

Analysis of Bitcoin Influence on Return on Risk

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ABSTRACT,

This study investigates whether the inclusion of Bitcoin in an already well-diversified portfolio can bring diversification benefits. This is done by investigating the risk-return properties of two identical portfolios, with the only difference between the portfolios being the inclusion of Bitcoin, during the period from the 1st of January 2016 to the end of 2021. A mean-variance spanning test together with a Wald test to assess spanning will be used in order to test whether the inclusion of Bitcoin provides a statistically significant difference. This study concludes that during the investigated period, the benchmark portfolio would have had better mean-variance tradeoffs if it included Bitcoin compared to without Bitcoin. In other words, Bitcoin has the potential to improve the Sharpe ratio and mean-variance tradeoffs of the benchmark portfolio while having low levels of correlation with the benchmark assets, which is beneficial for the portfolio's return on risk.

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Keywords

Finance, Cryptocurrency, Bitcoin, Return, Risk, Diversification, Mean-Variance Spanning

1. INTRODUCTION

Investors all over the world are constantly searching for assets that provide high returns while exposing their portfolios to minimal risk. In recent years, cryptocurrency has slowly become a more mainstream investment. This is largely thanks to the nearly unparalleled performance of cryptocurrencies. Many stockbrokers recently even started offering the option to invest in cryptocurrency Exchange Traded Funds, most of those ETFs tracking the price of the cryptocurrency Bitcoin.

Despite the tremendous performance of these cryptocurrencies over recent years, their prices have fluctuated quite a bit. The cryptocurrency Ethereum during the year 2021 is a strong example of these fluctuations. Starting the year at a price of around \$730, its price soared to an all-time high of \$4,865 on the 10th of November the same year. This success was not sustainable long term, since on the last day of the year Ethereum's price was already around \$3,700 (CoinMarketCap, 2022a). Because of this volatility, many investors wonder whether cryptocurrencies are a worthwhile investment.

Out of all these cryptocurrencies, Bitcoin is by far the most known asset. Consequently, it is also the most invested cryptocurrency, having a market capitalization of 695.55 billion U.S. dollars on 16 March 2022 (CoinMarketCap, 2022b). The worldwide interest in Bitcoin is the main reason that specific cryptocurrency is the one being investigated in this paper.

This research aims to explore the effects of adding Bitcoin to an investment portfolio. This in turn can provide valuable insights for investors, since many investors are interested in investing in cryptocurrency, despite its volatility. If this research is done correctly and yields representative results, investors will be able to use it as a tool when deciding whether to add Bitcoin to their portfolio or not. This could lead to a crossroads for investors. Bitcoin will either prove to be an asset with a viable return on risk or not. If it turns out to be that Bitcoin has no viable return on risk, investors could be protected from including Bitcoin in their portfolio, which would protect their portfolio from exposure to unnecessary risk. On the other hand, if Bitcoin turns out to provide a viable return on risk, more investors would be able to invest in Bitcoin, knowing it is not just a gamble, but a well-considered risk. Currently, Bitcoin is a popular asset among smaller, more speculative investors. However, investors with vast amounts of capital, such as various hedge funds on Wall Street, are also interested in investing in Bitcoin (Choe, 2021). One of the first questions they need an answer to before committing to such an investment however is, does Bitcoin improve the risk-return characteristic of our portfolio? A lot of research has previously been conducted on this subject, however recent events have seriously affected Bitcoin's performance and in turn its return on risk. Examples of these events include the ongoing pandemic, Bitcoin being introduced as legal tender in El Salvador, China's ban on all cryptocurrency transactions, and more. This research intends to measure the return on risk of Bitcoin, specifically investigating the asset from 2016 to 2021. This effort will contribute to existing research towards mapping Bitcoin's volatility and price action.

This paper will try to answer the aforementioned question regarding the improvement of risk-return characteristics in a portfolio and research the effect of adding Bitcoin to a well-diversified portfolio on that portfolio's return on risk using the more recent data of the period from 2016 to 2021. This has led to

the following research question, for which the answer will be sought during this research:

"To what extent does the inclusion of Bitcoin in a diversified portfolio affect that portfolio's return on risk?"

