Cryptocurrencies: Does their inclusion improve the risk-return characteristics of already well-diversified portfolios?

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ABSTRACT,
In this thesis, the extent to which the inclusion of Cryptocurrencies into already well-diversified portfolios can increase the portfolio´s overall risk-return characteristics is investigated. This is a relevant issue since previous research already showed risk-return benefits from the inclusion of Cryptocurrencies, but it remained questionable whether this holds true given more recent economic developments. A mean-variance spanning and intersection framework similar to those of Huberman and Kandel (1987) and Kan and Zhou (2008) was used to test this. The results did hereby show that Cryptocurrencies indeed improved the already well-diversified portfolio´s risk-return characteristics in the given sample period between March 18th, 2018, and May 20th, 2022. However, when divided into the two sub-periods of up to (and including) December 31st, 2019, and since January 1st, 2020, the results show slightly different benefits in the second sub-period. There investors with a specific level of risk-aversion did not reap benefits while all others did.

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1. INTRODUCTION

In 2008, Bitcoin was introduced as an unregulated digital currency without legal tender status and is based on Blockchain technology. This allows a decentralized network and ledger to confirm transactions of Bitcoin. Simultaneously only a limited, decreasing number of new Bitcoins can be generated, and they do not require any financial intermediaries or monitoring authority. (Eisl et al., 2015) Thus, was Bitcoin the first Cryptocurrency, and was at the time of its creation intended to be used solely as an alternative currency. More recently however has it been used more as an investment opportunity in addition to conventional assets. This was inter alia observed by showing that Bitcoin accounts were primarily being used as speculative investments instead of as an alternative currency or exchange medium. Following Bitcoin´s example, a variety of other Cryptocurrencies was developed that show similar uses as alternative investments. (Baur et al., 2015; Petukhina et al., 2020; Platanakis & Urquhart, 2019) Moreover, Cryptocurrencies tend to show larger mean returns and variances than more traditional financial assets. (Chuen et al., 2017) Portfolios of big institutional investors such as insurance companies and pension funds however are aimed to be very well-diversified. They are oftentimes required to have rather low-risk characteristics, but nonetheless also aim for higher returns given these conditions. (Andonov & Rauh, 2018) Hence, can the inclusion of Cryptocurrency investments be relevant for investors aiming for better risk-return characteristics and diversification, should the inclusion increase those dimensions. Investments in Cryptocurrencies might thus be interesting to them, especially given the latest economic developments. This does not only hold true for institutional investors but potentially every investor with a well-diversified portfolio looking to enhance his portfolio in these aspects.

Resulting from this arises the problem of determining whether the inclusion of Cryptocurrencies into already well-diversified portfolios makes economic sense. The research question of this paper was thus formulated to investigate the extent to which the inclusion of selected Cryptocurrencies into well-diversified portfolios (of institutional investors) improves the portfolio´s risk-return characteristics and Sharpe ratio.

The research of this paper will focus on a selection of Cryptocurrencies. This was done using the Royalton CRIX index to include a qualified selection of Cryptocurrencies in one financial asset. For representing a well-diversified portfolio, a variety of stock, commodity, and sovereign and corporate bond indices was used. The period investigated ranges hereby from March 18th, 2018, to May 20th, 2022. Furthermore, will the research distinguish between these influences in different sub-periods in the given time: during and after the crisis caused by the Covid-19 pandemic, and before the crisis on the other hand.

The influences of Cryptocurrencies on portfolio risk-return characteristics and mean-variance efficient portfolio allocation are tested using the mean-variance spanning framework of Huberman and Kandel. (Huberman & Kandel, 1987) Moreover was the step-down procedure of the spanning test used as introduced by Kan and Zhou. (Kan & Zhou, 2008) The theoretical framework about to be tested on Cryptocurrencies was already applied to this context in the past. However, have the economic conditions changed with recent large-scale economic developments. The recently most notable hereby is the mentioned Covid-19 pandemic which had influences on both traditional financial assets as well as Cryptocurrencies. (Vidal-Tomás, 2021) Thus have the established theories not been applied to the new investment opportunities in Cryptocurrencies in the most recent times. Namely, shortly before and after the crisis caused by the mentioned Covid-19 pandemic. Therefore, will the research fill a gap in the Cryptocurrency literature with a mean-variance perspective.

Section 2 starts by offering an overview of the already established research regarding Cryptocurrencies in a mean-variance analysis context. Following this, does Section 3 explain the methodology used in more detail while Section 4 presents the data used and its preliminary analysis. The results can be seen in Section 5 of this paper. There the inclusion of Cryptocurrencies showed benefits to already well-diversified portfolios in the entire sample period of the research. It is hereby notable that the results for the time periods before and after the Covid-19 crisis differ. For the period before the crisis, there were benefits shown for all investors regardless of their risk-aversion like in the full period. On the other hand, benefits were visible for only some investors in the period during and after the crisis. These results are discussed in Section 6 regarding their assumptions, restrictions, limitations, and implications. In Section 7 the results are finally concluded while the appendix in Section 9 provides additional information.

2. THEORY

2.1 Cryptocurrency market developments

With their rapidly rising market capitalization in recent times, investments into Cryptocurrencies have gotten more and more public attention over the last years. (Petukhina et al., 2020) Moreover were they classified as financial assets. However, has research also shown that Cryptocurrencies have an unclear fundamental value, and that their value is thus only represented by their exchange rates to for instance fiat currencies. (Giudici et al., 2020) This raises the question of whether their inclusion into already well-diversified portfolios can improve the corresponding risk-return characteristics, i.e., in the form of higher Sharpe ratio results.

2.1.1 Market behaviour

Furthermore, are for some cryptocurrencies growth rates or price developments visible that would not be possible to this extent in traditional financial assets or commodities. Although the average daily realized returns of Cryptocurrencies outperform those of traditional asset classes, they also show larger variances associated with them and have been identified to be highly volatile. (Chaim & Laurini, 2018; Chuen et al., 2017; Platanakis & Urquhart, 2019) Adding to this research identified several behavioural finance phenomena caused by three distinct features of Cryptocurrencies: a lot of non-institutional investors, high risk and volatility, and their unclear fundamental value. These phenomena are inter alia herding behaviour and bubbles. Among other factors, this might lead to price drops which can heavily affect returns. Moreover, this can lead to illiquidity in some cases which will be discussed later in this paper. (Giudici et al., 2020)

In recent times, some degree of homogeneity was shown for the market for Cryptocurrencies as a whole, whereby Bitcoin was identified to drive the market risk. Moreover, was it shown that the Cryptocurrency (or Bitcoin) market had comparably high levels of inefficiency, especially in its early years. (Giudici et al., 2020) Later on, studies showed that Cryptocurrency markets have been integrated progressively with other, traditional financial markets. (Petukhina et al., 2020)

2.1.2 Correlations to traditional financial assets

Recent studies have furthermore shown that Cryptocurrencies had remarkably low correlations with traditional financial markets. This was observed by inter alia determining the correlations to industry portfolios and bond indices.

(Akhtaruzzaman et al., 2020) Also for Bitcoin alone, several studies have shown low correlations with stocks, bonds, and furthermore commodities like gold and oil in the past. (Eisl et al., 2015) It was stated that the low correlations to other assets in the timeframe of the research will not necessarily hold in times of crisis. This is because it is typical for correlations to increase in times of crisis and because Bitcoin was still rather new as an investment opportunity. (Briere et al., 2015; Vidal-Tomás, 2021)

Later, during the economic crisis caused by the Covid-19 pandemic, this temporarily changed in 2020. Then traditional financial markets and Cryptocurrencies had strong positive correlations for the first time. This was driven by a bad performance of financial markets in general and investors, therefore, selling positions for cash. Consequently to this, the correlations vanished and Cryptocurrencies showed diversifying properties to traditional assets once again. (Petukhina et al., 2020)

2.1.3 Diversification benefits

Despite their highly volatile nature, Cryptocurrencies were shown to have diversification properties. Thus, they provide benefits to investors that go further than those of conventional well-diversified portfolios of global nature. These benefits are due to the mentioned low correlations to traditional assets as well as their high volatility and resulting risk being adequately represented by higher (expected) returns. (Briere et al., 2015; Petukhina et al., 2020)

2.2 Institutional Investors

As stated, are Institutional investors known for being rather risk-averse. Cryptocurrencies on the other hand were observed to be quite volatile, unstable, and unpredictable when compared to more traditional investments. They could therefore potentially be seen as (too) risky by institutional investors, particularly by those being exceptionally risk-averse such as insurances or pension funds. (Andonov & Rauh, 2018; Białkowski, 2020; Worzala et al., 2000) The portfolios managed by these institutions have oftentimes pre-determined rules, such as stop-losses, allocation rules, and levels of risk-aversion which might make Cryptocurrencies unsuitable for their uses. In addition, do Cryptocurrencies have a rather high risk-return profile compared to other financial assets and can thus influence portfolio performance to a greater extent. This could furthermore make them non-beneficial to include in institutional portfolios. (Białkowski, 2020)

Hence, the question comes up whether it makes economic sense to include investments in the forms of Cryptocurrencies in already well-diversified portfolios from a risk-return perspective of the owners of these portfolios.

