Commodity price co-movement: Comparing models and correlation measures

16.11.2023

Luca Kozian, s2355051, l.l.kozian@student.utwente.nl

University of Twente

MSc Business Administration – Financial management

1st Supervision: Prof. Dr. Jörg Osterrieder

2nd Supervision: Dr. Marcos Machado

Acknowledgements

Abstract

This thesis investigates how well different types of methodologies and co-movement measures can explain cross-commodity price co-movement using macroeconomic variables. Thereby, VAR, VARX, multiple regressions and Random Forest regressions are applied as models, and Pearson correlations and Gerber statistics as co-movement measures. They are implemented on a dataset of price co-movement of 20 major commodities and various macroeconomic factors from mid-2003 to early 2023. The findings show that VAR and VARX models outperformed Random Forests and multiple regressions, reaching R^2 values as high as 89%. Random Forest regressions, however, only showed slightly better performance metrics than multiple regressions. Moreover, did the use of Gerber statistics over Pearson correlations enhance model performance for VAR and VARX models, but for Random Forests and multiple regressions this is less clear. By examining and comparing different methodologies, this thesis contributes to existing literature on commodity price co-movement and lays a groundwork for further assessments of the performance of various methods in modelling this phenomenon.

Keywords: Commodities, Co-movement, Gerber statistic, VAR, Random Forest

Table of Contents

1. Introduction

Publicly traded commodities offer alternative investment opportunities to investors and have thus been a subject of research for academia and professionals alike. Thereby, the "co-movement" of the prices of various commodities is of special interest, given its implications in for instance portfolio construction. It has been investigated for more than 30 years by now, with Pindyck and Rotemberg (1990) introducing the hypothesis of the existence of excess co-movement, the remaining proportion of cross-commodity price co-movement that is not attributable to shared factors. Since then, various publications aimed at validating the theory of Pindyck and Rotemberg (1990), yet to this day research is still inconclusive about the existence of excess co-movement of commodity prices. Some studies rejected Pindyck and Rotemberg's hypothesis or found no evidence for it (Ai et al., 2006; Deb et al., 1996) while others did not (Le Pen & Sévi, 2017). This already raises the need for a comprehensive evaluation of different methodologies aimed at capturing and explaining the phenomenon of commodity price co-movement. The recent introduction of the Geber statistic as a robust alternative for measuring co-movement (Gerber et al., 2022), and the rising popularity of machine learning algorithms like Random Forest models (Breiman, 2001) in financial applications, underscore the need for this comparison. Thereby, their application may enhance prediction accuracy and hence minimize excess co-movement.

1.1. Research questions

To address this problem of a missing evaluation of methods to explain commodity price co-movement, the main research question guiding the research presented in this thesis was formulated as follows:

> *"How well can different types of co-movement measures and models with macroeconomic determinants as inputs explain commodity price co-movement between mid-2003 and early 2023?"*

The mentioned macroeconomic determinants are hereby specified in Section 2, while the co-movement measures and methodologies applied to model them are presented in Section 3. In pursuit of answering this question, several sub-questions are formulated. These are aimed at tackling the research problem described previously by helping to give an answer to the main research question. They are formulated as follows:

Sub-question 1. *Is there evidence of significant macroeconomic factors acting as determinants of commodity price co-movement during the considered sample period?*

Sub-question 1a: *Do direct macroeconomic factors acting as proxies for supply and demand have significant effects in explaining commodity price co-movement?*

Sub-question 1b: *Do indirect macroeconomic factors that are not directly related to supply and demand commodity pricing have significant effects in explaining commodity price co-movement?*

Sub-question 2.   *Is there evidence that the effects of macroeconomic determinants on commodity price co-movement change over time within the sample period?*

Sub-question 3.   *Are the magnitude of commodity price co-movement, its proportion that remains unexplained, and the effects of macroeconomic determinants different between hard commodities and soft commodities?*

Sub-question 4.   *Does applying novel co-movement measures and methodologies to model them allow for a more precise prediction of commodity price co-movement, thus minimizing unexplained excess co-movement?*

Sub-question 5.   *Is there evidence of significant structural changes in levels of commodity price co-movement during the considered sample period?*

Here, Sub-question 1 suggests that certain macroeconomic variables can have an impact on the co-movement level of commodity prices, indicating their importance in explaining the behavior of commodity markets. Sub-question 1a further extends this topic and specifically focuses on the significance of macroeconomic indicators "directly" related to supply and demand dynamics in predicting commodity price co-movement. Similarly, Sub-question 1b recognizes that "indirect" macroeconomic factors beyond traditional supply-demand dynamics may play a role in shaping commodity price co-movement. The difference between direct and indirect factors is elaborated on in Section 2. Moreover, as the sample period considers almost two decades with interesting economic developments such as the 2008 financial crisis and the Covid-19 pandemic, Sub-question 2 was introduced. It anticipates that the impact of macroeconomic determinants on commodity price co-movement may vary over time, suggesting the presence of evolving dynamics in commodity markets. With regards to previous studies on this topic that showed co-movement levels to spike during economic crises (Zaremba et al., 2021), the effects of macroeconomic determinants are especially interesting during the economic disruptions included in the research's sample period. Particularly, are there two subsamples characterized by the distinct economic events mentioned earlier. Additionally, co-movement levels of the prices of only hard and only soft commodities are investigated as part of Sub-question 3. Thereby, attention is not only paid to the effects of macroeconomic variables like in Sub-questions 1, 1a, and 1b, but also to levels of co-movement and the unexplained proportion of "excess co-movement" as defined in Section 2.1. Hence, does Sub-question 3 recognize that different commodity sectors, characterized as hard and soft commodities, may exhibit distinct patterns of co-movement. Moreover, it is aimed to give insights into whether the ability of macroeconomic determinants to explain co-movement varies when considering different commodity types. Lastly, it recognizes that the proportion of unexplained co-movement may also be dependent on whether hard or soft commodities are investigated. Furthermore, several methodologies and two distinct co-movement measures were used to examine the main research question and Sub-questions 1 to 3. Thus, Sub-question 4 investigates if an innovative co-movement measure (the Gerber statistic) and models new to the context of commodity price co-movement (Random Forest regression models) yield

superior results with regards to minimizing unexplained excess co-movement. In essence, Sub-question 4 examines if utilizing Random Forest regression models (See Breiman, 2001) in contrast to conventional methodologies, could enhance the accuracy of predicting commodity price co-movement with macroeconomic variables. Similarly, it is examined if the novel co-movement measure in forms of the Gerber statistic allows for higher prediction accuracy. Lastly, to complement the findings of the subsample models applied in this research, particularly with regards to Sub-question 2, and to extend the conclusions of previous research, Sub-question 5 was introduced. Is investigates if co-movement levels of commodity prices remain stable throughout the study period, of if they indicate significant structural changes. This is aimed to add to the conclusions of Zaremba et al. (2021), who found no structural changes in the 150 year sample period considered in their research.

1.2. Relevance

The research presented in this thesis holds significant relevance from both practical and academic perspectives. From a practical standpoint, the findings of this study can contribute to portfolio construction of institutional investors, helping them to make informed investment decisions. By understanding the macroeconomic determinants of commodity price co-movement, investors can better manage their portfolios and mitigate risks associated with commodity investments by being able to anticipate changes in co-movement. For instance, this would enable investors to foresee periods of high commodity price co-movement and adapt positions in commodity investments. The implications of co-movement for investors are important, since two correlated assets having high co-movement tend to move together in price. By predicting co-movement, investors can invest in assets that are not highly correlated with each other and may thus offer diversification benefits. Conversely, when two assets do have high co-movement, investing in both may not be beneficial for diversification purposes. For instance, Algieri et al. (2021) found that investing in both gold and silver during their sample period would be redundant. Hence, co-movement is important because it can affect portfolio diversification strategies, hedging techniques, as well as risk management decisions. Although it can for instance help to avoid overdiversification, it must be noted that low co-movement does not necessarily imply low risk, as the assets may be highly volatile on their own. (Gerber et al., 2022)

Academically, this research addresses a notable research gap, consisting of two crucial aspects: Firstly, it incorporates new data, capturing the full impact of the Covid-19 pandemic. More importantly however, this research explores the comparison of old and new methods for investigating commodity price co-movement. Pearson's correlation coefficients as an established co-movement measure have been widely used in previous studies to investigate co-movement levels of commodity prices. However, the introduction of Gerber statistics as a new co-movement measure offers an innovative approach to analyzing commodity price relationships, and was shown to provide certain benefits compared to Pearson's correlations (Gerber et al., 2022). Hence, is a direct comparison of the two needed in the context of commodity prices. Additionally, this research employs a Random Forest machine learning model for regression purposes, in contrast to traditional models used to investigate commodity price co-movement. Random Forests may hereby offer certain advantages over traditional, linear models due to for instance their abilities to capture nonlinearities in the data (See e.g., Schonlau & Zou, 2020). Such a comparison of methodologies provides a comprehensive

evaluation of the effectiveness of Random Forests in understanding the determinants of commodity price co-movement.

Hence, does the research presented in this thesis aim to give insights into the ability of Random Forest regressions to explain commodity price co-movement with macroeconomic variables. Thereby, their performance is compared to multiple regressions, VAR models, and VARX models. This is done considering pairwise averages of both traditional Pearson correlations as well as Gerber statistics over a 6-month rolling window. Hence, insights are also given about the performance of methodologies applying Gerber statistics to the topic. The study focusses on recent data, with monthly observations from mid-2003 to early 2023 and incorporates various potential determinants of co-movement levels. These macroeconomic variables include proxies for demand and supply, interest rates, exchange rates, inflation, and market volatility measures for uncertainty. The findings of the research presented in this paper indicate that the use of Gerber statistics over Pearson correlations helps in improving the accuracy of VAR and VARX models in predicting commodity price co-movement. Random Forest models on the other hand, performed worse than these linear models, only showing slightly better performance metrics than multiple regressions. Moreover, did the findings show that both the EUR/USD foreign exchange rate as well as commodity market volatility seem to be of importance in explaining commodity price co-movement.

The rest of this paper is organized as follows: Section 2 defines co-movement for this study and reviews the existing literature on the topic, while Section 3 specifies the methods applied. The data used and the exact application of said methodologies is explained in Section 4, while the results are provided in Section 5. Lastly, the findings are interpreted in the discussion of Section 6, while Section 7 puts them into context and offers concluding remarks.

2. Literature review

To illustrate the study selection process for the literature review conducted, Figure 4: Literature selection process presents six stages of filtering and searching for related academic articles. This was done in the style presented in Kumar et al. (2021) and can, together with a description, be found in Appendix A1: Study selection process. The literature review is structured in the following way: first, co-movement in the context of this study is defined. Consequently, the existing literature on the historical development of commodity price co-movement, and on its determinants is reviewed. Lastly, an overview of the different methods used in highly related past research is given.

2.1. Co-movement definition

In the literature on commodity price co-movement and co-movement in general, there is a lack of a universally used definition of co-movement. This results in only implicit definitions being used in many cases, yet, some papers aimed at providing a clearer definition. One of these papers is the work of Baur (2003) titled "What is Co-movement?". There, it is stated that co-movement refers to the degree to which two or more securities or financial assets move together in a specific period. Specifically, co-movement describes the degree to which two or more financial instruments move together in the same direction at the same time. It was defined in the given paper as the following:

> **Definition 1:** "Co-movement is the common movement of returns that is shared by all returns at time t". (Baur, 2003, p. 5)

Hence, when co-movement is mentioned in financial literature it is oftentimes referring to price co-movement, since asset prices are the basis for returns upon which a degree of co-mevement can be determined (see, e.g., Byrne et al., 2013; de Nicola et al., 2014; Janzen & Smith, 2012). Thus, does price co-movement refer to the extent to which the prices of two or more assets are moving in the same direction at the same time. It is a measure of the correlation between the price returns of two (or more) assets, with Pearson's correlation coefficients being a common measure of price co-movement and ranging from -1 to 1. A correlation of 1 means that the two assets move perfectly in tandem with each other and have perfectly positive co-movement, while a correlation of -1 means that the two assets move in perfectly opposite directions to each other and thus have perfectly negative co-movement. A correlation of 0 means that the two assets have no relationship to each other in terms of price movements and hence no co-movement is present. (Baur, 2003)

In mathematical terms it can thus be stated that, based on Pearson's correlation coefficients, the bivariate co-movement of two return series x and y behaves as follows:

Definition 2: For return series x and y, it can be said that:

1. (x, y) exhibits positive co-movement if $0 < \rho_{xy} \leq 1$,
2. (x, y) exhibits negative co-movement if $0 > \rho_{xy} \geq -1$,
3. (x, y) exhibits no co-movement if $\rho_{xy} = 0$,

   Whereby $\rho_{xy} = \frac{\Sigma_i[(x_i - \bar{x})(y_i - \bar{y})]}{\sigma_x \sigma_y}$, for all observations i, with

   $\bar{x}$ and $\bar{y}$ denoting means of the return series,

   $\sigma_x = \sqrt{\frac{\Sigma_i(x_i - \bar{x})^2}{N}}$ and $\sigma_y = \sqrt{\frac{\Sigma_i(y_i - \bar{y})^2}{N}}$ denoting standard deviations, and

   N denoting the total number of observations i.
   (see e.g., Ai et al., 2006; Baur, 2003; Zaremba et al., 2021)

Similarly, to conventional correlation coefficients such as Pearson´s correlation coefficients, other measures of co-movement (or correlation for that matter) may be used. The literature on commodity price co-movement hereby used a variety of established measures and novel co-movement measures to avoid drawbacks associated with certain measures. These are elaborated on in Section 3.1, where the co-movement measures used as part of this research are explained in more detail. However, a main limitation of many co-movement measures is that they are inherently static, and their computations are based on a specific period or point in time. To cope with this issue, time-varying co-movement was introduced by the literature. Hereby, time-varying (price) co-movement is a more sophisticated form of co-movement since it considers how the degree of co-movement between two or more asset prices changes over time (Algieri et al., 2021). One common method used to study time-varying co-movement is the use of rolling co-movement measures. Thereby, rolling correlation coefficients measure the correlation between two assets over a sliding window of time. Thus, they can help to identify trends in the co-movement between two assets and provide insights into the stability of their relationship over time. (Algieri et al., 2021; Le Pen & Sévi, 2017)

In Baur (2003) it is furthermore distinguished between "normal" and "extreme" co-movements which can be seen as excessive. In the literature, these extreme co-movements are thus oftentimes referred to as "excess co-movement". The study of Pindyck and Rotemberg (1990) investigated excessive co-movements in the context of commodity prices and gave further explanations to the definition of excess co-movement. There it is found that excess co-movement in the context of commodities refers to the phenomenon where prices of a diverse range of commodities move together, with little to no direct relation in terms of for instance the cross-price elasticities of demand and supply effects. This co-movement cannot be entirely attributed to common factors like inflation, aggregate demand fluctuations, interest rates, or exchange rates. Contrary to this, is argued that "normal" co-movement can indeed be explained by these underlying common factors. The research of Pindyck and Rotemberg (1990) has furthermore shown that latent variables, such as unobserved forecasts for inflation and industrial production, play a significant role in explaining commodity price movements. Nevertheless, even when accounting for these factors, there remains an unexplained degree of excess co-movement in their study. One potential explanation for excess co-movement is hereby the dependence of results on the length of the holding period. The paper has found that the amount of unexplained co-movement increases with a longer holding period. This is said to be possibly due to the exclusion of relevant macroeconomic variables that impact commodity

prices slowly over time. For instance, an "unusual monthly change in inflation might have to persist for some time before it affects perceptions about the future" and hence prices and co-movement levels (Pindyck & Rotemberg, 1990, p. 1186). These slowly acting effects of macroeconomic variables align with the observation that they explain more of the movements in commodity prices over extended holding periods. In consequence, the exclusion of any slow-acting macroeconomic variable would result in an increase in unexplained co-movement as the holding period lengthens (Pindyck & Rotemberg, 1990). Hence, are there implications on the lenght of the holding period for investors, as well as the length of time-varying co-movement measures and data frequency for researchers aiming to minimize unexplained excess co-movement of commodity prices.

Hereby must be stated however, that the existence of excess co-movement of commodity prices is contested in the litereature. While there are studies supporting the findings of Pindyck and Rotemberg (1990) (e.g., Le Pen & Sévi, 2017), other studies were able to explain a higher proportion of co-movement, resulting in close to no unexplainable, excessive price co-movements (e.g., Deb et al., 1996). As an example, did the study of Ai et al. (2006) investigate whether commodity price co-movements are indeed excessive as indicated by Pindyck and Rotemberg (1990). Thereby, they concluded that the co-movements should not be considered as excessive since the vast majority can be explained by their models.

2.2. Historical commodity price co-movement

The historical development of commodity price co-movement has been a subject of interest for researchers, especially in recent years given its considerable developments in the early 21st century (See Figure 1). According to Zaremba et al. (2021), the co-movement of commodity prices experienced a significant increase in recent years which may be due to commodity financialization. However, this increase in co-movement peaked in the 2000s and 2010s, and no significant structural increase was observed across two centuries in their sample period from 1850 to 2019. In fact, after reaching elevated levels in the early 2010s, the correlations measured in Zaremba et al. (2021) returned to the long-run average in more recent years. Interestingly, the peak in commodity price correlations during the early 2010s was not exceptionally high compared to historical levels. Pearson's correlation coefficients displayed similar levels in the 1860s, 1920s, 1930s, and 1940s. A closer examination of the results suggests that spikes in co-movement coincided with major crises and disruptions in the global economy, such as the American Civil War, the Great Depression, World War II, and the global financial crisis. Thus, the increase in commodity dependence in the 2000s to 2010s may simply reflect effects of major economic and financial disruptions linked to the 2008 financial crisis, rather than a structural change linked to commodity market financialization. (Zaremba et al., 2021)
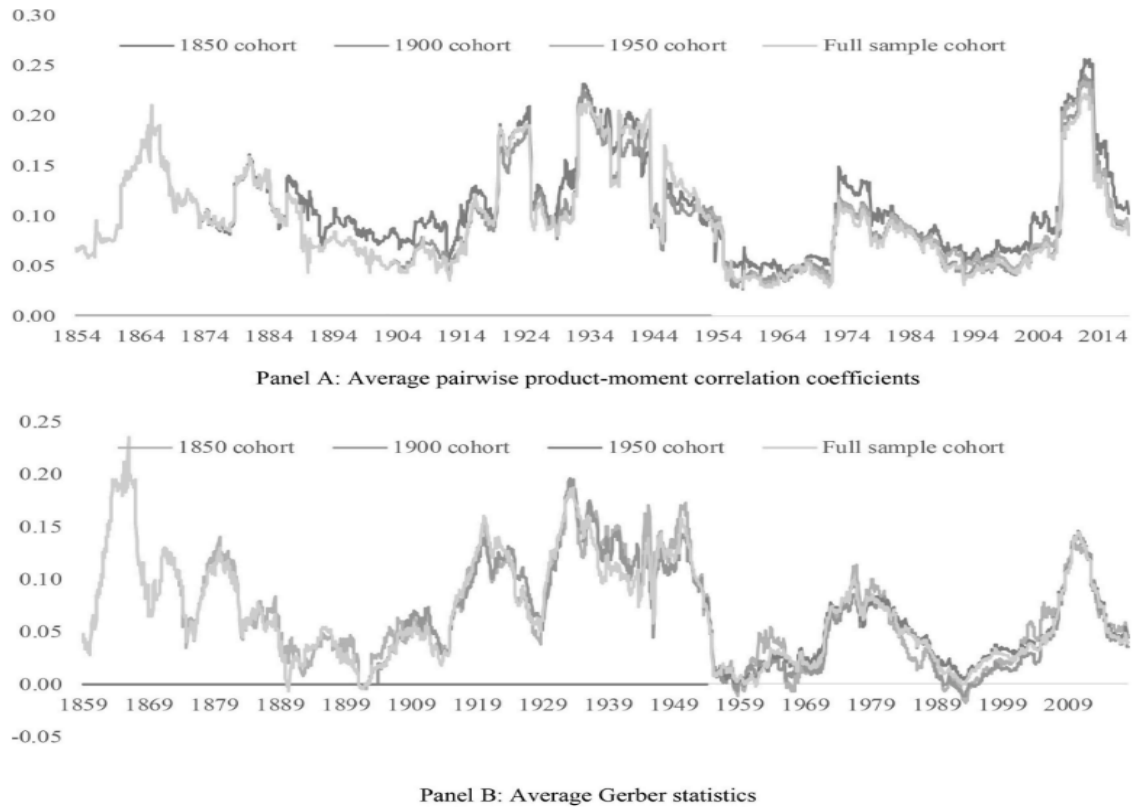
Panel A: Average pairwise product-moment correlation coefficients

Panel B: Average Gerber statistics

Figure 1: Historical commodity price co-movement levels (Zaremba et al., 2021, p. 3)

Figure 1 was taken from the paper of Zaremba et al. (2021, p. 3) and illustrates their findings described in the previous paragraph. It depicts historical commidity price co-movement levels during their sample period based on the two co-movement measures also used in the research presented in this thesis. In line with these findings, the study of Algieri et al. (2021) found evidence of cross-commodity return co-movements as well as evidence of changes in these co-movement levels over time. There, it was also stated that extreme market distress tends to lead to rising levels of commodity co-movement. This supports the findings of Zaremba et al. (2021) and moreover is consistent with the argumentation of for instance Bekaert et al. (2005) and Dornbusch et al. (2000), where it is argued that prices of seemingly unrelated assets may start to move together during crisis periods. Additionally, Alquist et al. (2020) found the highest degrees of co-movement in commodity prices occurring between 1973 and 1975, the early to mid-1980s, the late 1990s, and the mid- to late 2000s continuing to 2013. The time variation in co-movement was primarily explained by changes in the indirect factor (which will be elaborated on in Section 2.3.2.). Thereby, periods with strong co-movement coincided with those where commodity price changes were driven by the "endogenous response of commodity prices to non-commodity shocks" (Alquist et al., 2020, p. 2). These non-commodity shocks are likely to be for instance the consequences of economic disruptions and crises, as was also indicated by Zaremba et al. (2021) and Algieri et al. (2021).

Hereby is notable that the periods of exceptionally high commodity price co-movements identified by Alquist et al. (2020) and Zaremba et al. (2021) do not perfectly align. This can be attributed to different time periods being investigated, with some periods of especially high co-movement identified by Zaremba et al. (2021) not being included in the sample of Alquist et al. (2020). Moreover, these differences might be due to different commodities being used in

the respective analyses. This would be in line with the findings of Algieri et al. (2021), where it was found that co-movement levels differ between commodity pairs and that they are more noticeable for some pairs (e.g., wheat–corn, corn–soybeans, silver–gold, gold–oil). Further supporting this, did Cai et al. (2019) find that the degree of co-movement differs between commodity sectors. Thereby, several commodity sector pairs showed rather strong co-movement[1], while on the other hand, the pairs of agriculture, livestock, and precious metals only showed weak co-movement patterns. Hence, does this underline the importance of commodity selection for investigating co-movements of commodity prices. Furthermore, did Byrne et al. (2013) find that commodity sectors also react differently to macroeconomic shocks. These shocks include various influential determinants of commodity prices and their co-movement, which are elaborated on in the following section.

2.3. Determinants of commodity prices and their co-movement

Adding to the historical development of co-movement of commodity prices, research has investigated potential sources or determinants of the variation in the levels of co-movement. Hereby, was distinguished between "direct" supply and demand factors on the one hand, and "indirect" factors on the other in the FAVAR framework of Alquist et al. (2020). There, direct factors represent commodity-related shocks and hence factors directly shifting the supply and demand curves of commodities and thus their prices. Conversely, indirect factors do not directly shift supply and demand curves and thus only indirectly affect prices. This is said to occur for instance through their effects on aggregate output (Alquist et al., 2020). Although no factors were extracted for factor analysis purposes, this distinction was also made in the research presented in the following sections to classify potential determinants of commodity price co-movement.

2.3.1. Direct factors

Firstly, direct factors influencing the supply and demand of commodities are reviewed. They have been shown to have influential effects on the pricing of commodities, and hence on the amount of co-movement through changes in these prices (Ai et al., 2006; Byrne et al., 2013). As early as Pindyck and Rotemberg (1990) it was argued that both supply and demand factors have a major role in determining the prices of commodities and their co-movement. Thereby however, changes in supply and demand were only approximated into one index in the model. Later research on this topic compared supply and demand factors, such as the paper of Ai et al. (2006) where agricultural and energy commodities were investigated. Thereby, the findings suggested supply factors to have a greater influence on co-movement compared to demand factors. The framework of Alquist et al. (2020) furthermore suggests, that the impact of supply factors on commodity prices and consequently price co-movement is dependent on the substitutability between commodities. Thereby, for non-substitutable commodities, supply factors can have significant price implications, while contrary to this, supply factors may only have a limited influence for highly substitutable commodities. Moreover, the authors gave examples of direct factors, with supply factors being for instance productivity as well as input prices, such as labor and energy. Demand factors on the other hand are for example derived

---

[1] agriculture - energy, agriculture - industrial metals, energy - industrial metals, industrial metals - precious metals, energy - livestock, energy - precious metals, and industrial - livestock

from the relative need of commodities for the production of consumption goods. (Alquist et al., 2020)

To account for supply and demand factors, different approaches were taken in past literature. For instance, did Ai et al. (2006) use U.S. prices of commodities, inventory levels, and data on harvests or yields to fit an equilibrium model accounting for both supply and demand factors to commodity prices. Similarly, Bakas and Triantafyllou (2020) controlled for both aggregate and commodity specific demand using commodity price returns in the VAR model presented. Moreover, were proxies of both supply and demand used in the literature, with recent examples being the works of Byrne et al. (2013, 2019). There, the real U.S. GDP growth rate was used to proxy for demand effects, while real crude oil prices proxied for supply effects, since they (partly) represent energy input costs. There, both factors appear to positively relate to the common factor of commodity prices. Moreover, is noteworthy that the remaining results investigating indirect macroeconomic indicators are robust to their inclusion (Byrne et al., 2013). In a subsequent article by Byrne et al. (2019), U.S. industrial production was used as a proxy for demand instead of the U.S. GDP growth rate. Supply factors, on the other hand, were for instance also proxied for using energy indices by other authors (Poncela et al., 2014).

Lastly, must be stated that Alquist et al. (2020) pointed out that direct factors may have additional indirect effects on commodity prices through for example their general-equilibrium effects on aggregate income. Similarly, indirect factors are said to also influence commodity prices through either the standard "demand channel" or "supply-side channel" (Alquist et al., 2020, p. 42). This underlines the importance of also considering indirect factors, which make up the second group of influential determinants of commodity prices and their co-movement and are elaborated on in the following section.

2.3.2. Indirect factors

As indicated previously, indirect factors do not directly influence the supply and demand equilibrium, but still have important implications for commodity co-movement through their influence on prices. Hence, do indirect factors refer to the broader economic and financial conditions that influence commodity pricing. These include for instance interest rates, currency fluctuations, and financial and economic developments among others.

The paper of Alquist et al. (2020) found that the "primary source of commodity price movements is their endogenous response to non-commodity-related shocks" (Alquist et al., 2020, p. 2). Thus, was argued that most historical commodity price movements can be attributed to indirect factors, as changes or shocks in indirect factors lead to corresponding supply and demand equilibrium responses. Thereby, the indirect common factor accounted for about 60-70% of the variance in commodity prices in the model presented in Alquist et al. (2020). Nevertheless, there were several periods identified where direct factors at least had some influence on movements of commodity prices, with the most notable being around 1979-1980, the early 2000s where commodity prices increased, and 2008-2009 where commodity prices declined. Moreover, as mentioned earlier, were two channels of indirect factors affecting commodity prices identified in Alquist et al. (2020): Firstly, the standard demand channel has effects on commodity prices since it is argued that, for example, high aggregate economic activity leads to a high demand for commodities used in production. This, in return, raises the prices of the respective commodities. Moreover, it is argued that the supply-side channel also

raises commodity prices when aggregate income is high. The reasoning behind this is that income effects lead to a lower relative supply of commodities. This is since suppliers may be less willing to provide the inputs for the production of commdities, consequently raising prices. Thus, indirect factors should usually have both supply and demand side effects on commodity pricing. Additionally, it can be stated that both channels induce positive co-movement of commodity prices, as their effects described above can either influence prices to rise or to fall. (Alquist et al., 2020)

Several macroeconomic and financial factors considered to be "indirect" were shown to impact both the prices of commodities and their co-movement in recent literature. Hereby, different argumentations were made. For instance, Byrne et al. (2013) found significant cross-commodity price co-movement in their sample of 24 commodities. There, the research shows that this co-movement has a negative relationship with the macroeconomic variables of interest rates and stock market uncertainty within their FAVAR model. More precisely, they found that interest rate shocks have a significantly negative effect that is absorbed over a five-year period following the shock. This was further supported by a consequent study, which also showed significant negative effects of interest rate shocks on commodity co-movement (Byrne et al., 2019). The paper of Gruber and Vigfusson (2018) further supports this as it provides evidence of declining interest rates positively affecting commodity price co-movement. Contrary to these findings however, did the macroeconomic model of Ai et al. (2006) show no statistical significance for the effects of interest rates and only explained a comparably small proportion of the variance in commodity price returns. Hence, are the effects of interest rates on commodity price co-movement still somewhat contested within the literature.

Next to interest rates, the effects of foreign exchange rates on commodity prices and their co-movement were investigated, however, with less emphasis in the literature. Regarding exchange rates, the study of Ai et al. (2006) found no statistically significant evidence of effects on commodity price co-movement, similarly to interest rates. In support of this, did de Nicola et al. (2016) find little to no statistically important effects of both exchange rate and interest rate variations on commodity price co-movement. For exchange rates there was a weak statistical association found, but for interest rates none was detected. More recent research however, found that exchange rate fluctuations are correlated with commodity prices. For instance, Poncela et al. (2014) found exchange rates to have negative impacts on the prices of non-fuel commodities. Another example is the research of Chien et al. (2021), where co-movement between exchange rates and both gold and oil prices was found. There, however, only gold prices showed significance during the period of the Covid-19 pandemic while oil prices did not.

Additionally, uncertainty, as captured by the volatility of financial asset prices, was also employed to model commodity prices and their co-movement. Thereby, Byrne et al. (2013) have found a negative relationship between stock market uncertainty and commodity prices. It was shown that increases in volatility result in a significant, immediate decrease of commodity prices. This decrease is said to hold for about four years following the increase in risk as measured by stock market volatility. Adding to this, did Byrne et al. (2019) also detect a negative relationship between risk and commodity prices, with the effect being the most pronounced from mid-2007 on and the strongest in late 2008. Moreover, de Nicola et al. (2016) have found a positive association between stock market uncertainty and the co-movement of

commodity price returns. This, combined with the earlier mentioned findings of a negative relationship between stock market volatility and commodity prices, would imply that as volatility in stock markets rises, various commodity prices tend to fall, thereby exhibiting positive co-movement.

Although it can be considered an indirect factor since it does not directly influence the supply-demand equilibrium, inflation plays a special role in determining commodity price co-movement. To account for this, past literature oftentimes used the U.S. Consumer Price Index (CPI) to deflate commodity prices before applying methods to model them with explanatory variables. Examples of this are the studies of Ai et al. (2006), where commodity prices were CPI-deflated before their logarithmic returns are regressed on contemporary and lagged indicator variables, and Byrne et al. (2013) where the extracted principal component is deflated using the U.S. CPI. The same was applied in more recent studies as well, such as for instance in Alquist et al. (2020) where price series are normalized with regards to inflation. Moreover, did Byrne et al. (2019) not only deflate commodity prices, but also used the U.S. CPI to realize explanatory variables such as interest rates.

To sum up, the main determinants of the co-movement of commodity prices are summarized on the right, whereby two main categories of direct factors and indirect factors can be distinguished. Here, direct factors are represented by for instance supply and demand proxies, while indirect factors include e.g., interest rates, exchange rates, and uncertainty. Inflation is also considered to be an indirect factor but has additional, crucial effects in determining commodity prices and co-movement. In response, other variables may need to be adjusted to it.

Direct factors
i. Supply proxies
ii. Demand proxies
Indirect factors
iii. Interest rates
iv. Exchange rates
v. Uncertainty
vi. Inflation

Figure 2: Determinants of commodity price co-movement

2.4. Methods used in past literature

To conclude the literature review, an overview of the different co-movement measures and models used in highly related literature is given. The full overview can be seen in Appendix A2: Literature comparison, together with information on validation methods, datasets, commonly used variables, and model fit measures where applicable.

Regarding co-movement measures, different methods were used to capture commodity price co-movement. Thereby, both Pearson correlations (Ai et al., 2006; Cai et al., 2019; Zaremba et al., 2021) and R^2 measures are commonly seen. In the context of R^2 measures, excess co-movement is investigated in many cases (e.g., Pindyck & Rotemberg, 1990), but it was also used to capture overall price co-movements (Alquist et al., 2020; Zaremba et al., 2021). Moreover, dynamic conditional correlations (DCC) were used in some cases (de Nicola et al., 2014; Le Pen & Sévi, 2017), while in other papers commodity price co-movement was derived from extracted factors (Byrne et al., 2019; Poncela et al., 2014). Here is notable, that also Gerber statistics, which are used in the research presented in this thesis, show some uses in recent literature on the topic (Algieri et al., 2021; Zaremba et al., 2021). This diversity in co-movement measures underlines the lack of a universal definition of co-movement in related financial literature, as discussed earlier.

Similarly, a variety of different models were applied in past research to investigate the co-movement of commodity prices. Firstly, various types of regressions were used (Ai et al., 2006; Le Pen & Sévi, 2017; Pindyck & Rotemberg, 1990; Zhang et al., 2019). Moreover, did several authors use VAR models (Bakas & Triantafyllou, 2020; de Nicola et al., 2014; de Nicola et al., 2016; Janzen & Smith, 2012), oftentimes in combination with factor extraction as FAVAR models (Alquist et al., 2020; Byrne et al., 2019; Poncela et al., 2014). Additionally, it is noteworthy, that DCC models were also used in several cases (See Appendix A2: Literature comparison). Resulting from this, it can be stated that multiple regressions and VAR model types form a basis for evaluating the performance of novel methodologies in the context of commodity price co-movement. The same holds true for Pearson correlations, especially considering its comparability to Gerber statistics (See e.g., Zaremba et al., 2021).

3. Methods

This section presents the two co-movement measures and four types of models used in this research to investigate commodity price co-movement. The methodologies considered to be established are hereby Pearson's correlation coefficients and multiple regressions, reduced VAR models, and reduced VARX models. On the other hand, the methodologies that are considered novel to research on commodity price co-movement are Gerber statistics and Random Forest models used for regression purposes. Here these methods are specified, while their exact application to the research of this thesis is presented in Section 4.3.

3.1. Co-movement measures

To measure the magnitude of co-movement, which essentially refers to correlations between the return series of commodity prices considered, pairwise Pearson's correlations are used. These are defined as follows for return series x and y at time t for all observations i.

$$\rho_{xy,t} = \frac{\sum_{i=1}^{t}[(x_i - \bar{x})(y_i - \bar{y})]}{\sigma_x \sigma_y} \tag{1}$$

Thereby, σ_x and σ_y denote the respective standard deviations for return series x and y during period ($i=1, t$). Similarly, $\bar{x}$ and $\bar{y}$ denote the average or mean values observed for each series during the period considered. (Also see Definition 2)

However, recent research introduced a novel measure of co-movement that is said to provide certain benefits over traditional Pearson correlations. This so-called Gerber statistic was shown to be superior to Pearson's correlation coefficients for capturing characteristics of financial time series, as it is less distorted by for instance volatility clustering, leptokurtosis, as well as outliers. This is the case since it is insensitive to both extremely large co-movements and noise commonly found in financial time series. (Gerber et al., 2022)

For the purposes of this research, the Gerber statistic of return series x and y for a sample of t observations i is defined as presented in Algieri et al. (2021):

$$g_{xy,t} = \frac{\sum_{i=1}^{t} m_{xy}(i)}{\sum_{i=1}^{t} |m_{xy}(i)|} \tag{2}$$

Where:

$$m_{xy}(i) = I\left(r_{x,i} \geq Q_x\right) I\left(r_{y,i} \geq Q_y\right) + I\left(r_{x,i} \leq -Q_x\right) I\left(r_{y,i} \leq -Q_y\right)$$
$$- I\left(r_{x,i} \geq Q_x\right) I\left(r_{y,i} \leq -Q_y\right) - I\left(r_{x,i} \leq -Q_x\right) I\left(r_{y,i} \geq Q_y\right)$$

(See Algieri et al., 2021, p. 340)

Hereby, I is an indicator variable that is equal to 1 if the condition in the brackets following it is met, and equal to 0 otherwise. r_i is the observed return of its respective return series at observation i, while Q denotes the respective Gerber threshold. Resulting from this, $m_{xy}(i)$ is equal to 1 if both returns exceed their threshold simultaneously at observation i; is equal to -1 if both returns exceed their threshold in opposite directions simultaneously at observation i; and is equal to 0 if at least one absolute value of returns does not exceed the respective threshold. These thresholds are further defined in Section 4.3.

3.2. Multiple Regression

Firstly, to create base models for purposes of comparison, multiple regressions are specified. Regressions were deemed to be an appropriate model for comparison in this context because of their use as early as Pindyck and Rotemberg (1990), and their continued use in e.g., the models of Ai et al. (2006). They are specified as follows:

$$y_t = \alpha_0 + \beta_1 x_{1,t} + \cdots + \beta_n x_{n,t} + \varepsilon_t \tag{3}$$

Thereby, y_t refers to the dependent variable at time t, α_0 refers to the intercept in the model, while $x_{1,t}$ to $x_{n,t}$ denote the n number of independent variables at time t. β_1 to β_n are the estimated coefficients of each respective independent variable. Lastly, the error term is given as ε_t.

3.3. VAR model

Moreover, a reduced VAR model is applied and specified in this section. Hereby, VAR models were chosen to be appropriate for model comparison purposes because of their prominent use in related literature (See Section 2.4). In matrix notation the reduced VAR(p) models used in this research is given as follows:

$$Y_t = C + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t \tag{4}$$

Hereby, Y_t denotes a vector of endogenous variables included in the model at time t, while A_p is a matrix containing coefficients of lag p and Y_{t-p} is a vector containing the respective lagged variables. For each of the variables, the respective equations are defined as given here for y_{1t}:

$$y_{1t} = c_1 + \sum_{l=1}^{p} \sum_{j=1}^{n} a_{1j,l} y_{j,t-l} + \varepsilon_{1t} \tag{5}$$

$y_{j,t-l}$ are thereby the endogenous variables of lag l and $a_{1j,l}$ the respective coefficients. c_1 denotes the first equation's intercept, ε_{1t} the first equation's error term, and y_{1t} the endogenous variable being predicted by the equation at time t.

3.4. VARX model

Similarly, to the reduced VAR models from Section 3.3, reduced VARX models are introduced which include exogenous variables. Thereby, only one additional variable is used as an endogenous variable in the model along with the variable of interest to maintain a VAR structure, while all other predictor variables are included as exogenous variables. Hence, can the VARX model in matrix notation be defined as follows:

$$Y_t = C + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + B_0 X_t + B_1 X_{t-1} + \cdots + B_p X_{t-p} + \varepsilon_t \tag{6}$$

Hereby, Y_t again denotes the matrix of current values of the endogenous variables, Y_{t-p} their p^{th}-lagged values, and A_p the estimated coefficients at lag p. Similarly, matrix B_p denotes the coefficients of the exogenous variables at lag p while X_{t-p} denotes their respective lagged values. However, unlike in the VAR model of the previous section, the current values of exogenous variables, X_t, are included in the model along with their coefficient estimates B_0.

Resulting from this, the equation predicting the variable of interest, $y_{1,t}$, is given as the following:

$$y_{1t} = c_1 + \sum_{l=1}^{p} a_{11,l} y_{1,t-l} + \sum_{l=1}^{p} a_{12,l} y_{2,t-l} + \sum_{l=0}^{p} \sum_{j=1}^{n} b_{1j,l} x_{j,t-l} + \varepsilon_{1t} \qquad (7)$$

Adding to the definitions of terms given earlier for the VAR model, $a_{11,l}$ and $y_{1,t-l}$ denote the first endogenous variable's coefficient and value at lag l, while $a_{12,l}$ and $y_{2,t-l}$ denote the second endogenous variable's coefficient and value at lag l. Similarly, $b_{1j,l}$ and $x_{j,t-l}$ denote the coefficients and lagged values of the n number of exogenous variables. Hereby, however, current values are included.

