BSc Thesis Applied Mathematics

Crowd Modelling: Simulating the Evacuation of Pedestrians

Oisín Mc Inerney

Supervisor: G. Meinsma

June, 2023

Department of Applied Mathematics

UNIVERSITY OF TWENTE.

Preface

This paper is a bachelor's assignment of the study Applied Mathematics at the University of Twente. The subject of this assignment is crowd modeling. This field of research is inspired by the behaviour of starlings in flocks, bu this paper will focus on the behaviour of humans in crowds.

In this paper a model is proposed based on the Buckingham π-theorem. After long hours of modelling and coding, this is the result. I hope you enjoy reading it as I have enjoyed writing it.

I would like to thank my supervisor Gjerrit Meinsma for his help and ideas about the subject and specifically the idea for using the Buckingham π-theorem to determine an equation that can be used for crowd modelling.

Simulating the Evacuation of Pedestrians

Oisín Mc Inerney

June, 2023

Abstract

The title 'Crowd Modelling: Simulating the Evacuation of Pedestrians' pretty much summarizes the essence of the report. The goal of this report is to make a dynamic crowd model that does not produce "bouncing" effects like [2]. Each individual in the crowd has to abide by three rules: do not get too close to your neighbours, keep a distance from objects and walls and walk a regular pace towards the exit. In the model this translates to accelerations that guide people. It is assumed that these accelerations can be summed to obtain the total acceleration. The motion is described by differential equations.

The inspiration for this paper is the Buckingham π-theorem. The acceleration can be expressed as a differential equation of velocity and position. The most important result of this paper is that with dimensional analysis using the Buckingham π-theorem produces a realistic model, but it has its limitations.

Further research will mainly focus on flow time (time it takes for everyone to reach the exit). How do different floor plan designs affect the flow time of the simulation? It can seen that the width of the door influences the flow time. Also adding an extra exit decreases the flow time. However, adding an obstacle in front of the exit to create two separate queues for the exit, does not seem to influence the flow time at first, but after changing some parameters it does improve the flow time.

Contents

1 Introduction

The goal of this bachelor's assignment is to create a crowd model in which people accelerate and decelerate based on their velocity, position and surroundings. Furthermore, it is the aim to find out how the shape and design of a room can influence the flow of a crowd. Effective crowd evacuation is very important for the safety of pedestrians is public spaces, but also, though less of a priority, it can save time and prevent "traffic" jams.

Crowd modeling is a well-researched and widely discussed topic. In 1987, Craig Reynolds was one of the first researchers to propose a model on the behaviour of birds in a flock, where he calls the bird-like objects "boids", [4]. Many researchers have followed since then, and research on this topic is still ongoing. Most of these studies are based on some basic principles. For example, birds are modeled according to three basic rules. By using these rules, it is possible to simulate a large flock of birds that displays almost the same pattern as real birds. This indicates a reasonable assumption of the three basic rules. The first rule is that birds like to stay close to each other as they are social animals. In addition, birds do not want to collide, so they keep a certain distance. The last rule assumed is that every bird prefers to fly at the same speed as the rest of the flock. Birds change direction by banking, which means that their wings remain in line, and their speed does not change when making a turn.

Modeling humans can be done in a similar way, although there are some differences. First of all, it is assumed that humans do not really work together in a group, so they do not necessarily have the urge to stay close to each other. In some situations, this is the case, but in most cases, a person will do what is best for themselves. Therefore, human crowds are seen as a group of individuals. The second rule does apply to humans, as they try to avoid collisions. The third rule is also similar, as each person has a desired speed. As a situation becomes more and more chaotic, the desired speed will increase, and everyone wants to get out of the room or hallway as quickly as possible. Additionally, humans are modeled in a space with walls and other stationary obstacles, which must also be avoided in order to prevent collisions.

One way of modelling a human crowd can be found in [2]. Simulating a massive crowd in a small room at first seems to produce reasonable results. But when taking a closer look it shows that the people keep bouncing of each other, which is not realistic at all.

This paper proposes an alternative model based on the physical dimensionality property, as per example 4.7 in [3]. This bachelor's assignment aims to analyze this model and to extend it to crowds in venues with walls and other stationary obstacles.

This is not the first paper to do research on crowd evacuation. Still, a good argument can be made for the importance of this research. The problem with crowd modelling is that it is (at least for now) impossible to make a completely realistic model. However, if multiple different models all have roughly the same conclusion, then it might not matter that much if it is not completely realistic. By studying an alternative way of modelling a crowd, the results of previous crowd models can be taken more seriously. So, there remains enough useful research in this field.

The research question and sub question of this paper are a follows:

"Is the model based on the Buckingham π-theorem realistic?"

"How can a floor of a building be designed such that a crowd of people can leave through the exit doors as quickly as possible?"

In remainder of this paper both questions are aimed to be answered via simulations and data. But first, an explanation of the model is required.

2 Dimensional and scaling analysis

Imagine that a person is walking towards a wall. If that person wants to avoid walking into the wall, he might want to apply a certain acceleration in order to brake. It will now be demonstrated how the acceleration can be expressed as a function of the velocity and position.

The acceleration $\ddot{x}$ is assumed to be a function of position x and velocity $\dot{x}$. With respect to x and t, the dimensional matrix of this expression is as follows:

	$x(t)$	$\dot{x}(t)$	$\ddot{x}(t)$
x	1	1	1
t	0	-1	-2

The null-space of this matrix is spanned by (1, -2, 1). By the Buckingham π-theorem, the expression is equivalent to the dimensionless quantity below. Moreover, if the acceleration follows only from the velocity and position, and no other variables, the Buckingham π-theorem states that any policy that determines $\ddot{x}$ at any time t must be of the following form. [3]

$$\begin{bmatrix} x(t) & \dot{x}(t) & \ddot{x}(t) \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \frac{\ddot{x}(t)x(t)}{\dot{x}^2(t)}.$$

This quantity is unique, since $\ddot{x}(t)$ is assumed to follow uniquely from $\dot{x}(t)$ and $x(t)$, hence

$$\ddot{x}(t) = c\frac{\dot{x}^2(t)}{x(t)}, \qquad c \geq 0. \tag{2.1}$$

Here c is a constant without dimensions.

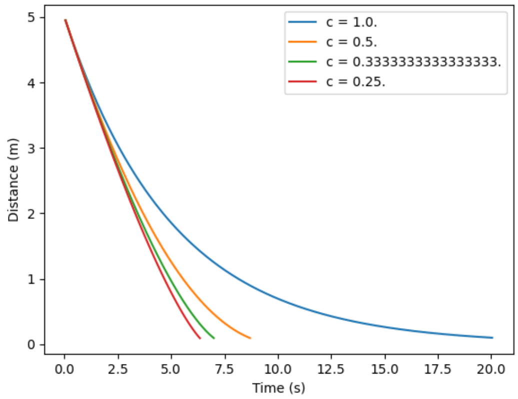
Equation (2.1) solves for $\ddot{x}$ at any moment in time. So, if someone were to push this person into another state $(x, \dot{x})$, (2.1) provides the correct solution for this new state.

