Placement of Electric Vehicle Charging Stations in Congestion Games

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This paper extends Rosenthal's [9] discrete congestion game model in which selfish, rational players minimize travel time. We extend the model by including electric vehicle charging stations. We study the influence of charging station placements on the efficiency of a Nash equilibrium. To that end, we built a simulation tool that can find the social optimum, the worst case, and a Nash equilbrium given an instance of the model.

We show that Braess's paradox [1] can occur when placing electric vehicle chargers, where extra charging facilities can paradoxically lead to higher costs. We show Braess's paradox in two ways: analytically in an example, and in our simulations. Furthermore, we show that good placement of charging stations can positively affect costs.

Additional Key Words and Phrases: Electric Vehicle Charging, Nash Equilibria, Game Theory, Routing Games, Congestion Games

1 INTRODUCTION

Over the past decades, it has become common knowledge that human activities negatively affect the Earth's climate. Of particular interest is our reliance on fossil fuels, which is a large contributor to climate change [5]. Electric vehicles are rapidly gaining ground as a response to this, but they also bring new challenges. Some of those are of a technical nature, such as the impact of charging on electricity networks [3]. Other challenges include changing human behaviour, as this is necessary for adoption of electric vehicles. For example, Mashhoodi [4] looks into the impact of walkability from the users' home to a charging station on the use of the charging station.

The driving range of vehicles is limited by the amount of energy they can carry. When a conventional petrol vehicle is out of energy, it can rapidly refill at a gas station, of which there are plenty. For electric vehicles, substantial charging infrastructure still needs to be built. It is paramount to place such infrastructure at strategic positions. [3]

In this paper, we extend a well-studied congestion game[1] model from Rosenthal [9] to include electric vehicle charging facilities. We show that correct placement of such charging stations can shift the resulting Nash equilibria towards the social optimum, whereas poor placement can worsen the situation in a manner similar to Braess's paradox [1]. The social optimum is the situation with the lowest total cost, where the cost for a player is equal to their travel time.

Furthermore, we developed a program to simulate instances of our model. The simulation tool can be used to plan new electric vehicle charging stations and to test different strategies for placing charging stations. We studied two example graphs, varying the capacity and the number of charging stations to study the impact on the costs.

[1]See Appendix B for a definition of congestion games and several other concepts from game theory

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2 LITERATURE REVIEW

The strategic placement of electric vehicle charging stations is investigated in this research. In a recent review of key challenges for the adoption of electric vehicles [3], charging station placement is considered an important factor. One of the reasons that it is important to strategically place charging stations, is that drivers may suffer from so called "range anxiety", where users are scared to adopt electric vehicles for fear of being stranded when vehicle charge runs out. [7] This fear needs to be taken away by providing ample chargers at the right locations.

In 1973, Rosenthal [9] introduced a discrete congestion game model that has been studied well since. In the same paper, Rosenthal shows that pure-strategy Nash equilibria exist for the model. The model from Rosenthal has been analysed, for example in Christodoulou [2] where the price of anarchy is studied. Rosenthal's model has also been extended, for example in Scheffler [10], where a model with edge capacities and priority based on the previous edge is considered.

There are insights from game theory in electric vehicle charging, for example in Xiong [11] where a game theoretic model is applied to a case study of Singapore. However, little research has been done into adapting Rosenthal [9] to include electric vehicle charging stations. In this paper, we research whether the placement of electric vehicle charging stations in a road network influences the Nash equilibrium in an associated congestion game.

3 OUR MODEL

Our model is based on the congestion game model in Rosenthal [9]. A road network is modelled as a directed graph $G = (V, E)$. The edges $e \in E$ all have an associated non-negative and non-decreasing cost function f_e: if n_e players take an edge e, then the cost for taking the edge is $f_e(n_e)$. Each edge e represents a road fragment, and the cost of taking that road equals the time spent to reach the other end.

The vertices can be charging stations, in which case it is possible for a player to charge there. Charging station vertices have an associated non-negative and non-decreasing cost function c_v: if n_v players charge at a vertex v, then the cost for charging at the vertex is $c_v(n_v)$. To reduce complexity, several assumptions are made. First, we assume that all players have identical charging times and that both the charging time and the monetary cost, if applicable, are included in the cost function. Furthermore, we assume that each player shares the same start node s and target node t, so the game is symmetric. Lastly, we assume that each player can travel from the start node to the target note over any path, by charging at exactly one charging station that is not the start or target node.

