Inventory strategies for the surgery department.

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Abstract

We take a look at the inventory problem for the disposables within the surgery department of the hospital. These disposables can only be used once before they are thrown away. There is a certain stock within the department for all the disposables that might be needed during a surgery. This is necessary for easy access to the products.

Part of the products are non scan-buy products. These products are also stored in the central warehouse of the hospital. All these products are ordered there and have a certain stock in the local warehouse. How many stock and when to order how many products from the central warehouse is the inventory problem that is solved in this paper. We describe two different models that can solve this inventory problem. The first model is for when there is only one order moment in the morning. In the second model the (s, S) inventory policy is used until it isn't optimal anymore. We describe a method that finds the time at which we should change the strategy.

The other disposables are the scan-buy products. These products are ordered immediately from the supplier. For this inventory problem the (R, s, Q) policy is used. The total cost function will differ, because the disposables are perishable. This means that the products that aren't used before their expiration date will be thrown away.

Preface

Before you lies my master thesis. With this thesis I will end my time as a student at the University of Twente. Years of hard work ended with this research. It's been years where I learned a lot. Years full of ups and downs. Sometimes it was a real struggle which made me consider quiting. But eventually I forced myself to come back and finish my master with this thesis as a result.

Several people helped me finishing this thesis. First of all, I'd like to thank the hospital Nij Smellinghe. They provided the interesting problem that resulted in this thesis. A big thanks to Sven Talman and Stefan Meijering for inviting me to the hospital and giving me guidance. And thanks to everyone else in the hospital who showed me around and answered my questions.

Furthermore, I'd like to thank my supervisor Richard Boucherie for all the effort and time he put in guiding me. Even in sickness and during busy periods. Finally, I would like to thank the remaining people from my graduation committee for reading and assessing my work.

Contents

1 Introduction.

In this section we are going to take a look at the inventory management in the hospital. There are multiple departments in the hospital. The one that we're going to focus on is the surgery department. To be able to do this, we'll also need to know more about the logistics department. In the surgery department they make use of different types of products. One of them is the disposables like bandages and syringes. These products are used only once before they are thrown away or gone. Since they absolutely cannot be used again, the hospital needs to have enough in stock. Not having enough stock might result in not being able to perform certain surgeries. Too much stock however is also not desirable. This will lead to unnecessary costs, a lack of storage space and eventually also to expired products.

The hospital divides the disposables into two categories. The first one consists of the disposables they have stored in the central warehouse. When ordering these products, they are collected in the warehouse and transported to the surgery department. The other category is the scan-buy products. These products aren't stored in the central warehouse and are only used in the surgery department. When they need more stock, they scan the products. Then the purchase department places an order at the supplier. After receiving these products, they get checked and then they are going straight to the surgery department.

All products that are coming in, are going to the logistics department. From there they are sorted out. Some products go straight to other departments and other products will be stored in the central warehouse. The central warehouse is divided into three storage rooms. We won't be looking at the first one which contains intravenous drips. The second one is for the sterilized products and the third one is for the remaining products. The logistics department is responsible for stocking the other departments in the hospital.

In Figure 1 we can see where the disposables in the surgery department come from and at

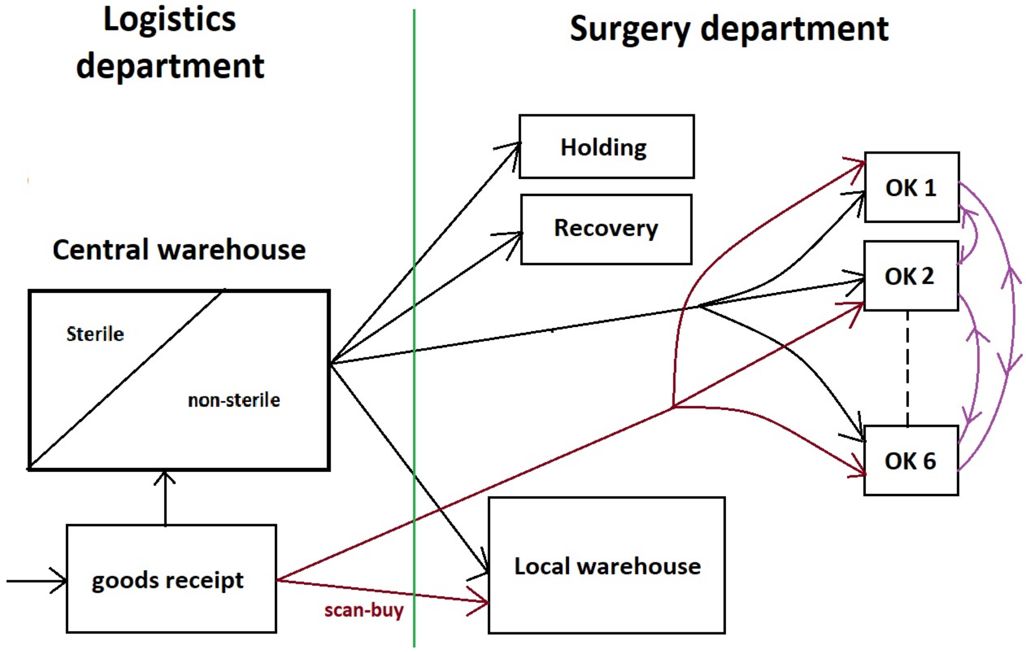

Figure 1: Current situation.

which places they are stored. Everything at the left side is part of the logistics department and the right side is the surgery department. The surgery department has a local warehouse with many disposables and sterile sets. The sterile sets will be sterilized after each use before they are stored again. There is also a storage room for the anesthetist and a closet with office supplies. We aren't going to look at the medicines that are also in the surgery department. There are six surgery rooms (OK 1 to 6) with a supply closet outside each one. The products in these rooms are different disposables than the ones stored in the local warehouse. Next there is a holding room. This room is used to prepare the patients for surgery. There are also products stored here. The last location is called the recovery. This is where the patients will be transported to after surgery. This is the last room in the surgery department with disposables stored.

Every morning at 7 am an employee of the logistics department checks the inventory levels in the surgery department. All the products at every official storage location have a card. For scan-buy products this card says at which inventory level it should be scanned. For the other disposables it says how many there should be in stock. The employee counts how many there are and when there isn't enough the card get scanned. In case of the non scan-buy products he also fills in how many need to be ordered. After having gone through every product and location the order list will be sent. All the products that are not scan-buy will be collected in the central warehouse and brought to the surgery department. Then they will be placed in the right location.

During the day there will be planned elective surgeries. The schedule for these surgeries is known beforehand. For each type of surgery they get a fixed collection of supplies from the local warehouse. These supplies are collected the day before the surgery. A part of it is always needed and will be used afterwards. The other part might be used during the surgery. These products must be available in case they need them. If they aren't used, they will be placed back in the local warehouse. There are also extra products from the local warehouse that they grab and use during the day. This demand is unknown beforehand. Even though most surgeries will be planned, it is also possible that emergency surgeries arrive, for example a broken leg or an unexpected C-section. These surgeries need to be performed within a day. The acute surgeries are emergency surgeries with the highest emergency level. These surgeries are done as soon as possible. The demand for all the unexpected emergency surgeries are also unknown during the reviewing moment.

There is one more department that deals with the inventories in the hospital. This is the purchasing department. This department is responsible for placing orders at the suppliers. They also keep track of the inventory levels within the central warehouse. They make use of the Min-Max model to determine at which inventory level (Min) a product will be reordered up to which level (Max). Whenever they expect a delivery problem, they might choose to reorder earlier to make sure there is enough stock.

This department also place the orders for the scan-buy products whenever the threshold is reached.

1.1 The problem

Because of all the needed items, it can become quite expensive to perform a surgery. The warehouses in the hospital are full of disposables. They are all perishable goods. Therefore, it doesn't only cost money to store, but it also might expire. When they expire they need to be thrown away and replaced with new products. Another important issue is the service level which indicates the probability of not having a shortage. The value of this service level determines the minimum safety stock that is needed. When restocking, the hospital tries to follow the first in, first out (FIFO) principle. With a high inventory level this can get more difficult. For this reason they also check all the expiration dates every three months. Products that are expired will be thrown away. And products with a relatively short time until expiration might be collected in one drawer. They can put on a label on this drawer that says to empty that drawer first. This is only possible with products that use up multiple drawers. When there are many small products in one single drawer, then they only check the date of one or two. With this current method it still occurs that they grab an expired product at the moment they need it. For simplicity we'll ignore the holding and recovering room for now.

1.2 Changes

Every product and every extra activity brings extra costs. The higher the total costs, the more expensive the medical treatments are. The hospital bill, minus the remaining own risk, will be paid by the health insurance companies. These companies pass this on by letting people pay insurance premium. The more expensive the health care, the higher this premium will be.

The hospital is planning some changes. At the moment they are building a new surgery location. There will be an extra surgery room and a larger local warehouse. They also plan on keeping track of the inventory levels within the surgery department. This makes scanning unnecessary and will result in an earlier restocking. Another change they consider is getting rid of the supply closets in front of each room and work with mobile carts. This means that these products will be stored at the local warehouse. The right column in Figure 1 will be removed. Expected is that this also stops the stashing of products outside the official supply closets. Since products on the mobile carts can be used at every location within the surgery department and can even be placed back in the local warehouse, we'll consider it to be part of the local warehouse inventory. The inventory level in this paper will consist of the amount in the local warehouse plus the amount in the mobile carts. How many products should be placed on these carts and how many times a day, are not part of this research.

One question the hospital has, is how much money they could save by optimizing the inventory levels under the new circumstances. Another question that is relevant is when you want to make a product a scan-buy product. In this paper we're going to create models for the disposables. For these models we will minimize the costs while meeting the service level.

The changed situation as mentioned above will be assumed.

1.3 Demand

The needed inventory level is dependent of the total demand. Since there is a surgery schedule known in advance, we also know the minimum needed products. The remaining demand is a stochastic process with a known mean. If certain disposables are used for the same surgeries we might have multiple types of products with the same expected demand. If for these products the life times and lead times are also expected to be the same then we can form groups of products that will each get the same inventory level.

1.4 Scan-buy products.

All the scan-buy products that are part of this research are only stored in the local warehouse of the surgery department. For these products there is a longer lead time that might differ a few days. Even though they scan every morning, the order placements occur only a few times a week.

Since these products will come straight from the supplier we'll need to take into account the costs. It might be cheaper to order more products at once, but you'll have more holding costs. Since they are also perishable it isn't desirable to order too many at once. It also isn't desirable to have stock-outs since the consequences will be larger for these products. It is possible to place an emergency order, but it costs more and still has a lead time. If these products aren't delivered in time and there is no alternative product, the surgery can't take place. Backorders aren't possible. A high service level is needed to prevent having too many lost surgeries which leads to a higher safety stock. A trade-off between the risks of rejecting the patient versus the costs of a higher safety stock should be made.

To get the optimal inventory level, we need to know how many are needed to meet the service level and when to order how many. The perishability of the products have to be taken into account.

1.5 Non scan-buy products.

Since non scan-buy products are stored in the central warehouse first, it is possible that they expire while still being stored there. In our research we're only going to focus on the inventory levels in the local warehouse and not the central warehouse. The longer the products stay in the central warehouse, the later the threshold for reordering will be reached. So, to have the highest shelf life for these products when they are at the local warehouse, we'll need the lowest possible inventory level. A low inventory level will also make FIFO more manageable.

In the current situation, the surgery department determines how many of each product they need at the local warehouse. For the non scan-buy products, they look at the possible maximum number of each surgery and what they would need for this. They also like to have enough stock for multiple days. Since the logistics department reorder and restock every day, this may lead to high inventory levels at each moment of the day which is the opposite of what we want.

Stock-outs are not desirable, but the consequences of a stock-out are smaller than for scan-buy products. Assuming that in the central warehouse all products are in stock, the products are never far away. During the day, when a request comes in, the central warehouse can be called. Assume that the product will immediately be collected and brought to the surgery department. This leads to a small lead time. There is also the option that an employee goes down to the warehouse to grab it.This process might disrupt the current activities, and are therefore not desirable. The necessary products for the elective surgeries for the next day are collected on carts, they will be stored in the hall. Since these products won't be used until the next day, it is possible to use these for an emergency surgery first, provided that they won't forget to place a new one back. This could potentially bring the lead time down, but it can still take time to search through all the carts. We assume in this paper that these lateral transshipments don't take place. During the weekend the logistics employees aren't working. They can be called, but it takes a bit more time for them to get to the hospital. Therefore it is preferred that an employee of the surgery departments collect the necessary products at the central warehouse.

1.6 Product flows

There are two different product flows within the surgery department. The first one consists of the disposables that are used by the surgeons. These are the surgery disposables. The second product flow are the disposables used by the anesthetist. Assume that these two sets of products are disjoint. In this paper we will mainly focus on the surgery disposables. However, the anesthesia disposables can also follow the same strategies.

1.6.1 Surgery disposables

These disposable are stored at the local warehouse. They are collected and set aside on mobile carts. For elective surgeries this takes place the day before the surgery. Whenever an emergency surgery arrives, the corresponding set of disposables for this surgery will also be collected and set aside. When the lists of products that are needed are optimized for each surgery, then there will be no extra demand besides the carts. The total demand D of the products will be equal to all the products that are set aside during the day and didn't come back unused. For products that won't be used for sure, there is a second option where they are placed on a separate cart in flexible drawers. This results in the drawer being unavailable during surgeries for which they might need it. And when they aren't used, they become available again immediately after the surgery and can be used during the next one. Depending on the usage probability and the number of surgeries for which it is needed, this option might result in a lower inventory level. The problem for this set is a two-echelon inventory problem with emergency shipments.

1.6.2 Example

One of product A is needed every surgery. We are currently reviewing. Tomorrow, there are s planned surgeries. Between now and the restocking moment tomorrow, the hospital performs X emergency surgeries. Then we get $D = s + X$. In order to meet the service level α, we'll need an inventory level IL such that $\mathbb{P}(D \leq IL) \geq \alpha$.

1.6.3 Anesthesia disposables

These disposables are the ones that are currently stored in the closets outside every OK. In the near future, these products will be stored in the local warehouse and the closets will be replaced by mobile carts. At certain moments these carts will be restocked at the local warehouse. Between restocking moments, it is possible that the cart runs out of stock. In that case an extra order takes place at the local warehouse. Lateral transshipment between these mobile carts is also possible. When a product isn't in stock at any of these locations, then an emergency shipment is necessary. In case of non scan-buy products, they'll need to come from the central warehouse. Otherwise an emergency shipment at the supplier is needed.

1.7 Research questions

The research objective in this thesis is to help the hospital to lower their stock and costs within the surgery department. The main research question is:
How can we calculate the optimal order size?

There are a few sub-questions that helps answering the main research question. These are:
How do we determine the optimal order-up-to level in the morning when this is the only order moment?
Is there another model and strategy possible which results in lower costs and less stock?
Which model and strategy should be used for each type of product?

2 Literature

Searching in research databases gives lots of different results. There are many papers en books about the inventory problem. The inventory problem is also very large. The most common inventory inventory policies are shown in both Silver [5] and Winston [3]. We will use Winston [3] for providing an overview of the most common inventory policies. In this book the method marginal analysis is used for solving part of the policies. The service level approach is used in some cases. The basic EOQ model is also elaborated in this book.

Silver [5] also uses the EOQ model. The iterative solution procedure is also used, which we will mention in the inventory policies section. Axsater [11] is another book that contains a lot of information for inventory problems. This book provides information about inventory models and inventory problems.

In our research we will use some of the inventory policies and methods that are used in the previous books.

So there is a lot of information, but none of those problems equal the complicated situation of the hospital. There aren't many relevant papers about the complete problem in the hospital which is a multi-echelon inventory problem for perishable disposables. However, there are papers that contain certain parts.

Ahmadi [12] and Baron [10] are about perishable goods. Ahmadi [12] uses interesting inventory models. There are perishable products that result in an age-dependent purchase price and lead time. The products in our problem however won't have this kind of age-dependent prices. Baron [10] solves inventory models with continuous review and where the products are perishable.

There are several researches done about joint-optimizations. Guerrero [4] is a paper that shows how joint-optimization of inventory policies can be done on a specific pharmaceutical system. Masache [9] shows a model with joint ordering. This paper gives a nonlinear inventory-programing model for a single planning period review. Nakunthod [2] gives an improvement for the joint ordering policy for multi product inventory problems. Boucherie [1] shows that joint ordering doesn't always lead to lower costs.

3 Inventory policies

There are multiple existing inventory policies that are mentioned in the literature. A part of these policies have continuous reviewing. It is also possible to have periodic reviewing moments where R is the time in days between reviews. In this section we will show a few policies and how they can be solved. We make use of the methods from Winston [3] unless stated otherwise.

First we give an overview of all the relevant costs that will be used in this paper. These costs except for the perishing costs are also used for the inventory policies.

Secondly we will show the basic EOQ model that is often used for the inventory policies.

3.1 Costs

There are several important and relevant cost. The costs that are mostly used in this paper will be explained in this section.

3.1.1 Holding costs

Having a higher stock also brings extra costs. The most important costs are the holding costs. Having a local warehouse and stocking items costs money. Let C_h be the holding costs per disposable A per day that it is stocked in the local warehouse.