2.1 Deceleration constant

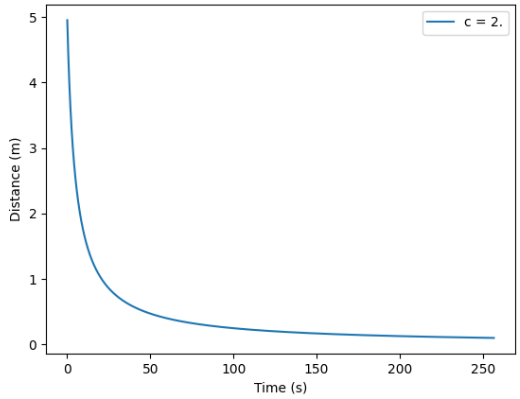
In (2.2) the deceleration constant c is used to condition how fast a person decelerates. Interestingly, if $c = 1/2$, a constant acceleration is applied.[1] Similarly, we might want to choose $c = 1/3$ in case we want a constant power $\ddot{x}\dot{x}(t)$ to be applied.

Figure 1 shows the consequences of varying the value of c.

[1] By separating $\dot{x}^2$, $\ddot{x}(t) = \dot{x}^2(t)/x(t)$ becomes $\ddot{x}(t)/\dot{x}(t) = \dot{x}(t)/x(t)$. Integrating that over t yields $\dot{x}(t) = \alpha x^c(t)$ for some constant α. It then follows that $\ddot{x}(t) = \alpha^2 c x^{2c-1}(t)$. Therefore $\ddot{x}$ is constant iff $c = 1/2$.



(A) Time to wall with $c \leq 1$.



(B) Time to wall with $c > 1$.

FIGURE 1: The time until a person reaches the wall for different values of c. The starting distance is 5 m and the initial speed is 1 m/s.

It is hard to show but figure 1 shows that for $c < 1$ the wall is reached within finite time, whereas for $c \geq 1$ the wall is approached but never reached.

2.2 One dimensional model

Say now we want to simulate people walking in a line towards some exit, now a person might have the brake for a moving person in stead of a wall. In case a person is nearing a moving object such as another person, equation (2.1) changes.

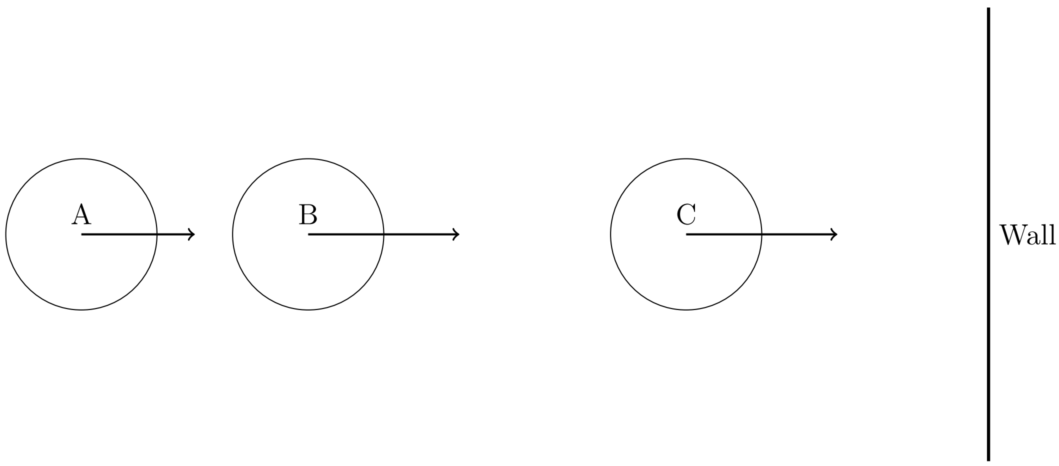


FIGURE 2: People in a 1 dimensional space walking towards a wall

We obtain the following formula for a person i walking into person j.

$$\ddot{x}_{ij} = -c\frac{(\dot{x}_i - \dot{x}_j)^2}{|x_i - x_j|}. \tag{2.2}$$

This acceleration only needs to be applied to person if[2] $x_j > x_i$ and $\dot{x}_j < \dot{x}_i$. In figure 2 person B will react to person C if C is going slower than B and B will react to the wall.

[2]Since the wall is on the right and the starting point $x = 0$ is on the left.

Person B will, however, not consider person A, since $x_A < x_B$.

The next section proposes a two dimensional model based on (2.2).

3 Pedestrian model

In this section a two dimensional model will be described. The model is in two dimensions, since humans cannot move up and down.

3.1 Model description

In this model people are modelled as circles with radius r, and the number of people is denoted with n. In the model, variables $\mathbf{x}, \dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ denote the position, velocity and acceleration of a person respectively, where $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}} \in \mathbb{R}^2$. The movement of a person will be described as the sum of accelerations caused by different factors:

$$\ddot{\mathbf{x}}_i = \ddot{\mathbf{x}}_{i.\text{exit}} + \sum_{k=1}^{K} \ddot{\mathbf{x}}_{ik.\text{wall}} + \sum_{j=1}^{n} \ddot{\mathbf{x}}_{ij}, \tag{3.3}$$

where $\ddot{\mathbf{x}}_{i.\text{exit}}$ is the acceleration that pushes the people towards the exit, $\ddot{\mathbf{x}}_{ik.\text{wall}}$ is the acceleration due to wall k, $k = 1, \ldots, K$ and $\ddot{\mathbf{x}}_{ij}$ is the acceleration caused by person j, $j = 1, \ldots, n$ and $j \neq i$.

In (3.3) it can be seen that multiple accelerations influence the movement of a person. It is assumed that a person abides by three rules. Firstly, a person does not want to bump into other people. Secondly, a person prefers not to hit a wall. And lastly, people want to reach the exit with a desired speed.

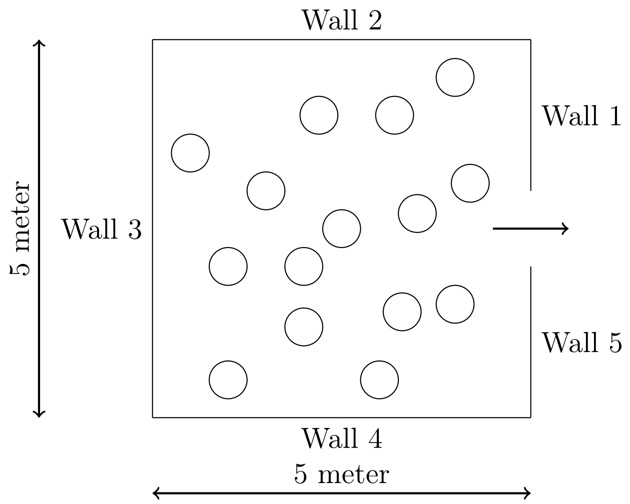


FIGURE 3: The simulated room which is 5 m by 5 m. The exit is in the middle of the right wall with width 1 m. Each circle represents a person with radius r.

To abide by those three rules, this model states that accelerations should be applied and summed according to (3.3). These accelerations will now be highlighted, but note that these accelerations are only active when it is required. Later in this section it will be shown when these equations become active in the simulation.

