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Performance of Dynamic Lot Sizing Order Policies - A Case Study on Lumpy Demand

Author
H.G.T. KOK

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University of Twente

Industrial Engineering and Management
Drienerlolaan 5
7522 NB Enschede

PO Box 217
7500 AE Enschede
The Netherlands

+31 (0)53 489 9111

info@utwente.nl
www.utwente.nl

VMI Holland BV

Supply Chain Innovation
Gelriaweg 16
8161 RK Epe

PO Box 161
8160 AD Epe
The Netherlands

+31 (0) 578 679 111

info@vmi-group.com
www.vmi-group.com/

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Student

H.G.T. Kok
Industrial Engineering and Management
University of Twente

Supervisors

University of Twente

Dr. M.C. Van der Heijden
Dr. P.C. Schuur

VMI Holland BV

Ir. B. Brummelhuis
Ir. H. Esveld

Management Summary

VMI is market leader in manufacturing production machinery, specialized in the tire, rubber, can, and care industries. VMI works project-orientated since all projects are (re)designed to suit the customers' requirements. The purchase process is characterized as project-oriented since 95% of the stock keeping units (SKUs) are purchased on a lot-for-lot (LFL) basis. This results in high ordering costs. VMI finds it hard to optimize and maintain its order policy to balance order and storage costs while taking discounts and risks regarding obsolescence into account. This study focuses on order policy improvements. This research answers the following research question:

What are appropriate order policies for VMI, and which order policy suits which SKU?

The current situation is analyzed in order to find a suitable order policy for the characteristics of VMI. VMI's engineering department (re)designs a standard machine to the customer's requirements. There are two groups of SKUs, SKUs which are part of the standard machine and customer-order-specific SKUs. The last group has a high risk of becoming obsolete. The purchase department has eight weeks of logistical time to purchase the SKUs after engineering is finished, so the demand is known at least eight weeks in advance. Next, the availability of discounts can enlarge the optimal order quantity. VMI uses a safety time of 10 business days to cope with delays in the supply chain. This research assumes deterministic lead times since 97% of the deliveries do not exceed this safety time. Lumpy demand characterizes 99% of the SKUs. The service level is based on the probability of no stock-out occurrence per replenishment cycle for SKUs with a lead time larger than eight weeks. The service level is between 95% and 99%, depending on the SKU price.

An order policy selection model selects a suitable order policy for every SKU. The decision is based on the lead time, prices, discounts, expected demand, order, and holding costs, and whether the SKU is a standard or customer-order-specific part.

SKUs with a lead time larger than the logistical time have partly stochastic demand and require a continuous review policy (s, Q) with fixed parameters since non-stationary demand is not included in the forecast. The reorder point is based on the demand during the effective lead time. The effective lead time is the lead time minus the eight weeks in which demand is known in advance. The known demand has been reserved and removed from the inventory position. Furthermore, discounts are incorporated in the decision of the order quantity.

Different dynamic lot sizing (DLS) methods are investigated to find an order policy that optimizes the orders using the known demand upfront. Forward dynamic programming, Silver-Meal, and maximum part-period gain are DLS methods of interest. The DLS methods are restricted to demand during the first eight weeks since the quality of the current forecast is lacking. The EOQ order policy is of interest when the optimal order quantity exceeds the known demand in the first eight weeks.

The choice between the different DLS order policies is based on a simulation. A simulation on representative demand data quantifies the performance of different order policies. The different order policies are used in a simulation of one year of historical data to quantify the performance. The order policy with the lowest costs during the simulation also has the lowest expected costs for future use and is therefore chosen as the order policy.

This research shows that the forward dynamic programming algorithm has the best performance of the different dynamic lot sizing methods. The algorithm combines the demand of the different production orders into an optimal purchase order while taking the discounts into account.

The order policy selection model is implemented in Python, tested, and validated using a toy problem. The current and proposed order policies of the selection model are compared. The relevant costs exist of the ordering plus holding costs minus the discount. The relevant costs decrease from €¿ million to €¿ million by switching to the proposed order policy. The cost reduction of €¿ million is mainly caused by reducing ordering costs using larger orders instead of ordering LFL. The inventory position increases, resulting in a rise in holding costs; the average capital on stock increases. An extrapolation of the savings for all the SKUs results in a cost reduction of between €¿ million and €¿ million a year for all SKUs needed in production in Europe.

The toy problem exists of 3274 different SKUs. This is 10% of the active SKUs for VMI. 11% of the SKUs have a lead time larger than eight weeks, so they have a (s, Q) order policy with a safety stock to deal with the stochastic demand. 61% have FDP as the optimal order policy. The remaining 28% of the SKUs benefit from a simple EOQ order policy.

The mechanical outsource (OS) and electrical original equipment manufacturer (OEM) commodities benefit the most from the FDP order policy. Mechanical OS contains SKUs with discounts, and those SKUs have demand concentrated over time. The FDP is capable of grouping the demand. The electric OEM commodity has discounts and high variability in demand. FDP excels in this situation.

The current ERP system should be modified in order to make use of the FDP. The cost saving relative to the current situation will be 20% less compared to when the FDP algorithm is not used but replaced by LFL and EOQ order policies already in the ERP system. Extending the ERP system with the FDP should be done if the savings exceed the implementation and maintenance costs.

The relevant costs decrease if the logistical time increases since there is less uncertainty, so less safety stock is required. Furthermore, more production orders can be combined into a single purchase order since the DLS methods can look further ahead. The relevant costs stabilize at a logistical time of 30 weeks, which indicates that the forecast should look up to 30 weeks ahead.

The order policy should be determined or reassessed when the input parameters change. The input parameters are costs, demand, lead time, and price for an SKU. The recommendation is to run the order policy selection model every month to update the order policy, including parameters, to prevent the order policy becomes outdated. The total computation time to determine the order policy, including parameters, for all active SKUs is 12 hours. Most of the time is spent simulating the different DLS methods to quantify their performance. A longer logistical time and more discounts result in lower costs but increase the computation time.

The implementation consists of three main steps. First, the supply chain engineers, in collaboration with the supply buyers, should verify if the data quality is sufficient. Second, the supply chain engineers are responsible for providing a user interface, loading data, and writing results to the system, as well as for the ease of use of the selection model. Third, extend the use by incorporating spare parts, multiple suppliers, and restriction of order quantities, such as a minimum order quantity.

Acknowledgments

This master thesis is the result of the final part of my master study Industrial Engineering and Management at the University of Twente. The thesis does research into order policies, especially the dynamic lot sizing methods how multiple production orders can be combined into a single purchase order. This study has allowed me to deepen my understanding of order policies, further specializing in this vital area.

I am deeply grateful to VMI for providing me with the opportunity to undertake my graduation project in its supply chain innovation department. I extend my heartfelt thanks to Bob Brummelhuis for his invaluable guidance and feedback during our regular meetings. I also acknowledge the immense contribution of Henk Esveld and his extensive knowledge about the company's processes, which greatly enriched my project.

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Rik Kok,
Epe, May 2024

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Glossary

Acronym	Description
ADI	Average Demand Interval
BOM	Bill of Materials
CTO	Configure to Order
CV	Coefficient of Variation
DLS	Dynamic Lot Sizing
EOQ	Economic Order Quantity
EPP	Economic Part-Period
ERP	Enterprise Resource Planning
ETO	Engineer to Order
FDP	Forward Dynamic Programming
LFL	Lot-For-Lot
LUC	Least Unit Costs
MOQ	Minimum Order Quantity
MPPG	Maximum Part-Period-Gain
MSM	Modified Silver-Meal
OEM	Original Equipment Manufacturer
OQI	Order Quantity Increment
OS	Outsource
PP	Part-Period
PTO	Purchase-to-Order
PTS	Purchase-to-Stock
QLTCS	Quality, Lead time, Technique, Costs and Sustainability
ROI	Return on Investment
SCI	Supply Chain Innovation
SCV	Squared Coefficient of Variation
SKU	Stock Keeping Unit
SM	Silver-Meal
VAR	Value at Risk
WW	Wagner-Whitin

1 Introduction

This master's thesis researches inventory management with a focus on order policies. The study is conducted at VMI. Section 1.1 introduces the company VMI. The subsequent section, Section 1.2, describes the problem that occurs at VMI. Section 1.3 states the research questions, including sub-questions. This section also describes the methodology. Section 1.4 defines the scope of this study. Section 1.5 specifies the deliverables.

1.1 Organization

VMI is a market leader in manufacturing production machinery, specialized in the tire, rubber, can, and care industries. VMI focuses on developing and innovating products and solutions to meet current and future manufacturing demands with excellent service [3]. VMI was founded by Jan de Lange in 1945, and its original name was Veluwse Machine Industrie. VMI started with repair and construction work for the Dutch railways. Now, the company has over 1800 employees in 9 countries. In Epe, the headquarters, research and development takes place. Epe is the main location and has over 1000 employees. The production facilities are located in Epe (Netherlands), Leszno (Poland), and Yantai (China). In addition, there are several sales offices around the world. VMI is part of the TKH Group since 1985. TKH Group is a listed holding which consists of multiple technological production companies.

VMI offers different types of machines, each available in a standard configuration that can be modified to customer wishes where needed. Figure 1.1 shows a tire-building machine that VMI produces. The quotation is based on the standard machine where customers have the possibility to add function packages and may even have specific wishes for redesign of a machine part. These functional packages can be extensions or are related to the machine's input and output processes. VMI's customer order decoupling point lies between configure to order (CTO) and engineering to order (ETO), resulting in a high mix and low volume material flow.

Figure 1.1: Tire Machine

1.2 Problem Statement

VMI operates project-oriented and purchases project-based whenever feasible. Every machine is redesigned to the customer's requirements, which results in all unique machines with their unique bill of materials (BOM). The current order policy of VMI is to purchase on a project basis whenever feasible. This enhances that the order policy is mainly lot-for-lot (LFL), resulting in high ordering costs. By optimizing the order settings, VMI can potentially reduce ordering costs, negotiate lower purchase prices through lot-sized-based discounts, and take advantage of economies of scale. However, it is important to consider that higher order quantities increase holding costs due to longer inventory holding periods. Therefore, the potential cost savings from bulk purchasing must outweigh the additional costs associated with storage. In light of this, VMI has posed the following question: *How can VMI effectively incorporate lot-sized based discounts and ensure that the implemented policies remain appropriate and relevant?*

VMI wants to improve its insight into the supply chain costs. VMI is not reviewing its order policy regularly and, therefore, does not adapt to changes in demand or prices by changing its order policy for a stock keeping unit (SKU). A demand increase does not trigger a deviation from the order policy. VMI faces optimization challenges due to a lack of detailed knowledge regarding the cost of purchasing, handling, and storing individual items. As a result, the consequences of procurement decisions are unknown on item level. VMI aims to have a more proactive approach to maintain an appropriate order policy that suits the current situation.

VMI's supply chain is highly complex due to several factors. First of all, VMI has a diverse product portfolio since it manufactures different kinds of machines. VMI has machines for the tire and rubber industry and the pharmacy. Moreover, production can be subject to delays or changes in production location. Consequently, these factors combined have led to a complex material flow within VMI, challenging VMI to adapt and maintain optimal order policies.

Lot-sized-based discounts add complexity to the problem as well. Suppliers offer discounts to increase order quantities for reasons like economies of scale, achieving cost reduction, and having a competitive advantage. Increasing the order quantity for VMI is necessary to avail of these discounts. Consequently, as the number of orders decreases, the associated ordering costs are also reduced. However, the consequences of increasing order quantity for the internal material flow are partly unknown to VMI, making it impossible to balance order and storage costs. VMI aims to save costs by balancing and optimizing the trade-off between order and storage costs.

So all these problems and aspects lead to the following core problem:

VMI needs to optimize and maintain its order policy to balance the order and storage costs.

1.3 Research Questions

The main research question aims to offer a solution to the core problem.

What are appropriate order policies for VMI, and which order policy suits which SKU?

This research aims to identify relevant order policies for VMI and then select the most suitable order policy for every SKU. The main research question is further divided into five sub-questions, corresponding to the five phases described below.

Analysis of Current Situation This phase examines the existing approach to describe the process and its characteristics in order to identify opportunities and possible threats. The process flow is described so it can be modeled at a later stage. The analysis of the current situation will be based on interviews with the relevant departments: warehousing, purchase, finance, and engineering. Next, historical data from the ERP system is analyzed to quantify the current performance and verify if assumptions are realistic. Moreover, the direction of interest is investigated by calculating the potential improvements of the order policy on holding and ordering costs.

What is VMI's current situation regarding the order policies of SKUs?

Literature Review Existing knowledge related to order policies is reviewed, and any research gaps are identified during the literature review if present. Solutions and approaches found in the literature will be used in the next phase to build the solution. This entails considering the effects of costs and determining the relevant parameters that should be incorporated into the solution.

What relevant order policies are described in the literature, and in which situations are they promising?

Solution Design The solution design phase connects the promising order policies identified in the literature to SKUs. Given the highly complex nature of the problem, the proposed solution is expected to be a selection model that provides the most suitable order policy for a SKU. The selection model selects the most appropriate order policy by outweighing the different cost components while comparing promising approaches identified from the literature revealing the performance of these different approaches.

Which method is appropriate for selecting the most suitable order policy for SKUs?

Result Analysis Verification and validation take place using historical data to optimize and calibrate the parameters for the order policy selection model. This phase analyzes the performance and defines experiments to test the performance under different circumstances. The consequences of the proposed policy are described. A sensitivity analysis assesses the solution's robustness.

What is the performance of the selection model?

Implementation Plan The final phase presents the implementation plan. This plan is a roadmap that outlines the necessary steps to integrate the research findings into VMI's operations and states the responsible departments. Additionally, it will propose enhancements to VMI's ordering process.

How can the research findings be implemented into the organization of VMI?

This study aims to provide valuable insights into optimizing the company's order policies while balancing the order and storage costs by structuring the research process into these five phases and corresponding sub-questions.

1.4 Research Scope

This research is conducted within VMI's Supply Chain Innovation (SCI) department. SCI oversees logistic processes and focuses on cost savings throughout the supply chain. The following paragraphs explain the scope of this research.

This research focuses on improving the order policy and excludes improvements in the forecast. Although a good forecast is required for the order policy to perform well, this research focuses on the core of order policy for the sake of time.

The process flow is defined from the purchase of SKUs to the end of storage in the warehouse before the SKUs are needed in production. This enhances that supplier selection is outside the scope. Furthermore, changes in production planning resulting in different required delivery dates for SKUs are also excluded from this research.

The happy material flow of production orders for Europe is included. Section 2.1 describes the happy flow in detail. The happy flow is the normal flow for production without exceptions. Spare parts are excluded from this research. Rush orders and interchangeability of SKUs are excluded from this research as well. The exceptions are excluded from this research since human intervention is required to review the possibilities for that specific situation. Next, the floor stock items are excluded since it is outsourced to an external company.

The purchase amount, ordering, inbound, and holding costs are included in the analysis. These costs impact the order policy and are included for this reason. Costs like transportation, packages, penalties for small orders, and degradation of items in storage are excluded from this analysis since they are unlikely to influence the order policy. Section 2.4 describes these costs in more detail, including a reason for their exclusion.

All units lot-sized based discounts per SKU are included. There are no discounts over multiple SKUs, with one exception. Some suppliers give a discount if VMI purchases more than a certain amount per year. Since demand is independent of an order policy, the order policy will not influence this discount. Therefore, this discount will not be incorporated into the research.

Every SKU has a preferred supplier based on the performance of QLTCS (quality, lead time, technique, costs, and sustainability). Only the preferred supplier is considered, resulting in one supplier per SKU. The costs related to the order policy apply to each order line instead of the entire order. Next, the order and handling costs are for every order line, not the order itself. Furthermore, the cost of transportation is not included in this research. So, the order policies for a single SKU are independent of the order policies of other SKUs since this research deals with a single SKU, single supplier problem.

There are no restrictions regarding the quantity of orders from suppliers. Only 1.5% of the SKUs have a minimum order quantity (MOQ) or order quantity increment (OQI). The OQI is a fixed quantity with which the order size can be increased, resulting in not all the order quantities being feasible. These are typically cheap SKUs where the MOQ and OQI correspond with the package quantity. The MOQ and OQI are out of scope since this occurs only for a small and relatively unimportant group of SKUs. Next, there are no suppliers which have a maximum order quantity.

Lead times are seen as deterministic since 88% of the deliveries are on time. Analysis is done in Section 2.5.4 regarding the lead time to determine if this assumption is valid. Moreover, most delays are due to COVID-19 for a particular product group.

Safety stock required from an order policy perspective is included. For example, a stochastic order policy requires safety stock to deal with uncertainty. Safety stock to cover the demand for spare parts or other reasons is excluded from this research.

The risk of SKUs becoming obsolete is included in this research to prevent SKUs from being stocked without future demand. The risk of item damage, degradation in storage, fire, etc., is excluded from this research since limited data is available on this topic.

1.5 Deliverables

This research has the following three deliverables:

Figure 1.2: Model Overview

Deliverable 1: Deliver a selection model that provides the best suitable order policy for every SKU, including the relevant parameters such as order quantity and safety stock. The model decides the order policy based on the SKU characteristics and general parameters resulting in the lowest expected costs.

Deliverable 2: Analyze and communicate the impact and consequences of implementing the new order policy, providing a clear understanding of its effects on the supply chain. Develop an implementation plan outlining improvements to the order policy based on the research findings, facilitating its enhancement.

Deliverable 3: Advise VMI how to improve the order policy in order to increase operational efficiency and minimize costs.

2 Current Situation

This chapter describes the current situation by answering the first research question: *What is VMI's current situation regarding the order policies of SKUs?* It starts with Section 2.1, offering insight into VMI's structure by describing the involved departments of the logistical process. Section 2.2 consists of a stakeholder analysis to describe the different interests regarding this subject. Section 2.3 describes the current order policy and the reasoning behind it. Section 2.4 describes the costs associated with the order policy. Section 2.5 provides insight into the current situation using descriptive statistics. Section 2.6 ends with a conclusion.

2.1 Involved Departments

This section describes the departments that influence the order policy, highlighting their respective roles and impact on the subject. This will provide a better understanding of the process and the influence of the different departments on it, making it possible to refer to the departments at a later stage.

Operation Control Department The operation control department is responsible for planning the production of the different processes and dictating the production location for all machine modules in order to balance the workload. Specifically, the operation control team takes charge of orchestrating the production of modules, carefully considering the timing and selecting the production locations where these modules will be constructed.

Engineering The engineering department develops and redesigns standardized machines to fulfill customer requirements. These customizations can range from selecting the exterior color of the machine to tailor-made in and output trajectories for the materials. Additionally, the company offers various machine packages. The order engineering department translates the customers' wishes and needs into a machine redesign. Machine redesigns ultimately give rise to a different machine with its unique BOM, resulting in the birth of new SKUs. Machine revisions occur in response to system improvements or new industry requirements. Consequently, the BOM, which includes standard and modified components, undergoes modifications. Next, the machine design is translated into various modules with the required materials to build the machine modular. Afterward, the SKU demand in ERP is available once the engineering step is finished.

Due to this way of working, VMI's SKUs can be divided into two groups: SKUs part of the standard and customer-order-specific SKUs. The last group is highly likely to become obsolete and should not be purchased-to-stock. The SKUs that are part of the standard and can be purchased-to-stock with a small risk of becoming obsolete.

Sourcing and Purchase Sourcing buyers select a preferred supplier for each SKU based on the supplier's QLTCS (quality, lead time, technique, costs, and sustainability) performance. Part of these negotiations, agreements may be reached on annual purchase amounts, allowing for the opportunity to secure competitive prices. Furthermore, agreements to stock SKUs at the supplier to decrease the lead time often also contain a minimum order quantity on an annual basis or extra costs. Purchasers place orders to procure the SKUs according to the order policy in response to demand requests from production. Section 2.3 describes the current order policy in detail. The order price is based on volume, number of orders, lead time, and the negotiation agreements.

The time between engineering is finished and production starts is at least 50 business days. SKUs should be delivered ten business days before the required date. These ten days are the safety time and do not raise any problems in production if a SKU is delivered within these ten days. The logistical time is the time between engineering being finished and the required date. This is at least 40 business days for VMI. This enhances that all SKUs with a logistical time shorter than 40 business days are exceptions to the standard flow.

The need for specialized expertise in each category drives the decision that purchasers focus on a specific commodity. A commodity is an SKU characteristic used to describe the type of industry from which the SKU comes. There are four different commodities.

- Mechanical: The opportunity for mechanical SKUs is consolidated purchasing. Production of mechanical outsourced SKUs often involves setup time; therefore, larger lot sizes are relatively cheaper since the setup costs can be divided over more items. A revision change is one of the causes of SKU obsolescence. Long lead time SKUs with special materials are hard to get and classified as critical SKUs.

- Electrical: The risk for electrical is extended lead times and sensitivity to worldly events, like pandemics or political decisions that impact trade regulations. For example, the COVID-19 pandemic has disrupted supply chains, resulting in scarcity, leading to long lead times and higher prices, creating uncertainty about future pricing trends. The market dynamics in the electrical sector are currently characterized by instability and unpredictability.

- Mechatronics: The risk of damaged goods if SKUs are in stock.

- Maintenance, repair, and operations (MRO), as well as non-product related (NPR) SKUs, are two categories of SKUs that are not directly related to products for the customers but related to internal processes. A part of MRO and NPR expenses are incidental in nature. While these expenses are included in the data analysis to provide a comprehensive understanding of the overall expenditure, it is essential to note that no specific order policy is required for managing these incidental expenses. They are not subjected to the same ordering considerations as regular inventory SKUs, as they are often ad-hoc or sporadic and do not follow a predefined procurement process.

SKUs can also be classified as outsourcing (OS) or original equipment manufacturer (OEM) SKUs next to the commodity classification. The first category, OS, consists of parts manufactured explicitly for VMI's unique requirements, made from a drawing made by VMI. Other companies cannot purchase these SKUs since they are manufactured according to a drawing made by VMI. The second category, OEM, are standard catalog SKUs from various suppliers commonly used by other companies.

Part of the SKUs is customer-order specific and cannot be used for other projects. Those SKUs are only needed for one order and will most likely not be needed in the future. They cannot be purchased-to-stock but should be purchased-to-order on a lot-for-lot basis.

Warehouse Department The warehouse department facilitates storing and handling materials. The warehouses take care of the inbound process, checking if the purchase order is complete and storing the SKUs before delivering them to production. There are three main warehouses. Two are in the Netherlands, and one is in Yantai, China. The warehouses receive SKUs from the suppliers and store those until they are required for production. Part of the demand goes from the warehouse directly to the customer. This flow consists predominantly of spares.

Determining whether certain SKUs have been phased out and are no longer needed poses a challenge. This is particularly true for SKUs with low demand since those SKUs can be required in the future. This results in occupied storage space for irrelevant SKUs. SKUs in storage without five years of demand are checked to see if they belong to storage or should be disposed of. There are no active system checks to see if the stored parts are still relevant.

Production The machine is manufactured and assembled in (sub)modules. Production produces the (sub)modules as the work preparation department has proposed on the date set by operation control. The final products are shipped to the client's factory, installed, and tested. The required materials for each module must arrive at the relevant production hall one day before the scheduled production. Delays in the supply chain caused by extended lead times can adversely affect production, leading to undesired production delays. Furthermore, the production location may change, resulting in the required SKUs being needed at a different location. Currently, the impact of varying production locations remains unknown to VMI.

Production orders are the orders which are issued to production. A production order contains only one type of SKU for one project required for a specific week.

Service VMI also provides service, which is mainly the delivery of spare parts. Most of the maintenance is done by the customers themselves. Service also consists of repairs if needed. Delivering excellent service is crucial to ensure optimal uptime for customers. Service has become a significant and expanding division within the company, accounting for a substantial part of the total turnover.

Essential spare parts with high replacement rates or crucial parts for performance are stocked at customer locations. This ensures that the machines experience minimal downtime. The expected demand for spare parts is determined based on historical data and insights from the engineering department. The service department decides which SKUs to stock based on the lead time and the past insights gained.

2.2 Stakeholder Analysis

Performing a stakeholder analysis reveals the interests related to SKU order policies. Appendix A contains the complete stakeholder analysis. The key findings of the stakeholder analysis are summarized below. The stakeholder interest regarding order policies represents potential assessment criteria for selecting the order policies. The findings are presented in order of importance.

Overall costs Efforts are directed towards achieving the lowest overall costs, aiming to maximize profit. The overall costs exist out of purchase amount, ordering, and storage cost.

Item availability On-time delivery of SKUs is crucial, ensuring that the required SKUs are available when needed.

Working capital A minimal inventory position is preferred since this enhances the minimal capital investment, resulting in capital being used for other purposes, like R&D investments.

Easy and smooth process A simple, efficient, and smooth procurement process is preferred to streamline the process and, adjust to new situations, and minimize required human resources. Fast determination of the order quantity is part of this to adapt quickly to changes in demand.

In consultation with the stakeholders, it is determined that the most relevant objective is to base safety stock on the probability of occurrence of a stock-out per replenishment cycle or costs of a stock-out. At first, the occurrence of a stock-out should be independent of the demand of a SKU. SKUs that are frequently used may not have more stock-outs compared to SKUs with low occurrence of demand. Next, problems arise if an SKU is one day too late in production. This results in production delays or rescheduling. Therefore, no distinction is made on how long the SKUs are too late.

2.3 Current Order Policy

The order policy determines the quantity and frequency of stock replenishment. Minimizing inventory is key for VMI, although maintaining some inventory is unavoidable. In particular, long lead time SKUs must be purchased in bulk and kept in stock. Long lead time SKUs exceed the logistical time of 40 business days, the time between engineering is finished and the required delivery date.

VMI has two different order policies: ‘to-order’ and ‘to-stock’. ‘To-stock’ SKUs are purchased in bulk and stocked, while those ‘to-order’ are purchased lot-for-lot. 94% of the SKUs are purchased ‘to-order’ and the remaining 6% to ‘to-stock’ with a continuous review policy and fixed order quantity.

VMI has categorized parts as ‘to-order’ or ‘to-stock’ based on demand volume, price, and criticality. Figure 2.1 shows how the current order policy for every SKU is determined. This figure is for explanatory purposes and is not required to be understood in full detail. Deviations from this decision tree can be made when the purchase buyer finds another order policy more appropriate. This can be based on information not included in the decision tree.

The reasoning behind this decision tree is as follows. SKUs with an MOQ given by the supplier are ‘to-stock’ since they need to be purchased in bulk. Next, long lead time SKUs must be ‘to-stock’ since the lead time extends the period in which the SKUs can be purchased. Stock is needed to fulfill demand during lead time in such situations. The buyer decides if the discount is worth the inventory position and the holding cost. There is no uniform way of making this decision, and the buyer makes the decision based on experience gained in the past. This decision is more based on experience and feeling than calculations since there is a lack of insight, as stated in Section 1.2. This is one of the reasons to investigate the order policy at VMI. All floor stock SKUs are ‘to-stock’. VMI aims to prioritize ‘to-order’ whenever possible and resort to ‘to-stock’ only when ‘to-order’ is not feasible. This approach aligns with VMI’s objective of minimizing inventory and working on a project basis whenever possible.

