Bachelor thesis

Backtesting the PD model for the Financial Lease and Hire Purchase portfolio of Volkswagen Bank GmbH branch NL.

July 1th, 2010

<table>
<thead>
<tr>
<th>Student</th>
<th>Internal supervisors</th>
<th>External commissioner</th>
</tr>
</thead>
</table>
| H.G. Hoeksema  
S0176877 | Dr. B. Roorda  
Assistant Professor  
University of Twente  
Ir. Drs. A.C.M. de Bakker  
Lecturer University of Twente  
& Credit Modeling Manager | Ir. D. Stienstra  
Basel II coordinator  
Volkswagen Pon Financial Services  
Terminalweg 20  
Amersfoort |
Management summary

Probability of default model
A research has been carried out for Volkswagen Bank NL. The main purpose of the report is to backtest the currently used probability of default model (PD model), which estimates the chance of default. A contract is in default when the banks considers that the obligor is unlikely to pay off his debts, or is over ninety days in arrears. A default can occur in the lease portfolio. The portfolio is either that of financial lease contracts for business clients or hire purchase contracts for private clients.

Backtesting
The main focus of this report lies on the backtest of the PD model. In addition, the model is calibrated and its backtest procedure is standardized, in order to simplify future testing. The purpose of backtesting is to verify whether the model accurately estimates the PD – which is in essence a logistic regression formula [1]. The PD is based on the variables of a contract, its weight ($\beta$) and value ($x$).

$$\text{PD} = \frac{1}{1 + e^{(\alpha + \gamma + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \ldots)}}$$ [1]

Purpose
The underlying purpose of backtesting lies in the compliance to internal and regulatory requirements of the Internal Rating Based Approach for retail exposure of Basel II. With this approach, Volkswagen Bank can determine credit risk in a more accurate manner than the traditional approach.

Defaults
The PD is dependent on the realized default rates. Unfortunately, a default increase was realized in the last year which was greatly influenced by the recession. This led to the bankruptcy of a great amount of companies. Needless to say, this leaded to an increase in the PD.

Performance
The PD has increased significantly. Performance of the model was rather good, nonetheless. The PD model on financial lease contracts, as well as the model on hire purchase contracts, have great discriminatory power, though, their precision is lacking; meaning that the model can accurately discern ‘bad’ from ‘good’ contracts, but the occurrence of a default is underestimated. Hence, remodelling is unnecessary because the variables are solid, save for the weights given to them, these need to be adjusted – by calibration.

Recalibration
<< Paragraph confidential >>

<< Figure confidential >>
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<th>Description</th>
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<tr>
<td>FL</td>
<td>Financial Lease</td>
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<tr>
<td>HK</td>
<td>Hire purchase</td>
</tr>
<tr>
<td>DK</td>
<td>Resolving Credit (‘Duurkoop’)</td>
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<tr>
<td>PL</td>
<td>Personal Loan</td>
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<td>VW</td>
<td>Volkswagen</td>
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<td>VWPFS</td>
<td>Volkswagen Pon Financial Services</td>
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<tr>
<td>AG</td>
<td>German business entity ‘Aktiengesellschaft’, similar to ‘NV’</td>
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<td>GMBH</td>
<td>German business entity ‘Gesellschaft mit beschränkter Haftung’, similar to ‘BV’</td>
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<tr>
<td>Risk SSC</td>
<td>Risk management, a shared service centre</td>
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<td>IRB</td>
<td>Internal Ratings Based approach (used in determining credit risk)</td>
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<td>SME</td>
<td>Small or Medium sized Enterprise</td>
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<td>CAP</td>
<td>Cumulative Accuracy Profile</td>
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<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic</td>
</tr>
<tr>
<td>CoC</td>
<td>Coefficient of Concordance (derived from the CAP chart)</td>
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<tr>
<td>GINI, KS</td>
<td>Gini-coefficient and Kolmogorov–Smirnov (derived from the ROC chart)</td>
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<td>SI</td>
<td>Stability Index</td>
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### Risk parameters

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<th>Parameter</th>
<th>Description</th>
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<tr>
<td>PD</td>
<td>Probability of Default - The expected probability a contract goes into default</td>
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<tr>
<td>LGD</td>
<td>Loss Given Default - Expected ratio of the loss given an EAD</td>
</tr>
<tr>
<td>EAD</td>
<td>Exposure At Default - Amount outstanding at the moment of default</td>
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Preface
International banks must be compliant conform financial regulations, which are currently under heavy discussion. Under the latest regulations, Volkswagen Bank GmbH would like to implement an advanced approach in determining credit risk. Therefore, internal models should be tested frequently and adjusted when necessary – it was my task to perform such an assessment, with regard to the PD model. The bachelor thesis is the final course of the bachelor ‘Industrial Engineering and Management’. Its assignment should be carried out from a rather practical point of view, and is commonly executed in an external company. Through the bachelor thesis administration, I came across Volkswagen Pon Financial Services. Without hesitation, I applied – motivated to work in the financial field, in order to see whether my future prospects should lie in finance.

During this period, I learned about financial regulations and model validation – from which I had no experience beforehand. Nevertheless, I accepted the challenge and gained a thorough understanding of quantitative validation throughout the internship.

I would like to express my gratitude to Mr. Roorda and Mr. de Bakker for providing feedback on my report. In addition, I would like to thank Mr. Stienstra for giving me the opportunity to take on this task. Furthermore, I appreciated my other colleagues who treated me like a fellow colleague in the risk management department, who undoubtedly will continue to manage the risks effectively.

Harm Hoeksema
Enschede, 2010
Chapter 1. Introduction

In this chapter an introduction is given to the research which has been carried out for the bachelor thesis. The main purpose is explaining the structure of the report. This is done first by examining the organization (1.1), followed by the problem description (1.2). Afterwards, the research questions are elaborated (1.3). In conclusion, a conceptual research model and planning are given (1.4).

1.1 Overview of Volkswagen Pon Financial Services

Volkswagen Pon Financial Services (VWPFS) is a group of financial orientated businesses concerning the automotive industry. Its primary activities are financing, leasing and insurance for consumers and corporate customers. For instance, one of the businesses within VWPFS – such as Volkswagen Leasing B.V. – provides leasing contracts for their cars. All the primary activities of the financial institutions are reflected in figure 1 (VWPFS, 2010).

The different financial services institutions were merged into the PON Holding in 1998, with the purpose of combining the financial knowledge while maintaining the flexibility for the automotive industry. In 2003, VWPFS came into existence, established as a joint-venture between Volkswagen Financial Services AG (60%) and PON Holdings B.V. (40%). When in 2004 Volkswagen Bank set up their Dutch facility, to provide financial services to clients for Volkswagen automobiles (VWPFS, 2010).