3.1.1 Avoiding the walls

One of the rules is that a person wants to avoid bumping into a wall. From (2.2) the following acceleration is derived that is applied to a person $i = 1, \ldots, n$ in order to not hit a wall $k = 1, \dots, 5$.

$$\ddot{\mathbf{x}}_{ik.\text{wall}} = -c \frac{||\dot{\mathbf{x}}_{ik}||^2}{||\mathbf{d}_{ik}||} \frac{\mathbf{d}_{ik}}{||\mathbf{d}_{ik}||}. \tag{3.4}$$

In (3.4) $\mathbf{d}_{ik}$ is defined as the distance vector from person i to wall k, i.e. $\mathbf{d}_{ik} = \mathbf{x}_{ik} - \mathbf{x}_i$, where $\mathbf{x}_i$ is the position of person i and $\mathbf{x}_{ik}$ is the projection of $\mathbf{x}_i$ on wall k.

Furthermore, $\dot{\mathbf{x}}_{ik}$ is the relative velocity of person i with respect to wall k. It is defined as $\dot{\mathbf{x}}_{ik} = \langle \mathbf{n}_k, \dot{\mathbf{x}}_i \rangle$, where $\mathbf{n}_k$ is the normal vector of wall k pointed inward.

Moreover, the dimensionless constant c is used from section 2.

The direction of the acceleration is perpendicular to and away from the wall.

3.1.2 Avoiding neighbours

The last part in the sum of equation (3.3) is given by $\ddot{\mathbf{x}}_{ij}$. This acceleration is defined for each individual in (3.5).

$$\ddot{\mathbf{x}}_{ij} = -c \frac{||\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j||^2}{||\mathbf{x}_i - \mathbf{x}_j||} \frac{\mathbf{d}_{ij}}{||\mathbf{d}_{ij}||}. \tag{3.5}$$

In (3.5) $\mathbf{d}_{ij}$ is the distance vector from person i to person j, defined by $\mathbf{d}_{ij} = \mathbf{x}_j - \mathbf{x}_i$. Equation (3.4) and (3.5) are quite similar, since they are both based on equation (2.2).

3.1.3 Steering towards the exit

One of the other wishes of the people is to reach the exit. Each person has a desired speed at which they prefer to walk. This acceleration is modelled by $\ddot{\mathbf{x}}_{i.\text{exit}}$ and is given by the following equation. [2]

$$\ddot{\mathbf{x}}_{i.\text{exit}} = \frac{1}{\tau} \left(v_{i.0} \frac{\mathbf{x}_{\text{exit}} - \mathbf{x}_i}{||\mathbf{x}_{\text{exit}} - \mathbf{x}_i||} - \dot{\mathbf{x}}_i \right) \tag{3.6}$$

In (3.6) two constants are included, namely $v_{i.0}$ and τ. The first denoted the desired speed of a person i, $i = 1, \dots, n$. The latter is the so called time characteristic, more will be explained in section 3.1.4.

3.1.4 Time characteristic

In equation (3.6) a time characteristic τ is used to control how fast a person reaches its desired speed. Figure 4 demonstrates the change in behaviour for different values of τ. In figure 4, (3.6) is simulated in one dimension to showcase the effect that τ has. Equation (3.6) in one dimension is just

$$\ddot{x}_{i.\text{exit}}(t) = \frac{1}{\tau} \Big(v_{i.0} - \dot{x}_i(t) \Big).$$

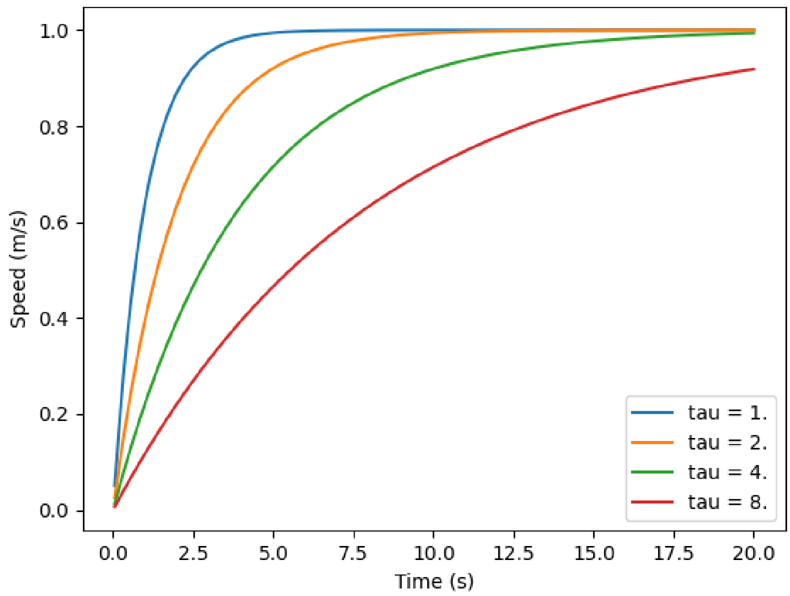


FIGURE 4: Speed progression for different values for the time characteristic τ.

We see that there exists an exponential relation between $\dot{x}$ and τ. Also, for larger values for τ, it takes the person longer to reach its desired speed.

3.1.5 Panic

Besides simulating a crowd in a regular situation it is important to simulate a crowd in an emergency. When people start to panic their patience reduces and their desire to reach the exit increases.

One way to simulate panic is by changing the desired speed in the direction of the exit according to the degree of panic. This means that the acceleration towards the door increases when people start to panic. This can be modelled by varying the desired speed $v_{i.0}$ in equation (3.6) in time.

The desired speed can be determined per time interval by a "panic parameter" $p_i(t)$. In [2] it is defined as per equation (3.7).

$$p_i(t + \Delta_t) = 1 - \frac{||\dot{\mathbf{x}}_i(t)||}{v_{i.0}(t)} \tag{3.7}$$

In this formula $p_i(t)$ indicates that panic level of person i at time t, where $0 \leq p_i(t) \leq 1$ by construction.[3] According to the formula $p_i(t)$ increases when the speed decreases, and it goes to zero when people reach their desired speed. However, the desired speed is also dependent on the panic level, since people want to walk faster when they are panicking. In [2] the desired speed is calculated as follows.

$$v_{i.0}(t + \Delta_t) = \big[1 - p_i(t)\big] v_{i.0}(0) + p_i(t) v_{i.\max} \tag{3.8}$$

[3] $||\dot{\mathbf{x}}_i(t)||$ cannot exceed $v_{i.\max}$

The initial desired speed is denoted by $v_{i.0}(0)$ and the maximum speed is given by $v_{i.\max}$.

Equations (3.7) and (3.8) make sure that the desired pace increases as the time it takes to exit the room increases. The level of panic grows and shrinks proportionate to the waiting time.

3.2 Motion model

From the previous subsection we know the different accelerations that influence a person's course. That means that the movements of person i can now be computed.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\mathbf{x}_i(t)\\ \dot{\mathbf{x}}_i(t)\end{bmatrix} = \begin{bmatrix}\dot{\mathbf{x}}_i(t)\\ \ddot{\mathbf{x}}_i(t)\end{bmatrix}. \tag{3.9}$$

Equation (3.9) is a state representation of the model, where $\ddot{\mathbf{x}}_i$ is calculated in equations (3.3), (3.6), (3.4) and (3.5).