3.1.2 Ordering costs

Every time there is an order it will result in ordering costs. Ordering products from the central warehouse have costs due to labor costs. Ordering from a supplier will have extra transport costs. There is also the option for emergency orders. These can have their own costs.

3.1.3 Shortage costs

When there is a shortage there will be emergency order costs. For simplicity there will be assumed that emergency surgeries occur after preparing the carts for the elective surgeries. If there is a shortage for the elective surgeries then there will be one emergency order. The amount of ordered products is equal to the shortage. The emergency order costs are independent of the ordered amount.

If there is a shortage for a emergency surgery then the costs depend on the surgery. When it is an urgent one then the shortage cost will be equal to the emergency order cost. However, when it is an acute emergency there will also be a penalty. The total expected shortage costs depend on the expected number of emergency orders and the probability of an emergency surgery being acute.

3.1.4 Costs for perishable goods

Since all disposables are perishable goods there is a possibility of having products expire. Let C_{priceA} be the price of disposable A and let $p_{exp}(i)$ be the probability that the ith one expires before it is used. $p_{exp}(i)$ equals the probability that the total demand until the expiration date is less than i. It is assumed that FIFO is used and that the same disposables that are ordered at the same day have the same time until expiration.

Let IL_t be the inventory level at the reviewing moment at time t. If $S_t > IL_t$ then the expected total expiration costs that are ordered at time t is

$$\mathbb{E}(C_{expire}^{S_t,t}) = C_{priceA} \times \sum_{i=IL_t+1}^{S_t} p_{exp}(i). \tag{1}$$

If $S_t = IL_t$, then $\mathbb{E}(C_{expire}^{t}) = 0$.

3.2 The EOQ model

The EOQ model is often mentioned when searching for inventory policies. The EOQ is the Economic Order Quantity.

Notation

K	= costs per order
h	= holding costs per product per unit of time
D	= demand rate per time unit
s	= cost per shortage for one time unit
q	= order quantity
q^*	= optimal order quantity
$TC(q)$	= total annual cost if q units are ordered
p	= purchasing cost per unit

The basic EOQ model

The basic model is determined under the condition that the demand is deterministic and there is no lead time. Shortages aren't allowed.

Because of the instant delivery we can order as soon as the inventory level becomes zero. If we order when the inventory level is higher, then we get unnecessary holding cost. Every time we want to place an order, we will have the same situation with a current inventory level equal to zero. Therefore, we should order the same optimal quantity every time. Let q be the order quantity. The optimal quantity $q*$ is the quantity q that minimizes the annual cost. We have

$$TC(q) = \text{annual order cost} + \text{annual purchasing cost} + \text{annual holding cost}$$

Since every order is of size q and the annual demand is D, we know that there will be $\frac{D}{q}$ orders per year. So

$$\frac{\text{Ordering cost}}{\text{Year}} = \frac{\text{ordering cost}}{\text{order}} \times \frac{\text{orders}}{\text{year}} = \frac{KD}{q}. \tag{2}$$

We purchase D units per year and the purchase cost per unit is p.

$$\frac{\text{Purchasing cost}}{\text{Year}} = \frac{\text{purchasing cost}}{\text{unit}} \times \frac{\text{units}}{\text{year}} = pD. \tag{3}$$

The annual holding costs depends on the inventory level. If the average inventory level is $\tilde{I}$ during a length of time T, then the holding cost during time interval

$$0, T$$

is $hT\tilde{I}$. Our model consists of cycles. Each cycle begins with an order arrival and it ends before the next order. The demand has a constant rate, so the average inventory level during each cycle is $\frac{q}{2}$. We get

$$\frac{\text{Holding cost}}{\text{Cycle}} = \frac{q}{2} \times \frac{q}{D} \times h = \frac{q^2 h}{2D}. \tag{4}$$

Then

$$\frac{\text{Holding cost}}{\text{Year}} = \frac{q^2 h}{2D} \times \frac{D}{q} = \frac{hq}{2}. \tag{5}$$

If we combine all the costs we get the total annual cost

$$TC(q) = \frac{KD}{q} + pD + \frac{hq}{2}. \tag{6}$$

The value q that minimizes $TC(q)$ can be found by setting $TC'(q)$ equal to zero

$$TC'(q) = -\frac{KD}{q^2} + \frac{h}{2} = 0. \tag{7}$$

Since a negative value for q doesn't make sense we get $q = (2KD/h)^{1/2}$. $TC''(q) = 2KD/q^3 > 0$ for all $q > 0$. So we know that $TC(q)$ is a convex function and any point where $TC'(q) = 0$ will minimize the cost function $TC(q)$. So the Economic Order Quantity is

$$EOQ = q^* = \frac{2KD}{h}^{1/2}. \tag{8}$$

3.3 Continuous review (r, q) policy

At any moment in time the inventory level is known and an order can be placed. If the demand arrives one at a time, there will always be a moment where the inventory level becomes equal to r. At this point q units will be ordered. The situation over time can be divided in cycles where one cycle is the time between two restocking moments. The first part of the cycle is from the start until the reorder point r is reached. During this time there are holding costs with an expected value equal to the expected average inventory level

$$\frac{(r + q - \mathbb{E}(D_L)) + (r - \mathbb{E}(D_L))}{2} \tag{9}$$

where $\mathbb{E}(D_L)$ is the expected demand during the lead time.

At the reorder point an order is placed which will give ordering costs. After that there are costs made during the lead time which consists of holding costs and shortage costs. Multiplying the expected total costs per cycle with the expected cycles per year results in the expected total costs per year TC.

$$TC = \frac{\text{expected total costs}}{\textit{cycle}} \times \frac{\text{expected number of cycles}}{\textit{year}}. \tag{10}$$

One method to find the optimal values q^* and r^* is by solving

$$\frac{\partial TC(q^*, r^*)}{\partial q} = \frac{\partial TC(q^*, r^*)}{\partial r} = 0. \tag{11}$$

If q is given, then another method of finding the optimal value of r is by marginal analysis.

3.3.1 Marginal Analysis

Marginal analysis is a method where the minimum costs can be found by repeatedly computing the effect of adding a marginal unit to a certain value. On pages 880-881 in Winston [3], the concept of this method is described. Let s be the decision and $\mathbb{E}(C(s))$ the expected costs given s. In most practical applications the cost function is convex. This means that there is one clear global minimum and no other local minima. If the optimum is greater or equal to one than starting from zero means that increasing s to $s + 1$ results in lower costs. So $\mathbb{E}(s + 1) - \mathbb{E}(s) < 0$. If we keep increasing s than there will be a point from where the new costs will be higher and $\mathbb{E}(s + 1) - \mathbb{E}(s) > 0$. Thus, the optimal decision value s^* is the smallest value of s for which

$$\mathbb{E}(s + 1) - \mathbb{E}(s) \geq 0. \tag{12}$$

Example with lost sales The news vendor problem is a problem that can be solved by marginal analysis. Instead of having to option of demand being backlogged all shortages result in lost sales. The complete problem with discrete demand can be found in Winston [3], p 881-883.

Consider a vendor that sells newspapers who needs to know how many papers he should order each day. Let D be the demand and q the ordering quantity. The overstocking cost c_o is the cost due to being overstocked by one extra newspaper. The understocking cost c_u is the per-unit cost of being understocked.

The extra costs for increasing q to $q+1$ depends on if that extra product will be sold or not. With probability $\mathbb{P}(D \leq q)$ the extra newspaper won't be sold. This results in expected extra costs

$$\mathbb{E}(q+1) - \mathbb{E}(q) = c_o \mathbb{P}(D \leq q) - c_u(1 - \mathbb{P}(D \leq q)). \tag{13}$$

Then $\mathbb{E}(q+1) - \mathbb{E}(q) \geq 0$ will hold if

$$\mathbb{P}(D \leq q) \geq \frac{c_u}{c_o + c_u}. \tag{14}$$

So the optimal value q^* is the smallest value for which the previous inequality applies.

3.3.2 The service level approach

Since it can be very difficult to determine the costs for a shortage, it's sometimes easier to use the service level approach. There are two basic measures of service level. The first one SLM_1 is the expected fraction of all demand that is met on time. And the second one $SLM_2$is the expected number of cycles per year during which there are shortages.

SLM_1 is calculated by

$$1 - SLM_1 = \frac{\text{expected shortages per year}}{\text{expected demand per year}}. \tag{15}$$

Working out this equation makes it possible to determine r given q. So the reorder point is then determined by the service level. SLM_2 can also be used since

$$SLM_2 = \mathbb{P}(D_L > r) \times \text{expected number of cycles per year}. \tag{16}$$

3.4 Continuous review (s, S) policy

In some cases it is possible that a demand for more than one unit can arrive. Then the reorder point r in the previous policy doesn't have to be reached. Instead of reordering when the inventory level is equal to r there will be an order when it's smaller or equal to s. And where the order size in the previous policy was q, it is replaced here by an order-up-to level S. So if the current inventory level is $x \leq s$ than there will be $S - x$ units ordered. The exact computation of the optimal (s, S) policy is difficult.

3.5 (R, S) policy

The (R, S) policy is widely used. Every reviewing moment we look at the current inventory levels i. R is the time period between the review moments. If at the reviewing time the inventory level i is less than S then we order-up-to level S. So as long as there has been demand during two reviewing moments, there will be an order every R days. The order quantity equals $S - i$. This policy in case of backlogs is explained on pages 907-920 in *Winston*. Here all the expected annual costs are calculated. Marginal analysis is used to determine the value of S that minimizes the costs for given R. R can also be determined. Often this interval is set equal to $\frac{EOQ}{\mathbb{E}(D)}$ where EOQ is the economic order quantity that is explained in Section 15.2 of Winston. We won't show this case in this paper.

3.6 (R, s, S) policy

With this policy we only order when the inventory level at the time of reviewing is smaller or equal to the reorder point s. If we have to reorder than we order up to level S. The order quantity

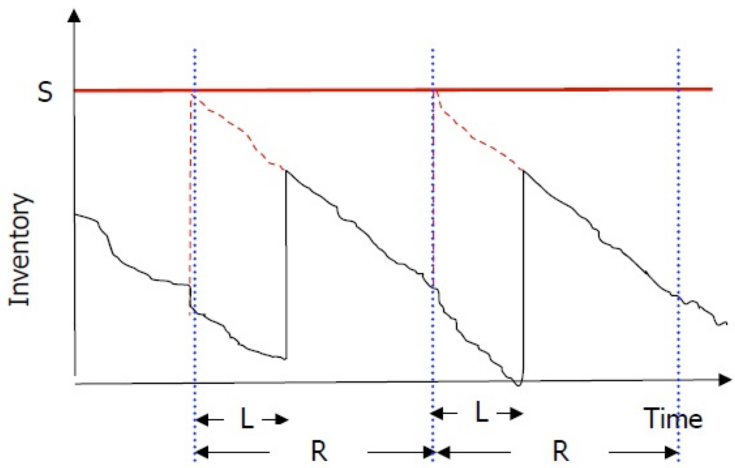

Figure 2: An example of the (R, S) policy.

equals $S - i$. This policy equals the (s, S) policy, but with a reviewing period R.

When using this policy, depending on the values of s and S, we will not necessarily have to reorder every time this product is used. The value s should be chosen high enough to meet the demand during the lead time L.

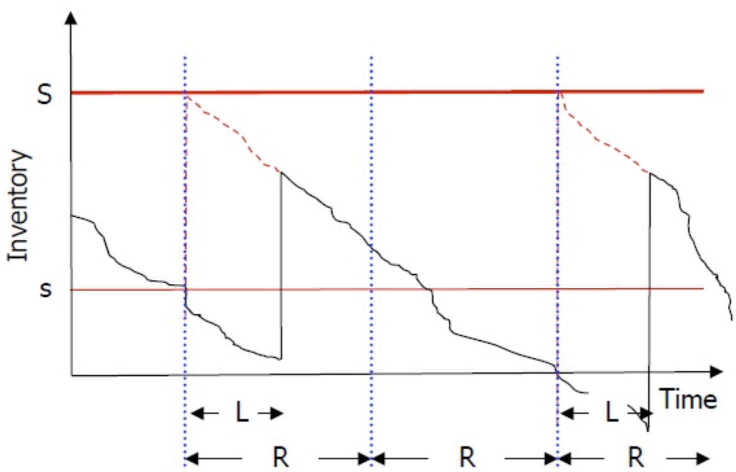

Figure 3: An example of the (R, s, S) policy.

3.7 (R, s, Q) policy

Again, the reorder point equals s. Whenever $i \leq s$ we order Q products. This policy can be used whenever you order packages with a fixed amount of products. If there are q products in one package, then we order a multiple of this amount every time we reorder. So, we get $Q = nq$ with n being the number of packages. If we take $s = Q - 1$ then we get the (R, Q) policy. And if $q = 1$, then we get the previous mentioned (R, s, S) policy.

There are many researches about this policy. If R is known as it is in our hospital case, then we only need to find the optimal values for s and Q. The function for the total cost will depend on both s and Q. One way of determining the reorder point is by the service level approach. In that case only Q will still be undetermined in the cost function. The optimal value for Q can be found by setting the differentiated cost function to zero. Another method is to use marginal analysis.

If s isn't determined with the service level approach, then this can't be done. We can calculate the partial derivatives of the cost function and set them to zero. This leads to two equations that calculates the s^* and Q^*. The equation for s^* will contain Q^* and the equation for Q^* contain s^*.

The golden section search can be used to find the minimum (s^*, Q^*). Silver [5] also uses an iterative solution procedure (p. 326) to solve this kind of problem. They make use of the cost function being convex and set the initial value of Q to the found EOQ. For this temporary value of Q, the temporary value for s can be found by. This temporary s can be used to calculate the new temporary Q and so on. This method converges to the minimum (s^*, Q^*).

3.8 Policies for the disposables

Every morning from Monday to Friday, there is an employee who determines which disposables should be ordered. Therefore, we have a discrete reviewing period. The review period R is the time between the reviews. This is one day on Monday to Thursday. In the weekend R is three days.

After reviewing the ordered non scan-buy products collected and delivered to the surgery department. The local warehouse and the storage rooms are restocked. Since there isn't a lot of uncertainty in this lead time, we're assuming that it's a constant value. The policy we use to model the current situation is the (R, S) policy.

In this paper we will also give an alternative model. The strategy for the whole day consists of different strategies at different times. If a certain time hasn't been reached yet, we use the (s, S) policy.

For scan-buy products we have to place an order at the supplier. This order placement only occurs at certain fixed moments. The delivery times may also vary, which results in longer and slightly more uncertain lead times.

Some scan-buy products will be ordered in boxes. When complete boxes are ordered, there need to be enough demand or you might risk getting expired products. For the scan-buy products we're going to use the (R, s, Q) policy.

4 Model 1: basic model for the non scan-buy surgery disposables

In this section we show the model for the current situation where we order once a day. We use the (R, S) policy with fixed R. The probability distribution for the demand is calculated. And we set the cost function. The optimal order-up-to value S is found by using marginal analyses.

Every workday t starts early in the morning with reviewing and ordering. So the reviewing moment is assumed to be fixed and known. On Saturday and Sunday there are no reviewing and ordering moments. Therefore, the Friday is extended and ends on Monday morning.

After reviewing and ordering it takes a lead time L until the restocking moment. For the non scan-buy products this often will take a couple of hours. After restocking, the carts for the elective surgeries will be prepared and set aside. Once set aside these products can't be used for other surgeries. Assume that all the unused products from the previous day will be back the next day right before reviewing and ordering again. Demand due to emergency surgeries can arrive at any time.

As a consequence, every morning from Monday to Friday there will be an order going to the central warehouse. So these ordering costs are assumed to be fixed and can be neglected in this model when trying to minimize the costs. There will be no advantage in ordering one type of disposables for multiple days instead of reordering this disposable every day. This makes an order-up-to strategy (R, S) with fixed R more preferable than the (R, s, S) policy. Each cycle consists of one day. From Monday until Thursday the cycle starts in the early morning when the workday starts and it ends the next morning. On Fridays the cycle again starts in the morning, but now it ends on Monday morning.

Each time there is a shortage, there will be an extra emergency order with the ordering size equal to the amount that is required for that specific surgery. Therefore this problem behaves as a lost sales problem as was mentioned in Section 3.3.1.

We consider a single disposable A. Below, we obtain the probability distributions of the demand, all the transition probabilities between the different moments of the day and the costs.

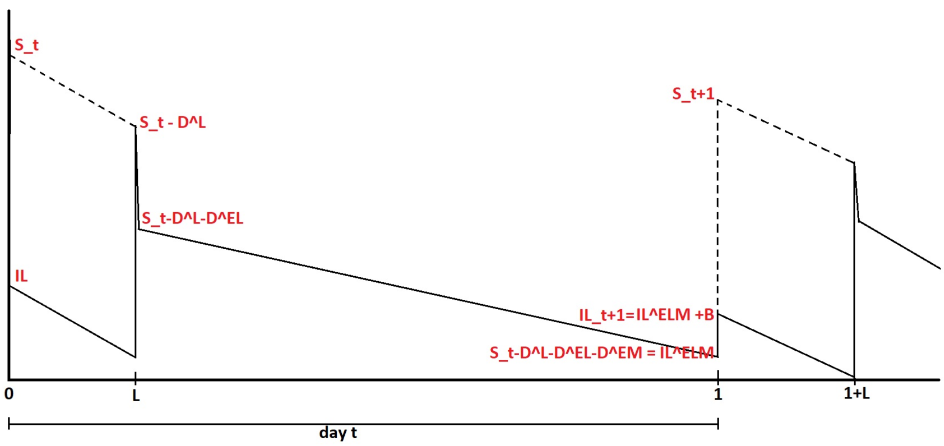

Figure 4: An example of one day without any stock-outs.

