Framework for Implementing the Take-Along Option in the Valuation of a Mortgage Portfolio

What is the impact of the take-along option on the portfolio value, especially in a raising interest rate environment?

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Abstract

FRAMEWORK FOR IMPLEMENTING THE TAKE-ALONG OPTION IN THE VALUATION OF A MORTGAGE PORTFOLIO

Financial institutions originate mortgages that are placed on their balance sheet and therefore need to be valued. In addition, these financial institutions offer multiple options for their customers on their mortgage that can suit their respective preferences. One of these options is the take-along option, where someone who relocates to a new home has the opportunity to take-along their mortgage and their respective mortgage rate as initially contractually agreed. In an increasing interest rate environment, it becomes more rational to take-along a mortgage rate that is below the current market rate and therefore it is expected that this option will be exercised more often.

Based on literature review, it appears that the take-along option is currently not factored in the valuation of a mortgage portfolio. In this thesis a framework is proposed how to incorporate the take-along option in the valuation of a mortgage portfolio. A literature review has been performed to investigate the methods used to value a mortgage portfolio. In literature the prepayment option is both valued using an option-based and econometric approach. The framework in this research is based on the econometric approach. The prepayment option is included by using the conditional prepayment rate ("CPR"). It is expected that the CPR will change due to the take-along option, given the borrower in case of relocation has either the option to exercise the take-along option or to repay the mortgage. If the market mortgage rate is higher than the coupon (i.e. negative prepayment incentive) the borrower would tend to exercise the take-along option instead of prepaying the mortgage, which would result in a lower CPR and thereby impacting the valuation.

The framework has been applied to a mortgage portfolio representing the Dutch mortgage market. The data has been constructed by public available data from de Nederlandsche bank ("DNB"), which hence represents a case study. In the results the cashflow changes and the NPV values are compared with the following options: no options involved, only the prepayment option (caused by relocating) included, the base take-along option and a scenario take-along option considering a representative example that takes into account specific terms and conditions applied by a financial institution in the Netherlands.

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List of abbreviations

ARM	Adjustable-Rate mortgage
CPR	Conditional Prepayment Rate
DNB	De Nederlandsche Bank
IAS	Index Amortizing Swap
IFRS	International Financial Reporting Standards
LTV	Loan to Value
NHG	Nationale Hypotheek Garantie
NPV	Net Present Value
NRMSE	Normalized Root Mean Squared Error
OAS	Option-Adjusted Spread
PDE	Partial Differential Equation
PSA	Public Securities Association
RMSE	Root Mean Squared Error
RWA	Risk Weighted Assets

1. Introduction

This first chapter describes the context of the research and provides background information on important topics to better understand the thesis. The Dutch housing market and mortgages are discussed. The chapter concludes with the objective of this thesis.

1.1 Background information

The Dutch housing market is overheated for a while now. The term overheated (also the term exuberance is used) refers to the situation of the high market demand on houses versus a limited number of properties offered. In recent years, the market was characterized by relatively low interest rates, hence making it cheaper (on a monthly basis) to take on a mortgage. This has caused (high) overbids on the house prices and an increase in house prices over the past years. This has resulted in lower loan to value ("LTV") for mortgages on these houses, resulting in fewer losses for banks which has been reflected in the calculations in their internal models (Verslag Financieel Stabiliteitscomité 2021). The maximum LTV ratio in the Netherlands is 100% since 2018, which is regulated by the mortgage code of conduct. The LTV ratio indicates the risk class of the mortgage for banks. A high LTV ratio denotes a riskier class, resulting in a higher mortgage rate (Sun, 2016), and vice versa.

De Nederlandsche Bank ("DNB") is committed to the stability of the Dutch financial system and economy. DNB creates, amongst others, measures to ensure that banks could absorb housing market shocks. This is driven by that mortgage loans are important due to their large share in banks' assets and their long credit lifetime and therefore an important driver for financial stability (Schütze, 2020). A decrease in house prices will affect both banks and borrowers, therefore a minimum capital buffer is set by the DNB. In January 2022 the overall capital requirements were set at 15.1% of risk-weighted assets ("RWAs") compared to 14.9% in 2020 (ECB, 2022). Setting higher buffers could result in higher mortgage rates, since the financing costs will increase (Rooijers, 2019).

Next to the higher capital requirements, the increased inflation has caused increasing interest rates on mortgages. Figure 1 displays the mortgage rate over time from 2013 until March 2022. The mortgage rate has doubled between October 2021 and March 2022 resulting in higher monthly costs for borrowers and subsequently in a lower maximum loan value for borrowers. However, loan providers (i.e. financial institutions) are hesitant in letting the borrowers pay higher rates, due to the competition in the mortgage market. The competition caused profit margins for financial institutions to flatten (Van Rein & Trappenburg, 2022).

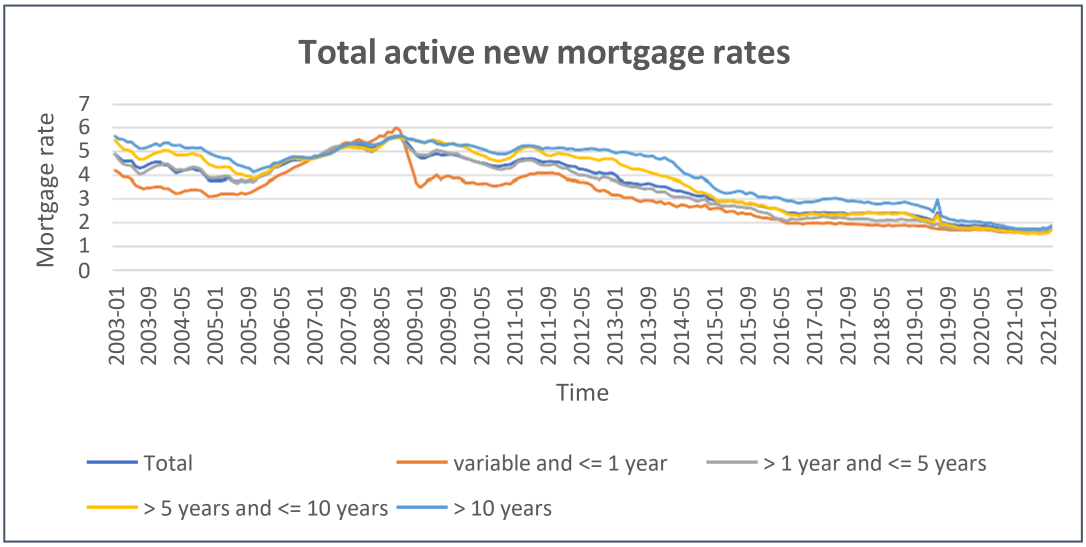


Figure 1: Total active new mortgage rates split over the different length of fixed rate periods (source: DNB)

The rise in house prices also provokes starters to take out a maximum loan based on their income, which exposes borrowers and banks to price and income fluctuations (verslag financieel stabiliteitscomité, 2021).

1.1.1 Mortgages

A mortgage is a long-term loan that is secured by a collateral. The two counterparties of a mortgage are the lender (the *mortgagee*), and the borrower (the *mortgagor*). In this thesis the Dutch housing market and mortgage market is of interest.

There are two main interest rate mortgages groups:

i. **Fixed-rate, level-payment, fully amortized mortgages**
For this group of mortgages the mortgagor pays each month an amount of fixed annual interest rate of the outstanding balance and a fraction of principaldepending on the mortgage type.

ii. **Adjustable-rate mortgages**
An adjustable rate mortgage ("ARM") has a more complicated cash flow scheme compared to fixed-rate mortgages, since the mortgage rate is based on an appropriate chosen reference rate instead of a fixed rate. On the other hand, the instrument was developed to deal with mismatch between the mortgage rate and the market rate, especially in a high interest rate environment. To encourage borrowers to choose for ARM over other types, mortgagees typically offer an initial fixed contract rate, which tends to be less than the current market mortgage rate and a variable rate that resets in predetermined periods. For ARM a mortgagor has the choice to either pay principal each month or at the end of the contract (Tong, 2009).

In this thesis only fixed-rate mortgages are considered since for ARM mortgages, the mortgage rate will adjust to the current market rate. This results in no difference between the agreed mortgage rate and the market mortgage rate for ARM mortgages. Therefore, limited interest rate risk appears (Casamassima et al., 2021). However, for fixed rate mortgages it results in a difference between the agreed coupon and the current market mortgage rate. Therefore, the take-along option can only be favorable for a fixed rate mortgage.

Banks offer mortgages with a fixed interest rates for a pre-set fixed period to consumers. The borrower pays the bank a monthly fixed interest rate and principal payments. In this case, the bank receives a predetermined interest income over the lifetime of the contract based on the fixed rate and the repayment scheme, whilst the market interest changes every day. In addition, banksoffer borrowers multiple options to reflect their individual situation such as (i) early repayment (or prepayment) and (ii) the take-along option. Early repayment of the loan refers to additional repayments prior to what is determined in the repayment schedule, meaning that the outstanding loan balance is decreasing faster than expected from the perspective of the lender. The take-along option refers to a situation where borrowers relocate to a new home and can transfer their outstanding loan with the same or similar rates as initially agreed (Boshuizen and Spreij, 2002). The unexpected cash flows result in prepayment risk for financial institutions, since they typically expect long-term payments periods by borrowers (Casamassima et al., 2021). A financial institution typically relies on long-term payment periods by the mortgagor, with associated interest rate payments. A change in the cash flows caused by early repayments could result in a substantial risk. The duration, interest payment and notional impacts the bank's costs and partly represents the bank's profits from selling the mortgage. Early repayment could result in losses due to the outstanding debt and the interest rate may get lower, and the loan duration may be reduced. When multiple clients suddenly repay their loan earlier, the early repayment may result for the bank for a significant difference, which is a risk that needs to be analyzed and hedged (Casamassima et al., 2022). In case banks have matched a liability to fund the newly originated mortgage, this would also impose an early repayment risk for banks.

Lending money for a fixed period at a fixed rate can create interest rate and liquidity risks that need to be managed by the mortgagee. Interest rate risk arises where the interest rate paid on the funding for a loan may change over time while the interest rate on the loan is fixed. Liquidity risk occurs where the funding for a loan has been borrowed for a time period that is shorter than the life of the loan (Perry at al., 2001).

Since a difference between the market rate and the fixed rate may occur over time, a hedging strategy is applied to cover this risk. This hedging strategy can for example include the use of Index amortizing swaps (IASs), which consist of a predetermined scheme as a function of a specific interest rate. Therefore, the IASs is effectively acting as an option on that specific interest rate and can replicate the floating rate of the market (Casamassima, 2021).

A financial institution can take out a mortgage insurance. The mortgage insurance is an insurance product to compensate the lender for the losses caused by mortgage payment defaults. The financial institute improves the risk class without changing the LTV ratios using this insurance product. In the Netherlands mortgages with the Notional Mortgage Guarantee scheme (in Dutch "Nationale hypotheek garantie" ("NHG'')) are designed to limit the financial risk for the mortgagee, given that the mortgagee receives 90% from the NHG if the NHG mortgage goes into default (Sun, 2016).

Mortgages valuation

It is important to value the mortgages of financial institutions as these are recorded on their balance sheet but also reflect the financial stability of these financial institutions. The valuation of these mortgages is derived using a discounted cash flow method (Tijsterman and Van Hees, 2017). The discounted cash flow method assumes a certain expected cash flow scheme based on their respective mortgage types. Clients have a number of options that they can exercise during the lifetime of their mortgage contract, as highlighted in Figure 2.

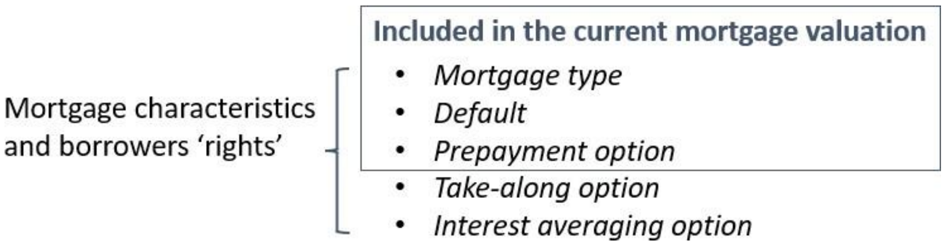


Figure 2: Schematic overview of mortgagor options on a mortgage contract.

The current valuation takes into account the mortgage type, default and the prepayment option as can be seen in Figure 2. The mortgage type, default and prepayment option influence the cash flow.

The mortgage types include the bullet mortgage, annuity mortgage, linear mortgage and savings mortgage. Default refers to the situation wherein the borrower cannot fulfil its obligations for the payments related to their respective loan. The prepayment option, however, refers to a situation wherein a borrower chooses to repay more compared to the initially agreed repayment scheme.

Based on literature review, the take-along option and interest averaging option are not included in the current mortgage valuation. The take-along option refers to the situation where a borrower relocates to a new home and has the option to take-along its current mortgage to finance its new home. The interest averaging option is an option in which the fixed rate period is not completed and the mortgagee and the mortgagor agree to replace the current contract by a new contract. The new contract consists of a new mortgage rate by an averaging method. The averaging method takes into account the coupon and the remaining duration to determine the new interest rate for the upcoming duration. Compared to the take-along option this option can be exercised whether the mortgagor relocates or not. Mortgage type, prepayment option and take-along option are further elaborated in the Literature Review section as these are most relevant when indicating the impact of the take-along option.

1.1.2 Expected interest rate rise

The interest rate forecasts of the ECB show an increase from 0.5% in the third quarter of 2022 to 1.3% in the first quarter of 2023 and reach an average interest rate raise of 1.8% in 2024 (ECB, 2022). The rising interest rates can impact the client behavior and subsequently can affect the interest rate risk measurement approach. Since it will become more likely that current mortgage rates will be higher than the initial agreed fixed rate as part the contract, it becomes less attractive for mortgagors to refinance their mortgage and therefore will also limit the incentive for mortgagors for prepayment as a result of market developments.

Since take-along options are expected to be more favorable to consider for clients in an increasing interest rate environment, financial institutions can potentially overestimate the prepayments and underestimate the interest rate risk and liquidity spread as lenders do not incorporate the take-along option in their models. These factors can result in incorrect pricing. Besides, not including the use of the take-along option could result in an underestimation of the option-adjusted spread (“OAS”), driven by an increasing interest rate environment. The OAS is the value of the outstanding mortgage times the difference between the market mortgage rate and the fixed rate of the mortgage. The incorrect pricing results in incorrect hedging, since the take-along option results in a change in the forecasted cash flows.

1.1.3 Inducement to this paper

The decreasing interest rate made it favorable for mortgagors to repay their mortgage. In case of relocation the mortgagor is not limited to a certain amount and is able to repay the entire outstanding mortgage amount without a penalty. However, this changes as a result of an anticipated increase of the interest rate, which is correlated to the mortgage rate. Therefore the impact of the take-along option on mortgage portfolio could be significant.

The problem owner is Deloitte, specifically the market risk team within the Financial Risk Management department. Deloitte has the aim to research the impact on the take-along option on a mortgage portfolio, hence it is expected that in an increasing interest rate environment the take-along option will be exercised more often. Furthermore, recent comments and questions from clients on how to incorporate the take-along option in their mortgage portfolio valuation were placed. It is therefore important to research whether the take-along option has a significant (financial) impact on the valuation of a mortgage portfolio.

Ultimately, banks and other lenders and financing parties of mortgages will bear most of the risk as the take-along option may impact the value as recorded on its balance sheet. It may also impact the amount of capital that financial institutions should have to comply with the rules and regulations as set out by DNB.

Problem context

Figure 3 provides an overview of (potential) challenges involved in the bigger picture of not including the take-along option in the mortgage portfolio valuation. Given that the take-along option is included in the mortgage contract, the models used in the valuation are not completely exhaustive. The models in the literature do not take into account the change in cash flows corresponding to the take-along option. The incorrect valuation could result in (unexpected) financial risks, which could result in a hedging strategy that is not aligned with the actual situation due to changing cash flows. In addition, the asset and liability management could be impacted as the resulting value of the mortgage portfolio could increase the total assets and hence the liabilities required for funding. Therefore, exercising the take-along options by mortgagors could affect the balance sheet of banks, since the value on the balance sheet is determined by the expected cash flows and an assumption on the prepayments.

Figure 3: Problem cluster

1.2 Research objective

The aim of this thesis is to investigate how to include the take-along option in the valuation of a mortgage portfolio, by complementing existing valuation techniques. Especially the impact of the take-along option is of interest. Finally, the aim is to compare the impact on the value of the mortgage portfolio considering the main mortgage types, by including and excluding the take-along option. For this research, a representative Dutch mortgage portfolio was constructed based on publicly available data. This therefore represents a case study to compare the mortgage portfolio value if the take-along option is included in the valuation method.

The research objective is formulated as following:

The goal is to provide a framework to include the take-along option and to be able to calculate the difference in value including and excluding the take-along option, which is applicable for different mortgage portfolios.

1.2.1 Research questions

To achieve the study objective, a main research question is formulated. This main question will be answered with the help of multiple sub-questions.

Main research question:	*What is the impact on a mortgage portfolio if the take-along option is included in the valuation (by implementing the econometric approach)?*

Hypothesis: It is assumed that the take-along option has an influence on the mortgage portfolio valuation as the prepayments will likely decrease when borrowers have the option to take-along their mortgage with a lower (fixed) rate compared to the current market rate. When prepayments decrease, it is likely that the total mortgage value will increase and hence it is expected that the take-along option has a positive contribution to the mortgage portfolio valuation. A different NPV is expected for different types of mortgages, hence the unequal cash flow scheme.

The results are compared for two types of fixed-rate mortgages and sensitivity analysis is performed for the (chosen) parameters.

Sub-question 1:	*How are the mortgages valued in scientific research?*

A literature research is conducted in order to get a broader overview of how mortgages are valued in scientific papers.

Sub-question 2:	*How does the take-along option impact a mortgage portfolio valuation?*

It is investigated how mortgages are valued and also the prepayment rate changes as a result of the utilization of the take-along option, given that people that relocate either prepay their mortgage or exercise the take-along option.

Sub-question 3:	*What effect has the take-along option structure on the mortgage portfolio valuation and what are the main sensitivities?*

A sensitivity analysis has been performed to investigate the impact of the parameters used.

1.2.2 Scope
The scope of this thesis is providing a framework for financial institutes that aim to include the take-along option in their mortgage portfolio valuation. As a result including the take-along option could potentially impact the risk management of financial institutions. In order to estimate the (potential) impact, a Dutch mortgage portfolio is derived based on publicly available data and the valuation is performed with and without the take-along option. The publicly available data does not contain information on NHG and LTV ratios of the mortgages, and aretherefore not included in the scope. In this research, the focus is on constructing a framework on howto include the take-along option and therefore hedging strategy is not in scope as the focus is on the impact of the valuation and not on how to hedge the differences in the valuation.

In the scope the contractual cash flows and the adjustments to these cash flows caused by the prepayment option (caused by relocating) and the take-along option are included. Other options, such as default and the interest averaging option, are not incorporated in the framework. It is assumed that financial institutions already have included the default option in their models and the take-along option has no influence on this option. For the interest averaging option, this is currently not included in the valuation model based on literature review, but is not included in scope as the main goal is to provide insights on how the take-along can be included and what the impact of including this option is.

As mentioned before the fixed rate mortgages are covered in this research, given that these mortgages are of interest regarding the take-along option. Furthermore, the two most common mortgage types are included in the model. Since the cash flow scheme is different for each mortgage type. The impact resulted from the mortgage type is included in this research.

1.2.3 Research Structure
The research structure is based on the research questions.

- A **literature review** is used for the definitions of main terms, the theoretical framework and the models used to value mortgage portfolios. In addition, a literature review is performed on the definition of the take-along option and its structure.
- **Modelling** the valuation of mortgage portfolio with and without the take-along option is constructed and, in both cases, includes the prepayment option.
- Building a **framework** on how to incorporate the take-along option in a mortgage valuation model.
- **Conclusion & discussion** are included to indicate the impact of the take-along option on mortgage portfolio valuations, including limitations of this research and suggestions for further research.

1.3 Software
Excel is used for the calculation and preparing the data. The data is used as input from Excel into the Python script for the valuation of the mortgage portfolio.

2. Literature review

There are many papers published for valuing mortgages. However, the take-along option is not yet included in papers. The option is occasionally mentioned but not incorporated in the valuation. Boshuizen and Spreij (2002) developed a prepayment model for the early repayment option based on an interest rate model to determine future interest rate values. The mathematical fundamentals are based on the Wiener-process/Brownian motion. In the paper Boshuizen and Spreij refer to the rights mentioned in the mortgage contract as options for borrowers. Executing these options depends on the interest rates and if the option is in the money. In this article, the take-along option is mentioned but is not modelled. Therefore, the next sections in the literature review will be based on the mortgage valuation technique(s) present in literature.

2.1 Mortgage valuation in literature

Based on literature, International Financial Reporting Standards ("IFRS") requirements and a document provided by the DNB 'valuation of mortgage loans in the adequacy test' (2015), the most common valuation method for mortgage portfolio is known as the fair value method based on a discounted cash flow method (Tijsterman and Van Hees, 2017). This method consists of two building blocks: the cash flows and market data. This is mostly based on assumptions for future cash flows based on historical performances and the most recent market outlook (Xu & Hsiao, 2019).

The mortgage type, the prepayment option and the discount curve are main drivers in the cash flows determination in literature.

2.1.1 Mortgage types

For each mortgage type the mortgagor and the mortgagee agree upon the following elements being the interest rate, the maturity and the redemption type. There are four main types of mortgages:

i. **Balloon/bullet mortgage**
Balloon mortgages are mortgages in which the mortgagor repays the total amount of debt at a specific date in the future. However, these mortgages are these days more of a hybrid version where the mortgagor pays each month an amount of principal and interest and at the specific date in the future the outstanding debt. The outstanding debt is usually significantly higher than the principal payments throughout the contract. These mortgages are typically attractive to mortgagors, since mortgages rates are significantly lower than other types of mortgages (Tong, 2009).

ii. **Annuity mortgage**
An annuity mortgage is a mortgage whereby the mortgagor pays a fixed monthly amount made up of interest and capital repayment (i.e. principal). The interest and the principal together are called the annuity. The monthly amount remains the same throughout the term of the mortgage. Advantages of the annuity mortgage are that the total monthly payments remain the same. Currently the typical annuity mortgage includes relatively high interest components versus repayment components, which switches over time.

iii. **Linear mortgage**
A linear mortgage is a mortgage whereby the mortgagor pays a fixed amount of monthly capital repayment and on top of this amount an interest payment based on the outstanding debt. This mortgage type can be attractive for people who, for example, want to stop working earlier. It is also interesting for people with a risky profession where there is a risk that their

income may be lower in the future due to unforeseen circumstances. A disadvantage is that monthly payments in the beginning of the contract are relatively high.

iv. **Savings mortgage**

A savings mortgage was a mortgage type without mandatory principal payment during the contract. The monthly payment consists of interest over the mortgage amount and the savings interest of the related savings account. The savings interest that a mortgagee receives is equal to the mortgage interest rate. The amount the mortgagor has to put into the savings account is basedon a value such that the value is equal to the original debt at expiration. The savings mortgage type originated towards the end of the 1980s. The mortgage became popular as mortgage rates reached up to thirteen percent and the AEX-index increased with 580 percent from 1993 to 2000 (Kalse & Voogt, 2020). In addition, the savings mortgage type enables mortgagors to make use of a maximum interest deduction, since it has not been common to make principal payments during the lifetime of the contract. In the financial crisis in 2008, the risks involved in this type of mortgages appeared. After the financial crisis the Dutch government started to discourage the savings mortgage by only offering tax reduction for annuity and linear mortgages. In addition, the low interest rates on saving accounts made it more profitable to repay or pay principal (Langenberg & Jonkers, 2022). As a result, the savings mortgage type is limitedly used, and can even not be originated at this moment (since 2013). Therefore, the savings mortgage type is not considered further as part of this research.

