MODELING LAPSE RATES

Investigating the Variables that Drive Lapse Rates

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Zeist, the Netherlands

Master Thesis

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Abstract
In the life insurance industry individually closed contracts are accompanied by risks. This report focuses on one of these risks, more specifically, the risk involving the termination of policies by the policyholders or, as it is called, “lapse” risk

The possibility of a lapse can influence the prices of contracts, necessary liquidity of an insurer and the regulatory capital which should be preserved. The possibility of a lapse is reckoned to account for up to 50% of the contract’s fair value and one of the largest components of the regulatory capital. For these reasons it is of great importance to prognosticate lapse rates accurately. These were the main reasons for conducting this research on behalf of Achmea and for investigating models and explanatory variables. The research question which functioned as the guide line for this research at Achmea is the following:
Can the current calculation model for lapse rates be improved1, while staying in compliance with the Solvency II directive, and which variables have a significant2 relation to the lapse rates?

The model applied and the explanatory variables analyzed are the result of a literature study. This study provided the Generalized Linear Model [GLM] to be a suitable choice and led to a list of 38 possible explanatory variables of which 9 were tested3. The GLM was applied to the data of CBA and FBTO corresponding to the years 1996 to 2010 and aggregated per product group. The seven product groups that were analyzed were: mortgages, risk, savings (regular premium), savings (Single premium), unit-linked (Regular premium), unit-linked (Single premium) and whole life & funeral. The aggregation of the data has been done using Data Conversion System and Glean, two products of Sungard, and the data were analyzed using SPSS 17, a product of IBM.

The research provided seven models, one for each product group, including variables as “buyer confidence”, “first difference in lapse rates”, “gross domestic product [GDP]”, “inflation”, “reference market rate” and “return on stock market”. Every model provided more accurate predictions than the application of the mean of the data would. It should be noted

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1 The performance of the model has been measured in terms of accuracy, on which it has also been compared.
2 The significance of the variables has been tested by statistical measures using a 5% level of significance.
3 Lagged values of these variables have been included as well, which led to a total of 14 analyzed variables.
that, due to lack of data, this comparison has been done on the training set. The performance of the models, when compared with the model provided by regulatory bodies (standard formula), is dependent on the level of expected lapse rates as well as the relative error of the predicted values. The level of the expected lapse rates greatly influences the standard formula, whereas the relative error of the predicted values is one of the great contributors to the prediction interval of the developed model.

Additional research showed that the choice for division of the data into several product groups is supported by the huge diversity in lapse rate developments amongst the product groups. Further analysis of the lapse rates with respect to the duration of policies also provided a reason for further research. The analysis indicated that the effect of macro-economic variables on lapse rates is dependent on its duration, indicating that the data per product group can be subdivided or duration can be used as explanatory variable.

Based on the research results it is recommended to analyze the possibility of generalizing the results by extending the research to other parts of Achmea. Next to that, it is recommended to investigate the data on a policy level in order to assess the significance of other variables. These additional researches will also increase the statistical strength and accuracy of the inferences that can be made. It is also recommended to clarify the importance of (accurate recording of) lapse rates and to denote a universal definition of a lapse, all to make sure that the lapse data become unpolluted. Finally, it is advised to monitor the models and to examine their performance and sensitivity to new data.
Preface

Six years ago I commenced studying mechanical engineering, a bachelor study, at Saxion University of Applied Sciences. After graduation I chose to enroll for the master’s study Financial Engineering and Management at the University of Twente to learn to apply my mathematical skills to problems which are more financial by nature. During this master I studied several courses which were (partly) focused on the insurance industry. These courses raised my curiosity for the actual insurance industry and led to the application at Group Risk Management of Achmea.

During the internship at Achmea I received help from several colleagues. Of these I would like to thank L. Menger for interesting me in the research topic, providing much relevant information and his guidance during the first weeks. For subsequent guidance and the final review of this report I would like to thank P. J. Meister and T. Delen. Naturally I am thankful for the opportunity to graduate at Achmea for which I have to thank M.R. Sandford. I would like to thank all other colleagues at Group Risk Management and Actuarial Reporting who provided time and information, of which special thanks goes out to R.A. Schrakamp for providing the research data.

From the University of Twente I would like to thank E.A. van Doorn for being prepared to act as my supervisor throughout this period and for his remarks and suggestions which have helped me to improve this thesis. Also from the University Of Twente is B. Roorda who I would like to thank for being prepared to act as additional/second supervisor and for his answers to difficult questions throughout my master’s program.

Finally, I would like to thank S.A. Leferink and Y.F.M. Michorius for their support and all their additional comments without which this thesis could not have been finalized.
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List of Important Definitions

The following definitions are those of terms which are used throughout this document.

An **internal model** is a model developed by an insurer to (partly) replace the Solvency II standard model.

A **lapse (event)** is the termination of coverage by the policy owner/insured.

Note: In this study a “lapse event” is said to occur if a personal contract is fully terminated by the policy holder and non-revivable, regardless of the refund, administrated at a divisional level.

The **lapse rate** of a particular product group in a particular time period is the fraction of lapses of the product group in that time period.

**Lapse risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level or volatility of the rates of policy lapses, terminations, renewals and surrenders.

The **Minimum Capital Requirement (MCR)** “is the minimum level of security below which the amount of financial resources should not fall. When the amount of eligible basic own funds falls below the Minimum Capital Requirement, the authorisation of insurance and reinsurance undertakings should be withdrawn.”(Solvency II Association, n.d.a)

The **outcome/response variable** is a dependent variable whose value is typically determined by the result of a set of independent and random variables.

The **predictor/explanatory variable** is an “independent” variable which is used to explain or predict the value of another variable.

The **risk-free rate (of return)** is the best rate that does not involve taking a risk. Both the return of the original capital and the payment of interest are completely certain. The risk free rate for a given period is taken to be the return on government bonds over the period. This is because a government cannot run out of its own currency, as it is able to create more than necessary.
Solvency 2 “is a fundamental review of the capital adequacy regime for the European insurance industry. It aims to establish a revised set of EU-wide capital requirements and risk management standards that will replace the current solvency requirements.” (Solvency II, Financial Services Authority).

The Solvency Capital Requirement (SCR) “should reflect a level of eligible own funds that enables insurance and reinsurance undertakings to absorb significant losses and that gives reasonable assurance to policyholders and beneficiaries that payments will be made as they fall due” (Solvency II Association, n.d.b) to ensure that an (re)insurance company will be able to meet its obligations over the next 12 months with a probability of at least 99.5%.

The standard model is the model prescribed by the Solvency II directive.

A surrender is a terminated policy, like a lapse, but when there is still a cash refund.

A termination is the cancellation of a life insurance policy by either the policyholder or the insurance company.
Chapter 1 Introduction

In the life insurance industry individually closed contracts are accompanied by risks. This report focuses on one of these risks, specifically the risk of termination of a policy by the policyholder.

Policyholders may exercise their right to terminate a contract; this event is called a lapse. One of the problems with policies that lapse (at an early stage) is that not enough premium payments have been made to cover the policy expenses. To diminish the negative effects of a lapse, the loss on lapsed contracts is included in the calculation of new policy prices. Consequently present and future policyholders will be held accountable for this risk by adjusting future premiums. According to Grosen & Jorgensen (2000) the option to lapse can, under certain conditions, account for up to 50% of the contract’s fair value.

The uncertainty that surrounds policy lapses is the source of yet other problems. This uncertainty or risk of loss due to the level and volatility of lapses is called lapse risk. To ensure an insurer’s continuity certain required buffers, or regulatory capital, are specified by regulatory bodies. This regulatory capital should be preserved in a manner which is considered to be risk free. Figures from Achmea (2010b) indicate that the increase in lapse risk during 2010 accounted for the largest increase in regulatory capital. In order to mitigate the negative effects of policy lapses it is important for an insurance company to develop reliable models for predicting lapse rates. The recorded study on lapse rates goes back to the beginning of the 20th century, where Papps (1919) tried to forecast lapse rates using an analytical formula. Soon afterwards theories were developed on the influences of variables on future lapse rates. Well-known hypotheses are the interest rate hypothesis and the emergency fund hypothesis. The interest rate hypothesis suspects interest rate to be an explanatory variable of lapse rates. It bases that suspicion on the thought that a change in relative profitability of alternative investments might arise from interest rate fluctuation. This hypothesis presumes that the market interest rate is seen as opportunity cost (Kuo, Tsai & Chen, 2003). Looking at insurances from a different angle the emergency fund hypothesis suggests that an insurance is seen “as an emergency fund to be drawn upon in times of personal financial crisis” (Outreville, 1990, p.249). Although many of the past studies focused on these hypotheses, recent research shifted to more complex predictors of lapse rates. The huge diversity in published researches demonstrates that characteristics or behavior of the
policyholder, insurance company and macro-economic environment can all experience a significant association with lapse rates.

Associated with the research on explanatory variables is the research on predictive modeling, in which explanatory variables can be included. Outreville (1990) published an article on predicting lapse rates by use of explanatory variables which stirred up the interest in and modeling of lapse rates. The number of publications on predictive models increased rapidly since then, covering a vast amount of models ranging from single- to multi-factor models. Even though many years of research have been conducted since then, no consensus has been established on the specific model which should be used. Publishing authors concur that it is due to the large variety of conditions per study that the insurance industry lacks a universally applicable model and universally significant drivers. Studies on models which are done by Kim (2005 and 2009) show that the choice for a model is, both, case as well as requirement dependent.

For Achmea it is necessary to determine a predicted value of the lapse rates for calculations, such as the pricing of insurances, and forecasting of cash flows. The goal of this research is to find those variables which are seen as significant drivers of lapse rates and to determine which model is most fit for forecasting with those variables. The model is subjected to requirements set by regulatory bodies as well as company specific requirements. The regulatory body, in this case De Nederlandsche Bank [DNB], employs the Solvency II which is developed by the European Insurance and Occupational Pensions Authority [EIOPA4] to improve the supervision of European insurers.

Summarizing, this research should comply with the requirements/legislation and has been conducted to provide an answer to the question:

Can the current calculation model for lapse rates be improved5, while staying in compliance with the Solvency II directive, and in particular, which variables have a significant 6 relation to the lapse rates?

---

4 EIOPA is one of three European Supervisory Authorities and is an independent advisory body to the European Parliament and the Council of the European Union.

5 The performance of the model will be measured in terms of accuracy, on which it will also be compared.

6 The significance of the variables will be tested by a well-chosen statistical measure and should render a not yet specified level of significance.
To answer this question a literature study has been conducted on Solvency II, lapse risk and lapse rates. Subsequent study has been done to provide a set of variables which were expected to have significant influence on the lapse rate. A similar comparison study has been performed for predictive models. Finally, using selection criteria and statistical methods, a model has been formed. The performance of the model is analyzed by comparing the model with the current lapse rate prediction method and a prediction method which is in development.

This thesis is organized as follows.

- Chapter two, lapse rates, starts with the rise of the life- and pension business as an introduction and provides an overview of the regulations and associated risk categories. Subsequently, the relationship between the regulations, risks and this research will be provided, which is done by illustrating the importance of lapse rates.
- Chapter three, explanatory variables, provides an overview of various insights and expectations on lapse rates which have been tested in literature. The chapter ends with a table in which all variables which have proven to experience or are expected to experience a significant relationship with lapse rates are listed.
- Chapter four, predictive models, provides an overview of the various types of models which are widely used in the life insurance industry. Model requirements as well as the results of researches which have been conducted internally are presented in this chapter. The chapter concludes with different recommended approaches dependent on the type of data which will be analyzed.
- Chapter five, methods, briefly covers the scope of the research, used tools, analysis procedure and (possible) limitations.
- Chapter six, data and analysis, provides information on the sources which constitute the used data set. The chapter continuous with a description of the data analysis and the obtained results for all product groups and of additional research.
- Chapter seven, conclusion and recommendation, provides some concluding remarks and recommendations for improvements.
- At the end of this thesis, after chapter seven, the list of literature sources and appendices can be found.
Chapter 2  Lapse Rates

In this chapter the concept lapse rate is elaborated and consequently the influence of a single lapse will be elaborated as well. To understand the influence that lapse rate have on the insurer this chapter provides a top-down elaboration of the risks accompanying an insurance agency. After it is indicated which risk categories are existent and into which category the lapse rates belong. Subsequently the operational definition of a “lapse event” is stated and the relations between lapses, regulations and costs are mentioned, which led to the research question.

2.1  The Rise of the Life- and Pensions Industry

In literature many stories are told about the rise of insurance industries all around the globe. Regardless of the origin of the insurance scheme, which could be in India (Smith, 2004) or in Rome (Robinson, 2009), they all served a similar purpose, namely; to hedge against the risk of a contingent, uncertain loss at the expense of a certain but relatively small loss in the form of a payment.

