Optimization of the Supply Chain for an Animal Feed Company in Vietnam

Master Thesis

Final version

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II. Preface

This thesis is the final product to obtain my Master's Degree in Industrial Engineering and Management at the University of Twente. Additionally, it marks the end of my student period in Enschede. During the last months of my master's program, I had the special opportunity to write my thesis for De Heus in Vietnam. Besides the invaluable experience gained from working within a company, I also had the chance to explore an entirely new country with a completely different way of life.

First, I would like to express my gratitude to my colleagues at De Heus Vietnam for their unwavering support in familiarizing me with the company's practices and showing me around various locations to gain a comprehensive understanding of the company in Vietnam. It was a privilege to have the opportunity from De Heus to visit multiple interesting places within the company, including different factories, a dealer, a chicken farm, a genetics farm, and the honor of joining several year-end parties to celebrate the lunar new year. In particular, I want to thank Arno Willemink for his supervision during this period. Thank you for the opportunity that you gave me for going to Vietnam and also you obtained my by various parts of the company. Arno has a lot of knowledge about the company and its processes, which made it effortless to find solutions when facing challenges during this this. This greatly assisted me in developing a model that closely reflects reality. I also want to thank Bo and Rick for their feedback and discussions, Tania for her assistance with arrangements, Patrick and Hank for their prompt and helpful responses to production-related questions, and Silva for being a great companion during visits to factories where Vietnamese was the primary language.

Next, I want to extend my thanks to Eduardo Lalla-Ruiz for his daily supervision from the University of Twente. It was engaging to discuss the model I wanted to create to address the company's problem, and your enthusiasm for creating something new was inspiring. Your valuable feedback always helped elevate the scientific level of this thesis. I also appreciate your flexibility regarding our online meetings, considering I was not located in the Netherlands for six months, but at the end is was great to meet in person in Enschede afterward.

I also want to thank Engin Topan for providing valuable feedback from someone not involved in the daily routine. Your insights provided fresh perspectives on what was necessary to add to the model and thesis.

Lastly, I want to express my gratitude to Corinne, Robert, Britt, and Lianne for their support during this thesis and throughout my study period in Enschede. It was invaluable to discuss both easy and challenging problems together and gain new energy to complete this thesis.

With this thesis, I aim to provide insight into the knowledge gained during my studies in Enschede and specifically showcase the type of optimization model that can be created to provide solutions easily implementable in reality

Niek van der Wijst

Utrecht, May 8, 2024

For this version values regarding costs, volumes, capacities and sales are multiplied by random non-integer numbers due to confidentiality. Specific customer and production information are also removed.

III. Summary

This research is conducted at an animal feed company called De Heus from the Netherlands, located in Vietnam. The company produces livestock feed for pigs, poultry, and ruminants for the local market. The customers are direct farmers and dealers (who sell the feed to their smaller customers) across the country, each with a certain demand per week for one or multiple products (SKUs). The finished goods are transported to direct farms by bulk truck, which directly delivers the feed from the factory to the customers where dealers pick up the feed themselves at a pick-up warehouse at a factory or depot. After a large acquisition of a major competitor, new problems have arisen within De Heus Vietnam. There is a lack of insight into the relationship between different costs. The acquisition has led to an increased number of SKUs, bringing many of the competitor's products into the assortment. Lastly, there is a lack of integrated optimization, causing De Heus to be uncertain about how changes in production planning affect transportation costs. This leads to the research question: What is the most cost-effective way to plan production, inventory, and transportation of finished goods? This question aims to determine where to produce which products in what quantity and how to supply the customers.

The research area for the problem is in the southeast region of Vietnam. This region has three factories: Dong Nai, Bien Hoa, and Binh Duong. Each of these factories has characteristics regarding production capacity, the types of products they can produce, bulk capacity, and production costs at these locations. The products that De Heus sells to customers are finished goods, each with its own SKU number. These SKUs contain a certain recipe created by nutrients, and multiple SKUs can have the same recipe. These recipes are planned in a production run, which consists of a certain amount of product in tons produced successively. After a production run, the machines need to be flushed, leading to a reduction in factory capacity. Once the finished goods are produced, they are directly transported by bulk trucks to the direct customers or stored in 25 kg bags in the warehouse of the factory, which also has a certain capacity limit. The bags wait in the warehouse until dealers pick up the feed at that location, or the finished goods are transported to depots or other factories where they are sold to the dealer. The types of transportation can be described in three forms: internal transportation from factory to another pick-up location, external transportation from the customer to the pick-up location, and bulk transportation to supply direct customers. External transportation refers to the extra kilometers that customers need to drive to a pick-up location that is not the closest one, resulting in extra driven kilometers. These extra driven kilometers can be translated into a cost per ton per kilometer or optimized using multi-objective optimization. Production, inventory, and transportation describe the finished goods supply chain, which is optimized using a model.

The model created is a production assignment problem based on the production-routing problem. The routing of the problem is replaced by by flow constraints and multiple products/factories relationships are implemented. Additionally, reduced production capacity due to setup times and joint replenishment are incorporated. For multi-objective optimization, the AUGMECON2 algorithm is chosen because of its impartial treatment of objective function importance and its efficiency.

The model optimizes the current case instance, which represents the actual situation in the southeast region of Vietnam with 3 factories, 3 depots, 1117 customers, and 494 SKUs derived from 194 recipes. Additionally, it optimizes artificial pseudo-randomly created data instances with varying numbers of customers. The experiments are conducted in two phases: In phase 1, the model is used to optimize both the current case and artificially created instances within the existing capabilities. In phase 2, a sensitivity analysis is performed on the current case, examining the effects of expanding production and/or warehouse capacities, testing new potential production locations, and investigating the impact of reducing the number of SKUs.

Artificial instances are utilized during both single and multi-objective optimization processes.

The experimental results of the current case optimization show a total cost reduction of -1.8%. This reduction can be attributed to several factors, including the shifting of multiple products to another factory, reducing stock levels, and assigning customers to more favorable locations for De Heus. Increasing the production capacity by +10% at the Dong Nai factory leads to a total cost reduction of -1.1% per month, achieved through lower production costs at this factory and serving customers from closer locations. The investment costs for this capacity expansion are $525,600, resulting in a payback period of 3.5 years. Testing new production locations results in significant cost reductions for the Binh Duong II and Binh Phuoc locations, with monthly savings of -9.4% and -7.9%, respectively. This is due to the lower production costs and increased flexibility offered by this new factory, along with its advantageous locations. The investment costs for a new factory are $13.87 million, with the location of Binh Duong II costing $4.38 million instead of $2.92 million for Binh Phuoc, making both options viable. Reducing the number of SKUs has a relatively minor impact on total costs, resulting in a reduction of only -0.3%. Given the considerable effort required to reduce SKUs, it may be more cost-effective to focus on merging or removing recipes. The analysis of the multi-objective optimization Pareto diagram reveals that larger data instances create smoother Pareto fronts, where costs increase gradually with the reduction of extra driven kilometers. De Heus can decide how much they want to pay for lowering the extra driven kilometers, as the costs remain the same regardless of the reduction steps. The current case optimization can be implemented without additional investments, next to that it is advised to invest in capacity extension at the Dong Nai factory and replace the factory in Bien Hoa with a new one located in Binh Duong II or Binh Phuoc. Instead of focusing solely on reducing SKUs, De Heus should also consider merging recipes to optimize production planning. Additionally, implementing forecasted demand could further improve the analysis.

IV. List of Mathematical Definitions

$\in$	in ($i \in I$: item i in set I
$\forall$	for all ($\forall i$: for all items i)
$\sum$	summation
$\mathbb{Z}^+$	= 0, 1, 2, ..., the set of all positive integers, including 0

V. List of Terms and Abbreviations

AUGMECON2	Multi-objective optimization method
Bien Hoa	Factory location in Bien Hoa province
Binh Duong	Factory location in Binh Duong province
CLSP	Capacitated Multi-item Lot-sizing Problem
Cross-production	Shifting production volumes from one to another location
De Heus	Animal feed company where research is conducted
DIO	Days Iventory Outstanding
Dong Nai	Factory location in Dong Nai province
FTE	Full-Time Equivalent
KPI	Key Performance Indication
MCNF	Multi-Commodity Network Flow Problem
POF	Pareto Optimal Front
Proconco/ANCO	Acquired company by De Heus
PRP	Production Routing Problem
SKU	Stock Keeping Unit
WACC	Weighted Average Costs of Capital

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1 Introduction

The first section of this thesis introduces De Heus as a company and provides an explanation of its operations in Vietnam. After reading this chapter, the reader will have an overview of De Heus in Vietnam, the research problem they are currently facing, and the corresponding motivated research questions aimed at solving the main problem. In Section 1.1, the description of the company in Vietnam is elaborated. In Section 1.2, the research problem is described, leading to the research goal in Section 1.3 and the corresponding research questions given in Section 1.4. Section 1.5 presents the research design and an overview of the thesis.

1.1 De Heus as a company

Royal De Heus is a global top-10 player in the production and sale of animal nutritional products. The company produces compound feed, premixes and feed specialties for livestock and aquaculture customers worldwide. In addition, they are actively involved in genetic research for pigs and poultry, managing breeding farms for chickens, and owning slaughter facilities. This enables De Heus to influence and control the entire value chain, from animal growth to slaughter, ensuring it is ready for consumption.

Royal De Heus was founded in 1911 in Barneveld by H.A. de Heus and remains under family ownership. During the 1990s, the company expanded its production capacity through the acquisition of other companies in the Netherlands. After acquiring the company 'Koudijs,' they also gained access to its export department, enabling them to expand operations beyond the Netherlands and begin exporting to other countries. De Heus maintains its interest in expanding to other countries, with more than 80 production locations in over 20 countries. Its products are distributed in 75 countries, and that number continues to grow every day (De Heus, 2023).

"Keep animals healthy and drive optimal production. It's what we call Powering Progress."

- De Heus (2023)

In November 2008, De Heus acquired its first two factories in Vietnam. They continued to expand their operations, increasing the number of factories to achieve a total sales volume of 1 million metric tons per year, with seven operating factories by 2016. Over time, De Heus secured the third position in the animal feed market in Vietnam. In 2021, one of De Heus's most significant acquisitions took place when the company purchased Proconco/ANCO, which held the second position in the market at that time. As a result, De Heus currently operates a total of 17 factories in Vietnam, with a total sales volume of more than 3 million tons. The map of Vietnam with the factory locations can be seen in Figure 1.1.

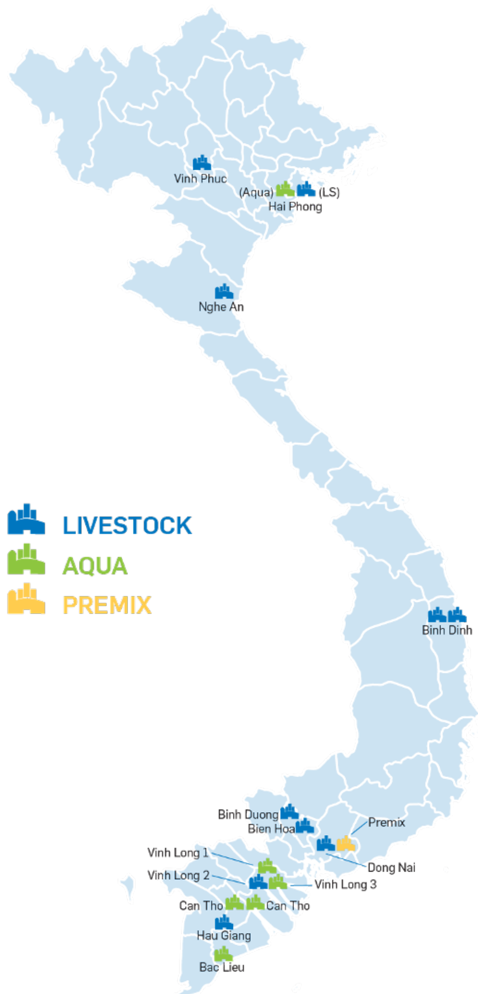


Figure 1.1: De Heus Vietnam factory locations.

1.2 Problem description

After the acquisition of Proconco/ANCO in 2021, De Heus is currently still busy integrating this company within De Heus. On an organizational level, the management and sales teams are already integrated, and the current phase is to integrate the production volume output across all the factories. This integration makes it possible to assign customers to more different production or pick-up locations or produce specific products only at certain locations to avoid startup costs. Currently, De Heus and Proconco/ANCO operate their production facilities independently. In practice, this means that De Heus factories exclusively produce feed and stock keeping units (SKUs) under the De Heus brand or related brands, while Proconco/ANCO produce feed and SKUs under their own brands. As a result, some customers are served from a location that is not the closest to their area, leading to potential savings in logistics costs when there is a possibility to serve customers from a less distant production location. There is a lack of insight into the relationship between costs of producing products in the different factories, the storage costs of the finished goods, and the transportation costs to the customers. This is explained in Section 1.2.1. Another problem arising from the acquisition of Proconco/ANCO is the increase in the number of SKUs, which leads to more different products that need to be produced and stored at the different locations, putting more pressure on production and inventory planning. This is explained in Section 1.2.2. Currently, De Heus is already starting to shift production volumes from one factory to another. However, this is only performed on an individual SKU basis and does not consider the consequences, which also include changing production costs, inventory costs, and transportation costs due to these changes. This is explained in Section 1.2.3.

1.2.1 Lack of insight into the relationship between different costs

At first, De Heus lacks insight into the relationship between the costs associated with producing a ton of feed at different locations, inventory costs, and transportation costs to transport the feed to its selling location. The production costs include, for example, inbound logistics costs for raw materials, labor costs at the factory, and transportation costs to a depot, among others. Currently, production costs are calculated by dividing the total cost of the factory (including labor, electricity, raw material imports, etc.) by the total amount of feed produced. These costs are termed 'semi-variable' rather than 'variable' because an increase of one ton in production does not necessarily correspond to an equal increase in these determined costs. So it is important to make the distinction between fixed and variable costs. Besides these costs, the relationship between the different costs is not known. De Heus is not calculating inventory holding costs for the finished goods in the warehouse, and it has no insights into the additional cost changes when production volumes are shifted to other locations. This shifting can, for example, lead to higher internal transportation costs from a factory to a depot when deciding to produce the product at a factory which is further away from the depot. Therefore, De Heus needs to gain insights into what happens with the different costs in production costs, inventory costs, and transportation costs when a change is made to optimize one of them.

1.2.2 Increased number of SKUs

Secondly, De Heus is facing a challenge with the increased number of SKUs that arise from the acquisition of Proconco/ANCO, which mostly consists of different brands of the same recipe. A recipe contains the ingredient list for a certain feed, where the recipe is put in bags with a different brand on it, which is called a SKU. For example, looking at De Heus, they use three different brand names on their bags: De Heus, Windmill, and Koudijs. The feed in these bags has exactly the same recipes, but the brands are used to sell to dealers in the same town, allowing De Heus to strike deals with various dealers in a single location, each granted exclusive rights to sell a particular brand. Initially, this branding strategy for its various brands did not result in production and storage issues. However, after acquiring two new main brands, Proconco and ANCO, each with its own sub-brands, the total number of SKUs increased from 1236 to 2045. The more SKUs are produced at a location, the more separate storage space is required in the finished goods warehouse of the factories, and the overall production capacity of the factory will decrease due to the turnover time associated with switching bag types or recipes. So it is important to search for a new optimal solution, where it is still possible to produce the SKUs within the capacity limitations of the factories. Another aspect of these SKUs is that De Heus is reducing SKUs by stopping production when the sales volume is low or merging them with other brands. However, De Heus does not know what the actual effect on production planning or warehouse occupation will be from reducing this number of SKUs. It is difficult to convince the sales team to reduce the number of SKUs because the customers also need to be convinced to switch products. Therefore, it is necessary to gain insights into the optimization that could be used when the number of SKUs is reduced.

1.2.3 Lack of integrated optimization

Thirdly, De Heus lacks integrated optimization, which can result in cost reduction on one side of the supply chain but may lead to increased costs or capacity problems on the other side, resulting in no net cost reduction. An example of this is De Heus shifting production volumes from one location to another, which reduces transportation costs as the product is produced closer to a depot. However, the impact on production planning or warehouse occupation is not calculated, potentially leading to increased costs. Therefore, it is important for De Heus to understand how changes in production planning affect warehouse occupation and customer assignment. Additionally, when a customer is assigned to another factory, understanding how the production schedule needs to be adjusted to fulfill that customer's demand is essential. Currently, these optimizations and changes are only measured in costs, but it could also be effective to optimize based on other variables important to De Heus. If these relationships become clear and integrated optimization is performed, it becomes possible to achieve the best overall solution instead of just optimizing production, inventory, or transportation costs in isolation.

1.3 Research goal

The problems identified reflect a lack of insight into the relationship between different costs, an increased number of SKUs, and lack of integrated optimization. Consequently, De Heus is currently not producing in the most cost-effective way due to the acquisition of Proconco/ANCO, which increased the number of SKUs, but it is also not able to optimize in the most cost-effective way because the relationship between costs and the influence on capacities in production, inventory, and transportation are not known. The research into these different aspects begins by gaining knowledge about the possibilities of integrated optimization, where the relationship in costs and influence on the different parts of the production, inventory, and production planning are used to optimize the new situation where the amount of SKUs is increased.

Integrated optimization consists of three parts: production, inventory, and transportation, where simultaneously optimizing them could outperform individual optimization. These parts describe the supply chain of the finished goods of De Heus, including where and in what quantities SKUs are produced, stored, and supplied to the customer. Therefore, the primary objective of this research is to optimize production planning, inventory occupation, and customer assignment simultaneously while taking the company's constraints into account in the most cost-effective manner. Understanding the possibilities and limits of the company is crucial; these need to be translated into variables and constraints, such as warehouse capacity and driving distances, used during the optimization process. These decisions rely on the network of factories, depots, and customers. The final output of this research includes a production plan for each factory, necessary transshipments between factories and/or depots, and the allocation of customers to specific factories or depots for goods collection. Consequently, there should be a reduction in production, inventory, and transportation costs, along with a better understanding of the changes made within the organization.

1.4 Research questions

During this research, a model has to be created to provide cost-effective production, inventory and transportation planning, which offers improved insights into the influences of integrated supply chain optimization. This leads to the following main research question:

"What is the most cost-effective way to plan production, inventory, and transportation of finished goods?"

To support the main research question, several research questions are defined. Each of these research questions has its dedicated chapter where the answer to the question is provided and is further supported by sub-questions to enhance readability.

The first research questions center around the current situation at De Heus in Vietnam. The primary goal of this set of research questions is to gain a deeper understanding of the problem context, analyze potential areas for improvement, and identify gaps in knowledge that can be addressed through a literature review.

1. How is the production, inventory and transportation of De Heus currently planned?
    (a) What are the characteristics of the demand, production, inventory and transportation of De Heus Vietnam?
    (b) What are the cost differences for demand, production, inventory and transportation at the various locations?
    (c) What kind of KPIs are used to measure the performance of production, inventory and transportation?
    (d) What assumptions have to be made to enable fully integrated cross-production?
    (e) What are the requirements for the production , inventory and transportation planning of De Heus?

To address the company's problem, it is essential to understand the supply chain, which comprises production, inventory, and transportation characteristics specific to De Heus' situation, and the various optimization techniques applied in these contexts.

2. What is proposed in the literature for solving the optimisation problem of De Heus?
    (a) What type of optimisation problem is suitable for the current situation at De Heus?
    (b) What types of characteristics match those of De Heus in production, inventory, and transportation?
    (c) Which solution methods are used to solve the supply chain optimisation problem?
    (d) Which multi-objective optimization methods are most appropriate for the case of De Heus?

After reviewing the literature, insights about the solution for solving the supply chain optimisation problem become clear. These insights help to design a solution that aligns with reality.

3. How should the solution approach be designed?

    (a) How is the optimization problem to be solved?

    (b) What are assumptions and requirements of the solution approach?

After the solution is designed, it needs to be tested and validated in the current situation. Various experimental settings are designed and used to assess the real-world solution. It needs to be investigated whether the optimized production, inventory and transportation planning outperforms the current situation.

4. How does the solution approach perform compared to the current situation?

    (a) How to test the performance of the solution approach?

    (b) What are the different experimental scenarios that should be tested?

    (c) How does the solution approach perform for the different scenarios?

    (d) What insights does the sensitivity analysis on the possibilities within the company provide?

Finally, the conclusions are drawn, and recommendations for De Heus are provided.

5. What are the conclusions and recommendations for De Heus?

    (a) What can be concluded from the supply chain optimisation for production, inventory and transportation planning?

    (b) What are the recommendations and future research for De Heus?

1.5 Research design, scope and methods

This research will primarily involve developing a mathematical model that can optimize the current production, inventory and transportation strategy. First, the current situation needs to be identified and literature research is needed to gain more knowledge about the mathematical formulation of the current situations. Combining this information creates the conceptual solution framework. Next, the data and the methods are combined, and a solution is designed, validated, and tested. The results of the experiments provide De Heus with new insights into optimizing their processes and offer information about opportunities due to the binding constraint analysis. Figure 1.2 provides an overview of the relationship between the research questions and the thesis structure, illustrating where the various questions are addressed. In Chapters 2 and 3, the problem context is presented with input from factory visits, expert opinions, and the ERP system, followed by a literature review. Chapter 4 outlines the solution approach with input from data obtained from de Heus. Chapters 5 and 6 validate the solution approach with the results of experiments. Chapter 7 concludes the thesis by addressing the main research question.

The scope of the research will be the South-East region of Vietnam, focusing on livestock, which includes pigs, poultry, and ruminants. This region will serve as a test area to keep computation times manageable. According to De Heus, this area also has the most potential for optimizing the production, inventory, and transportation planning because the factories are relatively close to each other. De Heus has three livestock factories in this region: Dong Nai, Bien Hoa, and Binh Duong. The locations of these factories can be seen in Figure 1.1.

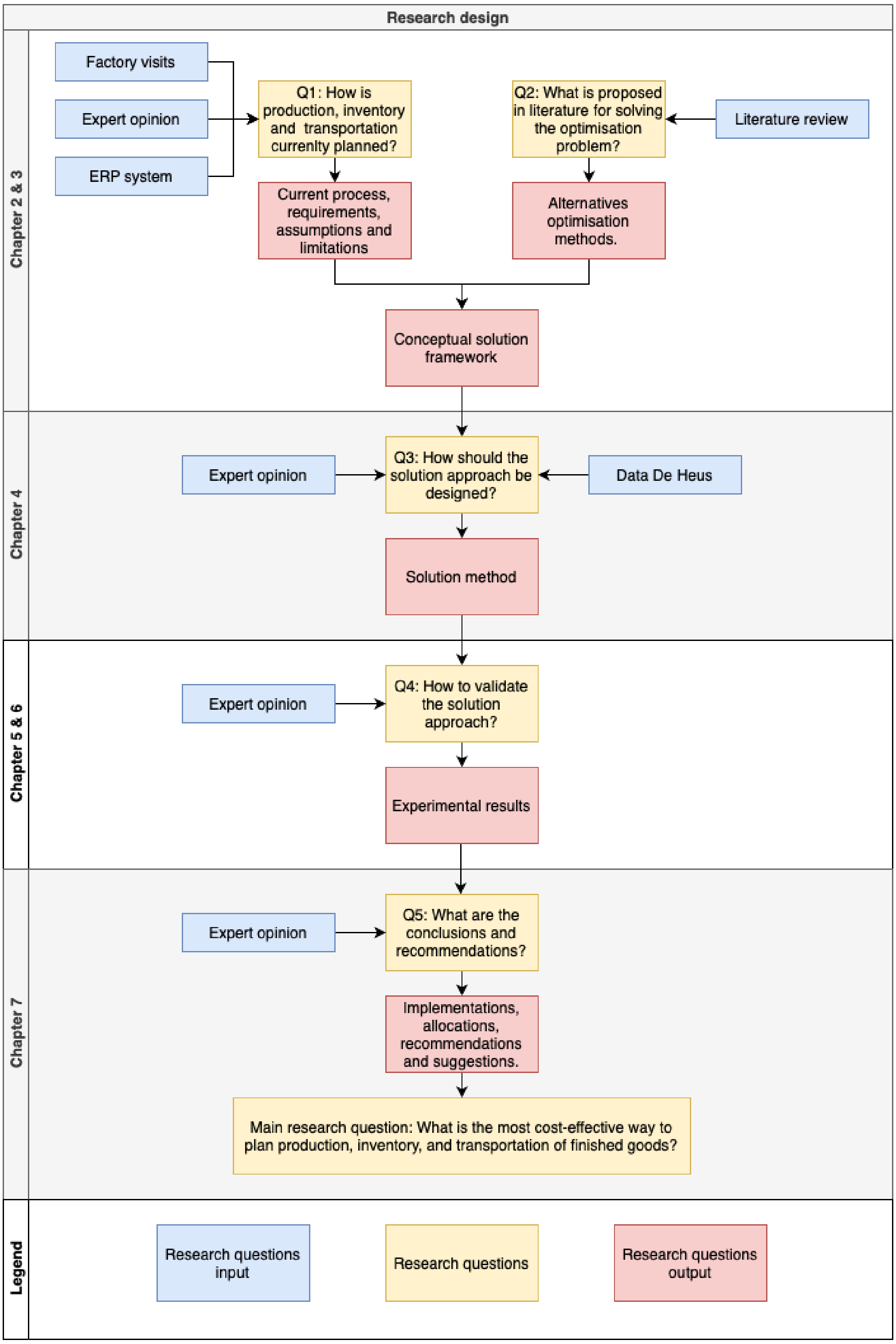


Figure 1.2: Research design. This figure shows the research questions (yellow) with the necessary input (blue) and the produced output (red). The thesis structure with chapters is represented by the different boxes.

2 Problem context

In this section, a detailed description of the problem and the current situation is given. Section 2.1 explains the current supply chain network and describes the different parts. Section 2.2 gives the characteristics of demand, production, inventory, and transportation. Section 2.3 outlines the different KPIs of De Heus. Section 2.4 outlines the pitfalls of implementing cross-production.

This section addresses the research question, *'How is the production, inventory, and transportation of De Heus currently planned?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(a) What are the characteristics of the demand, production, inventory and transportation of De Heus Vietnam?

(b) What are the cost differences for demand, production, inventory, and transportation at the various locations?

(c) What kind of KPIs are used to measure the performance of production, inventory, and transportation?

(d) What assumptions have to be made to enable fully integrated cross-production?

(e) What are the requirements for the production, inventory and transportation planning of De Heus?

2.1 Supply chain network

The problem described in Section 1 involves the production, inventory, and transportation issues related to the finished goods of De Heus. Finished goods are defined as products that have completed the manufacturing process, but have not yet been sold. The supply chain network for these finished goods is illustrated in Figure 2.1, depicting their flow from left to right, with blocks representing locations.

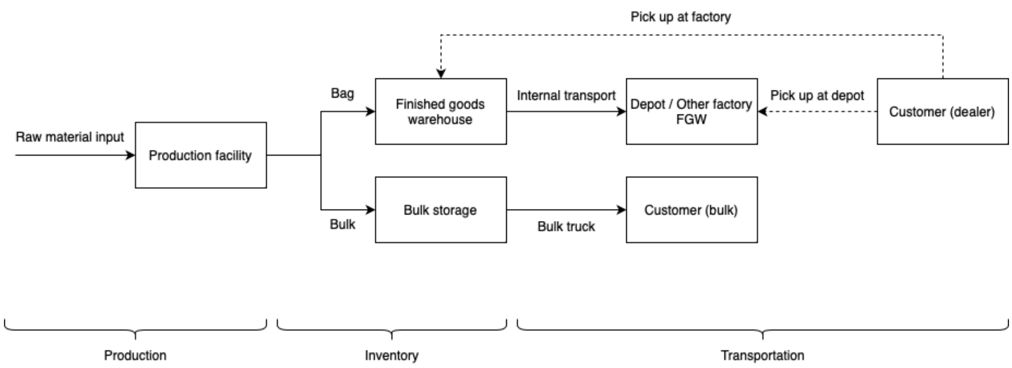


Figure 2.1: Supply chain network - De Heus Vietnam

At the beginning of the supply chain, raw materials such as corn, wheat, and nutritional additives are used to produce animal feed. In factories, these materials undergo processes like sifting, crushing, heating, and pressing to yield the finished product, typically pellets. This can be seen in the production part of the figure. Delivery to customers occurs in two ways: either in bags or in bulk volumes. Bagged goods are transported to the factory's finished goods warehouse, while bulk items are conveyed via a conveyor belt to bulk storage bins. This can be seen in the inventory part of the figure. Customer acquisition methods vary; for bagged products, customers collect their orders from factories or depots, aiming for the closest possible location. This may require internal transport from the production facility to a depot or another selling factory. Bulk feed loads onto trucks from bulk storage bins, directly serving farms, a process managed by De Heus's trucks. These different formats for bagged and bulk customers can be seen in the transportation part of the figure.

2.2 Characteristics

After defining the supply chain, it is separated into three main parts: production, inventory, and transportation. It is important to understand the characteristics and limitations of these different components. The demand is explained to better understand the various products delivered to the customers. Firstly, the production is detailed, including information about various production locations, their corresponding costs, and capacities. Secondly, the inventory capacities of the finished goods warehouses are provided. Lastly, the transportation routes are outlined, along with associated transportation costs

2.2.1 Demand

In Vietnam, customers of De Heus can be divided into two groups: dealers and direct farmers. Dealers own shops where they sell products from various brands and manufacturers, supplying medium and small farmers from their locations. The second group comprises direct farms, which are larger farms with larger sales volumes. Due to the higher sales volume of direct farms, it is economically beneficial for De Heus to deliver feed directly to these customers. Because of the different demands from small and medium-sized farmers through dealers and the larger demand from direct farms, De Heus delivers feed in two ways. Dealers are supplied with bags (typically 25 kg), while direct farms receive bulk feed delivered by bulk trucks. Dealers still have substantial sales volumes, representing 90% of the total sales volume, with direct farms accounting for the remaining 10%. A summary of the characteristics regarding the volume of sales, the feed product and the transport methods for the demand from the direct farms and dealers is provided in Table 2.1. Customer demand must always be fulfilled within the requested period, as animals require their feed. Therefore, back orders are not allowed.

	Direct farms	**Dealers**
Sales volumes	High	Medium & Small
Feed product	Bulk	Bag
Transportation method	Bulk trucks	Pick-up

Table 2.1: Characteristics of customers.

The distribution of demand throughout the month differs for bags and bulk products. When analyzing the sales volume of bagged and bulk products separately, it becomes evident that the patterns are distinct. The demand distribution over the month is illustrated in Figure 2.2, utilizing data from December 2023 for the sales volume of the South-East region. The figure reveals that the sales demand pattern is time dependent within the month. Direct farms, which own farms and maintain a constant number of livestock, require, on average, the same amount of feed every week, resulting in a consistent demand depicted by the blue line. In contrast, dealers exhibit an increasing trend throughout the month, represented by the green line. This trend is attributable to the discounts offered to dealers to boost sales volume toward the end of the month, resulting in a peak in demand during that period. Consequently, dealers tend to purchase fewer bags at the beginning of the month. These variations in demand can be represented as vectors for each month. For bulk products, the vector is evenly distributed throughout the month, resulting in [0.25, 0.25, 0.25, 0.25] for weeks 1 to 4. However, for bagged products, the distribution varies slightly, with the vector being [0.22, 0.24, 0.25, 0.29]. The uneven distribution throughout the week results in a higher workload at the end of the month, necessitating coverage from the beginning of the month to fulfill the demand. It is essential that the components of the supply chain are adjusted to one another to accommodate this fluctuating demand.