To make the computation more efficient, the movement will be modelled discretely. At each time step Δ_t the current position $\mathbf{x}_i$ and velocity $\dot{\mathbf{x}}_i$ of person i are calculated. Using equation (3.9) the following equating is implemented for the simulation:

$$\dot{\mathbf{x}}_i(t+\Delta_t) = \dot{\mathbf{x}}_i(t) + \ddot{\mathbf{x}}_i(t)\Delta_t \tag{3.10}$$

$$\mathbf{x}_i(t+\Delta_t) = \mathbf{x}_i(t) + \dot{\mathbf{x}}_i(t)\Delta_t \tag{3.11}$$

3.2.1 Potential collisions

An important question remains: When will a person collide? It is important for a person in the simulation to know when a potential collision is feasible, so it can activate the required acceleration.

It is easily derived when a person is going to hit a wall. If the distance is rather small and the relative velocity $\dot{\mathbf{x}}_{ik}$ is negative, the person will want to steer away from the wall. In other words, (3.4) is activated for the respective wall. But when is a person going to collide with another person, i.e. when must a person steer away from or break for another person?

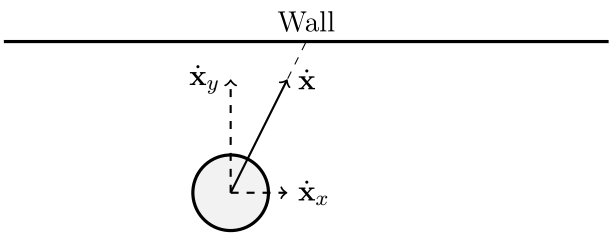


FIGURE 5: A person (circle) is going to hit the wall.

First of all, an individual will obviously not react to the movements of another at the other side of the room. So we must set a radius R for each person. If a person or obstacle comes within the circle with radius R, the person might have to react. Otherwise no acceleration needs to be applied.

Furthermore, a person will only slow down for or move away from a person if he or she can see him. The visual span of the human eye is approximately 120° [1]. However, there is also that what we call *peripheral vision*, which means that humans can sense objects and movement at a range from 200° to 220° [5].

Now there are two cases left, namely when two individuals come close to each other but do not collide, and of course when two individuals are on a collision course. But how does one decide whether they are on a collision course or not?

Say two people are both walking, person A and person B, where A is the 'walker' and B is seen as the obstacle. They both have a certain velocity. It is assumed that A is able to see B. Now we want to check whether A and B are on a collision course or not. This is possible by doing some simple trigonometry.

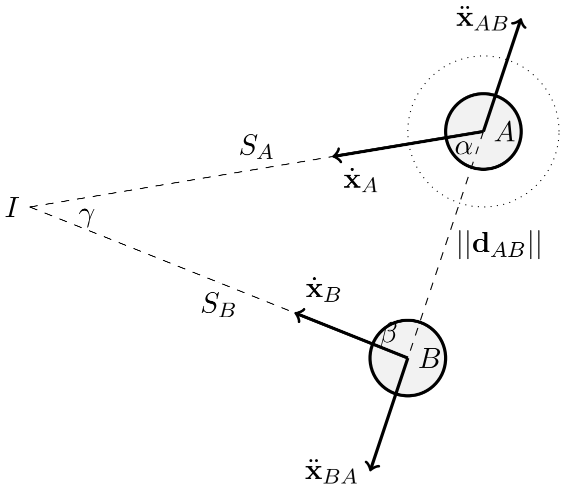


FIGURE 6: The paths of person A and B intersect.

See figure 6. The first step is to compute angles α and β. There is a distance vector between the two persons denoted by $\mathbf{d}_{AB}$, where $\mathbf{d}_{AB} = \mathbf{x}_B - \mathbf{x}_A$. From that and the two velocities $\dot{\mathbf{x}}_A$ and $\dot{\mathbf{x}}_B$ we can calculate α and β with equation (3.12) and (3.13).

$$\alpha = \cos^{-1}\left(\frac{\dot{\mathbf{x}}_A \cdot \mathbf{d}_{AB}}{||\dot{\mathbf{x}}_A||||\mathbf{d}_{AB}||}\right). \tag{3.12}$$

$$\beta = \cos^{-1}\left(\frac{\dot{\mathbf{x}}_B \cdot \mathbf{d}_{AB}}{||\dot{\mathbf{x}}_B||||\mathbf{d}_{AB}||}\right). \tag{3.13}$$

In equations (3.12) and (3.13) $\mathbf{d}_{AB}$ is the distance vector from point A to B.

In the case that[4] $\alpha + \beta < 180°$, we now have a so-called *angle-side-angle triangle*. We can calculate sides S_A and S_B in equation (3.15). But first we calculate γ in equation (3.14).

$$\gamma = 180° - \alpha - \beta. \tag{3.14}$$

Now we can compute the sides:

$$\frac{S_B}{\sin\alpha} = \frac{S_A}{\sin\beta} = \frac{||\mathbf{d}_{AB}||}{\sin\gamma}. \tag{3.15}$$

[4]Otherwise A and B do not converge.

These sides represent the distance each person has to walk in order to reach the intersection of paths, denoted by I. A collision occurs when they both reach this intersection at roughly the same time. This is all dependent on the speed (absolute value of the velocity) of each person.

Person A knows their own speed. We can calculate an interval of speeds for person B such that A and B collide or come uncomfortably close to each other. First we must calculate the time until A reaches the intersection and then we can compute interval. The formula for both these problems is given in equation (3.16).

$$s = vt. \tag{3.16}$$

Here s is the distance, v is the speed and t is the time.

The interval can be computed by taking $s \in [S_B - (r + r^*), S_B + (r + r^*)]$, where r^* is the distance people want to keep away from other people, and t from having computed the time it takes A to reach I. With those s's and t's, the interval of v's is computed.

If person $||\dot{\mathbf{x}}_B||$ falls within that interval, A must apply an acceleration $\ddot{\mathbf{x}}_{AB}$ given by equation (3.5), otherwise it will collide with person B. If B's speed does not fall within that interval, there will be no collision and therefore no acceleration need to be applied.

Similarly, B will also sense a collision and must also react, in the case that they are able to see A.

It becomes more complicated when there are more than two people. Then a person might have to change course for multiple different people. The accelerations will be added up as per equation (3.3).

3.3 Assumptions

Before the model can be simulated a few assumptions have to be made clear.

- Humans cannot apply infinite force, so there must be a limit on the acceleration. According to the article "Will Humans Ever Be Able to Outrun A Car?" by Rilind Elezaj, Usain Bolt accelerates at a maximum of 8 to 10 m/s^2. Assuming that in the simulation we do not have any Olympic athletes and no-one is sprinting, the maximum acceleration is given by a, where $a = 3$ m/s^2.
- The radius of a person is $r = 0.25$ m.

3.4 Obstacles

In this paper the influence of obstacles is also being researched. More specifically, we will add a pillar to the simulation on different coordinates. It will now be describes how a person interacts with a pillar.
First of all, a person must know if it is going to collide with the pillar.