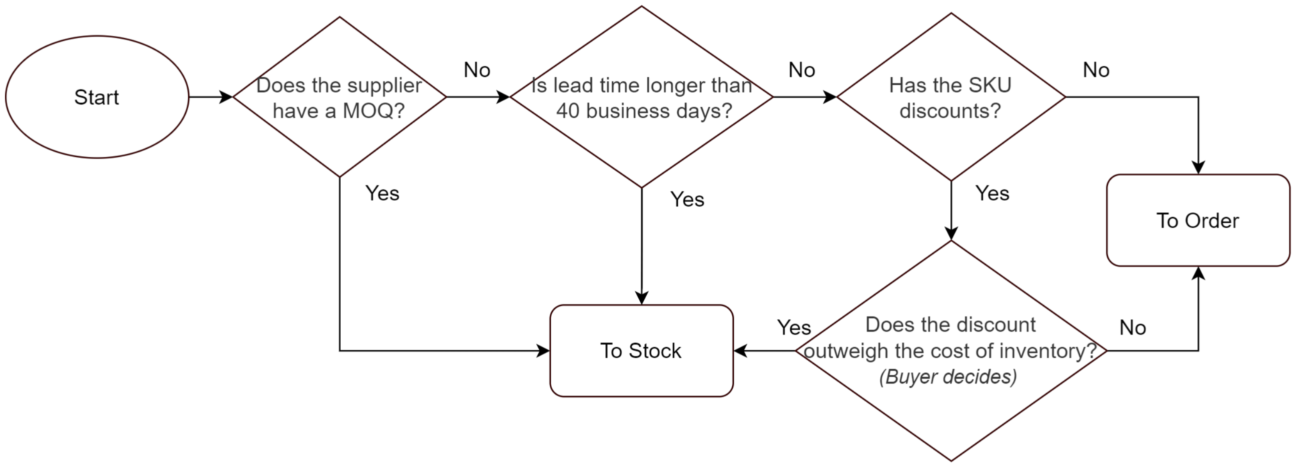

Figure 2.1: Current Order Policy Decision Tree

To-order SKUs that are purchased 'to-order' are purchased on a lot-for-lot (LFL) basis. Every production order has its purchase order. SKU demand recorded in the Enterprise Resource Planning (ERP) serves as the trigger for the buyers. These buyers place orders corresponding to the exact quantity the ERP system requires, scheduled with an arrival date of ten business days before the production date.

To-stock SKUs which are purchased 'to-stock' are purchased with a continuous review policy with a fixed order quantity. The reorder point equals the expected demand during lead time in addition to safety stock. The order quantity is set to the MOQ in cases where the SKU has a MOQ. The economic order quantity (EOQ) is used to determine the order quantity in cases without MOQ. An order is placed with the order quantity if the inventory position drops below the reorder point.

There is currently no systematic review of the order policies for SKUs, and a change in demand does not automatically prompt a reconsideration of the policies. There is one trigger to reassess the order policy for a SKU. The SKU order policy is reassessed if there has been no demand for them within the past five years. Section 2.5.7 shows the performance of the current order policy by comparing it to the upper and lower limits for the costs of an order policy.

2.4 Costs Related to Order Policies

This section describes all the costs influenced by the order policy. At first, the costs included in the scope are deliberated on containing the cost values so they can be used in Section 5.3 to test the model's performance. Next, an explanation is given why certain costs are out of scope with quantification as an elaboration of the scope in Section 1.4.

Purchase Amount

The purchase amount is the value spent at the supplier on purchasing the SKUs. Discounts are included. The price book includes all valid unit lot-sized-based discounts, which can be used to select new SKU order policies. The purchase price associated with the current order policy is used for the data analysis to quantify the current performance where the SKU purchase price is required. The price book contains only actual prices and no future price changes. All the purchase prices stated in this research exclude VAT.

Order Costs

The order costs occur for every line, not the order itself. VMI did research into ordering costs in 2018. The costs to place an order are estimated at €¿for every order line. After correction for inflation, the current ordering costs are estimated at €¿. Furthermore, there are inbound costs associated with placing an order. The inbound costs are estimated at €¿ per order line. To conclude, the order costs are €¿, which includes the costs of placing an order and the inbound costs.

Holding Costs

The holding costs are associated with the costs of the warehouse and the expenses related to the capital invested in inventory. For the last one, these costs consist of three primary components: the weighted average cost of capital (WACC), risk-related costs, and insurance expenses. For the sake of confidentiality, detailed calculations about these components are not disclosed within this research.

Transportation Costs

Transportation costs are included in the purchase price for the majority of SKUs. However, there are exceptions to this. Some suppliers charge extra transportation costs, and VMI needs to organize transportation by themselves in these instances. The cost of transportation, which is not included in the purchase price, is estimated to be €¿a year. The potential savings are limited to only €¿ at maximum, but the problem complexity will significantly increase. Moreover, VMI lacks insight into the transportation costs on the SKU level, making it challenging to incorporate these costs. The problem complexity will rise from a single SKU to a multi-SKU problem by including the transportation costs. To conclude, the limited savings are not worth the complexity; therefore, the transportation costs are out of scope.

Package Costs

The package costs are excluded from the analysis since it only accounts for ¿% of the total expenditure on suppliers. It is important to note that a portion of these package costs remains unaffected by the order size, thereby also independent of the order policy. These package costs represent a relatively small amount, and their inclusion in the analysis would introduce significant complexity to the problem. Therefore, it is anticipated that excluding package costs from further research would have minimal impact on the resulting solution. Therefore, the package costs will not be considered in the subsequent study.

Penalties for Small Orders

Penalties for small orders will not be considered in this research since exceptions mainly cause them, so they are not part of the happy flow. The penalties account for ¿% of the total purchase amount. Penalties for small orders are given to prevent small orders since the handling costs are too high compared with the product price. This most often occurs if VMI places a rush order or orders small quantities of spare parts. Both are out of scope, and the minimal costs are unlikely to alter the results but do introduce unnecessary complexity.

2.5 Descriptive Statistics

Data analysis is done to receive more insight into the current situation by analyzing different aspects of the supply chain. At first, the fraction of demand known upfront is described. This provides information on the fraction of the demand is stochastic and which part is deterministic. Afterward, an analysis regarding the number of SKUs and the division between 'to-order' and 'to-stock' provides insight into the amount of SKUs and describes the need for automation. This is followed by describing the discounts of the SKUs. The number of SKUs with discounts and the amount of discounts are described. Next, the lead time, including the lead time variability, is described to test the assumption that lead times are deterministic. The demand patterns of the SKUs are described in order to see how constant or variable the demand is. Lastly, the current order policies are described, followed by the performance of the order policies.

2.5.1 Process Information Known in Advance

VMI works project-oriented, which means that most of the orders are already known in advance. Figure 2.2 shows the percentage of demand known before the required date for production in Epe in 2022. The required date is ten business days before the production date. VMI works five days a week, so there are five business days in one week, and the logistical time equals 40 business days or eight weeks. There are three lines to see if there is a difference between larger orders or more expensive SKUs. The blue line resembles the percentage of orders that are known upfront. The green line represents the order quantity, whereas the red line represents the price for the orders known upfront. Since the lines are close to each other, it can be concluded that there is no relation between the order size or price of SKUs and how long the orders are known in advance.

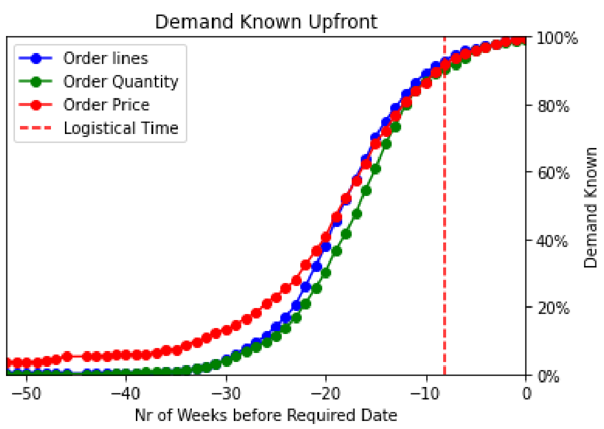

Figure 2.2: Weeks Demand Known Upfront

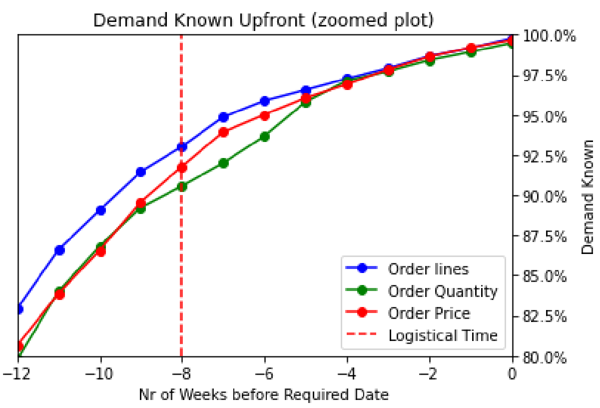

Figure 2.3: Zoom of Weeks Demand Known Upfront

Less than 3% of the demand is known 50 weeks upfront. Most demand information becomes available at 18 weeks in advance. Figure 2.3 shows the upper right corner from 12 weeks in advance. The vertical line at eight weeks resembles the logistical time of eight weeks. Approximately 93% of the demand is known eight weeks in advance. This means that 7% of demand is uncertain for eight weeks upfront. This part is the rush orders for spare parts or other exceptions from the happy flow. Both are out of scope.

To simplify the research, it is assumed that demand during the first eight weeks is fully known and deterministic, and demand after the eight weeks is fully stochastic. In reality, more information is known.

The production date may undergo alterations due to shifts in production schedules. 80% of production orders experience changes in the required date, which can be brought forward or backward.

2.5.2 Number of SKUs

VMI has 547,686 different SKUs. There are multiple ways to classify the SKUs. The SKUs can be divided into outsourcing (OS) and original equipment manufactured (OEM). The last group is the catalog SKUs, whereas OS SKUs are specially made for VMI from a drawing delivered by VMI. Table 2.1 shows the distribution of SKUs within those groups in combination with the order policy. 84% of the SKUs are OS. Next, 95% of the SKUs are purchased 'to-order'. The fraction of OEM SKUs purchased 'to-stock' (25%) is larger than that of OS SKUs purchased 'to-stock' (1.3%). OEM SKUs have, in general, higher demand, and it is, therefore, logical that these SKUs are relatively more purchased 'to-stock'.

Table 2.1: Order Policy Distribution of SKUs

	To order	**To stock**	**Total**
OEM	64,132	21,417	85,549
OS	455,875	6,262	462,137
Total	520,007	27,679	547,686

The number of SKUs is increasing since none of the SKUs are removed from the system, but new ones have been introduced. SKUs do not disappear from the VMI system since it is essential to see which SKUs are used in machines with all the information in cases of a machine breakdown. Figure 2.4 shows that, on average, 1200 SKUs are introduced monthly. The new SKUs are mainly (92%) OS SKUs corresponding with the current distribution of OS and OEM SKUs. As the machine designs are already established, the engineering efforts primarily revolve around tailoring the machine to meet specific customer requirements. Consequently, the newly introduced parts are often caused by customization, making them predominantly OS SKUs.

There is a high need for automation of selecting the optimal order policy for VMI with so many SKUs and the birth of 1200 SKUs each month. Next, changes in demand or machine production location require adapting the order policies. Moreover, price changes or new lot-sized-based discounts can change the optimal order policy.

Only a selection, around 35,000 to 45,000 SKUs, are used and purchased every year. Further data analysis is therefore done by selecting SKUs that have been in demand over the past three years. The selection consists of 58,000 SKUs. A subsection from these 58,000 SKUs is taken for small other analysis to see what the performance of the current order policy is. This sub-selection exists of 2,350 SKUs from a particular machine group that represent the happy flow.

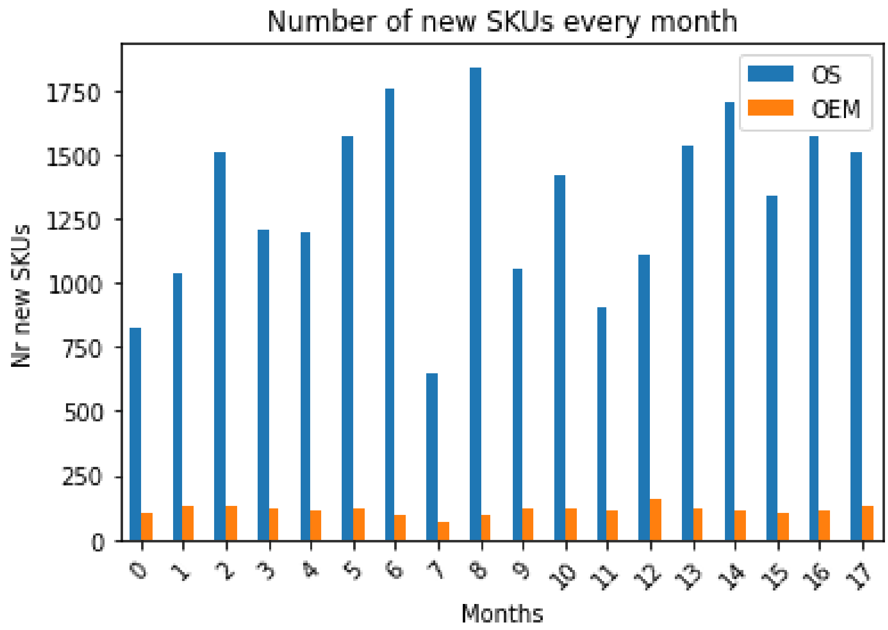

Figure 2.4: New SKUs per Month

2.5.3 Lot-Sized Based Discounts for SKUs

The purchase price of SKUs depends on the supplier and the quantity of the purchase orders. Larger orders require the same setup costs as smaller orders and are relatively cheaper, resulting in lot-sized-based discounts. 8% of the SKUs have lot-sized based discounts, from now on, shorted to discounts. 96% of the discounts are for OS SKUs. There are relatively more discounts for OS SKUs compared to OEM. After correcting the fraction of discounts between OS and OEM SKUs based on the number of OEM and OS SKUs, it can be concluded that OS SKUs have 4.3 times more frequent discounts than OEM SKUs. 35% of the SKUs get new prices or more discounts a year.

The discounts can be split into discounts for SKUs with a low and high obsolesce risk. SKUs in the standard BOM of machines have a negligible risk of becoming obsolete. 86.2% of the maximum achievable discounts a year come from SKUs in the standard BOM of machines. The other 13.8% corresponds with SKUs, not in the standard BOM of machines. The percentages are the upper limit of discounts that can be reached by placing an order with the demand of a full year and is unrealistic to achieve. Therefore, should the focus be on something other than including discounts for SKUs with a high risk of becoming obsolete since the cost reduction is limited.

To conclude, discounts exist for different SKU types and save up to ¿% of the purchase amount at maximum if you purchase with an order quantity of one year's demand.

2.5.4 Lead Time

Figure 2.5 shows the distribution of lead times of the SKUs. The red line at 40 business days resembles the logistical time to purchase the SKUs. 80% of the SKUs have a lead time smaller than 40 business days, meaning they do not require safety stock.

Not all deliveries are on time. Research is done to identify the percentage of orders delivered on time

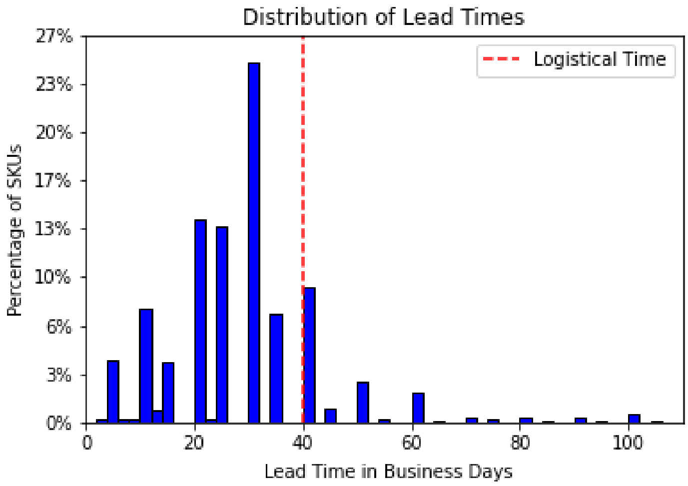

Figure 2.5: Distribution of Lead Times

and the number of days too late. Figure 2.6 shows the percentage of orders delivered on the required date and the days after for all orders in 2022. Zero business days after the required date means the order was not too late. The orders received after ten business days show the percentage of orders that are not more than ten business days too late. Figure 2.7 zooms in on the first 20 business days. 90% of the orders are delivered on time. Orders delivered too early are also considered on time in this plot since these orders do not cause problems in production. VMI has a safety time of 10 days. 97% of the orders are delivered within the safety time.

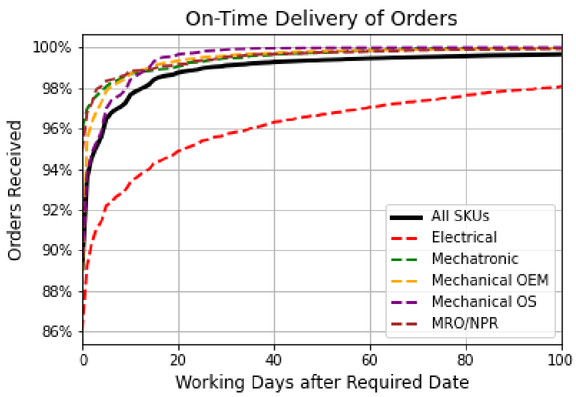

Figure 2.6: Lead Time Performance

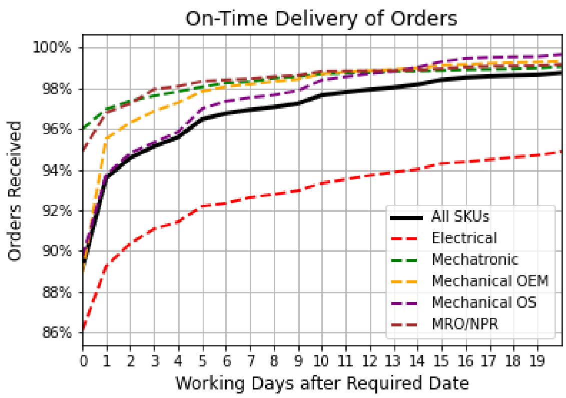

Figure 2.7: Zoom of Weeks Demand Known Upfront

The black line in Figure 2.6 and 2.7 shows the performance of all the SKUs. The electrical OEM performance is worse compared to the other commodities. This is still a result of the Covid pandemic. The other commodities have recovered, but electrical has not yet. The lead time performance of electrical OEMs started to improve later, and the analysis was done over the year 2022 when electrical showed terrible performance. Electrical OEM had the best lead time performance before COVID-19. Figure 2.8 shows the lead time performance during Covid (2021) versus (2022) when most commodities had already recovered from the COVID pandemic.

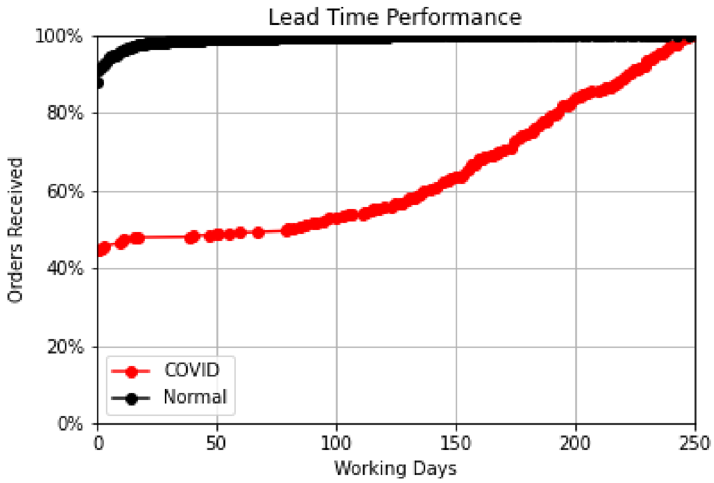

Figure 2.8: Comparison Lead Time Performance Covid and non-Covid

2.5.5 Demand Patterns

The demand patterns can be divided into four categories [4]. The classification is based on the variability of demand and the interval between the occurrence of demand. The variability of demand is expressed in the coefficient of variation of demand per week as defined in Equation 2.1. Equation 2.2 shows how the average demand interval (ADI) is calculated. The ADI indicates how often demand occurs. A time bucket of one week is used for this analysis. Figure B.1 in Appendix B shows an example of the four different demand patterns.

$$\text{Coefficient of Variation (CV)} = \frac{St.Dev. \text{ of demand per week}}{\text{Average demand per week}} \tag{2.1}$$

$$\text{Average Demand Interval (ADI)} = \frac{\text{Number of weeks}}{\text{Number of weeks in which demand occurs}} \tag{2.2}$$

99% of all SKUs can be classified as lumpy demand. Table 2.2 shows the number of SKUs with their demand pattern. Table 2.2 also states the cut-off values used for the demand patterns classification.

Table 2.2: Demand Pattern Classification and SKUs

	Number of SKUs		Demand Pattern Classification	
	Percentage	Count	CV^2	ADI
Erratic	1.0%	610	$CV^2 > 0.49$	$ADI \leq 1.32$
Lumpy	98.8%	57,656	$CV^2 > 0.49$	$ADI > 1.32$
Smooth	0.2%	106	$CV^2 \leq 0.49$	$ADI \leq 1.32$
Intermittent	0.0%	0	$CV^2 \leq 0.49$	$ADI > 1.32$

The largest part, 91% of the SKUs, have stationary demand. The Augmented Dickey-Fuller test is used to test whether the demand is stationary. The test's null hypothesis is that the demand pattern has no unit root, which indicates that the demand is non-stationary. The statistical properties like mean and variance change over time by non-stationary demand. Table 2.3 shows the number of stationary and non-stationary SKUs. There is no correlation between stationarity and the demand patterns. Appendix C shows a table with stationarity and demand patterns combined.

Table 2.3: Stationarity of SKUs

Count	Percentage	
Non Stationary	5,362	9.2%
Stationary demand	53,010	90.8%

2.5.6 Order Policies Distribution

This subsection discusses the current order policies. Figure 2.9 shows the different order policy types ('to order' and 'to stock') for the demand and price categories based on the demand of the past 12 months. The majority of the SKUs have had no demand in the past 12 months, which are omitted from the figure for readability. This figure indicates that most SKUs have low demand. More expensive SKUs tend to have more of a 'to-order' policy, while more demand results in higher chances of 'to-stock'. Appendix D analyzes all the different commodities.

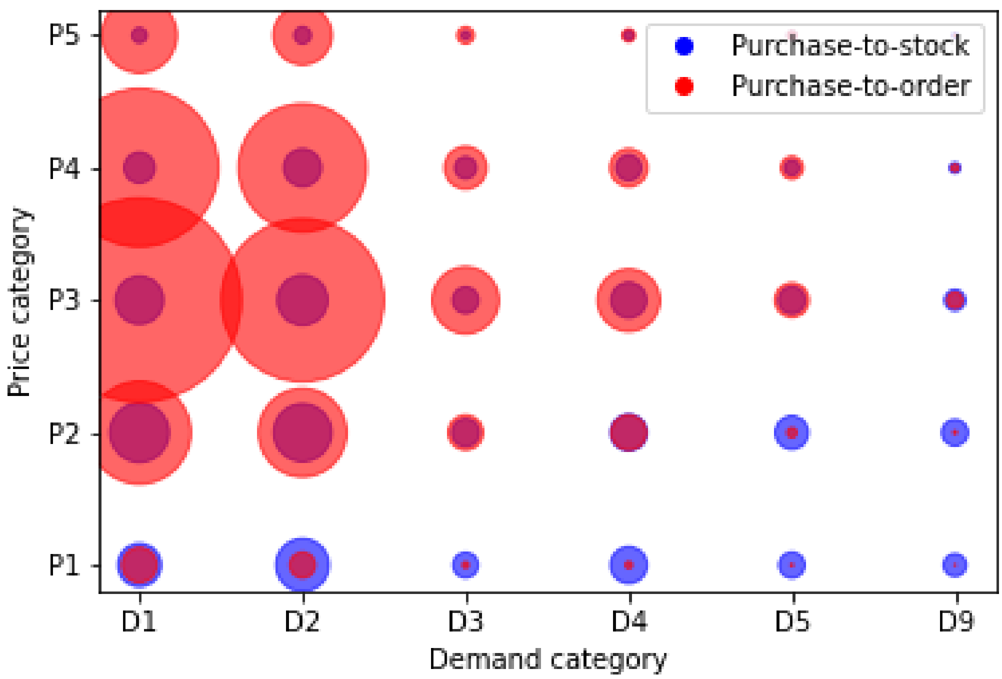

Figure 2.9: Order policy plot

VMI has divided the SKUs into different price ranges. Tables 2.4 and 2.5 display the categorization. The demand categories are based on the demand of the past 12 months. Note that demand categories D6 until D8 do not exist.

Table 2.4: Price Categories

Category	**Price**
P1	Cost price $<$ €1
P2	€1 $\leq$ Cost price $<$ €10
P3	€10 $\leq$ Cost price $<$ €100
P4	€100 $\leq$ Cost price $<$ €1000
P5	€1000 $\leq$ Cost price

Table 2.5: Demand Categories

Category	**Demand (12 months)**
D0	No demand
D1	Demand = 1
D2	$1 <$ Demand ≤ 7
D3	$7 <$ Demand ≤ 13
D4	$13 <$ Demand ≤ 53
D5	$53 <$ Demand ≤ 120
D9	$120 <$ Demand

2.5.7 Performance of the Current Order Policy

The current order policy operates mainly project-oriented as described in Section 2.3. The performance of this policy can be split into different costs: purchase amount, ordering, and holding costs. A selection of 2,350 SKUs with demand for one year is used to quantify the current performance. An upper and lower bound gives some perspective on where improvements can be made.

The lower bound for the holding costs is received by purchasing everything on a lot-for-lot basis. The upper bound is received by ordering the largest quantity to have the most items in stock. Limiting the order quantity to the demand of a year and a half a year is used as an upper bound.

The ordering costs decrease for larger order sizes. The same is valid for purchase amount in cases of lot-sized-based discounts. The maximum order quantity of demand for one year is used for a lower bound. Limiting the order quantity to the demand of half a year is the low-risk lower bound. Purchasing larger order quantities to cover a longer demand period has a higher risk.

The upper and lower bounds for the different costs are based on the deterministic case. No order policy can reach all the lower bounds for all the categories since the order quantity should be larger to obtain lot-sized discounts, and low holdings costs require small and frequent orders.

Table 2.6: Normalized Values for Lower Bound of Order Policy (for selection of 2350 SKUs for one year)

	Purchase Amount	Ordering Costs	Holding Costs
Purchase Lot-for-lot	101%	218%	64%
Purchase one year ahead	99%	4%	672%
Purchase a half year ahead	99%	8%	352%
Current order process	100%	100%	100%
Difference Current and Lower Bound	1%	96%	36%

The largest range between the upper and lower bound of costs is in the ordering costs. The current order policy is €¿ above the lower bound, which means that the largest potential is to improve the order policy to purchase more ‘to-stock’ and less ‘to-order.’ This will raise the holding costs. The maximum sum of discounts is €¿, where almost half is achieved. The total amount of discounts is around ¿% of the original purchase price.

2.6 Conclusion

The current situation can be described as:

- VMI has a logistical time of 40 business days since VMI works project-oriented and knows the demand of the coming 50 business days in advanced and uses a safety time of 10 business days.
- SKUs in the standard BOM have a low risk of becoming obsolete and SKUs needed for customization have a high risk of becoming obsolete.
- 99% of the SKUs have lumpy demand.
- All units lot-sized based discounts can reduce the purchase amount by ¿ at maximum.
- The largest cost reduction is at the order costs, which can be reduced by ¿% maximum.
- Lead times can be seen as deterministic since 90% of the orders arrive on the requested date.
- The stakeholders prefer low item costs, high item availability, low working capital required and an easy and smooth process.

