Abstract

Defaulting suppliers form a great risk for many firms. In this thesis we quantify the individual risks and losses of supplier defaults using structural credit risk models, tailored to the practitioner without subscribed database access. Supplier portfolios in the German automotive industry are used both to explore how such an implementation works, and how credit risk theory can be used to accommodate distinct characteristics of supplier default risk. Modeling elements based on the largest commercial application of structural credit risk models, Moody’s KMV model, are verified and tested for practicality when applied to supply chains. We provide examples of how commonly encountered assumptions from practice can be modeled with the use of the models we propose. We then construct a portfolio model using a multi factor dependency model in which loss distributions are estimated using Monte Carlo techniques with importance sampling. Finally we illustrate some settings and examples of modeling elements unique to supply chains.

Keywords: supply chain risk, supplier default, credit risk, structural credit risk, Merton Model
Preface

It was probably somewhere during the summer of 2009 when I first decided I wanted to do a semester abroad. The obvious choice for me was Switzerland and a mere six months later I found myself doing courses at ETH Zurich. Now, over a year after I left Switzerland to finish my courses, I am writing these words about one hundred meters from where I attended these very courses and I am happy I have the chance to do so.

Writing this thesis does not just mark the end of my master’s degree in Industrial Engineering and Management at the University of Twente, it also marks the end of my time as a student. Doing so at the Chair of Logistics Management at ETH has allowed me to write a thesis outside of my own university while nevertheless becoming familiar with the workings of a university and has broadened both my knowledge and scope.

I would like to thank Christoph Bode for the ideas and comments that have helped make this thesis possible. My gratitude goes to Berend Roorda who has adequately helped me structure my ideas and guided me through this thesis as well as Peter Schuur, who helped me question what I thought was clear. I would also like to thank Dirk Weenk for carefully checking it.

Last but certainly not least I would like to thank my family and friends, especially my parents and girlfriend for their support during this thesis. Without you it would not have been possible.

Zurich, January 10, 2012
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Chapter 1

Introduction

The last few decades have yielded a vast amount of literature and models in the area of financial engineering, mostly with the purpose of quantifying risks of derivative financial products. Only more recently researchers have begun to apply risk models to the field of operations research and, more specifically, to supply chain management.

It is no surprise that supplier defaults can cause severe problems regarding production, ranging from loss of sales to setbacks in reputation and the loss of customers. One can only hope to make informed decisions and use the means at hand effectively by quantifying these risks and subsequently dealing with them.

Recent research has picked up on this need (e.g. Bode & Wagner; Babich), but most advanced approaches are based on models for defaultable claims such as credit default swaps, and much less so for areas where such instruments are sparse as in operations management.

This thesis was written with the purpose of expanding the current body of literature to model risk within a supplier portfolio using structural credit risk models.
In this introductory chapter, we outline this thesis.

1.1 Background

This thesis is inspired by ongoing work by Bode and Wagner,\textsuperscript{11} carried out at the Swiss Federal Institute of Technology (ETH Zürich) in Switzerland. That work models the supplier portfolio using the CreditRisk+ model, a so-called reduced form model that makes no assumptions about the mechanism of defaults\textsuperscript{a}. Defaults in the Bode-Wagner application are characterized by the Poisson distribution, while dependencies are modeled using so-called systemic risk factors, which are common economic conditions among sets of suppliers. A distribution describing incurred losses is created by combining the probabilities of default, which are derived from the ratings from credit rating agencies, with estimates of losses based on survey data.

The available data applies to the German automobile industry. Data on the supplier portfolios of the Audi A6, the BMW 5-series and the Mercedes E-class is used. For some of these suppliers, credit ratings from rating agencies is available, as is access to Amadeus (a private firm database with financial information), Compustat and Bloomberg.

1.2 Relevance of research

Research on supply chain portfolio risk is still very recent and there are only a small number of papers on the topic. Aside from a few cases\textsuperscript{6,7,56} many focus on single supplier defaults from a static perspective, using only balance sheet information or data that is hard to obtain in practice (such as bond prices). As a result, there is no effective and efficient method of determining if a portfolio of suppliers is particularly risky. From this, combined with the promising results obtained from structural models,\textsuperscript{20} stems the origin of this thesis.

Most of the work done in this thesis is directly relevant for decision makers when trying to determine how to allocate their resources. Knowing where the default risk in a portfolio is buys a manager time to undertake action, such as re-sourcing, or perhaps prevention. Having a good model allows one to determine how risky using a supplier’s services is, determining the effect it will have on the portfolio while comparing it to the costs of alternatives. For example, effects of possible large swings in raw material prices or subsidies can be quantified and acted upon. Applying a simple yet effective model usable in practice is key to this thesis.