Through history many groups came up with a scheme to minimize exposure to a specific risk, like the loss of cargo at sea, by forming a “club”. Members of such a club would pay a premium at a specified frequency, of which the amount depended on factors as the coverage length and the desired type of insurance. One of the more popular insurance clubs was the burial club (Davis, 2006). Membership of a burial club ensured that the funeral expenses of the person insured would be covered. Even though these clubs existed for centuries it took the insurance industry a long time before the first legal privately owned insurer was founded.

Based on similar thoughts as the stories of the ancient sailors and burial clubs Achmea’s story started. It was in 1811 that a group of 39 Frisian farmers formed an insurance company called Achlum, to insure their possessions against fire (Achmea, 2010a). This group of farmers grew and merged with many other groups which all had their own founding story. The merger of many of such companies led to the present conglomerate which provides insurances and pensions as well as banking services. At the moment Achmea is the largest life insurer in the Netherlands and has based its operations within as well as across the Dutch border. Achmea employs around 22,000 employees and has a total of approximately 5 million policies
outstanding (Nederlandse Zorgautoriteit, 2011). The vast amount of policies in the product mix of Achmea may range from those that guarantee death benefits to retirement plans.

2.2 The Risks Which Are Associated with the Life- and Pensions Industry

The predecessors of present day insurers soon discovered that their simple insurance schemes exposed them to many risks. Some of the people that recognized the flaws in insurances would try to exploit them. Low to no entrance requirements was one of those flaws and led to adverse selection (Lester, 1866). People, aware of a child’s bad condition and low life expectancy, would enroll the child in more than one club knowing that only a few premium payments would provide a large death benefit. Such flaws in insurances led to the bankruptcy of many insurers. As time went by, most of these risks have been dealt with due to the evolution of the insurance industry and its products. The present life- and pensions business recognizes the following risk categories⁷:

- **Life underwriting risk**, also referred to as technical insurance risk, is the risk of a change in (shareholders’) value due to a deviation of the actual claims payments from the expected amount of claims payments (including expenses). Note: Life underwriting risk, like all the other risks, can be sub-divided into many risk types. One of the seven risks into which life underwriting risk can be divided is lapse risk. The seven underlying risks and especially lapse risk will be elaborated later on.

- **The financial risks**: Premiums that are paid by the insured are invested in financial assets backing the insurance liabilities. Just as underwriting risk, financial risks are very material for insurance companies. Financial risks can be categorized into the following risks:
  
  **Market risk** is the risk of changes in values caused by market prices or volatilities of market prices differing from their expected values.

  **Credit risk** is the risk of a change in value due to actual credit losses deviating from expected credit losses due to the failure to meet contractual debt obligations.

  **Liquidity risk** is the risk stemming from the lack of marketability of an investment that cannot be bought or sold quickly enough to prevent or minimize a loss.

⁷ All definitions are derived from the Solvency II glossary (Comité Européen des Assurances, 2007).
• **Operational risk** is the risk of a change in value caused by the fact that actual losses, incurred for inadequate or failed internal processes, people and systems, or from external events (including legal risk), differ from the expected losses.

A graphical representation of these risks is presented in appendix 1 “Risk map”.

### 2.3 The Intention of Solvency I and II

Solvency I is the first legislation for (re)insurance companies in the European Union that addressed rules on solvability requirements. It was introduced in the early 1970’s by the European Commission (Cox & Lin, 2006). The Solvency regulation compelled the (re)insurers within their jurisdiction to hold an amount of capital as a reserve in case an extreme event should occur.

A lot has changed since 1970. Risks have altered and become more diverse, the products have become more complex and more and more businesses have expanded their activities across their national border. This increase in complexity of the insurance industry is the reason why Solvency II has been developed.

Where Solvency I concentrated on insurance liabilities and sum at risk, Solvency II includes investment, financing and operational risk for the calculation of the capital requirement. This expansion provides a more risk-based measure. Apart from a better quantification of the risks, the regulation provides guidelines for insurers and their supervisors and disclosure and transparency requirements. All these adjustments have been made to achieve higher financial stability of the insurance sector and to regain customer confidence (European Central Bank, 2007).
### 2.4 Solvency II

The set-up of Solvency II was borrowed from Basel II\(^8\) and consists of three mutually connected pillars (International Centre for Financial Regulation, 2009):

- Quantitative requirements (Pillar 1)
- Qualitative requirements (Pillar 2)
- Reporting & disclosure (Pillar 3)

As part of the first pillar there are two Solvency requirements, the Minimum Capital Requirement (MCR) and the Solvency Capital Requirement (SCR). These measures are used to examine the level of the available capital\(^9\). If the available capital lies between both measures, capital reserves are said to be insufficient and supervisory action is triggered. In the worst case scenario, in which even the MCR level is breached, the supervisory authority can invoke severe measures; it can even prohibit the insurer to conduct any new business.

The value of the MCR and SCR can be calculated by prescribed formulas and models or (complemented) by a model developed by the insurance company itself; an internal model. It is up to the insurer to decide whether internal models are used. However, all internal models do need to be endorsed by the supervisory authority, which is the DNB.

Achmea, on the authority of whom this research has been conducted, has chosen to develop a partial internal model, which is a combination of prescribed and internal models. (Eureko Solvency II Project, 2009). The chosen internal models should account for most of the previously mentioned risk categories and, as mentioned, are only valid after they are approved by the DNB.

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\(^8\) Basel II is the second of the Basel Accords, which are issued by the Basel Committee on Bank Supervision. The purpose of Basel II was to create standards and regulations on how much capital financial institutions must put aside.

\(^9\) The available capital is closely related to the shareholders’ equity at the statutory balance sheet. The shareholders’ equity is adjusted with revaluations of assets and liabilities to obtain an economic (or market consistent) shareholders’ value.
2.5 Lapse Risk

Life underwriting risk, as previously mentioned, can be sub-divided into seven risks, which are\(^{10}\):

- **Mortality risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where an increase in the mortality rate leads to an increase in the value of insurance liabilities.

- **Longevity risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities.

- **Disability- and morbidity risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend or volatility of disability, sickness and morbidity rates.

- **Life expense risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of the expenses incurred in servicing insurance or reinsurance contracts.

- **Revision risk** is the risk of loss, or of adverse change in the value of insurance liabilities resulting from fluctuations in the level, trend, or volatility of the revision rates applied to annuities, due to changes in the legal environment or in the state of health of the person insured;

- **Life catastrophe risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from the significant uncertainty of pricing and provisioning assumptions related to extreme or irregular events.

  Note: In real life, catastrophes will have a direct effect on the profit, since settlements will be paid immediately.

- **Lapse risk** is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level or volatility of the rates of policy lapses, terminations, renewals and surrenders.

  Note: These types of cancellations together encompass policies cancelled or renewed by policyholders or insurers regardless of the surrender value.

\(^{10}\) All definitions are provided by the Committee of European insurance and Occupational Pensions Supervisors[CEIOPS](2009).
Lapse risk is the risk on which this research is focused; to be specific it is on the underlying cancellations which, together, are called lapses. This will be elaborated in the next section.

2.6 Lapse Events

A lapse event is the termination of a policy by the policyholder. For (scientific) analysis it becomes harder to be specific as the focus becomes narrower. In literature there seems to be no consensus about an operational definition of a “lapse event”, what a lapse is and consequently what is treated as a lapse in lapse analyses. Generally speaking, lapsing is recognized as the voluntary termination of a contract by the policyholder. But there are subtle differences between the used definitions.

Distinction can be made between

- Fully and partially terminated contracts;
- A cash refund(surrender)* and no refund after termination\(^\text{11}\); and
- Revivable and non-revivable contracts.

There is also a disagreement in literature as well as in reality on converted contracts. The debate discusses whether contracts which are converted, and remain within the division or within the company, should be counted as a lapse. Dependent on the level of analysis a conversion, transfer from one product to another, might be registered as the loss of a client.

The definition of a lapse will determine the magnitude as well as the development of the lapse pattern. The broader definitions will include more types of policy terminations and every type may show a different development of its total policy terminations.

Operational definition

In this study a “lapse event” is said to occur if a personal contract is fully terminated by the policy holder and is non-revivable. All contracts which satisfy these conditions are examined, regardless of the refund, and the lapse data is administrated at a divisional level, which means that the data is aggregated\(^\text{12}\).

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* A surrender is a terminated policy, like a lapse, but when there is still a cash refund. In which a cash refund refers to a predetermined amount of money which is refunded whenever the contract passes away.

11 See Kiesenbauer(2011).

12 A similar definition has been used by Renshaw & Haberman (1986).
2.7 Lapse Events and Solvency II

As element of the SCR and MCR calculations lapse rates influence the capital requirement that will be set by the regulatory bodies. As part of the required capital calculation the choice between an external and internal model has to be made for lapse events as well. The choice for an internal model to prognosticate the lapse rates and to estimate its variance is stimulated by the size of lapse risk compared to the overall size of the SCR and MCR. According to CEIOPS (2008) “life underwriting risk constitutes the second largest component of the SCR, lapse risk makes up for approximately 60 percent of the life underwriting risk module before diversification effects.” To comprehend that statement it is necessary to list some of the consequences that come with the uncertainty surrounding the estimation of the lapse rates. The uncertainty in the estimation of the lapse rates has effect on many calculations, such as the calculation of the:

- SCR and MCR
  Higher/lower outcomes for these measures will increase/decrease the regulatory capital\(^{13}\). This capital should be held (partially) in forms that are considered as risk-free and cannot be invested otherwise. The costs involved with the regulatory capital are equal to the opportunity costs, which are equal to the benefits which could have been received by taking an alternative action.

- Price of insurances
  Lapse rates higher/lower than expected will increase/decrease (dependent on insurance characteristics) prognosticated cash flows. It is often the case that the most substantial administrative and acquisition costs are incurred at the beginning of a contract. Hence, early lapses may cause negative cash flows. To remain at a certain level of profitability this will be reflected by premium increases/decreases.

- Liquidity assessment (Kuo, Tsai & Chen, 2003)
  Liquidity of products is desired when it is uncertain whether the product will have to be traded; an unexpected lapse is such an uncertainty. When lapsing is possible, the hedging portfolio should be flexible to some extent. Liquidity of products comes at a price, which means that the costs of a hedging portfolio are dependent on the lapse rate. This will eventually translate into an (il)liquidity premium (part of the total premium) to ensure cost coverage.

\(^{13}\) Kannan, Sarma, Rao & Sarma (2008) state that the sign of the influence of lapses on the statutory reserves deviates per product group as well as over time.
• Profitability assessments (Loisel & Milhaud, 2010)
  For the assessment of an insurance’s or division’s profitability it is of importance to
  know whether future cash flows are congruent with the long-term expectation/plans\textsuperscript{14}.

The possibility of a lapse can be avoided in several ways. The most straightforward solution is
to ensure no lapse occurs by prohibiting it, which can be mentioned in the financial leaflet.
Due to regulatory constraints such a solution will lead to a patchwork of rules, which will not
be pleasant looking nor will it be clear/transparent. Another action, permitted when it is
mentioned in the financial leaflet, is the increase/decrease of premiums under specific
conditions (Achmea, 2007). Actions such as the formation of a watertight financial leaflet and
the execution of the consequential rights or the increase of premiums do not occur in practice,
unless extreme events occur. Such actions, even though they are legitimate, may damage the
goodwill of a company for they are experienced as unfair or vague.

Summarizing the list of consequences of and possible remedies for lapses, it seems best to try
to forecast lapses accurately and to limit the number of rigorous measures.

\textsuperscript{14} Note: For the comparison of profitability the definition of a lapse should be similar for the units under
comparison, which is not always the case, see section 2.6.
Chapter 3  Explanatory Variables
Throughout literature many types of variables have been used, ranging from dummy to continuous variables. Whereas some variables receive much empirical support, such as product group, others receive contradicting remarks, such as unemployment rate (Carson & Hoyt, 1992). Even more variables are suggested in articles as possibly relevant, but lack empirical evidence. In this chapter the most important variables will be presented and this chapter ends with a list of all possible explanatory variables.

3.1 Explanatory Variables in Literature
Explanatory (or predictor) variables are variables which are used to explain or predict changes in the values of another variable. The latter is called the dependent or response variable.

According to Cerchiara, Edwards & Gambini (2008) the explanatory variables can be subdivided into two classes indicating either rational or irrational behavior. Rational lapse behavior is represented by the likely change in lapse frequency due to an evolution in the financial markets. Irrational lapse behavior is represented by a change in lapse frequency due to other changes than those in the characteristics of the financial markets. Irrational behavior, lapse rate developments which are not due to evolutions in the financial market, encompasses the explanatory variables such as gender and for instance the policy or policy holder its age.

The most used driver in predictive modeling of lapse rates is the insurance type. Examples of insurance types are; mortgages, unit-linked products and whole life insurances. The combination of guarantees, cash flows and other contract specifics that form an insurance is often used as categorical variable. The argument is that the type of insurance may affect the lapse behavior of an individual. Even though the choice for such a variable as a predictor variable is customary, it remains a combination of variables and as such provides no clear insight into the real drivers of the lapse rates.