2.1.2 Discount curve

In order to derive the fair value as per a certain valuation date, future cash flows are discounted back to this valuation date. In mortgage valuations, the discount rate is based on a discount curve that is related to the interest rate. For each time step included in the future cash flows (e.g. monthly cash flows in 2023), a discount curve corresponding to these dates need to be used (Tijsterman and Van Hees, 2017).

2.1.3 Prepayments

The expected cash flows will most likely differ from this contractually agreed pattern due to prepayments. Especially in the past low interest rate environment, borrowers frequently made prepayments on top of their scheduled principal payments. There are two methods to alternate the expected cash flows to take into account the prepayments, being backward looking and forward looking based on client behavior. Client behavior could be either dependent or independent on interest rates. The independent method uses an historical analysis, and hence constitutes the backward-looking element. On the other hand, the interest rate dependent behavior is forward looking and depends on the expected interest rate which can be derived using Monte Carlo simulations (Basel committee on banking supervision, 2016).

Another important factor in client behavior are prepayment penalties in case a contractually agreed threshold of maximum prepayment optionality is exceeded. In the Netherlands the threshold is mainly between ten and twenty percent per year on the outstanding mortgage (Kuijpers & Schotman, 2007).

In accordance, the behavior of mortgagors should be taken into account to value the mortgage portfolio. It is clear that mortgage borrowers do not exercise their options in the same way that owners of financial options exercise their options (Van Order, 2007). The behavior of each mortgagor may differ. Deng et al. (2000) included borrower heterogeneity to distinguish different types of borrowers, and indicated that they are either very active, passive and in between. The paper determined that heterogeneity is significant and has important effects on key elasticities explaining behavior, particularly with respect to repayment.

Mortgage option valuation methods

Different valuation methods are considered to calculate the effect for the prepayment option, which are the option-based and the econometric based approach (Chen et al., 2009).

i. **Option-based approach**

   Option-based models are based on a dataset consisting of individual mortgages. These models treat mortgages as a bond with embedded options: an American call option (the option to repay) and a series of European put options (the option to default). There are multiple models to price each individual loan (Manola & Urosevic, 2010). A partial differential equation ("PDE") with options to repay or default make use of two state variablesbeing the interest rate and the value of the underlying (i.e. house prices). Given a closed formsolution to the PDE results in an unfeasible solution, therefore numerical methods are generally used (Chen et al., 2009). A challenge which can be seen as a drawback is that formulating and solving option models can be difficult (L'Heureux & Coleman, 2004). Xu and Hsiao (2019) did not apply the option-based approach for the prepayments, since it is not able to capture the cross-sectional heterogeneity in prepayment rates for Dutch mortgages in an option-based model.

ii. **Econometric approach**

   An alternative to the option-based approach is the econometric approach, where the future discounted cash flows are taking into account all relevant market data for mortgages with certain risk characteristics. Xu and Hsiao (2019) proposed a valuation method that both meets the guidance on the fair value determination of Dutch mortgages for prudential purposes by the DNB (2015) and the IFRS standards. The risk characteristics taken into account are (i) the mortgage type, (ii) time to interest reset of fixed rate mortgages, (iii) the guarantee from NHG (if any) and (iv) the LTV of the mortgage as of January 2013. The assumption is made that the loans are valued at par or repay on the interest reset date, which is in line with the guidance by the DNB. The econometric approach includes the prepayment option in the cash flow with the conditional prepayment rate ("CPR"). The CPR specifies a particular relationship that captures mortgage termination patterns at a portfolio level. The CPR is calibrated to historical termination data and used to project cash flows over the life of the underlying assets, including an up-to-date CPR. Different variables can be used, such as the refinance incentive, burnout effect, seasonality, seasoning, housing prices, unemployment, recent developments of interest rates and GDP growth (Manola & Urosevic, 2010).

In this paper, the econometric approach is applied and altered to incorporate the take-along option. This is primarily driven by the fact that the dataset used did not include information on individual basis, and therefore the option-based approach was not feasible to determine the potential impact of including the take-along option in the valuation. In addition, the option-based approach is not able to capture the prepayment rates properly, given the heterogeneity behavior (Xu & Hsiao, 2019). The prepayment rates are of great importance for the take-along option given if a mortgagor moves the borrower has eighter the optionto repay or to take-along the mortgage (Figure 4). In other words, when relocating there are two options for the mortgagor, however only one option can be exercised and therefore the two options have an opposite relationship with each other. In this thesis it is assumed that the mortgagor either repays or take-alongtheir mortgage in case of relocating. The approach will be further explained in the chapter: Methodology.

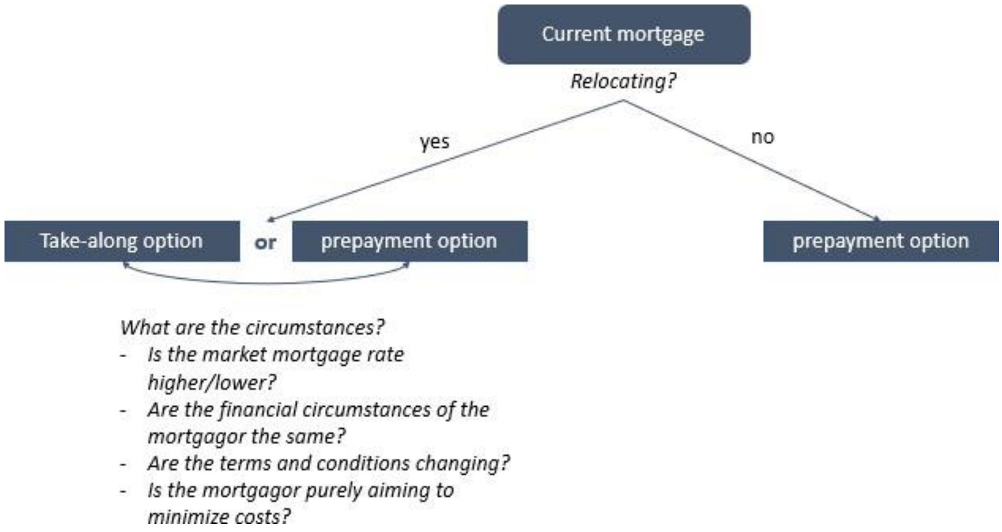


Figure 4: The relevance of prepayments related to the take-along option.

2.1.4 Conditional prepayment rate

The CPR refers to a percentage of the mortgage pool that are expected to be prepaid in that specific year. The reasons behind prepayments are diverse. According to Richard and Roll (2004) the conditional prepayment term consists of four important economic effects being the refinancing incentive, seasoning, seasonality and the burnout effect. All these effects are described below. The multiplicative model combines the four effects in a multiplicative formula to determine the prepayment rates at time t:

[1]

$$CPR(r,t) = (Refinancing\ incentive) \cdot \left(M_{Seasoning}\right) \cdot \left(M_{Month}\right) \cdot \left(M_{Burnout}\right)$$

The $CPR(r,t)$ is the yearly conditional prepayment rate and M the multiplier of the economic effects. The model requires a monthly prepayment rate. The following formula provides the monthly CPR (i.e. $SMM(r,t)$) :

[2]

$$SMM(r,t) = 1 - \left(1 - CPR(r,t)\right)^{\frac{1}{12}}$$

The refinancing incentive

The interest rate is one of the main reasons to prepay. Regarding the functional form of the incentive there are two main approaches in literature. Based on either a ratio or based on the difference between the rates will be used:

Variables	
$k(t)$	mortgage market rate
K	initial mortgage rate
$\epsilon(t)$	difference between mortgage rate and market rate.

[3]

$$\epsilon(t) = K - k(t)$$

This form has also been used by Kolbe, 2008, Hoda & Kee, 2007 and Perry at al., 2001. Clearly the smaller the market rate, the greater the difference, with for an at-the money mortgage $\epsilon(t) = \epsilon^* = 0$. It is well-known that people do not always act rational, meaning that they may not prepay when the opportunity arises, and mortgagors may repay when it is not optimal. If all mortgagors would act purely rational based on monetary motives, the response function would be represented by the blue line in the left-hand-side graph of Figure 5. This indicates that the maximum level of prepayment is reached as soon as the market rate is lower than the initial mortgage rate, which in case captured instantly also referred to as the step function.

In order to capture the heterogeneity, a S-shaped function is considered more reasonable to derive the expected mortgagor's reaction based on the incentive to prepay based on literature, as highlighted with the red line on the left graph in Figure 5. The S-shaped curves can be derived using a number of different functions, also referred to as sigmoid functions as highlighted in Figure 5 on the right-hand side where some examples of S-curves functions that capture nonlinear behavior are depicted. For modeling the nonlinear behavior, an arc-tangent function (as displayed in red in the right graph) has been widely applied (Bandic, 2004) and will therefore also be considered as part of this research.

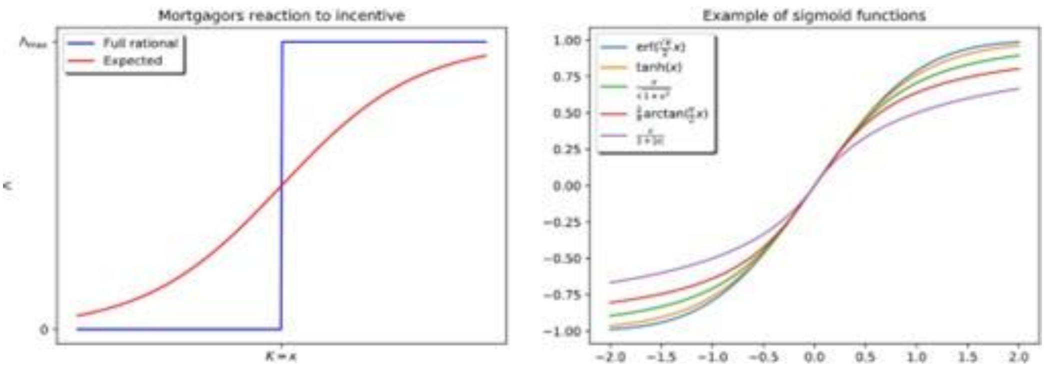


Figure 5: Real versus expected reaction of the people to the refinancing incentive (Bandic, 2004).

Seasoning or age of the mortgage

Mortgages generally display an age pattern. Refinancing pattern changes as mortgages are seasoned. It shifts upwards as the mortgagors are aging, meaning that mortgagors are more likely to refinance in case of an aging mortgage. For a new home owner, it is less likely to relocate for the first few months due to moving costs involved and the adaption time to a new environment including new social or occupational aspects.

Seasonality

Borrowers' behavior regarding the repayments is dependent on the month they occur. The beliefs stem from the mobility of mortgagors, time of housing construction, school year, and weather conditions. Usually, the seasonality pattern tends to be more active in the spring, rises to peak in the summer, decreases through the fall, and then turns to be slow in the winter. The pattern may be different geographically and demographically.

Burnout effect

Prepayments are likely to decrease in case interest rates decline, regardless if interest rates decline even further. Those who can benefit by taking advantages of refinancing will likely have done so already after the first declines and it is less likely that additional refinancing will occur from further declines. This prepayment behavior is called refinancing burnout. The burnout can be calculated using the following formula (Carlier and Bussel, 2001).

[4]

$$burnout = \max(refinancing\ incentive_t - max(refinancing\ incentive_{0,t}), 0)$$

2.1.5 CPR causes

In mortgage valuations, a CPR is used based on historically available data. However, the take-along option can only be exercised in case of a relocation of the mortgagor. Therefore, it is important to make a distinction between the causes of the CPR.

The CPR can be caused by three main elements why borrowers make unscheduled prepayments on their mortgage principal (Rabobank, 2015).

i. Curtailments (partial prepayments)
ii. Relocations
iii. Refinancing activity

There is no specific data available on Dutch CPRs by cause. However, there is a division made based on housing sales, new mortgage inscriptions and press statements by Dutch banks, which enabled the Rabobank to split the overall CPR by its three main causes as mentioned above (Figure 6). From Figure 6 it can be seen that the CPR caused by relocating is roughly between the 2.5% and 6.5% (indicated by CPR Relocation highlighted in orange). Since the relocating cause is only of interest provided that the take-along can only be exercised when relocating, the range from 2.5% to 6.5% is considered reasonable for this research.

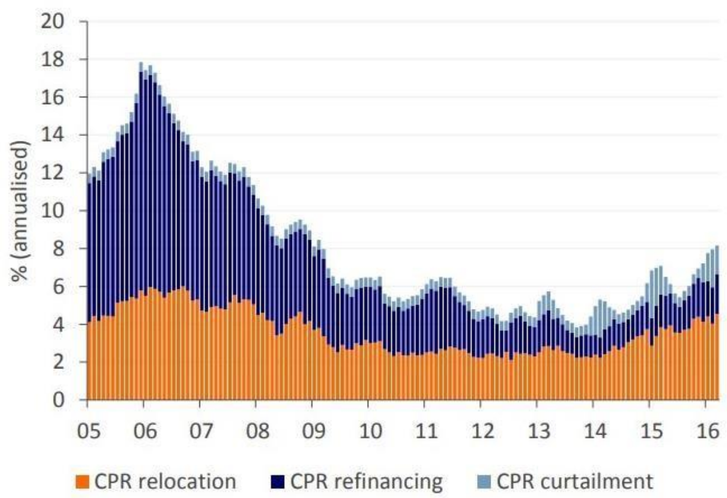


Figure 6: Average CPR Dutch RMBS per cause from 2005 – 2016, (Rabobank, 2015)

3. Methodology

This chapter provides a theoretical basis to the model. Below in Figure 7 the overview of the aspects related to mortgage portfolio valuation are highlighted. In this chapter, all aspects numbered in Figure 7 are further elaborated in the next sections.

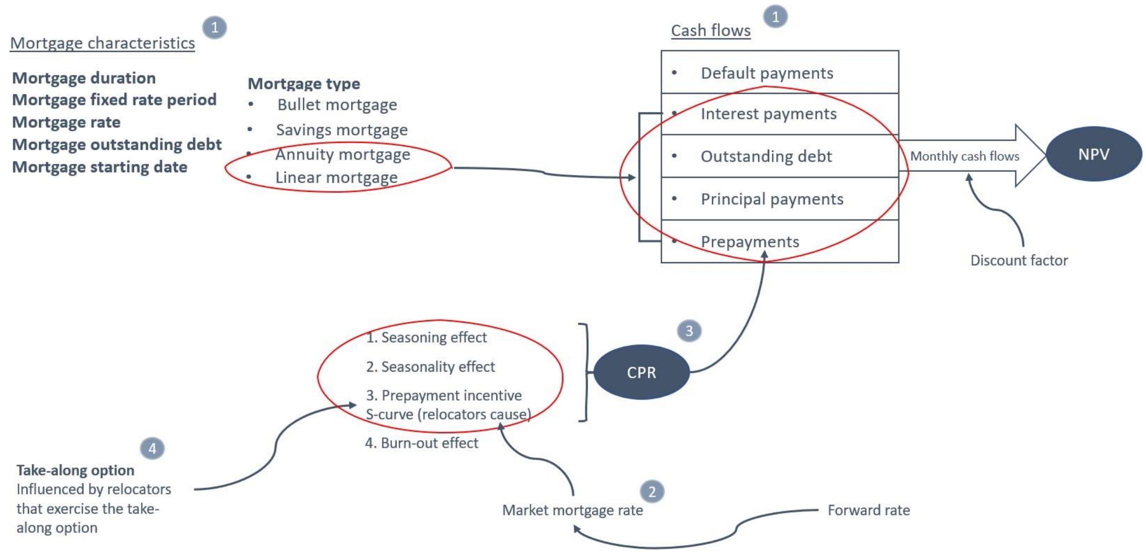


Figure 7: Mortgage valuation elements

3.1 Cash flow considering mortgage type

As elaborated in section 2.1 regarding the mortgage valuations, it was indicated that the valuation of a mortgage portfolio is the result of discounting future cash flows related to the mortgages. These cash flows consist of interest payment, outstanding debt, principal payments and prepayments. As indicated in section 1.2.2, default payments are considered out-of-scope.

The structure of the cash flows are highly dependent on the mortgage type. Since annuities and linear mortgages are the most common types of mortgages in the Netherlands after 2013 (Appendix I), these two types are considered in this research. The shifting in popularity could be caused by the restriction raised in 2013 in which the annuity and linear mortgage only offer tax deduction for starters. Both types resultin a monthly cash flow consisting of interest payment and principal, where the distribution differs amongst the two.

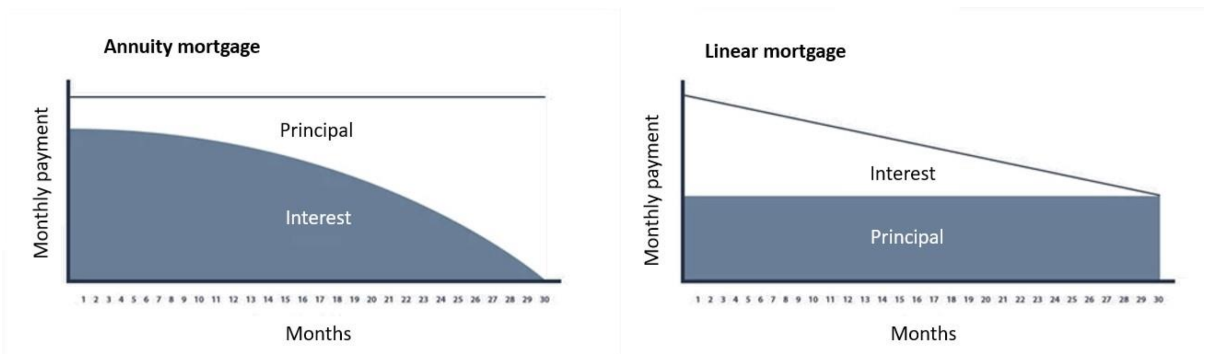
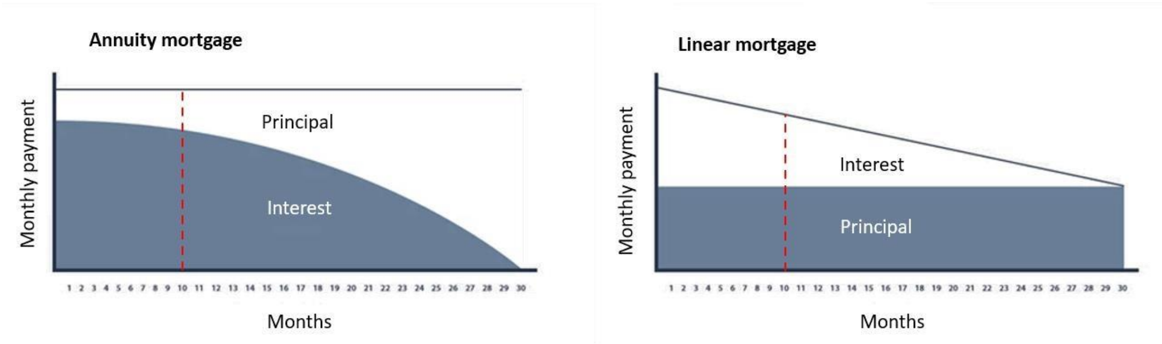


Figure 8: Mortgage types (left) Annuity mortgage and (right) Linear mortgage payment scheme.

Fixed interest period
A potential feature that is included in a mortgage contract is the fixed interest period, where the mortgagor is able to choose a fixed period from 1 month up until the entire duration of the contract. In the Dutch mortgage market, the most common fixed period is ten years.

The length of the fixed period influences the perspective from the lender. For example, if a mortgagor sets a fixed period of two years in a ten-year mortgage contract, the mortgage rate will be adjusted to market rates for the next fixed period (which is determined between the mortgagor and mortgagee). This is considered as new origination of a mortgage in mortgage valuations since it influences the future cash flows. Therefore, the outstanding debt at the end of the fixed period is hypothetically valued as a positive cash flow for the mortgagee.

Example fixed rate period
A mortgage with the following characteristics:
Mortgage rate $= 2.6\%$ for a fixed rate of 10 years and a maturity of 30 years. The cash flows are structured till and including the 120 month as mentioned before. The new mortgage rate (i.e. the market mortgage rate at 10 years) will be seen as a new mortgage with the outstanding debt. However, in the mortgage valuation new (future) mortgages are not considered resulting in the cash flows after the fixed rate period are not considered. Therefore, the outstanding debt at month 120 will be added to the cashflow of the principal of the 120th month. The outstanding debt depends on the mortgage structure as can be seen below.



Annuity mortgage
The sum of the interest and principal payments result in a constant monthly payment during the entire duration of the contract. In the first part of the lifetime of the contract, the interest payment represents the majority of the monthly payment compared to the principal, which is the other way around in the end of the lifetime of the contract (Figure 8). The monthly payment structure for an annuity mortgage has the following structure:

Variables	
AF	is the annuity factor
m	the periodically repayments in a given year
n	the maturity of the loan in years
i	the interest rate
MP	the monthly payment
B	the total debt at the start of the mortgage
I	interest that has to be paid over a monthly period
D_t	the outstanding debt at the beginning of the month
r	the annual interest rate
CD	is the capital down payment
t	point in time within the bounds of [0,n*m]

[5]

$$AF = \frac{\left(1 + \frac{i}{m}\right)^{m \cdot n} - 1}{\frac{i}{m} \cdot \left(1 + \frac{i}{m}\right)^{m \cdot n}}$$

$$MP = \frac{B}{AF}$$

$$I_t = D_t \cdot \frac{r}{m}$$

$$CD_{t+1} = MP - I_t$$

$$D_{t+1} = D_t - CD_{t+1}$$

Linear mortgage

The sum of the interest and principal payments result in a declining monthly payment towards the end of the contractual duration. Since the interest payment declines in a straight line from origination date until maturity and the principal payments are constant throughout the duration of the contract (Figure 8). The structure of a linear mortgage is as follows:

[6]

$$CD = \frac{B}{m \cdot n}$$

$$I_t = \big(B - (CD \cdot t)\big) \cdot \frac{r}{m}$$

$$MP_t = CD + I_t$$

In Appendix II, example calculations for the annuity and linear mortgage are presented.

3.2 Mortgage rate

The interest rate is a reasonable benchmark for the mortgage rate, which matches the maturity and the frequency of the mortgage payments. A spread is added to the interest rate in order to derive the mortgage rate. Banks typically derive the at-the money mortgage rate for new clients from the present level of the corresponding swap rates (Bandic, 2004).

Variables	
$k(t)$	mortgage market rate
$S_{t,T}(t)$	interest rate from simulation at time t for [0,T]
γ	spread

[7]

$$k(t) \equiv k(t; T, \gamma) \coloneqq S_{t,T}(t) + \gamma$$

3.3 CPR

For this research the focus lays on the relocation as this is required for the optionality of the take-along option. This implies that relocations are an important reason for prepayments, also displayed in Figure 6 where especially from 2013 onwards the majority of the CPR is driven by relocation (Rabobank, 2015). Figure 9 provides an illustrative graph of the prepayments with the cause of relocating. Comparing the values of the CPR relocation displayed in Figure 6 and thevalues given in Figure 9, a range from 2.5% to 6.25% is derived.

The prepayment incentive is calibrated on the data points in Figure 9. The other elements, the seasoning and seasonality, are not calibrated in this research. The main reason is that the seasoning factor and seasonality factor are not expected to change as a result of the take-along option. The inputs for these factors are represented in the next chapter: Data. The burnout effect is not included in the model for two reasons. Firstly, in equation [4] the burnout factor would be equal to zero for each time point in case of an increasing interest rate. Secondly, the burnout factor represents the effect of mortgagors leaving the pool if a prepayment took place, which is not considered to be reasonable for prepayments caused by relocating.