We consider a set of players $p_1..p_k \in P$. For each player p_i, we consider a set of strategies X_i, which consist of the combination of an s-t path E and a charging vertex Y. For a strategy $x \in X_i$, the cost that player i incurs from using that strategy is the sum of the cost for each edge plus the cost of charging at the charging vertex: $p_i(x) = \sum_{e \in E} f_e(n_e) + c_Y(n_Y)$.

Each player selfishly and rationally chooses a strategy, seeking to minimise their cost. The social optimum is a situation where the combined strategies of all players lead to the lowest possible total cost. A Nash equilibrium is a situation where no players have incentive to switch strategies.

4 RESULTS

4.1 Analytical results: Braess's paradox

Braess's paradox, where adding a road to a road network makes the equilibrium costs of the network higher, can also occur in our model. That is, adding a charger to a network can make costs higher. Intuitively, adding a charger sounds like a good idea. It should give motorists more freedom to choose their route, and less range anxiety. However, a similar intuition holds for adding a road, and as Braess has shown, this intuition is not always correct.

Consider the network in Figure 1. Charging at any charging node is assumed to have a cost of 0; such that the players are not influenced by the type of charger. The edge of cost 0 from node 1 to node 5 is not accessible when only the green nodes, 2 and 4, are charging nodes. However, adding brown node 1 as a charger in the network will make the edge from node 1 to 5 accessible.

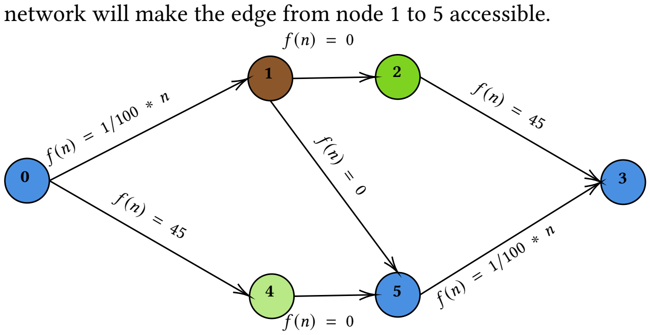

Fig. 1. A network where Braess's paradox occurs. We consider 4000 players travelling from node 0 to node 3.

If 4000 cars travel over the example network from node 0 to node 3, with chargers at nodes 2 and 4 only, then they will be at equilibrium by splitting up exactly over the upper path and the lower path. After all, the edge that connects node 1 and 5 for a third path is still inaccessible. This yields a cost of 2000/100 + 45 = 65 for all players, or 260.000 in total. Players have no incentive to switch paths, as both paths are as expensive as the other.

When we add the charger at node 1, players can travel on the edge from node 1 to node 5. This results in all cars using the newly feasible path, yielding a cost of 4000/100 + 4000/100 = 80 for each player, or 320.000 in total. Players will not switch to a different path either, because the cost would be higher. As such, the equilibrium now equals the worst cost, while the social optimum has not changed.

4.2 Numerical results

We developed a Python program to simulate scenarios. Given an instance of the model as described above, the program can calculate the social optimum, the worst case cost and an equilibrium. It is easy to change which vertices are charging stations, such that comparisons can be made.

The social optimum and the worst case cost are calculated by brute force. We calculate the costs for all combinations of strategies and find the combinations with the lowest and the highest cost.

An equilibrium is found using best response dynamics. All players select an initial strategy in turn. Next, players may switch strategies if this benefits them, a process that we repeat until no player switches strategies. Then, we have found an equilibrium.

In this section, we discuss the results of several simulations that were run. For every setup where a charging vertex v had a limited capacity C_v, the cost for charging at that vertex c_v was given by the charging time t_c if the number n_v of cars at v was smaller than the maximum capacity C_v. If the number of cars at a charger exceeded the charger's capacity, the cost was given by $c_v = t_c + t_c * (n_v - C_v)$ instead.

4.2.1 *Braess.*

We first tested the results from the analysis section about Braess's paradox against results generated by our Python program, giving some added validity to both of those results. We decided to calculate a Nash equilibrium only, because calculating the social optimum and the worst case by brute force was infeasible. The number of calculations needed to simulate 4000 cars over three possible paths is too large.

The graph used is the same as that in Figure 1, and chargers have a cost of 0 as before. Indeed, the results from the Python simulation are the same as those obtained in the analysis. This confirms our analytical results.

