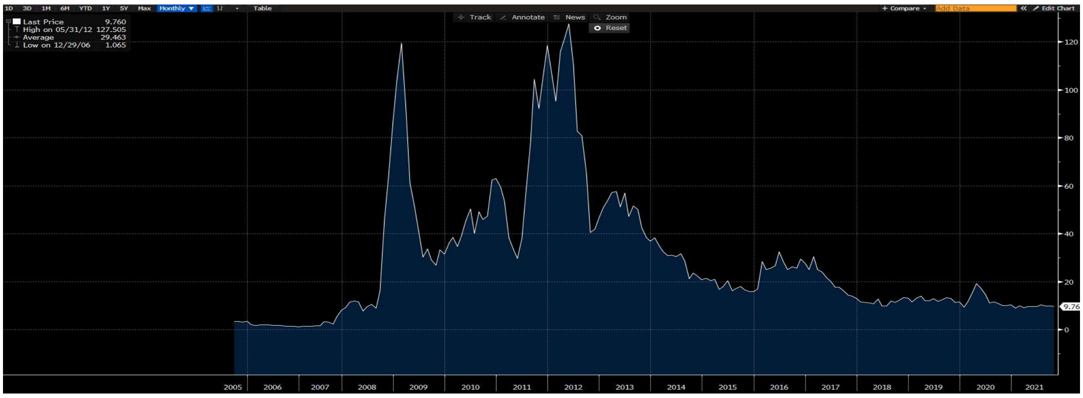

Figure 4-6 Dutch Credit Default Swap prices from 2005 to 2021 (retrieved from Bloomberg).

The graphs support what has been concluded, especially the credit risk. The peaks are at the same moment, however, the swap spread seems to return faster to a lower point. This might be explained by the illiquidity of the swaps. Unfortunately only the graphs of the CDS prices were available, which means that the numerical comparison is left open for future research. An additional variable to mention is the Dutch sovereign curve slope suggested by the literature. In this research, the slope was captured in a single variable by the Nelson-Siegel decomposition. The ARMAX fitting has shown that the parameter had significant values in the post-crisis period, but not in the pre- end mid-crisis periods. The slope could also be considered as a credit measure, which means that the creditworthiness of the Dutch has an impact on the Dutch swap spread, but only after the crisis period.

4.1.4 Further research and closing notes

The swap spread is a complex market variable that leaves a lot open for investigation. However, the results of this research show that there is some structure present in its past and future. Many paths have been taken to uncover this structure and just as much have been abandoned. The efforts did raise some questions, however, which deserve some mention.

The first major roadblock was the sovereign interest rate curve, more so which one to use and how to compute it. Its complexity and extensive literature lead to the decision to leave its computation to Bloomberg. Therefore a piece of advice to the DSTA would be to further explore different interest rate curves and their usages for the DSTA. The separation of the sovereign curve into cash and capital market also raises questions about the effect of demand and supply, which requires further research.

Another question encountered during the research was about risk aversion. One of the central aims of the DSTA is to manage the state debt with a justifiable balance between risks and costs. This poses a question that has not yet been answered: what is the risk aversion profile of the DSTA? This question should explore more numerically the risk profile of the DSTA and what limits should be put on constant costs in exchange for expected costs. This should further improve the decisions of when to use swaps and when not.

Finally, there are still different aspects of the swap spread left to explore. The first one is the relation between the Z-spread and swap spread since certain algebraic manipulations show that they are inverses of each other. A second aspect is the sampling frequency of the swap spread. This research used workdays as the sampling frequency, however, longer periods might give different results. One could also extend the ARMAX models with ARCH models to get a better explanation of the swap spreads temporal movement. However, another question would be if these ARMAX models are adequate to use at all since they filter out the important qualities of the time series, nor are they likely the real swap spread generating process. Reality is much more complex than the ARMAX models suggest, and its use merely moulds the swap spread into a temporal relationship format. The explaining variables could also be expanded upon, more so if the credit default swap data is available. Overall, there is still a lot to experiment with, but in the meantime, the DSTA is better informed on swap spreads and when it should use swaps to hedge interest rate risks.

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Appendix

A. Duration and Convexity

If a trader has a portfolio full of bonds, it is not a strange question to ask what happens to a bond's price when the underlying yield changes. Therefore, consider the first and second derivative with respect to the yield of the constant yield bond price equation:

$$\frac{\partial B}{\partial y} = -cP\sum_{t=1}^{T} t(1+y)^{-(t+1)} - PT(1+y)^{-(T+1)}$$

$$\frac{\partial^2 B}{\partial y^2} = cP\sum_{t=1}^{T} t(t+1)(1+y)^{-(t+2)} + PT(T+1)(1+y)^{-(T+2)}$$

Using Taylor expansions, an approximation of the change in the bond price, given a change in yield is given by:

$$\Delta B \approx \frac{\partial B}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 B}{\partial y^2}(\Delta y)^2$$

Dividing this by B introduces two concepts that are used a lot in interest risk management: duration and convexity (C. J. Hull, 2015, pp. 175–197). Thus:

$$\frac{\Delta B}{B} \approx -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Where Duration is defined as:

$$D = -\frac{1}{B}\frac{\partial B}{\partial y}$$

And Convexity is defined as:

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2}$$

Hence, a bond's Duration is a normalized measure of the bond's rate of change. To hedge the interest rate risk the trader would ideally like to have this measure equal to zero. The same goes for Convexity. A non-zero convexity would imply that Duration changes with changing yields. Thus, for longer-term interest rate hedging, the trader would like to have the Convexity at zero too. It is easy to see how this is reflected in the Taylor expansion above, where the change in the bond price would be zero, regardless of a changing yield.

An alternative definition and description of Duration is Macaulay's Duration (C. J. Hull, 2015, pp. 182–185):

$$D_M = \frac{1}{B}\sum_{t=1}^{T} t\frac{cP}{(1+y)^t} + \frac{T}{B}\frac{P}{(1+y)^T}$$

Which can be interpreted as the weighted average of the time till cash flows. It is easy to check that when continuous compounding is considered, the regular and Macaulay's Durations are the same.