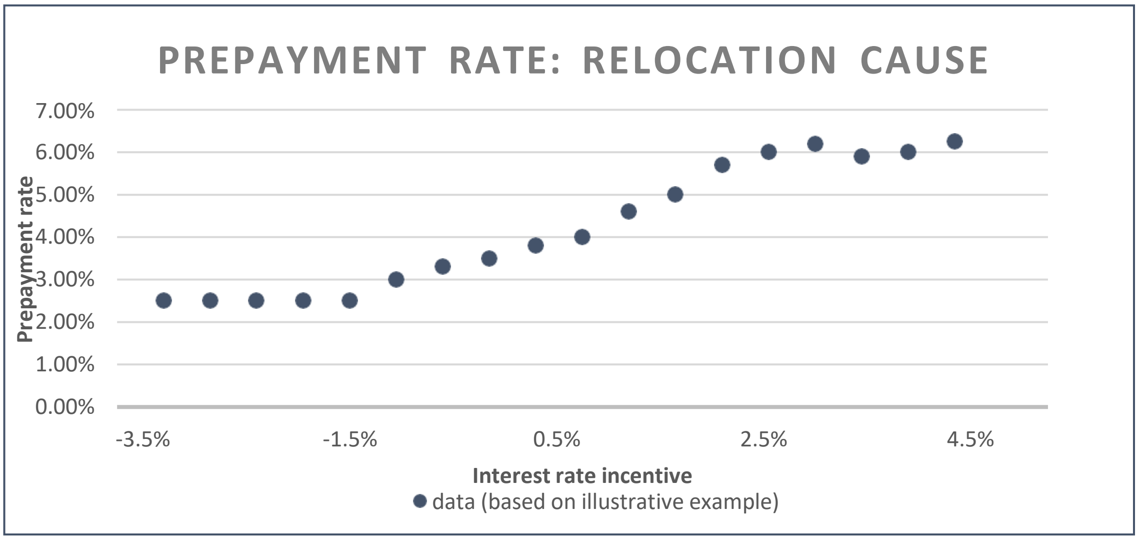


Figure 9: Illustrative prepayment caused by relocating.

3.3.1 Calibrating the S-curve

One of the S-curves in Figure 5 is based on an $\arctan(x)$ function (highlighted in red). This function has been widely applied (Bandic, 2004). In this thesis this function is calibrated to the sample data of the illustrative example. By calibrating the parameters a, b & c.

[8]

$$S_{curve} = a + arctan(\epsilon(t) - b) \cdot 100)/c$$

The values from the function are then calibrated to the values of the sample data. For the accuracy the square root of the mean square error ("RMSE") can be applied (Orlando et al., 2019):

Variables	
r_i^{data}	prepayment rate of sample data
r_i^{arctan}	prepayment rate of the S-curve line
e_i	residual value $e_i = r_i^{data} - r_i^{\text{arctan}}$
r_{max}	maximum value prepayment rate observed sample data
r_{min}	minimum value prepayment rate observed sample data

[9]

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2}$$

Since the RMSE depends on the scale of observed data, thus it is sensitive to outliers and large errors have a distortional large effect. For that reason, and the facilitate the comparison between data, Orlando & Bufalo (2021) adopt the normalized root mean squared error ("NRMSE") as well:

[10]

$$NRMSE = \frac{RMSE}{r_{max} - r_{min}}$$

3.4 Take-along option

The take-along option gives clients the option to take the terms and conditions for their current mortgage at their current mortgaged property and transfer it to their new home in case of relocation. Since the mortgage rates declined over the past decade, the take-along option was for most cases less attractive to exercise than to repay and take out a new mortgage, with a lower mortgage rate. However, the mortgages rates increased recently and is expected to increase further which could influence the attractiveness of the take-along option. Since the mortgage rates are increasing recently the attractiveness of the take-along option will be higher as the borrowers will have the ability to take-along their lower mortgage rate compared to entering in a loan with a higher mortgage rate.

Given that a take-along option can only be exercised when relocating (Figure 4), the amount of people that are allowed to exercise the option is expected to be limited. In the framework constructed, the prepayment option only relates to the CPR caused by relocation (Figure 6), as this is the only case when the take-along option can be exercised and influence the CPR.

3.4.1 Including the take-along option in a mortgage portfolio valuation

As mentioned before the take-along option in an increasing interest rate environment is expected to influence the prepayment rates. Therefore, in the model two S-curves are constructed. One S-curve representing the original S-curve, which only includes the CPR relocation (blue line in Figure 10). The second curve representing the change to the S-curve including a constant take-along rate (i.e. the expected percentage exercising the take-along option on a portfolio level) (orange line in Figure 10). The assumption is made that mortgagors are only exercising the take-along option if the option is in the money (i.e. the contractual interest rate is below the market rate). This is the case with a negative prepayment incentive. Therefore, the curve used in the model will consist of using the S-curve related to the take along option for values of the incentive represented by $\epsilon(t) \leq 0$ and for all other values the original S-curve (i.e. the curve that only includes the CPR relocation) is applied.

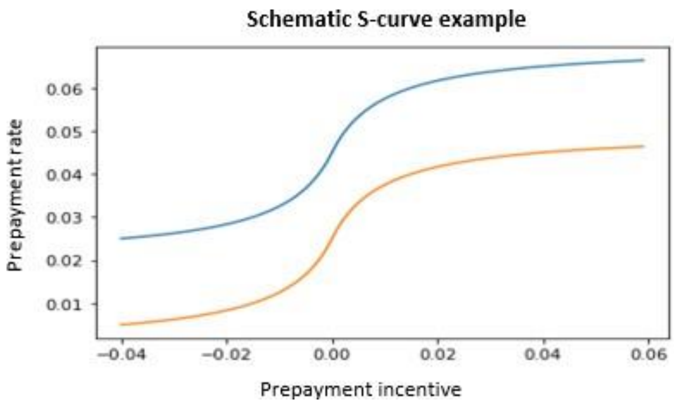


Figure 10: Example on how to implement the take-along option in the S-curve. The blue line indicates the S-curve for CPR relocation and the orange line indicates the shift including the take-along option in the S-curve.

3.4.2 Different take-along structures

The original idea of the take-along option is that the borrower can transfer the mortgage to a new property for the same mortgage rate and the outstanding loan (from now on referred to as 'base take-along structure'). Since the specific terms and conditions may differ per financial institution, also a specific example of a financial institution is included in this research. As an example, the second structure is inspired by the structure offered by ABN AMRO is used since the terms and conditions with respect to the take-along option are transparently described on their website.

The terms and conditions set by ABN AMRO

The ABN AMRO take-along option consists of a floating rate part and a fixed rate part, together representing the mortgage rate. The fixed rate part is set to a certain value at the origination date of the mortgage, also referred to as the basis rate. The basis rate remains the same throughout the entire contract even in case of relocation. The floating part is based on the market mortgage rate corresponding to the relevant LTV ratio, as explained in the example below. The floating part remains the same during the contract, however, with the exception of relocating. If the mortgagor relocates the floating part is adjusted to the market mortgage rate corresponding to the LTV ratio at the time of relocation (ABN-AMRO, 2019).

Example ABN AMRO structure

At origination date the basis rate is set to 3% and the property has a LTV ratio of 90%, therefore corresponding to a floating rate of 0.8%. Hence, the mortgage rate at origination date is 3.8%. In case of relocation and exercising the take-along option, the floating rate will be adjusted. If the LTV ratio decreases to 80%, the floating rate changes to 0.85% and thereby resulting in a mortgage rate on the loan of 3.85% on the new property.

LTV ratio	Floating rate	
	t=0	t=relocating
80%	0.7%	0.85%
90%	0.8%	1.0%

Application of the scenario take-along structure

For the scenario take-along structure the ABN AMRO terms and conditions serve as inspiration. The input data for the basis ratio and the LTV ratio over time are not known. The structure exists of a floor being the basis rate and a variable part changing over time according to market changes. For this scenario the basis rate is set to a percentage of the coupon, which is set to 60 percent. The other 40 percent will be the market mortgage rate of that specific moment in time when the take-along option will be exercised. The formulas below are implemented in the model to provide an estimation of another scenario than the base take-along option.

Variables

$k(t)$	mortgage market rate
b_{rate}	basis rate
r_m	mortgage rate at time t
r_{TA}	take-along rate
$r_{TAtotal}$	total take-along rate at time t
$debt_{out}$	Outstanding debt
$r_m(0)$	initial mortgage rate referred to as coupon

[11]

$$r_m(0) = coupon$$

$$r_m(t) = r_m(t-1) \cdot b_{rate} + k(t) \cdot (1 - b_{rate})$$

Thereafter the principal and the interest payments consist of two parts, one is still based on the coupon rate and one is based on the new $r_m(t)$, i.e. the mortgage rate at time t.

[12]

$$r_{TA_{total}}(t) = r_{TA_{total}}(t-1) + r_{TA}(t)$$

[13]

$$\text{interest payment} = r_{TA_{total}}(t) \cdot debt_{out}(t) \cdot r_m(t) + \left(r_{TA_{total}}(t)\right) \cdot debt_{out}(t) \cdot r_m(0)$$

For the linear mortgage the interest payment will only change. Since, for the annuity mortgage the principal payment schedule depends on the interest payments, the principal payment schedule will subsequently change. In Appendix IV it is shown how this is incorporated in the model.

4. Data

In this chapter provides an overview of the data used for the mortgage valuation model.

4.1 Mortgage portfolio

A mortgage portfolio has been derived representing a Dutch mortgage portfolio. Since there is no mortgage portfolio provided by a financial institution, the mortgage portfolio is composed with information gathered from databases provided by the DNB. Next to that, a portfolio of the Dutch market provide a good average impact for the Dutch financial institutions.

A portfolio changes overtime due to the inflow and outflow of mortgages, a schematic overview is displayed in Figure 11. The flows are divided into three groups:

i. **Inflow**

   The inflow is represented by new mortgages, these new mortgages originated due to buying a home or an additional loan. Buying a home could be the first house or relocating to another property. For an additional loan, clients take out another loan besides their existing one for example for renovation purposes. Also, the mortgagor is not allowed to transfer more than the outstanding balance to the new property when relocating as set out by the terms and conditions from the mortgagee. For this potential deficit, an additional loan must be taken out by the borrower.

ii. **Changes**

   The changes are represented by changes to existing mortgages, due to partial prepayments and full prepayments. The partial prepayment results in lower cash flows for the remaining duration due to a lower outstanding debt, but the mortgage remains. The full prepayment results in repaying the entire mortgage earlier than the contractual maturity date, which provides the mortgagor the opportunity to take out a new mortgage. Clients' motives can be that the characteristics of a new mortgage are more attractive compared to the current mortgage, most often based on the interest rates driven by potential monthly fixed costs.

iii. **Outflow**

   In case mortgages reach the end of the expiration, or the outstanding is fully repaid, or defaults (clients that are not able to pay the monthly payments anymore) result in the outflow of the mortgage. In case the mortgagor wants to open a new loan after repaying the loan, then the original mortgage is outflow and the new mortgage is inflow.

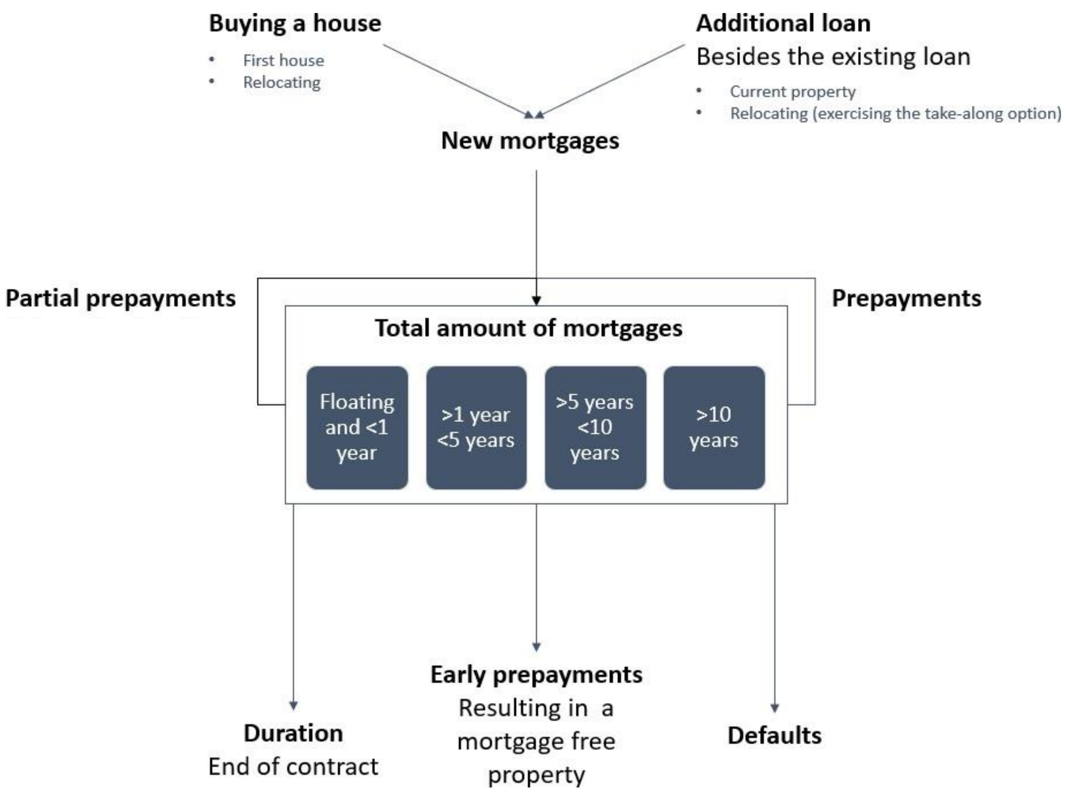


Figure 11: Schematic overview mortgage portfolio flows

The mortgage portfolio is displayed as the total amount of mortgages in Figure 11. These mortgages are split into buckets:

i. Floating and less than one-year fixed mortgage rate;
ii. From one year to five years fixed mortgage rate;
iii. From five years up to ten years fixed mortgage rate; and
iv. More than ten years fixed mortgage rate.

An existing portfolio as per January 2022 is used for the valuation. Mortgages that are originated after January 2022 are not taken into account.

The total mortgage portfolio is also referred as 'totale bancaire verstrekking' in the database of DNB (2022) using monthly data from December 2014 to January 2022. This timeframe was used since this refers to a period with relatively lower interest rates. In order to derive the total portfolio amount as per a certain month, the portfolio development has been included by considering the total portfolio amount and the fixed rate period. For example, in case EUR 20 million of loans were originated in March 2018 for a fixed period of 10 years, the loan balance is assumed to be fully repaid by March 2028. Please refer to the below section for further specifications.

4.1.1 Data preparation

In order to use the data provided by the DNB in the model in Appendix IV, the following data variables are used.

- **Mortgage rate**
  To convert the interest rate into the market mortgage rate a constant factor will be applied for each, since there is no information available for different LTV ratios in the portfolio an

approximation is made for each bucket based on the interest rate in January and the mortgage rates per bucket in January resulting in the following values:

i. Floating and existing less than one year: $\gamma = 2.24$ %;
ii. Existing up from one year to five years: $\gamma = 2.26$ %;
iii. Existing up from five years up to ten years: $\gamma = 2.15$ %; and
iv. Existing more than ten years: $\gamma = 2.38$ %.

- **Duration**
  The mortgages will not exist anymore after a (pre-set) amount of time as contractually agreed. The duration is assumed to be the same for each new mortgage in the same category resulting in the following durations:
  i. Floating and existing less than one year: 36 months;
  ii. Existing up from one year to five years: 72 months;
  iii. Existing up from five years up to ten years: 120 months; and
  iv. Existing more than ten years: 360 months.

- **Fixed interest rate period**
  The fixed interest rate period can differ as contractually agreed, however this data is not available. The duration is assumed to be the same for each new mortgage in the same category resulting in the following durations:
  v. Floating and existing less than one year: 8 months;
  vi. Existing up from one year to five years: 36 months;
  vii. Existing up from five years up to ten years: 72 months; and
  viii. Existing more than ten years 120 months.

- **Starting month**
  The starting month is determined since it is important to know in which month the mortgage originated for the CPR seasoning and seasonality factors. As well as the calculations for the past months and the remaining duration.

- **Past months**
  The past months are important for the seasonality factor and the remaining months for the calculations of the cash flows.

- **Remaining duration**
  Remaining duration is important for the cash flows and is the difference between the duration and the starting month.

- **Prepayments**
  Since the portfolio exists of mortgages originated before February 2022 and could already be originated up from December 2014. The prepayments are taken into account however a simplification is used. The assumption is made that for all four categories the prepayment is 10%, please note the difference between prepayments and prepayments based on relocating. Therefore, the prepayment exceeds the 6.5%.

- **Outstanding loan amounts**
  The outstanding at $t = 0$ is then taken as the original outstanding loan amounts minus the prepayments (assumed to be 10%). For the values of the outstanding loan amounts please see Appendix III.

4.2 CPR input

The CPR input would usually be calibrated based on an actual dataset consisting of a mortgage portfolio on a loan-by-loan basis. Since this data was not available for this research, fictive numbers are applied in order to provide an impact analysis.

- **Refinancing incentive**

The refinancing incentive is based on the S-curve in Figure 9. The following formula provides a S-curve the parameters are calibrated to this illustrative example with the solver in excel on the following formula:

[14]

$$S_{curve} = a + \arctan(\epsilon(t) - b) \cdot 100)/c$$

- **Seasoning effect**

The seasoning effect is included in the prepayment model used by the Public Securities Association ("PSA") which assumes that mortgage prepayment rates are increasing linearly from 0 to 100% PSA factor, i.e. 1. Based on this model, it is estimated that the seasoning effect increases linearly over the first 30 months thereafter it remains 1 after 30 months see Figure 12.

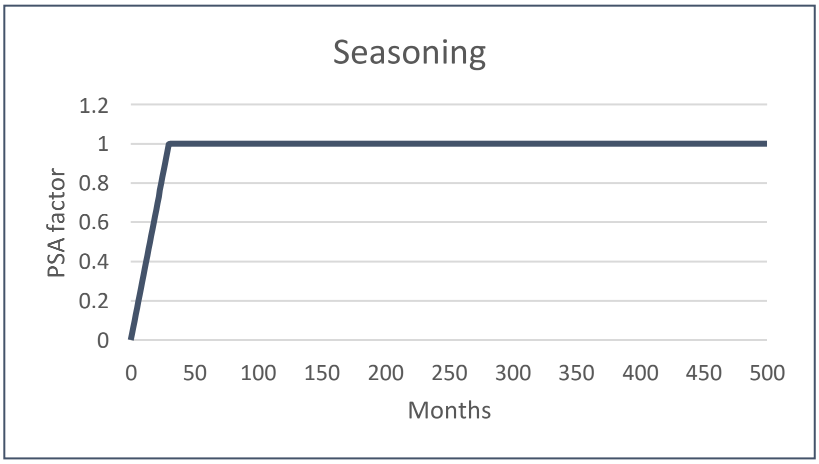


Figure 12: Seasoning factor

- **Seasonality**

The seasonality has been derived based on calculations from the dataset using monthly data and this incorporates the seasonality on the number of new originated loans using data from seven years.

Multiplier for each month											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0.94	0.76	0.73	0.96	0.98	0.92	0.99	1.10	1.18	1.21	1.23	0.97

Table 1: Seasonality table

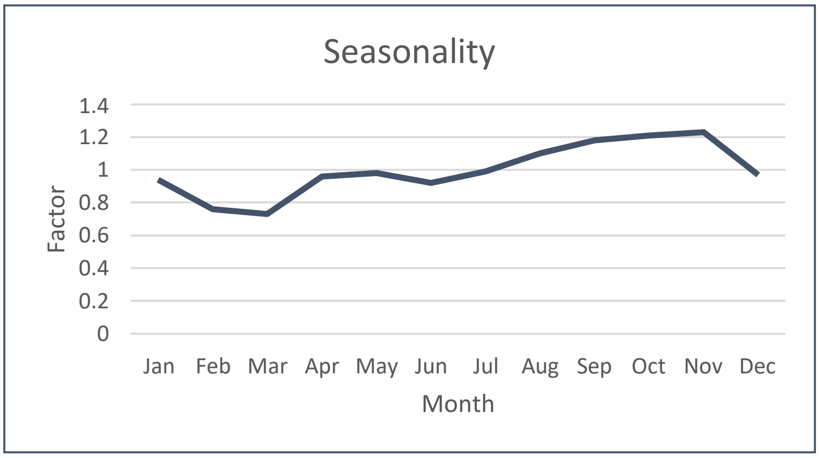


Figure 13: Seasonality graph

4.2.1 CPR input for relocating cause

From 2005 up until 2016 the causes are divided into three buckets (Rabobank, 2015, please refer to Figure 6). There is no specific data available on Dutch CPRs by cause, according to Rabobank. Therefore, the S-curve is shifted to represent the take-along option on the CPR relocation. For the take-along option the prepayment incentive curve is shifted by -2% representing that 2% will shift to exercising the take-along option, assuming that exercising the take-along option is favorable when the mortgage rate is lower than the market rate. As presented in Figure 10, the orange line will be used for negative rate incentive values and the blue line for positive rate incentives.

4.3 Discount curve & Forward rate

The discount curve and forward curve used for the case study are:

- The discount curve values are gathered from Bloomberg with the 6M EURIBOR tickers and settle date 2nd of January 2022 (see appendix for a print screen).
- The forward curve values are gathered from Bloomberg with the 6M EURIBOR tickers and settle date 2nd of January 2022, the zero curve has been used (see appendix for a print screen).

Bloomberg uses a risk neutral approach to construct the curves, and base their analysis on the risk-free rate. Ultimately, the forward rate can be determined in various ways but will eventually serve as an input in the framework.

4.3.1 Data preparation

The resulting cash flows are discounted based on the discount curve described above. However, there is no data available for each month required. Therefore, data preparation using the interpolation and extrapolation methodhas been used to determine the appropriate discount curve values for each month.

Interpolation method

Linear interpolation is a method of curve fitting using linear polynomials to construct new data points within the range of discrete set of known data points. The following formula is used in order to approximate the values between known values:

[15]

$$y = y_0 + (x - x_0)\frac{y_1 - y_0}{x_1 - x_0}$$

Extrapolation method

Since the values before July 2022 are not available in Bloomberg the following linear extrapolation method is used to approximate the values before July 2022:

[16]

$$y = y_0\left(\frac{x_1 - x}{x_1 - x_0}\right) + y_1\left(\frac{x - x_0}{x_1 - x_0}\right)$$

Applicable graphs

The data preparation resulted in the following graphs.

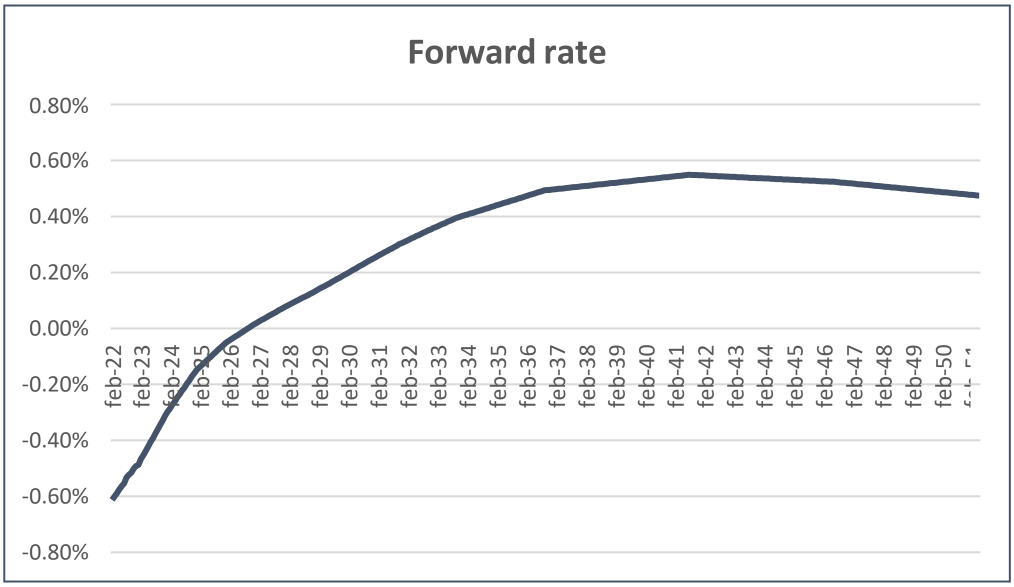


Figure 14: Forward curve

The discount factor to discount the future cash flows will decrease over time in order to reflect the time value of money.