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Braess 4000	2	4000	Skipped	Skipped	260000	[2, 4]
Braess 4000	3	4000	Skipped	Skipped	320000	[1, 2, 4]

Fig. 2. Results table for Braess's paradox

4.2.2 *Pigou.*

We ran simulations on a graph similar to Pigou's example [8], which is a well-studied network. The simulations were run four times, where the distribution of cost functions over the nodes varied throughout, while the cost functions for the edges remained fixed. We then calculated the costs for all combinations of charging nodes, using the previously appointed distribution of cost functions. So, taking turns, each node was either included or excluded from the set of chargers until we had calculated the cost of each combination of chargers.

The complete results from the simulations can be found in Appendix A. Summaries are also included in this section. A visualisation of the graph can be found in Figure 3.

The result summary tables include several statistics. Under 'chargers', is the number of nodes that are appointed as chargers during the simulation. Further, the tables include the average and minimum best case cost found for the given number of chargers in the network, and the maximum and average worst case cost for that number of chargers. Lastly, the average, minimum, and maximum cost at equilibrium for the number of selected chargers are included.

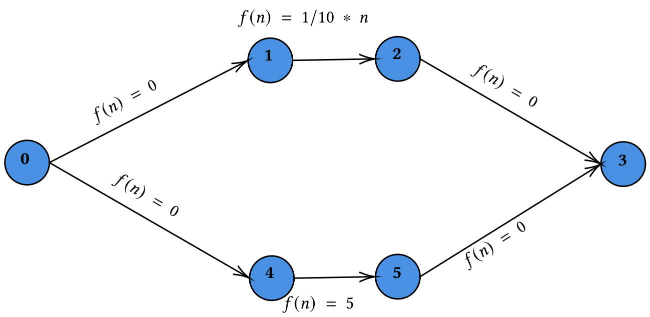

Fig. 3. The Pigou-like network. We consider 100 players travelling from node 0 to node 3.

The first simulation that we ran had chargers with unlimited capacity and a cost of 1. This example is called 'cheap charging'. We would expect that in this example, the equilibrium would be determined wholly by the costs of the edges. After all, there is no cost difference between the different chargers. The results show that this is correct and as long as a charger is available on both paths, the equilibrium is the same as it would be without charging. Naturally, when chargers are available on one of the paths only, all players take that path. Concretely, this shows that if ample charging is available on all paths, the equilibrium will not change.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	850	600	850	1100	850	600	1100
2	641.667	537.5	1016.67	1100	683.333	600	1100
3	537.5	537.5	1100	1100	600	600	600
4	537.5	537.5	1100	1100	600	600	600

Fig. 4. Summary table for Pigou with cheap charging

The next simulation that we ran, had high charging costs. In that simulation, there was a capacity of 20 and a charging time of 20. The limited capacity means that the cost increases rapidly for each additional car above the capacity. Moreover, the base cost of 20 is higher than the maximum total cost for the edges. Now, the equilibrium was determined entirely by the cost of charging, leading to a perfect split over the available chargers regardless of edge cost. This clearly shows that we can influence the routes taken by the placement of charging infrastructure in the system.

Figure 3 shows example output for this charger setup, with chargers at nodes 2, 4 and 5. The players are neatly divided over the chargers as this minimises each player's cost, even though the majority of players are on the more expensive path in terms of edges. This situation occurs due to the dominance of the charging cost over the driving cost in his scenario.

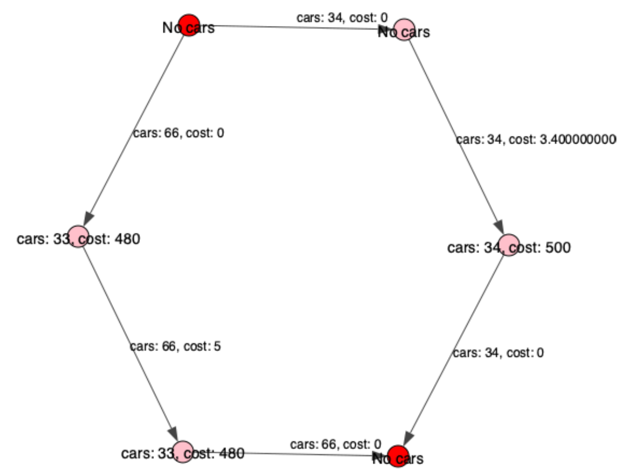

Fig. 5. Example output for Pigou expensive where nodes 2, 4 and 5 are chargers

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	182750	182500	182750	183000	182750	182500	183000
2	82583.3	82500	182917	183000	82583.3	82500	83000
3	49204.7	49123.9	183000	183000	49205.6	49125.6	49285.6
4	32500	32500	183000	183000	32500	32500	32500

Fig. 6. Summary table for Pigou with expensive charging

The third and fourth situations had asymmetric placements, that made the lower or upper paths as visualised in Figure 3 more attractive. The preferred path was assigned significantly cheaper chargers than the other, at a costs of 1 and 20 respectively.