Next to universally accepted drivers there are also some hypotheses formed which do not always receive significant support. Within the scientific communities there are two well-known hypotheses on lapse rates in the life insurance industry. The first one is the emergency fund hypothesis (Outreville, 1990) and contends that the surrender value of an insurance contract can be seen as an emergency fund in times of personal distress (Milhaud, Loisel & Maume-Deschamps, 2010). Different indicators are used for personal distress, such as
MODELING LAPSE RATES

(transitory) income and unemployment. Dependent on the scope, these variables are denoted as policyholder characteristics or macro-economic characteristics, using gross domestic product (GDP) and national unemployment rate as approximations. Whereas some studies support this hypothesis, others only evidence a low long-term or no relationship at all between unemployment and lapses. The second hypothesis is the interest rate hypothesis and contends that, in the eyes of an investor, the opportunity costs rise when the market interest rate increases. The logic behind this reasoning is that a rise in interest rates will decrease the equilibrium premium, the premium which is seen as adequate under present interest rates, and consequently increase the likelihood that a similar contract can be obtained at lower costs (Milhaud et al., 2010). Although Kuo, Tsai & Chen (2003) state that the second hypothesis is favored over the first one, the Financial Services Authority [FSA] (2004 and 2010) supports both hypotheses by stating that both the unaffordability - according for 60% of all lapses- and relative product performance are the main drivers of lapses.

Next to the traditional hypotheses some new and less popular hypotheses have been developed. One of these is the rational policy holder hypothesis which is based on the thought that there is a reference market rate at which it is optimal to lapse a policy (Milevsky & Salisbury, 2002). The authors base their optimal moment of lapsation on the Black-Scholes formula. The hypothesis is quite similar to the interest rate hypothesis. The mayor difference is in the chosen representation of the response variable. The interest rate hypothesis’ outcome is continuous, which was the likelihood of lapse, while the rational policy holder hypothesis models lapse as being either optimal or not; making that response variable binary.

Whereas interest rate alone can be selected as explanatory variable it is often a combination of variables that is used for predicting lapse rates. Some recent studies achieved high predictive power by applying completely different sets of variables; Milhaud et al. (2010) achieved an accuracy of 90%, whereas Briere-Giroux, Huet, Spaul, Staudt & Weinsier (2010) indicate that their model achieved an even higher accuracy. In their studies the authors used variables such as gender, premium size, premium frequency, surrender rate and the value of the insurance. Note that some lagged/forwarded variables are used in the analysis as well and are denoted as
being separate variables\textsuperscript{15}. The inclusion of such lagged/forwarded variables is done since a certain reacting/anticipating behavior is expected which is proven to be present by Kiesenbauer (2011).

Of the variables used in articles there are many variables that are strongly correlated. A nice example is the correlation between the age of a contract, its maturity (remaining duration) and the age of the insured. At the start of a contract period a contract’s age and maturity are each other’s’ opposites and they move in a perfect negatively correlated manner. When the insurance’s maturity reaches zero, its age will approximate the value the duration had at the commencement of the contract. The correlation between the age of a contract and that of a person is weaker, but still evident. They both age as time passes, depicting a similar absolute development. The difference is that the relative lifespan development per time period will often be much higher for a contract than for the insured, “time elapsed”/”total life span”. Whenever such a perfect correlation is noticed one of the variables is excluded from the analysis. Apart from this example there are many variables with high correlation coefficients, which is why it is possible to find so many models with so many combinations different combinations of variables.

To complement the list of variables deducted from articles a few extra variables were added to the list which is presented in table 1. One of these added variables is postal code, which might be an approximation of the social status or more precise the level of income. To add some variables that function as indicators of economic turbulence; inflation, some first-order differences of macro-economic variables and dummy (or binary) variables have been added. To account for the possibility of a trend in the lapse rates the lagged lapse rate is also examined on its explanatory possibility. Slow reactions or anticipating actions to the changing environment are modeled by adding lagged or prognosticated values of the chosen variables to the model. The mortality rate has been included because of the possibility of auto-selection. Auto-selection concerns the selection of insurances, by a person, which seems most beneficial. For instance, the choice to close a death benefit insurance contract when a death is expected, as in the burial club example, is a form of auto-selection. A negative correlation is

\textsuperscript{15} Whether a variable is lagged or forwarded is indicated by parentheses next to the name of the variable. A negative value between those parentheses indicates a lagged variable whereas a positive value represents a forwarded variable.
expected to exist between auto-selection and lapses. When a contract seems advantageous, the contract is not expected to be lapsed. However, this does regard the personal life expectancy which is difficult to measure and even hard to approximate.

A remark should be made on the great differences between the inferences which are derived in articles with respect to the mentioned variables. The researches are conducted in multiple regions and although they might use similar variables, the values and characteristics of the variables can be extremely different. Reasons for this deviation are differences in policy holder behavior, tax regimes, currencies and even by the subtle differences in definitions of variables.

### 3.2 Possible Explanatory Variables

The main conclusion to be derived from literature is that there can be correlation between lapse rates and policy holder, micro-/macroeconomic or company specific characteristics but that the relationship is case dependent. Lapse behavior can no longer be explained using simple models for it is not only subjected to rational behavior, but also by irrational behavior.

The following table consists of all (hypothesized) explanatory variables which came to the surface during this literature research. The presented variables have been used in various combinations in other studies.

<table>
<thead>
<tr>
<th>Possible explanatory variables(^{16})</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macro-Economic variables</strong></td>
</tr>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>Buyer confidence</td>
</tr>
<tr>
<td>Inflation</td>
</tr>
<tr>
<td>House price developments</td>
</tr>
<tr>
<td>Economical growth</td>
</tr>
<tr>
<td>Return on stock market</td>
</tr>
<tr>
<td>Unemployment</td>
</tr>
<tr>
<td>Equity market volatility</td>
</tr>
<tr>
<td>Interest rate volatility</td>
</tr>
<tr>
<td>Exchange rates</td>
</tr>
<tr>
<td>Crises variable (Binary)*</td>
</tr>
<tr>
<td>Growth in GDP</td>
</tr>
<tr>
<td><strong>Contract specific variables</strong></td>
</tr>
<tr>
<td>Type of product</td>
</tr>
<tr>
<td>Age of the contract</td>
</tr>
<tr>
<td>Lifetime of the contract</td>
</tr>
<tr>
<td>Premium frequency</td>
</tr>
<tr>
<td>Premium size</td>
</tr>
<tr>
<td>Value of the insurance</td>
</tr>
<tr>
<td>Surrender charge</td>
</tr>
<tr>
<td>Reference market rate</td>
</tr>
<tr>
<td>Optimal moment of lapsation</td>
</tr>
<tr>
<td>Saving premium (investment made by policy holder)</td>
</tr>
</tbody>
</table>

\(^{16}\) Lagged and forwarded values of these variables have been included in the analysis as well. These will be represented by the name of the variable and a value between parentheses which represents the “lag”.
### Table 1 Possible explanatory variables

<table>
<thead>
<tr>
<th>Policy holder specific variables</th>
<th>Company specific variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of policy holder</td>
<td>Division/part of the company</td>
</tr>
<tr>
<td>Gender</td>
<td>Distribution channel</td>
</tr>
<tr>
<td>Widowed</td>
<td>Negative publicity*</td>
</tr>
<tr>
<td>Marital status</td>
<td>Crediting rate</td>
</tr>
<tr>
<td>Postal code*</td>
<td></td>
</tr>
<tr>
<td>New Legislation*</td>
<td></td>
</tr>
<tr>
<td>Mortality rate</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time variables</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seasonal effects*</td>
<td>Number of new ordinary life insurances</td>
</tr>
<tr>
<td>Calendar year of exposure</td>
<td>Number of ordinary life insurances in force</td>
</tr>
<tr>
<td></td>
<td>Transitory income * working age population</td>
</tr>
<tr>
<td></td>
<td>Price deflator for disposable personal income</td>
</tr>
<tr>
<td></td>
<td>Number of term insurances</td>
</tr>
<tr>
<td></td>
<td>Number of life insurances</td>
</tr>
</tbody>
</table>

* Indicates that the variable is not mentioned in articles but is expected to be relevant.
Chapter 4  Predictive Models

“Generally, predictive modeling can be thought of as the application of certain algorithms and statistical techniques to a data set to better understand the behavior of a target variable based on the co-relationships of several explanatory variables.”\(^{17}\) For a predictive model to be appropriate it needs to meet certain criteria. This section starts with an introduction to predictive models which results in the choice for a Generalized Linear Model [GLM] as modeling technique. Subsequently the different modeling criteria, ranging from those set by Achmea to those set by the regulatory bodies, will be illustrated and the choice for a specific GLM is explicated.

4.1  Predictive Models

Predictive models are favored relative to traditional models for they capture more risks and can account for inter-variable correlation (Briere-Giroux et al., 2010). Whereas some variables can be accurately predicted with a predictive model, others are more complex and cannot be predicted with high accuracy. Depending on the amount of underlying drivers and the assumed relationship between drivers and variables there are various models which can be opted for.

The most basic type of predictive models with explanatory variables are the one-factor models. These models suggest a significant relationship between a single variable and lapse rates. A common predictor is the reference market rate, which is the interest rate provided by a competitor (Briere-Giroux et al., 2010). This choice is justified by suggesting that the policy holder (constantly) compares similar products and chooses its policy based on its relative costs. Examples of such functions which are currently used by insurance companies are the\(^{18}\):

- Arctangent model: \( r = a + b \times \arctan(m \times \Delta - n) \)
- Parabolic model: \( r = a + b \times \text{sign}(\Delta) \times \Delta^2 \)
- Exponential model: \( r = a + b \times e^{(m \times \text{CR}/\text{MR})} \)

In which

- \( r \) is the monthly lapse rate
- \( a, b, m, n \) are coefficients
- \( \Delta \) is the reference market rate minus crediting rate\(^{19}\) minus surrender charges

\(^{17}\) See Briere-Giroux et al.(2010, p.1).

\(^{18}\) See Kim (2005).

\(^{19}\) The crediting rate is the interest rate on an investment type insurance policy.
CR is the crediting rate
MR is the reference market rate
Sign ( ) is +1 if ( ) is positive, -1 if ( ) is negative and 0 when ( ) is zero

Even more complex are the models which select their components and their coefficients based on certain defined criteria and for which high mathematics/statistics is used to determine their coefficients. In literature there are two such models which are said to be applicable in the insurance industry. One is the classification and regression tree [CART] and the other is the Generalized Liner Model.

CART-models produce either classification or regression trees, dependant on whether the dependent variable is categorical or numeric. They are non-parametric forecasting methods. The main procedure of the CART-model is the step by step division of lapse data into smaller groups, based on binary rules. At each step the algorithm selects the variable or combination of variables which provides the greatest purity of data in order to form homogeneous data sets. The algorithm stops with dividing the data set as soon as an (arbitrarily) chosen criterion has been reached. Examples of such criteria are: Equality of observations of the explanatory variables in a given class, a minimum number of observations per node or a specific potential of increase in data purity (Loisel & Maume-Deschamps, 2010). The advantages of this method are that the results are easily interpretable, because of the tree structure, and that it is nonparametric and nonlinear (StatSoft, n.d.). Another advantage is the fact that CART-models can include continuous as well as categorical predictor variables, which is extremely useful with variables such as gender, division and product type.

Main disadvantages are the complexity of the CART algorithms and the instability of the model, a small change in data may lead to huge change in the outcome20. Guszcza (2005) states that the model does a poor job at modeling linear structure, but can be used as a pre-test to analyze which variables or combination of variables might be explanatory. The CART-model is just one type of tree model and by far not the most complex. Because of the nonlinearity and its subdivision of data, the trees can provide pretty accurate results.

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The second model which is mentioned in literature for the modeling lapse rates is the generalized linear model. This model combines a number of linear variables into one regression model and uses a link-function, a function which transforms the data distribution of the linear variables, to predict an outcome variable, which is expected to have a distribution from the exponential family of distributions. GLMs are favored for they can provide accurate results when applied to lapse data, similar to the CART-models, while remaining interpretable (Briere-Giroux et al., 2010).

A GLM is suited to model many different types of functions, due to its link-function, and can predict (with) continuous as well as (with) binary variables. This characteristic is of great use in the insurance industry since the lapse variable can be either zero or one on a policy level and continuous between zero and one on an aggregated level. For these reasons the GLM theory is investigated more thoroughly.

In the following two sections the requirements for lapse rate models and Achmea’s lapse rate model will be discussed, before going into more detail on GLMs

4.2 The Requirements of Regulatory Bodies (the DNB) for the Lapse Rate Model

With the implementation of Solvency II an amount of flexibility is given to the implementers, enabling them to shape the regulatory capital calculation, partly, as they see fit. The main flexibility is in the presented choice to use either a standard or an internal model. Since the standard model is based on industry averages it might be beneficial to develop internal model(s) whenever the company’s risk is expected to be below industry’s average, considering that a decrease in modeled risk leads to a decrease in regulatory capital. The requirements of the DNB for the model can be summarized as follows. The models should…

“…be able to provide an appropriate calculation of the SCR (Article 100);

…be an integrated part of the undertaking’s risk management process and systems of governance (Articles 43,110.5 and 118); and

…satisfy the tests and requirements as set out in Articles 118-123.”