3.5. Random Forest model

Next to the models presented, Random Forest regression models were applied. These Random Forest models can be specified as follows, loosely based on Breiman (2001): Firstly, for each of K number of trees at each point in time t, there are N randomly selected samples drawn from the original data D_t with replacement to create bootstrap data $D_t^{(k)}$.

$$D_t^{(k)} \subset D_t, \qquad k = 1,2,\dots,K; \qquad t = 1,2,\dots,T \qquad (8)$$

Based on these bootstrap datasets, decision trees are constructed on each one of them: $h_t^{(k)}(x_t, \theta^{(k)})$. At each node in these trees, m out of the total of M features are considered for splits. As the model is built for regression purposes, these features and split rules are determined to maximize the variance reduction. This process is repeated, letting the tree "grow" and creating and splitting new nodes, until the nodes contain less than the minimum number of observations to be split any further. This minimum number of observations to split nodes is hereby referred to as n. Consequently, the Random Forest prediction for a new data point, $\hat{y}_t(x_t)$, describing the target variable at time t with predictor variables x_t is obtained by averaging the ensemble of predictions of individual trees $h(x_t, \theta^{(k)})$:

$$\hat{y}_t(x_t) = \frac{1}{K_{OOB}} \sum_{k \in OOB} h(x_t, \theta^{(k)}) \qquad (9)$$

Since it is not spit into test and training sets, K_{OOB} is the number of trees where $\hat{y}_t(x_t)$ is an out-of-bag (OOB) sample. Averaging the predictions of K_{OOB} number of trees hence results in the Random Forest's prediction $\hat{y}_t(x_t)$. Resulting from this, the true observed value $y_t(x_t)$ in relation to the prediction $\hat{y}_t(x_t)$ and error term ε_t is specified along with the Mean Squared Error, MSE. They are defined as:

$$y_t(x_t) = \hat{y}_t(x_t) + \varepsilon_t , \qquad (10)$$

and

$$MSE = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \qquad (11)$$

This definition of the error term and MSE is crucial since the MSE is oftentimes used as the loss function in hyperparameter tuning, and as a performance metric to evaluate the Random

Forest models' accuracy and compare it to other models. Hyperparameter tuning hereby refers to the optimization process of so-called model hyperparameters. In the Random Forest regressions defined above, these are K, the number of trees, m, the number of splits considered at each node, and n, the minimum number of observations needed for splits, although there are several others one might consider.

Moreover, two distinct variable importance measures (VIMs) can be used to evaluate the effects of predictor variables on the target variable. Firstly, an impurity-based VIM is specified and secondly a permutation-based VIM. Impurity importance can be defined as follows: *"For the impurity importance, a split with a large decrease of impurity is considered important and as a consequence variables used for splitting at important splits are also considered important."* (Nembrini et al., 2018, p. 3712) In the Random Forest regression models specified before, this impurity measure refers to the variance of the target variable's observations included in the sample of the respective node. Hence is *MDI*, the "mean decrease in impurity", for predictor variable *j* defined as:

$$MDI_j = \frac{1}{K} \sum_{k=1}^{K} \Delta Var_j^{(k)} \tag{12}$$

There, $\Delta Var_j^{(k)}$ refers to the impurity (or variance) decrease for all nodes of tree k and predictor variable *j*. Additionally, permutation VIMs can be used to further evaluate the effects of predictor variables in the models. The computation of permutation VIMs was hereby defined in recent research as the following: "To calculate the permutation importance of the variable X_i, its original association with the response Y is broken by randomly permuting the values of all individuals for X_i. With this permuted data, the tree-wise OOB estimate of the prediction error is computed. The difference between this estimate and the OOB error without permutation, averaged over all trees, is the permutation importance of the variable X_i." (Nembrini et al., 2018, p. 3712) Hence, for the Random Forest regression models specified above, where MSE_{OOB} is used as the prediction error, the "permutation decrease in impurity", *PDI,* for predictor variable *j* can mathematically be defined as:

$$PDI_j = \frac{1}{K} \sum_{k=1}^{K} (MSE_{OOB}^{(k)} - MSE_{OOB,\ permute\ (j)}^{(k)}) \tag{13}$$

There, $MSE_{OOB}^{(k)}$ is the OOB-prediction error of tree k, and $MSE_{OOB,\ permute\ (j)}^{(k)}$ the OOB-prediction error of tree k with predictor variable *j* being permuted. For more details on the exact mechanics behind Random Forest models, please refer to Breiman (2001) and the user manual of `ranger`, the R package used for model implementations.[2]

[2] Wright, M. N., Wager, S., Probst, P. (2023, April 3). *Ranger: A fast implementation of random forests.*
    The Comprehensive R Archive Network. https://cran.r-project.org/web/packages/ranger/ranger.pdf

4. Experimental setup

The data to which the models presented in Section 3 are applied to can be split into two distinct parts: firstly, the price series of the 20 commodities that are used to calculate co-movement measures; and secondly, data on the macroeconomic determinants that may be assumed to be influential according to the literature review. These price series and macroeconomic indicators are collected for the sample period from 1st of July 2003 to 1st of March 2023. This sample is furthermore split into two parts: Subsample 1 including data up to the end of July 2013 and subsample 2 including data from August 2013 on.

4.1. Commodity price data

For commodity price series, monthly prices of continuous futures were collected from the Refinitiv Eikon database. The basis for the selection of commodities was hereby the combined list of commodities considered in the research of Alquist et al. (2020) and Zaremba et al. (2021). Moreover, several restrictions were put on the selection of commodities from that list, similarly but distinctly to the selection in Alquist et al. (2020). Thereby, commodities that are too vertically integrated were excluded. To give an example of this, including both soybeans and soybean meal would not be beneficial when investigating co-movement levels, as soybeans are used as the direct input in the production of soybean meal. Nonetheless, several commodities excluded in the research of Alquist et al. (2020) were included in the selection of commodities in this research. Examples of this are commodities that are primarily used for financial speculation purposes, such as several precious metals including gold and silver. To keep the commodity selection realistic and close to the investment profiles of institutional investors, the commodities considered in past literature are compared to those in AQR's Risk-Balanced Commodity Strategy Fund as of March 1st, 2023[3]. The commodities included in the final selection are all included in the Risk-Balanced Commodity Strategy Fund with two exceptions: Palladium and rubber.

Additionally, an emphasis was made to include commodities from different sectors to enable a cross-sector comparison as part of the analyses presented in Section 5. This resulted in an equal amount of hard and soft commodities being included in the selection of 20 commodities. Regarding hard commodities, four price series from the energy sector were included. These are: Brent crude oil, light sweet crude oil, natural gas, and heating oil. For the base metal sector, the continuous futures of copper and aluminum were collected. Lastly, to represent the precious metal sector, the price series of gold, silver, platinum, and palladium were included. For soft commodities, the prices of wheat (soft red winter wheat) and corn represent grains, while for the livestock sector lean hogs and live cattle are included. Moreover, are prices of oilseeds in forms of soybeans, and fibers in forms of cotton in the sample. Additionally, rubber, sugar, coffee, and cocoa, are included as other soft commodities. The exact continuous futures collected as price series of these commodities are given in Appendix B1: List of Commodity future continuations.

Prior to the calculations of the two co-movement measures considered, several data transformations were undertaken. Firstly, the price series of brent crude oil, heating oil, copper, and silver were corrected as the raw data from Refinitiv Eikon did not account for changes in

---

[3] AQR Funds. (2023, March 31). *AQR Funds Consolidated Schedule of Investments*.
https://connect.rightprospectus.com/AQRFunds/TADF/00203H180/QH1?site=AQRFunds

currency from U.S. Dollar cents (USc) to U.S. Dollar (USD). This was the case for brent crude oil, copper, and silver, while heating oil was denoted in tenth of a U.S. Dollar cent. Moreover, did the price series of both palladium and platinum show missing values. As this was more pronounced for closing prices compared to opening prices, the latter were used instead. For palladium, this resulted in 8 missing values instead of 32 for closing prices, while for platinum there were 10 missing values in opening prices compared to 25 in closing prices. These missing values were imputed using the "Last Observation Carried Forward" (LOCF) method. (See e.g., Little & Rubin, 2019) Additionally, regarding the price series of aluminum, it was converted to U.S. Dollar within Refinitiv Eikon to keep consistency as the initial data was given in Chinese Yuan. Consequently, prices were deflated using the U.S. CPI Index before their logarithmic returns were calculated. Based on these real returns of the continuous futures of the considered commodities, both co-movement measures were computed like specified in Sections 3.1 and 4.3.

4.2. Determining variable data

Moreover, data on macroeconomic indicators was collected to serve as determinants of commodity price co-movement. These data series can be classified into demand proxies, supply proxies, interest rates, exchange rates, market uncertainty, and inflation. Thereby, the selection of macroeconomic variables was based on the results of the literature review conducted in Section 2. Regarding demand proxies, GDP growth as captured by the Brave-Butters-Kelly Real GDP Index was used since it allows for a monthly data frequency. It was shown by for instance Byrne et al. (2013) that GDP growth can serve as a proxy for demand. Moreover, the index of industrial production was used as a demand proxy, similarly to Byrne et al. (2019). Both indices were thereby taken from the economic database of the Federal Reserve Bank of St. Louis (FRED).

Supply proxies on the other hand reflect shared input prices that producers of different commodities face. To account for this the United States Energy Price Return Index as available on Refinitiv Eikon was used. Similarly, did oil prices serve as a supply proxy in the research on commodity price co-movement of Byrne et al. (2013). These, however, were directly included in the calculation of co-movement levels in the research presented in this thesis, and hence not used as a proxy for energy input prices. Additionally, the producer price index for all commodities was introduced as a novel proxy for supply since it "measures the average change over time in the selling prices received by domestic producers for their output […] from the first commercial transactions"[4]. Hence, does it reflect a variety of shared input prices and supply-side effects faced by various producers of different commodities. Monthly values of the index are hereby available on FRED. For both indices as well as the industrial production index, logarithmic returns were calculated. Since the U.S. Energy Price Return Index is not inflation adjusted by nature, it was deflated using the U.S. CPI Index prior to return calculations.

Regarding interest rates, the monthly market yields on two distinct U.S. T-Bills were collected. Firstly, the yield on U.S. Treasury Securities with a 3-Month constant maturity was used to represent short term interest rates. Moreover, the yield on U.S. Treasury Securities with a 10-

---

[4] U.S. Bureau of Labor Statistics. (2022, February 22). *Producer Price Index Home*. https://www.bls.gov/ppi/

Year constant maturity was used to capture long term interest rates. Again, both series were collected from FRED.

Furthermore, exchange rates were captured using the ICE U.S. Dollar Index, as available on Refinitiv Eikon. The reasoning behind this is that the commodity prices used to determine co-movement levels are also (mostly) given in (and otherwise converted to) U.S. Dollars. Additionally, the foreign exchange spot rates of the U.S. Dollar to the Euro and the Chinese Yuan were collected as well. Both the index as well as the foreign exchange spot rates were collected from Refinitiv Eikon. For the U.S. Dollar Index, logarithmic returns were calculated, while the USD / CNY foreign exchange spot rate was converted to CNY / USD for consistency.

Moreover, uncertainty for both stock markets and commodity markets were used as determinants of commodity price co-movement. This uncertainty was captured by computing the monthly variances of two indices based on their daily prices. For stock markets, historical prices of the S&P 500 Index were used to calculate variances, while for commodity markets the Bloomberg Commodity Index was used. The price series of both indices were hereby taken from Refinitiv Eikon.

Lastly, data on inflation in the United States was needed both to serve as a determinant, and to deflate certain variables. The latter was done using the U.S. Consumer Price Index (CPI), which is also available on FRED. For commodity prices, a similar deflation using the U.S. CPI was done in the research of Ai et al. (2006); Alquist et al. (2020); and Byrne et al. (2013), while additionally several macroeconomic factors were deflated in past research (Byrne et al., 2019). All commodity prices and the U.S. Energy Price Return Index were deflated using the U.S. CPI Index (with a value of 100 in 2015), while the monthly U.S. CPI growth rate was used to deflate treasury bill yields and moreover used as a determining variable in the following analyses.

Table 1: Data sources and transformations

Variable/Factor	Name	Raw data series	Transformation	Data sources
Commodity prices	Pearson, Gerber	20 Commodity continuous future price series	Price-corrections; Deflation using U.S. CPI Index; Log return calculation	Refinitiv Eikon[5]
Demand proxies	GDP	U.S. GDP growth	% conversion	FRED[6]
	IP	U.S. Industrial production	Log return calculations	FRED[7]
Supply proxies	US Energy	U.S. Energy Price Return Index	Deflation using U.S. CPI Index; Log return calculation	Refinitiv Eikon[8]
	PPI	U.S. PPI	Log return calculations	FRED[9]
Interest rates	TBill 13w	13-week U.S. T-Bills	% conversion, deflation using U.S. CPI	FRED[10]
	TBill 10y	10-year U.S. T-Bills		FRED[11]
Exchange rates	USD Index	ICE U.S. Dollar index	Log return calculations	Refinitiv Eikon[12]
	EUR/USD	EUR/USD FX spot rate	-	Refinitiv Eikon[13]
	CNY/USD	USD/CNY FX spot rate	Conversion to CNY/USD	Refinitiv Eikon[14]
Uncertainty	SP500 Var	S&P 500	Monthly variance based on daily price series	Refinitiv Eikon[15]
	BCI Var	Bloomberg Commodity Index		Refinitiv Eikon[16]
Inflation	CPI	U.S. CPI	% conversion	FRED[17]
	-	U.S. CPI Index (2015=100)	% conversion, only used for deflation	FRED[18]

Table 1 above summarizes the sources of the raw data series, their transformations, and if applicable the variable they are representative of. Using these (transformed) data series, the co-movement measures and models specified in Section 3 are computed and implemented. The following section gives further explanations about the exact application of methods.

---

[5] See Appendix B1: List of Commodity future continuations

[6] FRED. (2023, October 30). *Brave-Butters-Kelley real gross domestic product* [Data set].
https://fred.stlouisfed.org/series/BBKMGDP

[7] FRED. (2023, October 17). *Industrial Production: Total index* [Data set].
https://fred.stlouisfed.org/series/INDPRO

[8] Thomson Reuters. (2023). *RIC: .TRXFLDUSPENE* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[9] FRED. (2023, October 11). *Producer Price Index by commodity: All commodities* [Data set].
https://fred.stlouisfed.org/series/PPIACO

[10] FRED. (2023, November 3). *Market yield on U.S. Treasury Securities at 3-Month Constant maturity, quoted on an investment basis* [Data set]. https://fred.stlouisfed.org/series/DGS3MO

[11] FRED. (2023, November 3). *Market yield on U.S. Treasury Securities at 10-Year Constant Maturity, quoted on an investment basis* [Data set]. https://fred.stlouisfed.org/series/DGS10

[12] Thomson Reuters. (2023). *RIC: .DXY* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[13] Thomson Reuters. (2023). *RIC: EUR=* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[14] Thomson Reuters. (2023). *RIC: CNY=* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[15] Thomson Reuters. (2023). *RIC: .SP500* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[16] Thomson Reuters. (2023). *RIC: .BCOM* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

[17] FRED. (2023, October 12). *Consumer Price Index: All items: total for United States* [Data set].
https://fred.stlouisfed.org/series/CPALTT01USM657N

[18] FRED. (2023, October 12). *Consumer Price Index: All items: total for United States* [Data set].
https://fred.stlouisfed.org/series/USACPIALLMINMEI

4.3. Method application

Firstly, overall commodity price co-movement measures are calculated based on the returns of the deflated commodity price series. The computation of Pearson is hereby specified in Equation (1) while Gerber statistics are specified in Equation (2). Regarding Gerber statistics, the Gerber threshold is defined as follows, similarly to both Gerber et al. (2022) and Algieri et al. (2021), with the fraction q being set equal to $\frac{1}{2}$. Resulting from this, the Gerber thresholds of return series x and y are defined based on their standard deviations σ_x and σ_y:

$$Q_x = q\sigma_x = \frac{\sigma_x}{2}; \ Q_y = q\sigma_y = \frac{\sigma_y}{2} \tag{14}$$

<div align="right">(See, e.g., Algieri et al., 2021; Gerber et al., 2022)</div>

For both co-movement measures considered, rolling arithmetic averages of all given pairwise measures over the last six observations (or months) are used. These are hereby denoted as $\overline{g_t}$ and $\overline{\rho_t}$ respectively and can be defined as follows for Pearson correlations:

$$\overline{\rho_t} = \frac{1}{\binom{n}{2}} \sum_{(x,y)\in XY} \rho_{xy,i=t-6} \tag{15}$$

And as follows for Gerber statistics:

$$\overline{g_t} = \frac{1}{\binom{n}{2}} \sum_{(x,y)\in XY} g_{xy,i=t-6} \tag{16}$$

Thereby, XY denotes the set of all possible unique combinations of return series pairs, while $\rho_{xy,i=t-6}$ and $g_{xy,i=t-6}$ denote their respective co-movement measure computed using the last six available data points. $\binom{n}{2}$ is hereby equal to the number of unique pairwise combinations of return series and used to calculate averages. Both co-movement measures are calculated in this manner for the whole sample of 20 commodities, as well as for the two subsamples of 10 hard commodities and 10 soft commodities. Using these measures as the variable of interest, the models specified in Section 3 are implemented.

In the multiple regression models, the two co-movement measures are linearly regressed on the macroeconomic predictor variables of Section 4.2. For Pearson's correlations, this results in a model as follows:

$$\begin{aligned}
\overline{\rho_t} = \ & \alpha_0 + \beta_1 GDP_t + \beta_2 IP_t + \beta_3 US\,Energy_t + \beta_4 PPI_t + \beta_5 TBill\,13w_t \\
& + \beta_6 TBill\,10y_t + \beta_7 USD\,Index_t + \beta_8 EUR/USD_t + \beta_9 CNY/USD_t \\
& + \beta_{10} SP500\,Var_t + \beta_{11} BCI\,Var_t + \beta_{12} CPI_t + \varepsilon_t
\end{aligned} \tag{17}$$

Similarly, for Gerber statistics, the model is defined as:

$$\begin{aligned}
\overline{g_t} = \ &\alpha_0 + \beta_1 GDP_t + \beta_2 IP_t + \beta_3 US\ Energy_t + \beta_4 PPI_t + \beta_5 TBill\ 13w_t \\
&+ \beta_6 TBill\ 10y_t + \beta_7 USD\ Index_t + \beta_8 EUR/USD_t + \beta_9 CNY/USD_t \\
&+ \beta_{10} SP500\ Var_t + \beta_{11} BCI\ Var_t + \beta_{12} CPI_t + \varepsilon_t
\end{aligned} \tag{18}$$

Here, independent variables x from Equation (3) are filled in with predictor variable names from Table 1, while all other terms are defined as mentioned previously. For the VAR models described next, these predictor variables are defined jointly in vector $y_{j,t-l}$ for lag l. Again, since two distinct co-movement measures are investigated, two different types of VAR(p) models are defined. Hence, y_{1t} from Equation (5) can be substituted by the respective averages of co-movement measures considered. For average Pearson's correlation coefficients, $\overline{\rho_t}$, this results in the following equation:

$$\overline{\rho_t} = c_1 + \sum_{l=1}^{p} a_{11,l} \overline{\rho_{t-l}} + \sum_{l=1}^{p} \sum_{j=2}^{13} a_{1j,l} y_{j,t-l} + \varepsilon_{1t} \tag{19}$$

In the second type of models considering the average Gerber statistic, $\overline{g_t}$, the equation of interest is given as:

$$\overline{g_t} = c_1 + \sum_{l=1}^{p} a_{11,l} \overline{g_{t-l}} + \sum_{l=1}^{p} \sum_{j=2}^{13} a_{1j,l} y_{j,t-l} + \varepsilon_{1t} \tag{20}$$

Hereby, $\overline{\rho_{t-l}}$ and $\overline{g_{t-l}}$ denote the lagged values of the endogenous variable of interest, while $y_{j,t-l}$ denotes the lagged values of predictor variables j from 2 to 13 and $a_{1j,l}$ their coefficients. p on the other hand denotes the number of lags included in the model. As this research is only aimed at investigating the two co-movement measures of interest, the other equations of the VAR models will not be further investigated nor specified.

Consequently, the number of lags p was determined based on the Akaike information criterion (AIC) to evaluate model fit with regards to losses in degrees of freedom caused by introducing additional lags. Thereby, AIC was chosen over the Bayesian information criterion (BIC), as it led to lower mean squared errors in the models. Hence, was model accuracy preferred over model simplicity. The reasoning behind this is that performance metrics of different models are compared in the analyses and should hence be optimized to allow for conclusions into which model is most accurate. Nonetheless, was the maximum number of lags considered set to six, since we are predicting 6-month rolling averages and since the dataset size is rather limited. For both VAR models of Pearson correlations and Gerber statistics, this resulted in a lag number of two being selected. The exact information criteria for different lag numbers are given in more detail in Appendix B3: Information criteria results and RF learning curves.

The implementation of VARX models was done similarly to the VAR models presented, however are all predictor variables except for the U.S. CPI Growth rate included as exogenous variables.[19] This was already indicated in Equations (6) and (7). Substituting for the co-movement measure of interest results in the following VARX models:

---

[19] In all VARX model implementations seen in the following sections, the U.S. CPI growth rate was used as the additional endogenous variable to maintain the models' VAR structure.

$$\bar{\rho}_t = c_1 + \sum_{l=1}^{p} a_{11,l}\overline{\rho_{t-l}} + \sum_{l=1}^{p} a_{12,l}y_{2,t-l} + \sum_{l=0}^{p}\sum_{j=1}^{11} b_{1j,l}x_{j,t-l} + \varepsilon_{1t} \tag{21}$$

for average Pearson correlations, $\bar{\rho}_t$, and:

$$\overline{g_t} = c_1 + \sum_{l=1}^{p} a_{11,l}\overline{g_{t-l}} + \sum_{l=1}^{p} a_{12,l}y_{2,t-l} + \sum_{l=0}^{p}\sum_{j=1}^{11} b_{1j,l}x_{j,t-l} + \varepsilon_{1t} \tag{22}$$

for average Gerber statistics, $\overline{g_t}$. Hereby, $y_{2,t-l}$ refers to the U.S. CPI growth rates lagged values with associated coefficients $a_{12,l}$, while $x_{j,t-l}$ denotes lagged values of all other exogenous predictor variables with associated coefficients $b_{1j,l}$. Regarding the selection of the number of lags p, the same procedure described previously for the VAR models also applies for VARX models. Again, a lag of 2 was selected and more details on the results of the AIC (and BIC for comparison) are given in Appendix B3: Information criteria results and RF learning curves.

Lastly regarding method application, the Random Forest regression defined in Section 3.5 are implemented to the same dataset to give insights into the performance of novel machine learning algorithms in the context of commodity price co-movement. There, MSEs are specified in Equation (11). However, to allow for an additional comparison metric, out-of-bag R^2 statistics are calculated for the Random Forest model. They can be defined as follows:

$$R_{OOB}^2 = 1 - \frac{\sum_{i\in OOB}(y_i - \hat{y}_i^{OOB})^2}{\sum_{i\in OOB}(y_i - \bar{y}_{OOB})^2} \tag{23}$$

There, y_i denotes the actual target variable at observation i of the OOB set $i \in OOB$. $\bar{y}_{OOB}$ denotes the mean value of the target variable in said OOB set, and $\hat{y}_i^{OOB}$ the Random Forest Prediction for observation i. Although its computation is close to that of traditional R^2 statistics where the sum of squared errors and the total sum of squares are used, it should not be the sole criterion to provide insights into the Random Forest's performance compared to traditional models like those of Sections 3.2 to 3.4. This is because of the differences in the models' nature, and since the R_{OOB}^2 statistic is only drawn from OOB observations. Hence, does the MSE hereby provide a more general performance measure in the scale of the variable of interest[20] and is commonly used in both linear and non-linear regression models. (See, e.g., Hastie et al., 2009; Murphy, 2012)

Crucial for the implementation of Random Forests is hyperparameter tuning. In the Random Forest regression models specified in Section 3.5, several key hyperparameters are considered. Firstly, K, the number of decision trees included in the Random Forest plays an important role in model performance. Because of the rather limited data size (see Sections 4.1 and 4.2), and since model performance with a higher number of trees did not further increase but rather stayed constant (or even decreased) beyond $K = 500$, it was set equal to 500 in all random

---

[20] Since MSE sums up squared values of the errors, using RMSE results in a scale that is directly comparable to the variable of interest. It is defined as: $RMSE = \sqrt{MSE}$

Forest models considered. This is visualized in the learning curves of K on MSE which can be found in Figure 5 of Appendix B3: Information criteria results and RF learning curves.

Moreover, automated hyperparameter tuning was performed using 10-fold cross-validations, where a grid search optimization algorithm was employed to explore possible combinations of the hyperparameters. Thereby, two hyperparameters were directly optimized to maximize model performance by minimizing the mean value of the earlier specified loss function (MSE) across all 10 folds:

1. The number of randomly selected features considered at each split (m), and
2. The minimum number of observations available for nodes to be split (n).

Thereby, m, the number of features considered at each split, ranged from 2 to $M = 12$ as this was the maximum number of features in the dataset.[21] It is an important hyperparameter to consider when building Random Forest models, since it controls for diversity in the trees. Smaller values of m can hereby reduce overfitting and improve generalization, but larger m-values lead to more powerful individual trees. Regarding n, the minimum number of observations needed for nodes to split, it ranged from 1 to 5 in the grid search hyperparameter tuning performed. It is also important to consider as it controls for how deep the trees in the model can grow and thus also mitigates overfitting. (Breiman, 2001)

Lastly regarding Random Forests, VIMs are obtained from the models. During hyperparameter tuning, only the MDI is extracted from the 10-fold cross-validation results of the model with the selected hyperparameters. Hence, to obtain both VIMs as described in Equations (12) and (13), the final model with the optimized hyperparameters is run again to compute PDI importance measures. All performance metrics ($MSE, OOB\ R^2, MAE$) given with the outputs of the Random Forest models in Sections 5 and 6 are thus computed as the average across both 10-fold cross-validations of the respective model.

In the following section, descriptive statistics of the data used for analyses are given as part of the preliminary analyses. There, it is elaborated on the data, before the results of the model implementations described before are presented in Section 5.

---

[21] $M = 12$ in this research since there are 12 explanatory variables in the dataset.

4.4. Preliminary analyses

Commodity price co-movement in the sample period
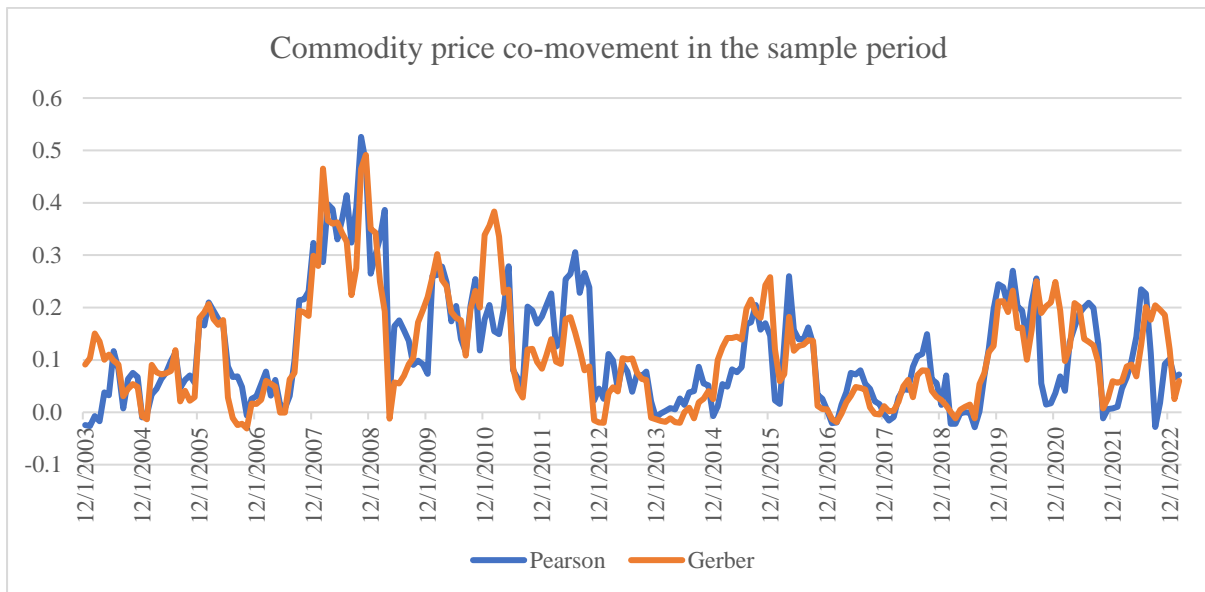
Figure 3: Commodity price co-movement in the sample period

Figure 3 depicts co-movement throughout the sample period measured by both average Pearson correlations and average Gerber statistics as outlined in Section 4.3. There, it can already be seen that in the considered time frame, co-movement spiked from late 2007 to early 2009, coinciding with the global financial crisis. In the following year up to the end of 2012, co-movement remained relatively high compared to values prior to the spike. Here it is noteworthy however, that co-movement directly after the spike in early 2009 dropped sharply to values around zero for both Pearson and Gerber. In the second half of the sample period, or subsample two, co-movement generally showed lower values than in the first half of the sample period. Nonetheless, there were several noteworthy periods of relatively high co-movement: In 2015 to 2016 co-movement rose before falling back to values comparable to those of 2013 to 2014, and in late 2019 co-movement spiked again, coinciding with the Covid-19 outbreak. In the years following this, co-movement levels had risen and fallen repeatedly until the end of the sample period.

Table 2: Descriptive statistics

Variables	Mean	SD	Min	Max	Range	Skewness	Kurtosis
Pearson	0.116	0.106	-0.029	0.526	0.554	0.116	0.796
Gerber	0.114	0.105	-0.031	0.491	0.522	0.114	0.810
Pearson HC	0.149	0.125	-0.088	0.474	0.563	0.149	-0.587
Gerber HC	0.153	0.116	-0.095	0.475	0.570	0.153	-0.306
Pearson SC	0.140	0.154	-0.084	0.612	0.696	0.140	0.310
Gerber SC	0.128	0.147	-0.073	0.544	0.617	0.128	0.186
GDP	1.85%	8.02%	-77.00%	45.99%	122.99%	-3.848	49.212
IP	0.05%	1.33%	-14.37%	6.30%	20.66%	-5.396	60.918
PPI	0.26%	1.28%	-5.48%	3.16%	8.64%	-0.890	2.952
TBill 13w	1.08%	1.62%	-0.88%	5.62%	6.50%	1.091	0.096
TBill 10y	2.65%	1.17%	0.12%	5.55%	5.43%	0.225	-0.564
Energy Index	0.22%	7.49%	-43.68%	26.73%	70.41%	-0.735	5.222
USD Index	0.07%	2.23%	-6.82%	7.50%	14.31%	0.092	0.718
EUR/USD	1.245	0.127	0.980	1.577	0.597	0.347	-0.521
CNY/USD	0.146	0.013	0.121	0.165	0.044	-0.697	-0.556
SP500 Var	0.01%	0.03%	0.00%	0.34%	0.34%	6.765	54.437
BCI Var	0.01%	0.01%	0.00%	0.08%	0.08%	2.929	11.530
CPI	0.21%	0.41%	-1.92%	1.37%	3.29%	-0.598	3.065
							n = 231
Pearson 1	0.151	0.119	-0.026	0.526	0.552	0.755	0.002
Gerber 1	0.141	0.119	-0.031	0.491	0.522	0.824	0.064
							n = 116
Pearson 2	0.081	0.078	-0.029	0.270	0.299	0.655	-0.651
Gerber 2	0.087	0.080	-0.020	0.258	0.278	0.404	-1.100
							n = 115

Table 2 contains the descriptive statistics of the full sample data used in the analysis as well as the co-movement measures in subsamples 1 and 2. Predictor variables are named according to Table 1, while "Pearson" and "Gerber" refer to full sample co-movement measures of all 20 commodities. "Pearson HC" and "Gerber HC" refer to those of only hard commodities throughout the full sample period, while "Pearson SC" and "Gerber SC" refer to those of only soft commodities. Similarly, "Pearson 1" and "Gerber 1" refer to the first subsample co-movement measures of all 20 commodities and "Pearson 2" and "Gerber 2" to those of the second subsample. Full descriptive statistics of predictor variables in the two subsamples as well as those of commodity prices used for measuring co-movement can be found in Appendix B2: Full descriptive statistics, along with histograms of all data used in the analyses.

In the table above, skewness and excess kurtosis values are depicted to give further insights into the distribution of Pearson correlations, Gerber statistics, and its determining variables. Regarding skewness for the full sample as well as hard and soft commodities, the co-movement measure distributions are approximately symmetric as values are close to zero. In the two subsamples however, the respective values for both measures are substantially higher, and their distributions are hence moderately skewed to the right. Regarding kurtosis, the distributions of the co-movement measures differ more: The full sample co-movement and that of soft commodities hereby show leptokurtic distributions (although this is less pronounced when only considering soft commodities). Hard commodities and subsample two on the other hand show platykurtic distributions, suggesting less extreme data points. In subsample one, both kurtosis values are close to zero and hence do their distribution have similar kurtoses to normal distributions. Lastly, do the descriptive statistics of several predictor variables indicate extreme values or distributions for some variables. Most notably, seem GDP, IP, SP500 Var, and BCI

Var far from normally distributed when looking at their skewness and kurtosis values (this also applies to the Energy Index, PPI, and CPI although to a lesser extent). Moreover, do the Energy Index and GDP show rather extreme values as minimums and maximums. Especially their minimums are exceptionally low, with -43.68% and -77% respectively. These minimums are both coinciding with the Covid-19 pandemic as they are recorded for 31st March 2020.

In Table 2, it can also be seen that the means of both Pearson correlations and Gerber statistics differ between subsample one and subsample two. For the full sample period, both means lie around 11.5%, while the first subsample shows mean co-movement measures of around 14%-15%. In the second subsample however, the respective means are between 8% and 9%. Moreover, it is noteworthy that the co-movement measures only considering either hard or soft commodities show higher means than those considering all commodities. Regarding hard commodities, the means are around 15% while for soft commodities they are slightly lower at 14% and 12.8% respectively for Pearson correlations and Gerber statistics. Moreover, do the standard deviations of the co-movement measures differ between the full sample, subsamples, and hard and soft commodities. Thereby, co-movement measures of soft commodities show the highest standard deviations, followed by hard commodities and subsample one. The standard deviations of co-movement measures considered in subsample 2 consequently lie below those of the full sample period.

This difference in co-movement levels is elaborated on at the start of the results in the following Section 5. There formal tests on the normality of Pearson correlations and Gerber statistics for the samples and commodity selections are given in Table 3, along with tests for differences in means. From here on, in the tables summarizing the results, all significances at the 1% level are denoted with ***, at the 5% level with **, and at the 10% level with *.

5. Results

In the descriptive statistics given in Table 2, it can already be seen that the mean values of the two co-movement measures considered in this research differ in subsamples 1 and 2. To test this statistically, Wilcoxon rank-sum tests were used. Since the data cannot be assumed to be normally distributed based on the outcomes of the Shapiro-Wilk tests in Table 3, Wilcoxon rank-sum tests were chosen as they do not assume normality. Table 3 hence presents the results of the tests for structural changes in the co-movement measures in the two given subsamples, as part of the research for Sub-question 5. There can be seen that both p-values lie below the significance level of 1%. Hence, do the tests indicate a statistically significant difference in the co-movement levels in the two periods. The same holds true for the Wilcoxon rank-sum tests of hard and soft commodities, but only at 10% significance when considering Pearson correlations. These differences in co-movement levels should be kept in mind when interpreting the results and performance of the respective models presented in the rest of the section.

Table 3: Preliminary tests

Tests	Variable	W
Shapiro-Wilk tests	Pearson	0.922***
	Gerber	0.927***
	Pearson HC	0.984**
	Pearson SC	0.927***
	Gerber HC	0.985**
	Gerber SC	0.911***
	Pearson 1	0.943***
	Pearson 2	0.928***
	Gerber 1	0.933***
	Gerber 2	0.929***
	Test	**W**
Wilcoxon rank-sum test	Pearson Subsamples	9080***
	Gerber Subsamples	8303***
	Pearson HC vs SC	29090*
	Gerber HC vs SC	31436***

Table 4: Multiple regression results

Panel A: Pearson										
	Full model		Hard commodities		Soft commodities		Subsample 1		Subsample 2	
Variable	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.
(Intercept)	-0.158		-0.446	***	-0.160		-0.519	**	0.331	
GDP	0.028		0.148		-0.066		-0.618	*	0.024	
IP	-0.373		-0.272		-1.338		-1.464		-0.120	
PPI	-1.239		-0.853		-1.493		-0.565		-1.066	
TBill 13w	0.637		3.916	***	0.663		0.485		-0.194	
TBill 10y	-4.175	***	-5.748	***	-3.461	*	-0.806		-3.920	**
Energy Index	-0.016		0.115		-0.122		-0.024		-0.030	
USD Index	0.046		0.630	*	-0.677		0.268		-0.480	
EUR/USD	0.374	***	0.475	***	0.402	***	0.272	**	-0.058	
CNY/USD	-0.969		0.478		-1.106		2.004		-0.780	
SP500 Var	-16.188		23.764		-96.748	*	22.828		-18.841	
BCI Var	565.798	***	410.997	***	643.776	***	371.541	***	336.619	***
CPI	-1.011		-2.138		-1.522		2.744		-3.594	
Multiple R^2	0.4264		0.2877		0.2360		0.5415		0.2939	
Adj. R^2	0.3949		0.2485		0.1940		0.4881		0.2108	
Panel B: Gerber										
	Full model		Hard commodities		Soft commodities		Subsample 1		Subsample 2	
Variable	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.
(Intercept)	0.081		0.023		-0.110		-0.675	***	0.062	
GDP	0.038		0.252	*	-0.075		-0.448		0.030	
IP	-0.707		-0.557		-1.290		-1.807		-0.275	
PPI	-0.390		-0.929		0.960		0.248		0.409	
TBill 13w	-1.335	**	-0.976		0.258		-2.766	***	1.678	
TBill 10y	-1.112		0.895		-4.486	***	6.218	***	-6.030	***
Energy Index	0.013		0.046		-0.081		-0.007		-0.003	
USD Index	-0.088		0.216		-0.466		0.279		-0.677	*
EUR/USD	0.182	***	0.028		0.439	***	0.225	*	-0.420	***
CNY/USD	-1.361	*	0.115		-1.510		2.188		4.065	**
SP500 Var	13.548		33.602		-42.909		116.090	**	2.761	
BCI Var	425.468	***	436.095	***	450.258	***	78.943		209.184	*
CPI	1.382		5.698	*	-6.398	*	9.636	**	-7.943	**
Multiple R^2	0.3136		0.2132		0.2296		0.5036		0.4062	
Adj. R^2	0.2758		0.1699		0.1872		0.4458		0.3363	

Firstly, the results of the multiple regression models are presented in Table 4, where Panel A shows the results of Pearson and Panel B those of Gerber as the dependent variable. There can be seen that the independent variables show different levels of significance depending on the sample of the model considered. Hereby, some variables show significance in close to all models considered, while others are only significant in a few or even one model. Most notable, does the monthly variance of the Bloomberg Commodity Index show positive effects at the 1% significance level for both co-movement measures in all models except for the subsamples of Gerber. There, it shows significance at the 10% level in the second subsample but none in the first. Moreover, does the EUR/USD foreign exchange rate show significant effects in most models considered. For Pearson in the second subsample no significance is seen and only significance at the 5% level is observed in the first subsample. For Gerber, no significance is seen for the model predicting the co-movement of hard commodity prices, and only a 10% significance in the first subsample model. All other estimates are significant at the 1% level and, moreover, all positive except for the second subsample of Gerber. Additionally, can be seen that 10-year U.S. treasury bill yields show highly significant effects for some models. For instance, it shows significance at the 1% level in the full model and hard-commodity model for Pearson, while showing none for Gerber in the respective models. For the remaining soft-commodity model and both subsample models, it shows significance at 1% for Gerber, but not for Pearson (only 5% for subsample 2 and 10% for the soft-commodity model). Thereby, all coefficient estimates are negative except for the first subsample model of Gerber. Lastly, it is notable that both 13-week U.S. treasury bill yields and the constant in the regression show significant effects at the 1% level in two cases. All other variables considered do not qualify for that significance level.