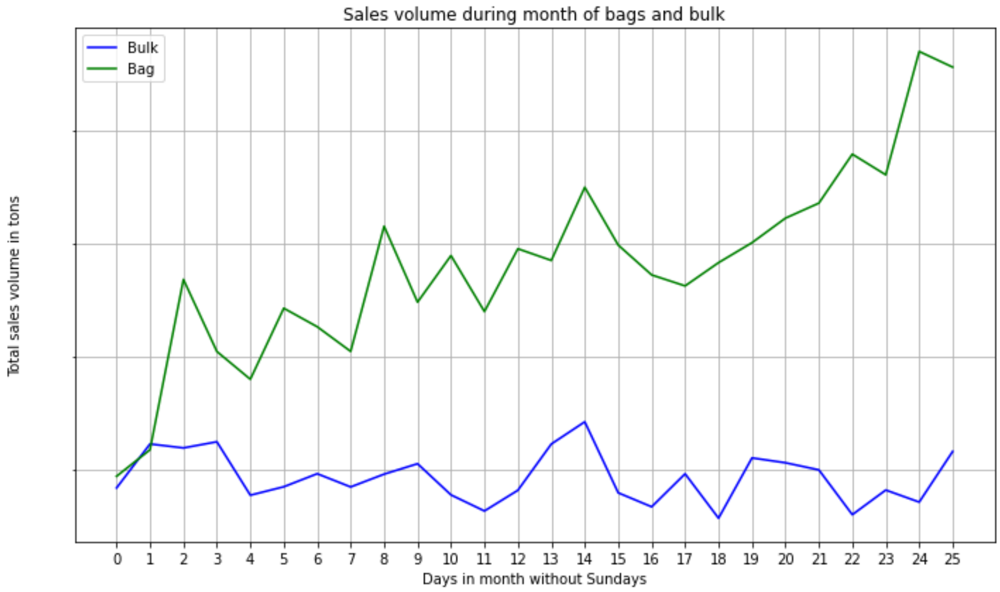


Figure 2.2: Demand distribution of South-East during month (Dec-2023)

In addition to categorizing the feed into bags and bulk with corresponding demand patterns, the feed is also categorized into product groups. The main product groups, with their subgroups for livestock, are: Pigs (breeding, fattening, growing, and piglets), Poultry layer (ducks and chickens), Poultry meat (white broilers and yellow broilers), and Ruminants (beef, dairy, and goats). Each of these specific groups has various growth phases. For example, to raise a pig, there is a need for piglet feed, starter feed and fattening feed are needed, each with its specific ingredients and formulations designed to meet animal requirements as best as possible. The demand volume for these different phases also varies. For instance, on average, a pig will require 30 kg of piglet feed, 50 kg of starter feed, and 150 kg of fattening feed, resulting in different demand volumes.

2.2.2 Production

In the focus area South-East, De Heus operates three factories. The Dong Nai and Binh Duong factories were already owned by De Heus, while Bien Hoa was acquired from Proconoco/ANCO. The Proconco/ANCO factories that were purchased were mostly very old, as the company did not invest in those factories in recent years. The different factories have varying production capacities and production costs. Some factories have newer machines capable of producing more tons per hour than others. The factories in Dong Nai and Binh Duong have bulk loading lines, while Bien Hoa produces only bags. All the factories have a finished goods warehouse where the bags are stored and supplied to dealers or depots. Each location has its own production planning, created on Fridays. Customers are required to place their orders by Thursday afternoon for the following week. Depots also forecast the sales volumes of different SKUs expected at various locations and place these as orders at the factory. After receiving the orders, the production manager examines inventory data and, based on safety stocks and incoming orders, creates the plan for the following week. This planning covers 24/6, meaning that production occurs 24 hours a day except for Sunday. Production planning is done per recipe, where a recipe consists of multiple bag and bulk SKUs. Each recipe contains the nutritional formula of the feed and can be sold in both bulk and bags, each with its own SKU number. Additionally, the same recipe may be sold under different brands, with each brand having its own bag and SKU number. For example, if 20 tons are produced from a certain recipe, it could be the case that 10 tons are bulk feed, while the remaining 10 tons are evenly divided between two brands, resulting in three different SKUs.

There are potentially 576 production hours per month, but each time a different recipe is produced, the machines need to be flushed. Flushing the machines cannot be considered a complete halt to the production process because not all machines are in series; there are also parallel machines that can continue production unless the other machine is busy flushing. The estimated flushing time resulting in a loss of production time is approximately 5 minutes. This is based on flushing the serial machines in the production process. The different SKUs have different brands on their bags, meaning it also takes 5 minutes to switch the bags and get a new pallet to stack the filled bags. After finishing production, the bags are stacked on pallets and transported by a forklift to the finished goods warehouse of the factories. The bulk is transported by a conveyor belt to the bulk storage bins.

The flushing time occurs between two production runs of two different recipes, where a production run consists of the amount of tons that are produced consecutively until the next production run of a new product starts. To calculate the average production run size of the feed, it is categorized into low, medium, and high sales volume products. To determine the classification, an ABC classification with the Pareto 80/20 rule is used (Ultsch, 2002). This classification assigns high volume (A), medium volume (B), and low volume (C) categories. Twenty percent of the products, responsible for 80% of the total volume, receive the high volume classification. Ten percent of the products, responsible for 10% of the volume, receive the medium classification, and the remaining 80% of products, responsible for 10% of the volume, receive the low classification. Each category is assigned an average production run size based on the average production volume per day when the product is produced. For this measurement, it is assumed that a production run is only produced once a day, as this theoretically provides the most efficient situation, requiring only one flush after production. The average production run for low volume is 12 tons, for medium volume 20 tons, and for high volume 36 tons. In reality, determining a specific production run size for a product is challenging because it depends on demand and can be easily adjusted to decrease or increase the production volume for each SKU and recipe. Ideally, using a variable lot size would be desirable, assuming a linear relationship between the total amount of production and the production run size. This relationship is depicted in Figure 2.3, where the horizontal axis represents the total production volume for 10 months, and the vertical axis represents the production run size, illustrating that the production run size increases with higher total volume. This is logical, as longer production runs are more efficient. Therefore, if the total volume increases and the production manager wishes to maintain the same number of production runs, the amount of tons per production run needs to increase. In addition to determining where and in what quantity to produce the product, decisions regarding production run sizes must also be made. In practice, production planning often relies on experience rather than calculated cost-effectiveness. Because of the increased number of SKUs resulting from the acquisition of Proconco/ANCO, the pressure on the production schedule has further increased, as more diverse SKUs need to be produced at each factory. This leads to longer flushing times, resulting in less efficient production planning. To address this issue and optimize the current situation, it may be possible to increase the production volume of a certain SKU at one factory and then transship it to other factories. Alternatively, it could be more optimal to produce products closer to the customers, but this would entail smaller production volumes, leading to increased flushing times. Therefore, the problem should be solved by optimizing all parts of the finished goods supply chain to make the most optimal decisions.

The factories of De Heus are unable to produce all the different types of feed and SKUs at every location. This differentiation depends on the product groups and the production quality of each factory. Animal feed

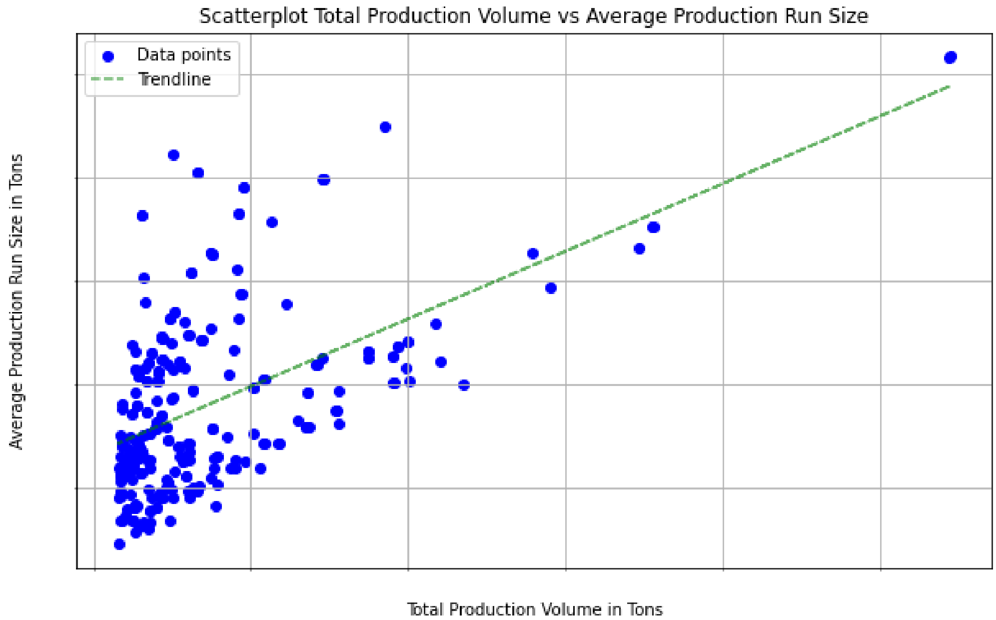


Figure 2.3: Scatter plot: Total Production Volume (10 months 2023) vs Average run size of recipe

can be categorized into main product groups such as pigs, ruminants and poultry, and subproduct groups like breeding pigs, piglets, growing pigs, and fattening pigs within the main group of pigs. Some product groups require additional cleaning measures due to safety and health concerns. For example, after the use of ruminant bone meal, it is not allowed to produce ruminant feeds without thorough cleaning due to the risk of 'Mad Cow Disease'. Piglet feed also requires a dedicated production line due to the highly stringent cleanliness requirements for this type of feed. For this reason, De Heus chooses not to produce this type of feed at all of its different factories, because this leads to inefficient production planning. To make it more optimal, the products can only be produced at dedicated locations to decrease the number of cleanings. Table 2.2 shows that Bien Duong is allowed to produce all kinds of feed, while Dong Nai and Bien Hoa are allowed to produce everything except for ruminant feed and piglet feed. The second differentiation in production type is production quality. After the acquisition of Proconco/Anco feedmills, the quality has already improved to meet De Heus standards. Previously, the feed quality was inferior, making cross-production not fully applicable. While most factories across Vietnam are now capable of producing all types of feed, it was not sustainable to invest in upgrading the Bien Hoa factory to meet these standards. Therefore, a restriction has been implemented stating that De Heus products can only be produced at factories that meet De Heus standards.

The costs of production are currently calculated using semi-variable costs. This means that the total costs of a factory are divided by the number of tons produced at a certain location. These costs consist of:

- Production costs(labour costs, outsource labour costs and cleaning/security)
- Maintenance (labour costs, outsource labour, consumables, repairs and the building)
- Warehouse (labour costs, outsource labour, vehicles and rentals)
- Utilities (Fuel for boilers, electricity, oil, water and other energy resources)

The semi-variable costs serve as key performance indicators (KPIs) for the company. During operational meetings, the actual variable costs are compared among different factories and with the budget set at the beginning of the year. Significant differences in semi-variable costs are observed when comparing De Heus

Product group	Product subgroup	abbr.	Dong Nai	Bien Hao	Binh Duong
Pigs	Breeding pigs	PB	✓	✓	✓
Pigs	Piglets	PP			✓
Pigs	Growing pigs	PG	✓	✓	✓
Pigs	Fattening pigs	PF	✓	✓	✓
Ruminants	Goat	RG			✓
Ruminants	Beef	RB			✓
Ruminants	Diary	RD			✓
Fur Animal	Rabbit	FA	✓	✓	✓
Poultry	Yellow Broilers	CYB	✓	✓	✓
Poultry	White Broilers	CWB	✓	✓	✓
Poultry	Duck	CD	✓	✓	✓
Poultry	Quail	CQ	✓	✓	✓
Poultry	Breeders	CB	✓	✓	✓
Poultry	Layer	CL	✓	✓	✓

Table 2.2: Production sub-group possibility factories

factories to Pronconco/ANCO factories. Specifically, the semi-variable costs for Dong Nai are $6.69 per ton, for Bien Duong $8.23 per ton, and for Bien Hoa $13.82 per ton. The most substantial variations are found in labor production costs, repairs, and electricity consumption. This discrepancy is attributed to the older machinery at the Bien Hoa factory, leading to increased labor needs and more frequent breakdowns. While these costs provide a useful indication of production costs per ton, the calculation is not entirely fair due to its 'semi' variable nature. Start-up costs for a factory become relatively smaller as production increases. When a factory is opened, it requires management, security, general electricity consumption, etc., which are independent of the production volume. Consequently, costs are relatively high with low production volumes but decrease as production volume increases.

It is not possible to use semi-variable costs alone to determine the cost of producing one ton of feed in a specific factory and compare it with other factories. Therefore, it is necessary to break down semi-variable costs into direct variable production costs, direct material costs, factory overhead costs and fixed production set-up costs. The direct production costs consist of the following components: labor costs production, outsourced production, labor costs maintenance, consumables maintenance, labor costs warehouse, fuel for boilers, electricity, and QC teams. The fixed production costs consist of: cleaning and guards labor costs, repair maintenance, M&R buildings, outsourced warehouse, Vehicles & machine lease, Do oil, others (water, ink, solvent, chemical, etc.) utilities, QC expenses, and HSE expenses. Not all costs are implemented as fully variable or fully fixed, so some assumption are made to make them usable. For example, electricity costs are not completely variable in reality because the general buildings of a factory also require electricity, which is not solely dedicated to an increase in production volume. It is also assumed that maintenance costs are fixed, while in reality, they may increase in a trapezoid form as production volume increases by a certain amount. However, it is not possible to make them fully variable because there is always maintenance when there is feed in production. In consultation with the company, this list of direct production costs and fixed (overhead) costs is made as realistic as possible, while some simplifications and assumptions are made to use the costs currently calculated directly in the model. Depreciation is not taken into account in the factory overhead costs because depreciation costs will always exist, even if the decision is made not to open the factory at all. The direct material costs are based on the logistic inbound costs and depend on the amount of tons produced at a factory. The direct material costs encompass expenses related to raw materials such as corn, nutrient and enzymes. Each recipe has its own composition of ingredients, which is similar across all factories. Therefore, it is assumed that there are no differences in material usage between factories, as it has only a small impact. However, transportation costs for materials differ for each factory due to variations in the supply chain. For instance, Bien Hoa is located near a river, which results in lower raw material costs due to economies of scale. These transportation costs are factored into the total direct material costs. The cost per production run is equal to the cleaning time of the machine, which is performed by the production team. The cleaning time is estimated to be an average of 5 minutes between production runs. Therefore, the cost of cleaning is calculated based on the machines' inability to produce during this time. Normally, the production speed is 60 tons per hour, so a loss of 5 minutes results in a loss of 5 tons. So for example for the Binh Duong factory it gives a labor cost per ton of \$1.55. Thus, the total cost is $5 \cdot \$2.13 = \$7.77$ per flush between two production runs.

The production capacity, bulk possibilities, and the direct, set-up, and overhead costs are provided in Table 2.3, expressed per factory. It can be observed that the direct material costs in Bien Hao are cheaper than in other locations, but the production costs are higher at this location. These costs serve as inputs to determine the appropriate factories for meeting demand.

	Dong Nai	**Bien Hao**	**Binh Duong**
Production capacity	36,500 ton	19,710 ton	16,425 ton
Bulk possibility	Yes	No	Yes
Direct variable production costs	4.88$	8.62$	6.18$
Direct material costs	4.4$	2.1$	4.9$
Fixed production set-up costs	0.82$	14.82$	7.77$
Factory overhead costs	34,997.40$	80,789.96$	57,688.24$

Table 2.3: Production location capacity and costs.

2.2.3 Inventory

The next step in the supply chain of finished goods after production is storing them in the finished goods warehouse of the factory. For bags, this means storing them on pallets which are transported to the finished goods warehouse with a forklift, with each pallet containing 2 tons of feed. Bulk feed is transported by a conveyor belt to the bulk storage bins. The capacity of the finished goods warehouse is limited by the amount of space available to store the pallets. The warehouse capacity for Dong Nai is 3,500 tons, for Bien Hao is 3,000 tons, and for Binh Duong is 2,000 tons. An overview of these capacities can be found in Table 2.4. The capacity of the bulk storage bins is determined by the maximum bulk capacity, which includes production, storage, and transportation possibilities, this capacity can be found in Section 2.2.4.

The finished goods warehouse at the factories are used for storing and supplying two types of inventory: the finished goods that are picked-up by the dealers and other depots/factories which place an internal order which means that the finished goods needs to be transshiped to another location. Upon loading orders, pallets are transported by forklifts to the loading station, where bags are loaded onto trucks using conveyor belts. Finished goods transshipped to depots or other factories spend one day in the warehouse for high-sales volumes, three days for medium volumes, while goods sold at the factory remain in the warehouse for two days for high volumes, seven days for medium volumes, and fourteen days for low volumes. Slow-moving products can pose an issue by occupying warehouse space where potential 6 (three pallets stacked) tons can be stored. The average stay durations are provided in Table 2.5. These average lengths of stay, in combination with the occupation of the warehouse space, determine the total usage of the warehouse capacity. In reality, achieving 100% warehouse utilization is not possible because the stored tonnage is not always a multiple of six. It is important to monitor the number of warehouse slots occupied by finished goods. For instance, if a product uses an average of seven tons of warehouse space, it requires two slots in the warehouse. In production planning, it is important to consider inventory capacity, as the inability to store finished goods means production cannot continue. With an increase in the number of SKUs, it has become even more challenging to store all the different SKUs in the warehouse, as each SKU with an average warehouse capacity < 6 occupies a slot. Therefore, optimization is necessary to determine where products should be produced and subsequently stored and sold.

The holding costs at the different warehouses are based on the working capital of the finished goods. If the finished goods are at the warehouse and waiting to be sold, it means that the value of money used to produce the finished goods cannot be utilized for investing in new projects of the company or placed in a savings account to accrue interest. These costs are referred to as Opportunity Costs of Capital. Other potential costs that could be included in the holding costs are the occupancy costs and depreciation costs. However, since the finished goods warehouse is always necessary after production, and it is assume that every bag gets sold, there is no depreciation. So it is assumed that the opportunity costs of capital are the only costs influencing the holding cost. The Weighted Average Costs of Capital (WACC) are used to approximate the opportunity cost of capital. If the average value per ton of finished goods is \$292, and the WACC is 17.53% it gives: Holding costs per ton per year $= 292\$ \cdot 17.53\% = 51.20\$$

	Dong Nai	**Bien Hao**	**Binh Duong**
Warehouse capacity	3,500 ton	3,000 ton	2,000 ton

Table 2.4: Warehouse: Finished goods warehouse capacity factories and depots

	Transshipments	**Selling warehouse**
High sales volume	1 day	2 days
Medium sales volume	3 days	7 days
Low sales volume	3 days	14 days

Table 2.5: Length of stay finished goods

2.2.4 Transportation

The last step in the supply chain is to fulfill the demand of the customers through transportation. There are two types of transportation from De Heus company to meet the customers' demand: the internal transportation of bags to depots and factories, and the bulk transportation of bulk feed to customers. After the finished goods are transported to the correct locations, the dealers pick up the bags themselves at the depots or factories, meaning that De Heus does not incur direct transportation costs for this part of the process.

For bag customers, the pick-up locations are the factories or depots. The factories can produce the feed themselves, but they can also be supplied by other factories. This is the case, for example, when a factory is unable to produce a certain type of feed or when it is more cost-effective to produce it at another location. Depots have capacities and cannot produce feed themselves; they need to be supplied by factories. In the south-east region, there are three depots: Dau Giang, Long An, and Ben Tre with corresponding capacities of 1,750 tons, 3,300 tons, and 1,700 tons, respectively. The primary purpose of these depots is to provide an additional service to customers. They are strategically situated near the customers, eliminating the need for them to travel all the way to the production locations. Currently, most dealers are assigned to the depot/factory that is closest to them. They pick up the feed at these locations and are not willing to pick up different SKUs at different locations. For that reason, it is only possible to assign a dealer to at most one pick-up location. The resupply of a certain SKU at a depot/factory can only be done by one and the same factory to ensure consistent quality for customers.

The direct customers who demand higher volumes are supplied by a bulk truck. When a bulk truck is scheduled for delivery to a customer, it loads its truck using a loading robot positioned above the truck, which deposits the right feed in the correct part of the tank. After loading, the truck can proceed to the customer and transfer the feed into a silo. One bulk truck is only able to go to one farm on each trip due to biosecurity measures. If the trucks were to drive to several customers in one run, it could potentially spread disease from one farm to another, which is not desired. It is possible to supply different SKUs from different factories to one direct farm. The bulk trucks have different capacities ranging from 5 to 24 tons, but most of the trucks are 14 tons, so this number is used to calculate the bulk transportation costs. Because the optimization will not assign a specific truck to a certain route, it is assumed that the total truck capacity is used as a constraint in the maximum amount of feed each location can supply in bulk. The total bulk capacity and bag capacity are provided in Table 2.6. Bien Hoa is unable to supply bulk feed, and the bag fleet is always infinite because it is managed by a third-party logistics company.

The different types of transportation can be categorized into three cost groups: internal transportation between factories and from factories to depots, external transportation caused by bag customers picking up the feed, and bulk transportation costs occurring when bulk feed is transported by bulk truck to the customer. Internal transportation is used to supply factories and depots and is carried out by a third-party logistics company, where the capacity of these third-party logistics is unlimited. A differentiation between internal transportation costs between factories and from factories to depots is made because potentially, the transportation costs from the factory to the depot could be forwarded to the customer who picks up the feed at a closer location. It could be more cost-effective for De Heus to deliver the feed at a depot location instead of the customer having to drive a longer distance because of scale advantages. The costs for this third-party logistics are \$0.03 per ton per kilometer. External transportation costs arise from the additional kilometers that customers must drive to pick up feed from their designated locations. Currently, these costs are not included in the total price calculation because there is uncertainty regarding customers' willingness to drive to alternative locations, and no incentives are offered to encourage them to do so. However, for optimization purposes, it is essential to include these external costs. They are determined based on available information, such as the customer's location and the assigned depot/factory. The definition of 'extra' kilometers is that these are the kilometers that the customer needs to drive additionally to a location that is not the closest one to the customer. For instance, if the closest pick-up location is 40 km away, but assigning the customer to another location results in a total driving distance of 60 km, then the additional 20 km incurs costs for De Heus. Customers are willing to travel to alternative locations if it is more cost-effective for them and to incentivize this behavior a discount is offered, which constitutes the external transportation costs. The external transportation costs amount to \$0.031 per ton per kilometer. Bulk transportation costs to the direct farms are currently calculated for each route to the customers separately by hand. These total transportation costs consist of toll costs, fuel costs, depreciation/salary costs, and depend on the amount of tons transported by the truck. Fuel costs are responsible for 80-90% of the total costs, so these costs are used to calculate the transportation costs from the factories to the direct farms. Transportation costs are given per ton per kilometer driven. The trucks' fuel consumption is on average 25 liters per 100 km. With an average truck weight of 14 tons, it gives Costs per kilometer per ton $= (0.25 \cdot \$0.62)/14\ ton = \$0.011$.

An overview of the transportation network for internal bag transportation and external transportation to the direct farms is provided in Figure 2.4. The internal transportation routes from a factory to a depot/factory are represented by dotted lines. The black trucks performing these routes are supplied by a third-party logistics company and operate at a fixed price per trip. The external transportation routes, depicted by the black lines, involve bulk transport from the factory to the direct customer. This transportation is handled by our own bulk trucks, and its capacity is constrained by the total bulk capacity of the factory, determined by production levels and the number of trucks available. Dealers (customers) collect their orders from the depot or factory, and their transportation costs are not incurred by us. However, they opt to travel to an alternate location only if it proves more cost-effective for them in the end. Thus, it remains crucial to consider the distances the dealers need to cover.

	Dong Nai	**Bien Hao**	**Binh Duong**
Bulk fleet capacity per month	12,410 ton	-	5,840 ton
Bag fleet capacity per month	∞	∞	∞

Table 2.6: Truck fleet capacity

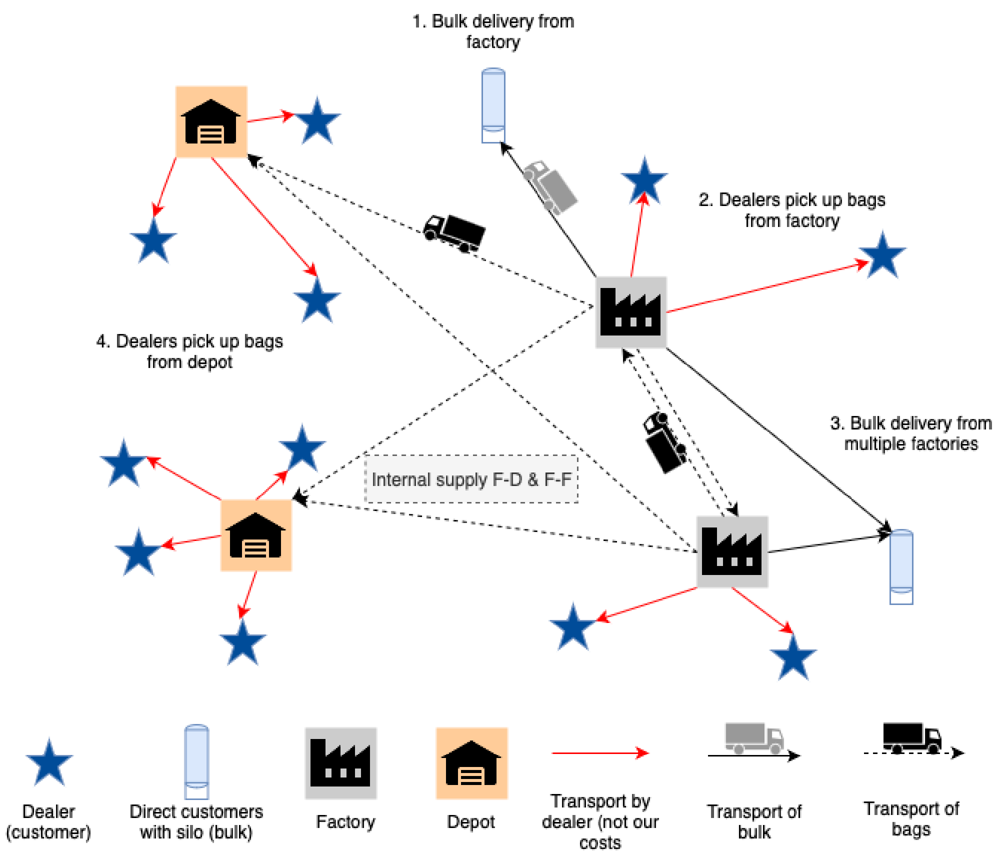


Figure 2.4: Transportation network: Network of bag and bulk delivery.

2.3 Key Performance Indicators (KPIs)

To measure the performance of the company, De Heus uses several KPIs. The most important KPI of the supply chain is costs, which consist of raw material costs, production costs across various factories and transportation costs for transshipment between them. Currently, production costs depend on the production volume at each factory because the total costs of a factory are divided by the production volume to calculate the cost per produced ton. Therefore, when analyzing these costs, it is important to consider various factors at each location, such as the number of SKUs produced and the total production volume. The previous Section describes the various components of these costs and the necessary transformations for cost calculation, including the impact of shifting an SKU to another factory and the impact of increasing the production volume of a SKU in a factory. Inventory costs are presently excluded from the company's total cost calculation for inventor where these costs are now integrated into the overall cost optimization strategy, leading to reduced total inventory levels.

Other KPIs crucial for the company encompass various production-level metrics: customer complaints. rework volume, operational utilization, full-time equivalent (FTE) count (which correlates with costs), tons per FTE, and energy consumption. In the warehouse, the Days Inventory Outstanding (DIO), indicating inventory in days based on average daily demand, stands as the primary metric. However, as it is measured at specific points in time, drawing conclusions on this KPI currently proves challenging. The transportation cost per ton and bulk truck utilization are important factors for transportation. Most of these KPIs directly relate to costs. For instance, increased factory utilization leads to lower production costs per ton. Similarly, reducing DIO in a factory lowers average inventory, subsequently reducing capital requirements. While the model primarily focuses on cost reduction, these KPIs are not individually considered; however, optimization positively impacts these indicators.

2.4 Cross-production

The process of shifting production volumes to different locations, where the optimization encompasses the overarching optimization of De Heus and Proconco/ANCO factories, is named cross-production. De Heus has already begun implementing cross-production, optimizing the production capacities of the factories from different entities, although on a smaller scale. Some recipes and SKUs are transferred to factories that are clearly located in better positions. However, cross-production also introduces new challenges when transferring products to other factories, as the quality and texture may differ if the same product is produced in a different factory. Each factory has its own layout and its own machines, so not every factory has the same type of machines. Machines can be from different manufacturers or different ages, which could lead to quality differences. Consequently, when producing the same recipe in different factories, variations may arise in factors such as hardness, pH, and pallet durability index (PDI). Maintaining consistent quality across different factories is crucial, as any discrepancies could lead to farmers rejecting the feed. This is especially critical in Vietnam, where many farmers still hand-feed animals and inspect feed for color, structure, and smell. If any of these aspects differ, there is a high likelihood that they will reject it and send a complaint to De Heus. To address this issue, De Heus is currently investing in its factories to enhance product quality. This investment aims to enable the production of all recipes in all factories with minimal differences in quality. The assumption that every recipe (and SKU) can be produced at every location is made in this research to optimize the situation to its fullest potential.

2.5 Conclusion

For the focus area of southeast Vietnam, encompassing the factories in Dong Nai, Bien Hoa, and Binh Duong, this chapter provides information on the characteristics of the production locations, inventory, and demand distribution. The supply chain network of finished goods begins at production, where raw materials are imported and processed into feed. Production occurs in factories, each with its own production planning that schedules recipes comprising multiple bag and/or bulk SKUs. This demand comes from dealers, serving smaller customers in bags, and from direct farms, which are larger customers served by bulk trucks. Bag demand fluctuates, with 24% of sales occurring in the first week and 29% in the last week of the month due to end-of-month discounts. To manage this peak, it is necessary to optimize the problem within a monthly time frame to respond to and prepare for these differences in earlier weeks.

To make production plans for the factories, recipes are scheduled. These recipes are produced in production runs, with the average tonnage determined based on the total production volume during the month. High, medium, and low volume products yield 12, 20, and 36 tons per run, respectively. Factories have a total production time of 567 hours, with capacity reduced due to 5-minute flushing times between production runs. Additionally, not all types of feed can be produced at every location due to safety and health concerns; former Proconco/Anco factories cannot produce De Heus feed due to machinery limitations affecting quality. Production costs depend on semi-variable costs of the factories, divided into setup and variable costs for recipes and SKUs. Direct material costs depend on inbound logistic costs, while production run costs are determined by cleaning costs. These production restrictions needs an integrated optimization approach between different factories to determine the best locations for producing certain recipes and SKUs.

After finished goods are produced, they are stored in the factory's finished goods warehouse, which has capacity constraints. From these locations, finished goods are sold or transshipped to other warehouses/depots. Warehouse occupancy is determined by the average length of stay of finished goods, with goods staying for 1 or 3 days when ordered for depots/factories, and 2, 7, or 14 days when sold at a location, based on volume (high, medium, or low). Occupancy is based on available slots, each capable of storing a maximum of 6 tons (3 stacked pallets), with full occupancy maintained even if less than 6 tons are present. Holding costs are $51.20 per ton, based on opportunity costs of capacity. Due to the increased number of SKUs, it becomes even more important to strategize optimization of production locations to efficiently utilize warehouse slots.