The test which is mentioned is the use test. That test will require firms “to demonstrate that the internal model is widely used in and plays an important role in their system of governance

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21 Loisel & Maume-Deschamps (2010) state that the GLM they analyzed provided more prudent results.

22 See FSA (2009a, p.15)
(art 41-49), and in particular, their risk management system, decision making processes and the ORSA (Own Risk & Solvency Assessment, ed.)” (Lloyd’s, n.d.) The other requirements are on statistical quality, calibration, profit & loss, validation and documentation standards.

Research presented by Deloitte (2010) states that it might be hard and too cost-/time-consuming to develop and implement a full model -a model composed out of multiple internal models for all risk categories. This is why the choice can be made to use the standard formula for one or more of the Solvency II’s risk categories. Such a combination is what is called a partial model. Achmea chose to develop a partial model with inclusion of an internal model for lapse rates.

4.3 Achmea’s Internal Model

The current lapse rate calculation

Currently there is no finished internal model which meets Achmea’s as well as the DNB’s requirements. This means that the prescribed standard lapse rate formula is adopted, for the time being, instead of an internal model. The calculation prescribed by the standard model for regulatory capital held for lapse events is as described below.

The standard formula

The capital requirement for lapse risk under the standard formula is the maximum loss resulting from one of three scenarios:\n
- A permanent increase of lapse rates by 50%.
- A permanent decrease of lapse rates by 50%.
- A mass lapse event where 30% of the policies are surrendered at once.

Per scenario the new lapse rate is used to calculate the loss corresponding to a loss event that, statistically speaking, occurs only once every two hundred years. The new lapse rate which is used and to which the shocks are applied is chosen by the insurer based on its own experience. The lapse rates which are expected, based on the insurer’s experience, need to be, amongst other things, plausible, testable and representative. The proposed lapse rate and the method for obtaining it are examined by the DNB.

\[^{23}\text{See CEIOPS (2009).}\]
Example
When the lapse rate is expected to remain steady at 2% for the coming five years, the lapse rates per scenario will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal situation</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>50% Increase</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>50% Decrease</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Mass lapse</td>
<td>30%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 2  The standard formula

While it is possible to compare the first two scenarios with the boundaries of a prediction interval [P.I.], the difference with the mass lapse event is dependent on product type and on the time period under consideration. The differences in total percentage of lapsed policies per scenario will decrease as the time horizon is further away.

For an internal model to be advantageous - in the regulatory capital sense - it should provide a, dependent on the cash flow characteristics, lower/higher lapse rate than is calculated using those three scenarios. Even when the model is not advantageous with respect to the regulatory capital it induces, it can provide meaningful insight into the lapse rate developments. To provide insight into lapse rate developments and in order to predict lapse rates as accurately as possible the choice has been made for a market based model.

The development of an internal model
Although not yet fully developed, Achmea has started already with the development of an internal model for lapse rates. The model will yield different results dependent on three predictor variables. The predictor variables are the policy’s product group (p), policy age (y) and the year of evaluation (t). The proposed internal model is represented by the following formula:\(^{24}\):

\[ l_p(y, t) = f_p(y) \times L_p(t) \]

In which

- \( l_p(y, t) \) is the stochastic variable representing the lapse rate for product group \( p \), policy age \( y \) and year of evaluation \( t \)
- \( L_p(t) \) is a stochastic variable representing the total lapse rate per product group

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\(^{24}\) This is a rewritten version of the formula Achmea (2010b) presented.


\[ f_p(y) \] is a deterministic scaling parameter for policy age \( y \) and product group \( p \)

The expected value of a stochastic variable, \( L_p(t) \), is estimated by means of autoregressive integrated moving average-models, ARIMA (0,0,0); ARIMA (0,1,0) and ARIMA (1,0,0), which are well-known time series models, the models correspond to, respectively; a constant with noise/trendless process, a random walk with drift and a first degree autoregressive process (Achmea, 2010b). The ARIMA-models have some limitations of which their focus on historical values is seen as the most disturbing for application in the life insurance industry. The scaling parameters might make up for some of its shortcomings, although the process and values of the scaling parameter remains unspecified. The effect of this focus on historic values makes the model sensitive to unique events.

The added value of the research that will be done in subsequent sections, to the standard and internal model, is the broad scope. In this research more variables are analyzed on their significance and distribution, which is expected to lead to a more accurate result.

### 4.4 Generalized Linear Models

Generalized linear models represent a class of regression models, which involves more than just linear regression models. One of the additional features is that the response variable can have a distribution other than Normal; it can be any distribution of the exponential family of distributions. Examples of probability distribution functions which belong to the exponential family of distributions are: the Gaussian, Bernoulli, Binomial, Gamma, Poisson and Weibull distribution. Another additional feature is that the relationship between the response variable and explanatory variables can be chosen to be non-linear\(^{25}\).

**The composition of a GLM**

A GLM consists of three components, which are (Anderson, 2010a):

1. A random component – This is the response variable (Y), it has to be identified and its probability distribution has to be specified/assumed. In this research Y denotes the lapse rate.

2. A systematic component – This is a set of explanatory variables (X) and corresponding coefficients set \( \beta \). A single observation is denoted as \( x_{jm} \) in which j

\(^{25}\) See Anderson (2010a) and Cerchiara et al. (2008).
indicates it is the $j^{th}$ observation and m indicates it regards the $m^{th}$ variable in the equation. The explanatory variables are entered in a linear manner, for example: $\beta_0 + x_{12}\beta_1 + x_{13}\beta_2 + \ldots + x_{1K}\beta_K$ in which $\beta_0$ represents a constant.

3. A link function – This is an important aspect of the model and is used to specify the relationship between the mean/expected value of the random component and the systematic component. The GLM is represented by the following equation:

$$g(Y) = X\beta + \epsilon,$$

which can rewritten as

$$Y = g^{-1}(X\beta + \epsilon)$$

In which

- $g(.)$ represents the link function
- $\epsilon$ represents the error term and is assumed to be $N(0, \sigma^2)$

The random component $Y$

Between the random components distinction is made based on the type of response variable. For this research the interest is in binary and continuous response variables, dependent on whether the data are aggregated or not.

The systematic component $X\beta$

In which

$$X = \begin{pmatrix} 1 & x_{12} & \ldots & x_{1K} \\ 1 & \vdots & & \vdots \\ 1 & x_{N2} & \ldots & x_{NK} \end{pmatrix}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_K \end{pmatrix}$$

$K$ is the number of explanatory variables and $N$ is the number of observations$^{26}$.

This component of the model is a linear function of a set of explanatory variables. While the function of variables is linear, the variables do not have to be so. A good example of the possibilities for the systematic component is the following function:

$$X_1\beta = \beta_0 + x_{12}^2 \cdot \beta_1 + \log(x_{13}) \cdot \beta_2 + x_{12}^2 \cdot \log(x_{13}) \cdot \beta_3$$

$^{26}$ See Verbeek (2001).
The link function $g(.)$

With the random component symbolizing the response variable and the systematic component symbolizing the explanatory variables, the only thing missing (apart from the error term) is the link between both sides; the relationship explaining how the explanatory variables relate to the response variable. That relationship is described by the link function. The relationship between the random component, systematic component and link-function is represented by the following equation:

$$g(Y_i) = \beta_0 + x_{i1}\beta_1 + x_{i2}\beta_2 + \ldots + x_{ik}\beta_k + \epsilon$$

Applying a GLM involves assuming certain conditions of the data. The congruence of the data with these assumptions should be checked before the model can be assessed. Assumptions are made with respect to, for instance, the distribution of the variables and on characteristics of the residuals.

Next to the assumptions that underlie GLM’s there are certain guidelines for applying a GLM. One guideline is to assume a linear relationship between the systematic component and the random component when there is no reason to assume another than linear link-function to be more appropriate. This is also the procedure which will be followed in this study. Nonetheless the next section will be on the most relevant link-functions for binary and continuous lapse data, to complete the theory on GLMs.

4.5 The Link Function

Generally there are two different methods used for modeling lapse rates. The difference in methods is in the link function and is mostly dependent on the aggregation level of the used data. In this section the link functions for lapse data on policy as well as aggregated level are presented.

Policy level data - Binary lapse variable

In many cases the lapse variable is recorded in such detail that its values are known per policy and are either zero or one, representing no lapse or a lapse. This binary outcome renders a couple of methods as inappropriate.

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27 See Wei (2002).
Researchers usually choose one of the following link-functions when dealing with binary data:

- **Logit function**, \( g(Y) = \ln\left(\frac{Y}{1-Y}\right) \)
- **Probit model**, \( g(Y) = \Phi^{-1}(Y) \)
  
  In which \( \Phi^{-1} \) is the inverse cumulative distribution function associated with the standard Normal distribution.

- **Log-linear/Poisson** \( g(Y) = \log(Y) \)

Application of one of these functions will provide an estimate which can be different from zero or one. This is why a threshold is chosen and it is examined whether the prediction is below or above that threshold which will determine whether the predicted value is rounded off to zero or one.

The mentioned Poisson model assumes, amongst other things, that the changes in rate of predictor variables have a multiplicative effect and that the variance is equal to the mean. The Poisson model is regarded to be a suited choice for predicting extreme events. The model is applied when the lapse rates are expected to be close to zero and when the results of the model are used qualitative rather than quantitative (Cerchiara et al., 2008).

As shown by Anderson (2010b) there is little difference in results between the logit- and probit-function, with the logit-function providing slightly more accurate results. Kim (2005) also shows that the logit-function is generally better than existing surrender models which only use a single parameter for lapse rate estimation. Congruent to those findings the logit-function appears to be the most favored method for the modeling of lapse rates. Guszcza (2005) compared some tree-models with logistic regression and concluded that logistic regression outperformed the relative simple CART-model but was inferior to the more sophisticated MARS-model. The low complexity and consequently high understandability of logistic regression are additional reasons for the logit-function to be a suited predictor.

One of the advantages of the logit-function is that it can be used to form the odds-ratio (Anderson, 2010b). The odds-ratio indicates how much an option is more likely than another option. This scaling factor increases the interpretability and applicability of the model.
Aggregated data - Continuous lapse variable

Not many articles on lapse rates in the insurance industry publish which link-function they use for analyzing a continuous lapse variable.

Common link-functions are\textsuperscript{28}:
\begin{itemize}
  \item Linear regression $g(Y) = Y$
  \item Logistic regression $g(Y) = \log (Y)$
    This function is the same as the one for binary data.
  \item Complementary log-log regression $g(Y) = \ln (-\ln (1 - Y))$
\end{itemize}

The difference with binary data is that the predicted values are not rounded off to zero or one. Which of the link-functions is most suited can be analyzed using either a graph, for instance a scatterplot, or a statistical measure, for instance Pearson’s correlation coefficient.

When the choice has been made to develop a GLM and it is hard to identify the data distribution it is practice to start without a link function\textsuperscript{29}. This specific GLM is, as is demonstrated above, known as the linear regression model.

Conclusion

On a policy level, where the lapse variable is either 0 or 1, the logit-link function is advised to use with the lapse data.

On a more aggregated level, where the recorded lapse data are averages, it should be tested which link-function suits the data the most, starting with linear regression.

\textsuperscript{28} See Fox(2008).

\textsuperscript{29} See Guyon & Elisseeff (2003).
Chapter 5  Methods

This chapter starts with a quick explanation of the subjects of interest and of programs which were used to analyze the data of those subjects. Subsequently the appliance of the linear regression model is discussed; the procedure for forming a linear regression model is explained, as well the assessment of certain model characteristics.

5.1  Scope of the Research

Due to several reasons lapse rates have not always been a field of study, neither in literature nor within Achmea. This is why the data on lapses do not go as far back in time as the company’s origin and accurately recording of the data did not always have the highest priority. That is also the reason why it is rather doubtful whether, in some cases, the recorded lapse rates are just unusual or the data have not been accurately managed. Next to the state and validity of the data, there is also the problem with obtainability of the data. Those obstacles influenced the choice to model lapse rates just for a part of Achmea’s portfolio. The companies included in the analysis are CBA (Centraal Beheer Achmea) and FBTO (Friesche Boeren en Tuinders Onderlinge). They constitute approximately 8% of Achmea’s provisions and are mainly oriented on the insurance market. Based on experience a difference between individual and group contracts is suspected, which is why the lapse rates of only the individually closed contracts are analyzed.

Since it cost too much time to get the needed authority to obtain the right data, only aggregated lapse rates are available for analysis instead of policy specific data. As a result of this unavailability of data a different modeling technique is applied than preferred, the amount of data is restricted and consequently a lot of explanatory variables cannot be tested on significance. The data available belong to the years 1996 to 2011 and were recorded on a yearly basis. The data are not modified before analysis.