2.3 Cryptocurrency portfolio allocation

In recent years have various studies been conducted about the implications for portfolio allocation and diversification of Bitcoin, and later Cryptocurrencies in general.

2.3.1 Bitcoin portfolio allocation

For Bitcoin alone, it was shown that there are diversification benefits to traditional financial assets (Bouri et al., 2017; Corbet et al., 2018) and moreover that Bitcoin improves the overall risk-return ratios when included in an already well-diversified portfolio. (Eisl et al., 2015) This was explained in previous literature by stating that although Bitcoin has a comparably high risk, the risk is compensated by low correlations to more traditional assets. (Briere et al., 2015)

In Eisl et al. (2015), it is also observable that Bitcoin was included in all portfolio allocation optimization frameworks examined. Its mean portfolio weights ranged thereby between 1.65% and 7.69%. Notable hereby is that portfolio weights for corporate bonds were on average approximately 15 percentage points higher in portfolios including Bitcoin, while the weights for the variety of securities were on average 21 percentage points lower if it was included. (Eisl et al., 2015)

Furthermore, was shown in more recent research that industry portfolios have higher returns and lower volatility if Bitcoin is being included. Nonetheless does its inclusion in a bond index increase both the returns and volatility when compared to the bond index on its own. This suggests that Bitcoin is suitable to increase the risk-adjusted performance of well-diversified portfolios made up of global industry portfolios and investment grade bond indices. Moreover, does this indicate that positions in Bitcoin act as an efficient hedge against certain industry sectors and bonds. (Akhtaruzzaman et al., 2020)

2.3.2 Broad Cryptocurrency portfolio allocation

Despite this, more recent research was conducted. Petukhina et al. (2020), a major paper in the field, concluded that from the perspective of investors not solely Bitcoin should be considered. Instead, they proposed considering a wider selection of Cryptocurrencies. (Petukhina et al., 2020)

The study focused on 52 Cryptocurrencies and traditional assets and tests their performance using various portfolio allocation strategies, including mean-variance spanning. The mean-variance spanning portfolio allocation was hereby improved by the inclusion of 7 out of the 52 Cryptocurrencies. Prior to this extensive study, earlier research findings already indicated diversification benefits from a broad selection of Cryptocurrencies. (Chuen et al., 2017) In line with this, did Petukhina et al.´s (2020) research show that investors should consider a broader selection of Cryptocurrencies instead of only a few or one (In past research this has been the case with Bitcoin for example). This is concluded since it is indicated that also diversification across Cryptocurrencies is beneficial to investors. However, is it also visible that only a small number of Cryptocurrencies improve the efficient frontiers. Moreover, is there the possibility that investors over-diversify, which can hinder wealth creation and is not necessary to represent the Cryptocurrencies' covariance. (Petukhina et al., 2020)

Moreover, was it stated in the paper that due to the volatility structure of Cryptocurrencies and their returns, their risk contributions are disproportionate. Thus, they affect portfolio values and changes to a higher degree than more traditional assets do. Nonetheless, it is possible to include Cryptocurrencies appropriately by balancing them and less risky traditional assets in the portfolios. Then Cryptocurrencies are beneficial in diversifying the portfolio while achieving higher target returns. Thus they are considered by the authors to add value to the investment universe. (Petukhina et al., 2020)

2.3.3 Benefits for different investors

Notable for the results of Petukhina et al. (2020) is that the benefits of including Cryptocurrencies in the portfolios differ with investor profiles. In their paper investors were defined as being risk-averse, return-maximizing, or diversification-seeking. This is the case since the benefits are subject to the investor's objectives, although Cryptocurrencies improve the risk-return profiles of the portfolios investigated in general. Hereby the research showed that Cryptocurrencies were included only to a very limited extent in risk-oriented strategies (minimizing variance, conditional value at risk). This means that investors with a high risk-aversion benefit the least from their inclusion in portfolio allocation. Thus, the portfolio allocation strategies need to have at least a risk-return orientation for Cryptocurrencies to really take part in portfolio allocation.

However, the study also concluded despite the risky and volatile nature of Cryptocurrencies leading to low weight allocations for risk-averse strategies, that there are still benefits since an inclusion improves portfolio diversification properties. Hence does the usefulness of Cryptocurrencies to portfolio allocation depend heavily on the investment objectives. Therefore, are investors with already well-diversified portfolios generally advised to consider an inclusion of Cryptocurrencies when they are willing to take some risk. Still, does the study point out that for certain highly risk-averse investors it might be too risky to pursue the benefits. (Petukhina et al., 2020)

2.3.4 Short selling

A standard assumption in the Cryptocurrency literature is the ruling out of short selling. This is because short positions in Cryptocurrencies are impossible or at least impractical in reality. An exception to this is Bitcoin, for which it is possible to trade futures since the end of 2017. (Petukhina et al., 2020) However, with the rising market capitalizations, trading volumes and following liquidity for Cryptocurrencies in general (and not only Bitcoin) this might change soon.

2.3.5 Mean-variance frontiers

Another result of the research conducted on Cryptocurrencies in a mean-variance context is their effect on minimum-variance frontiers. Thereby it was shown that their inclusion leads to an extension of the frontiers. This is because the portfolios on the efficient frontiers, with the inclusion of Cryptocurrencies, can achieve much higher expected returns for higher levels of risk. For low levels of risk however this is not the case, and they showed similar expected returns to the portfolios excluding Cryptocurrencies. Notable hereby is that this seems to be dependent on the performance of the Cryptocurrency market. For instance, did they not extend the efficient frontiers in the declining market of 2018 but did so once again in 2019 with the market consolidating. Lastly, is pointed out that mean-variance frontiers are in most cases shorter than the respective mean-CVar frontiers (conditional value at risk frontiers). This indicates that Cryptocurrencies´ risks are not adequately captured by their variance alone. (Petukhina et al., 2020)

2.3.6 Sharpe ratios

Regarding Sharpe ratios, previous research showed that most of the time the inclusion of Cryptocurrencies does not improve the portfolio risk-adjusted excess return. This indicates that Cryptocurrency markets are rather well integrated with traditional financial markets. Moreover, does this imply that their higher expected returns compensate for their higher volatilities. This supports the findings of Briere et al. (2015), but is contrary to the result of the mentioned mean-CVar frontier comparison. (Petukhina et al., 2020) Furthermore is it notable that research has shown that many portfolio optimization techniques using risk-return measures lead to similar or even lower Sharpe ratios than an equally-weighted portfolio. Thus should Sharpe ratio result of those techniques, including mean-variance spanning and intersection, be evaluated with caution. (DeMiguel et al., 2009)

2.4 Research question and economic hypothesis

Based on this literature review and the current and recent economic situations, the research question was formulated:

"To what extent does the inclusion of selected Cryptocurrencies into well-diversified portfolios of institutional investors improve the portfolio´s risk-return characteristics and Sharpe ratio?"

In line with this, is the title of this thesis formulated as the following: "Cryptocurrencies: Does their inclusion improve the risk-return characteristics of already well-diversified portfolios?"

Both the research question as well as the title of this paper are based on the hypothesis that the inclusion of Cryptocurrencies will have a positive effect on the well-diversified portfolios' overall risk-return characteristics and thus show benefits or improvements. This hypothesis is based on the already established research, which was discussed in the previous paragraphs and will be evaluated in the next sections.

3. METHODOLOGY

3.1 Background Literature

The research´s background literature will be the closely connected economic theories of Harry Markowitz, James Tobin, and William F. Sharpe that among others resulted in the Modern Portfolio Theory of 1952 (Markowitz, 1952). It argues that an investor can build a portfolio using many assets that results in no higher portfolio levels of risk, but greater portfolio returns. This is achievable if an optimal mix between high-risk high-return and low-risk low-return assets is determined based on the investor's risk aversion.