2. LITERATURE REVIEW

Despite cryptocurrencies' relatively short existence as an asset class, various types of research relating to both the return and risk of various cryptocurrencies have been conducted over the years. One tool often used by both investors and researchers when investigating the return and risk of an asset, is the Sharpe ratio. Originally developed by William F. Sharpe, the Sharpe ratio indicates the average return one obtains per unit of increase in risk. This gives an investor the ability to compare the performance of a particular asset against one or multiple benchmark assets. In turn, the investor will be able to decide whether the inclusion of the investigated assets in his/her portfolio will provide a worthwhile return on risk. However, while the Sharpe ratio is a useful tool for investors, it has its limitations. The ratio uses the standard deviation of assets to measure the total risk. This works when returns are distributed normally, but this is seldom the case (Gatfaoui, 2015). Furthermore, Sharpe ratios can be manipulated by portfolio managers in order to make their apparent performance improve. This can be done by increasing the measurement interval (Tavaga, 2022).

One key point of interest of researchers regarding cryptocurrencies is the asset class's level of volatility. All classes of investments are affected by volatility to an extent. Volatility, a measure of dispersion of returns (Hayes, 2021), affects most cryptocurrencies quite seriously, even more so than most other asset classes. The cryptocurrency researched in this essay, Bitcoin, follows this trend of displaying a high level of price volatility compared to other assets and a low correlation to these assets (Brière et al., 2015).

Despite this lower level of correlation to most assets, even Bitcoin has shown to be affected by "spillover". The spillover effect can be described as an instance in which an unrelated event in one country can affect the economies of other countries (Kenton, 2020). While Bitcoin is a decentralized currency, meaning that there is not one country in which Bitcoin or its developers are situated, it still experiences spillover from other markets at times. A clear example of this spillover effect taking place would be during March of 2020. During this period many countries established some type of lockdown, with the exact rules and regulations differing per country. These lockdowns shook up the international stock market, causing many stocks to plummet. To give an example, the S&P 500 (SPY) dropped from \$298.21 on the 1st of March to \$228.02 two weeks later on the 15th of March, a drop of roughly 23,5% (Yahoo Finance, 2022). During the same period, Bitcoin experienced a price decrease from \$8,562.45 to \$5,392.31, a drop of 37% (Coinmarketcap, 2022c). Furthermore, the European Central Bank (ECB) believes that cryptocurrencies are a poor diversification tool (Brasser, 2022). This belief is a result of the increase in crypto trading by hedge funds. Because of this, a drop in the price of certain stocks, most often those in the technology sector, can negatively affect cryptocurrencies and vice versa (Brasser, 2022). This unpredictable vulnerability to spillover only contributes to the volatile nature of Bitcoin.

Moreover, Bitcoin's volatile nature, Koutmos (2019) mentions that Bitcoin keeps switching between two regimes, one of high volatility and one of low volatility. While this "high volatility regime" does reward higher mean returns relative to the "low

volatility regime”, investing during a “high volatility regime” will not necessarily be more rewarding due to more exposure to higher moment risk. From that is deducted that Bitcoin is only useful as diversification of an investment portfolio if one is aware of time-dependent regime-shifting dynamics and how to exploit these regimes (Koutmos, 2019).

Furthermore, Bitcoin is often compared to gold. This might be thanks to Bitcoin’s nickname as “digital gold”. However, research comparing Bitcoin’s volatility and standard deviation of daily returns from the period January 2010 to February 2014 against that of gold showed that their utility as an asset is nothing alike. Especially the maximum standard deviation of gold and Bitcoin during that period showed a massive difference, being 2.2% and 16% respectively (Dwyer, 2015).

However, not all research on Bitcoin has come to a negative conclusion regarding the inclusion of the digital currency in an investment portfolio. Brière et al. (2015) found that Bitcoin provides high returns, despite the currency’s high level of risk. As mentioned before, Bitcoin has a low correlation with other assets. This low correlation combined with high returns can make Bitcoin a valuable diversification tool in a well-diversified portfolio, improving said portfolio’s return on risk (Brière et al., 2015). Reading this paragraph, one might think that Bitcoin is a good inclusion in a well-diversified portfolio. Two caveats have to be kept in mind however. Namely, the data used in the study by Brière et al. was from the period of 2010 to 2013. In other words, the data is over a decade old, which is a long time for an evolving asset class. This could mean that the data analyzed in Brière et al.’s research is not relevant to Bitcoin’s current price action. Furthermore, past performance of an asset is no guarantee of future results.