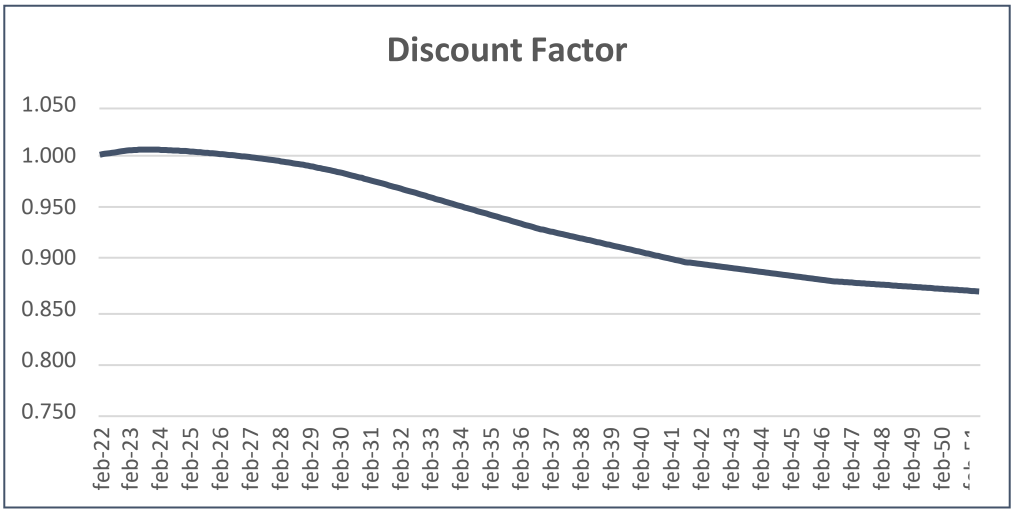


Figure 15: Discount curve

4.4 Assumptions

In this section the assumptions are discussed, which are applied in the model (Appendix IV).

i. It is assumed that the γ parameter is constant overtime.
ii. It is assumed that mortgagors will not exercise the take-along option when it is economically more favorable to exercise the prepayment option.
iii. It is assumed that the mortgagor in case of relocation either repays or exercises the take-along option.
iv. It is assumed that the illustrative data points for the prepayments with the cause relocation is representative.
v. It is assumed that the take-along rate is 2% for all negative prepayment incentives.
vi. It is assumed that the ABN AMRO structure a percentage of the coupon and a percentage of the market mortgage rate is a good approximation of the structure. As well as, the application in the model by the sum of the take-along-rate for the bucket of mortgages.
vii. It is assumed that the portfolio of mortgages, provides good insights for the potential impact on a mortgage portfolio valuation.
viii. it is assumed that financial institutes already have included other options not included in the model, such as the default option, in their models and the take-along option has no influence on the other options.
ix. It is assumed that the data of the CPR relocators is without the take-along option in Figure 6 and Figure 9.
x. It is assumed that the burnout should not be included in the CPR calculations.

5. Results

In this chapter the results of all tests and outputs are listed. Firstly, the calibrated results of the S-curve and the altered S-curve caused by the take-along option are discussed. The results of the calibrated S-curve is required for the CPR calculations, the CPR and the take-along rate are needed for estimating the cash flows and thereby deriving the value of the mortgage portfolio. Next, a sensitivity analysis is performed for the S-curve shape, the gamma, take-along rate, the discount and forward curve and the basis rate and the differences compared to the base result. Furthermore, the impact of the take-along option is compared to the prepayment option. The results are in further discussed in the following paragraphs.

5.1 S-curve

The S-curve is required for the refinancing incentive values for the CPR calculations. However, the historical data points provided in Figure 9 does not allow for values for the refinancing incentive between the data points. Therefore, a S-curve function is required, which is achieved by calibrating the $\arctan()$ function. Subsequently, the S-curve is altered to include the take-along option.

5.1.1. S-curve cause relocation only prepayments

The $\arctan()$ function is calibrated using the illustrative example as displayed in Figure 9. The $\arctan()$ function in Figure 16 is used in order to get values between the data points that were required to derive the S-curve function. The function for the S-curve is a result of minimizing the values of $RMSE$ and subsequently $NRMSE$.

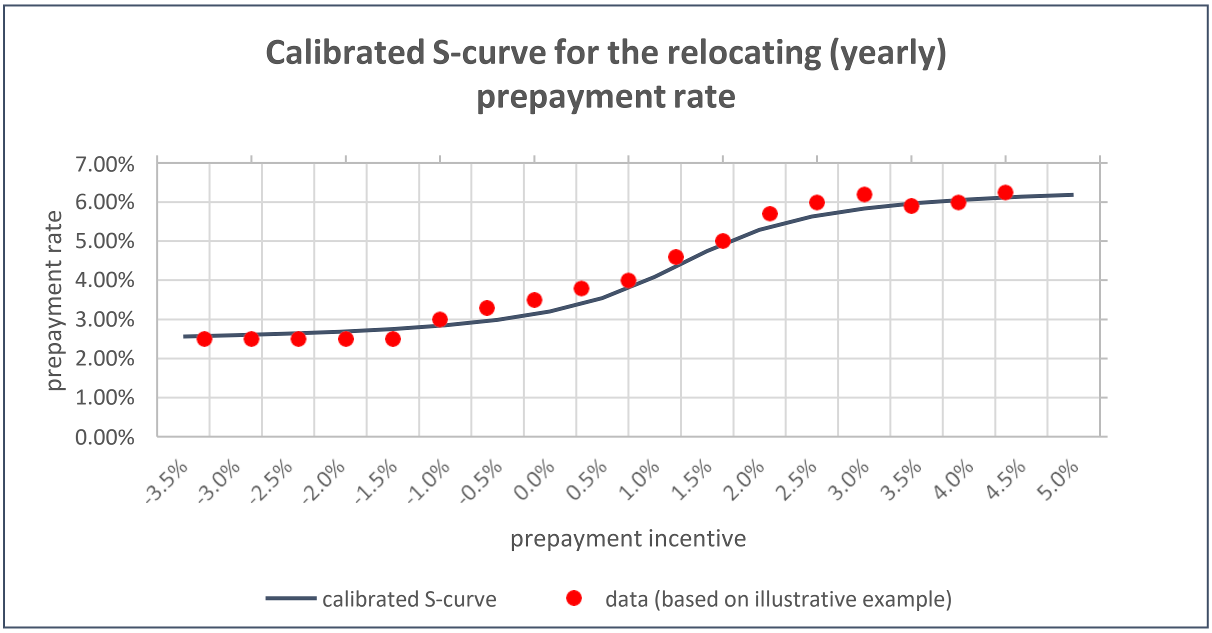


Figure 16: Calibrated S-curve on illustrative figure.

The calibration resulted in the following values for the parameters for the S-curve:

$a = 0.04409$
$b = 0.012435$
$c = 73.78206$

Resulting in the following S-curve:

[17]

$$S_{curve} = 0.04409 + \arctan((\epsilon(t) - 0.012435) \cdot 100)/73.78206$$

The error of the calibrated results:

$RMSE = 0.0019$
$NRMSE = 0.050827$

For the $RMSE$ and the $NRMSE$ the lower the value the more accurate the S_{curve} function. The $RMSE$ value of 0.0019 indicates that the residual value per data point is approximately 0.19%. This is relatively limited considering the total range of 2.5% to 6.25%. Given this total range $NRMSE$ is higher as expected.

5.1.1 Including the take-along option

Figure 17 represents the S-curve applied for including the take-along option in the model. Figure 17 shows the shift for the negative incentive downwards by 2%, i.e. the take-along rate is equal to 2%. The grey line indicates the line used for including the take-along option. This grey line is used in the model for including the take-along option. As mentioned before the behavior of mortgagors is not expected to be heterogenic, thereforethe prepayment rate is not set equal to 0% for the negative prepayment incentives.

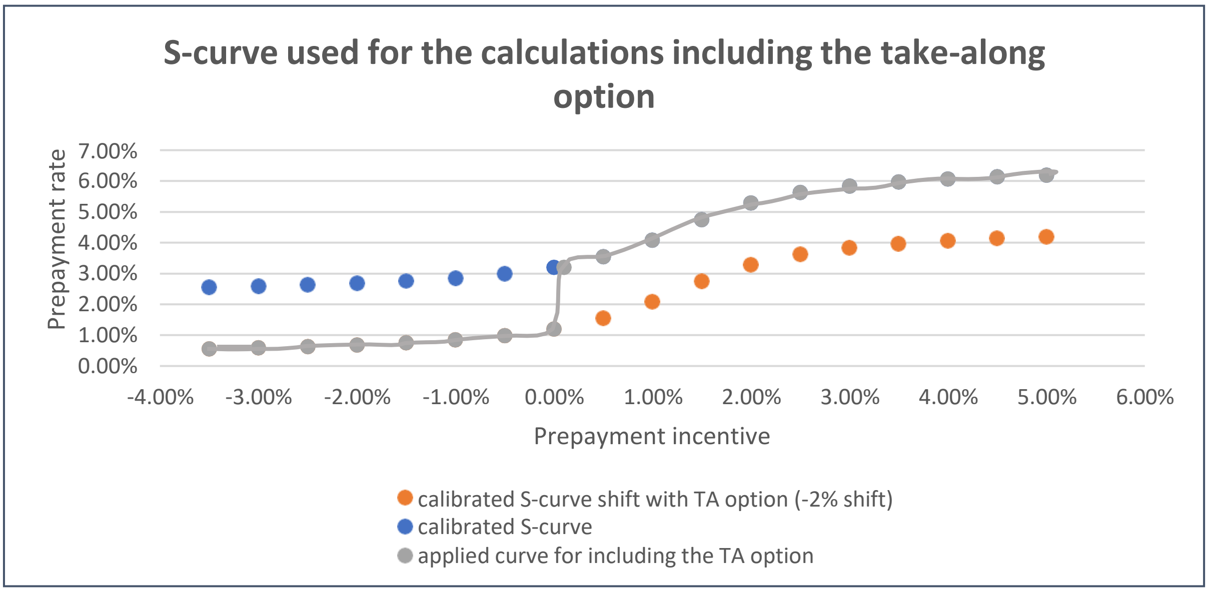


Figure 17: Prepayment rate with the relocating cause, changed with a -2% towards the take-along option for a negative incentive.

5.2 Base result

The base results are divided into two groups of mortgage types being: annuity and linear mortgages. For the base results the described parameters in the methodology and data sections are used. The results are provided in table format and in a figure. The percentage difference for excluding the option and other options is provided in the table. The sensitivity analysis performed is further explained in the next section. The base results are calculated based on the following input parameters:

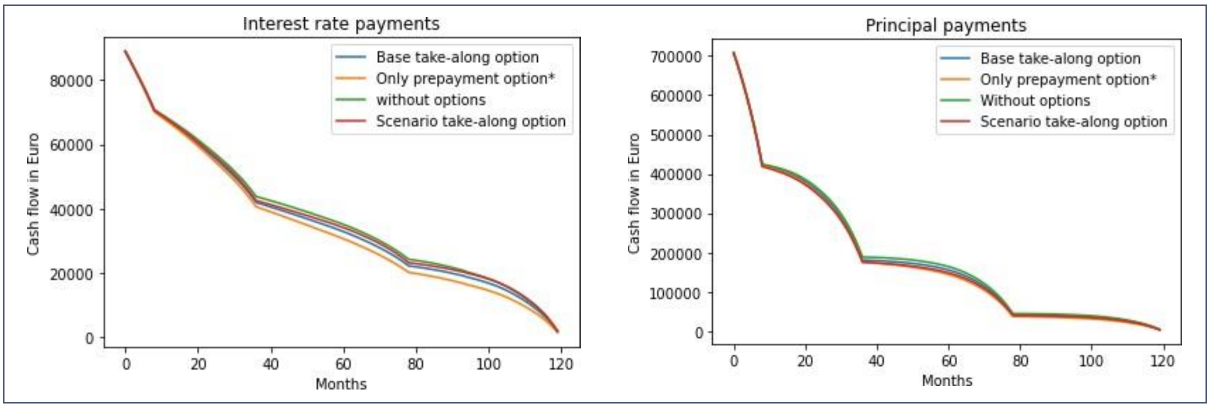
- The S-curve is based on Figure 16: resulting in equation [17].
- The gamma values are constant overtime: $\gamma_1 = 2.24\%, \gamma_2 = 2.26\%, \gamma_3 = 2.15\%$ and $\gamma_4 = 2.38\%$
- Take-along rate is constant and 2% for a negative prepayment incentive
- Forward and discount curve provided by Bloomberg
- Basis rate 60%

Table 2 for annuity and Table 3 for linear mortgages provide the NPV values of the discounted cash flows separated between prepayment, debt, interest and principal since these are the four main elements related to the mortgage cash flows. The debt represents the cash flow based on the ending period of the fixed rate period. Since either the prepayment option will be exercised (ascurrently included in the valuation of mortgages), the focus is on the comparison between the resulting difference when including either the base take-along option or the scenario take-along option. For the prepayment option[1], base take-along option and the scenario take-along option, the numerical values in euros are presented. Next to this, the relative percentage difference for either the base take-along option or scenario take-along option versus the prepayment option are displayed.In Figure 18 and Figure 19 for annuity and Figure 20 and Figure 21 for linear the monthly discounted cash flows are presented. The results are discussed below.

5.2.1 Annuity

base results	**Only prepayment***	**Base take-along**	**Δ%**	**Scenario take-along**	**Δ%**
Prepayment	€ 5,900,052.67	€ 2,622,580.00	(55.55)	€ 2,632,038.84	(55.39)
Debt	€ 33,544,822.67	€ 36,020,673.40	7.38	€ 36,538,804.72	8.93
Interest	€ 4,163,254.49	€ 4,350,955.97	4.51	€ 4,456,137.28	7.03
Principal	€ 21,498,350.50	€ 22,212,667.81	3.32	€ 21,682,087.71	0.85
Total	€ 65,106,480.33	€ 65,206,877.69	0.15	€ 65,309,068.56	0.31

Table 2: Base result for annuity mortgage type.



[1] The prepayment considered in the results is based on the only prepayments caused by relocating therefore denoted in the tables as only prepayments*

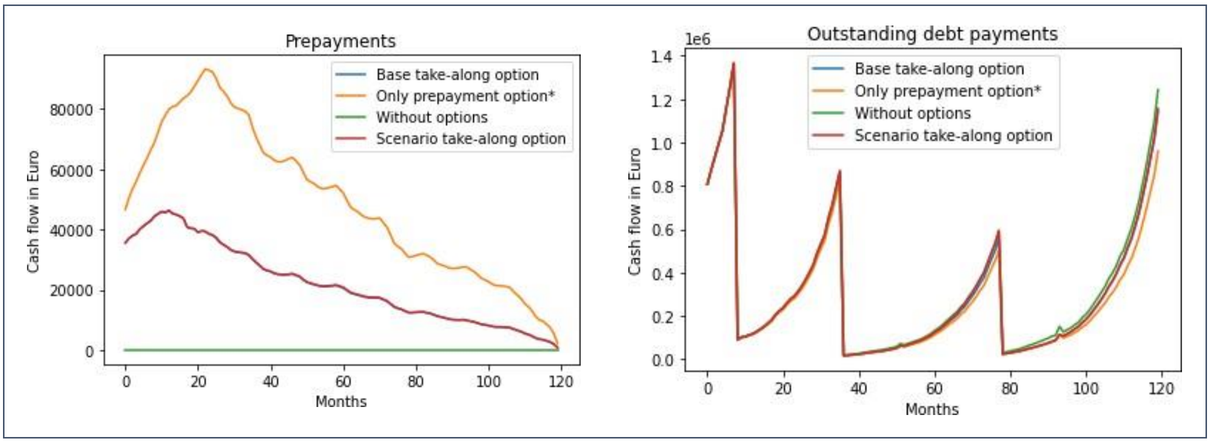


Figure 18: Discounted cash flow scheme for annuity mortgage.

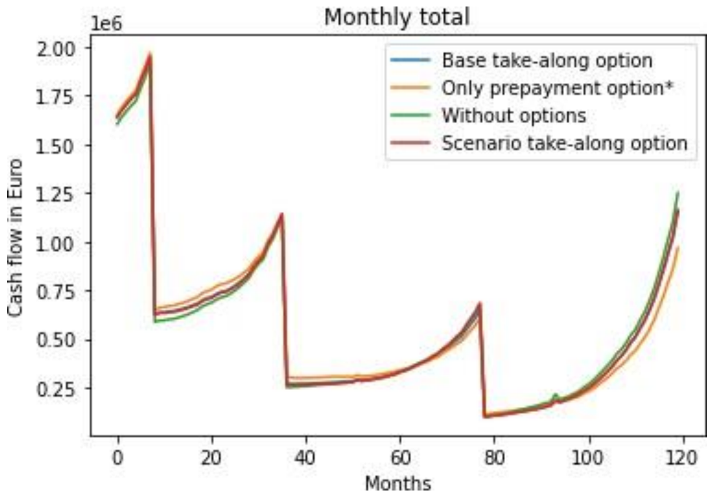


Figure 19: Total monthly discounted cash flows annuity.

5.2.2 Linear

base results	**Only prepayment***	**Base take-along**	**Δ%**	**Scenario take-along**	**Δ%**
Prepayment	€ 5,747,486.21	€ 2,562,324.07	(55.42)	€ 2,562,324.07	(55.42)
Debt	€ 31,937,277.45	€ 34,459,331.69	7.90	€ 34,459,331.69	7.90
Interest	€ 4,041,576.17	€ 4,224,985.14	4.54	€ 4,308,992.94	6.62
Principal	€ 23,213,982.45	€ 23,833,060.11	2.67	€ 23,833,060.11	2.67
Total	€ 64,940,322.28	€ 65,079,701.02	0.21	€ 65,163,708.82	0.34

Table 3: Base results for linear mortgage type.

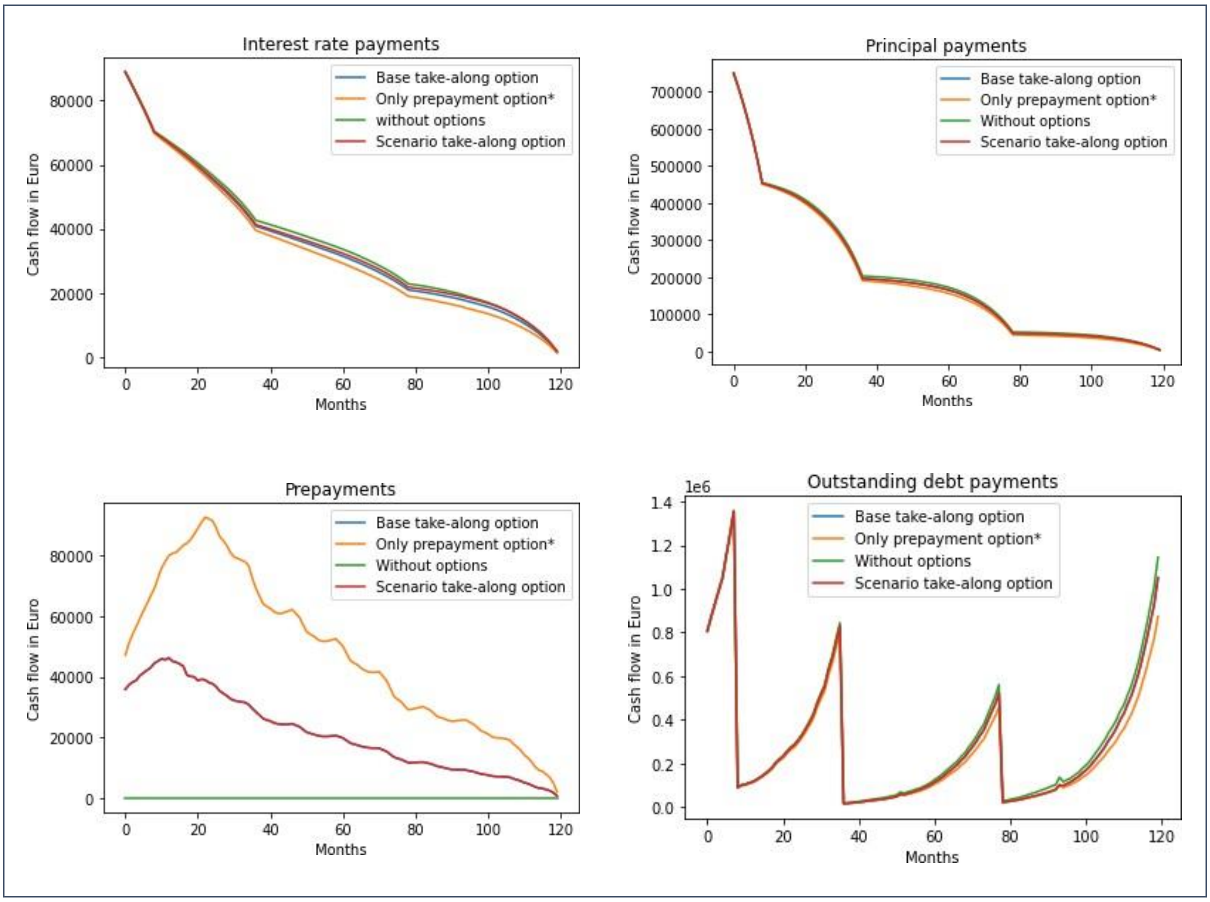


Figure 20: Discounted cash flow scheme for linear mortgage.

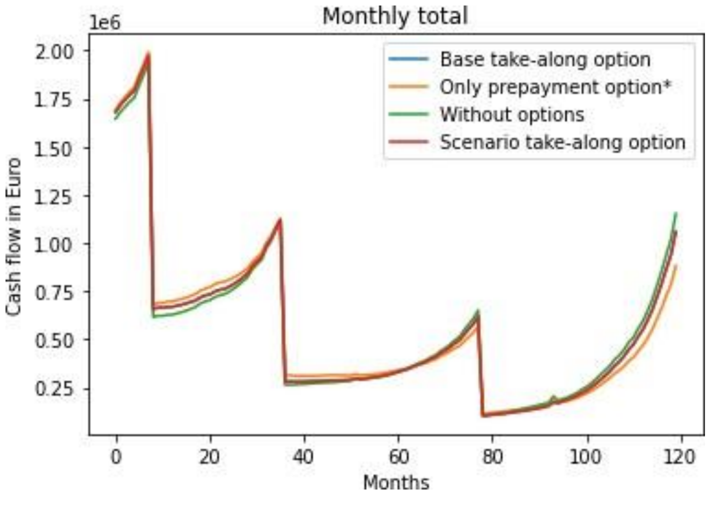


Figure 21: Total monthly discounted cash flows linear.

Interpretation of results

The patterns of the different cash flow components are relatively similar for annuities and linear mortgages, and therefore the interpretation of the results is often applicable for the two mortgage types. The difference between the two mortgage types is the volume as presented on the y-axis, and therefore yielding different percentage difference compared to the prepayment option.

As can be seen from the tables, there is a noticeable difference in each category (i.e. prepayment, debt, interest and principal NPV) for the annuity mortgage since the principal and the interest payments depend on each other as can be seen from Figure 8 and the corresponding formulasin [5]. On the other hand, the linear mortgage only shows a difference in interest payments, since theprincipal payments depend on the debt and not on the interest payments. This is in line with expectations given the formulas in [6].

The size of the cash flows is highest at the beginning of the timeframe, which is in line with expectation as there is no new origination assumed but rather only outflow. Furthermore, the peaks in the cash flow debt graphs are linked to the fixed rate period of the different buckets, and thereforefour peaks shown as expected.

The prepayment declines by approximately 55% with the take-along option included compared to the prepayments caused by relocators. However, considering the adverse impact on the debt interest and principal, the difference on total level is between 0.15% and 0.31% for annuity mortgages and between 0.21% and 0.34% for linear mortgages. This is partly explained by the different formulas for the different mortgage types. Furthermore, the scenario take-along option-based results show that an altered form of the take-along option structure results in a different outcome. Therefore, the specific applicable terms and conditions should be carefully considered when offering the take-along option as a mortgagee.