Note that the high worst case costs and maximum equilibrium costs result from the cases where only expensive chargers are available. When 3 or 4 chargers are active, that means that a charger with a cost of 1 must be available. Then, the charging cost is 1 for each car, as the charging cost of the expensive chargers dominates the edge cost in this example. This will cause all cars to select the path with cheap charging. The difference in cost between the situation with cheaper charging stations on the lower path (600), and the situation with cheaper charging on the higher path (1100), thus stems entirely from the difference in edge costs for both paths. As such, these situations show that by applying different types of chargers, players can be nudged towards a desired equilibrium regardless of prior preferences. Clearly, this is contingent on the charging being expensive enough to significantly impact the cost. In reality, the difference in charging cost would likely be smaller than in this example, allowing for useful load balancing over the edges.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	91800	600	91800	183000	91800	600	183000
2	14333.3	600	152600	183000	14333.3	600	83000
3	600	600	183000	183000	600	600	600
4	600	600	183000	183000	600	600	600

Fig. 7. Summary table for Pigou lower path preference

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	91800	1100	91800	182500	91800	1100	182500
2	14666.7	1100	152267	182500	14666.7	1100	82500
3	1100	1100	182500	182500	1100	1100	1100
4	1100	1100	182500	182500	1100	1100	1100

Fig. 8. Summary table for Pigou upper path preference

5 CONCLUSIONS

This research investigated the role of the placement of electric vehicle chargers on Nash equilibria in our adapted congestion game model. From our results, we can conclude that the placement of electric vehicle charging stations in a congestion game model can significantly influence the resulting equilibrium negatively or positively, depending on the particular graph and the chosen placement.

Wrongly placed chargers can negatively impact travel times. This is because placing a new charging station may redirect traffic towards different paths than before, and the resulting equilibrium can be worse than the prior equilibrium. We substantiated this claim both through analytical results, adapting Braess's paradox, and through numerical results.

On the other hand, placing chargers strategically can be beneficial. The possibility of redirecting traffic was shown in the *Pigou* example of our numerical results section. This could be useful for load balancing, preventing congestion, or for keeping cars away from certain areas where they are a nuisance. Enabling players to use preferred paths can yield shorter traffic times and save energy, which could improve people's daily lives and benefit the environment.

6 DISCUSSION

6.1 Models

It is important to note that this research makes use of a model of real world traffic networks. Models inherently and intentionally leave out certain details of the real world, allowing us to study particular features in isolation. This is likely to influence the results, meaning that they may not always completely align with the real world.

Our model, as specified in section 4, imposes some limitations on the numerical results. The Python program as we implemented it, imposes several more. All together, the limitations for situations that can be simulated are as follows:

- All cars have the same range, permitting them to take any path if, and only if, they charge at exactly one node
- All cars have the same starting node and destination node
- Graphs can have a limited size, number of cars, and number of charging stations, where if one grows considerably the others need to shrink

The third limitation mentioned, which is essentially a limitation on the scale of the simulations, stems from the method that is used to compute the Social optimum, the worst case cost and the equilibrium. Particularly, the social optimum and the worst case are currently calculated by brute force. This means that for each extra possible path, charging option, or car, the number of total options that are checked increases rapidly.

Several factors may be problematic. Arrival time at a charging station is critical for whether cars need to wait before they can charge. This is difficult to model in our case, as the cost function impacts all cars in the same manner. This makes sense for edges, but a congested edge will still have quite some spread in arrival times at charging stations. It is important to consider this when modelling the charging stations, and to try to come up with reasonable cost functions.

Regardless of the challenges regarding models presented above, Braess's paradox has been observed in the real world [6]. As such, the model presented in this paper may yet turn out to be useful in deciding where to place charging stations in the real world. For example, our model can help in gathering some suggested spots or even in pointing out where not to place a charging station. It could also help as part of checking a selected location for mistakes or against several other options.

7 FURTHER RESEARCH

7.1 Removing limitations of the model

Further research could try to solve some of the limitations in the model and in the Python program. For example, cars could be permitted to have different initial and maximum ranges, potentially on a per car basis. However, that brings a challenge considering that in reality the order of arrival at a charging station will now greatly impact the charging time at that station. Still, it would be interesting to see how the asymmetric feasibility of paths impacts the results.