Table 5: VAR model results

| | Panel A: Pearson | | | | | | | | | |
| | Full model | | Hard commodities | | Soft commodities | | Subsample 1 | | Subsample 2 | |
Variable	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.	Estimate	Sig.
Pearson.l1	0.743	***	0.666	***	0.822	***	0.482	***	0.858	***
GDP.l1	0.011		0.195		-0.238		-0.145		0.055	
IP.l1	0.099		-0.767		1.206		-2.528	**	1.557	***
PPI.l1	0.909		1.283		0.291		1.382		1.214	
TBill 13w.l1	0.494		3.348		3.790		-3.091		-2.750	
TBill 10y.l1	-4.800	**	-6.268	*	-2.715		-0.552		-3.651	
Energy Index.l1	-0.030		0.090		-0.093		-0.032		-0.050	
USD Index.l1	0.089		0.233		0.176		-0.391		0.490	*
EUR/USD.l1	0.594		0.462		0.219		1.619	**	-0.178	
CNY/USD.l1	2.400		7.808	*	-3.057		-9.605		-0.971	
SP500 Var.l1	-1.277		-7.419		-2.517		-60.679		68.837	*
BCI Var.l1	110.897		143.054		24.171		224.716	*	6.549	
CPI.l1	-7.487	**	-8.551		-0.188		-9.096		-10.186	*
Pearson.l2	-0.038		-0.031		-0.067		0.098		-0.249	**
GDP.l2	-0.071		-0.027		-0.020		-0.218		-0.168	
IP.l2	0.291		0.016		0.059		-0.913		0.319	
PPI.l2	-1.358	**	-0.377		-2.575	***	0.009		-1.566	*
TBill 13w.l2	-0.422		-2.039		-4.091		3.070		2.653	
TBill 10y.l2	3.322		4.156		2.049		0.375		1.883	
Energy Index.l2	-0.076		-0.020		-0.088		-0.262	*	0.047	
USD Index.l2	-0.987		-0.554		-0.556		-2.963	**	0.562	
EUR/USD.l2	-0.473		-0.303		-0.117		-1.660	**	0.099	
CNY/USD.l2	-2.891		-7.820	*	2.487		11.155		0.588	
SP500 Var.l2	1.761		16.375		7.802		37.792		18.322	
BCI Var.l2	34.982		2.380		35.125		-47.390		-78.157	
CPI.l2	7.175	**	3.682		5.190		8.270		5.991	
const	-0.019		-0.111		0.005		-0.110		0.220	
Multiple R²	0.7184		0.5879		0.7415		0.7722		0.7489	
Adj. R²	0.6822		0.5349		0.7082		0.7041		0.6730	

Panel B: Gerber

Variable	Full model Estimate	Sig.	Hard commodities Estimate	Sig.	Soft commodities Estimate	Sig.	Subsample 1 Estimate	Sig.	Subsample 2 Estimate	Sig.
Gerber.l1	0.915	***	0.787	***	1.027	***	0.691	***	0.974	***
GDP.l1	-0.013		0.106		-0.111		0.188		0.035	
IP.l1	-0.437		-0.631		-0.654		-3.604	***	0.736	
PPI.l1	0.426		0.306		-0.480		0.985		0.984	
TBill 13w.l1	-2.195		-2.680		-1.523		-11.236	***	3.098	
TBill 10y.l1	-0.515		-1.316		1.012		4.994	*	-4.304	
EnergyIndex.l1	-0.047		-0.009		-0.011		-0.389	***	0.079	
USD Index.l1	0.013		0.093		0.218		-0.553	**	0.096	
EUR/USD.l1	1.263	***	1.034	*	0.973	*	1.435	**	1.217	
CNY/USD.l1	0.763		1.329		-2.921		-13.050		1.202	
SP500 Var.l1	-29.903		-15.216		-71.090	**	-73.430	*	57.867	*
BCI Var.l1	104.544	*	120.164		117.514		162.928	*	-66.253	
CPI.l1	-2.839		-4.607		1.001		-7.774		-3.503	
Gerber.l2	-0.094		-0.049		-0.184	***	-0.025		-0.193	
GDP.l2	0.035		-0.008		0.076		-0.380		-0.046	
IP.l2	0.030		-0.235		0.228		-0.879		0.282	
PPI.l2	-0.425		0.072		-0.764		0.744		-0.684	
TBill 13w.l2	2.004		2.834		1.267		9.925	***	-2.745	
TBill 10y.l2	0.098		0.952		-1.888		-1.612		3.954	
EnergyIndex.l2	-0.068		-0.067		-0.036		-0.288	**	-0.026	
USD Index.l2	-1.909	***	-1.353		-1.503	*	-2.318	**	-1.748	
EUR/USD.l2	-1.214	***	-1.005	*	-0.875	*	-1.500	***	-1.348	*
CNY/USD.l2	-0.926		-1.301		2.566		14.954		-0.017	
SP500 Var.l2	11.272		-16.456		56.384	**	49.984		26.447	
BCI Var.l2	-4.427		5.371		-44.595		-143.280		24.048	
CPI.l2	4.087		5.113		2.733		10.887	**	1.591	
const	-0.014		-0.004		-0.033		-0.234		-0.007	
Multiple R^2	0.7960		0.6307		0.8423		0.8579		0.7961	
Adj. R^2	0.7698		0.5832		0.8220		0.8154		0.7344	

Table 5 presents the results of the reduced form VAR models for the considered samples and co-movement measures. l1 hereby refers to first lags of the respective variable, while l2 refers to second lags. There, the results are a lot more mixed compared to the multiple regression models. Nonetheless, do the first lags of the respective co-movement measures show positive, significant effects at the 1% level for all models. This significance however does not hold for the second lags, where only the soft-commodity model of Gerber shows significance at 1%, and the second subsample model of Pearson at the 5% level, but with both coefficients being negative.

The other variables considered in the models do not yield consistent results like this for neither lag. They do not show statistically significant effects across all or close to all models, at least not when considering models of both co-movement measures. Still, there are several noteworthy cases. Firstly, regarding the full model of Pearson in Panel A, CPI shows significant effects at 5% for both the first and second lag. This significance however is not supported by the other models[22], and moreover do the coefficients of the full model show negative effects in the first lag and positive in the second. Furthermore, do first-lag 10-year U.S. treasury bills and second-lag PPI show significant negative effects at the 5% level. For 10-year U.S. treasury bills this effect might be driven by hard commodities, since the respective model shows negative effects at 10% significance, while for PPI this effect might be driven by soft commodities where a negative effect at 1% significance can be seen. In addition, does the second subsample model of Pearson show positive effects of industrial production at the 1% level.

Regarding the equations of the VAR models predicting Gerber in Panel B, the effects of certain variables are more consistent compared to the Pearson equations. Here, the EUR/USD foreign exchange rate shows significant effects in both lags in the full model, although positive in the

---

[22] Only the Pearson model of subsample 2 shows negative, significant effects at the 10% significance level.

first lag and negative in the second. This is supported by positive effects of the first lag at 10% significance in both hard and soft commodity models, and at 5% in the first subsample. For the second lag, the results are supported by all other models with the first subsample model showing significance at 1% and all other models at 10%. Furthermore, it is notable that the U.S. Dollar index's second lag shows negative effects at the 1% level in the full model, which however is only supported by a 5% significance in subsample 1 and a 10% significance in the soft-commodity model. Lastly, compared to the other reduced VAR models of Gerber, the model of the first subsample shows significant effects for substantially more variables than all other models. Thereby, all variables except for GDP, PPI, and the CNY/USD exchange rate show effects significant at least at the 10% level. Moreover, does the model yield the highest multiple and adjusted R^2 values of all VAR models. Also, in the VAR models predicting Pearson, the subsample 1 model achieved slightly higher R^2 values than the subsample 2 model. Recalling from the results of Table 2 and Table 3 that co-movement levels in the first subsample were higher than in the second, this could indicate that co-movement is easier to predict when its level is higher.

Table 6: VARX models results Pearson

Variable	Full model Estimate	Sig.	Hard Commodities Estimate	Sig.	Soft Commodities Estimate	Sig.	Subsample 1 Estimate	Sig.	Subsample 2 Estimate	Sig.
Pearson.l1	0.705	***	0.641	***	0.772	***	0.546	***	0.827	***
CPI.l1	-4.302		-7.565		4.314		-7.429		-12.301	*
Pearson.l2	-0.041		-0.004		-0.040		0.057		-0.291	**
CPI.l2	0.740		-1.730		-1.787		3.449		4.186	
const	-0.035		-0.127		0.003		-0.050		0.019	
GDP	-0.075		0.079		-0.241		0.016		-0.266	
IP	-0.369		-0.970		0.095		-1.180		-0.329	
PPI	-0.372		-0.190		-0.553		-0.249		0.530	
TBill 13w	2.754		4.200		0.997		0.613		4.307	
TBill 10y	-2.725		-2.490		-2.561		-0.221		-3.797	
Energy Index	0.016		0.205	**	-0.078		0.024		-0.075	
USD Index	-0.034		0.519	*	-0.634	**	-0.260		-0.134	
EUR/USD	-0.681		-0.904		-0.765		0.172		-0.747	
CNY/USD	0.543		5.051		-0.949		1.064		0.097	
SP500 Var	-20.907		-7.681		-43.444		-61.830		-19.841	
BCI Var	339.794	***	250.650	**	421.647	***	426.290	***	170.977	
GDP.l1	0.183		0.195		0.108		-0.036		0.603	*
GDP.l2	-0.196		-0.088		-0.223		-0.164		-0.479	**
IP.l1	-0.407		-0.990		0.398		-2.666	**	0.283	
IP.l2	0.389		0.178		0.233		-0.954		0.209	
PPI.l1	0.885		0.923		0.540		1.843		1.035	
PPI.l2	-0.176		0.807		-1.522		0.658		-0.516	
TBill 13w.l1	-2.600		-1.946		3.417		-2.759		-10.272	
TBill 13w.l2	0.163		-0.495		-4.680		2.344		6.748	
TBill 10y.l1	0.663		-1.639		2.759		0.938		1.565	
TBill 10y.l2	0.218		1.247		-0.806		-1.337		-0.603	
Energy Index.l1	-0.106		0.073		-0.175		-0.164		-0.112	
Energy Index.l2	-0.109		-0.048		-0.113		-0.238		-0.014	
USD Index.l1	1.067		1.516		1.218		-0.731		1.456	
USD Index.l2	-0.807		-0.456		-0.154		-2.936	**	0.431	
EUR/USD.l1	1.261	**	1.369		0.839		1.525	*	0.540	
EUR/USD.l2	-0.464		-0.282		-0.015		-1.794	***	0.189	
CNY/USD.l1	-0.150		0.215		-4.519		-17.819		-0.951	
CNY/USD.l2	-0.704		-5.264		5.227		18.363		1.433	
SP500 Var.l1	-17.108		-32.871		-6.210		-58.776		80.955	**
SP500 Var.l2	-4.287		10.898		9.433		43.935		-18.736	
BCI Var.l1	38.451		125.778		-94.752		76.372		37.520	
BCI Var.l2	-58.141		-113.811		-73.403		-155.262		-73.403	
Multiple R^2	0.7664		0.6339		0.7764		0.8209		0.7815	
Adj. R^2	0.7211		0.5629		0.7331		0.7338		0.6736	

Table 7: VARX model results Gerber

Variable	Full model Estimate	Sig.	Hard Commodities Estimate	Sig.	Soft Commodities Estimate	Sig.	Subsample 1 Estimate	Sig.	Subsample 2 Estimate	Sig.
Gerber.l1	0.850	***	0.752	***	0.976	***	0.650	***	1.020	***
CPI.l1	-1.799		-2.094		-0.040		-5.234		-5.071	
Gerber.l2	-0.059		-0.024		-0.163	**	0.028		-0.251	*
CPI.l2	1.348		0.626		1.562		7.937	*	2.087	
const	-0.029		-0.052		-0.047		-0.236		-0.126	
GDP	-0.189		-0.087		-0.279		0.207		0.002	
IP	-0.602		-1.196		-0.203		-2.240	**	-0.170	
PPI	0.345		-0.308		1.170	*	0.292		1.895	**
TBill 13w	0.458		-4.143		3.161		-3.778		7.043	**
TBill 10y	-1.526		2.172		-3.035		2.509		-4.865	
Energy Index	-0.029		0.039		-0.127	*	-0.129		-0.094	
USD Index	-0.163		0.138		-0.366	*	-0.297		-0.314	
EUR/USD	-0.676	*	-0.765		-0.387		-0.343		-1.070	
CNY/USD	-1.163		0.138		-1.389		-3.027		0.036	
SP500 Var	-4.682		-14.080		-3.474		-29.236		25.562	
BCI Var	180.257	***	214.540	**	166.669	**	249.841	***	85.371	
GDP.l1	0.409	*	0.446		0.414		0.115		0.087	
GDP.l2	-0.192		-0.182		-0.223		-0.292		-0.066	
IP.l1	-1.496	**	-1.587	*	-1.841	**	-3.781	***	0.483	
IP.l2	0.073		-0.204		0.338		-0.501		-0.229	
PPI.l1	0.623		0.794		-0.374		1.512	*	0.040	
PPI.l2	0.214		1.133		-0.408		0.956		-0.865	
TBill 13w.l1	-3.381		1.658		-5.853		-6.188		-4.310	
TBill 13w.l2	2.647		2.676		2.378		8.609	**	-1.896	
TBill 10y.l1	2.942		-1.737		5.622		4.258		0.897	
TBill 10y.l2	-1.389		-0.621		-2.824		-2.739		3.759	
Energy Index.l1	-0.118	*	-0.091		-0.093		-0.494	***	0.084	
Energy Index.l2	-0.081		-0.076		-0.052		-0.251	**	-0.034	
USD Index.l1	1.013		1.214		0.799		-0.056		1.609	
USD Index.l2	-1.661	**	-0.874		-1.484	*	-1.957	**	-1.459	
EUR/USD.l1	1.813	***	1.574	*	1.357	**	1.565	**	1.966	*
EUR/USD.l2	-1.118	***	-0.779		-0.920	*	-1.376	**	-1.053	
CNY/USD.l1	-0.254		-1.497		-2.954		-15.549		-0.045	
CNY/USD.l2	1.535		1.657		4.400		21.094	**	2.121	
SP500 Var.l1	-24.562		-21.755		-54.945	*	-60.014		56.987	*
SP500 Var.l2	14.849		5.031		47.701	*	56.762		-8.413	
BCI Var.l1	34.452		45.849		40.490		53.551		-54.894	
BCI Var.l2	-84.171		-111.329		-102.644		-229.035	**	7.813	
Multiple R²	0.8266		0.6654		0.8613		0.8932		0.8304	
Adj. R²	0.7930		0.6006		0.8345		0.8412		0.7467	

In Table 6 and Table 7 the outcomes of the VARX models are depicted. There, the first lags of the respective co-movement measure considered show significant positive effects at 1% significance. Their second lags, however, do mostly not show significant effects. Exceptions here are the second lags in subsample two, with the Pearson model's estimate being negative at 5% and Gerber's estimate negatively significant at 10%. Also, the estimate of the soft-commodity model of Gerber is negatively significant at 5%. The inconsistency of significances of the remaining variables across models that was seen in the reduced VAR models mostly holds true for the VARX models as well, although there are two noteworthy exceptions: Firstly, the current values of the Bloomberg Commodity Index have significant positive effects for all models but the two models of subsample 2. These effects are significant at the 1% level for the full and subsample 1 models of Pearson and Gerber, and the soft-commodity model of Pearson. The hard-commodity models and the soft-commodity model of Gerber only show significant effects at a 5% significance level. Regarding the VARX models of Pearson, there are no other variables that show significant effects at the 1% level for the full models, and only the effect of the first lag of the EUR/USD exchange rate is positive and significant at a 5% level. This is somewhat supported by a 10% significance in the first subsample model, however, does its second lag show negative effects at 1% significance in the same sample.

In the VARX models of Gerber in Table 7, there are more variables showing significance. Here, the first lag of EUR/USD is significantly positive at 1% in the full model and positively significant at least at the 10% level for all remaining models. Its second lag, however, is

negatively significant at 1% in the full model which is supported by the respective first subsample and soft-commodity models. This switch in signs of the coefficient from the first lag to the second lag, might indicate that there is a short-term reversal of the effect. Moreover, its current values show negative effects at 10% significance, but only for the full model, which could indicate that changes in the EUR/USD rate take some time before they start to affect co-movement levels. Furthermore, does the first lag of industrial production have negative effects at 5% significance in the full model of Gerber. This extends to the respective hard-commodity (10%), soft-commodity (5%) and subsample one (1%) models. In the first subsample model, industrial production's current values also show negative significant effects at 5%. Furthermore, does the second lag of the U.S. Dollar index show negative coefficients at 5% significance for the full and subsample 1 models of Gerber, and at 10% significance for the soft-commodity model. Other than that, only the first lags of GDP and the U.S. Energy price return index show negative effects at 10% significance, which however, is only supported by a 1% significance in the first subsample model for the latter. Lastly regarding the VARX models, comparably more variables (and their lagged values) show significance in the first subsample model of Gerber and, although to a lesser extent, the soft-commodity model of Gerber. For the models predicting Pearson however, this does not hold true. Yet, did first subsample models (with higher co-movement levels) once again achieve higher R^2 values than subsample 2 models, and soft-commodity models higher R^2 values than hard-commodity models for both Pearson and Gerber.

Table 8: Random Forest results

Panel A: Variable Importance

	Pearson										Gerber									
	Full Model		Hard Commodity		Soft Commodity		Subsample 1		Subsample 2		Full Model		Hard Commodity		Soft Commodity		Subsample 1		Subsample 2	
Variable	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI	MDI	PDI
GDP	0.19472	0.00060	0.33018	0.00106	0.67847	0.00283	0.18244	0.00125	0.04799	0.0002	0.22625	0.00100	0.17900	0.00038	0.60752	0.00264	0.20828	0.00178	0.02909	0.00006
IP	0.15568	0.00032	0.18247	0.00018	0.40626	0.0010	0.17792	0.00038	0.03499	0.00015	0.12133	0.00031	0.19375	0.00008	0.51804	0.00142	0.19825	0.00061	0.01740	0.00003
PPI	0.15747	0.00040	0.18817	0.00030	0.26649	0.00063	0.02588	0.00005	0.05270	0.00018	0.13094	0.00038	0.22361	0.00058	0.31158	0.00094	0.12552	0.00081	0.04110	0.00036
TBill 13w	0.19777	0.00271	0.28480	0.00155	0.31734	0.0012	0.13141	0.00209	0.03180	0.00016	0.21778	0.00255	0.22307	0.00065	0.28147	0.00176	0.13387	0.00132	0.02766	0.00034
TBill 10y	0.16251	0.00105	0.26637	0.00110	0.23630	0.00077	0.02340	0.00007	0.15039	0.00209	0.15470	0.00109	0.27893	0.00169	0.23230	0.0015	0.07720	0.00089	0.18727	0.00397
Energy Index	0.09629	0.00004	0.15412	-0.00012	0.23769	0.00013	0.02627	0.00010	0.03744	-0.0001	0.06839	0.00000	0.12285	-0.00002	0.19717	-0.00005	0.05000	-0.00003	0.02456	0.00000
USD Index	0.09627	0.00001	0.13872	-0.00012	0.33979	0.00052	0.03810	0.00004	0.02735	-0.00014	0.09067	0.00000	0.15131	-0.00013	0.31730	0.00054	0.05193	0.00002	0.02485	0.00001
EUR/USD	0.49604	0.00371	0.71594	0.00441	0.97083	0.00521	0.10423	0.00062	0.04815	0.00036	0.55474	0.00335	0.31757	0.00128	1.12482	0.00774	0.23013	0.00211	0.07777	0.00096
CNY/USD	0.20858	0.00204	0.31989	0.00206	0.54685	0.00338	0.61338	0.01409	0.07468	0.00075	0.20657	0.0019	0.26206	0.00167	0.35784	0.00219	0.22525	0.00408	0.04010	0.00039
SP500 Var	0.19312	0.00074	0.45752	0.00135	0.31903	0.00105	0.12146	0.00057	0.03136	0.00004	0.18012	0.00058	0.32502	0.00061	0.24709	0.00048	0.11840	0.00029	0.02599	0.00014
BCI Var	0.52479	0.00231	0.30947	0.00054	0.85184	0.00299	0.10918	0.00056	0.10477	0.00058	0.39634	0.00152	0.53515	0.00239	0.38329	0.00117	0.12266	0.00077	0.16670	0.00214
CPI	0.10468	0.00021	0.17661	0.00024	0.26619	0.00053	0.04210	0.00017	0.04486	0.00027	0.13649	0.00024	0.21649	0.00033	0.25464	0.00055	0.10410	0.00035	0.03878	0.00026

Panel B: Model performance and Hyperparameters

Performance Metric	Pearson					Gerber				
	Full	Hard C.	Soft C.	First	Second	Full	Hard C.	Soft C.	First	Second
Mean RMSE	0.0755	0.102	0.1262	0.0708	0.066	0.0731	0.0995	0.1131	0.0834	0.057
Mean $R^{2\,(OOB)}$	0.5137	0.3343	0.3377	0.6462	0.3243	0.5167	0.2723	0.4178	0.6039	0.5353
Mean MAE	0.0614	0.084	0.0989	0.0581	0.0552	0.0578	0.0791	0.0911	0.0641	0.0471
m	12	11	10	12	11	11	5	6	5	11
n	1	4	1	2	1	4	3	4	1	4

Notes: Performance metrics are given as means across all samples of the two 10-fold cross-validations of the final models.

Table 8 presents the results of the Random Forest regression models. Panel A gives VIMs as discussed in Sections 3.5 and 4.4, while Panel B shows performance measures as well as the selected hyperparameters for each respective model. The *MDI* and *PDI* values of Panel A are hereby individually formatted to be shown in green for the highest value in each respective model, and in white for the lowest value. The performance metrics on the other hand are formatted in red for the worst (highest RMSE and MAE, lowest $R^{2\,(OOB)}$) and green for the best performance metric across all Random Forest models considered. Regarding the VIMs, the EUR/USD foreign exchange rate shows high importance scores across most models. Exceptions hereby are the subsample 1 and 2 models of Pearson and subsample 2 model of Gerber, where only relatively low scores are observed. Moreover, only moderate scores are seen in the subsample 1 model of Gerber, and the hard-commodity models of Gerber shows a high *MDI* but only moderate *PDI*. Otherwise, these observations hold true for both *MDI* and *PDI*. In both subsample 1 models however, CNY/USD showed exceptionally high values compared to its VIMs in the other models. For Pearson, the subsample 1 model shows by far the highest VIMs for CNY/USD compared to all other predictors, while for Gerber also EUR/USD, GDP, and IP showed relatively high values. In the Random Forest models of subsample 2 on the other hand, 10-year T-Bills showed the highest *MDI* and *PDI* values. In all other models, however, its VIMs were moderate at most. Furthermore, did the monthly variance of the Bloomberg Commodity Index show relatively high values in certain models: In the hard-commodity model of Gerber, it showed the highest values for both VIMs and in the full model of Pearson the highest *MDI*. Moreover, did both subsample 2 models, the full model of Gerber, and the soft-commodity model of Pearson show the second highest *MDI* for the monthly variance of the Bloomberg Commodity Index. Regarding the other predictor variables included in the models, both the U.S. Energy Price Return Index and U.S. Dollar Index showed extremely low VIMs across all models compared to the other variables. The monthly CPI growth and PPI growth have slightly higher VIMs, while the VIMs of GDP growth, the monthly variance of the S&P 500, and 13-week T-Bill returns show low to moderate values.

In Panel B performance metrics of the models are given, and it is evident that the subsample 2 models performed best considering RMSE and MAE. Regarding the $R^{2\,(OOB)}$ statistics, the first subsample models performed best. Additionally, can be said that considering all metrics, the full models performed moderately, while the hard and soft commodity model of both Pearson and Gerber performed worst. Moreover, it is noteworthy that on average the Pearson models selected higher values for m and lower values for n than the Gerber models during hyperparameter tuning. Nonetheless, did the models perform comparably in each considered sample across co-movement measures considered.

5.1. Validation

In Tables Table **4** to Table **8** the results of 40 models in total are presented. Regarding the linear models, one can see that several assumptions of multiple regressions, VAR, and VARX models are violated when looking at the variables' histograms in Appendix B2: Full descriptive statistics as well as the normality tests of Table 3**: Preliminary tests**. Random Forest regression models on the other hand are less reliant on assumptions like normality and are 10-fold cross-validated twice to obtain reliable performance metrics and VIMs. This cross-validation yields not only reliable results less prone to the random mechanics of the model, but also makes them reproduceable.

Nonetheless, are the results of both linear as well as non-linear models validated by comparing them to the results of similar models considering different samples or variables. More precisely, the results are validated across three dimensions: Firstly, the sample (or time) period included in the model is used to validate results by comparing results across the full sample, first subsample, and second subsample. Secondly, the selection of commodities upon which co-movement measures are computed serves similar purposes, as results are validated and compared across all 20 commodities, only hard commodities, and only soft commodities. Lastly, there are two distinct co-movement measures used as dependent or target variables in the models: Pearson correlations and Gerber statistics. These, although based on different calculations, measure essentially the same phenomenon. Hence, can results of different models (of e.g., the same sample) be validated by comparing significances or effects between models predicting Pearson and Gerber.

By comparing the effects of predictor variables across these three dimensions, they can be deemed validated to at least some extent if their effects hold across different types of models, samples, and co-movement measures. Similarly, model performance measures are validated across these dimensions. This approach is generally in line with previous literature on the topic, with for instance Zaremba et al. (2021) using different co-movement measures and subsamples for validation, and Alquist et al. (2020) comparing the results of different selections (or pairs) of commodities. Despite this, are there recommendations for further research on the validation of the results presented here given throughout the following sections, which discuss these results, and Section 7.3 which summarizes the recommendations.

6. Discussion

Crucial for a thorough interpretation of the findings presented in Section 5 is the performance of the considered models. Table 9 below summarizes two key performance metrics for all models considered and compares them across models: MSEs and R^2.

Table 9: Performance comparison of all models

	Model	VAR		VARX		Multiple Regression		Random Forest	
		MSE	Mult. R^2	MSE	Mult. R^2	MSE	Mult. R^2	MSE	R^2 (OBB)
Pearson	Full Model	0.00320	0.7184	0.00265	0.7664	0.00646	0.4264	0.0057	0.5137
	Hard Commodities	0.00644	0.5879	0.00573	0.6339	0.01105	0.2877	0.0104	0.3343
	Soft Commodities	0.00615	0.7415	0.00532	0.7764	0.01807	0.2360	0.01593	0.3377
	Subsample 1	0.00321	0.7722	0.00252	0.8209	0.00641	0.5415	0.00501	0.6462
	Subsample 2	0.00154	0.7489	0.00134	0.7815	0.00427	0.2939	0.00436	0.3243
Gerber	Full Model	0.00224	0.7960	0.00191	0.8266	0.00751	0.3136	0.00534	0.5167
	Hard Commodities	0.00494	0.6307	0.00447	0.6654	0.01047	0.2132	0.00990	0.2723
	Soft Commodities	0.00340	0.8423	0.00299	0.8613	0.01656	0.2296	0.01279	0.4178
	Subsample 1	0.00204	0.8579	0.00153	0.8932	0.00701	0.5036	0.00696	0.6039
	Subsample 2	0.00129	0.7961	0.00107	0.8304	0.00371	0.4062	0.00325	0.5353

Thereby, the values are formatted as follows: In green for the highest R^2 value and the lowest MSE, and in red for the lowest R^2 value and highest MSE across all models considered. Thereby, both linear and non-linear regression models are given, which makes a direct comparison harder. Hence, should the R^2 values be interpreted with caution as they are multiple R^2 for linear models and OOB R^2 for Random Forest regressions (See Section 4.3). As a more general performance metric applicable to various types of models, MSE is given.

In Table 9 can be seen, that on average, considering both performance metrics, the VAR and VARX models outperformed both multiple regression models and Random Forest models. Thereby, multiple regressions performed the worst, with R^2 values as low as 0.21 and MSEs as high as 0.018. Thereby is notable that the soft and hard commodity models performed worst overall, while subsample 1 models showed moderate performance. Moreover, did the subsample 2 models achieve the lowest MSE for multiple regression models considering each respective co-movement measure. The highest R^2 value however is observed for the respective subsample 1 models. The same observation about performance on different samples holds true for the Random Forest models as well, which outperformed multiple regression models in almost all cases. Solely in the subsample 2 model considering Pearson did the multiple regression model show a lower MSE than the Random Forest regression. Compared to the VAR and VARX models however, both multiple regressions and Random Forest models performed rather poorly. Across all models considering each respective sample and co-movement measure, the MSEs of VAR and VARX models were substantially lower and R^2 values substantially higher than those of the other models presented. Thereby, VARX models achieved even better performance metrics than VAR models in all cases. The best observed MSE of 0.01 was recorded for the VARX subsample 2 model of Gerber, while the best R^2 value is seen for the VARX subsample 1 model of Gerber. Nonetheless, did the VAR and VARX models also perform worse on the hard commodity models compared to models of other

samples, similarly to multiple regressions and Random Forest models. The respective soft commodity models however yielded the second worst MSEs across VAR or VARX models but rather high R^2 values.

Concluding this, the model performance comparison can be summarized as follows: Overall, VARX models performed the best, closely followed by VAR models. Multiple regressions performed the worst overall, while Random Forest regressions performed slightly better. Across all types of models, the hard commodity co-movement models performed comparably poorly, with soft commodity models performing even worse for multiple regressions and Random Forests but better than hard commodity VAR and VARX models. Regarding the co-movement measure predicted, the respective VAR and VARX models of the Gerber statistic outperformed those of Pearson correlations, while for multiple regressions and Random Forests the picture is less clear.

By establishing which models achieved the best results, the research questions formulated in Section 1.1 above can be answered more meaningfully using the findings of Section 5. Firstly, tests for difference in means between the two subsamples are performed. They show that there are statistically significant differences between mean co-movement levels in the two subsamples for both average Pearson correlations and average Gerber statistics. Furthermore, it was shown that on average, subsample 1 showed substantially higher levels of co-movement than subsample 2. These findings are further supported by the preliminary analyses of Section 4.4, in particular Figure 3: Commodity price co-movement in the sample period and Table 2. Resulting from this, we can answer Sub-question 5, since the evidence suggests that there is a significant structural change in levels of commodity price co-movement during the considered sample period.

Following this, the results of all models implemented in this research are given. Using this in combination with the model performance comparison above, Sub-questions 1 to 4 can be answered. To recall, Sub-question 1 addressed whether there is evidence of significant macroeconomic factors acting as determinants of commodity price co-movement in the sample period. Sub-questions 1a and 1b broke this down further into significant effects of direct and indirect factors respectively, as defined in Section 2.3. Regarding Sub-question 1, two aspects are important to consider: Firstly, that throughout all models, certain variables showed significant coefficients for linear models and comparably high VIMs for non-linear models, and secondly, that two of the four model types presented show relatively high adjusted R^2 values of about 0.7[23]. Hence, can Sub-question 1 be answered as the results suggest that there is evidence of significant macroeconomic factors acting as determinants of commodity price co-movement. In Sub-question 1a, only the effects of proxies for supply and demand which were specified as direct factors are investigated. There, the results showed mixed findings, with most coefficients of supply and demand proxies not being significant across most models. However, there were some cases in which their coefficients showed significant effects on commodity price co-movement. For instance, regarding demand factors, did the first lag of industrial production growth in the full VARX model predicting Gerber statistics show significantly negative effects. Throughout the other VARX models predicting Gerber statistics,

---

[23] Only the full VAR model of Pearson showed an adjusted R^2 value slightly below 0.7, while all other full VAR and VARX models showed R^2 values above that.

43

it also showed negative effects at differing significance levels, except for the second subsample model. Otherwise, it only showed significance in some subsample VAR models and in one subsample VARX model for Pearson correlations. GDP growth on the other hand, with a few scattered exceptions, did not yield statistically significant coefficients. For supply factors, PPI and the U.S. Energy Price Return Index were investigated. The second lag of PPI did hereby show negative effects in the full and soft commodity VAR models predicting Pearson, but otherwise only scattered significant effects. Similarly, the Energy Index showed negative effects at 10% significance in the full VARX model predicting Gerber, but only scattered significances in the other models. In the Random Forest models, direct factors showed comparably low VIMs, with demand factors having slightly higher values, and in multiple regressions direct factors showed no significance. As a result of these conflicting findings, we cannot trust the effects of direct factor on commodity price co-movement we found to be valid. Therefore, Sub-question 1a is answered as there is no clear evidence of proxies for direct factors related to supply and demand significantly affecting commodity price co-movement. To answer Sub-question 1b, indirect factors were investigated. Thereby, both the EUR/USD foreign exchange rate and the monthly Bloomberg Commodity Index variance showed high significance across most models. Both variables showed highly significant effects in the multiple regressions and VARX models, while EUR/USD also showed significance in the VAR models of Gerber (where BCI variance only showed effects at 10% significance). For EUR/USD, these effects are consistent across most Random Forest models as well, but for BCI variance they only hold for some Random Forests. Furthermore, did both CPI growth and 10-year U.S. T-Bill yields show significant effects in the VAR model of Pearson at a level of 5%. 10-year T-Bill yields moreover show high VIMs in the subsample 2 Random Forest models. Concluding from this, Sub-question 1b is answered as the findings suggest indirect macroeconomic factors, in particular foreign exchange rates (EUR/USD) and commodity market uncertainty (BCI Var), to have significant effects on commodity price co-movement levels.

To investigate Sub-question 2, the outcomes of the two subsample models are compared. This gives insights into how the effects of macroeconomic factors on commodity price co-movement change over time by investigating both subsequent periods separately. In multiple regressions, the significance of many coefficients changed over time, with many only being significant in the first subsample and some exclusively in the second. In case of significance in both subsamples, the signs of the coefficients changed, except for the monthly BCI variance which showed positive effects throughout both periods. For the VAR and VARX models considering the two subsamples, the significance of most effects differs between the two periods as well. Again, in case of significant effects in both respective models, the sign of the coefficients' changes.[24] Hence, do these results suggest the effects of macroeconomic determinants on co-movement levels to change over time in the period considered. While the effects of various variables change either sign or significance, those of the two variables considered to be valid as determinants should be considered as most important. In the most precise models, their effects changed in significance and cannot be regarded to be statistically significant in both subsamples. Resulting from this, we can answer Sub-question 2 and state

---

[24] An exception hereby is the second lag of the EUR/USD rate in the relevant VAR and VARX models of Gerber statistics, where the signs did not change. However, did the coefficients of the subsample 2 models only show effects at 10% significance.

that the effects of determinants vary over time. Additionally, these effects are mostly significant in the first subsample with higher co-movement. Hence, is indicated that macroeconomic factors influence commodity price co-movement more when it is exceptionally high.

To answer Sub-question 3, differences between the co-movement of hard and soft commodities were investigated. This can be split into three parts: co-movement levels, the proportion of excess co-movement that remains unexplained, and effects of macroeconomic predictors. In Table 3, Wilcoxon rank-sum tests show that the mean co-movement levels in subsamples 1 and 2 are statistically different. Measured by Geber statistics, this is significant at the 1% level, but measured by Pearson correlations it is only significant at 10%. Table 2 furthermore shows descriptive statistics, where hard commodities showed higher levels of co-movement of price returns than soft commodities. For Gerber statistics, this is more pronounced than for Pearson correlations. Moreover, do the performance metrics in Table 9 show that co-movement for exclusively hard or soft commodities was harder to explain by the models applied. In VAR and VARX models, soft commodities showed lower MSEs and higher multiple R^2 values than hard commodities. This is consistent with the adjusted R^2 values of Section 5.[25] Hence, were soft commodities predicted with more accuracy and show less excess co-movement based on the results presented earlier. Regarding the effects of macroeconomic predictor variables, there were some consistencies across hard and soft commodity models. For instance, did the monthly BCI variance show significant effects across relevant VARX models at 5% and multiple regression models a 1%. The EUR/USD exchange rate only showed this for multiple regressions of Pearson at a level of 1%, while all other consistencies were only significant at a 10% significance level. Moreover, the hard and soft commodity models show several significant effects of certain variables that are not seen in the respective other soft or hard commodity model. The Random Forests also show some differences in VIMs of hard and soft commodity models, with this being more pronounced for models predicting Gerber statistics. Considering these three points of focus, we can answer Sub-question 3 and conclude that the findings suggest the magnitude of co-movements, the proportion of unexplained excess co-movement, and the effects of macroeconomic factors to differ between hard and soft commodities. Moreover, does the co-movement of prices of exclusively hard or soft commodities seem to be harder to predict than that of various types of commodities. However, might the number of commodities considered in co-movement measures also have an influence on this.

In Sub-question 4 it is examined whether novel co-movement measures and methodologies allow to predict commodity price co-movement more accurately, resulting in less excess co-movement. Hence, is the performance of models applying Gerber statistics compared to that of models using Pearson correlations, and the performance of Random Forest models to traditional ones. Regarding Random Forest models, it can be seen in Table 9 that in terms of MSE values, both VAR and VARX models vastly outperform the Random Forest models that were applied. In fact, did Random Forest models show only slightly lower MSE values than multiple regression models. In one case, the second subsample model of Pearson, the multiple regression even showed a better MSE. This also holds true when considering R^2 values, however, with more differences between Random Forests and multiple regressions. These R^2

---

[25] For the worse performing models, multiple regressions, and Random Forest regressions, this does not hold true. In the context of excess co-movement in this case, however, only the best performing models are considered.

values should be interpreted with caution, however, since they are not directly comparable. To recall: The performance measures of Random Forests were hereby collected on out-of-bag samples across the two 10-fold cross-validations. Hence, do they represent the performance of the Random Forests on unforeseen data with some confidence, but it was not split into separate training and test sets. Therefore, MSEs are the more general and reliable measure to assess this. Moreover, the Gerber statistic was used as a novel measure of asset price co-movement, which was said to be superior to Pearson correlations in several aspects according to past research (Algieri et al., 2021). Again, its performance compared to Pearson correlations can be seen in Table 9. There, VAR and VARX models predicting 6-month moving average pairwise Gerber statistics outperformed those predicting Pearson correlations in all cases. Multiple regressions showed rather mixed results, with Gerber statistics overall showing lower MSE values but also lower R^2 values. In Random Forests, the models predicting Gerber statistics outperformed those of Pearson correlations across both performance metrics, except for the first subsample models.[26] Resulting from this, we can conclude two findings to answer Sub-question 4: Firstly, do the results suggest Gerber statistics to allow for better co-movement prediction accuracy, at least in the best performing models. Secondly, is there no evidence that the use of Random Forest regressions increases prediction accuracy. This, however, might this be due to the research's limitations mentioned in the Section 7.2 and should encourage further research.

---

[26] The hard commodity Random Forest regressions also showed a higher OBB R^2 for the Pearson model, but a lower MSE for the Gerber model.

7. Conclusion

To recall, the main research question of this thesis was formulated as follows: "How well can different types of co-movement measures and models with macroeconomic determinants as inputs explain commodity price co-movement between mid-2003 and early 2023?"

To answer it based on the findings of Sub-questions 1-4, several aspects are considered. Firstly, did VARX models followed by VAR models explain commodity price co-movement best in terms of performance metrics. Random Forests on the other hand only achieved slightly better results than multiple regressions, which overall performed worst. Moreover, at least in the two best performing types of models, Gerber statistics showed an edge over Pearson correlations regarding model performance. Additionally, do the results suggest two key findings closely related to the research question: Overall, does the co-movement of exclusively hard and soft commodities seem to be harder to model than that of various commodities, at least with the models applied. Furthermore, did macroeconomic determinants generally show more effects in the subsample with higher co-movement, where models also achieved higher R^2 values. This indicates that commodity price co-movement is easier to explain with macroeconomic variables and the considered methods when it is exceptionally high.