3 Literature Review

The literature review in this chapter answers the question: *What relevant order policies are described in the literature, and in which situations are they promising?* Section 3.1 starts with the distinction between purchase-to-order and purchase-to-stock to identify the situations in which they are applicable. Section 3.2 discusses the order policies suitable for stochastic demand. The next section, Section 3.3, describes different algorithms and heuristics regarding dynamic lot-sizing, suitable for deterministic demand. The characteristics of the different heuristics, along with their advantages and disadvantages, are described to indicate in which situations they can be applied. The section ends with a summary comparing the performance, advantages, and disadvantages of the different dynamic lot sizing methods. Section 3.4 elaborates on the risk associated with ordering and how it can be quantified. The subsequent section, Section 3.5, describes the added academic value of this paper by identifying the research gap. Section 3.6 ends with a conclusion summarizing the key findings in the literature.

3.1 Purchase-to-Order and Purchase-to-Stock

Engineering firms employing project customization face significant variability in SKU diversity and demand characteristics. It is crucial to identify suitable order policies for the required SKUs that fit the demand characteristics. SKUs only needed once in a customization project should be purchased differently as SKUs that have high and constant demand.

Order policies determine the timing and quantity of placing an order. The two main order policies are purchase-to-order (PTO) and purchase-to-stock (PTS) [5]. Stock balances supply and demand, copes with lead time uncertainty, handles supplier unreliability, and achieves economies of scale [6, 7]. PTO purchases on a lot-for-lot (LFL) basis. This policy, also known as JIT purchasing, aims to eliminate waste by achieving zero inventories. PTO is suitable for SKUs with low demand and reliable suppliers with the condition that replenishment lead time is short enough for orders to arrive on time. Next, the risk of being obsolete is negligible since SKUs are only PTO. Another reason to use PTS is to consolidate demand, increase volume, and leverage a stronger negotiation position. The appropriate supply strategy depends on profit impact and supply risk [8]. Liu and Nishi [9] state that PTO is, in most situations, effective, but PTS is a competitive alternative when inventory costs are low. This raises the question of what is the tipping point between PTO and PTS.

SKUs are purchased LFL for PTO. The SKUs must be procured with an arrival date scheduled before the required date. The term LFL is used instead of PTO in the rest of the research. Appendix F provides more information according to LFL and a numerical illustration of the order policy.

Various order policies can be applied for PTS. Figure 3.1 shows the discussed order policies, including the sections where the order policies are explained. Section 3.2 will discuss the order policies suitable for stochastic demand, referred to as (s,Q) order policies. Section 3.3 discusses various dynamic lot sizing (DLS) methods that can be used for deterministic demand. Multiple order policies are discussed for some groups within DLS. These order policies are depicted below each other in Figure 3.1. DLS methods are suitable for demand which exhibits variability. Dynamic lot sizing is advisable when the squared coefficient of variation (SCV) for the different demand periods exceeds 0.2 [10]. Demand periods of one week are used in this research. Otherwise, a simple EOQ approach within safety stock is recommended. Section 3.3 ends with a comparison of the different DLS methods. The terms DLS and (s,Q) order policies are used instead of PTS in the rest of the research.

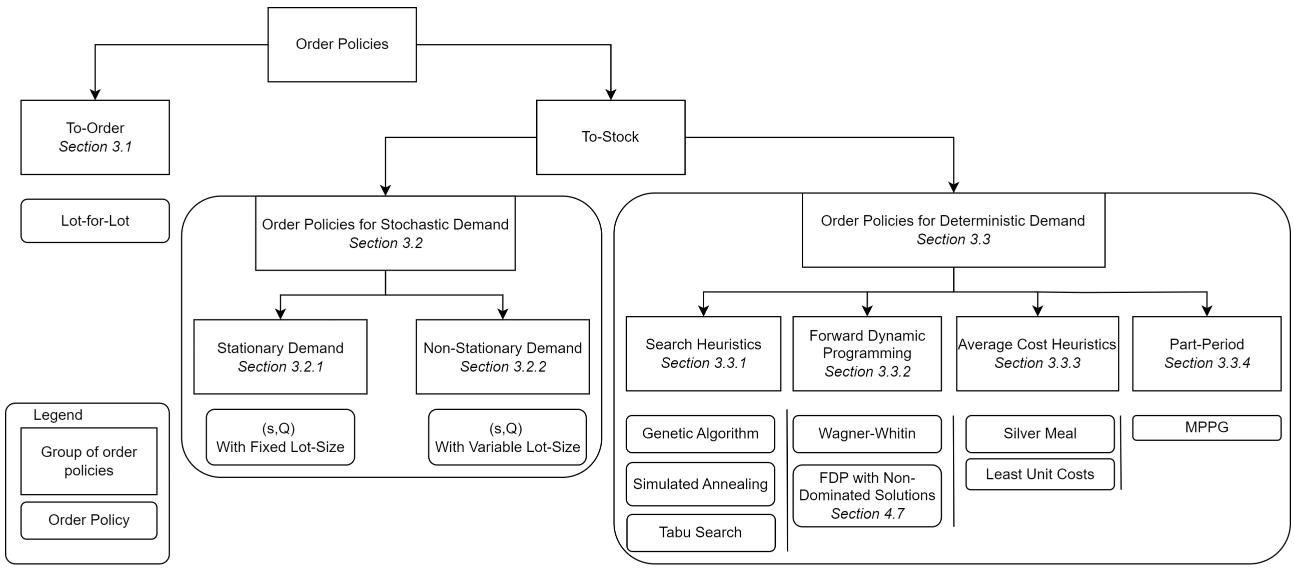

Figure 3.1: Overview of Discussed Order Policies

3.2 Order Policies for Stochastic Demand

There are two types of demand: stationary and non-stationary demand. Statistical properties, such as mean and standard deviation, remain constant over time in the case of stationary demand, unlike non-stationary demand. The order policy with fixed order quantity and reorder point handle stationary demand and is described first. A variable lot size is more effective in addressing non-stationary demand. The second half of this section discusses how a variable lot size can be obtained for non-stationary order policies.

The periodic review policy applies when the inventory is measured periodically or orders are placed regularly. In contrast, a continuous review policy constantly monitors the inventory position to place a new order whenever necessary. The main advantage of the continuous review system is the reduced safety stock, as orders can be placed earlier in this approach [10]. The choice between periodic and continuous depends mainly on the system's capabilities and the frequency of suppliers' deliveries. For example, if a supplier delivers once a month, the periodic review with a review period of one month should be used. Continuous order policies are most useful for VMI since VMI constantly monitors the inventory position, resulting in less inventory than periodic order policies. Therefore, the emphasis will be on the continuous order policies.

3.2.1 Order Policy with Fixed Lot-Size for Stationary Demand

Order policies using a fixed order quantity are well-suited for stationary demand, given that statistical properties remain constant over time. There is no reason to change the order quantity in such instances. Additionally, this order policy is applicable when a straightforward approach is necessary. For example, the least important SKUs are classified as C-type from the ABC classification. The ABC analysis divides the SKU into the important group (A), middle importance (B), and the least importance (C) based on the turnover. The C-type SKUs benefit from a simple procedure such as the two-bin system.

The continuous order policy with fixed order quantity uses the (s,Q) order policy system. The s is the reorder point and determines when a new order is placed, and the Q is the order quantity of the new order. It is important to clarify that the trigger for placing another order is the inventory position, not the net stock. This order policy is straightforward, easy to implement, and comprehensible. Appendix G presents the equations used to calculate the (s,Q) order policy parameters, which are determined based on the probability of a stock-out event or the associated costs of stock-outs.

A variation to the continuous review policy is the periodic review policy. The periodic review policy places an order of size Q at the end of the review period R if the inventory position is below the order point s. The periodic review policy is applicable when demand is periodically measured but results in a higher reorder point with higher holding costs. VMI monitors the inventory position constantly; therefore, the continuous order policy is the better choice.

Lot-Sized Based Discounts for Stationary Demand

Suppliers offer lot-sized-based discounts to encourage larger order quantities, using economies of scale to improve process efficiency and build stronger customer relations. Exploitation of discounts is a trade-off between maintaining a higher inventory level and reducing the number of orders placed to meet the criteria for the discount [11]. There are two types of discounts: all-unit quantity discounts and marginal unit quantity discounts. Only all unit quantity costs are described since this is the only type of discount in scope.

Discounts can increase the optimal order quantity. The optimal order quantity corresponds either with the EOQ or a price breakpoint (DL). The optimal order quantity under all unit quantity discounts can be calculated with the following four steps [6]. The price breakpoints are DL_i with $DL_o = 0$ as the base price. The price p_i is valid for an order quantity between DL_i and DL_{i+1}.

Step 1: Calculate the optimal order size for each price Q_i with the EOQ formula as follows:

$$Q_i = \sqrt{\frac{2DS}{hP_i}} \tag{3.1}$$

The EOQ is based on the yearly demand D, order costs S and holding cost rate h.

Step 2: Select the valid order quantity Q_i^* for price p_i

$$Q_i^* = \begin{cases} DL_i & \text{For: } Q_i < DL_i \\ Q_i & \text{For: } DL_i \leq Q_i \leq DL_{i+1} \\ \text{Ignore} & \text{For: } DL_{i+1} \leq Q_i \end{cases} \tag{3.2}$$

The last case can be ignored since it is considered for the Q_{i+1}.

Step 3: Calculate the total annual costs TC_i for each ordering quantity, Q_i^* as follows:

$$TC_i = \frac{D}{Q_i^*}S + \frac{Q_i^*}{2}hp_i + Dp_i \tag{3.3}$$

Step 4: Select the order quantity Q_i^* with the lowest total costs TC_i.

Graphical illustration The following example shows how the method works. Table 3.1 shows the parameters of this problem, including the variable value of the different steps. Figure 3.2 shows that the EOQs of the different prices are close to each other. The EOQ is outside the valid region for the price without discounts since $DL_1 < Q_1$. Therefore, this point is ignored since the first discount price will check the optimal order quantity in this price range. For the first discount, Q_2 fits within the valid region and equals Q_2^*. Since Q_3 is smaller than DL_2, Q_3^* is set to the lower price breakpoint (DL_2). The total costs of Q_2^* and Q_3^* are compared, and the price breakpoint of 100 results in the lowest costs; this point is assigned with a black dot in the figure.

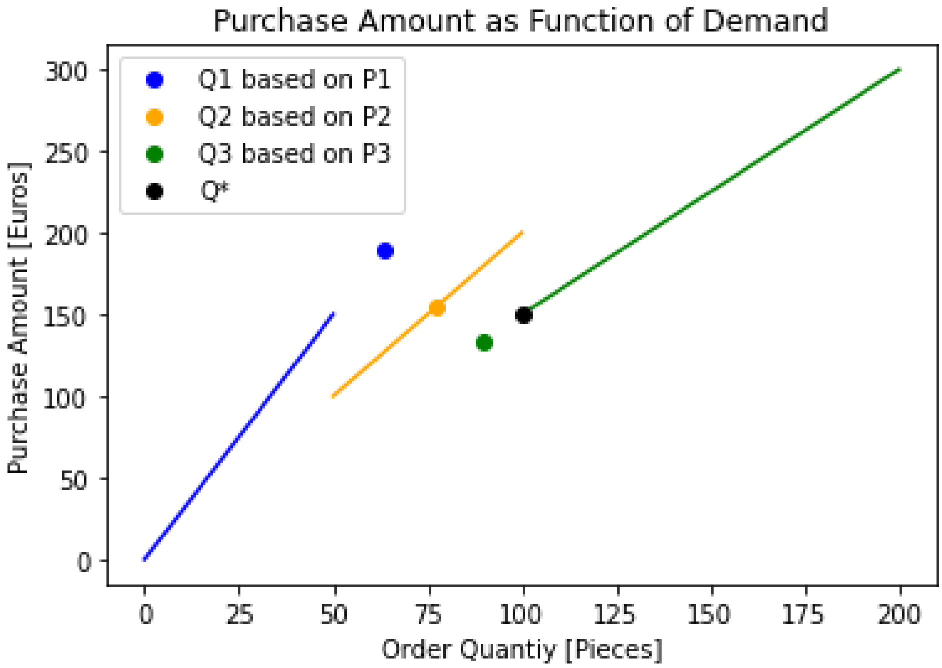

Figure 3.2: EOQ and Price Breakpoints

Table 3.1: Parameters for Graphical Illustration Discounts.

Variable	Value	Variable	Value
D	1,200 units	Q_1	63 units
S	€1	Q_2	77 units
h	20% (a year)	Q_3	89 units
p_1	€3	Q_2^*	77 units
p_2	€2	Q_3^*	100 units
p_3	€1.5	TC_2	€2,431
q_0	0 units	TC_3	€1,827
q_1	50 units	Q^*	100 units
q_2	100 units		

3.2.2 Order Policy with Variable Lot-Size for Non-Stationary Demand

Order policies using a variable lot size are well-suited for non-stationary demand since the statistical properties vary over time. The (s, Q) policy can be altered to variable lot sizes as described below.

The reorder point is the expected demand during the effective lead time (EL), the safety stock, and the expected undershoot $(E[Z])$.

$$s = \hat{X}_{t,t+EL} + ss + E[Z] \tag{3.4}$$

The forecast can be used to determine the expected demand during effective lead time. Fluctuations in demand can be incorporated in this way. The safety stock (ss) is based on the forecast error of demand during the effective lead time.

$$ss = k\sigma_{t,t+EL} \tag{3.5}$$

The forecast error can change over time since demand is non-stationary. The demand is non-stationary, so the forecast error is not independent of time. An estimation of the standard deviation of the forecast error for the lead time can be made in different ways. At first, the smoothing constants α and β can be used for a simple exponential smoothing model. Equation 3.6 shows how the forecast error for one period ahead can be converted to the forecast error for the effective lead time ahead [10]. Unfortunately, this method cannot incorporate the effect of seasonal factors in the forecast error.

$$\sigma_{t,t+EL} = \sigma \sqrt{\sum_{j=0}^{EL-1} [1 + j\alpha + j(j+1)\beta/2]} \tag{3.6}$$

Another option is to relate the standard deviation of forecast error during effective lead time demand to the forecast of demand. In other words, express the forecast error in the forecast demand. Equation 3.7 shows how the standard deviation of forecast error during effective lead time can be expressed in the forecast of demand and two constants.

$$\sigma_{t,t+EL} = C_1 \left(\hat{X}_{t,t+EL}\right)^{C_2} \tag{3.7}$$

Historical data is required to estimate the constants C_1 and C_2. The constant C_2 can be set to 1 if the forecast error is assumed to be proportional to the demand forecast [12]. The maximum likelihood method can be used to estimate C_1. The data point of the forecast, $\hat{X}_1$ to $\hat{X}_n$, and the observed forecast errors, e_1 to e_n are used. Equation 3.8 shows the likelihood function with the assumption of unbiased forecast.

$$L(e_1, \ldots, e_n | C_1) = \prod_{i=1}^{n} \frac{1}{\sigma(x_i)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{e_i}{\sigma(x_i)}\right)^2} \tag{3.8}$$

Equation 3.9 shows how the value of C_1 can be found by setting the derivative of the log-likelihood over C_1 to zero.

$$C_1 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{e_i}{x_i}\right)^2} \tag{3.9}$$

The safety stock can be expressed in a fraction of the period forecast using the method above [10].

The order quantity (Q) can also vary over time due to the non-stationary. Equation 3.10 calculates the number of periods an order should cover. The EOQ is divided by the average demand for a single period, $\bar{D}$. Equation G.1 shows the EOQ calculation. The number of periods (T_{EOQ}) are rounded upwards to the next integer for small periods [10]. Q equals the demand of T_{EOQ} periods ahead.

$$T_{EOQ} = \frac{EOQ}{\bar{D}} \tag{3.10}$$

The (s,Q) order policy can be used for non-stationary demand when the reorder point is based on the expected demand of the forecast, and the order quantity corresponds with the forecast demand of T_{EOQ} ahead.

Lot-Sized Based Discounts for Non-Stationary Demand

Discounts can increase the optimal order quantity. The order quantity for non-stationary demand is expressed in forecasted demand for a number of periods ahead. The consequence of this method is that the order quantity can fall slightly short to qualify for the discount price. On the other hand, the order quantity is much larger than the price breakpoint, where the optimal order quantity would equal the price breakpoint. Therefore, the optimal order quantity under discount is not able to be defined in the forecast demand for several periods ahead.

Different papers discuss the influence of discounts on stochastic non-stationary order policies. For example, Güder and Zydiak [13] describe how order policies can be determined under non-stationary demand for a multi-SKUs system. However, this research deals with a single SKU problem; therefore, the approach from Güder and Zydiak is irrelevant to this case. Purohit et al. [14] present a model for non-stationary stochastic demand. This model is not a variation on the (s,Q) order policy but a complete model to determine the optimal order quantity based on non-stationary and stochastic demand conditions. Wang et al. [15] present an adaptation to the (s,S) order policy in order to incorporate discounts. The optimal order policy has a generalized Q-jump structure and can handle multi-price breakpoints. Unfortunately, the derivation is based on the lost-sales case. To conclude, no adaptation is found in the literature on including discounts for non-stationary demand for the (s,Q) order policy when Q is expressed in a number of periods of forecast demand.

3.2.3 Incorporate Knowledge of Demand

The demand for the first 40 business days is known within VMI. This enhances that the demand is not fully stochastic, but the first part is deterministic. The knowledge can be used to set the parameters for the order policy. The effective lead time can be considered instead of the lead time from the supplier. The effective lead time is the lead time of the SKU minus the logistical time. The logistical time is the time between engineering being finished and the start of production. For the case of VMI: effective lead time = lead time − 40 business days. The effective lead time can never be smaller than zero and, therefore, is zero for SKUs with a shorter lead time than 40 business days. A shorter effective lead time reduces the safety stock and, thus, lowers the reorder point for SKUs. To the best of my knowledge, no methods exist for the order quantity to be adapted to demand known in the first eight weeks other than dynamic lot sizing methods.

3.3 Dynamic Lot-Sizing for Deterministic Demand

Dynamic lot-sizing (DLS) determines the order quantity by considering the impact of accumulating demand across periods to determine the optimal order quantity for each replenishment order [16]. DLS methods need all knowledge of the different demand periods and can, therefore, only be used for deterministic demand. The future demand is known and is divided into distinct periods. If new information becomes available, DLS allows for adjustments to previous plans. DLS incorporates more information and adopts the order policy to the knowledge of the coming demand.

Several different DLS methods exist. This section presents different groups of DLS methods and explains in which situations they are useful, including their strong points and drawbacks. The promising DLS groups are described in more detail at the end of this section.

Lot-sizing problems with practical implications are often NP-hard due to their constraints [17]. Various techniques are developed and presented to solve these problems. Dynamic programming uses insight into the structure of the optimal solution, with the major drawback being that it is designed for single-SKU problems. Next, multiple algorithms exploit specific knowledge about the substructure of solutions. For example, Dantzig-Wolfe, Lagrange relaxation, and cutting planes find tighter bounds for the problem. Dantzig-Wolfe provides an upper and lower bound but has slow convergence to the end. Lagrange is often done for a limited number of iterations without guarantee that the optimal bound will be found. These algorithms are usually extended with a branch and bound to obtain feasible solutions [18].

3.3.1 Search heuristics

Meta-heuristics such as genetic algorithm, simulated annealing, and tabu search are used for combinatorial problems. Order policies where the order policy for one SKU is not independent of the order policy of another SKU is an example of a combinatorial problem. These search heuristics can be used for complicated multi-SKU lot-sizing problems [18, 19]. Search heuristics do not necessarily give the optimum solution and require good parameter settings for the cooling scheme in order to provide good results. Since this research does not deal with coordinated discounts and it is challenging to set the cooling parameters settings right, it is chosen not to exploit this algorithm further. A small explanation of the genetic algorithm is provided below to provide insight into the possibilities of incorporating discounts over multiple SKUs. The literature often refers to genetic algorithms, which are frequently used in performance comparisons; therefore, this method is described in more detail as to the possibilities for coordinated problems. Search heuristics can solve complex problems, and it would be interesting to investigate if the problem would be a coordinated problem with discounts per supplier for multiple SKUs.

Genetic Algorithm

The genetic algorithm (GA) represents a heuristic search process for optimization to solve combinatorial problems. The GA is well-suited for tackling complex problem domains by employing the evaluation concept. This algorithm draws inspiration from nature's evolution and uses fitness values to identify and select the most promising solutions. Near-optimal solutions can be reached for lot-sizing problems with quantity discounts [20]. Despite its effectiveness, the efficiency is closely tied to various parameters, and finding appropriate settings for these parameters can be challenging.

Additionally, the computational cost of evaluating fitness values can pose a practical constraint.

At its core, the genetic algorithm operates through the iterative generation of a population of potential solutions represented as chromosomes. These chromosomes are analogous to genetic code and undergo operations such as selection, crossover, and mutation, mimicking the principles of natural selection. Over successive generations, the GA continuously refines and improves the population, ultimately converging towards better solutions. This process allows GA to navigate complex solution spaces and efficiently search for near-optimal outcomes.

3.3.2 Forward Dynamic Programming

Different DLS methods use a form of forward dynamic programming (FDP). The basics of FDP are explained below. The Wagner-Whitin algorithm is the most well-known within FDP. It is well known for finding the optimal order policy with the lowest cost and is explained in more detail in the subsequent section. Different FDPs are published since the Wagner-Whitin algorithm is not capable of including discounts. These will be discussed in this section.

Explanation of FDP in General

The forward dynamic programming uses the optimal value function of Equation (3.11). $F(j)$ are the minimum costs for the first j periods, and $c(j,k)$ is the cumulative stage cost function between two consecutive regeneration points j and k.

$$F(k) = min_{j=1,\dots,k}\{F(j)+c(j,k)\} \tag{3.11}$$

FDP starts at the beginning of the problem and uses the value function and the stage cost function to determine the minimum costs to get to the next point in the future. The lowest costs to reach the last state are defined in this way.

Wagner-Whitin Algorithm

The Wagner-Whitin algorithm computes the optimal order policy based on demand over future periods. It requires knowledge of the demand for each period until the end of the planning horizon and the desired inventory position at the horizon's end. The Wagner-Whitin algorithm has the following property: orders will only arrive in periods where the inventory level is zero. Therefore, the order quantity is zero or equals the demand of one or multiple periods as defined in Equation 3.12. Q_t is the order quantity in period t and d_i is the demand of period i with N is the last period [21].

$$Q_t = 0 \; or \; \sum_{j=t}^{k} d_j \text{ for some } k,\; t \leq k \leq N \tag{3.12}$$

The algorithm makes decisions regarding the placement of replenishment orders from the current period to the end of the horizon, with the number of periods for demand consolidation determined through a forward dynamic programming problem.

The main advantage of the Wagner-Whitin algorithm lies in its ability to establish an optimal order policy for scenarios involving irregular or seasonal demand. The drawback of the Wagner-Whitin algorithm is that it cannot include discounts. Next, the algorithm works with a finite planning horizon.

This means that the demand should terminate at the end of the horizon or when the inventory position at the end is known [10, 16].

FDP with Discounts

There are other methods presented in the literature which can include discounts. The methods are shortly described in which situations they are useful.

Chung et al. [22] describe a model for coordinated replenishment with quantity discounts. This is a forward dynamic programming approach that builds on solutions to sub-problems. Building further on an optimal algorithm for the quantity discount problem [23]. Although this approach can be used, inserting a solution that optimizes over multiple SKUs is unnecessary since this research handles an uncoordinated problem. Therefore, Chung's method is not elaborated further in this research.
Chyr [24] introduces a recursive method to construct a dynamic programming model for solving a lot-sized problem with quantity discounts. Unfortunately, the recursion is only built on one discount level. Unfortunately, the recursive relation cannot handle the complexity of multiple discounts.
Mizutani and Trista [25] describe an improved standard FDP by limiting the decisions to either price breakpoints or equal to the demand of one or more adjacent periods.

The above-described methods are taken as inspiration for an FDP with non-dominated solutions that can handle multiple price levels for multiple discounts and are presented in Section 4.7.

3.3.3 Average Costs Heuristics

Silver-Meal and Least Unit Costs are forward-based heuristics that compromise solution quality but require less computational time. These heuristics perform adequately.

Silver-Meal Heuristic

The Silver-Meal (SM) heuristic determines the optimal order quantity by consolidating demand over multiple periods. It involves selecting the number of demand periods for a single order based on the average cost per period. The heuristic starts with a single period and gradually incorporates the demand for subsequent periods to increase the order size. If the average costs per period decrease with the inclusion of each additional demand period, the heuristic continuous this process. Conversely, if the average costs rise, the heuristic merges demand over the period where the average SKU costs were minimal [10].

Note that the Silver-Meal heuristic offers a more straightforward approach and requires less computation time compared to the Wagner-Whitin method. However, this heuristic sacrifices accuracy in exchange for its simplicity [26].

Hu [27] presents a modified Silver-Meal heuristic, which can handle quantity discounts by including the opportunity costs of not taking advance of the price discount. The modified heuristic calculates the total relevant costs per period with equation (3.13). The relevant costs include fixed costs, opportunity costs of price discounts, and holding costs.

$$C^p(T) = \frac{S + (v_a - v_m)Q(T) + Iv_a \sum_{j=1}^{T} (j-1)D_j}{T} \tag{3.13}$$

Where D_j is the demand of period j, S is the order costs, v_a is the average purchase price associated with the order quantity $Q(T) = \sum_{j=1}^{T} D_j$ and v_m is the minimum price related to order quantity of demand over the complete horizon. I are the holding costs per unit per time as a fraction of the production value. Alfares and Turnadi [28] provide an adapted version where multiple suppliers are incorporated.

Least Unit Costs

The least unit costs is a similar heuristic compared to the Silver-Meal heuristic with the difference that the average costs per unit are considered instead of the average cost per period [10]. Equation (3.14) shows how the average costs per unit are calculated. The heuristic starts with an order size of one period and increases the order size by considering the demand of multiple periods until the average costs per unit rise.

$$C(T) = \frac{S + Iv_a \sum_{j=1}^{T} (j-1) D_j}{\sum_{j=1}^{T} D_j} \tag{3.14}$$

Least unit costs can be modified in the same manner as Silver-Meal to include the price discounts.

3.3.4 Part-Period Algorithms

The part-period algorithm is an alternative order policy that does not have a forward approach like most heuristics have. The principle behind the part-period algorithm is as follows. The part-period algorithm increases the order size by accumulating orders, starting from the smallest order. Part-period is the sum of parts multiplied by the number of periods stored. The part-period algorithm combines the demand of different periods based on the economic part period (EPP). The EPP represents a break-even point between order and holding costs and is the number of part-periods at which order costs and holding costs are equal. Equation (3.15) shows the EPP [29, 30].

$$EPP = \frac{\text{Order Costs}}{\text{Holding Costs per Unit per Period}} = \frac{S}{h} \tag{3.15}$$

An order is placed with the largest part-period, which is not larger than the EPP. Equation. (3.16) shows the condition for placing an order.

$$F = PP_k = \sum_{k=i}^{j} (k-i) d_k \leq EPP \tag{3.16}$$

An order is placed in period k with the order quantity, which satisfies the demand d until period j, where the last order was placed in period $i-1$. The highest value of j is taken, which satisfies this equation. The part-period PP_k is the number of units times the number of periods they are in stock.

Different variations of the part-period exist. For example, DeMatteis [29] describes a variation looking backward and forward. The maximum part-period gain (MPPG) has the best performance of the different part-period methods [31] and is therefore described in detail. Unfortunately, no version of the part-period that is able to include lot-sized discounts has been found.

Maximum Part-Period Gain

Karni [32] introduces the maximum part-period gain, which consists of the following steps:

Step 1: Set the PP_k to d_K for all periods.

Step 2: Select the smallest PP_k.