B. Spot curve estimation

Ideally, the spot rate curve is derived from the price of zero-coupon bonds. But what if no such bonds are available in the market? The problem with coupon-bearing bonds is that if yields are variable over the maturities, solving for them using the bond price is a challenge because of the degrees of freedom. For example, a 2-year bond price with present value equation:

$$B = \frac{c}{1 + r(1)} + \frac{c + 1}{\left(1 + r(2)\right)^2}$$

Has two free variables to solve: $r(1)$ and $r(2)$. This proposes the challenge, however, if $r(1)$ can be derived from some other bond, then so can this one in succession. If, for example, there is some coupon-bearing bond with only a single final payment left, then $r(1)$ can be implied from that bond's price. This principal underlies the bootstrap method laid out by Hull (2018, pp. 106–108). However, for this method to work, a bond price of every added coupon period and before is required. The size of the coupon does not matter in this case. However, considering the Z-spread proposed in section 2.1.2, the bootstrap method ignores possible differences in zero curves of coupon-bearing bonds. Suppose we have bonds with maturities for every date in the set $T \coloneqq \{T_1, T_2, \dots, T_N\}$, i.e., we have chained subsets T^i of T where

$$T^1 = \{T_1\}\ and\ T^i = T^{i-1} \cup \{T_i\}\ \forall\ i \in \{2,3,\dots,N\}$$

From the bond price equation

$$B_{c_i}\left(t, T^i\right) = \sum_{\tau \in T^i : t \le \tau} cD(\tau - t) + D(T_i - t)$$

with

$$D(t) = \frac{1}{\left(1 + r(t)\right)^t}$$

Then the bootstrap method proposes to solve

$$\begin{bmatrix} c_1 + 1 & 0 & \cdots & 0 \\ c_2 & c_2 + 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_N & c_N & \cdots & c_N + 1 \end{bmatrix} \begin{bmatrix} D(T_1) \\ D(T_2) \\ \vdots \\ D(T_N) \end{bmatrix} = \begin{bmatrix} B_{c_1} \\ B_{c_2} \\ \vdots \\ B_{c_N} \end{bmatrix}$$

The leftmost matrix needs to be invertible for this method to work. However, reality rarely lends itself to such simple calculations because the bonds on the market don't have maturities that are nicely distributed to make the matrix invertible. There are periods with "holes" that need to be dealt with before the method could work. Therefore, it is wise to exchange the bootstrap method to account for situations where bonds in between periods are missing and the periods between payments shift over time. An article by David Cox (1995) suggests using interpolation and splining methods. At its easiest, it proposes linear interpolation, but the more advanced methods propose the fitting of non-linear models. The cubic splines method differs from linear interpolation by relaxing the linear constraint but adding the constraint that there are no "kinks" in the curve. This means that the first and second derivatives ought to be equal at known points. Nawalkha & Soto (2017) suggest another model: the Nelson and Siegel model. This model proposes a simpler exponential model with parameters that describe the changes in the term structure. In contrast to the other methods, this model is not easily solved by OLS methods and requires a non-linear solving method, like regular least squares. Therefore, the methods are computational wise more "expensive."

To shape this incidental literature exposition into a more structured form, ScienceDirect is once again consulted. The query used is: "term structure models" AND "interest rate" AND "short rate models". This gives 98 results. To make the results more containable and recent, the constraint of the years 2011 to 2021 is added, which narrows the results down to 47. This research doesn't focus on modelling the zero curve perfectly but requires a computable method to create zero curves from data. Therefore, most articles are deemed irrelevant, not because they don't propose a good zero curve model technique, but their focus on detail and complexity exceeds the goal of the method required. After filtering for comprehensive papers, the paper that piqued some interest was: Interest rate term structure modelling (Schmidt, 2011). Just like the two papers used before, this paper introduces multiple models that are used for term structure modelling over time. The paper also mentions the splines methods; however, it further proposes an extension of the Nelson-Siegel model mentioned before. The paper further explores certain arbitrage-free models, where a perfect fit with historical data is incorporated to mitigate pricing errors of derivatives, and, hence lowering arbitrage. It further lays out market models, like the Libor market model and the swap market model. These models are called market models, because they are more driven by market observable values, making them easier to use in practice. These models are meant for pricing interest rate derivatives and are calibrated using the volatilities of existing derivatives with the same underlying interest rate. Because derivatives based on the Dutch sovereign bonds are not actively traded on the market, these models are put back on the shelf.

The Bank of International Settlements (2005) has also done its fair share of investigation in the past. The bank investigated zero-coupon curve models and their usage by central banks. It also presents the spline-based, Nielson-Siegel, and Svensson extension models. What is interesting, however, is to what degree these models are used by central banks. The obvious majority there uses the Nelson-Siegel-Svensson model, with the minority resorting to spline like methods. Unfortunately, this paper is quite outdated and there are no more recent comparable papers available.

The question now remains: Nelson-Siegel-(Svensson) or splines? The overall choice by central banks seems obvious, however, to tip the balance a bit further, the results of one more paper are considered. Lorenčič (2016) pondered the same question and therefore set out to compare both models. They used 1 to 13 year Austrian Government bond data of October 8, 2013. Comparison concludes that Nelson-Siegel outperforms cubic spline in the short term up to 2 years. However, for the longer-term maturities, both models are comparable. This conclusion once again favours Nelson-Siegel, however, it is hard to call decisive, because only the curve of a single day is approximated. If a longer period was considered, then the conclusion might have had a lot more substance.

Nonetheless, a decision needs to be made and a model needs to be chosen for further use. The papers presented are far from recent or decisive, however, the weak conclusions point towards the Nelson-Siegel model. The advantage of this model is that it gives an intuitive interpretation of the variables and that they can be used for studying the composition of zero curves. Splines, in contrast, have no such representation. The non-linearity of calibrating the Nelson-Siegel model is unfortunate, however, it is a necessary evil on the expedition to understanding the swap spread.

C. Implied swap and spot rates

Because the swap contracts are created at fair value (free), the swap rate must encompass the seller's perspective on the floating rates in the future. It is expressed by the relation:

$$\sum_{\tau \in T: t \leq \tau} \frac{V(\tau - \Delta\tau)}{(1 + r(\tau - t))^{\tau - t}} = \sum_{\tau \in T: t \leq \tau} \frac{F(T_N)}{(1 + r(\tau - t))^{\tau - t}}$$

Yasuoka (2018, pp. 1–10) solves this implied forward rate using the non-arbitrage argument that the difference between an n-year rate and an (n+m)-year rate implies an m-year rate n years in the future. Thus:

$$\left(1 + r(n + m)\right)^{n+m} = \left(\left(1 + r(n)\right)\right)^{n} \left(1 + V_m(n)\right)^{m}$$

Solving for the future rate gives:

$$V_m(n) = \left(\frac{\left(1 + r(n + m)\right)^{n+m}}{\left(\left(1 + r(n)\right)\right)^{n}}\right)^{\frac{1}{m}} - 1$$

Now define a discount function using variable yields like:

$$D(t) = \frac{1}{\left(1 + r(t)\right)^{t}}$$

Making the future rate function:

$$V_m(n) = \left(\frac{D(n)}{D(n + m)}\right)^{\frac{1}{m}} - 1$$

And the swap rate/floating rate relation becomes:

$$F(T_N) \sum_{\tau \in T: t \leq \tau} D(\tau - t) = \sum_{\tau \in T: t \leq \tau} V_{\Delta\tau}(\tau - \Delta\tau) D(\tau - t)$$