In this light, the Merton model and its variants offer some clear advantages over other

\textsuperscript{a}Terminology is explained in the next chapter.
instruments. The most prominent industrial application, Moody’s KMV model, has proven to have power of default prediction with ample warning.\textsuperscript{52} Using this class of models, which incorporate equity prices reflecting (at least partially) the future state of the firm, means not having to rely on the availability of (possibly outdated or biased) credit ratings. Such a warning might prove invaluable in operations management. Indirectly the models can be used to analyze various assumptions and situations and examine their influence on the portfolio of suppliers in terms of risk. This cannot be done when default probabilities are derived from credit ratings.

The application by Bode and Wagner maps credit ratings to default probabilities to obtain default probabilities for single firms. It then uses an approximating distribution to construct a portfolio model. While this has the advantage of not having to explicitly model the causality of a default, the suggested structural approach (an approach that does model the causality of default) has been shown to have its merits such as computing default probabilities using data other than credit ratings or bond spreads, the modeling of various assumptions and explicit definitions of default. The portfolio version of such a structural model can be constructed in a simple fashion, using the individual results (constructed including specific assumptions) as input, but can also come to depend on common factors. The data requirements for structural models are especially comforting, as credit ratings and bonds are not available for the vast majority of firms.

1.3 Research objectives

The risk we are modeling in this thesis is default risk. This is in sharp contrast with operational disturbances and we should not confuse the two. Regular operational disturbances occur when a supplier does not deliver its products due to bad planning, either by itself or by a second tier supplier, but not caused by default. What we do not attempt to do is a develop a grand unifying theory of supply risk. Inventory management has the mathematics of safety stocks, echelon stocks and even theories on multiple-tier supply networks (e.g. the bullwhip effect). Such theory can be found in for example Silver, Pyke and Peterson (1998) and Axsäter (2006) and is not part of this thesis.

The approach by Bode and Wagner to obtain default losses and probabilities for a supplier portfolio assumes the availability of credit ratings or bond prices and does not allow for the testing of various assumptions. It also assumes the availability of credit ratings. To counter data requirements and allow for the modeling of specific assumptions we therefore propose to model the problem using a structural model (like the Merton model) as a basis, which does allow for the modeling of these assumptions in order to address this gap in research.

Resulting default probabilities and loss distributions in this thesis are used for risk management as opposed to instrument pricing. This in turn is used as decision support:
acting upon the information provided by this analysis. It is important to note that we are not trying to quantify exact costs because an order of magnitude value for risk is sufficient for the purpose of risk management. A mere relative ordering would however be insufficient, as large differences can occur that mean action should be undertaken as opposed to small differences (at a low absolute risk level).

This thesis is then twofold, as are its goals.

1. The first goal is to provide a way of using Merton’s model to quantify order-of-magnitude risks of default and losses for individual suppliers, including specific assumptions: to produce default probabilities verified by testing the model’s default probabilities against the ratings of rating agencies and devise a method of estimating the corresponding losses.

2. The second goal is to develop a model that is able to quantify the risks of a portfolio of suppliers by incorporating the individual probability distributions, thereby creating a default probability and loss distribution on a portfolio level. We then examine the effects of changes in individual levels and supply structures on the portfolio.

1.4 Outline

From here on forward, we start by introducing some of the basics of credit risk modeling and examine a selection of the current literature in chapter 2. In this chapter, we outline which models are useful applications to model supply chain risk.

In chapter 3, we address modeling the single supplier problem in order to approximate the probability of default values implied by the credit ratings of rating agencies.

In chapter 4 we provide Monte Carlo simulation techniques to serve as a basis for chapter 5, but also to keep the flexibility of adding complicated components and to provide intuitive solutions that are easily understood. Having done so, we provide examples of how different assumptions might be modeled.

In chapter 5 we expand the individual model to obtain a probability of default and loss distribution at the portfolio level. Here too we use advanced Monte Carlo simulation techniques with the ones in chapter 4 as a basis. We include dependencies and provide examples showing how different aspects unique to supply chains might be modeled.

In chapter 6, we conclude this thesis and comment on further research directions.
Chapter 2

Credit risk models and the probability of default

It is clear that modeling supplier default risk is first of all about finding the supplier’s probability of default before we can even consider using more advanced methods. We use credit risk theory to accomplish this, with an emphasis on so-called structural credit risk models.

As models are never perfect, we should be careful in how we use them and what their assumptions are. In this chapter we explore what kind of models and extensions one might use to model the probability of default. Portfolio modeling itself as well as the theory is introduced in chapter 5.

We conclude by selecting a basis to continue with in the next chapters.
2.1 Modeling supply risk: Determining the probability of default

When modeling credit default risk, or in this case supplier default risk, what we mean is that we model the risk of a default and seek an expected value of the losses (denoted by EL). For the sake of clarity, let us introduce some terminology.

2.1.1 Credit risk terminology

The probability of default (PD) is the probability that a default of the firm of interest occurs within a fixed time span. Usually this is one year. In this chapter, PD is the value of interest.