7.1. Implications and contribution to the literature

The contribution of this thesis to the existing literature on commodity price co-movement can be summarized as follows: Firstly, it gives further insights into the effects of macroeconomic variables on overall co-movement levels, adding to the findings of previous studies. Moreover, it employs the Gerber statistic and hence contributes to the growing body of literature on the application of this co-movement measure in the context of commodity prices. It does so especially, by relating its use to model performance and the usefulness of Gerber statistics in minimizing excess co-movement. Lastly, this thesis newly introduced Random Forest regressions to the concept of commodity price co-movement, giving novel insights into their applicability in this context.

Regarding the determinants of commodity price co-movement investigated in this paper, it is notable that overall, variables classified as indirect factors in Section 2 showed more significance throughout the models considered. This adds to the findings of Alquist et al. (2020), who showed that most movements in commodity prices can be attributed to indirect factors. For interest rates, no relationship was detected in most models, which is in line with the research of for instance Ai et al. (2006) and de Nicola et al. (2016). The full sample VAR model predicting Pearson correlations, shows a significant, negative relationship between 10-year Treasury bill yields and co-movement levels, which would add to the findings of Byrne et al. (2013) and Gruber and Vigfusson (2018). This relationship, however, cannot be validated across the other models considered (except for the full sample multiple regression of Pearson). Regarding exchange rates, we found statistically significant effects on co-movement level, especially for the EUR/USD exchange rate. This somewhat complements the findings of Poncela et al. (2014), who found increases in exchange rates to negatively impact commodity prices. Nonetheless, it is contrary to the results presented in Ai et al. (2006) and de Nicola et al. (2016) where little to no significant effects are seen. Lastly, for stock market uncertainty, no consistent significance across models is seen. This is contradicting the positive relationship

found by de Nicola et al. (2016). However, did commodity market uncertainty, as measured by monthly BCI variances, show significant, positive effects in many cases.

Following the argumentation made in Ai et al. (2006), who concluded no excess co-movement to be present with adjusted R^2 values between 0.864 and 0.912 in their equilibrium model, the same could be concluded for certain results of this research. Thereby is notable, however, that pairwise co-movement was investigated in Ai et al. (2006) and that additional tests were performed. Nonetheless, did the subsample 1 VARX model of Gerber statistics, for instance, achieve an R^2 of 0.84 while the respective full model showed an R^2 of 0.77. Hereby is considered that the included macroeconomic variables are far from exhaustive, and that real world limitations such as measurement error might make it impossible to predict commodity price co-movement with perfect accuracy. Regardless of the existence of excess co-movement, it was established that Gerber statistics showed superior model accuracy compared to Pearson correlations in the most relevant models. In demonstrating this, the research presented here provides a valuable contribution to the literature on commodity price co-movement and should encourage future research to apply Gerber statistics in this context. Adding to this, Random Forest regressions were applied to model commodity price co-movement for the first time in existing literature. These however, contrary to the use of Gerber statistics, did not increase prediction accuracy. Hence, this research does not indicate their usefulness in the given context, but further research and academic literature on the applicability of Random Forests to this topic is needed because of the limitations mentioned in Section 7.2.

The results of this paper have several implications, both for future research and for professional investors. For future research, some implications were already mentioned as part of the contribution to literature. To recall: most notably, should academia consider the use of the Gerber statistic in future work on the topic of commodity price co-movement. Its performance should be further validated by comparing it to other co-movement measures like multiple R^2, and by investigating the effects of altering Gerber thresholds. Moreover, the results on impacts of macroeconomic variables on commodity price co-movement presented here should be further validated. For instance, this could be done with additional robustness checks such as additional explanatory variables and a broader consideration of commodities. Moreover, it should be checked whether the results hold across different countries and not only the United States. Regarding VAR models and other time-series models, the results should be reinvestigated using more lags in larger datasets. More importantly, however, future research should reinvestigate the performance of Random Forest models using larger datasets, as they did not show to be effective in predicting commodity price co-movement according to the results presented earlier. Nonetheless, offer Random Forests a novel perspective on the topic and may provide certain benefits to linear models, e.g., since they are not reliant on as many data assumptions, which are oftentimes violated when dealing with financial data. Thus, should they be further considered and not excluded from future research on this topic.

To professional investors, especially the insight of enhanced model performance when applying Gerber statistics as the measure of co-movement for commodities might prove beneficial. This can have implications on e.g., quantitative portfolio management strategies applied to commodities. Here is notable however, that Gerber statistics are already used by

certain institutional investors.[27] Moreover, do the results on determinants of commodity price co-movement suggest further real-world implications for investors: When modeling co-movement of commodity prices, two variables should be considered particularly. Firstly, the EUR/USD exchange rate, and secondly commodity market volatility.

7.2. Limitations

To highlight this research's main limitations, residual analyses are given firstly, with other limitations being discussed consequently. Table 10 gives the results of the residual analyses, while additionally impulse response functions of the full sample VAR models are given in Appendix B4: Impulse response functions.

Table 10: Residual analyses

Model			Shapiro-Wilk	Durbin-Watson	Breusch-Pagan	Ljung-Box
Pearson	Full	Multiple Regression	0.987**	0.711	21.033**	
		VAR	0.937***		0.852	0.016
		VARX	0.958***		1.162	0.001
		Random Forest	0.992			
	Hard C.	Multiple Regression	0.991	0.768	11.626	
		VAR	0.980***		0.001	0.046
		VARX	0.987**		0.186	0.099
		Random Forest	0.994			
	Soft C.	Multiple Regression	0.991	0.768	11.626	
		VAR	0.98***		0.001	0.046
		VARX	0.987**		0.186	0.099
		Random Forest	0.994			
	Sub. 1	Multiple Regression	0.975**	0.804	14.112	
		VAR	0.985		1.740	0.018
		VARX	0.987		0.144	0.060
		Random Forest	0.978*			
	Sub. 2	Multiple Regression	0.987	0.692	27.010***	
		VAR	0.960***		1.147	0.000
		VARX	0.968***		0.746	0.442
		Random Forest	0.987			
Gerber	Full	Multiple Regression	0.980***	0.471	34.621***	
		VAR	0.980***		9.593***	0.049
		VARX	0.992		7.593***	0.006
		Random Forest	0.981***			
	Hard C.	Multiple Regression	0.991	0.625	12.330	
		VAR	0.994		5.513**	0.007
		VARX	0.992		0.337	0.077
		Random Forest	0.995			
	Soft C.	Multiple Regression	0.949***	0.360	10.881	
		VAR	0.984**		6.344**	0.397
		VARX	0.983***		3.305*	0.507
		Random Forest	0.966***			
	Sub. 1	Multiple Regression	0.994	0.684	21.883**	
		VAR	0.995		0.619	0.313
		VARX	0.992		0.044	0.547
		Random Forest	0.982			
	Sub. 2	Multiple Regression	0.980*	0.530	13.642	
		VAR	0.969***		2.542	0.194
		VARX	0.974**		3.128*	0.062
		Random Forest	0.978*			

---

[27] McElhaney, A. (2022*). A hedge fund manager reluctantly challenged - then collaborated with - Harry Markowitz*. Institutional Investor.
https://www.institutionalinvestor.com/article/2bstnue49xqx5peulg2yo/culture/a-hedge-fund-manager-reluctantly-challenged-then-collaborated-with-harry-markowitz

In Table 10, Shapiro-Wilk tests to check for normality in the residuals are given for all models, and Durbin-Watson tests to check for autocorrelation in multiple regression models. For VAR and VARX models, Ljung-Box Q-tests were used to check for autocorrelation in the equations of interest. Moreover, Breusch-Pagen tests investigating heteroscedasticity are given for all linear models. Thereby, next to test statistics, applicable p-value significances are given as in Section 5. In the column depicting Shapiro-Wilk tests, many model residuals do not qualify to be considered normally distributed, as their p-values lie below the given significance levels. For the full models, only the Random Forest residuals show normality when predicting Pearson, while when predicting Gerber statistics only the VARX residuals do so. In the remaining Pearson models, VAR and VARX model residuals did not show normality, except for in subsample 1. For multiple regressions and Random Forests of Pearson, the inverse is true, however. The remaining models predicting Gerber statistics, on the other hand, did not show normally distributed residuals in soft commodity and subsample 2 models, but normally distributed residuals in the hard commodity and subsample 1 models, at least at 10% significance.[28]

Moreover, Breusch-Pagan tests were applied to check for heteroscedasticity in all linear models presented. Considering Pearson correlation models, the VAR and VARX models can be considered to have homoscedastic residuals, while multiple regressions showed unequal variance in residuals in the full and subsample 2 models. For Gerber statistics, all linear models of the full sample showed heteroscedasticity in error terms at a 1% significance level. Also, across subsample 2 and hard and soft commodity models, VAR and VARX models showed heteroscedasticity in some cases. Multiple regressions on the other hand only showed this in subsample 1. Hence, do the findings indicate that, with regards to heteroscedasticity of residuals, Pearson correlations might have an edge over Gerber statistics when modelling commodity price co-movement. Lastly, to investigate autocorrelation, Durbin-Watson tests are used for multiple regressions and Ljung-Box Q-tests (Q-tests) for the equations of interest in VAR and VARX models. These tests indicate for multiple regressions, since all test statistics are smaller than 2, that there is positive autocorrelation present in all models. The Q-tests for autocorrelation applied to VAR and VARX models, however, do not suggest autocorrelation in residuals of the equation of interest in all models considered. Hence, should the results of the multiple regressions be interpreted carefully with regards to autocorrelation, since it can lead to biased estimated and standard errors, and can indicate the presence of spurious relationships between the variables (See e.g., Hayashi, 2011). The VAR and VARX models, however, do not seem to have this limitation.

Additionally, to these tests, IRFs are given in Appendix B4: Impulse response functions for the full sample VAR models, to investigate the behavior of variables used as determinants of commodity price co-movement in VAR frameworks. They give further insights into the direction and duration of responses. There can be seen, considering both Pearson correlations and Gerber statistics, that several variables show persistent effects as their IRFs do not return to zero within the 10 periods shown. This can indicate that VAR(2) models like the ones used in this research might not consider enough lags to capture all relevant relationships between the macroeconomic variables and co-movement levels. The results of the Q-Tests, on the other

---

[28] Hereby is notable, that Random Forest models do not necessarily rely on normality in input variables nor residuals. This was still investigated, however, to get insights into the distribution of their residuals.

hand, do not suggest so as no autocorrelation is present. Here needs to be considered, that the lag number of 2 was determined using the AIC, but only values up to 6 were investigated as this is the time frame of the moving averages predicted. In larger datasets with moving averages of longer periods for instance, information criteria might thus suggest other lag numbers.

This leads us to another crucial limitation of the research presented here: its data. Thereby must be noted firstly, that the overall dataset size is rather limited. Hence, do the models only have a rather small number of observations to be trained with. This has implications especially on Random Forest models, since they usually work with significantly larger datasets (See e.g., Breiman, 2001). In conjunction with the dataset size, the time horizon investigated in this study is rather limited as well. It encompasses almost the last 20 years, but commodity price co-movement was shown to reach high levels prior to this in recent research (Zaremba et al., 2021) and those periods should also be considered for a more thorough analysis. Moreover, only a monthly data frequency is considered, because of the availability of data on macroeconomic factors. Daily data, however, might reveal additional relationships between the variables that are not seen when investigating commodity price co-movement on a monthly basis. This would also result in drastically more datapoints, which might enhance the performance of Random Forest applications to this context. Lastly, regarding data limitations, the variables used in this research must be mentioned. For co-movement, it is crucial to note that only 20 commodity price series were considered in the co-movement measures, and only 10 for co-movement of exclusively hard or soft commodities. Hence, could co-movement measures incorporating more commodities yield different results. Adding to this, must be noted that also the list of macroeconomic determinants considered here is far from exhaustive. There are several factors not considered in this research, such as for instance daily market liquidity (Zhang & Ding, 2021; Zhang et al., 2019), and latent variables such as forecasts (Pindyck & Rotemberg, 1990). Regarding supply and demand factors, more precise estimates than simple proxies could be used as well. In past research, this was for instance done using inventory levels (Ai et al., 2006). Lastly, another relevant limitation to this research's results is the restricted amount of robustness checks performed here. There are the subsample models and models investigating only hard and soft commodities to validate the results of the full sample models to some extent, but additional robustness checks should be considered in further investigations of co-movement. Some possible robustness checks for validation of the results are presented in the following section, along with other suggestions for further research.

7.3. Future research

Regarding robustness checks for additional validation of the results, several aspects are to be considered in future research. As mentioned earlier: More explanatory variables should be included in models of commodity price co-movement to reinvestigate effects of certain factors. Similarly, should co-movement of different or more extensive selections of commodities be investigated. Moreover, should longer time spans be considered in future work on the topic, perhaps with daily data frequency.

Furthermore, are there several other suggestions for future research. Firstly, did the research presented in this thesis only focus on commodity prices in U.S. Dollar and macroeconomic variables in or directly related to the United States. Hence, should the findings robustness across different countries be considered in validations of future research. Moreover, should be investigated whether the co-movement of only hard and soft commodities is indeed harder to

predict than that of various types of commodities, since the research presented here suggests this. Adding to this, should commodities not only be classified into hard and soft commodities but rather into sectors to decompose this further and give insights into differences between commodity sectors.[29] The research also suggests that when co-movement levels are high, macroeconomic variables show more significant effects. As a result, it should be examined whether this higher co-movement is indeed due to responses to macroeconomic variables unique to commodity markets, or whether it is because of broader market responses like contagion as the period coincides with the 2008 financial crisis. Most importantly, however, does this research hold two direct implications for further research: The superiority of Gerber statistics over Pearson correlations and the relatively bad performance of Random Forest regressions in the context of commodity price co-movement. For research on Gerber statistics, the threshold of the Gerber statistic might be adapted, as was indicated in Gerber et al. (2022). This could be done by considering a different fraction of the returns' standard deviation or a different, more robust measure as the basis of the threshold. Regarding its comparison to traditional measures of co-movement, other metrics like multiple R^2 should be investigated, as was done in e.g., Zaremba et al. (2021). Random Forest models on the other hand should be applied to larger datasets, and it should be evaluated in further research whether they offer benefits over linear models due to for instance less reliance on assumptions.

7.4. Concluding remarks

The research presented in this thesis aims to give insights into the applicability of various methods and co-movement measures in the context of commodity price co-movement. The findings show that commodity price co-movement can be explained rather well with macroeconomic determinants, depending on the methodology used. VAR and VARX models applying Gerber statistics thereby outperformed those applying Pearson correlations and showed the highest prediction accuracy overall, questioning the existence of excess co-movement when drawing similar conclusions to past research. Random Forest models on the other hand, did not prove very effective in predicting common movements of commodity prices in this research. Across all models considered, the EUR/USD exchange rate seemed to be the most influential determinant, followed by commodity market uncertainty as measured by monthly commodity index variances.

The main contribution of this thesis is the comparison of both traditional co-movement measures with novel ones, and established, linear models with non-linear machine learning algorithms. In doing so, this research provided a groundwork for further evaluation of methodologies to investigate commodity price co-movement. With more thorough comparisons of this and more complex applications of the best suited methods, the existence of excess co-movement as proposed more than 30 years ago by Pindyck and Rotemberg (1990), may soon be revisited more systematically.

---

[29] For instance, see Table 12.

Appendix A

Appendix A1: Study selection process

The database Semantic Scholar[30] was used as the source of the academic papers initially searched for in Phase 1. There, it was searched for the search terms presented in the figure below under "search terms", whereby must be noted that the first and second terms are the same except for the spelling of co-movement. Neither of the ways of spelling ("co-movement" and "comovement") is correct or false and both are used in academic literature. Hence, do the search results differ and are relevant for both search terms, although some articles are shown in either case. The right-hand side of the figure depicts the number of articles found for each search term in the order they are being presented in. These numbers however may not be simply added up because of the duplicate articles that are not dependent on the spelling of co-movement. To restrict the search results as part of Phases 2, 3, and 4, filters available on the Semantic Scholar´s website were used. Firstly, papers older than 2010 were filtered out in Phase 2, since the scope of this thesis is on recent years and since there were interesting developments in commodity price co-movements in recent years which are discussed in the literature review. Consequently, the database's results were filtered by their study field in Phase 3, whereby only papers within the study fields of business and economics were selected. As a final restriction based on the papers' importance, all results with less than three citations were filtered out in Phase 4. This was done, since there was a too large number of potentially relevant papers to investigate effectively after the restriction of Phase 3. The restrictions of these selection phases resulted in 56, 75, and 50 search results for each of the search terms respectively, including a few duplicates. From there on, papers were selected based on the fit to the research of this thesis. Moreover, the remaining duplicates were excluded which resulted in 29 key articles to be considered for the literature review at the end of Phase 5. Additionally, several exceptions which are the product of snowballing and further searches were added to this selection in Phase 6. This resulted in a total of 36 papers, although not all are directly cited.
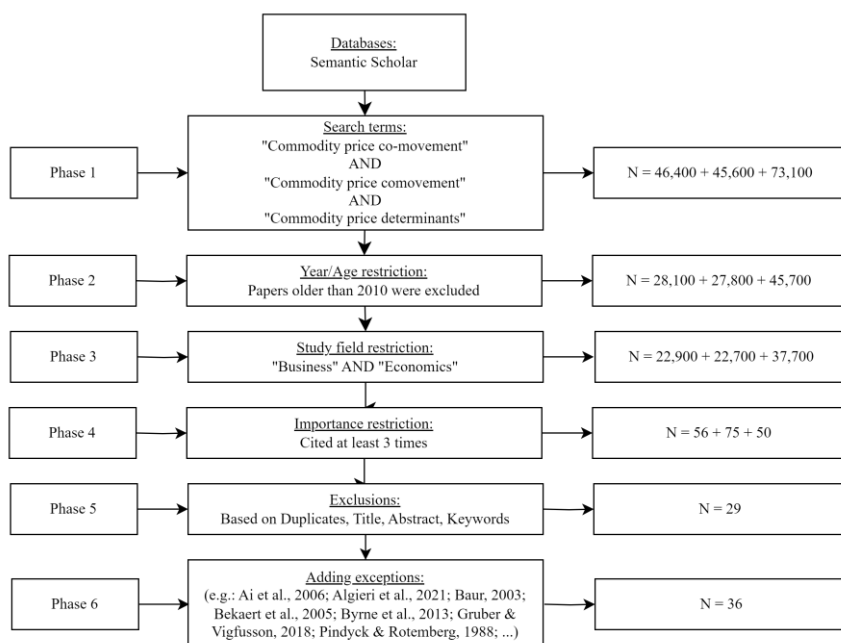
Figure 4: Literature selection process

[30] Semantic Scholar. (2023). *A free, AI-powered research tool for scientific literature*. https://www.semanticscholar.org/

Appendix A2: Literature comparison

Table 11: Overview of highly connected literature

Article	Main methods used	Validation methods	Dataset sizes	Co-movement measure	Commonly used variables	Model fits
Ai et al. (2006)	Regression (OLS, 2SLS), 'Equilibrium model', 'Macro model'	Cross-validation approach	Q1/1957-Q4/2002, quarterly	Pearson's correlation, Spearman rank correlation	Equilibrium model: U.S. prices, inventories, harvests/yields Macro model: IP, GDP, CPI, 3m T-bills, Dollar index (FXR)	Change in log price (Adj.R^2): Macro model (OLS): 0.016-0.108 Equil. model (2SLS): 0.308-0.542 Price levels (Adj.R^2): Macro: 0.58-0.719 Equil.: 0.864-0.912
Algieri et al. (2021)	CARML model, DCC model, (historical simulation)	Brier (skill) score, Robustness checks (Gerber thresholds)	N = 3500	Time-varying Gerber correlations	(Recession phases identified by the National Bureau of Economic Research)	Brier skill scores CARML: 0.22-3.245 DCC: -0.512-4.242 FHS: -3.201-1.152
Alquist et al. (2020)	Factor model, FAVAR	Forecasting application, Robustness checks, Moment condition length	Jan 1968 - Jan 2013, monthly, (missing cases)	Rolling R squared of 1-year price changes	Indirect common factor, Direct common factor, U.S. CPI, IP (OPEC production shocks, U.S. economic conditions)	MSPE Ratios: 0.921-1.106
Antwi et al. (2021)	Empirical mode decomposition (EMD), Variational mode decomposition (VDM)	-	N = 1277	-	High frequency, low frequency and trend components	-
Bakas and Triantafyllou (2020)	VAR model	Alternative volatility measures, alternative IRF decomposition, excluding observations	Q1/1996-Q1/2020	-	World pandemic uncertainty index (WPUI), World industrial production index (WIP), Geopolitical risk index (GPR)	-
Baur (2003)	Multinominal logit model, multivariate GARCH model	Simulation study	N = 389	Introduced in the paper	-	-
Byrne et al. (2013)	FAVAR, Panel methods (Panel analysis of nonstationary and idiosyncratic components, PANIC)	Robustness checks (Alternative risk measures/GDP, additional explanatory variables)	Time dimension T = 109 cross sectional dimension = 24	Correlation methodology of Ng (2006)	U.S. CPI, real interest rate, risk (stock market uncertainty), demand (real U.S. GDP growth), supply shocks (real crude oil prices)	R^2 of real commodity prices and macroeconomic variables: 0.13 (real interest rates), 0.18 (risk), 0.07 (output growth), 0.02 (oil prices)
Byrne et al. (2019)	Dynamic Hierarchical Factor model, Time varying parameter FAVAR	Alternative proxies as model variables	Q1/1974-Q3/2014	Modeled by factors	Demand (U.S. IP), interest rate (3m T-bills), uncertainty (Variances of S&P GSCI, S&P 500), U.S. CPI	Between 42.4% and 86.6% unexplained commodity price variance depending on sector
Cai et al. (2019)	three-dimensional continuous wavelet transform method, ARFIMA-FIGARCH, copula model	Sub-sample and model comparison	Jan 1991 - Dec 2018, weekly	Pearson correlation	-	-

Study	Models/Methods	Robustness checks	Data period	Co-movement measure	Variables	Results
Chien et al. (2021)	Wavelet coherence method, wavelet-based Granger causality tests	Sensitivity analysis wavelet-based Granger causality	31st December 2019 - 1st August 2020, daily	Derived from Wavelet approach	COVID-19 observations, oil and gold prices, US-EPU (news-based index), GPR (USA-geopolitical risk index), U.S. stock price index, FDI, gross domestic income, trade integration, emission intensity, exchange rates	-
Janzen and Smith (2012)	SVAR model	Robust standard errors, Wild bootstrap procedure, considering other commodities	01/1968-12/2011, monthly	Price of crude oil to represent co-movement effects	Global economic activity, current and expected supply and demand conditions, financial speculation (to observed cotton prices)	-
de Nicola et al. (2014)	VAR, DCC models, rolling regression procedure, Structural break test, uniform spacing analysis	Newley-West correction, BIC, alternative measures of returns	01/1970–05/2013, monthly	Rolling unconditional correlations, Dynamic Conditional Correlations, Constant Conditional Correlations	Standard deviation of S&P 500 returns, real interest rate, nominal prices of commodities	-
de Nicola et al. (2016)	Unrestricted VAR model, Ng's (2006) methodology, DCC models, Rolling regression approach of Tang and Xiong (2012)	Standardized spacings variance ratio test statistic (SVR), breakpoint analysis, Uniform-spacings analysis	520 monthly observations, subsamples	Ng's (2006) methodology, consideration of second mixed moments	Trade-weighted U.S. Dollar index, 13w U.S. T-Bills, CPI, S&P 500 returns	-
Le Pen and Sévi (2017)	Factor models, Principal components method, linear regression using factors	Hansen's (1982) J-test	02/1993-11/2013, monthly (N=250)	Conditional correlations (DCC)	184 real and nominal macro variables from both emerging and developed countries,	Component R^2 ranging from 0.093 to 0.3739, regression R^2 from 0.0107 to 0.2858
Pindyck and Rotemberg (1990)	Instrumental variables regression, OLS regression, Latent variable models	unrestricted contemporaneous covariance matrix	01/1960-12/1985, different frequencies	R-squared values of regressions for excess co-movement	Various macroeconomic variables (IP, interest, inflation, etc.), latent variables	R^2 ranging from 0.09 to 0.39 (latent variable model), higher when including prices of other commodities
Poncela et al. (2014)	Dynamic factor model, FAVAR	different Cholesky ordering of the variables in the VAR	02/1992-12/2012, monthly	(Residual) factor correlations	44 commodity prices, VIX, MSCI World, exchange rate, world IP, Energy index	
Zaremba et al. (2021)	Pearson correlations, Gerber statistic, Multiple R^2	alternative estimation periods, prices in GBP instead of USD, including zero-return months, number of principal components R^2	01/1850-05/2019	Pearson correlations, Gerber statistic, Multiple R^2	Monthly commodity returns, Principal component of commodity returns	-
Zhang et al. (2019)	Regression, Dick-Fuller tests for residuals	-	01/2005-12/2013	Correlation matrices	Spot and future prices of commodities, liquidity measures, CRB commodity index	F-values of regressions

Appendix B

Appendix B1: List of Commodity future continuations

Table 12: Commodity Continuous Future price series

Commodity Type	Sector	RIC	Exchange	Commodity	Future Type	Notes
Hard Commodities	Energy	CLc1[i]	NYMEX	Light Sweet Crude Oil (WTI)	Electronic Energy Future Continuation 1	
		LCOc1[ii]	ICE	Brent Crude Oil	Electronic Energy Future	Price correction
		NGc1[iii]	NYMEX	Henry Hub Natural Gas	Electronic Energy Future Continuation 1	
		HOc1[iv]	NYMEX	NY Harbor ULSD	Electronic Energy Future Continuation 1	"Heating oil"; Price corrections
	Base Metals	HGc1[v]	COMEX	Copper	Composite Commodity Future Continuation 1	Price corrections
		SAFc3[vi]	SHFE	Aluminium	Commodity Future Continuation 3	in CNY, converted to USD
	Precious Metals	GCc1[vii]	COMEX	Gold	Composite Commodity Future Continuation 1	
		SIc1[viii]	COMEX	Silver	Composite Commodity Future	Price corrections
		PLc1[ix]	NYMEX	Platinum	Electronic Commodity Future Continuation 1	Imputation of missing values
		PAc1[x]	NYMEX	Palladium	Electronic Commodity Future Continuation 1	Imputation of missing values
Soft Commodities	Grains	Wc1[xi]	CBoT	Wheat	Composite Commodity Future Continuation 1	
		Cc1[xii]	CBoT	Corn	Composite Commodity Future	
	Oilseeds	Sc1[xiii]	CBoT	Soybeans	Composite Commodity Future Continuation 1	
	Livestock	LCc1[xiv]	CME	Live Cattle	Electronic Commodity Future Continuation 1	
		LHc1[xv]	CME	Lean Hogs	Electronic Commodity Future Continuation 1	
	Fibres	CTc2[xvi]	ICE	Cotton No. 2	Futures Electronic Commodity Future Continuation 2	
	Other softs	SBc1[xvii]	ICE	Sugar No. 11	Futures Electronic Commodity Future Continuation 1	
		KCc2[xviii]	ICE	Coffee C	Futures Electronic Commodity Future Continuation 2	"Coffee Arabica"
		CCc2[xix]	ICE	Cocoa	Futures Electronic Commodity Future Continuation 2	
		STFc1[xx]	SGX	TSR20 Rubber	Commodity Future Continuation Month 1	

Appendix B3: Information criteria results and RF learning curves

Table 13: AIC (and BIC) results for VAR and VARX models

		Sample	Lag number	1	2	3	4	5	6
VAR models	**Pearson**	All	AIC	-140.79	-142.21	-142.14	-141.75	-141.67	-141.45
			BIC	-138.02	-136.88	-134.24	-131.29	-128.64	-125.86
		HC	AIC	-140.05	-141.38	-141.27	-140.87	-140.78	-140.56
			BIC	-137.28	-136.05	-133.38	-130.41	-127.76	-124.97
		SC	AIC	-140.02	-141.56	-141.38	-141.05	-140.96	-140.75
			BIC	-137.26	-136.23	-133.48	-130.59	-127.93	-125.15
	Gerber	All	AIC	-141.11	-142.51	-142.48	-142.14	-142.05	-141.83
			BIC	-138.35	-137.18	-134.58	-131.68	-129.03	-126.24
		HC	AIC	-140.27	-141.63	-141.50	-141.32	-141.22	-141.04
			BIC	-137.50	-136.30	-133.61	-130.86	-128.20	-125.45
		SC	AIC	-140.56	-142.07	-141.93	-141.62	-141.60	-141.30
			BIC	-137.80	-136.74	-134.04	-131.16	-128.57	-125.71
VARX models	**Pearson**	All	AIC	-140.79	-142.21	-142.14	-141.75	-141.67	-141.45
			BIC	-138.02	-136.88	-134.24	-131.29	-128.64	-125.86
		HC	AIC	-140.05	-141.38	-141.27	-140.87	-140.78	-140.56
			BIC	-137.28	-136.05	-133.38	-130.41	-127.76	-124.97
		SC	AIC	-140.02	-141.56	-141.38	-141.05	-140.96	-140.75
			BIC	-137.26	-136.23	-133.48	-130.59	-127.93	-125.15
	Gerber	All	AIC	-141.11	-142.51	-142.48	-142.14	-142.05	-141.83
			BIC	-138.35	-137.18	-134.58	-131.68	-129.03	-126.24
		HC	AIC	-140.27	-141.63	-141.50	-141.32	-141.22	-141.04
			BIC	-137.50	-136.30	-133.61	-130.86	-128.20	-125.45
		SC	AIC	-140.56	-142.07	-141.93	-141.62	-141.60	-141.30
			BIC	-137.80	-136.74	-134.04	-131.16	-128.57	-125.71

Figure 5: Random Forest regression - Number of trees learning curves
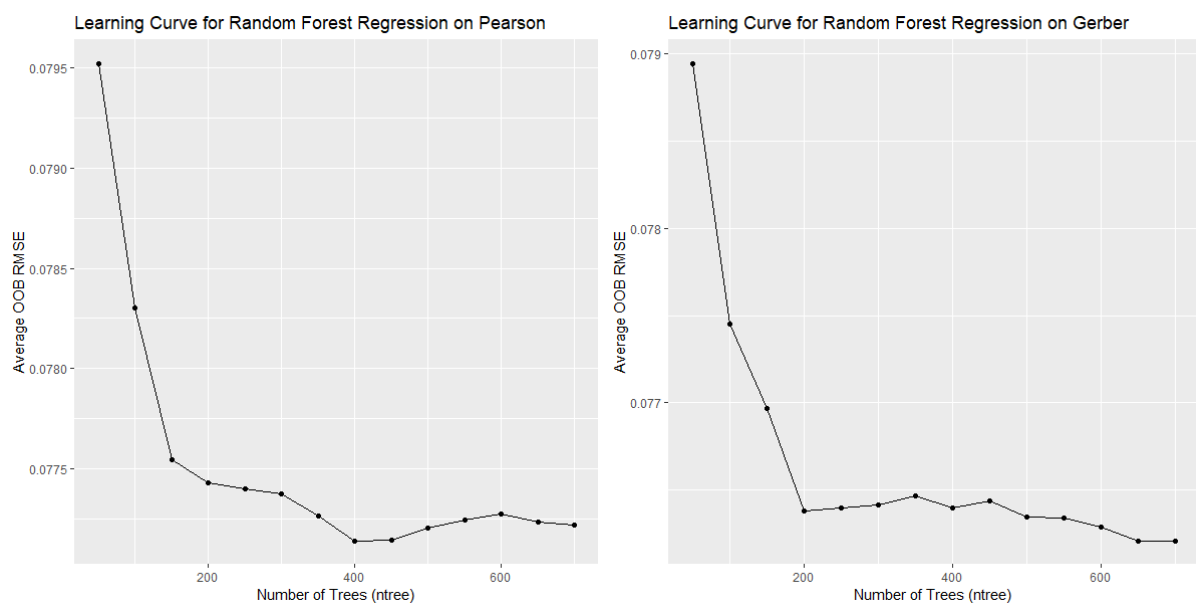
Appendix B2: Full descriptive statistics

Table 14: Descriptive statistics of subsamples 1 and 2

Variable	Mean	SD	Min	Max	Range	Skewness	Kurtosis
Panel A: Subsample 1							
Pearson	0.151	0.119	-0.026	0.526	0.552	0.755	0.002
Gerber	0.141	0.119	-0.031	0.491	0.522	0.824	0.064
GDP	1.64%	3.11%	-11.09%	7.25%	18.34%	-1.544	3.398
IP	0.07%	0.81%	-4.48%	1.36%	5.84%	-2.344	9.122
PPI	0.33%	1.35%	-5.48%	2.94%	8.42%	-1.417	4.277
TBill 13w	1.43%	1.87%	-0.87%	5.62%	6.49%	0.770	-0.875
TBill 10y	3.32%	1.06%	1.12%	5.55%	4.43%	-0.116	-0.819
Energy Index	0.68%	6.20%	-18.65%	16.14%	34.80%	-0.583	0.906
USD Index	-0.09%	2.53%	-6.82%	7.50%	14.31%	0.271	0.663
EUR/USD	1.332	0.094	1.179	1.577	0.399	0.653	-0.165
CNY/USD	0.140	0.014	0.121	0.163	0.042	-0.079	-1.486
SP500 Var	0.02%	0.03%	0.00%	0.24%	0.24%	4.662	24.927
BCI Var	0.01%	0.01%	0.00%	0.08%	0.07%	2.680	9.374
CPI	0.21%	0.45%	-1.92%	1.22%	3.14%	-1.121	3.713
							N = 116
Panel B: Subsample 2							
Pearson	0.081	0.078	-0.028	0.270	0.299	0.655	-0.651
Gerber	0.087	0.079	-0.020	0.258	0.278	0.404	-1.100
GDP	2.07%	10.95%	-77.00%	45.99%	122.99%	-3.055	27.377
IP	0.03%	1.70%	-14.37%	6.30%	20.66%	-4.865	43.698
PPI	0.20%	1.21%	-4.02%	3.16%	7.18%	-0.160	0.922
TBill 13w	0.73%	1.25%	-0.88%	4.68%	5.56%	1.198	1.037
TBill 10y	1.97%	0.83%	0.12%	4.00%	3.88%	-0.083	-0.518
Energy Index	-0.25%	8.60%	-43.68%	26.73%	70.41%	-0.677	5.304
USD Index	0.22%	1.89%	-5.13%	4.90%	10.03%	-0.154	-0.095
EUR/USD	1.157	0.090	0.980	1.387	0.407	0.952	0.528
CNY/USD	0.152	0.007	0.137	0.165	0.028	0.033	-1.140
SP500 Var	0.01%	0.03%	0.00%	0.34%	0.34%	9.010	87.715
BCI Var	0.01%	0.01%	0.00%	0.05%	0.05%	3.218	13.032
CPI	0.22%	0.37%	-0.67%	1.37%	2.04%	0.408	0.604
							N = 115

Table 15: Descriptive statistics of commodity price returns

Commodity	Mean	SD	Min	Max	Range	Skewness	Kurtosis
LCOc1	0.23%	10.27%	-79.15%	32.97%	112.12%	-2.108	14.734
CLc1	0.18%	11.12%	-77.52%	62.78%	140.30%	-0.963	13.078
NGc1	-0.59%	14.85%	-51.68%	48.52%	100.20%	-0.129	1.418
HOc1	0.32%	9.96%	-38.04%	30.40%	68.44%	-0.487	1.764
HGc1	0.51%	7.56%	-42.93%	28.82%	71.75%	-0.712	4.949
SAFc3	-0.03%	4.69%	-14.38%	14.59%	28.97%	0.017	0.949
GCc1	0.52%	4.81%	-17.92%	14.03%	31.94%	-0.193	0.599
SIc1	0.49%	9.25%	-32.58%	26.28%	58.86%	-0.231	0.772
PLc1	-0.04%	7.83%	-60.64%	31.52%	92.16%	-1.696	14.869
PAc1	0.66%	10.26%	-49.00%	46.63%	95.63%	-0.644	4.824
Wc1	0.14%	9.09%	-29.19%	35.16%	64.35%	0.126	0.847
Cc1	0.24%	8.94%	-30.96%	26.32%	57.28%	-0.325	0.749
Sc1	0.16%	7.92%	-39.90%	17.10%	57.00%	-0.959	2.753
LCc1	0.14%	6.17%	-29.12%	33.13%	62.25%	-0.433	6.245
LHc1	-0.14%	10.92%	-30.50%	32.03%	62.52%	-0.108	0.207
CTc2	-0.06%	8.27%	-28.66%	20.03%	48.69%	-0.425	0.415
KCc2	0.22%	8.49%	-25.57%	34.24%	59.81%	0.361	1.000
CCc2	0.03%	7.75%	-22.62%	21.29%	43.91%	-0.123	-0.155
SBc1	0.32%	9.12%	-37.64%	26.98%	64.62%	-0.251	1.638
STFc1	-0.01%	8.27%	-35.08%	21.98%	57.05%	-0.449	1.333
							N = 237