The cause of the similarities and differences cannot be explained single relations amongst the factors but are rather explained by effects from multiple factors, such as the S-curve shape, gamma, take-along rate, the curves and the basis rate. The impact and relationships of these multiple factors are further elaborated in the next section.

5.3 Sensitivity analysis

For the sensitivity analysis the base results of both types annuity and linear mortgages are compared to the values when changing the parameters. The sensitivity analysis has been performed on the following parameter: the S-curve, gamma, take-along rate, discount factor and forward rate, and basis rate. These parameters have been selected for the following reasons. Firstly, the S-curve has been selected since this curve is altered to include the take-along option in the valuation. Secondly, the gamma and the curves (discount factor and forward curve) are market developments of which the interest rate is the main reason to exercise the take-along option. Lastly, the basis rate has direct influence on the scenario take-along option value. The duration and fixed rate period have not been included in the sensitivity analyses because these elements are more portfolio specific and therefore dependent on the case study used.

I. The S-curve

The S-curve as applied in the base results ranges from 2.5% to 6.25%. As a sensitivity analysis on the S-curve, a parallel shift of 1% compared to the base curve has been considered to derive the sensitivity on this specific parameter. In Table 4 and Table 5, the results for respectively annuity and linear mortgages are depicted. The results in these tables are presented as a percentage change compared to the base result. In Figure 22, the difference on a total level compared to the base result are displayed. For example, for the annuity a difference for the base take-along on a total level was 0.15%. When considering the base take-along sensitivity below, a downward step of 1% yield a percentage change of 0.11%. Subsequently, a downward step of 1% yield a percentage change of 0.11% for only prepayment for relocators. Therefore, the difference in Table 2 remains approximately 0.15%, resulting in a projected value of less than 0.01% in Figure 22. A similar approach has been applied for all other presentations of the sensitivity analyses.

Annuity

S-curve	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-1% (in %)	base	+1% (in %)	-1% (in %)	base	+1% (in %)	-1% (in %)	base	+1% (in %)
Prepayment	(31.07)	€ 5,900,052.67	29.41	(77.05)	€ 2,622,580.00	72.93	(77.16)	€ 2,632,038.84	73.02
Debt	4.09	€ 33,544,822.67	(3.85)	4.25	€ 36,020,673.40	(3.99)	4.26	€ 36,538,804.72	(4.01)
Interest	2.75	€ 4,163,254.49	(2.64)	2.81	€ 4,350,955.97	(2.70)	2.87	€ 4,456,137.28	(2.75)
Principal	1.93	€ 21,498,350.50	(1.87)	1.98	€ 22,212,667.81	(1.91)	1.95	€ 21,682,087.71	(1.89)
Total	0.11	€ 65,106,480.33	(0.10)	0.11	€ 65,206,877.69	(0.11)	0.12	€ 65,309,068.56	(0.11)

Table 4: Annuity sensitivity S-curve

Linear

S-curve	Only prepayment*			Base take-along			Scenario take-along		
	-1% (in %)	base	+1% (in %)	-1% (in %)	base	+1% (in %)	-1% (in %)	base	+1% (in %)
Prepayment	(31.03)	€ 5,747,486.21	29.36	(76.84)	€ 2,562,324.07	72.68	(76.84)	€ 2,562,324.07	72.68
Debt	4.35	€ 31,937,277.45	(4.09)	4.50	€ 34,459,331.69	(4.23)	4.50	€ 34,459,331.69	(4.23)
Interest	2.76	€ 4,041,576.17	(2.65)	2.83	€ 4,224,985.14	(2.72)	2.88	€ 4,308,992.94	(2.76)
Principal	1.60	€ 23,213,982.45	(1.55)	1.64	€ 23,833,060.11	(1.59)	1.64	€ 23,833,060.11	(1.59)
Total	0.14	€ 64,940,322.28	(0.13)	0.14	€ 65,079,701.02	(0.14)	0.15	€ 65,163,708.82	(0.14)

Table 5: Linear sensitivity to S-curve

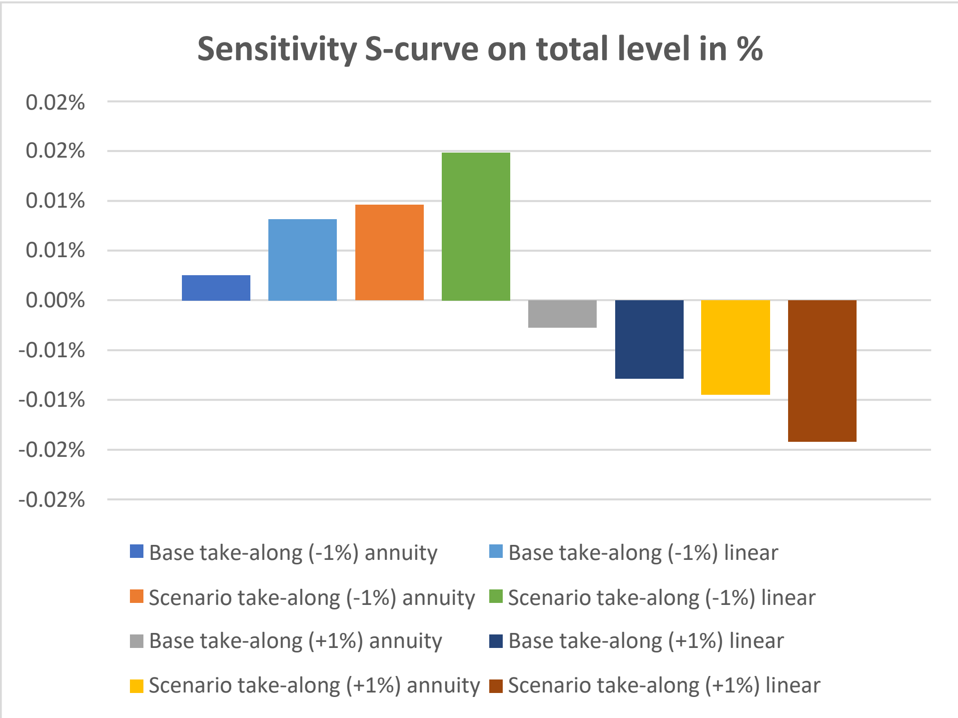


Figure 22: Sensitivity on the S-curve shift.

Interpretation of results

The results indicate that the level of prepayments is predominantly influenced by a change in the S-curve. Which can be explained by the formula of the CPR, the corresponding values of the prepayment incentive shift with 1% meanwhile the incentive itself stays the same. The other percentage changes are a result of the change in prepayments, caused by the shifted S-curve. In this case study these percentages and the total percentage change are relatively limited compared to the change in the prepayment. This was also visible in the base results, and therefore as expected.

When looking at Figure 22, the scenario take-along option appears to be most impacted by the change in the S-curve for both linear and annuity mortgage types, with the largest difference for the linear mortgage type. This is the result of a difference in the cash flow scheme driven by the prepayment cash flow change caused by the CPR value changes.

II. Gamma

The sensitivity analysis on gamma included a parallel step of 0.5% compared to the base gamma. In Table 6 and Table 7, the results of the sensitivity analysis are presented for annuity and linear mortgage types, respectively. In Figure 23, a graphical presentation for the sensitivity analysis for gamma is depicted, in line with the way of presenting of the S-curve.

Annuity

Gamma	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)
Prepayment	7.33	€ 5,900,052.67	(4.82)	73.51	€ 2,622,580.00	(25.26)	73.00	€ 2,632,038.84	(25.22)
Debt	(0.94)	€ 33,544,822.67	0.62	(3.73)	€ 36,020,673.40	1.32	(4.45)	€ 36,538,804.72	1.33
Interest	(0.76)	€ 4,163,254.49	0.50	(3.37)	€ 4,350,955.97	1.19	(5.22)	€ 4,456,137.28	3.08
Principal	(0.50)	€ 21,498,350.50	0.32	(2.45)	€ 22,212,667.81	0.78	(1.17)	€ 21,682,087.71	0.77
Total	(0.03)	€ 65,106,480.33	0.02	(0.16)	€ 65,206,877.69	0.06	(0.29)	€ 65,309,068.56	0.19

Table 6: Annuity sensitivity to gamma

Linear

Gamma	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)
Prepayment	7.34	€ 5,747,486.21	(4.82)	73.74	€ 2,562,324.07	(25.35)	73.74	€ 2,562,324.07	(25.35)
Debt	(1.00)	€ 31,937,277.45	0.66	(3.97)	€ 34,459,331.69	1.38	(3.97)	€ 34,459,331.69	1.38
Interest	(0.76)	€ 4,041,576.17	0.50	(3.40)	€ 4,224,985.14	1.19	(4.97)	€ 4,308,992.94	3.07
Principal	(0.42)	€ 23,213,982.45	0.27	(2.06)	€ 23,833,060.11	0.71	(2.06)	€ 23,833,060.11	0.71
Total	(0.04)	€ 64,940,322.28	0.03	(0.18)	€ 65,079,701.02	0.07	(0.28)	€ 65,163,708.82	0.19

Table 7: Linear sensitivity to gamma

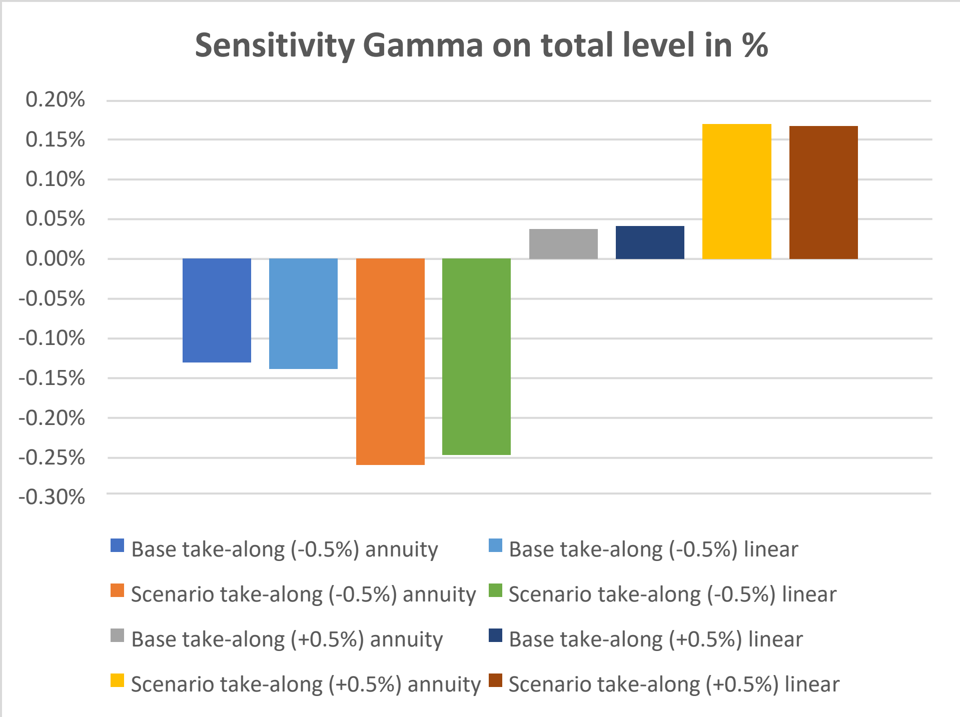


Figure 23: Sensitivity on the gamma shift.

Interpretation of results

Gamma is correlated with the market mortgage rate since it is a (constant) parameter that is added up to the market interest rates (i.e. the forward curve). The market mortgage rate is included in the model in two ways. Firstly, the prepayment incentive refers to the difference between the market mortgage rate and the coupon. Secondly, the market mortgage rate is included in the calculations of the scenario take-along since it consists partly of the coupon rate (60%) and partly of the market mortgage rate (40%).

The effect on the prepayment incentive by a positive shift of 0.5% is a shift to the left on the x-axis in Figure 17 resulting in lower prepayment incentive and therefore lower prepayments and the take-along rate will increase by 2% if the prepayment incentive changes from a positive value towards a negative value, and vice versa.

Considering that the market mortgage rate is only partly included in the scenario take-along option, the relative effect on the scenario take-along is bigger (Figure 23). This is as expected given the above.

III. Take-along rate

The sensitivity analysis on the take-along rate includes a parallel step of 0.5% compared to the take-along rate of 2% of the S-curve. In Table 8 and Table 9, the results of the sensitivity analysis on the take-along rate are presented for the annuity and linear mortgage type, respectively. In Figure 24, the results of the sensitivity analysis are graphically presented in line with the methodology described under the S-curve sensitivity analysis.

Annuity

Take-along rate	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)
Prepayment	-	€ 5,900,052.67	-	32.49	€ 2,622,580.00	(33.36)	32.42	€ 2,632,038.84	(33.38)
Debt	-	€ 33,544,822.67	-	(1.79)	€ 36,020,673.40	1.85	(2.16)	€ 36,538,804.72	2.23
Interest	-	€ 4,163,254.49	-	(1.11)	€ 4,350,955.97	1.13	(1.72)	€ 4,456,137.28	1.78
Principal	-	€ 21,498,350.50	-	(0.82)	€ 22,212,667.81	0.84	(0.19)	€ 21,682,087.71	0.18
Total	-	€ 65,106,480.33	-	(0.04)	€ 65,206,877.69	0.04	(0.08)	€ 65,309,068.56	0.08

Table 8: Annuity sensitivity to take-along rate

Linear

Take-along rate	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)	-0.5% (in %)	base	+0.5% (in %)
Prepayment	-	€ 5,747,486.21	-	32.33	€ 2,562,324.07	(33.21)	32.33	€ 2,562,324.07	(33.21)
Debt	-	€ 31,937,277.45	-	(1.91)	€ 34,459,331.69	1.97	(1.91)	€ 34,459,331.69	1.97
Interest	-	€ 4,041,576.17	-	(1.12)	€ 4,224,985.14	1.14	(1.62)	€ 4,308,992.94	1.67
Principal	-	€ 23,213,982.45	-	(0.66)	€ 23,833,060.11	0.67	(0.66)	€ 23,833,060.11	0.67
Total	-	€ 64,940,322.28	-	(0.05)	€ 65,079,701.02	0.06	(0.09)	€ 65,163,708.82	0.09

Table 9: Linear sensitivity to take-along rate

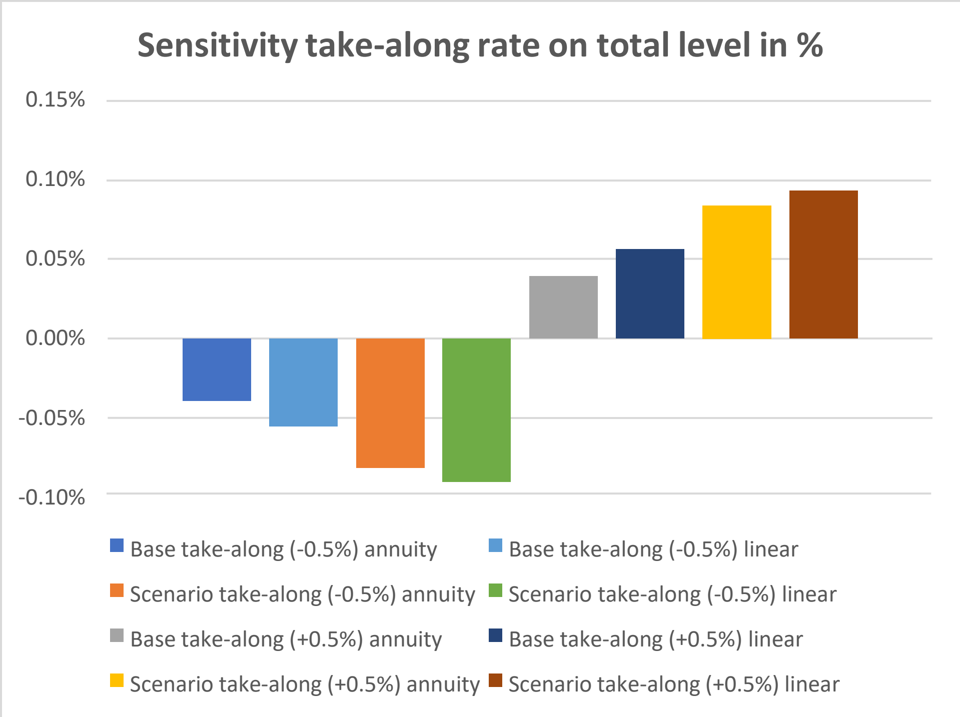


Figure 24: Sensitivity on the take-along rate shift.

Interpretation of results

In the tables above the change to the base results for the only prepayment caused by relocating remains the same. This is expected since the take-along rate does not have an effect as the scenario considering only prepayment does not contain a take-along option.

The results indicate that an increase in the take-along rate would result in lower prepayments. This is as expected given that it is assumed that more people will take-along their mortgage instead of prepaying. On a total level, there is a positive impact when the take-along rate increases, both for annuity and the linear mortgage type and also both for the base take-along option and the scenario take-along option. This indicates that the take-along rate applied is important to assess correctly when deriving the value of the mortgage portfolio.

The results for the linear mortgage type are higher compared to the annuity mortgage type. This appears to be driven by the differences in the outstanding debt and the remaining principal. In addition, the impact of the scenario take-along is higher compared to the base take-along which is driven by a difference in interest rates since 60% of the coupon is considered in the interest calculation.

IV. Discount curve & forward rate

The sensitivity analysis on the discount curve and the forward rate includes a 5-basis point shift in both directions on the base values of the curves. In Table 10 and Table 11, the results of the sensitivity analysis on the discount curve and the forward rate are presented for the annuity and linear mortgage type, respectively. In Figure 25 the results of the sensitivity analysis are graphically presented in line with the methodology described under the S-curve sensitivity analysis.

Annuity

Curves	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-5bp (in %)	base	+5bp (in %)	-5bp (in %)	base	+5bp (in %)	-5bp (in %)	base	+5bp (in %)
Prepayment	0.55	€ 5,900,052.67	(0.53)	4.18	€ 2,622,580.00	(3.77)	4.13	€ 2,632,038.84	(3.74)
Debt	(0.13)	€ 33,544,822.67	0.12	(0.25)	€ 36,020,673.40	0.24	(0.34)	€ 36,538,804.72	0.27
Interest	(0.11)	€ 4,163,254.49	0.11	(0.26)	€ 4,350,955.97	0.24	(0.52)	€ 4,456,137.28	0.45
Principal	(0.09)	€ 21,498,350.50	0.09	(0.21)	€ 22,212,667.81	0.19	(0.06)	€ 21,682,087.71	0.13
Total	(0.05)	€ 65,106,480.33	0.05	(0.06)	€ 65,206,877.69	0.06	(0.08)	€ 65,309,068.56	0.07

Table 10: Annuity sensitivity to the discount curve and forward rate

Linear

Curves	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-5bp (in %)	base	+5bp (in %)	-5bp (in %)	base	+5bp (in %)	-5bp (in %)	base	+5bp (in %)
Prepayment	0.55	€ 5,747,486.21	(0.53)	4.23	€ 2,562,324.07	(3.81)	4.23	€ 2,562,324.07	(3.81)
Debt	(0.13)	€ 31,937,277.45	0.13	(0.27)	€ 34,459,331.69	0.24	(0.27)	€ 34,459,331.69	0.24
Interest	(0.11)	€ 4,041,576.17	0.11	(0.26)	€ 4,224,985.14	0.24	(0.49)	€ 4,308,992.94	0.43
Principal	(0.08)	€ 23,213,982.45	0.08	(0.19)	€ 23,833,060.11	0.18	(0.19)	€ 23,833,060.11	0.18
Total	(0.05)	€ 64,940,322.28	0.05	(0.06)	€ 65,079,701.02	0.06	(0.08)	€ 65,163,708.82	0.07

Table 11: Linear sensitivity to the discount curve and forward curve

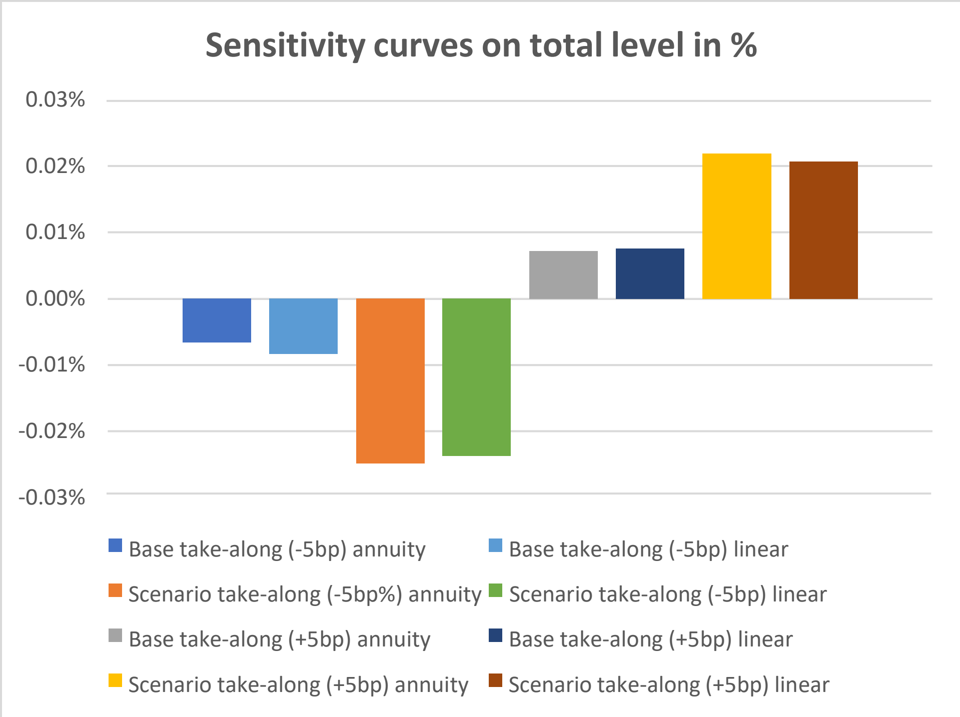


Figure 25: Sensitivity on the discount curve and forward rate shift.

Interpretation of results

The scenario take-along considers the forward rate in two ways whilst only prepayment and the base take-along option only consider the forward rate in the S-curve for the prepayment incentive.

Lowering the discount curve would normally increase the present value of future cash flows, however in this sensitivity analysis this is not the case as the present value (on a total level) is approximately 5 basis points lower compared to the base case with a downward step of 5 basis points in the discount curve, and vice versa. This is explained by the relative shift on the prepayment incentive, which results in the take-along option to be exercised less in case the forward rate is shifted downwards. This is as expected as a lower forward rate indicates that interest rates will decline over time, which would result in fewer people to exercise the take-along option since there are relatively less scenarios in which the incentive becomes negative.

It is noted that both the forward curve and discount curve are more subject to market developments.

V. Basis rate

The sensitivity analysis on the basis rate includes a parallel step of 0.1 compared to the base values of the basis rate of 0.6. In Table 12 and Table 13, the results of the sensitivity analysis on the basis rate are presented for the annuity and linear mortgage type, respectively. In Figure 26 the results of the sensitivity analysis are graphically presented in line with the methodology described under the S-curve sensitivity analysis.