3. HYPOTHESIS

Dwyer (2015) has done research into Bitcoin’s volatility and compared the monthly standard deviation of daily log-returns of Bitcoin to that of gold. He found that during the period of January 2010 to February 2014, both the mean and median standard deviation of gold were lower than that of Bitcoin. Especially the maximum standard deviation of gold and Bitcoin during that period showed a massive difference, being 2.2% and 16% respectively. Dwyer’s (2015) research on Bitcoin volatility has led to the following hypothesis:

“Including Bitcoin in a diversified portfolio will negatively affect that portfolio’s return on risk.”.

4. METHODOLOGY

In order to measure whether the inclusion of Bitcoin in an established portfolio improves said portfolio’s performance, a mean-variance spanning test similar to the test used by Brière et al. (2015) will be used. This test was originally proposed by Huberman and Kandel (1987) with one of the purposes of this test being a means to test whether the inclusion of an investigated asset in an existing portfolio can bring diversification benefits (Zhou & Kan, 2000). This makes mean-variance spanning useful for this research in order to determine whether the inclusion of Bitcoin in an existing portfolio will bring benefits.

In order to determine the distribution of weights for each of the portfolios, the optimal weight distribution for maximization of the Sharpe ratio has been calculated. Short positions were excluded when calculating these weights. The Sharpe ratio indicates the average return one obtains per unit of increase in risk. A part of calculating a Sharpe ratio is subtracting the current

risk-free rate from the return of the investigated asset or portfolio. This gives an investor insight into the potential profits associated with risk-taking activities (Fernando, 2022). The Sharpe ratio can be calculated using the following formula:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

In this formula, R_p is the return of the portfolio or the asset being investigated, R_f is the risk-free rate and σ_p is the standard deviation of the portfolio’s or asset’s excess return. Investors are interested in maximizing this Sharpe Ratio, which maximizes return on risk.

The first step consists of conducting a regression of the weekly return of the portfolio consisting of the benchmark assets and Bitcoin on the weekly returns of the portfolio with just the benchmark assets. The following formula is necessary for this regression:

$$r_{BTC} = \alpha + \beta R_{\text{Benchmark}} + \varepsilon$$

With the necessary and sufficient condition for spanning being:

$$H_0: \alpha = 0 \quad \text{and} \quad \sum_{k=1}^k \beta_k = 1$$

In this formula, the returns of the portfolio consisting of just the benchmark assets is denoted by the vector $R_{\text{Benchmark}}$, the returns of Bitcoin are denoted by the vector r_{BTC} , and ε is the error. In this case, the expected value of ε is zero.

The next step would be to run a Wald test, also known as the Wald-Chi Squared Test, to check for spanning. The Wald test statistic has the following distribution (Berndt and Savin, 1977):

$$W = T(\lambda_1 + \lambda_2) \sim \chi^2,$$

Where:

$$\lambda_1 = \max_r \frac{1 + \hat{\theta}_2^2(r)}{1 + \hat{\theta}_1^2(r)} - 1, \quad \lambda_2 = \min_r \frac{1 + \hat{\theta}_2^2(r)}{1 + \hat{\theta}_1^2(r)} - 1$$

In this formula, $\hat{\theta}_1^2(r)$ is the Sharpe ratio of the risk-free rate and $1 + \hat{\theta}_2^2(r)$ is the Sharpe ratio of the portfolio consisting of both the benchmark assets and Bitcoin. This statistic can give insight into which variables contribute something significant to the model.

Lastly, the efficient frontier of both portfolios will be calculated in order to provide a visual representation of what returns each of the portfolios can provide at which level of risk. In other words, it shows how much risk the portfolio is exposed to at every level of expected return.