At the present time VWPFS contains the following companies: Volkswagen Bank Branch NL, DFM (Dealer Financierings Maatschappij), Dutchlease, LeasePlus, Volkswagen Leasing, VVS (Volkswagen Verzekeringen Services). To display, the organizational structure is shown in figure 2 (SSC Risk Management, 2008, p. 5).
1.1.2 Key figures
Volkswagen Pon Financial Services B.V. was definitely affected by the crisis in numerous ways, as a great proportion of dealers did not make it through 2009, and the total market for automobiles shrunk by more than 20%. Nevertheless, the results of the recent year were decent: still 400,000 Volkswagen passenger cars were sold in the Netherlands and an increasing profit was realized. VWPFS’s total assets on 31 Dec, 2009 reached over << Confidential >>. (PricewaterhouseCoopers, 2010)

1.1.3 Risk management
Risk management is an important department of a financial institution – especially for VWPFS because of its large range of shareholders. Within risk management, the Basel II regulations are essential and do apply for VWPFS. For this reason, various risk models are in place which ought to be compliant to both internal and external requirements. For Volkswagen Bank NL, the internal rating based approach has been implemented and a PD-model (Probability of Default) and a LGD-model (Loss Given Default) have been developed. With these models, expected losses are calculated for the portfolio of Volkswagen Bank NL. In addition, the minimum required capital is determined and allocated, to cope with potential losses. Next to these models, also credit score models are in place to estimate the creditworthiness of the customers at the moment of application. (SSC Risk Management, 2008)

1.1.4 Portfolio information
Volkswagen Bank the Netherlands provides financial services for the automotive consumer, in terms of financing, insuring and leasing. The portfolio mainly consists of financial lease (FL) contracts for business clients and hire purchase (HP) contracts for private clients; roughly << confidential >><% based on the contract snapshot of January 2009. The other two types of contracts are personal loans and resolving credits, however, for these models a standardized approach is used concerning credit risk. Regarding the FL and HP contracts, the risk management department prefers to use the internal rating approach (IRB) for modelling credit risk. An overview is given in figure 3 (SSC Risk Management, 2008, p. 17).

Figure 3 – Volkswagen Bank products and approach under the Solvency Ordinance1

1.2 Problem description

1.2.1 Problem background
Volkswagen Pon Financial Services needs – just as any financial institution – to be compliant with the financial and risk management regulations, in particular, Basel-II. Basel-II’s main objective is to realise a sufficient facility for expected and unexpected losses. In this case, Volkswagen Bank Netherlands uses an internal ratings based (IRB) approach, which uses internal models to measure credit risk. Credit risk can be measured by calculating the expected loss, where an important element in its calculation is the probability of default (PD) – which estimates the chance of a default on a contract. The PD model must be backtested2 regularly to maintain high accuracy; in order to make solid predictions according to internal standards and regulations. The backtest assesses the predictive and dis-

1 Regulation governing the capital adequacy of institutions groups of institutions and financial holding groups (Des Bankenfachverbandes, 2009).
2 Backtesting refers to testing a model, in this case the PD model, internally, and on historic data.
Criminatory power of the model in the current situation, next to these, confirming data stability is a part of backtesting as well. Backtesting was historically done by Capgemini, however, to improve the responsiveness it is currently done internally.

The external commissioner stated the assignment as follows: “A study has to be done how the risk model can be optimized (if necessary), and to develop a standard validation process. The validation process and PD model have to be compliant to the Basel II regulations and the internal requirements of Volkswagen Financial Services AG.” (Stienstra, 2010)

In brief, the PD model is essential in calculating the expected losses, and thus, should be tested regularly. Therefore, research is necessary to determine if the current PD model is accurate – if not, changes must be made. In addition, an automated procedure for backtesting should be set up.

### 1.2.2 Problem statement

According to the ABP[^3] the problem statement can be distinguished as soon as there is a difference between the reality and a desirable situation (School, TSM Business, 2005). In this case, the PD model has not been reviewed for the current year, while the regulators require annual testing. The second discrepancy is that the current PD model is perhaps performing in an insufficient way; the norm is that the PD model is modelled in a way that it can predict defaults in an accurate manner. The third problem is that the model cannot be tested quickly by the department.

Out of these discrepancies the problem statement results, whereas its focus lies on backtesting the PD model. The executing problem[^4], that defines the discrepancy between the reality and the desirable situation for the risk management department, Volkswagen Bank and the regulators is as followed:

**Backtesting, calibrating, and standardizing the backtest procedure regarding the PD model designed for the retail portfolio of Volkswagen Bank NL.**

### 1.2.3 Research scope

The research is limited to the Volkswagen Bank NL portfolio for financial lease and hire purchase contracts. The model that will be studied is the PD model, its indicators will be measured, which may lead to the conclusion to optimize the model.

The model itself is subject to certain constraints, such as the Basel-II regulations and internal requirements. How these regulations and internal requirements for the PD model are implemented is represented in an internal document from Germany: ‘Business Blueprint Monitoring / Validation (Retail)’ (Volkswagen GmbH, 2010).

### 1.2.4 Research study

The research can be defined as a descriptive study (Cooper & Schindler, 2008). The objective of such a study is to find the answers to the who, what, when, where and how questions[^5]. These studies focus on describing a specific set of variables by a detailed analysis. In this case, describing the adoption and performance of the PD model within the retail portfolio.

Out of the executing problem, knowledge problems arise; for example, more information is needed concerning the PD model. Furthermore, the interpretation of the statistical tests used in backtesting must be explained – these tests are mentioned as performance indicators in this report. In addition, all important terms in the problem statement are treated in the research questions, such as backtesting, calibrating and standardizing.

[^3]: Abbreviation for ‘Algemene bedrijfskundige probleemaanpak’
[^4]: With an executing problem an ‘Handelingsprobleem’ is meant (Heerkens, 2010)
[^5]: One drawback of these studies is that why questions are commonly left unanswered.
1.2.5 Research questions

The research questions are the structure of the research report and are answered throughout the report at the end of a chapter.

1. What is a PD model according to literature and company information?
2. What regulatory and internal requirements are there for backtesting?
3. Which criteria can be defined for the performance indicators?
4. How does the current PD model perform in terms of predictive power? | Backtesting
   a. How well does the model discriminate between contracts?
   b. How stable are the used model variables?
5. In what ways can the PD model be improved? | Calibrating
6. How can the process of backtesting be standardized? | Standardizing

1.3 Research approach

In the following section, activities, information sources and possible choices will be explained per research question.

1.3.1 What is a PD model according to literature?
Understanding the PD model concerns the knowledge needed for fulfilling the research objective. One would need to understand what PD models are, why, when and how they are used. The answers are specifically valuable for additional – possibly very important – information in performing the research, moreover, it could provide new practical information for the company. On information sources, my expectation is that there is likely plenty of research done on the IRB approach of Basel-II and the usage of the PD model; it should therefore be relatively easy to find within the university library.