Annuity

Basis rate	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.1 (in %)	base	+0.1 (in %)	-0.1 (in %)	base	+0.1 (in %)	-0.1 (in %)	base	+0.1 (in %)
Prepayment	-	€ 5,900,052.67	-	-	€ 2,622,580.00	-	-	€ 2,632,038.84	-
Debt	-	€ 33,544,822.67	-	-	€ 36,020,673.40	-	-	€ 36,538,804.72	-
Interest	-	€ 4,163,254.49	-	-	€ 4,350,955.97	-	0.01	€ 4,456,137.28	(0.02)
Principal	-	€ 21,498,350.50	-	-	€ 22,212,667.81	-	-	€ 21,682,087.71	-
Total	-	€ 65,106,480.33	-	-	€ 65,206,877.69	-	-	€ 65,309,068.56	-

Table 12: Annuity sensitivity to the basis rate

Linear

Basis rate	**Only prepayment***			**Base take-along**			**Scenario take-along**		
	-0.1 (in %)	base	+0.1 (in %)	-0.1 (in %)	base	+0.1 (in %)	-0.1 (in %)	base	+0.1 (in %)
Prepayment	-	€ 5,747,486.21	-	-	€ 2,562,324.07	-	-	€ 2,562,324.07	-
Debt	-	€ 31,937,277.45	-	-	€ 34,459,331.69	-	-	€ 34,459,331.69	-
Interest	-	€ 4,041,576.17	-	-	€ 4,224,985.14	-	0.01	€ 4,308,992.94	(0.02)
Principal	-	€ 23,213,982.45	-	-	€ 23,833,060.11	-	-	€ 23,833,060.11	-
Total	-	€ 64,940,322.28	-	-	€ 65,079,701.02	-	-	€ 65,163,708.82	-

Table 13: Linear sensitivity to the basis rate

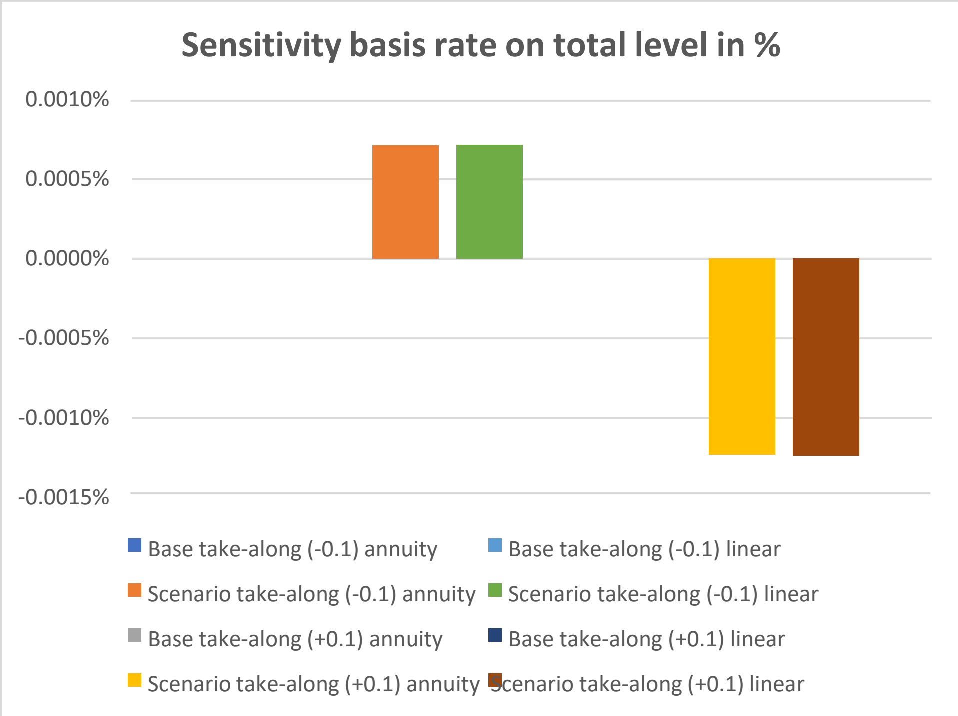


Figure 26: Sensitivity on the basis rate shift.

Interpretation of results

Since the basis rate is only applicable to the scenario take-along option, there are no results for the other options. The results in both the tables and the figure indicate that the impact of a shift in the basis rate is limited.

A positive shift meaning that the basis rate increases and the coupon rate increases and the market mortgage rate decreases, results in a negative shift and vice versa. This is expected since the forward rate increases in this research, hence the mortgage rate would increase more as formulated in formula [11].

6. Conclusion

In this chapter the conclusion and a reflection on the research questions is given. The sub-questions are first discussed and thereafter concluded with the main research question.

Sub-question 1:	*How are the mortgages valued in scientific research?*

Literature research provided insights for the calculations of mortgages, and consists of various components being discounting and underlying assumptions for cash flows such as prepayments incorporated with the CPR. The valuation of mortgage portfolio is often performed using a discounted cash flow method, which thereby calculates the present value as per a certain valuation date. For this research the change in NPV value is used in the valuation method.

Based on the literature research performed, the take-along option has been noted but the take-along option appears not be considered in the valuation of a mortgage portfolio as a separate item.

Sub-question 2:	*How does the take-along option impact a mortgage portfolio valuation?*

A mortgage portfolio can be valued using the option-based approach or the econometric approach. Since the option-based approach requires a dataset consisting of individual items (such as the mortgage of an individual) which was not available for this research, the econometric method has been applied in order to include the take-along option in the mortgage valuation. In addition, the option-based approach does not capture the heterogeneity behavior of the prepayment option, whereas the econometric approach does capture this.

Since it is expected that the take-along option will be exercised more often in an increasing interest rate environment, the prepayment incentive becomes more negative, including the take-along option has an influence on the CPR caused by relocation.

Sub-question 3:	*What effect has the take-along option structure on the mortgage portfolio valuation and what are the main sensitivities?*

In order to estimate the effect of the take-along option, a mortgage portfolio was constructed based on data from 2015 to 2022 based on the relative monthly new originations in that period and the respective fixed term period. As mentioned above, the NPV was calculated for this portfolio under four scenarios being (i) exercising no option, (ii) the prepayment option only for relocators, (iii) exercising the base take-along option and (iv) exercising the scenario take-along option.

Subsequently, a sensitivity analysis was performed on the S-curve shape, gamma, take-along rate, the curves (discount rate and forward rate) and the basis rate. Based on the sensitivity analysis the scenario-based take-along option appears to be most sensitive in all sensitivity analyses. In addition, the gamma sensitivity analysis indicates that both take-along options are more sensitive to a downward movement of the gamma than an upward movement of the gamma.

Main research question:	*What is the impact on a mortgage portfolio if the take-along option is included in the valuation (by implementing the econometric approach)?*

In the study objective the hypothesis for the main research question was formulated as follows: *"It is assumed that the take-along option has an influence on the mortgage portfolio valuation as the prepayments will likely decrease when borrowers have the option to take-along their mortgage with a lower (fixed) rate compared to the current market rate. When prepayments decrease, it is likely that the total mortgage value will increase and hence it is expected that the take-along option has a positive contribution to the mortgage portfolio valuation. A different NPV is expected for different types of mortgages, hence the unequal cash flow scheme."*

The base results indicate that the take-along option will be exercised more often as a result of the increasing interest rate environment. As a result, the level of prepayments are decreasing for both mortgage types. In Table 2 and Table 3, the results indicate that prepayments caused by relocation decrease by approximately 55% compared to the scenario with only prepayment caused by relocating. Based on historical results provided by Rabobank (Figure 6), the CPR caused by relocation ranges from approximately 2.5% to 6.25%. Hence, a decrease of approximately 50% would result in a decrease of 1.25% to 3.175% for CPR. Given that the total CPR can be approximately 5% to 18%, the effect on total level can be substantial.

On a total level it indeed appears that the value of including the take-along option the NPV values are higher. The different mortgage types result in different cash flows as expected for both on prepayment, debt, interest and principal level as on total level. The difference in valuation is higher for the linear mortgage type. The actual differences will depend on the underlying assumptions and characteristics of the mortgage portfolio included as a case study. In addition, market developments will also impact the relative outcomes of the valuation.

7. Discussion

Provided that there has been relatively limited scientific research was present on how to treat a take-along optionin mortgage valuations, certain approximates needed to be used in order to be able to perform calculations.

Firstly, the take-along option is only available for relocators that can transfer their respective mortgage to their new home. The number of relocators is difficult to estimate as this is highly dependent on demographic and geographical characteristics, but also personal preferences and personal developments. Based on research conducted by Rabobank (2015) it was determined that the prepayments as a result of relocation constitute approximately 2.5% to 6.25% of the total CPR (refer to Figure 6). These percentages have been derived on a dataset from 2005 till 2016. It is, however, unknown what the current percentage would be let alone the percentage going-forward.

Secondly, not all factors considered in the mortgage valuation were considered to derive the NPV in this research. Factors that are not included in scope have been default, the interest rate averaging, NHG and LTV, hedging (and associated costs) and the total CPR. In order to estimate the true impact of the take-along option on mortgage valuations, the full scope of all aspects included in mortgage valuations should be considered. This was not done in this research, given that only dataset for a mortgage portfolio on an aggregate level was used instead of an actual mortgage portfolio including all individual characteristics.

Thirdly, some factors included in the valuation model were simplified in order to be able to estimate the effect of the take-along option. The simplifications included bucketing of certain characteristics of the portfolio and treat the full bucket as the same. The bucketing of characteristics, such as setting gamma equal to approximately 2%, results in less representative results as gamma may differ from time to time. The same holds for the take-along rate applied. In addition, the S-curve may deviate over time as a result of changing consumer behavior. In addition, for a mortgagor it is only possible to exercise the take-along option if the mortgagee agrees. The mortgagee is able to decline the request based on the mortgage conditions but also if current law or regulations change overtime. Despite this, it not expected that a financial institution would decline the request for multiple reasons such as brand reputation, customer loyalty and customer retention.

The econometric approach uses historical data to derive the S-curve and the other elements of the CPR. However, microeconomic factors that drive the mortgage origination market can change dramatically, even if analysts continuously update their prepayment models. The models of the analysts will always lag behind shifts in the microeconomic structure of the mortgage market (Chen et al., 2009). As a result, the econometric approach fails to capture the true risk. The global financial crisis in 2008 is mentioned in this context to prove its inaccuracy, and at that time resulted in underestimating the possibility of default (Manola & Urosevic, 2010). It is important to consider recent market developments in estimating the value of a mortgage portfolio.

8. Suggestions for further research

In this section, the recommendations for further research are elaborated on. Up to this point, there has not been much literature research on the take-along option and the impact of the option on the valuation of a mortgage portfolio, and hence there are several opportunities for further research. Firstly, the improvements are discussed covering the assumptions or/and limitations based on the approach in this thesis, the econometric approach. Secondly, suggestions are made for the option-based approach fur further research.

8.1 Econometric approach

As mentioned earlier the econometric approach is applied given the available data. For the econometric approach, multiple assumptions are made. These assumptions are further elaborated on in this part of the further research, since further research could either avoid the usage of assumptions or a better estimation for the assumptions made.

8.1.1 Take-along rate

The amount percentage transferring from prepaying the mortgage for negative incentives is assumed to be equal to 2% of the total amount of prepayments. This assumption is based on that people do not always act as we aspect them to, since mortgagors are not paying attention, are not in a situationthey can exercise the option (due to financial changes) or do not act rational. A better approximation could be made by deriving a S-curve for each incentive instead of a static value (of 2%). Besides, it canbe expected that borrowers even exercise the option if the incentive for prepaying is positive.

8.1.2 S-curve

The emphasis of this research lays on the refinancing incentive and the corresponding S-curve. In this research a S-curve has been used calibrated on an illustrative example. These elements should be calibrated on the data of the financial institution, corresponding to their portfolio to capture a better estimation of the impact of the take-along value and the prepayment rate. The elements for the CPR should be calibrated frequently in order to reflect the current estimate of the take-along option.

Dua (2004) investigated the determinants of consumers' buying behavior for houses. The mortgage interest rate is expected to significantly influence an individual's decision to purchase a house. An increase in the mortgage rate or mortgage payment discourages people from buying a house and thus decreases the demand of houses. The buying behavior of consumers is divided over three potential determinants being:

i. housing sector factors such as house prices and mortgage rates;
ii. factors that measure general economic conditions such as the real disposable income; and
iii. factors that measure future expected housing-related and general economic conditions.

Johansen's test technique is used to test for cointegration between the buying index, interest rates, real disposal income, house prices, and indices of real income. The conclusion of this paper is that consumers' perception of buying conditions for houses are cointegrated with current and expected interest rates, current and expected disposable income and current prices of homes. Therefore, further research is required to capture the impact on the S-curve for prepayments regarding the take-along rate.

8.1.3 Gamma

Gamma is set to a value based on the market interest rate and the mortgage rate in January for the different buckets. However, as can be seen in Figure 1 the mortgage rates are not running parallel therefore a fixed rate for gamma for each bucket is not truly representative. For further research it is recommended to use the correlation between the interest rate and the γ values, for better estimations for the market mortgage rate.

8.2 Option based approach

As mentioned in the introduction and background information the econometric approach of valuation the take-along option results in a lag behind shifts in the microeconomic structure of the mortgage market. An option-based approach could solve this problem, however, for the option-based approach data is required on single mortgage level instead of mortgage portfolio/bucket level.

8.3 Interest averaging option

In the introduction the interest averaging option has been mentioned and briefly explained. The interest averaging option, could also result in a higher OAS. Therefore, it seems appropriate to consider for mortgage valuations.

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Appendix I: Data

Distribution of types of mortgages

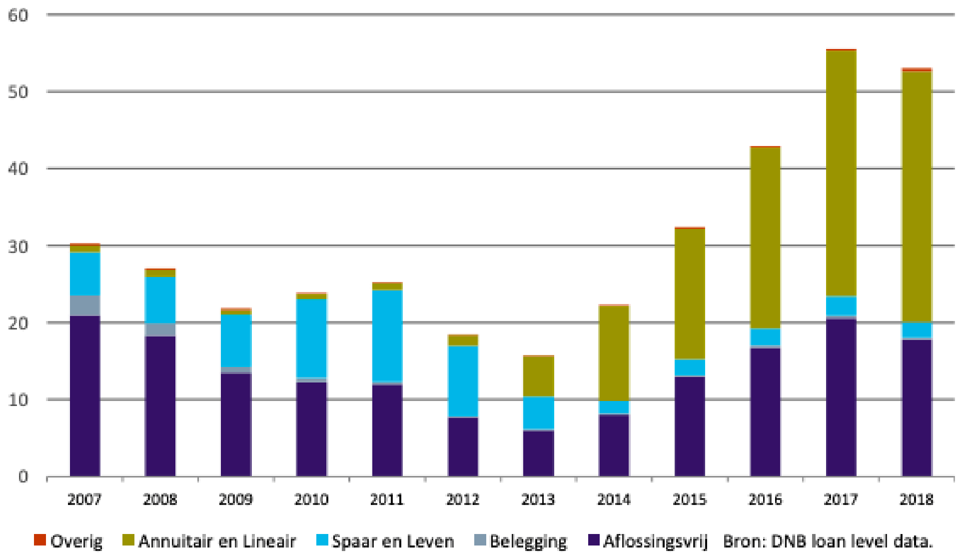
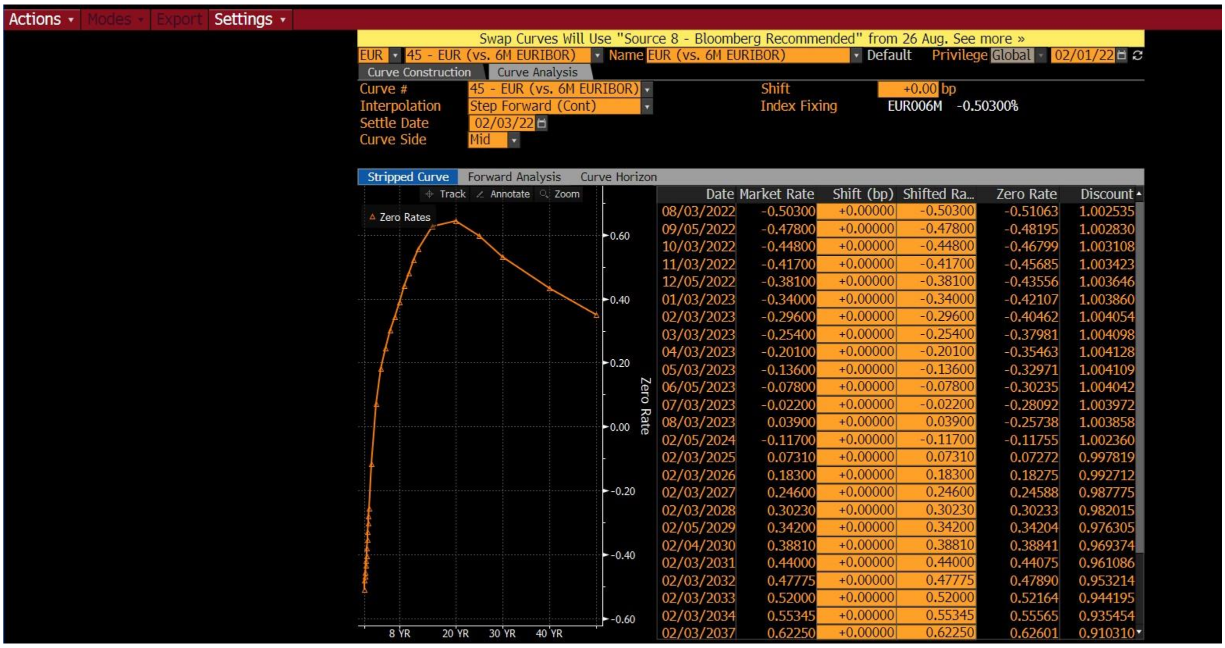


Figure 27: Mortgage types from 2007 to 2018 in the Netherlands: Source DNB

Forward and discount curve



Date	Market Rate	Shift (bp)	Shifted Ra...	Zero Rate	Discount
08/03/2022	-0.50300	+0.00000	-0.50300	-0.51063	1.002535
09/05/2022	-0.47800	+0.00000	-0.47800	-0.48195	1.002830
10/03/2022	-0.44800	+0.00000	-0.44800	-0.46799	1.003108
11/03/2022	-0.41700	+0.00000	-0.41700	-0.45685	1.003423
12/05/2022	-0.38100	+0.00000	-0.38100	-0.43556	1.003646
01/03/2023	-0.34000	+0.00000	-0.34000	-0.42107	1.003860
02/03/2023	-0.29600	+0.00000	-0.29600	-0.40462	1.004054
03/03/2023	-0.25400	+0.00000	-0.25400	-0.37981	1.004098
04/03/2023	-0.20100	+0.00000	-0.20100	-0.35463	1.004128
05/03/2023	-0.13600	+0.00000	-0.13600	-0.32971	1.004109
06/05/2023	-0.07800	+0.00000	-0.07800	-0.30235	1.004042
07/03/2023	-0.02200	+0.00000	-0.02200	-0.28092	1.003972
08/03/2023	0.03900	+0.00000	0.03900	-0.25738	1.003858
02/05/2024	-0.11700	+0.00000	-0.11700	-0.11755	1.002360
02/03/2025	0.07310	+0.00000	0.07310	0.07272	0.997819
02/03/2026	0.18300	+0.00000	0.18300	0.18275	0.992712
02/03/2027	0.24600	+0.00000	0.24600	0.24588	0.987775
02/03/2028	0.30230	+0.00000	0.30230	0.30233	0.982015
02/05/2029	0.34200	+0.00000	0.34200	0.34204	0.976305
02/04/2030	0.38810	+0.00000	0.38810	0.38841	0.969374
02/03/2031	0.44000	+0.00000	0.44000	0.44075	0.961086
02/03/2032	0.47775	+0.00000	0.47775	0.47890	0.953214
02/03/2033	0.52000	+0.00000	0.52000	0.52164	0.944195
02/03/2034	0.55345	+0.00000	0.55345	0.55565	0.935454
02/03/2037	0.62250	+0.00000	0.62250	0.62601	0.910310

Figure 28: Bloomberg data for the discount curve

Appendix II: examples

Example Annuity

A borrower has a CPR rate of ten percent one year after origination date. The original contract consist out of a mortgage of 200,000.00 euro at a mortgage rate of 1.8 (fixed for 10 years). The borrower pays every month.

The original constant MP is then

$$AF = \frac{\left(1+\left(\frac{1.8\%}{12}\right)\right)^{12*30} - 1}{\left(\frac{1.8\%}{12}\right) * \left(1+\left(\frac{1.8\%}{12}\right)\right)^{12*30}} = 278.01$$

$$MP = \frac{200{,}000.00}{278.01} = 719.40$$

However the constant MP changes after a repayment to a new constant MP payment of

$$AF = \frac{\left(1+\left(\frac{1.8\%}{12}\right)\right)^{12*29} - 1}{\left(\frac{1.8\%}{12}\right) * \left(1+\left(\frac{1.8\%}{12}\right)\right)^{12*29}} = 270.96$$

$$MP = \frac{(200{,}000.00 - \sum_{t=1}^{m*n} CD_t) * (1 - 10\%)}{270.96} = \frac{194925.5}{647.46}$$

The interest and the capital down payment changes subsequently.

Example Linear

A borrower has a CPR rate of ten percent one year after origination date. The original contract consist out of a mortgage of 200,000.00 euro at a mortgage rate of 1.8 (fixed for 10 years). The borrower pays every month.

The original constant CD is then

$$CD_t = \frac{200{,}000.00}{12 * 30} = 555.56$$

However the constant CD changes after a repayment to a new constant CD payment of

$$CD_t = \frac{200{,}000.00(1 - 10\%) - \sum_{t=1}^{m*n} CD_{t-1}}{12 * 29} = \frac{(200{,}000.00 - 555.56 * 12)(1 - 10\%)}{12 * 29} = 500$$

The interest and the capital down payment changes subsequently.