Figure 6: Full sample histograms
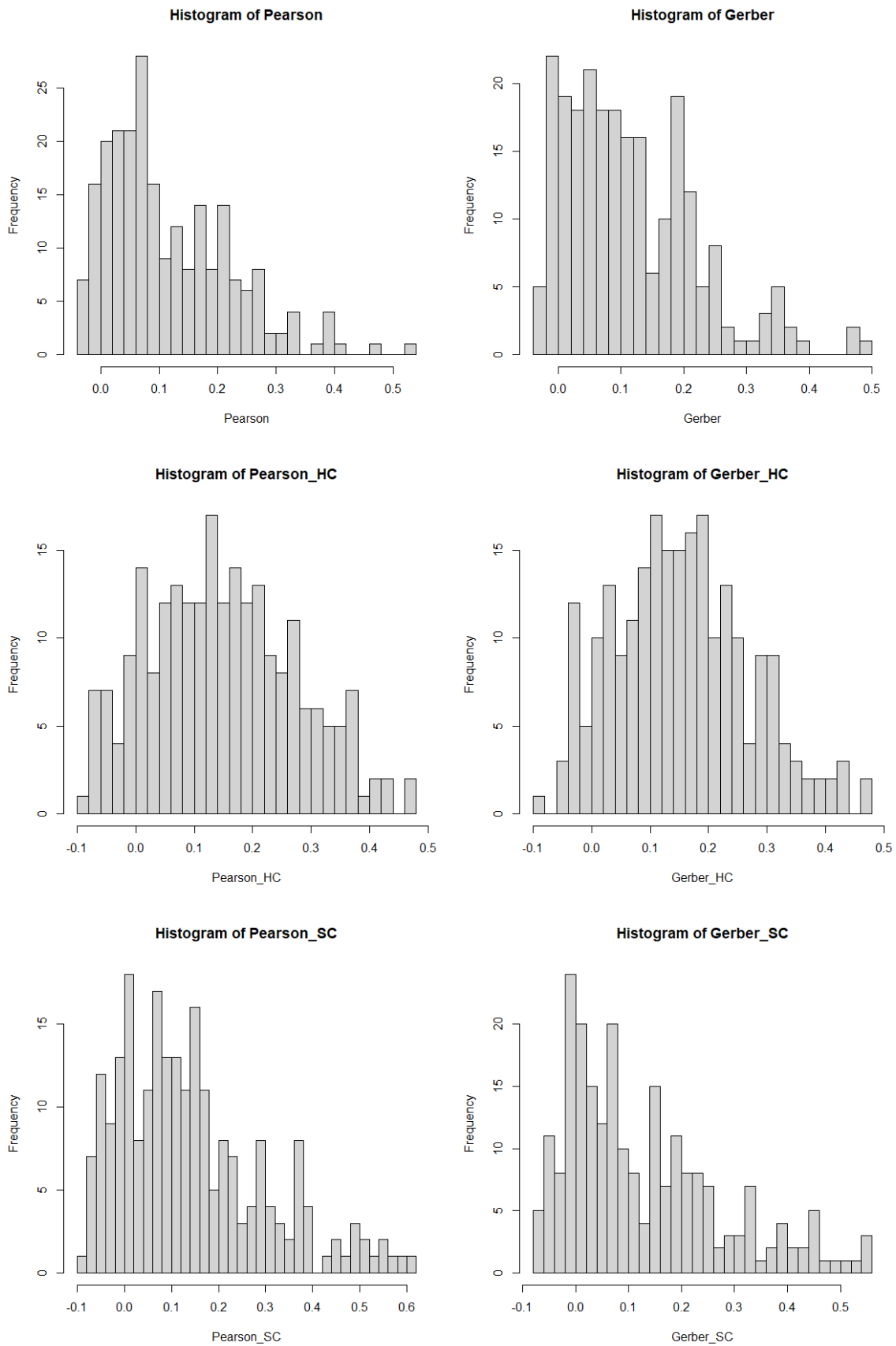
Histogram of GDP
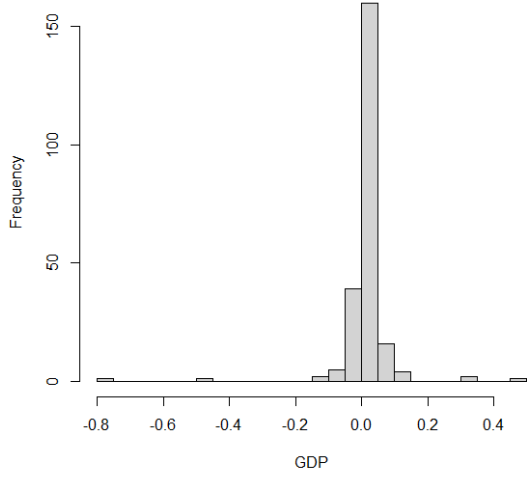
Histogram of IP
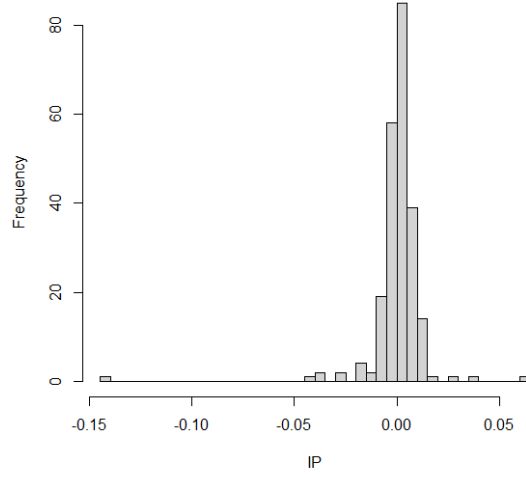
Histogram of PPI
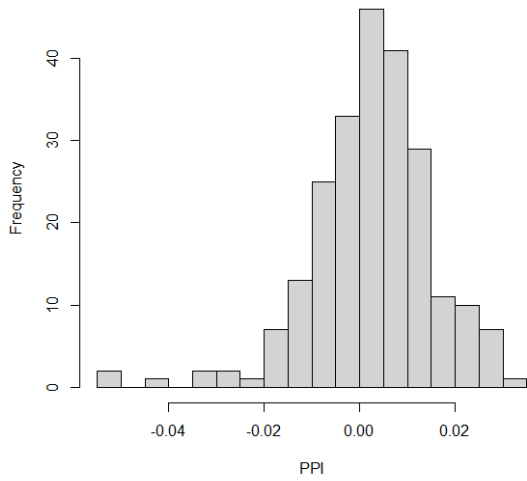
Histogram of TBill_13w
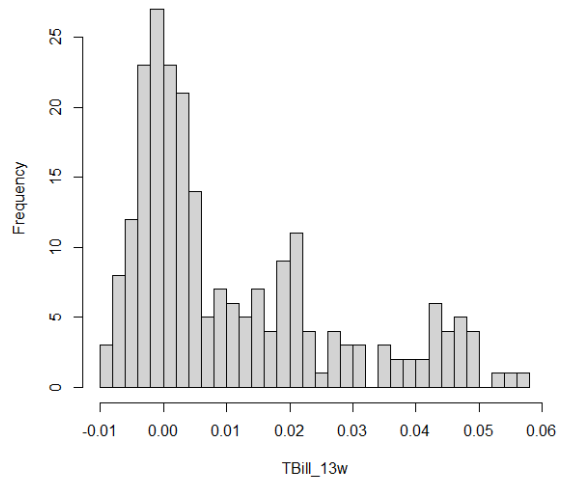
Histogram of TBill_10y
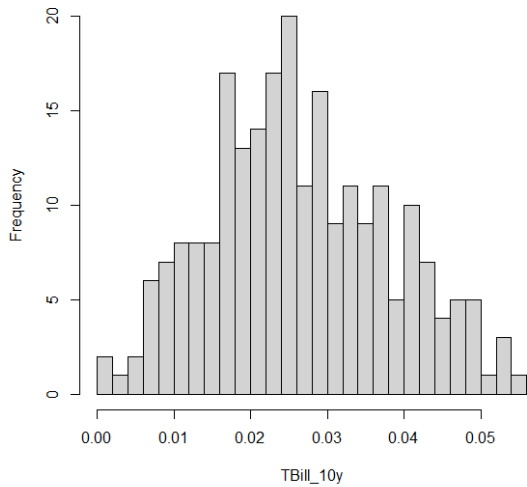
Histogram of EnergyIndex
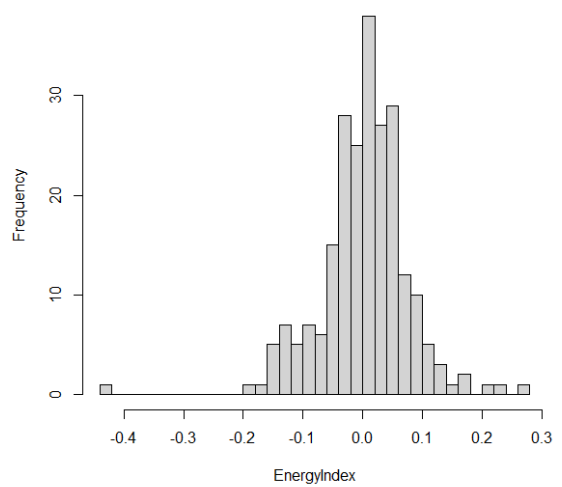
Histogram of USDIndex
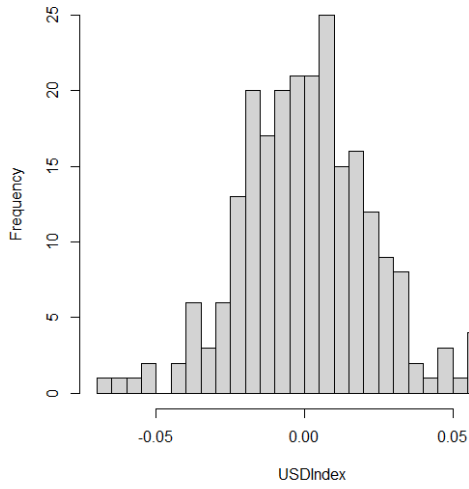
Histogram of EURUSD
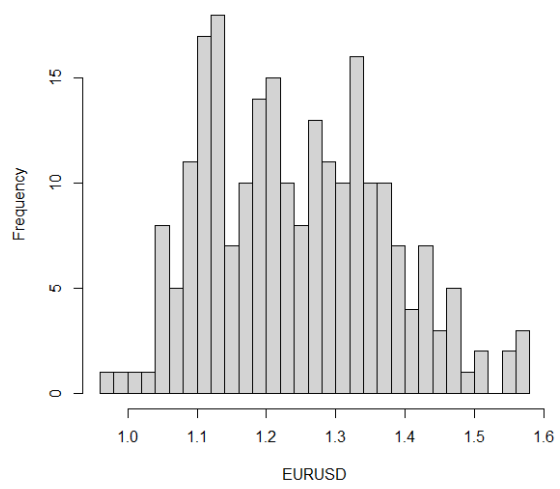
Histogram of CYNUSD
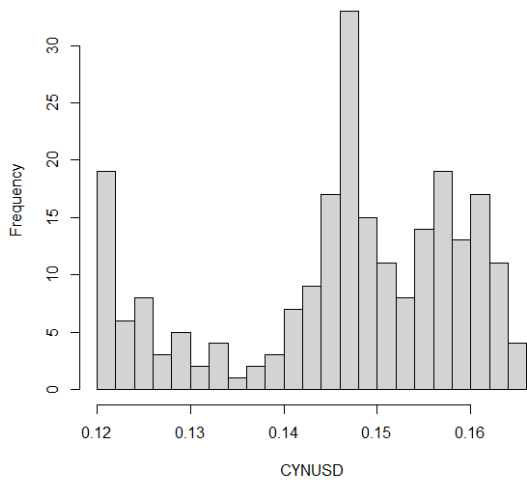
Histogram of SP500Var
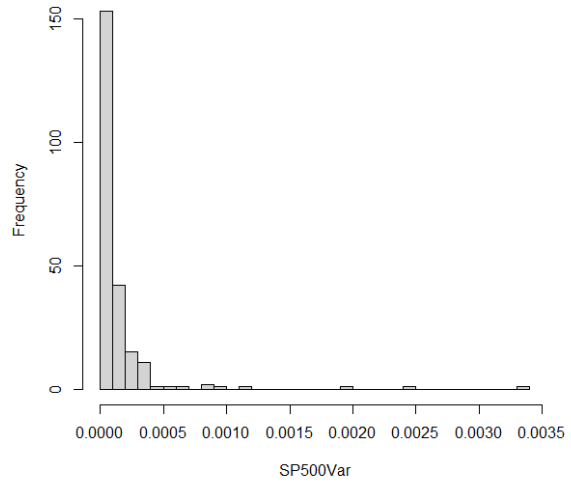
Histogram of BCIVar
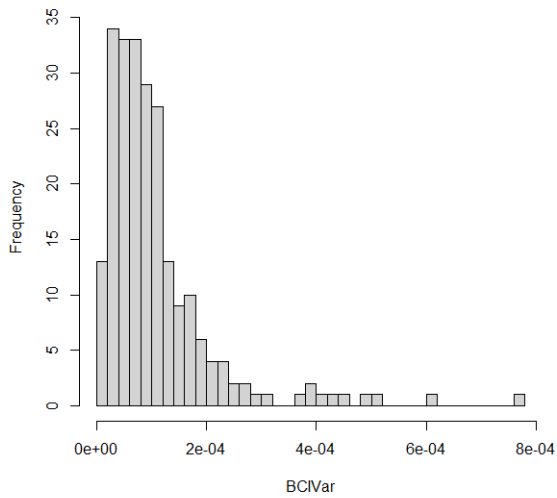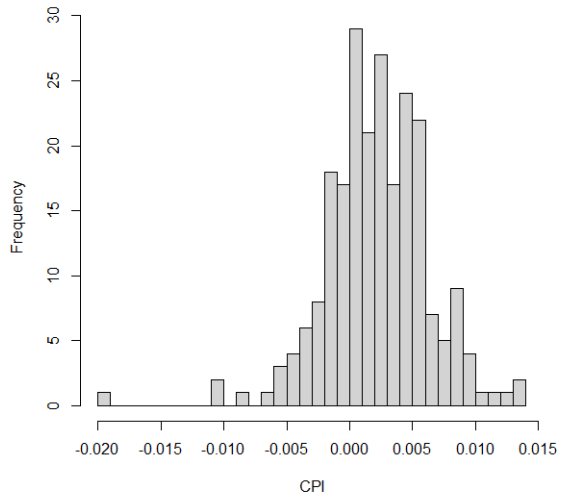
Histogram of CPI

Figure 7: First subsample histograms
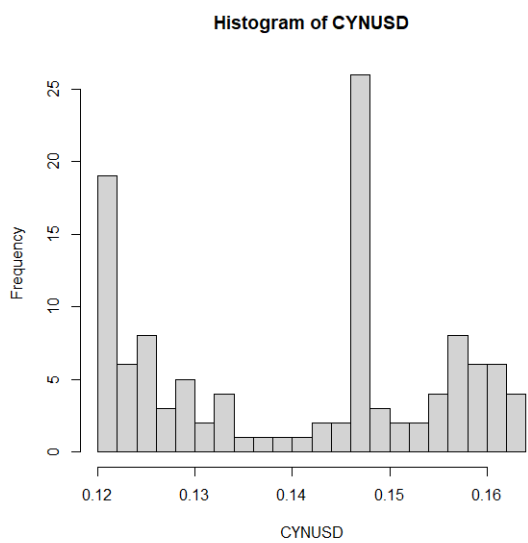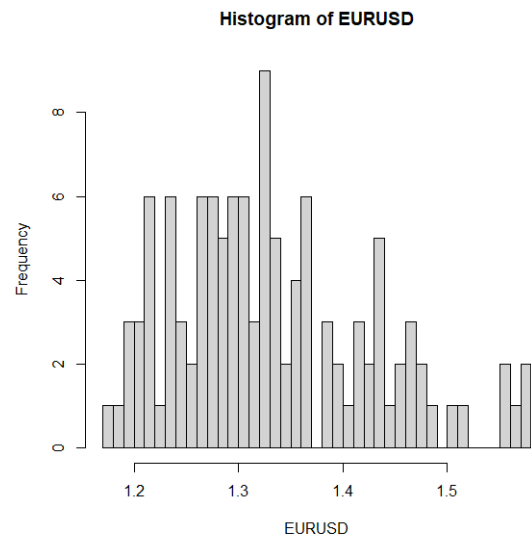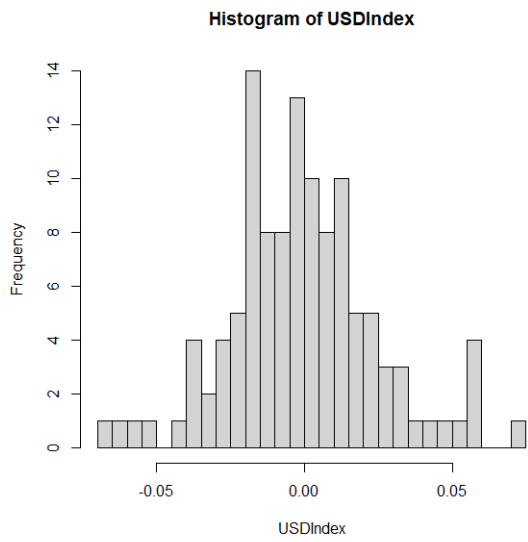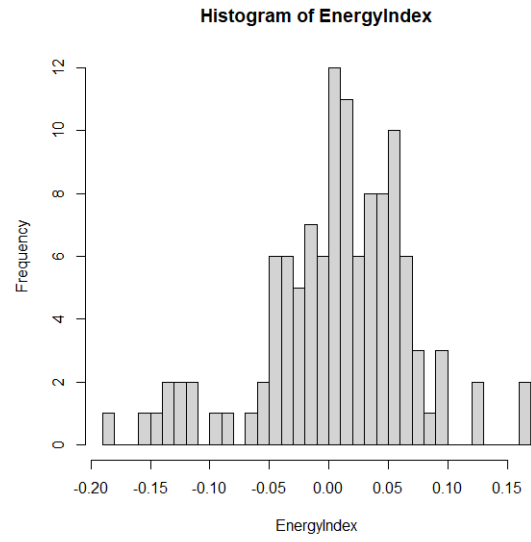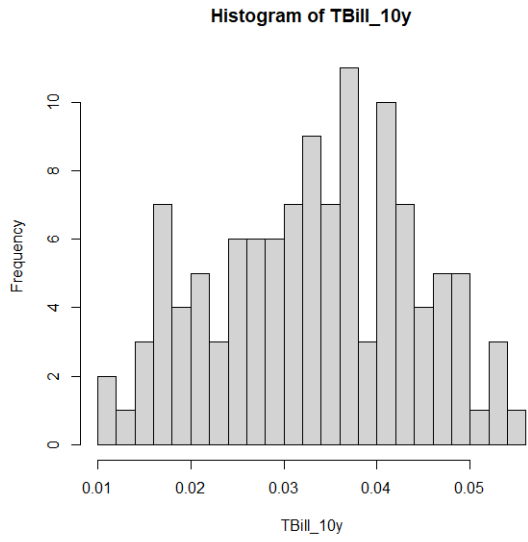
Histogram of BCIVar
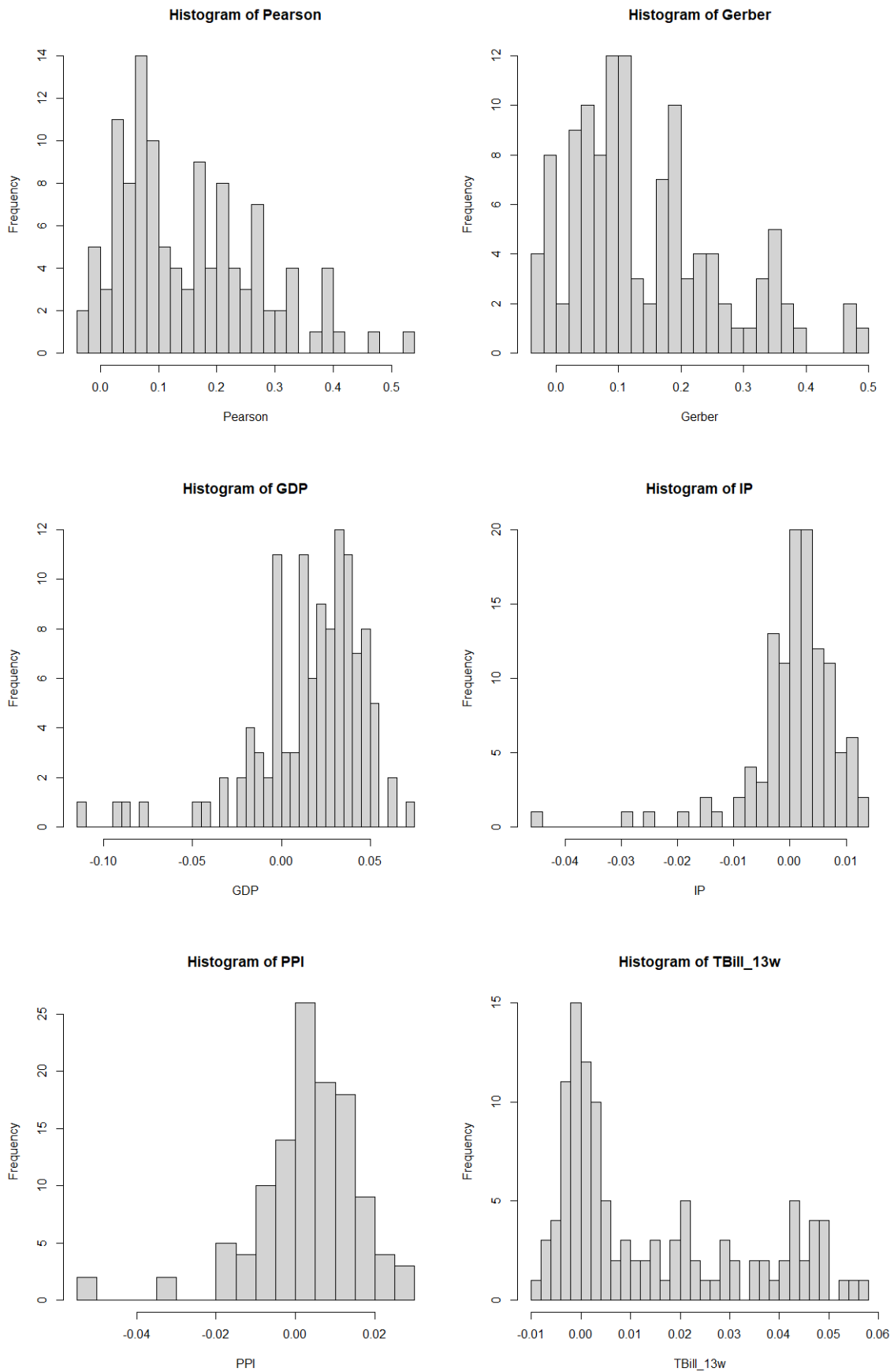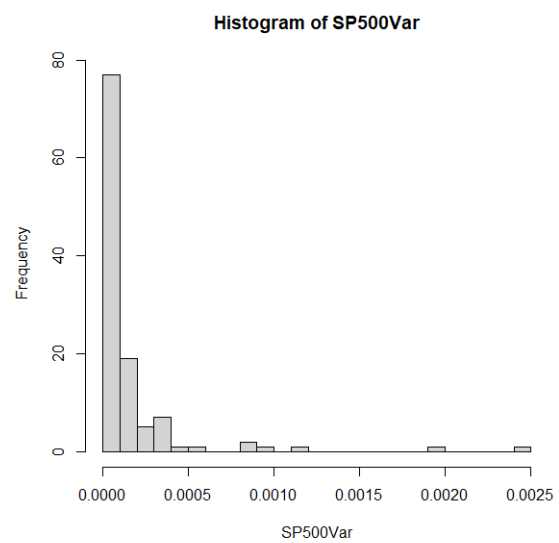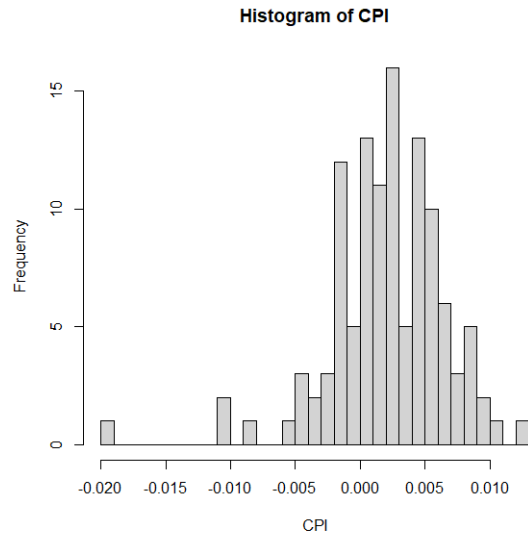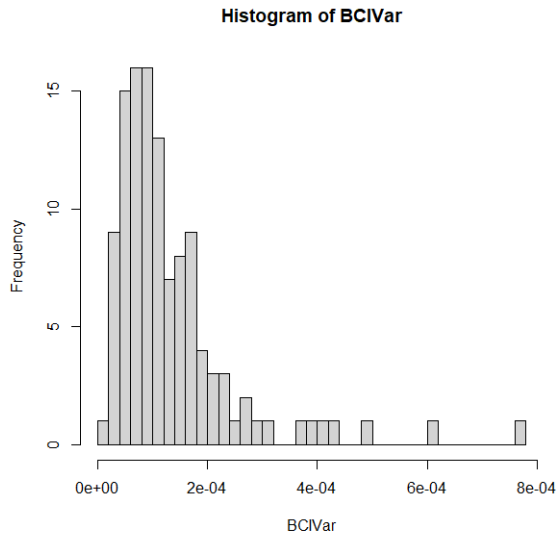
Histogram of CPI

Figure 8: Subsample 2 Histograms

Histogram of Pearson

Histogram of Gerber

Histogram of GDP
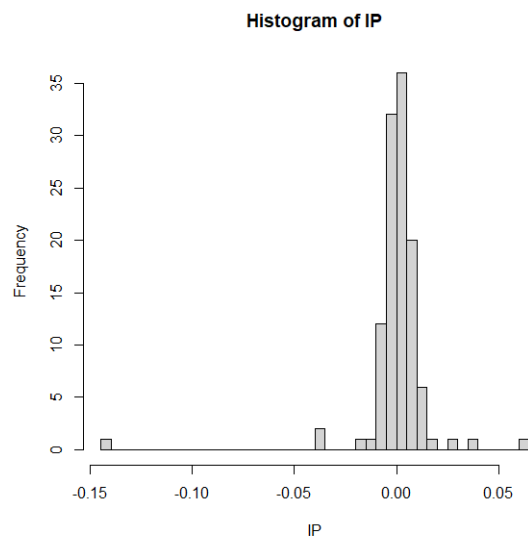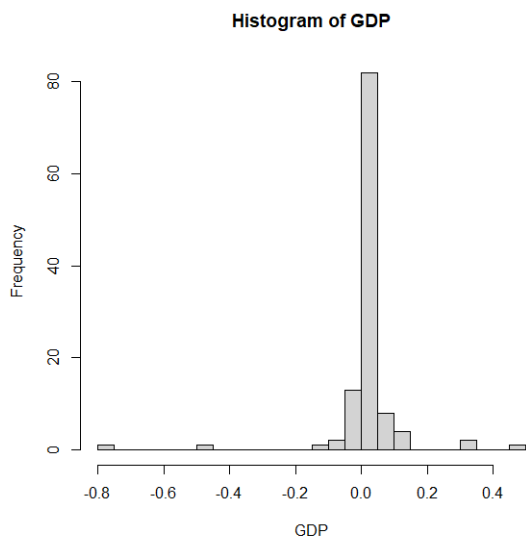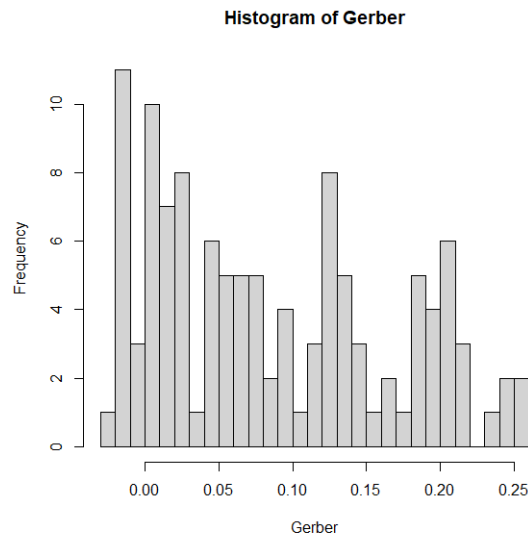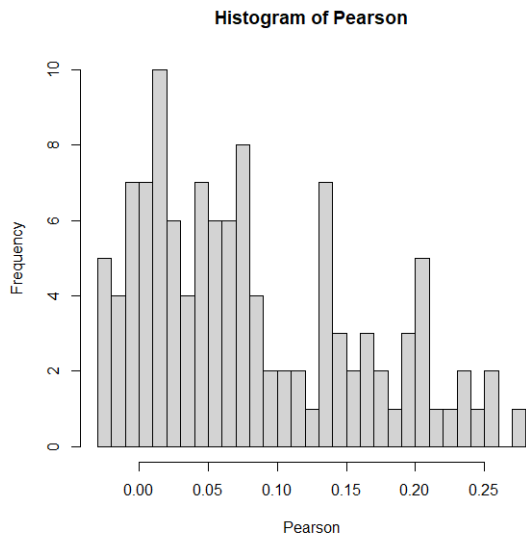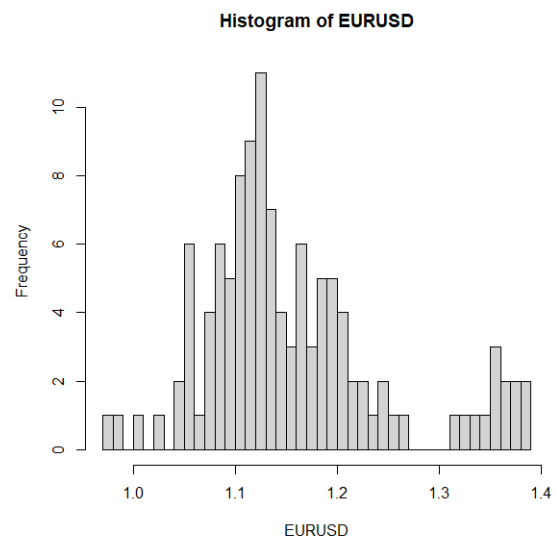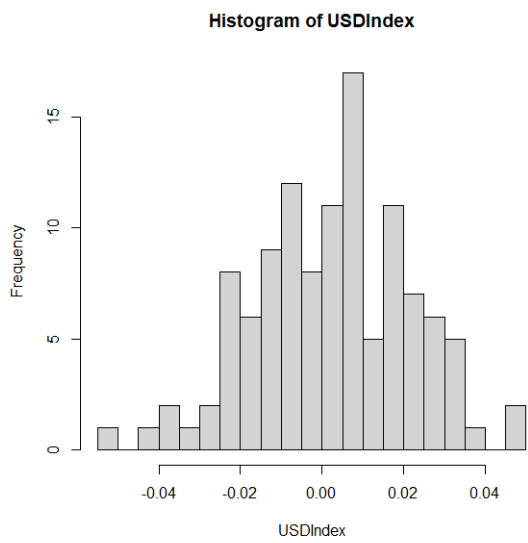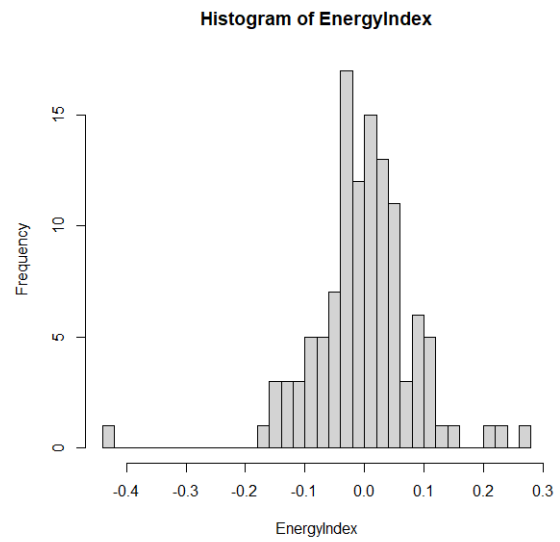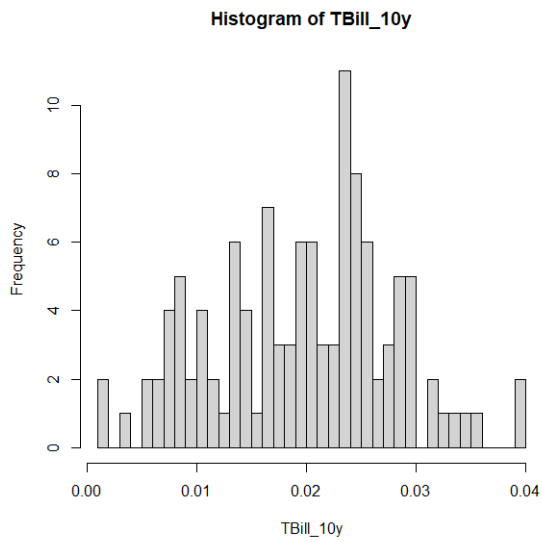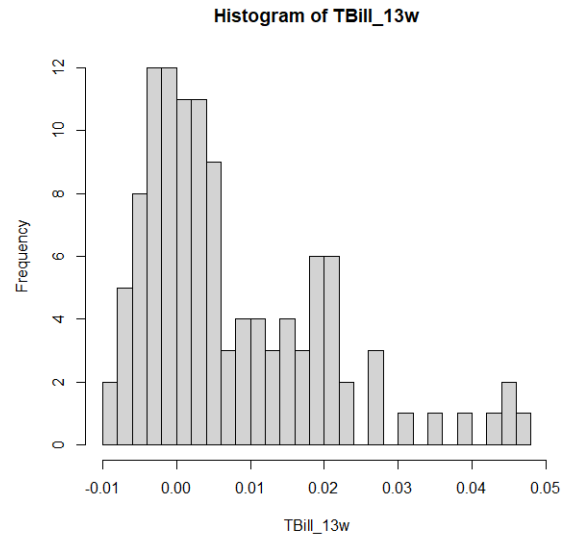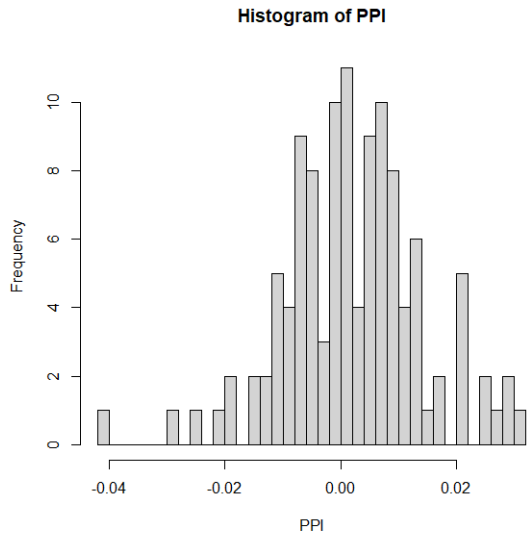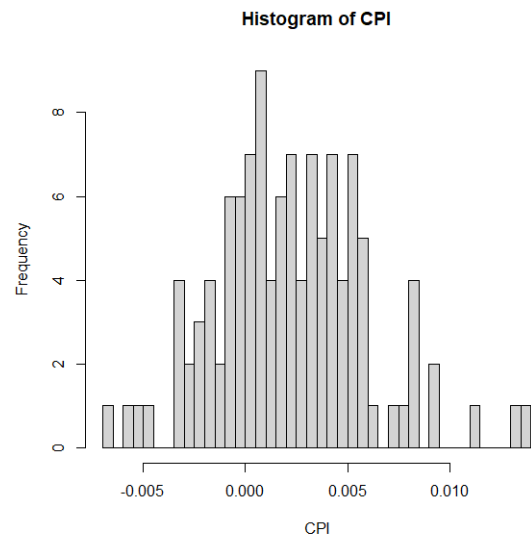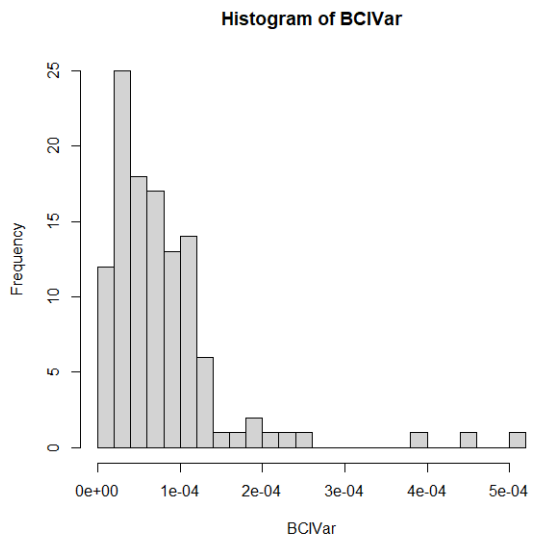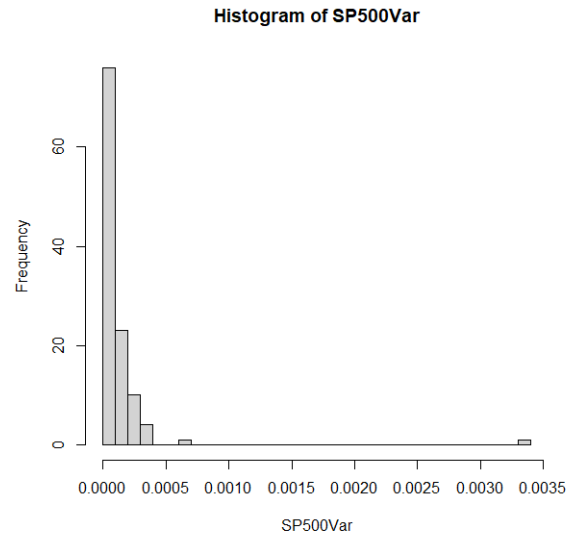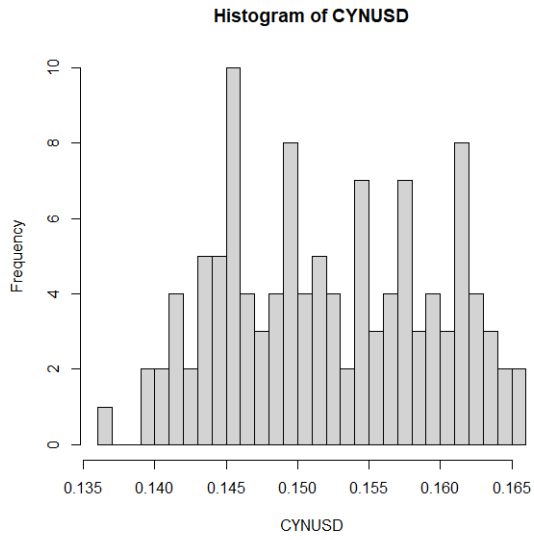
Histogram of IP