We define the probability of a default as the probability that a default will take place within one year. In the case of single firms this is usually a Bernoulli distribution as shown in equation 2.1. In some cases, such as the case of a portfolio, it might also be a random variable with an underlying distribution.

\[ Y_i \sim \text{Bernoulli}(p_i) : Y_i = \begin{cases} 1 & \text{with probability } p_i \\ 0 & \text{with probability } 1 - p_i \end{cases} \text{ if supplier } i \text{ defaults} \]

Exposure at default (EAD) is the amount of money or effort we are in for with the other party. The fraction of EAD lost when that party defaults is called the loss given default (LGD). In the case of supply chain risk, the loss given default is less relevant than in credit risk, because we estimate losses directly. Should a default take place and a supplier stops delivering, we usually replace this supplier. We do not retrieve anything, but just incur the costs of the replacement. In such cases, the loss given default is simply 1.

Expected loss is defined as below.

\[ EL = EAD \times LGD \times PD \]

Risk managers are often interested in obtaining distributions of the losses or exposures that may occur. Such a distribution is called a loss distribution and we will come back to this in chapter 5.

2.1.2 Obtaining the probability of default

The models in this chapter are built around finding the probability of default (PD). We might stop and ask ourselves why such an elaborate method is used when there are tools such as balance-sheet based models and rating agencies.

Bankruptcy prediction models based on balance sheet information have been around for a long time and are simple. Examples are Altman’s Z-score, a linear combination
of balance sheet ratios, and Ohlson’s O-score. Because these depend on the numbers found on balance sheets, updates occur rather infrequently. Some are known to have a bias, which is why Shumway proposed a hazard model of a more dynamic nature.

Ratings from agencies such as Moody’s face the downside of infrequent updating too. More problematic is that many firms are deemed too small to be rated by the big agencies and that smaller agencies use ambiguous rating methods. This is one of the key arguments to perform this analysis: simply because there is no other data available.

Besides the above it might be useful to test various assumptions, such as big lawsuits, pending patents or takeovers and see what effect these may have on supplier default risk. The structural models would allow for such testing, whereas the other methods (especially those based on credit ratings) do not. Directly linked to that is the lack of understanding in what constitutes a credit rating, meaning ratings may be subjective or inconsistent over time. This may be less of a concern when using data from big credit rating agencies, though, as it does not seem likely for an individual or unspecialized group to outperform them.

2.1.3 Classification of credit risk models

Credit risk models can be divided in two main categories: structural models and reduced form models. The structural models explicitly assume a mechanism for the defaults that occur, while reduced form models are based on statistical relationships between variables without any assumptions regarding the mechanism. Mixture models are a mix of the previous two classes.

This thesis focuses on the application of structural models, but we briefly review an example of reduced form models to provide some contrast.

2.2 Structural credit risk models

Structural models are models in which a default is triggered by a stochastic variable falling below a certain threshold at some point in time. These models therefore explicitly specify the mechanism of default through this stochastic variable. Sometimes they are also called threshold models because default is triggered by a threshold, or alternatively firm-value models because defaults depend on the value of the firm.

The performance of these structural models has been shown to be robust: even the simplest models are often not significantly outperformed by more complicated models. Below we discuss several structural models, all of which are extensions of the basic Merton model.
2.2.1 The Merton Model

The Merton model is one of the most famous structural models and is illustrated in figure 2.1. It assumes that liabilities are claims on the assets (denoted by $V$) of the firm. Default depends on the level of debt (denoted by $D$) to be repaid within a certain time period. If $V$ is below $D$ at time $T$, a bankruptcy occurs.

Asset values are all assumed to be market prices. Note that such prices are not directly observable because balance sheets contain book values, while debt is often not publicly traded and certainly not very liquid. We therefore need to determine the value of the firm $V$. For now it is important to keep in mind that we do so by modeling equity ($E$) as a call option on a firm's assets. This is shown in equation 2.3, where $T$ denotes the end of the time interval of one year. This means we set the value of $E$ equal to the value of a call option on the assets $V$, with $D$ as the strike price. By inverting this problem we determine the unobservable value $V$ and this lies at the core of the model. Details can be found in the next chapter where we provide an application.

$$E_T = \max(V_T - D, 0)$$ (2.3)

We use stock prices and the number of stocks outstanding to determine the value of equity ($E$). Stock prices are by nature very much forward-looking because no-one should be willing to buy stocks with bad prospects, let alone ones that will cease to exist. Using stock prices containing such forward-looking information for the purpose of predicting defaults is an important part of the Merton model. Theory supporting this comes in the form of the efficient market hypothesis, which in its strongest form states that a price includes all possible public and private information. We do not claim this hypothesis holds, but do claim that it is very hard to outperform the stock market on average. Therefore instead of using historical data, we include future information using stock market data.