Similarly, cars could have different start and end nodes from each other, to allow for modelling of real world traffic which is probably not symmetric. This would also allow for simulations on more realistic road networks. For example, a small road network such as that of Luxembourg seemed interesting at first and is feasible in terms of complexity. However, it is not sensible to simulate traffic with symmetric start and end nodes on such a network considering that the roads spread towards many directions.

7.2 Adding user profiles and multiple cost functions

An extension to the model where different user profiles are considered could be interesting too. For example, some users could prioritise the monetary cost over the time consumed on a road. Including such factors in the model may yield new implications. However, this would only be beneficial in real world applications if some statistics are known on preferences of drivers in a modeled area. More generally, even without user profiles, it would be valuable to extend the model to include separate cost functions for energy usage and time. After all, some roads will allow higher speeds than others, yielding lower cost in terms of time. However, driving at higher speeds also tends to consume more energy.

7.3 Accounting for the electric grid

Our model optimizes for a particular goal: total travel time. This may not always be the only goal to keep in mind. Charging stations also impact the electrical grid [3], which could lead to issues for other energy consumers. It may not always be trivial or feasible to compensate such shortages, and this may not always be possible in a green and renewable manner.

7.4 Larger networks and more examples

It could be beneficial to have a way of simulating larger networks. This could be accomplished through applying various optimizations to the Python program, for example using approximation or improved pre-processing. Additionally, more example graphs could be tried in general, yielding a larger variation in results and potentially some useful suggestions for real-world charger placement.

8 ACKNOWLEDGEMENTS

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A FULL RESULTS

A.1 Braess

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Braess 4000	2	4000	Skipped	Skipped	260000	[2, 4]
Braess 4000	3	4000	Skipped	Skipped	320000	[1, 2, 4]

Fig. 9. Results table for Braess's paradox

A.2 Pigou

A.2.1 Pigou cheap.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	850	600	850	1100	850	600	1100
2	641.667	537.5	1016.67	1100	683.333	600	1100
3	537.5	537.5	1100	1100	600	600	600
4	537.5	537.5	1100	1100	600	600	600

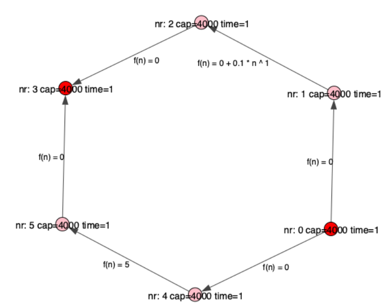
Fig. 10. Summary table for Pigou cheap

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Pigou cheap	1	100	1100	1100	1100	[1]
Pigou cheap	1	100	1100	1100	1100	[2]
Pigou cheap	1	100	600	600	600	[4]
Pigou cheap	1	100	600	600	600	[5]
Pigou cheap	2	100	1100	1100	1100	[1, 2]
Pigou cheap	1	100	1100	1100	1100	[1]
Pigou cheap	2	100	537.5	1100	600	[1, 4]
Pigou cheap	2	100	537.5	1100	600	[1, 5]
Pigou cheap	1	100	1100	1100	1100	[2]
Pigou cheap	2	100	537.5	1100	600	[2, 4]
Pigou cheap	2	100	537.5	1100	600	[2, 5]
Pigou cheap	1	100	600	600	600	[4]
Pigou cheap	1	100	600	600	600	[5]
Pigou cheap	2	100	600	600	600	[4, 5]
Pigou cheap	2	100	1100	1100	1100	[1, 2]
Pigou cheap	1	100	1100	1100	1100	[1]
Pigou cheap	2	100	537.5	1100	600	[1, 4]
Pigou cheap	2	100	537.5	1100	600	[1, 5]
Pigou cheap	1	100	1100	1100	1100	[2]
Pigou cheap	2	100	537.5	1100	600	[2, 4]
Pigou cheap	2	100	537.5	1100	600	[2, 5]
Pigou cheap	1	100	600	600	600	[4]
Pigou cheap	1	100	600	600	600	[5]
Pigou cheap	2	100	600	600	600	[4, 5]
Pigou cheap	2	100	1100	1100	1100	[1, 2]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 4]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 5]
Pigou cheap	2	100	537.5	1100	600	[1, 4]
Pigou cheap	2	100	537.5	1100	600	[1, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 4, 5]
Pigou cheap	2	100	537.5	1100	600	[2, 4]
Pigou cheap	2	100	537.5	1100	600	[2, 5]
Pigou cheap	3	100	537.5	1100	600	[2, 4, 5]
Pigou cheap	2	100	600	600	600	[4, 5]
Pigou cheap	2	100	1100	1100	1100	[1, 2]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 4]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 5]
Pigou cheap	2	100	537.5	1100	600	[1, 4]
Pigou cheap	2	100	537.5	1100	600	[1, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 4, 5]
Pigou cheap	2	100	537.5	1100	600	[2, 4]
Pigou cheap	2	100	537.5	1100	600	[2, 5]
Pigou cheap	3	100	537.5	1100	600	[2, 4, 5]
Pigou cheap	2	100	600	600	600	[4, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 4]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 5]
Pigou cheap	4	100	537.5	1100	600	[1, 2, 4, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 4, 5]
Pigou cheap	3	100	537.5	1100	600	[2, 4, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 4]
Pigou cheap	3	100	537.5	1100	600	[1, 2, 5]
Pigou cheap	4	100	537.5	1100	600	[1, 2, 4, 5]
Pigou cheap	3	100	537.5	1100	600	[1, 4, 5]
Pigou cheap	3	100	537.5	1100	600	[2, 4, 5]
Pigou cheap	4	100	537.5	1100	600	[1, 2, 4, 5]
Pigou cheap	4	100	537.5	1100	600	[1, 2, 4, 5]