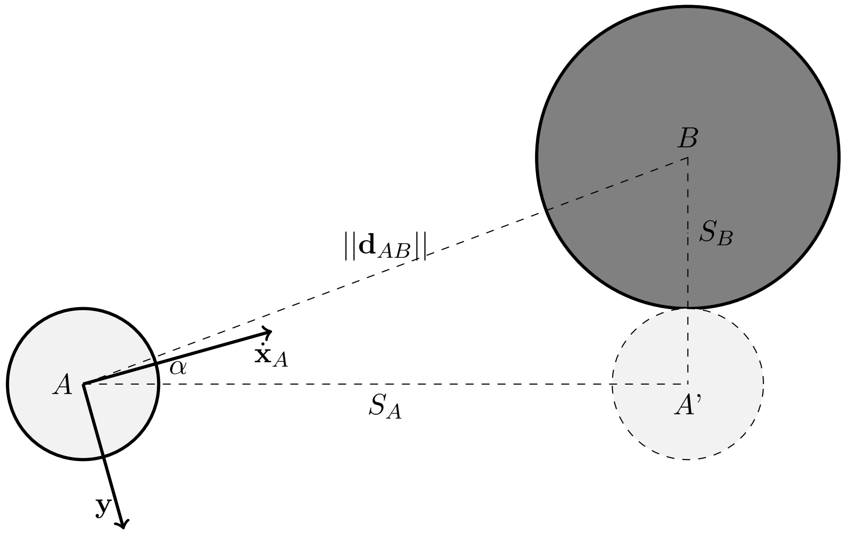


Figure 7: A person (grey) waling towards a pillar (dark grey).

See figure 7. If the angle between $\dot{\mathbf{x}}$ and $\mathbf{d}_{AB}$ is smaller than α, we can see that the person will run into the pillar. If that is the case, the person must take action.

For optimal efficiency, the person wants to steer away from the pillar such that it just does not run into the pillar. In figure 7 this is shown by the circle with the dashed outline. A' is the position that person A wants to steer to, such that it does not hit the pillar. Therefore an acceleration must be applied to push the person in that direction. The following equation describes the acceleration.

$$\ddot{\mathbf{x}}_{i.\text{pillar}} = -c \frac{||\dot{\mathbf{x}}_{i.\text{relative}}||^2}{||\mathbf{d}_{i.\text{pillar}}|| - r_i - r_{\text{pillar}}} \frac{\mathbf{y}_i}{||\mathbf{y}_i||} \tag{3.17}$$

First of all, $\dot{\mathbf{x}}_{i.\text{relative}}$ is the relative velocity[5] of person i with respect to the distance to the pillar. Furthermore, $\mathbf{y}_i$ is the direction vector that is perpendicular to $\dot{\mathbf{x}}_i$, defined as:

$$\mathbf{y}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \dot{\mathbf{x}}_i \text{ if } \dot{\mathbf{x}}_i \text{ is below } \mathbf{d}_{i.\text{pillar}}. \text{ (See figure 8.)}$$

$$\mathbf{y}_i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \dot{\mathbf{x}}_i \text{ if } \dot{\mathbf{x}}_i \text{ is above } \mathbf{d}_{i.\text{pillar}}.$$

And r_A and r_B denote the radii of the person and pillar respectively.

[5] In figure 7 $\dot{\mathbf{x}}_{A.\text{relative}}$ is the projection of $\dot{\mathbf{x}}_A$ onto $\mathbf{d}_{AB}$, where $\mathbf{d}_{AB} = \mathbf{x}_B - \mathbf{x}_A$.

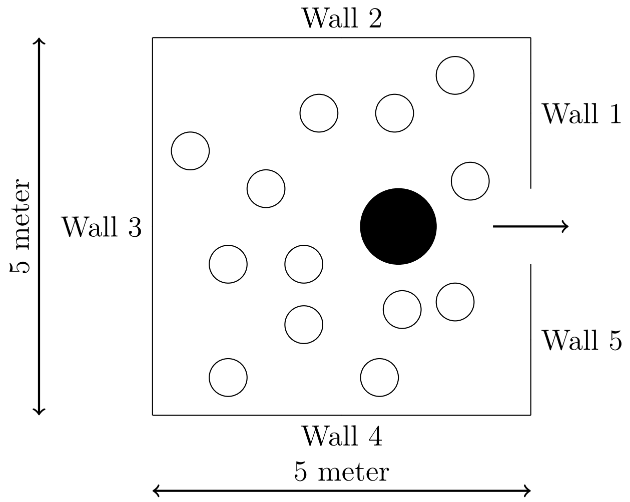


FIGURE 8: The simulated room which is 5 m by 5 m. The exit is in the middle of the right wall with width 1 m. Each circle represents a person with radius $r = 0.25$ m. A pillar (black circle) with a radius of $r_{\text{pillar}} = 0.5$ m is added 1.25 m in front of the exit.

As a result equation (3.3) now becomes

$$\ddot{\mathbf{x}}_i = \ddot{\mathbf{x}}_{i.\text{exit}} + \sum_{k=1}^{K} \ddot{\mathbf{x}}_{ik} + \sum_{j=1}^{n} \ddot{\mathbf{x}}_{ij} + \ddot{\mathbf{x}}_{i.\text{pillar}}. \tag{3.18}$$

4 Results

After the model is finished it is time to make simulations. This section provides the results obtained from the simulations. Moreover, the validity of the model is checked and it is shown how a change in choice for the parameters affects the model.

For the simulations the room is always 5 m by 5 m. If not specified, the width of the exit is 1 m. The simulations will be done for different number of people, but that will be specified with the result. Finally, for every instance, the simulation will be run 10 times, since the initial positions of the people are random. Furthermore $\dot{\mathbf{x}}_i = 0$ for every $i = 1, \ldots, n$.

4.1 Validation

A model has to be validated in order to confirm whether it is realistic. The best way to do this is by comparing the results of the model to field data. For this project, however, that is not possible. Therefore the only comparison that can be made is with the expectations of human behaviour in a crowd.

In section 3 three important things were highlighted for a crowd model: do not collide with walls, do not collide other people, walk towards the exit. This model seems to do all those things well. People make way for other people and all move towards the exit at the same speed. It is clear from the simulations that people try to avoid the walls and other people.

This model is also a relatively smart model: people change their path in order to not collide. In that sense it is a realistic projection of real life. However, it seems that people will give way too much for other people even in panic situations, whereas one could expect people would rush to reach the exit.

4.2 Analysis of different floor plans

In this section the sub question *"How can a floor of a building be designed such that a crowd of people can leave through the exit doors as quick as possible?"* will be answered. There are three specific instances that are researched. The first is how the width of the exit influences the flow time. The second is whether adding a pillar in front of the exit somehow influences the flow time. Finally, it will be researched what happens when the door is split in two, such that there are two exits in the same wall.

4.2.1 Variation of door width

Intuitively, the bigger the door, the more people can fit through, the shorter the flow time is. This reasoning is not hard to believe. However, nothing can yet be said about the exact effect in door width. This paragraph will showcase the influence of the width of the exit on the flow time.

The door width ranges from 80 cm to 2 m. The optimal flow time would of course occur when we take a much larger width, but that is not realistic in building design. For these simulations the intitial positions are completely random.