There is an established way of determining the value of the call option described above and the probability that it will be exercised. Obtaining a value for this call option on a firm's assets requires both the standard assumption of no arbitrage (the so-called 'no free lunch' argument) as well as the assumption that in case of no arbitrage the return is the risk-free (usually triple-A rated) interest rate $r$. The resulting differential equation is called the Black-Scholes(-Merton) differential equation. Its solution is written down in equation 2.4, where $N$ denotes the cumulative Normal distribution. Proof of the solution can be found in either of Hull’s (2009, 2010) books.

$$E_0 = V_0N(d_1) - De^{-rT}N(d_2)$$ (2.4)

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
\[ d_2 = d_1 - \sigma_v \sqrt{T} \]

\( T \) denotes the length of the time horizon that is considered, \( \sigma_v \) denotes the volatility of the assets. Subscript denotes time zero or time \( T \) values. \( N(d_2) \) is the (risk-neutral, more on that later) probability that the option is executed, something a rational mind would only do if the value of assets exceeds the value of debt.\(^4\) Exactly the opposite, where asset value does not cover debt, is our definition of default. The probability of default is then given as below.

\[ PD = N(-d_2) \quad (2.5) \]

Mathematically, the process pictured by the curvy line in figure 2.1 follows geometric Brownian motion, as seen in equation 2.6. This is the process on which the above solution is based and is what we use for the Monte Carlo simulations in later chapters. Many have argued that a random walk is not a good representation of how equity prices behave\(^43\) and while there seems to be some truth to that, we and many others\(^52\) consider it a reasonable assumption for the purposes of supplier default risk valuation.

\[ dV_t = \mu V_t dt + \sigma V_t dW_t \quad (2.6) \]

Important to note is that in the Merton model, the value of the firm at time \( T \) is considered to determine defaults. That means a firm can in fact recover from bankruptcy and it has not defaulted as long as its value is not below the threshold value at time \( T \). This ignores any values between 0 and \( T \).

An assumption in this model is that the asset-value process is not changed over this valuation period and that volatility\(^b\) and interest rates are stable. The notion of a stable capital structure also implies that when default probabilities are extremely high, times when firms are prone to changing their capital structure by for instance borrowing, the model may not work as well. As mentioned above the entire analysis is based on the premise of an arbitrage-free European call option.

As a result of the mathematical analysis above we have computed the risk-neutral probability of default by determining the probability that the option will not be exercised (in a risk-neutral world). Risk-neutral means we disregard any premium paid for extra risk, meaning a US Treasury bill offers the same return as a Saab stock\(^c\). The model is assumed to provide an accurate ranking of the default probabilities, but what

\(^a\)Unfortunately \( d_1 \) does not have such a simple interpretation. It is the factor by which the present value of contingent receipt of the stock exceeds the current stock price.\(^47\)

\(^b\)This assumption can be relaxed by the use of stochastic volatility, for instance GARCH-models. We will come back to this later.

\(^c\)At the time of writing, Saab is nearly bankrupt.
Figure 2.1: Four sample paths in the Merton model. The distribution on the right is measured at time $T$ and is a density function. $D$ represents the barrier, $V$ the firm value. Graph taken from Kay Giesecke.

is still missing is a real-world mapping of these risk-neutral probabilities\(^4\) which is why our application is slightly more complicated. We have a more elaborate discussion of risk-neutral probabilities later in this chapter.

2.2.2 The first-passage approach

The last paragraph illustrates a potential problem in the sense that firm value can go down to having almost no money without triggering a default. This is often not the case, as firms might be restructured as soon as they seem bankrupt even between time 0 and time $T$.

A way of including this is the first-passage approach. In this approach, a default barrier $B$ ($0 < B < D$) is defined and as soon as the assets break through the barrier at any point in time, a default is triggered immediately regardless of the value at time $T$.\(^{23}\) The process is illustrated in figure 2.2. Such a situation is common among exotic options and is referred to as a down-and-out option, which is a type of knock-out option.\(^{40}\)

The result below is taken from Giesecke.\(^{28}\) The barrier must be below the face value of debt ($B < D$). Otherwise bond holders run no default risk and should hence obtain no

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\(^{4}\)Interestingly, the price of the ’option’ is valid in both worlds. Moving to the real world, the growth of stocks change and the discount rate changes. In option pricing these two offset each other.\(^{39}\)
Figure 2.2: Almost identical to the Merton model, with the major exception that defaults now occur as soon as a sample paths dives below the barrier $B$. Graph taken from Kay Giesecke.

risk premium, because they can force the firm to bankruptcy while the money is still there. There is another peculiarity here. Although bond holders might force the firm to bankruptcy at some level below the face value of the bond, at time $T$ (when the value of assets $V$ is below the default point $D$ while it is above the barrier $B$) the firm should still default on the grounds of not being able to pay off its debt ($V < D$) to bond holders. If we only consider the barrier $B$ (which is below $D$), we ignore the fact that debt must in fact be paid off when it is due. The probability of default is in that particular case is as in equation 2.7.28

$$PD = N\left(\frac{\ln(D/V_0) - (r - .5\sigma^2)T}{\sigma \sqrt{T}}\right) + \left(\frac{B}{V_0}\right)^{\frac{2(r - .5\sigma^2)}{\sigma^2}} N\left(\frac{\ln(B^2/DV_0) + (r - .5\sigma^2)T}{\sigma \sqrt{T}}\right)$$