1.3.2 What regulatory and internal requirements are there for backtesting?
Backtesting is needed recurrently because of regulatory and internal requirements for determining credit risk, though, the underlying questions is how the company should realise its backtest. In essence, on what basis is the PD model validated. It is also interesting if there are recent updates in the internal or external requirements which could improve the current model.

1.3.3 Which criteria can be defined for the performance indicators?
Research should be done to which specific tests should be carried out. The testing of the performance of the PD model is split in three parts: precision, discriminatory power and data stability tests. Hence, I need sufficient knowledge on specific statistical tests (such as binomial, chi-square and Hosmer Le- meshow). Furthermore, information is needed on calculating the discriminatory power of variables (such as CoC, GINI, KS). Information on this subject can probably be found in internal documents.

1.3.4 How does the PD model perform?

Out of the performance criteria, which assess the performance of the PD model, the results of the current PD model can be concluded. The realized performance of the PD model gives reason to take steps accordingly. In essence, this question focuses on how the results of the tests should be interpret.

1.3.5 In what ways can the PD model be improved?

After the PD model is tested, conclusions can be made on the fact that the model might need calibration or complete remodelling. In this scenario, the model should be calibrated or modelled in the best possible way. The results of the backtest might also give reason to propose other improvements.

1.3.6 How can the process of backtesting be standardized?
The process of backtesting is very time-consuming, hence the commissioner asked if the procedure can be standardized; so that, when new data arrives, results can be interpret in a matter of days or even hours. The underlying reason, is that the backtest is preferably done quarterly in a consistent fashion. By standardizing the backtesting procedure, it can always be carried out – even in busy periods.
1.4 Research structure

1.4.1 Conceptual research Model

The conceptual research model explains graphically in what order research activities are done, corresponding to the research questions (figure 4). First, the problem background is worked out so that the objective of the research clearly follows. Research questions are formulated and elaborated in the research approach. Secondly, the literature concerning the PD model is researched, followed up by demonstrating all the realized defaults. Third, the criteria of the performance indicators and data stability tests are explained. Then, the backtest calculations are carried out: how the portfolio fits within the masterclasses, how well the performance indicators perform, as well as testing the individual variables on data stability. Fifth, possible improvements will be made to the PD model, in addition, the calculations carried out in chapter two and four are to be standardized. The thesis finishes with conclusions and recommendations. Normally in research, an evaluation is carried out – however, because of the time span of the research this phase will be left out.

![Research structure of the bachelor thesis](image)

1.4.2 Evaluating information sources

In most descriptive studies, there is a great amount of information used which originated from within the organization. An important aspect of the information used within a research is its creditability. Information sources can be evaluated on five factors according to (Cooper & Schindler, 2008) – namely, its purpose, scope, authority, audience and format.

At VWPFS, internal documentation has a clear internal goal and no hidden agenda (purpose), includes specific information which is regularly updated (scope), has clear references to its author(s) (authority), mostly meant for employees or managers (audience) and in a presentable manner (format). It can be said that most internal documents and scientific articles have decent creditability for usage within the research. However, the credibility of other references, such as websites, should be thoroughly evaluated, conform these five factors.

1.4.3 Planning

To conduct the research, and its components, in eleven weeks requires a strict schedule. For that reason, a detailed planning has been made for the time span, this planning can be found in Appendix B.
Chapter 2. The Probability of Default model

In this chapter a detailed outlook will be given of the PD model. First, the Basel capital requirements will be globally explained (2.1). Within Basel the PD formula is used (2.2). The PD model must be validated regularly (2.3). An default overview is essential in PD backtesting (2.4). Finally, the conclusion of this chapter will be given (2.5).

2.1 An overview of credit capital requirements

2.1.1 Modern risk management

Risk, the effect of uncertainty on one’s objectives, have to be identified, assessed and prioritized; so that the impact of unfortunate events is minimized, monitored and controlled – as is done by risk management.

Traditionally, companies that issued bonds have been graded in their riskiness, to assist investors in their decision-making process. This grading was done by both hard and soft facts. On the other hand, credit institutions have been scoring their borrowers to assist their decisions in lending money. Which was most of the time done by statistically based score variables. (Tasche, 2006)

In traditional risk management, rating and scoring was a completely separate occurrence. Conversely, modern risk management focuses more on a unified view on rating and scoring alike – combining both in regulatory capital requirements.

2.1.2 A Basel-II overview

Since 1988, the Basel Accord is in existence, which improved the way on which capital requirements were determined. Since then, a great number of revisions were made to the Basel Accord. Nowadays, the latest version of ‘Basel II’ consists of over three hundred pages of regulations, applying to all ‘internationally active’ banks. Hence, all European banks, large or small, are regulated under Basel II. The regulation is separated unto three ‘pillars’: minimum capital requirements, supervisory review and market discipline. In the first pillar, credit risk is calculated in a new approach which takes into account the credit rating of counterparties. In the second pillar, risks, that are not covered in pillar one, must be evaluated by the regulator. In the third pillar, additional information must be given about the manner on which capital is allocated. (Hull, 2007)

2.1.3 Basel-II capital requirements

Banks are obliged to fulfil their capital charge on the risks they are taking. In the latest international capital standards for credit institutions – Basel II – it is permitted to use internal rating systems for calculating the weighted risk, so that banks are allowed to reserve less money for certain portfolios under the condition that it is supported by solid quantitative and qualitative validation. Formerly, credit ratings had no impact in regulatory standards, which provides a framework to ensure ongoing monitoring of the internal ratings models used. (Deutsche Bundesbank, September 2003)

2.1.4 Basel-II credit rating approaches

The Basel II credit rating approaches are supported by the following quote: “The Committee proposes to permit banks a choice between two broad methodologies for calculating their capital requirements for credit risk. One alternative will be to measure credit risk in a standardised manner, supported by external credit assessments. The alternative methodology, which is subject to the explicit approval of the bank’s supervisor, would allow banks to use their internal rating systems for credit risk.” (Basel Committee on Banking Supervision, 2010, 50 & 51)

With credit risk, the standardized approach uses external ratings determined by credit rating agencies. Formerly, in Basel I, credit risk was always set at a uniform 8% – in spite of the ratings. At present, in

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6 Definition according to the ISO 31000 standard on the implementation of risk management.
7 The bank must, however, implement the IRB approach throughout the whole organization on every portfolio.
8 Within Europe, only a small amount of businesses actually have external ratings.
Basel II, companies can additionally apply for the foundation IRB approach and the advanced IRB approach, which uses internal ratings, such as the PD, LGD and EAD, derived by the company itself. However, in the case of retail exposure, the choice is limited to the standardized approach and the IRB approach – which is similar to the advanced IRB approach in the case of corporate exposure. (Hull, 2007)