Notation

S_t	= order-up-to level for day t.
L	= lead time.
$q_t(j)$	= probability of an emergency surgery being of type j during time t.
$\nu_t(j)$	= the necessary amount for emergency surgery type j.
t	= time in days (excluding weekends)
λ_t	= the mean of the number of emergency surgeries between two reviewing moments.
D_t^L	= demand during the lead time of time t due to emergency surgeries.
D_t^{EL}	= demand for the elective surgeries during time t.
D_t^{EM}	= demand after restocking for the emergency surgeries on time t.
D_t^{ELM}	= demand of the elective surgeries and emergency surgeries combined outside the lead time.
IL_t	= inventory level during reviewing after the unused products returned.
I_t^L	= inventory level after the order arrived.
I_t^{EL}	= remaining inventory level after the carts for the elective surgeries are prepared.
I_t^{EM}	= remaining inventory level after all the elective and emergency surgeries and before reviewing.
$u_{t-2}(i)$	= probability of not using the product during an elective surgery of type i on day $t-1$.
$a_{t-2}(i)$	= amount of surgeries of type i that were prepared on day $t-2$ and performed on day $t-1$.
$v_{t-1}(i)$	= probability of getting a product back for an emergency surgery of type i.
B_t	= the amount of unused products that come back before the reviewing moment of day t.
$EO_{S_t,t}$	= the amount of shortages during restocking and reviewing if S_t is chosen.
$EO_{S_t,t}^L$	= the amount of shortages during lead time L of day t.
C_{short}^L	= the costs per shortage during the lead time.
C_{EO}	= costs for an emergency order.
$C_{penalty}$	= extra penalty costs for having a shortage when there is an incoming acute surgery.
$p_{a,t}$	= probability of an emergency surgery is acute.
C_h	= holding costs per product per day.

4.1 The transition probabilities and costs

4.1.1 Start of day t until restocking.

Each workday t starts at time 0 with a new cycle. Given the current inventory level IL_t and the known demand D_{t-1} of the previous day $t-1$, the expected costs for ordering up to level S_t can be calculated at this point.

In this section there is made a difference between the on-hand inventory level and the on-order inventory level. The on-hand inventory is the amount of products in the local warehouse that is available. The on-order inventory level is the on-hand inventory level plus the amount of products that are in order.

Let I_t denote the on-hand inventory level at the reviewing moment of day t If $I_t < S_t$ then there will be an order placed of size $S_t - I_t$ which makes the on-order inventory level equal to S_t. Otherwise there will be no extra products ordered and the on-order inventory level is the same as the on-hand inventory level.

In some cases if there is a stock-out it is possible to wait until the order arrives. But in other cases when the incoming surgery is acute, it might be necessary to get the product sooner. Therefore, in case of a stock-out the needed product is grabbed from the central warehouse. If D_t^L is the demand during the lead time than the on-hand inventory level at time L becomes

$$I_t^L = (I_t - D_t^L)^+(S_t - I_t)^+. \tag{17}$$

This inventory level has to be high enough for the demand during the rest of the cycle time. The leftover products together with the returning products are used for the demand during the next lead time

After a time L after the reviewing moment, there is a restocking moment. Only emergency surgeries can occur during this time. The corresponding probabilities can be calculated in the same way as before as compound Poisson process. Formula 27 becomes

$$Y_t(L) = \sum_{i=1}^{X_t(L)} D_i^t. \tag{18}$$

The probability distribution for this demand D_t^L can be calculated by

$$\mathbb{P}(D_t^L = i) = \mathbb{P}(Y_t(L) = i) = \sum_{n=0}^{\infty} \mathbb{P}(Y_t(L) = i | X_t(L) = n)\mathbb{P}(X_t(L) = n). \tag{19}$$

4.1.1.1 Shortage costs It is possible to have shortages in this period which give shortage costs. Let $EO_{I_t,t}^L$ be the amount of shortages between reviewing and restocking on day t given I_t. Let $\hat{\ell}$ be the expected amount of needed products for a demand.

$$\mathbb{E}(EO_{I_t,t}^L) = \sum_{\ell=1}^{\infty} \mathbb{P}(D_t^L = I_t + \ell)\frac{\ell}{\hat{\ell}}. \tag{20}$$

Let C_{short}^L be the shortage costs per shortage during the lead time. The expected total shortage costs can be calculated by

$$\mathbb{E}(C_t^{short}(I_t, L)) = C_{short}^L \times \mathbb{E}(EO_{I_t,t}^L) + C_{penalty} \times p_{a,t} \times \mathbb{E}(EO_{I_t,t}^L). \tag{21}$$

4.1.1.2 Holding costs The expected holding costs can be determined by

$$\mathbb{E}(C_t^{hold}(L)) = \frac{2I_t - \mathbb{E}(D_t^L)}{2} LC_h. \tag{22}$$

4.1.2 Elective surgeries

After the arrival of the ordered products, the carts for the elective surgeries are prepared.
The schedule for the elective surgeries for day t is known at the time of reviewing. Therefore, the demand for this part of the surgeries D_t^{EL} is known. Assuming that these carts are prepared before the rest of the demand arrives gives the following new inventory level afterwards

$$I_t^{EL} = (IL_t^L - D_t^{EL})^+. \tag{23}$$

4.1.3 Emergency surgeries

Emergency surgeries arrive as a Poisson process with mean λ_t. Let $X_t(h)$ denote the number of emergency surgeries in $(0, h)$, then

$$\mathbb{P}(X_t(h) = x) = \frac{(h\lambda_t)^x}{x!} e^{-h\lambda_t}. \tag{24}$$

Let F be the set of all the emergency surgery types. If an emergency surgery is of type $j \in F$,then $\nu(j)_t$ is the amount of product A that is required for this specific surgery at day t. Let $q(j)_t$ be the probability that an emergency surgery during day t is of type j. Let K_t be the set of surgeries that need k products

$$K_t = \{j | \nu(j)_t = k\}. \tag{25}$$

The probability of needing k products for an emergency surgery at day t equals

$$p_t(k) = \sum_{i \in K_t} q_t(i). \tag{26}$$

The emergencies now behave as a compound Poisson process where the surgeries(jumps) arrive randomly. The size of the jumps are also random with a known probability distribution $p_t(k)$.

Let $D_t(i)$ be the demand of emergency surgery i in $(0, h)$, $i = 1, \cdots, X_t(h)$ with probability distribution $\mathbb{P}(D_t(i) = k) = p_t(k)$. Then, the demand $Y_t(h)$ in $(0, h)$ is given by

$$Y_t(h) = \sum_{i=1}^{X_t(h)} D_t(i). \tag{27}$$

The expected value can be calculated using equality

$$\mathbb{E}(Y_t(h)) = \mathbb{E}(D_t(1) + \cdots + D_t(X_t(h))) = \mathbb{E}(X_t(h))\mathbb{E}(D_t(1)) = \lambda h \mathbb{E}(D_t). \tag{28}$$

The probability of having a total demand of i in $(0, h)$ can be calculated by

$$\mathbb{P}(Y_t(h) = i) = \sum_{n=0}^{\infty} \mathbb{P}(Y_t(h) = i | X_t(h) = n)\mathbb{P}(X_t(h) = n) \tag{29}$$

$$= \sum_{n=1}^{\infty} \mathbb{P}\left(\sum_{j=1}^{n} D_t(j) = i\right) \frac{(h\lambda_t)^x}{x!} e^{-h\lambda_t}. \tag{30}$$

The demand due to emergency surgeries between restocking and reviewing the following day is given by

$$D_t^{EM} = Y_t(1 - L). \tag{31}$$

The inventory level before the next reviewing moment and right before the unused products return to the warehouse equals

$$I_t^{EM} = (I_t^{EL} - D_t^{EM})^+ \tag{32}$$

The total demand between restocking and the end of day t equals

$$D_t^{ELM} = D_t^{EL} + D_t^{EM} \tag{33}$$

4.1.4 Reviewing moment at $t + 1$

The transition probabilities between the two reviewing moments follow from the demand throughout day t and the returned products. At this point all unused products return to the warehouse and day t ends.

First consider the situation in which it is only possible to have zero or one return per surgery. Let U_{t-1} be the amount that return at day $t+1$ due to the elective surgeries at day $t-1$. Let $u_{t-1}(i)$ be the probability of getting one product back unused after an elective surgery of type $i \in I$ where I is the set of elective surgery types and let $a_{t-1}(i)$ be the amount of surgeries of type i that were prepared on day $t-1$. The probability distribution for the returned products can be calculated by using the binomial distribution since there are only two options. Let $Z_t(i)$ be the amount that come back from all the surgeries of type i.

$$\mathbb{P}(Z_t(i) = x_i) = \binom{a_{t-1}(i)}{x_i} u_{t-1}(i)^{x_i}(1 - u_{t-1}(i))^{a_{t-1}(i) - x_i} \tag{34}$$

$$U_{t-1} = \sum_{i \in I} Z_t(i). \tag{35}$$

Let $v_t(i)$ be the probability of getting one back after the emergency surgery type i that was performed on day t. Let V_t be the amount that come back unused from the emergency surgeries from day t and $V_t(x)$ the returning amount given array X as the array for the emergency surgeries. If $\tilde{X}_t(i) \in X$ is the number of emergency surgeries of type i at day t and $W(x, i)$ the amount of returning products from surgery type i when $\tilde{X}_t(i) = x$

$$\mathbb{P}(W(x_i, i) = w_i) = \binom{x_i}{w_i} v_t(i)^{w_i}(1 - v_t(i))^{x_i - w_i} \tag{36}$$

$$V_t(x) = \sum_{i \in I} W(x_i, i) \tag{37}$$

$$\mathbb{P}(V_t = y) = \sum_{x=0}^{\infty} \mathbb{P}(V_t(x) = y)\mathbb{P}(X = x). \tag{38}$$

It might also be possible to get multiple products back from a surgery type. In that case the same method can be used, but by including these possibilities with corresponding probabilities. Let $u^2_{t-1}(i)$ be the probability of getting two products back from surgery type i. When it is also still possible to get only one back, it isn't possible anymore to use the binomial distribution. Instead of the binomial distribution the multinomial distribution is used. Let X^2_t be the number of times that two products return and X^1_t the number of times that one product return.

$$\mathbb{P}(X^1_t = x_1, X^2_t = x_2) = \frac{a_{t-1}!}{x_1!x_2!x_3!} u_{t-1}(i)^{x_1} \times u^2_{t-1}(i)^{x_2} \times (1 - u_{t-1}(i) - u^2_{t-1}(i))^{a_{t-1} - x_1 - x_2}. \tag{39}$$

With this probability there will be $x_1 + 2x_2$ returning products from all the surgeries of type i. To find the complete probabilities it is necessary to calculate these probabilities for all the possible outcomes. All the probabilities with the same amount of returned products are added together. This results in probability distribution functions for the returned products from the elective surgeries and from the emergency surgeries.

The probability of getting a total of b products back is calculated by

$$\mathbb{P}(B_{t+1} = b) = \sum_{x=0}^{b} \mathbb{P}(U_{t-1} = x)\mathbb{P}(V_t = b - x). \tag{40}$$

The new inventory level IL_{t+1} equals

$$\begin{aligned} I_{t+1} &= I^{EM}_t + B_{t+1} && (41)\\ &= (\max(I_t, S_t) - D^L_t - D^{EL}_t - D^{EM}_t)^+ + B_{t+1}. && (42) \end{aligned}$$

At this point the order-up-to level S_{t+1} is determined. If $I_{t+1} < S_{t+1}$ then an order of size $S_{t+1} - I_{t+1}$ takes place.

With these formulas the costs between restocking at time t and reviewing time $t + 1$ can be calculated. Add the expected costs $C_t(L)$ during the lead time and the costs for the whole cycle can be calculated. However, there are also expected costs in the future due to choosing S_t. These costs are made during reviewing and restocking on day $t + 1$. These expected future costs can be calculated in the same way as the costs $C_t(L)$.

4.1.4.1 Shortage costs Let $EO_{S_t,t}$ be the amount of emergency orders for disposable A between the restocking moments of day t and the start of day $t+1$ given S_t. Let $\hat{\ell}$ be the average amount of products that is needed per demand. The expected value is

$$\mathbb{E}(EO_{S_t,t}) = \sum_{\ell=1}^{\infty} \mathbb{P}(D^{ELM}_t = I^L_t + \ell)\frac{\ell}{\hat{\ell}}. \tag{43}$$

The cost per shortage depends on the type of surgery. Part of the emergency surgeries is acute. This means that the surgery needs to happen immediately and therefore the product also has to be available immediately. Let C_{EO} be the costs per emergency order and let $C_{penalty}$ be the extra penalty costs for not having disposable A when there is an acute surgery. If $p_{a,t}$ is the probability of an emergency surgery being acute at time t, then the expected total shortage costs given decision S_t for time t are

$$\mathbb{E}(C_{short}^{S_t,t}) = C_{EO} \times \mathbb{E}(EO_{S_t,t}) + C_{penalty} \times p_{a,t} \times \mathbb{E}(EO_{S_t,t}). \tag{44}$$

4.1.4.2 Holding costs Let C_h be the holding costs per product per day. The expected holding costs depends on the distribution of the surgeries. If it is assumed that the carts for the elective surgeries are prepared almost instantly after restocking than the expected holding costs between restocking and reviewing can be approximated by

$$\mathbb{E}(C_t^{hold}) = \frac{\mathbb{E}(I_t^{EL}) + \mathbb{E}(I_t^{EM})}{2}(1-L)C_h. \tag{45}$$

There is an alternative method where the elective surgeries are evenly spread throughout the workday. Let wt be the time in days between restocking and the end of the workday. Then the expected holding costs becomes

$$\mathbb{E}(C_t^{hold}) = \frac{\mathbb{E}(I_t^{EL}) + \mathbb{E}(I_t^{EM})}{2}(1-L)C_h + \frac{D_t^{EL}}{2} \times wt \times C_h. \tag{46}$$

After preparing the carts the carts go back to a separate place where they will wait until it's time for that surgery. Any holding costs at this point after preparation will always be the same regardless of the strategy. Therefore, these costs can be neglected in determining the optimal strategy.

4.1.5 After day t.

When day t is done a new day starts. This happens during the reviewing moment after the return of the unused products. Although day t is over, the consequences and costs due to choosing S_t aren't. Therefore it is necessary to also look at future costs. The extra costs after day t occur between reviewing and restocking on day $t+1$. These costs $C_{t+1}(L)$ can be calculated in the same way as the costs $C_t(L)$ for day t. Only when calculating these extra costs on day t means that the inventory level at day $t+1$ isn't known yet.

After a time L after the reviewing moment, there is again a restocking moment. The costs made during this lead time are all expected extra costs due to choosing S_t.

Only emergency surgeries can occur. The corresponding probabilities can be calculated in the same way as before as compound Poisson process.

$$Y_{t+1}(L) = \sum_{i=1}^{X_{t+1}(L)} D_i^{t+1}. \tag{47}$$

The probability distribution for this demand D_{t+1}^L can be calculated by

$$\mathbb{P}(D_{t+1}^L = i) = \mathbb{P}(Y_{t+1}(L) = i) = \sum_{n=0}^{\infty} \mathbb{P}(Y_{t+1}(L) = i | X_{t+1}(L) = n)\mathbb{P}(X_{t+1}(L) = n). \tag{48}$$

The new inventory level at the restocking moment without the new ordered products equal

$$\begin{aligned} I_{t+1}^L &= (I_{t+1} - D_{t+1}^L)^+ && (49)\\ &= ((\max(I_t, S_t) - D_t^L - D_t^{EL} - D_t^{EM})^+ + B_{t+1} - D_{t+1}^L)^+. && (50) \end{aligned}$$

4.1.5.1 Shortage costs

It is possible to have shortages in this period which cause shortage costs. Let $EO^L_{S_t,t+1}$ be the amount of shortages between reviewing and restocking on day $t+1$ given S_t. Then

$$\mathbb{E}(EO^L_{S_t,t+1}) = \sum_{\ell=1}^{\infty} \mathbb{P}(D^L_{t+1} = I_{t+1} + \ell)\ell. \tag{51}$$

Let C^L_{short} be the shortage costs per shortage during the lead time. The expected total shortage costs can be calculated by

$$\mathbb{E}(C^{short}_{t+1}(S_t, L)) = C^L_{short} \times \mathbb{E}(EO^L_{S_t,t+1}) + C_{penalty} \times p_{a,t+1} \times \mathbb{E}(EO^L_{S_t,t+1}). \tag{52}$$

4.1.5.2 Holding costs

Because of the risk aversion of the hospital it can be assumed that the expected value at this point is non-negative. Therefore, the expected holding costs can be determined by

$$\mathbb{E}(C^{hold}_t(L)) = \frac{\mathbb{E}(I_{t+1}) + \mathbb{E}(I^L_{t+1})}{2} LC_h. \tag{53}$$

4.2 Optimization

To find the optimal strategy this model is solved. For each disposable A, the expected total costs will be minimized such that the service level α is met. The expected total costs during cycle t plus future costs if S_t has been chosen is

$$\mathbb{E}(C^{total}_t(S_t)) = \mathbb{E}(C_t(L)) + \mathbb{E}(C^{short}_t(S_t)) + \mathbb{E}(C^{hold}_t) + \mathbb{E}(C^{short}_{t+1}(S_t, L)) + \mathbb{E}(C^{hold}_t(L)). \tag{54}$$

4.2.1 Analysis

The problem is to find the optimal order-up-to strategy. Whenever $I_t < S_t$ it is necessary to order-up-to value S_t. When I_t is larger, there will be no order since every extra product will add extra costs as will be shown later in this section. The current inventory level I_t only determines whether this product will be ordered or not. And if there is an order then the order size will $S_t - I_t$. The inventory level isn't relevant in finding the optimal order-up-to value S_t.