(2.7)

2.2.3 Jump diffusion models

The notion of a perfectly smooth and continuous market following the Normal distribution has been recognized to be a somewhat strong assumption.43 By dismissing large jumps often found in real data as anomalies, one runs the risk of underestimating the size of the fat tails of the distribution. An attempt to fix this is the jump diffusion model.
The first of such models was proposed by Merton himself,\textsuperscript{45} the timing of jumps are modeled using a (compound) Poisson distribution. Between these discrete points in time, a random walk is most commonly used to determine the sizes of the jumps. The use of a random walk means jump sizes are still independent. Downsides of the Merton jump diffusion model are that the distribution of the jumps is assumed to be Normal for its tractability\textsuperscript{e}. Double-exponential\textsuperscript{41} and mixed-exponential\textsuperscript{15} distributions have been suggested to allow for a wide array of modeling possibilities, with the idea of accurately replicating the volatility smile (a graph of the volatility that is implied by option prices, which shows volatility is not stable as implied by the Black-Scholes model\textsuperscript{40}) found in option pricing which is due to a skewness in the distribution.

Such models are mainly useful when reproducing more accurate credit spreads on bonds (see section on reduced-form models). The continuous modeling assumptions we use means the process slowly creeps towards default, which means that a default is largely predictable. This is obviously not the case in the real world. While this shortcoming might not be an issue when the maturity $T$ is longer (small chance of big enough jumps), it is certainly a problem when the maturity is short. Since our time window is always one year because of our application of risk analysis, we do not encounter this problem and have no real need for such models. We therefore do not use a jump diffusion model but feel the reader should be aware of these.

2.2.4 The KMV-Model and mapping risk-neutral to real-world probabilities

The probabilities we obtain in chapter 3 are risk-neutral and can not directly be used to take the expected value of losses. The KMV model has addressed this issue by using a database of historical default information. Before we explain this, let us take a closer look at what kind of results we obtain from the models above.

Risk-neutral probability measures explained

A risk-neutral probability is actually just the number of cents you lose on the euro because of the risk of default, in other words a value impact. It does not represent the actual probability of default and is only a probability due to its mathematical properties. It does reflect these actual probabilities up to some degree. This may sound fairly useless, but it is in fact not. In option pricing we simply do not wish to estimate the average growth of a stock since it is very hard to do so. In order to prevent having to estimate this, we have an artificially constructed probability measure where we hedge the derivative (in this case an option) by simultaneously holding the asset. By doing so we remove any exposure to the direction of the stock as these movements cancel out.\textsuperscript{57}

*Especially in first-passage models, it is to our knowledge impossible to estimate the exact distribution of the jumps under the barrier, unless there is a memoryless property such as in the exponential distribution.
By removing such a risk-premium from our considerations every asset should grow at the risk-free rate of interest, denoted by \( r \). The resulting ‘probabilities’ used are risk-neutral and merely used to arrive at the correct derivative price.

A way of illustrating risk-neutral probabilities is one in which payoffs can be replicated using a linear combination of other securities, which imposes a no-arbitrage condition.\(^2^9\) We present a simple example following Gisiger (2010). Assume a firm can take on three different values a year from now, those being low, medium and high. We have three derivatives whose payoffs are stochastic, where the word derivative is in the financial sense: it is an instrument which value depends on another asset. One derivative pays a euro if the state of the firm is low (called derivative one), another (derivative two) pays one euro when it is medium and a third derivative pays one euro when it is high. Arbitrage-free pricing, a condition enforced within these models, says pricing should be consistent in the sense that if a derivative that can be replicated using other items it has its price dictated. To illustrate, say we have a fourth derivative, one that pays one euro either when the future state is medium or high. Its price should be the sum of the 2nd and 3rd derivative since the linear combination of these does the same. If this is not the case, market forces (supply and demand) ensure that this is the case soon enough. This is the reason why we enforce such a situation. A risk-neutral price is simply one in which prices are determined by the unique solution of such no-arbitrage conditions.

Now our derivatives are dependent on the future states low, medium and high and each have a price. These prices of course will reflect (but not exactly represent) actual probabilities and are thus not independent of the actual values. Investors would be mad to pay money for a future state that never happens (0 percent chance), or to pay none for one that has a probability of occurring (nonzero probability). In this sense the actual probability and the risk-neutral probability are called equivalent.\(^2^9\) However, these need not represent the actual probabilities because investors are collectively risk-averse. This means they demand a premium for taking on risk (which means the chance of ending up in an undesirable future state) and exactly this is what we leave out in risk-neutral valuation. Perhaps they prefer to have the one euro when the state of the firm is low because their other assets are positively correlated with state of the economy, and it is worth more to them. If we normalize these state prices, formally called Arrow prices, to sum to one and let these be positive, they behave like probabilities and we can apply the math as such. Note that often we do not have the state prices and invert the problem, but what matters is the intuition behind risk-neutral probabilities. Now we see that if the price of state ‘low’ (the price of derivative one) in our example is 50 cents, the actual probability of ending up in state ‘low’ is usually lower than 50 percent because risk preferences are included. A risk-neutral probability is actually the number of cents you lose because of the risk of a default. In other words when considering Monte Carlo simulation: if we assume that there is no premium for risk, a risk-neutral probability

\(^2^9\)In the Black-Scholes-Merton model above we discount all future prices and this cancels out the rate of return, which is why prices are real-world.
is one in which more negative future paths are generated than might actually occur because investors fear these future states so much more.