2.1.5 Regulatory background

Regulation is an important factor in PD modelling, as the regulators need to approve the model used in determining capital requirements. Nevertheless, Basel II has no direct legal consequences on German banks; in contrast to the national policy which do have legal consequences. Basel II is a basis for future regulation, and is therefore a set of practises, instead of a direct legal force. The best practices of Basel II are implemented in the European Union (EU) directive documents, which is legally binding to EU members. The directives are adopted per country in the so-called Solvency Ordinance. Specifically for VW Bank, the German Solvency Ordinance report – the Solvabilätsverordnung – is the legally binding document. To illustrate how Basel II influences the legal consequences for banks, refer to figure 5. (Des Bankenfachverbandes, 2009)

![Figure 5 – Legal documents and their effect on Volkswagen Bank (Heeres, 2009)](image)

2.2 The PD model

“For retail exposures, banks must provide their own estimates of PD, LGD and EAD. There is no distinction between a foundation and advanced approach for this asset class.” (Basel Committee on Banking Supervision, 2010, §252)

2.2.1 Retail exposure portfolio

The retail portfolio of VW Bank branch NL is subject to the Solvabilätsverordnung and internal requirements. To be included in the regulatory retail portfolio, it must satisfy the retail definition of Basel II, simply put it the contract has been made with a person, persons or a SME\(^9\). At the moment, there is no IRB accreditation, so the capital requirement is at a regular eight percent. In order to apply for a IRB accreditation in the near future, records are needed of internal implantation in the organization for five years\(^10\). For that reason, annual PD backtesting is essential for the company.

2.2.2 Rating grades

A number of rating grades must be composed in advance, to facilitate the assignments of grades according to qualitative descriptions. In order to comply to Basel II, paragraph 410:\(^b\): “The grade descriptions and criteria must be sufficiently detailed to allow those charged with assigning ratings to consistently assign the same grade to borrowers or facilities posing similar risk. This consistency should exist across lines of business, departments and geographic locations. If rating criteria and procedures differ for different types of borrowers or facilities, the bank must monitor for possible inconsistency, and must alter rating criteria to improve consistency when appropriate”.

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\(^9\) Additional criteria for contracts in order to be considered in the retail portfolio can be found in §70 of Basel II

\(^10\) The transition period is three years, but at least five years of data is needed to implement the IRB approach for retail exposure. (Basel Committee on Banking Supervision, 2010, §264)
Specifically for PD model backtesting, it mentions in paragraph 501th: “Banks must regularly compare realised default rates with estimated PDs for each grade.” (Basel Committee on Banking Supervision, 2005).

2.2.3 Masterclasses
According to Basel II, paragraph 464th contracts must be assigned to certain pools, and in the case of retail exposure, internal data is the primary source of information for estimating loss characteristics. The method of VW Bank is the distribution of contracts in a ‘masterclass’ based on the PD percentage (which is calculated on internal data). These classes are represented later on, for the specific portfolios of FL and HK alike, in the default overview. The estimated and realized default rates will be distributed among the masterclasses.

2.2.4 Risk parameters
The IRB approach is composed out of the risk parameters ‘probability of default’ (PD), ‘loss given default’ (LGD) and ‘exposure at default’ (EAD)\(^\text{11}\). These risk parameters play an important role in determining the minimal required capital for banks, their estimation needs to be done with care. The minimal required capital is in essence needed for expected losses, in Basel II terms, so that the capital can cope with 99% confidence. The expected loss formula is used in the IRB approach [1]. It also contains the LGD and EAD, which are not validated in this report.

\[
\text{Expected Loss} = \text{PD} \times \text{LGD} \times \text{EAD} \quad [1]
\]

Risk drivers are the factors that influence risk parameters (i.e. influences the PD). These risk drivers are presumed to have a causal effect on the risk parameter value.

2.2.5 PD formula
The PD is a crucial determinant in calculating credit risk, which is clearly noticeable in the expected loss formula [1]. As a result, it has a great impact in the allocation of the required capital in the IRB approach. Logically, the PD stands for the probability a default occurs within the next twelve months. This probability must be determined per contract, which the PD model computes on the basis of certain variables (risk drivers). In essence, the PD model calculates the estimated PD of a contract, which in turn can be checked against realised defaults. The PD formula is given below (Capgemini, 2008).

\[
\text{PD} = \frac{1}{1 + e^{-z}} \quad [2]
\]

Where \(z\), consists of a constant, plus the different variables multiplied by their weights [3]. The exact values are given in appendix D for both the FL and HK model.

\[
\text{z} = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \cdots \quad [3]
\]

2.3 Validation of the PD model
“Banks must have a robust system in place to validate the accuracy and consistency of ratings systems, processes and the estimation of all relevant risk components. A bank must demonstrate to its supervisor that the internal validation process enables it to assess the performance of internal risk estimation systems consistently and meaningfully.” (Basel Committee on Banking Supervision, 2010, §501)

2.3.1 Overview
Basel II paragraph §501\(^\text{th}\) states that banks using an IRB approach must at least have an annual validation of its models – in order to determine their usefulness, and to demonstrate this to the supervisor.

\(^{11}\) Additional information on risk parameters can be found in Appendix A.
The term “validation” constitutes the following: (Basel Committee on Banking Supervision, 2005)
  o Range of processes that contribute to the assessment of the ratings adequately differentiating risk
  o Whether the estimates of PD appropriately characterise the relevant aspects of risk.

Validation maintains to be a major challenge for financial institutions. The statistical models used in the quantitative validation can give insight in undesirable developments. However, qualitative validation is needed to assess the suitability of the rating system. (Deutsche Bundesbank, September 2003)

A rating system serves to determine the credit risk of individual borrowers – in this case, a PD estimation. This is done by using a variety of discriminatory indicators which in turn can put them in a certain “risk class”\textsuperscript{12}. On a side note, the refinement of the credit rating models is ongoing.

2.3.2 Validation principles
In the fourth newsletter of Basel in 2005 the principles of the validating process under IRB approach are clarified. First of all, validation is fundamentally about assessing the predictive ability and the use of ratings. Meaning that, it should discriminate and calibrate risk effectively. When the outcomes diverge, the weighting of the IRB parameters must be renewed, as long as the variables still discriminate in a sufficient way. Second, the bank has primary responsibility for the validation. Third, validation is an iterative process. Fourth, there is no single validation method. Fifth, validation should encompass both quantitative and qualitative elements. Last, validation processes and outcomes should be subject to independent review. (Basel Committee on Banking Supervision, 2005)

2.3.3 Quantitative aspects
The validation process greatly depends on the character of the rating system in use. A line is drawn between model-based systems and expert judgement based systems. Whereas, model-based systems uses statistical models to capture a number of risk factors, an expert judgements based systems relies on the view of an expert (which most of the time means a standardized procedure). Either models can be quantitatively tested when there are enough defaults. Typically, this can be done in the case of retail business or to individuals. The principal criteria for the quantitative validation are its discriminatory power, data stability and calibration which are explained later in detail. (Deutsche Bundesbank, September 2003)

2.3.4 Backtesting
The testing of the PD formula, in order to determine if the model estimates the PD accurately, is called ‘backtesting’, because it is based on historic data. The PD model calculates the estimated PD of a contract, which in turn can be checked against realised defaults – to determine if the formula holds a ground on the true estimation. Elaboration on this subject will given in chapter three.