Transportation comprises three distinct parts: internal transportation from production factories to depots and other factories, handled by a third-party logistics company; external transportation costs, seen as the 'discount' given to customers for longer driving distances; and bulk transport to direct farms. Internal transportation costs are predetermined at $0.03 per ton per kilometer. External transportation costs depend on the additional driving costs for customers, estimated at $0.031 per ton per kilometer. Bulk transportation costs are based on fuel costs, accounting for 80-90% of total costs, at $0.011 per ton per kilometer. Cross-production, where production volumes are shifted to other locations, has already begun, but ensuring consistent feed quality at all locations remains a significant implementation challenge due to quality differences between factories. Currently, not all costs are yet incorporated into the cost calculations of De Heus. When all costs of production, inventory, and transportation are known, the most cost-effective solution can be determined where a model provide a solution that includes customer assignment to specific locations, in combination with production planning per week detailing which SKUs need to be produced at which location and how it reached the customer.

3 Literature Review

In this section, the existing literature on supply chain optimization, the corresponding mathematical models, and the solution methods are analyzed. In Section 3.1, supply chain management is explained. Section 3.2 examines various production scheduling models, Section 3.3 delves into inventory management control systems, and Section 3.4 focuses on transportation management, particularly routing explanations. Following the examination of each segment, the review proceeds to analyze their integrated optimization. Section 3.5 elaborates on the Lot-Sizing problem, addressing the combined optimization of production and inventory. Another aspect discussed is the simultaneous optimization of inventory and transportation, presented in Section 3.6 as the Inventory Routing Problem. To serve multiple customers, Section 3.7 introduces the Multi-Commodity Network Flow problem. Lastly, Section 3.8 introduces the concept of optimizing the entire supply chain, encompassing production, inventory, and transportation, potentially resulting in significant cost savings. Section 3.9 provides a literature review overview of the papers used. Section 3.10 discusses the different methods needed to solve larger instances of the given models. Section 3.11 provides different multi-objective optimization techniques. Section 3.12 summarizes the findings and concludes which literature is most suitable for De Heus.

This section addresses the research question, *'What is proposed in the literature for solving the optimisation problem of De Heus?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(a) What type of optimisation problem is suitable for the current situation at De Heus?

(b) What types of characteristics match those of De Heus in production, inventory, and transportation?

(c) Which solution methods are used to solve the supply chain optimisation problem?

(d) Which multi-objective optimization methods are most appropriate for the case of De Heus?

3.1 Supply chain management

A supply chain may be defined as an integrated process wherein various business entities work together in one effort: acquiring raw materials, converting them into a final product, and delivering this to the customer. In this supply chain, materials flow forward, while information flows backward. This means that products flow to the customers, but the demand, which can be seen as information, is customer-related and is sent to production (Beamon, 1998). The supply chain is comprised of two basic integrated processes: the Production Planning and Inventory Control process and the Distribution and Logistics process. The Production Planning and Inventory Control process designs the manufacturing process, including raw material scheduling, production planning, and design, while inventory control describes the storage policies and procedures. The Distribution and Logistics process uses input from inventory management to determine how products are retrieved and transported to warehouses and customers. Simultaneously optimizing the Production Planning and Inventory Control process and the Distribution and Logistics process could lead to promising results. Most of the time, these optimizations are cost-driven. However, it is also possible to focus on customer satisfaction, which is derived from on-time delivery or the shortest distance (Eksioglu, Vural, & Reisman, 2009). Chandra and Fisher (1994) showed that optimizing production scheduling and the vehicle routing problem simultaneously can result in a cost reduction of 3 to 20%. An integrated supply chain planning system is a tool to jointly optimize several planning decisions and, thereby, attempt to adjust the decisions to profit from the benefits (Adulyasak, Cordeau, & Jans, 2015). The supply chain usually consists of different parts: production, inventory, and distribution. This system includes a factory that produces goods, stores them and distributes them to other warehouses (Haq, Vrat, & Kanda, 1991). It is important to understand the different parts and know their characteristics before the optimization models can be explained. The different characteristics lead to different choices and modeling approaches.

The next sections aims to explore optimization models related to the production, inventory, and transportation segments within the supply chain, as well as their integrated optimization. Initially, the individual segments are analyzed separately, and subsequently, the simultaneous optimization of these parts is investigated.

3.2 Production

Before optimizing production planning and scheduling, it is needed to understand the various types of production processes and their specific characteristics. These diverse types possess specific attributes that result in particular constraints and input parameters that must be incorporated into the optimization model. It can be distinguished that there are four broad classes of processes: Job shop, batch flow, assembly line, and continuous process (Silver, Pyke, & Thomas, 2016). Job shops manufacture customized products, for example, a specially made part for a machine. Batch flow produces larger quantities at once before transferring to the next phase, for example, a bakery that produces different kinds of bread. An assembly line puts different parts together to produce the final product, for example, a car. The continuous process has a continuous flow, like chemical processes in the oil industry. These processes already provide characteristics for a high number of customers and products for job shops and a low number for continuous processes. Job shops are able to produce for customization, while continuous processes have more standardized output. In planning, it is easier to plan continuous processes because they require less optimization than job shop processes (Silver et al., 2016). The characteristics of the jobs that need to be planned are important for the design of the model. Key considerations include: How many production locations and lines are there? Are they able to produce all the different types of item? For items, do you have one or multiple items, and are there relationships between these items? Are there start-up costs for the different item groups? Is backlogging allowed, and does an increase in volume lead to discounts? Jans and Degraeve (2008).

3.3 Inventory

In inventory management, there are six broad decision categories in controlling inventory: cycle stock, congestion stock, safety or buffer stock, anticipation inventories, pipeline inventories, and decoupling stock (Silver et al., 2016). Cycle inventory results from producing batches instead of one unit at a time, where the amount on hand at any point is the cycle inventory. Congestion stocks are inventories due to limited production capacity, for example, if the same machine is used to produce the products. Safety stocks are used to cover uncertainties in demand. Anticipation stocks are produced in advance to accumulate an expected peak in sales, due to promotion. Pipeline stocks are in transit from one location to another, for example, from the factory to the depot. Decoupling stocks are used in multi-echelon situations to permit decision-making at different echelons, allowing decision-making without direct impact. Inventory needs to play one of these roles to have a significant impact on inventory control (Silver et al., 2016). The inventory is stored in warehouses with capacity restrictions. Therefore, it is important to understand the role of inventory in the different warehouses and depots before determining the actual usage of the capacity. A part of the total capacity of the warehouses of De Heus is used for the pipeline inventory that is transshipped to other depots or factory warehouses, while the other part of the capacity is used for selling the finished goods and consists of the cycle inventory and congestion stocks. These different types of inventory have varying lengths of stays; the pipeline inventory stays shorter in the warehouses compared to the finished goods that are sold at that location. In the model, it is important to distinguish between the pipeline and cycle inventory/congestion stocks.

3.4 Transportation

In transportation management, assigning a customer with a demand to a factory or warehouse capable of supplying this demand is crucial. This assignment ensures that the customer's demand is met and could be expanded to implement routing for optimizing supply to customers. The most basic form of routing involves assigning customers to a factory where a truck travels to the customer, delivers the supplies, and returns to the factory. Supplying customers depends on truck characteristics and routing possibilities. If visiting different customers in one trip is feasible, the Vehicle Routing Problem (VRP) could be applied (Bektas, 2006). The characteristics used in a VRP are also significant to customer assignment without routing. Eksioglu et al. (2009) defined various situations modelled in the VRP. Key demand characteristics include determinism and splittability. Time considerations involve the need for time windows and multiple periods. In addition, details about vehicles are crucial, whether they have capacity constraints, can reach only one point at a time, or form a homogeneous fleet. Although routing to customers is not integrated into the optimization model for De Heus, understanding these characteristics remains important due to bulk transport to direct farms and transshipments between factories and depots. These transportation characteristics describe aspects similar to the VRP.

3.5 Lot-Sizing Problem: Production-Inventory

The first possibility of gaining advantages in optimizing two parts simultaneously is by optimizing the production and inventory. Optimizing the production costs is balancing the production costs with the inventory cost. One of the first problems that defined it was a lot sizing problem named the Economic Order Quantity (EOQ) by Harris (1913). The EOQ model uses a constant demand rate of a single single item and searches for the optimal order point when the inventory and production costs are balanced. This model is later on translated to a model with discrete time periods and can be seen as the simplest form of the dynamic lot sizing problem is the Single-Item Uncapacitated Problem. This problem was initially discussed by Wagner and Whitin (1958), and Zangwill (1969) subsequently demonstrated that it can be viewed as a fixed-charge network problem. An extension of this problem, which provides a more realistic model, is the Capacitated Multi-Item Lot Sizing Problem (CLSP). Depending on the factory's characteristics, it can be either a large bucket model, where several items can be produced on the same machine in the same time period, or a small time bucket model, where only one product can be produced by a machine per time unit. The mathematical formulation of the large bucket CLSP Jans and Degraeve (2008).

In this formulation, each period is given by t, and each product is given by i. It has the following decision variables: production level x_{it}, setup decision y_{it}, and inventory variable s_{it}. The associated costs are vc_{it}, sc_{it}, and hc_{it}, respectively representing the production, setup, and holding costs of product i in period t. T is the set of all periods, with m as the last period. The known demand for each period is given by d_{it}, and cumulative from period t until k is given by d_{itm}. cap_t gives the production capacity, where vt_i shows how many units of capacity product i consume. The 'big M' is a large number, typically set equal to $min\{cap_t/vt_i, sd_{itm}\}$.

$$\min \sum_{i \in P} \sum_{t \in T} (sc_{it} + y_{it} + vc_{it}x_{it} + hc_{it}s_{it}) \tag{3.1}$$

$$\text{s.t. } s_{i,t-1} + x_{it} = d_{it} + s_{it} \qquad \forall i \in P, \forall t \in T \tag{3.2}$$

$$x_{it} \leq My_{it} \qquad \forall i \in P, \forall t \in T \tag{3.3}$$

$$\sum_{i \in P} vt_i x_{it} \leq cap_t \qquad \forall t \in T \tag{3.4}$$

$$x_{it}, s_{it} \geq 0 \qquad \forall i \in P, \forall t \in T \tag{3.5}$$

$$y_{it} \in \{0, 1\} \qquad \forall i \in P, \forall t \in T \tag{3.6}$$

The objective function 3.1 minimizes the total costs of start ups, set ups, variable production and inventory. The demand balance equation is given in constraint 3.2. Constraint 3.3 gives the set up. Constraint 3.4 gives the capacity restriction. Constraints 3.5 and 3.6 gives the non-negativity and binary variables.

The research explores various alternatives. On one hand, lot-sizing formulations incorporate more operational and scheduling issues to model the production process, costs, and demand more accurately. On the other hand, the model could also focus more on tactical and strategic problems, where operational lot-sizing decisions constitute a core substructure (Jans & Degraeve, 2008). On the operational level, the setup costs and times could not only apply to individual items but also to joint or major setups (Veinott Jr., 1969). (Hindi, 1995) consider setup times for CLPS, viewing them as capacity lost due to the time needed for tasks like cleaning, preheating, and adjustments. Production may occur in batches, where y_{it} is not a binary value anymore but a multiple of the number of batches produced (Van Vyve & Ortega, 2004). These batches, for example, could be limited by the capacity of a production tank. Sometimes, it is assumed that either production happens or it does not, and no half batch production is possible (Van Vyve & Ortega, 2004). Stowers and Palekar (1997) and Bhatia and Palekar (2001) consider a variant of production called the joint replenishment lot sizing problem, where products belong to the same family. This is also known as strong setup interaction and is common in industries like oil refineries (Persson, Göthe-Lundgren, Lundgren, & Gendron, 2004). Production costs may change when producing in larger volumes to obtain discounts. These discounts are often determined by a piecewise linear cost function (Shaw & Wagelmans, 1998). Inventory can be bounded by upper and lower limits at a customer (Jaruphongsa, Çetinkaya, & Lee, 2004). It can also be extended with perishable inventory, it considers the uncapacitated single item lot sizing problem with age-dependent inventory, which is lost when carried over to the next period. In some models, backlogging is allowed for demand, which means that demand can be met at a later time, resulting in negative inventory levels (Zangwill, 1966), (Federgruen & Tzur, 1993). This leads to a situation where lot sizing models with stock outs are an alternative to backlogging and result in lost sales (Sandbothe & Thompson, 1990), (Aksen, Altinkemer, & Chand, 2003). The demand time window for a single item uncapacitated dynamic lot sizing problem is introduced by C.-Y. Lee, Çetinkaya, and Wagelmans (2001). In this approach, each demand has an earliest and latest delivery date. When analyzing the time horizon of the schedules, a rolling horizon approach is commonly used, where the first period is implemented, and the demand forecast is used to look at one period further. The Wagner-Within algorithm still outperforms other heuristics (Simpson, 2001). The rolling horizon approach can result in planning nervousness because schedules need to be changed frequently, and these changes can result in extra costs (Kazan, Nagi, & Rump, 2000).

On the tactical and strategic level, hierarchical production planning is a procedure that addresses production planning at different levels of aggregation (Bitran, Haas, & Hax, 1981). First, a decision is made at a higher level, which provides restrictions to the lower level. Items are aggregated into families, and families are grouped into types. A type represents a set of items with a similar demand pattern. Within families, the items share the same setup. The decision is to determine the capacity for a type, with the objective of minimizing the total setup costs for all families within a type (Liberatore & Miller, 1985).

For De Heus, the lot-sizing decision regarding how much should be produced in each period and what inventory level is the most efficient is of interest. Flow constraints for the produced finished goods, inventory demand, stock levels, and end-of-period considerations are applied across various time steps. Specific cases discussed in several papers, such as capacity reduction due to flushing time, grouped set-up times, and joint replenishment times, are relevant to the De Heus model.

3.6 Inventory Routing Problem: Inventory-Transportation

The second way to simultaneously solve two parts of the supply chain is by combining inventory and transportation. The inventory routing problem (IRP) is a fusion of inventory and transportation management, first introduced by (Bell et al., 1983). This example aimed to demonstrate the impact on transportation costs when making joint decisions about whom to serve, how much to deliver, and the routes to take to reach customers Bertazzi and Speranza (2012). The problem entails a set of customers which are served by direct shipping from a single depot. Transportation connections are provided between different nodes and from the depot to the customers, each with specific costs based on distance. There are unlimited vehicles with a certain capacity, known daily customer demands, and maximum inventory levels at the customers. The objective is to minimize costs while providing a plan to serve the customers without stock-outs within the given capacity restrictions.

The shipping times and planning horizon for these shipments could be seen as continuous replenishment or discrete replenishment, where continuous means replenishment can occur at any time, while discrete can only be performed at certain times. The time horizon can be long-term for an infinite horizon or finite to address specific situations Bertazzi and Speranza (2012). The mathematical optimization model was introduced by (Bertazzi & Speranza, 2002). It has a finite time horizon, H, with possible shipping times in $T = \{0, 1, .., H-1\}$. The model aims to determine the quantity s_{it} of each product $i \in I$ to ship at each time $t \in T$, starting inventory levels d_i^A and d_i^B at suppliers A and customer B, and y_t representing the number of vehicles. The optimization model aims to minimize costs.

$$\min \sum_{i \in I} h_i(d_i^A + d_i^B) + \sum_{t \in T} \frac{c}{H} y_t \tag{3.7}$$

$$\text{s.t.} \sum_{t \in T} s_{it} = q_i H \qquad \forall i \in I \tag{3.8}$$

$$\sum_{i \in I} v_i s_{it} \leq y_t \qquad \forall t \in T \tag{3.9}$$

$$d_i^A + q_i t - \sum_{k=0}^{t} s_{ik} \geq 0 \qquad \forall i \in I, \forall t \in T \tag{3.10}$$

$$d_t^B + \sum_{k=0}^{t} s_{ik} - q_i(t+1) \geq 0 \qquad \forall i \in I, \forall t \in T \tag{3.11}$$

$$d_i^A, d_i^B \geq 0 \qquad \forall i \in I \tag{3.12}$$

$$s_{it} \geq 0 \qquad \forall i \in I, \forall t \in T \tag{3.13}$$

$$y_t \geq 0 \qquad \forall t \in T \tag{3.14}$$

The objective function 3.7 minimizes the total inventory and transportation costs. Constraint 3.8 ensures complete shipment for each product, while equation 3.9 determines the number of trucks required. Inequalities 3.10 and 3.11 represent the stock-out constraints, and equations 3.12 to 3.14 define the non-negative decision variables. In the context of De Heus, routing also occurs with direct shipping due to bag customers picking up feed at one location and bulk customers that can be served with only one trip. This mirrors the problem described by (Bertazzi & Speranza, 2002), where only one customer and one depot are used. What is introduced are the time frame and the inventory at the depot level which can also be applied to the model of De Heus. The De Heus model should be able to serve multiple customers with direct shipping, so network-flow constraints need to be added to enable this capability.

3.7 Multi-Commodity Network Flow Problem

After determining the necessity of network-flow constraints for implementing direct shipment to multiple customers, the solution involves utilizing the multi-commodity network flow problem (MCNF problem). This problem describes a network where multiple commodities need to be transported from designated origin nodes to destination nodes, with the arcs connecting these nodes restricted by capacities (Salimifard & Bigharaz, 2022). The node-arc model, connecting nodes through arcs to form a network as described by (Wang, 2003), entails several components. N represents a non-empty set of n nodes, while A signifies a non-empty set of all arcs, and K denotes the non-empty set of all commodities, defining their respective origins (s_k, supply points) and destinations (t_k, demand points). The capacity of arc (i, j) in the network is denoted by u_{ij}, with c_{ij} representing the cost per unit flowing through the arc (i, j). Furthermore, x_{ij}^k signifies the flow from node i to node j, and d^k represents the total demand for commodity k.

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{3.15}$$

$$\text{s.t.} \sum_{(i,j) \in K} x_{ij}^k - \sum_{(i,j) \in K} x_{ji}^k = \begin{cases} 0 & \text{if } i \in N \backslash \{s_k, t_k\}, \\ +d^K & \text{if } i = s_k, \\ -d^K & \text{if } i = t_k. \end{cases} \qquad \forall i \in N, \forall k \in K \tag{3.16}$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \qquad \forall (i,j) \in A \tag{3.17}$$

$$x_{ij}^k \geq 0 \qquad \forall (i,j) \in A, \forall k \in K \tag{3.18}$$

The objective function 3.15 aims to minimize the total shipping costs. Constraint 3.16 represents the supply and demand constraint, with positive values at the origin nodes and negative values at the destination nodes in the network. Constraint 3.17 addresses the capacity constraint, while Constraint 3.18 imposes non-negativity. These supply and demand constraints can be applied to the De Heus model, where factories serve as supply nodes and customers as demand points. Depots, however, illustrate commodities that are neither demand nor supply points, requiring incoming and outgoing arcs to balance each other.

3.8 Production Routing Problem: Production-Inventory-Transportation

After discussing the lot-sizing problem and the inventory routing problem separately, combining these models offers an additional advantage due to the optimized flow of items through the supply chain (Chandra & Fisher, 1994). The lot-sizing problem does not incorporate the routing decision, while the inventory routing problem does not address decisions at the production site. The production routing problem (PRP) integrates these models to optimize the entire supply chain, aiming to minimize total costs by integrating tactical and operational decisions on lot-sizing, inventory, distribution, and vehicle routing (Lei, Liu, Ruszczynski, & Park, 2006). In the PRP, the plant must decide in each period whether or not to make the product and determine the corresponding lot size. Production incurs fixed setup costs and a cost per product. The lot size is constrained by the production factory's capacity. Deliveries are made from the plant using a limited number of vehicles with specific capacities. Efficient routing of these vehicles fulfills demand and incurs transportation costs. Storing products in capacitated warehouses leads to inventory costs (Adulyasak et al., 2015).

The base model of PRP is given by Bard and Nananukul (2009a). It describes a model with one factory that produces a single product and has a capacity restriction on maximum production. The inventory replenishment policy is based on the maximum level (ML), which means that customer demand needs to be fulfilled but cannot be exceeded. The warehouse also has capacity restrictions. In the end, the distribution is done by a limited-capacity homogeneous fleet of trucks, all with the same capacity. The PRP network is defined in the complete directed graph $G = (N, A)$, where N is the set of plants and customers, and A is the set of arcs. The plant has node 0, and the customers are in the set $N_c = N\backslash 0$. T is the total time period, and K is the number of vehicles. The unit production cost is given by u, and there is a fixed production setup cost of f. h_i represents the holding costs at node i, and c_{ij} represents the transportation costs from node i to node j. The customer demand d_{it} represents the demand of customer i in period t. The constraints on capacity are given by C for production and Q for vehicles. L_i is the maximum or target inventory level, and I_{i0} is the initial available inventory. The decision variables are as follows: p_t for the production quantity in period t, I_{it} for the inventory at the end of period t, y_t equals 1 if there is production at the plant in period t; otherwise, it is 0. z_{0t} represents the number of vehicles leaving the plant, and z_{it} equals 1 if a customer is visited. x_{ijt} equals 1 if the route from i to j is used in period t, q_{it} represents the quantity delivered to customer i, W_{it} represents the load of a vehicle before making a delivery to customer i in period t. Further M_t and $\tilde{M}_{it}$ are given by $M_t = \min\left\{C, \sum_{j=t}^{l} \sum_{i \in N_c} d_{ij}\right\}$ and $\tilde{M}_{it} = \min\left\{L_i, Q, \sum_{j=t}^{l} d_{ij}\right\}$.

$$\min \sum_{t \in T} \left(up_t + fy_t + \sum_{i \in N} h_i I_{it} + \sum_{(i,j) \in A} c_{ij} x_{ijt} \right) \tag{3.19}$$

$$\text{s.t. } I_{0,t-1} + p_t = \sum_{i \in N_c} q_{it} + I_{0t} \qquad \forall t \in T \tag{3.20}$$

$$I_{i,t-1} + q_{it} = d_{it} + I_{it} \qquad \forall i \in N_c, \forall t \in T \tag{3.21}$$

$$p_t \leq M_t y_t \qquad \forall t \in T \tag{3.22}$$

$$I_{0t} \leq L_0 \qquad \forall t \in T \tag{3.23}$$

$$I_{i,t-1} + q_{it} \leq L_i \qquad \forall i \in N_c, \forall t \in T \tag{3.24}$$

$$q_{it} \leq \tilde{M}_{it} z_{it} \qquad \forall i \in N_c, \forall t \in T \tag{3.25}$$

$$\sum_{j \in N} x_{ijt} = z_{it} \qquad \forall i \in N_c, \forall t \in T \tag{3.26}$$

$$\sum_{j \in N} x_{jit} + \sum_{j \in N} x_{ijt} = 2Z_{it} \qquad \forall i \in N, \forall t \in T \tag{3.27}$$

$$z_{0t} \leq m \qquad \forall t \in T \tag{3.28}$$

$$w_{it} - w_{jt} \geq q_{it} - \tilde{M}_{it}(1 - x_{ijt}) \qquad \forall (i,j) \in A, \forall t \in T \tag{3.29}$$

$$0 \leq w_{it} \leq Q z_{it} \qquad \forall i \in N_c, \forall t \in T \tag{3.30}$$

$$p_t, I_{it}, q_{it} \geq 0 \qquad \forall i \in N, \forall t \in T \tag{3.31}$$

$$y_t, x_{ijt} \in \{0, 1\} \qquad \forall i, j \in N, \forall t \in T \tag{3.32}$$

$$z_{it} \in \{0, 1\} \qquad \forall i \in N_c, \forall t \in T \tag{3.33}$$

$$z_{0t} \in \mathbb{Z}^+ \qquad \forall t \in T \tag{3.34}$$

The objective function 3.19 minimizes the total production, setup, inventory, and routing costs. Constraints 3.20 and 3.21 represent the inventory flow balance at the plant and customer levels. Constraint 3.22 enforces the setup and production capacity. Constraints 3.23 and 3.24 limit the maximum inventory levels at the plants and customers, controlled by the Maximum Level (ML) policy (Archetti, Bertazzi, Laporte, & Speranza, 2007). Constraint 3.25 allows only positive delivery quantities, while constraint 3.26 ensures that each customer is visited only once. Constraints 3.27 maintain vehicle flow conservation. Constraints 3.28 restrict the number of trucks, and constraints 3.29 involve vehicle loading restrictions and sub-tour elimination based on the Miller-Tucker-Zemlin inequalities (Miller, Zemlin, & Tucker, 1960). Lastly, constraints 3.30 impose vehicle capacity constraints. Constraints 3.31, 3.32, 3.33 and 3.34 are the non-negativity, binary and positive integer variables.

The PRP has various papers focusing on differentiations at the production, inventory, or transportation levels. Different solution methods have also been used to solve the PRP within reasonable time frames. Chandra and Fisher (1994) introduced the PRP with capacitated single plant and multi-product characteristics. Instances were solved using a decomposition model, resulting in promising cost reductions. However, research progress on the integrated problem was relatively slow due to its complexity and limitations, leading to high computational time. Fumero and Vercellis (1999) introduced a Lagrangian relaxation method that could obtain a lower bound of the feasible solution, marking a significant step in the right direction. Another substantial advancement came from Lei et al. (2006), who developed a two-phase approach with a load consolidation algorithm. This approach tackled the problem of multiple plants producing a single item and transporting it to multiple distribution centers using a homogeneous fleet. When discussing the PRP under various inventory policies, namely the maximum level (ML) and the order-up-to level (OU), Y. Lee et al. (2022) introduces the PDP. They solve the oxygen supply chain problem using both their own trucks and third-party logistic trucks, employing a decomposition heuristic. This heuristic breaks down the problem into higher and lower-level problems, optimizing one problem and using its output as input for optimizing the other. The De Heus problem can depend on the constraints regarding the flow of finished goods, allowing for inventory buildup during different time steps. The objective function aims to minimize total costs in production, inventory, and transportation, which aligns with De Heus's goals. Routing constraints are not relevant for De Heus because only direct delivery is considered.

3.9 Literature overview

An overview of the literature for this thesis is presented in Table 3.1. The studies on the lot-sizing problem, the inventory routing problem, and the production routing problem are consolidated in a single table, analyzed based on the characteristic aspects of the supply chain. The different characteristics of the various models are combined at the end to define the situation at De Heus as accurately as possible. First, the meanings of the different symbols are explained, followed by the presentation of the table. A discussion is provided at the end of the section.

For the production aspect, the number of plants (*N.Plants*) is denoted as a single plant (S) or multiple plants (M). Similarly, the number of products (*N.Products*) optimized in the model is indicated as single product (S) or multiple products (M). Various models of the Lot-Sizing Problem offer specific extensions (*Extension*) that could be integrated into the model. Extensions in the production aspect encompass joint setup time and costs (JST), reduced production capacity due to setup times (SUT<Cap), batch production (BP), joint replenishment by family groups of products (JR), and piecewise linear costs based on different production amounts (PLC). A checkmark(✓) denotes whether the production is capacitated (*Cap.*). Regarding the inventory aspect, the inventory policy (*Policy*) is described by a maximum level (ML) or an order-up-to

level (OUL). Extensions in inventory management include time windows for delivery (TW), allowance of backlogging (BO), and the rolling horizon (RH) of the models. A checkmark indicates if the inventory is capacitated (*Cap.*).The distribution segment is characterized by the fleet (*Fleet*), classified as homogeneous (Hom.) when all trucks are identical or heterogeneous (Het.) when trucks have different characteristics. The number of trucks can be unlimited (∞), limited (Lim.), a specific multiple amount of trucks (M), or a single truck (S). A checkmark is provided if the trucks are capacitated (*Cap.*).

The solution method can be exact (E), heuristic (H), or computing a lower bound (L). Various methods include Decomposition, Lagrangian relaxation, Variable Neighborhood Search (VNS), Branch-and-Bound, dynamic programming, Wagner-Whitin, Silver-Meal, Mixed-Integer Programming (MIP), Branch-and-Cut, and Tabu Search. These solution methods are given in the next Section 3.10

Table 3.1: Literature review

	Production				**Inventory**			**Transportation**			**Solution method**	
Author	*N.Plants*	*N.Products*	*Extension*	*Cap*	*Policy*	*Extension*	*Cap*	*Fleet*	*N.Vehicles*	*Cap*	*Type*	*Approach*
Veinott Jr. (1969)	M	S	JST		ML		✓				H	Decomposition
Hindi (1995)	S	M	SUT<Cap		ML		✓				L/H	Lagrangian relaxation/VNS
Van Vyve and Ortega (2004)	S	S	BP		ML		✓				H	Dynamic programming
Stowers and Palekar (1997)	S	M	JR		ML		✓				L/H	Lagrangian relaxation/Decomposition
Bhatia and Palekar (2001)	S	M	JR		ML		✓				L/H	Lagrangian relaxation/VNS
Shaw and Wagelmans (1998)	S	S	PLC		ML		✓				H	Dynamic programming
Jaruphongsa et al. (2004)	S	S			ML	TW	✓				H	Dynamic programming
Zangwill (1966)	S	S			ML	BL	✓				H	Dynamic programming
Federgruen and Tzur (1993)	S	M			ML	BL	✓				E/H	Branch-and-Bound/Decomposition
C.-Y. Lee et al. (2001)	S	S			ML	TW	✓				H	Dynamic programming
Kazan et al. (2000)	S	S			ML	RH	✓				H	Wagner-Whitin/Silver-Meal/MIP
Bitran et al. (1981)	S	M			ML	RH	✓				H	Decomposition
Bertazzi and Speranza (2002)					ML		✓	Hom.	∞	✓	E	Branch-and-Bound
Bertazzi and Speranza (2012)					ML/OUL		✓	Hom.	∞	✓	H	*(proposed)*
Chandra and Fisher (1994)	S	M		✓	ML			Hom.	∞	✓	H	Decomposition
Lei et al. (2006)	M	S		✓	ML		✓	Het.	Lim.	✓	H	Decomposition
Adulyasak et al. (2015)	S	S		✓	ML/OUL			Hom.	M	✓	E/H	Branch-and-Cut/ALNS
Bard and Nananukul (2009b)	S	S		✓	ML		✓	Hom.	Lim.	✓	H	Tabu Search
Fumero and Vercellis (1999)	S	M		✓	ML			Hom.	Lim.	✓	L/H	Lagrangian relaxation
Archetti et al. (2007)	S	S			ML/OUL			Hom.	S	✓	E/H	Branch-and-Cut/MIP
Y. Lee et al. (2022)	M	S			ML			Hom.	M	✓	H	Decompostion
This work	M	M	JST/SUT<Cap/JR	✓	ML	RH	✓	Hom.	∞	(✓)	H	(Decomposition)

Table 3.1 provides information on the lot-sizing model, inventory routing problem, and the production routing problem, which combines the first two models. The table includes information about the production, inventory, and transportation aspects. The last column provides information about the solution methods used to solve the models.

The production part, the model of this Thesis involves multiple plants (M) and different SKUs for animal species with various growth phases at different production locations. Extensions required include joint set-up times (JST) for SKUs belonging to the same recipe, reduced production capacity due to increased SKUs causing overtime (SUT<CAP), and joint replenishment for SKUs within the same recipe (JR). De Heus factories have limited production capacity. For inventory, De Heus operates a finished goods warehouse limited by a specific area translated into a certain tonnage capacity. Inventory levels can be as high as possible as long as the warehouse's total capacity is not exceeded (ML). De Heus follows a recurring monthly cycle where the output inventory from week 4 becomes the input inventory for week 1, requiring consideration of a rolling horizon (RH). Time windows are not considered due to undefined delivery times and no backordering allowance. For transportation, De Heus focuses on vehicle characteristics used for transshipments and direct

farm deliveries. Transshipments utilize a third-party logistics company, implying an uncapacitated system with an unlimited fleet of vehicles. Direct deliveries prioritize bulk capacity, already inclusive of truck capacity. In both cases, there is a homogeneous fleet (Hom.) with unlimited vehicles (∞), wherein transshipments have no capacity restrictions, while direct deliveries align with bulk capacity. Finally, the proposed solution method for this thesis is a heuristic. If it is not possible to solve the problem within a reasonable time frame, a decomposition heuristic is employed. This approach is explained in Section 3.10

3.10 Solution methods

If solving the De Heus problem within a reasonable time frame is not feasible, solution methods can be employed. Various solution methods, detailed in Table 3.1, are utilized for determining lower bounds, providing exact solutions, or employing heuristics to solve larger instances. Not all of the solution methods are elaborated upon; only the models most likely to be used in solving the problem for De Heus are discussed.