From risk-neutral to real probabilities

Moody’s KMV model is an adaption of the Merton model, commonly referred to as the Vasicek-Kealhofer EDF expected default frequency (EDF) model. Actual estimates of 1-year default rates for firms with a similar distance to default (DD) are used to estimate actual default rates. The DD is the number of standard deviations by which assets exceed liabilities and is similar to using risk-neutral default probabilities. The mapping is shown in figure 2.3. The reason for mapping is that the Merton model (as stated before) produces risk-neutral default probabilities and as explained above, it is assumed to provide a default ranking that holds up in the real world, but not default probabilities that do. Moody’s KMV maps these DD scores depending on factors such as industry or region to produce actual default probabilities, using a database of historical defaults. The exact determination of distance to default and its mapping are proprietary, as is the data used to create such empirical EDFs.

Because of this empirical EDF, KMV now has real world default probabilities. It no longer assumes the return to be the risk-free rate, but uses the expected return instead. While this would work for our purposes, KMV’s customers often need risk-neutral probabilities for pricing purposes. The empirical results are adjusted by using Kealhofer’s model, in which the systematic return is estimated using the CAPM model. Instead of using the risk-neutral probabilities by the model (which assumes the random walk), the model is tweaked to reflect reality more closely.

Differences with the regular Merton model are present too. Moody’s model is known to use the first-passage approach. Furthermore, the modeled option is a perpetual call option instead of an expiring one and there are more classes of liabilities besides short-term liabilities and stocks. Other factors are that it uses two types of volatility (empirical and modeled) to prevent overestimation of the asset volatility of new firms for which equity is always very volatile. We explore some of these elements in the next chapter.

2.2.5 Other modeling ingredients

We briefly show some counterexamples of the assumptions of the Merton model. A Gaussian random walk assumes independence across the differentials between values, that these are normally distributed and continuous. Whether such assumptions hold is questionable, as some have shown. Single firms have private information affecting its future state that can largely be contained within the firm for quite some time, as illustrated by cases such as Enron. Introducing elements such as a thicker-tailed distribution (a larger probability of big changes in V), (large) discontinuous changes and
Credit risk models and the probability of default

a long-term memory can be implemented by the use of fractals, but its mathematics rapidly become complex and we have not found good applications.

There are somewhat simpler and more tractable ways to include some of the above. Elements such as modeling an increasingly uncertain state of a firm can be modeled by making volatility stochastic instead of constant, for which GARCH(1,1) is the most common model. The fractional Merton model, which models a long-term memory, includes a Hurst component for the modeling of dependencies and serves as a kind of memory component. When using this for option pricing a form of arbitrage is possible, unless proportional transaction costs are included (or of course assuming other market imperfections). We do not use such models in this thesis.

2.2.6 Indications of the performance of the models

Because we cannot apply every model found in literature, we briefly present some of the results others have found to provide a sense of direction. There is evidence both in favor of the default predicting capabilities of more complex structural models such as in Saunders (2010) and Chen (2006), while others such as Bharath and Shumway have found them to be useful but not even much more than heavily simplified versions. Because of the success of simple models, the need for an understandable application for practitioners and limited data availability, we start with a simple model in chapter 3 and expand it as needed.
2.3 Reduced form models

As opposed to structural models, reduced form models do not specify a mechanism for defaults.\textsuperscript{14} Instead, a default occurs without warning at a default rate, an intensity, that is usually calibrated using market prices. For this reason, such models are also referred to as ‘intensity models’. At the core is a statistical relationship, which eliminates the structural equations and leaves us with a so-called ‘reduced form’. Research preceding this thesis is based on the CreditRisk+ model, which is a reduced-form model.

We only discuss obtaining the probability of default of a single supplier. Portfolio models are discussed in chapter 5.

2.3.1 Default probabilities in the Bernoulli and CreditRisk+ Model

One of the well-known portfolio models of this class is the CreditRisk+ model as applied by Bode and Wagner (2011). No assumptions are made about the causes of default.\textsuperscript{14} In terms of the default probability distribution, default rates and their volatilities are considered inputs, as opposed to the structural models. We immediately see why these models are said not to explicitly model the causes of risk.