2.3.5 Requirements
There are several requirements to the PD modelling and backtesting: (Section 128, Solvabilitätsverordnung)
  • Incorporate all relevant data, information and methods
  • Empirical evidence, not only on judgmental considerations
  • Plausible and intuitive, based on material drivers of risk.

2.4 Defaults in the PD model
“Banks must regularly compare realised default rates with estimated PDs for each grade.” (Basel Committee on Banking Supervision, 2010, §501).

\textsuperscript{12} Also known as masterclasses
2.4.1 Definition
The PD substitutes for ‘probability of default’ (within twelve months), therefore it must be specified when a contract is officially in default, in order to arrange a consisting PD report. The definition of a default is given within company information (SSC Risk Management, 2008):

- Automatic default when a obligor is more than ninety days in arrears
- Manually in default when the bank considers the obligor will be unlikely to pay the total amount (as a result of an unlikeliness to pay: E.g. death of client, total loss, bankruptcy client13)

2.4.2 Defaulted contracts
The purpose of backtesting the PD model is to check if the PD model accurately calculates the number of defaults. The PD formula of the previous year, 2009, is given within the company data. The formula differs per contract type – one for financial lease and one for hire purchase contracts. With these contracts, the average estimated probability of default is calculated per contract type; to realize this, the following data is needed:

- All healthy contracts14 with relevant variables on snapshot date 31/1/2009
- All realized defaults within the period of 31/1/2009 to 31/1/2010
- All realized counter-entries15 within the period of 31/1/2009 to 31/1/2010

2.4.3 Types of contracts
Volkswagen Bank offers four types of contracts to its customers (SSC Risk Management, 2008):

1. Financial lease is the only available credit facility for business clients. This type of contract is primarily for financing a car. A contract between the bank and the client is made based on the value of the car, its expected trade-in value at the end of the contract and the duration of the contract. Usually, a final instalment has to be paid at the end of the contract, because the contract does not fully compensate the cars price. To minimize risk, the resale value of the car should cover the final instalment in value. During the life of the contract the car acts as collateral. In case the customer stops making the required payments, the car can be impounded and sold.

2. Hire purchase is only for consumers and is technically similar to financial lease in terms of the product offered. There is one important legal difference for Volkswagen Bank; hire purchase contracts with a credit amount up to 40,000 euro are subject to the ‘Wet op het consumentenkrediet’. This law aims to protect consumers and restricts the possibility of the lender to seize the collateral in case the client stops paying. In case 75% of the credit amount has been repaid it is not allowed to impound the car (stated in article 41th of the ‘Wet op het consumentenkrediet’). The lender still has a claim on the collateral, but to seize it requires costly legal procedures.

3. Personal loan is a credit facility for consumers in the form of a regular loan where the financed amount will be fully paid off by monthly instalments. Possibly a car is used as collateral.

4. Revolving credit is also for consumers, and most of the time a car is used as collateral. When the client uses the credit facility, the client has to pay at least one monthly payment to pay off the borrowed amount before making use of the credit facility again. This re-usage of the credit is subject to the manual acceptance of the credit acceptation department.

2.4.4 Portfolio size and default developments
A detailed overview of the portfolio of Volkswagen Bank is given in table 1, note that the amount of contracts consists of healthy contracts on the snapshot date. The amounts of defaults occurred on that portfolio is set in percentages (exclusive of repeating defaults in the same year). The size of the portfolios has remained relatively stable, in exception for the financial lease portfolio which shows a steady increase in the amount of contracts. In defaults, a clear negative trend can be distinguished in the rela-

---

13 Default reasons are given in appendix A
14 Contracts which are not defaulted on the snapshot date
15 In Dutch: ‘Storneringen’
tive amount of defaults. For additional information, a historic representation of the total amount of defaults is given in Appendix C.

<< Table confidential >>
Table 1 – Overview of contract set and occurred defaults

2.4.5 Realized amount of defaults

The PD model will be backtested by the estimated PDs versus the default rate from the recent year. Figure 6 represents these defaults, which occurred in 31-01-09 to 31-01-10 on the backtest set (healthy and existing contracts). Financial lease contracts have a higher probability on default, which is rational, because businesses have a higher chance of going in default than individual persons.

<< Figure confidential >>
Figure 6 – Amount of defaults of Volkswagen Bank branch Netherlands (healthy contracts on snapshot date 31/1/09)

2.4.6 Young contract defaults

It was advised in the previous backtest report to further investigate the effect of young contracts\(^{16}\) on PD (figure 7). (Capgemini, 2009). It is quite exceptional that in November over half of the defaults consisted of young contracts, this is however a particular event, which does not give reason to further investigate. As stated in the previous backtest report, one should keep a close eye on the effects of young contracts on the PD in the future.

<< Figure confidential >>
Figure 7 - Backtest set defaults, divided in normal and young contracts

2.5 Conclusion PD model

Besides the default overviews, two research questions have been handled throughout this chapter.

Q1. What is a PD model?
The PD model is utilized in calculating expected losses. However, the PD model might be inaccurate, hence, financial institutions who use the IRB approach are obliged to backtest the model regularly. This is done by using realized default data in quantitative validation. Contracts are positioned into different risk classes and tested if they discriminate correctly, along with testing its precision and data stability.

Q2. What regulatory and internal requirements are there for backtesting?
The PD model is utilized in calculating expected losses. However, the PD model might be inaccurate, hence, financial institutions who use the IRB approach are obliged to backtest the model regularly. This is done by using realized default data in quantitative validation. Contracts are positioned into different risk classes and tested if they discriminate correctly, along with testing its precision and data stability.

\(^{16}\) A young contract is a contract non-existent on the snapshot date
Chapter 3. PD model backtest performance indicators

In this chapter the PD backtest performance indicators (processes and activities used in backtesting) are explained. First, the framework of tests is given (3.1). Second, precision analysis is treated (3.2). Third, the discriminatory tests are explained (3.3). Fourth, the used model variables and data stability tests are elaborated upon (3.4). Finally, an overview is given (3.5).

3.1 Performance indicators

3.1.1 Framework

Performance indicators are tools on which one can measure the quality of the PD model. Hence, the goal of this chapter is to provide a statistical background, a unifying framework, for most of the technical considerations in validation techniques. As mentioned in the paragraph on the subject of validation (2.3), validation consists of both quantitative and qualitative aspects. Whereas, the first focuses on quantitative aspects such as backtesting and benchmarking; the qualitative focuses on non-statistical methods executed before the quantitative part. Even so, the main focus of validating a PD model lies in quantitative validation. To illustrate, these aspects are shown in figure 8. (Deutsche Bundesbank, September 2003)

![Figure 8 – Aspects of validation (Deutsche Bundesbank, September 2003, p. 60)](image)

3.1.2 Backtesting

Backtesting is the main focus of PD model validation, and has the overwhelming attention in this report. The tests used in backtesting are based on the literature of international banks, risk management groups and the tests previously used by Capgemini.