3.10.1 Lower bounds

Lagrangian relaxation is an approach used to obtain lower bounds by dualizing constraints with Lagrangian multipliers and decomposing (Adulyasak et al., 2015). In a modified version of the PRP, Fumero and Vercellis (1999) presented Lagrangian relaxation. They decomposed the PRP into smaller subproblems: Production, Inventory, Distribution, and Routing. Production and inventory problems were addressed using inspection, while a LP solver was employed for the distribution problem. The lower bound of routing was calculated by minimizing the cost network flow problem. Despite moderate problem sizes—8 periods, 12 customers, and 10 different products—the solution was achievable with an average gap of 5.5%

3.10.2 Exact solution

To achieve an exact solution for the PRP problem, a Branch-and-Cut algorithm can be utilized. Within the literature, few other methods exist to attain an exact solution (Adulyasak et al., 2015). The branch-and-cut algorithm presented by Archetti et al. (2007) involves a single uncapacitated vehicle, while the algorithm from Adulyasak et al. (2015) handles multiple capacitated vehicles. In the model by Archetti et al. (2007), an additional inequality is introduced, requiring that the inventory level in period $t-s-1$ must be sufficient to fulfill the demand in that period if no delivery occurs in the period $(t-s,t)$. This strengthens the lot-sizing part of customer replenishment, resulting in a better lower bound. The algorithm was tested on an instance with 14 customers and 6 periods, producing a solution within seconds.Adulyasak et al. (2015) extends the model of Archetti et al. (2007) by incorporating multiple vehicles. They also refined the formulation of the routing part and added the vehicle index, enhancing the symmetry-breaking constraints to disallow alternative solutions due to the homogeneous fleet of trucks. This algorithm successfully solved instances with up to 35 customers, 3 periods, and 3 vehicles within 2-hour.

3.10.3 Heuristics

To obtain a solution when encountering various problem instances, it is possible to design a heuristic method. The decomposition method, often utilized in several papers, acts as such a heuristic. Chandra and Fisher (1994) introduced this method by optimizing the lot-sizing problem and employing a 3-opt procedure for its enhancement. In a different approach, Lei et al. (2006) assumed direct shipments from plants to customers. In their second phase, decisions from the initial phase are fixed within the optimization model of the VRP heuristic. The paper by Y. Lee et al. (2022) presents a distinctive strategy involving a two-level solution approach. An upper level concentrates on determining optimal production, inventory decisions, and customer allocation to plants, disregarding routing decisions and considering only single customer routing. The lower level addresses multi-customer routing decisions, assuming fixed production and inventory decisions. For the upper level, the assumption is made that only one customer can be visited on each route. These models were solved using Gurobi. Archetti et al. (2007) designed a mixed-integer problem that decomposes the PRP into an uncapacitated lot-sizing problem and an inventory routing sub problem. They developed an algorithm that matches production amounts to demand and solves the inventory routing problem based on this input. After an initial solution is found, it attempts to shift production to different times to reduce inventory and transportation costs, restarting the algorithm if the best possible solution is reached. Bard and Nananukul (2009b) devised a tabu search where each iteration moves from the current solution to the best neighboring solution. They applied a reactive tabu search (RTS), creating the initial solution by solving the PRP without routing constraints, assuming equal transportation costs for the round trip. Subsequently, the routing was constructed using swap and move operators. Adulyasak et al. (2015) employed the adaptive large neighborhood search (LNS). They first created an enumeration scheme using two decomposed problems. In the second stage, two operators were chosen, alternating combinations of customer-period and removing/inserting them elsewhere. Thirdly, a minimum cost flow was used to minimize total costs. The algorithm halted upon reaching the maximum number of enumerations.

3.11 Multi-objective optimization

In addition to prioritizing cost minimization, it is also enticing to explore an alternative objective function. Customer satisfaction is one such objective that could be taken into account. However, optimizing costs and customer satisfaction simultaneously is not directly possible with the current model. Therefore, it is necessary to find a method capable of handling this situation These problems that require more than one objective function are called multi-objective optimization (Gunantara, 2018). Three different methods of multi-objective optimization are explained. First, the weighted sum method is analyzed because it is simple and easy to use. Second, the lexicographic method is analyzed because it prioritizes the objective functions. The third method is the ϵ–constraint method because it optimizes one problem while treating the other objective function as a constraint and it is capable of solving problems that are not convex (Gunantara, 2018).

These multi-objective methods are used to create a Pareto front. With the Pareto method, it is possible to keep each part of the solution separate and not mix them up during optimization. By examining dominant points, it is possible to determine which solutions are better and which ones are not. Dominant solutions usually occur when one objective cannot increase without reducing or increasing the other objective function (Gunantara, 2018). The solutions in this set are called Pareto optimal solutions. In the Pareto method, two terms need to be noted: the anchor point, which is obtained as the best of an objective function, and the utopia point, which is obtained as the combination of the best objectives from both objective functions. However, this utopia point is infeasible. The optimization of two objective functions can be described in a Pareto optimal front (POF) on a two-dimensional surface (Chong & Żak, 2013). An example of a POF can be seen in Figure 3.1. In this figure, there are dominated and non-dominated points on a two-dimensional axis. The non-dominated points are connected with a line called the Pareto front. The anchor points and the utopia point are also given. The methods provided are used to create this Pareto front, so in the next paragraphs, an analysis is conducted on which methods perform best in creating this front.

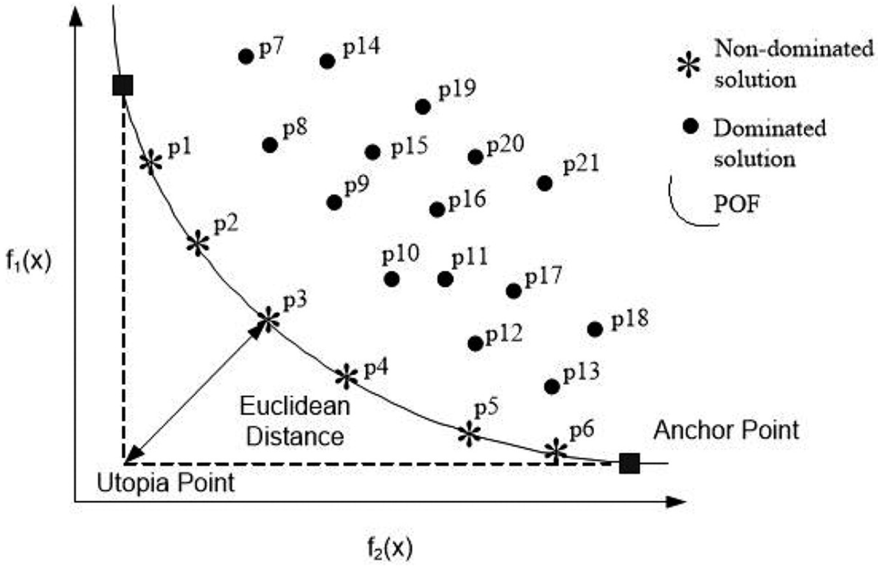


Figure 3.1: Pareto Optimal Front (POF) example (Gunantara, 2018)

The weighted sum method (WSM) is a traditional approach commonly used for resolving multi-objective models (Boyer, Sai Hong, Pedram, Mohd Yusuff, & Zulkifli, 2013). WSM combines two objectives into one by multiplying each objective by a weight provided by the decision maker. This technique transforms the objectives into a single dimension through a convex combination. Since $f_1(x)$ and $f_2(x)$ have different units, they are normalized, and a unified objective function is established by adding the weighted normalized objectives. This process converts the multi-objective model into a single-objective model, as given by Equations 3.35 and 3.36. The advantages of using WSM are its ease of application and implementation into your model. However, this method also has disadvantages. It is challenging to determine the weights, and the outcome of the optimization heavily depends on the chosen weights.

$$\min f(x) = \frac{w_1 \cdot f_1(x)}{f_1^{\cdot}(x) \cdot w_1 + w_2} + \frac{w_2 \cdot f_2(x)}{f_2^{\cdot}(x)} \tag{3.35}$$

$$w_1 + w_2 = 1 \tag{3.36}$$

The Lexicographic method, prioritizes objectives in a feasible set according to a lexicographic order, meaning that lower-priority objectives are optimized as long as they do not interfere with the optimization of higher-priority objectives (Isermann, 1982). Initially, the objective function with the highest priority, denoted as $f_1(x)$, is optimized. Subsequently, the solution of this optimization $f_1(X^{\cdot})$, is utilized in optimizing $f_2(x)$ under the condition $f_1(x) \leq f_1(X^{\cdot})$. An advantage of this method is that objective functions do not require normalization for comparability, and it is possible to create a Pareto front by optimizing the objective functions in the designated order. However, a disadvantage is the need for repeated optimization for each objective function, and the order of the objective functions in a lexicographic method is important for the optimization. It is more interesting to observe the relationship between the objective functions without any preferences.

The ϵ-constraint method is a technique in which one objective is optimized while the remaining objectives are translated into constraints and incorporated into the optimization process (Deb, 2001). Instead of altering the objective function alone to optimize multiple objectives simultaneously, this method involves optimizing multiple objective functions while restricting the optimization by adding constraints. This allows multi-objective optimization without a predefined sequence of importance. The simplest form of the ϵ-constraint method is represented by Equations 3.37 and 3.38.

$$\min f_2(X) \tag{3.37}$$

$$\text{s.t.} \quad f_1(X) \leq \epsilon, \quad X \in S \tag{3.38}$$

An improvement upon the ϵ-constraint method was introduced by Mavrotas (2009), termed AUGMECON. The conventional ϵ-constraint method calculates the bounds by simply maximizing or minimizing both functions and looking at the objective value of the other objective functions. The AUGMECON method proposes to create a payoff table, aiming to find non-dominant solutions for the other objective functions as well. This approach leads to narrower bounds, improving the density of the solution area and providing more meaningful results. It serves as an enhancement to the original ϵ-constraint method, which, alongside the weighting method, stands out as one of the most commonly utilized approaches for constructing representations of the Pareto front. The ϵ-constraint method offers certain advantages over the weighting method, particularly in scenarios involving discrete variables (such as Mixed Integer or Pure Integer problems). The AUGMECON method has been further enhanced, resulting in the introduction of AUGMECON2, which builds upon AUGMECON by incorporating information from slack variables in each iteration (Mavrotas & Florios, 2013). These enhancements primarily focus on reducing computation time by avoiding numerous redundant iterations. To outline the steps necessary for AUGMECON2, a simplified flowchart of the original AUGMECON2 algorithm is provided in Figure 3.2. The extended version, designed for scenarios with more than two objective functions, can be found in Appendix A

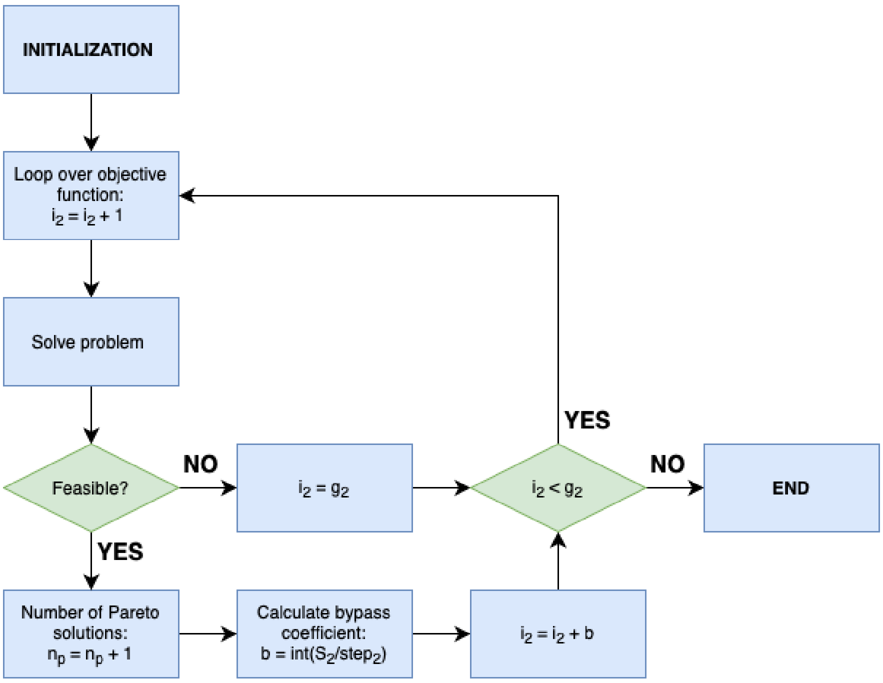


Figure 3.2: AUGMECON 2 flowchart with two objective functions.

The first step involves initialization. During this phase, a payoff table must be created. This table displays the maximum or minimum optimal value per objective function and the corresponding outcome of the other objective function, depending on whether minimization or maximization is the goal. These values serve as anchor points. Objective function $f_1(x)$ is utilized in the objective function, where $f_k(x)$ is transformed into a constraint. The minimum value is represented by fmin_k. With these anchor points established, the ranges r_k for the objective function can be determined. The $stepsize_k$ is computed by dividing the range r_k by the number of intervals g_k. To complete the initialization, the counters i_k and n_p need to be set to zero. In the second step, the loop over i_k is initiated. Since there are only two objective functions, it loops only over i_2. The multi-objective equations of AUGMECON2 that need to be solved during the iteration are given by equations 3.39 and 3.40.

$$\min \{f_1(x) + \text{eps} \cdot (S_2/r_2 + 10^{-1} \cdot S_3/r_3 + \cdots + 10^{-(p-2)} \cdot S_p/r_p)\} \tag{3.39}$$

$$\begin{aligned} \text{s.t. } & f_2(x) - S_2 = e_2 \\ & f_3(x) - S_3 = e_3 \\ & \vdots \\ & f_p(x) - S_p = e_p \qquad \forall p \in K \\ & x \in S \quad \text{and} \quad S_i \in \mathbb{R}^+ \end{aligned} \tag{3.40}$$

Where $f_1(x)$ is the objective function, e_p the parameters of the RHS for the specific iteration drawn from the grid points of the objective functions p. The parameters r_p are the ranges of the respective objective functions. S_p are the surplus variables of the respective constraints and $\epsilon \in \left[10^{-6}, 10^{-3}\right]$. For each objective function p, we calculate the objective function range. The RHS of the corresponding constraint in the i_k-th iteration in the specific objective function will be:

$$e_k = \text{fmin}_k + i_k \times \text{step}_k$$

where fmin_k is the minimum from the payoff table and i_k the counter for the specific objective function (Mavrotas & Florios, 2013).

If the problem is feasible the number of Pareto solution n_p is counted. After that it is checked if the surplus variable corresponds to the innermost objective function. In this case, it is the objective function with $k = 2$. Then, we calculate the bypass coefficient as:

$$b = \text{int}\left(\frac{S_2}{\text{step}_2}\right)$$

where *int()* is the function that gets a real number to its integer part. If the surplus variable S_2 surpasses the value of step_2, it implies that in the next iteration, the solution will remain identical except for the surplus variable, which will decrease by step_2. This makes the iteration redundant, allowing us to bypass it as it does not generate any new Pareto optimal solutions. The bypass coefficient b signifies the number of consecutive iterations we can skip. After the bypass calculation, the new value of i_2 is known. If the total number of iterations g_2 has not been reached yet, the algorithm continues until its value is reached, and the algorithm stops.

The AUGMECON2 method offers a systematic approach for applying the ϵ-constraint method more efficiently, thereby minimizing unnecessary iterations. Consequently, the results obtained are more meaningful, stemming from a comprehensive representation of the efficient set. AUGMECON2 achieves this by searching within the ranges defined in the payoff table. Depending on the running times of the De Heus model, it proves advantageous when the multi-objective optimization method efficiently reaches an optimal solution, free from biases introduced by manual inputs.

3.12 Conclusion

he problem at De Heus represents a variant of the production routing problem, where the routing aspect is replaced with direct delivery to customers. The model is based on the production routing problem, incorporating joint set-up times, reduced capacity, and joint replenishment from papers related to lot-sizing problems. Flow constraints from the multi-commodity network flow problem govern the flow of finished goods from factories to customers, with the option to store finished goods in warehouses at different time frames. The ability to make decisions on production levels, including stock numbers and transportation arcs, leads to a model that most accurately reflects the situation at De Heus.

Literature research on production, inventory, and transportation characteristics reveals that the problem faced by de Heus is closely related to the base model of the Production Routing Problem introduced by Bard and Nananukul (2009b). This problem involves determining the production volume for each week, managing inventory positions for every week, and planning the routing to customers. While Bard's model deals with a single factory, de Heus operates multiple factories. The study by Lei et al. (2006) involves multiple factories and pick-up locations, making its constraints applicable to our case. While the inventory component is based on Bard's model, which uses only a single SKU, de Heus incorporates multiple SKUs, as seen in Fumero and Vercellis (1999). Regarding production characteristics, insights from Lot-sizing problems are relevant, such as joint replenishment from Stowers and Palekar (1997) and total production capacity reduction from Hindi (1995). Capacity constraints for production and warehouse, as well as flow constraints. Transportation, particularly for transshipments to depots and factories, remains relatively unrestricted, whereas bulk trucks are limited by their capacity. The routing aspect of Bard's model can be omitted and replaced by additional flow constraints based on the principles outlined by Wang (2003).

For multi-objective optimization, three different methods are analyzed. Firstly, the weighted sum method is introduced due to its simplicity, where the objective functions are assigned weights and optimized simultaneously. However, because determining the weights heavily influences the optimization outcome, this method is less suitable. The lexicographic method prioritizes the objective functions based on their importance. An advantage is that it does not require normalization for comparability. However, implementing a dedicated order of importance can be challenging for multi-objective optimization, particularly for de Heus, as they aim to investigate the relationship between both objective functions, which are valued evenly. Lastly, the ϵ-constraint method is introduced, which presents the objective functions as constraints. The AUGMECON2 method enhances the ϵ-constraint method by reducing running time to avoid unnecessary iterations and narrowing the solution space by considering non-dominating anchor points. Notably, the AUGMECON2 method does not impose any order of importance, making it the most suitable multi-objective optimization method for de Heus.

It needs to be investigated whether the problem instance of De Heus is too big to execute the optimization within a reasonable time. If it is not possible to solve it exactly within this timeframe, the design of a heuristic becomes necessary. The decomposition heuristic appears well-suited to this situation, as it enables breaking down the problem into smaller parts. These parts are optimized individually in sequence to derive the best possible overall solution.

4 Problem description and solution approach

In this section, the conceptual model is explained, the mathematical model is presented, a toy problem is illustrated to explain the mathematical model and the algorithm and the objective function of the multi-objective optimization are introduced. Section 4.1 explains the conceptual model in words, first providing general remarks. Afterward, the incorporation of the company requirements for demand, production, inventory, and transportation is explained. At the end, a list of constraints, requirements, and assumptions is provided. Section 4.2 introduces the sets, indices, parameters, and decision variables. The objective function is presented, and each constraint is explained. Section 4.3 illustrates the outcome of the mathematical model. Section 4.4 provides the objective functions and algorithm necessary for multi-objective optimization. Section 4.5 summarizes the findings and concludes what is necessary from the problem description for the experimental design and results.

This section addresses the research question, *'How should the solution approach be designed?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(a) How is the optimization problem to be solved?

(b) What are assumptions and requirements of the solution approach?

4.1 Conceptual model

The problem faced by De Heus can be described as a production assignment problem, where the routing aspect of the original production routing problem is simplified to customer assignment and includes characteristics from lot-sizing problems such as joint replenishment and capacity reduction due to flushing times. This problem is defined on a complete directed graph $G = (N, A)$, where N represents the set of factories I, transshipment warehouses J, pick-up warehouses K, and customers L. A is the set of arcs defined as $(i, j) : i \in I, j \in J, i = j$; $(j, k) : j \in J, k \in K$; $(k, l) : k \in K, l \in L$. Over a finite set of time periods W, there are P different productions which are produced from one of the N recipe product groups. The products can be classified as P^{bu} for bulk products and P^{ba} for bag products, and further classified as P^{dh} for De Heus products and P^{ndh} for Proconco/ANCO products.

Figure 4.1 illustrates the transportation route of the finished goods. The process begins with production at factory i, where the feed is exclusively produced for its corresponding transshipment warehouse j (where $i = j$). From the transshipment warehouse, the finished goods are transported to any pick-up warehouse k, where they are subsequently allocated to a customer l. These products may be either bag products, picked up by the customers themselves, or bulk products, delivered to the customers. Consequently, bulk products can only be supplied from the factory locations. The distance between factories and transshipment points is 0 because they are located at the same position, whereas the distance between the transshipment warehouse and the pick-up warehouse is represented by the internal distance matrix idm_{jk}, and from the pick-up warehouse to the customer by the external distance matrix edm_{kl}. These distances are determined based on the shortest traveling distance.

Each customer l has a demand d_{lp}^{w} for each product p for every week w in tons, which is divided over the week according to a certain percentage of the total sales volume sv_{p}^{w}. The production at the factories is capacitated c_i, as is the bulk capacity cb_i. This production capacity decreases as the amount of production increases; the amount of production runs is based on the average run size bs_n, which helps to determine the number of runs, multiplied by the capacity loss los to calculate the remaining capacity. In the factories, it is not possible to produce all types of feed; certain product groups are allowed at a factory $pgaf_{ip}$, and the brand that is allowed in the factory is indicated by $bpaf_i$. The warehouse capacity at the factories and depots is given by whc_k and is calculated based on the average length of stay of goods that are only transshipped to other locations $lost_p$ and the average length of stay of goods that are sold from that warehouse $losp_p$.

The model aims to minimize the total cost, which consists of the sum of direct variable costs $fdlc_i$, direct material costs $fdmc_i$, factory overhead costs foc_i, fixed production set-up costs $fpsc_i$, holding costs h, internal transportation costs $tcif$ / $tcid$, external transportation costs tce, and bulk transportation costs tcb. This minimization takes place while considering the requirements and assumptions described in Section 4.1.1.

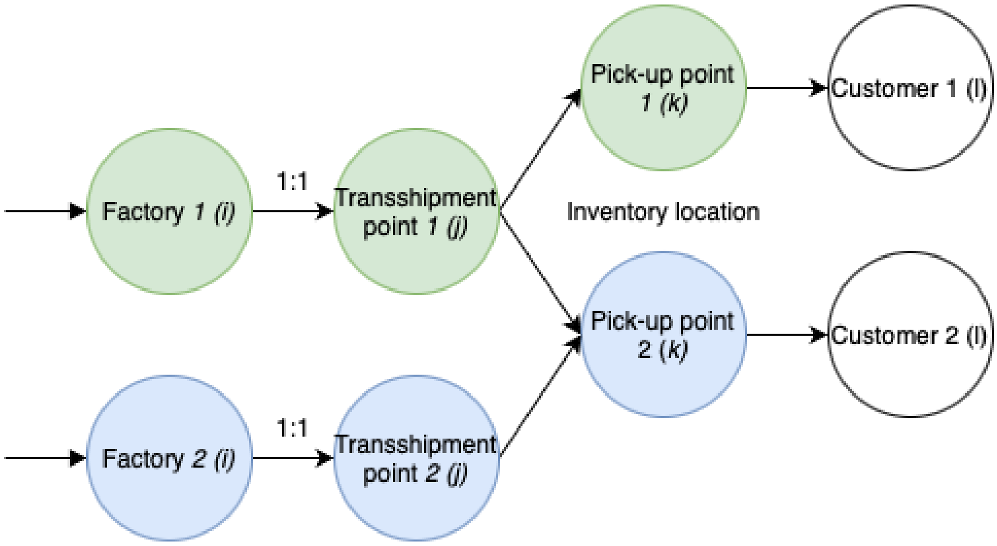


Figure 4.1: Diagram netwerk factory, transshipment and pick-up points

4.1.1 Requirements & assumptions

Before creating the mathematical model, it is useful to provide an overview of the requirements, and assumptions made to facilitate the model. The requirements are given by:

- The monthly total demand is divided over four weeks. Every week, the demand needs to be met within the period. So no backorders are allowed.
- Customers are served from the same location every week to maintain constant quality.
- Customers are only assigned to one pick-up location or are only served from one factory by bulk truck to maintain constant quality.
- A product sold from a location can only be produced in one factory to maintain constant quality.
- De Heus products can only be produced in De Heus factories.
- Specific product groups can only be produced in specific factories.
- Bulk can only be transported directly from the factory and cannot be stored for the next period.
- Production and warehouse capacities are limited.
- Bulk can only be produced at specific factories and is directly sold from these locations.
- Transshipment warehouses are used to keep track of products that are produced at a factory and are directly moved to another factory/depot.

- The model needs to calculate the number of production runs required to fulfill the demand and eventually add feed to stock.

The assumptions required for the model are as follows:

- It is assumed that the production is first-time right, which means that the model does not take rework into account.
- A fixed production run size is employed to estimate the number of production runs, resulting in them being merely counted rather than optimized. Consequently, the model does not aim to optimize production runs to achieve maximum capacity utilization, as it is relatively straightforward to extend the run length in practice but quite complex to do so during optimization in a model. One drawback of this approach is that small production runs are not considered to be full production runs (e.g., one run), requiring a flush. Thus, based on the total volume of high, medium, and low volume products at the factory, the model is capable of estimating the number of production runs necessary to produce this total volume.
- The capacity for transshipments between the transshipment warehouses and pick-up warehouses is unlimited because they are performed by a third-party logistics company.
- All customers can be convinced with a discount to drive to another location.
- SKU setup times are negligible because the delay at the bagging stage is insignificant, owing to the presence of multiple filling stations. So there is no time loss between the switch of different SKUs within the recipe.
- There are always enough raw materials to produce the feed.
- If a customer has both bag and bulk demand, all demand can only be fulfilled from a factory where bulk can be produced. Since a customer can only be assigned to one location, it must be a location where bulk feed can be produced.

4.2 Mathematical formulation

The mathematical formulated gives the production assignment problem which is based on the problem of Bard and Nananukul (2009b), which serves as the foundational model for the production routing problem. However, the routing components are transformed to direct customer assignment. Extensions have been developed to address specific cases, such as those related to De Heus.

4.2.1 Sets & Indices

The indices are created to refer to different elements. These distinct sets represent specific nodes in transportation, defining the product or recipe product group, or indicating the time period. These sets consist of elements representing the transportation network, products, product groups, recipes, and time periods. The subsets cover specific parts of a set based on particular characteristics.

Sets

I	Set of factories i where $i = \{1, .., I\}$
D	Set of depots d where $d = \{1, .., D\}$
J	Set of transshipment warehouses j where $j = \{1, .., J\}$
K	Set of pick-up warehouses k where $k = \{1, .., I \cup D\}$
L	Set of customers l where $l = \{1, .., L\}$
P	Set of products p where $p = \{1, .., P\}$
N	Set of recipe product groups n where $n = \{1, .., N\}$
W	Set of time periods in week w where $w = \{1, .., W\}$

Subsets

P^{bu}	Subset of products p which are bulk p^{bu} where $p^{bu} \subseteq \{1, .., P\}$
P^{ba}	Subset of products p which are bag p^{ba} where $p^{ba} \subseteq \{1, .., P\}$
P^{dh}	Subset of products p which are De Heus p^{dh} where $p^{dh} \subseteq \{1, .., P\}$
P^{ndh}	Subset of products p which are Proconco/ANCO p^{ndh} where $p^{ndh} \subseteq \{1, .., P\}$

4.2.2 Parameters

The parameters provide input for the model. All the necessary information is known in advance and is either directly stored in the model or used to calculate the required data.

Parameters

d_{lp}^{w}	$l \in L$, $p \in P$, $w \in W$	Demand from customer l for product p in week w in tons
$fdlc_i$	$i \in I$	Direct variable costs per ton at factory i
$fdmc_i$	$i \in I$	Direct material costs per ton at factory i
foc_i	$i \in I$	Factory overhead costs at factory i
$fpsc_i$	$i \in I$	Fixed production set-up costs at factory i
ii_pk	$p \in P$, $k \in K$	Initial inventory of product p at pick-up location k
sv_p^w	$p \in P$, $w \in W$	Percentage of sales volume from total volume for product p in week w
h		Holding costs
$tcif$		Transportation costs intern from factory to factory per km
$tcid$		Transportation costs intern from factory to depot per km
tce		Transportation costs external per km
tcb		Transportation costs bulk per km
idm_{jk}	$j \in J$, $k \in K$	Internal distance matrix from from transshipment warehouse j to pick-up warehouse k in km
edm_{kl}	$k \in K$, $l \in L$	External distance matrix from pick-up warehouse k to customer l in km
$sedm_{kl}$	$k \in K$, $l \in L$	Subtracted shortest distance external distance matrix from pick-up warehouse k to customer l in km
c_i	$i \in I$	Capacity of factory i in tons
cb_i	$i \in I$	Bulk capacity of factory i in tons
los		Capacity loss due to flushing time between production runs
pgp_p	$p \in P$	Product group of product p
$pgaf_{ip}$	$i \in I$, $p \in P$	Product group allowance of product p at factory i
$bpaf_i$	$i \in I$	Brand production allowance in factory i

$lost_p$	$p \in P$	Average length of storage period before transshipped to other warehouse of product p
$losp_p$	$p \in P$	Average length of storage period before complete sold out at pick-up warehouse of product p
whc_k	$k \in K$	Warehouse capacity at pick-up warehouse k in tons
bs_n	$n \in N$	Average production run size of recipe n
pnr_{np}	$n \in N$, $p \in P$	Recipe relationship matrix of recipe n and product p
M		'Big M' is a very large number

4.2.3 Decision variables & Auxiliary decision variables

The decision variables constitute the output of the model. The model's choices are utilized to provide advice to De Heus on enhancing their performance. The central decision within the model revolves around determining the quantity of product to produce at each factory and to which location the customers need to be assigned. While most variables are directly used to calculate the total cost of the objective function, other variables function as constraints to accommodate De Heus' specified limitations; these types of variables are referred to as auxiliary decision variables.