Obtaining PD's for every firm is commonly achieved by mapping credit ratings from rating agencies to default probabilities. It is important to observe that in the highest category (triple-A), there have historically been no defaults.\textsuperscript{10} This does not necessarily mean that this investment grade rating is riskless\textsuperscript{8}.

A calibration method by Bluhm, Wagner and Overbeck\textsuperscript{10} revolves around fitting a line through the means of all rating classes (using regression on a logarithmic scale). This assigns a small positive probability even to triple-A investments. We use this technique in a different setting in the next chapter.

2.3.2 Common methods to obtain reduced-form default probabilities

For the sake of explanation and completeness we outline another commonly used method to illustrate reduced form models. These are in sharp contrast with the structural models that are mainly used in this thesis.

At the basis of the common approach lies the notion that without additional risk of default, instruments should earn the risk-free rate of return. Knowing the price and face value of a 1-year (risky) bond, we can then solve for the probability of default. Say

\textsuperscript{8}This is false by definition, as these are calibrated to two basis points:\textsuperscript{10} on average, a default once every 10,000 years.
the unadjusted return equals $y$, whereas the risk-free interest rate is $r$. The PD is then defined as in the following equation.

$$PD = \frac{y - r}{1 + y}$$

(2.8)

Such a result can of course be expanded to include multiple years, multiple states of creditworthiness and time-varying loss-given default. None of these is particularly useful to us since all of them require the input of bond prices and we do not have those. Especially for smaller suppliers these prices are simply unavailable because the instruments do not even exist. Note that in this section we do not make assumptions as to why a default occurs, we simply derive the intensity from the state prices.

These probabilities are again risk-neutral since the prices are exactly the state prices we discussed in section 2.2.4. Risk preferences are, as before, included in the price. Whereas usually we do not have such prices, here we assume we do. There is much more to these models than this section may lead one to think. For a good overview see McNeil, Frey and Embrechts.44

2.4 Conclusion

The Merton models we proposed above have some downsides, such as the assumption of a flat volatility surface and a flat interest-rate curve. We know this is unrealistic simply from the fact that practitioners use notions such as implied volatility and more complex interest rate models. But do not attempt to reproduce reality, but instead need a satisfying way to obtain default probabilities.

We conclude from results from others8 that a simple version of the structural model is sufficient and take the KMV model as a basis, because it is both simple and because it is the most important industrial application of the structural model. This model’s output is used to price credit derivatives and seems to perform sufficiently well,52 so it is certainly good enough for our application.
In this chapter we model supply risk using the previously explained Merton model as a basis, a model that quantifies the default risk of a company by modeling it as a European call option on its assets. As concluded in the previous chapter we use the elements of Moody’s KMV model as a basis for modifications.

One of the strengths of the Merton model is its simplicity and as we have seen it can easily be expanded to address much more complex issues. In this chapter we consider only simple individual default probabilities.
Chapter 3

3.1 Input data for the structural credit risk model

What exactly is needed depends on the complexity of the model we wish to apply. We have chosen the basic structural model and components of Moodys KMV. At the very least, this means we need each of the following items:

1. losses in case of default (exposure at default);
2. the time horizon;
3. debt values;
4. the default barrier;
5. the risk-free interest rate;
6. asset values;
7. asset volatilities.

3.2 Obtaining data to quantify losses in case of default

The exposure-at-default (EAD) is probably hardest to measure. We spend a considerable piece of this chapter on it, because it is one of the key input values.

When a bankruptcy occurs there are generally three ways in which firms respond.27 Obviously there are many possible combinations, of which we discuss some of the most common ones. One of the simplest solutions that comes to mind is to just immediately find a different supplier. When multi-sourcing is used, this is not a big challenge (though we may want to restore the situation by replacing the bankrupt supplier anyway). In the case of single sourcing it is often time-consuming to find a different supplier, let alone when for instance the tools and machinery are proprietary or products are patented. Another way of dealing with bankruptcies is to help the supplier through the difficult time by providing financial support. This type of aid is not always as voluntary as it seems. An example of this involves Edscha, one of the suppliers analyzed in this thesis that supplied sun roofs for BMW, which found itself supporting Edscha financially to ensure the delivery of sun roofs for its soon-to-be-released Z4. Lastly, the customer can choose to operate the supplier’s equipment itself. All of these are usually temporary measures to bridge the time needed to find a new supplier. Then there are cases where customers have the time to help restructure the supplier.

With so many possibilities it is hard to decide what the exposure-at-default is. We have considered a number of methods to model such losses using the information above. In some cases stochastic components such as geometric distributions (modeling the first
Modeling the Merton model for a single supplier

failure) were considered, and we considered game theory (how much would other cus-
tomers of this supplier contribute) to incorporate the actions of other actors. After
discussing such models with practitioners these were found to be based on assumptions
that were too strong and we did not have access to the resources to research these.

We therefore define exposure-at-default as the costs incurred when doing nothing to
prevent a default. In that case you must find a new supplier. This is often the worst
case scenario since it requires building up a new relationship with a new supplier, for
which the associated costs are high. What we are measuring are switching costs while
assuming there is always another supplier (albeit at high costs in some cases). This
serves as the basis for the next section.