As expected, the flow time increases as the door becomes less wide. However, the flow time does not decrease by much when the door is wider than 1.6 m.

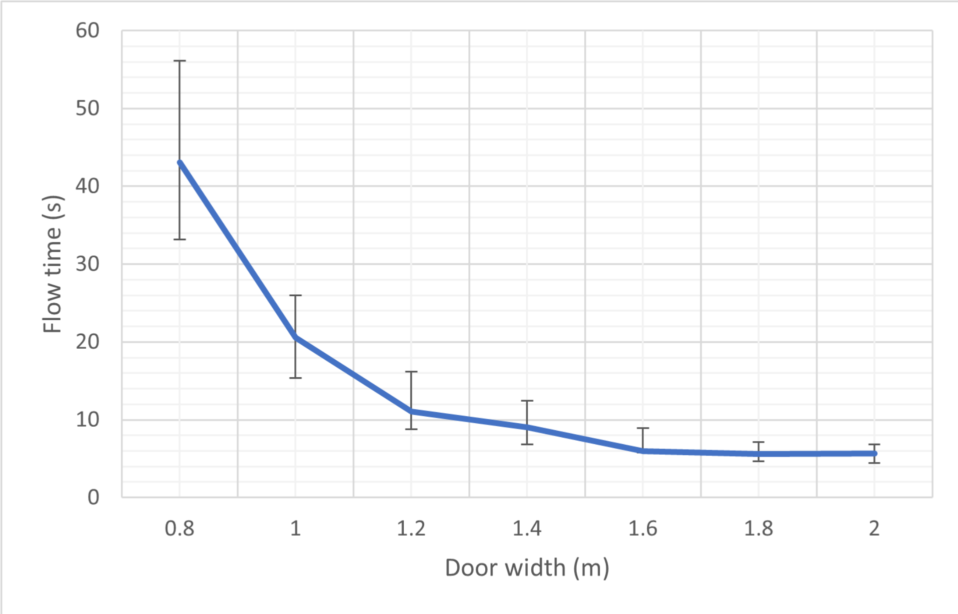


FIGURE 9: Flow time for different widths of the exit door where the maximum and minimum flow time is also shown. The simulation was done in a room of 5 m by 5 m. Here, $\Delta_t = 1/20$, $n = 10$, $\tau = 1$, $v_{i.0}(0) = 1$ m/s for all i, $R = 2$ m, $r^* = 0.5$ m and $c = 0.5$.

4.2.2 Addition of a pillar

Since obstacles and walls exert forces on individuals, the design of the space plays an important role in the flow time of people. There are many variations in space design, some more efficient than others. An example of this is placing a pillar in front of the exit. This could reduce the pressure on the exit, potentially decreasing the flow time. In this paragraph, this is tested using the model explained in section 3.
By using a pillar, a dual exit is created in a sense that people go through the exit from two different sides. The influence of the pillar's placement on the flow time will be examined. Only the x-coordinate of the pillar is varied (this is the distance from the exit to the pillar). The distance is varied from 0.75 m to 2 m and is compared to a simulation without a pillar. For the simulations, the people start at random positions close to the leftmost wall (furthest away from the exit) such that they all have to avoid the pillar. This way the influence of the pillar can be best seen. The results can be found in figure 10.

There does not seem to be any influence of the pillar, especially with the door width set to 1 m. When the width of the door is 1.5 m it is even the case that the flow time slightly increases with the addition of a pillar. This might be due to the fact that people cannot walk in a straight line to the exit anymore. They have to go around the pillar. That means that the relieving effect on the exit is not as significant as before speculated.

(A) Door width is 1 m in these simulations.

(B) Door width is 1.5 m in these simulations.

FIGURE 10: Flow time for different positions of a pillar to the exit door where the maximum and minimum flow time is also shown. The simulation was done in a room of 5 m by 5 m. Here, $\Delta_t = 1/20$, $n = 10$, $\tau = 1$, $v_{i,0}(0) = 1$ m/s for all i, $R = 2$ m, $r^* = 0.5$ m and $c = 0.5$.

4.2.3 Addition of an extra exit

Instead of using a pillar to create a sort of dual exit, why not actually make two exits? By adding an extra exit, the pressure on one exit is halved. Intuitively this must decrease the flow time. In this section the effect of two exits on the flow time will be shown. The two exits are always on the same wall, but they will be placed at different distances from each other, ranging from 0.5 m to 3 m. In figure 12 the doors are placed 1 m apart. Both doors are 1 m wide.

For these simulations the people start at random positions close to the leftmost wall.

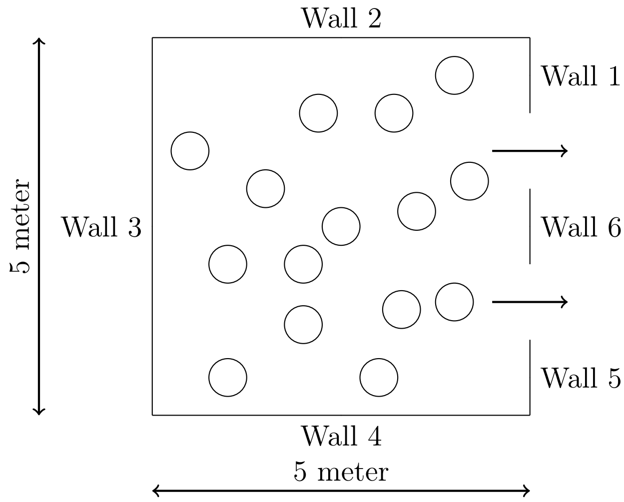


FIGURE 11: The simulated room which is 5 m by 5 m. The rightmost wall contains two exits of 1 m in width, placed 1 m apart. The circles represent people with radius $r = 0.5$.

As can be seen in figure 12, the spacing between the two exits is not of great influence. However, there is a small peak when the distance is 2 m and 2.5 m. Comparing the results from figure 12 to the results from figure 9 it can be seen that two separate exits does not decrease the flow time as much as making one bigger exit.

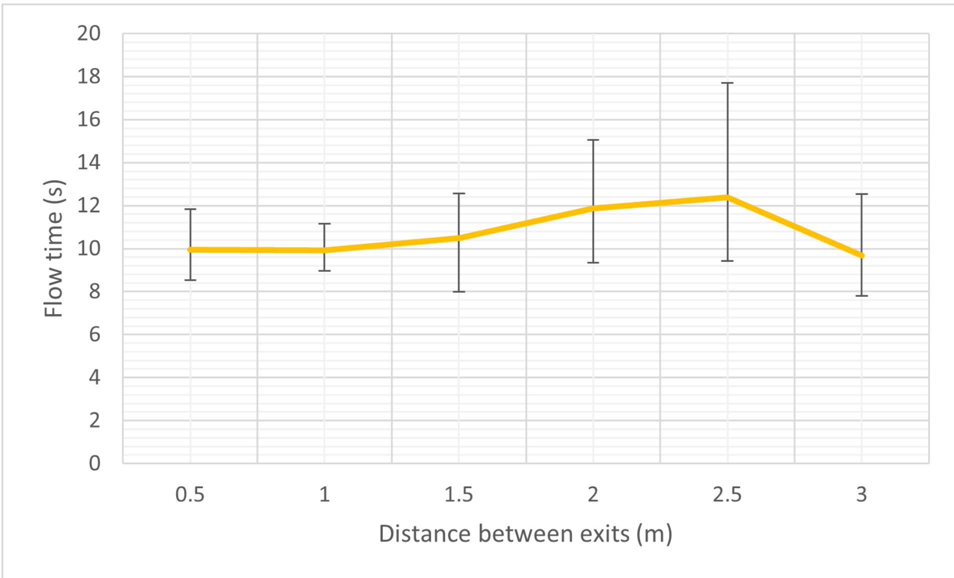


FIGURE 12: Flow time for different between the two exits where the maximum and minimum flow time is also shown. The simulation was done in a room of 5 m by 5 m. Here, $\Delta_t = 1/20$, $n = 10$, $\tau = 1$, $v_{i,0}(0) = 1$ m/s for all i, $R = 2$ m, $r^* = 0.5$ m and $c = 0.5$.