Variables

t^w_{ijp}	$i \in I$, $j \in J$, $p \in P$, $w \in W$	Transport from factory i to transshipment warehouse j of product p in week w in tons
x^w_{jkp}	$j \in J$, $k \in K$, $p \in P$, $w \in W$	Transport from transshipment warehouse j to pick-up warehouse k of product p in week w in tons
q^w_{klp}	$k \in K$, $l \in L$, $p \in P$, $w \in W$	Transport from pick-up warehouse k to customer l of product p in week w in tons
pd^w_{ip}	$i \in I$, $p \in P$, $w \in W$	Production amount in factory i of product p in week w in tons
bp^w_{in}	$i \in I$, $n \in N$, $w \in W$	Amount of produced production runs in factory i of recipe n in week w in tons
ip^w_{kp}	$k \in K$, $p \in P$, $w \in W$	Amount of inventory at pick-up point warehouse k at the end of week w in tons
y_i	$i \in I$	Indicator binary variable for overhead costs at factory i
r^w_{jkp}	$j \in J$, $k \in K$, $p \in P$, $w \in W$	Indicator binary variable for using transportation route from transshipment warehouse j to pick-up warehouse k of product p in week w
s^w_{klp}	$k \in K$, $l \in L$, $p \in P$, $w \in W$	Indicator binary variable for using transportation route from pick-up warehouse k to customer l of product p in week w
$slots^w_{kp}$	$k \in K$, $p \in P$, $w \in W$	Amount of slots used at pick-up point k for product p in week w

4.2.4 Objective function

The objective function 4.1 aims to minimize total costs throughout the entire supply chain, extending from production to customer transportation. The first segment calculates the following: (i) production costs per ton, aggregated across all factories i; (ii) holding costs for all inventory at pickup locations k; (iii) internal transportation costs per kilometer per ton for supplying pickup points (factory-factory and factory-depot) k from a transshipment warehouse j; (iv) external transportation costs, accounting for discounts offered to customers k who must travel to a pickup location l that is not the closest one; (v) bulk transportation costs per ton. The second segment calculates setup costs for production runs per week w, considering recipes under n associated with factory i. The third segment concludes the objective function by incorporating the overhead cost per factory i.

$$\begin{aligned}
\min \sum_{w\in W}\sum_{p\in P}\Bigg(&\sum_{i\in I}(fdlc_i + fdmc_i)\cdot pd^w_{ip} + h\cdot\Big(\sum_{k\in K} ip^w_{kp} + \sum_{k\in K}\sum_{l\in L} q^w_{klp}\cdot losp_p\cdot 0.5\\
&+\sum_{j\in J}\sum_{k\in K} x^w_{jkp}\cdot lost_p\cdot 0.5 - \sum_{k\in I} x^w_{kkp}\cdot lost_p\cdot 0.5\Big) + \sum_{j\in J}\sum_{k\in J} tcif\cdot x^w_{jkp}\cdot idm_{jk}\cdot 2\\
&+\sum_{j\in J}\sum_{k\in K\setminus J} tcid\cdot x^w_{jkp}\cdot idm_{jk}\cdot 2 + \sum_{k\in K}\sum_{l\in L} tce\cdot q^w_{klp}\cdot sedm_{kl}\cdot 2\cdot bb_p\\
&+\sum_{k\in K}\sum_{l\in L} tcb\cdot q^w_{klp}\cdot sedm_{kl}\cdot 2\cdot(1-bb_p)\Bigg) + \sum_{w\in W}\sum_{n\in N}\sum_{i\in I} fpsc_i\cdot bp^w_{in} + \sum_{i\in I} foc_i\cdot y_i
\end{aligned} \tag{4.1}$$

The basis of the objective function is derived from the model of Bard and Nananukul (2009b). The production, setup, inventory, and routing costs are adapted to the situation of De Heus. The production run costs are developed logically.

4.2.5 Constraints

Multiple constraints used to model the case of de Heus. These constraints serve various purposes within the model, encompassing limitations (such as production capacity), conditions (ensuring fulfillment of demand), and auxiliary elements (for instance, quantifying production run amounts). This section provides an explanation for each constraint, detailing its purpose and real-world significance.

Constraint 4.2 ensures that all products p produced at factory i are transported from the factory to the transshipment warehouse j. This condition must hold true for every week w.

$$pd^w_{ip} = \sum_{j\in J} t^w_{ijp} \qquad \forall i\in I, \forall p\in P, \forall w\in W \tag{4.2}$$

Constraint 4.3 ensures that all products entering transshipment warehouse j must be transported to a pickup point warehouse k. As per the model, inventory at the transshipment warehouse is not permissible. However, constraint 4.14 will account for the occupation of the warehouse by these goods. This condition must be satisfied for every week w.

$$\sum_{i\in I} t^w_{ijp} = \sum_{k\in K} x^w_{jkp} \qquad \forall j\in J, \forall p\in P, \forall w\in W \tag{4.3}$$

Constraint 4.4 ensures that bulk goods can only be delivered to the pickup warehouse k of factory i. Bulk feed can only originate from the factory where it is produced. Since this rule is implemented within the same model handling bagged feed (which can be supplied from various locations), a constraint is necessary to enforce this limitation. This constraint applies exclusively to products p categorized as bulk; these products are in the subset P^{bu}. The variable x^w_{jjp} indicates that transshipment warehouse j is identical to the pickup warehouse j (normally k). This condition must be met for every week w.

$$\sum_{i\in I} t^w_{ijp} = x^w_{jjp} \qquad \forall j\in J, \forall p\in P^{bu}, \forall w\in W \tag{4.4}$$

Constraints 4.5 and 4.6 describe the flow equations at the pickup warehouses k. Incoming goods originate from transshipment warehouse j and are supplemented by inventory from the previous week w. For week 0, the inventory is initially set in the optimization as ii_{pk} and is specified in Constraint 4.5. For subsequent weeks w, the inventory is represented as $ip_{kp}^{(w-1)}$ and is detailed in Constraint 4.6. Outgoing goods are directed to customer l, while any remaining goods are retained as inventory ip_{kp}^{w}. These conditions must be fulfilled for every product p.

$$ii_{pk} + \sum_{j \in J} x_{jkp}^{w} = \sum_{l \in L} q_{klp}^{w} + ip_{kp}^{w} \qquad \forall k \in K, \forall p \in P, w = 0 \tag{4.5}$$

$$ip_{kp}^{(w-1)} + \sum_{j \in J} x_{jkp}^{w} = \sum_{l \in L} q_{klp}^{w} + ip_{kp}^{w} \qquad \forall k \in K, \forall p \in P, w > 0 \tag{4.6}$$

Constraint 4.7 ensures that all products p produced at factory i are sent to the transshipment warehouse i (originally j), which corresponds to the factory of production. The transshipment warehouse serves as the hub from which products are distributed to other warehouses or transferred to the finished goods warehouse at the factory location, and it is essential for calculating warehouse occupation. This condition must be met for every week w.

$$pd_{ip}^{w} = t_{iip}^{w} \qquad \forall i \in I, \forall p \in P, \forall w \in W \tag{4.7}$$

Constraints 4.8 and 4.9 ensure that the demand d_{lp}^{w} from customer l is met from pick-up point k. The demand in week w is contingent on the sales volume sv_{p}^{w} and results in stable demand for bulk products and increasing demand for bag products. Customers with bagged products P^{ba} can be supplied from any pick-up location, as outlined in 4.8. However, customers purchasing bulk products P^{bu} can only be supplied directly from the factory, restricting the pick-up location to the set of factories I within the summation, as detailed in 4.9.

$$\sum_{k \in K} q_{klp}^{w} \geq d_{lp}^{w} \cdot sv_{p}^{w} \qquad \forall l \in L, \forall p \in P^{ba}, \forall w \in W \tag{4.8}$$

$$\sum_{k \in I} q_{klp}^{w} \geq d_{lp}^{w} \cdot sv_{p}^{w} \qquad \forall l \in L, \forall p \in P^{bu}, \forall w \in W \tag{4.9}$$

Constraint 4.10 defines the production capacity constraint for factory i. The total production at a factory must not exceed its capacity c_i. This capacity is influenced by the number of production runs bp_{in}^{w} produced at the factory due to flushing los. The factory's capacity begins at 100% and decreases through the multiplication of the production run quantity and the associated loss per production run. This condition applies to every week w.

$$\sum_{p \in P} pd_{ip}^{w} \leq c_i \cdot (1 - \sum_{n \in N} bp_{in}^{w} \cdot los) \qquad \forall i \in I, \forall w \in W \tag{4.10}$$

Constraint 4.11 ensures that factory i can only produce products p permitted within the product group of the product pgp_p. The binary variable $pgaf_{ip}$ is set to 1 if the product group can be produced in the factory; otherwise, it is set to 0. The large number M represents an arbitrary large value. This condition applies to every week w.

$$pd_{ip}^{w} \leq M \cdot pgaf_{ip} \qquad \forall i \in I, \forall p \in P, \forall w \in W \tag{4.11}$$

Constraint 4.12 ensures that products p with the De Heus brand that are in subset P^{dh} can only be manufactured in De Heus factories, denoted by $bpaf_i$ (set to 1). Due to quality distinctions between factories, De Heus aims to maintain high quality, achievable only by producing their products in a De Heus factory. This condition applies to every week w.

$$pd_{ip}^{w} \leq M \cdot bpaf_{i} \qquad \forall i \in I, \forall p \in P^{dh}, \forall w \in W \tag{4.12}$$

Constraint 4.13 defines the bulk capacity constraint for factory i. The total bulk production at a factory must not exceed its capacity cb_i. Only the bulk products, represented by bb_p (set to 0 for bulk), are summed to calculate the utilized capacity. This condition applies to every week w.

$$\sum_{p \in P} q_{klp}^{w} \cdot (1 - bb_p) \leq cb_i \qquad \forall i \in I, \forall w \in W \tag{4.13}$$

Constraints 4.14, 4.15, 4.16,and 4.17 specify the warehouse capacity at the pick-up warehouse k. The total warehouse capacity for bag products P^{ba} at the pick-up location must not exceed its limit whc_k which is given the maximum amount of slots $slots_{kp}^{w}$. Warehouse occupation arises from two factors: the average storage duration of products at the pick-up warehouse $losp_p$ and the average duration of transshipment goods staying there $lost_p$. The total occupation of a warehouse adjacent to the factory $k \in I$ comprises three parts: (i) Inventory ip_{kp}^{w} at the finished goods warehouse; (ii) products sold at the factory warehouse q_{klp}^{w} (iii) products transshipped from the factory's transshipment warehouse j to pick-up location p in x_{kjp}^{w}; (iV) minus the products transshipped from the factory's transshipment warehouse to the same factory's pick-up warehouse x_{kkp}^{w}. Subtracting these products is necessary as they are already accounted for in part (i), as detailed in 4.14. Depot warehouses that are situated at a factory side $k \in D$ do not contain transshipment goods; hence, only inventory contributes to the occupation, as stated in 4.15. The translation from the used amount of slots to the warehouse capacity is given in 4.16. Additionally, assuming that bulk feed P^{bu} cannot be stored implies that bulk products cannot have inventory, as described in 4.17. These conditions apply to every week w.

$$\sum_{p \in P^{ba}} \left(ip_{kp}^{w} + \sum_{l \in L} q_{klp}^{w} \cdot losp_p \cdot 0.5 + \sum_{j \in K} x_{kjp}^{w} \cdot lost_p \cdot 0.5 - x_{kkp}^{w} \cdot lost_p \cdot 0.5 \right)$$
$$\leq slots_{kp}^{w} \cdot 6 \qquad \forall k \in I, \forall w \in W \tag{4.14}$$

$$\sum_{p \in P^{ba}} \left(ip_{kp}^{w} + \sum_{l \in L} q_{klp}^{w} \cdot losp_{p} \cdot 0.5 \right) \leq slots_{kp}^{w} \cdot 6 \qquad \forall k \in D, \forall w \in W \tag{4.15}$$

$$\sum_{p \in P^{ba}} slots_{kp}^{w} \leq whc_{k}/6 \qquad \forall k \in K, \forall w \in W \tag{4.16}$$

$$\sum_{p \in P^{bu}} ip_{kp}^{w} \leq 0 \qquad \forall k \in K, \forall w \in W \tag{4.17}$$

Constraint 4.18 ensures that the factory overhead costs in the objective function are applied only when there is production pd_{ip}^{w} at factory i. In this scenario, the binary variable y_i is set to 1. M represents a large number. The overhead costs are applied for the entire planning horizon.

$$\sum_{w \in W} \sum_{p \in P} pd_{ip}^{w} \leq M \cdot y_{i} \qquad \forall i \in I \tag{4.18}$$

Constraint 4.19 computes the minimum quantity of production runs bp_{in}^{w}, each consisting of bs_n tons of feed per production run, required to fulfill the production quantity of the products in the recipe pd_{ip}^{w} and pnr_{np}. The production run size depends on the overall production volume of recipe n, resulting in larger production run sizes for higher volumes and smaller production run sizes for lower volumes. These conditions are applicable to every factory i and for every week w.

$$bs_{n} \cdot bp_{in}^{w} \geq \sum_{p \in P} pd_{ip}^{w} \cdot pnr_{np} \qquad \forall i \in I, \forall n \in N, \forall w \in W \tag{4.19}$$

Constraints 4.20 and 4.21 ensure that the demand for a product p from a customer l can only be fulfilled by a single pick-up warehouse k. De Heus aims to maintain consistent feed quality, achievable by supplying the customer from the same factory. The auxiliary variable s_{klp}^{w} is employed in Constraint 4.20 to tally the number of different pick-up warehouses, ensuring it does not exceed one in Constraint 4.21. These conditions apply for every week w.

$$\sum_{w \in W} q_{klp}^{w} \leq M \cdot \sum_{w \in W} s_{klp}^{w} \qquad \forall k \in K, \forall l \in L, \forall p \in P \tag{4.20}$$

$$\sum_{w \in W} \sum_{k \in K} s_{klp}^{w} \leq 1 \qquad \forall l \in L, \forall p \in P \tag{4.21}$$

Constraints 4.22 and 4.23 ensure that products p needed at pick-up warehouse k can only be fulfilled by a single transshipment warehouse j. The auxiliary variable r_{jkp}^{w} is employed in Constraint 4.22 to tally the number of different transshipment warehouses, ensuring it does not exceed one in Constraint 4.23. These conditions apply for every week w.

$$\sum_{w \in W} x_{jkp}^{w} \leq M \cdot \sum_{w \in W} r_{jkp}^{w} \qquad \forall j \in J, \forall k \in K, \forall p \in P \tag{4.22}$$

$$\sum_{w \in W} \sum_{j \in J} r^w_{jkp} \leq 1 \qquad \forall k \in K, \forall p \in P \qquad (4.23)$$

Constraints 4.24, 4.25 and 4.26 provides the definition of the variables. Where Constraint 4.24 gives the non-negativity, Constraint4.25 gives the binary and Constraints 4.25 give the positive integer.

$$t^w_{ijp}, x^w_{jkp}, q^w_{klp}, pd^w_{ip}, bp^w_{in}, ip^w_{kp} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall p \in P, \forall w \in W, \forall k \in K, \forall l \in L, \forall n \in N \qquad (4.24)$$

$$y_i, r^w_{jkp}, s^w_{klp} \in 0, 1 \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall p \in P, \forall w \in W, \forall l \in L \qquad (4.25)$$

$$slots^w_{kp} \in \mathbb{Z}+ \qquad \forall k \in K, \forall p \in P, \forall w \in W \qquad (4.26)$$

Constraints 4.2, 4.3, and 4.4 are derived from the flow constraint of the model by Bard and Nananukul (2009b) and Wang (2003). However, they are extended from 2 to 3 constraints because transshipment warehouses are added. The concept of handling multiple products in these and subsequent constraints is derived from Fumero and Vercellis (1999), while the inclusion of multiple factories originates from the model of Fumero and Vercellis (1999) as well. Constraints 4.5 and 4.6 are based on Bard and Nananukul (2009b) but adjusted to account for the added locations. Constraint 4.7 is implemented by myself because it is necessary for the characteristics of transshipment warehouses. The demand in 4.8 and 4.9 is based on the distribution throughout the month, and in the previous constraints, it is replaced by q^w_{klp}, which represents the arcs from the pick-up warehouses to the customers. This implementation is developed by myself. Constraint 4.10 is a capacity constraint with reduction due to set-up times and is based on Hindi (1995), with the relationship with joint set-up times coming from Stowers and Palekar (1997) and adjusted by myself for the structure with an average run size based on the classification of sales volume and recipes. Constraints 4.11, 4.12, and 4.13 are implemented by myself and are necessary for the specific characteristics of the problem at De Heus. The warehouse capacity constraints 4.14, 4.15, 4.16, and 4.17 are developed by myself. After analyzing the situation at De Heus with transshipment goods and selling goods, the length of stay is determined and used to calculate the occupation. Additionally, working with slots instead of capacity is implemented by myself. Constraint 4.18 for overhead costs comes from Bard and Nananukul (2009b), while Constraint 4.19 is designed by myself to count the number of production runs. Constraints 4.20, 4.21, 4.22, and 4.23 are designed by myself to meet the product quality requirements at De Heus.

4.3 Toy problem illustration

An illustrative solution of the model is presented in Figure 4.2. This toy problem shows three factories, two depots, four bag customers, and two bulk customers. The problem presents a solution for a recipe consisting of two products, one bag product, and one bulk product. The factories are represented by the blue blocks, the depots by the green diamonds, the bag customers by red circles, and the bulk customers by orange circles. The dotted lines represent internal transportation routes, while solid lines represent customers picking up the feed at the location or bulk trucks driving to customers. The reason for not switching the direction of the lines where customers pick up the feed is that it also represents the flow of the finished goods. If the direction were changed, it would not be clear anymore how the finished goods flow. All possible solutions that could be used by the optimization model are presented in the figure.

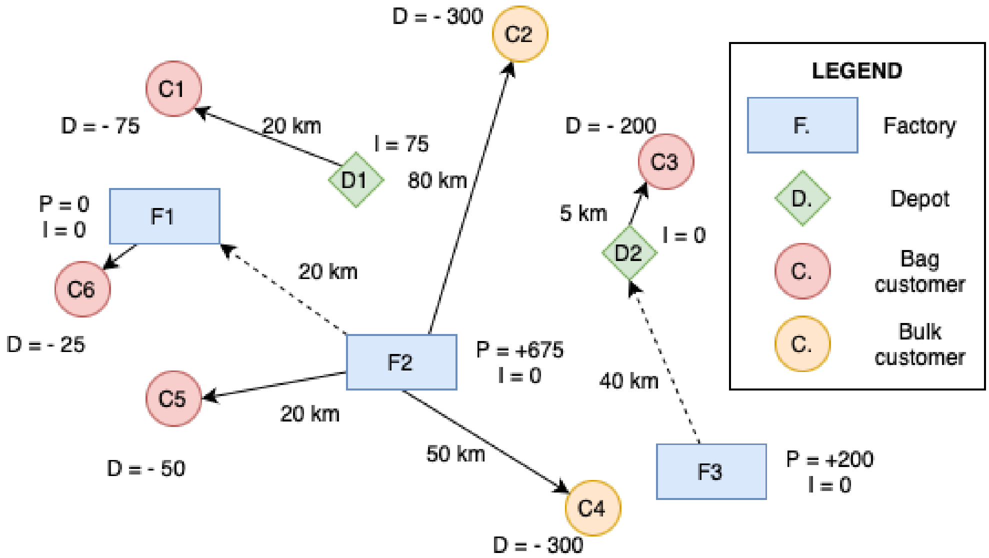


Figure 4.2: An illustration of the toy instance solution of the model

This model presents the solution for a specific week of the month, where each customer exhibits a particular solution within the given requirements and constraints. Customer 1 demonstrates a scenario where demand is fulfilled from the inventory of depot 1. In period $w-1$, the model maintained some stock in the depot to meet the demand in week w. Although Factory 1 is closer to Customer 1 than Depot 1, due to the optimization being for a single product, it may be necessary for this product to also come from Depot 1 if other products already originate from there. This constraint arises because only one factory/depot is allowed to serve the customer, even if Factory 1 is closer for that specific product. Customer 2, being a bulk customer, is served directly from a factory. If this customer were a bag customer, it would have been possible to serve them from Depot 1, but bulk customers can only be served directly from a factory. Customer 3 is served from Depot 2, which is restocked by Factory 3. This situation may occur if the warehouse capacity limit of Factory 2 is already reached, and because the average warehouse occupancy is lower for transshipment goods, it could be advantageous to transfer the goods to a depot for sale from that location. Customer 4 is served from Factory 2, even though Factory 3 is closer and capable of producing the same product. However, Factory 3 cannot produce bulk feed, so it is not possible to serve Customer 4 from that factory. Customer 5 is served directly from the factory. Additionally, a customer can be served from a factory warehouse. Customer 6 is served from the warehouse of Factory 1. Although Factory 1 itself is unable to produce this product, perhaps due to limitations in the production of the type of feed or the brand, it is still possible to serve the customer from that location because the factory has a warehouse that can be supplied by another factory.

Calculating the costs of the objective function is challenging due to the difficulty in determining warehouse occupation for a specific week. Nonetheless, it is still possible to compute the costs for the other components of the objective function. From this solution, we know that 675 products were produced at factory 2 and 200 at factory 3. Assuming that the direct variable costs and the direct material costs together are \$10 per ton for factory 2 and \$12 per ton for factory 3, the production costs are calculated as follows: $675 \cdot \$10 + 200 \cdot \$12 = \$9,150$. Factory overhead costs are only taken into account if the factory was in production; for this product, factory 1 was not producing, indicating that it was not in production at all. The overhead costs for factory 2 are \$5000 and for factory 3 are \$7500, totaling \$12,500. The fixed production set-up costs depend on the production run size, which, in turn, depends on the sales volume. Assuming that this recipe is a high-volume recipe, the average run size is 36 tons. If the flushing costs are \$8 at factory 2 and \$10 at factory 3, the calculation is as follows: $(675/36) \cdot \$8 + (200/36) \cdot \$10 = \$205.56$. The internal transportation costs are calculated as $(20 \cdot 25 \cdot 2 + 40 \cdot 200 \cdot 2) \cdot \$0.03 = \$510$, and the external transportation costs are $(20 \cdot 75 \cdot 2 + 5 \cdot 200 \cdot 2 + 20 \cdot 50 \cdot 2 + 25 \cdot 3 \cdot 2) \cdot \$0.031 = \$221.65$. The bulk transportation costs are calculated as $(80 \cdot 300 \cdot 2 + 50 \cdot 300 \cdot 2) \cdot \$0.011 = \$858$. Since it is difficult to determine the holding costs for one week for one product, this part is not included in this toy calculation. Adding all the costs together, the total cost for this product for one week is calculated as follows: $\$9,150 + \$12,500 + \$205.56 + \$510 + \$221.65 + \$858 = \$23,445.21$.

The different customer assignments to factories or depots provide insights into the choices and limitations of the model. The cost calculation also provides an overview of how the costs are minimized by the model.

4.4 Multi-objective optimization

The current model focuses solely on minimizing total costs. In this model, external transportation kilometers are translated into costs and added to the cost function. The total number of kilometers is multiplied by a cost factor, assumed to represent the discount needed to persuade customers to drive longer distances. These costs are also estimated based on the scale advantage that De Heus has when transporting finished goods closer to the customer. However, for De Heus, it is difficult to estimate the exact factor needed because it could vary depending on the situation; some customers are easier to convince than others. To address this issue, external transportation costs could be excluded from the objective cost function and reinterpreted as a new objective function called 'Customer Satisfaction.' Customers prefer to minimize the distance traveled as it is more cost-effective for them, but this preference conflicts with cost minimization because it requires accepting less optimal production and transportation planning. This is due to having fewer opportunities to assign customers to pick-up locations, which results in higher costs for internal transportation, production costs, etc. To simultaneously optimize cost minimization and minimize the extra kilometers driven by customers, a multi-objective optimization approach is required. For this analysis, the AUGMECON2 method for multi-objective optimization is utilized. To apply this method, two objective functions are considered: 1. The cost function excluding the 'external' driving costs of the customers, and 2. The quantity of 'extra' kilometers. These objective functions are represented by Equations 4.27 and 4.28.

$$
\begin{aligned}
f_1(x) : \min \; & \sum_{w \in W} \sum_{p \in P} \Bigg(\sum_{i \in I} (fdlc_i + fdmc_i) \cdot pd^w_{ip} + h \cdot \Big(\sum_{k \in K} ip^w_{kp} + \sum_{k \in K} \sum_{l \in L} q^w_{klp} \cdot losp_p \cdot 0.5 \\
& + \sum_{j \in J} \sum_{k \in K} x^w_{jkp} \cdot lost_p \cdot 0.5 - \sum_{k \in I} x^w_{kkp} \cdot lost_p \cdot 0.5 \Big) + \sum_{j \in J} \sum_{k \in J} tcif \cdot x^w_{jkp} \cdot idm_{jk} \cdot 2 \\
& + \sum_{j \in J} \sum_{k \in K \setminus J} tcid \cdot x^w_{jkp} \cdot idm_{jk} \cdot 2 + \sum_{k \in K} \sum_{l \in L} tcb \cdot q^w_{klp} \cdot sedm_{kl} \cdot 2 \cdot (1 - bb_p) \Bigg) \\
& + \sum_{w \in W} \sum_{n \in N} \sum_{i \in I} fpsc_i \cdot bp^w_{in} + \sum_{i \in I} foc_i \cdot y_i
\end{aligned} \tag{4.27}
$$

$$
f_2(x) : \min \; \sum_{w \in W} \sum_{p \in P} \sum_{k \in K} \sum_{l \in l} q^w_{klp} \cdot sedm_{kl} \cdot 2 \cdot bb_p \tag{4.28}
$$

In Section 3.11 a flowchart is given of the AUGMECON2 that needs to be followed to apply AUGMECON2 on two objective function. This flowchart is translated into an algorithm that needs to be defined during the experimental phase in the next chapter. This algorithm is given in Algorithm 1, the steps of these algorithm and the bypass value are explained in Section 3.11.

The problem solved during the multi-objective optimization in Algorithm 1 is defined in equations 4.29 and 4.30. In these equations, ϵ has a value of 10^{-5}, and the surplus S_2 is a variable.

Algorithm 1 AUGMECON2 algorithm

Initialization of loop and parameters

while $i \; < g_k$ **do**

$e_2 = \text{fmin}_k - i \cdot \text{step}_k$

Solve Problem with $f_1(x)$ *and* $f_2(x)$

$b = \text{int}\left(\frac{S_2}{\text{step}_2}\right)$

$i+ = 1 + b$

end while

$$\min \{f_1(x) + \epsilon \cdot (S_2/r_2\} \tag{4.29}$$

$$\text{s.t. } f_2(x) - S_2 = e_2 \tag{4.30}$$

4.5 Conclusion

The model developed is a production assignment problem based on the production routing problem from the literature, adjusted with characteristics from the lot-sizing problem, including joint replenishment and capacity reduction due to flushing times. The objective function aims to minimize the total costs of production per ton, production run cost, factory overhead costs, holding costs, internal transportation costs, external transportation costs, and bulk transportation costs. To accurately reflect the situation at De Heus, constraints are added to the model. Flow constraints secure the transportation lines from the production facilities to the customer, along with inventory opportunities. It ensure that bulk customers are only served from a factory, each customer can only receive finished goods from one location, and the resupply of a certain finished good can only be done by one factory. Capacity constraints for production, warehouse, and bulk are included, as well as limitations for factories in producing all types of feed and brands. Customer demand must be fulfilled every week of the month, with no backorders allowed. Some necessary assumptions include no rework being performed, incorporating a fixed production run size, and assuming customers are willing to go to every assigned location when compensated. This formulation of the objective function and constraints enables modeling of the actual case of De Heus, resulting in realistic results suitable for experimental analysis.

The multi-objective optimization objective functions focus on customer satisfaction, expressed in the extra kilometers customers need to drive. The first objective function aims to minimize costs without considering external transportation costs. The second objective function aims to minimize the sum of extra kilometers driven per ton.

5 Experimental design

In this section, the data instances are defined, and the experiments are designed. Section 5.1 introduces the current case dataset and data instances. Section 5.2 presents the experimental setup design, which is divided into two phases: Phase 1 optimizes the base model and Phase 2 contains the sensitivity analysis and multi-objective analysis. Section 5.3 summarizes the findings and provides a clear overview of the experiments.

This section addresses the first part of answering the research question, *'How does the solution approach perform compared to the current situation?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(a) How to test the performance of the solution approach?

(b) What are the different experimental scenarios that should be tested?

5.1 Data instances

The model is designed to solve the problem instance given by the actual situation of De Heus in Vietnam. The current case of De Heus consists of the customers and factories located in the South-East region of Vietnam. This scenario comprises 3 factories, 3 depots, 1117 customers, and 494 SKUs, which are derived from 194 recipes. The customer locations are obtained from the ERP system and translated into latitude and longitude coordinates. Demand data is sourced from December 2023, as this provides the most realistic case for optimization. Using the average demand of 2023 would require including customers who are no longer ours and implementing SKUs that have been replaced by newly developed or combined SKUs, which makes it more valuable to use the latest known demand. For all experiments, a computer with an Apple M2 processor and 8GB of RAM is used. The experiments are performed using Gurobi version 11.0.0 in Python 3.11.5.

The model is designed to solve the problem instance of the actual situation of De Heus in Vietnam. Constraints are used to make the model applicable to this specific situation, including distinctions between bag and bulk customers, restrictions for different brands/products, and the use of depots. Currently, it is applied to the southeast region, but with new customer input from the north, middle, and Mekong regions, optimization could be extended to these areas as well. Additionally, it may be possible to apply the model to other business units of De Heus in different countries, although adjustments may be necessary due to differing characteristics. For example, in the Netherlands, all feed is supplied by bulk trucks to customers, and these trucks are allowed to visit multiple customers in a single trip, requiring adjustments to the model for that scenario. Business units in countries undergoing similar economic development processes bear the closest resemblance to the situation of De Heus in Vietnam, making them candidates for applying the model to other realistic scenarios, particularly those with smaller farms that have mostly bag demand.

To assess the model's performance under varying problem sizes, artificial instances from different regions are deployed. These instances simulate scenarios where the problem size differs, either smaller or larger than the original instance. These instances vary in the number of customers and factories. The problem instances are artificially created by generating random numbers to obtain X and Y coordinates and assigning pseudo-random demand to the customers, based on the actual demand of December 2023. The number of customers varies from 10, 20, 50, 100, 200, 500, 750, 1000, 1200, to 1300. The number of factories varies from 3 to 4, resulting in a total of $10 \cdot 2 = 20$ instances. The actual scenario consists of 1117 customers, 3 factories, and 494 SKUs. The problem instances differ from the actual situation in that they are plotted in an area ranging from -500 to 500 for the X and Y dimensions. The actual situation is based on the latitude and longitude coordinates of the factories, depots, and customers. The optimization concept remains the same, but the distances from customers to factories/depots are calculated using the Euclidean distance formula ($\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$) instead of driving distances determined by the shortest route in Google Maps. The problem instances provide an idea of the solution quality and solving time if the actual situation of De Heus is scaled up or applied to a different region/business unit

5.2 Experimental set-up

The data sets that contain the current case of De Heus and artificial data instances are created. An experiment setup needs to be designed to conduct a proper analysis of the current situation and also explore future opportunities that could be tested using the mathematical model. This experimental setup design is divided into two phases:

- Phase 1: Optimizing base case.
- Phase 2: Sensitivity analysis and multi-objective optimization.

An overview of the different phases and their components can be seen in Figure 5.1.

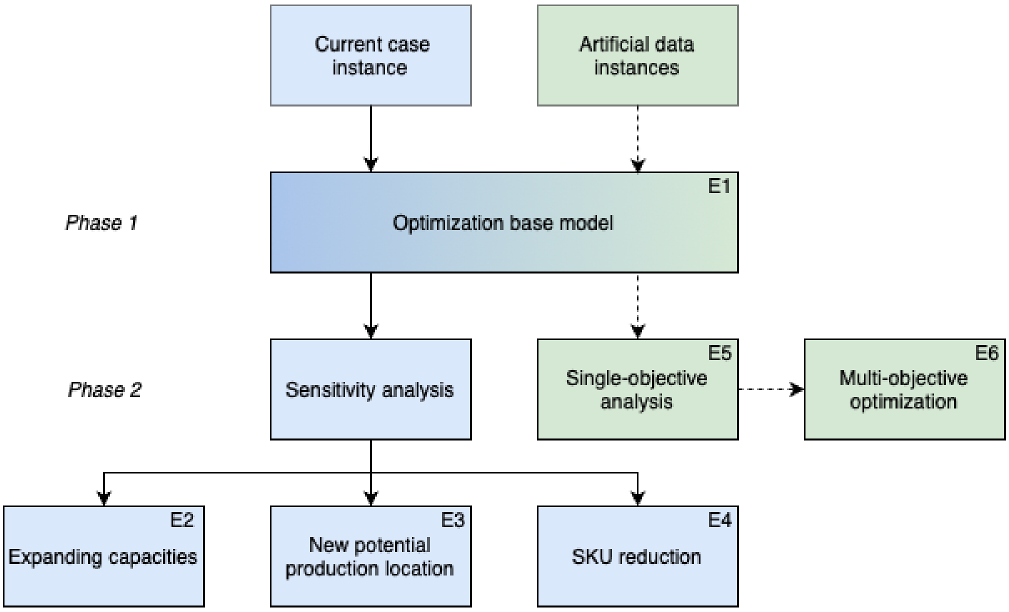


Figure 5.1: Experimental set-up

Within the phases, there are five different experiments labeled from *E1* to *E6*. Experiments *E1*, *E2*, *E3*, and *E4* are performed on the current case instance of De Heus, optimizing the base case and conducting a sensitivity analysis. This means that the outcome of these experiments could potentially be implemented directly. Experiments *E1*, *E5*, and *E6* are performed on artificial data related where customer locations and demand are artificial. These experiments show the performance of the model if the data instances change in the future for the actual situation. They also demonstrate the influence of using multi-objective optimization in comparison with single-objective optimization. The different experiments are listed in 5.1, with detailed explanations provided in 5.2.1 and 5.2.2.