Step 3: Satisfy the smallest PP_k from the previous $k-1$ period. Update the value of PP_{k-1} and delete PP_k.

Step 4: Return to step 2 until all part-periods exceed the EPP.

The advantage of the maximum part-period gain is that it is not based on forward procedures. The disadvantage is that the maximum part-period gain heuristic cannot include discounts. Part-period is suited for situations with low demand and demand fluctuations. Maximum part-period gain is effective and the most promising variant of the part-period group.

3.3.5 Performance of Different DLS Methods

The performance of the different DLS methods is compared to each other in order to select the most promising methods. The search heuristics are not included in this comparison since those are suitable for combinatorial problems, and this research deals with a single SKU problem.

There are different FDP algorithms. The Wagner-Whitin (WW) algorithm is well known for finding the optimal order policy by minimizing the objective function. The WW works with a finite planning horizon and a running time of $O(n^2)$, with n the number of periods. The drawback of the WW algorithm is that it cannot include discounts. The FDP with non-dominated solutions based on the WW algorithm can handle discounts.

Silver-Meal (SM) and least unit costs (LUC) are both heuristics that minimize the average costs. They both sacrifice accuracy for simplicity and running time. The running time of both heuristics is $O(n)$. The algorithms can be modified so that discounts can be included in the heuristic without changing the order of running time. SM is known for its simplicity, ease of implementation, and good performance. The (modified) SM algorithm is faster and can solve problems with many variables and practical constraints within a minute. In contrast, mixed integer programming (MIP) needs more than an hour for it [33]. The LUC is sensitive to the standard deviation of the demand and shows poor performance when the average demand comes close to the EPP. Next, the performance of the LUC heuristic is better when the time between orders is low. The SM heuristic shows better performance compared to the LUC. At least in 25% of the cases, SM resulted in the optimal solution, even in scenarios with higher standard deviations of demand. Where the LUC shows good performance with low variation in demand, SM offers better performance with more variation in demand [31]. The SM is therefore chosen over LUC since this case deals with high variation in demand.

Part-period is a non-forward procedure that does not need a finite planning horizon. The running time is in $O(n)$. The maximum part-period gain found the optimum solution in 90% of the cases where the average order is smaller than the EPP in the experiments done by Baciarello et al. [31].

The methods can also be assessed on return on investment (ROI) performance in addition to finding the optimal value of the objective function. Although WW can minimize the objective function, it does not necessarily have the highest ROI. MPPG has a higher return on investment for high purchase prices than the WW algorithm. At the same time, Silver Meal (SM) shows good performance for low purchase prices [26]. Unfortunately, the LUC is not discussed in the paper, so there is no information regarding the ROI.

Baciarello [31] compares different lot-sizing heuristics under different conditions and concludes that MPPG has the best results of the various heuristics. Discounts were not included in the analysis of Baciarello.

Simpson [1] investigates the performance of DLS methods under a rolling time horizon. The total problem length is 300 periods, with a maximum length of planning horizon equal to 20 periods. Figure 3.3 shows the different DLS methods' performance given the optimal solution's average cost gap. The WW algorithm offers the best performance, followed by the MPPG. The performance of the SM is again better than the LUC.

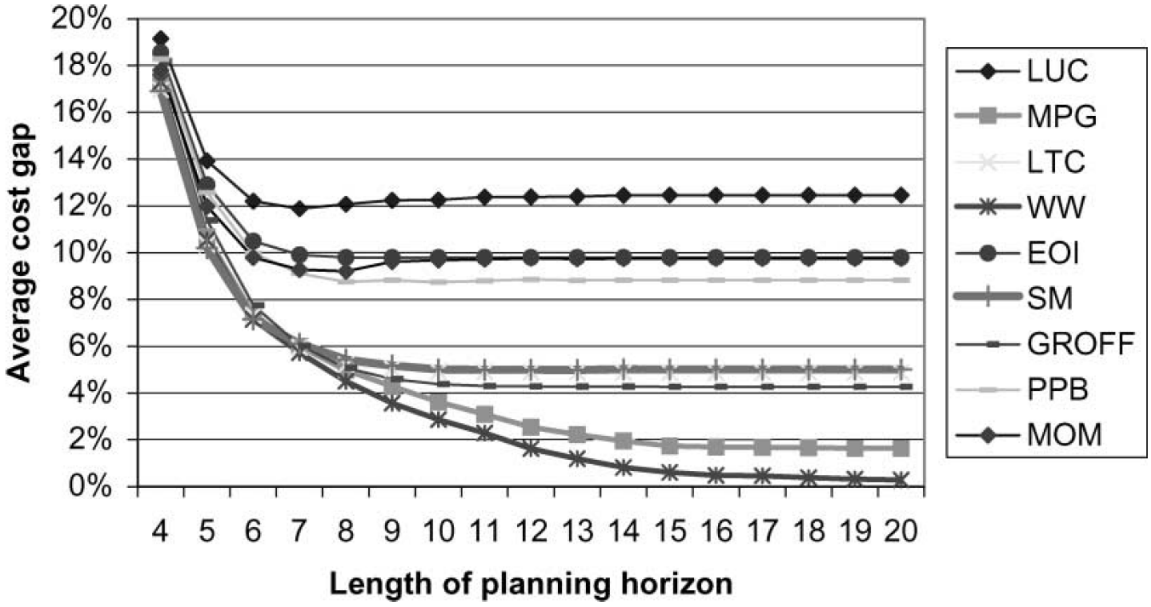

Figure 3.3: Average Cost Gap Between Scheduling Costs and Optimal Solution [1]

To conclude, the FDP algorithm with non-dominated solutions is further explored since this function can give the lowest costs for a selected time window and can be used to find the lower bound. However, it is unknown if the FDP algorithm can solve the problem for multiple discount levels in a reasonable time. The modified Silver-Meal heuristic is also included to see how much accuracy will be lost compared to the optimal situation. Although maximum part-period gain cannot include discounts, it shows good performance on a rolling horizon and ROI, and is interested for SKUs with no discounts.

Table 3.2: Comparison of DLS Methods

Method	Performance Optimizing Objective	Running Horizon Performance	ROI	Running Time	Discounts Included
WW	Optimality	High	High	$O(n^2)$	No
FDP	Optimality	High	Unknown	$O(n^2)$	Yes
MSM	Good for high demand variability	Moderate	Unknown	$O(n)$	Yes
LUC	Good for low demand variability	Low	Unknown	$O(n)$	Yes
MPPG	Good	High	Good for expensive SKUs	$O(n)$	No

3.3.6 Incorporate Uncertainty of Demand in DLS

Expected demand can also be incorporated and used to feed the DLS methods by extending the number of periods with demand. Different papers describe the consequences of incorporating the forecast in the order quantity decision. However, no literature has been found on whether the near forecast or forecast with low forecast error should be treated differently as forecasts far away with a larger forecast error.

Uncertainty can be related to the demand quantity and the timing of the demand or with lead time uncertainty [34]. Demand uncertainty results in drastic costs increase. Already small forecast errors affect the cost-effectiveness of DLS methods [35, 36]. A continuous ordering policy might be the better choice for situations with high forecast errors since forecast errors have a high impact on the performance of the DLS methods [37].

Callerman and Hamrin [38] study the performance of DLS methods with stochastic demand. They conclude that the maximum part-period gain is the best method regarding cost-based and computational running time performance.

3.4 Risk of Obsolescence

Larger order quantities result in reduced ordering costs and can result in discounts. The downside is that the SKUs in stock become obsolete if the demand drops or disappears. The following is found in the literature regarding risk management.

Quantitative risk management addresses the design of representative and traceable risk measures. Stochastic programming is frequently used to determine the optimal order policy if different scenarios with their risk are known. Alem et al. [39] illustrate a risk-averse approach using the conditional value at risk. This method detects the worst scenarios by evaluating the expected costs for (1-α)*100% worst scenarios and optimizes over them. Alem et al. [39] also describe the mean-risk model and stochastic dominance constraints, which prove beneficial when dealing with known demand fluctuations but may be less suitable for handling disappearing demand. Ali et al. [40] offer insights into coping with perishable products, focusing on fast-deteriorating SKUs like vegetables and fruits. This differs from the slow-deteriorating, more uncertain deterioration typically observed in electrical components. In conclusion, there are multiple methods in the literature available on how to cope with risk, but the literature does not describe how to quantify risk.

All the approaches described above need scenarios, including the probability that a scenario will appear. Unfortunately, VMI has no insight into future scenarios of demand occurrence with associated probability. VMI has categorized SKUs as customer-order-specific or part of standardized machines. The risk regarding obsolesce can be based on this categorization.

3.5 Gap in Literature

The existing literature offers valuable insights into various aspects of order policies. However, this research contributes to the academic literature in two distinct gaps in this literature.

The literature elaborates on the differences between LFL, DLS, and EOQ-based order policies. The advantages and disadvantages of the different methods are described. However, the literature does not explain the tipping point between these different methods. In particular, the influence of discounts brings uncertainty about the optimal order policy method.

Furthermore, the literature lacks guidance on which DLS method best determines the order policy in cases with multiple lot-size-based discount levels. Research is needed to determine which DLS method suits the different SKUs.

3.6 Conclusion

From the literature, it can be concluded that the relevant order policies are:

- Lot-for-lot (LFL) for SKUs with low demand, lead time smaller than the logistical time, and where the discounts do not outweigh the holding costs.
- Continuous review policy (s,Q) for SKUs with stochastic demand, lead time larger than logistical time. (s,Q) order policy with fixed order quantity for SKUs with stationary demand where a variable order size suits SKUs with non-stationary demand.
- Various dynamic lot sizing (DLS) methods for SKUs with lead time smaller than the logistical time and high variability in demand. Each dynamic lot sizing method has its advantages and disadvantages.
    - Forward dynamic programming (FDP) calculates the optimal orders over the horizon demand is known but requires most computational time of all the DLS methods.
    - Silver Meal (SM) and least unit costs (LUC) are heuristics that can include discounts, which do not necessarily result in the lowest costs but take less time than FDP.
    - Maximum part-period gain (MPPG) shows good results under the rolling horizon but cannot include discounts.
- An EOQ for SKUs with high demand, lead time smaller than the logistical time where the optimal order quantity is larger than the demand during the logistical time.

The following questions remain:

- What is the tipping point when DLS methods are more suitable than a continuous review policy for SKUs?
- Which DLS method is most appropriate for multiple discount levels?

4 Solution Design

This chapter addresses the question: *Which method is appropriate for selecting the most suitable order policy for SKUs?* Section 4.1 starts with stating the assumptions. Section 4.2 states the solution's outline where the decision between a DLS, EOQ, or (s,Q) order policy is made. The rest of the chapter describes parts of the solution in more detail. Section 4.3 deals with the decision between DLS methods or an EOQ order policy. Section 4.4 describes the selection of the most suitable DLS method. Section 4.5 describes the performance indicators. Section 4.6 shows how the appropriate order policy is determined based on a comparison with the performance indicator defined in the previous section. Section 4.7 describes FDP with non-dominated solutions since the applied FDP is adapted from the one found in the literature. The other DLS methods are described in Section 3.3 during the literature review. Section 4.8 ends with a conclusion.

4.1 Assumptions

The assumptions for all the order policies are:

- The price decreases or remains constant if the order quantity increases.
- Demand for the coming 40 business days is known and demand after that period is stochastic.
- SKUs with lead time larger than logistical time have stationary demand.
- Holding costs are only included when SKUs are in stock. This means that holding costs between the order's placement and the order's arrival are zero.
- Historical data is representative of future demand.
- All discounts are all unit lot-sized-based discounts on SKU level. There are no coordinated discounts over multiple SKUs as stated in the scope in Section 1.4.
- SKU lead times are deterministic as defined in the scope in Section 1.4.
- There are no SKUs with a minimum order quantity (MOQ) or order quantity increment (OQI) as defined in the scope in Section 1.4.
- There is enough storage space and capital available to purchase all orders.

The DLS methods are restricted to consider only the known demand, i.e., the forecast is not considered. This is done since the quality of the forecast is unknown. VMI uses a point forecast based on the average historical demand, which is equal for every period in the future.

The DLS methods work with periods of demand. A smaller period expands the solution space, possibly leading to better solutions. However, this approach also demands more computational time. Additionally, the length of each period corresponds with the frequency of model executions and the rate at which information is communicated to the purchasing department. A period length of one week is considered an optimal interval for running the model effectively. Next, the holding costs are charged on a weekly basis, which corresponds to the length of the period.

4.2 Solution Outline

The literature study in the previous chapter identifies five relevant order policies for VMI. Each other policy has its advantages and disadvantages. Table 4.1 shows the different order policies. Only the (s,Q) order policy is suitable for stochastic demand and can be used when the SKU's lead time is larger than the logistical time. The logistical time is the time between engineering and the required date. The performance of the deterministic order policies is indicated based on the information from the literature. Next, the table shows the performance regarding optimizing the order policy to have the lowest relevant costs for the deterministic order policies. Furthermore, an indication of the computational time is given. Forward dynamic programming (FDP) and Silver-Meal (SM) are able to incorporate discounts into the decision of the order policy, whereas lot-for-lot (LFL) and maximum part-period gain (MPPG) are not. The current ERP system cannot handle the different dynamic lot sizing (DLS) methods.

Table 4.1: Relevant Order Policies Found in Literature

Order Policy	Demand	Performance	Computational Time	Discount	ERP System
LFL	Deterministic	Low	Low	No	Yes
FDP	Deterministic	High	High	Yes	No
SM	Deterministic	Moderate	Moderate	Yes	No
MPPG	Deterministic	Moderate	Moderate	No	No
EOQ	Deterministic	-	Low	Yes	Yes
(s,Q)	Stochastic	-	Low	Yes	Yes

Figure 4.1 shows how an order policy can be selected for every SKU. Below is an explanation regarding the selection of order policies, which explains the different boxes, including the coloring. Appendix K shows a more general approach with the possibility of extending the DLS methods with a forecast.

The flowchart starts by checking if the SKU lead time exceeds the logistical time. VMI has a logistical time of 40 business days in the current situation. SKUs with a lead time larger than the logistical time have stochastic demand and require safety stock. The first subsection describes the left side of the flowchart for SKUs without safety stock. The right side for SKUs with a safety stock is described in the subsequent subsection.

4.2.1 SKUs without Safety Stock

This subsection describes the left part of the flowchart for SKUs with a lead time shorter than the logistical time. These SKUs have deterministic demand for the first 40 business days. Demand after that period is stochastic. This means that no safety stock is required.

DLS methods are restricted to the known demand, and no forecast is used to expand the input. The EOQ is based on the average historical demand and is not restricted to known demand. There are multiple reasons why the order quantity would exceed the deterministic demand. Section 4.3 describes these different reasons. However, SKUs with a high risk of becoming obsolete should not be purchased more than the demand known because the potential benefit will not outweigh the risk.

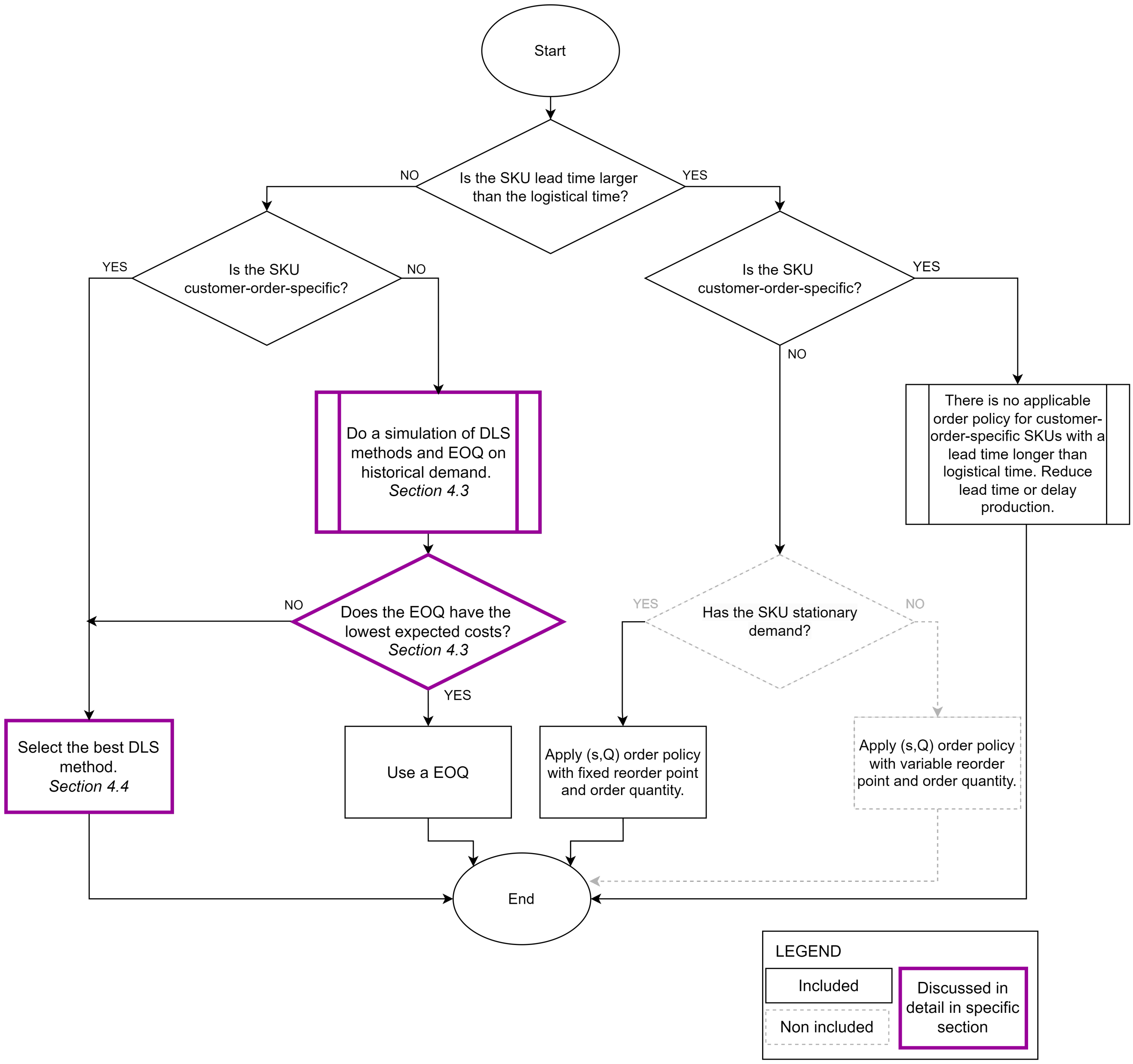

Figure 4.1: Select Order Policy

These SKUs should be purchased using the best DLS method.

SKUs with lead time under the logistical time and a low risk of becoming obsolete can also be purchased-to-stock using a simple EOQ order policy. For example, the availability of discounts may enlarge the optimal order quantity. Section 4.3 describes the decision between an EOQ and DLS methods.

The best DLS method is unknown at this stage. Section 4.4 describes in detail how the best DLS method is determined. The boxes which are uncertain at this stage are colored in purple.

4.2.2 SKUs with Safety Stock

This subsection describes the right part of the flowchart for SKUs with a lead time larger than the logistical time. These SKUs have stochastic demand.

SKUs with a lead time larger than the logistical time which are customer-order-specific raise problems. These SKUs cannot be stocked to cope with demand since it is impossible to forecast the demand for SKUs needed once in production. An inventory model cannot solve the problem since it is impossible to cope with such demand uncertainty. Other options need to be examined to solve the issue. For example, consult with the suppliers if the lead time can be decreased to 40 business days so these SKUs can be purchased LFL. The use of semi-finished products can reduce the lead time. A possibility is to produce the machine later in time so the logistical time increases. Another option is to add the SKU later in the production process. Exotic SKUs should be avoided in customization projects to prevent problems for the purchase department.

A (s,Q) order policy fits SKUs with stochastic demand. SKUs with non-stationary demand benefit from a (s,Q) order policy where the parameters change. A variable reorder point and lot-size cope with the non-stationarity effect. A fixed lot-size can handle stationary demand. The Augmented Dickey-Fuller test indicates if demand is stationary. The Augmented Dickey-Fuller test requires the demand per period per SKU as input. A period length of one week is used since non-stationary within the week is not of interest. The Augmented Dickey-Fuller test is introduced in Section 2.5.5.

The demand during the logistical time is known and is used to lower the reorder point. An order is placed when the inventory drops below the reorder point. The inventory position is the on-hand inventory plus on-order minus back orders minus committed demand. The committed demand is the demand known during the logistical time. Safety stock is needed to cope with the demand uncertainty during the effective lead time. The effective lead time is the lead time minus the logistical time. Using the information of demand during the logistical time decreases the uncertainty and lowers the required safety stock. This benefits the service level and reduces the holding costs. Appendix G shows the (s,Q) order policy in more detail. Section 3.2 describes the influence of discount for the order quantity.

Section 3.2 describes the (s,Q) order policy with fixed lot-size for stationary demand. Appendix G describes this order policy in more detail. Section 3.2 describes the (s,Q) order policy with variable lot-size for non-stationary demand. Unfortunately, no forecast, which includes the non-stationarity effect of demand, is available. Therefore, only the (s,Q) with fixed lot-sizes is used during the implementation. The flow of non-stationarity is depicted in gray for this reason.

4.2.3 Examples

This subsection describes the selection of order policies using the flowchart for different SKUs. Four different SKUs are taken in order to provide insight into the solution approach. Figure 4.2 to 4.5 shows drawings of the example SKUs.

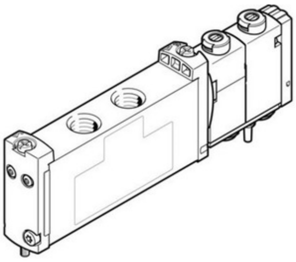

Figure 4.2: SKU *A*

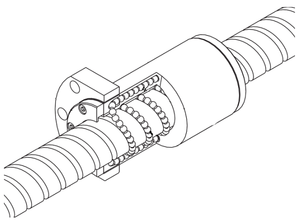

Figure 4.3: SKU *B*

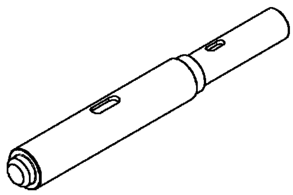

Figure 4.4: SKU *C*

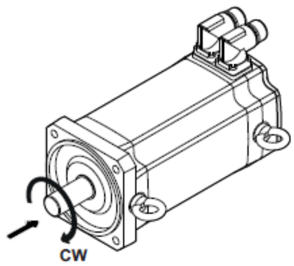

Figure 4.5: SKU *D*

Table 4.2: Order Policies for example SKUs

SKU	Lead Time	Standard Part	Discounts	Order Policy
SKU *A*	55 days	Yes	No	(s,Q)
SKU *B*	20 days	Yes	Yes	EOQ
SKU *C*	10 days	Yes	No	FDP
SKU *D*	125 days	No	No	-

SKU *A* is a pneumatic valve with a lead time of 55 business days and has, therefore, partly stochastic demand. The SKU is a standard part. No distinction has been made between SKUs with stationary and non-stationary demand. The SKU has a (s,Q) order policy. Appendix G shows how the parameters for a (s,Q) order policy are determined.

SKU *B* is a ball screw drive with a lead time of 20 business days. The SKU is part of the standard and has a discount of 20% when purchased with more than 25 pieces. Section 4.3 shows how the performance of the EOQ and DLS methods are compared. A simulation is done to quantify the performance of the EOQ and DLS methods. The simulation of SKU *B* is not described in detail since this would take up too much space. The relevant costs as the output of the simulation for EOQ, FDP, SM, and MPPG are €1500, €1550, €1560, and €1565, respectively. The EOQ outperforms the DLS methods for this SKU. Therefore, SKU *B* has an EOQ order policy.

SKU *C* is a shaft with a keyway. The SKU's lead time is ten business days, and it is part of the standard machine. The demand exhibits some variability between one and four pieces per week. The SKU has no discount. The performance of the EOQ and DLS are compared by a simulation like SKU *B*. The FDP has the lowest costs during the simulation and is chosen as order policy.

SKU *D* is an electric motor with a purchase price of €100. The lead time is 125 days. This electrical motor is stronger than the one used in the standard machine and is customer-order-specific. Unfortunately, the lead time, in combination with the fact that the SKU is not a standard part, makes it impossible to get the SKU on time in production. Therefore, an alternative way is searched. In consultation with the customer, the standard electric motor is used in the production process and when the machine is delivered. The electric motor is switched at the customer's location to the stronger one when the stronger motor becomes available.

4.3 DLS Methods versus EOQ

This section deals with whether a simple EOQ outperforms the different DLS methods for a SKU. First, the EOQ order policy is described. There are different reasons why an EOQ can outperform DLS methods. The advantages and disadvantages of both groups are described to provide insight into their needs. The last part of the section deals with how the decision between DLS methods and EOQ is made.

4.3.1 EOQ Description

The EOQ order policy is an order policy that purchases-to-stock. The SKU lead time is smaller than the logistical time of 40 business days, so there is no need for safety stock. The reorder point equals the demand during the lead time. The order quantity equals the EOQ or a price breakpoint. The parameters of this order policy can be calculated using the (s,Q) order policy information by setting the safety stock to zero.

4.3.2 Advantages and Disadvantages of EOQ and DLS Methods

The DLS order policies are limited to the demand known, so the order quantity cannot exceed the demand of the first 40 business days. Multiple reasons exist to increase the demand and use an EOQ order policy.

First, the stock position covers demand during the SKU's lead time. This means that the order quantity for DLS methods is limited to the demand of 40 business days minus the lead time. Only demand for a short time window can be purchased if the lead time is close to the logistical time. For example, if the SKU lead time is 39 business days, the order quantity is limited to the demand of one day.

Secondly, the availability of discounts can increase the order quantity. The demand within the first 40 business days, minus the lead time, might not be high enough to be eligible for discounts. Switching to the EOQ can mean incorporating discounts and reducing the purchase amount.

Thirdly, the economic order quantity can simply be larger than the demand during the logistical time minus the lead time. Especially if the demand is low and the order costs are high. This results in an optimal lot-size larger than the demand during the logistical time. Therefore, the EOQ outperforms the DLS methods. The order costs are lower using an EOQ instead of a DLS method.

4.3.3 Decision Between EOQ and DLS

The demand after 40 business days is stochastic. The DLS methods are restricted to the first 40 business days and have no risk that inventory becomes obsolete, unlike EOQ. Therefore, the EOQ order policy is only possible for SKUs that are part of the standard machine since those SKUs have a low chance of becoming obsolete. For this reason, customer order-specific SKUs with a high risk of becoming obsolete are not purchased by an EOQ order policy.

An order can cover the demand of one or multiple weeks. An indication of the number of weeks the optimal order covers is required to select the appropriate order policy. The question arises of how many weeks of demand an order should cover. This answer can be answered in two ways.

The first option is calculating the number of weeks an order should cover based on the EOQ order policy. The number of weeks of demand is the economic order quantity (EOQ) divided by the average weekly demand. This procedure provides an indication of the number of weeks an order covers. If the number of weeks is short enough that the optimal order quantity is determined on the deterministic demand, use DLS methods. Otherwise, the EOQ order policy can be used.

The second option is to base the decision on the performance of a representative data set of both order policies. Therefore, stochastic and deterministic order policies are tested on historical data based on the assumption that the data represents the future demand. The performance of both order policies is compared, and the best performance is chosen. Section 4.6 shows how the order policies can be compared.

Multiple SKUs have lumpy or non-stationary demand, meaning demand might fluctuate over time. An indication about the number of weeks that an order covers is a less suitable approach for non-stationary demand and, therefore, is the second option preferred to base the decision on the performance of both order policies. However, the first option should be used if there is no historical data, which is the case for new SKUs.