From the future yearly rate function:

$$V_{\Delta\tau}(\tau - \Delta\tau) D(\tau)^{\frac{1}{\Delta\tau}} = D(\tau - \Delta\tau)^{\frac{1}{\Delta\tau}} - D(\tau)^{\frac{1}{\Delta\tau}}$$

When this is substituted in the swap/floating rate equation:

$$F(T_N) \sum_{\tau \in T: t \leq \tau} D(\tau - t) = \sum_{\tau \in T: t \leq \tau} \left(D(\tau - \Delta\tau)^{\frac{1}{\Delta\tau}} - D(\tau)^{\frac{1}{\Delta\tau}}\right)$$
$$= D(T_1 - \Delta\tau)^{\frac{1}{\Delta\tau}} - D(T_N)^{\frac{1}{\Delta\tau}}$$

The last result only works when $\Delta\tau$ is considered a constant spread between the settlement dates. Thus, making the swap rate function:

$$F_{imp}(T_N) = \frac{D(T_1 - \Delta\tau)^{\frac{1}{\Delta\tau}} - D(T_N)^{\frac{1}{\Delta\tau}}}{\sum_{\tau \in T: t \leq \tau} D(\tau - t)}$$

From here on the swap rate seems to be only dependent on an underlying spot rate curve, therefore using this curve, a swap rate can be implied. This relation could of course work both ways, and using the recursive relation one could also compute a spot rate curve from the swap rates:

$$F(T_N) = \frac{D(T_1 - \Delta\tau)^{\frac{1}{\Delta\tau}} - D(T_N)^{\frac{1}{\Delta\tau}}}{\sum_{\tau \in T \setminus \{T_N\}: t \leq \tau} D(\tau - t) + D(T_N - t)}$$

Assuming $t = 0$ and $\Delta t = 1$, (meaning that present time is considered, and a period is the same as the general discount period) then solving for the discount function:

$$D(T) = \frac{D(T_1 - 1) - F(T_N)\sum_{\tau \in T \setminus \{T_N\}: t \leq \tau} D(\tau)}{1 + F(T_N)}$$

This discount function can then be reversed back to its spot rate equivalent. The resulting equation describes the discount function to have an autoregressive like structure.

To demonstrate the usefulness of the implied swap rate it can simply be regarded as the difference between the market swap rate $F(M)$ and government bond YTM $y(M)$ or spot rate $r(M)$:

$$SS(M) = F(M) - y(M) \ or \ SS(M) = F(M) - r(M)$$

It is generally considered a measure of the risk & cost premium that swaps incorporate on top of the risk- “free” benchmark. Figure 0-1 visualizes the swap spread for different maturities M.

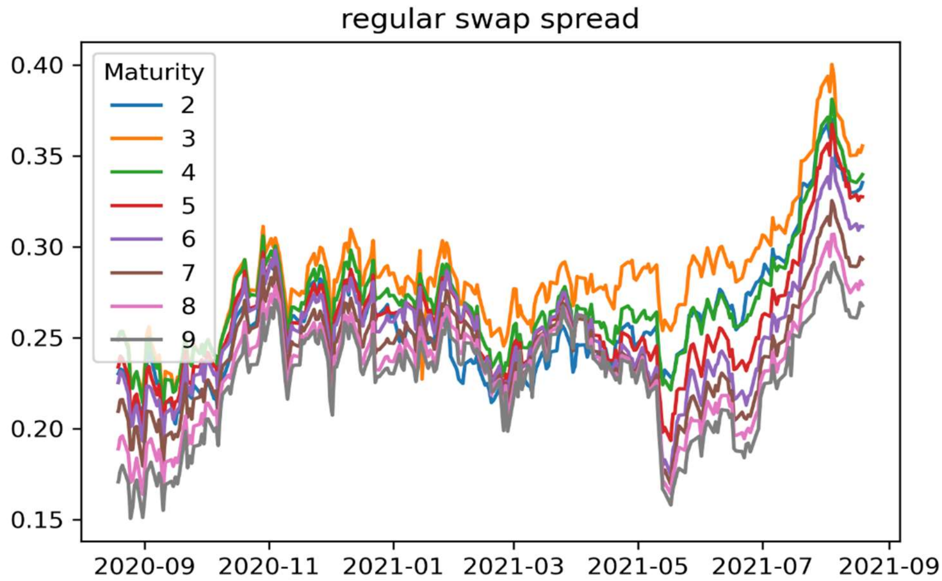

Figure 0-1 The Swap Spread of Dutch zero-coupon bonds and Euribor swaps (19-8-2021 to 19-8-2021)

Similar like swap spread, the implied swap spread is a different relation between the market swap rate and the implied swap rate:

$$S_F(T) = F(T) - F_{imp}(T)$$

Noteworthy is that the implied swap rate is built up from the same yield curve used in the swap spread. Therefore, to get the implied swap rate spread, the yield curve is transformed from its yield form to its swap rate form. Just like the swap spread, this spread is a measure of how the rate should be, according to some underlying yield, and how it is observed to be. Figure 0-2 visualizes the implied swap rate spread. The same yield curve data was used as in Figure 0-1. The layers of different maturities are better visible in this spread, combined with the observation that these time series seem to be more stationary.

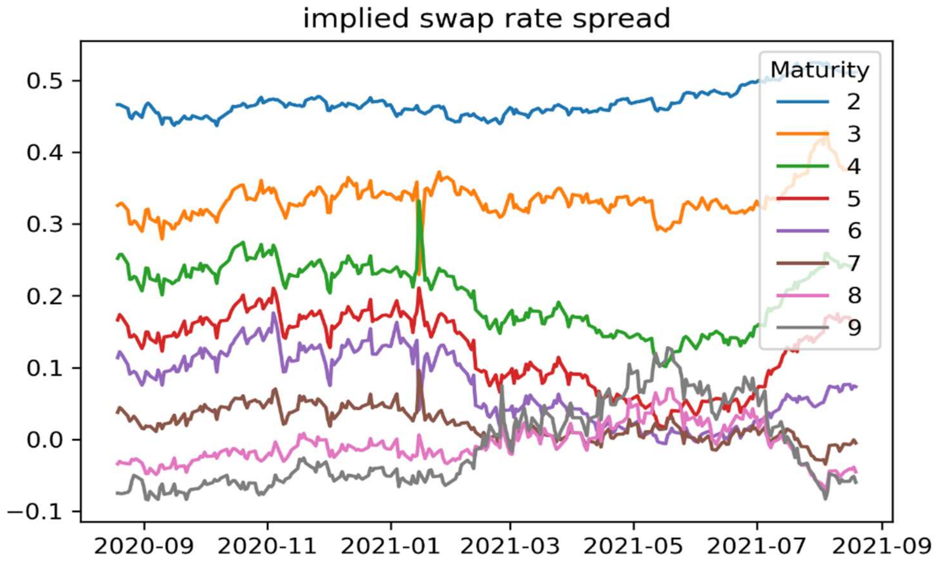

Figure 0-2 The Implied Swap Rate Spread of Dutch zero-coupon bonds and Euribor swaps (19-8-2021 to 19-8-2021)