5. DATA DESCRIPTION

The study sample includes Bitcoin and a variety of indices as benchmarks. The weekly close data of these assets during the period from 2016 to 2021 will be analyzed for this research. The reasoning behind the choice for weekly data is that most stock exchanges are closed during weekends, while crypto markets remain open. This makes it difficult to compare daily data between stocks and cryptocurrencies, making weekly data a good alternative. Furthermore, Brière et al. (2015) conducted their research on Bitcoin using the weekly data of the asset. By also using weekly data, this research will be conducted in a similar way. The choice for the time period of 2016 to 2021 is made in order to investigate recent data, compared to Brière et al.’s data frame of 2010 to 2013, and to investigate a data set that is somewhat homogeneous. At the start of 2022, many cryptocurrency prices began to decrease drastically, including the price of Bitcoin. Including this data in the research might

negatively affect the homogeneity of the data. An example of these unproportionally negative results can be found in the charts below.

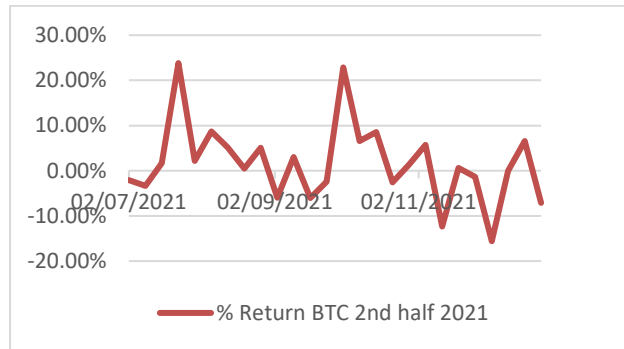


Chart 1. Weekly returns data Bitcoin in % during the second half of 2021

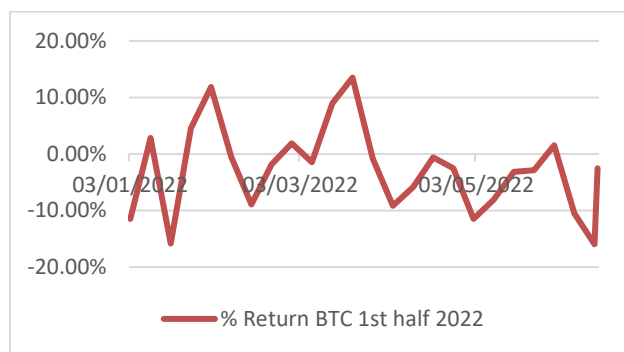


Chart 2. Weekly returns data Bitcoin in % during the first half of 2022

The indices that will be investigated are Standard & Poor's 500 index, the Bloomberg US Aggregate Bond Index, a gold index, and a general agricultural commodities index. These indices are United States-based in order to ensure data availability. For the risk-free rate the average yield of the US 5 Year Treasury Rate during the investigated time period was used, which was 1.55%.

The specific ETFs that will be used are listed in table 1. Furthermore, table 2 provides some standard information on the weekly returns of the investigated assets during the period of 2016 to the end of 2021. Lastly, table 3 displays the correlation between Bitcoin and the other benchmark assets.

Symbol	ETF name	Type of asset
SPY	SPDR S&P 500 ETF Trust	Stocks
AGG	iShares Core U.S. Aggregate Bond ETF	Bonds
GLD	SPDR Gold Shares	Commodity (gold)
DBA	Invesco DB Agriculture Fund	Commodity (agricultural)

Table 1. Benchmark Assets

Symbol	Average	Max	Min	St. Dev.
BTC	2.10%	74.91%	-45.25%	11.21%
SPY	0.35%	10.47%	-17.97%	2.29%
AGG	0.06%	5.63%	-5.68%	0.65%
GLD	0.17%	11.01%	-6.69%	2%
DBA	0.02%	5.03%	-7.63%	1.79%

Table 2. Data on weekly returns of investigated assets during the period 01-01-2016 to 31-12-2021

	BTC	SPY	AGG	GLD	DBA
BTC	1				
SPY	0.21	1			
AGG	0.12	0.31	1		
GLD	0.08	0.20	0.57	1	
DBA	0.08	0.37	0.11	0.19	1

Table 3. Correlation coefficients between the returns of the investigated assets during the period 01-01-2016 to 31-12-2021

When looking at table 3, especially the first column is of interest. It shows that BTC has only a slight positive relationship with each of the investigated assets. This indicates that Bitcoin is not really affected by the performance of the other assets. However, if a large event were to happen to the stock market (e.g. the COVID-19 pandemic in 2021) and the whole market would suffer due to such an event, Bitcoin will be affected as well. It is worth noting that the correlation between the other assets is low to moderate as well. If there were substantial levels of correlation between the benchmark assets, they would not be able to perform the role of a well-diversified portfolio.