3.2 Theoretical Framework

Building on this, mean-variance spanning regressions as developed by Huberman and Kandel (1987) will be used as the main theoretical framework for the tests to be conducted. There the authors presented methods for analyzing the effect that the inclusion of additional *test assets* has on the mean-variance frontier of both another set of *benchmark assets* and the added test assets. (Huberman & Kandel, 1987) In the following research, the set of assets of Cryptocurrencies was referred to as the test assets and the set representing a well-diversified portfolio as the benchmark assets.

Hereby it is tested whether the minimum-variance frontier of one set of assets K coincides, intersects, or does neither with the minimum-variance frontier of both the same set of assets and another set N. The set of assets N hereby represents the test assets whose influences are to be investigated. K on the other hand denotes the set of benchmark assets. Spanning hereby refers to the two frontiers coinciding while intersection means they have exactly one point in common which will be elaborated on in the following paragraphs.

Minimum-variance frontiers are hereby convex curves of the expected returns and variances of minimum-variance portfolios. They are made up of the point with the lowest variance, the global minimum-variance portfolio (*GMVP*), as well as an efficient and inefficient frontier. The former lies above the global minimum-variance portfolio and thus offers higher returns with higher variances. Portfolios lying on the efficient frontier are thus all seen as mean-variance efficient and can be differentiated based on different levels of risk-aversion. It includes the tangency portfolio (*TP*) which is the combination of assets offering the highest Sharpe ratio. The inefficient frontier on the other hand lies below the global minimum variance portfolio and is thus inefficient since it offers lower returns with higher variances compared to the global minimum-variance portfolio. The mean-variance spanning and intersection tests conducted have therefore examined the risk-return characteristics of a variety of hypothetical portfolios with or without different levels of inclusions of the risky assets.

To investigate if the frontier of the K set of assets intersects, coincides with, or does neither with the frontier of the N+K set of assets, several hypotheses were put up and consequently tested in Huberman and Kandel´s paper (1987).

- H1: R spans (R, r)
- H2: R intersects (R, r)

R hereby denotes the returns of benchmark assets K while r denotes the returns of test assets N. The larger N+K set of assets is denoted with (R, r).

Hereby it is notable that Huberman and Kandel (1987) introduced two additional hypotheses to test for intersection of the frontier of R with the frontier of R, r, and the risk-free rate r_f, which was referred to as (R, r, r_f). The initial framework introduced by Huberman and Kandel (1987) did thus not intend an inclusion of the risk-free asset in the set of assets K. However, was this done in previous research and was also done in the following research. The reasoning behind this is elaborated on in Section 6.2.1.

If the intersection hypothesis is not rejected, this means that the frontiers of the set of K assets and the frontier of the set of N+K assets have exactly one point in common. Hence, a portfolio $w*$ exists which is mean-variance efficient for both the K and N+K sets of assets. Therefore, intersection means economically that there is exactly one level of risk aversion for which a mean-variance efficient portfolio cannot be improved by including the test assets. This exact level of risk-aversion results in portfolio $w*$ with weight zero in the test assets.

On the other hand, if the spanning hypothesis is not rejected, the two frontiers coincide and thus have every point in common. In this case, does the set of test assets therefore not provide any risk-return benefits when included in the initial set of benchmark assets. The test assets can thus only add to the variance of the virtual portfolio and not the expected return, meaning there is no mean-variance efficient portfolio that can be improved by their inclusion. Therefore, do they all have a weight of zero in the test assets. Put in economic terms, spanning means that no investor is better off by including the test assets, regardless of their level of risk aversion.

To test these hypotheses Huberman and Kandel proposed multivariate tests based on a regression of the vector of test asset returns on the vector of the benchmark asset returns.

$$\underline{r_t} = \underline{\alpha} + \beta \underline{R_t} + \underline{e_t} \qquad \text{(Eq. 1)}$$

Hereby does $\underline{r_t}$ represent the N×1 vector of test asset returns in period t, while $\underline{R_t}$ represents the K×1 vector of benchmark asset returns in the same period. Moreover does β denote the N×K matrix of regressions coefficients, and $\underline{\alpha}$ the N×1 vector of constants or intercepts under which the regression holds. Residuals are lastly given by the N×1 vector $\underline{e_t}$.

The spanning and intersection hypotheses impose parameter restrictions on the estimates of the regression coefficients in Equation 1. To not reject intersection, it needs to be statistically determined whether the estimate of $\underline{\alpha}$ ($\underline{\hat{\alpha}}$) is equal to the product given in the following equation. There, the scalar η represents the zero-beta rate or the risk-free rate if it is available. This is the case for the research of this paper and will be elaborated on in Section 4. It is multiplied by the difference of a vector of ones of size N ($\underline{i_N}$) and the product of the estimate of β and a vector of ones of size K ($\hat{\beta}\underline{i_k}$).

$$\underline{\hat{\alpha}} = \eta\ (\underline{i_N} - \hat{\beta}\underline{i_k}) \qquad \text{(Eq. 2)}$$

On the other hand, for the spanning hypothesis to not be rejected, the equations

$$\underline{\hat{\alpha}} = \underline{0} \qquad \text{(Eq. 3)}$$

and

$$\hat{\beta}\underline{i_k} = \underline{i_N} \qquad \text{(Eq. 4)}$$

need to hold simultaneously. Thus, does spanning put differently mean that intersection holds for any value given to the scalar η.

It was added to this by Kan and Zhou (2008) that the test for spanning if broken down into separate tests, can give more statistical and economical insights. Thus, is their 'step-down approach' a test for Equation 3 followed by a test for Equation 4 conditional on Equation 3 holding. More insights can be given since the step-down approach shows on which step the test was rejected. The first step of testing $\underline{\hat{\alpha}}$ equaling zero is hereby a test of the tangency portfolio having a weight of zero in the test assets. The second step on the other hand tests if the product of $\hat{\beta}$ and $\underline{i_k}$ equals $\underline{i_N}$, and thus if the global minimum-variance portfolio has a weight of zero in the test assets. Given that both tests of the step-down procedure are not rejected, it means that the two mean-variance efficient portfolios are on the efficient frontier of both the K and larger N+K sets of assets. It was shown that if it is the case that the frontiers share two points in common, they share all points in common. (Kan & Zhou, 2008; Tobin, 1958) Following from this, two more hypotheses adding to H1 are introduced:

- H1a: r has a weight of zero in the tangency portfolio of (R, r) (Eq. 3 holding)
- H1b: r has a weight of zero in the global minimum-variance portfolio of (R, r) (Eq. 4 holding)

Several methods were proposed in the literature to test for intersection and spanning using the restrictions in Equations 2 to 4, such as the likelihood-ratio test initially given by Huberman and Kandel (1987). More recent research however has concluded that this test is only valid for N≥2 and needs correction for N=1. In this research, this is the case (since there is only one test asset) and thus Wald tests will be used instead, which also showed further benefits in more recent research. (Kan & Zhou, 2008) Furthermore, does N being 1 implicate the one-dimensionality of all vectors of size N as well as matrix β, which is thus a row vector. Resulting from this, Equation 1 can be regressed linearly as all vectors of size N are scalars. (Briere et al., 2015; Scholtens & Spierdijk, 2010)

3.3 Performance Evaluation

Several performance measures were proposed by the literature to evaluate mean-variance spanning and intersection tests. The framework hypotheses H1 to H2 of Huberman and Kandel (1987) and Kan and Zhou (2008) act as the first measure to differentiate between spanning, intersection or neither. They are seen as the hard results of this research. The significance levels α for this research were determined to be 0.10 for each test, given the relatively small sample size (See Section 4). Notable for the step-down test is that each test needs to be not rejected for the spanning hypothesis to be not rejected. The combined level of significance for the step-down test is thus equal to $\alpha_{H1a} + \alpha_{H1b} - \alpha_{H1a}\alpha_{H1b} = 0.19$ with individual significance levels of 0.10. (Kan & Zhou, 2008)

Moreover, can the risk-return characteristics of the assets being investigated, with the use of risk-free asset rates, be expressed as Sharpe ratios. This can also be done for combinations of those assets and thus portfolios positioned on for example the minimum-variance frontier since the frontiers and the portfolios positioned on them are composed of both returns and variances. (Sharpe, 1998) The differences in the Sharpe ratio between the portfolios including and excluding the test assets are thus the weak results of this research and add further insights to the framework hypotheses. DeRoon and Nijman (2001) showed, that if the spanning hypothesis or the intersection hypotheses with an unknown zero-beta rate is not rejected, this implies that the maximum attainable Sharpe ratios cannot be improved by the inclusion of the test assets to the benchmark assets. With a given risk-free rate for the intersection test however this is only the case for a specific level of risk aversion. (DeRoon & Nijman, 2001)

3.4 Statistical hypothesis

Based on this framework we can conclude what the hypothesis inherent to the research question expects of the tests in this research. Improvements in the risk-return characteristics of the larger N+K set of assets (or the well-diversified portfolio including Cryptocurrencies) compared to the K set of assets (or the well-diversified portfolio excluding Cryptocurrencies) implies that spanning and hence H1 and/or both H1a and H1b are rejected. On the other hand, for intersection or if H2 is not rejected it depends on an investor's level of risk-aversion since there is one portfolio w^* or level of risk-aversion for which there are no benefits. Spanning thus means that all mean-variance efficient portfolios have a weight of zero in the test assets since they offer no risk-return improvements or benefits. For intersection, the weights of the test assets are zero only at exactly the point of intersection of the frontiers. Furthermore, should an improvement in risk-return characteristics be visible in a positive difference between the Sharpe ratios of the larger set of assets N+K compared to the smaller set of assets K. Whether this difference is significant however is determined by the previous Wald tests for H1a and H1b, but also H1 and H2 might rule these differences out.