Fig. 11. Results table for Pigou cheap

Visualisation:

Fig. 12. Visualisation of Pigou cheap



A.2.2 Pigou expensive.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	182750	182500	182750	183000	182750	182500	183000
2	82583.3	82500	182917	183000	82583.3	82500	83000
3	49204.7	49123.9	183000	183000	49205.6	49125.6	49285.6
4	32500	32500	183000	183000	32500	32500	32500

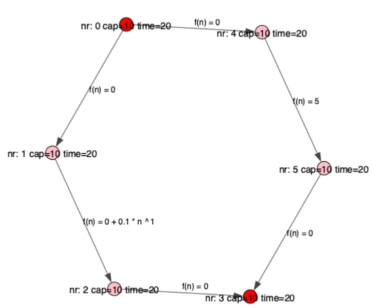
Fig. 13. Summary table for Pigou expensive

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Pigou expensive	1	100	183000	183000	183000	[1]
Pigou expensive	1	100	183000	183000	183000	[2]
Pigou expensive	1	100	182500	182500	182500	[4]
Pigou expensive	1	100	182500	182500	182500	[5]
Pigou expensive	2	100	83000	183000	83000	[1, 2]
Pigou expensive	1	100	183000	183000	183000	[1]
Pigou expensive	2	100	82500	183000	82500	[1, 4]
Pigou expensive	2	100	82500	183000	82500	[1, 5]
Pigou expensive	1	100	183000	183000	183000	[2]
Pigou expensive	2	100	82500	183000	82500	[2, 4]
Pigou expensive	2	100	82500	183000	82500	[2, 5]
Pigou expensive	1	100	182500	182500	182500	[4]
Pigou expensive	1	100	182500	182500	182500	[5]
Pigou expensive	2	100	82500	182500	82500	[4, 5]
Pigou expensive	2	100	83000	183000	83000	[1, 2]
Pigou expensive	1	100	183000	183000	183000	[1]
Pigou expensive	2	100	82500	183000	82500	[1, 4]
Pigou expensive	2	100	82500	183000	82500	[1, 5]
Pigou expensive	1	100	183000	183000	183000	[2]
Pigou expensive	2	100	82500	183000	82500	[2, 4]
Pigou expensive	2	100	82500	183000	82500	[2, 5]
Pigou expensive	1	100	182500	182500	182500	[4]
Pigou expensive	1	100	182500	182500	182500	[5]
Pigou expensive	2	100	82500	182500	82500	[4, 5]
Pigou expensive	2	100	83000	183000	83000	[1, 2]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 4]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 5]
Pigou expensive	2	100	82500	183000	82500	[1, 4]
Pigou expensive	2	100	82500	183000	82500	[1, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[1, 4, 5]
Pigou expensive	2	100	82500	183000	82500	[2, 4]
Pigou expensive	2	100	82500	183000	82500	[2, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[2, 4, 5]
Pigou expensive	2	100	82500	182500	82500	[4, 5]
Pigou expensive	2	100	83000	183000	83000	[1, 2]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 4]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 5]
Pigou expensive	2	100	82500	183000	82500	[1, 4]
Pigou expensive	2	100	82500	183000	82500	[1, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[1, 4, 5]
Pigou expensive	2	100	82500	183000	82500	[2, 4]
Pigou expensive	2	100	82500	183000	82500	[2, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[2, 4, 5]
Pigou expensive	2	100	82500	182500	82500	[4, 5]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 4]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 5]
Pigou expensive	4	100	32500	183000	32500	[1, 2, 4, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[1, 4, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[2, 4, 5]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 4]
Pigou expensive	3	100	49285.6	183000	49285.6	[1, 2, 5]
Pigou expensive	4	100	32500	183000	32500	[1, 2, 4, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[1, 4, 5]
Pigou expensive	3	100	49123.9	183000	49125.6	[2, 4, 5]
Pigou expensive	4	100	32500	183000	32500	[1, 2, 4, 5]
Pigou expensive	4	100	32500	183000	32500	[1, 2, 4, 5]