Figure 9: Commodity price return histograms
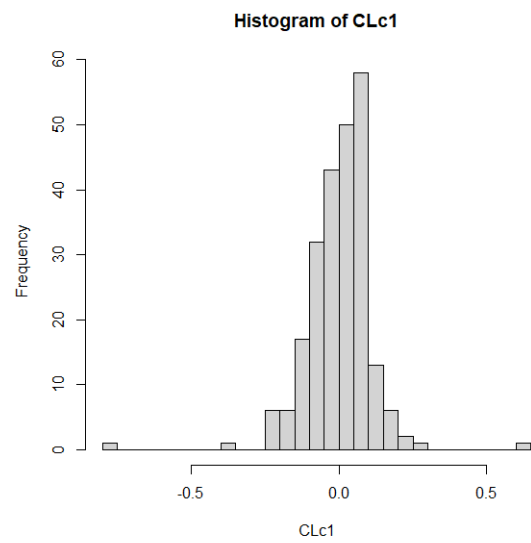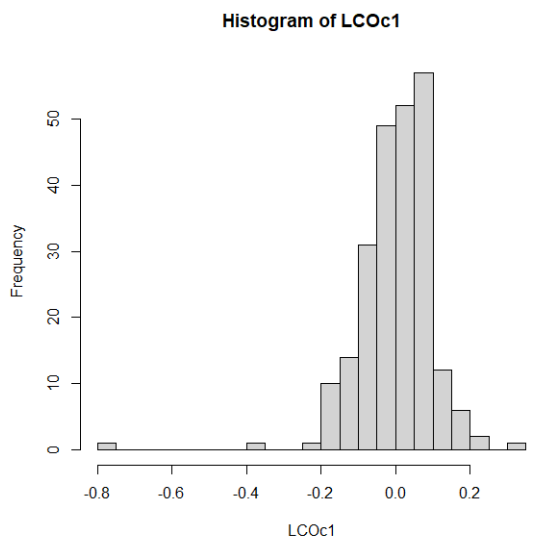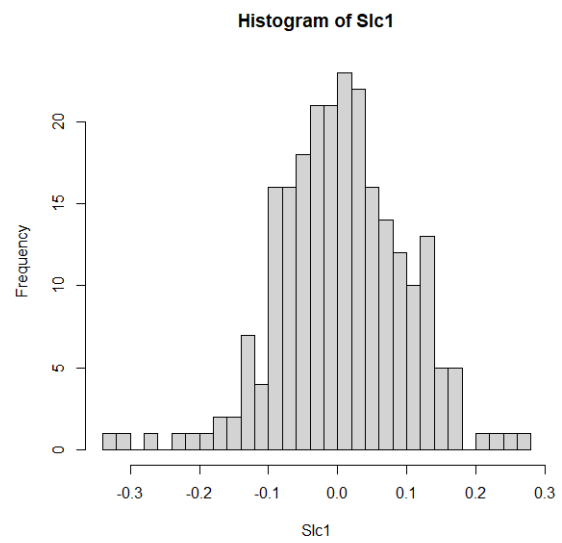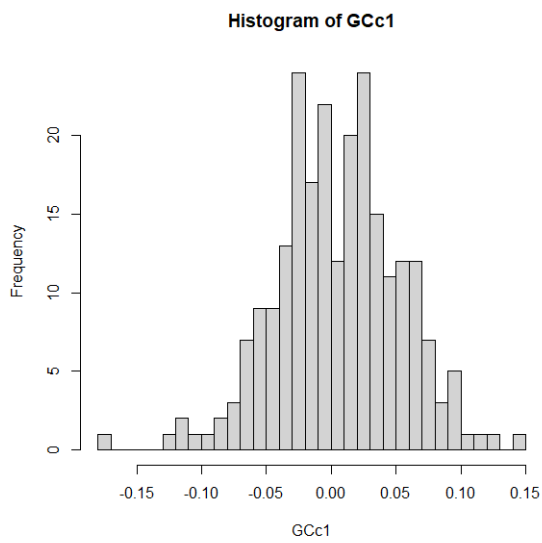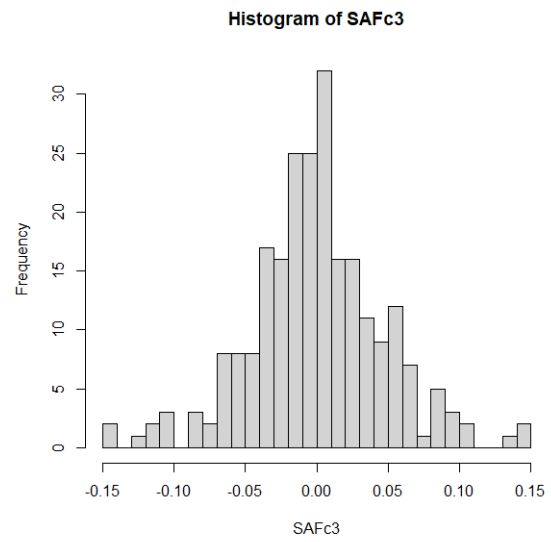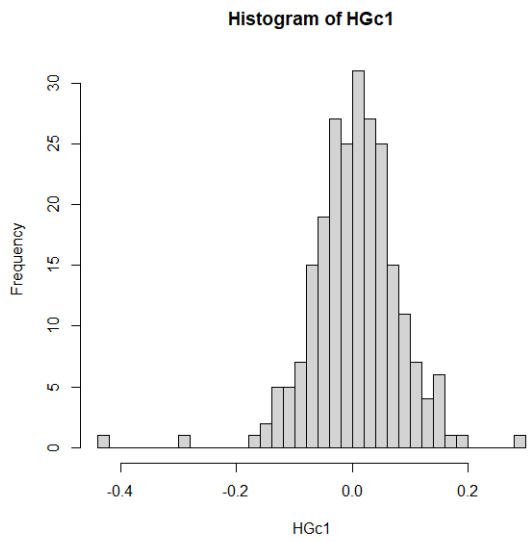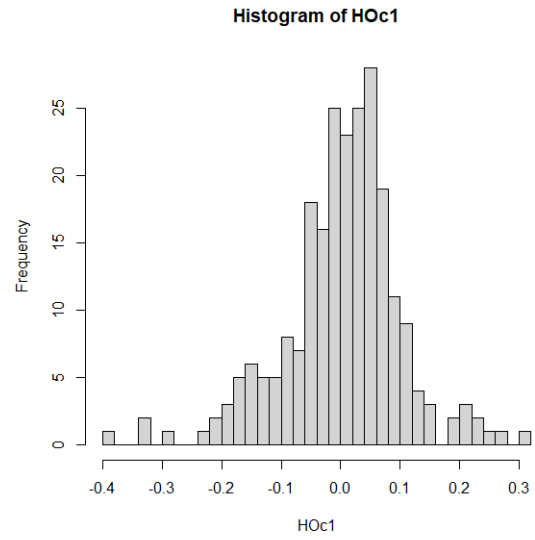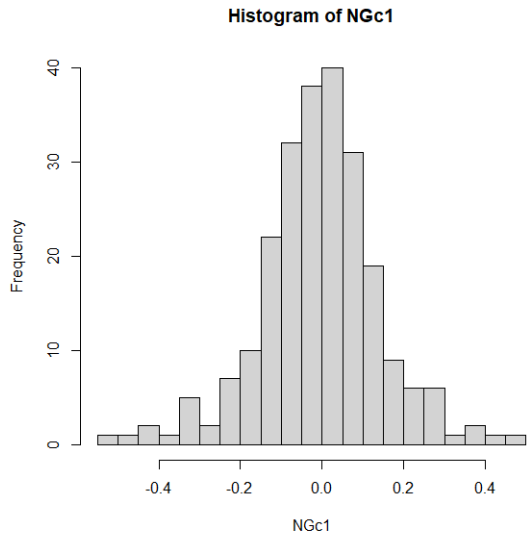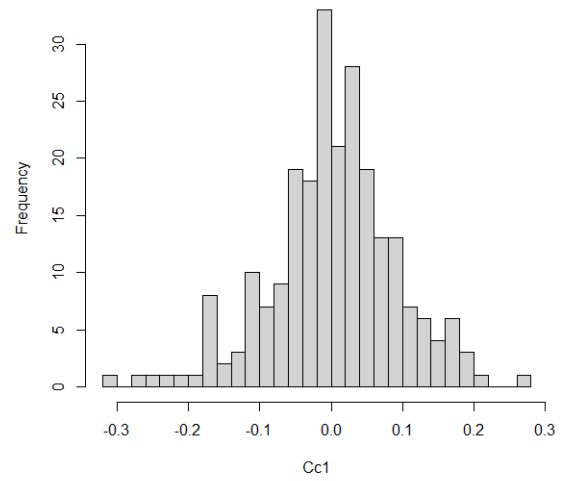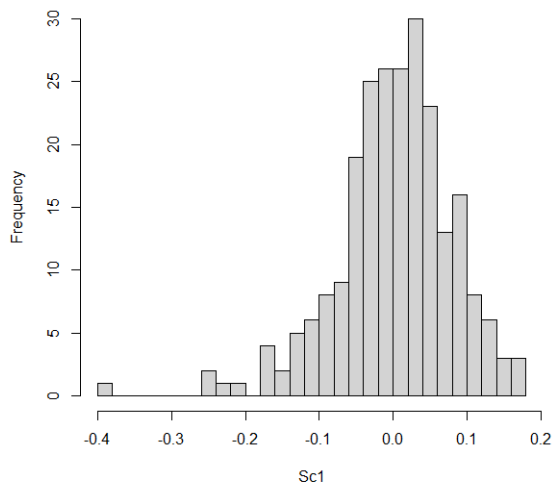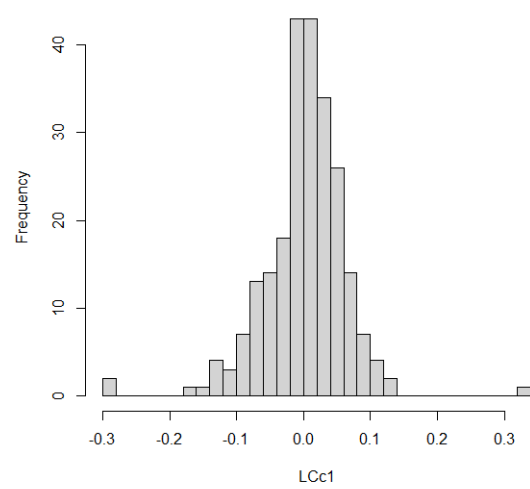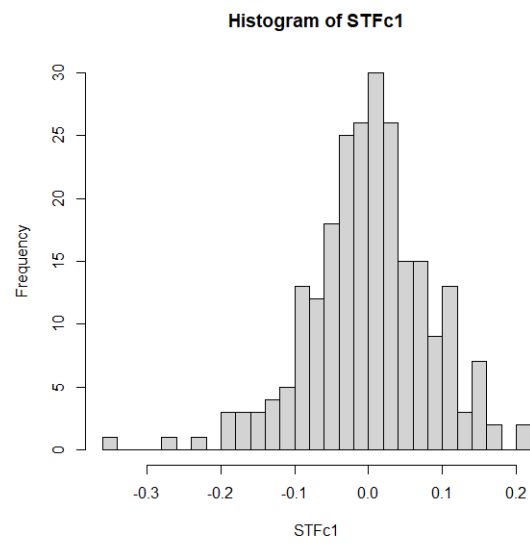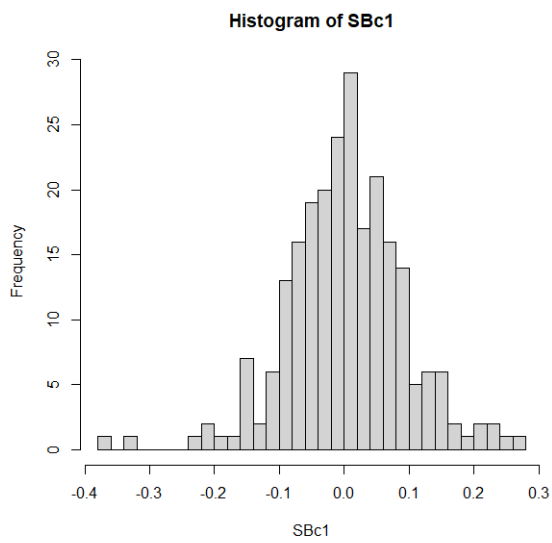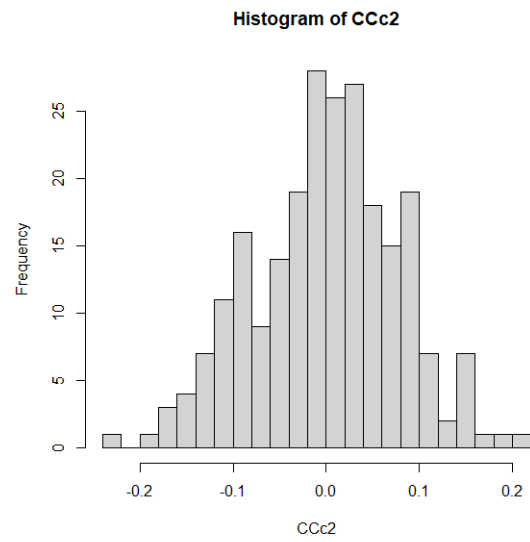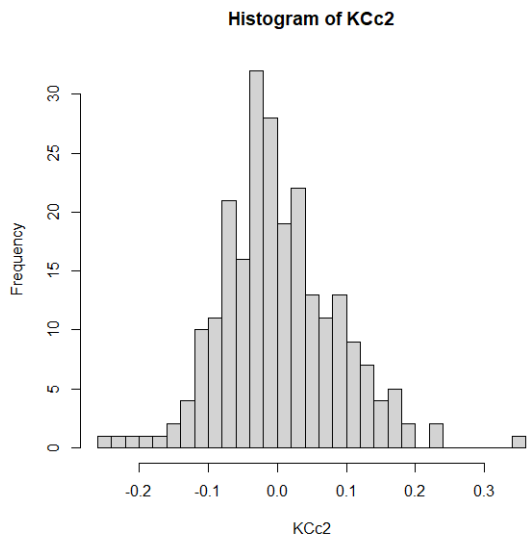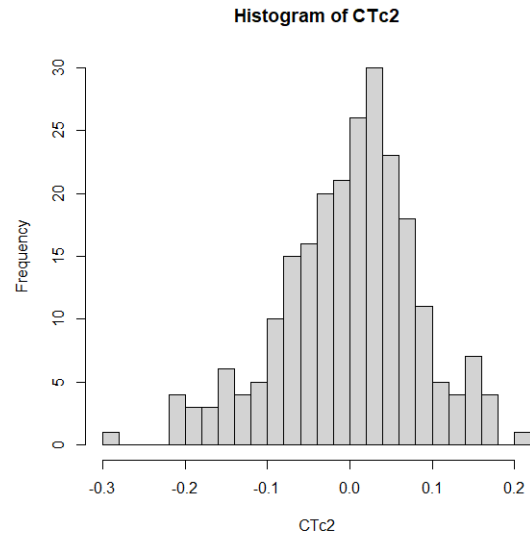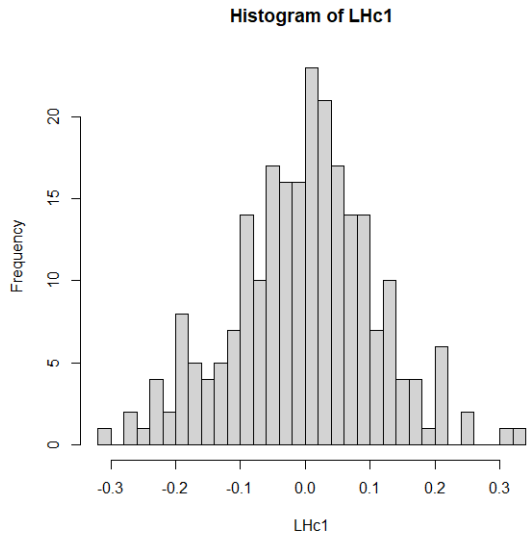
Appendix B4: Impulse response functions

Figure 10: Impulse Response Functions VAR Pearson

IRF of Pearson to GDP

95 % Bootstrap CI, 100 runs

IRF of Pearson to IP

95 % Bootstrap CI, 100 runs

IRF of Pearson to PPI

95 % Bootstrap CI, 100 runs

IRF of Pearson to EnergyIndex

95 % Bootstrap CI, 100 runs

IRF of Pearson to TBill_13w

95 % Bootstrap CI, 100 runs

IRF of Pearson to TBill_10y

95 % Bootstrap CI, 100 runs

71
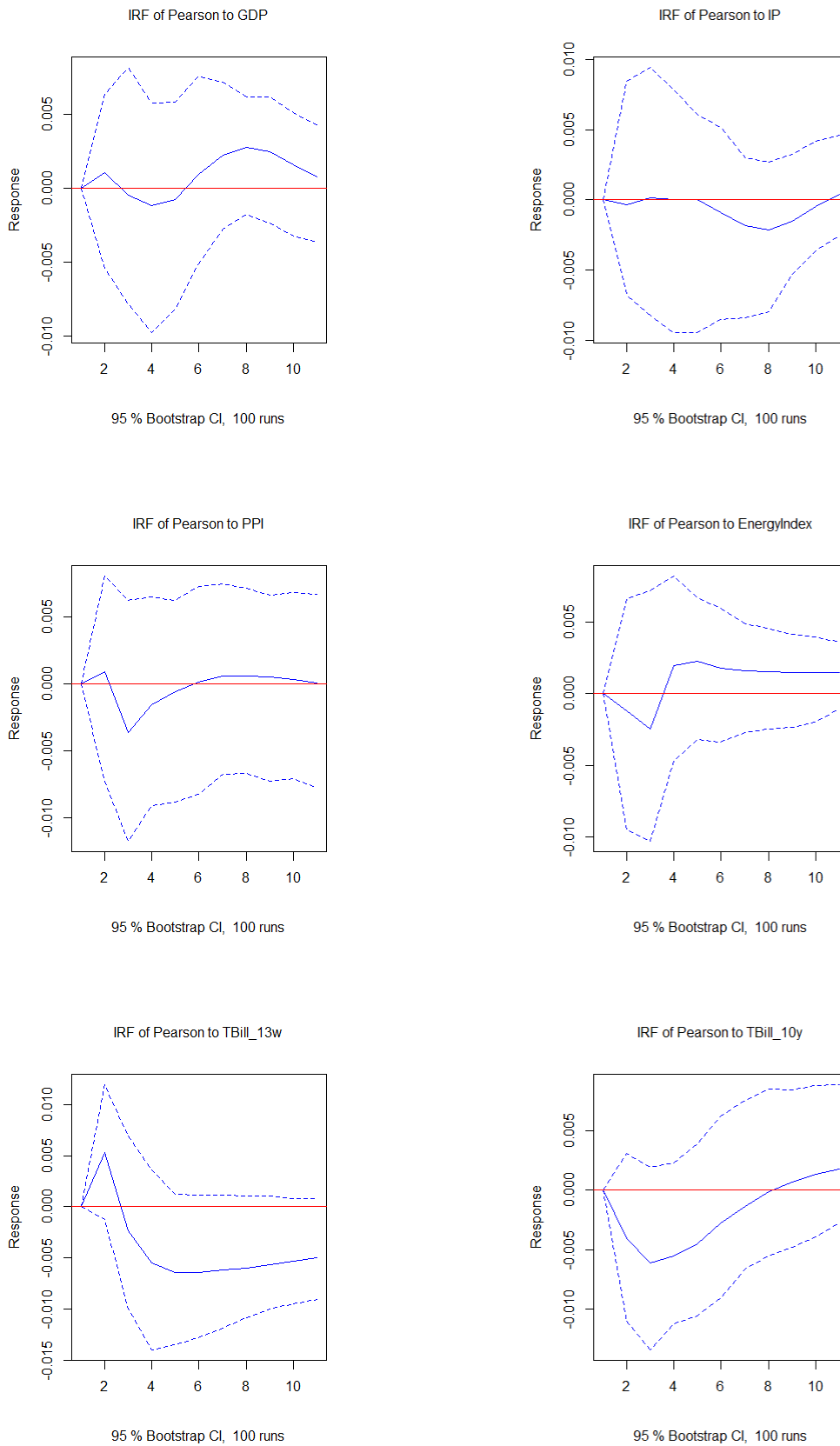
IRF of Pearson to USDIndex
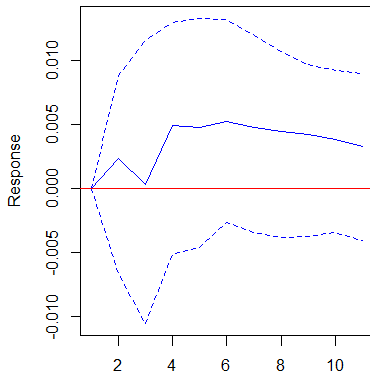
95 % Bootstrap CI, 100 runs

IRF of Pearson to EURUSD
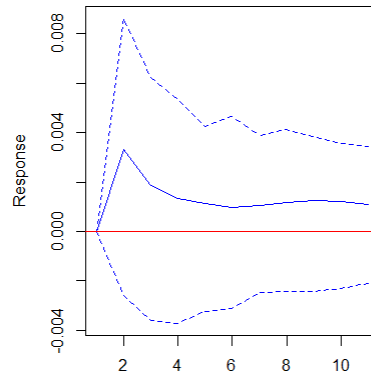
95 % Bootstrap CI, 100 runs

IRF of Pearson to CYNUSD
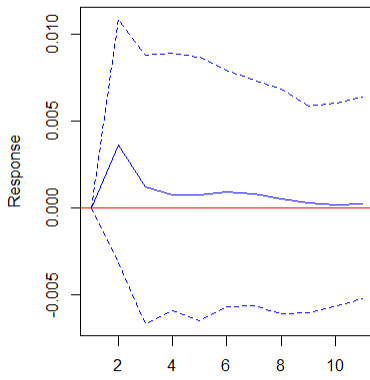
95 % Bootstrap CI, 100 runs

IRF of Pearson to SP500Var
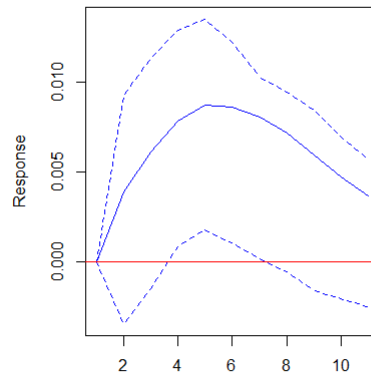
95 % Bootstrap CI, 100 runs

IRF of Pearson to BCIVar
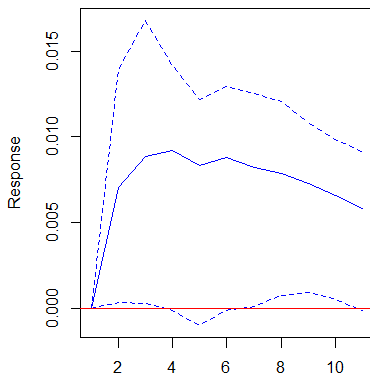
95 % Bootstrap CI, 100 runs

IRF of Pearson to CPI
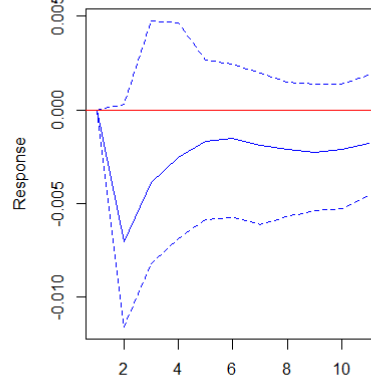
95 % Bootstrap CI, 100 runs

Figure 11: Impulse response functions VAR Gerber

IRF of Gerber to GDP

95 % Bootstrap CI, 100 runs

IRF of Gerber to IP

95 % Bootstrap CI, 100 runs

IRF of Gerber to PPI

95 % Bootstrap CI, 100 runs

IRF of Gerber to TBill_13w

95 % Bootstrap CI, 100 runs

IRF of Gerber to TBill_10y

95 % Bootstrap CI, 100 runs

IRF of Gerber to EnergyIndex

95 % Bootstrap CI, 100 runs

IRF of Gerber to USDIndex

95 % Bootstrap CI, 100 runs

IRF of Gerber to EURUSD

95 % Bootstrap CI, 100 runs

IRF of Gerber to CYNUSD

95 % Bootstrap CI, 100 runs

IRF of Gerber to SP500Var

95 % Bootstrap CI, 100 runs

IRF of Gerber to BCIVar

95 % Bootstrap CI, 100 runs

IRF of Gerber to CPI

95 % Bootstrap CI, 100 runs

[i] Thomson Reuters. (2023). *RIC: CLc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[ii] Thomson Reuters. (2023). *RIC: LCOc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[iii] Thomson Reuters. (2023). *RIC: NGc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[iv] Thomson Reuters. (2023). *RIC: HOc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[v] Thomson Reuters. (2023). *RIC: HGc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[vi] Thomson Reuters. (2023). *RIC: SAFc3* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[vii] Thomson Reuters. (2023). *RIC: GCc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[viii] Thomson Reuters. (2023). *RIC: SIc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[ix] Thomson Reuters. (2023). *RIC: PLc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[x] Thomson Reuters. (2023). *RIC: PAc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xi] Thomson Reuters. (2023). *RIC: Wc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xii] Thomson Reuters. (2023). *RIC: Cc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xiii] Thomson Reuters. (2023). *RIC: Sc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xiv] Thomson Reuters. (2023). *RIC: LCc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xv] Thomson Reuters. (2023). *RIC: LHc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xvi] Thomson Reuters. (2023). *RIC: CTc2* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xvii] Thomson Reuters. (2023). *RIC: SBc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xviii] Thomson Reuters. (2023). *RIC: KCc2* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xix] Thomson Reuters. (2023). *RIC: CCc2* [Data set]. Eikon. https://eikon.refinitiv.com/index.html
[xx] Thomson Reuters. (2023). *RIC: STFc1* [Data set]. Eikon. https://eikon.refinitiv.com/index.html