D. Swap spread statistics

Table 0-1 Swap spread statistics for full 20-year period.

	count	mean	std	min	25%	50%	75%	max
1	5218	3.61E-01	2.66E-01	-7.55E-02	2.09E-01	2.99E-01	4.56E-01	1.79E+00
2	5218	3.41E-01	1.85E-01	3.20E-02	2.09E-01	2.97E-01	4.65E-01	1.14E+00
3	5218	3.12E-01	1.56E-01	-7.91E-02	1.95E-01	2.73E-01	4.36E-01	9.73E-01
4	5218	2.81E-01	1.40E-01	-2.44E-01	1.70E-01	2.57E-01	3.86E-01	9.12E-01
5	5218	2.47E-01	1.33E-01	-2.76E-01	1.41E-01	2.39E-01	3.27E-01	9.03E-01
6	5218	2.22E-01	1.37E-01	-3.00E-01	1.12E-01	2.14E-01	3.08E-01	8.81E-01
7	5218	1.89E-01	1.49E-01	-4.40E-01	7.65E-02	1.87E-01	2.94E-01	8.03E-01
8	5218	1.57E-01	1.60E-01	-5.61E-01	5.00E-02	1.63E-01	2.76E-01	6.95E-01
9	5218	1.36E-01	1.65E-01	-5.93E-01	3.09E-02	1.50E-01	2.68E-01	6.27E-01
10	5218	1.19E-01	1.70E-01	-5.98E-01	1.46E-02	1.39E-01	2.51E-01	5.79E-01
15	5218	8.54E-02	2.11E-01	-6.97E-01	-5.67E-02	1.04E-01	2.64E-01	5.11E-01
20	5218	6.78E-02	2.16E-01	-8.13E-01	-8.52E-02	7.62E-02	2.44E-01	5.42E-01
30	5218	3.01E-03	1.94E-01	-7.15E-01	-1.51E-01	-1.05E-02	1.56E-01	4.22E-01

Table 0-2 Swap spread statistics for pre-crisis period.

	count	mean	std	min	25%	50%	75%	max
1	1527	1.32E-01	1.09E-01	-7.55E-02	3.03E-02	1.30E-01	2.35E-01	3.90E-01
2	1527	1.70E-01	7.29E-02	3.20E-02	1.18E-01	1.50E-01	2.17E-01	4.29E-01
3	1527	1.76E-01	6.77E-02	-4.25E-02	1.29E-01	1.58E-01	2.18E-01	4.34E-01
4	1527	1.76E-01	6.59E-02	2.78E-02	1.33E-01	1.59E-01	2.07E-01	4.35E-01
5	1527	1.73E-01	6.67E-02	5.97E-02	1.16E-01	1.59E-01	2.13E-01	4.34E-01
6	1527	1.69E-01	7.43E-02	3.50E-02	1.02E-01	1.61E-01	2.20E-01	4.47E-01
7	1527	1.52E-01	8.49E-02	-8.20E-03	7.74E-02	1.34E-01	2.06E-01	4.51E-01
8	1527	1.37E-01	9.07E-02	-1.86E-02	5.85E-02	1.14E-01	1.90E-01	4.29E-01
9	1527	1.32E-01	9.39E-02	-2.10E-02	5.26E-02	1.06E-01	1.88E-01	4.52E-01
10	1527	1.28E-01	9.45E-02	-2.89E-02	4.59E-02	1.01E-01	1.93E-01	4.31E-01
15	1527	8.53E-02	1.26E-01	-1.63E-01	-2.94E-02	5.51E-02	1.86E-01	4.16E-01
20	1527	6.78E-02	1.29E-01	-2.23E-01	-5.40E-02	5.32E-02	1.79E-01	3.92E-01
30	1527	1.90E-02	1.71E-01	-3.64E-01	-1.28E-01	5.72E-03	2.19E-01	3.71E-01

Table 0-3 Swap spread statistics for mid-crisis period.

	count	mean	std	min	25%	50%	75%	max
1	1784	5.92 E-01	2.94E-01	1.91E-01	3.57E-01	5.45E-01	7.50E-01	1.79E+00
2	1784	4.86 E-01	1.92E-01	1.40E-01	3.18E-01	4.81E-01	6.27E-01	1.14E+00
3	1784	3.91 E-01	1.77E-01	-7.91E-02	2.39E-01	4.09E-01	5.27E-01	9.73E-01
4	1784	3.08 E-01	1.70E-01	-2.44E-01	1.74E-01	3.04E-01	4.35E-01	9.12E-01
5	1784	2.38 E-01	1.73E-01	-2.76E-01	1.19E-01	2.07E-01	3.49E-01	9.03E-01
6	1784	1.82 E-01	1.76E-01	-3.00E-01	6.94E-02	1.35E-01	2.55E-01	8.81E-01
7	1784	1.19 E-01	1.85E-01	-4.40E-01	7.99E-03	7.56E-02	1.91E-01	8.03E-01
8	1784	6.45 E-02	1.92E-01	-5.61E-01	-4.94E-02	2.32E-02	1.41E-01	6.95E-01
9	1784	2.76 E-02	1.95E-01	-5.93E-01	-9.06E-02	-1.89E-02	1.03E-01	6.27E-01
10	1784	2.27 E-03	1.98E-01	-5.98E-01	-1.25E-01	-4.94E-02	8.30E-02	5.79E-01
15	1784	-8.04 E-02	2.03E-01	-6.97E-01	-2.31E-01	-1.45E-01	3.74E-02	4.04E-01
20	1784	-1.11 E-01	1.88E-01	-8.13E-01	-2.27E-01	-1.55E-01	3.47E-02	3.84E-01
30	1784	-1.23 E-01	1.83E-01	-7.15E-01	-2.35E-01	-1.76E-01	-1.98E-02	3.90E-01

Table 0-4 Swap spread statistics for post-crisis period.