With calculating the costs and finding the optimal order-up-to value let $I_t \leq S_t$ and

$$I^L_t = (S_t - I_t) + (I_t - D^L_t)^+. \tag{55}$$

Simplified model

For the analysis the model will first be simplified. First the probability distribution of the demand is assumed to be the same each day. This results in one strategy and order-up-to value for each day. Now the expected costs are also the same each day and marginal analysis can be used to find the optimal value for S. Since reordering and restocking happens every day and FIFO is used, the expiration costs will be negligible.

If S is increased by 1 than the costs change. There is one extra product available each day. This means that if there are shortages than there will be one less shortage after the increase. Therefore, the expected shortage costs will decrease. One extra products also means higher expected holding costs. If the product is used than the expected holding costs increase by $\frac{C_h}{2}$. If the extra product isn't used than the holding costs increase by C_h. So the expected extra costs can be calculated by

$$\begin{aligned}\mathbb{E}(C^{total}(S+1) - C^{total}(S)) = \big(1 - \mathbb{P}(D^{ELM} \leq I^L)\mathbb{P}(D^L \leq I)\big)\big(\frac{C_h}{2} - C_{short} - C_{penalty} \times p_a\big) + \\ \mathbb{P}(D^{ELM} \leq I^L)\mathbb{P}(D^L \leq I)C_h.\end{aligned} \tag{56}$$

The marginal analysis method can now be used to find the optimal value $\tilde{S}$ that minimizes the costs. If the extra security of having a service level α is added, then this optimal value gets a minimum m. m can be determined by finding the smallest non-negative integer for which

$$\mathbb{P}(D^{ELM} \leq I^L)\mathbb{P}(D^L \leq I) \geq m. \tag{57}$$

These probabilities have to be determined from data.

The optimal value $\tilde{S}$ that meet the service level and gives the smallest expected total costs is the smallest non-negative integer larger or equal to m for which

$$C^{total}(S+1) - C^{total}(S) > 0. \tag{58}$$

Strategy dependent on t

If each day isn't the same, then the probability distributions of the demand and returning products need to be recalculated every reviewing moment. Every day t starts with reviewing. At this point it is known what the current inventory level is and what the demand of the previous day $t-1$ for the elective surgeries was. This demand will determine the probability distribution function of the returning products due to elective surgeries that arrive at the end of day t.

Every day t the probability distributions for the demand need to be recalculated if a value changes. All the necessary input in the model are now dependent on t. When these values for day t are used, then the model can be used again to calculate all the probabilities and costs for day t. Then it is also possible to calculate the extra costs for increasing S_t by one. So marginal analysis can again be used in the same way to find the optimal order-up-to value S_t for day t.

The decision depends on the calculated optimal value S_t and the current inventory level. When $I_t < S_t$ an order takes place of size $S_t - I_t$. When $I_t = S_t$ there will be no order since the optimal value is already reached. When $I_t > S_t$ there will also be no order. Every extra product will result in higher expected costs. Therefore, it is best to keep the current inventory level and not order more.

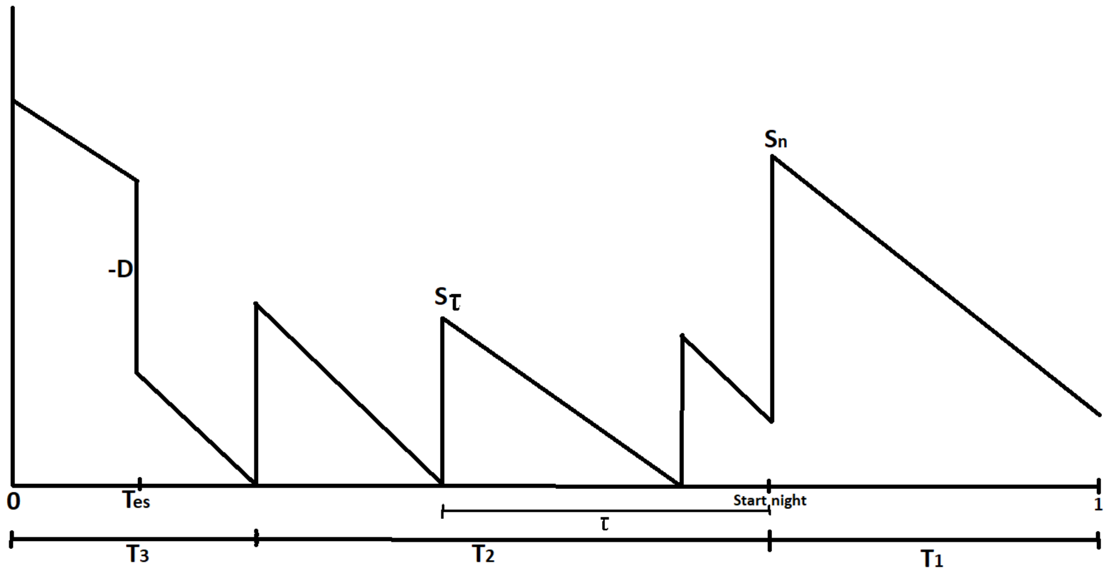

Figure 5: An example for one day.

5 Model 2: model with extra orders during the day

In this section we will be looking at a different model. In this model there can be multiple orders during the workday. This makes for a more complex model with more possibilities. Every day starts with a workday. There will always be a reviewing and restocking moment in the morning. However, it isn't necessary that every type of product is ordered. After the restocking moment it takes a time T_{es} before the known demand of the elective surgeries arrive. During the whole day it is possible for emergency surgeries to arrive. The number of emergency surgeries have a Poisson distribution. During the workday as soon as a certain threshold s is reached there will be an extra order. This makes it possible to have less stock during the day when the ordering costs are small enough. In this section the model for one day is split into three time intervals.

The first interval is the night. This interval starts at the end of the workday. At the start it is possible to already have enough stock and otherwise an extra order takes place. During this period the hospital doesn't focus on the complete inventory in the local warehouse. If there is a shortage, then they only get the needed products from the central warehouse. If they would take more then the remaining products won't end up in the local warehouse and get lost.

The second interval doesn't necessarily occur. When it occurs it starts as soon as the threshold s is reached and the second order of the day will be placed. And the interval ends when interval 1 starts.

Interval 3 starts at the reviewing moment in the morning which is also the beginning of the day. In this interval the first order of the day takes place. Which products will be ordered at this point depends on the current inventory level and the optimal strategy.

The optimal strategies during each time interval will lead to an optimal strategy for day t. Assume first that every day is identical with the same probability distribution for the demand and the same costs structure. First we look at the model with instant delivery and give the costs functions. Since the cost function turns out to be too complex, we will need to find another way to determine the optimal strategy. Later on in this section we will take a look at what would change if the demand and values depend on t and when there are lead times.

Notation

D_n	= the demand during the night.
S_n	= the order-up-to level at the beginning of the night.
T_1	= the duration of the night interval.
$C^n(x)$	= the expected nightly costs when the night is started with x products.
C_h	= daily holding costs per product.
C^n_{EO}	= costs per shortage during the night.
C_{EO}	= costs for extra orders during the workday.
s_n	= threshold for ordering at the end of the workday.
s^*_n	= point from where it is more cost efficient to not order extra products for the night.
s^{min}_n	= the minimum stock that is needed for the night to meet the service level.
α	= the needed service level.
η	= the inventory level at the end of the workday before the optional extra order for the night.
s	= the order point or threshold. It equals 0 when there is instant delivery.
τ	= the remaining time until the beginning of the night.
S_τ	= the order-up-to level when there is τ time left until the start of the night.
T_2	= the time of this whole period.
η	= the inventory level at the end of this period
D_τ	= a stochastic variable for the demand during given τ.
t_{es}	= remaining time until the arrival of the demand for the elective surgeries.
T_{es}	= the time when the demand of the elective surgeries arrive.
S^*	= the optimal order-up-to level for the first order of the day.
S	= the optimal order-up-to level in an infinite horizon.
$\tilde{T}_x$	= the expected time until a demand of x is reached.
C_{short}	= expected shortage costs per shortage during the workday.
p_{em}	= the probability that an incoming emergency surgery is acute (surgery takes place right away).
$C_{penalty}$	= extra penalty for not having any stock when there is an acute surgery.
L_1	= lead time of the first order at the start of the day.
L_2	= lead time for any extra order.
D_{L_1}	= demand during lead time L_1.
D_{L_2}	= demand during lead time L_2.

Table 1: All the used notation in model 2.

5.1 Time interval 1: The night

The first interval is the night period. During this period there are no employees present from the logistics department. Therefore, there will be no planned extra orders. Every demand after reaching zero stock will result in a shortage which will cost C^n_{EO} per shortage.

This period starts at the end of the workday with the final reviewing moment and possible restocking moment of that day. The period ends the next morning at the reviewing moment. This situation is similar as the situation in the previous model without elective surgeries and with a smaller time window. Therefore, it can also be solved in a similar way.

5.1.1 Optimal strategy

Before we can find the optimal order-up-to level we need to know more about the costs. The consequences for not having the product in stock when an emergency surgery arrives is calculated in the shortage costs.

The expected total costs for this period when starting with x products are:

$$C^n(x) = \sum_{j=0}^{x} \mathbb{P}(D_n = x - j) \left(\frac{x+j}{2} T_1 C_h \right) + \sum_{j=1}^{\infty} \mathbb{P}(D_n = x + j) \left(\frac{x}{2} \left(\frac{x}{x+j} \right) T_1 C_h + j C_{EO}^n \right). \tag{59}$$

Finding the optimal order-up-to value for S_n is now determined by marginal analysis.

Increasing S_n by 1

If S_n is increased by 1, then the expected extra costs equal

$$\begin{aligned} C^n(S_n+1) - C^n(S_n) =& \mathbb{P}(D_n \leq S_n) T_1 C_h - (1 - \mathbb{P}(D_n \leq S_n)) C_{EO}^n \\ &+ \sum_{d=S_n+1}^{\infty} \mathbb{P}(D_n = d) \frac{2S_n + 1}{2d} T_1 C_h. \end{aligned} \tag{60}$$

The smallest value $S_n > 0$ for which the extra costs is greater than zero is the optimal order-up-to value if you place an order at the end of the workday.

Now we know how many to order when there is an order. What is left is to decide when to order. Equation (60) increases when S_n increases. So, it is best to not order when the inventory level at the end of the workday is equal or larger than S_n. If your inventory level is less then S_n then the decision to order depends on the expected nightly costs plus the order costs. If we don't order then there will be no order costs. But if the inventory level is less than S_n then the nightly costs will be higher. It's possible that not ordering for every inventory level beneath S_n gives extra expected costs. In that case threshold $s_n^* = S_n$. If not ordering is more cost efficient for certain inventory levels then there is a smallest value $s_n^* < S_n$ for which

$$C^n(s_n^*) < C_{EO} + C^n(S_n). \tag{61}$$

If there are two options that have the same expected total costs, then we always choose the option that gives the lowest probability of shortages. So in this case we would choose to order.

The decision to place an order at the end of the workday is decided by the inventory level. Let η be the inventory level at that time. If $\eta < s_n^*$ then it is best to order-up-to S_n. If $\eta \geq s_n^*$ then not ordering extra for the night gives the smallest expected costs.

5.1.2 Optional extra safety

Previously in this chapter we let the order point s_n^* be determined by the expected costs. A lower order point results in a lower safety stock and makes it possible to start the night with less stock. If the night starts with less stock, then the expected shortage costs increases while the holding costs decreases. Therefore, the shortage costs per shortage prevents the model from getting more shortages than is wanted. However, it can be quite difficult to translate the consequences of having a shortage into costs. For this reason the hospital might prefer to have an extra option to make sure that there won't be too much shortages. This can be done by adding a minimum needed stock. This minimum will force the model to meet a fixed service level α. The service level equals the probability of not having any shortages. The value of this fixed minimum s_n^{min} equals the smallest number of products for which

$$\mathbb{P}(D_n > s_n^{min}) \leq \alpha. \tag{62}$$

The order point for the night s_n is the value greater or equal to s_n^{min} that minimizes the expected costs.

$$s_n = \max(s_n^{min}, s_n^*). \tag{63}$$

With the addition of the service level, the order-up-to value S_n also need to be greater or equal to s_n^{min}. If the inventory level $\eta < s_n$, then we order-up-to S_n. If $\eta \geq s_n$, then no order will be placed for the night.

5.2 Time interval 2: Between the first stock out and the night

The second interval starts during the working hours as soon as there are less than s left and an extra order is necessary. For now $s = 0$ due to instant delivery. The size of this second time interval will be zero when the threshold 0 is never reached. If the threshold is reached, then it can either happen before the demand of the elective surgeries, after or during this time. The interval ends when time interval 1 starts. As soon as s is reached we will order S_τ products. Every order within this interval will cost C_{EO}. Because there is instant delivery there are no shortage costs.

5.2.1 Start after the elective surgeries

In this subsection we will take a look at the expected costs when time interval 2 starts after the elective surgeries. It turns out that the cost function contains a recursion which makes it hard to calculate. Therefore, it won't be possible to minimize the cost function in this section. In subsection 5.4 we will use a different approach to determine the optimal strategy.

The expected costs during the night depends on the inventory level at the end of time interval 2. These nightly costs plus the costs during time interval 2 decides the best strategy during the day. There are three different situation. In the first situation the demand D_τ is less or equal than the order-up-to level S_τ and $\eta = S_\tau - D_\tau < s_n$. In this case there will be no more extra orders during time interval 2, but there is an order for the night. The expected costs are

$$f_1(S_\tau) = \frac{S_\tau + \eta}{2}\tau C_h + C_{EO} + C^n(S_n). \tag{64}$$

In the second case we have no more orders during interval 2 and no order for the night. So $D_\tau \leq S_\tau$ and $\eta \geq s_n$. The the expected costs in this case equal

$$f_2(S_\tau) = \frac{S_\tau + \eta}{2}\tau C_h + C^n(\eta). \tag{65}$$

In the third and last case there will be an extra order during time interval 2. So $D_\tau > S_\tau$ and the threshold s is reached again before the start of the night. The costs in this case are much harder to calculate. The time at which the next order occurs is a random variable. Let random variable $\tilde{\tau}$ denote the remaining time between the next order and the start of the night. The expected costs are

$$f_3(\tau, \tilde{\tau}, S_\tau) = \frac{S_\tau}{2}(\tau - \tilde{\tau})C_h + C_{EO} + (\text{Costs after the next order}). \tag{66}$$

The costs after the next order depends on the remaining demand. The order-up-to level of the next order equals $S_{\tilde{\tau}}$ and the demand after the next order equals $D_{\tilde{\tau}}$. If $S_{\tilde{\tau}} - s_n < D_{\tilde{\tau}} \leq S_{\tilde{\tau}}$, then the situation during the remaining time corresponds to the first case. The second case occurs when $D_{\tilde{\tau}} \leq S_{\tilde{\tau}} - s_n$. If the demand after the next order is greater than the ordered number of products, then the threshold is reached again and another order will take place with the new remaining time until the night $\tilde{\tau}_2$. The expected costs after the next order $Cost(S_{\tilde{\tau}})$ that was mentioned in equation (66) equal

$$\begin{aligned} Cost(S_{\tilde{\tau}}) =& \mathbb{P}(S_{\tilde{\tau}} - s_n < D_{\tilde{\tau}} \leq S_{\tilde{\tau}}) \times f1(S_{\tilde{\tau}}) + \mathbb{P}(D_{\tilde{\tau}} \leq S_{\tilde{\tau}} - s_n) \times f2(S_{\tilde{\tau}}) \\ &+ \mathbb{P}(D_{\tilde{\tau}} > S_{\tilde{\tau}}) \times f3(\tilde{\tau}, \tilde{\tau}_2, S_{\tilde{\tau}}). \end{aligned} \tag{67}$$

Since $\tilde{\tau}$ is a random variable the costs can't be calculated and minimized like we did previously. Therefore, another method for finding the optimal strategy will be used in subsection 5.4.