Backtesting is split into three parts:
- Precision analysis
- Discriminatory analysis
- Univariate analysis

They all evaluate the different aspects of the PD model. These aspects are explained in the following paragraphs and performed in chapter four.
3.2 **Precision analysis**

“The burden is on the bank to satisfy its supervisor that a model or procedure has good predictive power and that regulatory capital requirements will not be distorted as a result of its use...” (Basel Committee on Banking Supervision, 2010, §536).

3.2.1 **Overview**

Precision analysis (also called calibration analysis) is used in the determining if the estimate PDs deviates only marginally from the actual default rates. By using statistical tests it can be determined if the PD model is accurate in calculating the estimated PDs. If not accurate, the model has to be calibrated, meaning that only the weights – not the whole model – are adjusted.

3.2.2 **Binomial test**

The binomial test contrasts the forecasted defaults of rating; i.e. predicted PD versus observed default rate. For a specific risk class, it specifies if the realized defaults are within reasonable limits. According to Castermans et al., the test is carried out under the following hypothesis:

- Null-hypothesis: The observed PD is equal to the predicted PD
- Alternative hypothesis: The observed PD is larger than the predicted PD

The hypothesis is distributed under a normal distribution [4], because of the large amounts of contracts involved; $n$. This distribution is accurate under two assumptions: that defaults occur independently and $H_0$ is true. It tests the probability that the actual number of defaults occurred under the assumption that the PD is correct. When $P(Z = z) < 0.05$, then a significant difference is discerned.

$$PD \sim \left(\frac{\bar{PD}(1-\bar{PD})}{n}\right)$$

To test the hypothesis, one then arrives at the following test statistic [5].

$$Z = \frac{E(D) - D_{obs}}{\sqrt{\text{Var}(D)}}$$

Where,

$E(D) = n*p$ (predicted mean of defaults)

$\text{Var}(D) = n * p * (1-p)$ (predicted variance of defaults)

3.2.3 **Hosmer-Lemeshow test**

The previous test is appropriate for checking a single PD forecast, however, when fifteen classes are used, the likelihood is great that at least one of these PD forecast will be erroneously rejected. These tests are stand-alone, in order to control these arbitrary deviations one must use a joint test for multiple risk classes.

The Hosmer-Lemeshow test can determine if the hypothesis is accurate for all classes.

$$H = \sum_{i=1}^{k} \frac{(n_i p_i^{(1-p_i)})^2}{n_i p_i^{(1-p_i)}}$$

Where, $p_1,...,p_k$ is the estimated probability of default for rating grades 1,...,$k$. Let $n_i$ denote the number of contracts with grade $i$, and $d_i$ denote the number of defaulted borrowers with grade $i$. The formula is the sum of the squared differences of forecasted and realized defaults, divided by the theoretical variances of default amounts. This results in the Hosmer-Lemeshow statistic $H$. (Tasche, 2006)

The Hosmer-Lemeshow statistic has a chi square distribution with $k$ freedom grades under the hypothesis that all the PD forecasts match the true PDs. The test is under the assumption of normal approximation (independence and large enough sample size). The assumption of independence is crucial, because a bad approximation may be the case when a rating grade consists of a small numbers of borrowers, where a small change in rating grade distribution can have a large impact on the Hosmer-Lemeshow statistic.
3.3 Discriminatory analysis

"...The variables that are input to the model must form a reasonable set of predictors." (Basel Committee on Banking Supervision, 2010, §536).

3.3.1 Discriminatory power

The second part of backtesting consists of researching the so-called discriminatory power of a model, meaning the essence of the model to discriminate in ‘bad’ and ‘good’ contracts in terms of realised defaults – so that, its tested if the ‘bad contracts’ actually consist of the bad contracts and vice versa. (Tasche, 2006)

3.3.2 Cumulative Accuracy Profile

The Cumulative Accuracy Profile (CAP) is a graphical tool for investigating the discriminatory power. Its horizontal axis consist of the fraction of the contracts; the vertical axis determines the so-called ‘hit rate’ – the fraction of defaulters included in the current set of contracts.

Figure 9 shows an example CAP curve, the rating model line illustrates a model which could occur in practice. The other lines demonstrate a perfect random model based on a set with zero discriminatory power, whereas the perfect model has perfect discriminatory power.

3.3.3 Accuracy Ratio

From the illustration in figure 9 it is intuitively clear that the area between the random model and the rating model is a measure for discriminatory power. This area can be calculated by taking the integral below the curve minus \( \frac{1}{2} \). However, in estimating its discriminatory power, one must compare it with the perfect model. The result is the so-called accuracy ratio.

The accuracy ratio (also called Gini-coefficient) is defined as the area between the perfect and rating model divided by the area between the perfect model and rating model. Where \( p \) denotes the total probability of defaults [7].

\[
\text{Accuracy Ratio} = \frac{2 \int_0^1 \text{CAP}(u) \, du - 1}{1 - p} \quad [7]
\]

3.3.4 Receiver Operating Characteristic

The Receiver Operating Characteristic (ROC) is essentially a CAP curve without the fraction of defaulted contracted in the horizontal axis. As such, a perfect model would move through the coordinates \((0,1)\) as all default are occurred in the fraction before the non-defaulted contracts.

The Coefficient of Concordance (CoC) is the area under the curve in the ROC chart. Logically, the Gini-coefficient and COC have a relation with each other, given by [8],

\[
\text{Accuracy Ratio} = 2 \times (\text{CoC} - 0.5) \quad [8]
\]

In addition, the Kolmogorov–Smirnov (KS) statistic gives us the maximum difference between the curve and random model. A very steep line would result in a good KS statistic and thus good discriminatory performance. In company data critical bounds are given for these statistics\(^{17}\).

\(^{17}\) (Business Blueprint Monitoring / Validation (Retail).doc, p. 34)
### 3.3.5 Discriminatory interpretation

In the previous backtest report by Capgemini, a set of rule-of-thumb conclusions has been set. These is no reason to doubt this perception, and are thus reused (table 2).

<< Table confidential >>

Table 2 – Rule of thumb conclusions based on the performance indicators (Capgemini, 2009)

### 3.4 Univariate analysis

"Banks must demonstrate that quantitative validation methods and data are consistent through time. Changes in estimation methods and data (both data sources and periods covered) must be clearly and thoroughly documented." (Basel Committee on Banking Supervision, 2010, §533).