	count	mean	std	min	25%	50%	75%	max
1	1907	3.27 E-01	1.04E-01	1.15E-01	2.55E-01	3.17E-01	4.00E-01	6.52E-01
2	1907	3.42 E-01	1.09E-01	1.73E-01	2.53E-01	3.11E-01	4.30E-01	6.59E-01
3	1907	3.48 E-01	1.11E-01	1.75E-01	2.51E-01	3.22E-01	4.40E-01	6.38E-01
4	1907	3.39 E-01	1.00E-01	1.40E-01	2.53E-01	3.13E-01	4.20E-01	6.14E-01
5	1907	3.14 E-01	8.76E-02	8.99E-02	2.48E-01	2.91E-01	3.86E-01	5.19E-01
6	1907	3.02 E-01	8.85E-02	5.65E-02	2.37E-01	2.82E-01	3.78E-01	4.86E-01
7	1907	2.84 E-01	9.34E-02	-2.09E-03	2.27E-01	2.71E-01	3.62E-01	4.68E-01
8	1907	2.61 E-01	9.82E-02	-5.32E-02	2.04E-01	2.55E-01	3.44E-01	4.61E-01
9	1907	2.41 E-01	1.02E-01	-8.57E-02	1.84E-01	2.46E-01	3.21E-01	4.35E-01
10	1907	2.21 E-01	1.08E-01	-1.23E-01	1.60E-01	2.29E-01	2.91E-01	4.30E-01
15	1907	2.41 E-01	1.45E-01	-2.15E-01	1.43E-01	2.68E-01	3.48E-01	5.11E-01
20	1907	2.35 E-01	1.54E-01	-2.05E-01	1.21E-01	2.60E-01	3.37E-01	5.42E-01
30	1907	1.08 E-01	1.47E-01	-2.40E-01	-2.28E-02	1.18E-01	2.05E-01	4.22E-01

Table 0-5 Differenced swap spread statistics for full 20-year period.

	count	mean	std	min	25%	50%	75%	max
1	5217	4.22E-05	2.38E-02	-2.25E-01	-7.84E-03	-1.10E-04	7.57E-03	2.57E-01
2	5217	2.02E-05	2.08E-02	-2.28E-01	-7.38E-03	-5.00E-05	7.46E-03	2.27E-01
3	5217	1.72E-05	2.24E-02	-2.40E-01	-8.58E-03	-1.10E-04	8.33E-03	2.16E-01
4	5217	1.53E-05	2.25E-02	-2.23E-01	-8.79E-03	-1.20E-04	8.99E-03	1.86E-01
5	5217	1.24E-05	2.31E-02	-2.20E-01	-9.29E-03	9.00E-05	9.37E-03	2.03E-01
6	5217	9.93E-06	2.21E-02	-2.13E-01	-9.06E-03	3.00E-05	9.35E-03	2.05E-01
7	5217	3.05E-06	2.16E-02	-2.37E-01	-9.10E-03	-1.00E-05	9.42E-03	2.34E-01
8	5217	6.46E-06	2.16E-02	-2.53E-01	-9.16E-03	-5.00E-05	9.44E-03	2.70E-01
9	5217	9.34E-06	2.17E-02	-2.45E-01	-9.35E-03	7.00E-05	9.66E-03	2.73E-01
10	5217	6.05E-06	2.20E-02	-2.26E-01	-9.44E-03	8.00E-05	9.44E-03	2.62E-01
15	5217	2.60E-05	2.32E-02	-2.55E-01	-9.56E-03	-1.10E-04	9.67E-03	2.88E-01
20	5217	4.32E-05	2.34E-02	-1.90E-01	-1.01E-02	3.00E-05	1.04E-02	2.48E-01
30	5217	4.33E-05	2.51E-02	-3.63E-01	-1.04E-02	-6.00E-05	1.08E-02	2.70E-01

Table 0-6 Differenced swap spread statistics for pre-crisis period.

	count	mean	std	min	25%	50%	75%	max
1	1526	1.96E-04	2.82E-02	-1.56E-01	-1.21E-02	0.00E+00	1.20E-02	1.62E-01
2	1526	1.50E-04	1.62E-02	-1.18E-01	-6.85E-03	2.90E-04	7.46E-03	1.14E-01
3	1526	1.32E-04	2.19E-02	-1.38E-01	-8.02E-03	-2.55E-04	8.53E-03	2.16E-01
4	1526	1.25E-04	1.64E-02	-7.92E-02	-8.39E-03	-9.00E-05	9.39E-03	7.78E-02
5	1526	1.18E-04	1.58E-02	-7.65E-02	-8.65E-03	-1.00E-05	8.96E-03	7.02E-02
6	1526	1.22E-04	1.51E-02	-6.17E-02	-8.37E-03	1.30E-04	8.31E-03	6.81E-02
7	1526	1.20E-04	1.60E-02	-8.86E-02	-8.19E-03	-2.70E-04	8.81E-03	7.21E-02
8	1526	1.38E-04	1.44E-02	-7.63E-02	-7.61E-03	-3.50E-04	8.25E-03	5.69E-02
9	1526	1.71E-04	1.47E-02	-6.76E-02	-7.85E-03	-1.50E-04	8.47E-03	5.31E-02
10	1526	1.55E-04	1.43E-02	-7.10E-02	-7.72E-03	1.40E-04	8.20E-03	6.13E-02
15	1526	2.15E-04	1.49E-02	-7.64E-02	-7.51E-03	-1.25E-04	7.98E-03	9.50E-02
20	1526	2.33E-04	1.70E-02	-1.13E-01	-8.35E-03	-1.25E-04	8.85E-03	1.35E-01
30	1526	3.11E-04	1.91E-02	-1.72E-01	-8.77E-03	-2.20E-04	9.04E-03	1.77E-01

Table 0-7 Differenced swap spread statistics for mid-crisis period.

	count	*mean*	*std*	*min*	*25%*	*50%*	*75%*	*max*
1	1784	-3.77E-05	2.83E-02	-2.25E-01	-9.79E-03	-6.60E-04	9.09E-03	2.57E-01
2	1784	-1.15E-04	3.01E-02	-2.28E-01	-1.26E-02	-2.80E-04	1.29E-02	2.27E-01
3	1784	-1.26E-04	3.03E-02	-2.40E-01	-1.38E-02	-1.10E-04	1.39E-02	1.49E-01
4	1784	-1.50E-04	3.32E-02	-2.23E-01	-1.57E-02	-2.90E-04	1.53E-02	1.86E-01
5	1784	-1.78E-04	3.42E-02	-2.20E-01	-1.60E-02	-2.35E-04	1.67E-02	2.03E-01
6	1784	-2.06E-04	3.28E-02	-2.13E-01	-1.63E-02	-2.90E-04	1.59E-02	2.05E-01
7	1784	-2.40E-04	3.14E-02	-2.37E-01	-1.58E-02	3.40E-04	1.49E-02	2.34E-01
8	1784	-2.53E-04	3.16E-02	-2.53E-01	-1.58E-02	3.35E-04	1.53E-02	2.70E-01
9	1784	-2.86E-04	3.14E-02	-2.45E-01	-1.63E-02	2.10E-04	1.52E-02	2.73E-01
10	1784	-3.00E-04	3.20E-02	-2.26E-01	-1.61E-02	-3.25E-04	1.52E-02	2.62E-01
15	1784	-3.27E-04	3.37E-02	-2.55E-01	-1.57E-02	-5.65E-04	1.58E-02	2.88E-01
20	1784	-3.11E-04	3.29E-02	-1.90E-01	-1.62E-02	-4.60E-04	1.65E-02	2.48E-01
30	1784	-3.25E-04	3.51E-02	-3.63E-01	-1.66E-02	-4.10E-04	1.73E-02	2.70E-01