5.2.1 Phase 1 - E1: Optimization base case

The optimization of the base case provides the most common solution for the current case, because it aims to use the current resources in the most cost-effective way, so without any further investments. This optimization is also performed on artificial data instances to demonstrate the quality of the model. The base case

Experiment ID	Title	Goal
E1	Optimization base case	The experiment consists of two parts: optimizing the base case with the real dataset and testing the performance of the base model on different artificial data instances. The outcome of optimizing the base case provides De Heus with the most common solution, which represents the most cost-effective way to utilize the current resources. The artificial cases are used to assess the performance of the model; these cases vary in the number of customers, factories, and SKUs. The goal is to obtain the optimal solution for the base case and gain insights into the model's performance.
E2	Expanding capacities	This experiment evaluates the expansion of the capacities of one of the factories in the current case situation. It provides insights into the expansion of production and warehouse capacities and also investigates a combination of both. The goal is to determine if it is cost-effective to invest in a capacity extension.
E3	New potential production location	This experiment evaluates three potential production locations to replace the old factory of Bien Hoa. The goal is to determine which location would be the most suitable as a new site for the future.
E4	SKU reduction	This experiment evaluates the influence of reducing the number of SKUs to two options: either two bag SKUs and one bulk SKU, or one bag SKU and one bulk SKU. The goal is to determine if reducing the number of SKUs alleviates production planning and leads to a more cost-effective situation.
E5	Single objective analysis	This experiment optimizes the different data instances for the individual objectives and shows the differences in computational time for the various sizes of the instances. The goal is to set up the single-objective optimization results to compare them with the multi-objective optimization.
E6	Multi-objective optimization	This experiment performs multi-objective optimization using the AUGMECON2 algorithm. This algorithm provides a Pareto front diagram for the different instances. The goal is to gain insights into the relationship between the two objective functions.

Table 5.1: Experimental design

optimization results in better utilization of resources, leading to lower total costs. This is also the goal of this optimization: to provide a solution that includes customer assignment to specific locations, in combination with production planning per week detailing which SKUs need to be produced at which location and how they reach the customer. In addition to modeling the actual situation, artificially created scenarios of customers and demand are used to assess the limitations of the model and provide insights into its performance. These scenarios instances differ from the base case in terms of size, location, and customer demand.

5.2.2 Phase 2 - Sensitivity analysis and multi-objective optimization

The second step is to utilize the optimized base case as the current optimum and conduct a sensitivity analysis to assess the influence of changes in the current situation. Questions from the company are translated into experiments, offering the opportunity to test if the solution found in Phase 1 remains effective and to determine what actions the company should take if the situation changes in the future. The last two experiments of this phase involve conducting single-objective analysis and multi-objective analysis to obtain insights into the relationship between the objective functions of total costs and customer satisfaction.

- *E2*: Expanding capacities
- *E3*: New potential production location
- *E4*: Reducing SKUs
- *E5*: Single-objective analysis
- *E6*: Multi-objective analysis

E2: Expanding capacities

In the current situation, some factories are reaching their limitations in production and warehouse capacity. For this reason, this experiment investigates the influence of expanding capacity at locations where it is possible. The production capacity of a factory is increased by a certain percentage to measure the impact on the final solution. Additionally, the warehouse capacity is extended to assess if the combination leads to even better performance. The different extensions of factory production or warehouse capacity will only be applied to the Dong Nai factory. The reasons for not applying it to the Bien Hoa and Bien Duong factories are that the Bien Hoa factory will be closed within four years, meaning that De Heus is not willing to make large investments in this factory anymore. As for the Bien Duong factory, it is not possible to expand at the current location due to limited space. Therefore, the experiments include capacity increments of 5% and 10%, which can be combined with a warehouse capacity increment of 250 tons (from 3,500 to 3,750 tons) or 500 tons (to 4,000 tons). The costs are evaluated and compared with the potential investment required to reach the proposed capacity.

E3: New potential production location

Currently, De Heus operates three different factories in the South-East region: Dong Nai, Bien Hoa, and Binh Duong. The factory in Bien Hoa is located in an area that the government of Vietnam plans to redevelop for a more high-tech industry in the coming years. Consequently, De Heus needs to find a new location to provide feed to customers because, due to capacity constraints, handling the task with only two existing factories is not feasible. Typically, there are two opportunities for De Heus to acquire a new factory location: acquiring an existing company and utilizing its factories to produce feed, or constructing a new factory in a chosen location. The first solution is unpredictable so the second opportunity is likely to be evaluated using the created model, as it enables the easy implementation of new potential factory locations and comparison of solutions across different factories to determine the best location. These factory locations are not randomly chosen; the government has already designated industrial zones where De Heus can build a new factory. Three of these locations are selected and analyzed in sub-experiments a, b, and c. Alongside the production manager of De Heus, potential locations for a new factory are examined, and the following provinces are chosen based on the current situation and anticipated future shifts in demand: Binh Duong, Binh Phuoc, and Dak Nong. While De Heus already has a factory in the Binh Duong province, it is currently operating at full capacity, making expansion unfeasible due to space constraints. Consequently, a new factory in that area could cater to customers in the region instead of transporting feed from factories located farther away.

The characteristics of the newly built factory can be found in 5.2. The capacity and production costs are the same for all factory locations except for the direct material costs. The direct material costs depend on the driving distance from the harbor to the production location (supply by barge is not possible for all locations). The Binh Duong province has a direct material costs of 4.91$ per ton, Binh Phuoc has 5.62$ per ton and Dak Nong has 17.23$. The location of Dak Nong is much farther from a harbor, resulting in a significant increase in inbound logistic costs. All costs are considered in consultation with the production managers, logistics managers, and operations officers

Factory	**Production capacity**	**Warehouse capacity**	**Bulk possibility**	**Direct var. prod. costs**	**Direct material costs**	**Fixed prod. set-up costs**	**Factory overhead costs**
New factory	18,250 ton	2,500 ton	3,650 ton	5.48$	4.91-17.23$	7,3$	45,990$

Table 5.2: New factory location capacity and costs.

E4: Reducing SKUs

Currently, De Heus produces 494 different SKUs due to the acquisition of Proconco/ANCO. This situation poses challenges in production planning and warehouse capacity, prompting De Heus to initiate efforts to reduce some SKUs by removing low-volume SKUs from their assortment. However, the actual impact of these reductions remains unclear, as only several SKUs are removed without calculating the overall results in total costs or observing it in optimized production planning. This experiment explores the possibility of merging SKUs that share the same recipe, effectively consolidating products with identical feeds. Collaboration with the nutritionist clarifies the relationship between recipes and SKU numbers, enabling a reduction in the number of SKUs per recipe. For example, a recipe that initially has five different SKU numbers (indicating the same feed for all SKUs) is reduced to only two or one SKU number to experiment with SKU reduction. It's important to note that no recipes are merged, as this requires well-considered decisions in consultation with the sales and nutrition teams, which is beyond the scope of this project. The presence of numerous SKUs increases warehouse occupancy because each SKU requires its own slot in the warehouse and production. Therefore, reducing the number of SKUs allows testing if this reduction in occupancy is cost-effective. Additionally, it can be observed if the reduction in the number of SKUs leads to changes in production planning. The experiments to be conducted involve two bag SKUs and one bulk SKU, as well as one bag SKU and one bulk SKU.

E5: Single-objective optimization

Before the multi-objective optimization can be applied to the artificial instances, it is necessary to test the artificial data instances on the individual objective functions used in the multi-objective optimization. Instead of optimizing based solely on total costs, customer satisfaction could be included as a factor. This would involve considering the inconvenience to customers, such as longer driving distances, not only in terms of cost but also in terms of overall satisfaction. Therefore, the objective functions would be: minimizing the total costs (without external transportation costs) and minimizing the extra kilometers driven by the customers. The artificial data instances that are used in the multi-objective optimization are also employed in the single-objective optimization. These instances vary the number of customers from 10, 20, 50, 100 to 200, while maintaining consistency in the number of factories and SKUs, with 3 factories and 494 SKUs, mirroring the actual case.

E6: Multi-objective optimization

For the multi-objective optimization, both objectives of total costs and minimization of the extra kilometers driven by the customers are optimized using the AUGMECON2 algorithm. During this algorithm the total extra driven kilometers per ton of feed are expected to be zero in theory. This is because all bag feed can be transported as close as possible to the customer, as there are no limitations on that part. However, in the model, there are still extra kilometers that need to be driven for some customers. The reason for this is that all customers can only receive feed from one location, which results in extra kilometers for customers with bulk and bag demand. The bulk feed demand can only be covered by factories capable of producing bulk, so customers can only be assigned to one of these locations. This means that bag feed demand is also assigned to that location, even if it is not necessarily the closest location for the customer. For example, a closer location could be a depot or a factory that is unable to produce bulk feed. This situation only occurs when a customer has bulk/bag demand. The restriction of De Heus and Proconco/ANCO feed can be resolved by transportation between the factories.

The optimization for the current case takes approximately 3 hours, which is impractical for generating a Pareto front with several iterations (around 10). Consequently, the AUGMECON2 multi-objective optimization is conducted on artificial data instances introduced in experiment *E5*, focusing on single-objective optimization. The payoff table necessary for initializing the AUGMECON2 algorithm is created during experiment *E5*, which involves single-objective optimization. In this experiment, the problem instances are solved with either $min\ f_1(x)$ or $min\ f_2(x)$, providing the values for the payoff table when the other objective is also minimized.

5.3 Conclusion

For the design of the experiments, the data instances are introduced. The real data, the current case instance, consists of 3 factories, 3 depots, 1117 customers, and 494 SKUs derived from 194 recipes. This real data instance is used to perform experiments with practical impact. The artificial data instances are used to test the performance of the model and to perform multi-objective optimization.

The first phase is about the optimization of the base model and it contains the first experiment, *E1*. In this first experiment, the base model is optimized. This means that within the given resources, the total costs are optimized to their minimum. For the current case, the objective function, the minimization of the total costs, is performed once on the current case dataset. For the artificial data instances, the datasets are used to show how the model is performing in comparison with the current case.

The second phase contains the sensitivity analysis and the multi-objective optimization. For the sensitivity analysis, the current case is used to perform experiments which have a relevant contribution to practice. Experiment *E2* involves expanding the production and/or warehouse capacity of the factory in Dong Nai. Experiment *E3* tests three new potential production locations to replace the factory in Bien Hoa. Experiment *E4* tests the influence of reducing the number of SKUs. These experiments yield outputs which could directly be used in the current organization (after some modifications) or could be used as a basis for an investment plan. The next part of phase two contains the single-objective optimization and the multi-objective optimization, which are performed on artificial data instances. Experiment *E5*, the single-objective optimization, provides the optimization of the total costs (without external transportation costs) and the extra kilometers driven by the customers separately. In experiment *E6*, multi-objective optimization, these single objectives are optimized in a multi-objective optimization using AUGMECON2. This multi-objective function is not performed on the current case because the optimization of the current case takes around 3 hours to get a solution, as AUGMECON2 requires several iterations (around 10) to create a Pareto front. Therefore, the algorithm is tested on smaller artificial instances which provide insights into its performance if the problem instances are increased.

Different experiments are conducted, where the basic experiment is to solve the current case, providing insights into how De Heus can improve itself with the current resources. The first part of phase two gives insights into the influence of changes that could result from an impactful change. The second part of phase two shows how the model would react if the focus is not only on minimizing the total cost but also on customer satisfaction. These experiments provide a wide and specific overview of the applications of the model.

6 Experimental results

In this section, the experimental results are analyzed, and conclusions are drawn based on these findings. Section 6.1 provides an explanation of the outcome data from the optimization process. Section 6.2 outlines the validation and verification procedures. Subsequently, Section 6.3 presents the results of experiment *E1*, which includes optimizing the base case for both the current scenario and artificial instances. Section 6.4 delves into the sensitivity analysis conducted in experiments *E2*, *E3*, and *E4*, along with the results of the single-objective optimization from experiment *E5* and the multi-objective optimization from experiment *E6*. Section 6.5 summarizes the findings and draws conclusions regarding the next steps.

This section addresses the research question, *'How does the solution approach perform compared to the current situation?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(b) How does the solution approach perform for the different scenarios?

(c) What insights does the sensitivity analysis on possibilities within the companies provide?

6.1 Results of the optimization

The optimization results are solved by the optimization model implemented in Python. The output provides an Excel sheet where the results are presented. This file contains the following information: total costs per factory and depot, a list per SKU per factory indicating how much should be produced in each week and in which factory, the assigned pick-up locations for the customers, the transshipments of SKUs from each factory to other factories or depots for every week, and the average warehouse occupation based on the amount of tons and the number of slots. This information is used to create three different overviews for comparing the different situations: Overview of total costs, overview of variables, overview of production, and a map showing customer assignments.

6.2 Validation and verification

The validation of the model occurs during its development. Each constraint added to the model is individually tested in an artificial environment to ensure that the expected changes are implemented correctly. For example, it is verified whether all bulk customers are assigned to factories capable of producing bulk products. To validate the larger model, an Excel file is created containing information about production volumes at factories, transshipment of finished goods, inventories, and customer assignments for each week. This data allows for a comprehensive review of the solution, verifying whether the model's decisions align with the given constraints and if they are logical. In addition to the Excel file, the model generates a map illustrating customer assignments as output. This visualization enables direct assessment of the logical coherence of customer assignments. The step-by-step approach of adding and verifying constraints, along with detailed data exports and customer assignment visualizations, facilitates ongoing validation of the model both before and after optimization.

The verification of the model is conducted in consultation with the production managers, the COO, and CEO. Initial results are discussed during individual meetings. After incorporating their feedback, the model and results are once again verified through a presentation covering the experiments that impact the current case. The optimization results of the base model closely match the actual situation in terms of production amounts, and the customer assignment provides logical options. Therefore, the model produces results that can be implemented in the real situation.

6.3 Phase 1 - Optimizing base case

E1: Optimizing base case - current case

The optimization for the base case is performed on the current case. This data instance is listed in Table 6.1 and provides information about the objective function, computational time, the gap, and the extra kilometers driven. The objective function represents the total costs of the optimization, while computational time refers to the running time of the model, limited to 3 hours (10,800 seconds). The gap indicates the percentage by which the objective function deviates from the lower bound, and the extra kilometers driven account for the additional distance customers travel to pick up feed from a location that is not the closest to them.The objective function is $1,097,765.73, which will be detailed explained in the coming paragraphs. The computational time is 10,800 seconds, indicating that the optimization was stopped before an optimal solution was reached, with a gap of 0.0193%. The extra amount of driven kilometers is 1,390,082.4 and is used to calculate the external transportation costs.

Data instance	**Objective function ($)**	**Computational time(s)**	**gap (%)**	**Extra driven KMs (km)**
'Current case'	$1,097,765.73	10,800 (max)	0.0193%	1.390.082,48

Table 6.1: Experiment *E1* - optimization performances (current case)

The differences in costs between the current situation and the optimized current case can be seen in Table 6.2. The production costs and factory overhead costs are almost the same. This is because the amount of tons produced in the different factories is similar for both situations. The factory overhead costs are the same for both situations because all factories are necessary in the total production volume, so it is not possible for the model to close one of the factories. The production set-up costs are reduced from $25,014.87 to $21,939.83 , which is a reduction of -12.3%. The reason for this reduction is that the feed is better distributed over the factories, which means that volumes are bundled together to reduce the amount of production runs, resulting in less switching time. The holding costs are reduced from $23,009.60 to $15,431.18, which is a reduction of -32.9%. Currently, the company is producing as much as possible to be sure that the warehouses are as full as possible, which leads to a service level where De Heus is always able to serve the customer as quickly as possible. This high availability in the amount of goods in stock leads to high holding costs, but according to the model, it is not necessary to have these high inventory levels to still meet all demand in time. Internal transportation costs can be reduced from $40,565.06 to $27,235.43, which gives a reduction of -32.9%, but the external transportation costs increase from $41,178.33 to $45,365.42 with 10.2%. The combination of the internal and external transportation costs is reached by assigning the customers to a more central pickup location. In general, this leads to the fact that customers need to drive a longer distance to pick up their feed, but they get compensated with a discount. A more optimal assigning of the customers to a pickup location leads to less internal transportation because the necessary feed is produced closer to the demand. The bulk transportation costs are decreased from $59,948.33 to $58,796.20, which gives a reduction of -1.9%. This reduction is obtained with a more optimal customer assignment. This gives in total that the costs are reduced from $1,118,028.47 to $1,097,765.94, which is effective -$20,262.53 (-1.8%) per month for the southeast region of Vietnam.

The current customer assignment and the optimized customer assignment can be seen in Figures 6.1a and 6.1b, respectively. It is evident that customers are currently assigned closer to pick-up locations than the proposed optimized situation suggests. This observation is supported by the transportation costs, wherein

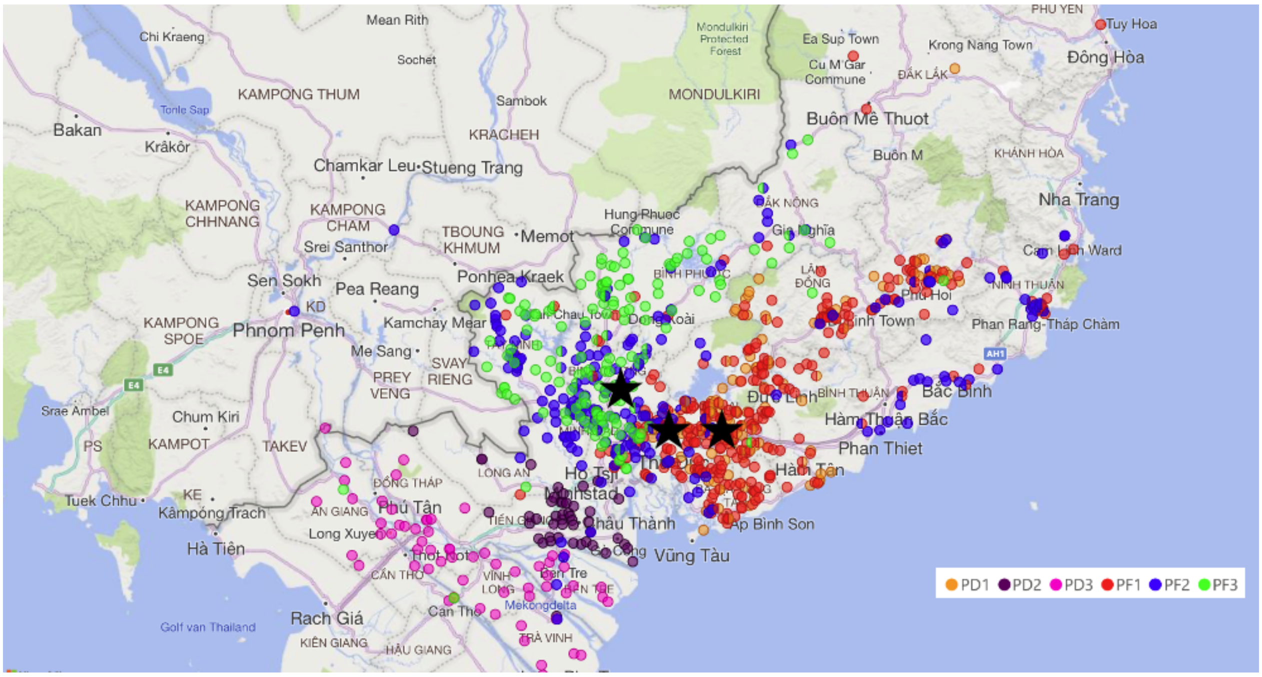
Costs name	Total costs (current) ($)	Total costs (optimized) ($)	dif ($)	%
Production costs	$754,827.45	$755,513.02	$685.57	0.1%
Factory overhead costs	$173,484.62	$173,484.62	-	0.0%
Production setup costs	$25,014.87	$21,939.82	-$3,075.04	-12.3%
Holding costs	$23,009.60	$15,431.18	-$7,578.41	-32.9%
Internal transportation costs	$40,565.06	$27,235.43	-$13,329.62	-32.9%
External transportation costs	$41,178.33	$45,365.42	$4,187.09	10.2%
Bulk transportation costs	$59,948.33	$58,796.20	-$1,152.12	-1.9%
TOTAL COSTS	$1,118,028.26	$1,097,765.72	-$20,262.53	-1.8%

Table 6.2: Overview total costs: Optimizing base case

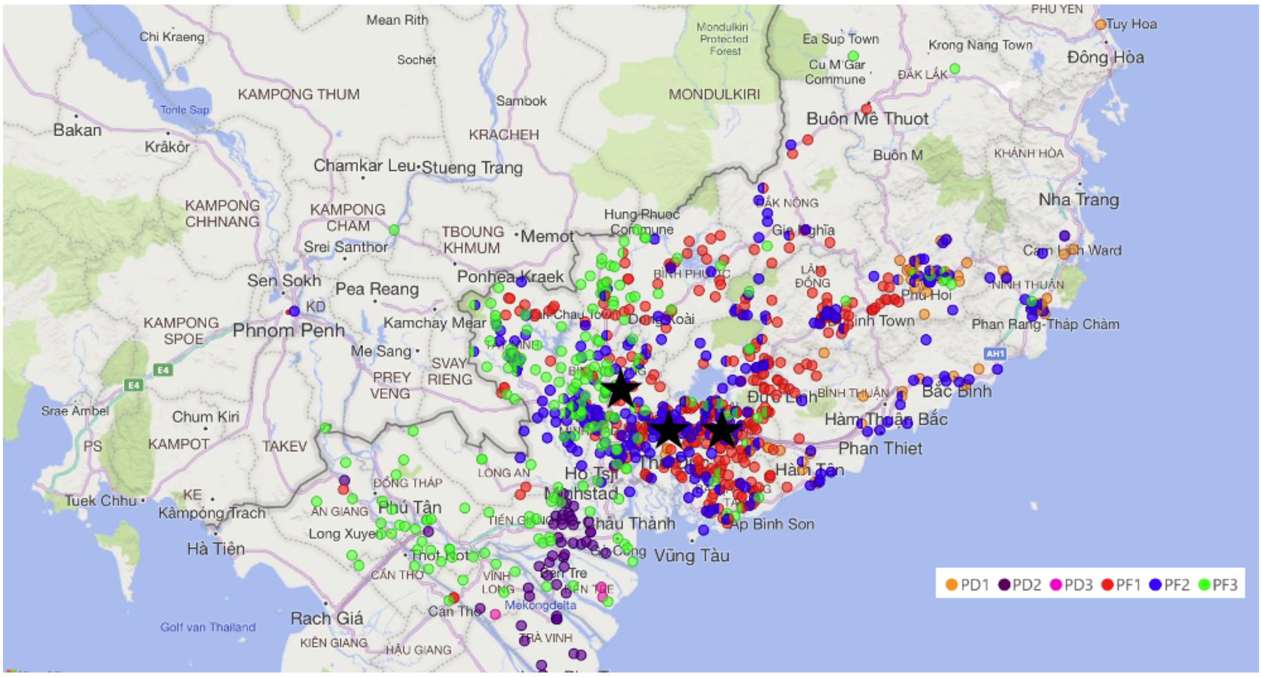
internal transportation costs decrease while external transportation costs increase, indicating that customers are being assigned to locations farther away. Currently, De Heus aims to deliver feed as close as possible to the customer in almost all cases. However, in an optimal situation, it could be more beneficial to produce specific types of feed at certain locations closer to the customer. This may result in some customers, who are farther away from these locations, needing to drive a longer distance. Not all data points in the figure represent the same demand. Therefore, for the model, it is more advantageous to relocate relatively small customers to pick-up locations further away, instead of larger customers, because the cost for driving distance is calculated per kilometer per ton of feed.

The most significant differences in volumes that are shifted to other locations can be observed in pigs (PB, PP, PG, and PG) and chicken white broiler (CWB) feed. For the model, it is only possible to produce piglet feed at the Binh Duong factory. In reality, some small volumes are still produced in the Dong Nai factory and Bien Hoa factory. However, in the Dong Nai factory, it only occurs when the Binh Duong factory is unable to fulfill all the demand, and in the Bien Hoa factory, only Proconco/ANCO feed can be produced. Therefore, as it is likely to only produce piglet feed in Binh Duong. Because all piglet feed is shifted to the Binh Duong factory, it is also more advantageous to produce the other pig feed in Binh Duong. The reason for this is that customers can only have one pick-up location. Therefore, if the customers already have piglet feed demand, the customer needs to be assigned to the Binh Duong factory, or the piglet feed needs to be transhipped to the pick-up location from the customer. As the option of selling the piglet feed from the Binh Duong factory is already more attractive, it becomes more optimal to produce the other pig feed at this location to fulfill the other demand for pig feed.

From Dong Nai 7,469 tons of feed is shifted to Bien Hoa and Binh Duong, with Binh Duong receiving 6,626 tons of pig feed. As determined during the initialization of the feed types producible in different factories, ruminant feed can also exclusively be produced in Binh Duong, resulting in a production increase of 1,138 tons. Binh Duong, located favorably for reaching customers in the northern part of the southeast region, is nearing its production capacity limits. This is due to its advantageous location, which assigns more customers. Therefore, its capacity limit is exceeded if no feed is shifted to other locations. In the optimization, almost all chicken white broiler feed, 10,038 out of 11,159 tons, is shifted from the Binh Duong to the Dong Nai factory. Additionally, since most large customers are bulk customers, similar to those for pig feed, and bulk feed customers are assigned to the Binh Duong factory, there is a need to shift bulk production to another factory, with Dong Nai being the only viable option.



(a) Current case situation



(b) current case optimization

Figure 6.1: Optimization customer assignment: Optimizing current case.

(a) Depots: Orange = Dau Giang, Purple = Long An, Pink = Ben Tre

(b) Factories: Red = Dong Nai, Blue = Bien Hoa, Green = Binh Duong

Finally, the variables: Total production volume, number of recipes in production, number of SKUs in production, number of SKUs in the warehouse, Total factory costs, and costs per ton are used to compare the current situation with the optimized situation. These variables can also serve as KPIs which the company uses to compare the factories with each other. The overview of the variables of the current situation can be seen in Table 6.3, and the overview of the variables of the optimized situation can be seen in Table 6.4. The total production in the different factories has not changed significantly when comparing the current situation with the optimized situation. This is logical because the factories in Dong Nai and Binh Duong are already operating at their limitations due to production capacity restrictions. For the Dong Nai factory, the production volume cannot reach its full capacity due to changes in overtime that decreased the production capacity because of flushes between production runs. However, because the volumes are almost the same for both situations, it can also be said that the optimized situation proposed a solution within the possibilities of the real capacities. The number of different recipes and SKUs produced has decreased for Dong Nai but increased for Binh Duong. The optimization proposes to produce more products close to the customer. The number of SKUs in the warehouse can increase for all factories because the warehouses can store more efficiently due to the model decreasing the average inventory. The total factory costs are increasing with the optimization for Binh Duong in comparison with the current situation. This is caused by the increased holding costs and production set-up costs. Binh Duong is assigned to large pig bulk customers which have a long driving distance, so these costs are shifted from the Dong Nai factory to Binh Duong. Additionally, the amount of recipes that are produced is increased which also leads to more flushing times between the production runs, resulting in higher costs. The cost per ton is the most important KPI for De Heus to measure the performance of the factories. Currently, the transportation and holding costs are not incorporated into these costs, but to make the costs comparable, the holding costs are calculated based on the average warehouse occupation and the transportation costs are calculated based on the transportation that is currently necessary to fulfill the demand. In total, the costs per ton are decreased from $14.82 to $14.53, which is achieved by reducing the costs incurred at the Dong Nai and Bien Hoa factories. The costs of the Binh Duong factory are increased because they are assigned some new customers with piglet bulk feed demand who are further away.

Variables	**Total**	**Dong Nai**	**Bien Hoa**	**Binh Duong**
Total production volume	54,967	29,946	10,364	14,657
# recipes in production	225	136	61	69
# SKUs in production	494	287	277	132
# SKUs in warehouse	494	283	306	244
Total factory costs ($)	$1,116,526.26	$495,941.50	$287,301.66	$333,283.10
$ per ton (incl holding/transportation costs)	$14.82	$12.08	$20.49	$16.60

Table 6.3: Overview variables: Current situation

At the end, it is possible to save $27,756.90 by optimizing the current situation by the use of the model. However, it is important to take some remarks into account. The demand implemented into the model is already known beforehand, making it possible for the model to make decisions based on information that is not precisely known at that moment. Because the production time is relatively short and flexibility in production planning is high, the given information being known beforehand does not vary significantly. De

Variables	Total	Dong Nai	Bien Hoa	Binh Duong
Total production volume	54,967	29,635	10,551	14,781
# recipes in production	225	126	61	103
# SKUs in production	494	271	132	217
# SKUs in warehouse	494	287	216	261
Total factory costs ($)	$1,094,320.09	$475,543.76	$278,554.61	$340,221.69
$ per ton (incl holding/transportation costs)	$14.53	$11.71	$19.27	$16.80

Table 6.4: Overview variables: Optimizing base case

Heus is already working with demand forecasts and experience also helps in learning patterns, which can be used to predict demand very accurately. Also no safety stock is incorporated into the production schedule. In reality, some safety stock is needed to cover uncertainties, but because the demand is already known, it is not necessary. In the production schedule per factory, no differences are necessary to incorporate the safety stocks because the total production remains the same, but with a buffer produced at an earlier stage. The safety stocks can lead to a higher occupation of the warehouse, so if De Heus wants to implement safety stocks, the warehouse capacity can be reduced by the percentage of the safety stocks. Another point leading to different outcomes is that according to the model, all feed needs to come from one factory to guarantee stable quality. In reality, there are cases where big bulk customers get their feed from two different factories. For example, piglet feed may come from Binh Duong, and other pig feed from Dong Nai. In our model, everything needs to come from Binh Duong. Implementing this throughout the model could lead to solutions that are unrealistic because sourcing from two locations is more of an exception than a rule. These examples of big customers could be even more optimal if they are allowed to be supplied from two locations. The last remark concerns the production runs, which are only counted and are not variable. Investigating the variable run sizes of the production runs would also be interesting for the company, but for this model, it is not possible to incorporate that due to conflicting situations on linearity. Because the model can only count by dividing the total production by the average run size, the small production runs are also counted with a small part. In practice, small production runs lead to problems because the number of flushing times compared to the production volume is high. However, since this only affects smaller volumes, which also have a small part in the total production planning, the estimation of the number of production runs is assumed to remain valid for this model.