#### 3.4.1 Model variables overview

The univariate analysis describes the variables used in the modelling process – either used by FL, HK or both. Any changes in the individual variables used in the PD model must be noticed in order to discover the cause of changes within the PD model. It is important that the aspects of variable, such as its distribution remain reasonable consistent. The following risk drivers are used (table 3). The implementation of these variables is examined in appendix D.

<< Figure confidential >>

Table 3 – Overview of variables in the PD model

#### 3.4.2 Bubble charts

Univariate analysis is intended to provide an insight in the contribution of individual independent variables to the risk of default – it is based on the realized defaults. The analysis is conducted by assessing the default percentage for different values of the variable being assessed. ‘Bubble charts’ are used for the purpose of this analysis. An example of a bubble chart for the variable ‘age’ is illustrated and explained below (a fictional example). (Capgemini, 2008)

The horizontal axis of the bubble chart shows the age; the vertical axis shows the observed PD as a percentage. Each ‘bubble’ represents an age category. The size of the bubble is dependent on the amount of clients in that age category. The horizontal position of a bubble corresponds to the upper limit of the age category, the vertical position shows the observed average default risk for all clients in the relevant bubble. These aspects can be interpret in numerous ways, here, the risk of default is relatively low for clients close to age category eight.

#### 3.4.3 Univariate criteria

The value and suitability of a variable in the PD model can be measured by three criteria in the bubble chart. (Capgemini, 2008)

- Sufficient spread

---

18 Independent variables which are dependent in the modelling of the risk parameter.
A sufficient spread in population in the different categories is needed. Suppose a variable has two categories which are distributed for one and nine-ninety percent. Then the variable would have very little impact – and thus relevance – in the PD model. Ideally, the bubbles are of the same size.

- **Distinctive capacity**
  A variable’s distinctive capacity must be high, meaning there must be a sufficient amount of difference in the variable to discriminate ‘bad’ and ‘good’ contracts. Without that, the variable would not contribute in the PD model (E.g. the bubbles would lay on a horizontal line).

- **Explicable progress**
  A third more complicated criteria is the progress of one variable. Its function of default risk should be explicable, i.e. it should follow a clear pattern such as a linear line. For example, the diagram in figure 10 has a fluctuating default risk (indicated by a red line), making modelling obviously more difficult.

### 3.4.4 Stability Index

The stability index (SI) tests the variable’s population for changes. Stability checks for population, comparison between year t against the model set on which the model is calibrated. The SI is calculated using the formula below [9].

\[
SI = \sum_{i=1}^{m} (D_i - B_i) \times \ln\left(\frac{D_i}{B_i}\right)
\]

Divided into m buckets. \(D_i\) denotes the percentage of cases in bucket \(i\) in the development set, and \(B_i\) denotes the percentage in bucket \(i\) in the back test set.

The SI statistic is by no means an absolute measure for data stability, it should be interpreted by humans if a significant shift has taken place. Commonly applied rules in its interpretations are (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007, p. 8):

- SI \(\leq 0.1\): no significant shift
- \(0.1 \leq SI \leq 0.25\): minor shift
- SI > 0.25: major shift

### 3.4.5 Test of independence

The chi-square test of independence is a widely used non-parametric test. The statistic works very similar to the stability index, in the manner it that it checks for shifts in data. It compares two categories, the realized value of the model set and backtest set, by the expected (calculated) values. Both the contract distribution, as well as the default frequencies are tested with the chi-square under the assumption that the rating and default frequencies are independent – the more the values between observed and expected differ, the more likely the assumption of independence is invalid.

The theoretical expected value under the independence assumption can be derived from the number of defaults per rating, \(N\), and the total number of observations, \(OD\). The theoretical expected value is given by [10]: (Cooper & Schindler, 2008)

\[
TD_{ij} = N \times \frac{\sum_{k=1}^{C} OD_{ik}}{N} \times \frac{\sum_{l=1}^{R} OD_{kl}}{N}
\]

Simplified:

\[
TD_{ij} = \frac{\sum_{k=1}^{C} OD_{ik} \sum_{l=1}^{R} OD_{kl}}{N}
\]

The theoretical number \(TD_{ij}\) is the total number of observed defaults multiplied by the chance the default appears in rating \(i\), multiplied by the chance the default appears in specific set. Hence, if there \(r\) rows (ratings), \(c\) columns (model and backtest set) and total number \(N\) (which is the sum of the whole matrix). The degrees of freedom are given by: \((r - 1) \times (c - 1)\). The following hypothesis is tested [12]:

Null-hypothesis: There is no association between the two variables

Alternative hypothesis: There is an association between the two variables
The \( \chi^2 \) gives an indication to which the extent the observed and expected frequency deviate from each other. When the chi square is smaller than \( \alpha \), the test is rejected – stating that the relation is significant. (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007)

The advantage of this test is that the value of the statistic is clearly statistically defined, because critical values are determined by the chi-square distribution. However, the chi-square could depend on the number of buckets used – leading to a different conclusion altogether. (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007, p. 4)

In essence, the chi-square test is very strict in its acceptance when it comes to very large groups, whereas the SI is based on percentages, and is very loose in its acceptance.
3.5 Conclusion performance indicators

In this chapter, one research question has been dealt with: explaining the performance indicators which are used in the backtest of the PD model.

Q3. Which criteria can be defined for the performance indicators?

The performance indicators are the criteria which assess the different aspects of the PD model. The tests take a view on the following aspects of the PD model: prediction, discriminatory power and data stability. When the model is lacking in precision and other aspects it gives reason to remodel the PD, it is however important, that all that the statistics are interpreted by human judgement, as the automatic results are not absolute.

An overview of the performance indicators, used in backtesting the PD model, is given in table 2.

<table>
<thead>
<tr>
<th>Tested element</th>
<th>Performance indicator</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predictive</td>
<td>Binomial test</td>
<td>$Z = \frac{E(D) - DF_{obs}}{\sqrt{Var(D)}}$</td>
</tr>
<tr>
<td></td>
<td>Hosmer-Lemeshow test</td>
<td>$H = \sum_{i=1}^{k} \frac{(n_i p_i - d_i)^2}{n_i p_i (1 - p_i)}$</td>
</tr>
<tr>
<td>Discriminatory power</td>
<td>Cumulative Accuracy Profile (CAP) – Gini-coefficient</td>
<td>$AR = 2 \int_0^1 CAP(u) du - 1$</td>
</tr>
<tr>
<td></td>
<td>Receiver Operating Characteristic (ROC) – COC, KS</td>
<td>$AR = 2 \times (COC - 0.5)$</td>
</tr>
<tr>
<td>Data stability</td>
<td>Stability Index (SI)</td>
<td>$SI = \sum_{i=1}^{m} (D_i - B_i) \times ln \left( D_i B_i \right)$</td>
</tr>
<tr>
<td></td>
<td>Chi-square (test of independence)</td>
<td>$\chi^2 = \sum_{i=1}^{m} \frac{(OD_i - TD_i)^2}{TD}$</td>
</tr>
</tbody>
</table>

Table 4 – Overview of performance indicators
Chapter 4. PD model backtest results

In this chapter the results of the backtest are presented, based on the tests explained in chapter three. The analysis is done separately for the FL and HK portfolio. First the precision is tested (4.1 & 4.4), followed by discriminatory tests (4.2 & 4.5), and concluded with the potential causes of change, by testing data instability (4.3 & 4.6). Finally, a summary of the results will be given (4.7).