5.2  Apparatus

The policy data of Achmea’s subsidiaries are not all recorded in a similar format. This is why the program Data Conversion System (DCS) is used. DCS is a conversion system that can be coded to transform the input’s format. The conversed data are then used as input by Glean, an analysis system. Glean is specifically designed to analyze policy data. Both programs, DCS

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30 This expectation is supported by the report on persistency of the FSA (2010).

31 The little data set makes the relative influence of an observation quite high and, consequently, making the availability of data one of the important factors which influence the results.
and Glean, are developed by SunGard, a major software and technology services company and are part of the regular data analysis process of Achmea.

After the data have been processed by Glean they are extracted and used as input for SPSS 17, which is used for the formation of models. SPSS is a well-known statistical software program which is developed by IBM. SPSS is promoted by IBM for its predictive analytics, the function which is used in this research.

The graphical images which are not developed by SPSS 17 or Glean are made by programs from the Microsoft Office 2010 package. Of this package the programs Excel and Word have been used. It should be noted that these programs have not been included in the mathematical part of the data-analysis.

5.3 Procedure & Measurement

GLMs rest on certain assumptions about the data, assumptions which justify the use of the regression models for purpose of prediction. There are four principal assumptions which are linearity of the relationship between the dependent and independent variables and independence, homoscedasticity and normality of the errors. Apart from those principal assumptions there are conditions which should be tested to justify the application of statistical measures and goodness-of-fit of the model. The procedure of the data-analysis can be described by the following steps:

Conditions which can be checked before formation of a model
1. No outlier distortion
2. A linear relationship between dependent and independent variables
3. Independence of observations

Condition which can be checked after formation of a model
4. No multicollinearity
5. Representativeness of the sample and proper specification of the model (no variables were omitted)

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32 The following procedure is applicable to continuous lapse data. The procedure described might differ from those actions which should be taken when analyzing binary data.
33 Homoscedasticity states that all error terms have the same variance.
34 No multicollinearity refers to the assumption that none of the explanatory variables is an exact linear combination of the other explanatory variables, which would render the variable redundant.
Selection of a model (in case of multiple possible models)

Conditions which can be checked after selection of the model

6. Normality of the residuals
7. Equality in variance of the errors (homogeneity of residual variance)
8. No auto-correlation of the errors

In the following subsections the procedure of verifying whether the assumptions/requirements are fulfilled is described as well as the procedure for assessing the model.

5.3.1 Conditions which can be checked before formation of a model.

Assumption 1 No outlier distortion

Outliers are observations which are classified as being abnormal. Outliers are said to influence the model in a negative way, since a model is fitted to the observation’s data using the least squares formula. The least squares formula can be used when there are more equations than unknowns; it provides the overall solution which minimizes the sum of the squares of the errors made in solving every single equation.

When data are expected to be erroneous they can be ignored, excluded from the analysis, transformed or the variable can be modified/excluded. The benchmark for outliers is chosen rather arbitrarily. Usually observations are classified as outliers based on their deviation from the mean, expressed in standard deviations of the sample. Outliers can be detected using a boxplot. A boxplot calculates a measure called interquartile range (IQR), which is the difference between the third and first quartile. Subsequently an outlier is classified, in this case, as all values which are $1.5 \times IQR$ below the first or above the third quartile.

However, when an outlier did not arise due to an error or an event that is known to occur once and once only, the exclusion of the data might have negative consequences. The exclusion of the data will make the model sensitive to the particular event. For this reason exclusion of outliers is not done in this research unless it can be underpinned by a better reason than the $1.5 \times IQR$ measure.

Assumption 2 A linear relationship between dependent and independent variables

One of the main assumptions is the existence of a linear relationship between the dependent and independent variables. This linearity can be assessed in two manners, namely:
mathematical and visual. The scientific approach is to use mathematical procedures to address the fit between variables. For this the Pearson’s correlation coefficient, also denoted as “r”, is used. Pearson’s correlation coefficient is a measure for the linear dependence between two variables, represented by:

\[ r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2(Y_i - \bar{Y})^2}} \]

In which

\( X_i, Y_i \) represents the \( i^{th} \) observation of variable \( X, Y \)

\( \bar{X}, \bar{Y} \) represent the sample mean of \( X, Y \);

The sample mean is said to be a good estimator of the population mean. Subsequently, the expected values of the random variables are said to be equal to the population mean. This assumption will be used throughout the research. The correlation coefficient which results from the formula will have a value between -1 and 1 of which the sign indicates the direction of the linear relationship. The closer r’s value is to \(( -1 )\) the more perfect the linear relationship.

The visual assessment of the linear relationship can be done by plotting both variables in a single graph. Preferably as a scatterplot, which plots dots of which the x-value corresponds to the value at an index of a variable and the y-value corresponds to the value of another variable with a similar index. The data are said to be correlated when the dots show a clear pattern and uncorrelated when there seems to be no pattern within the data cloud.

A different than linear relationship, low r or low pattern of dots, indicates that the data need to be transformed or that the model might not be a suited choice. The result of continuing the analysis with non-normal data might be a badly estimated model, since the fit is not optimal and test-statistics might assume normality of the data.

**Assumption 3** The observations are independent

When there is a relationship of a variable with one of its historical/lagged values the two values are said to be inter-related/autocorrelated. A huge advantage of that autocorrelation is
that past values give an indication of future values. Hence, autocorrelation figures of significant magnitude show that past values can be used as explanatory variables.

The test statistic for autocorrelation is the Ljung-Box statistic. The test statistic has the following formula:

\[ Q_{LB} = n(n + 2) \sum_{j=1}^{h} \frac{p^2(j)}{n-j} \]

In which

- \( Q_{LB} \) is the Ljung-Box test statistic
- \( n \) is the sample size
- \( p(j) \) is the sample autocorrelation at lag \( j \)
- \( h \) is the number of lags being tested
- \( j \) is the lag under consideration

The critical region, given a level of significance, is represented by \( Q_{LB} > \chi^2_{1-\alpha,h} \) in which \( \chi^2 \) is the percent point function of the chi-square distribution. The corresponding null hypothesis states that there is no dependence amongst values of a variable analyzing a specific lag. Values below 5% indicate that there is significant evidence of autocorrelation.

### 5.3.2 Model formation.

SPSS accepts linear regression models based on a selection, elimination and “acceptance” criterion. The selection criterion is used to analyze whether variables can be entered into the model of analysis or not. The elimination method does the opposite and analyzes which variable, of those present in the model, are not significant and can be eliminated from the model. Both criteria together are called the stepwise selection criterion and is applied in this article, as suggested by Raghavendra & Simha (2010). The acceptance criterion represents the significance of the total model and validates whether the model, based on its significance, should be accepted. The criteria are used as follows:

- The entry criterion. The variable which has the smallest probability of \( F^{36} \), when \( F \) is below 5%, is included in the model.
- The removal criterion. The variable in the model which has the highest probability of \( F \), when \( F \) is above 10%, is removed from the model.

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\(^{36}\) The \( F \) is an abbreviation for the underlying Fisher-Snedecor distribution (Mun, 2010).
- The model acceptance criterion. Every new model, unique set of accepted explanatory variables, will be presented as an outcome when the model has a probability of $F$ below 5%.

The $F$-value (Nau, 2005) is the ratio of the explained-variance-per-degree-of-freedom-used to the unexplained-variance-per-degree-of-freedom-unused. Basically, the significance which is described at each bullet point is expressed as the relative increase in explained variance. Hence, a variable/model is significant when the amount of extra explanation of the variance which it provides is above a certain margin. The calculation of the $F$ values is done by SPSS according to the description in the ANOVA\textsuperscript{37}-table below. For convenience the indices of some cells are denoted between brackets and are used in other cells to abbreviate the formula.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression Explained variance (A1)</td>
<td>Number of variables (B1)</td>
<td>A1/B1 (C1)</td>
<td>C1/C2 (D1)</td>
<td>F(D1,B1,B2)</td>
</tr>
<tr>
<td></td>
<td>Residual Unexplained variance (A2)</td>
<td>Number of observations-B1-1 (B2)</td>
<td>A2/B2 (C2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Total variance (A3)</td>
<td>Number of observations-1 (B3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textit{Table 3 ANOVA}

The $F$-test gives information about the vertical deviation of the regression line from the empirical data. The null hypothesis of the $F$-test is that there is no statistically significant association between the dependent and independent variables or, literally, that the coefficients of all explanatory variables are equal to zero. Significance values below 5% indicate that the null hypothesis can be rejected and that is evidenced, using a 5% level of significance, that the explained variance did not occur due to chance. When no results are provided it means that no variable is significant at the chosen significance levels, with the applied tests.

The $F$-statistic is a parametric measure which is extremely sensitive to non-normality. To see whether the normality assumption is a justifiable assumption, the residuals need to be

\textsuperscript{37} ANOVA stands for Analysis Of Variance.
analyzed and checked on normality. If the data pass the normality test the F-statistic can be used. Otherwise, with all things remaining equal, another test statistic should be chosen. An alternative which is more robust is Levene’s test (National Institute of Standards and Technology & Sematech, n.d.). This test can be thought of when the normality assumption is questioned. The model can be accepted when the significance is sufficient and the assumptions and results are acceptable.

Next to the mathematical test there is again a visual test. The historical values of the explanatory variables can be used to predict the lapse rates and a comparison can be made by plotting real and predicted values for the year 2011 in combination with a 99.5% prediction interval. The upper and lower bound of the 99.5% P.I. can be compared with the 50% in- and decrease of the standard formula. Whether the results of the model are more advantageous for the calculation of capital requirements et cetera will be dependent on the effects of lapses on the profitability of product groups.

Before any inferences can be made there are a few other assumptions which should be checked, the corresponding tests are described below.

5.3.3 Condition which can be checked after formation of a model.

Assumption 4 No multicollinearity

When the term multicollinearity is used in the context of multiple regression it concerns the event of strong correlation amongst the predictor variables (Doorn, 2006). When there are variables included in a model which experience high multicollinearity their individual effect on the dependent variable becomes an arbitrary value. This might make the exclusion of one of the mutual dependent variables inevitable. Another issue with multicollinearity is that some independent variables are redundant with one and another. As a consequence one or more of the variables do not add any predictive value, while they do cost a degree of freedom. This will consequently weaken the analysis. Logically this type of multicollinearity can only occur when more than one variable is present in the model. When the model consists of only one variable the term multicollinearity has a totally different meaning. In that case the term multicollinearity refers to the problem of having multiple different values of the response variable related to one value of the independent variable. This type of multicollinearity would provide difficulties in assessing the best regression coefficient for the regression line.
The multicollinearity will be assessed based on the “variance inflation factor” (VIF). The VIF can be calculated by (Yaffee, 2004):

\[
VIF = \frac{1}{1 - R_i^2}
\]

In which \(R_i^2\) is the coefficient of determination of the regression equation.

While there are no clear boundaries for the VIF-values, it is known that low values- with a minimum of 1- indicate low multicollinearity. As a rule of thumb Yaffee (2004) suggests to use a VIF-value of 10 as a boundary, with which values above 10 indicate alarming multicollinearity. Next to the VIF-value SPSS also provides other measures for intercorrelation among independent variables like tolerance and the variance proportion. Both measures are related to the variance inflation factor.

**Assumption 5** The sample is representative and the model has been properly specified (no variables were omitted)

The representativeness of the sample is usually based on the randomness in the selection of the sample. The sample used, FBTO and CBA data from 1996 to 2011, encompasses many product groups and many policies. However, it is not known whether the lapse rate levels or developments are representative for Achmea as a holding. This is why the research will only help to make inferences on the lapse rates of this particular sample while leaving the generalizability to be a question mark. Eventually, after accepting a model, the congruence of the predicted lapse rates with the real lapse rates can be assessed. The deviation between both rates will be an indication of the accuracy with which the rates can be predicted. The deviation will indicate whether the model is a fine predictor, variables are missing or that the deviation in prediction comes down to an unpredictable error term. In advance it can be noted that the lack of many policy (holder) related data/variables might ensure that some relevant variables are omitted, since those variables would probably help explaining individual behavior more extensively.

**5.3.4 Selection of a model.**

Whenever more than one model is accepted using the prescribed test statistics the different models should be compared. Next to the already explained statistics there is another measure, the adjusted r square, which penalizes the model for the inclusion of extra variables. This effect is created by adding the degrees of freedom to the calculation of the by the model
accounted for proportion of the variation in the dependent variable (SAP, n.d.). In other
words, the additional explanatory power which is created by adding a variable to the model is
estimated. As a consequence the value of the measure decreases when the addition of an
explanatory variable does not increase the residual sum of squares more than would be
expected by chance. So, the model with the highest adjusted r square model is chosen to
represent the model with the highest goodness-of-fit.