There does seem to be an improvement when the exits are spaced 3 m apart. This is because the exits now border the top and bottom wall which means that people can only come from one side. As a consequence, there is less of a queue at the exit.

4.3 Variation of the deceleration constant

For the results above, it was chosen that $c = 1/2$. In this section, it will be shown what happens when c is chosen to take on a different value.

In figure 13 it can be seen that the flow time is significantly higher for $c = 1/2$ than for other values of c. The higher the value of c, the quicker a person brakes. An explanation for the results in figure 13 could be that for $c = 1/4$, the people decelerate very little and push themselves through the exit, and for $c = 1, 2$ it could be due to the fact that a higher deceleration constant results in more space at the exit, which reduces a congestion at the exit.

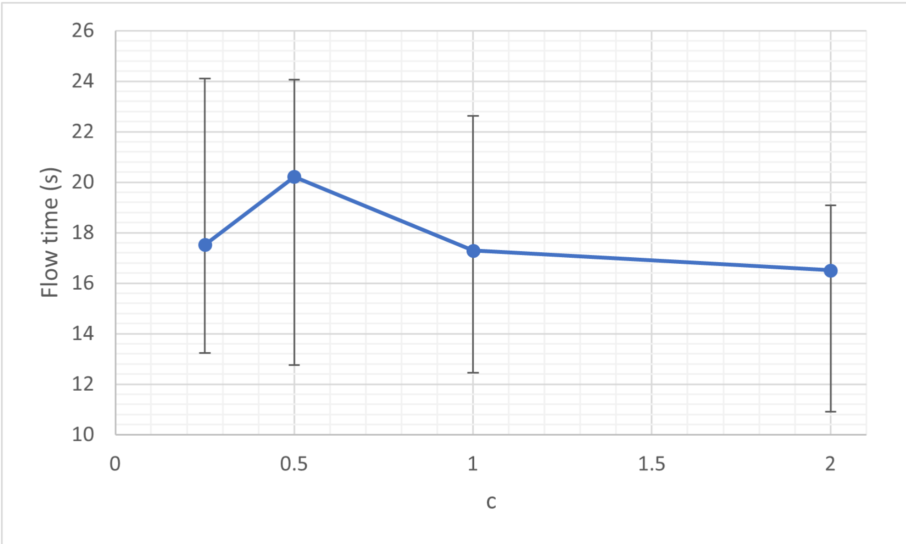


FIGURE 13: Flow time for different values of c where the maximum and minimum flow time is also shown. The simulation was done in a room of 5 m by 5 m. Here, $\Delta_t = 1/20$, $n = 10$, $\tau = 1$, $v_{i.0}(0) = 1$ m/s for all i, $R = 2$ m and $r^* = 0.5$ m.

In section 4.2.2 it was suggested that adding a pillar in front of the exit might reduce the congestion at the exit, but, against expectations, it turned out this is not the case. However, this was only simulated for $c = 1/2$. It is interesting to see what will happen for the pillar simulation when $c = 1$. Since the people are more cautious, more space should be created which could give the addition of a pillar the desired effect.

Figure 14a shows that the flow time reduces significantly when the pillar is place around 1.25 m in front of the exit. This is the result that was expected in section 4.2.2. In figure 14b it is less easy to see the influence of the distance, but it somewhat agrees with the data from the graph above it. However, at no position does the pillar reduce the flow time compared to the simulation with no pillar when the door is 1.5 m wide. When the width of the door is 1.5 m, the congestion at the exit is not that big to start with, hence the lack of decrease in flow time.

(A) Door width is 1 m in these simulations.

(B) Door width is 1.5 m in these simulations.

FIGURE 14: Flow time for different positions of a pillar to the exit door where the maximum and minimum flow time is also shown. The simulation was done in a room of 5 m by 5 m. Here, $\Delta_t = 1/20$, $n = 10$, $\tau = 1$, $v_{i.0}(0) = 1$ m/s for all i, $R = 2$ m, $r^* = 0.5$ m and $c = 1$.

5 Discussion

Most of the time the simulation looks realistic. There are instances, though, where the model does not seem to produce a realistic simulation. In this section, some of the limitations of the model are highlighted and discussed, as well as some problems encountered along the way for which a solution has been found

5.1 Collisions and behaviour at the boundaries

Firstly, it is of course important that people cannot move through the walls. In the first few simulations people would avoid all the walls perfectly, except at the wall where the exit is located. This is due to $\ddot{\mathbf{x}}_{i.\text{exit}}$. To make sure that does not happen, the state representations below are implemented.

$$\begin{cases} \mathbf{x}_i(t+\Delta_t) = \text{proj}_{\text{wall}}(\mathbf{x}_i(t)); \\ \dot{\mathbf{x}}_{ik.\text{wall}}(t+\Delta_t) = \mathbf{0}. \end{cases} \tag{5.19}$$

These formulas are only used if a person happens to be outside of the room without having gone through the exit. In (5.19) the first equation says that the position of person i should be projected onto the wall such that the person finds itself inside the room again whilst not jumping to a completely different position. In the second equation $\dot{\mathbf{x}}_{ik.\text{wall}}$ is the relative velocity[6] of person i.

Furthermore, when the model was first simulated, the people would sort of morph into each other (as in the distance between the two mid points of the people would be smaller than their combined radii), and they would walk towards the exit that way (stuck while morphed into each other). This is of course very non-realistic. To get rid of that, an extra acceleration was implemented.

$$\ddot{\mathbf{x}}_{ij} = -c\frac{\mathbf{d}_{ij}}{||\mathbf{d}_{ij}||}, \quad \text{if } ||\mathbf{d}_{ij}|| < r_i + r_j. \tag{5.20}$$

Here, $\mathbf{d}_{ij}$ is the distance vector from the mid point of person i to the mid point of person j, and r_i and r_j denote the radii of i and j respectively.

After implementing this solution people are still slightly morphing into each other, but they quickly push each other away. Still, this may not be the most realistic representation of a human crowd, since people in real life cannot physically move into each other's bodies, but it does give the idea of people of being squashed against each other. It is deliberately chosen not to implement the same solution as for the walls, because that would result in a simulation where people bounce off of each other, which is not the desired result.