4. DATA SELECTION

As mentioned, mean-variance spanning and intersection tests were used as the main tool in determining the risk-return characteristics of the sets of assets investigated. In addition to examining the minimum-variance frontiers of the differing sets of assets, Sharpe ratios were calculated for the respective tangency and global minimum-variance portfolios with and without the test asset. In this context, those sets of assets are used to form virtual representations of well-diversified portfolios including or not including Cryptocurrencies. The research will hereby not focus on theoretical portfolios including individual Cryptocurrencies or traditional assets. Instead, an index to represent several Cryptocurrencies and other indices to represent a well-diversified portfolio will be used. Those portfolios do not aim at imitating a real-world portfolio but rather represent a portfolio with high diversification and risk aversion, such as those of for instance institutional investors. The use of indices in this context was inter alia proposed in previous research investigating the portfolio diversification properties of Bitcoin (Eisl et al., 2015), and furthermore helps to keep this research contained.

4.1 Test assets

For the representation of the test assets to be investigated the Cryptocurrency index Royalton CRIX will be used since it composes the most relevant Cryptocurrencies according to the CRIX concept developed by Härdle. It is based on both market capitalization and trading volume which will add to eliminating liquidity problems associated with less popular Cryptocurrencies (Trimborn & Härdle, 2018). As of May 2022, the Royalton CRIX index is composed of Bitcoin (58.25%), Ether (24.47%), Binance Coin (5.08%), Ripple (2.74%), Luna (2.47%), Solana Token

Table 1: Descriptive Statistics of Returns

Period	Index	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
		Statistic	*Statistic*	*Statistic*	*Statistic*	*Statistic*	*Statistic*
Full sample period	Royalton CRIX	-23.8566%	20.8513%	0.1992%	4.6564%	-0.0516	2.6991
	MSCI ACWI	-9.5133%	8.3953%	0.0227%	1.0948%	-1.0558	16.5892
	S&P USTBI	-0.0374%	0.0709%	0.0045%	0.0072%	1.7717	10.2304
	S&P GDSBI	-1.8158%	1.6512%	-0.0069%	0.3074%	-0.3383	4.8783
	S&P 500BI	-2.8036%	2.0810%	0.0093%	0.3439%	-1.3533	14.1673
	S&P ICBI	-4.5484%	2.9914%	-0.0081%	0.4799%	-1.1917	13.2411
	S&P GSCI	-11.7708%	7.9860%	0.0650%	1.6021%	-1.0102	9.1363
First sample period	Royalton CRIX	-20.1315%	20.8513%	-0.0316%	4.7639%	0.2723	2.8106
	MSCI ACWI	-2.5202%	2.6180%	0.0201%	0.7013%	-0.4590	1.2866
	S&P USTBI	-0.0042%	0.0296%	0.0088%	0.0062%	0.8650	0.1388
	S&P GDSBI	-0.7640%	0.8231%	0.0052%	0.2435%	0.1660	0.6247
	S&P 500BI	-0.7543%	0.7284%	0.0301%	0.2121%	-0.1141	0.9171
	S&P ICBI	-1.0237%	1.1739%	0.0040%	0.3256%	0.1509	0.7209
	S&P GSCI	-4.5616%	7.9860%	0.0029%	1.1958%	0.0007	5.5274
Second sample period	Royalton CRIX	-23.8566%	20.4195%	0.3717%	4.5707%	-0.3157	2.7325
	MSCI ACWI	-9.5133%	8.3953%	0.0247%	1.3147%	-1.0188	13.2804
	S&P USTBI	-0.0374%	0.0709%	0.0014%	0.0062%	4.1280	40.1352
	S&P GDSBI	-1.8158%	1.6512%	-0.0160%	0.3475%	-0.4129	4.7567
	S&P 500BI	-2.8036%	2.0810%	-0.0062%	0.4155%	-1.2492	10.7428
	S&P ICBI	-4.5484%	2.9914%	-0.0172%	0.5686%	-1.2432	11.0794
	S&P GSCI	-11.7708%	7.3739%	0.1115%	1.8480%	-1.1968	8.0324

Notes: Full period (N statistic = 1045, Skewness Std. error = 0.076, Kurtosis Std. error = 0.151); First sub-period (N statistic = 447, Skewness Std. error = 0.115, Kurtosis Std. error = 0.230); Second sub-period (N statistic =598, Skewness Std. error = 0.100, Kurtosis Std. error = 0.200), Values rounded to 4 decimals

(2.07%), Cardano (2.04%), Polkadot (1.47%), and Avalance (1.42%). Note that the CRIX composition is adjusted monthly and that these weights show the end of the sample period. The composition on the start date of the sample period is therefore different.

Price data for the Royalton CRIX crypto index can be found publicly on the S&P Global website as it was published by S&P Dow Jones Indices daily since December 27th, 2020, when the index was launched. Moreover, can data on the index's hypothetical performance previous to its launch date also be found on the S&P Global website and reaches back until March 16th, 2018, the start of this research's sample period.

4.1.1 Sample periods

Notable hereby is that the history of Cryptocurrencies is relatively recent. Hence, it is impractical to cover periods of time reaching much further back than this effectively with only one test asset (that includes not only Bitcoin for instance) and thus the given index was used. The end of the sample period is hereby May 20th, 2022, when the data was accessed. Nonetheless, did the use of daily data still yield a big enough data set for research to be conducted in this rather short sample period. This sample period furthermore allowed the research to investigate results in different periods of time and thus different economic states of nature. Therefore, the research compares both pre-and post-crisis results. Given the sample period, possible crises to be observed are the Covid-19 pandemic as well as the start of the current Russia-Ukraine conflict. To illustrate potential differences, the full sample period was divided into two sub-periods. The *first sub-period* ranges from March 16th, 2018, to December 31st, 2019, and the *second sub-period* from January 1st, 2020, to May 20th, 2022. The two sub-periods together make up the *full sample period* and will in this research be referred to as the first and second sub- or sample period, respectively.

4.2 Benchmark assets

Moreover, did the research require assets to form an already well-diversified portfolio, whereby inspiration was taken from Eisl et al. (2015). There, indices were used to represent the price developments of a broad selection of stocks, corporate and sovereign bonds, and commodities in the context of Cryptocurrency mean-variance analysis. (Eisl et al., 2015)

To represent stocks in well-diversified portfolios the exchange traded fund MSCI ACWI was made use of. More precisely, was its standard version used containing large- and mid-cap firms. It was to be included in the benchmark assets as it comprises equity investments in various industry sectors as well as countries of origin. Therefore, it represents stocks in a global, well-diversified portfolio like that of institutional investors rather accurately. Furthermore, does the portfolio representation include U.S. Treasury bills as well as other sovereign bonds. Again, indices were used, and the assets are represented in the forms of the S&P U.S. Treasury bill index (*S&P USTBI*) and the S&P Global Developed Sovereign Bond Index (*S&P GDSBI*). This allowed for easier representation of multiple U.S. treasury bills with different maturities as well as the representation of bonds issued by other governments. Corporate bonds were likewise represented using indices, the S&P 500 Bond Index for the U.S. market (*S&P 500BI*), and the S&P International Corporate Bond Index for other international markets (*S&P ICBI*). Moreover, were commodities needed to fully diversify the portfolio, which were included by means of the S&P GSCI. The index represents commodity price developments rather well since it is composed of a variety of different commodity types, including precious metals like gold among others such as industrial metals, energy, and soft commodities.