5.3.5 Conditions which can be checked after selection of the model.

Assumption 6 The residuals are normally distributed
For this assessment the Kolmogorov-Smirnov (K-S) test is used. More specifically it is the
one-sample K-S test which is used. This test compares a sample with a reference probability
distribution, in this case with the Normal distribution. The hypotheses of this test are (Gate,
2000):

- $H_0$: There is no difference between the distribution of the data set and a normal one $38$
- $H_A$: There is a difference between the distribution of the data set and normal $39$

In the case of normal distributed residuals the outcome of the K-S test will have to be a level
of significance above 5%. A significance above 5% leads to the inference that $H_0$ should be
accepted, so the hypothesis that the residuals are normally distributed is accepted. As a visual
verification of the outcome of the K-S test the residuals can be shown by a histogram. For
convenience the normal-curve can be superimposed as a line, so that the normality of the data
can be observed by looking at its deviation from the superimposed curve.

Assumption 7 The errors have equal variances (homogeneity of residual variance)
Assessment of the equality of the variance of the errors is conducted by plotting the
standardized residuals versus the standardized predicted values. Homogeneity of residual
variance requires that the residuals per observation remain approximately constant as the
predicted value changes. An approximately constant level of variances would indicate that the
variance of the residuals is independent of the predicted value. Heteroscedasticity might occur
when some variables are skewed while others are not. This means that when the data has
passed the test for normality the chance of the data being homoscedastic will increase.

$38$ $H_0$ is the abbreviation of the null hypothesis.
$39$ $H_A$ is the abbreviation of the alternative hypothesis.
Although violation of the assumption of homoscedasticity will not invalidate the regression model, it will weaken it.

**Assumption 8** The errors are not autocorrelated

This assumption is checked in a similar manner as assumption 3.

The independence of the residuals is tested mathematically and visually verified. The mathematical interpretation of the level of dependence amongst the residuals is done using the Ljung-Box statistic. The null-hypothesis of this test is that the data are random/not related to their historical values, while rejecting this hypothesis would mean that the data are dependent on historical values.

The independence of the residuals can be verified by plotting the residuals over time. Dependence is demonstrated by the display of a pattern in the residuals over time.

**Other model statistics**

Closely related to r is the r square(d) statistic. R squared ($r^2$) is the proportion of variation in the dependent variable which is explained by the model. As r squared is literally the square of r, it can be calculated by multiplying r by itself. The result will be a positive figure between 0 and one. A $r^2$-value of one indicates a perfect explanation of the variance of the dependent variable by the model.

**5.3.6 Product groups for which models are formed.**

For every product group, of which data is available, a different model will be formed, since their underlying parameters are assumed to deviate significantly.

The product groups are provided by Achmea and the list is composed out of the following groups:

- **Mortgages.** A mortgage is a security interest in real property held by a lender as a security for a debt, usually a loan of money. A mortgage will give profits from investment fees and related risk products. Lapses of a mortgage are expected to be sensitive to macroeconomic changes, such as market interest.

- **Risk.** The product group Risk contains term life insurance products (term assurance), that is a life insurance which provides coverage at a fixed rate of payments for a
limited period of time, the relevant term. After this term the policy expires and the assured life is not covered anymore. It is the least expensive insurance type because savings are not financed.

Term insurance is therefore expected not to be very sensitive to market interest, but is probably sensitive to the liquidity position of the assured.

- **Savings (regular premium)**. Non-linked (traditional) insurance savings products financed by regular premiums [RP] are comparable to bank savings. The difference with bank savings is that insurance products must have an element of risk for the insurance company or the insured.

  Therefore a term insurance is attached to the savings policy, negative or positive risk can both be attached. Savings policies can have a clause (tax related) that will prevent lapses.

- **Savings (single premium)**. Non-linked (traditional) insurance savings products financed by single premiums [SP] are comparable to the savings products with regular premium payments. Since there is no obligation for the policyholder to pay future premiums the emergency fund hypothesis seems most appropriate.

- **Unit-Linked (regular premium)**. Unit-linked insurance savings products financed by regular premiums are comparable to Savings. The fund development of a unit-linked product is very volatile due to the market returns (stock and interest returns). Consequently lapses are expected to be negatively correlated with stock market developments.

- **Unit-Linked (single premium)**. Unit-linked insurance savings products financed by single premiums are comparable to Savings.

- **Whole life & Funeral**. The whole life insurance (permanent insurance) and funeral products have the same coverage as the risk products (term insurance) with a main difference in the coverage term. Whole life and funeral products have a permanent coverage until the insured deceases. Therefore the products must include a savings part because the sum assured will be paid in the end. The savings part (technical reserve) will in most policy agreements not be paid to the insured when the policy is lapsed. Lapses are expected to have the same pattern as term insurance products.
5.3.7 Additional research.

Two analyses have been chosen as additional research topics, namely:

1. The assumption of dissimilarity of the lapse rate development per product group is tested, to verify that a different model per product group is essential.

2. For the group “whole life & funeral” the data of 1 and 5 year old insurances will be analyzed, apart from the aggregated “whole life & funeral” data. This test is performed to evidence the hypothesis that the age, or duration, of an insurance has influence on the corresponding lapse behavior.

5.4 (Possible) Limitations

- Since the data set is small and the list of possible explanatory variables is only composed of macro-economic variables there is a possibility that no model will pass the significance tests.

- The lack of policy data on policyholder characteristics might limit the insight the model will provide in individual behavior, which might have consequence for the goodness-of-fit of the model.

- The data set does not include many observations. This absence of a long history will limit the generalization of the model and might make it hard to obtain a model with a high statistical significance.

- Some of the measures which test the satisfaction of underlying assumptions might disapprove the model(ing) choice. It will depend on the measure whether the incorrect assumptions will jeopardize the results.

- It is not analyzed whether the lapse data of CBA and FBTO deviate from Achmea’s aggregated lapse data; therefore it is not known whether this data set is representative for the holding.

- Due to a maximum availability of two future values of the explanatory variables the lapse rate estimations can only be done for the near future.

Note: The standard model predicts lapse rates and assumes subsequent lapse rates to be equal to the one-year future rate. The assumption of constant future lapse rates can be applied to the models which will be developed as well.
Chapter 6  Data and Analysis

This chapter is divided into two parts. Part one is on the used data set and explains which data sources have been chosen. Part two consists of a description of the actual data-analysis for one product group, which is followed by the corresponding results of the other product groups and ends with the results of the two additional researches.

6.1 Data Set

The specific range of available lapse data is the data from 1996, on a yearly basis, until 2011. The rate corresponding to the year 2011 is omitted from the analysis. These data have been omitted since the year is not yet finished and it is not assumed that the number of lapse events which have occurred during the first months is representative for the number of lapses during the last months of 2011. As indicated before it is not analyzed whether the lapse data of CBA and FBTO deviate from Achmea’s aggregated lapse data.

The data on Gross Domestic Product [GDP] are provided by the European Central Bank [ECB] (2011) which collects its data in cooperation with the DNB. As data on the reference rate the 5-years treasury rate has been used. This rate is quoted by the Federal Reserve System [FED] (n.d.). The choice has been made for data of the FED instead of the ECB, this has been done since ECB recordings of Dutch treasury rates do not go back further than 1999.

As a stock market for comparison the Amsterdam Exchange Index [AEX] has been chosen, which represents a maximum of 25 of the most actively traded securities that trade on Euronext Amsterdam. The data on this variable are provided by Yahoo (n.d.). It should be noted that there is some criticism on the relevance of a national index due to the internationalization of investors their operations. Other criticism is on the AEX for not representing the whole Dutch market and for including multinationals which lack significant bonding with the Netherlands.

The data on growth in GDP, buyer confidence, inflation and unemployment are extracted from the database of the Centraal Planbureau [CPB] (2011). It is mentioned in articles, and apparent from the data, that the data of the CPB differ greatly from those of the World Bank’s database. This deviation is, however, no longer present in the years which are of interest for this research.
Finally there were two macro events chosen which represented moments of extreme circumstances. The first year in which an extreme situation happened was 1999, in which a huge increase in lapse rates occurred, probably, due to the immense upcoming popularity of unit linked mortgages (Achmea, 2010b). The second was the stock market crash during the subprime mortgages crisis which is expected to have had its impact in 2008. The dot-com bubble which took place in 2000 does not show any visual influence on the lapse rates. Since it is hard to predict the occurrence of an extreme event, let alone the impact of the specific event, it is unadvisable to include such dummy variables in the model. Because of the uncertainty surrounding the variable it will only make sense when the events are expected to have similar impact and future extreme events have similar influence and are predictable.

6.2 Analysis
The analysis of lapse data has been conducted a total of 9 times. Seven models have been formed for the different product groups and two for the additional studies. That is why the procedure, which is explained in the previous section, will be described only once using the whole life & funeral data. This section will end with a summary of the important statistics and conclusions regarding the other 8 models. It is important to note that all macro-economic variables have been standardized, using their first value, since the values of some variables were of immense magnitude.
6.2.1 Formation of the model.
In this subsection the steps described in chapter 5 will be elaborated with use of the actual data.

Assumption 1 No outlier distortion
Figure 1 displays the boxplot based on the whole life & funeral data.

Based on the box plot the events 4 and 5 are classified as outliers, these points correspond to the lapse rates of 1999 and 2000. SPSS uses two kinds of indicators for outliers, dots and stars. Of these indicators stars are “extreme” outliers, those which are more than 3*IQR below the first or above the third quartile, and the dots represent “normal” outliers, those which are more than 1.5*IQR, but less than 3*IQR below the first or above the third quartile. When further research indicates that the data appear too skewed to be normal these events can be investigated on their influence on the data set. For now there is no reason to exclude the observations.

Note that exclusion of the years 1999 and 2000 will decrease the small data set even further. A boxplot without the two observations indicates that the skewness is diminished and also shows that there are still outliers detectable in the remaining data set.
**Assumption 2** A linear relation between the dependent and independent variables

The two manners in which the dependency is analyzed is by using the r-measure and a scatterplot. The following table lists the scatterplots of the lapse rates for Whole life and Funeral insurances and each of the predictor variables. The figures are arranged on a descending order based on their r (squared) statistic. Each figure is composed out of lapse rate, represented on the y-axis, and a specific variable, represented on the x-axis. The specific variable is one of macro-economic variables from the list of possible explanatory variables and is each time denoted above the scatterplot.

<table>
<thead>
<tr>
<th>Expl. Variable</th>
<th>GDP (+1)</th>
<th>Buyer Confidence (-1)</th>
<th>Lapse rate (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-statistic</td>
<td>-0.916</td>
<td>0.895</td>
<td>0.772</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.839</td>
<td>0.801</td>
<td>0.595</td>
</tr>
</tbody>
</table>

Scatterplot

<table>
<thead>
<tr>
<th>Expl. Variable</th>
<th>Return on Stock Market (-2)</th>
<th>Reference Market Rate</th>
<th>Return on Stock Market (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-statistic</td>
<td>0.750</td>
<td>0.666</td>
<td>0.615</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.562</td>
<td>0.444</td>
<td>0.378</td>
</tr>
</tbody>
</table>

Scatterplot

<table>
<thead>
<tr>
<th>Expl. Variable</th>
<th>First difference in lapse rates</th>
<th>Unemployment (+1)</th>
<th>Growth in GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-statistic</td>
<td>0.551</td>
<td>-0.440</td>
<td>0.426</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.303</td>
<td>0.194</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Scatterplot
Table 4: Scatterplots of the dependent and independent variables

The regression line is fitted in such a way that the error sum is minimized by means of least squares regression. In the case of perfect correlation, r equal to (-1), the dots will lie on the regression line, the average dot will deviate more and more as r approaches zero. The r squared statistic of the one term forwarded GDP is the one which is closest to (-1). The difference between the statistic’s value and 1 indicates that the variable on its own may predict a large part of the variation in the lapse rate, but that the model will need more than one variable to “perfectly” predict the lapse rate. The negative sign of the r measure suggests that an increase in next year’s GDP lowers present lapse rate.

Unemployment and lagged inflation are the variables with the lowest r squared statistics. Congruently their dots seem rather scattered over the plotted area, making the variables seem uncorrelated.

Since some of the variables appear to be linearly related to the lapse rates it is assumed that the linear dependence assumption is fulfilled. Although a spline or other line might provide a closer approximation of the data set, there is no reason to reject the assumption that the data

<table>
<thead>
<tr>
<th>Expl. Variable</th>
<th>Lapse rate (-2)</th>
<th>Inflation</th>
<th>Macro Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-statistic</td>
<td>0.372</td>
<td>0.347</td>
<td>0.275</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.139</td>
<td>0.121</td>
<td>0.076</td>
</tr>
<tr>
<td>Scatterplot</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expl. Variable</th>
<th>Inflation (-1)</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-statistic</td>
<td>0.258</td>
<td>-0.201</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.067</td>
<td>0.04</td>
</tr>
<tr>
<td>Scatterplot</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
experience linear dependency. A double-check of the linear relationship assumption will occur in the next step, the model formation.

**Assumption 3** The observations are independent

The test for autocorrelation amongst observations is performed using the Ljung-Box test. The significance values which correspond to the Ljung-Box statistic are presented in the last column of the table which is presented below.