Appendix C: R code

```
#   MSc BA Thesis – Financial management – University
of Twente

#   Title: Commodity price co-movement: Comparing
models and correlation measures
#   Appendix C: R codes
#   Author: Luca Kozian
#   Date: 24/09/2023


# Packages and data
  {
  library(psych)
  library(broom)
  library(flextable)
  library(purrr)
  library(dplyr)
  library(vars)
  library(urca)
  library(caret)
  library(readxl)
  library(lmtest)
  library(pdp)
  library(ranger)
  library(ggplot2)

  MScBA_Data_R <- read_excel("C:/ INSERT PATH HERE
/MScBA_Data_R.xlsx",
                            col_types = c("date",
"numeric", "numeric", "numeric", "numeric", "numeric",
"numeric", "numeric", "numeric", "numeric", "numeric",
"numeric", "numeric", "numeric", "numeric", "numeric",
"numeric", "numeric", "numeric"))

  Commodiy_returns <- read_excel("C:/ INSERT PATH HERE
/Commodiy_returns.xlsx")


}

# Multiple Regression
  {
  # All commodities


  # Multiple Regression with Pearson as the dependent
variable
  regression_pearson <- lm(Pearson ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R)
```

```
  summary(regression_pearson)

  # Multiple Regression with Gerber as the dependent
variable
  regression_gerber <- lm(Gerber ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R)
  summary(regression_gerber)



  # Hard/Soft commodities


  # Hard commodities

  regression_pearson_hc <- lm(Pearson_HC ~ GDP + IP +
PPI + `TBill_13w` + `TBill_10y` + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
data = MScBA_Data_R)
  summary(regression_pearson_hc)

  regression_gerber_hc <- lm(Gerber_HC ~ GDP + IP + PPI
+ `TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R)
  summary(regression_gerber_hc)


  # Soft commodities
  regression_pearson_sc <- lm(Pearson_SC ~ GDP + IP +
PPI + `TBill_13w` + `TBill_10y` + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
data = MScBA_Data_R)
  summary(regression_pearson_sc)

  regression_gerber_sc <- lm(Gerber_SC ~ GDP + IP + PPI
+ `TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R)
  summary(regression_gerber_sc)


  # Sub-sample multiple regressions


  # Split the data based on Date
  MScBA_Data_R$Date <- as.Date(MScBA_Data_R$Date,
format = "%d/%m/%Y")
  MScBA_Data_R_1 <- filter(MScBA_Data_R, Date <=
as.Date("2013-07-31"))
  MScBA_Data_R_2 <- filter(MScBA_Data_R, Date >=
as.Date("2013-08-31"))
```

```
  # Multiple Regression with Pearson as the dependent
variable on data before 31/08/2013
  regression_pearson_1 <- lm(Pearson ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R_1)
  summary(regression_pearson_1)

  # Multiple Regression with Pearson as the dependent
variable on data after 31/08/2013
  regression_pearson_2 <- lm(Pearson ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R_2)
  summary(regression_pearson_2)

  # Multiple Regression with Gerber as the dependent
variable on data before 31/08/2013
  regression_gerber_1 <- lm(Gerber ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R_1)
  summary(regression_gerber_1)

  # Multiple Regression with Gerber as the dependent
variable on data after 31/08/2013
  regression_gerber_2 <- lm(Gerber ~ GDP + IP + PPI +
`TBill_13w` + `TBill_10y` + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI, data =
MScBA_Data_R_2)
  summary(regression_gerber_2)


}

# VAR
  {
  # Create separate data frames for each dependent
variable
  data_pearson <- MScBA_Data_R[, c("Pearson", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]
  data_gerber <- MScBA_Data_R[, c("Gerber", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]

  data_pearson_hc <- MScBA_Data_R[, c("Pearson_HC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
  data_gerber_hc <- MScBA_Data_R[, c("Gerber_HC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
```

```r
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]

  data_pearson_sc <- MScBA_Data_R[, c("Pearson_SC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
  data_gerber_sc <- MScBA_Data_R[, c("Gerber_SC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]

  data_pearson_1 <- MScBA_Data_R_1[, c("Pearson",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
  data_pearson_2 <- MScBA_Data_R_2[, c("Pearson",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]

  data_gerber_1 <- MScBA_Data_R_1[, c("Gerber", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]
  data_gerber_2 <- MScBA_Data_R_2[, c("Gerber", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]

  # Rename 'Pearson_HC' and 'Gerber_HC' to 'Pearson'
and 'Gerber' in data_pearson_hc and data_gerber_hc
  colnames(data_pearson_hc)[colnames(data_pearson_hc)
== 'Pearson_HC'] <- 'Pearson'
  colnames(data_gerber_hc)[colnames(data_gerber_hc) ==
'Gerber_HC'] <- 'Gerber'

  # Rename 'Pearson_SC' and 'Gerber_SC' to 'Pearson'
and 'Gerber' in data_pearson_sc and data_gerber_sc
  colnames(data_pearson_sc)[colnames(data_pearson_sc)
== 'Pearson_SC'] <- 'Pearson'
  colnames(data_gerber_sc)[colnames(data_gerber_sc) ==
'Gerber_SC'] <- 'Gerber'


  # All commodities

  # Pearson

  # Perform BIC-based lag selection for Pearson
  bic_results_pearson <- VARselect(data_pearson,
lag.max = 6, type = "const")
  print(bic_results_pearson)

  # Determine the optimal number of lags using BIC for
Pearson
  max_lag_pearson <- VARselect(data_pearson, lag.max =
6, type = "const")$selection[1]
  print(max_lag_pearson)

  # Fit the VAR model for Pearson with the selected lag
  var_model_pearson <- VAR(data_pearson, p =
max_lag_pearson, type = "const")

  # Summary of the VAR model for Pearson
  summary(var_model_pearson)$var$"Pearson"



  # Gerber

  # Perform BIC-based lag selection for Gerber
  bic_results_gerber <- VARselect(data_gerber, lag.max
= 6, type = "const")
  print(bic_results_gerber)

  # Determine the optimal number of lags using BIC for
Gerber
  max_lag_gerber <- VARselect(data_gerber, lag.max = 6,
type = "const")$selection[1]
  print(max_lag_gerber)

  # Fit the VAR model for Gerber with the selected lag
  var_model_gerber <- VAR(data_gerber, p =
max_lag_gerber, type = "const")

  # Summary of the VAR model for Gerber
  summary(var_model_gerber)$var$"Gerber"



  # Hard commodities

  # Pearson_HC

  # Perform BIC-based lag selection for Pearson
  bic_results_pearson_hc <- VARselect(data_pearson_hc,
lag.max = 6, type = "const")
  print(bic_results_pearson_hc)

  # Determine the optimal number of lags using BIC for
Pearson
  max_lag_pearson_hc <- VARselect(data_pearson_hc,
lag.max = 6, type = "const")$selection[1]
  print(max_lag_pearson_hc)

  # Fit the VAR model for Pearson with the selected lag
  var_model_pearson_hc <- VAR(data_pearson_hc, p =
max_lag_pearson_hc, type = "const")

  # Summary of the VAR model for Pearson
  summary(var_model_pearson_hc)$var$"Pearson"


  # Gerber_HC

  # Perform BIC-based lag selection for Gerber
  bic_results_gerber_hc <- VARselect(data_gerber_hc,
lag.max = 6, type = "const")
  print(bic_results_gerber_hc)

  # Determine the optimal number of lags using BIC for
Gerber
  max_lag_gerber_hc <- VARselect(data_gerber_hc,
lag.max = 6, type = "const")$selection[1]
  print(max_lag_gerber_hc)

  # Fit the VAR model for Gerber with the selected lag
  var_model_gerber_hc <- VAR(data_gerber_hc, p =
max_lag_gerber_hc, type = "const")

  # Summary of the VAR model for Gerber
  summary(var_model_gerber_hc)$var$"Gerber"



  # Soft commodities

  # Pearson_SC

  # Perform BIC-based lag selection for Pearson_SC
  bic_results_pearson_sc <- VARselect(data_pearson_sc,
lag.max = 6, type = "const")
  print(bic_results_pearson_sc)

  # Determine the optimal number of lags using BIC for
Pearson
  max_lag_pearson_sc <- VARselect(data_pearson_sc,
lag.max = 6, type = "const")$selection[1]
  print(max_lag_pearson_sc)

  # Fit the VAR model for Pearson with the selected lag
  var_model_pearson_sc <- VAR(data_pearson_sc, p =
max_lag_pearson_sc, type = "const")

  # Summary of the VAR model for Pearson
  summary(var_model_pearson_sc)$var$"Pearson"


  # Gerber_SC

  # Perform BIC-based lag selection for Gerber
  bic_results_gerber_sc <- VARselect(data_gerber_sc,
lag.max = 6, type = "const")
```

```r
    print(bic_results_gerber_sc)

    # Determine the optimal number of lags using BIC for
Gerber
    max_lag_gerber_sc <- VARselect(data_gerber_sc,
lag.max = 6, type = "const")$selection[1]
    print(max_lag_gerber_sc)

    # Fit the VAR model for Gerber with the selected lag
    var_model_gerber_sc <- VAR(data_gerber_sc, p =
max_lag_gerber_sc, type = "const")

    # Summary of the VAR model for Gerber
    summary(var_model_gerber_sc)$var$"Gerber"


    # Sub samples

    # Pearson subset 1
    var_model_pearson_1 <- VAR(data_pearson_1, p =
max_lag_pearson, type = "const")
    # Summary of the VAR model for Pearson subset 1
    summary(var_model_pearson_1)$var$"Pearson"

    # Pearson subset 2
    var_model_pearson_2 <- VAR(data_pearson_2, p =
max_lag_pearson, type = "const")
    # Summary of the VAR model for Pearson subset 2
    summary(var_model_pearson_2)$var$"Pearson"

    # Gerber subset 1
    var_model_gerber_1 <- VAR(data_gerber_1, p =
max_lag_gerber, type = "const")
    # Summary of the VAR model for Gerber subset 1
    summary(var_model_gerber_1)$var$"Gerber"

    # Gerber subset 2
    var_model_gerber_2 <- VAR(data_gerber_2, p =
max_lag_gerber, type = "const")
    # Summary of the VAR model for Gerber subset 2
    summary(var_model_gerber_2)$var$"Gerber"


}

# VARX
  {
  # Define the exogenous variables list
  exog_vars_list <- c("GDP", "IP", "PPI", "TBill_13w",
"TBill_10y", "EnergyIndex", "USDIndex", "EURUSD",
"CNYUSD", "SP500Var", "BCIVar")


    # All commodities


    # Pearson
    {
      # Perform BIC-based lag selection for Pearson with
exogenous variables
      bic_results_pearson_exog <- VARselect(data_pearson,
lag.max = 6, type = "const")
      print(bic_results_pearson_exog)
      # Determine the optimal number of lags using BIC
for Pearson with exogenous variables
      max_lag_pearson_exog <- VARselect(data_pearson,
lag.max = 6, type = "const")$selection[1]
      print(max_lag_pearson_exog)

      # Number of lags you want for each exogenous
variable
      n_lags_pearson <- max_lag_pearson_exog  # Change
this to the number of lags you want
      # Create lags for the exogenous variables
      exog_vars_pearson <- exog_vars_list
      all_exog_vars_pearson <- exog_vars_pearson  #
Initialize with original exogenous variables
      for (var in exog_vars_pearson) {
        for (lag in 1:n_lags_pearson) {
          new_var_name <- paste0(var, ".l", lag)
          data_pearson <- data_pearson %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
          all_exog_vars_pearson <-
c(all_exog_vars_pearson, new_var_name)  # Add the new
lagged variable to the list
        }
      }

      # Fit the VAR model with original and lagged
exogenous variables
      var_model_pearson_exog <- VAR(data_pearson[,
c("Pearson", "CPI")],
                                    p =
max_lag_pearson_exog,
                                    type = "const",
                                    exogen =
data_pearson[, all_exog_vars_pearson])
      # Summary of the model
      summary(var_model_pearson_exog)
    }

    # Gerber
    {
      # Perform BIC-based lag selection for Gerber with
exogenous variables
      bic_results_gerber_exog <- VARselect(data_gerber,
lag.max = 6, type = "const")
      print(bic_results_gerber_exog)
      # Determine the optimal number of lags using BIC
for Gerber with exogenous variables
      max_lag_gerber_exog <- VARselect(data_gerber,
lag.max = 6, type = "const")$selection[1]
      print(max_lag_gerber_exog)

      # Number of lags you want for each exogenous
variable
      n_lags_gerber <- max_lag_gerber_exog  # Change this
to the number of lags you want
      # Create lags for the exogenous variables
      exog_vars_gerber <- exog_vars_list
      all_exog_vars_gerber <- exog_vars_gerber  #
Initialize with original exogenous variables
      for (var in exog_vars_gerber) {
        for (lag in 1:n_lags_gerber) {
          new_var_name <- paste0(var, ".l", lag)
          data_gerber <- data_gerber %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
          all_exog_vars_gerber <- c(all_exog_vars_gerber,
new_var_name)  # Add the new lagged variable to the
list
        }
      }

      # Fit the VAR model with original and lagged
exogenous variables
      var_model_gerber_exog <- VAR(data_gerber[,
c("Gerber", "CPI")],
                                   p =
max_lag_gerber_exog,
                                   type = "const",
                                   exogen = data_gerber[,
all_exog_vars_gerber])
      # Summary of the model
      summary(var_model_gerber_exog)
    }


  # Hard commodities


    # Pearson
    {
      # Perform BIC-based lag selection for Pearson with
exogenous variables
      bic_results_pearson_exog_hc <-
VARselect(data_pearson_hc, lag.max = 6, type = "const")
      print(bic_results_pearson_exog_hc)
      # Determine the optimal number of lags using BIC
for Pearson with exogenous variables
      max_lag_pearson_exog_hc <-
VARselect(data_pearson_hc, lag.max = 6, type =
"const")$selection[1]
      print(max_lag_pearson_exog_hc)
```

```r
    # Number of lags you want for each exogenous
variable
    n_lags_pearson_hc <- max_lag_pearson_exog_hc  #
Change this to the number of lags you want
    # Create lags for the exogenous variables
    exog_vars_pearson_hc <- exog_vars_list
    all_exog_vars_pearson_hc <- exog_vars_pearson_hc  #
Initialize with original exogenous variables
    for (var in exog_vars_pearson_hc) {
       for (lag in 1:n_lags_pearson_hc) {
         new_var_name <- paste0(var, ".l", lag)
         data_pearson_hc <- data_pearson_hc %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
         all_exog_vars_pearson_hc <-
c(all_exog_vars_pearson_hc, new_var_name)  # Add the
new lagged variable to the list
       }
    }

    # Fit the VAR model with original and lagged
exogenous variables
    var_model_pearson_exog_hc <- VAR(data_pearson_hc[,
c("Pearson", "CPI")],
                                    p =
max_lag_pearson_exog_hc,
                                    type = "const",
                                    exogen =
data_pearson_hc[, all_exog_vars_pearson_hc])
    # Summary of the model
    summary(var_model_pearson_exog_hc)
  }

  # Gerber
  {
    # Perform BIC-based lag selection for Gerber with
exogenous variables
    bic_results_gerber_exog_hc <-
VARselect(data_gerber_hc, lag.max = 6, type = "const")
    print(bic_results_gerber_exog_hc)
    # Determine the optimal number of lags using BIC
for Gerber with exogenous variables
    max_lag_gerber_exog_hc <- VARselect(data_gerber_hc,
lag.max = 6, type = "const")$selection[1]
    print(max_lag_gerber_exog_hc)

    # Number of lags you want for each exogenous
variable
    n_lags_gerber_hc <- max_lag_gerber_exog_hc  #
Change this to the number of lags you want
    # Create lags for the exogenous variables
    exog_vars_gerber_hc <- exog_vars_list
    all_exog_vars_gerber_hc <- exog_vars_gerber_hc  #
Initialize with original exogenous variables
    for (var in exog_vars_gerber_hc) {

       for (lag in 1:n_lags_gerber_hc) {
         new_var_name <- paste0(var, ".l", lag)
         data_gerber_hc <- data_gerber_hc %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
         all_exog_vars_gerber_hc <-
c(all_exog_vars_gerber_hc, new_var_name)  # Add the new
lagged variable to the list
       }
    }

    # Fit the VAR model with original and lagged
exogenous variables
    var_model_gerber_exog_hc <- VAR(data_gerber_hc[,
c("Gerber", "CPI")],
                                   p =
max_lag_gerber_exog_hc,
                                   type = "const",
                                   exogen =
data_gerber_hc[, all_exog_vars_gerber_hc])
    # Summary of the model
    summary(var_model_gerber_exog_hc)
  }


# Soft commodities

  # Pearson
  {
    # Perform BIC-based lag selection for Pearson with
exogenous variables
    bic_results_pearson_exog_sc <-
VARselect(data_pearson_sc, lag.max = 6, type = "const")
    print(bic_results_pearson_exog_sc)
    # Determine the optimal number of lags using BIC
for Pearson with exogenous variables
    max_lag_pearson_exog_sc <-
VARselect(data_pearson_sc, lag.max = 6, type =
"const")$selection[1]
    print(max_lag_pearson_exog_sc)

    # Number of lags you want for each exogenous
variable
    n_lags_pearson_sc <- max_lag_pearson_exog_sc  #
Change this to the number of lags you want
    # Create lags for the exogenous variables
    exog_vars_pearson_sc <- exog_vars_list
    all_exog_vars_pearson_sc <- exog_vars_pearson_sc  #
Initialize with original exogenous variables
    for (var in exog_vars_pearson_sc) {
       for (lag in 1:n_lags_pearson_sc) {
         new_var_name <- paste0(var, ".l", lag)
         data_pearson_sc <- data_pearson_sc %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
         all_exog_vars_pearson_sc <-
c(all_exog_vars_pearson_sc, new_var_name)  # Add the
new lagged variable to the list
       }
    }

    # Fit the VAR model with original and lagged
exogenous variables
    var_model_pearson_exog_sc <- VAR(data_pearson_sc[,
c("Pearson", "CPI")],
                                    p =
max_lag_pearson_exog_sc,
                                    type = "const",
                                    exogen =
data_pearson_sc[, all_exog_vars_pearson_sc])
    # Summary of the model
    summary(var_model_pearson_exog_sc)
  }

  # Gerber
  {
    # Perform BIC-based lag selection for Gerber with
exogenous variables
    bic_results_gerber_exog_sc <-
VARselect(data_gerber_sc, lag.max = 6, type = "const")
    print(bic_results_gerber_exog_sc)
    # Determine the optimal number of lags using BIC
for Gerber with exogenous variables
    max_lag_gerber_exog_sc <- VARselect(data_gerber_sc,
lag.max = 6, type = "const")$selection[1]
    print(max_lag_gerber_exog_sc)

    # Number of lags you want for each exogenous
variable
    n_lags_gerber_sc <- max_lag_gerber_exog_sc  #
Change this to the number of lags you want
    # Create lags for the exogenous variables
    exog_vars_gerber_sc <- exog_vars_list
    all_exog_vars_gerber_sc <- exog_vars_gerber_sc  #
Initialize with original exogenous variables
    for (var in exog_vars_gerber_sc) {
       for (lag in 1:n_lags_gerber_sc) {
         new_var_name <- paste0(var, ".l", lag)
         data_gerber_sc <- data_gerber_sc %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
         all_exog_vars_gerber_sc <-
c(all_exog_vars_gerber_sc, new_var_name)  # Add the new
lagged variable to the list
       }
    }

    # Fit the VAR model with original and lagged
exogenous variables
    var_model_gerber_exog_sc <- VAR(data_gerber_sc[,
c("Gerber", "CPI")],
```

```r
                                    p =
max_lag_gerber_exog_sc,
                                    type = "const",
                                    exogen =
data_gerber_sc[, all_exog_vars_gerber_sc])
    # Summary of the model
    summary(var_model_gerber_exog_sc)
  }


  # Subsamples


  # Pearson subsamples
  {
    # Create lags for the exogenous variables for
Pearson: Split 1
    all_exog_vars_pearson_1 <- exog_vars_list
    for (var in exog_vars_list) {
      for (lag in 1:max_lag_pearson_exog) {
        new_var_name <- paste0(var, ".l", lag)
        data_pearson_1 <- data_pearson_1 %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
        all_exog_vars_pearson_1 <-
c(all_exog_vars_pearson_1, new_var_name)
      }
    }


    # Create lags for the exogenous variables for
Pearson: Split 2
    all_exog_vars_pearson_2 <- exog_vars_list
    for (var in exog_vars_list) {
      for (lag in 1:max_lag_pearson_exog) {
        new_var_name <- paste0(var, ".l", lag)
        data_pearson_2 <- data_pearson_2 %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
        all_exog_vars_pearson_2 <-
c(all_exog_vars_pearson_2, new_var_name)
      }
    }


    # Run VAR models for Pearson: Split 1 and Split 2
    var_model_pearson_exog_1 <- VAR(data_pearson_1[,
c("Pearson", "CPI")], p = max_lag_pearson_exog, type =
"const", exogen = data_pearson_1[,
all_exog_vars_pearson_1])
    var_model_pearson_exog_2 <- VAR(data_pearson_2[,
c("Pearson", "CPI")], p = max_lag_pearson_exog, type =
"const", exogen = data_pearson_2[,
all_exog_vars_pearson_2])
    summary(var_model_pearson_exog_1)
    summary(var_model_pearson_exog_2)
  }

  # Gerber subsamples


  {
    # Create lags for the exogenous variables for
Gerber: Split 1
    all_exog_vars_gerber_1 <- exog_vars_list
    for (var in exog_vars_list) {
      for (lag in 1:max_lag_gerber_exog) {
        new_var_name <- paste0(var, ".l", lag)
        data_gerber_1 <- data_gerber_1 %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
        all_exog_vars_gerber_1 <-
c(all_exog_vars_gerber_1, new_var_name)
      }
    }

    # Create lags for the exogenous variables for
Gerber: Split 2
    all_exog_vars_gerber_2 <- exog_vars_list
    for (var in exog_vars_list) {
      for (lag in 1:max_lag_gerber_exog) {
        new_var_name <- paste0(var, ".l", lag)
        data_gerber_2 <- data_gerber_2 %>%
mutate(!!new_var_name := lag(!!sym(var), n = lag))
        all_exog_vars_gerber_2 <-
c(all_exog_vars_gerber_2, new_var_name)
      }
    }

    # Run VAR models for Gerber: Split 1 and Split 2
    var_model_gerber_exog_1 <- VAR(data_gerber_1[,
c("Gerber", "CPI")], p = max_lag_gerber_exog, type =
"const", exogen = data_gerber_1[,
all_exog_vars_gerber_1])
    var_model_gerber_exog_2 <- VAR(data_gerber_2[,
c("Gerber", "CPI")], p = max_lag_gerber_exog, type =
"const", exogen = data_gerber_2[,
all_exog_vars_gerber_2])
    summary(var_model_gerber_exog_1)
    summary(var_model_gerber_exog_2)
  }



  # Restore data frames
  {
    data_pearson <- MScBA_Data_R[, c("Pearson", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]
    data_gerber <- MScBA_Data_R[, c("Gerber", "GDP",
"IP", "PPI", "TBill_13w", "TBill_10y", "EnergyIndex",
"USDIndex", "EURUSD", "CNYUSD", "SP500Var", "BCIVar",
"CPI")]
    data_pearson_hc <- MScBA_Data_R[, c("Pearson_HC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
```

```r
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_gerber_hc <- MScBA_Data_R[, c("Gerber_HC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_pearson_sc <- MScBA_Data_R[, c("Pearson_SC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_gerber_sc <- MScBA_Data_R[, c("Gerber_SC",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_pearson_1 <- MScBA_Data_R_1[, c("Pearson",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_pearson_2 <- MScBA_Data_R_2[, c("Pearson",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_gerber_1 <- MScBA_Data_R_1[, c("Gerber",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    data_gerber_2 <- MScBA_Data_R_2[, c("Gerber",
"GDP", "IP", "PPI", "TBill_13w", "TBill_10y",
"EnergyIndex", "USDIndex", "EURUSD", "CNYUSD",
"SP500Var", "BCIVar", "CPI")]
    colnames(data_pearson_hc)[colnames(data_pearson_hc)
== 'Pearson_HC'] <- 'Pearson'
    colnames(data_gerber_hc)[colnames(data_gerber_hc)
== 'Gerber_HC'] <- 'Gerber'
    colnames(data_pearson_sc)[colnames(data_pearson_sc)
== 'Pearson_SC'] <- 'Pearson'
    colnames(data_gerber_sc)[colnames(data_gerber_sc)
== 'Gerber_SC'] <- 'Gerber'
  }
}


# Random Forest
  {
  # Set a random seed for reproducibility
  set.seed(123)

  # Define the control using a grid with 10 values of
mtry
  ctrl <- trainControl(method = "cv", number = 10)

  # Define the grid to search for mtry and
min.node.size
  grid <- expand.grid(.mtry = 2:12, .splitrule =
"variance", .min.node.size = 1:5)
```

```r
  # Full sample

  # Pearson
  {
    # Tune the random forest model for 'Pearson'
    rf_model_Pearson <- caret::train(Pearson ~ GDP + IP
+ PPI + TBill_13w + TBill_10y + EnergyIndex + USDIndex
+ EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                     data =
data_pearson,
                                     method = "ranger",
                                     trControl = ctrl,
                                     tuneGrid = grid,
                                     importance =
"impurity")

    # Define hyperparameters for Pearson
    grid_pearson <- data.frame(.mtry =
rf_model_Pearson$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Pearson$bestTune$min.node.size)

    # Ger permutation importance measures
    rf_model_Pearson_ <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_pearson,
                                      method =
"ranger",
                                      trControl = ctrl,
                                      tuneGrid =
grid_pearson,
                                      importance =
"permutation")

    # Print the model summary
    print("Pearson")
    print(rf_model_Pearson)
    print(rf_model_Pearson_)
    print(rf_model_Pearson$finalModel)
    print(rf_model_Pearson_$finalModel)
    rf_model_Pearson_importance <-
flextable(data.frame(
      Variable =
names(rf_model_Pearson$finalModel$variable.importance),
      Impurity =
rf_model_Pearson$finalModel$variable.importance,
      Permutation =
rf_model_Pearson_$finalModel$variable.importance))
    print(rf_model_Pearson_importance)
  }

  # Gerber
  {
    # Tune the random forest model for 'Gerber'
    rf_model_Gerber <- caret::train(Gerber ~ GDP + IP +
PPI + TBill_13w + TBill_10y + EnergyIndex + USDIndex +
EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                    data = data_gerber,
                                    method = "ranger",
                                    trControl = ctrl,
                                    tuneGrid = grid,
                                    importance =
"impurity")

    # Define hyperparameters for Gerber
    grid_gerber <- data.frame(.mtry =
rf_model_Gerber$bestTune$mtry, .splitrule = "variance",
.min.node.size =
rf_model_Gerber$bestTune$min.node.size)

    # Get permutation importance measures
    rf_model_Gerber_ <- caret::train(Gerber ~ GDP + IP
+ PPI + TBill_13w + TBill_10y + EnergyIndex + USDIndex
+ EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                     data =
data_gerber,
                                     method = "ranger",
                                     trControl = ctrl,
                                     tuneGrid =
grid_gerber,
                                     importance =
"permutation")

    # Print the model summary
    print("Gerber")
    print(rf_model_Gerber)
    print(rf_model_Gerber_)
    print(rf_model_Gerber$finalModel)
    print(rf_model_Gerber_$finalModel)
    rf_model_Gerber_importance <- flextable(data.frame(
      Variable =
names(rf_model_Gerber$finalModel$variable.importance),
      Impurity =
rf_model_Gerber$finalModel$variable.importance,
      Permutation =
rf_model_Gerber_$finalModel$variable.importance))
    print(rf_model_Gerber_importance)
  }


# Hard/Soft commodities

# Pearson_HC
{
  # Tune the random forest model for 'Pearson_HC'
  rf_model_Pearson_hc <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_pearson_hc,
                                      method =
"ranger",
                                      trControl =
ctrl,
                                      tuneGrid =
grid,
                                      importance =
"impurity")

  # Define hyperparameters for Pearson_hc
  grid_pearson_hc <- data.frame(.mtry =
rf_model_Pearson_hc$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Pearson_hc$bestTune$min.node.size)

  # Get permutation importance measures
  rf_model_Pearson_hc_ <- caret::train(Pearson ~ GDP
+ IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                       data =
data_pearson_hc,
                                       method =
"ranger",
                                       trControl =
ctrl,
                                       tuneGrid =
grid_pearson_hc,
                                       importance =
"permutation")

  # Print the model summary
  print("Pearson_HC")
  print(rf_model_Pearson_hc)
  print(rf_model_Pearson_hc_)
  print(rf_model_Pearson_hc$finalModel)
  print(rf_model_Pearson_hc_$finalModel)
  rf_model_Pearson_hc_importance <-
flextable(data.frame(
    Variable =
names(rf_model_Pearson_hc$finalModel$variable.importanc
e),
    Impurity =
rf_model_Pearson_hc$finalModel$variable.importance,
    Permutation =
rf_model_Pearson_hc_$finalModel$variable.importance))
  print(rf_model_Pearson_hc_importance)
}
```

```r
# Gerber_HC
{
  # Tune the random forest model for 'Gerber_HC'
  rf_model_Gerber_hc <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                     data =
data_gerber_hc,
                                     method =
"ranger",
                                     trControl =
ctrl,
                                     tuneGrid = grid,
                                     importance =
"impurity")

  # Define hyperparameters for Gerber_hc
  grid_gerber_hc <- data.frame(.mtry =
rf_model_Gerber_hc$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Gerber_hc$bestTune$min.node.size)

  # Get permutation importance measures
  rf_model_Gerber_hc_ <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_gerber_hc,
                                      method =
"ranger",
                                      trControl =
ctrl,
                                      tuneGrid =
grid_gerber_hc,
                                      importance =
"permutation")

  # Print the model summary
  print("Gerber_HC")
  print(rf_model_Gerber_hc)
  print(rf_model_Gerber_hc_)
  print(rf_model_Gerber_hc$finalModel)
  print(rf_model_Gerber_hc_$finalModel)
  rf_model_Gerber_hc_importance <-
flextable(data.frame(
    Variable =
names(rf_model_Gerber_hc$finalModel$variable.importance
),
    Impurity =
rf_model_Gerber_hc$finalModel$variable.importance,
    Permutation =
rf_model_Gerber_hc_$finalModel$variable.importance))
  print(rf_model_Gerber_hc_importance)
}
```

```r
# Pearson_SC
{
  # Tune the random forest model for 'Pearson_SC'
  rf_model_Pearson_sc <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_pearson_sc,
                                      method =
"ranger",
                                      trControl =
ctrl,
                                      tuneGrid =
grid,
                                      importance =
"impurity")

  # Define hyperparameters for Pearson_sc
  grid_pearson_sc <- data.frame(.mtry =
rf_model_Pearson_sc$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Pearson_sc$bestTune$min.node.size)

  # Get permutation importance measures
  rf_model_Pearson_sc_ <- caret::train(Pearson ~ GDP
+ IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                       data =
data_pearson_sc,
                                       method =
"ranger",
                                       trControl =
ctrl,
                                       tuneGrid =
grid_pearson_sc,
                                       importance =
"permutation")

  # Print the model summary
  print("Pearson_SC")
  print(rf_model_Pearson_sc)
  print(rf_model_Pearson_sc_)
  print(rf_model_Pearson_sc$finalModel)
  print(rf_model_Pearson_sc_$finalModel)
  rf_model_Pearson_sc_importance <-
flextable(data.frame(
    Variable =
names(rf_model_Pearson_sc$finalModel$variable.importanc
e),
    Impurity =
rf_model_Pearson_sc$finalModel$variable.importance,
    Permutation =
rf_model_Pearson_sc_$finalModel$variable.importance))
  print(rf_model_Pearson_sc_importance)
}
```

```r
# Gerber_SC
{
  # Tune the random forest model for 'Gerber_SC'
  rf_model_Gerber_sc <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                     data =
data_gerber_sc,
                                     method =
"ranger",
                                     trControl =
ctrl,
                                     tuneGrid = grid,
                                     importance =
"impurity")

  # Define hyperparameters for Gerber_sc
  grid_gerber_sc <- data.frame(.mtry =
rf_model_Gerber_sc$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Gerber_sc$bestTune$min.node.size)

  # Get permutation importance measures
  rf_model_Gerber_sc_ <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_gerber_sc,
                                      method =
"ranger",
                                      trControl =
ctrl,
                                      tuneGrid =
grid_gerber_sc,
                                      importance =
"permutation")

  # Print the model summary
  print("Gerber_SC")
  print(rf_model_Gerber_sc)
  print(rf_model_Gerber_sc_)
  print(rf_model_Gerber_sc$finalModel)
  print(rf_model_Gerber_sc_$finalModel)
  rf_model_Gerber_sc_importance <-
flextable(data.frame(
    Variable =
names(rf_model_Gerber_sc$finalModel$variable.importance
),
    Impurity =
rf_model_Gerber_sc$finalModel$variable.importance,
    Permutation =
rf_model_Gerber_sc_$finalModel$variable.importance))
  print(rf_model_Gerber_sc_importance)
}
```

```
   # Subsamples

   # Pearson_1
   {
      # Tune the random forest model for 'Pearson_1'
      rf_model_Pearson_1 <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                         data =
data_pearson_1,
                                         method =
"ranger",
                                         trControl =
ctrl,
                                         tuneGrid = grid,
                                         importance =
"impurity")

      # Define hyperparameters for Pearson_1
      grid_pearson_1 <- data.frame(.mtry =
rf_model_Pearson_1$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Pearson_1$bestTune$min.node.size)

      # Get permutation importance measures
      rf_model_Pearson_1_ <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                          data =
data_pearson_1,
                                          method =
"ranger",
                                          trControl =
ctrl,
                                          tuneGrid =
grid_pearson_1,
                                          importance =
"permutation")

      # Print the model summary
      print("Pearson_1")
      print(rf_model_Pearson_1)
      print(rf_model_Pearson_1_)
      print(rf_model_Pearson_1$finalModel)
      print(rf_model_Pearson_1_$finalModel)
      rf_model_Pearson_1_importance <-
flextable(data.frame(
         Variable =
names(rf_model_Pearson_1$finalModel$variable.importance
),
         Impurity =
rf_model_Pearson_1$finalModel$variable.importance,
         Permutation =
rf_model_Pearson_1_$finalModel$variable.importance))
      print(rf_model_Pearson_1_importance)
   }

   # Gerber_1
   {
      # Tune the random forest model for 'Gerber_1'
      rf_model_Gerber_1 <- caret::train(Gerber ~ GDP + IP
+ PPI + TBill_13w + TBill_10y + EnergyIndex + USDIndex
+ EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                        data =
data_gerber_1,
                                        method =
"ranger",
                                        trControl = ctrl,
                                        tuneGrid = grid,
                                        importance =
"impurity")

      # Define hyperparameters for Gerber_1
      grid_gerber_1 <- data.frame(.mtry =
rf_model_Gerber_1$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Gerber_1$bestTune$min.node.size)

      # Get permutation importance measures
      rf_model_Gerber_1_ <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                         data =
data_gerber_1,
                                         method =
"ranger",
                                         trControl =
ctrl,
                                         tuneGrid =
grid_gerber_1,
                                         importance =
"permutation")

      # Print the model summary
      print("Gerber_1")
      print(rf_model_Gerber_1)
      print(rf_model_Gerber_1_)
      print(rf_model_Gerber_1$finalModel)
      print(rf_model_Gerber_1_$finalModel)
      rf_model_Gerber_1_importance <-
flextable(data.frame(
         Variable =
names(rf_model_Gerber_1$finalModel$variable.importance)
,
         Impurity =
rf_model_Gerber_1$finalModel$variable.importance,
         Permutation =
rf_model_Gerber_1_$finalModel$variable.importance))
      print(rf_model_Gerber_1_importance)
   }

   # Pearson_2
   {
      # Tune the random forest model for 'Pearson_2'
      rf_model_Pearson_2 <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                         data =
data_pearson_2,
                                         method =
"ranger",
                                         trControl =
ctrl,
                                         tuneGrid = grid,
                                         importance =
"impurity")

      # Define hyperparameters for Pearson_2
      grid_pearson_2 <- data.frame(.mtry =
rf_model_Pearson_2$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Pearson_2$bestTune$min.node.size)

      # Get permutation importance measures
      rf_model_Pearson_2_ <- caret::train(Pearson ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                          data =
data_pearson_2,
                                          method =
"ranger",
                                          trControl =
ctrl,
                                          tuneGrid =
grid_pearson_2,
                                          importance =
"permutation")

      # Print the model summary
      print("Pearson_2")
      print(rf_model_Pearson_2)
      print(rf_model_Pearson_2_)
      print(rf_model_Pearson_2$finalModel)
      print(rf_model_Pearson_2_$finalModel)
      rf_model_Pearson_2_importance <-
flextable(data.frame(
         Variable =
names(rf_model_Pearson_2$finalModel$variable.importance
),
```

```r
        Impurity =
rf_model_Pearson_2$finalModel$variable.importance,
        Permutation =
rf_model_Pearson_2_$finalModel$variable.importance))
    print(rf_model_Pearson_2_importance)
  }

  # Gerber_2
  {
    # Tune the random forest model for 'Gerber_2'
    rf_model_Gerber_2 <- caret::train(Gerber ~ GDP + IP
+ PPI + TBill_13w + TBill_10y + EnergyIndex + USDIndex
+ EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                      data =
data_gerber_2,
                                      method =
"ranger",
                                      trControl = ctrl,
                                      tuneGrid = grid,
                                      importance =
"impurity")

    # Define hyperparameters for Gerber_2
    grid_gerber_2 <- data.frame(.mtry =
rf_model_Gerber_2$bestTune$mtry, .splitrule =
"variance", .min.node.size =
rf_model_Gerber_2$bestTune$min.node.size)

    # Get permutation importance measures
    rf_model_Gerber_2_ <- caret::train(Gerber ~ GDP +
IP + PPI + TBill_13w + TBill_10y + EnergyIndex +
USDIndex + EURUSD + CNYUSD + SP500Var + BCIVar + CPI,
                                       data =
data_gerber_2,
                                       method =
"ranger",
                                       trControl =
ctrl,
                                       tuneGrid =
grid_gerber_2,
                                       importance =
"permutation")

    # Print the model summary
    print("Gerber_2")
    print(rf_model_Gerber_2)
    print(rf_model_Gerber_2_)
    print(rf_model_Gerber_2$finalModel)
    print(rf_model_Gerber_2_$finalModel)
    rf_model_Gerber_2_importance <-
flextable(data.frame(
      Variable =
names(rf_model_Gerber_2$finalModel$variable.importance)
,
```

```r
        Impurity =
rf_model_Gerber_2$finalModel$variable.importance,
        Permutation =
rf_model_Gerber_2_$finalModel$variable.importance))
    print(rf_model_Gerber_2_importance)
  }

}

# Tables
  {
  # Multiple Regressions
  {

    # Extract coefficients and p-values from the models
    lm_tidy_pearson <- tidy(regression_pearson) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_pearson_hc <- tidy(regression_pearson_hc)
%>% dplyr::select(term, estimate, p.value)
    lm_tidy_pearson_sc <- tidy(regression_pearson_sc)
%>% dplyr::select(term, estimate, p.value)
    lm_tidy_pearson_1 <- tidy(regression_pearson_1) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_pearson_2 <- tidy(regression_pearson_2) %>%
dplyr::select(term, estimate, p.value)

    lm_tidy_gerber <- tidy(regression_gerber) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_gerber_hc <- tidy(regression_gerber_hc) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_gerber_sc <- tidy(regression_gerber_sc) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_gerber_1 <- tidy(regression_gerber_1) %>%
dplyr::select(term, estimate, p.value)
    lm_tidy_gerber_2 <- tidy(regression_gerber_2) %>%
dplyr::select(term, estimate, p.value)

    # Merge data frames
    lm_merged_pearson <- reduce(list(lm_tidy_pearson,
lm_tidy_pearson_hc, lm_tidy_pearson_sc,
lm_tidy_pearson_1, lm_tidy_pearson_2), full_join, by =
"term")
    lm_merged_gerber <- reduce(list(lm_tidy_gerber,
lm_tidy_gerber_hc, lm_tidy_gerber_sc, lm_tidy_gerber_1,
lm_tidy_gerber_2), full_join, by = "term")

    # Rename columns for clarity
    colnames(lm_merged_pearson) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")
    colnames(lm_merged_gerber) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")
```

```r
    # Create flextables
    lm_pearson_flextable <-
flextable(lm_merged_pearson)
    lm_gerber_flextable <- flextable(lm_merged_gerber)
    print(lm_pearson_flextable)
    print(lm_gerber_flextable)

  }

  # VAR models
  {# Function to get tidy summary of VAR model for a
specific dependent variable (e.g., Pearson, Gerber)
    get_var_tidy <- function(var_model, dependent_var)
{
      var_summary <-
summary(var_model)$var[[dependent_var]]
      data.frame(
        term = rownames(var_summary$coef),
        estimate = var_summary$coef[, 1],
        std.error = var_summary$coef[, 2],
        p.value = var_summary$coef[, 4]
      )
    }

    # Create tidy summaries for VAR models
    var_tidy_pearson <- get_var_tidy(var_model_pearson,
"Pearson") %>% dplyr::select(term, estimate, p.value)
    var_tidy_pearson_hc <-
get_var_tidy(var_model_pearson_hc, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_pearson_sc <-
get_var_tidy(var_model_pearson_sc, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_pearson_1 <-
get_var_tidy(var_model_pearson_1, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_pearson_2 <-
get_var_tidy(var_model_pearson_2, "Pearson") %>%
dplyr::select(term, estimate, p.value)

    var_tidy_gerber <- get_var_tidy(var_model_gerber,
"Gerber") %>% dplyr::select(term, estimate, p.value)
    var_tidy_gerber_hc <-
get_var_tidy(var_model_gerber_hc, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_gerber_sc <-
get_var_tidy(var_model_gerber_sc, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_gerber_1 <-
get_var_tidy(var_model_gerber_1, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_tidy_gerber_2 <-
get_var_tidy(var_model_gerber_2, "Gerber") %>%
dplyr::select(term, estimate, p.value)
```

```r
    # Merge data frames for Pearson and Gerber
    var_merged_pearson <- reduce(list(var_tidy_pearson,
var_tidy_pearson_hc, var_tidy_pearson_sc,
var_tidy_pearson_1, var_tidy_pearson_2), full_join, by
= "term")
    var_merged_gerber <- reduce(list(var_tidy_gerber,
var_tidy_gerber_hc, var_tidy_gerber_sc,
var_tidy_gerber_1, var_tidy_gerber_2), full_join, by =
"term")

    # Rename columns for clarity
    colnames(var_merged_pearson) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")
    colnames(var_merged_gerber) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")

    # Create flextables
    var_pearson_flextable <-
flextable(var_merged_pearson)
    var_gerber_flextable <-
flextable(var_merged_gerber)

    # Output the tables
    print(var_merged_pearson)
    print(var_pearson_flextable)

    print(var_merged_gerber)
    print(var_gerber_flextable)
  }

  # Recursive VAR models
  {# Create tidy summaries for recursive VAR models
    var_exog_tidy_pearson <-
get_var_tidy(var_model_pearson_exog, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_pearson_hc <-
get_var_tidy(var_model_pearson_exog_hc, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_pearson_sc <-
get_var_tidy(var_model_pearson_exog_sc, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_pearson_1 <-
get_var_tidy(var_model_pearson_exog_1, "Pearson") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_pearson_2 <-
get_var_tidy(var_model_pearson_exog_2, "Pearson") %>%
dplyr::select(term, estimate, p.value)

    var_exog_tidy_gerber <-
get_var_tidy(var_model_gerber_exog, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_gerber_hc <-
get_var_tidy(var_model_gerber_exog_hc, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_gerber_sc <-
get_var_tidy(var_model_gerber_exog_sc, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_gerber_1 <-
get_var_tidy(var_model_gerber_exog_1, "Gerber") %>%
dplyr::select(term, estimate, p.value)
    var_exog_tidy_gerber_2 <-
get_var_tidy(var_model_gerber_exog_2, "Gerber") %>%
dplyr::select(term, estimate, p.value)

    # Merge data frames for Pearson and Gerber
    var_exog_merged_pearson <-
reduce(list(var_exog_tidy_pearson,
var_exog_tidy_pearson_hc, var_exog_tidy_pearson_sc,
var_exog_tidy_pearson_1, var_exog_tidy_pearson_2),
full_join, by = "term")
    var_exog_merged_gerber <-
reduce(list(var_exog_tidy_gerber,
var_exog_tidy_gerber_hc, var_exog_tidy_gerber_sc,
var_exog_tidy_gerber_1, var_exog_tidy_gerber_2),
full_join, by = "term")

    # Rename columns for clarity
    colnames(var_exog_merged_pearson) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")
    colnames(var_exog_merged_gerber) <- c("Variable",
"Estimate_Full", "P-Value_Full", "Estimate_HC", "P-
Value_HC", "Estimate_SC", "P-Value_SC", "Estimate_1",
"P-Value_1", "Estimate_2", "P-Value_2")

    # Create flextables
    var_exog_pearson_flextable <-
flextable(var_exog_merged_pearson)
    var_exog_gerber_flextable <-
flextable(var_exog_merged_gerber)
    set_flextable_defaults(digits = 3)
    # Output the tables
    print(var_exog_merged_pearson)
    print(var_exog_pearson_flextable)

    print(var_exog_merged_gerber)
    print(var_exog_gerber_flextable)
  }

# MSE for linear models
{
  # Calculate MSE for VAR models

  MSE_pearson <-
mean(residuals(var_model_pearson$var$"Pearson")^2)
  MSE_gerber <-
mean(residuals(var_model_gerber$var$"Gerber")^2)
  MSE_pearson_1 <-
mean(residuals(var_model_pearson_1$var$"Pearson")^2)
  MSE_gerber_1 <-
mean(residuals(var_model_gerber_1$var$"Gerber")^2)
  MSE_pearson_2 <-
mean(residuals(var_model_pearson_2$var$"Pearson")^2)
  MSE_gerber_2 <-
mean(residuals(var_model_gerber_2$var$"Gerber")^2)
  MSE_pearson_hc <-
mean(residuals(var_model_pearson_hc$var$"Pearson")^2)
  MSE_gerber_hc <-
mean(residuals(var_model_gerber_hc$var$"Gerber")^2)
  MSE_pearson_sc <-
mean(residuals(var_model_pearson_sc$var$"Pearson")^2)
  MSE_gerber_sc <-
mean(residuals(var_model_gerber_sc$var$"Gerber")^2)

  mse_var <- flextable(data.frame(
    Model = c("var_model_pearson",
"var_model_pearson_hc", "var_model_pearson_sc",
              "var_model_pearson_1",
"var_model_pearson_2",
              "var_model_gerber",
"var_model_gerber_hc", "var_model_gerber_sc",
              "var_model_gerber_1",
"var_model_gerber_2"),
    MSE = c(MSE_pearson, MSE_pearson_hc,
MSE_pearson_sc,
            MSE_pearson_1, MSE_pearson_2,
            MSE_gerber, MSE_gerber_hc, MSE_gerber_sc,
            MSE_gerber_1, MSE_gerber_2)
  ))

  mse_var


  # Calculate MSE for recursive VAR models
  MSE_pearson_exog <-
mean(residuals(var_model_pearson_exog$var$"Pearson")^2)
  MSE_gerber_exog <-
mean(residuals(var_model_gerber_exog$var$"Gerber")^2)
  MSE_pearson_exog_1 <-
mean(residuals(var_model_pearson_exog_1$var$"Pearson")^
2)
  MSE_gerber_exog_1 <-
mean(residuals(var_model_gerber_exog_1$var$"Gerber")^2)
  MSE_pearson_exog_2 <-
mean(residuals(var_model_pearson_exog_2$var$"Pearson")^
2)
  MSE_gerber_exog_2 <-
mean(residuals(var_model_gerber_exog_2$var$"Gerber")^2)
```

```r
    MSE_pearson_exog_hc <-
mean(residuals(var_model_pearson_exog_hc$var$"Pearson")
^2)
    MSE_gerber_exog_hc <-
mean(residuals(var_model_gerber_exog_hc$var$"Gerber")^2
)
    MSE_pearson_exog_sc <-
mean(residuals(var_model_pearson_exog_sc$var$"Pearson")
^2)
    MSE_gerber_exog_sc <-
mean(residuals(var_model_gerber_exog_sc$var$"Gerber")^2
)

    mse_var_exog <- flextable(data.frame(
        Model = c("var_model_pearson_exog",
"var_model_pearson_exog_hc",
"var_model_pearson_exog_sc",
                  "var_model_pearson_exog_1",
"var_model_pearson_exog_2",
                  "var_model_gerber_exog",
"var_model_gerber_exog_hc", "var_model_gerber_exog_sc",
                  "var_model_gerber_exog_1",
"var_model_gerber_exog_2"),
        MSE = c(MSE_pearson_exog, MSE_pearson_exog_hc,
MSE_pearson_exog_sc,
                MSE_pearson_exog_1, MSE_pearson_exog_2,
                MSE_gerber_exog, MSE_gerber_exog_hc,
MSE_gerber_exog_sc,
                MSE_gerber_exog_1, MSE_gerber_exog_2)
    ))

    mse_var_exog


    # Calculate MSE for regression models
    MSE_lm_pearson <-
mean(residuals(regression_pearson)^2)
    MSE_lm_gerber <-
mean(residuals(regression_gerber)^2)
    MSE_lm_pearson_hc <-
mean(residuals(regression_pearson_hc)^2)
    MSE_lm_gerber_hc <-
mean(residuals(regression_gerber_hc)^2)
    MSE_lm_pearson_sc <-
mean(residuals(regression_pearson_sc)^2)
    MSE_lm_gerber_sc <-
mean(residuals(regression_gerber_sc)^2)
    MSE_lm_pearson_1 <-
mean(residuals(regression_pearson_1)^2)
    MSE_lm_gerber_1 <-
mean(residuals(regression_gerber_1)^2)
    MSE_lm_pearson_2 <-
mean(residuals(regression_pearson_2)^2)
    MSE_lm_gerber_2 <-
mean(residuals(regression_gerber_2)^2)

    mse_lm <- flextable(data.frame(
        Model = c("lm_pearson", "lm_pearson_hc",
"lm_pearson_sc", "lm_pearson_1", "lm_pearson_2",
                  "lm_gerber", "lm_gerber_hc",
"lm_gerber_sc", "lm_gerber_1", "lm_gerber_2"),
        MSE = c(MSE_lm_pearson, MSE_lm_pearson_hc,
MSE_lm_pearson_sc, MSE_lm_pearson_1, MSE_lm_pearson_2,
                MSE_lm_gerber, MSE_lm_gerber_hc,
MSE_lm_gerber_sc, MSE_lm_gerber_1, MSE_lm_gerber_2)
    ))

    mse_lm
}

# Random Forest models
{
    # Performance overview


    # Initialize a dataframe with Pearson model results
    final_df <- data.frame(
        "Performance Measure" = c("Mean_RMSE",
"Mean_Rsquared", "Mean_MAE"),
        Pearson = c(
            mean(mean(rf_model_Pearson_$resample$RMSE),
mean(rf_model_Pearson$resample$RMSE)),
            mean(mean(rf_model_Pearson_$resample$Rsquared),
mean(rf_model_Pearson$resample$Rsquared)),
            mean(mean(rf_model_Pearson_$resample$MAE),
mean(rf_model_Pearson$resample$MAE))
        )
    )

    # Manually add Pearson_hc model results as a new
column
    final_df$Pearson_hc <- c(
        mean(mean(rf_model_Pearson_hc_$resample$RMSE),
mean(rf_model_Pearson_hc$resample$RMSE)),

mean(mean(rf_model_Pearson_hc_$resample$Rsquared),
mean(rf_model_Pearson_hc$resample$Rsquared)),
        mean(mean(rf_model_Pearson_hc_$resample$MAE),
mean(rf_model_Pearson_hc$resample$MAE))
    )

    # Manually add Pearson_sc model results as a new
column
    final_df$Pearson_sc <- c(
        mean(mean(rf_model_Pearson_sc_$resample$RMSE),
mean(rf_model_Pearson_sc$resample$RMSE)),

mean(mean(rf_model_Pearson_sc_$resample$Rsquared),
mean(rf_model_Pearson_sc$resample$Rsquared)),
        mean(mean(rf_model_Pearson_sc_$resample$MAE),
mean(rf_model_Pearson_sc$resample$MAE))
    )

    # Manually add Pearson_1 model results as a new
column
    final_df$Pearson_1 <- c(
        mean(mean(rf_model_Pearson_1_$resample$RMSE),
mean(rf_model_Pearson_1$resample$RMSE)),
        mean(mean(rf_model_Pearson_1_$resample$Rsquared),
mean(rf_model_Pearson_1$resample$Rsquared)),
        mean(mean(rf_model_Pearson_1_$resample$MAE),
mean(rf_model_Pearson_1$resample$MAE))
    )

    # Manually add Pearson_2 model results as a new
column
    final_df$Pearson_2 <- c(
        mean(mean(rf_model_Pearson_2_$resample$RMSE),
mean(rf_model_Pearson_2$resample$RMSE)),
        mean(mean(rf_model_Pearson_2_$resample$Rsquared),
mean(rf_model_Pearson_2$resample$Rsquared)),
        mean(mean(rf_model_Pearson_2_$resample$MAE),
mean(rf_model_Pearson_2$resample$MAE))
    )

    # Manually add Gerber model results as a new column
    final_df$Gerber <- c(
        mean(mean(rf_model_Gerber_$resample$RMSE),
mean(rf_model_Gerber$resample$RMSE)),
        mean(mean(rf_model_Gerber_$resample$Rsquared),
mean(rf_model_Gerber$resample$Rsquared)),
        mean(mean(rf_model_Gerber_$resample$MAE),
mean(rf_model_Gerber$resample$MAE))
    )

    # Manually add Gerber_hc model results as a new
column
    final_df$Gerber_hc <- c(
        mean(mean(rf_model_Gerber_hc_$resample$RMSE),
mean(rf_model_Gerber_hc$resample$RMSE)),
        mean(mean(rf_model_Gerber_hc_$resample$Rsquared),
mean(rf_model_Gerber_hc$resample$Rsquared)),
        mean(mean(rf_model_Gerber_hc_$resample$MAE),
mean(rf_model_Gerber_hc$resample$MAE))
    )

    # Manually add Gerber_sc model results as a new
column
    final_df$Gerber_sc <- c(
        mean(mean(rf_model_Gerber_sc_$resample$RMSE),
mean(rf_model_Gerber_sc$resample$RMSE)),
        mean(mean(rf_model_Gerber_sc_$resample$Rsquared),
mean(rf_model_Gerber_sc$resample$Rsquared)),
```

```r
    mean(mean(rf_model_Gerber_sc_$resample$MAE),
mean(rf_model_Gerber_sc$resample$MAE))
    )

    # Manually add Gerber_1 model results as a new
column
    final_df$Gerber_1 <- c(
    mean(mean(rf_model_Gerber_1_$resample$RMSE),
mean(rf_model_Gerber_1$resample$RMSE)),
    mean(mean(rf_model_Gerber_1_$resample$Rsquared),
mean(rf_model_Gerber_1$resample$Rsquared)),
    mean(mean(rf_model_Gerber_1_$resample$MAE),
mean(rf_model_Gerber_1$resample$MAE))
    )

    # Manually add Gerber_2 model results as a new
column
    final_df$Gerber_2 <- c(
    mean(mean(rf_model_Gerber_2_$resample$RMSE),
mean(rf_model_Gerber_2$resample$RMSE)),
    mean(mean(rf_model_Gerber_2_$resample$Rsquared),
mean(rf_model_Gerber_2$resample$Rsquared)),
    mean(mean(rf_model_Gerber_2_$resample$MAE),
mean(rf_model_Gerber_2$resample$MAE))
    )

    # Print the final data frame
    print(flextable(final_df))
  }
  {
    print(rf_model_Pearson_importance)
    print(rf_model_Pearson_hc_importance)
    print(rf_model_Pearson_sc_importance)
    print(rf_model_Pearson_1_importance)
    print(rf_model_Pearson_2_importance)
    print(rf_model_Gerber_importance)
    print(rf_model_Gerber_hc_importance)
    print(rf_model_Gerber_sc_importance)
    print(rf_model_Gerber_1_importance)
    print(rf_model_Gerber_2_importance)
  }
}

# Descriptive statistics and preliminary tests
  {
    # Descriptives
    {
    descriptives <- flextable(cbind(VariableNames =
row.names(describe(MScBA_Data_R)),
describe(MScBA_Data_R)))
      descriptives

    descriptives1 <- flextable(cbind(VariableNames =
row.names(describe(MScBA_Data_R_1)),
describe(MScBA_Data_R_1)))
```

```r
      descriptives1

    descriptives2 <- flextable(cbind(VariableNames =
row.names(describe(MScBA_Data_R_2)),
describe(MScBA_Data_R_2)))
      descriptives2

    descriptivesCR <- flextable(cbind(VariableNames =
row.names(describe(Commodiy_returns)),
describe(Commodiy_returns)))
      descriptivesCR

    # Histograms
    breaks <- 30

    for(variable in names(MScBA_Data_R)[-1]) {
      # Create a histogram for each variable
      hist(MScBA_Data_R[[variable]], breaks = breaks,
main=paste("Histogram of", variable), xlab=variable)
    }
    for(variable in names(MScBA_Data_R_1)[-1]) {
      # Create a histogram for each variable
      hist(MScBA_Data_R_1[[variable]], breaks =
breaks, main=paste("Histogram of", variable),
xlab=variable)
    }
    for(variable in names(MScBA_Data_R_2)[-1]) {
      # Create a histogram for each variable
      hist(MScBA_Data_R_2[[variable]], breaks =
breaks, main=paste("Histogram of", variable),
xlab=variable)
    }
    for(variable in names(Commodiy_returns)[-1]) {
      # Create a histogram for each variable
      hist(Commodiy_returns[[variable]], breaks =
breaks, main=paste("Histogram of", variable),
xlab=variable)
    }

    }

    # Normality tests
    {
    # Function to perform Shapiro-Wilk test for
normality
    test_normality <- function(data, var_name) {
      shapiro_test <- shapiro.test(data[[var_name]])
      list(
      Variable = var_name,
      W = shapiro_test$statistic,
      p_value = shapiro_test$p.value
      )
    }

    # Variables of interest in MScBA_Data R
```

```r
    variables <- c("Pearson", "Gerber", "Pearson_HC",
"Pearson_SC", "Gerber_HC", "Gerber_SC")

    # Perform tests of normality on the variables
    results <- lapply(variables, function(var)
test_normality(MScBA_Data_R, var))

    # Perform tests of normality on subsamples
    results_pearson_1 <-
test_normality(data_pearson_1, "Pearson")
    results_pearson_2 <-
test_normality(data_pearson_2, "Pearson")
    results_gerber_1 <- test_normality(data_gerber_1,
"Gerber")
    results_gerber_2 <- test_normality(data_gerber_2,
"Gerber")

    # Combine all results
    all_results <- c(results, list(results_pearson_1,
results_pearson_2, results_gerber_1, results_gerber_2))

    # Convert to a data frame
    df_results <- do.call(rbind, lapply(all_results,
as.data.frame))

    # Display the outcomes in a flextable
    flextable(df_results)
    }

    # Difference in means tests
    {
    # Subsamples
    wilcox_test_pearson <-
wilcox.test(data_pearson_1$Pearson,
data_pearson_2$Pearson)
    wilcox_test_gerber <-
wilcox.test(data_gerber_1$Gerber, data_gerber_2$Gerber)
    wilcox_test_pearson_HC_SC <-
wilcox.test(MScBA_Data_R$Pearson_HC,
MScBA_Data_R$Pearson_SC)
    wilcox_test_gerber_HC_SC <-
wilcox.test(MScBA_Data_R$Gerber_HC,
MScBA_Data_R$Gerber_SC)

    # Create a data frame with the results
    results_df <- data.frame(
      Test = c("Pearson Subsamples", "Gerber
Subsamples", "Pearson HC vs SC", "Gerber HC vs SC"),
      W_statistic = c(wilcox_test_pearson$statistic,
wilcox_test_gerber$statistic,
wilcox_test_pearson_HC_SC$statistic,
wilcox_test_gerber_HC_SC$statistic),
      p_value = c(wilcox_test_pearson$p.value,
wilcox_test_gerber$p.value,
```

```
    wilcox_test_pearson_HC_SC$p.value,
    wilcox_test_gerber_HC_SC$p.value)
      )

      # Display the results in a flextable
      flextable(results_df)

    }

    # Other
    {
      # AIC results
      combined_results <-
rbind(bic_results_gerber$criteria,

bic_results_gerber_exog$criteria,

bic_results_gerber_hc$criteria,

bic_results_gerber_exog_hc$criteria,

bic_results_gerber_sc$criteria,

bic_results_gerber_exog_sc$criteria,

bic_results_pearson$criteria,

bic_results_pearson_exog$criteria,

bic_results_pearson_hc$criteria,

bic_results_pearson_exog_hc$criteria,

bic_results_pearson_sc$criteria,

bic_results_pearson_exog_sc$criteria
                                )

      print(flextable(as.data.frame(combined_results)))
      # RF learning curve


    }

    # Learning curves
    {
      # Pearson learning curve
      {
        # Assuming 'data_pearson' is your dataset with
the appropriate structure
        # Define the formula for the model
        formula <- Pearson ~ GDP + IP + PPI + TBill_13w
+ TBill_10y + EnergyIndex + USDIndex + EURUSD + CNYUSD
+ SP500Var + BCIVar + CPI

        # Define a sequence of 'ntree' values for the
learning curve
        ntree_values <- seq(50, 700, by = 50)

        # Initialize a data frame to store the results
        learning_curve_results <- data.frame(ntree =
integer(), Avg_RMSE = numeric())

        # Loop over 'ntree' values
        for (ntree in ntree_values) {
          rmse_values <- numeric(5) # To store RMSE
values for each run

          # Run the model 5 times for each 'ntree'
value
          for (run in 1:5) {
            set.seed(run) # Set a seed for
reproducibility

            # Fit the random forest model
            model <- ranger(
              formula,
              data = data_pearson,
              num.trees = ntree,
              mtry = floor(sqrt(12)), # mtry as the
square root of the number of predictors
              min.node.size = 3, # minimum node size
              importance = 'impurity',
              seed = run # Ensure reproducibility of
the random process within ranger
            )

            # Calculate the OOB RMSE from the model
            oob_rmse <- sqrt(model$prediction.error)

            # Store the RMSE value
            rmse_values[run] <- oob_rmse
          }

          # Calculate the average RMSE for the current
'ntree' value
          avg_rmse <- mean(rmse_values)

          # Add the results to our data frame
          learning_curve_results <-
rbind(learning_curve_results, data.frame(ntree = ntree,
Avg_RMSE = avg_rmse))
        }

        # Plot the learning curve using ggplot2
        ggplot(learning_curve_results, aes(x = ntree, y
= Avg_RMSE)) +
          geom_line() +
          geom_point() +
          xlab("Number of Trees (ntree)") +
          ylab("Average OOB RMSE") +
          ggtitle("Learning Curve for Random Forest
Regression on Pearson")
      }

      # Gerber learning curve
      {
        # Assuming 'data_pearson' is your dataset with
the appropriate structure
        # Define the formula for the model
        formula <- Gerber ~ GDP + IP + PPI + TBill_13w
+ TBill_10y + EnergyIndex + USDIndex + EURUSD + CNYUSD
+ SP500Var + BCIVar + CPI

        # Define a sequence of 'ntree' values for the
learning curve
        ntree_values <- seq(50, 700, by = 50)

        # Initialize a data frame to store the results
        learning_curve_results <- data.frame(ntree =
integer(), Avg_RMSE = numeric())

        # Loop over 'ntree' values
        for (ntree in ntree_values) {
          rmse_values <- numeric(5) # To store RMSE
values for each run

          # Run the model 5 times for each 'ntree'
value
          for (run in 1:5) {
            set.seed(run) # Set a seed for
reproducibility

            # Fit the random forest model
            model <- ranger(
              formula,
              data = data_gerber,
              num.trees = ntree,
              mtry = floor(sqrt(12)), # mtry as the
square root of the number of predictors
              min.node.size = 3, # minimum node size
              importance = 'impurity',
              seed = run # Ensure reproducibility of
the random process within ranger
            )

            # Calculate the OOB RMSE from the model
            oob_rmse <- sqrt(model$prediction.error)

            # Store the RMSE value
            rmse_values[run] <- oob_rmse
          }
```

```r
        # Calculate the average RMSE for the current
'ntree' value
        avg_rmse <- mean(rmse_values)

        # Add the results to our data frame
        learning_curve_results <-
rbind(learning_curve_results, data.frame(ntree = ntree,
Avg_RMSE = avg_rmse))
      }

      # Plot the learning curve using ggplot2
      ggplot(learning_curve_results, aes(x = ntree, y
= Avg_RMSE)) +
        geom_line() +
        geom_point() +
        xlab("Number of Trees (ntree)") +
        ylab("Average OOB RMSE") +
        ggtitle("Learning Curve for Random Forest
Regression on Gerber")
    }
  }

  }

# Residual analysis
  {
  # Pearson
  {
    # Table
    {
      # Extract residuals from each model
      resid_regression_pearson <-
residuals(regression_pearson)
      resid_var_pearson <-
residuals(var_model_pearson)[, 1]  # First equation
      resid_varx_pearson <-
residuals(var_model_pearson_exog)[, 1]  # First
equation
      resid_rf_pearson <-
rf_model_Pearson$trainingData$.outcome -
rf_model_Pearson$finalModel$predictions

      # Perform Shapiro-Wilk test for normality
      shapiro_regression_pearson <-
shapiro.test(resid_regression_pearson)
      shapiro_var_pearson <-
shapiro.test(resid_var_pearson)
      shapiro_varx_pearson <-
shapiro.test(resid_varx_pearson)
      shapiro_rf_pearson <-
shapiro.test(resid_rf_pearson)

      # Perform Durbin-Watson test for autocorrelation
(for regression model)

      dw_regression_pearson <-
dwtest(regression_pearson)

      # Perform Breusch-Pagan test for
heteroscedasticity for all models
      bp_regression_pearson <-
bptest(regression_pearson)
      bp_var_pearson <- bptest(resid_var_pearson ~
fitted(var_model_pearson)[, 1])
      bp_varx_pearson <- bptest(resid_varx_pearson ~
fitted(var_model_pearson_exog)[, 1])

      # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
      lb_var_pearson <- Box.test(resid_var_pearson,
type = "Ljung-Box")
      lb_varx_pearson <- Box.test(resid_varx_pearson,
type = "Ljung-Box")

      # Summarize results in a flextable
      resid_pearson <- data.frame(
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_pearson$statistic,
shapiro_var_pearson$statistic,
shapiro_varx_pearson$statistic,
shapiro_rf_pearson$statistic),
        Shapiro_P_Value =
c(shapiro_regression_pearson$p.value,
shapiro_var_pearson$p.value,
shapiro_varx_pearson$p.value,
shapiro_rf_pearson$p.value),
        DW_Statistic =
c(dw_regression_pearson$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_pearson$statistic,
bp_var_pearson$statistic, bp_varx_pearson$statistic,
NA),
        BP_P_Value = c(bp_regression_pearson$p.value,
bp_var_pearson$p.value, bp_varx_pearson$p.value, NA),
        LB_Statistic = c(NA, lb_var_pearson$statistic,
lb_varx_pearson$statistic, NA),
        LB_P_Value = c(NA, lb_var_pearson$p.value,
lb_varx_pearson$p.value, NA),
      )

      flextable(resid_pearson)
    }

    # VAR IRFs
    {
      # List of predictor variables in the VAR model
      predictors <- colnames(var_model_pearson$y)

      # Create an empty list to store IRF data for each
predictor
      irf_list <- list()

      # Loop through predictors and generate IRFs
      for (pred in predictors) {
        irf_data <- irf(var_model_pearson, impulse =
pred, response = "Pearson", n.ahead = 10, ortho = TRUE)
        irf_list[[pred]] <- irf_data
      }

      # Plot IRFs
      par(mfrow = c(ceiling(length(predictors)/4), 2))
      for (pred in predictors) {
        plot(irf_list[[pred]],
            xlab = "Periods", ylab = "Response",
            main = paste("IRF of Pearson to", pred),
col = "blue")
      }
    }

  # Gerber
  {
    #Table
    {
      # Extract residuals from each model
      resid_regression_gerber <-
residuals(regression_gerber)
      resid_var_gerber <- residuals(var_model_gerber)[,
1]  # First equation
      resid_varx_gerber <-
residuals(var_model_gerber_exog)[, 1]  # First equation
      resid_rf_gerber <-
rf_model_Gerber$trainingData$.outcome -
rf_model_Gerber$finalModel$predictions

      # Perform Shapiro-Wilk test for normality
      shapiro_regression_gerber <-
shapiro.test(resid_regression_gerber)
      shapiro_var_gerber <-
shapiro.test(resid_var_gerber)
      shapiro_varx_gerber <-
shapiro.test(resid_varx_gerber)
      shapiro_rf_gerber <-
shapiro.test(resid_rf_gerber)

      # Perform Durbin-Watson test for autocorrelation
(for regression model)
      dw_regression_gerber <- dwtest(regression_gerber)

      # Perform Breusch-Pagan test for
heteroscedasticity for all models
      bp_regression_gerber <- bptest(regression_gerber)
      bp_var_gerber <- bptest(resid_var_gerber ~
fitted(var_model_gerber)[, 1])
```

```
        bp_varx_gerber <- bptest(resid_varx_gerber ~
fitted(var_model_gerber_exog)[, 1])

        # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
        lb_var_gerber <- Box.test(resid_var_gerber, type
= "Ljung-Box")
        lb_varx_gerber <- Box.test(resid_varx_gerber,
type = "Ljung-Box")

        # Summarize results in a flextable
        resid_gerber <- data.frame(
            Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
            Shapiro_W =
c(shapiro_regression_gerber$statistic,
shapiro_var_gerber$statistic,
shapiro_varx_gerber$statistic,
shapiro_rf_gerber$statistic),
            Shapiro_P_Value =
c(shapiro_regression_gerber$p.value,
shapiro_var_gerber$p.value,
shapiro_varx_gerber$p.value,
shapiro_rf_gerber$p.value),
            DW_Statistic =
c(dw_regression_gerber$statistic, NA, NA, NA),
            BP_Statistic =
c(bp_regression_gerber$statistic,
bp_var_gerber$statistic, bp_varx_gerber$statistic, NA),
            BP_P_Value = c(bp_regression_gerber$p.value,
bp_var_gerber$p.value, bp_varx_gerber$p.value, NA),
            LB_Statistic = c(NA, lb_var_gerber$statistic,
lb_varx_gerber$statistic, NA),
            LB_P_Value = c(NA, lb_var_gerber$p.value,
lb_varx_gerber$p.value, NA)
        )

        flextable(resid_gerber)
    }
    # VAR IRFs
    {
        # List of predictor variables in the VAR model
        predictors <- colnames(var_model_gerber$y)
        # Create an empty list to store IRF data for each
predictor
        irf_list <- list()

        # Loop through predictors and generate IRFs
        for (pred in predictors) {
            irf_data <- irf(var_model_gerber, impulse =
pred, response = "Gerber", n.ahead = 10, ortho = TRUE)
            irf_list[[pred]] <- irf_data
        }

        # Plot IRFs
```

```
        par(mfrow = c(ceiling(length(predictors)/4), 2))
        for (pred in predictors) {
            plot(irf_list[[pred]],
                xlab = "Periods", ylab = "Response",
                main = paste("IRF of Gerber to", pred),
col = "blue")
        }
    }

    # 1, 2, HC and SC models
    {
        # 1
        {
            # Extract residuals from each model
            resid_regression_pearson_1 <-
residuals(regression_pearson_1)
            resid_var_pearson_1 <-
residuals(var_model_pearson_1)[, 1]  # First equation
            resid_varx_pearson_1 <-
residuals(var_model_pearson_exog_1)[, 1]  # First
equation
            resid_rf_pearson_1 <-
rf_model_Pearson_1$trainingData$.outcome -
rf_model_Pearson_1$finalModel$predictions

            # Perform Shapiro-Wilk test for normality
            shapiro_regression_pearson_1 <-
shapiro.test(resid_regression_pearson_1)
            shapiro_var_pearson_1 <-
shapiro.test(resid_var_pearson_1)
            shapiro_varx_pearson_1 <-
shapiro.test(resid_varx_pearson_1)
            shapiro_rf_pearson_1 <-
shapiro.test(resid_rf_pearson_1)

            # Perform Durbin-Watson test for autocorrelation
(for regression model)
            dw_regression_pearson_1 <-
dwtest(regression_pearson_1)

            # Perform Breusch-Pagan test for
heteroscedasticity for all models
            bp_regression_pearson_1 <-
bptest(regression_pearson_1)
            bp_var_pearson_1 <- bptest(resid_var_pearson_1 ~
fitted(var_model_pearson_1)[, 1])
            bp_varx_pearson_1 <- bptest(resid_varx_pearson_1
~ fitted(var_model_pearson_exog_1)[, 1])

            # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
            lb_var_pearson_1 <- Box.test(resid_var_pearson_1,
type = "Ljung-Box")
            lb_varx_pearson_1 <-
Box.test(resid_varx_pearson_1, type = "Ljung-Box")
```

```
            # Summarize results in a flextable
            resid_pearson_1 <- data.frame(
                Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
                Shapiro_W =
c(shapiro_regression_pearson_1$statistic,
shapiro_var_pearson_1$statistic,
shapiro_varx_pearson_1$statistic,
shapiro_rf_pearson_1$statistic),
                Shapiro_P_Value =
c(shapiro_regression_pearson_1$p.value,
shapiro_var_pearson_1$p.value,
shapiro_varx_pearson_1$p.value,
shapiro_rf_pearson_1$p.value),
                DW_Statistic =
c(dw_regression_pearson_1$statistic, NA, NA, NA),
                BP_Statistic =
c(bp_regression_pearson_1$statistic,
bp_var_pearson_1$statistic,
bp_varx_pearson_1$statistic, NA),
                BP_P_Value = c(bp_regression_pearson_1$p.value,
bp_var_pearson_1$p.value, bp_varx_pearson_1$p.value,
NA),
                LB_Statistic = c(NA,
lb_var_pearson_1$statistic,
lb_varx_pearson_1$statistic, NA),
                LB_P_Value = c(NA, lb_var_pearson_1$p.value,
lb_varx_pearson_1$p.value, NA)
            )

            flextable(resid_pearson_1)
        }
        {
            # Extract residuals from each model
            resid_regression_gerber_1 <-
residuals(regression_gerber_1)
            resid_var_gerber_1 <-
residuals(var_model_gerber_1)[, 1]  # First equation
            resid_varx_gerber_1 <-
residuals(var_model_gerber_exog_1)[, 1]  # First
equation
            resid_rf_gerber_1 <-
rf_model_Gerber_1$trainingData$.outcome -
rf_model_Gerber_1$finalModel$predictions

            # Perform Shapiro-Wilk test for normality
            shapiro_regression_gerber_1 <-
shapiro.test(resid_regression_gerber_1)
            shapiro_var_gerber_1 <-
shapiro.test(resid_var_gerber_1)
            shapiro_varx_gerber_1 <-
shapiro.test(resid_varx_gerber_1)
            shapiro_rf_gerber_1 <-
shapiro.test(resid_rf_gerber_1)
```

```r
    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_gerber_1 <-
dwtest(regression_gerber_1)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_gerber_1 <-
bptest(regression_gerber_1)
    bp_var_gerber_1 <- bptest(resid_var_gerber_1 ~
fitted(var_model_gerber_1)[, 1])
    bp_varx_gerber_1 <- bptest(resid_varx_gerber_1 ~
fitted(var_model_gerber_exog_1)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_gerber_1 <- Box.test(resid_var_gerber_1,
type = "Ljung-Box")
    lb_varx_gerber_1 <- Box.test(resid_varx_gerber_1,
type = "Ljung-Box")

    # Summarize results in a flextable
    resid_gerber_1 <- data.frame(
      Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
      Shapiro_W =
c(shapiro_regression_gerber_1$statistic,
shapiro_var_gerber_1$statistic,
shapiro_varx_gerber_1$statistic,
shapiro_rf_gerber_1$statistic),
      Shapiro_P_Value =
c(shapiro_regression_gerber_1$p.value,
shapiro_var_gerber_1$p.value,
shapiro_varx_gerber_1$p.value,
shapiro_rf_gerber_1$p.value),
      DW_Statistic =
c(dw_regression_gerber_1$statistic, NA, NA, NA),
      BP_Statistic =
c(bp_regression_gerber_1$statistic,
bp_var_gerber_1$statistic, bp_varx_gerber_1$statistic,
NA),
      BP_P_Value = c(bp_regression_gerber_1$p.value,
bp_var_gerber_1$p.value, bp_varx_gerber_1$p.value, NA),
      LB_Statistic = c(NA, lb_var_gerber_1$statistic,
lb_varx_gerber_1$statistic, NA),
      LB_P_Value = c(NA, lb_var_gerber_1$p.value,
lb_varx_gerber_1$p.value, NA)
    )

    flextable(resid_gerber_1)
  }
  # 2
  {
    # Extract residuals from each model

    resid_regression_pearson_2 <-
residuals(regression_pearson_2)
    resid_var_pearson_2 <-
residuals(var_model_pearson_2)[, 1]  # First equation
    resid_varx_pearson_2 <-
residuals(var_model_pearson_exog_2)[, 1]  # First
equation
    resid_rf_pearson_2 <-
rf_model_Pearson_2$trainingData$.outcome -
rf_model_Pearson_2$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_pearson_2 <-
shapiro.test(resid_regression_pearson_2)
    shapiro_var_pearson_2 <-
shapiro.test(resid_var_pearson_2)
    shapiro_varx_pearson_2 <-
shapiro.test(resid_varx_pearson_2)
    shapiro_rf_pearson_2 <-
shapiro.test(resid_rf_pearson_2)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_pearson_2 <-
dwtest(regression_pearson_2)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_pearson_2 <-
bptest(regression_pearson_2)
    bp_var_pearson_2 <- bptest(resid_var_pearson_2 ~
fitted(var_model_pearson_2)[, 1])
    bp_varx_pearson_2 <- bptest(resid_varx_pearson_2
~ fitted(var_model_pearson_exog_2)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_pearson_2 <- Box.test(resid_var_pearson_2,
type = "Ljung-Box")
    lb_varx_pearson_2 <-
Box.test(resid_varx_pearson_2, type = "Ljung-Box")

    # Summarize results in a flextable
    resid_pearson_2 <- data.frame(
      Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
      Shapiro_W =
c(shapiro_regression_pearson_2$statistic,
shapiro_var_pearson_2$statistic,
shapiro_varx_pearson_2$statistic,
shapiro_rf_pearson_2$statistic),
      Shapiro_P_Value =
c(shapiro_regression_pearson_2$p.value,
shapiro_var_pearson_2$p.value,
shapiro_varx_pearson_2$p.value,
shapiro_rf_pearson_2$p.value),
      DW_Statistic =
c(dw_regression_pearson_2$statistic, NA, NA, NA),
      BP_Statistic =
c(bp_regression_pearson_2$statistic,
bp_var_pearson_2$statistic,
bp_varx_pearson_2$statistic, NA),
      BP_P_Value = c(bp_regression_pearson_2$p.value,
bp_var_pearson_2$p.value, bp_varx_pearson_2$p.value,
NA),
      LB_Statistic = c(NA,
lb_var_pearson_2$statistic,
lb_varx_pearson_2$statistic, NA),
      LB_P_Value = c(NA, lb_var_pearson_2$p.value,
lb_varx_pearson_2$p.value, NA)
    )

    flextable(resid_pearson_2)
  }
  {
    # Extract residuals from each model
    resid_regression_gerber_2 <-
residuals(regression_gerber_2)
    resid_var_gerber_2 <-
residuals(var_model_gerber_2)[, 1]  # First equation
    resid_varx_gerber_2 <-
residuals(var_model_gerber_exog_2)[, 1]  # First
equation
    resid_rf_gerber_2 <-
rf_model_Gerber_2$trainingData$.outcome -
rf_model_Gerber_2$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_gerber_2 <-
shapiro.test(resid_regression_gerber_2)
    shapiro_var_gerber_2 <-
shapiro.test(resid_var_gerber_2)
    shapiro_varx_gerber_2 <-
shapiro.test(resid_varx_gerber_2)
    shapiro_rf_gerber_2 <-
shapiro.test(resid_rf_gerber_2)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_gerber_2 <-
dwtest(regression_gerber_2)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_gerber_2 <-
bptest(regression_gerber_2)
    bp_var_gerber_2 <- bptest(resid_var_gerber_2 ~
fitted(var_model_gerber_2)[, 1])
```

```r
    bp_varx_gerber_2 <- bptest(resid_varx_gerber_2 ~
fitted(var_model_gerber_exog_2)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_gerber_2 <- Box.test(resid_var_gerber_2,
type = "Ljung-Box")
    lb_varx_gerber_2 <- Box.test(resid_varx_gerber_2,
type = "Ljung-Box")

    # Summarize results in a flextable
    resid_gerber_2 <- data.frame(
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_gerber_2$statistic,
shapiro_var_gerber_2$statistic,
shapiro_varx_gerber_2$statistic,
shapiro_rf_gerber_2$statistic),
        Shapiro_P_Value =
c(shapiro_regression_gerber_2$p.value,
shapiro_var_gerber_2$p.value,
shapiro_varx_gerber_2$p.value,
shapiro_rf_gerber_2$p.value),
        DW_Statistic =
c(dw_regression_gerber_2$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_gerber_2$statistic,
bp_var_gerber_2$statistic, bp_varx_gerber_2$statistic,
NA),
        BP_P_Value = c(bp_regression_gerber_2$p.value,
bp_var_gerber_2$p.value, bp_varx_gerber_2$p.value, NA),
        LB_Statistic = c(NA, lb_var_gerber_2$statistic,
lb_varx_gerber_2$statistic, NA),
        LB_P_Value = c(NA, lb_var_gerber_2$p.value,
lb_varx_gerber_2$p.value, NA)
    )

    flextable(resid_gerber_2)
}
# HC
{
    # Extract residuals from each model
    resid_regression_pearson_hc <-
residuals(regression_pearson_hc)
    resid_var_pearson_hc <-
residuals(var_model_pearson_hc)[, 1]  # First equation
    resid_varx_pearson_hc <-
residuals(var_model_pearson_exog_hc)[, 1]  # First
equation
    resid_rf_pearson_hc <-
rf_model_Pearson_hc$trainingData$.outcome -
rf_model_Pearson_hc$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_pearson_hc <-
shapiro.test(resid_regression_pearson_hc)
    shapiro_var_pearson_hc <-
shapiro.test(resid_var_pearson_hc)
    shapiro_varx_pearson_hc <-
shapiro.test(resid_varx_pearson_hc)
    shapiro_rf_pearson_hc <-
shapiro.test(resid_rf_pearson_hc)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_pearson_hc <-
dwtest(regression_pearson_hc)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_pearson_hc <-
bptest(regression_pearson_hc)
    bp_var_pearson_hc <- bptest(resid_var_pearson_hc
~ fitted(var_model_pearson_hc)[, 1])
    bp_varx_pearson_hc <-
bptest(resid_varx_pearson_hc ~
fitted(var_model_pearson_exog_hc)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_pearson_hc <-
Box.test(resid_var_pearson_hc, type = "Ljung-Box")
    lb_varx_pearson_hc <-
Box.test(resid_varx_pearson_hc, type = "Ljung-Box")

    # Summarize results in a flextable
    resid_pearson_hc <- data.frame(
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_pearson_hc$statistic,
shapiro_var_pearson_hc$statistic,
shapiro_varx_pearson_hc$statistic,
shapiro_rf_pearson_hc$statistic),
        Shapiro_P_Value =
c(shapiro_regression_pearson_hc$p.value,
shapiro_var_pearson_hc$p.value,
shapiro_varx_pearson_hc$p.value,
shapiro_rf_pearson_hc$p.value),
        DW_Statistic =
c(dw_regression_pearson_hc$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_pearson_hc$statistic,
bp_var_pearson_hc$statistic,
bp_varx_pearson_hc$statistic, NA),
        BP_P_Value =
c(bp_regression_pearson_hc$p.value,
bp_var_pearson_hc$p.value, bp_varx_pearson_hc$p.value,
NA),
        LB_Statistic = c(NA,
lb_var_pearson_hc$statistic,
lb_varx_pearson_hc$statistic, NA),
        LB_P_Value = c(NA, lb_var_pearson_hc$p.value,
lb_varx_pearson_hc$p.value, NA)
    )

    flextable(resid_pearson_hc)
}
{
    # Extract residuals from each model
    resid_regression_gerber_hc <-
residuals(regression_gerber_hc)
    resid_var_gerber_hc <-
residuals(var_model_gerber_hc)[, 1]  # First equation
    resid_varx_gerber_hc <-
residuals(var_model_gerber_exog_hc)[, 1]  # First
equation
    resid_rf_gerber_hc <-
rf_model_Gerber_hc$trainingData$.outcome -
rf_model_Gerber_hc$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_gerber_hc <-
shapiro.test(resid_regression_gerber_hc)
    shapiro_var_gerber_hc <-
shapiro.test(resid_var_gerber_hc)
    shapiro_varx_gerber_hc <-
shapiro.test(resid_varx_gerber_hc)
    shapiro_rf_gerber_hc <-
shapiro.test(resid_rf_gerber_hc)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_gerber_hc <-
dwtest(regression_gerber_hc)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_gerber_hc <-
bptest(regression_gerber_hc)
    bp_var_gerber_hc <- bptest(resid_var_gerber_hc ~
fitted(var_model_gerber_hc)[, 1])
    bp_varx_gerber_hc <- bptest(resid_varx_gerber_hc
~ fitted(var_model_gerber_exog_hc)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_gerber_hc <- Box.test(resid_var_gerber_hc,
type = "Ljung-Box")
    lb_varx_gerber_hc <-
Box.test(resid_varx_gerber_hc, type = "Ljung-Box")

    # Summarize results in a flextable
    resid_gerber_hc <- data.frame(
```

```r
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_gerber_hc$statistic,
shapiro_var_gerber_hc$statistic,
shapiro_varx_gerber_hc$statistic,
shapiro_rf_gerber_hc$statistic),
        Shapiro_P_Value =
c(shapiro_regression_gerber_hc$p.value,
shapiro_var_gerber_hc$p.value,
shapiro_varx_gerber_hc$p.value,
shapiro_rf_gerber_hc$p.value),
        DW_Statistic =
c(dw_regression_gerber_hc$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_gerber_hc$statistic,
bp_var_gerber_hc$statistic,
bp_varx_gerber_hc$statistic, NA),
        BP_P_Value = c(bp_regression_gerber_hc$p.value,
bp_var_gerber_hc$p.value, bp_varx_gerber_hc$p.value,
NA),
        LB_Statistic = c(NA,
lb_var_gerber_hc$statistic,
lb_varx_gerber_hc$statistic, NA),
        LB_P_Value = c(NA, lb_var_gerber_hc$p.value,
lb_varx_gerber_hc$p.value, NA)
    )

    flextable(resid_gerber_hc)
  }
  # SC
  {
    # Extract residuals from each model
    resid_regression_pearson_sc <-
residuals(regression_pearson_sc)
    resid_var_pearson_sc <-
residuals(var_model_pearson_sc)[, 1]  # First equation
    resid_varx_pearson_sc <-
residuals(var_model_pearson_exog_sc)[, 1]  # First
equation
    resid_rf_pearson_sc <-
rf_model_Pearson_sc$trainingData$.outcome -
rf_model_Pearson_sc$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_pearson_sc <-
shapiro.test(resid_regression_pearson_sc)
    shapiro_var_pearson_sc <-
shapiro.test(resid_var_pearson_sc)
    shapiro_varx_pearson_sc <-
shapiro.test(resid_varx_pearson_sc)
    shapiro_rf_pearson_sc <-
shapiro.test(resid_rf_pearson_sc)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_pearson_sc <-
dwtest(regression_pearson_sc)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_pearson_sc <-
bptest(regression_pearson_sc)
    bp_var_pearson_sc <- bptest(resid_var_pearson_sc
~ fitted(var_model_pearson_sc)[, 1])
    bp_varx_pearson_sc <-
bptest(resid_varx_pearson_sc ~
fitted(var_model_pearson_exog_sc)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_pearson_sc <-
Box.test(resid_var_pearson_sc, type = "Ljung-Box")
    lb_varx_pearson_sc <-
Box.test(resid_varx_pearson_sc, type = "Ljung-Box")

    # Summarize results in a flextable
    resid_pearson_sc <- data.frame(
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_pearson_sc$statistic,
shapiro_var_pearson_sc$statistic,
shapiro_varx_pearson_sc$statistic,
shapiro_rf_pearson_sc$statistic),
        Shapiro_P_Value =
c(shapiro_regression_pearson_sc$p.value,
shapiro_var_pearson_sc$p.value,
shapiro_varx_pearson_sc$p.value,
shapiro_rf_pearson_sc$p.value),
        DW_Statistic =
c(dw_regression_pearson_sc$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_pearson_sc$statistic,
bp_var_pearson_sc$statistic,
bp_varx_pearson_sc$statistic, NA),
        BP_P_Value =
c(bp_regression_pearson_sc$p.value,
bp_var_pearson_sc$p.value, bp_varx_pearson_sc$p.value,
NA),
        LB_Statistic = c(NA,
lb_var_pearson_sc$statistic,
lb_varx_pearson_sc$statistic, NA),
        LB_P_Value = c(NA, lb_var_pearson_sc$p.value,
lb_varx_pearson_sc$p.value, NA)
    )

    flextable(resid_pearson_sc)
  }

  {
    # Extract residuals from each model
    resid_regression_gerber_sc <-
residuals(regression_gerber_sc)
    resid_var_gerber_sc <-
residuals(var_model_gerber_sc)[, 1]  # First equation
    resid_varx_gerber_sc <-
residuals(var_model_gerber_exog_sc)[, 1]  # First
equation
    resid_rf_gerber_sc <-
rf_model_Gerber_sc$trainingData$.outcome -
rf_model_Gerber_sc$finalModel$predictions

    # Perform Shapiro-Wilk test for normality
    shapiro_regression_gerber_sc <-
shapiro.test(resid_regression_gerber_sc)
    shapiro_var_gerber_sc <-
shapiro.test(resid_var_gerber_sc)
    shapiro_varx_gerber_sc <-
shapiro.test(resid_varx_gerber_sc)
    shapiro_rf_gerber_sc <-
shapiro.test(resid_rf_gerber_sc)

    # Perform Durbin-Watson test for autocorrelation
(for regression model)
    dw_regression_gerber_sc <-
dwtest(regression_gerber_sc)

    # Perform Breusch-Pagan test for
heteroscedasticity for all models
    bp_regression_gerber_sc <-
bptest(regression_gerber_sc)
    bp_var_gerber_sc <- bptest(resid_var_gerber_sc ~
fitted(var_model_gerber_sc)[, 1])
    bp_varx_gerber_sc <- bptest(resid_varx_gerber_sc
~ fitted(var_model_gerber_exog_sc)[, 1])

    # Perform Ljung-Box test for autocorrelation (for
VAR and VARX)
    lb_var_gerber_sc <- Box.test(resid_var_gerber_sc,
type = "Ljung-Box")
    lb_varx_gerber_sc <-
Box.test(resid_varx_gerber_sc, type = "Ljung-Box")

    # Summarize results in a flextable
    resid_gerber_sc <- data.frame(
        Model = c("Multiple Regression", "VAR", "VARX",
"Random Forest"),
        Shapiro_W =
c(shapiro_regression_gerber_sc$statistic,
shapiro_var_gerber_sc$statistic,
shapiro_varx_gerber_sc$statistic,
shapiro_rf_gerber_sc$statistic),
        Shapiro_P_Value =
c(shapiro_regression_gerber_sc$p.value,
```

```
shapiro_var_gerber_sc$p.value,
shapiro_varx_gerber_sc$p.value,
shapiro_rf_gerber_sc$p.value),
        DW_Statistic =
c(dw_regression_gerber_sc$statistic, NA, NA, NA),
        BP_Statistic =
c(bp_regression_gerber_sc$statistic,
bp_var_gerber_sc$statistic,
bp_varx_gerber_sc$statistic, NA),
        BP_P_Value = c(bp_regression_gerber_sc$p.value,
bp_var_gerber_sc$p.value, bp_varx_gerber_sc$p.value,
NA),
        LB_Statistic = c(NA,
lb_var_gerber_sc$statistic,
lb_varx_gerber_sc$statistic, NA),
        LB_P_Value = c(NA, lb_var_gerber_sc$p.value,
lb_varx_gerber_sc$p.value, NA)
      )

    flextable(resid_gerber_sc)
    }
  }

}
```