The representation of the portfolios is thus well-diversified and of global nature since a variety of traditional (and less traditional) assets from different countries are included in the indices. All the price data related to the well-diversified portfolio can be found on the MSCI and S&P Global websites and is available daily since at least March 16th, 2018, the start of the full sample period.

4.2.1 Risk-free rates

Lastly, were rates for risk-free assets used as part of the mean-variance regression tests and calculations of Sharpe ratios. This was done using the mean returns of the previously mentioned S&P U.S. Treasury Bills index for the respective sample periods. For the full sample period, the daily risk-free rate was thus assumed to be 0.0045%, for the first sub-period 0.0088%, and for the second sub-period 0.0014%. (see Table 1)

4.3 Preliminary data analysis

As the data for both the test asset and benchmark assets is given in prices in USD, returns needed to be calculated which was done using the standard formula for simple returns ($Return_t = [Price_t - Price_{t-1}]/Price_{t-1}$). Moreover, is it notable that days on which at least one of the indices did not trade were left out completely. Descriptive statistics of the return data can be found in Table 1. Notable hereby is that the Kurtosis statistics are shown with the constant three being subtracted and are thus zero for a normal distribution.

Table 1 shows that stocks and commodities in the forms of the MSCI ACWI and S&P GSCI had larger minimum and maximum values than the treasury bill and bond indices. This is visible also in their larger standard deviations as well as their higher mean

Table 2: Parameter Estimates with Robust Standard Errors

Parameter	B	Robust Std. Error[a]	t	p-value	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	0.237	0.167	1.423	0.155	-0.090	0.564
Return MSCI ACWI	1.048	0.219	4.791	0.000	0.619	1.477
Return S&P USTBI	-11.703	19.046	-0.614	0.539	-49.076	25.670
Return S&P GDSBI	1.722	1.181	1.458	0.145	-0.596	4.040
Return S&P 500BI	-0.893	0.749	-1.193	0.233	-2.362	0.576
Return S&P ICIB	-0.177	0.631	-0.281	0.779	-1.415	1.061
Return S&P GSCI	0.155	0.116	1.338	0.181	-0.072	0.383

Notes: Dependent Variable: Return CRIX, a - HC3 method used to calculate robust Standard errors, Values rounded to 3 decimals
Multiple R-squared: 0.07047, Adjusted R-squared: 0.0651

values. Cryptocurrencies in the forms of the CRIX however showed minimum and maximum values roughly twice the size of those of the stock and commodity indices. The given mean values and exhibited variances are furthermore the largest of all the indices. In the first sub-period, it is notable that the CRIX showed a negative mean value. Three of the indices used to represent bonds, the S&P GDSBI, S&P 500BI and S&P ICBI, also showed negative means in either one of the sub-periods alone or one of the sub-periods and the full sample period. Lastly, can be seen that the S&P U.S. T-Bills Index showed both the lowest values in means, standard deviations as well as minimum and maximum values. Figures 1 and 2 show the daily simple returns of the indices throughout the full sample period and can be found in the appendix in Section 9. Additionally, are the rows of the correlation matrix depicting values for the CRIX given there as well in Table 6.

Table 3: Wald test statistics

Period	Hypothesis	Test	Wald	p-value
Full sample period	H1	Spanning	1.0325	0.3565
	H1a	Spanning (TP)	2.0244	0.1551
	H1b	Spanning (GMVP)	0.3314	0.5650
	H2	Intersection	2.0236	0.1552
First sub-period	H1	Spanning	0.2232	0.8001
	H1a	Spanning (TP)	0.3855	0.5350
	H1b	Spanning (GMVP)	0.4312	0.5117
	H2	Intersection	0.3858	0.5348
Second sub-period	H1	Spanning	1.9231	0.1471
	H1a	Spanning (TP)	3.7990	0.0518
	H1b	Spanning (GMVP)	0.0605	0.8058
	H2	Intersection	3.7987	0.0518

Note: Mean returns of S&P USTBI as rf: rf = 0.0045% (full period); rf = 0.0088% (first period); rf = 0.0014% (second period); Values rounded to 3 decimals

5. RESULTS

5.1 Framework hypotheses

Recalling from earlier in this paper, the returns of the test asset (Royalton CRIX) was regressed on the returns of the benchmark assets representing a virtual well-diversified portfolio. Based on the outcomes of this regression, Wald tests were run to reject or not reject the given parameter restrictions and thus the mean-variance framework hypotheses. This is shown in Table 3, while the parameter estimates of the regression can be seen in Table 2 for the full period and Table 5 for the first and second sub-periods, respectively. As seen in Table 2, was the regression hereby run using HC3 robust standard errors of the parameter or coefficient estimates. This was done to allow the regression to fit the earlier described data more accurately. A model without robust standard errors might have heteroskedastic residuals because of the financial data used. The parameter estimates would in this case not be biased or wrong, however, could they be less precise which might have crucially altered the results of the tests on parameter restrictions.

The p-values in Table 3 show the outcomes of the Wald tests. There the given p-values show whether the test hypothesis (H1 to H2) of the parameter equaling its restriction is rejected or not rejected. Thus, does a rejection of the test equal a rejection of the respective mean-variance framework hypothesis and vice versa.

It shows that H1 and thus spanning was rejected for the full sample period on the overall spanning test as well as both parts of the step-down test. Thus were H1a and H1b also rejected. Moreover, was H2 and thus intersection with the same set of assets rejected as well. The same holds true for the first sub-period, although with significantly higher p-values of rejection given by the tests for H1, H1a, and H2. The p-value of the test for H1b did not change by much on the other hand.

In the second sub-period however, both H2 and H1a (the tangency portfolio part of the step-down test for spanning) are not rejected. H1b as well as H1 and thus the overall spanning test on the other hand are still rejected. Hence is there not enough evidence to reject the intersection hypothesis H2, while the spanning hypothesis H1 is rejected. However, for spanning it is notable that H1a was not rejected and thus does there seem to be no significant difference in the frontiers´ tangency portfolios.

5.2 Sharpe ratios and portfolio weights

Moreover, complementary to the hard results of the Wald tests, were the improvements in risk-adjusted excess returns investigated, which are given by Sharpe ratios. This was done for both the global minimum-variance portfolios and tangency portfolios. Furthermore, was once again distinguished between the full period and both sub-periods, with the results being shown in Table 4.

The last column of Table 4 gives the differences between the portfolios including the test assets and the respective portfolios not including them. The difference in basis points (bp) is hereby given by the raw difference multiplied by the factor 10,000 and is thus in scale with the e-05 notations of the corresponding Sharpe ratios (and expected returns). For the full sample period, it is visible that the Sharpe ratio of the global minimum-variance portfolio increased marginally by 0.0183 bp, while the Sharpe ratio of the tangency portfolio increased more notably by 1.4226 bp.

In the first sub-period however, no improvements in Sharpe ratios for both portfolios were seen. Moreover, were there also no improvements in the Sharpe ratio of the global minimum-variance portfolio in the second sub-period. The Sharpe ratio of the tangency portfolio however increased rather markedly by 3.2154 bp in the second period. This is the biggest Sharpe ratio increase visible in this research.

Table 4: Portfolio weights and Sharpe ratios

Portfolio		Portfolio weights CRIX	Expected return [e-05]	Standard deviation	Sharpe ratio [e-05]	ΔSharpe ratio [bp]
Full sample period	GMVP	0.0008%	4.5426	0.7041 %	6.0554	0.0183
	TP	1.0505 %	8.4361	7.1933 %	54.719	1.4226
First sample period	GMVP	0.0000 %	8.8096	0.6127 %	1.5637	0.0000
	TP	0.0000 %	29.0681	18.6952 %	108.4138	0.0000
Second sample period	GMVP	0.0000 %	1.4605	0.5963 %	10.1397	0.0000
	TP	0.6789 %	5.1032	3.9949 %	92.6981	3.2154

Notes: Values rounded to 4 decimals, bp difference equal to raw difference times factor 10,000

Lastly, does Table 4 also show the weight allocations in the test asset of both mean-variance efficient portfolios for all periods. There can be observed that the weights allocated to the test asset by the global minimum-variance portfolios are zero for both sub-periods. The same holds true for the tangency portfolio of the first sub-period. The tangency portfolio of the second sub-period however allocated a respective weight of 0.68%. In the full period on the other hand the global minimum-variance portfolio showed a weight of 0.0008% in the test asset, while the tangency portfolio allocated a weight of 1.05%.