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Std. Error(^a)</th>
<th>Ljung-Box Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Value</td>
</tr>
<tr>
<td>1</td>
<td>0.683</td>
<td>0.234</td>
<td>8.492</td>
</tr>
<tr>
<td>2</td>
<td>0.335</td>
<td>0.226</td>
<td>10.690</td>
</tr>
<tr>
<td>3</td>
<td>0.117</td>
<td>0.217</td>
<td>10.983</td>
</tr>
<tr>
<td>4</td>
<td>-0.042</td>
<td>0.208</td>
<td>11.023</td>
</tr>
<tr>
<td>5</td>
<td>-0.070</td>
<td>0.198</td>
<td>11.147</td>
</tr>
<tr>
<td>6</td>
<td>-0.072</td>
<td>0.188</td>
<td>11.294</td>
</tr>
<tr>
<td>7</td>
<td>-0.223</td>
<td>0.177</td>
<td>12.884</td>
</tr>
<tr>
<td>8</td>
<td>-0.316</td>
<td>0.166</td>
<td>16.527</td>
</tr>
<tr>
<td>9</td>
<td>-0.358</td>
<td>0.153</td>
<td>21.967</td>
</tr>
<tr>
<td>10</td>
<td>-0.329</td>
<td>0.140</td>
<td>27.485</td>
</tr>
<tr>
<td>11</td>
<td>-0.221</td>
<td>0.125</td>
<td>30.596</td>
</tr>
<tr>
<td>12</td>
<td>-0.094</td>
<td>0.108</td>
<td>31.345</td>
</tr>
<tr>
<td>13</td>
<td>0.023</td>
<td>0.089</td>
<td>31.412</td>
</tr>
</tbody>
</table>

\(^a\) The underlying process assumed is independence (white noise).

\(^b\) Based on the asymptotic chi-square approximation

**Table 5 Autocorrelation of the lapse rates**

The figures indicate that all lagged values, apart from lag 6 and 7, experience autocorrelation which is significant at a 5% level of significance. Based on the significant autocorrelation, all lagged values, apart from those of lag 6 and 7 can be included in the data analysis. A consequence is that every included lagged value will decrease the amount of observations, which is already little. For this reason the choice has been made to include the first and second lag only\(^{40}\).

\(^{40}\) Subsequent tests indicated that the autocorrelation at 3, 4 and 5 lags did not appear to be significant enough to be included in the model.
Model formation

Based on the entry, removal and acceptance criterion SPSS provided the following models:

Model 1: \( \hat{Y} = 0.054 - 0.018 \times GDP (+1) \); and

Model 2: \( \hat{Y} = 0.041 - 0.011 \times GDP (+1) + 0.002 \times Buyer\ Confidence (-1) \)

Model 3: \( \hat{Y} = 0.030 - 0.008 \times GDP (+1) + 0.004 \times Buyer\ Confidence (-1) + 0.009 \times Unemployment \)

The F-value and probability of F are presented in the following table.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Regression</td>
<td>0.00018</td>
<td>1</td>
<td>0.000184</td>
<td>57</td>
<td>(11*10^-6)^a</td>
</tr>
<tr>
<td>Residual</td>
<td>0.00004</td>
<td>11</td>
<td>0.000003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.00022</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Regression</td>
<td>0.00020</td>
<td>2</td>
<td>0.000100</td>
<td>50</td>
<td>(6*10^-6)^b</td>
</tr>
<tr>
<td>Residual</td>
<td>0.00002</td>
<td>10</td>
<td>0.000002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.00022</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Regression</td>
<td>0.00021</td>
<td>3</td>
<td>0.000070</td>
<td>61</td>
<td>(3*10^-6)^c</td>
</tr>
<tr>
<td>Residual</td>
<td>0.00001</td>
<td>9</td>
<td>0.000001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.00022</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Predictors: (Constant), GDP (+1)
b. Predictors: (Constant), GDP (+1), Buyer Confidence (-1)
c. Predictors: (Constant), GDP (+1), Buyer Confidence (-1), Unemployment
d. Dependent Variable: Lapse rate

Table 6 ANOVA of model 1, 2 and 3

As previously mentioned the null hypothesis of the F-test is that there is no statistically significant association between the dependent and independent variables. All three models have level of significance far below 5%, meaning that the null hypothesis, that all of the coefficients of the explanatory variables are equal to zero, is rejected. Hence, it can be concluded that, using a 5% level of significance, at least one of the coefficients is not equal to zero.
**Assumption 4** No multicollinearity

Multicollinearity is measured using the variance inflation factor. The table below provides the VIF-values per model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Collinearity Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(Constant)</td>
</tr>
<tr>
<td></td>
<td>GDP (+1)</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>(Constant)</td>
</tr>
<tr>
<td></td>
<td>GDP (+1)</td>
</tr>
<tr>
<td></td>
<td>Buyer Confidence (-1)</td>
</tr>
<tr>
<td></td>
<td>2.84</td>
</tr>
<tr>
<td>3</td>
<td>(Constant)</td>
</tr>
<tr>
<td></td>
<td>GDP (+1)</td>
</tr>
<tr>
<td></td>
<td>Buyer Confidence (-1)</td>
</tr>
<tr>
<td></td>
<td>3.41</td>
</tr>
<tr>
<td></td>
<td>Unemployment</td>
</tr>
<tr>
<td></td>
<td>1.68</td>
</tr>
</tbody>
</table>

*a. Dependent Variable: Lapse Rate*

**Table 7** The VIF calculation

The VIF-values presented in the table above show that the VIF-values of the variables of every model are far below 10. With one as indication of no multicollinearity and 10 as indication of an alarming level of multicollinearity it is evident that the highest VIF-value, of 4.52, does not fail this test. Hence, there is no alarming level of multicollinearity.

**Assumption 5** The sample is representative and the model has been properly specified (no variables were omitted)

Although the sample’s representativeness is hard to determine, the r square measure can be calculated. With r square being the proportion of variation in the dependent variable which is explained by the model it can be used to calculate the proportion of variation which remains unexplained. The r square statistic for the third model is 0.953. The standard error of the estimates is 0.0011. The r square value is fairly close to 1, especially the value of the second model. That value in combination with the corresponding standard error shows that, although there is some variation left to explain, the possible gains in accuracy is minimal. The associated conclusion is that: If the data set is representative and relationships remain existent, the model will provide an accurate prediction of future lapse rates.
Selection of a model

Three models are accepted based on the first criteria and not one of them experiences significant multicollinearity. In the figure below some comparable statistics of the models are presented.

<table>
<thead>
<tr>
<th>Model</th>
<th>R</th>
<th>R Square</th>
<th>Adjusted R Square</th>
<th>Std. Error of the Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.916a</td>
<td>0.839</td>
<td>0.824</td>
<td>0.0018</td>
</tr>
<tr>
<td>2</td>
<td>0.954b</td>
<td>0.910</td>
<td>0.892</td>
<td>0.0014</td>
</tr>
<tr>
<td>3</td>
<td>0.976c</td>
<td>0.953</td>
<td>0.938</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

- a. Predictors: (Constant), GDP(+1)
- b. Predictors: (Constant), GDP(+1), Buyer Confidence (-1)
- c. Predictors: (Constant), GDP(+1), Buyer Confidence (-1), Unemployment
- d. Dependent Variable: Lapse rate

Table 8  Model summary

Based on all of the values in the model summary it can be concluded that the third model outperforms the first two regarding explanatory power. The adjusted r square measure is higher and, logically, the standard error of the estimate is lower. The standard deviation of the lapse variable is 0.0043. The reduction in the standard error of approximately 75%, from 0.0043 to 0.0011, indicates that the model has a better goodness-of-fit than the mean of the sample does (Kiesenbauer, 2011). Figure 5 displays the actual lapse rates in combination with the third model. The figure strengthens the conclusion that the model is preferred above using the mean of the sample as prediction.

The tests which follow are all performed using model 3.
**Assumption 6** The residuals are normally distributed

The Kolmogorov-Smirnov test [K-S] compares the data selected with the expected data of a selected distribution. The null-hypothesis of the one-sample K-S test below is that the data are normally distributed.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0,000</td>
<td>0,866</td>
</tr>
<tr>
<td>Most Extreme Differences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absolute</td>
<td>0,144</td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>0,144</td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td>-0,109</td>
<td></td>
</tr>
</tbody>
</table>

Kolmogorov-Smirnov Z 0,521
Asymp. Sig. (2-tailed) 0,949

a. Test distribution is Normal.
b. Calculated from the standardized residuals.

Table 9  The one-sample Kolmogorov-Smirnov

The significance value of 0.949 indicates that the null-hypothesis cannot be rejected using a 5% level of significance. Consequently it is accepted that, using the 5% level of significance, the residuals are normally distributed. Figure 2 is the histogram of the residuals with superimposed normal curve. Based on this figure a different than normal, for example exponential distribution, is expected as well.

![Histogram of standardized residuals and superimposed normal curve](image)

Further research showed that the different null hypothesis stating that the data were exponentially distributed was accepted. If the data were actually exponentially distributed a transformation of the data or non-normal model might be a better choice.
**Assumption 7** The errors have equal variances (homogeneity of residual variance)

In the following figure the standardized residuals are plotted versus the standardized predicted values.

*Figure 3  Scatterplot of standardized residuals and predicted values*

The scatterplot does not include that many dots that a clear pattern becomes visible, but the dots appear to deviate more as the standardized predicted value moves further away from zero. As mentioned, the violation of the assumption of homoscedasticity will not invalidate the regression model, but it will weaken it.
Assumption 8 The errors are not autocorrelated

In the table below the figures for the Ljung-Box test applied to the unstandardized residuals are presented.

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Std. Error&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Ljung-Box Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Value</td>
</tr>
<tr>
<td>1</td>
<td>-0.166</td>
<td>0.248</td>
<td>0.449</td>
</tr>
<tr>
<td>2</td>
<td>-0.423</td>
<td>0.238</td>
<td>3.623</td>
</tr>
<tr>
<td>3</td>
<td>0.264</td>
<td>0.226</td>
<td>4.983</td>
</tr>
<tr>
<td>4</td>
<td>0.086</td>
<td>0.215</td>
<td>5.142</td>
</tr>
<tr>
<td>5</td>
<td>-0.228</td>
<td>0.203</td>
<td>6.404</td>
</tr>
<tr>
<td>6</td>
<td>-0.152</td>
<td>0.189</td>
<td>7.045</td>
</tr>
<tr>
<td>7</td>
<td>0.188</td>
<td>0.175</td>
<td>8.191</td>
</tr>
<tr>
<td>8</td>
<td>-0.007</td>
<td>0.160</td>
<td>8.193</td>
</tr>
<tr>
<td>9</td>
<td>-0.392</td>
<td>0.143</td>
<td>15.695</td>
</tr>
<tr>
<td>10</td>
<td>0.231</td>
<td>0.124</td>
<td>19.149</td>
</tr>
<tr>
<td>11</td>
<td>0.229</td>
<td>0.101</td>
<td>24.264</td>
</tr>
</tbody>
</table>

<sup>a</sup> The underlying process assumed is independence (white noise).
<sup>b</sup> Based on the asymptotic chi-square approximation.

Table 10 Autocorrelation of unstandardized residuals

From the significance which is presented in the last column it can be deducted that there is no reason to reject the null hypothesis using the 5% level of significance for the first 9 lags. Autocorrelation is detected at lag 10 and 11. However, since not one of the other tests suggested the data to be not linearly related the assumption is not expected to be violated. The detection of autocorrelation in the last lags might be due to chance, since there were only a few observations of these lags. To visually analyze the autocorrelation figure 4 presents the residuals per year. Note that most of the autocorrelation which was present in the lapse data is removed by the explanatory variables.

The visual representation of the autocorrelation of the residuals is presented in figure 4.
Figure 4 shows that all residuals are positioned around zero, with a maximum deviation of approximately 0.0016. The dots do not exhibit a clear pattern, which is in congruence with the results of the Ljung-Box test.

### 6.2.2 Comparison of the developed with the standard model.

A prediction interval can be calculated for the future lapse rates. The prediction interval is an estimated range in which the unknown future lapse rate will fall, with a specific chance. Part of the P.I. is the predicted value. The predicted value can be calculated for the year 2011 by using model 3, which results in:

\[
\hat{Y}_{2011} = 0.030 - 0.008 \cdot GDP(2012) + 0.004 \cdot Buyer\ Confidence(2010) + 0.009 \cdot Unemployment(2011) = 0.022
\]

The predicted lapse rate will serve as the mean of the P.I. The formula for the prediction interval is:

\[
\hat{Y}_{2011} + s_n \cdot t_{1-\frac{\alpha}{2}, n-1} \sqrt{1 + \frac{1}{n}}
\]

With

\[
s_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \bar{Y}_n)^2
\]

In which

\(\hat{Y}_{2011}\) is the estimated value of the lapse rate of the year 2011.