Appendix III: Input mortgage portfolio data

Starting month	Duration fixed rate	Remaining months for fixed rate	Outstanding debt	Mortgage rate / coupon	Original debt	Total duration for expiration mortgage
6	8	1	779623.95	1.63	1630000.00	36
7	8	2	871563.24	1.64	1640000.00	36
8	8	3	962498.70	1.63	1630000.00	36
9	8	4	1056321.00	1.61	1610000.00	36
10	8	5	1151820.00	1.58	1580000.00	36
11	8	6	1304100.00	1.61	1610000.00	36
12	8	7	1458000.00	1.62	1620000.00	36
1	8	8	1630000.00	1.63	1630000.00	36
2	36	1	53817.84	2.15	2150000.00	72
3	36	2	58963.22	2.12	2120000.00	72
4	36	3	64896.62	2.1	2100000.00	72
5	36	4	72450.73	2.11	2110000.00	72
6	36	5	80500.81	2.11	2110000.00	72
7	36	6	87749.70	2.07	2070000.00	72
8	36	7	96557.64	2.05	2050000.00	72
9	36	8	103622.83	1.98	1980000.00	72
10	36	9	109903.00	1.89	1890000.00	72
11	36	10	125991.10	1.95	1950000.00	72
12	36	11	131375.33	1.83	1830000.00	72
1	36	12	145174.93	1.82	1820000.00	72
2	36	13	154215.12	1.74	1740000.00	72
3	36	14	171350.14	1.74	1740000.00	72
4	36	15	190389.04	1.74	1740000.00	72
5	36	16	211543.38	1.74	1740000.00	72
6	36	17	236399.05	1.75	1750000.00	72
7	36	18	264166.56	1.76	1760000.00	72
8	36	19	303524.71	1.82	1820000.00	72
9	36	20	335396.65	1.81	1810000.00	72
10	36	21	362368.39	1.76	1760000.00	72
11	36	22	402631.55	1.76	1760000.00	72
12	36	23	439742.79	1.73	1730000.00	72
1	36	24	468833.03	1.66	1660000.00	72
2	36	25	511511.27	1.63	1630000.00	72
3	36	26	561372.29	1.61	1610000.00	72
4	36	27	619872.78	1.6	1600000.00	72
5	36	28	684442.86	1.59	1590000.00	72
6	36	29	755709.10	1.58	1580000.00	72
7	36	30	860934.42	1.62	1620000.00	72
8	36	31	950688.90	1.61	1610000.00	72

9	36	32	1036638.00	1.58	1580000.00	72
10	36	33	1195560.00	1.64	1640000.00	72
11	36	34	1320300.00	1.63	1630000.00	72
12	36	35	1485000.00	1.65	1650000.00	72
1	36	36	1650000.00	1.65	1650000.00	72
8	78	1	875.10	2.92	2920000.00	120
9	78	2	962.34	2.89	2890000.00	120
10	78	3	1058.17	2.86	2860000.00	120
11	78	4	1179.85	2.87	2870000.00	120
12	78	5	1265.27	2.77	2770000.00	120
1	78	6	1431.23	2.82	2820000.00	120
2	78	7	1573.34	2.79	2790000.00	120
3	78	8	1704.29	2.72	2720000.00	120
4	78	9	1907.58	2.74	2740000.00	120
5	78	10	2065.39	2.67	2670000.00	120
6	78	11	2234.71	2.6	2600000.00	120
7	78	12	2502.11	2.62	2620000.00	120
8	78	13	2705.85	2.55	2550000.00	120
9	78	14	2900.39	2.46	2460000.00	120
10	78	15	3104.75	2.37	2370000.00	120
11	78	16	3391.50	2.33	2330000.00	120
12	78	17	3752.16	2.32	2320000.00	120
1	78	18	4097.18	2.28	2280000.00	120
2	78	19	4632.29	2.32	2320000.00	120
3	78	20	5191.36	2.34	2340000.00	120
4	78	21	5866.78	2.38	2380000.00	120
5	78	22	6491.26	2.37	2370000.00	120
6	78	23	7121.21	2.34	2340000.00	120
7	78	24	7980.08	2.36	2360000.00	120
8	78	25	8754.05	2.33	2330000.00	120
9	78	26	9768.47	2.34	2340000.00	120
10	78	27	10807.47	2.33	2330000.00	120
11	78	28	12162.91	2.36	2360000.00	120
12	78	29	13399.82	2.34	2340000.00	120
1	78	30	14761.43	2.32	2320000.00	120
2	78	31	16684.38	2.36	2360000.00	120
3	78	32	18459.64	2.35	2350000.00	120
4	78	33	20597.99	2.36	2360000.00	120
5	78	34	23080.61	2.38	2380000.00	120
6	78	35	26291.64	2.44	2440000.00	120
7	78	36	28853.76	2.41	2410000.00	120
8	78	37	31793.68	2.39	2390000.00	120
9	78	38	35178.50	2.38	2380000.00	120
10	78	39	39087.22	2.38	2380000.00	120
11	78	40	43612.73	2.39	2390000.00	120
12	78	41	48053.08	2.37	2370000.00	120

1	78	42	53392.31	2.37	2370000.00	120
2	78	43	60075.73	2.4	2400000.00	120
3	78	44	66472.69	2.39	2390000.00	120
4	78	45	72004.35	2.33	2330000.00	120
5	78	46	77944.62	2.27	2270000.00	120
6	78	47	83934.49	2.2	2200000.00	120
7	78	48	90717.08	2.14	2140000.00	120
8	78	49	99383.72	2.11	2110000.00	120
9	78	50	108856.31	2.08	2080000.00	120
10	78	51	118043.97	2.03	2030000.00	120
11	78	52	149250.99	2.31	2310000.00	120
12	78	53	137836.41	1.92	1920000.00	120
1	78	54	153949.24	1.93	1930000.00	120
2	78	55	164850.65	1.86	1860000.00	120
3	78	56	182182.62	1.85	1850000.00	120
4	78	57	194765.80	1.78	1780000.00	120
5	78	58	215190.68	1.77	1770000.00	120
6	78	59	237749.90	1.76	1760000.00	120
7	78	60	262665.61	1.75	1750000.00	120
8	78	61	296853.83	1.78	1780000.00	120
9	78	62	326131.55	1.76	1760000.00	120
10	78	63	360309.48	1.75	1750000.00	120
11	78	64	402631.55	1.76	1760000.00	120
12	78	65	442284.65	1.74	1740000.00	120
1	78	66	482954.51	1.71	1710000.00	120
2	78	67	524063.70	1.67	1670000.00	120
3	78	68	582292.99	1.67	1670000.00	120
4	78	69	627621.19	1.62	1620000.00	120
5	78	70	701661.55	1.63	1630000.00	120
6	78	71	765275.04	1.6	1600000.00	120
7	78	72	839676.78	1.58	1580000.00	120
8	78	73	927069.30	1.57	1570000.00	120
9	78	74	1003833.00	1.53	1530000.00	120
10	78	75	1129950.00	1.55	1550000.00	120
11	78	76	1255500.00	1.55	1550000.00	120
12	78	77	1377000.00	1.53	1530000.00	120
1	78	78	1540000.00	1.54	1540000.00	120
12	120	35	492.81	3.82	3820000.00	360
1	120	36	556.16	3.88	3880000.00	360
2	120	37	603.63	3.79	3790000.00	360
3	120	38	649.46	3.67	3670000.00	360
4	120	39	686.23	3.49	3490000.00	360
5	120	40	734.07	3.36	3360000.00	360
6	120	41	798.65	3.29	3290000.00	360
7	120	42	871.20	3.23	3230000.00	360
8	120	43	973.99	3.25	3250000.00	360

9	120	44	1092.21	3.28	3280000.00	360
10	120	45	1228.36	3.32	3320000.00	360
11	120	46	1360.74	3.31	3310000.00	360
12	120	47	1466.25	3.21	3210000.00	360
1	120	48	1659.62	3.27	3270000.00	360
2	120	49	1810.19	3.21	3210000.00	360
3	120	50	1929.86	3.08	3080000.00	360
4	120	51	2151.25	3.09	3090000.00	360
5	120	52	2374.81	3.07	3070000.00	360
6	120	53	2612.89	3.04	3040000.00	360
7	120	54	2931.87	3.07	3070000.00	360
8	120	55	3236.41	3.05	3050000.00	360
9	120	56	3513.48	2.98	2980000.00	360
10	120	57	3825.26	2.92	2920000.00	360
11	120	58	4250.29	2.92	2920000.00	360
12	120	59	4641.68	2.87	2870000.00	360
1	120	60	5049.60	2.81	2810000.00	360
2	120	61	5690.53	2.85	2850000.00	360
3	120	62	6389.37	2.88	2880000.00	360
4	120	63	7050.00	2.86	2860000.00	360
5	120	64	7942.89	2.9	2900000.00	360
6	120	65	8947.16	2.94	2940000.00	360
7	120	66	10144.18	3	3000000.00	360
8	120	67	11271.31	3	3000000.00	360
9	120	68	12565.42	3.01	3010000.00	360
10	120	69	14054.34	3.03	3030000.00	360
11	120	70	15461.33	3	3000000.00	360
12	120	71	16950.19	2.96	2960000.00	360
1	120	72	18388.16	2.89	2890000.00	360
2	120	73	20643.38	2.92	2920000.00	360
3	120	74	22701.43	2.89	2890000.00	360
4	120	75	25136.54	2.88	2880000.00	360
5	120	76	27541.57	2.84	2840000.00	360
6	120	77	30817.25	2.86	2860000.00	360
7	120	78	33642.77	2.81	2810000.00	360
8	120	79	37114.80	2.79	2790000.00	360
9	120	80	41238.66	2.79	2790000.00	360
10	120	81	46313.43	2.82	2820000.00	360
11	120	82	51641.85	2.83	2830000.00	360
12	120	83	56771.57	2.8	2800000.00	360
1	120	84	63304.80	2.81	2810000.00	360
2	120	85	71089.62	2.84	2840000.00	360
3	120	86	79822.85	2.87	2870000.00	360
4	120	87	87146.90	2.82	2820000.00	360
5	120	88	95113.04	2.77	2770000.00	360
6	120	89	104918.12	2.75	2750000.00	360

7	120	90	113608.30	2.68	2680000.00	360
8	120	91	123876.38	2.63	2630000.00	360
9	120	92	135023.69	2.58	2580000.00	360
10	120	93	142466.86	2.45	2450000.00	360
11	120	94	192540.24	2.98	2980000.00	360
12	120	95	163680.74	2.28	2280000.00	360
1	120	96	177081.50	2.22	2220000.00	360
2	120	97	189666.88	2.14	2140000.00	360
3	120	98	208771.43	2.12	2120000.00	360
4	120	99	226497.31	2.07	2070000.00	360
5	120	100	254095.21	2.09	2090000.00	360
6	120	101	276924.60	2.05	2050000.00	360
7	120	102	306193.06	2.04	2040000.00	360
8	120	103	341882.22	2.05	2050000.00	360
9	120	104	378016.12	2.04	2040000.00	360
10	120	105	415900.09	2.02	2020000.00	360
11	120	106	455248.17	1.99	1990000.00	360
12	120	107	505831.30	1.99	1990000.00	360
1	120	108	545089.01	1.93	1930000.00	360
2	120	109	593102.03	1.89	1890000.00	360
3	120	110	655515.47	1.88	1880000.00	360
4	120	111	701231.09	1.81	1810000.00	360
5	120	112	774840.98	1.8	1800000.00	360
6	120	113	841802.54	1.76	1760000.00	360
7	120	114	930021.75	1.75	1750000.00	360
8	120	115	1027452.60	1.74	1740000.00	360
9	120	116	1148175.00	1.75	1750000.00	360
10	120	117	1275750.00	1.75	1750000.00	360
11	120	118	1417500.00	1.75	1750000.00	360
12	120	119	1557000.00	1.73	1730000.00	360
1	120	120	1770000.00	1.77	1770000.00	360

Table 14: Input data for python model

Appendix IV: python code

Annuity

```
#code for annuity mortgages

# initial set-up
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import numpy_financial as npf

#gathering data from excel file
df = pd.read_excel (r'C:\Users\Stefanie\Documents\Python Scripts\portfolio.xlsx')
list_starting_months = df['Month'].values.tolist()
list_duration_fixed_rate = df['Duration'].values.tolist()
list_remaining_months = df['Remaining_Months'].values.tolist()
list_outstanding_debt = df['Outstanding_Debt'].values.tolist()
list_mortgage_rate = df['Mortgage_Rate'].values.tolist()
list_total_duration = df['Total Duration'].values.tolist()

#loading in data for forward and discount rates from Bloomberg
df = pd.read_excel (r'C:\Users\Stefanie\Documents\Python Scripts\Discount.xlsx')
list_forward_rate = df['Zero_rate'].values.tolist()       #sensitivity 'Zero_rate_min/plus'
list_discount_rate = df['Discount_rate'].values.tolist()  #sensitivity 'discount_rate_min/plus'

#parameters for the sensitivity analysis
take_along_rate_shock = 0           #-0.005, 0.005 as sensitivity values
gamma_shock = 0                     #-0.005, 0.005 as sensitivity values
S_curve_shock = 0                   #-0.01, 0.01 as sensitivity values
base_shock = 0                      #-0.1, 0.1 as sensitivity values

#take-along option array with no take-along option (i.e. 0) and 2%
take_along_rates = [0.0, (0.02 + take_along_rate_shock), 0.0, (0.02 + take_along_rate_shock)]

#the portfolio total is 208 items and months taken into account are 360.
entire_portfolio = 208
total_months = 360

for n in np.arange(0,4,1):
    take_along_rate = take_along_rates[n]
    if n == 0:
        print ('Only prepayment option')
    if n == 1:
        print ('Take-along option')
    if n == 2:
        print ('Without options')
    if n == 3:
        print ('ABN AMRO take-along option')

    #initialize arrays
    total_array_interest_payments = []
    total_array_principal_payments = []
    total_array_prepayment_payments = []
    total_array_outstanding_debt = []

    #characteristics of each item m denotes the mortgages
    for m in np.arange(0, entire_portfolio, 1):
        months = list_total_duration[m] - list_duration_fixed_rate[m] + list_remaining_months[m]

        # loan characteristics
        starting_month = list_starting_months[m]
        fixed_duration = list_duration_fixed_rate[m]
        remaining_months = list_remaining_months[m]
```

```
original_balance = list_outstanding_debt[m]
debt = original_balance
past_time = fixed_duration - remaining_months
coupon = (list_mortgage_rate[m] / 100)

#initialize arrays
array_interest_payments = []
array_principal_payments = []
array_prepayment_payments = []
array_outstanding_debt = []
future_mortgage_rate = []

#1. Refinancing incentive element
C_M = np.arange(-0.04, 0.06, 0.001)             #x-axis for S-curve

#after calibrating the arctan function these parameters are determined
a = 0.04409 + S_curve_shock
b = 0.012435
c = 73.78206

#S-curves (i) is the S-curve of CPR relocators and (ii) is the one that
#changes the S-curve with the take-along rate.
Curve_Refi_cpr = a + np.arctan((C_M - b) * 100) / c                     #(i)
Curve_Refi_ta = (a - take_along_rate) + np.arctan((C_M - b) * 100) / c  #(ii)

#in order to change market interest rate to market mortgage rates
if m in range (0,8):
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0224 + gamma_shock)
if m in range(8,44):
    for iinterest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0226 + gamma_shock)
if m in range(44, 123):
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0215 + gamma_shock)
if m >= 123:
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0238 + gamma_shock)

#2. Seasoning element
x = np.array(np.arange(0, 500, 1))   #on a horizon from 0 to 500.
y = [1] * len(x)                     #horizontal line at 1.
x_1 = x[:31] / 30                    #representing the 0.02 per month.
x_2 = y[31:500]                      #representing the straight line after 30 months.
seasoning_factor = np.concatenate((x_1, x_2))

#3. Seasonality element
seasonality_factor = [0.94, 0.76, 0.73, 0.96, 0.98, 0.92, 0.99, 1.1, 1.18, 1.21, 1.23, 0.97]*18

#specific for the mortgage (m)
if remaining_months == 1:
    seasoning = seasoning_factor[past_time]
    seasonality = seasonality_factor[starting_month - 1]
    applicable_mortgage_rate = future_mortgage_rate[remaining_months]
    rate_incentive = coupon-applicable_mortgage_rate

    #if statement for the take along option for values for the rate
    #incentive lower or equal to zero.
    if rate_incentive <= 0:
        prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_ta)
```

```
        original_prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_cpr)
        TAR_yearly = original_prepayment_incentive - prepayment_incentive
    else:
        prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_cpr)
        TAR_yearly = 0
    CPR_yearly = prepayment_incentive * seasonality * seasoning
    CPR = 1 - (1 - CPR_yearly) ** (1/12)
    TAR = 1 - (1 - TAR_yearly) ** (1/12)


else:
    seasoning = seasoning_factor[past_time:((past_time+remaining_months))]
    seasonality = seasonality_factor[starting_month - 1 :starting_month - 1 + remaining_months]

    applicable_mortgage_rate = future_mortgage_rate[1 : (remaining_months + 1)]
    mortgage_rate_array = [coupon] * len(applicable_mortgage_rate)
    rate_incentive = [coupon - applicable_mortgage_rate for
                      applicable_mortgage_rate, mortgage_rate_array
                      in zip(applicable_mortgage_rate, mortgage_rate_array)]
    rate_incentive_array = np.array(rate_incentive)
    length_rate_incentive_array = len(rate_incentive_array)

    #for/if statement for the take along option for values for the rate
    #incentive lower or equal to zero.
    prepayment_incentives = []
    take_along = []
    for r in np.arange(0, length_rate_incentive_array, 1):
        if rate_incentive_array[r] <= 0:
            c_m = rate_incentive_array[r]
            prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_ta)
            original_prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_cpr)
            TAR_yearly = original_prepayment_incentive - prepayment_incentive
            prepayment_incentives.append(prepayment_incentive)
            take_along.append(TAR_yearly)
        else:
            c_m = rate_incentive_array[r]
            prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_cpr)
            TAR_yearly = 0
            prepayment_incentives.append(prepayment_incentive)
            take_along.append(TAR_yearly)

    float_take_along = (np.float_(take_along))
    float_prepayment_incentives = (np.float_(prepayment_incentives))
    CPR_yearly = float_prepayment_incentives * seasonality * seasoning
    CPR = 1 - (1 - CPR_yearly) ** (1/12)
    TAR = 1 - (1 - float_take_along) ** (1/12)


#adjust length prepaymentrates
if m <= 122:
    if m == 0 or m == 8 or m == 44:
        array_CPR = CPR
        length_CPR = 1
        Array_CPR = [CPR] + [0] * (total_months - length_CPR)
        applied_CPR_array = Array_CPR

        TAR_array = TAR
        TKRs = [TAR] + [0] * (total_months - length_CPR)
        applied_TAR_array = TKRs
    else:
```

```
        array_CPR = np.array(CPR)
        length_CPR = remaining_months
        Array_CPR = [CPR] + [0] * (total_months - length_CPR)
        Array_CPRs = Array_CPR[0]
        Array_CPRS = list(Array_CPRs)
        applied_CPR_array = Array_CPRS + [0] * (total_months - length_CPR)
        applied_CPR_array = applied_CPR_array[:360]

        TAR_array = np.array(TAR)
        TKRs = [TAR_array] + [0] * (total_months - length_CPR)
        Array_TKRs = TKRs[0]
        Array_TKRS = list(Array_TKRs)
        applied_TAR_array = Array_TKRS + [0] * (total_months - length_CPR)
else:
    array_CPR = np.array(CPR)
    length_CPR = remaining_months
    Array_CPR = [CPR] + [0] * (total_months - length_CPR)
    Array_CPRs = Array_CPR[0]
    Array_CPRS = list(Array_CPRs)
    applied_CPR_array = Array_CPRS + [0] * (total_months-length_CPR)
    applied_CPR_array = applied_CPR_array[:360]

    TAR_array = np.array(TAR)
    TKRs = [TAR_array] + [0] * (total_months - length_CPR)
    Array_TKRs = TKRs[0]
    Array_TKRS = list(Array_TKRs)
    applied_TAR_array = Array_TKRS + [0] * (total_months - length_CPR)

#Preypayment rate
if n == 2:
    conditional_prepayment_rate = [0] * 360
else:
    conditional_prepayment_rate = applied_CPR_array

#take-along rate
if n == 3:
    take_along_rate_list = applied_TAR_array
else:
    take_along_rate_list = [0] * 360

#market rates for the ABN AMRO take-along option.
market_mortgage_rate_array = np.array(applicable_mortgage_rate)
if remaining_months == 1:
    MMR = market_mortgage_rate_array + [0] * 360
else:
    MMR = list(market_mortgage_rate_array) + [0] * 360


debt = original_balance
outstanding = original_balance
base_percentage = 0.6 + base_shock
take_along_rate_interest = 0
ABN_market_rate = coupon

array_months = np.arange(1, months + 1, 1)
array_months_amending = array_months
array_months_decending = np.arange(months, 0, -1)
array_months_ascending = np.arange(1, months, 1)

length = len(array_months_decending)
```

```
remaining_months_mortgage = list_remaining_months[m] - 1

array_interest_2 = []
array_principal_2 = []
array_debts_inbetween = []
array_debts_2 = []
array_prepayments_2 = []

for i in np.arange(1, months + 1, 1):
    if n == 3:
        ABN_market_rate = (ABN_market_rate * base_percentage) +\
            (MMR[i-1] * (1 - base_percentage))
        take_along_rate_interest += take_along_rate_list[i-1]
        interest_payment = (npf.ipmt(
            rate=coupon / 12, per = array_months, nper = array_months_decending[i-1],
            pv = - original_balance) * (1 - take_along_rate_interest)) +\
            (take_along_rate_interest * npf.ipmt(
                rate = ABN_market_rate / 12, per = array_months,
                nper = array_months_decending[i-1], pv = - original_balance))
        principal_payment = (npf.ppmt(
            rate = coupon / 12, per = array_months, nper = array_months_decending[i-1],
            pv = - original_balance) * (1 - take_along_rate_interest)) +\
            (take_along_rate_interest * npf.ipmt(
                rate = ABN_market_rate / 12, per = array_months,
                nper = array_months_decending[i-1], pv = - original_balance))

        # pandas float formatting
        pd.options.display.float_format = '{:,.2f}'.format

        # cash flow table
        cf_data = {'Interest': interest_payment, 'Principal': principal_payment}
        cf_table = pd.DataFrame(data = cf_data, index = array_months_amending)
        cf_table['Payment'] = cf_table['Interest'] + cf_table['Principal']
        cf_table['Ending Balance'] = original_balance - \
                    cf_table['Principal'].cumsum()
        cf_table['Beginning Balance'] = [original_balance] + \
                    list(cf_table['Ending Balance'])[:-1]
        cf_table = cf_table[['Beginning Balance', 'Payment', 'Interest',
            'Principal', 'Ending Balance']]
        cf_table.head(8)

        #use values from table that are needed for the cashflows
        array_interest = np.array(cf_table['Interest'])
        array_principal = np.array(cf_table['Principal'])
        array_interest = [array_interest[0]]
        array_interest_2.append(array_interest[0])
        array_principal = [array_principal[0]]
        array_principal_2.append(array_principal[0])
        debt_array_from_table = np.array(cf_table['Ending Balance'])
        array_debts_inbetween.append(debt_array_from_table[0])
        array_debts_2 = np.array([array_debts_inbetween])

        #determines the values required for the prepayement and the take-along
        if i <= (length - 1):
            original_balance = cf_table['Beginning Balance']\
                [array_months_ascending[i-1] + 1] * (1 - conditional_prepayment_rate[i-1])
            prepaid_amount = cf_table['Beginning Balance']\
                [array_months_ascending [i-1] + 1] * conditional_prepayment_rate[i-1]
            array_months = np.arange(1,(array_months_decending[i] + 1), 1)
            array_months_amending = np.arange(array_months_ascending[i-1] + 1, months + 1, 1)
```

```
        else:
            break
        prepaid = prepaid_amount
        array_prepayments_2.append(prepaid)
        list_prepayments_remaining_months = array_prepayments_2[:remaining_months]


    else:
        interest_payment = npf.ipmt(rate=coupon / 12, per = array_months,
            nper = array_months_decending[i-1], pv = - original_balance)
        principal_payment = npf.ppmt(rate = coupon / 12, per = array_months,
            nper = array_months_decending[i-1], pv = - original_balance)

        # pandas float formatting
        pd.options.display.float_format = '{:,.2f}'.format

        # cash flow table
        cf_data = {'Interest': interest_payment, 'Principal': principal_payment}
        cf_table = pd.DataFrame(data=cf_data, index=array_months_amending)
        cf_table['Payment'] = cf_table['Interest'] + cf_table['Principal']
        cf_table['Ending Balance'] = original_balance - \
            cf_table['Principal'].cumsum()
        cf_table['Beginning Balance'] = [original_balance] + \
            list(cf_table['Ending Balance'])[:-1]
        cf_table = cf_table[['Beginning Balance', 'Payment', 'Interest',
             'Principal', 'Ending Balance']]
        cf_table.head(8)

        array_interest = np.array(cf_table['Interest'])
        array_principal = np.array(cf_table['Principal'])
        array_interest = [array_interest[0]]
        array_interest_2.append(array_interest[0])
        array_principal = [array_principal[0]]
        array_principal_2.append(array_principal[0])
        debt_array_from_table = np.array(cf_table['Ending Balance'])
        array_debts_inbetween.append(debt_array_from_table[0])
        array_debts_2 = np.array([array_debts_inbetween])


        if i <= (length - 1):
            original_balance = cf_table['Beginning Balance']\
                [array_months_ascending[i-1] + 1] * (1 - conditional_prepayment_rate[i-1])
            prepaid_amount = cf_table['Beginning Balance']\
                [array_months_ascending[i-1] + 1] * conditional_prepayment_rate[i-1]
            array_months = np.arange(1, (array_months_decending[i] + 1), 1)
            array_months_amending = np.arange(array_months_ascending[i-1] + 1, (months + 1), 1)

        else:
            break
        prepaid = prepaid_amount
        array_prepayments_2.append(prepaid)
        list_prepayments_remaining_months = array_prepayments_2[:remaining_months]

length_remaining_months_prepayments = len(list_prepayments_remaining_months)
array_prepayments_remaining_months = np.asarray(list_prepayments_remaining_months)
list_prepaid_totalmonths = list(list_prepayments_remaining_months) + [0] * (
    total_months - length_remaining_months_prepayments)
total_array_prepayment_payments.append(list_prepaid_totalmonths)

list_principals_remaing_months = array_principal_2[:remaining_months]
```

```
    length_remaining_months_prepayments = len(list_principals_remaing_months)
    list_principals_totalmonths = list(list_principals_remaing_months) + [0] * (
        total_months - length_remaining_months_prepayments)
    total_array_principal_payments.append(list_principals_totalmonths)

    list_interests_remaing_months = array_interest_2[:remaining_months]
    length_remaining_months_interests = len(list_interests_remaing_months)
    list_interests_totalmonths = list(list_interests_remaing_months) + [0] * (
        total_months - length_remaining_months_interests)
    total_array_interest_payments.append(list_interests_totalmonths)

    array_debts_remaining_months = (array_debts_2[0, remaining_months_mortgage])
    if m == 0:
        list_debts_totalmonths = [array_debts_remaining_months] + [0] * (total_months - 1)
        total_array_outstanding_debt.append(list_debts_totalmonths)
    else:
        list_debts_totalmonths = [0] * (remaining_months_mortgage) + [
            array_debts_remaining_months] + [0] * (total_months - remaining_months_mortgage - 1)
        total_array_outstanding_debt.append(list_debts_totalmonths)

#To add up all the interest payments made in the same month.
list_interests=total_array_interest_payments[0]
for q in np.arange(1, entire_portfolio, 1):
    list_interest_a = list_interests
    list_interest_b = total_array_interest_payments[q]
    list_interests = [w + e for w, e in zip(list_interest_a, list_interest_b)]
    list_interest_a = list_interests

#To add up all the debt payments made in the same month.
list_debts = total_array_outstanding_debt[0]
for q in np.arange(1, entire_portfolio, 1):
    list_debt_a = list_debts
    list_debt_b = total_array_outstanding_debt[q]
    list_debts = [w + e for w, e in zip(list_debt_a, list_debt_b)]
    list_debt_a = list_debts

#To add up all the principal payments made in the same month.
list_principals = total_array_principal_payments[0]
for q in np.arange(1, entire_portfolio, 1):
    list_principal_a = list_principals
    list_principal_b = total_array_principal_payments[q]
    list_principals = [w + e for w, e in zip(list_principal_a, list_principal_b)]
    list_principal_a = list_principals

#To add up all the prepayments made in the same month.
list_prepayments = total_array_prepayment_payments[0]
for q in np.arange(1, entire_portfolio, 1):
    list_prepayment_a = list_prepayments
    list_prepayment_b = total_array_prepayment_payments[q]
    list_prepayments = [w + e for w, e in zip(list_prepayment_a, list_prepayment_b)]
    list_prepayment_a = list_prepayments

#DISCOUNTED CASH FLOWS

#timesteps and corresponding discount factor Bloomberg
discount = list_discount_rate[:total_months]
discount_array = np.array(discount)

#discounted prepayment payments
array_prepayments = np.array(list_prepayments)
```

```
array_discounted_prepayments = array_prepayments * discount_array
sum_discounted_prepayments = sum(array_discounted_prepayments)
print("NPV prepayments  = %.2f" %(sum_discounted_prepayments))


#discounted debt payments
array_debts = np.array(list_debts)
array_discounted_debts = array_debts * discount_array
sum_discounted_debts = sum(array_discounted_debts)
print("NPV debt         = %.2f" %(sum_discounted_debts))


#discounted interest payments
array_interests = np.array(list_interests)
array_discounted_interests = array_interests * discount_array
sum_discounted_interests = sum(array_discounted_interests)
print("NPV interest     = %.2f" %(sum_discounted_interests))


#discounted principal payments
array_principals = np.array(list_principals)
array_discounted_principals = array_principals * discount_array
sum_discounted_principals = sum(array_discounted_principals)
print("NPV principal    = %.2f" %(sum_discounted_principals))
print()

total_monthly = array_discounted_principals + array_discounted_interests +\
    array_discounted_debts + array_discounted_prepayments
total_all = sum_discounted_prepayments + sum_discounted_debts +\
    sum_discounted_interests + sum_discounted_principals

print("NPV total        = %.2f" %(total_all))
print()

if n == 0:
    prepayment_array_prepayments = array_discounted_prepayments[:120]
    prepayment_array_debts = array_discounted_debts[:120]
    prepayment_array_interests = array_discounted_interests[:120]
    prepayment_array_principals = array_discounted_principals[:120]
    prepayment_total_monthly = total_monthly[:120]

if n == 1:
    take_along_array_prepayments = array_discounted_prepayments[:120]
    take_along_array_debts = array_discounted_debts[:120]
    take_along_array_interests = array_discounted_interests[:120]
    take_along_array_principals = array_discounted_principals[:120]
    take_along_total_monthly = total_monthly[:120]

if n == 2:
    no_option_array_prepayments = array_discounted_prepayments[:120]
    no_option_array_debts = array_discounted_debts[:120]
    no_option_array_interests = array_discounted_interests[:120]
    no_option_array_principals = array_discounted_principals[:120]
    no_option_total_monthly = total_monthly[:120]

if n == 3:
    ABN_array_prepayments = array_discounted_prepayments[:120]
    ABN_array_debts = array_discounted_debts[:120]
    ABN_array_interests = array_discounted_interests[:120]
    ABN_array_principals = array_discounted_principals[:120]
```

```
        ABN_total_monthly = total_monthly[:120]

dates = np.arange(0,120,1)
plt.plot(dates, take_along_array_interests, label = "Take-along option")
plt.plot(dates, prepayment_array_interests, label = "Only prepayment option")
plt.plot(dates, no_option_array_interests, label = "without options")
plt.plot(dates, ABN_array_interests, label = "ABN AMRO take-along")
plt.title('Interest rate payments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_prepayments, label = "Take-along option")
plt.plot(dates, prepayment_array_prepayments, label = "Only prepayment option")
plt.plot(dates, no_option_array_prepayments, label = "Without options")
plt.plot(dates, ABN_array_prepayments, label = "ABN AMRO take-along")
plt.title('Prepayments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_principals, label = "Take-along option")
plt.plot(dates, prepayment_array_principals, label = "Only prepayment option")
plt.plot(dates, no_option_array_principals, label = "Without options")
plt.plot(dates, ABN_array_principals, label = "ABN AMRO take-along")
plt.title('Principal payments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_debts, label = "Take-along option")
plt.plot(dates, prepayment_array_debts, label = "Only prepayment option")
plt.plot(dates, no_option_array_debts, label = "Without options")
plt.plot(dates, ABN_array_debts, label = "ABN AMRO take-along")
plt.title('Outstanding debt payments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_total_monthly, label = "Take-along option")
plt.plot(dates, prepayment_total_monthly, label = "Only prepayment option")
plt.plot(dates, no_option_total_monthly, label = "Without options")
plt.plot(dates, ABN_total_monthly, label = "ABN AMRO take-along")
plt.title('Monthly total')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()
```