5.3 Results by period

To sum it up, does the full period show neither intersection nor spanning as H1 to H2 were rejected. Furthermore, are risk-return benefits also visible in the increase in Sharpe ratios of the two mean-variance efficient portfolios investigated. This is visible as well in the positive weights allocated to the test assets by the two portfolios. Hereby are benefits visible for all investors, although the tangency portfolio showed more benefits than the global minimum-variance portfolio.

The findings of the first sub-period before the Covid-19 crisis also show neither intersection nor spanning. Now however, the differences in Sharpe ratios of the mean-variance efficient portfolios and their respective test asset weights are all equal to zero. Hence, the complementary weak results do not support the hard results hereby. The latter would indicate no risk-return benefits for investors, while the rejection of the framework hypotheses indicates the opposite. Thus do the findings for the first sub-period still show risk-return benefits for investors by the inclusion of the test asset as this is indicated by the hard results.

Lastly, does the second sub-period investigate the time of the Covid-19 crisis and afterwards. There the tests do not reject the intersection hypothesis. In addition to this is H1a, the tangency portfolio part of the step-down spanning test, not rejected as well. In combination do those results indicate that intersection takes place at portfolio *w**, where the test asset´s inclusion does not bring any significant benefits. Thus, needs this point to be the same point as the tangency portfolio of the frontiers since it has a weight of zero in the test asset. Resulting from this can be stated, that the test asset does not add to the frontier at exactly that point of intersection at the tangency portfolio. On the other hand, it does add to the frontier on all other points or portfolios than *w** or the tangency portfolio. This is contradicted by the research´s weak results again like in the first sub-period. The differences in Sharpe ratios in the second period showed no difference for the global minimum-variance portfolio (and a weight of zero), while the largest difference in risk-adjusted excess return (and a positive weight) was seen for the tangency portfolio. Therefore, do the differences in Sharpe ratios show no benefits for very high risk-aversion at the global minimum-variance portfolio and rather large benefits for less risk-aversion at the tangency portfolio. The not rejected intersection hypothesis H2 however indicates that there are only no risk-return benefits for investors with a specific level of risk aversion resulting in *w**. Moreover, does H1a state that there is a weight of zero in the test assets in the tangency portfolio, which is in the weak results not the case for the tangency portfolio but the global minimum-variance portfolio.

6. DISCUSSION

6.1 Previous research

These results support some findings of the literature discussed earlier in Section 2 of this paper. There it was stated that Petukhina et al. (2020) had shown that the benefits of including Cryptocurrencies differ based on an investor's risk-aversion. A high risk-aversion resulted thereby in less benefits than a low risk-aversion. This research has found the same in the full period since the global minimum-variance portfolios with higher risk-aversion showed fewer benefits with a 3.11% increase than the tangency portfolios with a lower risk-aversion and a 35.13% increase. As H2 and H1a were not rejected in the second sub-period, we can furthermore conclude that the economic benefits depend on the level of risk aversion in this period (and can be none at the exact risk-aversion of portfolio *w**).

In general, do the results differ quite a bit based on the sample period investigated. In this research, the hard results of the full and first sample periods are different from those of the second sample period. This is rather commonly found in the mean-variance spanning literature and was for instance also seen in the example of Huberman and Kandel (1987). (Huberman & Kandel, 1987)

Moreover, is it notable that the weak results in the form of Sharpe ratio differences only constitute the hard results in the full period. In the two sub-periods, they either show no economic benefits where there should be some or vice versa. In the first period, this might be caused by the ruling out of short selling in the calculation of mean-variance efficient portfolios in combination with negative mean returns of -0.0316% for the test asset. However, has previous research shown that the Sharpe ratio results of portfolio optimization techniques including mean-variance analysis should be interpreted with caution. Thereby did equally-weighted portfolio allocations show higher Sharpe ratios than for instance mean-variance efficient portfolios. (DeMiguel et al., 2009) In the results of the research conducted in this paper it thus seems to be the case that the Sharpe ratio results are also somewhat unreliable, especially for the smaller sample sizes of the two sub-periods.

6.2 Critical reflection

6.2.1 Assumptions and restrictions

Several assumptions were made in the used model to investigate the effects of Cryptocurrencies on a well-diversified portfolio. Firstly, was the S&P U.S. Treasury Bill Index used as a risk-free rate. The descriptive statistics of Table 1 however show that its returns still show some variance (standard deviation) and that the minimum return value was negative with -0.04%, which is remarkable. However, are U.S. Treasury bills oftentimes the risk-free rate in financial literature including mean-variance analysis literature. Hence is an index of treasury bills therefore nonetheless an appropriate risk-free asset, since it includes not only one type of treasury bills but a variety with differing maturities.

Moreover, was the original model of Huberman and Kandel (1987) modified to include the S&P U.S. Treasury Bill Index in the set of assets K. Spanning and intersection are therefore tested for the frontier (R, r) which includes the risk-free rate. Consequently, is it also included in the global minimum-variance portfolios and tangency portfolios. As a result, big parts of the mean-variance efficient portfolios are composed of the risk-free asset. This was done to be able to assume the given index as a risk-free rate. If the risk-free asset was to be excluded from the set of assets K (and thus R) it would not be possible to form tangency portfolios. This is since the risk-free rate would then be higher than the expected return on the global minimum-variance portfolio.

6.2.2 Limitations

A limitation to this research´s results is that the regression used is not fully unbiased since the assumptions for linear regression were not all investigated. As an example, could the residuals be non-normally distributed because of the non-normal returns of both the dependent and independent variables given the data´s financial nature. Nonetheless are ordinary least squares still consistent, and the model is robust towards heteroskedasticity since HC3 standard errors were used.

Still, the model is not fitting perfectly by far, which can be seen in the adjusted R squared values in Tables 2 and 5. Especially in the first sub-period the model fits very badly, with a negative adjusted R squared of -0.01145. Moreover, is it notable that the model for the second sub-period fits the data better than the full period model. Resulting from this are estimation errors in the regression parameters are far more likely to occur, which also might explain the conflicting hard and weak results. Nonetheless is the power of the spanning and intersection tests for H1 to H2 more meaningful while the differences in Sharpe ratios are complementary.

In general, would a robustness test of the model be advisable for further research on this topic. This extends not only to the (linear) regression used but also to for instance the statistical tests on parameter restrictions. This research used Wald tests, however, would the difference to for example GMM (Generalized Method of Moments) tests be interesting given the relatively small sample size. (DeRoon & Nijman, 2001) Moreover, has previous research proposed for instance CVar models which have less assumptions on the data. (Eisl et al., 2015; Petukhina et al., 2020)

6.2.3 Data and practical implications

Regarding the practical implications of the results, several issues arise. Firstly, might the low liquidity of some Cryptocurrencies make larger trades impossible in real life at all or at least without heavily affecting prices. Thus, did previous research impose liquidity constraints on Cryptocurrencies when used in a portfolio allocation context to ensure feasibility in reality. (Petukhina et al., 2020) Moreover was stated that relatively low and stable portfolio weights (for Bitcoin in that context) further limit liquidity implications. (Eisl et al., 2015) The weights of mean-variance efficient portfolios in this research are rather low as well, although their development over time was not investigated. Moreover, did research into this topic show that market capitalizations and trading volumes can proxy for liquidity (Amihud, 2002) and that Cryptocurrencies have grown substantially on those dimensions since the study of Petukhina et al. (2020). (See e.g., Coin Market Cap) In addition, was the CRIX index used to represent Cryptocurrencies instead of individual ones. This index is composed of assets based on their high values of market capitalization and trading volume and will thus furthermore limit liquidity problems. (Trimborn & Härdle, 2018) Because of this, no liquidity constraints were built into the model despite previous research advising this.

Strongly connected to problems associated with liquidity is the ruling out of short selling for Cryptocurrencies in most portfolio allocation models in the literature. Nonetheless, is it as of now possible to trade in options of some Cryptocurrencies (e.g., Bitcoin) and on specific exchanges. With rising market capitalizations and trading volumes, this might become more common practice for Cryptocurrencies soon. (Krafft et al., 2018)

Lastly, does the impact of trading frequencies have relevant practical implications. Thereby, has previous research shown that there is statistically no significant difference in portfolio returns between daily, weekly, and monthly rebalancing frequencies. Whether this still holds true in the context of Cryptocurrencies in the given sample period however was not investigated in the research conducted in this paper. (Petukhina et al., 2020)

6.2.3.1 Policy recommendations

Still, there are some notable remarks about these practical implications given. Firstly, are the results based on historical data of about the last four years. Thus, do neither the framework hypotheses tests nor the differences in Sharpe ratios have predictive power for future performance of the assets investigated. Rather do they solely describe their properties in the given sample period or time frame. Moreover, are these results based only on mean-variance efficiency. This does not imply that (institutional) investors should or will act like this since there are a variety of other factors to be considered. Adding to this, were different investment horizons for the time assets are held not considered. These might vary between investors and furthermore alter the results´ feasibility.