Table 0-8 Differenced swap spread statistics for post-crisis period.

	count	mean	std	min	25%	50%	75%	max
1	1907	-6.02E-06	1.26E-02	-2.08E-01	-4.75E-03	6.00E-05	4.93E-03	2.01E-01
2	1907	4.24E-05	1.11E-02	-7.23E-02	-5.13E-03	-1.40E-04	5.21E-03	6.81E-02
3	1907	5.88E-05	1.14E-02	-8.24E-02	-5.87E-03	0.00E+00	5.80E-03	5.87E-02
4	1907	8.19E-05	1.17E-02	-7.46E-02	-5.84E-03	-8.00E-05	6.01E-03	9.05E-02
5	1907	1.06E-04	1.27E-02	-6.58E-02	-6.38E-03	2.20E-04	6.64E-03	9.42E-02
6	1907	1.22E-04	1.23E-02	-6.37E-02	-6.58E-03	6.00E-05	6.84E-03	6.80E-02
7	1907	1.37E-04	1.23E-02	-6.67E-02	-6.41E-03	2.00E-05	6.95E-03	7.40E-02
8	1907	1.44E-04	1.33E-02	-8.89E-02	-7.08E-03	6.00E-05	7.36E-03	8.53E-02
9	1907	1.56E-04	1.41E-02	-8.93E-02	-6.98E-03	9.00E-05	7.36E-03	8.94E-02
10	1907	1.74E-04	1.42E-02	-1.12E-01	-7.17E-03	1.90E-04	7.66E-03	8.29E-02
15	1907	2.05E-04	1.51E-02	-7.44E-02	-7.61E-03	1.10E-04	8.23E-03	9.79E-02
20	1907	2.23E-04	1.59E-02	-9.50E-02	-7.82E-03	4.60E-04	8.68E-03	9.77E-02
30	1907	1.74E-04	1.69E-02	-1.41E-01	-8.41E-03	4.20E-04	8.99E-03	1.25E-01

E. Model parameters and p-values

Table 0-9 ARIMA constant parameter values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.00018	-0.00004	-0.00001	0.00004
2	0.00014	-0.00011	0.00004	0.00002
3	0.00012	-0.00012	0.00006	0.00002
4	0.00013	-0.00014	0.00007	0.00002
5	0.00011	-0.00017	0.00010	0.00001
6	0.00011	-0.00020	0.00012	0.00001
7	0.00011	-0.00023	0.00014	0.00000
8	0.00013	-0.00024	0.00014	0.00001
9	0.00016	-0.00028	0.00016	0.00001
10	0.00014	-0.00029	0.00017	0.00001
15	0.00019	-0.00032	0.00021	0.00003
20	0.00021	-0.00030	0.00022	0.00004
30	0.00029	-0.00032	0.00017	0.00005
Total	0.00015	-0.00021	0.00012	0.00002

Table 0-10 ARIMA moving average parameter values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	-0.61511	0.24152	-0.01343	-0.12901
2	-0.63595	-0.50461	-0.52156	-0.55404
3	-0.73288	-0.60486	-0.65250	-0.66341
4	-0.69815	-0.53684	-0.92599	-0.72033
5	-0.73897	-0.34499	-0.74359	-0.60918
6	-0.74992	-0.44327	-0.64955	-0.61425
7	-0.76528	-0.28289	-0.86445	-0.63754
8	-0.74779	-0.33189	-0.82773	-0.63580
9	-0.71574	-0.31469	-0.49995	-0.51013
10	-0.73807	-0.37229	-0.37765	-0.49600
15	-0.65073	-0.20981	-0.16131	-0.34062
20	-0.55996	-0.20656	-0.34270	-0.36974
30	-0.47817	-0.47775	-0.03796	-0.33129
Total	-0.67898	-0.33761	-0.50910	-0.50856

Table 0-11 ARIMA autoregressive parameter values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.10213	-0.39548	-0.01343	-0.10226
2	0.00396	0.28690	0.46530	0.25205
3	-0.03963	0.44273	0.56004	0.32104
4	0.10539	0.33944	0.85747	0.43410
5	0.12316	0.11795	0.62270	0.28794
6	0.14736	0.22216	0.53068	0.30007
7	0.17923	0.02709	0.75547	0.32060
8	0.16485	0.05667	0.69139	0.30430
9	0.18596	0.02909	0.31900	0.17801
10	0.23729	0.07842	0.20980	0.17517
15	0.18989	-0.08880	0.04722	0.04944

20	0.07048	-0.09094	0.24166	0.07373
30	-0.03833	0.16539	-0.04804	0.02634
Total	0.11013	0.09159	0.40302	0.20158

Table 0-12 ARMA residual variance parameter values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.000626	0.000188	0.000301	0.000191
2	0.000779	0.000861	0.000887	0.001056
3	0.000159	0.000122	0.000128	0.000135
4	0.000521	0.000391	0.000439	0.000460
5	0.000626	0.000188	0.000301	0.000191
6	0.000779	0.000861	0.000887	0.001056
7	0.000159	0.000122	0.000128	0.000135
8	0.000521	0.000391	0.000439	0.000460
9	0.000626	0.000188	0.000301	0.000191
10	0.000779	0.000861	0.000887	0.001056
15	0.000159	0.000122	0.000128	0.000135
20	0.000521	0.000391	0.000439	0.000460
30	0.000626	0.000188	0.000301	0.000191
Total	0.000779	0.000861	0.000887	0.001056

Table 0-13 ARIMA constant parameter p-values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.51240	0.94630	0.96900	0.80923
2	0.29289	0.82414	0.86015	0.65906
3	0.29560	0.81409	0.78148	0.63039
4	0.29909	0.79633	0.60620	0.56721
5	0.29544	0.77193	0.60149	0.55628
6	0.27126	0.71868	0.56671	0.51888
7	0.28117	0.66811	0.37255	0.44061
8	0.18546	0.63951	0.39966	0.40821
9	0.17809	0.59297	0.51185	0.42764
10	0.21120	0.56625	0.50033	0.42593
15	0.21217	0.57606	0.50092	0.42972
20	0.26517	0.57827	0.48127	0.44157
30	0.18736	0.53124	0.63655	0.45171
Total	0.26825	0.69414	0.59909	0.52049