Fig. 14. Results table for Pigou expensive

Visualisation:

Fig. 15. Visualisation of Pigou expensive



A.2.3 Pigou lower path preference.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	91800	600	91800	183000	91800	600	183000
2	14333.3	600	152600	183000	14333.3	600	83000
3	600	600	183000	183000	600	600	600
4	600	600	183000	183000	600	600	600

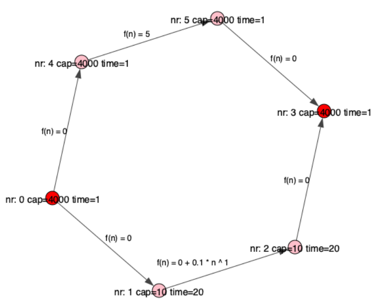
Fig. 16. Summary table for Pigou lower path preference

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Pigou asym lower pref	1	100	183000	183000	183000	[1]
Pigou asym lower pref	1	100	183000	183000	183000	[2]
Pigou asym lower pref	1	100	600	600	600	[4]
Pigou asym lower pref	1	100	600	600	600	[5]
Pigou asym lower pref	2	100	83000	183000	83000	[1, 2]
Pigou asym lower pref	1	100	183000	183000	183000	[1]
Pigou asym lower pref	2	100	600	183000	600	[1, 4]
Pigou asym lower pref	2	100	600	183000	600	[1, 5]
Pigou asym lower pref	1	100	183000	183000	183000	[2]
Pigou asym lower pref	2	100	600	183000	600	[2, 4]
Pigou asym lower pref	2	100	600	183000	600	[2, 5]
Pigou asym lower pref	1	100	600	600	600	[4]
Pigou asym lower pref	1	100	600	600	600	[5]
Pigou asym lower pref	2	100	600	600	600	[4, 5]
Pigou asym lower pref	2	100	83000	183000	83000	[1, 2]
Pigou asym lower pref	1	100	183000	183000	183000	[1]
Pigou asym lower pref	2	100	600	183000	600	[1, 4]
Pigou asym lower pref	2	100	600	183000	600	[1, 5]
Pigou asym lower pref	1	100	183000	183000	183000	[2]
Pigou asym lower pref	2	100	600	183000	600	[2, 4]
Pigou asym lower pref	2	100	600	183000	600	[2, 5]
Pigou asym lower pref	1	100	600	600	600	[4]
Pigou asym lower pref	1	100	600	600	600	[5]
Pigou asym lower pref	2	100	600	600	600	[4, 5]
Pigou asym lower pref	2	100	83000	183000	83000	[1, 2]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 4]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 5]
Pigou asym lower pref	2	100	600	183000	600	[1, 4]
Pigou asym lower pref	2	100	600	183000	600	[1, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 4, 5]
Pigou asym lower pref	2	100	600	183000	600	[2, 4]
Pigou asym lower pref	2	100	600	183000	600	[2, 5]
Pigou asym lower pref	3	100	600	183000	600	[2, 4, 5]
Pigou asym lower pref	2	100	600	600	600	[4, 5]
Pigou asym lower pref	2	100	83000	183000	83000	[1, 2]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 4]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 5]
Pigou asym lower pref	2	100	600	183000	600	[1, 4]
Pigou asym lower pref	2	100	600	183000	600	[1, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 4, 5]
Pigou asym lower pref	2	100	600	183000	600	[2, 4]
Pigou asym lower pref	2	100	600	183000	600	[2, 5]
Pigou asym lower pref	3	100	600	183000	600	[2, 4, 5]
Pigou asym lower pref	2	100	600	600	600	[4, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 4]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 5]
Pigou asym lower pref	4	100	600	183000	600	[1, 2, 4, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 4, 5]
Pigou asym lower pref	3	100	600	183000	600	[2, 4, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 4]
Pigou asym lower pref	3	100	600	183000	600	[1, 2, 5]
Pigou asym lower pref	4	100	600	183000	600	[1, 2, 4, 5]
Pigou asym lower pref	3	100	600	183000	600	[1, 4, 5]
Pigou asym lower pref	3	100	600	183000	600	[2, 4, 5]
Pigou asym lower pref	4	100	600	183000	600	[1, 2, 4, 5]
Pigou asym lower pref	4	100	600	183000	600	[1, 2, 4, 5]