6.3 Further research

Lastly, some recommendations for future research are given. Regarding the data used, an investigation of more recent times than the 20th of May 2022 would make sense. This is on the one hand because of turbulent developments in the Cryptocurrency market, and on the other because of general economic developments. The former includes for instance the crash (and 'recovery') of the Luna coin, which is included in the CRIX and only partly in the sample period. (Kampakis, 2022) Moreover, did the prices of traditional financial assets move radically with the consequences of the current Russia-Ukraine conflict (Carlomagno & Albagli, 2022). In addition, would research starting from the launch date of the CRIX, 27th December 2020, be interesting since its results would have been appliable using the CRIX in reality. Regarding the well-diversified portfolio, further benchmark assets might be taken into consideration in future research. For instance, would including real estate and money market assets make sense to further diversify. This can be done using indices like in this research which was for example seen in Scholtens and Spierdijk. (Scholtens & Spierdijk, 2010)

Moreover, as previously indicated, might a model allowing for short selling make sense in the future because of the rising market capitalizations, trading volumes and liquidity. In addition, would an investigation into what asset classes are replaced by Cryptocurrencies make sense. This was done for instance in Eisl et al. (2015) and gives investors further insights into the (over-) diversifying properties of Cryptocurrencies. (Eisl et al., 2015) The same holds true for the effect of trading frequencies on cumulative wealth, where research had shown the findings presented earlier. Yet remains the question whether this holds true in the most recent times. (Petukhina et al., 2020)

Another suggestion for further research is measuring the exact distance between the frontiers of the two sets of assets. For the full period this was given in this research, but for the two sub-periods not since the weak results were determined to be unreliable. However, given that spanning was not rejected in these periods, would an investigation of this be interesting.

Moreover, did the model used in this paper include the risk-free rate in the set of benchmark assets K. Thereby the S&P U.S. Treasury Bill Index was used, which still showed some variance. Thus, might a new model excluding that index be interesting. Notable hereby is that another risk-free rate would need to be determined then because of the reasons given in Section 6.2.1. For instance, did previous research set the risk-free rate equal to zero which would also be possible in this case. However, is it then not possible to form real Sharpe ratios but only risk-adjusted returns.

Furthermore, did the model fit rather poorly to the given data in general (See adjusted R squared in Tables 2 and 5). Thus, would the use of different models than mean-variance spanning regressions make sense if they fit the data better. This was already indicated in previous research, where for instance the use of CVar models was proposed. An investigation of the same sets of test and benchmark assets in the given time frame with this or another model is another suggestion for future research. Thereby could also be examined whether the CVar frontiers are still longer than the minimum-variance frontiers and whether Cryptocurrency risks are adequately captured by their variance in the given sample period. (Eisl et al., 2015; Petukhina et al., 2020)

7. CONCLUSION

Recalling from Section 2.4, it was investigated to which extent the inclusion of selected Cryptocurrencies into well-diversified portfolios improves the portfolio´s risk-return characteristics. In addition to mean-variance spanning and intersection tests were mean-variance efficient portfolios´ Sharpe ratios and test asset weights investigated to answer this question.

Between March 18th, 2018, and December 31st, 2019, the hard results showed that there are risk-return benefits from the inclusion of Cryptocurrencies in the form of the CRIX on the representation of the well-diversified portfolio. There are benefits for all investors regardless of their risk-aversion, however, the research´s weak results did not quantify those benefits. Later, between January 1st, 2020, and May 20th, 2022, the hard results showed no benefits for a specific level of risk-aversion resulting in the tangency portfolio. This means that there are still benefits for all investors with other levels of risk-aversion. Once again however, the weak results did not quantify those benefits since they did not constitute the hard results.

In the full period investigated, ranging from March 18th, 2018, to May 20th, 2022, the research´s hard results also showed benefits for all levels of risk-aversion. The Sharpe ratio of the global minimum-variance portfolio increased hereby by 0.0183 bp, which equals a 3.11% increase. Simultaneously the Sharpe ratio of the tangency portfolio increased by 3.2154 bp and thus 35.13%. Therefore, does the extent of the risk-return characteristic improvements seem to depend on the risk-aversion also for the full sample period, although benefits are visible for all levels of risk-aversion.

To sum it up, the extent of the risk-return improvements is hereby dependent on the level of risk aversion, with a higher risk-aversion generally resulting in more benefits. Especially during and shortly after the Covid-19 pandemic (and thus during times of crises) do the benefits depend heavily on risk-aversion as specific levels showed no benefits at all. Nonetheless, can be concluded that the inclusion of selected Cryptocurrencies into well-diversified portfolios (of for instance institutional investors) indeed improves the portfolios´ risk-return characteristics.

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9. APPENDIX

The appendix includes Figure 1, Figure 2, Table 5 and Table 6.