Table 0-14 ARIMA moving average parameter p-values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.00000	0.00241	0.98519	0.32920
2	0.00000	0.00000	0.00232	0.00077
3	0.00000	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000	0.00000
5	0.00000	0.00000	0.00000	0.00000
6	0.00000	0.00000	0.00000	0.00000
7	0.00000	0.00000	0.00000	0.00000
8	0.00000	0.00000	0.00000	0.00000
9	0.00000	0.00000	0.00000	0.00000

10	0.00000	0.00000	0.00000	0.00000
15	0.00000	0.00035	0.27026	0.09020
20	0.00000	0.00004	0.01820	0.00608
30	0.00000	0.00000	0.81369	0.27123
Total	0.00000	0.00022	0.16074	0.05365

Table 0-15 ARIMA autoregressive parameter p-values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0.00779	0.00000	0.98520	0.33100
2	0.90185	0.00000	0.00874	0.30353
3	0.04745	0.00000	0.00000	0.01582
4	0.00479	0.00000	0.00000	0.00160
5	0.00026	0.07024	0.00000	0.02350
6	0.00001	0.00017	0.00000	0.00006
7	0.00000	0.65605	0.00000	0.21868
8	0.00000	0.31376	0.00000	0.10459
9	0.00000	0.59900	0.00000	0.19967
10	0.00000	0.11940	0.00459	0.04133
15	0.00000	0.15913	0.75192	0.30369
20	0.02316	0.07012	0.10991	0.06773
30	0.11413	0.00058	0.76834	0.29435
Total	0.08457	0.15296	0.20221	0.14658

Table 0-16 ARMA residual variance parameter p-values per period and per maturity.

Maturity	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	***Total***
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
15	0	0	0	0
20	0	0	0	0
30	0	0	0	0
Total	0	0	0	0

F. Linear regression results

Table 0-17 Mean linear regression intercept of GDP, AEX, and Inflation.

	Pre-crisis	*Mid-crisis*	*Post-crisis*	*Mean*
GDP	5.08E-04	-1.41E-05	9.77E-04	4.90E-04
AEX	1.42E-05	-1.88E-05	-9.14E-06	-4.56E-06
Inflation	1.13E-04	-5.74E-05	-1.36E-05	1.41E-05

Table 0-18 Mean linear regression intercept of YTM bid-ask spreads.

Maturities BA spread:	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	8.17E-06	-1.53E-05	6.38E-06	-2.45E-07
2	9.35E-06	-2.49E-05	8.93E-06	-2.20E-06
3	3.60E-06	-2.70E-05	1.33E-05	-3.36E-06
4	6.03E-06	-3.69E-05	1.83E-05	-4.19E-06
5	5.45E-06	-4.04E-05	1.99E-05	-5.01E-06
6	6.27E-06	-4.16E-05	1.86E-05	-5.58E-06
7	6.46E-06	-4.43E-05	1.90E-05	-6.29E-06
8	4.92E-06	-4.68E-05	1.87E-05	-7.74E-06
9	7.27E-06	-4.53E-05	1.81E-05	-6.65E-06
10	7.80E-06	-4.77E-05	1.69E-05	-7.67E-06
15	8.57E-06	-4.28E-05	1.56E-05	-6.21E-06
20	8.41E-06	-4.28E-05	1.36E-05	-6.94E-06
30	8.80E-06	-3.53E-05	1.12E-05	-5.10E-06

Table 0-19 Mean linear regression intercept of Netherlands and German zero-coupon spreads.

Maturities 0-cpn spread:	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	7.89E-06	-6.74E-06	4.27E-06	1.81E-06
2	7.70E-06	-6.78E-06	4.21E-06	1.71E-06
3	4.75E-06	-6.34E-06	3.74E-06	7.15E-07
4	6.20E-06	-4.03E-06	-3.97E-06	-5.99E-07
5	1.50E-05	4.64E-06	-3.07E-06	5.53E-06
6	9.74E-06	1.09E-05	-9.01E-06	3.87E-06
7	7.03E-06	9.56E-06	-1.08E-05	1.91E-06
8	1.46E-05	1.86E-05	-1.27E-05	6.83E-06
9	1.23E-05	2.22E-05	-1.29E-05	7.18E-06
10	8.21E-06	1.85E-05	-9.62E-06	5.71E-06
15	1.00E-05	1.22E-05	-5.78E-06	5.47E-06
20	8.74E-06	1.11E-05	-4.71E-06	5.05E-06
30	7.17E-06	-5.20E-07	3.96E-06	3.54E-06

Table 0-20 Mean linear regression intercept of swap rate bid-ask spreads.

Maturities swap BA spread	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	1.03E-03	-6.71E-06	-1.43E-05	3.37E-04
2	-9.68E-06	-2.95E-06	-1.65E-05	-9.71E-06
3	-8.92E-06	-3.85E-06	-1.70E-05	-9.94E-06

4	-9.29E-06	-3.30E-06	-1.32E-05	-8.61E-06
5	-8.86E-06	-1.11E-06	-1.85E-05	-9.49E-06
6	-8.69E-06	1.30E-06	-1.81E-05	-8.48E-06
7	-8.32E-06	5.58E-06	-2.00E-05	-7.57E-06
8	-8.43E-06	7.91E-06	-2.02E-05	-6.92E-06
9	-8.13E-06	1.24E-05	-2.33E-05	-6.35E-06
10	-8.15E-06	1.49E-05	-2.29E-05	-5.37E-06
15	1.11E-03	2.21E-05	-2.24E-05	3.70E-04
20	5.08E-04	2.57E-05	-2.30E-05	1.70E-04
30	-1.05E-05	2.14E-05	-2.01E-05	-3.09E-06

Table 0-21 Mean linear regression intercept of liquidity spreads.

Maturities liq. spread:	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	1.03E-03	-6.77E-06	3.93E-06	3.42E-04
2	-8.35E-06	-7.06E-06	4.02E-06	-3.80E-06
3	-8.38E-06	-6.47E-06	3.86E-06	-3.66E-06
4	-8.48E-06	-6.74E-06	4.11E-06	-3.70E-06
5	-8.15E-06	-6.00E-06	3.78E-06	-3.46E-06
6	-7.02E-06	-4.96E-06	3.85E-06	-2.71E-06
7	-8.57E-06	-7.11E-06	4.17E-06	-3.84E-06
8	-8.17E-06	-6.72E-06	4.54E-06	-3.45E-06
9	-8.31E-06	-6.51E-06	4.09E-06	-3.58E-06
10	-8.61E-06	-6.10E-06	4.06E-06	-3.55E-06
15	1.06E-03	-6.65E-06	4.06E-06	3.51E-04
20	5.08E-04	-5.55E-06	4.15E-06	1.69E-04
30	-8.63E-06	-6.33E-06	4.09E-06	-3.62E-06

Table 0-22 Mean linear regression coefficient of GDP, AEX, and Inflation.