5.2.2 Start before elective surgeries

It is also possible that the first stock-out occurs before the demand for the elective surgeries arrives. The cost function for time interval 2 changes when this occurs.

Let T_{es} denote the time from the start of the day until the demand for the elective surgeries. The remaining time until the start of the night is still τ. The remaining time until the arrival of the elective surgeries is te. If interval 2 starts at time $h < T_{es}$, then $te = T_{es} - h$ and $\tau = 1 - T_1 - h$. In this situation we have $\tau > 1 - T_1 - T_{es}$. The demand D for the elective surgeries is known.

There are three options. Option 1 is where there isn't another stock-out before or at time T_{es}. This means that the products for the elective surgeries are present until time T_{es} and have holding costs during this time $te = T_{es} - 1 + T_1 + \tau$. Let the random variable D_{te} denote the demand during time te. The inventory level at time T_{es} after the demand of the elective surgery becomes $S_\tau - D_{te} - D$. And remaining time until the night becomes $\tilde{\tau} = 1 - T_1 - T_{es}$. The expected costs when $D_{te} < S_\tau - D$ are

$$f_4(S_\tau) = D \times (\tau - 1 + T_3 + T_{es})C_h + Cost(S_{\tilde{\tau}}). \tag{68}$$

The second option is where there is a stock-out at time T_{es}. This is the case when $(S_\tau - D)^+ \leq D_{te} < S_\tau - D$. If there isn't enough stock for the elective surgeries, then the $D + D_{te} - S_\tau$ remaining needed products will be ordered and instantly leave the warehouse after receiving. Because of the instant delivery this all happens at time T_{es} without any shortage costs or extra holding costs. The expected costs in this situation becomes

$$f_5(S_\tau) = \frac{2S_\tau - D_{te}}{2} te \times C_h + C_{EO} + Cost(S_{1-T_1-T_{es}} - (D + D_{te} - S_\tau)). \tag{69}$$

The third option is where there will be another stock-out before the elective surgeries. The expected costs consists of the holding costs until the next stock-out, the order cost for the next stock-out and the costs after the next stock-out. These last costs will again depend on the demand after the next stock-out. And on whether there will be more orders after the next stock-out and at what time. The time between the next stock-out and the night is $\tilde{\tau}$ and the new time until the elective surgeries is $\tilde{te} = \tilde{\tau} - 1 + T_1 + T_{es}$). The expected costs are

$$f_6(\tau, \tilde{\tau}, S_\tau) = \frac{S_\tau}{2}(\tau - \tilde{\tau})C_h + C_{EO} + Cost2(S_{\tilde{\tau}}), \tag{70}$$

$$Cost2(S_{\tilde{\tau}}) = \mathbb{P}(D_{\tilde{te}} < S_{\tilde{\tau}} - D) \times f_4(S_{\tilde{\tau}}) + \mathbb{P}((S_{\tilde{\tau}} - D)^+ \leq D_{\tilde{te}} < S_{\tilde{\tau}} - D) f_5(S_{\tilde{\tau}}). \tag{71}$$

This cost function also includes a recursion and isn't easily solvable.

5.3 Time interval 3: Start of the workday

The third and last time interval starts in the morning at the reviewing moment. The current inventory level is known. The time until the arrival of the elective surgeries is T_{es} and the time until the night is $\tau = 1 - T_1$. The situation in this interval is the same as in interval 2 when we are still before the elective surgeries. Therefore, the same possibilities and cost functions apply. Only now with $\tau = 1 - T_1$ and $te = T_{es}$. Now we can see that the cost function for the daily expected costs can't be minimized.

5.4 Finding the optimal strategy

We have shown that the strategy for the night can be found by minimizing the costs. The optimal threshold s_n can be calculated as was shown in the previous subsection. And the optimal order-up-to value S_n can be found with marginal analysis. The strategy throughout the workday can't be found that way since the cost functions are too complex. Therefore, we need another approach for finding the optimal strategy. We will describe the method and strategy in this section and prove the optimality.

5.4.1 Optimal strategy in an infinite horizon

First we take a look at the optimal strategy when the workday never ends and there are no elective surgeries. Let S be the optimal order-up-to value. The time between two stock-outs is one cycle. If x is the order-up-to value, then the total expected cost per cycle divided by x gives the expected cost per product $q(x)$.

$$q(x) = \frac{1/2 \times x \times \tilde{T}_x \times C_h^{day} + C_{EO}}{x}, \tag{72}$$

where $\tilde{T}_x$ is the expected cycle time. The optimal value S is the value that minimizes the expected costs per product. This results in $\mathbb{E}(q(S)) < \mathbb{E}(q(x))$ for $x \neq S$. Rewriting the inequality $\mathbb{E}(q(S)) < \mathbb{E}(q(S+1))$ gives

$$C_h > \frac{2}{S(S+1)(\tilde{T}_{S+1} - \tilde{T}_S)} C_{EO}. \tag{73}$$

So if the day is endless, then it's optimal to keep ordering S products per order.

5.4.2 Prove: ordering more than S results in higher expected costs

We know that when the $\tau = \infty$ the optimal order-up-to level equals S. But is S also optimal for $\tau = \tilde{T}_S + \tilde{T}_1$. And what will that tell us about the best strategy for other values of τ.

When $\tau = \tilde{T}_S + \tilde{T}_1$, we have an expected demand $D_\tau = S + 1$. We first take a look at the extra expected costs when we order $S+1$ products instead of S. If $D_\tau \leq S$, then the last product won't be used and the extra holding costs are τC_h. If $D_\tau = S+1$, then the expected extra holding costs are

$$\tilde{T}_S \times C_h - (S-1) \times \tilde{T}_1 C_h = \tilde{T}_1 C_h. \tag{74}$$

There will also be no extra order with costs C_{EO} in this case.

If $D_\tau > S+1$, then we get the same extra holding costs, but instead of $\tilde{T}_1$ we get a different expected time until one arrival that depends on the value of D_τ. These extra costs are always positive. Without the extra costs for $D_\tau > S+1$ we get extra total costs of at least

$$\begin{aligned} \text{Extra costs} &= \mathbb{P}(D_\tau \leq S)\tau C_h + \mathbb{P}(D_\tau = S+1)\tilde{T}_1 C_h - \mathbb{P}(D_\tau = S+1)C_{EO} && (75)\\ &= \mathbb{P}(D_\tau \leq S)\tilde{T}_S C_h + \mathbb{P}(D_\tau \leq S+1)\tilde{T}_1 C_h. - \mathbb{P}(D_\tau = S+1)C_{EO}. && (76) \end{aligned}$$

If the extra cost is larger than zero, then ordering S products results in lower expected costs.

Because of inequality (73), we know that the extra costs are greater than

$$\mathbb{P}(D_\tau \leq S)\frac{2}{S+1}C_{EO} + \mathbb{P}(D_\tau \leq S)\frac{2}{S(S+1)}C_{EO} - \mathbb{P}(D_\tau = S+1)C_{EO}. \tag{77}$$

We made use of $\tilde{T}_S = S \times \tilde{T}_1$, which is the case when the demand has a Poisson distribution.

When $\tau = \tilde{T}_S + \tilde{T}_1$ and $S = 1$, the extra costs are positive. If the demand has a Poisson distribution, then the probability of having a demand equal to it's expected value $\mathbb{P}(D_\tau = S+1)$ decreases when S increases. The probability of having a demand less than the expected value increases when S increases. This leads to positive extra costs for all values of S.

So if $\tilde{T}_S = S\tilde{T}_1$ and $\mathbb{P}(D_\tau = S+1)$ decreases and $\mathbb{P}(D_\tau < S+1)$ increases, then ordering more than S products leads to higher expected total costs.

If $\tau > \tilde{T}_S + \tilde{T}_1$, then $\mathbb{P}(D_\tau = S+1)$ is greater than when $\tau = \tilde{T}_S + \tilde{T}_1$. This means that the expected saved order costs when you order $S+1$ decreases. The probability of having extra order costs increases. Therefore, ordering $S+1$ products leads to higher expected total costs. If $\tau < \tilde{T}_S + \tilde{T}_1$, then the probability $\mathbb{P}(D_\tau = S+1)$ is greater than when $\tau = \tilde{T}_S + \tilde{T}_1$. And the probability $\mathbb{P}(D_\tau \leq S)$ is larger.

We can now conclude that ordering more than S always result in higher expected total costs when the demand has a Poisson distribution. This is also the case with a compound Poisson distribution when $\tilde{T}_S = S\tilde{T}_1$ and $\mathbb{P}(D_\tau = S+1)$ decreases and $\mathbb{P}(D_\tau < S+1)$ increases for $\tau = \tilde{T}_S + T_1$.

5.4.3 Prove: ordering less than S results in higher expected costs

We still have to show that ordering less than S also gives higher expected total costs. We do this by showing that ordering $S-1$ when $\tau = \tilde{T}_{S-1}$ and S is large, leads to higher expected costs. If this is true, then ordering $S-1$ is better than ordering $S-2$ when $\tau = \tilde{T}_{S-2}$. Therefore, ordering S gives the lowest expected costs for every $S \geq 1$ and for every $\tau > 0$.

Let τ be equal to $\tilde{T}_{S-1}$. If $D_\tau < S$, then the last product isn't used and the extra costs for ordering S instead of $S-1$ becomes $T_{s-1}C_h$. If $D_\tau = S$, then the extra expected holding costs until the stock-out equals $\frac{s-1}{s}\tau C_h = \frac{s-1}{s}\tilde{T}_{S-1}C_h$. In this case the extra order with cost C_{EO} won't take place. The expected extra costs are negative when $D_\tau > S$. Now the remaining expected extra costs also have to be negative. So

$$\mathbb{P}(D_\tau < S)T_{s-1}C_h + \mathbb{P}(D_\tau = S)\frac{s-1}{s}\tilde{T}_{S-1}C_h - \mathbb{P}(D_\tau = S)C_{EO} < 0. \tag{78}$$

We know that $q(S) < q(S-1)$. Rewriting this inequality gives

$$\tilde{T}_1 C_h < \frac{2}{S(S-1)}C_{EO}. \tag{79}$$

If we use this inequality and let S be equal to 999, then inequality (78) is true. Therefore, ordering S products gives the lowest expected costs.

5.4.4 When to order the products for the night

We have proven that if the order for the night takes place at the end of the workday, then order-up-to S is the best strategy. However, it is also possible that it's better to order extra products for the night. Then an extra order at the end of the workday isn't necessary. Let $\hat{\tau}$ be the remaining time at which both strategies give equal expected total costs.

We know that $\hat{\tau} < \tilde{T}_S$. Otherwise S wouldn't give the lowest expected cost per product. If we choose to order S products, then we might get another stock-out before the end of the workday. If this happens, then that will be the last order of the day. However, we don't expect any stock-out.

τ is a continuous random variable and the cost function for the expected costs is too complex to calculate and use. If you order for the night, then the optimal order-up-to level goes to S_n when τ goes to zero. And the extra expected costs for ordering for the night will be $(S_n - S)\tau C_h - C_{EO}$. If these costs are zero, then both strategies result in equal expected costs for the remaining workday. However, if τ gets larger, then ordering S_n leads to a smaller service level. Therefore it is necessary to check if ordering S_n products is possible for τ. If $\mathbb{P}(D_{\tau+T_1} \leq S_n) > \alpha$, then $\hat{\tau} = \tau$. Otherwise we need at least an order size of $S_n + 1$. A higher order size results in a higher value for $\hat{\tau}$. This possible value can be determined by solving $(S_n + 1 - S)\tau C_h - C_{EO} = 0$. This value of τ will be the time of ordering for the night if $\mathbb{P}(D_{\tau+T_1} \leq S_n + 1) > \alpha$. If τ turns out to be too large, then we need to keep continuing this search and keep increasing the order-up-to value. So the next method will find the value $\hat{\tau}$.

Step 1) $k = 0$
Step 2) $(S_n + k - S)\tau C_h - C_{EO} = 0$
Step 3) If $\mathbb{P}(D_{\tau+T_1} \leq S_n + k) \geq \alpha$, then $\hat{\tau}$. Else $k = k + 1$ and return to **Step 2**.

So if $\tau \leq min(\tilde{T}_S, \hat{\tau})$, then it is best to also order immediately for the night. If $\tau > min(\tilde{T}_S, \hat{\tau})$, then we order S products.

This method fails when $S_n + k \leq S$. If $S_n + k \leq S$, then the needed products for the remaining workday and night is less or equal to S. Therefore, ordering to S will already contain the products for the night.

5.4.5 Before the elective surgeries.

In the previous subsections we found the best strategy for during the workday when there are no elective surgeries. However, the hospital does have elective surgeries. If $\tau < 1 - T_1 - T_{es}$, then there will be no more demand for the elective surgeries. In this situation we can use the strategies as was found in the previous subsection. If $\tau = 1 - T_1 - T_{es}$, then there might still be a remaining demand for the elective surgeries. Ordering S plus the remaining demand leads to the same situation as when we order S products after the elective surgeries demand. Therefore, an order of size S plus the remaining elective surgery demand is the optimal order-up-to level. If $\tau > 1 - T_1 - T_{es}$, then the time until the arrival of the elective surgeries te is positive. This changes the expected costs which may change the strategy.

This situation with the elective surgeries has strong similarities with the situation before the end of the workday. We have shown that before a certain time it is optimal to order-up-to S. But if an order is placed after a certain time it will be better to also order the extra products for the elective surgeries. Let D determine the known demand for the elective surgeries. The difference with the situation for the night is that the demand for the elective surgeries all arrives at time T_{es}. Therefore, the situation after and before T_{es} behaves the same as for the situation without elective surgeries. The two possible optimal strategies are to either order-up-to S or to order-up-to $S + D$.

If $S \leq D$, then there will be an extra order at time T_{es} and there will be no holding costs for the D products. Therefore, ordering D extra products lead to less or equal expected costs when

$$te \leq \frac{C_{EO}}{D \times C_h}. \tag{80}$$

If $S > D$, then there won't be an order at time T_{es} with probability $\mathbb{P}(D_{te} < S - D)$. In this case all the D products have expected costs $te \times C_h$. The remaining $S - D$ have a higher expected cost per product than when strategy $S + D$ is chosen. Ordering $S + D$ is the best strategy when the expected extra costs for choosing $S + D$ are less or equal to zero. So we order $S + D$ when $te \leq te^*$ where te^* is calculated by solving the equation

$$\mathbb{P}(D_{te^*} < S - D)\left((S - D)(q(S - D) - q(S))\right) + \mathbb{P}(D_{te^*} \geq S - D)(te^* D C_h - C_{EO}). \tag{81}$$

5.5 Constant lead times

We proved and found an optimal strategy for when the demand has a Poisson distribution. We know that under certain conditions this strategy is also optimal for a compound Poisson process.

However, proving whether this strategy is always optimal for a compound Poisson process isn't part of this research. While using method 2, we assume that the demand has a Poisson distribution. This will also be the case for most of the disposables in the hospital.

Until now there was instant delivery. In the real case there will be lead times for all the orders. There have two different lead times since there is a larger lead time in the morning. These lead times in combination with shortage costs and possibly a service level, determine a minimum needed stock. Previously we ordered as soon as there were zero products. Now we order as soon as the minimum stock, better known as the threshold or reorder point, is reached.

The cost functions will be different from the situation with no lead time. During the lead time there are no holding costs for the ordered products. But there are extra costs due to the possibility of having shortages. The same strategy can be used as before with these updated cost functions.

5.5.1 Safety stock during the day

Let L_2 be the lead time in hours during time interval 2. This lead time is equal to the lead time for an emergency order. So, if there is an incoming demand during the lead time, then the waiting time for the ordered products is less or equal to the waiting time for an emergency order. Therefore, every shortage during L_2 is a backorder.

As soon as the threshold s_2 is reached, a new order is placed. We order-up to S_τ with τ being the time left until the reviewing time for the night. s_2 is the number of products that should meet the during lead time L_2. The cost of a shortage depends on the type of surgery for which the product is. Some surgeries won't take immediately and the consequences aren't that large. However, there is a possibility of having an emergency where you need the product as soon as possible. In these cases the consequences of not having the product available can be really large. Therefore, there is a penalty $C_{penalty}$ for these shortages. Let p_{em} be the probability of an incoming demand being for such an emergency case and C_{out} the basic costs for being out of stock when demand is incoming. The expected shortage costs per shortage C_{short} can now be determined by

$$C_{short} = C_{out} + p_{em} \times C_{penalty}. \tag{82}$$

The expected demand during the lead time L_2 is equal to $\lambda \times L_2$. The probability that the demand D_{L_2} during the lead time equals x is calculated by

$$P(D_{L_2} = x) = \frac{(\lambda \times L_2)^x}{x!} e^{-\lambda \times L_2}. \tag{83}$$

With this equation the probability of meeting the demand during L_2 if the threshold equals s_2 can now easily be calculated. If s_2 is increased by one then the costs changes. Ir the demand is more than s_2, there will be one less shortage. Else the extra product wasn't necessary and only increases the holding costs for that cycle. The inventory level after restocking is $S_\tau - D_{L_2}$.