\(n\) is the number of observations.
MODELING LAPSE RATES

\[ t_{1-\frac{\alpha}{2}, n-1} \] is the \((1 - \frac{\alpha}{2}) \times 100\) percentile of the t-distribution with \(n-1\) degrees of freedom.

\(\alpha\) is 5%

Based on these formulae the prediction interval for the lapse rate of 2011 is \(0.022 \pm 0.005\) or \((0.017; 0.027)\) The P.I. which is prescribed by the standard formula can be calculated by multiplying the predicted value by either 50 or 150%. The resulting P.I. of the standard formula is from 0.011 to 0.033. This leads to the conclusion that the internal formula prescribes smaller regulatory capital than the standard formula does.

Figure 5 provides the curve which is constructed by applying the developed model (gray color) which can be compared with the actual lapse rates (blue colored line). The P.I. of the developed model and the results of the standard formula for year 2011 are presented in, respectively, green and red.

![Figure 5](image)

**Figure 5  Comparison of the model results and standard formula for the whole life & funeral data**

As is illustrated in the figure above the lapse rates are quite accurately predicted using the developed model. Since all the available data have been used to construct the model no real validation is possible. Application of the model to the data on which it is based (training data) might ensure overrating of the model when the results look promising.
6.2.3 **Results of the other product groups.**

Following the same steps and using the same methods to interpret data the other product groups have been analyzed. The results of all the analyses are provided in the following table:

<table>
<thead>
<tr>
<th>Product Group</th>
<th>Outliers (obs)</th>
<th>Link-function</th>
<th>Multi-collinearity</th>
<th>Autocor. Lapse rate (obs)</th>
<th>Normality Of residuals</th>
<th>Suggested model</th>
<th>Std. Error of the model</th>
<th>Std. error of the lapse var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life &amp; Funeral</td>
<td>4 &amp; 5</td>
<td>Linear</td>
<td>No</td>
<td>1-5 &amp; 8-10</td>
<td>Yes</td>
<td>$\hat{Y} = 0.020 - 0.008 \times GDP (+1) + 0.004 \times Buyer Confidence (-1) + 0.009 \times Unemployment$</td>
<td>0.0011</td>
<td>0.0043</td>
</tr>
<tr>
<td>Mortgage</td>
<td>No</td>
<td>Linear</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>$\hat{Y} = 0.055 - 0.021 \times Buyer Confidence (-1)$</td>
<td>0.0249</td>
<td>0.0292</td>
</tr>
<tr>
<td>Risk</td>
<td>1 &amp; 15</td>
<td>Linear</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>$\hat{Y} = 0.060 - 0.025 \times Reference Market Rate$</td>
<td>0.0056</td>
<td>0.0073</td>
</tr>
<tr>
<td>Savings (regular premium)</td>
<td>No</td>
<td>Linear</td>
<td>No</td>
<td>1-4</td>
<td>Yes</td>
<td>$\hat{Y} = 0.002 - 0.003 \times Buyer Confidence (-1) + 0.004 \times Return on Stock Market (-1)$</td>
<td>0.0012</td>
<td>0.0037</td>
</tr>
<tr>
<td>Savings (single premium)</td>
<td>No</td>
<td>Linear</td>
<td>No</td>
<td>1 &amp; 5-10</td>
<td>Yes</td>
<td>$\hat{Y} = 0.002 - 0.613 \times First difference in Lapse Rates$</td>
<td>0.0009</td>
<td>0.0011</td>
</tr>
<tr>
<td>Unit linked (regular premium)</td>
<td>No</td>
<td>Linear</td>
<td>No</td>
<td>1-3</td>
<td>Yes</td>
<td>$\hat{Y} = 0.100 - 0.036 \times GDP (+1) - 0.010 \times Inflation$</td>
<td>0.0041</td>
<td>0.0077</td>
</tr>
<tr>
<td>Unit linked (single premium)</td>
<td>4</td>
<td>Linear</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>$\hat{Y} = -0.009 + 0.022 \times Return on Stock Market (-2) - 0.010 \times Inflation$</td>
<td>0.0056</td>
<td>0.0099</td>
</tr>
</tbody>
</table>

**Table 11 Important results per product group**
6.2.4 Results of the additional research.

Hypothesis 1 - The lapse rate development per product group deviates

In order to analyze whether the lapse rates per product group deviate from each other all the curves are fitted into one figure, figure 6. As becomes evident from these lines there is a huge difference between magnitude and development of the lapse rate per product group.

![Figure 6 Lapse rates per product group over time](image)

Next to the graph of the lapse rates there is also the huge difference in significant variables and corresponding coefficients per model. This difference also indicates that the implementation of one model for all the different product groups will provide inaccurate results.

Hypothesis 2 – The age of an policy, also called its duration, has influence on lapse behavior. The duration is expected to be a driver of lapse rates. Hence, lapse rates are expected to in/decrease as time passes by. This hypothesis is strengthened by figure 7 in which every line represents the lapse rate development belonging to the contracts with a specified duration. The “summary”-curve represents the average of the data, which are similar to the data which are used throughout chapter 6.
As is evident from the graph there is a huge deviation in lapse rate developments between the different durations. Without further tests it can be concluded that the developments are different, which is congruent with the results of, among others, the FSA (2004) and Cox & Lin (2006). For comparison the table below depicts the lapse rates for the 1-, 9-year and summary curve.

<table>
<thead>
<tr>
<th>Product Group</th>
<th>Outliers (obs)</th>
<th>Link-function</th>
<th>Multi-collinearity</th>
<th>Autocor. Lapse rate (obs)</th>
<th>Normality of residuals</th>
<th>Autocor. Residuals</th>
<th>Suggested model</th>
<th>Std. Error of the model</th>
<th>Std. error of the lapse variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life &amp; Funeral (1)</td>
<td>6</td>
<td>Linear</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>$\hat{\beta} = 0.048 + 0.017 \times \text{Inflation}$</td>
<td>0.0086</td>
<td>0.0114</td>
</tr>
<tr>
<td>Whole life &amp; Funeral (9)</td>
<td>15</td>
<td>Linear</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>$\hat{\beta} = 0.025 - 0.008 \times \text{Inflation}$</td>
<td>0.0042</td>
<td>0.0056</td>
</tr>
<tr>
<td>Whole life &amp; Funeral (summary)</td>
<td>4&amp;5</td>
<td>Linear</td>
<td>No</td>
<td>1-5 &amp; 8-10</td>
<td>Yes</td>
<td>Only at lag 10,11</td>
<td>$\hat{\beta} = 0.030 - 0.008 \times \text{GDP} (\text{+1}) + 0.004 \times \text{Buyer Confidence} (\text{-1}) + 0.009 \times \text{Unemployment}$</td>
<td>0.0011</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

Table 12 Important results for the 1- and 10 year duration of whole life & funeral lapse rates
When the model is applied to the whole life & funeral lapse data per duration, the first two cases in table 13, the fit is less accurate than when the average of all durations is modeled. For that reason, and based on these observations, deviation from the summary model is only suggested when the lapse rates per duration need to be estimated. The better fit can be caused by individual behavior which might influence the lapse data per duration while the summary data are more influenced by macro-economic factors.

Noticeable is that the models for durations 1 and 9 use the same variable, which are different from those used for the aggregated lapse data. The other remarkable result is that both models include inflation but with different signs. While an increase in inflation will increase the lapse rate of products with low durations, it will decrease the lapse rate for products with high durations.

### 6.3 Limitations

- Although the data set was small there were models, which proved to be significant, formulized for every product group. These models should be looked upon with caution for the model might be sensitive to new data.
- The generalizability of the models over time and within the company remains uncertain.
- The missing policy data, as is indicated by the $r^2$ statistic and analysis of lapse rate per policy duration, might help decrease the unexplained variance since the additional data will increase the number of observations and variables.
- The research focused on linear regression models, the appropriateness of this choice compared to other models has not been analyzed. The small amount of data might make it impossible to reject other distributions, such as the uniform or exponential distribution.
Chapter 7  Conclusion and Recommendation

7.1 Conclusion
This thesis started with a research question on explanatory variables and predictive models for lapse rates in the life insurance industry, which was:
Can the current calculation model for lapse rates be improved\(^{41}\), while staying in compliance with the Solvency II directive, and which variables have a significant\(^{42}\) relation to the lapse rates?

To answer the first part of the question examination had to be done to discover which predictive models were available as well as applicable to this data set. To answer the second part of the question it had to be examined which variables could have exercised influence on the lapse rates, which was tested using data of CBA and FBTO on individually closed contracts. With the theory in scientific papers as a basis, complemented by company experience and logic, a list of 38 possible explanatory variables had been composed. The available data were the aggregated data of CBA and FBTO on a yearly basis from 1996 to 2011.

Literature research on predictive modeling of lapse rates in the insurance industry led to the choice for predicting with Generalized Linear Models. There are different types of GLMs with applications dependent on the type and distribution of the included variables. For the modeling of lapse rates two GLMs are recited. The logistic regression model has been chosen to model binary lapse data and the linear regression model has been chosen, as a starting point, for the modeling of continuous lapse data. Application of the linear regression model provided formulae for forecasting future lapse rates.

A total of 14 variables - including lagged variables, the first difference of lapse rates and lagged lapse rates – have been analyzed on their relationship with the average lapse rate. Of these variables “buyer confidence”, “first difference in lapse rates”, “gross domestic product”, “inflation”, “reference market rate” and “return on stock market” have proven to be valuable for modeling. Buyer confidence was also of significance in the research of Kiesenbauer

\(^{41}\) The performance of the model has been measured in terms of accuracy, on which it has also been compared.

\(^{42}\) The significance of the variables has been tested by well-chosen statistical measures using a 5% level of significance.
(2011) which is also one of the only studies which took lagged explanatory variables into account. The inclusion of reference market rate is analogous to the results of Cox & Lin (2006) who included the variable complemented by growth in GDP and unemployment in their predictive model for the aggregated lapse data of the US market. The FSA (2010) stated that inflation accounted for about 60% of the lapses in their research, which was experienced to be one of the most significant variables throughout the research. The exclusion of the binary crises variable indicates that the shocks in 1999 and 2008, if present, were (significantly) accounted for by the macro-economic variables, analogous to the expectancy of Kim (2005).

A difference between the developed model and those posted by quoted researchers is in the number of significant drivers. The general amount of drivers used in literature appeared to be between 4 and 8, which is quite different from the one or two variable models of this research. The number of used variables and the r squared statistics were lower than those of Milhau et al. (2010) and Briere-Giroux et al. (2010), this indicates that there might be room for improvement. Although there is room for improvement of the models, every model provides understandable results and they are all more accurate than the prediction of future lapse rates by assuming the mean of the lapse data to be a good predictor.

The performance of the model, when compared with the standard formula, is dependent on the level of expected lapse rates as well as the relative error of the predicted values. The level of the expected lapse rates greatly influences the standard formula, whereas the relative error of the predicted values is one of the great contributors to the prediction interval of the developed model. The influence of the mass lapse event remains unknown and should be analyzed as well.

Additional research indicated that the assumed difference in lapse rate (development) per product group is justifiable and that the lapse rate per policy duration deviates in level as well as in development. These results correspond to those of Cerchiara et al. (2008) who used lapse data of an Italian banc-assurer of the period 1991 to 2007 and came up with, amongst other things, policy duration and product class as explanatory variables. They also found policyholder age and competitiveness of prices to be important, which were not tested. Other research with congruent results is that of the he FSA (2010) which indicated that a difference exists between the lapse behavior of contracts with single and regular premiums.
All these findings should be interpreted with caution since assumptions have been made to justify the applied linear regression model. One of the assumptions which have not been extensively discussed is that the relations between relevant variables are assumed to remain existent at future moments. The standard formula avoids this kind of problems since it does not use any explanatory variables, which is also its weakness. The main difference between the linear regression model and the time-series model which is in development is in the type of variables which are included. Whereas the time-series model focuses on past lapse rate values, the regression model may also includes (future) values of other variables.

### 7.2 Recommendation

The main limitation of this research was the small amount of data and their level of detail. Since the results of this research are promising it is recommended to extend the research to other parts of the company. To increase the statistical strength and accuracy of the inferences that can be made it is recommended to examine the lapse rates at a policyholder level. This will provide the option to include more variables in the modeling in order to explain individual behavior. It should also be verified whether the data are pure and not contaminated, since some data is expected to be contaminated due to policy mutations (Achmea, 2010b).

The promising results do stimulate monitoring of the model to check its reliability and accuracy. One of the ways to analyze the model is to compare the results with the time series model which is under development. If the results are satisfactory it should be investigated how the model can be implemented in such a way that it fulfills the Solvency II use test.

Finally it should be noted that some confusion about the definition of lapses remains. This is why it is advised to define and implement a general lapse rate definition to facilitate future research and minimize the exposure of lapse rates to contamination.
REFERENCES


APPENDICES

Appendix 1: Risk map
Appendix 1: Risk Map

In figure 8 most of the (sub-) risk categories which are mentioned in Solvency II and internal reports are denoted. The tree structure provides a clear example of which variables fall into which risk category.