	Pre-crisis	*Mid-crisis*	*Post-crisis*	*Mean*
GDP	-0.02769	0.00103	-0.04345	-0.02337
AEX	-0.00009	-0.00018	0.00007	-0.00007
Inflation	-0.02013	0.00719	0.00252	-0.00347

Table 0-23 Mean linear regression coefficient of YTM bid-ask spreads.

Maturities BA spread:	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	-0.00105	-0.01948	-0.03834	-0.01962
2	-0.00227	-0.03127	-0.02327	-0.01893
3	0.00344	-0.02903	-0.02806	-0.01788
4	0.00099	-0.03840	-0.03322	-0.02355
5	0.00183	-0.03978	-0.03133	-0.02309
6	0.00086	-0.04004	-0.02661	-0.02193
7	0.00065	-0.04347	-0.02679	-0.02320
8	0.00298	-0.04656	-0.02571	-0.02310
9	-0.00055	-0.04500	-0.02442	-0.02332
10	-0.00139	-0.04779	-0.02209	-0.02376
15	-0.00279	-0.04341	-0.02011	-0.02210
20	-0.00280	-0.04648	-0.01746	-0.02225
30	-0.00395	-0.03992	-0.01383	-0.01923

Table 0-24 Mean linear regression coefficient of Netherlands and German zero-coupon spreads.

Maturities 0-cpn spread::	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	0.03087	-0.01688	-0.00263	0.00379
2	0.05840	0.01141	-0.00620	0.02120
3	0.03081	0.08577	0.00866	0.04175
4	0.01260	0.16010	0.13386	0.10219
5	-0.09409	0.19561	0.14017	0.08056
6	-0.05059	0.22428	0.21424	0.12931
7	-0.00114	0.19794	0.20252	0.13311
8	-0.10366	0.20164	0.17206	0.09002
9	-0.12701	0.21595	0.17182	0.08692
10	-0.13302	0.22532	0.17470	0.08900
15	-0.07519	0.19759	0.15392	0.09211
20	-0.04935	0.19656	0.13184	0.09302
30	-0.01614	0.14876	0.08363	0.07208

Table 0-25 Mean linear regression coefficient of swap rate bid-ask spreads.

Maturities swap BA spread	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	0.02398	0.00000	0.00781	0.01059
2	-0.00377	0.03909	0.01831	0.01788
3	-0.00138	0.04570	0.01670	0.02034
4	-0.00224	0.04999	0.00090	0.01622
5	-0.00131	0.05217	0.01482	0.02189

6	-0.00101	0.05114	0.01149	0.02054
7	0.00014	0.05410	0.01367	0.02264
8	-0.00032	0.05184	0.01268	0.02140
9	0.00155	0.05651	0.01666	0.02490
10	0.00207	0.05575	0.01484	0.02422
15	0.01551	0.05037	0.01164	0.02584
20	0.00208	0.05554	0.01193	0.02318
30	0.00843	0.05117	0.00871	0.02277

Table 0-26 Mean linear regression coefficient of liquidity spreads.

Maturities liq. spread:	*Pre-crisis*	*Mid-crisis*	*Post-crisis*	*Mean*
1	-0.02756	0.02650	-0.01864	-0.00657
2	-0.00992	-0.06079	-0.00755	-0.02609
3	-0.00380	-0.02017	-0.01185	-0.01194
4	0.03521	0.03613	-0.00271	0.02287
5	0.02930	-0.07765	-0.01990	-0.02275
6	0.07554	-0.09480	-0.01574	-0.01167
7	0.04060	-0.02894	-0.00020	0.00382
8	0.09833	-0.03966	0.02443	0.02770
9	0.07063	-0.05368	-0.00437	0.00419
10	0.04805	0.04490	-0.01571	0.02575
15	0.08774	0.09847	-0.00708	0.05971
20	0.05853	0.06985	-0.01111	0.03909
30	0.08238	0.04757	-0.04037	0.02986

G. ARMA model conditional expectation and error

Suppose the change in the swap spread follows ARMA(1,1) model with autoregressive variable a, moving average variable b, residual variance σ_ε^2, initial swap spread change Y_0, and initial residual value ε_0. It follows the process with initial conditions:

$$Y_k = a^k Y_0 + a^{k-1}\varepsilon_0 + (a+b)\sum_{i=1}^{k-1} a^{k-1-i}\varepsilon_i + \varepsilon_k$$

Consider the conditional expectation as the future estimate.

$$Y_k^* = E[Y_k|Y_0,\varepsilon_0] = a^k Y_0 + a^{k-1}\varepsilon_0$$

The variance of this estimate is

$$E\left[Y_k^2|Y_0,\varepsilon_0\right] = a^{2k}Y_0^2 + a^{2(k-1)}\varepsilon_0^2 + (a+b)^2\sigma_\varepsilon^2\sum_{i=1}^{k-1} a^{2(k-1-i)} + \sigma_\varepsilon^2$$

$$Var[Y_k|Y_0,\varepsilon_0] = \sum_{i=1}^{k-1} a^{2(k-1-i)}(a+b)^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2$$

Now consider the error of this variable, which has expectation and variance

$$E[Y_k - Y_k^*|Y_0,\varepsilon_0] = 0$$

$$Var[Y_k - Y_k^*|Y_0,\varepsilon_0] = \sigma_\varepsilon^2 + (a+b)^2\sigma_\varepsilon^2\sum_{i=0}^{k-2} a^{2i}$$

The last process considered changes in the swap spread, the sum of the random variables constitutes the swap spread itself. Redefine that process as:

$$X_k = X_0 + \sum_{i=1}^{k} Y_k = X_0 + Y_0\sum_{i=1}^{k} a^i + \varepsilon_0\sum_{i=1}^{k-1} a^i + \sum_{i=1}^{k-1}\varepsilon_i\left(1 + (a+b)\sum_{j=0}^{k-1-i} a^j\right) + \varepsilon_k$$

With the conditional estimate

$$X_k^* = E[X_k|X_0,Y_0,\varepsilon_0] = X_0 + (Y_0+\varepsilon_0)\sum_{i=1}^{k} a^i - \varepsilon_0 a^k$$

This process has an estimation error expectation and variance of

$$E[X_k - X_k^*|Y_0,\varepsilon_0] = 0$$

$$Var[X_k - X_k^*|Y_0,\varepsilon_0] = \sigma_\varepsilon^2\left(1 + \sum_{i=1}^{k-1}\left(1 + (a+b)\sum_{j=0}^{k-1-i} a^j\right)^2\right)$$

$$= \sigma_\varepsilon^2\left(k + (a+b)\sum_{i=1}^{k-1}\left(2\sum_{j=0}^{k-1-i} a^j + (a+b)\left(\sum_{j=0}^{k-1-i} a^j\right)^2\right)\right)$$