Let $C_{L_2}(s_2)$ denote the total expected costs between ordering and restocking.

$$\begin{aligned} C_{L_2}(s_2) = &\sum_{j=0}^{s_2} \mathbb{P}(D_{L_2} = s_2 - j) \left(\frac{s_2 + j}{2} L_2 C_h \right) \\ &+ \sum_{j=1}^{\infty} \mathbb{P}(D_{L_2} = s_2 + j) \left(\frac{s_2}{2} \left(\frac{s_2}{s_2 + j} \right) L_2 C_h + j C_{short} \right). \end{aligned} \tag{84}$$

The total extra costs $C_{L_2}(s_2) - C_{L_2}(s_2)$ when we increase s_2 by one is calculated by

$$\begin{aligned} C_{L_2}(s_2 + 1) - C_{L_2}(s_2) = &P(D_{L_2} \leq s_2) L_2 C_h + \sum_{k=s_2+1}^{\infty} \mathbb{P}(D_{L_2} = k) \frac{s_2 + 1}{k} C_h \\ &- (1 - P(D_{L_2} \leq s_2)) C_{short}. \end{aligned} \tag{85}$$

The smallest nonnegative value of s_2 for which $C_{L_2}(s_2+1) - C_{L_2}(s_2) > 0$ is the optimal value that leads to the lowest expected costs. This is also the optimal threshold for reordering.

The hospital likes to be sure to have a small possibility of having a shortage. Therefore we add the option to specify a minimum for the value s_2. This minimum is the smallest value of s_2 for which

$$\mathbb{P}(D_{L_2} \leq s_2) \geq \alpha, \tag{86}$$

where α is the service level.

5.5.2 Strategy during the day

The model and strategy are mostly the same as when there is no lead time. The difference is that there is another cost function for the total expected costs.. The new costs for one cycle are

$$C_{cycle}(S_\tau) = C_{EO}^{day} + C_{L_2}(s_2) + \frac{1}{2}(S_\tau - D_{L_2} + s_{L_2})(\tilde{T}_{S_\tau} - L_2)C_h^{day}. \tag{87}$$

$\tilde{T}_{S_\tau}$ is the expected time until there has been a demand of $S_\tau - s_2$. S_τ is large enough such that the expected inventory level after restocking is nonnegative. The expected costs per product can be calculated by

$$C_{pp}(S) = \frac{C_{cycle}(S)}{S}. \tag{88}$$

The positive integer S for which the costs in 88 are the lowest is the optimal order-up-to value in an infinite horizon

The method we used for when there was no lead time can also be used to show that the same strategy applies when there is a lead time. Inequality (73) becomes

$$C_h > \frac{2}{X(X-1)\tilde{T}_1}(C_{L_2}(s_2) + C_{EO}), \tag{89}$$

where $X = (S - s_2)$. To prove that ordering more than X products we have to show that ordering $X+1$ when $\tau = \tilde{T}_{X+1}$ leads to higher expected costs. Assume that $L_2 < \tilde{T}_1$. Equation (75) changes a little. The probabilities stay the same. τ in the first term will become $\tilde{T}_{X+1} - L_2$. The second term after the probability changes into $\tilde{T}_1 C_h + (X-1)L_2 C_h$. The new equation for the extra costs are

$$\mathbb{P}(D_\tau \leq X)(\tilde{T}_{X+1} - L_2)C_h + \mathbb{P}(D_\tau = X+1)(\tilde{T}_1 + (X-1)L_2)C_h - \mathbb{P}(D_\tau = X+1)C_{EO}. \tag{90}$$

Inequality (89) can be used to substitute C_h and to get an inequality for the extra costs. Filling is this inequality for the case $X = 1$, $L_2 < \tilde{T}_1$ results in positive extra costs. If this specific case leads to higher expected costs, then this will be the case for all values of X and L_2. And ordering more than X products always lead to higher expected costs.

Ordering less than X products also turns out to lead to higher costs. The method in the previous section can again be followed to see this.

The time after which we will order for the night will have to be recalculated. At point τ we will either order-up-to S or order-up-to $S_n + k$ for some value k. Without lead time the extra holding costs for choosing $S_n + k$ equals $(S_n - S)\tau C_h$. With a lead time L_2 the extra holding costs are

$$(S_n - S)(\tau - L_2)C_h. \tag{91}$$

The same steps can be followed to find the moment after which you order extra for the night. But now it is also necessary to meet the service level during the morning lead time.

Step 1) $k = 0$
Step 2) $(S_n + k - S)(\tau - L_2)C_h - C_{EO} = 0$
Step 3) If $\mathbb{P}(D_{\tau+T_1} \leq S_n + k) \geq \alpha$ and $\mathbb{P}(D_{\tau+T_1+L_1} - B \leq S_n + k) \geq \alpha$, then $\hat{\tau}$.
Else $k = k + 1$ and return to **Step 2**.

5.6 The beginning of the workday

The lead time L_1 at the start of the day may be greater than L_2, since now all the products will be reviewed and ordered at once. The number of products that is left for during the lead time is determined by the inventory level before the start of the night, the demand during the night and the number of products that return. Since the unused products return right before reviewing, it is possible to use these for the demand during this lead time. If the order-up-to level for the night is S_n, then the probabilities for having a specific inventory level at the time of reviewing were previously shown in the model description. Combining these probabilities with the expected costs during the lead time for each possible inventory level leads to the expected costs during the lead time given S_n.

The costs during the lead time given a starting inventory level I_1 can be determined in the same way as during the day. The difference is that if the lead time is much larger, then only the demand won't be back ordered. In case of a real emergency that can't wait, there will be an emergency order where they only get the amount of the product that is needed for that surgery. Therefore, the expected inventory level after restocking when ordering-up-to S_1 becomes

$$S_1 - (1 - p_{em}) \times \mathbb{E}(D_{L_1}), \tag{92}$$

where D_{L_1} is the demand during lead time L_1.

This leaves the question how many products we should order. Let S denote the order-up-to value that minimizes the cost per product. We first need to check whether we should order for the night. If this isn't the case, then we need to check if ordering S or if ordering $S + D$ products lead to smaller costs. How to do this was shown in the previous section.

There will be another order when the inventory level becomes s and the night hasn't started yet. If the current time is after the arrival of the elective surgeries, then we need to check if $\tau \leq min(\tilde{T}_S, \tilde{\tau})$. As long as this isn't true, we will order-up-to S. If there is an order after this time, then we will also order the products for the night. This optimal value can be equal to S. Otherwise the optimal order-up-to value can be calculated in the same way as we calculated S_n, but with a larger time period.

6 Results

We have two different models and strategies for the same inventory problem. In section 4 the first model was described. In this model there was only one order in the morning. In section 5 another model was described. In this second model there can also be orders throughout the workday. In this section both models will be tested for two different example products. For this it is necessary to take a look at the given data and determining input.

6.1 Data

Some values for the input can be estimated by the data from the past. Part of this data was easily come by, but the data that is used in this paper is limited. Therefore, it is necessary to choose and estimate the other values. The situation will also need to be simplified. The data that is used are:
- An overview of the performed surgeries in 2019.
- The amount of ordered products per month in 2019.
- Which products are scan-buy products.
- The current minimum amount of products in stock.
- The supplier for each product and the lead-time.
- For each supplier the days when possible orders are placed.
- The prices for the products which won't be mentioned due to confidentiality.

The models that will be used are for stock products and not scan-buy products. The prices for the products are a small part of the holding costs. Furthermore, the first four points are used.

6.1.1 Estimated number of surgeries

An overview of the values mentioned in this subsection can be found in table 2.

# (semi-)elective surgeries	32
# surgeries on request	1.8
# (sub)acute surgeries	3.53
# acute surgeries	0.79
# acute per night	0.49
Arrival rate elective	33.8
Arrival rate emergency	3.53

Table 2: Estimated number of surgeries per day unless stated otherwise.

The average amount of surgeries are estimated by looking at the given amount of surgeries during the first part of 2019.
The average amount of (semi-)elective surgeries that are known in advance per workday equals 32. All the days that are used for this estimation are regular workdays outside of holidays.
The average amount of surgeries on request per workday equals 1.8.
The average amount of (sub)acute surgeries per day is 3.53. This number contains the acute surgeries that need to occur immediately and the sub-acute surgeries. In those surgeries there is a bit more time. Both type of surgeries are emergency surgeries.
The average amount of acute surgeries that need to be occur immediately per day is 0.79. It is assumed that the acute surgeries have the same arrival rate throughout the whole day. So the average amount of acute surgeries per night is 0.49, since the workday takes 9 hours.

For the calculations the arrival rate of elective surgeries are set to equal 33.8. This is under the assumption that all the surgeries on request for that day are known during the reviewing moment in the morning.

The daily arrival rate of emergency surgeries are set to 3.53.

6.2 Estimated costs

There isn't much known about the costs. The ordering costs for the first order in the morning won't be used and are therefore irrelevant at this point. The ordering costs throughout the day for model 2 consists of labour costs. The time that it takes to get new products are set to 10 minutes. The average monthly salary in the Netherlands is around 2150 euro per month. With four weeks holiday and 40 hours of work per week, this means an average salary of around 13.38 per hour. If the extra costs for the employer is set to 30%, this means an hourly cost of 17.40 euro. With a 10 minute lead time the ordering costs becomes 2.90 euro. The holding costs are difficult to estimate. It depends partly on unknown costs for power, maintenance and how much space the product takes up. Due to a low interest, these missed income is close to zero. Because of the estimation difficulty the models will be run with a few example holding costs C_h. The shortage costs are also difficult to estimate. The probability that an incoming emergency surgery is acute is $\frac{0.79}{3.53} = 0.2238$. The shortage costs per shortage are calculated by $C_{EO} + 0.2238 \times C_{penalty}$.

6.3 Simplification of the models

The first model is the model for when there is only one reviewing and ordering moment each workday. The second model is the model in which there is a reviewing and ordering moment in the morning with the addition of possible extra orders throughout the workday.

With the available data, the demand will need to be estimated from the amount of ordered products. This downside of this is that it isn't known for which these products were used. The products could have been used in any of the surgeries that might or will need it. They could also have been used for the flexible drawers. But it's also possible that some products were lost or out of date. In this section it is assumed that the amount of ordered products equals the total demand. The flexible drawers will also be neglected at this point. Any returning products arrive right before reviewing. The lead time in the morning are set to three hours. And the demand for the elective surgeries arrive immediately after the arrival of the order.

6.4 Product A

The first product that will be solved is the catheter females CH 12 -S-. This it a product that isn't used a lot, but still quite regularly. For this product it is assumed that the probability of needing one is the same for each type of surgery. The probability of needing more than one catheter of this type for one surgery equals zero. To solve the problem for this product the input is estimated for this situation.

6.4.1 Input

In 2019 there were 327 of this product ordered. Therefore, the yearly demand has been set to 327. In 2019 there were 52 weekends and 261 remaining days. Using the estimated rate of arrivals, the expected amount of surgeries for the whole year is 10110.25. The average usage per surgery is $\frac{327}{10110.25} = 0.0323434$. Since it's possible for the product to come back unused, the demand will be higher.

Let u be the probability of not using the product during a random surgery. Part of all the prepared products come back unused. The amount of necessary products depends on the type of surgery. For now this probability is assumed to be the same for all types of surgery. A regular random surgery seem to need around the 20 products and a couple or a handful of products return. Therefore, the probability of a product to return is set to either 0.2 or 0.1.

The arrival rate of the demand is $\frac{0.03234}{1-u}$. The elective surgeries are fixed so this demand is set to the closest integer of $\frac{0.03234}{1-u} \times 33.8$. The complete input can be found in table 3. The

Probability of demand 0 for random surgery	$1 - \frac{0.0323434}{1-u}$
Probability of demand 1 for random surgery	$\frac{0.0323434}{1-u}$
Leadtime morning	2 hours
Leadtime extra orders	10 minutes
Order costs for extra orders	2.9
Shortage costs per shortage	$2.9 + 0.2238 \times C_{penalty}$
Time between restocking and the arrival of elective demand	0

Table 3: Set input for product A

output is calculated for three different situations. These situations are displayed in table 4.

Situation	u	α	Daily holding costs C_h	$C_{penalty}$	# electives
1	0.2	0.95	5	1000	1
2	0.1	0.9	5	1000	1
3	0.1	0.9	20	100	0

Table 4: Input for three different situations.

6.4.2 Output

There are a few relevant values that we want to know. Firstly, the optimal order-up-to level will be calculated. For method 1, there is only one order-up-to value for in the morning. Other than that, we want to know the expected costs for this order-up-to value. Information about the probability of having shortages can also be relevant for making decisions. There are two different values for this. α_1 is the probability of having no shortages between restocking and reviewing the next day. α_2 is the probability of having no shortages between reviewing and restocking the next day.The outcome for all three different situations can be found in table 5. For method 2, the optimal

Situation	Order-up-to value S	Expected costs	α_1	α_2
1, Monday-Thursday	2	7.5291	0.9907	0.9991
1, Friday	3	29.8514	0.9905	0.9998
2, Monday-Thursday	2	7.2302	0.9926	0.9992
2, Friday	3	29.8822	0.9931	0.9998
3, Monday-Thursday	1	19.3897	0.9926	0.9991
3, Friday	1	50.0887	0.9436	0.9988

Table 5: Output for product A with method 1.

order-up-to value for the night S^n can be calculated. For this nightly situation the expected costs and the probabilities α_1 and α_2 will also be calculated. On Fridays the night starts on Friday at the end of the workday and it ends on the reviewing moment on Monday. If there is an extra order and the time until the end of the workday is less than τ hours, it is best to also order for the night.

Another optimal value that will be calculated is the optimal order-up-to value S and threshold s for ordering when the workday is endless. This optimal cycle also has an expected duration $T1$ and expected costs C_{cycle}. Every extra order throughout the day has a safety of α_{cycle} during the lead time. The outcome for all three different situations can be found in table 6. The calculated value τ is the time in hours without the lead time. So if $\tau \geq 7$, then it's better to only order once in the morning for the whole day. If $\tau < 7$, then the products for the night should be ordered when the remaining time is less or equal to $\tau + \frac{1}{6}$. If $\tau < 7$, then the optimal order-up-to level in

Situation	S^n	Nightly costs	τ	α_1	α_2	S	s	$T1$	C_{cycle}	α_{cycle}
1, M-Th	1	4.6605	0	0.9962	0.9994	1	0	168.167	423.1084	0.9990
1, Friday	2	26.7584	13.92	0.9933	0.9998					
2, M-Th	1	4.4382	0	0.9970	0.9995	1	0	189.1879	475.6357	0.9991
2, Friday	2	26.5279	13.92	0.9952	0.9998					
3, M-Th	0	2.8005	-	0.923	0.9926	1	0	189.1879	1893.1	0.9991
3, Friday	1	48.2318	0	0.923	0.9926					

Table 6: Output for product A with method 2.

the morning is

$$S + \text{elective demand} \tag{93}$$

If $\tau \geq 7$, then the optimal order-up-to level in the morning equals the optimal order-up-to level from method 1.

6.5 Product B

The second product that we're testing is product B. This product is known as called set universal. This product is used quite a lot. The yearly usage in 2019 was 2096. There were also two extra orders, but it isn't known how many were taken in those orders. So these possible extra demand is neglected. With this yearly usage it is one of the most used products. There are a few outliers, but otherwise this product is pretty representative for a highly used product.

6.5.1 Input

The input and situations are mostly the same as for product A. The only difference is the usage. The average usage per surgery is $\frac{2096}{10110.25} = 0.2073144$. The probability of demand 1 for a random surgery is $\frac{0.2073144}{1-u}$. Needing more than one isn't possible. The demand of the elective surgeries is 9 in situation 1, 8 in situation 2 and 7 in situation 3.

6.5.2 Output

The same values are calculated for this product as was done for product A. The output for method 1 can be found in table 7 The output for method 2 can be found in table 8. In the discussion

Situation	Order-up-to value S	Expected costs	α_1	α_2
1, Monday-Thursday	12	31.1718	0.9858	0.9995
1, Friday	16	102.4091	0.9927	0.9999
2, Monday-Thursday	11	28.9869	0.9904	0.9989
2, Friday	14	87.3487	0.9874	0.9996
3, Monday-Thursday	9	34.1365	0.9507	0.9952
3, Friday	12	227.5467	0.9619	0.9988

Table 7: Output for product B with method 1.

section we will take a further look at how the results can be improved.

Situation	S^n	Nightly costs	τ	α_1	α_2	S	s	$T1$	C_{cycle}	α_{cycle}
1, M-Th	3	11.2526	6.96	0.9971	0.9998	1	0	26.236	69.4975	0.9937
1, Friday	6	69.5309	2.78	0.9927	0.9999					
2, M-Th	3	10.7454	6.96	0.9981	0.9996	1	0	29.5155	77.536	0.9944
2, Friday	6	69.2622	2.78	0.9935	0.9998					
3, M-Th	1	15.2902	0	0.9056	0.9904	1	0	29.5155	296.4663	0.9944
3, Friday	4	162.8841	1.16	0.9338	0.9980					

Table 8: Output for product B with method 2.