When a person bumps into a pillar, however, a different method was implemented as to not have people morph into the pillar. The following state representations make sure that people cannot move through the pillar.

$$\begin{cases} \mathbf{x}_i(t+\Delta_t) = \mathbf{x}_{\text{pillar}} - (r_i + r_{\text{pillar}})\frac{\mathbf{d}_{i.\text{pillar}}}{||\mathbf{d}_{i.\text{pillar}}||}; \\ \dot{\mathbf{x}}_i(t+\Delta_t) = 0; \\ \ddot{\mathbf{x}}_{i.\text{pillar}}(t+\Delta_t) = -c\frac{\mathbf{y}_i}{||\mathbf{y}_i||} \end{cases} \quad \text{if } ||\mathbf{d}_{i.\text{pillar}}|| < r_i + r_{\text{pillar}}. \tag{5.21}$$

[6] The relative velocity with respect to a wall is the projection of $\dot{\mathbf{x}}_i$ onto the normal vector of the respective wall.

In (5.21) $\mathbf{d}_{i.\text{pillar}}$ denotes the distance vector from $\mathbf{x}_i$ to $\mathbf{x}_{\text{pillar}}$, and $\mathbf{y}_i$ is a direction vector defined as:

$$\mathbf{y}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \dot{\mathbf{x}}_i \text{ if } \mathbf{x}_i \text{ is below } \mathbf{x}_{\text{pillar}};$$
$$\mathbf{y}_i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \dot{\mathbf{x}}_i \text{ if } \mathbf{x}_i \text{ is above } \mathbf{x}_{\text{pillar}}.$$

This is not a smooth solution, since $\dot{\mathbf{x}}_i$ abruptly goes to 0. The difference with how Helbing's model [2] works is that the model described in this paper does not bounce of the pillar whilst keeping the same speed. In the simulations, when people hit the pillar, they do not travel back the way they came from, but accelerate away from and around the pillar.

5.2 Two people heading straight for each other

See figure 15 and recall section 3.2.1. If two people are heading straight for each other, that is $\alpha = \beta = 0°, \gamma = 180°$, (3.15) produces an error in the simulation caused by dividing by zero. Therefore, if that is the case, the steps laid out in section 3.2.1 can be skipped. If two people are directly headed for each other, it is obvious that they will collide, hence (3.5) shall be applied, in figure 15 these accelerations are depicted by the dashed arrows.

FIGURE 15: Two people i (left) and j (right) are directly headed for each other.

6 Conclusion

How can a floor of a building be designed such that a crowd of people can leave through the exit doors as quickly as possible? That was one of the research questions of this paper, and the one most easily answered, since it can be concluded from section 4.

The results showed a decrease in flow time (the time it took until everyone had left the room became shorter) when the door was made wider. The decrease is large ranging from a width of 0.8 m to 1.5 m, whereas even wider exits did not decrease the flow time significantly. It was then simulated how adding a pillar in front of the exit would influence the flow time. It can be concluded that for $c = 1/2$ it did not seem to affect the time at all, while for $c = 1$ it improved the flow time rather significantly for certain distances to the exit. Finally, it was researched what adding an extra exit would do. Based on the results, it did a lot better than adding a pillar, but compared to the wider doors, the flow time was still much larger. Also, the distance between the two exits does not matter according to the data.

To answer this question, a wider door is the best strategy to decrease flow time according to this model.

Is the model based on the Buckingham π-theorem realistic? That is the main research question of this paper. This one is harder to answer, since there is not empirical evidence to support the case. Though based on sections 4.1 and 5, it can be concluded that the model has its limitations and is therefore not completely realistic. It does, however, abide by the three rules mentioned in section 3.1, and based on that, the model does exactly what it is meant to do, disregarding of its flaws.

7 Recommendations

Unfortunately, since this is a bachelor's thesis, the time spent on this paper is limited. For that reason some ideas were not able to be researched or implemented. In this section a few of these ideas will be highlighted so that they may be looked at and perhaps used in future research in crowd modelling.

7.1 Prioritising

In the model, the acceleration of a person is determined by a sum of different accelerations each caused by a specific person or object. The consequence is that the resulting total acceleration does not guarantee that the person will not hit for example a wall or a neighbour. This problem could be fixed by setting priorities.

What that means is that, instead of summing all different acceleration in order to obtain a total acceleration, the acceleration is determined by what obstacle is closest to the person, and only taking that specific acceleration. That way it can be guaranteed that the person does not bump into its closest obstacle.

There might be some faults in prioritising. For example, if a person only focuses on one obstacle, it could become blind to other obstacles. A consequence is that the person will not be able to react in time. Another thing that could happen is that the person will steer away from an object and than get too close to another object and steering back and forth and so on. This would create not per se a bouncing effect, but something similar, which is not realistic.

7.2 Sensitivity analysis

In sensitivity analysis, it is examined how sensitive the model is to small changes. If a model yields significantly different results when constants and variables are slightly altered, it may indicate that the model is not valid. Additionally, this analysis reveals the outcomes when certain variables are varied. In this paper is has already been shown how variation in c influences the simulations, but there are still some parameters that could be examined. The expectation is that the varying these parameters will not influence the outcome a lot, the model will still work the same way. However, it is always better to support this with actual data.

7.2.1 Increasing n

One parameter that is interesting to examine is the total number of people n, especially increasing n. The flow time will of course increase with more people in the simulation. What is unknown however, is whether the simulation will change a lot now that one person is affected by more people than before.

7.2.2 Varying $v_{i.0}$ and $v_{i.\max}$

It could be interesting to see how the desired speed affects the simulations. The assumption is that a higher value for desired speed correlates to panic, which has a negative affect on the flow time, that is it will take the crowd longer to leave the room.

7.2.3 Varying $\dot{\mathbf{x}}_i(0)$

Finally, for all the simulations people started from rest. It has not been researched if the simulations change significantly if a different initial speed is chosen. There are many variations possible. For example, each person $i = 1, \ldots, n$ could have a random value for $\dot{\mathbf{x}}_i(0)$.

References

[1] Riad I. Hammoud. *Passive Eye Monitoring: Algorithms, Applications and Experiments.* Tech. rep. Springer, 2008. DOI: 10.1007/978-3-540-75412-1.

[2] Dirk Helbing, Illés Farkas, and Tamás Vicsek. “Simulating dynamical features of escape panic”. In: *Nature* 490 (July 2000), pp. 487–490. DOI: 10.1038/35035023.

[3] Gjerrit Meinsma. *Dimensional and scaling analysis.* Tech. rep. 1. SIAM Review, 2019, pp. 159–184. DOI: 10.1137/16M1107127.

[4] Craig W Reynolds. *Flocks, Herds, and Schools: A Distributed Behavioral Model.* Tech. rep. 4. 1987. DOI: 10.1145/37401.37406.

[5] Martin Szinte and Patrick Cavanagh. “Apparent Motion from Outside the Visual Field, Retinotopic Cortices May Register Extra-Retinal Positions”. In: *PLoS ONE* 7.10 (Oct. 2012). ISSN: 19326203. DOI: 10.1371/journal.pone.0047386.