Fig. 17. Results table for Pigou lower path preference

Visualisation:

Fig. 18. Visualisation of Pigou lower path preference



A.2.4 Pigou upper path preference.

chargers	avg best	min best	avg worst	max worst	avg eq	min eq	max eq
1	91800	1100	91800	182500	91800	1100	182500
2	14666.7	1100	152267	182500	14666.7	1100	82500
3	1100	1100	182500	182500	1100	1100	1100
4	1100	1100	182500	182500	1100	1100	1100

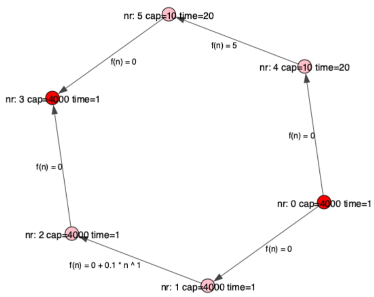
Fig. 19. Summary table for Pigou upper path preference

Name	chargers	cars	Best case	Worst case	Equilibrium	chargers
Pigou asym upper pref	1	100	1100	1100	1100	[1]
Pigou asym upper pref	1	100	1100	1100	1100	[2]
Pigou asym upper pref	1	100	182500	182500	182500	[4]
Pigou asym upper pref	1	100	182500	182500	182500	[5]
Pigou asym upper pref	2	100	1100	1100	1100	[1, 2]
Pigou asym upper pref	1	100	1100	1100	1100	[1]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 5]
Pigou asym upper pref	1	100	1100	1100	1100	[2]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 5]
Pigou asym upper pref	1	100	182500	182500	182500	[4]
Pigou asym upper pref	1	100	182500	182500	182500	[5]
Pigou asym upper pref	2	100	82500	182500	82500	[4, 5]
Pigou asym upper pref	2	100	1100	1100	1100	[1, 2]
Pigou asym upper pref	1	100	1100	1100	1100	[1]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 5]
Pigou asym upper pref	1	100	1100	1100	1100	[2]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 5]
Pigou asym upper pref	1	100	182500	182500	182500	[4]
Pigou asym upper pref	1	100	182500	182500	182500	[5]
Pigou asym upper pref	2	100	82500	182500	82500	[4, 5]
Pigou asym upper pref	2	100	1100	1100	1100	[1, 2]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 4]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 5]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 4, 5]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[2, 4, 5]
Pigou asym upper pref	2	100	82500	182500	82500	[4, 5]
Pigou asym upper pref	2	100	1100	1100	1100	[1, 2]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 4]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 5]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[1, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 4, 5]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 4]
Pigou asym upper pref	2	100	1100	182500	1100	[2, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[2, 4, 5]
Pigou asym upper pref	2	100	82500	182500	82500	[4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 4]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 5]
Pigou asym upper pref	4	100	1100	182500	1100	[1, 2, 4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[2, 4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 4]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 2, 5]
Pigou asym upper pref	4	100	1100	182500	1100	[1, 2, 4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[1, 4, 5]
Pigou asym upper pref	3	100	1100	182500	1100	[2, 4, 5]
Pigou asym upper pref	4	100	1100	182500	1100	[1, 2, 4, 5]
Pigou asym upper pref	4	100	1100	182500	1100	[1, 2, 4, 5]

Fig. 20. Results table for Pigou upper path preference

Visualisation:

Fig. 21. Visualisation of Pigou upper path preference



B GLOSSARY

- Strategy: Some way of playing for a player in a game, in this paper a path that a player can take through a graph representation of a road network and a charging station to make that path feasible
- Nash equilibrium: A situation in a game where none of the players can gain anything from switching strategies, which is thus a stable situation
- Cost: The result a player playing a certain strategy, given the chosen strategies of the other players in the game
- Social Optimum: The lowest possible total cost for all players, choosing all players' strategies to minimize not their own cost but the total
- Price of Anarchy: The difference between the social optimum and the worst possible Nash equilibrium
- Braess's paradox: A paradox where adding a road segment may lead to more congestion in a road network
- Congestion games: A class of games where players travel through a graph. The players incur a certain cost depending on their strategy, which is a path from their start node to their end node.