E1: Optimizing base case - artificial instances

The optimization for the base case is also carried out on the artificial instances which vary in the amount of customers and factories. The instances with 3 factories are listed in Table 6.5 and with 4 factories are listen in Table 6.6 and provide information about the objective function, computational time, the gap, and the extra kilometers driven. The instances vary in the number of customers and factories but remain consistent with the actual capacities to ensure comparability. The table shows that the computational times increase as the number of customers increases. The reason for this is that the model needs to consider more customers, making the solution more complex. In Table 6.5 The amount of extra driven kilometers from 50 to 100 customers is decreasing, the reason for that is that the model makes the decision to open an extra factory, this results in higher costs because of the factory overhead costs, but it decreases the extra driven kilometers by the customers because it became possible to serve the customers from a location which is closer for them. If Table 6.5 and Table 6.6 are compared, it can be seen that the computational time increases more rapidly when an extra factory is added. The reason for this is that the model has more possible solutions to check, leading to higher computational times. Additionally, the data instance with 4 factories is able to serve more customers; therefore, it is able to obtain a solution with 1200 customers, whereas the instance with 3 factories cannot achieve this.

Based on the data instances, it is still hard to conclude how many customers could be served by the model because this depends on the input capacities of the different factories. To make the data instances comparable, the real factory characteristics are used to test the different computational times against each other. This reveals that the computational times increase when the number of customers increases, with the most significant increases occurring when the capacity restrictions are almost reached. In such cases, the model needs to perform more calculations to determine if a solution is both optimal and feasible. Additionally, attempts were made to solve for a larger number of customers with higher factory capacities. While the model can still find a solution, the various possibilities for assigning customers to pick-up locations when capacities are increased make it challenging to compare computational times. For example, with 2500 customers and an overall capacity increase (warehouse, production, and bulk) of a factor of 2.5, the model can find a solution within 2,375 seconds. However, this does not align with the trend of increasing computational time with a higher number of customers. Therefore, the increase in the number of customers is not the primary reason for the increased computational times. Consequently, it is difficult to conclude how many customers can be served by the current model due to its dependence on customer demand and factory capacities. In conclusion, the optimization of the base case demonstrates that computational time increases with the number of customers. As factory capacities approach their limits, it becomes more challenging for the model to find feasible solutions, resulting in higher computational times. Therefore, it is not possible to precisely determine how many customers the model can handle in the current and future situations, as this depends on customer demand, which may exceed capacity limits.

Data instance with 3 factories	**Objective function ($)**	**Computational time(s)**	**gap (%)**	**Extra driven KMs (km)**
10 customers	$272,071.92	1.08 s	0.0000%	2,384,290.76
20 customers	$293,955.14	1.61 s	0.0000%	2,571,898.22
50 customers	$326,748.63	1.89 s	0.0000%	2,702,049.24
100 customers	$456,469.64	15.53 s	0.0096%	2,445,141.44
200 customers	$655,792.03	118.22 s	0.0039%	3,694,305.42
500 customers	$1,057,407.20	99.22 s	0.0097%	4,976,664.14
750 customers	$1,805,279.74	3,563.21 s	0.0074%	8,555,871.043
1000 customers	$2,622,564.52	4,735.78 s	0.0065%	13,933,407.81
1100 customers	$2,842,588.54	10,120.97 s	0.0998%	15,659,483.01
1200 customers	$ -	- s	- %	-
1300 customers	$ -	- s	- %	-

Table 6.5: Experiment *E1* - optimization performances with 3 factories (artificial)

Data instance with 4 factories	Objective function ($)	Computational time(s)	gap (%)	Extra driven KMs (km)
10 customers	$145,868.09	1.77 s	0.0000%	631,698.56
20 customers	$192,825.47	2.17 s	0.0000%	840,865.88
50 customers	$245,287.57	3.13 s	0.0000%	1,623,764.30
100 customers	$371,942,3552	5.13 s	0.0000%	1,101,479.93
200 customers	$517,646,04	50.28 s	0.0000%	2,526,813.66
500 customers	$863,015.28	789.97 s	0.0000%	5,396,705.10
750 customers	$1,385,099.22	10,800 s	0.0120%	6,093,258.02
1000 customers	$1,994,996.31	10,800 s	0.0844%	7,797,565.09
1100 customers	$2,096,194.93	10,800 s	0.0255%	8,068,570.57
1200 customers	$2,318,551.39	10,800 s	0.0310%	9,763,982.92
1300 customers	$ -	- s	- %	-

Table 6.6: Experiment *E1* - optimization performances with 4 factories (artificial)

6.4 Phase 2 - Sensitivity analysis and multi-objective optimization

In this section, the sensitivity analysis and multi-objective optimization are performed, and the results are explained.

6.4.1 E2: Expanding capacities

The first experiment within the sensitivity analysis involves expanding the capacities of the production and/or warehouse. The different instances are listed in Table 6.7, providing information about the objective function, computational time, gap, and extra driven kilometers. In the experiment aimed at expanding capacities, the warehouse and production capacity of the Dong Nai factory are extended. Four different scenarios are tested: +5% or 10% production capacity, and +5% production capacity with +250 or +500 warehouse capacity. Tables 6.8 and 6.9 present the total costs for these scenarios. These modifications are compared with the optimized current case because the scenario changes should lead to an improvement over the already optimized situation based on the current conditions.

Data instance	Objective function ($)	Computational time(s)	gap (%)	Extra driven KMs (km)
'Current case'	$1,097,765.72	10,800 s	0.0193%	1.390.082,48
'Current case' with +5% prod	$1,091,807.94	10,800 s	0.1422 %	1,295,689.66
'Current case' with +5% prod + 250 WH	$1,091,077.30	10,800 s	0.0207 %	1,316,525.72
'Current case' with +5% prod + 500 WH	$1,091,076.45	10,800 s	0.0191 %	1,316,525.72
'Current case' with +10% prod	$1,085,220.45	10,800 s	0.1518 %	1,223,891.02

Table 6.7: Experiment *E2* - optimization performances

In the first scenario, the +5% production capacity is optimized. The total costs for this scenario decrease from $1,097,765.729 to $1,091,807.944, resulting in a monthly difference of -0.6%. These cost savings are achieved by reducing total production setup costs, because of the increased production capacity at Dong Nai, it becomes more efficient in terms of switching times to produce more feed at one location. Internal transportation costs increase by 5.9% because some feed previously produced at other, more expensive locations, like Bien Hoa, is now produced at Dong Nai and transported back to be sold there. Additionally, because Dong Nai is near most bag customers, more customers can be assigned to this factory, reducing external transportation costs by 10.3%.

In addition to increasing production capacity, it is also interesting to investigate whether expanding the warehouse in combination with production capacity leads to even lower total costs. The current warehouse capacity of Dong Nai is 3,500 tons, increased to 3,750 and 4,000 tons. Additional savings could be achieved: -$730.64 if the capacity is increased by 250 tons, and an extra $0.84 if the capacity is increased with 500 tons. Increasing the warehouse capacity changes the decision-making process of the model, instead of increasing internal transportation costs by supplying feed to another pick-up location, more feed can be sold at the factory itself. However, to fully utilize production capacity, goods need to be transshipped to other warehouses for sale, reducing warehouse occupation in the warehouses at the factories but increases the internal transportation costs. There is an optimum in available warehouse space for maximum cost-effectiveness, as seen from the minimal difference between 3,750 and 4,000 tons, indicating that this extension does not lead to further savings.

Lastly, optimizing production expansion of + 10% results in a total cost reduction from $1,097,765.47 to $1,085,220.45, yielding a percentage decrease of -1.1%. With expanded capacity, it is even more feasible to assign customers to closer locations, resulting in additional cost savings for external transportation, reduced by 15.3%.

Therefore, expanding production and/or warehouse capacity leads to cost savings, as production in the Dong Nai factory is cheaper, and the location is near bag customers. The cost for expanding the factory with + 10% is estimated at $525,600, while the expansion of the warehouse by 500 tons is estimated to be $343,100. The factory overhead costs do not change with these investments because, for example, if the production capacity is increased, an extra pelletizing line is added, and an old hammermill machine is replaced by a newer, bigger one. This does not lead to higher overhead costs because the new machines replace old ones and are likely to use less energy. Therefore, the total overhead costs and production costs do not change much when the investments are made, so for comparability, it is assumed that they will remain the same. For the expansion of the warehouse, it can be concluded that it is not cost-effective, as the extra warehouse capacity does not lead to enough savings to recoup the investment. However, the investment in the extra production capacity of +10% can be earned back within a reasonable time of 3.5 years. The price for the expansion of +5% is estimated to be half of that price, but because the cost savings are also half of the savings of +10%, so the payback period remains the same. Therefore, both investments in expanding the production capacities are economically attractive with the same payback period. De Heus could decide how much they want to invest to reach the amount of savings they aim to achieve.

6.4.2 E3: New potential production location

The next experiment within the sensitivity analysis involves the new potential production locations of De Heus. For optimizing the testing of new potential production locations, various instances are listed in Table 6.10, providing information about the objective function, computational time, gap, and extra driven kilometers. Instead of expanding existing capacities with an expansion, it is also possible to build a new factory at a location that maximizes customer assignment potential. This option becomes even more attractive because

Costs name	Optimized current case	+ 5% prod	%	+5% prod + 250 WH	%
Production costs	$755,512.77	$753,107.36	-0,3%	$753,081.31	-0,3%
Factory overhead costs	$173,484.61	$173,484.61	0,0%	$173,484.61	0,0%
Production setup costs	$21,939.83	$21,351.39	-2,7%	$21,365.41	-2,6%
Holding costs	$15,431.18	$15,545.86	0,7%	$15,540.47	0,7%
Internal transportation costs	$27,235.43	$28,850.79	5,9%	$27,483.52	0,9%
External transportation costs	$45,365.42	$40,671.70	-10,3%	$41,325.74	-8,9%
Bulk transportation costs	$58,796.20	$58,796.20	0,0%	$58,796.20	0,0%
TOTAL COSTS	$1,097,765.47	$1,091,807.94	-0,5%	$1,091,077.30	-0,6%
dif		*-$5,957.53*		*-$6,688.17*	

Table 6.8: Overview total costs: Expanding capacities part 1

Costs name	Optimized current case	+ 5% prod + 500 WH	%	+ 10% prod	%
Production costs	$755,512.77	$753,081.31	-0,3%	$750,831.73	-0,6%
Factory overhead costs	$173,484.61	$173,484.61	0,0%	$237.650,16	0,0%
Production setup costs	$21,939.83	$21,365.41	-2,6%	$20,715.06	-5,6%
Holding costs	$15,431.18	$15,539.62	0,7%	$15,669.45	1,5%
Internal transportation costs	$27,235.43	$27,483.52	0,9%	$27,305.43	0,3%
External transportation costs	$45,365.42	$41,325.74	-8,9%	$38,417.93	-15,3%
Bulk transportation costs	$58,796.20	$58,796.20	0,0%	$58,796.20	0,0%
TOTAL COSTS	$1,097,765.47	$1,091,076.45	-0,6%	$1,085,220.45	-1,1%
dif		*-$6,689.01*		*-$12,545.02*	

Table 6.9: Overview total costs: Expanding capacities part 2

the Bien Hoa factory needs to close in the coming years. Section 5.2.2 proposes three potential locations: Binh Duong II, Binh Phuoc, and Dak Nong.

Data instance	Objective function ($)	Computational time(s)	gap (%)	Extra driven KMs (km)
'Current case'	$1,097,765.72	10,800 s	0.0193%	1.390.082,48
'Current case' with Binh Duong II	$994,363.39	511.76 s	0,0090 %	312,372.30
'Current case' with Binh Phuoc	$1,011,247.69	10,800 s	0,0201 %	520,556.71
'Current case' with Dak Nong	$1,209,423.63	5,380.29 s	0,0090 %	985,320.83

Table 6.10: Experiment *E3* - optimization performances with new location instead of Bien Hoa

The first potential location is in the Binh Duong region: Binh Duong II. This factory is close to the existing Binh Duong factory (24.4 km), which is the most northern factory, and is capable of producing all kinds of feed, including bulk feed. As a replacement for Bien Hoa, this results in a new situation where all factories are able to produce De Heus and Proconoco/ANCO feed and bulk feed. It also provides an extra option to produce piglet and ruminant feed at this location. The total costs of this new factory in comparison with the already optimized situation can be seen in Table 6.11. The total costs can be reduced from $1,097,765.72 to $994,363.39, which is a decrease of 9.4%. This significant reduction in costs has several reasons:

- The production costs per ton are lower at the new location because this factory can be designed by De Heus engineers, which leads to lower production costs per ton compared to the Bien Hoa factory. Thus, the production costs with a new factory are obviously lower than with the old Bien Hoa factory, gives a reduction of 0.9%.
- The factory overhead costs at the Bien Hoa factory are currently relatively high. A new factory requires less maintenance, uses less energy, and can be designed with fewer employees needed, resulting in reduced overhead costs from 20.1%.
- The production is more flexible in the new situation, allowing for the combination of production runs at different locations, resulting in a reduction in production setup costs of 21.6%. Holding costs can be reduced because the products can be produced closer to the customer, and there is more capacity to produce all the needed products per week, so less work in advance is necessary to meet all demand. Additionally, more SKUs can be sold at the factory warehouse location, reducing the number of transshipments and, consequently, lowering the average occupation in total at all warehouses due to the sold feed.
- The internal, external, and bulk transportation costs are also reduced by 39.5%, 78.4%, and 15.1%, respectively, because the factory is more flexible in production and able to serve customers located in the northern part of the southeast region.

Costs name	**Total costs (opt) ($)**	**Total costs (Binh Duong II) ($)**	**dif ($)**	**%**
Production costs	$755,512.77	$748,487.83	-$7,025.19	-0.9%
Factory overhead costs	$173,484.61	$13,8675.64	-$34,808.96	-20.1%
Production setup costs	$21,939.83	$17,208.08	-$4,731.74	-21.6%
Holding costs	$15,431.18	$13,770.77	-$1,660.40	-10.8%
Internal transportation costs	$27,235.43	$16,484.50	-$10,750.92	-39.5%
External transportation costs	$45,365.42	$9,805.36	-$35,560.05	-78.4%
Bulk transportation costs	$58,796.20	$49,931.17	-$8,865.03	-15.1%
TOTAL COSTS	$1,097,765.47	$994,363.39	-$103,402.33	-9.4%

Table 6.11: Overview total costs: New potential production location - Binh Duong II

The second potential location is the Binh Phuoc area. This factory has the same characteristics as explained for the Binh Duong II factory, which means that this factory is also flexible, leading to cost reduction at almost every part. The Binh Phuoc factory has a driving distance of 42.0 km to the Binh Duong factory. The total costs are decreasing from $1,097,765.72 to $1,011,247.69 which gives a reduction of -7.9%. The differences between Binh Phuoc and the current situation has several reasons:

- The Binh Phuoc factory is located at a further distance from the harbor in comparison with Binh Duong II, which means that the direct material costs are $5.62 per ton instead of the $4.90 per ton. In comparison with the current case optimization, this results in an increase of 0.8%.
- The bulk transportation costs in this case decrease with 21.8%. There is already a significant reduction for the current customer situation, which aligns with expectations for the future, where farmers from the southern part move to the northern part to build larger farms requiring more bulk resources. Thus, it holds even more potential for the future.
- The other costs have similar explanations as already mentioned at the Binh Duong II factory.

The last potential location is the Dak Nong location. This location is positioned far to the northeast part of the southeast region. The driving distance to Binh Duong is 290.7 km, which if you compare it with the Binh Duong II factory (24.4 km) is completely at a different location. The total costs are increasing from $1,097,765.72 to $1,209,423.63 which is an increase of 14.1%. The differences between Dak Nong and the current situation has several reasons:

- The direct material costs for Dak Nong are higher than those for the Binh Duong II and Binh Phuoc factories because the location is even further from the harbor. This results in direct material costs of $17,22 per ton. This is already $12,32 per ton higher than the Binh Duong II factory location. These significant cost increases are evident in the production costs, which increase of 22.9%.
- The bulk transportation costs are not decreasing because the customers are not located in areas that would lead to a more efficient customer allocation if this factory location were added.
- The factory overhead, production setup, internal, and external transportation costs are still decreasing in comparison with the current case optimization and have similar explanations as Binh Duong II and Binh Phuoc.

The Binh Duong II factory will take over most of the piglet and ruminant feed from the Binh Duong factory because this factory has lower production costs per ton produced (when adding up the direct variable costs and direct material costs, the difference is $10,56 for Binh Duong and $10,38 for Binh Duong II). Due to the transfer of that feed, capacity at Binh Duong is once again available to deliver the feed of chicken white broiler (CWB) close to the customer. Production planning is changed to the optimal situation for every new factory, and this differs greatly due to the differences in customer assignment, which depend on the location of the new factory. The variables of the new production location can be seen in Table 6.12. Because the new factory is able to produce more different products closer to the customer, the number of SKUs produced per factory increases. The costs per ton are cheaper in Binh Duong compared to Binh Phuoc because the production volume is higher at this location. The total factory costs are high for Dak Nong because the direct material costs are high at this location.

In conclusion, there are three possible locations where it is possible to build a factory: Binh Duong II, Binh Phuoc, and Dak Nong. Optimization shows that the location in Dak Nong is not economically viable because it costs De Heus $133,750.73 extra per month to operate this factory. This is caused by the long transportation route to the nearest harbor and the lower customer density in that area. Both Binh Duong II and Binh Phuoc are options to build a factory, with the total cost reduction per month for Binh Duong II being $103,402.33 and for Binh Phuoc being $86,518.03, making the option to choose Binh Phuoc $16,884.30 per month more expensive. It would suggest that the Binh Duong II location is the best option, but based on

Variables	Total	Binh Duong II	Binh Phuoc	Dak Nong
Total production volume	54,967	15,319	11,625	10,566
# recipes in production	225	154	145	166
# SKUs in production	494	307	283	296
# SKUs in warehouse	494	272	242	250
Total factory costs ($)		$318,446.44	$256,132.71	$426,169.70
$ per ton (incl holding/transportation costs)		$15.17	$16.08	$29.44

Table 6.12: Overview variables: New potential production location

the differences in building costs and possible customer movements, it has not been decided yet. The estimated building costs for a new factory are $13.87 million, where the land prices in Binh Duong II are $4.38 million and in Binh Phuoc are $2.92 million. Depending on the total investment costs and the forecasted demand for the different areas for the coming 10, 20, and 30 years, De Heus can decide where the factory should be built.

6.4.3 E4: Reducing SKUs

The last experiment within the sensitivity analysis involves reducing the amount of SKUs. For the optimization of reducing the amount of SKUs, the different instances are listed in Table 6.13, providing information about the objective function, computational time, gap, and extra driven kilometers. he next experiment performed aims to reduce the number of SKUs to two bags and one bulk SKU, and to one bag and one bulk SKU. The overview of the total cost optimized current case and the two different experiments can be seen in Table 6.14.

Data instance	Objective function ($)	Computational time(s)	gap (%)	Extra driven KMs (km)
'Current case'	$1,097,765.72	10,800 s	0.0193%	1.390.082,48
'Current case' with 2 bag, 1 bulk SKUs	$1,096,557.96	5,854.53 s	0.0100 %	1,372,950.55
'Current case' with 1 bag, 1 bulk SKUs	$1,094,735.06	6,853.02 s	0.0203 %	1,249,024.52

Table 6.13: Experiment *E4* - optimization performances

For the reduction of the number of SKUs to two bags and one bulk SKU, the holding costs can be reduced with 24.0%. With fewer SKUs, it becomes easier to produce the demand in higher volumes and transport it to the customers. While the internal transportation costs and bulk transportation costs increase slightly, these increased costs are offset by lower holding and external transportation costs. This results in a decrease in total costs from $1,097,765.72 to $1,096,557.96, which is a reduction of -0.1%.

Reducing to one bag SKU and one bulk SKU yields better results than reducing to two bulk SKUs. The holding costs decrease 31.7%. Although the internal transportation costs increase further, the decrease in external transportation costs, combined with lower holding costs, results in a more cost-effective solution.

This could be attributed to the cost-effectiveness of assigning customers to locations closer to them, although it requires transportation from De Heus to that pick-up location to fulfill the demand. The total costs decrease from $1,097,765.72 to $1,094,735.06, which is a reduction of -0.3%.

Before this experiment, it was expected that reducing SKUs would lead to more cost reduction, as the company is actively reducing the number of SKUs. However, the results of this experiment indicate that reducing SKUs alone does not optimize the production schedule. This is because reducing only the number of SKUs still requires producing the same volume of recipes, with corresponding flushing times between the recipes. The only advantage of reducing the number of SKUs is that warehouse occupancy becomes more efficient, due to fewer different SKUs in storage which leads to less 'empty' occupied space at a warehouse slot. To increase production schedule efficiency, it is important to reduce the number of recipes that need to be produced. Thus, the company should consider combining recipes to increase total production volume and corresponding longer production runs. Longer runs in the factory require less flushing time.

Overview total costs	**Total costs (optimized)**	**2 SKUs bag 1 SKU Bulk**	%	**1 SKU bag 1 SKU Bulk**	%
Production costs	$755,512.77	$755,494.36	0.0%	$755,464.61	0.0%
Factory overhead costs	$173,484.61	$173,484.61	0.0%	$173,484.61	0.0%
Production setup costs	$21,939.83	$21,932.52	0.0%	$21,941.39	0.0%
Holding costs	$15,431.18	$11,732.27	-24.0%	$10,542.33	-31.7%
Internal transportation costs	$27,235.43	$29,057.13	6.7%	$32,334.89	18.7%
External transportation costs	$45,365.42	$43,096.91	-5.0%	$39,206.87	-13.6%
Bulk transportation costs	$58,796.20	$61,760.13	5.0%	$61,760.13	5.0%
TOTAL COSTS	$1,097,765.47	$1,096,557.96	-0.1%	$1,094,735.06	-0.3%

Table 6.14: Overview total costs: Reducing SKUs

6.4.4 E5: Single-objective optimization

The second part of phase two is multi-objective optimization. Before this multi-objective optimization is performed, the experiment *E5* includes single-objective optimization. This optimization displays objective values for cost minimization ($f_1(x)$) and minimization of extra kilometers driven by the customer ($f_2(x)$). The data instances used are artificial and vary in the number of customers: 10, 20, 50, 100, and 200. Each of these data instances is optimized based on one of these objective functions. The costs, total extra kilometers, computational time, and the gap are presented in Table 6.15. The data instances are defined by the number of customers, the number of factories, and the number of SKUs. Therefore, C10-F3-S494 means there are 10 customers, 3 factories, and 494 different SKUs.

The costs and the extra driven kilometers increase as the problem instance grows larger. Optimization of the extra driven kilometers is performed more quickly in terms of computational time compared to the optimization of costs. The reason for this is that the objective function for minimizing extra kilometers is relatively simple. It minimizes the extra driven kilometers by assigning the customer to the closest possible location that meets the constraints of the model. This is not always the closest possible location because a bulk customer can be close to a factory that is not able to produce bulk feed, so it must be assigned to a location that is not the closest possible. This results in extra driven kilometers, which are not zero. Since the model optimizes only one of the two objective functions, the other objective still has the potential to be reduced. This reduction in both objective functions establishes the lower and higher bounds. These bounds

Data instance	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C10-F3-S494 ($min\ f_1(x)$)	$168,753.11	2,403,129.72	1.05 s	0.0000 %
C10-F3-S494 ($min\ f_2(x)$)	$197,316.28	1,788,731.39	1.55 s	0.0000 %
C20-F3-S494 ($min\ f_1(x)$)	$180,261.89	2,645,184.13	1.75 s	0.0000 %
C20-F3-S494 ($min\ f_2(x)$)	$360,777.37	1,097,649.78	0.92 s	0.0000 %
C50-F3-S494 ($min\ f_1(x)$)	$200,073.51	2,952,039.04	2.38 s	0.0000 %
C50-F3-S494 ($min\ f_2(x)$)	$393,637.01	1,097,649.78	1.64 s	0.0000 %
C100-F3-S494 ($min\ f_1(x)$)	$257,150.32	4,876,539.08	10.80 s	0.0000 %
C100-F3-S494 ($min\ f_2(x)$)	$646,896.63	1,097,649.78	2.18 s	0.0000 %
C200-F3-S494 ($min\ f_1(x)$)	$339,459.86	8,342,603.75	31.04 s	0.0000 %
C200-F3-S494 ($min\ f_2(x)$)	$739,407.38	1,097,649.78	2.77 s	0.0000 %

Table 6.15: Experiment *E5* - optimization performances

are utilized in the payoff table, which is necessary to determine the step size for the AUGMECON2 multi-objective optimization. An example illustrating the differences when a second optimization is performed can be seen in Table 6.16.

Data instance	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C200-F3-S494 ($min\ f_1(x)$)	$339,459.86	8,342,603.75	31.04 s	0.0000 %
C200-F3-S494 ($min\ f_1(x) \rightarrow min\ f_2(x)$)	$339,460.85	8,342,579.73	51.92 s	0.0000 %
C200-F3-S494 ($min\ f_2(x)$)	$739,407.38	1,097,649.78	2.77 s	0.0000 %
C200-F3-S494 ($min\ f_2(x) \rightarrow min\ f_1(x)$)	$669,559.88	1,097,649.78	10.25 s	0.0000 %

Table 6.16: Experiment *E5* - payoff table

Table 6.16 illustrates an example of single optimization and additional minimization of the second objective function for data instance C200-F3-S494. It shows that after minimizing the cost amount, the kilometers cannot be further reduced. This limitation arises because every kilometer incurs extra costs. Thus, if the minimum cost remains unchanged, the extra kilometers cannot decrease further. However, when minimizing the extra kilometer first, there are still opportunities to minimize the total costs. By optimizing the costs of the solution obtained from minimizing the extra kilometer, the total costs decrease from $739,407.38 to $669,559.88. This reduction is possible because for example production planning and internal transportation costs can still be optimized while maintaining the same amount of extra kilometers driven by the customers. The objective values obtained in the $min\ f_1(x) \rightarrow min\ f_2(x)$ and $min\ f_2(x) \rightarrow min\ f_1(x)$ optimizations serve as the bounds in the payoff table used for multi-objective optimization.

6.4.5 E6: Multi-objective optimization

For the multi-objective optimization, the payoff values obtained from experiment *E5*, the single-objective optimization, are utilized to determine the stepsize. The multi-objective optimization constains data instances with 10, 20, 50, 100, and 200 customers, while maintaining the same number of factories and SKUs. A Pareto front is generated for each data instance using the AUGMECON2 algorithm. In the Pareto diagram, specific points on the graph are identified with capital letters. These points are further analyzed in the corresponding table, which provides the iteration number, costs $f_1(x)$, extra driven kilometers $f_2(x)$, computation time, and gap. Following the diagram and table, an explanation of the values is provided. The data instances 20, 50 and 100 are given and explained in Appendix A.2.

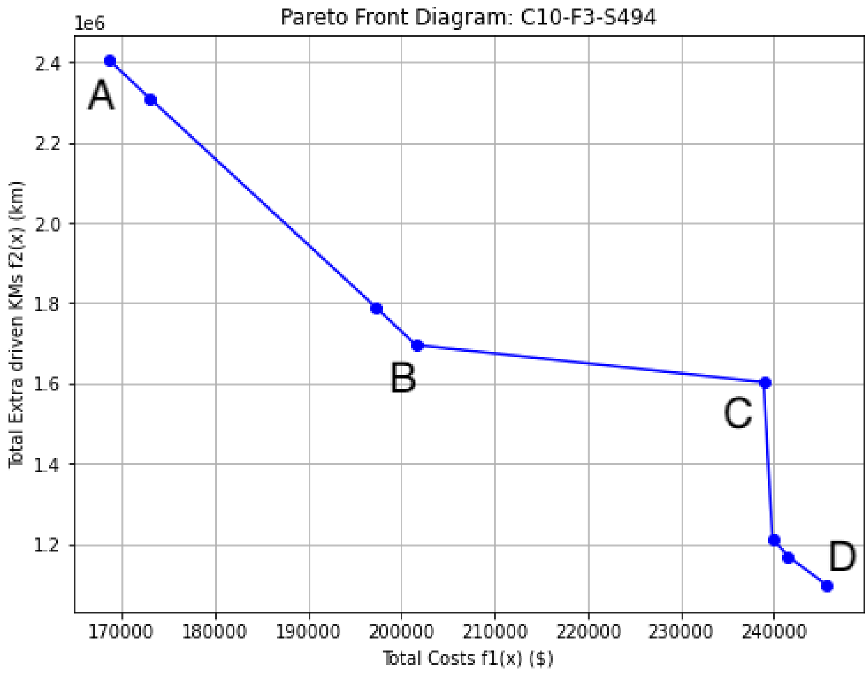


Figure 6.3: Pareto front: C10-F3-S494

Data instance	#Iteration	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C10-F3-S494 (A)	0	$168,753.12	2,403,129.72	1.05 s	0.0000 %
C10-F3-S494 (B)	4	$201,578.95	1,695,515.54	1,51 s	0.0000 %
C10-F3-S494 (C)	8	$238,883.47	1,603,128.71	1.86 s	0.0000 %
C10-F3-S494 (D)	15	$245,657.43	1,097,649.78	0.91 s	0.0000 %

Table 6.17: Experiment *E6* - Pareto front values C10-F3-S494

Figure 6.3 and Table 6.17 shows the Pareto results of 10 customers. The Pareto front shows only 8 data points, although the algorithm is capable of generating 16 data points. The algorithm is designed to avoid unnecessary iterations; if no improvement is expected in upcoming iterations, these iterations are skipped due to the bypass coefficient. The most interesting data points are provided in the table. The difference in the amount of costs needed to achieve a small reduction in extra driven kilometers is significant between solutions B and C. The differences in costs are $\$238,883.47 - \$201,578.95 = \$37,304.52$ for only $1,695,515.54 - 1,603,128.71 = 92,386$ kilometers, resulting in \$2.48 per reduced kilometer. Previously, this rate was $\$201,578.95 - \$168,753.12 = \$32,825.83$ for $2,403,129.72 - 1,695,515.54 = 707,614.18$ kilometers, giving \$21.56 per reduced kilometer. The closer the amount of kilometers gets to the minimum amount, the higher the price per reduced kilometer becomes. This trend can already be observed in this Pareto front, but because the number of customers is limited, it is challenging to create a smooth line due to the limited possibilities in customer assignment and finding a better solution within the given bounds.

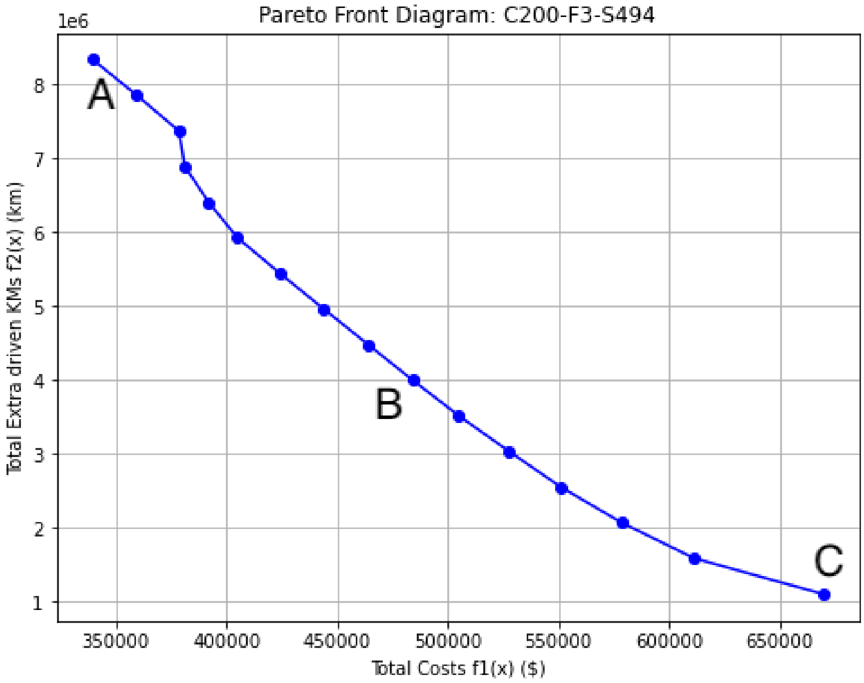


Figure 6.4: Pareto front: C200-F3-S494

Data instance	#Iteration	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C200-F3-S494 (A)	0	$339,460.86	8,342,579.73	43.10 s	0.0000 %
C200-F3-S494 (B)	9	$484,173.63	3,995,491.31	246.02 s	0.0052%
C200-F3-S494 (C)	15	$669,559.90	1,097,649.78	9.86 s	0.0000 %

Table 6.18: Experiment *E6* - Pareto front values C200-F3-S494

Figure 6.4 and Table 6.18 show the Pareto results for 200 customers: This Pareto front displays all 16 potential data points, resulting in a smooth curve. Compared to the other data instances, this Pareto front diagram is the smoothest. The reason for this is that the model has more optimization options within the new bounds provided at every point. Therefore, the optimal points become more evident in the middle of the Pareto front as the number of customers in the data instance increases.