New introduced SKUs might not have a forecast or a representative data set for demand. Such SKUs with a lead time shorter than the logistical time receive standard the FDP order policy. SKUs with a lead time longer than the logistical time need a forecast before the order policy can be determined.

Appendix L describes how the DLS method can enlarge the order quantity by incorporating a forecast. Currently, VMI has no forecast, including an accuracy level, but only has a point forecast. Therefore, this is not incorporated into the current model.

4.4 DLS Method Selection

This section describes how the best DLS method is selected. Firstly, the characteristics and capabilities of the different DLS methods. Afterward, this section explains how comparing the order policies can help determine the best DLS method.

The literature identifies three relevant DLS methods: forward dynamic programming (FDP), Silver Meal (SM), and maximum part-period gain (MPPG), each with advantages and disadvantages. FDP can find the optimal solution for a limited horizon. Section 4.7 describes FDP in detail. Since FDP checks all relevant order quantities, the computational time rises exponentially with the number of periods and discount levels. The computational time of FDP can be too high to make decisions in time, necessitating the use of heuristics with a lower computational time. SM and MPPG are two promising heuristics. Next, the performance difference between the heuristics and the FDP algorithm is unknown. The performance of the heuristics might equal the performance of the optimal algorithm.

It is unknown if the overall performance improves by optimizing for a limited period ahead. FDP finds the solution by optimizing to the optimal solution for the limited period ahead. However, the order quantity found based on this limited window is not necessarily the optimal solution given the complete information. It is, therefore, uncertain if optimizing to the optimal solution for a limited time window improves the overall performance or if a heuristic shows equal or even better performance.

Moreover, there is uncertainty about which heuristic has the best performance. The SM heuristic incorporates discounts, but the MPPG performs well under the rolling horizon. These questions are answered by comparison of the performance under a rolling time horizon.

Although different relevant order policies are identified from the literature, it is still uncertain which DLS order policy should be used for which SKU. This has the following causes: the performance is not quantified, so whether computational time provides issues is unknown. The performance of the heuristics under a rolling time horizon is unknown. The performance can be quantified by simulating the different order policies under a rolling horizon. Section 4.6 shows how such a simulation can be done. A decision based on quantitative information can be made in this way.

Customer order-specific SKUs with low demand benefit from a LFL purchase approach. Purchase on a LFL basis is a possible outcome of the DLS order methods. Therefore, the LFL order policy will never outperform DLS methods since the DLS methods can purchase on a LFL basis, but the LFL order policy can never enlarge the order. Analysis in a later stage of the research validates that the LFL never performs better than the DLS methods. In case of equal performance between LFL and FDP, it is chosen to select the FDP order policy in the state of the LFL. This is because FDP will outperform LFL in case demand increases and LFL will never outperform FDP. Given that the optimal order quantity of the past was always LFL, this does not mean that multiple production orders should never be combined into one purchase order in the future. The FDP is returned as an order policy for such SKUs since this order policy has the best-expected performance.

Example SKU *E* is a brake block. This is a new SKU since the material quality of this brake block is different from the one used in the standard machine. The SKUs lead time is 30 days. The combination of 30 days lead time and not in the standard makes the order policy a DLS method. There is no expectations of demand in the future and therefore has this SKU the standard DLS method FDP as order policy.

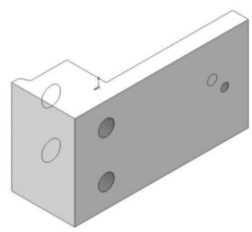

Figure 4.6: SKU *E* brake block

4.5 KPIs

This section describes which key performance indicator (KPI) is used to evaluate order policies. At first, the different assessment criteria given by the stakeholder analysis are evaluated to explain how these are used in the decision of the appropriate order policy. The stakeholder analysis, in Section 2.2, defines four possible assessment criteria. These different criteria are described below in detail.

Relevant Costs The relevant costs of an order policy are the sum of the ordering costs and holding costs minus the obtained discounts. The relevant costs as a result of the order policy is the most important performance indicator.

Item Availability The stakeholders prefer a high item availability. The lead time is assumed to be deterministic. The item availability depends on the safety stock of the SKUs with a nonzero effective lead time. The effective lead time is the replenishment lead time minus the logistical time of 40 business days. Only SKUs with a nonzero effective lead time have a safety stock. The (s, Q) order policy is the only order policy for stochastic demand. Therefore, the height of the safety stock does not influence the decision between the order policies. The item availability is not used as a KPI to select the order policy; it is only used to determine the service level.

Average Capital in Stock VMI wants to reduce its inventory position in order to minimize capital investment. The holding costs exist partly out of the weighted average costs of capital (WACC), so the costs of capital on stock are part of the holding costs. Therefore, the inventory position or capital on stock is not defined as a KPI since it is already included in the overall costs and otherwise will be incorporated twice.

Easy and Smooth Process VMI prefers an easy and smooth order process over a complicated one. An order policy that the ERP system can handle is quantified to be a simple order policy that results in an easy and smooth process. Next, the computation time indicates how complicated the order policy is. VMI does not care about the computational time as long as the results are on time. The order policy is unusable if the computational time is too high, such that the results will be outdated if the order policy is finished. So, the computational time should not exceed one week for all the SKUs since orders should be placed weekly. The constraint that the ERP can handle the order policy is neglected at first sight when looking at the possibilities with these order policies. Section 5.5.1 analyses if VMI should neglect this constraint and use order policies outside of the ERP system. Next, the computational time of the different order policies is a constraint and is not used as a performance indicator.

To conclude, although there are multiple performance indicators, only one remains relevant. The appropriate order policy is based on the relevant costs of the different order policies, given that it provides results within a reasonable time frame.

4.6 Comparison between Order Policies

A comparison of the different order policies is required to decide which order policy is most appropriate. The order policies are tested by means of simulation to make a quantified decision of the most appropriate order policy. A historical data set of two years is used in the simulation to test the performance. The first year is used to determine the parameters. The second year of data is used to test the order policies. The performance under a rolling horizon is tested in this way. The overall costs of the different order policies are compared to each other, and the order policy with the lowest overall costs is selected.

It is possible that there are multiple order policies with the same, lowest overall costs. The order policy is selected based on preference among the different order policies. LFL is preferred over EOQ-based order policy, which is preferred over DLS methods. DLS methods are the least preferable since they are outside the ERP system. The LFL requires less capital investment and has less risk regarding obsolesce; therefore, it is preferred over EOQ-based order policies.

4.7 Forward Dynamic Programming with Non-Dominated Solutions

The literature introduces the different order policies. The forward dynamic programming (FDP), presented during the literature review, is changed to handle multiple discount levels. This section describes how FDP from the literature is altered to this FDP with non-dominated solutions. The other order policies can be found in Chapter 3 and are described in detail in Appendices F to J for LFL, SM, and MPPG, respectively. Appendix E provides an overview of all the parameters used for the different order policies.

This section describes forward dynamic programming (FDP) with non-dominated solutions. FDP can find the optimal order moments and quantities for the periods over which demand is known. The Wagner-Whitin algorithm [21] is taken as the basis for this algorithm but is changed so discounts are included. This algorithm incorporates discounts by building a solution based on non-dominated solutions. The basic principle of the FDP with non-dominated solutions is to examine all relevant order quantities in a structured order and select non-dominated solutions to decrease the number of options and reduce computational time.

First, the relevant order quantities (ROQs) are discussed. The algorithm is then adapted to include all ROQs. Next, logic is included to limit the solutions to only non-dominated solutions to save computational time. Afterward, the algorithm is described in mathematical form. Finally, the influence of the adaptation of discounts on computational time is discussed.

Relevant Order Quantities

The optimal order quantity equals the demand of one or more adjacent periods or price breakpoints. Appendix H.1 shows proof of this theorem and provides more insight into the optimal order quantity. The consequence of having multiple potential order quantities is that there are multiple ROQs to go from one stage to the next stage. Multiple ROQs should be considered when moving from one stage to another stage, resulting in multiple solutions. Each solution has its own costs, ending inventory position, and order quantities.

ROQs are all order quantities that should be considered and checked in the solution. Equation (4.1) shows all the relevant order quantities for the period j to period k. The start inventory of period j is I_j. The set DL denotes all price breakpoints. No order is placed if the inventory position is high enough to fulfill demand.

$$ROQ_j = \begin{cases} \{0\} & \text{if } d_j - I_j \leq 0 \\ \{\alpha \in DL \mid \alpha > d_j - I_j\} \cup \{d_{j,k} - I_j \text{ for } k = j+1, \ldots, T-1\} & \text{if } d_j - I_j > 0 \end{cases} \tag{4.1}$$

$$d_{j,k} = \sum_{i=j}^{k-1} d_i \quad \text{for} \quad k \geq j \quad \text{with} \quad d_{j,j+1} \equiv d_j \tag{4.2}$$

Non-Dominated Solutions

Each intermediate solution exists out of three components. First, the costs associated with the solution. This is the value of the solution to which the optimal value function is referring to. Secondly, the end inventory position of a solution. This is the state of the solution. At last, the order quantities of the solution define the solution.

There are different ROQs to go from one stage to the next stage. Different ROQs result in different solutions, each with its own costs and ending inventory. Furthermore, a stage can be reached in different ways. In other words, different order moments can be used to end up in the same period. The best solution out of the different solutions should be chosen.

A solution is non-dominated if there is no other solution for the same stage with lower costs and the same or higher inventory. Solutions with the same costs and ending inventory are of equal quality and are indifferent. The order policy where the first order quantity is the smallest is chosen for indifferent solutions (solutions of the same quality) since they have the least risk of SKUs becoming obsolete.

Next, there are solutions where the inventory position does not justify the higher costs. For example, there are two solutions. Solution A has an overall cost of €100 and no end inventory. The other solution, solution B, has an overall cost of €125 and an end inventory position of five units. At first sight, both solutions are non-dominated since none is strictly better. The cost for an order, existing out of both purchase amount and order costs, of 5 units is 20 euros. This enhances the fact that choosing solution A and buying the five units in the next period is always cheaper than solution B, independent of future demand. Therefore, a solution is non-dominated if there is no other solution with lower costs with an equal or larger inventory position.

FDP with Non-Dominated Solutions Algorithm

The stage corresponds to the beginning of period k with T as the length of the planning horizon. The state, denoted as I_k, represents the inventory level at the end of period k. The decision is to choose the preceding regeneration point j. Equation (4.3) shows the optimal value function, where $F(k)$ is the value at the start of period k when considering the previous regeneration point in period j. The value is the overall costs made until period k with the ending inventory I_k.

$$F(k) = \min_{j=1,\ldots,k-1} \{F(j) + c(j,k)\} \tag{4.3}$$

The boundary condition is defined as $F(1) = 0$. The solution is represented by $F(T+1)$, which exist of all non-dominated solutions for stage $T+1$.

The cumulative stage-cost function, $c(j,k)$, represents the total cost between two successive regeneration points. Equation (4.4) presents the stage-cost function. Q is the order quantity and S is the order cost. The purchase price $(P(Q))$ is a function of the order quantity since discounts reduce the item price for larger quantities. The price is independent of the order quantity if there are no discounts.

$$c(j,k) = P(Q) \cdot Q + \delta(Q) \cdot S + \sum_{i=j}^{k-1} [I_j + Q - d_{j,i}] h_w \tag{4.4}$$

Equation 4.1 determines all the ROQs. An order is only placed if the initial inventory is insufficient to meet the demand until period k. The order costs are only included if an order is placed, with δ defined in Equation (4.5). The holding costs are based on the inventory position at the end of the week. The holding costs are charged every week an item is in stock. h_w is the holding costs in euros per item per week in storage.

$$\delta(Q) = \begin{cases} 0 & \text{if } Q = 0 \\ 1 & \text{if } Q > 0 \end{cases} \tag{4.5}$$

$F(k)$ exists out of all non-dominated solutions for period k. All these non-dominated solutions are incorporated to find the optimal solutions for the next period. In the end, the optimal solution should be chosen for $T+1$. Multiple non-dominated solutions exist for $T+1$ in the case of discounts. There are multiple ways to choose the solution. Option 1 is used for the implementation.

1. Choose the solution with the lowest ending inventory. This solution has the lowest costs and risk of obsolete SKUs.

2. Calculate the costs without inventory position. Equation 4.6 shows the costs without inventory. The question is which price to use since the next order quantity will be unknown, thereby the item price. The holding costs are also unknown; thus, an expected value should be taken. The item price is based on the order quantity of the last order. Next, the holding costs are calculated based on the expected time on storage. The expected periods in storage are half the inventory position divided by the average demand per period.

$$\text{Costs without IP} = \text{Costs} - \text{Inventory} \cdot \text{Item Price} + \text{Expected Holding Costs} \tag{4.6}$$

Appendix H shows the pseudo-code for the FDP with non-dominated solutions.

Computation Time for FDP with Non-Dominated Solutions

The running time for the case with no discounts is order $(periods)^2$ since there is one ROQ to go from one stage to another stage. The ROQs are based on the number of non-dominated solutions. Given that the maximum number of non-dominated solutions is $(1+\#DL)^{\text{periods}}$ as defined in Theorem 2 (in Appendix H.2) with $\#DL$ the number of discounts for the SKU. The number of ROQs for every non-dominated solution is $(1+\#DL)$. Combining this information leads to a computational time in order $\left(periods^2 \cdot (1+\#DL)^{periods+1}\right)$. The computational time increases exponentially with the number of periods and discount levels. Appendix H.2 describes the maximum number of non-dominated solutions and more information regarding the computational time of the FDP algorithm.

Numerical illustration

Table 4.3 shows the production orders for SKU B grouped per week since there is a time bucket of one week. Table 4.4 shows the relevant prices for this numerical illustration. The start inventory is ten units.

Table 4.3: Demand SKU *B*

Week	Quantity
~~0~~	~~10~~
1	15
2	9
3	30
4	24

Table 4.4: Prices SKU *B*

Price	Description
€ 10	$Q < 25$
€ 8	$Q \geq 25$
€ 25	Order costs
€ 1	Holding costs per unit per period

The lead time of SKU *B* is five days, and this toy problem starts in week zero. The inventory position equals the demand during lead time. Therefore, the problem is analyzed from week 1 with no lead time and a start inventory position of zero units to simplify the calculations.

$$F(1) = 0 \tag{4.7}$$

The stage cost function $c(1,2)$ includes the following relevant order quantities: $Q = \{15,25\}$. The 15 units cover the demand from the start of period 1 to the start of period 2. Additionally, the price breakpoint of 25 units is considered as a ROQ. The stage cost is calculated as defined in Equation 4.4.

$$c(1,2) = \begin{cases} 10 \cdot 15 + 1 \cdot 25 + (0 + 15 - 15)(1) = €175 \text{ (0 units)} \\ 8 \cdot 25 + 1 \cdot 25 + (0 + 25 - 15)(1) = €235 \text{ (15 units)} \end{cases} \tag{4.8}$$

$$F(2) = \min\{F(1) + c(1,2)\} = \begin{cases} 0 + 175 & = €175 \text{ (0 units)} \quad [15]* \\ 0 + 235 & = €235 \text{ (10 units)} \quad [25]* \end{cases} \tag{4.9}$$

At the start of period 2, there are two non-dominated solutions. The costs, inventory position and order quantities are provided. The order quantities are between square brackets. Both should be considered in further calculations when evaluating $F(2)$.

$$F(3) = \min\{F(1) + c(1,3),\ F(2) + c(2,3)\} \tag{4.10}$$

$$= \begin{cases} 0 + 10 \cdot 24 + 25 + (9) \cdot 1 & = €274 \text{ (00 units)}[24] \\ 0 + 8 \cdot 25 + 25 + (11) \cdot 1 & = €236 \text{ (01 units)}[25]* \\ 175 + 10 \cdot 9 + 25 + (0) \cdot 1 & = €290 \text{ (00 units)}[15,9] \\ 175 + 8 \cdot 25 + 25 + (16) \cdot 1 & = €416 \text{ (16 units)}[15,25]\circ \\ 235 + 10 \cdot 0 + 0 + (1) \cdot 1 & = €236 \text{ (01 units)}[25]* \end{cases} \tag{4.11}$$

Solutions 2 and 5, indicated with $*$, are identical since both correspond to the same price breakpoint. Both solutions order the same quantity of 25 units for the first period. Solutions 1 and 3 are dominated solutions since these solutions do not have end inventory, but the costs are higher compared to solutions 2 and 5. At first sight, solution 4, indicated with $\circ$, is also non-dominated since no solution has an ending inventory larger than 16 units with costs equal to or lower than 416 euros. However, solution 4 is dominated by solution 2. Solution 4 has $16 - 1 = 15$ units higher ending inventory than solution 2, costing $416 - 236 = €180$ more. An order of 15 units costs: $10 \cdot 15 + 25 = €175$. Therefore, solution 4 is dominated by solution 2 since the cost difference is larger than the costs associated with an order of 15 units. Stage 2 has only one non-dominated solution since solutions 2 and 5 are identical, and these will consistently outperform solutions 1,3 and 4. The non-dominated solutions are indicated with $*$. Note that the number of non-dominated solutions can decrease in later stages.

$$F(4) = \min\{F(1)+c(1,4),\ F(2)+c(2,4),\ F(3)+c(3,4)\} \tag{4.12}$$

$$= \begin{cases} 0+8\cdot 54+25+(39+30)\cdot 1 & = €526 \text{ (0 units)}[54] \\ 175+8\cdot 39+25+(30)\cdot 1 & = €542 \text{ (0 units)}[15-39] \\ 235+8\cdot 29+25+(1)\cdot 1 & = €468 \text{ (0 units)}[25-29]* \\ 236+8\cdot 29+25+(0)\cdot 1 & = €468 \text{ (0 units)}[25-29]* \end{cases} \tag{4.13}$$

Solutions 3 and 4 are identical since they are built on the same solution of 25 units. However, multiple solutions with identical paths of order quantities might exist. This is because the algorithm systematically checks all relevant solutions, and some solutions can be reached in various ways due to the discount levels.

$$F(5) = \min\{F(1)+c(1,5),\ F(2)+c(2,5),\ F(3)+c(3,5),\ F(4)+c(4,5)\} \tag{4.14}$$

$$= \begin{cases} 0+8\cdot 78+25+(62+54+24)\cdot 1 & = €789 \text{ (0 units)}[78] \\ 175+8\cdot 63+25+(54+24)\cdot 1 & = €782 \text{ (0 units)}[15-63] \\ 235+8\cdot 53+25+(1+24)\cdot 1 & = €709 \text{ (0 units)}[25-53]* \\ 236+8\cdot 53+25+(24)\cdot 1 & = €709 \text{ (0 units)}[25-53]* \\ 493+10\cdot 24+25+(0)\cdot 1 & = €758 \text{ (0 units)}[25-29-24] \\ 493+8\cdot 25+25+(1)\cdot 1 & = €719 \text{ (1 units)}[25-29-25]* \end{cases} \tag{4.15}$$

There are two different non-dominated solutions. Order $[25-53]$ or $[25-29-25]$ where the last solution has an ending inventory of 1 unit. It is chosen to always select the solution with the lowest cost and, therefore, is the first non-dominated solution. In this case, all non-dominated solutions have the same first-order quantity; thus, this does not make a difference. The solution has an order quantity of 25 pieces for the first period. The order quantity of 25 units can cover the demand of 2 weeks. Since more demand is known, the FDP algorithm is used in two weeks to determine the next order quantity.

4.8 Conclusion

The model design can be summarized as follows:

- The model's primary objective is determining the most appropriate ordering policy for each SKU, including the relevant parameters based on the overall costs.
- SKUs with stochastic demand and purchased by means of a (s,Q) order policy.
- SKUs with deterministic demand and a high risk of becoming obsolete are purchased by means of a DLS method.
- SKUs with deterministic demand with a low risk of becoming obsolete are purchased by means of a DLS method or an EOQ.
- The model compares the performance of DLS and EOQ order policies on a historical and representative data set to the proposed order policy of the future.
- Forward dynamic programming with non-dominated solutions finds the optimal lot-sizes, but the computational time grows exponentially with the number of periods and discounts.

5 Results Analysis

This chapter consists of an analysis of the result in order to answer the question: *What is the performance of the selection model?* Section 5.1 starts with an explanation of the choice of software and hardware usage. Section 5.2 describes how the model code is verified and validated. The following section, Section 5.3, describes which input data is used for the model and in which format the model results are presented. Section 5.4 compares the model's results to the current situation. The comparison shows the improvement of the order policy if VMI implemented the new order policies relative to the current situation. Section 5.5 presents the sensitivity analysis. Different experiments show the impact of the constraints of the selection model, and the sensitivity analysis shows the influence of parameters. Section 5.6 ends with a conclusion.

5.1 Python Implementation and Hardware Setup

The model is implemented in Python for several reasons. Python is capable of handling the model complexity. Furthermore, Python is an open-source language that allows other users to run the code without buying licenses. Next, Python enjoys widespread popularity and boasts a wealth of documentation and freely available packages. This accessibility makes it feasible for other VMI employees to implement and expand upon the model. Python has the downside of being limited to a single core. This means the model is executed on a single core and does not fully utilize the hardware's capabilities.

The hardware configuration is as follows: The model runs on a notebook equipped with a 12th-generation Intel i5 processor running at 1.6 GHz and 16 GB of RAM. This setup is commonly used within VMI and delivers results within a reasonable time frame for small experiments. The running time for a single run for 3274 SKUs is around two hours, depending on the specific parameter setting.

5.2 Verification and Validation of the Model

The selection model is verified to confirm the reliability of the model and the code. This verification entails a thorough examination of all the steps that the model takes during its operation. The examination process exists out of different SKUs to assess the model's functioning and confirm its correct. Furthermore, the model is executed under various settings for the input parameters. The output is analyzed and found to be reasonable and logical. During the sensitivity analysis, part of these experiments are described in Section 5.5.

A selection of 3274 SKUs is used to validate the model. These SKUs consist of a single machine. 2350 of the SKUs have an effective lead time of zero days, making them suitable for the DLS order policies. A selection of SKUs is chosen to reduce the computation time and make it possible to do multiple experiments. Unfortunately, the real-world costs for this selection of SKUs are not available. Therefore, the model's results with the current parameter settings are compared to the expected costs of experts within VMI. The experts are the managers of the supply chain and the purchase department. The relevant cost for the current situation of the model deviates 3% by the expert opinion. This deviation is accepted. Furthermore, individual SKUs are analyzed, and results are found to be logical and realistic. The result is that the model is seen as valid.

5.3 Input and Output Data

This section presents the model's input and output data. The model selects the most appropriate order policy for every SKU. The model's input consists of SKU characteristics and general parameter settings. The model uses a rolling horizon to test the different order policies.

Figure 5.1: Input and Output Model

The SKU characteristics exist out of the following:

- The SKU's lead time in days is required to calculate the number of weeks in which demand can be combined and calculate the reorder point.
- The base price and discounts, if applicable, are input for the model.
- A representative data set of 2 years is needed for parameter estimation and used for testing the order policies. The parameter estimation is done in the first year of data. The second year is used to test the performance of the different order policies.

The general parameter settings exist of parameters that are independent of the SKU:

- The order cost is €¿ per order line as defined in Section 2.4.
- The holding costs are ¿% of value in stock per year as defined in Section 2.4.
- The period length is set to one week, as explained in Section 4.1.
- The logistical time determines how many deterministic weeks there are. VMI has eight weeks of logistical time, so the demand for eight weeks upfront is known.
- The safety time is set to ten business days before the start of production. The required date equals the production date minus the safety time to cope with extended lead times. So, the required date is ten days before the start of production.
- The service level for SKUs influences the safety stock when the SKUs have a nonzero effective lead time. The probability that no stock-out occurs is based on the service level and is explained in detail below.

Service Level

The service level is based on the probability of a stock-out. The stakeholder analysis in Section 2.2 shows that this is the most appropriate service level for VMI since VMI wants to reduce the number of disruptions in the production process.

This research assumes deterministic lead times. Therefore, all SKUs with a lead time smaller than the logistical time of 40 business days will always be available on time for production. Only the availability of SKUs with a lead time larger than 40 business days can be influenced. The service level influences the height of the safety stock for the (s, Q) order policies by setting the probability of no stock out per order cycle. The service level does not influence the choice between the order policies.

A high service level is typically essential for critical SKUs; however, the distinction between critical and non-critical SKUs is unknown. Therefore, no distinction can be made regarding the criticality of SKUs, and the distinction is made on the purchase price of a SKU. Maintaining a continuous production process is crucial, and it is undesirable to stop the production process for cheap SKUs that are not in stock. Cheap SKUs are less costly to stock compared to expensive SKUs. Cheap SKUs receive a high service level. The service level for expensive SKUs is lower. The management of VMI sets the probability that no stock-out occurs per order cycle. Table 5.1 shows the probability of no stock-out per order cycle for the different price categories.

Table 5.1: Probability of No Stock-out per Order Cycle

SKU Category	Probability	Categorization Definition
Cheap SKUs	99%	Purchase Price $\leq$ €50
Mid-range SKUs	98%	€50 $<$ Purchase Price $\leq$ €200
Expensive SKUs	95%	Purchase Price $>$ €200

In the ideal case, the service level depends on the criticality, item price, and the risk of obsolescence. Critical SKUs require a higher service level compared to less critical SKUs. Next, expensive SKUs are less preferred to stock and have high holding costs, so they should have a lower service level. Finally, SKUs with a high probability of becoming obsolete should also have a lower service level. The current service level only depends on the item price.

Output Data

The model returns the most appropriate order policy, including the relevant parameters for all the SKUs. Next, the cost components for the different order policies are given as output to provide more insight into the solution.

5.4 Analysis Results Selection Model

The proposed order policies by the model are compared to the current order policies to place the proposed order policies in perspective. Table 5.2 shows the number of SKUs for the different order policies. The ‘to-stock’ order policies for the current situation are split into two categories. SKUs with a lead time smaller than 40 business days are EOQ since they do not require a safety stock. The SKUs with a lead time larger than 40 business days are (s, Q).

5.4.1 Division Order Policies

Around two-thirds is purchased LFL for the current situation, whereas the LFL is not used in the proposed situation. The number of SKUs purchased-to-stock is larger in the proposed situation. In the current situation, there are SKUs purchased LFL with a lead time larger than 40 business days. This can be done for SKUs that are needed in a later stage for production and have, therefore, a larger logistical time. However, detailed knowledge of the SKU level is required for this decision, which the model does not have. Next, the FDP appears to be the best DLS method. The SM and MPPG are not used as order policies.

Table 5.2: Order Policies Distribution of Current and Proposed Order Policies

Order policy	Current Situation	Proposed Situation
LFL	2064 SKUs	0 SKUs
FDP	0 SKUs	1996 SKUs
SM	0 SKUs	0 SKUs
MPPG	0 SKUs	0 SKUs
EOQ	890 SKUs	906 SKUs
(s,Q)	320 SKUs	372 SKUs
Total	3274 SKUs	3274 SKUs

5.4.2 Costs Order Policies

The overall costs, consisting of purchase amount, ordering costs, and holding costs, are €¿ million and €¿ million for the current and proposed situations, respectively. The purchase amount is a substantial part of the overall costs. The relevant costs provide more insight so is used in the rest of this chapter. Table 5.3 shows the relevant costs. The relevant costs are ordering and holding costs minus the obtained discount. Absolute values are used to compare the cost reduction between the current and proposed situation.

Table 5.3: Normalized Values of Relevant Costs of Current and Proposed Order Policies

Order policy	Current Situation	Proposed Situation	Cost Reduction
Ordering Costs	97%	113%	84%
Holding Costs	16%	18%	-2%
Discount	-27%	-31%	19%
Relevant Costs	100%	100%	100%

The relevant cost is reduced by 45% for the proposed situation. The largest part of the cost reduction is in the ordering costs. The new order policy has larger order quantities, which reduces the number of order moments. Therefore, the proposed order policy is much better able to include discounts in its decision. The increase in order quantities leads to a higher inventory level and, consequently, an increase in holding costs. The average capital on stock increases from €¿ million to €¿ million.