6.6 Differences between the two models

We have given some results about our two models for two different products. For both models we used the same input. In theory the second method has the possibility to lower the average inventory level and the expected total costs. In the previous section both methods were used for two example products in three different situations. In these situation the outcome for product A gives an indication for a more average and regularly used product. From the used dataset it follows that the arrival rate of the demand is small.

In table 6 we see that some optimal S are the same value as the S^n. This means that if there is only one extra product ordered in the morning for emergency surgeries, then there will only be an extra order when this product is used. An order for the night won't be needed as long as the product is still available. The only difference left between model 1 and model 2 is that there will be an extra order whenever this product is used. In model 1, there remains zero products. And there will always be one product available for the night. These differences from method 1 gives extra safety in the case that the product will be used throughout the workday. However, the extra ordered product after that has a pretty big possibility of not being needed.

The expected time between two emergency arrivals $T1$ is around 7 or 8 days. This means that besides the elective surgeries we expect one demand every 7 to 8 days. This exceeds the time between two regular workday mornings. Therefore, it might not be optimal to order only 1 at first. The probability of not having a demand during the remaining workday is very high. But since there is a preference for having at least 1 in stock, there isn't the option to order nothing when the current inventory level is 0. This means that having no demand during the workday and one demand during the night results in lower expected holding costs for the first method. This makes comparing the expected costs between the two models hard.

Besides an extra safety, the second model isn't much better in these situations and will return the same strategy in the morning as in the first model. This seems to be mostly the case because of the very small arrival rate. The expected demand from emergency surgeries throughout the workday is close to zero and the arrival rate between two demands is very high. This makes method 2 less relevant since this method adds the possibility of having extra orders throughout the day which most likely won't happen. However, product A does seem to be an average product. There are many products with the same or even much smaller expected demand. . A smaller expected demand will result in a higher probability of not having any demand from emergency surgeries. If the holding and shortage costs stay the same, then the needed stock for the night will either stay the same or it will decrease. Therefore, the time at which it is better to order for the night will be earlier when the expected demand decreases. We can conclude that model 2 also wouldn't add much for any of the products that has less expected demand than product A but the same costs. In these cases it should be decided if you want to keep track of the inventory level throughout the day and reorder if necessary or not.

Product B is one of the more highly used products, but the expected demand during the workday is still small with zero still being the closest integer. This also show in high values for $T1$ in table 8. The τ are a lot smaller here. This means that a later order for the night gives less costs. The holding costs for having the extra night products during the day are higher. Using model 2 results in order costs of at least 2.9 for the products for the night. Looking at situation 1 on a Friday we can see that method 1 gives extra costs of $102.4091 - 69.5309 - 2.9 = 29.9782$. If the demand during the workday is 0, then the extra holding costs during the workday in method 2 becomes $\frac{7}{24}C_h = 1.4583$. If there is one demand in the middle of the day, then the extra costs become $1.5 \times \frac{3.5}{24}C_h + 2.9 = 3.9937$. So the expected costs in method 1 can only be reached with method 2 when there is high demand during the workday. However, if that happens then there will be shortages with method 1. Therefore, method 2 will give lower costs. This shows that a product with higher demand like product B can profit from using method 2 depending on the input. This profit may be worth the extra effort.

7 Scan-buy products

Scan-buy products aren't stored in the central warehouse, but will be ordered directly at the supplier. For these products there will be a lead time of one or more days. There is an option for emergency orders, but they still have a lead time and results in extra costs. The question is when to order how many products/boxes.

Ordering multiple types of products at the same time from the same supplier results in shared order costs. For this reason we might want consider joint ordering. However, this won't be done in this paper. All the notation that we are going to use in this section can be found in the overview below.

Notation

s	= ordering threshold.
$Q = nq$	= ordering size.
C_{order}	= ordering costs per order.
$Price_A$	= price of product A.
UB	= the amount of days before the product expires.
C_h	= holding costs per product per day.
λ	= average daily demand.
λ_{week}	= average daily demand on weekdays.
$\lambda_{weekend}$	= average daily demand in the weekend.
$p_{week}(k)$	= probability of having a demand k on a weekday.
$p_{weekend}(k)$	= probability of having a demand k on a Saturday or Sunday.
L	= lead time for a regular order.
L_{EM}	= lead time for an emergency order.
α	= regular service level.
C_{short}	= expected shortage costs per shortage.
p_{EM}	= probability of needing an emergency order to fulfill.
C_{EO}	= the costs for an emergency order.
C_{pen}^{wait}	= penalty for delaying/waiting for an (emergency)order.
C_{pen}	= penalty when waiting isn't possible.
p_{wait}	= probability of needing to delay the surgery.
p_{late}	= probability of the order coming too late.
$p^i(k)$	= probability of having a total demand of k in the next i days.
D_L	= demand during the lead time.
D_1	= demand during two reviewing moments.
$D_{\frac{1}{2}}$	= demand during half of the time between reviewing moments.
$I(s)$	= the expected inventory level during the order moment.
TC_L	= total costs during lead time L.
I_{stock}	= the inventory level after restocking.

7.1 Basic model for one scan-buy product with constant lead times.

In this basic model we are going to show the model that makes it possible to find the optimal strategy for disposable A. Assume that this product is the only product from that supplier, so shared costs can be neglected at this point. There will be one reviewing moment every day from Monday to Friday. Only at these moments it is possible to place a regular order. Emergency orders can take place at any time. We have a constant lead time. Scan-buy products aren't ordered every day. This method is used by the hospital for the products that aren't used a lot. In this section we make use of an average daily arrival rate for every day. So we use

$$\lambda = \frac{5\lambda_{week} + 2\lambda_{weekend}}{7}. \tag{94}$$

Since we want to determine the optimal moment to place a new order and to determine how many products to order, the (R, s, Q) policy is used here with fixed R. This policy was described in subsection 3.7.

7.2 The (R, s, Q) policy

Some scan-buy products can only be ordered in specific amounts. Instead of finding the optimal order-up-to level it is necessary to find the optimal number of boxes n that need to be ordered. Let q denote the number of products per box. Then for each disposable this value for q is fixed and known. $Q = nq$ is the total number of products that will be ordered. R is known and fixed. s is the reorder point. It is already known how to solve this policy for a basic problem. The difference with our problem is that we will calculate a minimum for the reorder point s with the service level.

The basic method of finding the optimal value for Q can also be used for our model. However, the cost function for the total expected costs is different. Therefore, we will need to find this cost function before we can calculate Q.

7.2.1 Determine s

Since continuous reviewing and ordering isn't possible, the value of s has to be determined a little bit different. In the previous models, the threshold s should be large enough to meet the demand during the lead time. In this problem everything larger than s should be enough to meet the demand until the next reviewing moment D_1 plus the demand during the lead time D_L. Therefore it is necessary that

$$\mathbb{P}(D_L + D_1 \leq s + 1) \geq \alpha. \tag{95}$$

The minimum s that is necessary to meet the service level is the smallest value of s for which inequality (95) is true. Increasing s results in more holding costs and less shortage costs during the lead time. It also results in possibly ordering earlier. therefore if increasing s above the minimum results in lower costs during lead time, the costs for the whole cycle should be looked at.

7.2.1.1 Costs during lead time The main costs during the lead time consists of the holding costs and the shortage costs.
Then it is expected to reach the threshold s in the middle of two reviewing moments. Let $D_{\frac{1}{2}}$ be the demand during half of the time between reviewing moments. Than the expected inventory level at the ordering moment I equals

$$I(s) = s - \mathbb{E}(D_{\frac{1}{2}}). \tag{96}$$

Let C_h be the holding costs per product per day and let C_{short} be the shortage costs per shortage. The expected total costs $\mathbb{E}(TC)$ during the lead time L when the threshold is s can be calculated by

$$\begin{aligned}\mathbb{E}(TC_L(s)) = &\sum_{j=0}^{x} \mathbb{P}(D_L = I(s) - j) \left(\frac{I(s) + j}{2} LC_h \right) \\ &+ \sum_{j=1}^{\infty} \mathbb{P}(D_L = I(s) + j) \left(\frac{I(s)}{2} \left(\frac{I(s)}{I(s) + j} \right) LC_h + jC_{short} \right).\end{aligned} \tag{97}$$

The expected extra costs for increasing s by one are

$$\mathbb{E}(TC_L(s+1)) - \mathbb{E}(TC_L(s)) = \mathbb{P}(D_L \leq I(s))LC_h + \mathbb{P}(D_L > I(s))(\frac{1}{2}LC_h - C_{short}). \tag{98}$$

The smallest value of s for which this is positive is the value that minimizes the expected total costs during the lead time.

7.2.1.2 Expected shortage costs per shortage. The shortage costs depends on the type of surgery for which the product is needed. We will look at the different costs and the probability of getting such a cost. This allows us to calculate the expected shortage costs.

We take a look at two different surgery categories and at the possibility of an emergency order. The first category consist of the surgeries for which there are no large consequences. There is no life threatening situation and it's possible to wait for the regular ordered products.This is independent of the time left until restocking. The probability of a demand belonging in this category is p_{wait}. Since every shortage is unwanted we still let this category have costs C_{pen}^{wait}.

The second category are the acute surgeries. These are of the highest emergency level. If there is no stock at the moment that the demand arrives, then a shortage in this category always lead to a lost sale. The probability of a surgery being acute is p_{late}. For every lost sale there is a penalty cost C_{pen}.

There is also the probability of placing an emergency order. Let p_{EM} denote the average part of the shortages that leads to an emergency order with costs C_{EO}. The expected value for the shortage cost per shortage can be approximated by

$$\mathbb{E}(C_{short}) = p_{EM}C_{EO} + p_{wait}C_{pen}^{wait} + p_{late}C_{pen}. \tag{99}$$

This approximated value is independent of the time and the surgery type.

7.2.2 Determine Q

To get an optimal strategy it is necessary to calculate the amount to order Q that results in the lowest expected costs per time period or per product. If the inventory level at the time of reviewing is less or equal to s than an order has to be placed. It then takes time L before these products arrive and become available for usage. It is assumed that with an emergency order only the necessary products are ordered. The expected inventory level after restocking I_{stock} is

$$\mathbb{E}(I_{stock}(s)) = Q + (\mathbb{E}(I(s)) - \mathbb{E}(D_L))^+. \tag{100}$$

Because of the high service level and because the expected values are used this is the same as

$$\mathbb{E}(I_{stock}(s)) = Q + \mathbb{E}(I(s)) - \mathbb{E}(D_L). \tag{101}$$

These are the only products left before the next restocking moment.

7.2.2.1 Expected total costs per product It is assumed that the price per product is the same for every ordering amount. Therefore these costs can be neglected in the ordering costs and only the fixed order costs per order C_{order} are left.

The expected average inventory level is

$$\frac{\mathbb{E}(I_{stock}(s)) + \mathbb{E}(I(s) - D_L)}{2}. \tag{102}$$

Therefore, the expected holding costs per cycle $\mathbb{E}(C^{cycle}_{hold})$ is

$$\mathbb{E}(C^{cycle}_{hold}) = \frac{\mathbb{E}(IL_{stock}(s)) + \mathbb{E}(I(s) - D_L)}{2}\mathbb{E}(T(Q))C_h. \tag{103}$$

In the previous formula $\mathbb{E}(T(Q))$ is the expected time per cycle for Q. This is equal to the expected time for the demand to equal Q

$$\mathbb{E}(T(Q)) = Q \times \frac{1}{\tilde{\lambda}}. \tag{104}$$

$\tilde{\lambda}$ is the expected demand for a random day so

$$\tilde{\lambda} = \frac{5\lambda_{week} + 2\lambda_{weekend}}{7}. \tag{105}$$

The expected shortage costs $\mathbb{E}(C^{cycle}_{short})$ are calculated with formula

$$\mathbb{E}(C^{cycle}_{short}) = \sum_{i=1}^{\infty} \mathbb{P}(D_L = I(s) + i)iC_{short}. \tag{106}$$

The remaining costs are made due to the products being perishable. If a product need to be thrown away than the costs equal the purchasing costs $Price_A$. So the expected extra costs for one specific product equals the probability of the product not being used within UB days times the price. The expected total perishable costs per regular order can be calculated by

$$\mathbb{E}(C^{order}_{perish}) = \mathbb{P}(D_{UB} \leq I(s))Q \times Price_A + \sum_{i=1}^{Q-1} \mathbb{P}(D_{UB} = I(s) + i)(Q - i) \times Price_A. \tag{107}$$

The expected total costs per product can now be calculated using the next formula.

$$\mathbb{E}(TC_{pp}(Q, s)) = \frac{\mathbb{E}(C^{cycle}_{hold})}{Q} + \frac{\mathbb{E}(C^{cycle}_{short})}{Q} + \frac{C_{order}}{Q} + \frac{\mathbb{E}(C^{order}_{perish})}{Q}. \tag{108}$$

7.2.2.2 Minimizing the expected total costs Now that the expected total costs per product is calculated it is possible to minimize these costs. If s was already determined by the service level than marginal analysis can be used to find the optimal value for Q.

Let Q increase by one box of size q. Than Q becomes $Q + q$ and the expected extra costs are

$$\begin{aligned} \mathbb{E}(TC_{pp}(Q + q)) - \mathbb{E}(TC_{pp}(Q)) = \frac{q}{2\tilde{\lambda}}C_h - \frac{q\mathbb{E}(C^{cycle}_{short})}{Q^2 + qQ} - \frac{q\mathbb{E}(C_{order})}{Q^2 + qQ} + \\ \mathbb{P}(D_{UB} \leq Q)q \times Price_A + \sum_{i=1}^{q-1} \mathbb{P}(D_{UB} = Q + i)(q - i) \times Price_A. \end{aligned} \tag{109}$$

The smallest value of Q for which this equation is positive is the optimal order quantity Q.

It is also possible that the optimal value for s wasn't determined yet. In that case the costs also depends s. The alternative method is to determine the optimal values by using the partial derivatives which are zero at the minimum.

$$\frac{\partial TC_{pp}(Q, s)}{\partial Q} = \frac{\partial TC_{pp}(Q, s)}{\partial s} = 0. \tag{110}$$

The downside of this method is that the optimal value of Q isn't necessary of the form nq and s doesn't have to be an integer. Since the costs function is convex it is known that the best ordering quantity and threshold is one of the most nearby options. The costs for these options can be calculated separately to find the best strategy.

7.3 Random lead time

In the basic model we assumed that the lead time is constant. However, in practice the lead time might vary. Assume that the probability distribution of the lead time stays the same throughout the time. Then these probabilities can be approximated by looking at the past with $\mathbb{E}(L)$ as expected value. Now $\mathbb{E}(D_L)$ becomes equal to $\mathbb{E}(L)\lambda$. Let m be the maximum lead time possible in days.

The main difference with the basic model is that the threshold s might need to be higher to still meet the service level. The probability $p(s)$ is the probability of being able to meet the demand if s is chosen as threshold.

$$p(s) = \sum_{i=1}^{m} \mathbb{P}(D_L \leq I(s)|L = i)\mathbb{P}(L = i). \tag{111}$$

The lowest value of s for which $p(s) \geq \alpha$ is the minimum for the threshold s that is needed to meet the service level α. The method of calculating the costs and finding the optimal values of s and Q are done in the same way as with constant lead time.

8 Conclusion

In this thesis we developed two different models for the non-scan buy products and one model for the scan-buy products. The first model for the non scan-buy products is a model where there is one planned order per workday. This model can be used to determine the optimal order-up-to level in the morning. This order-up-to level results in the lowest expected costs, but still meet the service level.

We also made an alternative model. In this second model it is possible to have more orders during the regular working hours. If the reorder point s is reached then we order-up-to S as long as the remaining time until the end of the workday is large enough. The optimal strategy for this model is found and proven for when the demand has a Poisson distribution. This strategy will also be optimal for a compound Poisson process under certain conditions. For products with a small arrival rate this model will most likely give the same strategy as model 1. Products with a higher arrival rate can result in a different strategy. In that case the average inventory level and the costs will be smaller than in model 1. So if the demand has a Poisson distribution, than model 2 will be the best option for the non scan-buy products.

We created a separate model for the scan-buy products. For these products we use the (R, s, Q) inventory policy, but with different cost functions.

9 Discussion

In this paper we showed three basic models for the inventory problem in the hospital. These models can be used to find good strategies. For the models we made some assumptions and simplifications. There are still more possibilities for these models than we showed. There are also some disposables within the surgery department that behaves differently from the average surgery disposable.

In this section we mention a few of these possibilities and exceptions. We answer some remaining questions from the hospital and give some advise about how the remaining problems can be approached with the use of the models in this paper. We also give some advise about how to get better results.

9.1 Further research for model 2

In this paper we proved that the strategy we found in model 2 is optimal when the demand has a Poisson distribution. It will also be optimal for a compound Poisson process under certain conditions.

Further research is necessary to find out if this strategy is optimal for every compound Poisson process. Or when it won't be optimal.

9.2 Products return as a Poisson process

In this paper we let unused products return to the local warehouse right before reviewing in the morning. The optimal strategy might change by letting the returning products be a Poisson process during the workday. No products return during the nighttime and the nightly situation stays the same as before. The number of returning products right before reviewing in the morning will now have a Poisson distribution. Assume that the demand also has a Poisson distribution.