<< Chapter confidential >>
Chapter 5. PD model calibration and standardization

In this chapter the PD model is calibrated for the FL and HK portfolio. First, an overview will be given about calibration and logistic regression (5.1). Second, the FL model, as well the HK model, needs calibration (5.2 & 5.3). Third, an outline of the performed standardization (5.4). Fourth, the used data files will be shown (5.5), and the conclusion on the research question will be given (5.6).

<< Chapter confidential >>
Chapter 6. Conclusions and recommendations

*In this chapter the conclusions (6.1) and recommendations (6.2) are presented.*

<< Chapter confidential >>
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Appendix A : Definitions and default reasons

Definitions

<table>
<thead>
<tr>
<th>Definition</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Validating</td>
<td>Validation is the combined term for both quantitative and qualitative validation. It encompasses a much broader spectrum of testing than just backtesting, as the process itself is reviewed</td>
</tr>
<tr>
<td>Backtesting</td>
<td>Evaluating process of a model applied on historic data. It involves comparing the model output with an independent set of data to determine whether the model accurately estimates the realized defaults</td>
</tr>
<tr>
<td>Calibrating</td>
<td>Changing values of a models input parameters in an attempt to improve it</td>
</tr>
<tr>
<td>Standardizing the procedure</td>
<td>Making easy-to-use guidelines and calculation steps for backtesting</td>
</tr>
<tr>
<td>Default</td>
<td>Failure to make debt required payments or to comply with other conditions of an obligation</td>
</tr>
<tr>
<td>Corporate exposure</td>
<td>An amount outstanding to businesses subject to risk</td>
</tr>
<tr>
<td>Retail exposure</td>
<td>An amount outstanding to private clients or SME’s subject to risk</td>
</tr>
<tr>
<td>External ratings</td>
<td>Ratings from external credit rating agencies such as Moody’s.</td>
</tr>
<tr>
<td>Internal ratings</td>
<td>Ratings based from internal data?</td>
</tr>
<tr>
<td>Risk driver</td>
<td>Independent variables which are dependent in the modelling of the risk parameter. For example, the variable ‘Married’ is a dependent variable in the PD modelling.</td>
</tr>
<tr>
<td>Performance indicators</td>
<td>Processes and activities used in backtesting</td>
</tr>
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</table>

Default reasons (SSC Risk Management, 2008)

<table>
<thead>
<tr>
<th>Reason</th>
<th>Description</th>
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<tbody>
<tr>
<td>BSL</td>
<td>Notification of confiscation by tax authorities</td>
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<tr>
<td>BTR</td>
<td>Individual payment deadline granted</td>
</tr>
<tr>
<td>COV</td>
<td>Obligor untraceable, without known place of housing</td>
</tr>
<tr>
<td>FAI</td>
<td>Bankruptcy of the obligor</td>
</tr>
<tr>
<td>ING</td>
<td>Repossession of collateral</td>
</tr>
<tr>
<td>OPZ</td>
<td>Cancellation by VW Bank</td>
</tr>
<tr>
<td>OVC</td>
<td>Deace of the contractor</td>
</tr>
<tr>
<td>TLS</td>
<td>Loss of an object, actual loss</td>
</tr>
<tr>
<td>VBL</td>
<td>Migration of contractor to a foreign country</td>
</tr>
<tr>
<td>VIG</td>
<td>Committed forgery (obligor gave false information)</td>
</tr>
<tr>
<td>VRL</td>
<td>Loss of an object, embezzlement</td>
</tr>
<tr>
<td>WSP</td>
<td>Obligor goes into debt restructuring</td>
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## Appendix B: Planning

### Planning bachelor thesis - April 26th, 2010

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tr>
<td>Month</td>
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<td>Questions (Q), Theories (T) &amp; Activities (A)</td>
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<tr>
<td>Q1. What is a PD model according to literature?</td>
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<td>T1: Business PD model development data</td>
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<td>T2: PD model literature</td>
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<td>A1: Data collection &amp; Preparation</td>
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<td>Q2. How can a PD model be backtested?</td>
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<td>T3: PD model backtest internal documents</td>
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<td>A2: Defaults overview</td>
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<td>A3: Univariate Analysis</td>
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<td>Q3. Which criteria for the performance indicators?</td>
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<td>T4: Literature concerning testing performance indicators</td>
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<td>A4: Data stability tests</td>
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<td>A5: Discriminatory power (GINI, KS, COC)</td>
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<td>Q4. What are the conclusions of the backtest?</td>
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<td>A8: Back test calculations</td>
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<td>A9: Conclusions back test</td>
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<td>Q5. How should the PD model be calibrated?</td>
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<td>T5: On what basis can the model be calibrated?</td>
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<td>A10: Weights adjustment</td>
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<td>Q6. How can the process of backtesting be standardized?</td>
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<td>A11: Procedure method</td>
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### Milestones and Deadlines

1. 30th of April - Introduction completed (M1)
2. 21st of May - Back test report finished (M2-H4)
3. 4th of June - Procedure completed (M5-H6)
4. 18th of June - Bachelor report finished (M1-H7)
5. 25th of June - Bachelor colloquium (M1-H7)

**Conclusions**

Complete report

Bachelor colloquium
Appendix C: Total defaults
<< Confidential >>
Appendix D: PD formula of 2009

The estimated probability of default is calculated by entering the sum of PD in the PD formula [2].

\[
PD = \frac{1}{1 + e^{-\left(\alpha + \gamma + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots\right)}}
\]

<< Table confidential >>
Appendix E : FL PD estimates
In the figure below an overview of the masterclasses is given.

<< Table confidential >>
Appendix F : FL Data stability

A graphical representation of the different variables used in the FL modelling is given below.

<< Figures confidential >>
Appendix G : HK PD estimates

In the figure below an overview of the masterclasses is given.

<< Figure confidential >>
Appendix H : HK Data stability
A graphical representation of the different variables used in HK modelling is given below.

<< Figures confidential >>
Appendix I: Correlation matrices

The matrices correspond to the correlation of the variables for the chosen model set, calculated by SPSS with logistic regression.

Financial lease, three-year weighted model:

<< Table confidential >>

Hire purchase, three-year weighted model:

<< Table confidential >>
Appendix J : New FL PD model backtest

<< Appendix confidential >>
Appendix K: New HK PD model backtest
<< Appendix confidential >>
Appendix L: Files overview

<< Appendix confidential >>