Data regarding the performance of the investigated assets will be collected from open sources, such as Yahoo Finance and CoinMarketCap. If necessary, Refinitiv Eikon can be used as a source as well. Access to this source is provided by the University of Twente.

6. RESULTS

The following tables showcase various potential weight distributions of the two investigated portfolios and their respective return, standard deviation, and Sharpe ratio.

	<i>Equal w</i>	<i>Max μ</i>	<i>Min σ</i>	<i>Max Sharpe</i>
SPY	25%	100%	0%	44%
AGG	25%	0%	91%	43%
GLD	25%	0%	0%	14%
DBA	25%	0%	9%	0%
Sum w	100%	100%	100%	100%
μ	0.15%	0.35%	0.06%	0.20%
σ	1.15%	2.29%	0.63%	1.23%
Sharpe	0.131898	0.153297	0.096664	0.165861

Table 4. Weight distributions benchmark portfolio and respective performance indicators

	<i>Equal w</i>	<i>Max μ</i>	<i>Min σ</i>	<i>Max Sharpe</i>
BTC	20%	100%	0%	12%
SPY	20%	0%	0%	39%
AGG	20%	0%	91%	31%
GLD	20%	0%	0%	18%
DBA	20%	0%	9%	0%
Sum w	100%	100%	100%	100%
μ	0.54%	2.10%	0.06%	0.44%
σ	2.58%	11.19%	0.63%	1.91%
Sharpe	0.209881	0.187401	0.096664	0.228138

Table 5. Weight distributions portfolio including Bitcoin and respective performance indicators

The two tables above provide insight into what Bitcoin provides for an investor. When looking at the maximum return of the portfolio including Bitcoin, it is shown that that level of return can only be achieved by investing 100% of the funds in Bitcoin. The opposite is shown when looking at the minimum variance portfolio, Bitcoin will always be excluded in that case. Furthermore, Bitcoin is invested when creating the mean-variance portfolio of the benchmark assets and Bitcoin, unlike the Agriculture Fund DBA. This means that adding Bitcoin can help improve the Sharpe ratio properties of a portfolio. In short, Bitcoin provides unconventionally high returns at the cost of higher volatility.

The following table displays the result of the spanning test that was conducted. In other words, it shows the statistic of the test that checks whether Bitcoin spans the assets included in the benchmark portfolio.

	<i>Wald statistic Huberman & Kandel Spanning test</i>
Portfolio of benchmark assets	3.35

Table 6. Spanning test results

From the statistic provided in table 6 can be deduced that Bitcoin significantly spans the portfolio of the benchmark assets during the period from the 1st of January 2016 to the 31st of December 2021. This implies that during that period, the benchmark portfolio would have had better mean-variance tradeoffs if it included Bitcoin compared to without Bitcoin.

In appendices A and B the efficient frontiers of each of the portfolios can be found. While the two scatterplots look similar, there is a distinct difference. As shown in appendix A, the portfolio including Bitcoin is much steeper than the scatterplot seen in appendix B, which showcases the frontier of the portfolio without Bitcoin. This means that the portfolio including Bitcoin has the ability to provide higher returns when compared to the portfolio without the cryptocurrency, albeit at the cost of having a higher standard deviation and thus volatility.

7. DISCUSSION

The research question to which answer was sought during this research was as follows:

“To what extent does the inclusion of Bitcoin in a diversified portfolio affect that portfolio’s return on risk?”.

When looking at the results, Bitcoin does display the ability to improve the return on risk properties of a diversified portfolio, since Bitcoin yields high returns while simultaneously exhibiting high levels of volatility. This high volatility can be compensated by investing in other assets that have a low correlation with Bitcoin (Brière et al., 2015). This is in line with Brière et al.’s (2015) findings, who conclude that Bitcoin can be useful as a diversification tool in an already diversified portfolio and that Bitcoin deserves to be taken seriously by investors. While this research yields very similar results to that of Brière et al. (2015), this research investigates a largely unresearched period. This is helpful for better charting the performance of Bitcoin in the long, since it remains a new and continuously evolving asset.