Analysis of the current order policies resulted in the following insights. First, 17% of the SKUs with a lead time of more than 40 business days have non-stationary demand. These SKUs benefit from a variable lot-size and reorder point, which they do not have since there is no forecast available that includes the non-stationarity effect. Second, 60% of the long lead time SKUs are electrical OEM. The electrical OEM commodity has the longest lead times in general. 95% of the SKUs of the mechanical OEM and OS commodities have a lead time smaller than 40 business days and do not need any safety stock. Mechanical SKUs have FDP and EOQ as order policies for 49% and 46% of the SKUs, respectively. Third, the most discounts in relative terms occur for SKUs with a price between €1 and €10. The most discount in absolute terms for SKUs with prices between €100 and €1,000. Last, SKUs with more demand often have an EOQ instead of FDP as an order policy, which makes sense.

5.4.3 Computational Time

The computational time for the new order policy is 105 minutes for the selection of SKUs. The major part, 88 minutes, is required for the simulation of the different DLS order policies. The simulation takes up most of the computational time. The simulation tests the order policies on one year of demand data. The DLS methods select the optimal order quantity for every week, which can be zero. The simulation costs around 1.1 seconds per SKU and depends on the demand and the number of price breakpoints for every SKU. This means a computational time of around a second is needed to select the order policy. Calculating the order quantity for the current period takes a maximum of 0.1 seconds per SKU. Extrapolation to all SKUs used in a year (40,000 SKUs) gives an expected computational time of 12 hours for determining the order policy and parameters. This is which SKUs have FDP, EOQ, or (s,Q) order policy and the parameters of the reorder point and order quantity. It takes less than one hour to determine the orders for one week for 40.000 SKUs. This makes it possible to evaluate and update the order policy for every SKU once a month. More often is possible, but the difference in order policy from one week to another is expected to be marginal. The results are within a reasonable time frame, so FDP is accepted as a possible order policy.

5.4.4 Performance DLS Methods

The proposed situation consists only of FDP, (s,Q), and EOQ order policies. This means that FDP outperforms SM and MPPG. A selection of SKUs is taken to test the new situation. All these SKUs have demand in 2021 and 2022. This means there are no customer-order-specific SKUs with one moment of demand during the SKU lifetime. The result is that FDP always performs at least as well as LFL since none of the SKUs have only demand once.

There are C-type SKUs, the least important group of SKUs, that have a FDP order policy. This is because the ABC classification is based on the turnover of SKUs. Type-C SKUs generally receive a simple order policy such as the EOQ. Expensive SKUs with low demand are type-C SKUs since the demand is low, so is the turnover. These SKUs often have FDP as the proposed order policy since FDP is able to choose the right order moment and quantity. The first experiment in Section 5.5.1 provides insight into the difference in performance between FDP and LFL or EOQ order policies.

The performance of the MPPG and SM heuristics is almost equal. The MPPG has equal solutions for 99% of the SKUs, and the relevant costs are €325 higher for 2450 SKUs for one year. The MPPG and SM have 16% higher relevant costs for 2450 SKUs than the FDP algorithm. FDP performs mainly better in electrical OEM and mechanical outsourcing commodities. Experiment *Restriction to Order*

Policies Inside ERP System in Section 5.5.1 describes the consequences if there is no FDP order policy. This provides insight into how much better the FDP algorithm is compared to the LFL and EOQ.

The current situation is simulated, and only demand is known during the first eight weeks. In reality, more information is known, so the DLS methods can look further ahead. Further research can be done to identify the consequence when more demand is known.

5.4.5 Extrapolation to all SKUs

5.4.6 Service Level

A simulation is done on the SKUs with an (s,Q) order policy to test the service level. The other SKUs have a lead time shorter than the logistical time, meaning an order can always be placed on time. Since delays in lead time are out of scope, SKUs with zero effective lead time have a service level of 100% and are therefore not included in this analysis. The service level obtained in the simulation experiment is compared to the target service level of the different categories.

Table 5.4: Obtained Service Level for Proposed Situation

SKU Category	Service Level of Experiment	Target Service Level
Cheap SKUs	98.7%	99%
Mid-range SKUs	98.1%	98%
Expensive SKUs	95.6%	95%

Table 5.4 shows the obtained service level during the simulation. The obtained service level for expensive SKUs is slightly higher than the target value, and the cheap SKUs underperform. The SKUs with low performance are mainly SKUs with non-stationary demand where the (s,Q) order policy assumes stationary demand.

5.5 Sensitivity Analysis

The sensitivity analysis analyzes the influence of parameter changes. The reason for conducting these experiments is twofold. Firstly, the model's input parameters are based on assumptions. The impact of these assumptions is tested in the experiments. Good parameter estimation is required if the results exhibit high fluctuations if the input parameter is slightly altered. Conversely, if the results remain unchanged despite variations in the input parameter, it suggests that no further research is required to redefine the parameter estimate. Secondly, these experiments provide valuable insights into the influence of constraints on the model. The consequence of addition constraints provides insight into the model.

5.5.1 Restriction to Order Policies Inside ERP System

VMI prefers a simple policy over a complicated one and prefers to handle all the order policies within the ERP system. This constraint is released during this research to determine the maximum achievable. However, this experiment includes the constraint of restricting the order policies to choose which the ERP system can handle. This means that all DLS order policies are excluded, and a new order policy must be found for the SKUs with FDP as the proposed order policy.

Table 5.5 shows the distribution of order policies when FDP is impossible. 87% of the SKUs with an order policy of FDP are transferred to LFL, and the other 13% have an EOQ order policy.

Table 5.5: Order Policies Distribution of Experiment no DLS Methods

Order policy	Exclude DLS	Including DLS	Current Situation
LFL	1719 SKUs	0 SKUs	2064 SKUs
FDP	0 SKUs	1996 SKUs	0 SKUs
EOQ	1183 SKUs	906 SKUs	890 SKUs
(s,Q)	372 SKUs	372 SKUs	320 SKUs
Total	3274 SKUs	3274 SKUs	3274 SKUs

Table 5.6 shows the relevant costs increase when FDP is impossible. The relevant costs increase with €¿, of which is 68% an increase in ordering costs.

Table 5.6: Normalized Values of Relevant Costs of Experiment no DLS Methods

	Exclude DLS Order Policies	Proposed Situation	Current Situation
Ordering Costs	107%	113%	97%
Holding Costs	18%	18%	7%
Discount	-25%	-31%	-4%
Relevant Costs	100%	100%	100%

Limited order policies to that the ERP system can handle lead to a cost increase of €¿. The ordering and holding costs increase by 10% and 12.3%, respectively. The increase in ordering costs is caused by SKUs being purchased by LFL instead of by FDP. A group of SKUs purchased by an EOQ system instead of FDP also causes an increase in holding costs. This experiment shows the strength of the FDP algorithm in balancing the holding and ordering costs.

Figure 5.2 shows the increased costs for the 1996 SKUs with FDP as the proposed order policy. There are 1066 SKUs where the costs rise by less than €50. It is questionable if the complexity of the FDP outweighs the cost reduction for these SKUs. There are 930 SKUs with a cost increase of more than €50. The SKUs with a relevant cost increase larger than €100 are mainly those from electrical OEM and mechanical OS. These commodities benefit the most from implementing FDP, so the implementation should start with these commodities.

5.5.2 Holding Costs Experiment

This experiment involves different holding cost rates and is conducted considering dual objectives. Firstly, holding costs can fluctuate over time, and decisions based on the current holding costs may

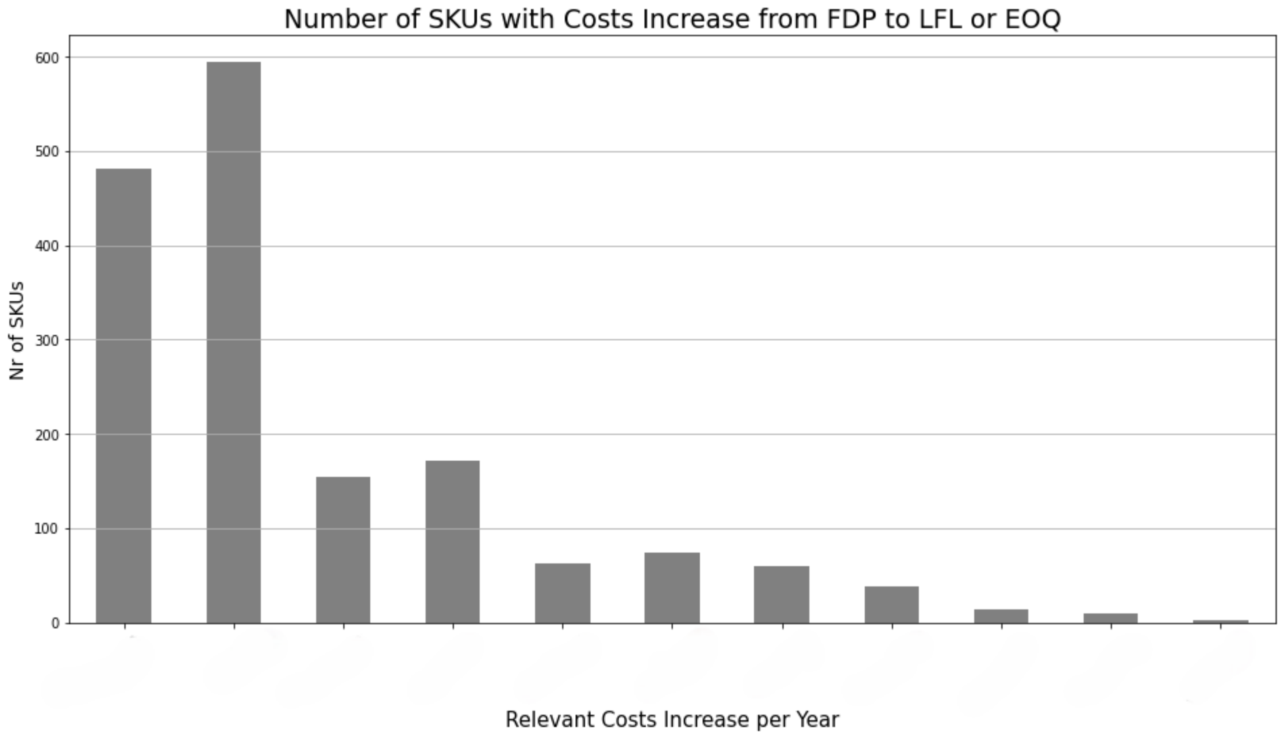

Figure 5.2: Relevant Costs Increase

not remain optimal for future prices. This experiment shows the influence of price changes regarding the holding costs on the order policy. Secondly, the storage costs are based on an average for all the SKUs regarding storage location. The storage represents the warehouse’s costs and is about one-third of the holding costs. The holding costs are an estimate. This experiment shows if a better estimate of the holding costs is required.

This experiment uses different percentages of the holding cost rates. The original holding costs rate is ?’% of the stock value per year. The costs are normalized based on the costs for the original situation.

Table 5.7: Normalized Costs for Holding Costs Experiment

Holding Costs Rate	Relevant Costs	Ordering Costs	Holding Costs	Discounts
80%	96.1%	99.5%	82.1%	100.3%
93%	98.6%	99.9%	94.0%	100.3%
100%	100.0%	100.0%	100.0%	100.0%
107%	101.2%	100.1%	106.0%	100.0%
120%	103.7%	100.4%	117.8%	99.9%

The order policies are relatively nonsensitive to changes in the holding costs since there are only small changes. The general trend is that the orders become smaller if the holding costs rise. This can be concluded from the fact that discounts are reduced and the ordering costs rise. Moreover, the sum of holding costs rises less than the input holding costs parameter rises. The holding costs rise by 17.8%

Table 5.8: Order Policies Count for Holding Costs Experiment

Holding Costs Rate	Nr FDP	MPPG	Nr EOQ	Nr (s,Q)	Total
80%	1987	0	915	372	3274
93%	1994	0	908	372	3274
100%	1996	0	906	372	3274
107%	1996	1	905	372	3274
120%	2004	0	898	372	3274

if the holding costs rate is increased by 20%. The holding costs rise by 18%, which is less, so there is less capital on the stock. This means that the inventory position must decrease to make it possible for the total holding costs to increase less.

SKUs change from EOQ to FDP as the holding costs increase. Only four SKUs change from EOQ to FDP if the holding costs increase by three percentage points. The deviation in order policies is small with respect to the increase in holding costs.

There is one SKU for which MPPG has the lowest cost when the holding costs are 107% of the original holding costs. The relevant costs are €2 cheaper compared to the FPD order policy. It is more of a coincidence that the MPPG performs better for this SKU since the FDP generally performs better, and the difference is small.

To conclude, the reduction in holding costs generally increases the order quantity and reduces the costs. Deviations in the holding costs have limited influence on the order policy selection but do affect the order quantity.

5.5.3 Order Costs Experiment

The order costs are based on research in 2018. Although the order costs are corrected for inflation, no research is done to determine if the order process has the same efficiency; therefore, the order costs of €¿ are still accurate and adequate. Furthermore, the order cost might not be equal for every order line. Some orders are more labor-intensive than others. Therefore, this experiment is done to see the influence of the order costs on the order policy.

Table 5.9: Normalized Costs for Order Costs Experiment

Order Costs	Relevant Costs	Ordering Costs	Holding Costs	Discounts
64%	61.2%	66.3%	95.3%	99.5%
80%	78.5%	81.3%	97.6%	99.9%
100%	100.0%	100.0%	100.0%	100.0%
120%	121.5%	118.8%	102.1%	100.3%
160%	166.2%	157.7%	107.1%	101.0%

Table 5.10: Order Policies Count for Order Costs Experiment

Order Costs	Nr FDP	Nr MPPG	Nr EOQ	Nr (s,Q)	Total
64%	2020	1	881	372	3274
80%	2007	0	895	372	3274
100%	1996	0	906	372	3274
120%	1988	0	914	372	3274
160%	1970	0	932	372	3274

The order policies are relatively nonsensitive to changes in the order costs since there are only small changes. Tables 5.9 and 5.10 illustrate a trend: as the order costs increase, there is a greater tendency for SKUs to shift to an EOQ order policy. This phenomenon is logical because higher order costs make procuring SKUs in larger quantities more advantageous. Consequently, the discount experiences a slight increase due to the larger order size, which makes more discounts possible. On the other hand, ordering costs exhibit an increase influenced by the experimental input. However, it's important to note that the rise in ordering costs is not proportional to the increase in order costs, as fewer orders are placed with higher order costs. Holding costs also rise, as the optimal strategy involves storing larger quantities.

An increase in order efficiency reduces the order costs. For example, a portal where suppliers can update the SKU characteristics and prices, and VMI can use to place orders at suppliers increases efficiency. Reducing order costs to 64% of the original value decreases results in smaller lot-sizes, reduced holding costs, and lower discounts. 24 more SKUs use FDP instead of an EOQ order policy, reducing the risk of obsolete SKUs.

Approximately 10 SKUs change their order policy when the order costs are altered by 20%. Additionally, alterations in the order costs impact the lot-size due to the parameter change. However, the holding cost and discount change marginal. This suggests that the shift in order policy is motivated by the fact that the new policy is slightly more cost-effective. Consequently, a slight increase in order costs does not substantially alter the order policy.

Lastly, the inbound costs are set at 160%, providing insight into the stability of the results and resembling the transportation costs. 26 SKUs shifted from FDP to EOQ order policy. Next, the average lot-size increases, increasing holding costs and fewer orders.

To conclude, an increase in order costs increases the average order quantity. There are more SKUs with an EOQ-based order policy with higher order costs.

5.5.4 Service Level Experiment

The service level is based on the fact that no stock-out per replenishment cycle occurs. The probability of no stock-out influences the safety inventory for the (s, Q) order policy since the height of the safety stock depends on the probability of a stock-out. The service level does not influence which order policy a SKU has. All SKUs with non-zero effective lead time have a (s,Q) order policy. This experiment uses one service level for all the SKUs. The service level is the change that no stock-out per replenishment cycle occurs.

The probability that no stock-out per replenishment cycle occurs changes the safety stock but does not influence the order quantity choice. The order costs and discounts remain equal over the experiments. Only SKUs with an effective lead time larger than zero days have a safety stock. DLS methods are impossible since the effective lead time is larger than zero, so no order policies switch due to the different service levels.

Table 5.11: Normalized Costs for Service Level Experiment

Service Level	Relevant Cost	Holding Costs
90%	97,0%	83,6%
95%	99,4%	96,7%
96%	100,1%	100,5%
97%	101,0%	105,2%
98%	102,1%	111,5%
99%	103,9%	121,3%
99.9%	108,9%	148,8%

Table 5.11 shows the normalized relevant and holding costs. The costs are normalized on the costs of the proposed institution. These experiments set the service level equal for all the SKUs, independent of the price of risk of obsolescence of a SKU.

The safety stock copes with demand uncertainty and disruptions in the supply chain. The height of a safety stock is a trade-off between holding costs, the risk of SKUs becoming obsolete and, service level. VMI's management can base the decision regarding the service level on these results.

5.5.5 Influence Logistical Time Experiment

There are eight weeks between the engineering department is finished and the SKUs are required for production. This means that the purchase department has eight weeks to purchase the SKUs. This time is called the logistical time. This experiment shows the consequences of what happens if the logistical time is altered.

This experiment adjusts the logistical time, the duration in which demand information is known in advance is adjusted. This experiment offers insights into situations where VMI structure changes and the time between the end of engineering and the start of production increases or decreases. Additionally, it helps to identify the preferred forecast window for decision-making.

Figure 5.3: Relevant Costs as Function of Logistical Time

Figure 5.4: Distribution of Order Policies Experiment Logistical Time

This experiment executes various scenarios, each characterized by a distinct duration of the logistical time. Figure 5.3 shows the relevant costs for the different scenarios. Figure 5.4 shows the distribution of order policies for these different scenarios. These scenarios directly impact dynamic lot-sizing because this order policy considers a look-ahead window equal to the logistical time. Additionally, it also affects the reorder point for EOQ-based policies. An increase in logistical time reduces safety stock due to the shortened effective lead time. Increasing the logistical time enables more SKUs to be classified as DLS, as their lead times are shorter than the period for which demand is fully known.

The computational time of the FDP with non-dominated solutions increases exponentially if the look-ahead window is increased. Therefore, the FDP is limited to looking 16 weeks ahead in order to maintain the computational time within reasonable boundaries.

Figure 5.3 shows a decrease in relevant costs if the logistical time increases. The relevant costs stabilize after 30 weeks; this indicates that a forecast of the first 30 weeks is important. Figure 5.4 shows the order policies for the different scenarios. SM heuristic outperforms FDP for a small horizon. This is caused by optimizing over a short window, which does not necessarily result in a better solution for the overall problem. The FDP is limited to a look-ahead window of 16 weeks in order to limit the computational time for this experiment. SKUs can choose between FDP, limited to 16 weeks ahead, or SM and MPPG, which looks further ahead for the scenarios with a long logistical time. Therefore, after 16 weeks, the number of order policies starts with a SM order policy. There are no SKUs with LFL or MPPG in any scenario.

5.5.6 Demand Increase

This experiment analyzes what would happen if the demand increases. The model is based on the assumption that all units are pieces. Therefore, only a multiple of demand can be used. One scenario is tested with twice the amount compared to the normal situation. The parameters are based on the increased demand.

Table 5.12: Order Policies Distribution for Demand Increase

Order policy	Double Demand	Proposed Situation	Current Situation
LFL	0 SKUs	0 SKUs	2064 SKUs
FDP	2023 SKUs	1996 SKUs	0 SKUs
SM	2 SKUs	0 SKUs	0 SKUs
MPPG	3 SKUs	0 SKUs	0 SKUs
EOQ	874 SKUs	906 SKUs	890 SKUs
(s,Q)	372 SKUs	372 SKUs	320 SKUs
Total	3274 SKUs	3274 SKUs	3274 SKUs

Table 5.13: Relevant Costs for Demand Increase

Order policy	Double Demand	Proposed Situation	Current Situation
Ordering Costs	123%	113%	97%
Holding Costs	19%	18%	7%
Discount	-42%	-31%	-4%
Relevant Costs	100%	100%	100%

Tables 5.12 and 5.13 show the experiment's results with double demand. There are 5 SKUs where FDP does not result in the lowest costs. The relevant costs increase is between 0.02% and 0.84% for

these SKUs. The lot-size increases by 11% on average. The discounts are more than double. The result is that the relevant costs increase by 64.4% instead of double.

5.6 Conclusion

- The model is implemented in Python since Python is open-source and has a wealth of documentation.
- The overall costs of the model for the current situation is validated by comparing the results with the exceptions of an expert within VMI. The results for the current situation of the model deviates less than 3% of the expectations by an expert within VMI.
- The relevant costs of the order policy can be reduced by ¿ million euros by changing to the proposed order policy.
- The relevant costs increase by ¿ million euros if DLS order policies are excluded as possible order policies.
- The holding and order costs have minimal impact on the order policies type.
- The relevant costs decrease if the logistical time increases, the relevant costs stabilizes after 30 weeks of logistical time.
- FDP is the best DLS method.

6 Implementation Plan

This chapter answers the questions: '*How can the research findings be implemented into the organization of VMI?*' This chapter presents the implementation plan consisting of six steps. Some of the steps are necessary to take; other steps are optional. Section 6.1 describes the first step, where decisions and assumptions must be validated before the research can be implemented in the organization. Section 6.2 describes which data needs to be improved in order to acquire accurate results. Section 6.3 describes how the data should be loaded into the model. Section 6.4 discusses a pilot where the model is tested in the organization. Section 6.5 describes how the usage of the model can be simplified using an user interface. Section 6.6 discusses how the model can be extended. Section 6.7 ends with a conclusion.

6.1 Step 1: Reconsider Input Parameters

The assumptions made during this research should be reconsidered if they are correct based on the gained insight. Parameters such as safety logistical time are seen as fixed during this research and altering these was out of scope. The management of VMI and the SCI department must reconsider if the following input parameters are still adequate for the entire organization.

- Reconsider the decision if deterministic lead times are reasonable. The safety time can be increased, or the safety stock should be enlarged to cover the lead time uncertainty.
- Reconsider if the safety time of ten days should be maintained or altered.
- Validate if the material flow globally requires other parameter settings. For example, the order and holding costs can differ from Epe (Netherlands) to Yantai (China).

6.2 Step 2: Improve Data Quality

The order policies are based on the input data. The input data into the model should be correct in order to provide correct and accurate results.

At first, a correct and up-to-date price book is required. The current price book does not contain price information for all the SKUs. Moreover, different SKUs with non-concave price functions are found. The purchase department is responsible for the correct and complete price book.

The lead times should be correct. The quality of the current lead time data is lacking, making it impossible to determine the optimal value for the safety stock. The SCI department must ensure the correct lead time and order policy parameters. Data analysis on the past lead time performance has to be done to see which SKUs have incorrect lead times in the system. The purchasing department should contact the suppliers who fail to adhere to their promised delivery times.

Next, improvements in the forecast are required. VMI lacks a forecast with confidence levels — the current forecast is a point forecast. A forecast with a confidence level is required. This makes it also possible to look further ahead and utilize the possibilities for FDP. Moreover, a forecast with a lower forecast error reduces the safety stock required. Next, a longer forecast window increases the knowledge of future demand. The operations control department is responsible for a high-quality forecast on a machine level. The SCI is responsible for delivering a forecast on SKU level based on the machine forecast.

6.3 Step 3: Automatically Import Data

Up-to-date data is required to make order policies that fit the current situation. Recent data can be obtained in different ways. One option is to make regular data dumps to CSV files so the model always uses recent data. Another option is to provide direct access to the data for the selection model. A direct link between the ERP system and the selection model ensures up-to-date data and accurate decisions. The SCI department is responsible for the input data of the selection model.

6.4 Step 4: Test by Means of a Pilot

The model should be tested in a pilot to see if the order policies make sense. Furthermore, a pilot is needed to see how the model works in practice and if there are elements the model should incorporate to deliver appropriate results. A pilot can point out problems and provide more insight and input for further revisions. The pilot should be done in collaboration between the SCI department and the suppliers.

The first step is to set the order policy to optimize the order policy to the capabilities of the ERP system. This means that the best order policy between LFL, EOQ, and (s,Q) should be selected. The DLS methods are excluded from possible order policies as in experiment *order policy restriction* described in Section 5.5.1. The next step is to combine different planned purchase orders into one manually until the FDP algorithm is implemented in the ERP system.

6.5 Step 5: Simplify the Use

The selection model is written in Python and works with different CSV files as input data. The results are presented in a CSV file, which should be manually transferred into the ERP system. The use can be simplified by providing an user interface.

The use of the model can be simplified by writing the results directly into the ERP system. This should be done after testing the selection model by a pilot to ensure the model results are correct and do not need human validation.

An implementation into the ERP system that automatically sets the correct order policy and places orders is the ultimate goal. Human intervention is not needed if this is the case.

The SCI department is responsible for a properly functioning and usable selection model.

6.6 Step 6: Extend the Functionality

Extending the functionality of the model is more optional. The following model extensions are suggested. Firstly, the demand for spare parts should be incorporated by increasing the safety stock. Secondly, multiple suppliers can be incorporated into the model. Thirdly, the suppliers' MOQ and OQI can be incorporated into the model. Next, more functionality and complexity can be added using a step-wise price function for the occupancy costs as part of the holding costs. The SCI department is responsible for further extension of the model.

Spare Parts VMI also sells spare parts. The demand for spare parts is not known upfront as the demand for production and the consumption of spare parts is excluded in the order policies. The demand for spare parts might require higher safety stocks. There are two possible ways to handle safety stock for spare parts: have one stock position to handle demand for production and spare parts or have different stock positions for spare parts. Research should be done to identify the most appropriate approach. The management of VMI in combination with the spare part division should decide if the spare parts have their own inventory position and safety stock or are combined with the material flow for production.

Multiple Suppliers The current model only considers one preferred supplier. There are SKUs with multiple preferred suppliers, and VMI is indifferent between them. The model can be extended by incorporating multiple suppliers by extending the price book of the extra suppliers. Be aware that the price function should be concave when working with a price book of various suppliers.

MOQ and OQI The minimal order quantity (MOQ) and order quantity increment (OQI) are excluded from this research since they do not occur for important SKUs. The model can implement the MOQ and OQI by rounding (up) the order quantity to a feasible quantity. In the case that MOQ and OQI frequently occur for more important SKUs, they can be included in the decision of the order policies. The SCI department is responsible for implementing of the MOQs and OQIs of suppliers.

Occupancy Cost Change the holding costs to a step-wise price function. The current holding costs are a fraction of the purchase value. By splitting the holding costs into capital-based and location-based costs, the location-based costs can be better defined. The costs based on location are incremental in reality and require a step-wise cost function. The occupancy cost reflects the incremental change in space cost due to changing cycle inventory. VMI has no direct occupancy costs since VMI is not charged based on the actual number of units held in storage but leases a number of storage locations. The incremental occupancy cost is zero as long as a marginal change in cycle inventory does not change the space requirements. Occupancy costs take the form of a step-wise function, with a sudden increase in cost when capacity is fully utilized and new space must be acquired.

6.7 Conclusion

The implementation plan exists out of the following steps to implement the model into the organization:

1. The SCI department should revise the decisions in collaboration with the management.
2. The SCI department should improve the quantity of input data to get accurate results.
3. The SCI departments should provide the selection model with up-to-date and correct data.
4. The SCI and purchase department should test the selection model in a pilot.
5. The SCI department should simplify the use of the selection model by providing an user interface.
6. The SCI department can increase functionality of the selection model by by incorporating spare parts, multiple suppliers and include MOQs and OQIs.