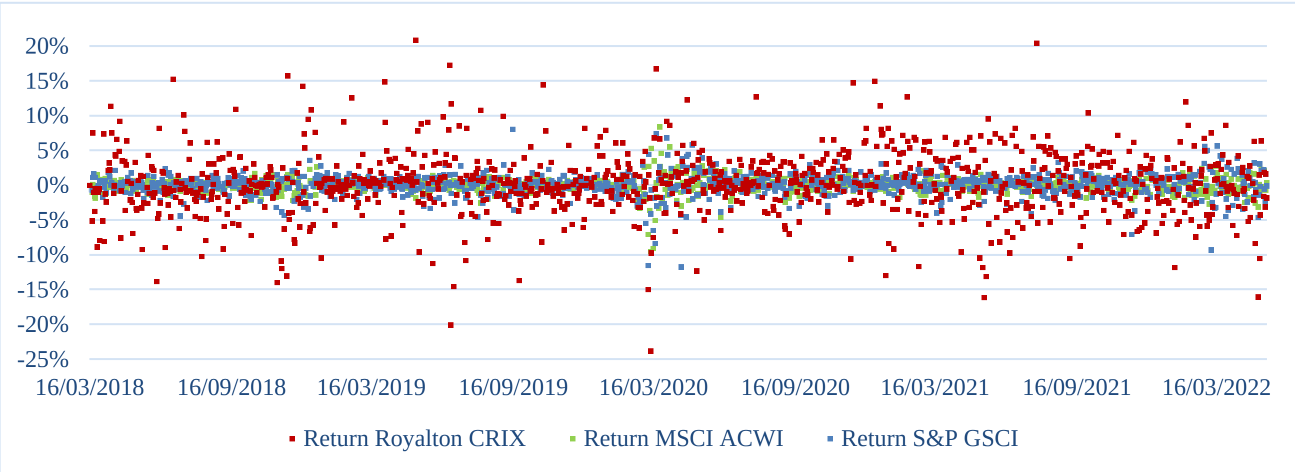

Figure 1: Daily Returns – CRIX, Stocks, and Commodities

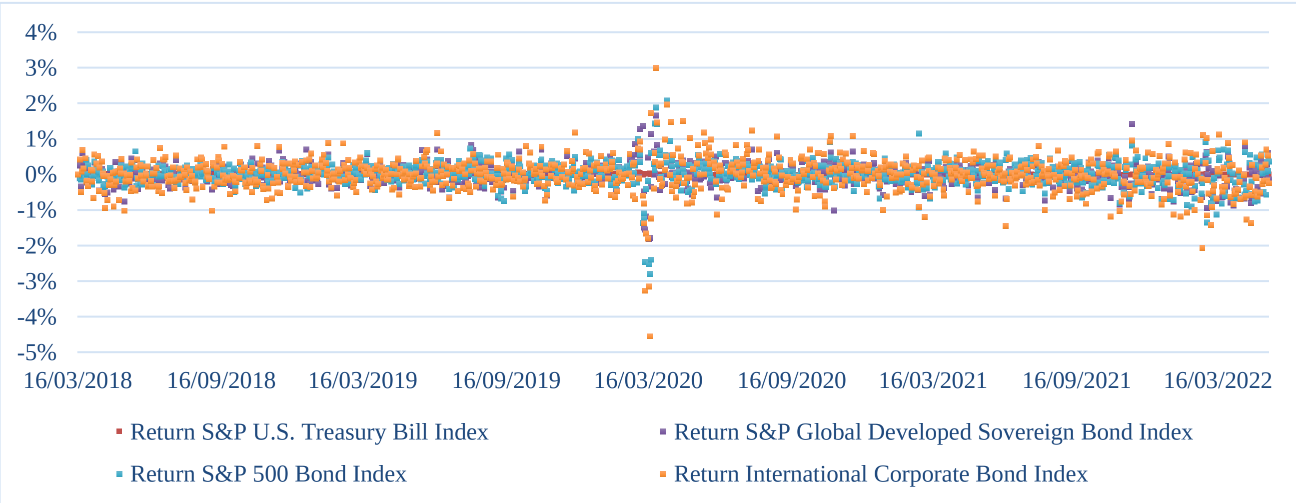

Figure 2: Daily Returns - Bond indices

Table 5: Sub-period Parameter Estimates with Robust Standard Errors

Period	Parameter	B	Robust Std. Error[a]	t	p-value	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
First sample period	Intercept	-0.264	0.426	-0.621	0.535	-1.100	0.572
	Return MSCI ACWI	-0.147	0.471	-0.311	0.756	-1.072	0.779
	Return S&P USTBI	25.342	36.776	0.689	0.491	-46.937	97.621
	Return S&P GDSBI	-1.037	1.611	-0.644	0.520	-4.203	2.129
	Return S&P 500BI	0.593	1.215	0.488	0.625	-1.795	2.982
	Return S&P ICIB	0.273	1.130	0.242	0.809	-1.948	2.495
	Return S&P GSCI	-0.028	0.211	-0.132	0.895	-0.442	0.386
Second sample period	Intercept	0.352	0.181	1.949	0.052	-0.003	0.707
	Return MSCI ACWI	1.342	0.247	5.436	0.000	0.857	1.827
	Return S&P USTBI	-9.147	32.137	-0.285	0.776	-72.263	53.970
	Return S&P GDSBI	2.814	1.515	1.857	0.064	-0.162	5.791
	Return S&P 500BI	-1.681	0.818	-2.055	0.040	-3.287	-0.075
	Return S&P ICIB	-0.388	0.758	-0.512	0.609	-1.877	1.101
	Return S&P GSCI	0.238	0.138	1.720	0.086	-0.034	0.510

Notes: Dependent Variable: Return CRIX, a - HC3 method used to calculate robust Standard errors, Values rounded to 3 decimals

First sample period: Multiple R-squared: 0.00216, Adjusted R-squared: -0.01145

Second sample period: Multiple R-squared: 0.1689, Adjusted R-squared: 0.1605

Table 6: Correlation Matrix rows for CRIX and benchmark asset daily returns

Period		MSCI ACWI	S&P USTBI	S&P GDSBI	S&P 500 BI	S&P ICBI	S&P GSCI
Full sample period	CRIX	0.2504	-0.0493	0.0341	0.0179	0.1196	0.1386
First sample period	CRIX	-0.0198	0.0299	-0.0083	0.0153	-0.0104	-0.0141
Second sample period	CRIX	0.3795	-0.0803	0.0606	0.0236	0.1859	0.2189

Note: Values rounded to 4 decimals

10. REFERENCES

Akhtaruzzaman, M., Sensoy, A., & Corbet, S. (2020). The influence of Bitcoin on portfolio diversification and design. *Finance Research Letters*, *37*, 101344.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of financial markets*, *5*, 31-56.

Andonov, A., & Rauh, J. D. (2018). The return expectations of institutional investors. *Erasmus University Stanford GSB, Hoover Institution, and NBER.–2017*.

Baur, A. W., Bühler, J., Bick, M., & Bonorden, C. S. (2015). Cryptocurrencies as a disruption? empirical findings on user adoption and future potential of bitcoin and co. Conference on e-Business, e-Services and e-Society,

Białkowski, J. (2020). Cryptocurrencies in institutional investors' portfolios: Evidence from industry stop-loss rules. *Economics Letters*, *191*, 108834.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters*, *20*, 192-198.

Briere, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management*, *16*, 365-373.

Carlomagno, G., & Albagli, E. (2022). Trade wars and asset prices. *Journal of International Money and Finance*, *124*, 102631.

Chaim, P., & Laurini, M. P. (2018). Volatility and return jumps in bitcoin. *Economics Letters*, *173*, 158-163.

Chuen, D. L. K., Guo, L., & Wang, Y. (2017). Cryptocurrency: A new investment opportunity? *The journal of alternative investments*, *20*, 16-40.

Corbet, S., Meegan, A., Larkin, C., Lucey, B., & Yarovaya, L. (2018). Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters*, *165*, 28-34.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *The review of Financial studies*, *22*, 1915-1953.

DeRoon, F. A., & Nijman, T. E. (2001). Testing for mean-variance spanning: a survey. *Journal of Empirical Finance*, *8*, 111-155.

Eisl, A., Gasser, S. M., & Weinmayer, K. (2015). Caveat emptor: Does Bitcoin improve portfolio diversification? *Available at SSRN 2408997*.

Giudici, G., Milne, A., & Vinogradov, D. (2020). Cryptocurrencies: market analysis and perspectives. *Journal of Industrial and Business Economics*, *47*, 1-18.
Huberman, G. U. R., & Kandel, S. (1987). Mean-Variance Spanning. *The Journal of Finance*, *42*, 873-888.
Kampakis, S. (2022). Auditing Tokenomics: A Case Study and Lessons from Auditing a Stablecoin Project. *The Journal of The British Blockchain Association*, 34696.
Kan, R., & Zhou, G. (2008). Tests of mean-variance spanning. AFA 2001 New Orleans Meetings, OLIN Working Paper,
Krafft, P. M., Della Penna, N., & Pentland, A. S. (2018). An experimental study of cryptocurrency market dynamics. Proceedings of the 2018 CHI conference on human factors in computing systems,
Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*, 77-91.
Petukhina, A., Trimborn, S., Härdle, W. K., & Elendner, H. (2020). Investing with Cryptocurrencies–evaluating their potential for portfolio allocation strategies. *Quantitative Finance*, *2021*, 1-29.
Platanakis, E., & Urquhart, A. (2019). Portfolio management with cryptocurrencies: The role of estimation risk. *Economics Letters*, *177*, 76-80.
Scholtens, B., & Spierdijk, L. (2010). Does money grow on trees? The diversification properties of US timberland investments. *Land Economics*, *86*, 514-529.
Sharpe, W. F. (1998). The sharpe ratio. *Streetwise–the Best of the Journal of Portfolio Management*, 169-185.
Tobin, J. (1958). Liquidity preference as behavior towards risk. *The review of economic studies*, *25*, 65-86.
Trimborn, S., & Härdle, W. K. (2018). CRIX an Index for cryptocurrencies. *Journal of Empirical Finance*, *49*, 107-122.
Vidal-Tomás, D. (2021). Transitions in the cryptocurrency market during the COVID-19 pandemic: A network analysis. *Finance Research Letters*, *43*, 101981.
Worzala, E., Sirmans, G., & Zietz, E. (2000). Risk and return perceptions of institutional investors. *Journal of Real Estate Portfolio Management*, *6*, 153-166.

10.1 Data sources

10.1.1 S&P Global

Royalton CRIX

https://www.spglobal.com/spdji/en/custom-indices/royalton-partners-ag-rpag/royalton-crix-crypto-index/#overview

S&P U.S. Treasury Bill Index

https://www.spglobal.com/spdji/en/indices/fixed-income/sp-us-treasury-bill-index/#overview

S&P Global Developed Sovereign Bond Index

https://www.spglobal.com/spdji/en/indices/fixed-income/sp-global-developed-sovereign-bond-index/#overview

S&P 500 Bond Index

https://www.spglobal.com/spdji/en/indices/fixed-income/sp-500-bond-index/#overview

S&P International Corporate Bond Index

https://www.spglobal.com/spdji/en/indices/fixed-income/sp-international-corporate-bond-index/#overview

S&P GSCI (Commodities)

https://www.spglobal.com/spdji/en/indices/commodities/sp-gsci/#overview

10.1.2 MSCI

MSCI ACWI

https://www.msci.com/end-of-day-history?chart=regional&priceLevel=0&scope=R&style=C&asOf=Apr%2014,%202022&currency=15&size=36&indexId=106

10.1.3 Other

Coin Market Cap Website

https://coinmarketcap.com/all/viels/all/