There are some limitations to this study, however. Firstly, all of the benchmark assets are United-States based funds. While the correlation between the benchmark assets was largely limited (see table 3), there remains a possibility that these assets influence each other more than a portfolio using assets from all over the world would. Secondly, more competitive benchmark assets could have been chosen. In table 4, various weight distributions of the benchmark portfolio excluding Bitcoin are displayed. However, it shows that DBA is not even included in the portfolio that maximizes the Sharpe ratio. In other words, the Invesco DB Agriculture Fund does not contribute much to this research, since it is not even included in the mean-variance efficient portfolio. Thirdly, the timeframe of this research is only up to the 31st of December 2021, but Bitcoin displayed real poor performance in the first half of 2022. This data was left out in order to improve the homogeneity of the researched data, but research including this data might yield interesting and maybe different results than this research.

8. CONCLUSION

The purpose of this study was to investigate whether the inclusion of Bitcoin in an already well-diversified portfolio could improve said portfolio’s return on risk properties. The hypothesis that was formed was as follows:

“Including Bitcoin in a diversified portfolio will negatively affect that portfolio’s return on risk.”.

The findings reveal that Bitcoin has the potential to improve the Sharpe ratio and mean-variance tradeoffs of the benchmark portfolio while having low levels of correlation with the benchmark assets. This means that we fail to support the previously formed hypothesis.

This study is limited to an extent by the choice of benchmark assets and the choice of timeframe. Both could be more extensive in order to yield more representative results.

By looking at unexplored time periods regarding the performance of Bitcoin, this research is able to add value to previous studies regarding the return on risk properties of Bitcoin.

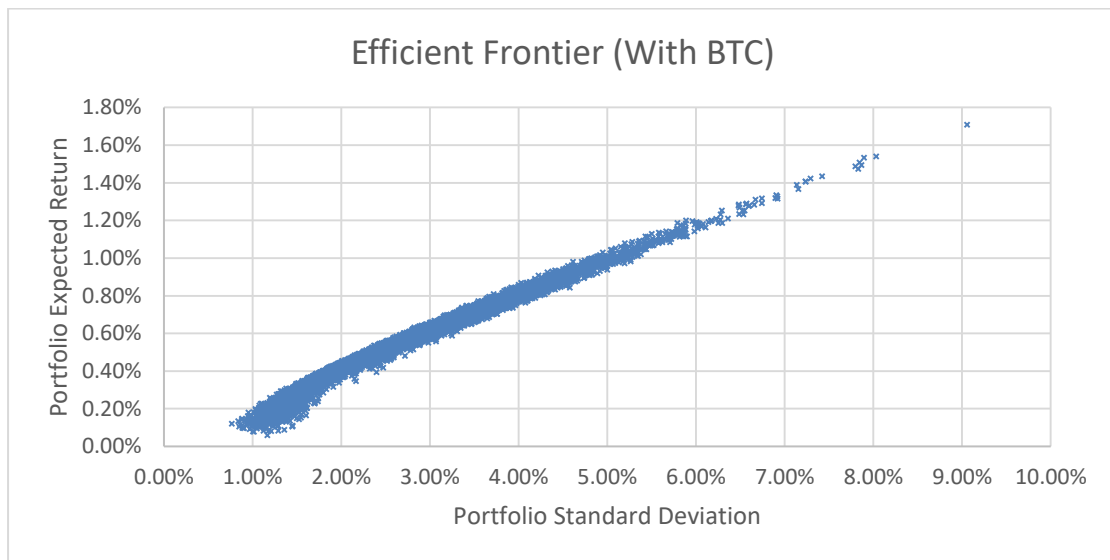
For future research, the timeframe could be expanded more, both further in the past and the future. With this, a more extensive analysis of Bitcoin’s performance could be performed. Furthermore, it might be interesting to compare the performance of Bitcoin to that of other cryptocurrencies. While Bitcoin is currently by far the most popular crypto, other digital currencies might yield superior results compared to Bitcoin and should therefore be researched.

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10. APPENDIX

10.1 Appendix A



10.2 Appendix B