7 Conclusion and Discussion

This section presents the conclusions, discussion, recommendations, and further research of this research.

7.1 Conclusions

The relevant costs decrease from €¿ million to €¿ million by changing from the current to the proposed situation. The relevant costs exist of the ordering costs plus the holding costs minus the obtained discount. The largest costs reduction is in the ordering costs since VMI purchases a lot on LFL basis. The number of orders can be halved and the holding costs rises with 18%.

The (s, Q), EOQ, and FDP are the relevant order policies for VMI. The (s, Q) order policy suits SKUs with a lead time longer than the logistical time, these SKUs have stochastic demand. The EOQ and FDP suit SKUs with a lead time shorter than the logistical time. The FDP method is restricted to the known demand since no forecast is incorporated. An EOQ can order larger quantities than FDP since an EOQ is not restricted to the known demand.

The relevant cost-saving decreases by €¿ million a year when the order policies are restricted to the current ERP system. The current ERP system cannot handle the FDP order policy, and therefore, SKUs with a FDP order policy should switch to LFL or EOQ order policies. This means that the costs for implementing and maintaining the FDP in the ERP system should be less than €¿ million per year.

The relevant costs decrease if the logistical time increases and stabilizes after 30 weeks of logistical time. An increase in the logistical time means less uncertainty and a reduction in the safety stock. Next, more SKUs can have FDP as an order policy since there are more SKUs with a zero effective lead time. At last, the possible order quantity for FDP increases since this method can look further ahead. No significant gains are made if the logistical time increases from 30 weeks of logistical time.

7.2 Discussion

The effect of non-stationary demand is excluded in this research. There is no forecast available that includes the non-stationarity. Therefore, the (s, Q) order policy is based on stationary demand. The effect of incorporating the non-stationarity on the service level is unknown.

This research is based on fully deterministic lead times, which is not the case in reality. Although this assumption is based on the current lead time performance in combination with the safety time of 10 business days, some deliveries exceed the safety time and cause problems. This research does not investigate the effect of non-deterministic lead times on the service level. However, the (safety) stock positions are expected to rise.

No future price changes are included in the model. The selection model assumes that prices will remain identical in the future. A price decrease or increase is not included in the order policy decision. SKUs subjected to a lot of innovation tend to decrease in value, whereas SKUs characterized by scarcity are expected to increase in price. Expectations in price changes are excluded in the selection model and parameter decision.

The effect of larger or smaller period length is excluded in this research This research works with a period length of one week since the planning of VMI works in weeks, and the holding costs are charged on a weekly basis. A smaller period length means an increase in the solution space, possibly leading to a better solution. No research has been done on the effect of the period length on the solution quality and computational time.

The storage capacity is excluded in this research. The order policies do not take into account whether there is enough storage capacity in the warehouse and enough money to purchase the SKUs to stock. This means that the optimal order policy can result in a solution that is not feasible in practice in case the warehouse is too small or there is not enough capital for investments in inventory.

Order policies outside Europe might deviate. This research is scoped to cope with demand in Europe. Demand and warehouses outside Europe are excluded from this research. Research can be done if demand outside Europe requires a different approach or parameter settings. However, demand for SKUs is expected to require similar order policies outside Europe.

7.3 Recommendations

Extend the DLS methods with the forecast. The DLS methods become more powerful if they are extended with the forecast so they can look further ahead into the future. The quality of the forecast should be improved so that it can be used as input data. A forecast for the first 30 weeks is important since the relevant costs decrease if more information becomes available. Research into cost reduction and increased risk need to be done before the DLS methods are extended with a forecast. Next, the increase in the look-ahead window increases the computational time, which can lead to the decision of other DLS methods.

Do a pilot with the selection model. The selection model is tested on SKUs of a single machine. A test with more SKUs can provide more insight. A pilot can identify any issues before implementing the selection model into the organization.

Implement the FDP order policy in the ERP system. Implementing the FDP order policy into the ERP system simplifies its use. Changing to multiple cores during the implementation process decreases the computational time.

Use the selection model on a monthly basis to update the order policy for SKUs. Use the selection model monthly to update the order policies so that the parameters are still optimal. In case of a cost, demand, or price change, update the order policy earlier.

7.4 Further Research

Extend the model functionality by incorporating multiple suppliers and order quantity restrictions. The purchase amount can decrease if the model can choose between multiple suppliers. Order quantity restrictions such as minimum order quantity and order quantity increment can be incorporated by rounding (up) to a feasible order quantity. This increases the computational time.

Effect of more information known. This research assumes that only eight weeks of demand is known, and demand after that is fully stochastic. More information is known in reality; see Figure 2.2. Most information comes available 20 weeks upfront. Further research can be done on how this information can be included in the order policies and the consequences of using this information.

Incorporate lead time uncertainty. This research assumes deterministic lead times. This assumption is made since 97% of the orders do not exceed the safety time. More research can be done to determine how lead time uncertainty can be incorporated and what the consequences of this are. The safety stocks increase by incorporating more uncertainty, so the holing as the relevant costs increase.

Work with a step-wise price function for the storage costs. The storage costs are based on the number of occupied storage locations and the number of SKUs in storage. The more specific the costs are defined, the better the model costs corresponds with the cost in the real world.

Incorporate sustainability into the decision. The production and transport of SKUs influence the footprint of the production. By including the sustainability aspects in the order policy selection, lower carbon emissions can be reached, so the footprint of the VMI product is lowered. This results in a more sustainable product.

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Appendices

A Stakeholder Analysis

The following internal stakeholders are identified: VMI management, source and supply buyers, SCI engineers, finance department, head of material management, product engineers, suppliers, and mechanics. Suppliers are the external stakeholders. The stakeholders are listed in order of importance and influence regarding this topic.

Management of VMI

The management of VMI wants a financially stable company and is determined to maintain the company's position as the market leader. The management has decided to prioritize customization to meet customers' specific requirements. Additionally, the management emphasizes ensuring the entire process, from quotation to final delivery at the customer's location, is executed efficiently and quickly without sacrificing quality.

Source and Supply Buyers

Sourcing is responsible for specifying the item requirements and selecting and establishing contractual agreements for new items. The purchase department is responsible for ordering, expediting, and aftercare. The purchase department workload is proportional to the number of orders. The purchase department prefers larger orders and stock-based. A stock-based policy is preferable for long lead time items and reduces the change on emergency requests.

SCI Engineers

Supply chain innovation endeavors to keep an overview of the logistics processes within VMI and optimize where possible. Simplicity is the preference of SCI engineers.

Finance Department

The finance department aims to reduce the amount of capital invested in inventory, as it is perceived as unproductive capital that ties up working capital. Unsold items lead to potential financial losses. Additionally, the finance department wants to ensure that the end product is manufactured at the lowest possible cost.

The Head of Material Management

The head of material management is responsible for the material flow from the inbound process until the delivery to production. Given the limited availability of storage spaces, the head of material management prioritizes the efficient utilization of storage facilities and emphasizes a high throughput speed.

Personnel is required for material handling. Peaks necessitate allocating additional personnel or may lead to a heightened workload. The preference lies in maintaining a consistent process flow to ensure a stable workload for the personnel. Moreover, reducing the number of materials held in stock would result in a decreased requirement for storage locations, aligning with the objectives set forth by the head of material management.

Product Engineers

Product engineers strive to create optimal products without being constrained by predetermined materials or items. They seek the freedom to select components that best suit the machine's requirements without considering the existing inventory position. The engineers prioritize the ability to design and engineer the best possible product, unencumbered by concerns related to inventory constraints.

The Suppliers

Suppliers generally prefer receiving order information well in advance, allowing them to plan their production processes effectively. Additionally, suppliers often prefer larger batch sizes, as larger quantities enable greater operational efficiency. Some suppliers actively promote larger order quantities by offering discounts for bulk purchases. However, it is essential to note that suppliers do not possess a direct influence on the order policy.

Mechanics

The mechanics are responsible for the assembly of the machines. The work preparation department has already determined the optimal sequence of tasks for the assembly process. If the required items arrive late, it can cause delays in the production process. There are instances where the entire assembly process is halted because it is only possible to proceed with the missing part. However, it is common for the assembly to continue, with the missing part added later. Unfortunately, this often necessitates the disassembly of specific components to accommodate the installation of the previously missing item, resulting in additional work.

B Examples of Demand Patterns

Figure B.1 shows the examples of the four different demand patterns.

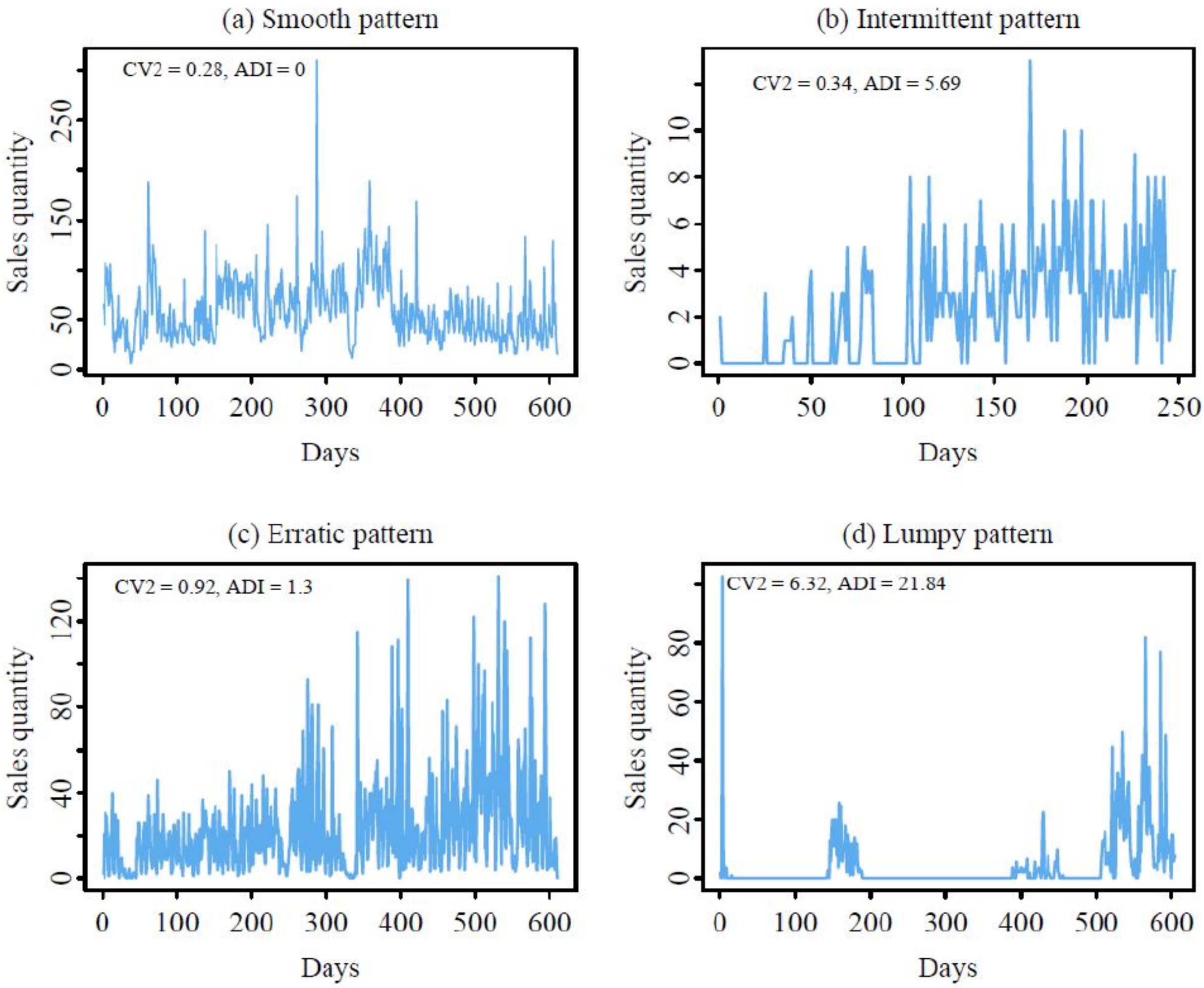


Figure B.1: Example of Different Demand Patterns [2]

C Correlation Between Demand Patterns and Stationarity

Table C.1 shows the correlation between the demand pattern and stationarity of demand. It can be concluded that the majority of demand is stationary, independent of the demand pattern.

Table C.1: Demand Patterns and Stationarity

	# Non-Stationary	% Non-Stationary	# Stationary	% Stationary
Erratic	90	14.8%	520	85.2%
Lumpy	5805	9.2%	57496	90.8%
Smooth	14	13.2%	92	86.8%
Intermittent	0	0%	0	0%
Total	5909	9.2%	58108	90.8%

D Detailed Commodity Information

This appendix describes the order policies of the different commodities.

Mechanical OS SKUs

Mechanical OS is the commodity with the most number of SKUs. Figure D.1 shows the distribution where it can be seen that most of the SKUs are purchased 'to-order'. This is logical with outsourcing items. Furthermore, most SKUs are in price category 3, €10 - €100.

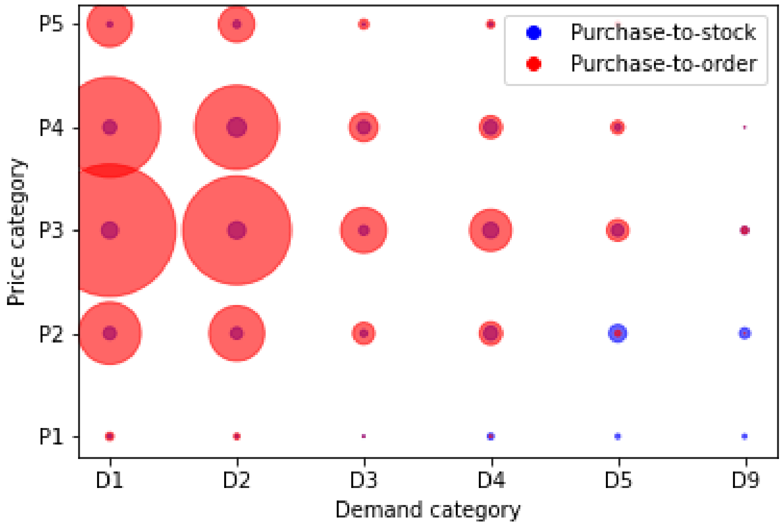


Figure D.1: Order Policy Plot of Mechanical OS SKUs

Mechanical OEM SKUs

The mechanical OEM SKUs are distributed over the entire range.

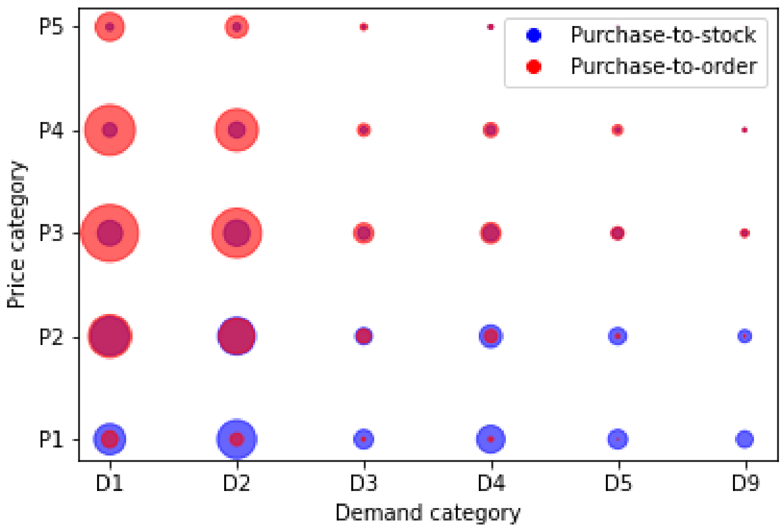


Figure D.2: Order Policy Plot of Mechanical OEM SKUs

Electrical OEM SKUs

For electrical OEM, the more demand there is, the more purchase-to-stock there is for SKUs.

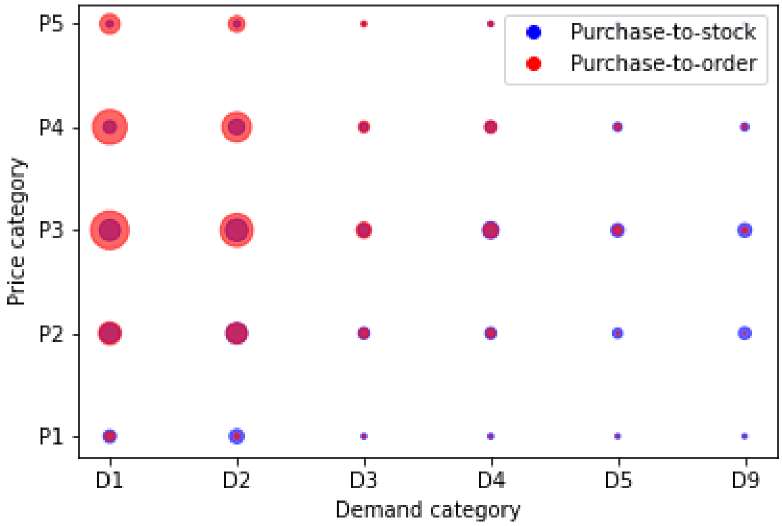


Figure D.3: Order Policy Plot of Electrical OEM SKUs

Mechatronic SKUs

Mechatronic is the smallest commodity. The majority are purchase-to-stock only; there are two large groups of purchase-to-order for price category P3 with low demand.

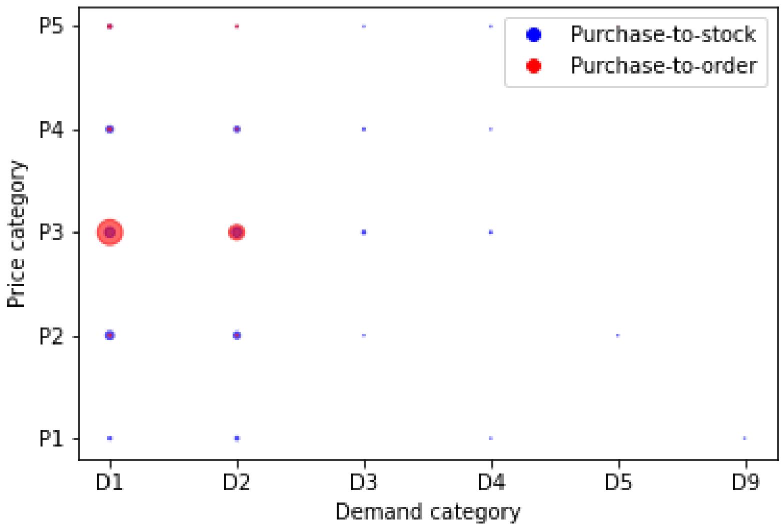


Figure D.4: Order Policy Plot of Mechatronic SKUs

E Overview of all Parameters

Table E.1 shows all the parameters used in this research.

Table E.1: Description of Variables

Variable	Description	Unit
$c(j,k)$	Cost Stage Function from stage j to k	€
D	Yearly demand	units
d_t	Demand for period t	units
DL	Price breakpoint	units
$E[Z]$	Expected undershoot	units
EL	Effective Lead time	days
$F()$	Value Function	€
h	Holding cost fraction per year	%/year
h_w	Holding cost per SKU per week	€/unit
k	Safety factor	-
LT	Lead time	days
p	Item price	€/unit
Q	Order quantity	units
s	Reorder point	units
S	Order Costs	€
σ_{EL}	Forecast error during effective lead time	units
ss	Safety Stock	units
X_{EL}	Demand during Effective lead time	units
$\hat{X}_{t,t+EL}$	Expected Demand from t to $t+EL$	units

F Lot-For-Lot

Lot-for-lot (LFL) is the only order policy within purchase-to-order, in literature, also referred to as just-in-time (JIT) purchasing. The SKU lead time should be shorter than the logistical time to make it possible to purchase a SKU to order. In the case of VMI, this means that the lead time must be smaller than 40 business days. Typical SKUs falling into this type include customer-order-specific SKUs, which are exclusively designed for a particular project and cannot be used in other projects. Therefore, there is a high risk for SKUs to become obsolete, and the SKUs can better not be purchased-to-stock. Next, this policy applies to SKUs with low demand for which maintaining stock is not beneficial.

LFL places an order for every production order, which results in a new purchase order with the same amount for every production order. If two production orders for the same SKU with identical required dates are received simultaneously, each order will be placed individually. The order is placed with an expected arrival date before the production date. There is a safety time of ten days, so the required date is ten days before the production date. This means the purchase orders are placed with an expected arrival date equal to the required date.

Numerical illustration SKU F has a LFL order policy with a lead time of 15 business days. This means that LFL is a possible order policy for SKU F. Table F.1 shows the production orders, and table F.2 shows cost parameters. Two weeks of safety time are used for LFL.

Table F.1: Production Orders of SKU F

Week	Day	Quantity
1	1	10
1	4	20
2	1	20
2	1	10
2	5	30
3	2	10
3	3	10

Table F.2: Cost for SKU F

Price	Description
€ 10	$Q < 25$
€ 9	$Q \geq 25$
€ 12.50	Order Costs
0.4%	Holding Costs per Week
2 weeks	Safety Time

All orders will be placed separately, so both orders for day 1 on week 2 are placed separately. Equation (F.1) shows the expected costs of LFL and Equations (F.2) to (F.4) show the different costs components.

$$\text{Total expected costs} = \text{Purchase Amount} + \text{Ordering costs} + \text{Holding costs} \tag{F.1}$$

$$\text{Purchase Amount} = \sum_{\text{Orders}} \text{Order quantity} \cdot \text{Purchase price} = 80 \cdot 10 + 30 \cdot 9 = €1070 \tag{F.2}$$

$$\text{Ordering costs} = \text{Order costs} \cdot \text{Nr Orders} = 12.50 * 7 = €87.5 \tag{F.3}$$

$$\begin{aligned}\text{Holding Costs} &= \text{Purchase Amount} \cdot \text{Weeks in Storage} \cdot \text{Holding Costs per Week} \\ &= 1070 \cdot 2 \cdot 0.004 = €8.56\end{aligned} \tag{F.4}$$

The total expected costs are € 1166.06 if SKU F is purchased by a lot for lot order policy.

G Continuous Review with Fixed Order Quantity (s,Q)

The continuous review with fixed order quantity (s,Q) is suitable for SKUs with stochastic, stationary demand or those that benefit from a 'simple' order policies as type-C SKUs. Orders are initiated when the inventory position reaches or falls below the reorder point and are purchased with a fixed order quantity Q.

Order Quantity

The order quantity Q equals the EOQ, as defined in Equation G.1 for the SKUs without discounts. The order quantity is based on the yearly demand (D), order costs (S), holding cost (h) as a fraction a year and the SKU price (p) per item.

$$EOQ = \sqrt{\frac{2DS}{hp}} \tag{G.1}$$

The order quantity for SKUs with discount is determined as in Section 3.2 explained where the total costs of the different EOQs and price breakpoints are compared, and the order quantity corresponding to the lowest total costs is taken.

Reorder Point

The inventory position is the on-hand inventory plus items on order minus back orders minus committed demand. The committed demand is the demand known during the first eight weeks, and it is removed from the inventory position. The reorder point is based on the demand during the effective lead time since the demand during the logistical time is already reserved and excluded from the inventory position. Equation G.2 illustrates how the reorder point is calculated, factoring in demand during the effective lead-time $(\hat{X}_{EL})$, safety stock (ss) and undershoot $(E[Z])$. The effective lead time is the lead time minus the logistical time. The effective lead time is zero in case the logistical time is larger than the lead time. No safety stock is required for an effective lead time of zero days. Therefore, the reorder point is zero units for SKUs with an effective lead time of zero days. An order is placed as the inventory position drops below the reorder point.

$$s = \hat{X}_{EL} + ss + E[Z] \tag{G.2}$$

The demand during the effective lead time is unknown; therefore, the expected demand is used. The expected demand during lead time is based on the forecast.

Undershoot

The undershoot is the drop of inventory position below the reorder point before placing an order. Equation G.3 shows the calculation of the expected undershoot [10]. Y is the distribution of the order size.

$$E[Z] = \frac{1}{2}\left[\frac{E(Y)^2}{E(Y)} - 1\right] \tag{G.3}$$

Safety Stock

The safety stock ss can be determined in multiple ways. At first, the safety stock is based on the probability of the occurrence of a stock-out. The probability that no stock-out occurs is the most relevant performance measure based on the stakeholder analysis in Section 2.2 P is the probability that no stock-out occurs for every replenishment cycle. σ_{EL} is the standard deviation of the forecast error during the effective lead time.

$$ss = k\sigma_{EL} \tag{G.4}$$

The safety factor k is based on the probability of no stock-out P with the normal standard distribution as defined in Equation G.5.

$$k = \Phi^{-1}[P] \tag{G.5}$$

The safety stock (ss) can also be based on the costs of a stock-out. The cost for every stock-out is defined as B with units € per stock-out. Calculate the safety factor using Equation G.6. In case the safety factor k is lower than the value specified by the management, set safety factor k to the value specified by the management.

$$k = \sqrt{2ln\left(\frac{DB}{\sqrt{2\pi}Qhp\sigma_{EL}}\right)} \tag{G.6}$$

$$Q = EOQ\sqrt{1+\frac{B}{A}\Phi(k)} \tag{G.7}$$

The reorder point and order quantity can be optimized if the safety stock is based on the costs of a stock-out. Equation G.7 defines the order quantity Q, which depends on the safety factor. The new order quantity updates the safety factor in Equation G.6. This iterative procedure continues until the order quantity Q remains constant. The reorder point is calculated based on the most recent value of k. The reorder point and order quantity are simultaneously optimized in this way.

Numerical illustration An example shows how the (s,Q) order policy works. Table G.1 displays the relevant parameters for computing the order policy of SKU A. SKU A is a pneumatic valve with a lead time of 55 business days with partly stochastic demand. The effective lead time is $55-40=15$ days.

Equation (G.8) calculates the reorder point of SKU A using the same formula as provided in Equation G.2. The safety factor is 1.64 based on a probability of stock-out of 5%.

$$s = 15 + 1.64 \cdot 5 + 5 = 28 \text{ units} \tag{G.8}$$

The relevant order quantities are the EOQ, 65 units, or the price breakpoint of 100 units. The total costs of both promising order quantities are compared in Equation (G.9).

$$\begin{aligned} TC(65) &= 10 \cdot 250 + 12.50\frac{250}{65} + \left(\frac{65}{2}+28\right)0.15*10 = €2600.83 \\ TC(100) &= 7.50 \cdot 250 + 12.50\frac{250}{100} + \left(\frac{100}{2}+28\right)0.15*7.50 = €1950.50 \end{aligned} \tag{G.9}$$

Table G.1: SKU A Characteristics

Variable	Name	Value
EL	Effective lead time	15 days
D	Demand of full year	250 units
X_{EL}	Demand during effective lead time	15 units
σ_{EL}	St. Dev. during effective lead time	5 units
$E[Z]$	Expected undershoot	5 units
K	Safety factor	1.64
h	Holding costs rate	15%
F	Order costs	€ 12.50
Price($Q < 100$)	Base price	€ 10
Price($Q \geq 100$)	Base price	€ 7.50
EOQ	EOQ for base price	65 units
EOQ	EOQ for discount price	75 units

An order quantity of 100 units is used since this results in the lowest expected costs. The increase in holding costs is worth the price discount for this SKU.