Let λ_D denote the rate parameter for the demand during the day. And let λ_B denote the rate parameter for the returning products. This means that the inter-arrival times between two demands and between two returns have an exponential distribution. Due to the memoryless property this also applies for the time until the next demand and until the next return.

If $\lambda_D > \lambda_B$ it is expected that there will be more demand than returns over time. The probability of having a demand before the next return is

$$\mathbb{P}(T_D < T_B) = \frac{\lambda_D}{\lambda_D + \lambda_B}. \tag{112}$$

T_D is the time until the next demand and T_B is the time until the next return. With probability $\frac{\lambda_B}{\lambda_D + \lambda_B}$ there will be a returned item before a demand. With this probability the next demand can make use of a product that will return before the demand arrives. Therefore, with this change it might be possible that less stock is needed.

9.3 Flexible drawers

There is also the possibility to add flexible drawers to the models. The resulting changes are given in this section.

The idea of the mathematical model with flexible drawers is mostly the same as without these drawers. Every flexible drawer is used for a known set of elective surgeries and contains a known set of products. The products in the flexible drawer have a smaller possibility of being used during the allocated surgeries. There is always one of each necessary product in the drawer. As soon as a product is used, it will be replaced by a new one unless the remaining surgeries don't require that specific product.

Let A^{flex} be a product in drawer i and let W^i be the list of all the surgeries that uses drawer i and might need product A^{flex}. Let w^i_j denote the j-th element of vector W^i. Probability $\mathbb{P}(w_j)$ is the probability of using product A^{flex} during surgery $w_j \in W_i$. Let X_i be the number of products that will be used. The probability of using x products can be calculated as follows.

$$\mathbb{P}(X_i = 0) = \prod_{j=1}^{|W^i|} (1 - \mathbb{P}(w^i_j)), \tag{113}$$

$$\mathbb{P}(X_i = 1) = \sum_{j=1}^{|W^i|} \left(\mathbb{P}(w^i_j) \prod_{z \in W^i, z \neq w^i_j} (1 - \mathbb{P}(z)) \right), \tag{114}$$

$$\mathbb{P}(X_i = x) = \sum_{j_1=1}^{|W^i|-x} \cdots \sum_{j_x=j_{x-1}+1}^{|W^i|} \left(\prod_{n=1}^{x} \mathbb{P}(w^i_{j_n}) \prod_{z \in W^i, z \neq w^i_{j_1}, \cdots, w^i_{j_x}} (1 - \mathbb{P}(z)) \right). \tag{115}$$

The product won't be restocked after the last surgery that needs product A^{flex} to be available. Therefore, the number of products that are needed on the drawer during the day depends on whether the product is used during the last surgery or not. If not, then there was one extra demand without that last product being used.

Let f_i be the last surgery that will use the flexible drawer i and possibly need product A^{flex}. $f_i = false$ if the last product isn't used during the last surgery and $f_i = true$ otherwise. Let random variable Y_i be the total demand of product A^{flex} for drawer i. The probabilities for the demand are calculated by

$$\begin{aligned} \mathbb{P}(Y_i = x) &= \mathbb{P}(X_i = x - 1; f_i = false) + \mathbb{P}(X_i = x; f_i = true) && (116)\\ &= \mathbb{P}(X_i = x - 1 | f_i = false)\mathbb{P}(f_i = false) + \mathbb{P}(X_i = x | f_i = true)\mathbb{P}(f_i = true). && (117) \end{aligned}$$

When $f_i = false$, there is one product left that will be returned before the next reviewing moment with probability 1. Therefore, the probability of getting one returned equals $p^{false}_i = \mathbb{P}(f_i = false)$.

All the flexible drawers with their demand are independent. Let D^{flex}_t be the total demand of product A^{flex} for all the flexible drawers, then

$$D^{flex}_t = \sum_{i=1}^{m} Y_i, \tag{118}$$

$$\mathbb{P}(D^{flex}_t = z) = \sum_{x_1=0}^{z} \sum_{x_2=0}^{z-x_1} \cdots \mathbb{P}(Y_1 = x_1)\mathbb{P}(Y_2 = x_2) \cdots \mathbb{P}(Y_m = z - x_1 - x_2 - \cdots). \tag{119}$$

9.3.1 Returning products

The method of calculating the returning products is unchanged for the remaining part of the elective surgeries and the emergency surgeries. The only addition are the unused products from the flexible drawers that return. Let F be the number of products that return from all the flexible drawers together and let N denote the number of flexible drawers. Following the previous calculations gives

$$\mathbb{P}(F=0)=\prod_{i=1}^{N}(1-p_i^{false}), \tag{120}$$

$$\mathbb{P}(F=1)=\sum_{i=1}^{N}p_i^{false}\prod_{z\neq i}(1-p_i^{false}), \tag{121}$$

$$\mathbb{P}(F=k)=\sum_{i_1=1}^{N-k}\cdots\sum_{i_k=i_{k-1}}^{N}\prod_{m=i_1}^{i_k}p_m^{false}\prod_{z\in\{k+1,\cdots,N}(1-p_{i_z}^{false}). \tag{122}$$

9.3.2 Changes for model 1

The inventory level I^{ELM} after all the normal elective surgeries and emergency surgeries can be calculated by using the first basic model. Let I^{ELMF} be the inventory level after all the demand including for the flexible drawers.

$$I_t^{ELMF}=(I_t^{ELM}-D_t^{flex})^+. \tag{123}$$

The total amount of product A that will return can be calculated by using formula (40) and

$$B_{t+1}^{flex}=B_t+F. \tag{124}$$

The costs and optimal value for S_t when there are flexible drawers can be calculated in the same way as for the basic model, but with using $D_t^{ELMF}=D_t^{ELM}+D_t^{flex}$ as total demand until reviewing. And the number of returning products is replaced by B_{t+1}^{flex}.

9.3.3 Changes for model 2

The possibility of flexible drawers results in an extra form of demand with a certain probability distribution. Since these drawers are only used for elective surgeries it has no influence on the arrival of the remaining demand.

It is assumed that this demand is uniformly spread throughout the workday. The demand during the day will now be the convolution of the demand for the emergency orders and the demand for the flexible drawers. This might result in a smaller $\tilde{T}_{S_\tau^*}$ which leads to a smaller value of $\mathbb{E}(\tilde{T}_{S_\tau^*})$. The optimal strategy can be calculated in the way as before only with a different probability distribution for the demand and a different expected time per cycle.

9.4 Improvement suggestions for better results

We showed a few results in this paper for two different products. To imitate reality it is important to use the right input. Most improvements can be made with more data about the real situation. Some costs has been guessed. With better approximated costs, there can be made a better trade-off between the real holding costs and the shortage costs.

The approximation that was made in this section for the demand was from a small data set that didn't include all information. Using a bigger set gives a better approximation for the average amount of surgeries. For an even better result, it would also be possible to take a look at the types of surgeries. With some surgeries the probability of needing the product will be zero. When it is known which surgeries these are, this can be used to determine the probability of having a demand of zero for a random emergency surgery. The probabilities of needing one or more can also be approximated in the same way.

The approximation for the amount of returning products can also be improved by keeping track of how many items return. Here it is also possible to differ in types of surgeries. There might be a set of surgeries for which the product won't ever return. There might also be a set of surgeries for which the product returns with a certain probability. This makes it possible to get a more accurate probability distribution for the amount of returning products given the elective surgeries of the current day.

Besides improving the data it is also possible to improve the models. Some of these ways were already mentioned in the corresponding section. So would it be possible to let the returning products arrive throughout the day with a Poisson distribution. However, this would add an extra uncertainty which might be unwanted. Adding flexible drawers is also an option. For this it would be necessary to collect data about this first.

9.5 Joint ordering for scan-buy products

When there are multiple different products coming from the same supplier, you might want to consider ordering these products at the same time. This results in shared order costs and could possibly lead to lower costs per product.

We consider two different products A and B from the same supplier. Let the strategy for A be equal to the strategy without joint ordering. Let $IL_B > s_B$ denote the inventory level of B whenever an order for A is placed. If we also order B, then the order cost per product type goes from C_{order} to $\frac{C_{order}}{2}$. If we decide to order B, then the current cycle will be cut short. This leads to a higher cost per demand in the current cycle. It is also possible to get a higher stock after restocking. That depends on IL_B and the possible order sizes nq. This would lead to higher expected perishable cost and holding cost.

Boucherie [1] shows that strategies for joint ordering don't always lead to less costs. More research is necessary to find out what the best strategy is for products from the same supplier.

9.6 Anesthesia disposables

The anesthesia disposables are stored separately from the surgical disposables. From there the carts will be stocked with products. There should always be one cart present during a surgery and all the surgery rooms could be occupied at the same time. Therefore, there should be at least one cart for each surgery room. This would mean that the new situation with the carts won't change much from the old situation where there were still closets.

The amount of products that are needed on each cart depends on the elective surgeries and the probability distribution of the demand. The main difference with the surgery disposables is that the necessary anesthesia disposables aren't necessarily known beforehand. It's possible that some of these demand occurs during the surgery itself. Because walking in and out of a surgery room during the surgery is difficult and highly unwanted, it is extra important to have everything in the cart when the demand arrives.

The previous models and strategies can also be used for this storage room and the carts. These carts are currently stocked in the morning with enough products to last the whole day. Therefore, there is no need to have more orders throughout the day for the storage room which corresponds with using method 1. All the demand now arrives around the same time. The holding costs can best be calculated in the same way as for the elective surgeries in the previous sections. The necessary products in the carts can also be calculated with the previous methods. However, it might be recommended to add an extra safety. There are products that aren't used a lot, but when it is used it is needed to have at least x of these. For these cases it might be best to add an extra minimum for the order-up-to value.

9.7 Rarely used disposables

There is a set of disposables that are rarely used. Here rarely means less than once a year on average. The probability of these products going out of date is very high. Which means that most of these products end up being thrown away. It is however necessary to have one of these products in stock. The current method that the hospital uses for these products is to make them scan-buy and only buy a new one whenever the product is used or out of date. Having no stock during the lead time is acceptable.

Because of the strict minimum of one product it isn't possible to find a strategy that gives less stock and lower holding costs. Ordering a second product will have an even higher probability of getting out of date. This extra product will also give extra holding costs. It won't lower the average shortage cost per year. And there will still be the same amount of orders over time when these products will be out of date.

For these reasons it is clear that for these products the current strategy is the optimal one. It is possible to use the method in the previous section. With one as the minimum ordering amount and needing a order when there is no stock, this method will result in the same strategy. Therefore, using the method in this paper is unnecessary for these products. It would only take time to find the input and run the program.

9.8 When to make a product a scan-buy product

The products that are used less often are made scan-buy by the hospital. However, it is possible to make all the disposables scan-buy. But when would making a product scan-buy result in lower expected costs? What is the advise for the hospital?

It's possible for products to also be used in other departments within the hospital. For these products there will already be stock in the central warehouse. Therefore, the central warehouse can supply these products and making them scan-buy isn't necessary. Making one of the other products scan-buy would result in not needing any stock in the central warehouse.

Using the scan-buy model results in expected costs per product $TC_{pp}(Q^*, s^*)$ where (Q^*, s^*) is the optimal strategy. Multiplying these costs with the expected yearly demand D_{year} results in the expected costs per year TC_{year}.

$$TC_{year} = TC_{pp}(Q^*, s^*) \times D_{year}. \tag{125}$$

These costs consists of all the expected holding costs, order costs, shortage costs and costs due to the items being perishable.

In the models for the non scan-buy products there are also costs calculated. Model 1 calculates the expected daily costs. These costs consists of the holding costs and the shortage costs. The expected daily costs for an average workday and the expected costs for a weekend can be used to calculate the expected yearly costs.

The non scan-buy products costs the hospital already money before they arrive at the surgery department. These costs are the order costs at the supplier including the prizes, the handling fee within the central warehouse and the holding costs. The central warehouse has their own (R, s, Q) policy. The expected yearly holding costs and order costs corresponding this policy can be calculated.

Making a product scan-buy leads to lower costs whenever

$$\text{yearly costs made in the central warehouse} + \text{yearly costs from model 1} > TC_{year}. \tag{126}$$

Looking at the results it can be expected that model 1 is used for the less often used products like the current set of scan-buy products. The expected costs for model 2 are too hard to calculate. The expected costs for when the demand equals the average can be calculated and used as indication.

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Appendix A: overview of the used notation.

S_t	= order-up-to level for day t.
L	= lead time.
$q_t(j)$	= probability of an emergency surgery being of type j during time t.
$\nu_t(j)$	= the necessary amount for emergency surgery type j.
t	= time in days (excluding weekends)
λ_t	= the mean of the number of emergency surgeries between two reviewing moments.
D_t^L	= demand during the lead time of time t due to emergency surgeries.
D_t^{EL}	= demand for the elective surgeries during time t.
D_t^{EM}	= demand after restocking for the emergency surgeries on time t.
D_t^{ELM}	= demand of the elective surgeries and emergency surgeries combined outside the lead time.
IL_t	= inventory level during reviewing after the unused products returned.
I_t^L	= inventory level after the order arrived.
I_t^{EL}	= remaining inventory level after the carts for the elective surgeries are prepared.
I_t^{EM}	= remaining inventory level after all the elective and emergency surgeries and before reviewing.
$u_{t-2}(i)$	= probability of not using the product during an elective surgery of type i on day $t-1$.
$a_{t-2}(i)$	= amount of surgeries of type i that were prepared on day $t-2$ and performed on day $t-1$.
$v_{t-1}(i)$	= probability of getting a product back for an emergency surgery of type i.
B_t	= the amount of unused products that come back before the reviewing moment of day t.
$EO_{S_t,t}$	= the amount of shortages during restocking and reviewing if S_t is chosen.
$EO_{S_t,t}^L$	= the amount of shortages during lead time L of day t.
C_{short}^L	= the costs per shortage during the lead time.
C_{EO}	= costs for an emergency order.
$C_{penalty}$	= extra penalty costs for having a shortage when there is an incoming acute surgery.
$p_{a,t}$	= probability of an emergency surgery is acute.
C_h	= holding costs per product per day.

Table 9: Used notation in model 1.

D_n	= the demand during the night.
S_n	= the order-up-to level at the beginning of the night.
T_1	= the duration of the night interval.
$C^n(x)$	= the expected nightly costs when the night is started with x products.
C_h	= daily holding costs per product.
C^n_{EO}	= costs per shortage during the night.
C_{EO}	= costs for extra orders during the workday.
s_n	= threshold for ordering at the end of the workday.
s^*_n	= point from where it is more cost efficient to not order extra products for the night.
s^{min}_n	= the minimum stock that is needed for the night to meet the service level.
α	= the needed service level.
η	= the inventory level at the end of the workday before the optional extra order for the night.
s	= the order point or threshold. It equals 0 when there is instant delivery.
τ	= the remaining time until the beginning of the night.
S_τ	= the order-up-to level when there is τ time left until the start of the night.
T_2	= the time of this whole period.
η	= the inventory level at the end of this period
D_τ	= a stochastic variable for the demand during given τ.
t_{es}	= remaining time until the arrival of the demand for the elective surgeries.
T_{es}	= the time when the demand of the elective surgeries arrive.
S^*	= the optimal order-up-to level for the first order of the day.
S	= the optimal order-up-to level in an infinite horizon.
$\tilde{T}_x$	= the expected time until a demand of x is reached.
C_{short}	= expected shortage costs per shortage during the workday.
p_{em}	= the probability that an incoming emergency surgery is acute (surgery takes place right away).
$C_{penalty}$	= extra penalty for not having any stock when there is an acute surgery.
L_1	= lead time of the first order at the start of the day.
L_2	= lead time for any extra order.
D_{L_1}	= demand during lead time L_1.
D_{L_2}	= demand during lead time L_2.

Table 10: Used notation in model 2.

s	= ordering threshold.
$Q = nq$	= ordering size.
C_{order}	= ordering costs per order.
$Price_A$	= price of product A.
UB	= the amount of days before the product expires.
C_h	= holding costs per product per day.
λ	= average daily demand.
λ_{week}	= average daily demand on weekdays.
$\lambda_{weekend}$	= average daily demand in the weekend.
$p_{week}(k)$	= probability of having a demand k on a weekday.
$p_{weekend}(k)$	= probability of having a demand k on a Saturday or Sunday.
L	= lead time for a regular order.
L_{EM}	= lead time for an emergency order.
α	= regular service level.
C_{short}	= expected shortage costs per shortage.
p_{EM}	= probability of needing an emergency order to fulfill.
C_{EO}	= the costs for an emergency order.
C_{pen}^{wait}	= penalty for delaying/waiting for an (emergency)order.
C_{pen}	= penalty when waiting isn't possible.
p_{wait}	= probability of needing to delay the surgery.
p_{late}	= probability of the order coming too late.
$p^i(k)$	= probability of having a total demand of k in the next i days.
D_L	= demand during the lead time.
D_1	= demand during two reviewing moments.
$D_{\frac{1}{2}}$	= demand during half of the time between reviewing moments.
$I(s)$	= the expected inventory level during the order moment.
TC_L	= total costs during lead time L.
I_{stock}	= the inventory level after restocking.

Table 11: Used notation in the scan-buy model.