References

Ai, C., Chatrath, A., & Song, F. (2006). On the Comovement of Commodity Prices. *American Journal of Agricultural Economics*, *88*(3), 574.

Algieri, B., Leccadito, A., & Toscano, P. (2021). A Time-Varying Gerber Statistic: Application of a Novel Correlation Metric to Commodity Price Co-Movements. *Forecasting*, *3*(2), 339-354.

Alquist, R., Bhattarai, S., & Coibion, O. (2020). Commodity-price comovement and global economic activity. *Journal of Monetary Economics*, *112*, 41-56.

Antwi, E., Gyamfi, E. N., Kyei, K., Gill, R., & Adam, A. M. (2021). Determinants of Commodity Futures Prices: Decomposition Approach. *Mathematical Problems in Engineering*, *2021*.

Bakas, D., & Triantafyllou, A. G. (2020). Commodity Price Volatility and the Economic Uncertainty of Pandemics. *ERN: Other Econometric Modeling: International Financial Markets - Volatility & Financial Crises (Topic)*.

Baur, D. (2003). What is Co-movement? *EUR Working Paper No. 20759 EN*.

Bekaert, G., Harvey, Campbell R., & Ng, A. (2005). Market Integration and Contagion. *The Journal of Business*, *78*(1), 39-69.

Breiman, L. (2001). Random forests. *Machine learning*, *45*, 5-32.

Byrne, J. P., Fazio, G., & Fiess, N. (2013). Primary commodity prices: Co-movements, common factors and fundamentals. *Journal of Development Economics*, *101*, 16-26.

Byrne, J. P., Sakemoto, R., & Xu, B. (2019). Commodity price co-movement: heterogeneity and the time-varying impact of fundamentals. *European Review of Agricultural Economics*, *47*(2), 499-528.

Cai, G.-f., Zhang, H., & Chen, Z.-y. (2019). Comovement between commodity sectors. *Physica A: Statistical Mechanics and its Applications*.

Chien, F., Sadiq, M., Kamran, H. W., Nawaz, M. A., Hussain, M. S., & Raza, M. (2021). Co-movement of energy prices and stock market return: environmental wavelet nexus of COVID-19 pandemic from the USA, Europe, and China. *Environmental Science and Pollution Research International*, *28*, 32359 - 32373.

de Nicola, F., De Pace, P., & Hernandez, M. A. (2014). Co-Movement of Major Commodity Price Returns: Time-Series Assessment. *Capital Markets: Asset Pricing & Valuation eJournal*.

de Nicola, F., De Pace, P., & Hernandez, M. A. (2016). Co-movement of major energy, agricultural, and food commodity price returns: A time-series assessment. *Energy Economics*, *57*, 28-41.

Deb, P., Trivedi, P. K., & Varangis, P. (1996). The excess co-movement of commodity prices reconsidered. *Journal of Applied Econometrics*, *11*(3), 275-291.

Dornbusch, R., Yung, C. P., & Claessens, S. (2000). Contagion. How it spreads and How it can be stopped. *World Bank Research Observer*, *15*(2), 177-197.

Gerber, S., Markowitz, H. M., Ernst, P. A., Miao, Y., Javid, B., & Sargen, P. (2022). The Gerber statistic: a robust co-movement measure for portfolio optimization. *The Journal of Portfolio Management*, *48*(3), 87-102.

Gruber, J. W., & Vigfusson, R. J. (2018). Interest rates and the volatility and correlation of commodity prices. *Macroeconomic Dynamics*, *22*(3), 600-619.

Hastie, T., Tibshirani, R., Friedman, J. H., & Friedman, J. H. (2009). *The elements of statistical learning: data mining, inference, and prediction* (Vol. 2). Springer.

Hayashi, F. (2011). *Econometrics*. Princeton University Press.

Janzen, J. P., & Smith, A. (2012). Commodity Price Comovement: The Case of Cotton.

Kumar, S., Kar, A. K., & Ilavarasan, P. V. (2021). Applications of text mining in services management: A systematic literature review. *Int. J. Inf. Manag. Data Insights*, *1*.

Le Pen, Y., & Sévi, B. (2017). Futures Trading and the Excess Comovement of Commodity Prices. *Econometric Modeling: Commodity Markets eJournal*.

Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.

Murphy, K. P. (2012). *Machine learning: a probabilistic perspective*. MIT press.

Nembrini, S., König, I. R., & Wright, M. N. (2018). The revival of the Gini importance? *Bioinformatics*, *34*(21), 3711-3718.

Pindyck, R. S., & Rotemberg, J. J. (1990). The excess co-movement of commodity prices. *The Economic Journal*, *100*(403), 1173-1189.

Poncela, P., Senra, E., & Sierra, L. (2014). Common dynamics of nonenergy commodity prices and their relation to uncertainty. *Applied Economics*, *46*, 3724 - 3735.

Schonlau, M., & Zou, R. Y. (2020). The random forest algorithm for statistical learning. *The Stata Journal*, *20*(1), 3-29.

Zaremba, A., Umar, Z., & Mikutowski, M. (2021). Commodity financialisation and price co-movement: Lessons from two centuries of evidence. *Finance Research Letters*, *38*, 101492.

Zhang, Y., & Ding, S. (2021). Liquidity effects on price and return co-movements in commodity futures markets. *International Review of Financial Analysis*, *76*, 101796.

Zhang, Y., Ding, S., & Scheffel, E. M. (2019). A key determinant of commodity price Co-movement: The role of daily market liquidity. *Economic Modelling*, *81*, 170-180.