H Forward Dynamic Programming with Non Dominated Solutions

This appendix deals with the details of forward dynamic programming (FDP). First, the steps to define the relevant order quantities (ROQs) are described. Afterward, an expectation of the computational time is given. The pseudo-code is provided at the end.

H.1 Theorems Regarding ROQ

Theorem 1: The optimal order quantity equals the demand of one or more adjacent periods or price breakpoints.

Explanation 1: Wagner and Whitin [21] prove that no inventory is left when an order is placed. Consequently, the order size equals the demand of one or multiple coming periods. However, this is not valid in the case of discounts. A price breakpoint can also be a relevant order quantity. Orders are placed if the inventory is insufficient to fulfill demand for the next period. The order quantity equals the demand of one or more periods or a price breakpoint.

Proof 1: Wagner states: $I_p * x_p = 0$; there is no inventory (I) in the same period (p) as an order (x) is placed. Below follows a proof by contradiction.

There is demand over two periods with one price breakpoint. Demand in period one is 99 units, and demand in period two equals 20 units. The purchase price is €1, and a discount is valid for 100 units or more. The discount price is €0.90. The holding costs are 1 euro per unit per period, and the order costs are 5 euro.

The order quantities that satisfy $I_p * x_p = 0$ are 119 units in the first period or 99 units in the first and 20 units in the second period. The costs of both options are compared with the costs of ordering the price breakpoint in the first period and the required units in the second period.

$$TC[x_1, x_2] = \text{Purchase Amount} + \text{Ordering Costs} + \text{Holding Costs} \tag{H.1}$$

$$TC[99, 20] = 119 * 1 + 10 + 0 = €129 \tag{H.2}$$

$$TC[100, 19] = 100 * 0.9 + 19 * 1 + 10 + 1 = €120 \tag{H.3}$$

$$TC[119] = 119 * 0.9 + 5 + 1 = €125 \tag{H.4}$$

This example shows that it is better to purchase the price breakpoint. The price breakpoint results in a lower purchase amount of €10 and increases the holding costs by €1 compared to order in periods one and two. It is worth increasing the order quantity to a price breakpoint if the reduction of the purchase amount is larger than the increase in holding costs.

An increase larger than the price breakpoint and smaller than the next order quantity does not make sense. In this case, an order quantity is between 101 and 118 units. There are two options: the decrease in purchase costs per unit is larger than the holding cost for the next period. This is the case in this example. The reduction per unit is €0,10 for the purchase amount, and the increase in holding costs is €1 per unit. The other case is that the reduction in purchase amount is smaller than the increase in holding costs. In that case, the complete demand for the following period should be ordered.

H.2 Computational Time

Theorem 2: The maximum number of non-dominated solutions is: $(1 + \text{nr. price breakpoints })^{\text{Periods}}$.

Proof 2: The possible order quantities to move from one stage to another stage are one plus the number of price breakpoints, as shown by Theorem 1. The maximum number of order moments corresponds to the number of periods since an order can be placed in each period. Every order quantity can result in a non-dominated solution, and all order quantities can be used for each solution. Therefore, the maximum number of non-dominated solutions is the ROQs to the power of the maximum number of order moments.

Theorem 3: The end inventory will never exceed the highest price breakpoint.

Proof 3: No order is placed if the order can be postponed to the next period. Waiting until the next period decreases the holding costs and is therefore always cheaper. Given heorem 1, the order quantity corresponds to the demand of one or multiple periods or price breakpoints. This enhances that the end inventory can never exceed the maximum price break point.

Theorem 4: The maximum number of non-dominated solutions equals the quantity of the highest price breakpoint.

Proof 4: By definition, only one non-dominated solution can exist for every inventory position. Theorem 3 states that the inventory position can never exceed the highest price breakpoint. Combining these two leads to the maximum number of non-dominated solutions that can never exceed the highest price breakpoint.

The maximum number of non-dominated solutions for period k is:

$$\text{max nr. solutions} = \max\left((1 + \#\text{DL})^{\text{period}}, \max(DL)\right).$$

With $\#DL$ as the number of price breakpoints and $\max(DL)$ as the highest price breakpoint.

The running time for the case with no discounts is order $(periods)^2$. The ROQs are based on the number of non-dominated solutions. Given that the maximum number of non-dominated solutions is $(1 + \#DL)^{\text{periods}}$ as defined in Theorem 2. The number of ROQ for every non-dominated solution is $(1 + \#DL)$. Combining this information leads to a computational time in order $\left(periods^2 \cdot (1 + \#DL)^{periods+1}\right)$.

The number of non-dominated solutions increases exponentially over time and with the number of price breakpoints and periods. Therefore, the running time also increases exponentially with increased periods and discounts.

The fundamental concept of this FDP approach is to explore relevant order quantities. These relevant order quantities are either the demand for one or multiple periods ($d_{j,k}$) minus the inventory level or the price discount level (DL). Only these potential order sizes are considered. Only non-dominated solutions are selected to decrease the number of options and computational time.

H.3 Pseudocode of FDP with Non-Dominated Solutions

Algorithm 1 states the pseudocode of the forward dynamic programming with non-dominated solutions.

Algorithm 1 FDP with non-dominated solutions

```
function FDP(ProductionOrders)
    OptimalValueFunction[1] = 0
    GetPrice_Breakpoints
    for k in range(2, len(ProductionOrders) + 1) do
        Initiate CandidateSolutions to empty list
        for j in range(1, k − 1) do
            F_j ← NonDominatedSolutions[j]        ▷ (F_j are all Non-dominated solutions of stage j)
            Demand_j_k ← demand from stage j to k
            for prev_sol in F_j do
                prev_costs, prev_ip, prev_orders ← prev_sol
                MinOrderQuantity ← max(0, Demand_j_k − prev_ip)
                Select ROQs based on Price_Breakpoints and MinOrderQuantity
                for OrderQuantity in ROQs do
                    NewCosts, NewIP ← cumulative_stage_costs_function(j, k, prev_ip, OrderQuantity)
                    Save new solution to list of CandidateSolutions
        Select NonDominatedSolutions from CandidateSolutions
        F_k = NonDominatedSolutions  ▷ Add NonDominatedSolutions to OptimalValueFunction
    Select optimal OrderQuantity for stage K + 1
    return OrderQuantity
```

I Modified Silver-Meal

The modified Silver-Meal (MSM) heuristic is an average cost heuristic that increases the order quantity until the average cost per period rises. SM is a forward-based heuristic that can handle discounts. While the MSM heuristic may not yield the absolute optimal solution, it is expected to provide a high-quality solution with reduced computational time.

MSM only places an order if the current inventory level is insufficient to fulfill demand during lead time plus one week. This is identical to FDP. An order is initiated if the inventory level falls short of meeting the demand during the lead time plus one week. At first, the relevant cost for an order size of the first week is calculated. Afterward, the heuristic calculates the relevant cost by increasing the order quantity until the relevant costs per period increase.

Equation (I.1) defines the concept of relevant costs, considering the demand for the lead time plus one week ahead and subtracting the current inventory position. The order quantity is then increased by the demand of the subsequent period, and the relevant costs are recalculated. This process continues by adding demand for more weeks until the lookahead window expires or the relevant costs per period rise. In this context, the lookahead window spans the time demand is known minus the lead time. Ultimately, the order quantity with the lowest relevant costs is selected.

$$RC = \frac{S + (p(Q(T)) - p_{\min}) \cdot Q(T) + h_w \sum_{j=1}^{T} (j-1) D_j}{T} \tag{I.1}$$

The relevant costs, denoted as RC for the order quantity Q, which is based on the number of periods T, are determined by considering the ordering costs represented by F, the difference between the piece price for the order quantity and the minimum piece price $p_{\min}$ based on the order quantity required for the entire lookahead window and the holding costs. The lookahead window and planning horizon are identical. The holding costs per unit per period symbolized as h_w, are calculated for each period j based on the demand D_j. T represents the number of periods corresponding to the number of weeks over which the demand is consolidated. The relevant costs represent the average weekly costs, forming the basis for determining the order quantity.

Numerical illustration The same demand pattern and SKU characteristics for SKU B in Tables I.1 and I.2 are used to show the working of the modified SM heuristic.

Table I.1: Demand SKU B

Week	Quantity
1	15
2	9
3	30
4	24

Table I.2: Prices SKU B

Price	Description
€ 10	$Q < 25$
€ 8	$Q \geq 25$
€ 25	Order Costs
€ 1	Holding Costs per Unit per Period

Equation I.2 shows the relevant cost calculation for the first week based on Equation I.1. Table I.3 shows the relevant costs for the different periods. The average costs per period increase from week three to week three and therefore equals the order quantity of the demand of the first three weeks. The order quantity is 54 units.

$$RC = \frac{25 + (10 - 8) \cdot 15 + 1 \cdot 0 \cdot 15}{1} = €55 \text{ per week} \tag{I.2}$$

Table I.3: Relevant Costs

Length of Order Period	Relevant Costs (€/per week)
1	55.00
2	41.00
3	31.33
4	41.50

I.1 Pseudocode of Modified Silver-Meal

Algorithm 2 states the pseudo code of the modified Silver-Meal.

Algorithm 2 Modified SM Heuristic

```
function SM(ProductionOrders)
    Periods ← 0
    MaxOrderQuantity ← sum(ProductionOrders)
    NewRelevantCosts ← High Number
    RelevantCosts ← High Number
    while NewRelevantCosts ≤ RelevantCosts and Periods ≤ len(ProductionOrders) do
        Periods ← Periods + 1                                        ▷ Select the next period
        OrderQuantity ← ProductionOrders[1:periods].sum()
                                                     ▷ Select ProductionOrders for periods
        NewRelevantCosts ← Calculate new Relevant Costs per Period     ▷ Use Equation I.1
        if NewRelevantCosts ≤ RelevantCosts then
            RelevantCosts ← NewRelevantCosts
            OptimalOrderQuantity ← OrderQuantity
    return OptimalOrderQuantity
```

J Maximum Part-Period Gain

The maximum part-period gain (MPPG) is a part-period heuristic that aggregates small orders into larger ones according to the part-period of orders in order to optimize the order quantity. The part period is not capable of including discounts.

The maximum part-period gain is implemented as described in Section 3.3 in the literature review. Algorithm 3 in Section J.1 provides the pseudo-code of MPPG.

Multiple orders may have the same part-period (PP) value. Therefore, the possibility exists that multiple orders have the lowest PP. Orders 1, 3, and 4 in the numerical illustration below have a PP of 1. One of these needs to be selected. The last order with the smallest PP has been chosen to be brought forward. The last order with the smallest PP will never increase since no order is brought forward to this order moment. On the other hand, a later order with the same PP may enlarge an earlier order. The numerical illustration below shows this for orders 3 and 4.

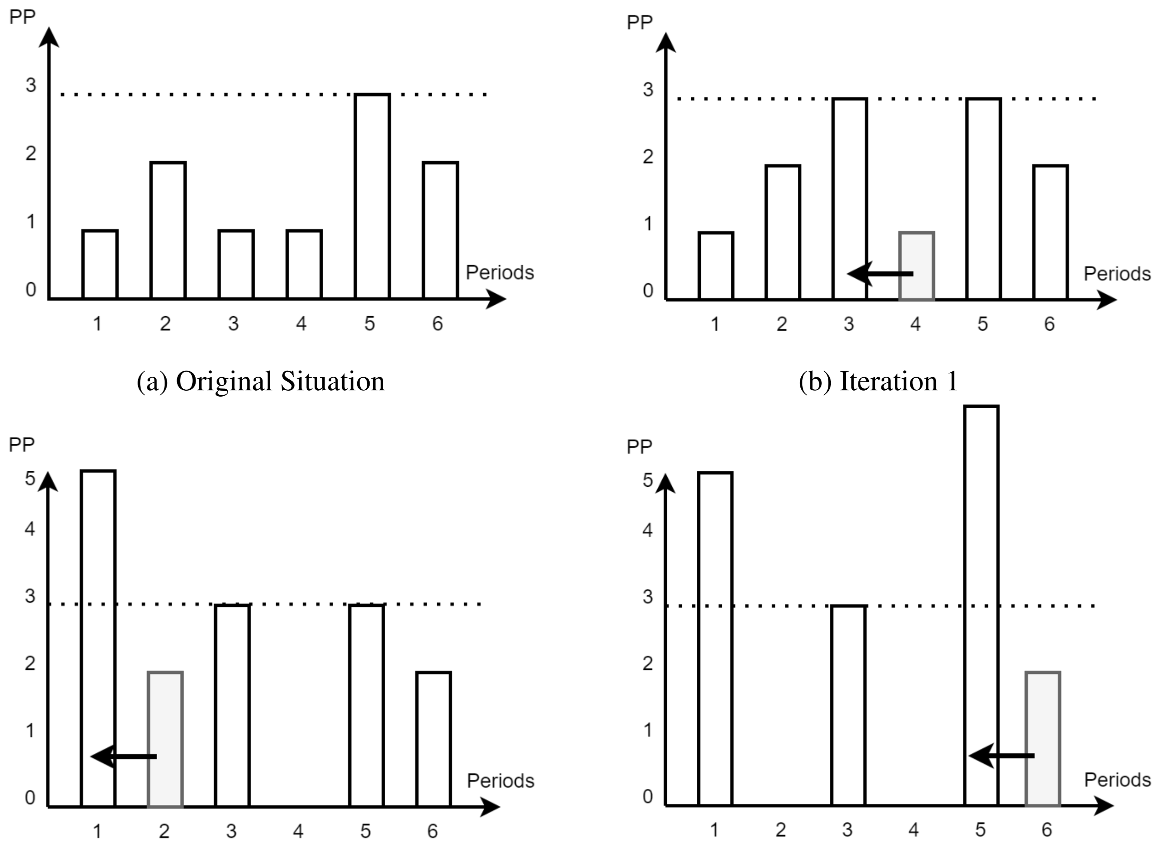
The first period can also have the smallest PP. Bringing the first order to the front is impossible since there are no previous orders to combine it. Next, delaying the first order is impossible since the demand for the first period should be met on time. Therefore, it is chosen to bring the second order to the front to increase the PP of the first order.

Numerical illustration Table J.1 shows the demand for SKU *C*. The EPP is 3 PP for SKU *C*. Figure J.1 gives the graphical representation of the different steps.

Table J.1: Demand SKU *C*

Week	Quantity
1	1
2	2
3	1
4	1
5	4
6	2

At first, the PP is set to the demand for all the periods. Multiple orders have a PP smaller than the EPP, so the heuristic is unfinished. The EPP of three is depicted with a dotted line. Multiple orders have the smallest PP, and the last order moment is chosen. The demand for order 4 is brought to order 3. The $PP_3 = 1*1+2*1 = 3$ since the quantity demand of period 3 is one period in stock and the demand of period 4 is two periods in stock. Next, the first order is identified as the smallest order. Since bringing the first order to the front is impossible, the second order is brought to period 1. All the orders equal the EPP or are larger, so the heuristic is stopped. The first order has a quantity of 3 units.



(a) Original Situation

(b) Iteration 1

(c) Iteration 2

(d) Iteration 3

Figure J.1: Graphical Representation of MPPG for SKU *C*

J.1 Pseudocode of Maximum Part-Period Gain

Algorithm 3 states the pseudo-code for maximum part-period gain. The steps correspond to the steps defined in the literature in Section 3.3.

Algorithm 3 MPPG Heuristic

```
function MPPG(ProductionOrders)
    // Step 1
    Periods ← 0
    EPP ← Order Costs / Holding Cost per Unit per Period    ▷ Calculate the Economic Part-Period (EPP)
    Set PP_k to D_k for all periods
    Finished ← False
    while not Finished do
        // Step 2
        Select the order with smallest PP_k                  ▷ Last period with lowest PP is chosen
        if first order has lowest PP then                    ▷ First order cannot be brought forward
            Select second order
        // Step 3
        Select previous order
        Increase PP_{k−1} and OrderQuantity from order of period k
        Delete period k
        if all PP_k ≥ EPP then
            Finished ← True
        OrderQuantity ← Quantity of first period
    return OrderQuantity
```

K General Solution Approach

This appendix presents a solution to choose the order policy for a more general case. The DLS method in this research is limited to the known demand and is not extended with a forecast. Furthermore, 99% of the SKUs within VMI have lumpy demand, so there is a high variability. Therefore, the decision between constant and variable demand is excluded from the solution approach, and the variable demand method is used.

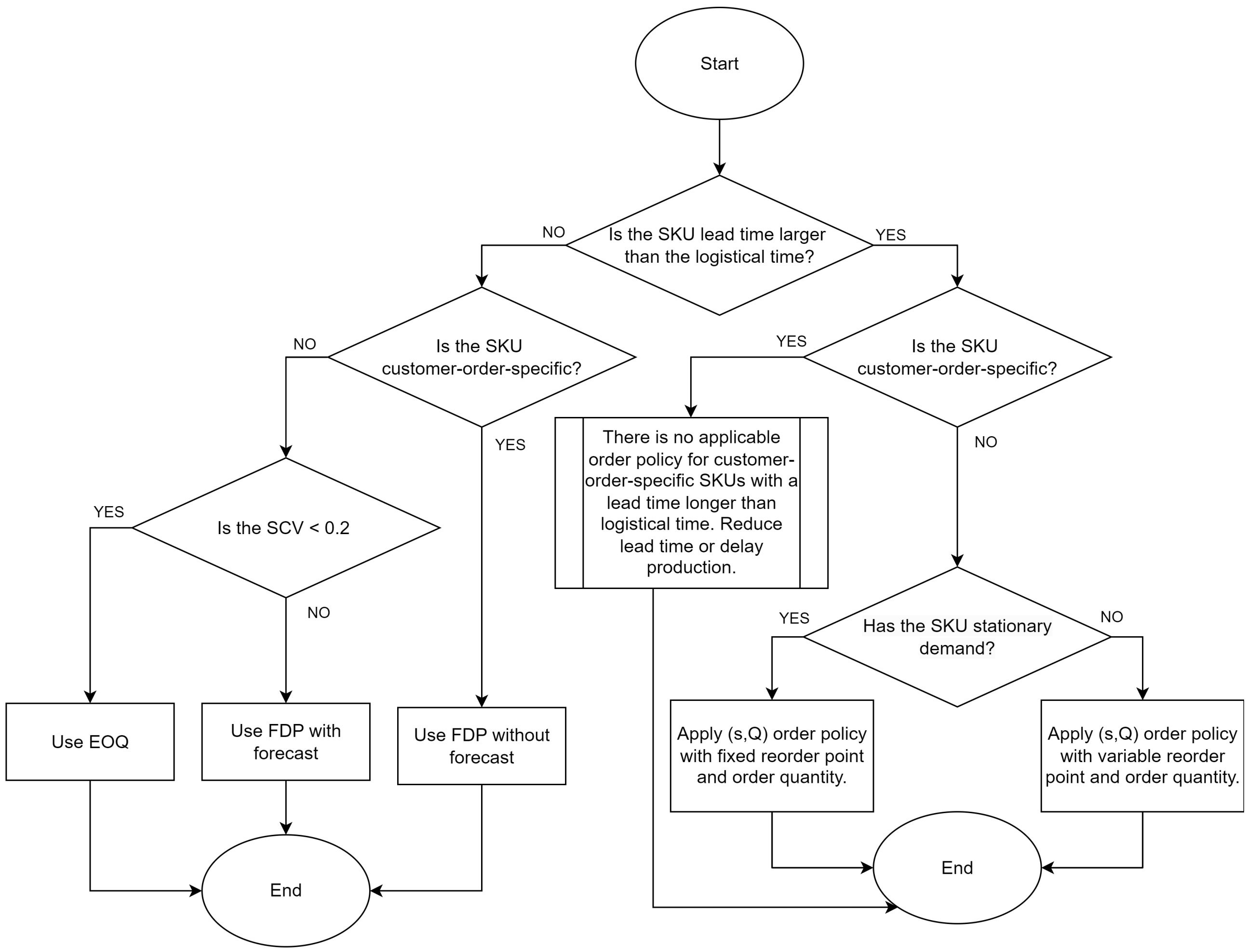


Figure K.1: Select Order Policy

Figure K.1 shows the general approach to selecting an order policy. The right side of the figure is for stochastic demand. This side is identical to the approach used in this research. Customer-order-specific SKUs with a lead time shorter than the logistical time have a FDP without a forecast as order policy. SKUs in the standard, with a lower risk of becoming obsolete, can have an EOQ or FDP where the forecast is included. The decision between FDP and EOQ is based on the variability of demand. EOQ is used for the SKUs with a squared coefficient of variance per period lower than 0.2.

L Optimize Order Quantity Based on Risk Obsolescence

There is a risk regarding obsolescence when SKUs are purchased-to-stock. If the demand disappears completely, SKUs will become obsolete and must be scrapped. This appendix only deals with the risk regarding obsolescence, and the risk of increased holding costs is neglected. Furthermore, there are no costs for the scrapping process itself. These costs are unknown for VMI before the scrap process starts. Sometimes, obsolete SKUs can be sold; sometimes, there are costs to discard these SKUs.

This appendix describes how risk can be incorporated into the order policies if a forecast with a confidence level is available. First, the probability density function of the forecast is explained. Next, the concept of value at risk is presented. Afterward, potential cost reduction is defined. This is followed by the order quantity decision, where the cost reduction is balanced to the value at risk.

L.1 Probability Density Function of Forecast

Figure L.1 shows a possible situation. There are eight weeks where demand is known and a forecast of four weeks. Let us assume that the forecasts of the different periods can be combined into one distribution of expected future demand. The sum of all the forecasts of the individual periods is a probability density function $P(x)$ as shown on the right side. The probability density function provides the probability that demand of quantity x occurs during the forecast period.

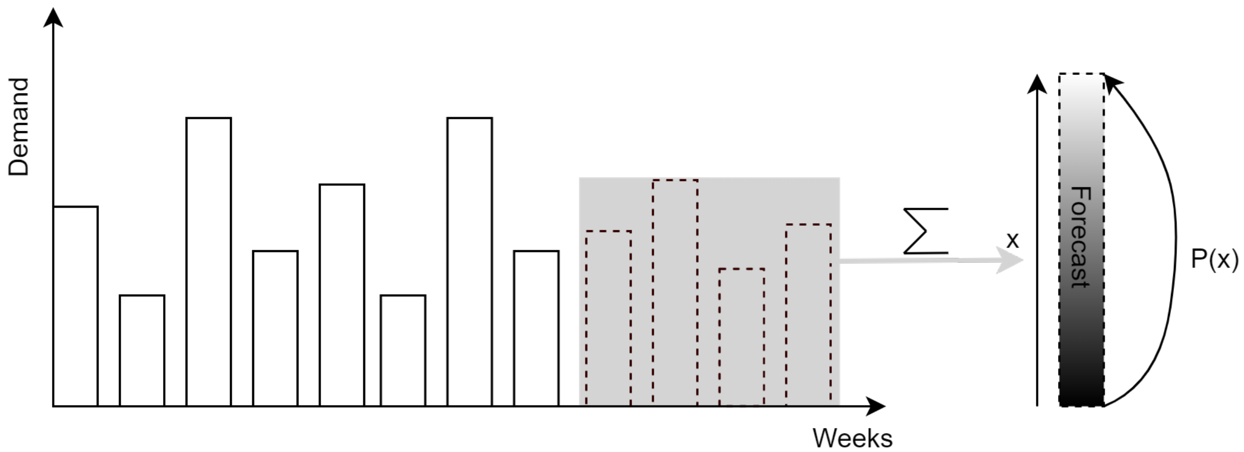


Figure L.1: Visualization of Demand and Forecast

Equation L.1 defines the forecast demand calculation based on the continuous probability function $P(x)$ of the total forecast. Negative demand does not occur; therefore, the integral starts at zero and ends at infinity. Equation L.2 shows the expected demand of the forecast for a discrete probability function.

$$E(x) = \int_0^\infty x \cdot P(x)\,dx \tag{L.1}$$

$$E(x) = \sum_x x \cdot P(x) \tag{L.2}$$

L.2 Value at Risk

The value at risk (VAR) defines the order quantity value for which demand is not certain. Equation L.3 shows the calculation of the VAR based on the order quantity (Q), demand which is known (d_k), forecast demand (d_f), reorder point (s) and the item price (p). The max function ensures that the VAR cannot be negative.

$$VAR = [s + Q - d_k - d_f]^+ \cdot p \tag{L.3}$$

$$[I]^+ = max(I, 0) \tag{L.4}$$

There are three ways to define the forecast demand (d_f). The risk-averse option, the low-risk option, and the normal risk incorporation.

The risk-averse version does not include the forecast by any means and only purchases items for which demand is known. The (d_f) and VAR are zero in this case. This risk-averse version is used for the SKUs with a high risk of becoming obsolete in this research.

The low-risk version is only willing to take a limited risk. For now, let's assume that we only want to purchase items for which we are 90% certain that demand during the forecast period will consume all those items. In other words, we are willing to take a 10% risk that we still have inventory left at the end of the forecast period. We need to solve Equation L.5 to find the 10th percentile to find the forecast demand quantity for which we are 90% certain that demand occurs. The risk we are willing to take depends on the SKU characteristics and the forecast length. If the forecast length is four weeks, but we expect demand to occur after the four weeks, we are willing to take a higher risk. The risk that we are willing to take is an input parameter and not further discussed in this appendix.

$$\int_0^{d_f} p(x)\,dx = 0.10 \tag{L.5}$$

The third version sets $d_f = E(x)$ where the demand forecast equals the expected demand of the forecast. The low-risk approach with a probability of 50% of an unskewed probability distribution gives the same result.

L.3 Potential Costs Reduction

By now, we have defined the VAR for a single order quantity. Let's look at the other side of the cost reduction of the larger order. The cost reduction (CR) exists out of the decrease in the ordering costs (S), potential more discount, and the increased holding costs (HC).

$$CR = \Delta S + \Delta Discount - \Delta HC \tag{L.6}$$

The order cost reduction is based on the fraction of forecast demand over the known demand. This shows the part of the order costs that is reduced. The order cost reduction is based on the fraction of forecast demand over the known demand. Equation L.7 shows how the order cost reduction calculation.

$$\Delta S = S\frac{Q_s}{Q_d} \tag{L.7}$$

$$Q_d = [d_k - s]^+ \tag{L.8}$$

$$Q_s = [s + Q - d_k]^+ \tag{L.9}$$

The discount is the difference in price if the order quantity is increased to more than the demand is known.

$$\Delta Discount = (P(Q) - P(Q_d)) \cdot Q \tag{L.10}$$

The holding costs increase because the items are stocked longer. The increase in holding costs is the difference between the new and old holding costs. Equation L.11 shows the calculation based on the assumption of constant demand with (D_w) as the average weekly demand.

$$\Delta HC = 0.5 \left[Q \frac{Q}{D_w} - Q_d \frac{Q_d}{D_w} \right]^+ \cdot h_w \tag{L.11}$$

L.4 Choose Order Quantity

To summarize, the VAR and cost reduction depend on the order quantity size (Q). The order quantity should be chosen to maximize the cost reduction (CR), subject to the constraint that the VAR is smaller than the cost reduction.

$$\begin{aligned} &\underset{Q}{\text{argmax}} \quad \text{CR}(Q) \\ &\text{subject to} \quad \text{VAR}(Q) < \text{CR}(Q) \end{aligned} \tag{L.12}$$

The VAR is limited to a single order since no new order would be placed if demand disappeared. Controversially, the cost reduction is per order, and multiple orders can be placed. We might be willing to take a risk that the VAR is larger than the cost reduction of one single order but should be smaller than the cost reduction of 2 orders, for example. The constraint can be altered to $\text{VAR}(Q) < A \cdot \text{CR}(Q)$ where A is the number of orders for which the cost reduction is incorporated. The value of A can be based on the expected number of orders until the next revision, for example.