Linear

```
#code for linear mortgages

# initial set-up
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

#gathering data from excel file
df = pd.read_excel (r'C:\Users\Stefanie\Documents\Python Scripts\portfolio.xlsx')
list_starting_months = df['Month'].values.tolist()
list_duration_fixed_rate = df['Duration'].values.tolist()
list_remaining_months = df['Remaining_Months'].values.tolist()
list_outstanding_debt = df['Outstanding_Debt'].values.tolist()
list_mortgage_rate = df['Mortgage_Rate'].values.tolist()
list_total_duration = df['Total Duration'].values.tolist()

#loading in data for forward and discount rates from Bloomberg
df = pd.read_excel (r'C:\Users\Stefanie\Documents\Python Scripts\Discount.xlsx')
list_forward_rate = df['Zero_rate'].values.tolist()         #sensitivity 'Zero_rate_min/plus'
list_discount_rate = df['Discount_rate'].values.tolist()    #sensitivity 'discount_rate_min/plus'

#parameters for the sensitivity analysis
take_along_rate_shock = 0       #-0.005, 0.005 as sensitivity values
gamma_shock = 0                 #-0.005, 0.005 as sensitivity values
S_curve_shock = 0               #-0.01, 0.01 as sensitivity values
base_shock = 0                  #-0.1, 0.1 as sensitivity values

#take-along option array with no take-along option (i.e. 0) and 2%
take_along_rates = [0.0, (0.02 + take_along_rate_shock), 0.0, (0.02 + take_along_rate_shock)]

#the portfolio total is 208 items and months taken into account are 360.
entire_portfolio = 208
total_months = 360

for n in np.arange(0,4,1):
    take_along_rate = take_along_rates[n]
    if n == 0:
        print ('Only prepayment option')
    if n == 1:
        print ('Take-along option')
    if n == 2:
        print ('Without options')
    if n == 3:
        print ('ABN AMRO take-along option')

    #initialize arrays
    total_array_interest_payments = []
    total_array_principal_payments = []
    total_array_prepayment_payments = []
    total_array_outstanding_debt = []

    #characteristics of each item m denotes the mortgages
    for m in np.arange(0, entire_portfolio, 1):
        months = list_total_duration[m] - list_duration_fixed_rate[m] + list_remaining_months[m]

        # loan characteristics
        starting_month = list_starting_months[m]
        fixed_duration = list_duration_fixed_rate[m]
        remaining_months = list_remaining_months[m]
        original_balance = list_outstanding_debt[m]
```

```
debt = original_balance
past_time = fixed_duration - remaining_months
coupon = (list_mortgage_rate[m] / 100)

#initialize arrays
array_interest_payments = []
array_principal_payments = []
array_prepayment_payments = []
array_outstanding_debt = []
future_mortgage_rate = []

#1. Refinancing incentive element
C_M = np.arange(-0.04, 0.06, 0.001)          #x-axis for S-curve

#after calibrating the arctan function these parameters are determined
a = 0.04409 + S_curve_shock
b = 0.012435
c = 73.78206

#S-curves (i) is the S-curve of CPR relocators and (ii) is the one that
#changes the S-curve with the take-along rate.
Curve_Refi_cpr = a + np.arctan((C_M - b) * 100) / c                 #(i)
Curve_Refi_ta = (a - take_along_rate) + np.arctan((C_M - b) * 100) / c  #(ii)

#in order to change market interest rate to market mortgage rates
if m in range (0,8):
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0224 + gamma_shock)
if m in range(8,44):
    for iinterest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0226 + gamma_shock)
if m in range(44, 123):
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0215 + gamma_shock)
if m >= 123:
    for interest_rate_markt in list_forward_rate:
        future_mortgage_rate.append(interest_rate_markt + 0.0238 + gamma_shock)

#2. Seasoning element
x = np.array(np.arange(0, 500, 1))   #on a horizon from 0 to 500.
y = [1] * len(x)                     #horizontal line at 1.
x_1 = x[:31] / 30                    #representing the 0.02 per month.
x_2 = y[31:500]                      #representing the straight line after 30 months.
seasoning_factor = np.concatenate((x_1, x_2))

#3. Seasonality element
seasonality_factor = [0.94, 0.76, 0.73, 0.96, 0.98, 0.92, 0.99, 1.1, 1.18, 1.21, 1.23, 0.97]*18

#specific for the mortgage (m)
if remaining_months == 1:
    seasoning = seasoning_factor[past_time]
    seasonality = seasonality_factor[starting_month - 1]
    applicable_mortgage_rate = future_mortgage_rate[remaining_months]
    rate_incentive = coupon-applicable_mortgage_rate

    #if statement for the take along option for values for the rate
    #incentive lower or equal to zero.
    if rate_incentive <= 0:
        prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_ta)
        original_prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_cpr)
```

```
        TAR_yearly = original_prepayment_incentive - prepayment_incentive
    else:
        prepayment_incentive = np.interp(rate_incentive, C_M, Curve_Refi_cpr)
        TAR_yearly = 0
    CPR_yearly = prepayment_incentive * seasonality * seasoning
    CPR = 1 - (1 - CPR_yearly) ** (1/12)
    TAR = 1 - (1 - TAR_yearly) ** (1/12)


else:
    seasoning = seasoning_factor[past_time:((past_time+remaining_months))]
    seasonality = seasonality_factor[starting_month - 1 :starting_month - 1 + remaining_months]

    applicable_mortgage_rate = future_mortgage_rate[1 : (remaining_months + 1)]
    mortgage_rate_array = [coupon] * len(applicable_mortgage_rate)
    rate_incentive = [coupon - applicable_mortgage_rate for applicable_mortgage_rate,
                      mortgage_rate_array in zip(applicable_mortgage_rate, mortgage_rate_array)]
    rate_incentive_array = np.array(rate_incentive)
    length_rate_incentive_array = len(rate_incentive_array)

    #for/if statement for the take along option for values for the rate
    #incentive lower or equal to zero.
    prepayment_incentives = []
    take_along = []
    for r in np.arange(0, length_rate_incentive_array, 1):
        if rate_incentive_array[r] <= 0:
            c_m = rate_incentive_array[r]
            prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_ta)
            original_prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_cpr)
            TAR_yearly = original_prepayment_incentive - prepayment_incentive
            prepayment_incentives.append(prepayment_incentive)
            take_along.append(TAR_yearly)
        else:
            c_m = rate_incentive_array[r]
            prepayment_incentive = np.interp(c_m, C_M, Curve_Refi_cpr)
            TAR_yearly = 0
            prepayment_incentives.append(prepayment_incentive)
            take_along.append(TAR_yearly)

    float_take_along = (np.float_(take_along))
    float_prepayment_incentives = (np.float_(prepayment_incentives))
    CPR_yearly = float_prepayment_incentives * seasonality * seasoning
    CPR = 1 - (1 - CPR_yearly) ** (1/12)
    TAR = 1 - (1 - float_take_along) ** (1/12)


#adjust length prepaymentrates
if m <= 122:
    if m == 0 or m == 8 or m == 44:
        array_CPR = CPR
        length_CPR = 1
        Array_CPR = [CPR] + [0] * (total_months - length_CPR)
        applied_CPR_array = Array_CPR

        TAR_array = TAR
        TKRs = [TAR] + [0] * (total_months - length_CPR)
        applied_TAR_array = TKRs
    else:
        array_CPR = np.array(CPR)
        length_CPR = remaining_months
```

```
        Array_CPR = [CPR] + [0] * (total_months - length_CPR)
        Array_CPRs = Array_CPR[0]
        Array_CPRS = list(Array_CPRs)
        applied_CPR_array = Array_CPRS + [0] * (total_months - length_CPR)
        applied_CPR_array = applied_CPR_array[:360]

        TAR_array = np.array(TAR)
        TKRs = [TAR_array] + [0] * (total_months - length_CPR)
        Array_TKRs = TKRs[0]
        Array_TKRS = list(Array_TKRs)
        applied_TAR_array = Array_TKRS + [0] * (total_months - length_CPR)
else:
    array_CPR = np.array(CPR)
    length_CPR = remaining_months
    Array_CPR = [CPR] + [0] * (total_months - length_CPR)
    Array_CPRs = Array_CPR[0]
    Array_CPRS = list(Array_CPRs)
    applied_CPR_array = Array_CPRS + [0] * (total_months-length_CPR)
    applied_CPR_array = applied_CPR_array[:360]

    TAR_array = np.array(TAR)
    TKRs = [TAR_array] + [0] * (total_months - length_CPR)
    Array_TKRs = TKRs[0]
    Array_TKRS = list(Array_TKRs)
    applied_TAR_array = Array_TKRS + [0] * (total_months - length_CPR)

#Preypayment rate
if n == 2:
    conditional_prepayment_rate = [0] * 360
else:
    conditional_prepayment_rate = applied_CPR_array

#take-along rate
if n == 3:
    take_along_rate_list = applied_TAR_array
else:
    take_along_rate_list = [0] * 360

#market rates for the ABN AMRO take-along option.
market_mortgage_rate_array = np.array(applicable_mortgage_rate)
if remaining_months == 1:
    MMR = market_mortgage_rate_array + [0] * 360
else:
    MMR = list(market_mortgage_rate_array) + [0] * 360


debt = original_balance
outstanding = original_balance
base_percentage = 0.6 + base_shock
take_along_rate_interest = 0
ABN_market_rate = coupon
for i in np.arange(1, months + 1, 1):
    if n == 3:
        ABN_market_rate = (ABN_market_rate * base_percentage) + (MMR[i-1] * (1 - base_percentage))
        take_along_rate_interest += take_along_rate_list[i-1]
        interest_payment = debt * (coupon / 12) * (1 - take_along_rate_interest) + debt *\
            (ABN_market_rate / 12) * take_along_rate_interest
        principal_payment = outstanding / (months)
        prepayment_value = debt * (conditional_prepayment_rate[i - 1])
        outstanding = outstanding - prepayment_value
```

```
            debt = debt - principal_payment - prepayment_value

            array_interest_payments.append(interest_payment)
            array_principal_payments.append(principal_payment)
            array_outstanding_debt.append(debt)
            array_prepayment_payments.append(prepayment_value)
        else:
            interest_payment = debt * (coupon / 12)
            principal_payment = outstanding / (months)
            prepayment_value = debt * (conditional_prepayment_rate[i - 1])
            outstanding = outstanding - prepayment_value
            debt = debt - principal_payment - prepayment_value

            array_interest_payments.append(interest_payment)
            array_principal_payments.append(principal_payment)
            array_outstanding_debt.append(debt)
            array_prepayment_payments.append(prepayment_value)


    array_interest_payments = array_interest_payments[:remaining_months]
    array_principal_payments = array_principal_payments[:remaining_months]
    array_prepayment_payments = array_prepayment_payments[:remaining_months]
    length_interest = len(array_interest_payments)
    length_principal = len(array_principal_payments)
    length_prepayment = len(array_prepayment_payments)

    array_interest_payments = array_interest_payments + [0] * (total_months - length_interest)
    array_principal_payments = array_principal_payments + [0] * (total_months - length_interest)
    array_prepayment_payments = array_prepayment_payments +[0] * (total_months - length_interest)

    total_array_interest_payments.append(array_interest_payments)
    total_array_principal_payments.append(array_principal_payments)
    total_array_prepayment_payments.append(array_prepayment_payments)

    array_outstanding_debt = array_outstanding_debt[:remaining_months]
    length_outstanding_debt = len(array_outstanding_debt)
    array_outstanding_debt = [array_outstanding_debt[ - 1]]
    array_outstanding_debt = [0] * (length_outstanding_debt - 1) + array_outstanding_debt +\
        [0] * (total_months - length_outstanding_debt)
    total_array_outstanding_debt.append(array_outstanding_debt)

#To add up all the interest payments made in the same month.
list_interests=total_array_interest_payments[0]
for q in np.arange(1, entire_portfolio, 1):
    list_interest_a = list_interests
    list_interest_b = total_array_interest_payments[q]
    list_interests = [w + e for w, e in zip(list_interest_a, list_interest_b)]
    list_interest_a = list_interests

#To add up all the debt payments made in the same month.
list_debts = total_array_outstanding_debt[0]
for q in np.arange(1, entire_portfolio, 1):
    list_debt_a = list_debts
    list_debt_b = total_array_outstanding_debt[q]
    list_debts = [w + e for w, e in zip(list_debt_a, list_debt_b)]
    list_debt_a = list_debts

#To add up all the principal payments made in the same month.
list_principals = total_array_principal_payments[0]
for q in np.arange(1, entire_portfolio, 1):
```

```
    list_principal_a = list_principals
    list_principal_b = total_array_principal_payments[q]
    list_principals = [w + e for w, e in zip(list_principal_a, list_principal_b)]
    list_principal_a = list_principals

#To add up all the prepayments made in the same month.
list_prepayments = total_array_prepayment_payments[0]
for q in np.arange(1, entire_portfolio, 1):
    list_prepayment_a = list_prepayments
    list_prepayment_b = total_array_prepayment_payments[q]
    list_prepayments = [w + e for w, e in zip(list_prepayment_a, list_prepayment_b)]
    list_prepayment_a = list_prepayments

#DISCOUNTED CASH FLOWS

#timesteps and corresponding discount factor Bloomberg
discount = list_discount_rate[:total_months]
discount_array = np.array(discount)

#discounted prepayment payments
array_prepayments = np.array(list_prepayments)
array_discounted_prepayments = array_prepayments * discount_array
sum_discounted_prepayments = sum(array_discounted_prepayments)
print("NPV prepayments  = %.2f" %(sum_discounted_prepayments))


#discounted debt payments
array_debts = np.array(list_debts)
array_discounted_debts = array_debts * discount_array
sum_discounted_debts = sum(array_discounted_debts)
print("NPV debt         = %.2f" %(sum_discounted_debts))


#discounted interest payments
array_interests = np.array(list_interests)
array_discounted_interests = array_interests * discount_array
sum_discounted_interests = sum(array_discounted_interests)
print("NPV interest     = %.2f" %(sum_discounted_interests))


#discounted principal payments
array_principals = np.array(list_principals)
array_discounted_principals = array_principals * discount_array
sum_discounted_principals = sum(array_discounted_principals)
print("NPV principal    = %.2f" %(sum_discounted_principals))
print()

total_monthly = array_discounted_principals + array_discounted_interests +\
    array_discounted_debts + array_discounted_prepayments
total_all = sum_discounted_prepayments + sum_discounted_debts +\
    sum_discounted_interests + sum_discounted_principals

print("NPV total        = %.2f" %(total_all))
print()

if n == 0:
    prepayment_array_prepayments = array_discounted_prepayments[:120]
    prepayment_array_debts = array_discounted_debts[:120]
    prepayment_array_interests = array_discounted_interests[:120]
    prepayment_array_principals = array_discounted_principals[:120]
```

```
        prepayment_total_monthly = total_monthly[:120]

    if n == 1:
        take_along_array_prepayments = array_discounted_prepayments[:120]
        take_along_array_debts = array_discounted_debts[:120]
        take_along_array_interests = array_discounted_interests[:120]
        take_along_array_principals = array_discounted_principals[:120]
        take_along_total_monthly = total_monthly[:120]

    if n == 2:
        no_option_array_prepayments = array_discounted_prepayments[:120]
        no_option_array_debts = array_discounted_debts[:120]
        no_option_array_interests = array_discounted_interests[:120]
        no_option_array_principals = array_discounted_principals[:120]
        no_option_total_monthly = total_monthly[:120]

    if n == 3:
        ABN_array_prepayments = array_discounted_prepayments[:120]
        ABN_array_debts = array_discounted_debts[:120]
        ABN_array_interests = array_discounted_interests[:120]
        ABN_array_principals = array_discounted_principals[:120]
        ABN_total_monthly = total_monthly[:120]

dates = np.arange(0,120,1)
plt.plot(dates, take_along_array_interests, label = "Take-along option")
plt.plot(dates, prepayment_array_interests, label = "Only prepayment option")
plt.plot(dates, no_option_array_interests, label = "without options")
plt.plot(dates, ABN_array_interests, label = "ABN AMRO take-along")
plt.title('Interest rate payments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_prepayments, label = "Take-along option")
plt.plot(dates, prepayment_array_prepayments, label = "Only prepayment option")
plt.plot(dates, no_option_array_prepayments, label = "Without options")
plt.plot(dates, ABN_array_prepayments, label = "ABN AMRO take-along")
plt.title('Prepayments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_principals, label = "Take-along option")
plt.plot(dates, prepayment_array_principals, label = "Only prepayment option")
plt.plot(dates, no_option_array_principals, label = "Without options")
plt.plot(dates, ABN_array_principals, label = "ABN AMRO take-along")
plt.title('Principal payments')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_array_debts, label = "Take-along option")
plt.plot(dates, prepayment_array_debts, label = "Only prepayment option")
plt.plot(dates, no_option_array_debts, label = "Without options")
plt.plot(dates, ABN_array_debts, label = "ABN AMRO take-along")
plt.title('Outstanding debt payments')
plt.ylabel('Cash flow in Euro')
```

```
plt.xlabel('Months')
plt.legend()
plt.show()

plt.plot(dates, take_along_total_monthly, label = "Take-along option")
plt.plot(dates, prepayment_total_monthly, label = "Only prepayment option")
plt.plot(dates, no_option_total_monthly, label = "Without options")
plt.plot(dates, ABN_total_monthly, label = "ABN AMRO take-along")
plt.title('Monthly total')
plt.ylabel('Cash flow in Euro')
plt.xlabel('Months')
plt.legend()
plt.show()
```