The analysis of the Pareto fronts for different artificial data instances reveals that the Pareto front becomes smoother as the model's options increase. For example, the Pareto front for 10 customers consists of only 8 points, whereas the Pareto front for 200 customers has the maximum of 16 points. The reduction in the number of data points occurs because there are no further improvements possible within a new iteration. This is logical because the options are limited in assigning a customer to a better location, requiring waiting until the amount of extra kilometers allows for a new optimal solution that reduces total costs. Applying this conclusion to the current case of De Heus, which involves 1117 customers, it can be assumed that it is also possible to create a smooth Pareto front for this amount because the number of possibilities to assign a customer is even higher, leading to smoother steps in the Pareto front. The price difference between the steps in reducing the amount of extra kilometers will be estimated to be equal. However, it is also expected that the last reduction of the amount of kilometers (which are close to the lower bound) will result in the highest costs. Therefore, De Heus can decide, based on the total costs they are currently incurring, how much they are willing to pay to reach the customer satisfaction level in terms of extra driven kilometers by the customers. The costs of these steps will only differ significantly at the beginning or at the end of the Pareto front but are approximately the same between these points.

6.5 Conclusion

Before the experiments in phases one and two can be executed, the model is validated by checking the outcomes of the constraints individually. Additionally, the outcomes can be followed step by step due to the visualization and output of every step during optimization. The verification process involves experts from De Heus, and after some adjustments, the model yields realistic results.

Experiment *E1*, optimizing the base case, reduces the total costs from $1,118,028.47 to $1,097,765.94, resulting in a decrease of -1.8%. De Heus is currently overly focused on filling warehouses to capacity, which leads to high holding costs. However, high inventory positions are unnecessary if better production planning could be made based on demand forecasts. The internal and bulk transportation costs are reduced, while external transportation costs increase. The model decides to assign customers to locations that are further away from them but cheaper for De Heus to supply. These customers are compensated for driving longer distances with discounts on their feed. The total number of SKUs in the warehouse and production remains the same for Dong Nai, decreases for Bien Hoa due to its expensive location, and increases for Binh Duong because it can produce all products and is favorably located. The remarks on solutions that could result in less positive outcomes are that the demand is already known at the beginning of the optimization, and no safety stock is incorporated. However, this could also lead to better results in the real world by not having variable run sizes currently. These remarks can be addressed by implementing proper demand forecasts and reducing warehouse capacity if safety inventory is needed. The variable run size could lead to a more optimal situation because fewer production runs are needed. Thus, the new optimized situation, where decisions about production locations and customer service are made, leads to cost savings, and the changes are manageable for De Heus because no investments are needed.

Experiment *E2*, examining the expansion of the capacities of the Dong Nai factory, demonstrates a further cost reduction from $1,097,765.72 to $1,091,807.94 for a 5% production expansion, and a reduction to $1,085,220.45 for a +10% expansion. The investment in extra warehouse capacity is not attractive due to the low savings against high investment costs. The investment costs for +10% production capacity are estimated at $525,600, so it has a payback period of around 3.5 years. This period remains the same for +5% production capacity because the savings and investment costs are half of those for +10% production. Thus, the expansion of the production capacity is an attractive option for De Heus, where they need to decide how much they are able to invest right now.

Experiment *E3*, exploring new potential production locations to replace Bien Hoa in the future, reveals potential reductions from $1,097,765.72 to $994,363.39, a reduction of 9.4% for a new location at Binh Duong II. This significant reduction is attributed to lower production and factory overhead costs due to a newer factory. Planning production, inventory, and transportation becomes more efficient and closer to the customer with the new factory, which can produce all types of products unlike the limited capabilities of Bien Hoa. Binh Phuoc also offers potential savings, with a reduction from $1,097,765.72 to $1,011,247.69, resulting in a reduction of -7.9%. This location is favorable due to its proximity to the northern part of the southeast region. Dak Nong is currently not viable due to high transportation costs, resulting in additional monthly expenses.

The investment required for Binh Duong II and Binh Phuoc differs by \$1.46 million, so De Heus needs to forecast future demand in the region and update the model to optimize the new scenario for even better results. However, based on current information, both locations yield significant savings, making building a new factory is an attractive option.

Experiment *E4* provides information on reducing the number of SKUs. It shows that reducing the number of SKUs does not significantly decrease costs. Reducing to 2 bag SKUs and 1 bulk SKU reduces costs from \$1,097,765.72 to \$1,096,557.96, a reduction of -0.1%. This occurs because reducing the number of SKUs does not optimize production planning, as the same number of recipes are still produced in the factory. To make a positive impact on production planning and cost reduction, De Heus needs to consider reducing or merging recipes instead of only reducing the number of SKUs.

Experiment *E5* provides the single-objective optimization function, where the payoff table is created by further optimizing the second objective after already minimizing the first objective. Experiment *E6* demonstrates the multi-objective optimization. As the problem instance used to generate the Pareto front increases, the number of data points also increases, resulting in a smoother curve. This is because the model has more options to find new optimal solutions within the given bounds. The cost of decreasing the extra driven kilometers of De Heus is approximately the same for every step. Based on that, De Heus can decide how much they want to increase the cost until they reach a certain level of customer satisfaction for the current case.

Concluding, the different experiments provide information about the performance of the model and how multi-objective optimization can be applied in the future. It also gives information about how to optimize the current situation and advises on which changes are worth considering and which are not effective.

7 Conclusion and Recommendations

In the last section of this thesis, insights are provided about the conclusion, contribution, and recommendations. This chapter summarizes the most important findings, but also gives advice to De Heus on what they should do with the results. Section 7.1 provides the conclusions found by the model. Sections 7.2 and 7.3 detail the contribution of this research to the literature and to practice. Section 7.4 provides recommendations for De Heus. Section 7.5 outlines the limitations of this research and offers advice for further research.

This chapter addresses the research question, *'What are the conclusions and recommendations for De Heus?'* along with its sub-questions, which represent smaller components of the overall inquiry.

(a) What can be concluded from the supply chain optimisation for production, inventory and transportation planning?

(b) What are the recommendations and future research for De Heus?

7.1 Conclusions

The research aims to find the most cost-effective way to plan production, inventory, and transportation of finished goods. After the acquisition of Pronco/ANCO, De Heus is busy integrating this company within its operations. De Heus have already started the integration at the organizational level, and now they are optimizing production and supply to customers, deciding where to produce which products and how to supply them. During this integration, De Heus faces the problems of a lack of relationship between costs, an increased number of SKUs, and a lack of integrated optimization between different parts of the finished goods supply chain. To solve this problem, a model was built, which can make decisions at the production, inventory, and transportation levels, covering different time frames.

The model used to solve the problem is the production assignment problem, based on the production routing problem, where the routing part is replaced by direct customer assignment. Network flows are used to ensure the supply from factory to customer, and specific characteristics of lot-sizing problems, like capacity reduction due to flushing time and joint set-up and replenishment time, are implemented. This model can make monthly decisions split into four weeks, deciding which SKUs are produced, how they are stored at each location, and how they are supplied to customers to meet the demand of every week. The model successfully solved the current case for the southeast region of Vietnam, which includes three factories, three depots, 1117 customers, and 494 SKUs derived from 194 recipes. Total costs were reduced from $1,118,028.26 to $1,097,765.72 , a monthly saving of -1.8%. This solution was reached within the maximum computational time of 10,800 seconds and a gap of 0.0193%. The main reason for these savings is the more cost-effective customer assignment; currently, most customers are assigned to the closest possible location, whereas it would be more cost-effective for De Heus to offer discounts to customers and assign them to other locations. Additionally, De Heus is currently focusing too much on maintaining high stock levels; however, with better demand planning, these high levels are unnecessary. Therefore, within the current case, De Heus is already able to save a significant amount of money by adjusting production volumes, inventory positions, internal transportation, and customer assignment decisions. The only drawbacks of the model are that it has fixed demand input, no safety stock, and no variable run size. However, these can be addressed by using properly forecasted demand and reducing warehouse capacity. A variable run size would lead to even better optimization, as it would result in fewer production runs compared to the current optimization.

In addition to optimizing the current case, the model was used to test three different situations within the sensitivity analysis of the current case. The first experiment tested the expansion of production capacity by +5% and +10% for the Dong Nai factory, as well as the expansion of warehouse capacity by +250 tons or +500 tons. The results showed that both capacity increases led to significant cost savings of -$5,957.53 and -$12,545.02 per month, with one-time investment costs of $525,600 for +10% production capacity and half of that for +5%. Both investments have a payback period of 3.5 years, making them advisable for De Heus. The second experiment tested potential new production locations in the Binh Duong (II), Binh Phuoc, and Dak Nong regions. A new factory in Binh Duong led to monthly savings of -$103,402.33, a cost reduction of 9.4% compared to the optimized current situation. Similarly, a new factory in Binh Phuoc showed savings of $86,517.99, making it a viable option due to its lower land price. The third experiment tested the reduction of SKUs within a recipe to only two bags and one bulk SKU, resulting in cost reductions of $1,207.76 and $3,030.66 per month, respectively. Less variation in SKUs reduces warehouse needs and increases utilization, thus reducing holding costs. However, drastic reductions in SKUs may lead to challenges with marketing and sales, outweighing the savings.

Additionally, multi-objective optimization was performed to optimize total costs and customer satisfaction in terms of extra driven kilometers to pick up feed. The AUGMECON2 algorithm was used, as it reduces runtime by skipping unnecessary iterations. The multi-objective optimization showed the Pareto front, which became smoother with larger problem instances, indicating more options for cost-effective customer assignment. Costs increased gradually for reducing extra driven kilometers, with the first extra kilometers being the cheapest and the last being the most expensive. Therefore, De Heus can decide how much it wants to pay for increasing customer satisfaction, with the price remaining approximately the same for each increment.

In conclusion, the optimization model created reflects the situation of De Heus and is capable of optimizing the current case and conducting experiments on future events, leading to better results. The model allows testing of these cases and presenting implications on production, inventory, and transportation. The multi-objective optimization provides insights into costs necessary to meet customer satisfaction. Without the model, obtaining information about the entire finished goods supply chain would have been challenging, relying solely on trial and error. Thus, the model enables informed decision-making.

7.2 Contribution to theory

The literature review discusses various models that optimize production, inventory, and transportation individually. The lot-sizing problem, focusing on production-inventory optimization, was addressed by Jans and Degraeve (2008). Bertazzi and Speranza (2012) introduced the inventory-routing problem, which optimizes inventory and transportation, while Bard and Nananukul (2009b) presented the production-routing problem, aiming to optimize all three aspects simultaneously. However, applying these models directly to the problem at De Heus revealed challenges, as certain characteristics of the situation were not addressed in the existing literature. To address this, the thesis simplified the routing aspect to customer assignment and incorporated additional elements such as multiple products and factories (Fumero & Vercellis, 1999), reduced production capacity due to setup times (Hindi, Fleszar, & Charalambous, 2003), and recipe relationships (Stowers & Palekar, 1997). The transportation routes of the finished goods, from production, to a transshipment warehouse, to a pick-up location, to the customers are based on the multi-commodity network flow problem given by Wang (2003). Moreover, specific constraints unique to De Heus, such as product types and bulk capacities, were added to the model.

The decision to base the model on the production-routing problem was driven by the lack of available models optimizing production, inventory, and transportation simultaneously. However, the model's flexibility allows for adaptation to different routing characteristics, such as scenarios where only one customer is served at a time, by adjusting constraints accordingly. This dynamic approach not only makes the model suitable for other business units within De Heus but also for companies with similar routing characteristics.

7.3 Contribution to practice

The research outcomes are easily understandable for De Heus, allowing for direct implementation in real-world scenarios. The experiments with current case data utilize recently gathered monthly data, ensuring realistic results. The optimization of the base case provides weekly production schedules, inventory planning, and more efficient customer assignments, presented through detailed Excel files and visualizations for easy comprehension. Experiments such as capacity extension, new production locations, and SKU reduction were designed based on questions or considerations raised by the company. The model is able to prove the existing insights and experiences within the company, facilitating discussions about improvements and expansions. For instance, the model for a new production location can support the development of a business model for a new factory, while the focus on SKU reduction can be expanded to include the reduction of recipe variety. Due to the modularity of the model and its high level of comprehensibility, this research provides valuable insights for both present and future decision-making.

7.4 Recommendations

The different experiments conducted on the current situation of De Heus reveal that significant cost savings are already possible when the current case is optimized. For De Heus, it does not require any additional investments or cultural changes to achieve these savings, so it is advised to implement the results of the current case. The model offers a broader perspective on shifting feed between locations, assuming customers are willing to change pickup locations for cost-effectiveness. However, currently, De Heus does not encourage customers to switch locations, despite indications from discussions with regional customers that they would be open to the idea. De Heus should initiate negotiations with customers in this regard. Additionally, De Heus is currently focusing on producing as much as possible to keep stock levels high. This results in high inventory levels in the warehouse, decreasing the flexibility of slots in the warehouse and leading to unnecessarily high inventory costs. De Heus is currently only focusing on reducing production costs, without considering inventory costs or the impact on transportation costs when making production decisions. Therefore, it is advised to also track these costs, as it allows for better analysis of changes in the finished goods supply chain.

In the experiments regarding expansion, it is advisable for De Heus to expand the Dong Nai factory and construct a new factory in the Binh Duong II or Binh Phuoc area soon. A new location would reduce production costs and provide flexibility for further optimization due to the new factory's production capabilities. Regarding the number of SKUs, it is recommended to continue reducing them, especially those with low volumes. De Heus should evaluate whether these SKUs are profitable. Moreover, De Heus should actively reduce or merge recipes to minimize flushing times. A smaller number of recipes would increase production run sizes and reduce total flushing time. It is suggested that De Heus conducts the optimization process annually to assess if any changes are necessary in production shifts or customer assignments. The model could serve as support for these decisions.

7.5 Limitations and future research

For the optimization of the model, it was necessary to implement a certain 'external transportation cost' to force the model to also optimize the distance for customers to drive to their pick-up points. In theory, it does not cost De Heus anything if customers need to drive longer distances than before to pick-up the feed. Thus, if no costs were implemented, the model would always choose to minimize costs for De Heus instead of also considering the driving distances for customers. To address this in the current case optimization, an 'external transportation cost' was added. This cost is estimated based on the internal transportation costs provided by the third-party logistics company and must be interpreted as a discount given to customers to convince them to drive longer distances. The model assumes that this discount is the same for every customer, but in reality, it takes time to convince customers to drive longer distances, leading to negotiations where the discount per ton per kilometer may vary for each customer. Therefore, while this fixed discount approximates the actual situation, the cost savings depend on the amount of discount given to customers. Changing the fixed value currently used would lead the model to make different decisions based on the new most cost-effective solution. To improve the model, De Heus could examine these external transportation costs and investigate the appropriate discount to offer customers. Multi-objective optimization is also optimize the extra driven kilometers without the need to implement specific costs during the optimization. In this case, De Heus only needs to decide the maximum amount of extra kilometers customers need to drive, and the model can minimize total costs based on that.

Another limitation is that the model optimizes based on fixed input data. In reality, demand forecasts and customer orders mean factories have approximate expectations for upcoming weeks, but this is not as fixed as the data currently used to optimize the model assumes. For now, demand data from December 2023 is used because it was the most recent available. Within this data, the demand for week 4 was already known at the beginning of week 1, allowing the model to plan to fulfill all demand in week 4. In reality, this demand is not known, but because demand patterns are predictable, the model still generates a valuable solution. However, De Heus also wants to optimize the forecasted situation for a future period. To improve optimization results, it would be better to implement forecasted demand and customer data into the model to make decisions, for example, regarding building a new factory based on forecasted demand data for the next 10, 20, or 30 years.

During the process of analyzing the structure of supplying depots from factories, the logistics department noted that the costs of transporting finished goods from the factory to the depot are fully passed on to customers. According to the department, transferring feed to that location is cheaper for De Heus because they have a scale advantage. they can combine multiple orders of feed and transport them in one truck. Theoretically, this would result in all feed being transported as close as possible to the customer, as it would be the most cost-effective solution. However, when analyzing current customer assignments and corresponding sales volumes, it was noted that sales at the depots were relatively low compared to sales at the factories. Additionally, the company is currently closing most depots due to excessive costs. Therefore, in consultation with the COO and production managers, it was decided not to implement this in the model, as proposed by the logistics department, because the results would not have been comparable to the current case. Nonetheless, for future research, it would be interesting to examine how these internal transportation costs are passed on to customers and how this could lead to win-win situations if De Heus could use the economies of scale advantage in supplying pick-up points.

The model currently cannot use variable run size during optimization. This means that the model estimates the amount of production runs based on product categorization (low, medium, or high), each with its own average production run size. In practice, production run size varies, allowing production planning to vary the tons produced in one run. Due to linearity restrictions, it was not possible to implement this in the current model. To implement it, it would have been interesting to focus more on production (and inventory planning) rather than also on transportation routes. In that case, customer assignment and transportation routes would be assumed to be fixed, while a detailed production planning per day would be created, varying the length of production runs. Most customers were assigned to the closest possible pick-up location, optimal in terms of customers but possibly less cost-effective for De Heus. However, this actual situation could have been used as fixed input for customer assignment, providing an opportunity to make more precise production planning simultaneously optimizing three factories. Currently, De Heus makes a production planning per factory, whereas it would be more efficient to plan production per region to see where demand occurs and how to meet it. Therefore, for future research, it would be interesting to examine production planning in detail to build a planning model able to plan demand per day in different factories, offering the opportunity to decide if longer or shorter production runs are necessary.

In the end, the model was able to solve the current case within a reasonable time of 3 hours. This was achieved without creating a heuristic to tackle theoretically larger problems with a systematic approach. Since this model could solve the current case, the decision was made to perform experiments providing relevant solutions to the company, rather than investing more time in developing a heuristic capable of solving the problem faster and potentially addressing larger problem instances. This decision was made in consultation with the company and supervisors because it was important to deliver results analyzable in detail. Unfortunately, it could be possible for further research to develop a heuristic capable of solving even larger problem instances, as the analysis on artificial data instances showed that computational time increased when capacity restrictions were almost met. If this heuristic were developed, the literature review of this thesis provides valuable information about different heuristics. The decomposition heuristic appears well-suited to this situation, as it enables breaking down the problem into smaller parts, which are then optimized individually in sequence to derive the best possible overall solution.

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A Appendix

A.1 AUGMECON2 flowchart

This part of the appendix containts the flowchart of the AUGMECON2 algorithm proposed by (Mavrotas & Florios, 2013).

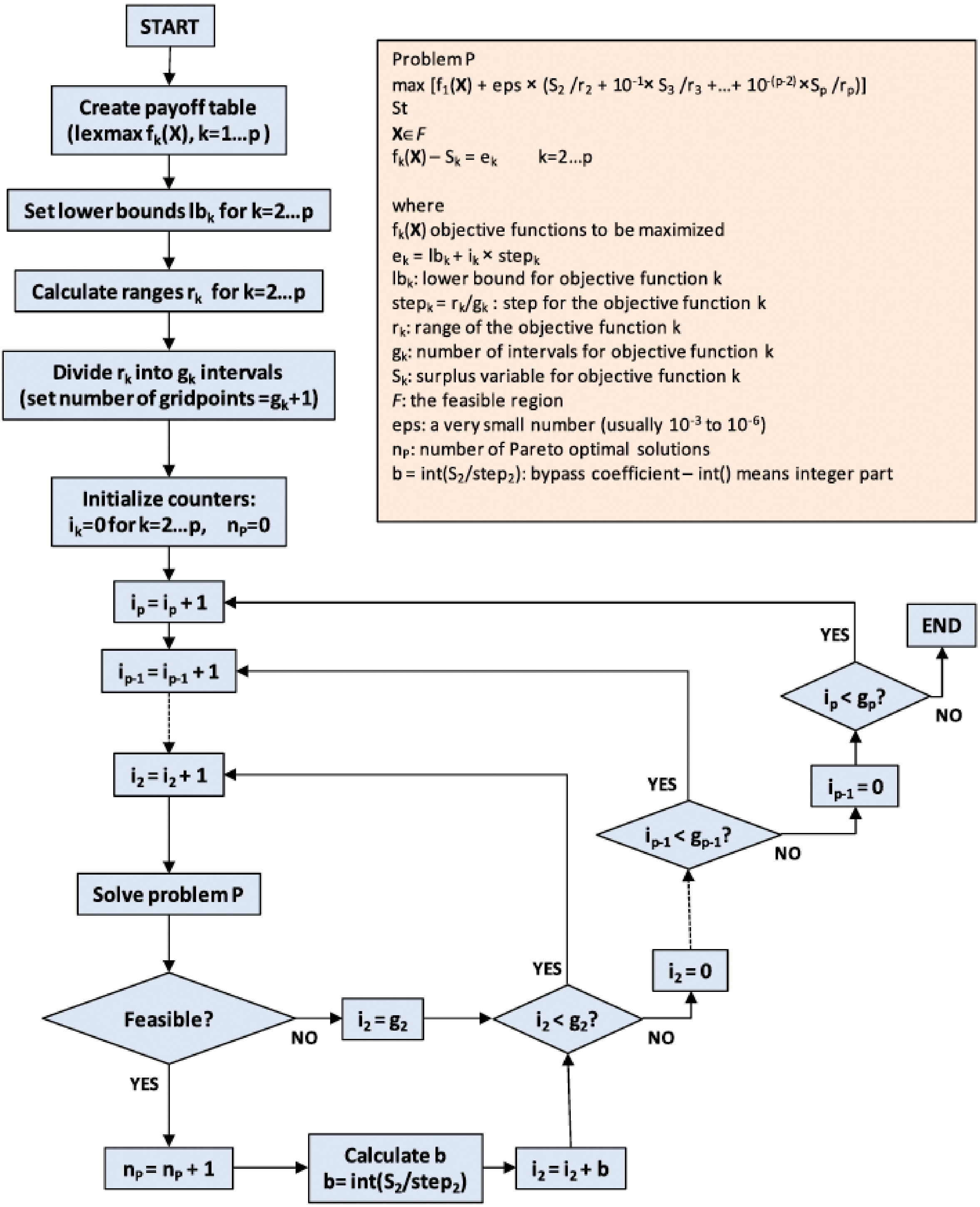

Figure A.1: AUGMECON2 flowchart

A.2 Experimental results figures and tables

This part of the Appendix containts the results of the experiments in additional figures and tables.

The Figures provides the following information:

- Paretro front diagram for 20, 50 and 100 customers: Figure A.2, A.3 and A.4.

The tables provides the following information:

- Multi-objective optimization experiments for 20, 50 and 100 customers. Table: A.1, A.2 and A.3

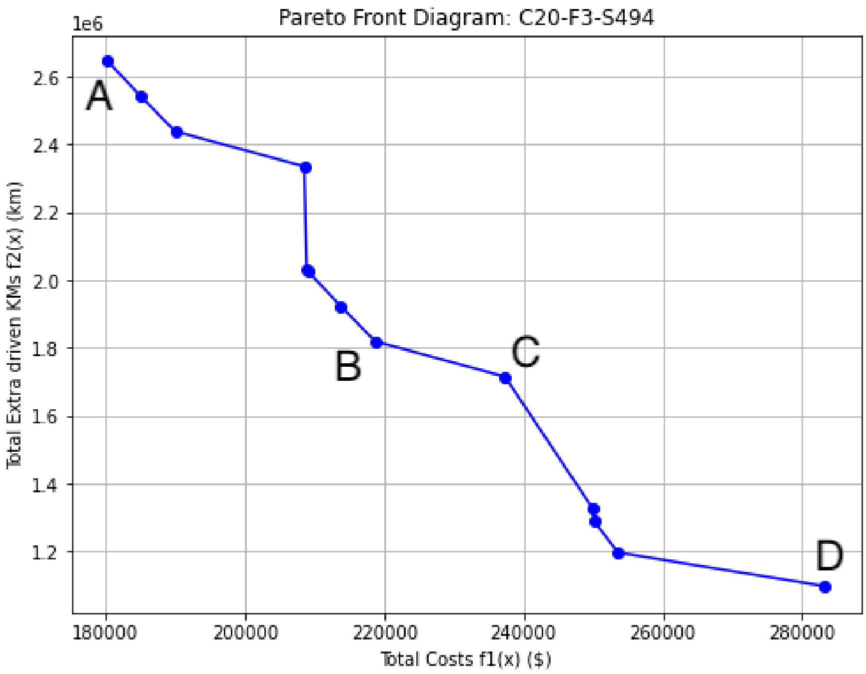

Figure A.2: Pareto front: C20-F3-S494

Data instance	#Iteration	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C20-F3-S494 (A)	0	$180,261.89	2,645,184.13	1.72 s	0.0000 %
C20-F3-S494 (B)	7	$213,712.77	1,922,891.99	4.57 s	0.0027 %
C20-F3-S494 (C)	8	$218,708.47	1,819,189.90	7.48 s	0.0000 %
C20-F3-S494 (D)	15	$283,290.18	1,097,649.78	1.07 s	0.0000 %

Table A.1: Experiment *E6* - Pareto front values C20-F3-S494

Figure A.2 and Table A.1 show the Pareto results for 20 customers. This Pareto front shows 12 of the potential 16 data points. Data point C is interesting because the costs increase significantly to reduce fewer extra driven kilometers. This observation was also made in the problem instance with 10 customers, where at a certain point, the cost per kilometer reduction became very high. However, it can also be observed that after point C, the reduction becomes cheaper again.

Figure A.3 and Table A.2 show the Pareto results for 50 customers. This Pareto front displays all 16 potential data points, resulting in a smooth Pareto front. Notably, from point B to C and onwards to point D,

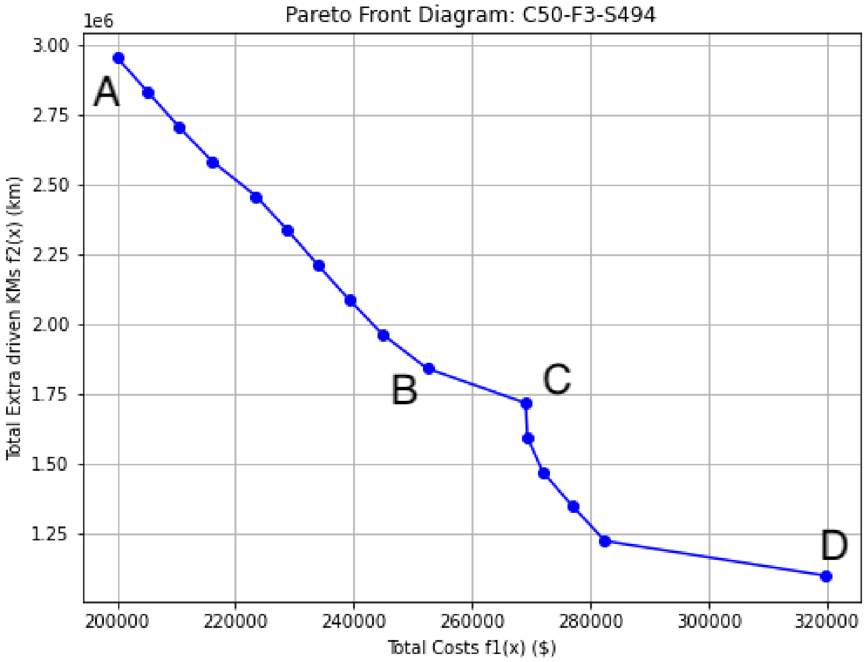

Figure A.3: Pareto front: C50-F3-S494

Data instance	#Iteration	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C50-F3-S494 (A)	0	$200,073.52	2,952,039.04	4.14 s	0.0000 %
C50-F3-S494 (B)	9	$252,423.23	1,839,163.051	15.60 s	0.0088%
C50-F3-S494 (C)	10	$269,057.93	1,715,514.11	18.81 s	0.0000 %
C50-F3-S494 (D)	15	$319,662.99	1,097,649.78	1.55 s	0.0000%

Table A.2: Experiment *E6* - Pareto front values C50-F3-S494

the graph gives an interesting trend. Between B and C, it becomes increasingly costly to reduce the amount of kilometers, but beyond point C, the cost decrease accelerates, leading to a steeper downward slope. Toward point D, it becomes evident that the last kilometers are the most expensive.

Data instance	#Iteration	Costs ($) - $f_1(x)$	Extra driven KMs (km) - $f_2(x)$	Computational time(s)	gap (%)
C100-F3-S494 (A)	0	$257,150.33	4,876,539.08	14.64 s	0.0000 %
C100-F3-S494 (B)	9	$344,415.52	2,609,122.45	45.43 s	0.0077%
C100-F3-S494 (C)	14	$413,251.93	1,349,554.92	36.17 s	0.0099 %
C100-F3-S494 (D)	15	$465,506.15	1,097,649.78	4.27 s	0.0000%

Table A.3: Experiment *E6* - Pareto front values C100-F3-S494

Figure A.4 and Table A.3 show the Pareto results for 100 customers. This Pareto front displays all 16 potential data points, leading to a smooth curve. Moreover, the costs between points C and D increase rapidly compared to the reduction in the number of extra kilometers.

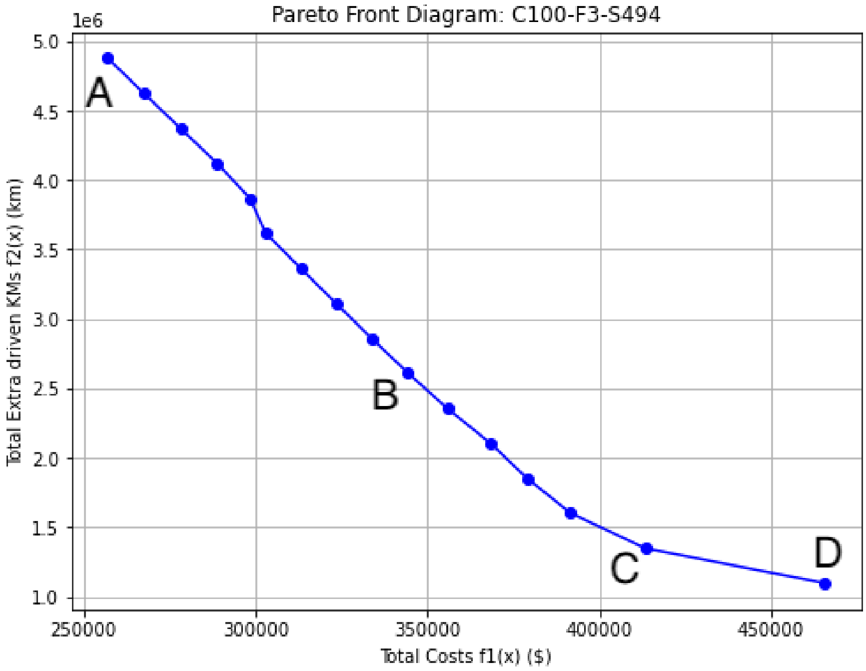

Figure A.4: Pareto front: C100-F3-S494