3.2.1 Measuring exposures

From the previous paragraphs it becomes clear why this is distinctly different from credit
risk. Whereas in credit risk you (hopefully) know exactly for how much money you are
in, in supplier default risk you do not know the costs of resourcing, restructuring and
possible bridge period payments to accurately determine switching costs. Such data is
not kept and can at best be estimated. We hold little hope of obtaining precise numbers,
but an order of magnitude estimate is sufficient to judge the impact for the purpose of
risk management.

Bode and Wagner approach

The easiest way that comes to mind is to simply ask experts to rate the switch costs on a
simple scale. This is exactly what was done in a survey carried out by Bode and Wagner
in 2007. For every item or product that is supplied, there is an estimate regarding the
hardness to replace this supplier, ranging from 1 through 5. The assumption is that
these values are a good indicator for the costs of switching, so all of these are on the
same scale. Estimates are provided per component due to the fact that one supplier can
deliver multiple products. In case of bankruptcy, we must of course find a supplier for
each of these components and it is very likely that no single supplier can deliver the same
portfolio of components. The good news is that under the assumption we can always
find another supplier, we have a worst case bound for our costs. We use the replacement
costs, our worst-case bound, as an approximation for the costs.

Such an approach has its problems. Not unsurprisingly, category 5 (meaning experts
rated it ‘5’) switch costs are nearly unimaginable for managers up to the point where
they have no idea of the costs, while category 1 switch costs appear to be quite com-
mon. From this perspective, this scale is clearly not linear. Some research suggests that
humans think logarithmically instead of linearly. That means the ratings should
Chapter 3

Figure 3.1: A rating scale to measure exposure. Intervals can be smaller as long as they are logarithmic.

probably be interpreted as order-of-magnitude estimates instead, but how exactly remains unclear. The question is how we should map these and there appears to be no clear answer.

In addition to that, humans are generally not very good at relatively ranking many things at the same time. The portfolio contains 150 suppliers, which means these should be considered in relation to each other: A is better than B, B is better than C, so where does that put D? Such complexity makes the accuracy of the data questionable. The Analytic Hierarchy Process (AHP)\textsuperscript{51} is a good tool to handle small problems and to verify the choices, but is still quite cumbersome in this case due to the problem size.

Our approach

We suggest the use of a base-10 logarithmic scale on which a selection of knowledgeable individuals rate the exposures on a component basis\textsuperscript{a}. In case of doubt, a midpoint can be selected. Such a scale looks like the one in figure 3.1. This scale has some advantages over the other methods. First of all, it is a very simple means of estimating without much analysis. Secondly, it makes clever use of the logarithmic thinking inherently present in humans.\textsuperscript{21,46} Thirdly, it avoids many comparisons to be made among all of the options, as the figure relates to the physical world and is an absolute estimate. Fourthly, by asking individuals we can produce a confidence interval regarding the estimates.

A disadvantage is that there is no way of telling what experts are thinking of when making these judgments. Perhaps they overrate an external effect (market size), or an internal effect (complexity of the product, volume). By asking more than one expert, the chance of having such a bias is lower. If a bias to one factor is suspected, one could ask ratings on such factors and see how strongly each value correlates with the switching cost value.

We provide an alternative that we think is too cumbersome to apply here, but might help some who have difficulties quantifying exposures. One might list the historical supplier defaults that have occurred in the past and determine the switching costs of these. We can then compare the situation of current suppliers to these cases and assign them to these cost estimates according to their likeness.

\textsuperscript{a}As it turns out from interviews, people in the industry are accustomed to thinking on a component basis, hence the choice.
Example. Intier supplies (among other items) door handles, pillars and carpets. Based on the absolute logarithmic scale above it is estimated that carpets cost €10,000 to replace, door handles €100,000 due to some redesign and pillars €1,000,000 because there is a small supply market and the parts are very specific.

3.2.2 Aggregation depending on the cost driver

We have suggested a way to measure component exposure by estimating the cost to find a different supplier on a logarithmic scale, but it is not a component or product but its supplier that runs the risk of default. There are many possible ways to aggregate these component scores to supplier scores. For the sake of simplicity, we discern resource use from time as the main cost driver.

In case of resource use, we have differences in the both the number of items (basic search and replacement costs per item), as well as in the values themselves (hardness to replace). We respectively refer to these as portfolio size and portfolio hardness. The sum and the arithmetic mean assume both linearity of the scale and additivity. We are working with absolute values, so we can sum all of the components in the portfolio to obtain an indication for the supplier replacement costs. By summing we incorporate both portfolio size (more products lead to a higher score) and ‘hardness to replace’ (some values are higher than others). Denoting $c_{i,\text{supp}}$ as supplier $i$’s total exposure and $c_{i,c,\text{comp}}$ as component $c$ of supplier $i$’s exposure, we get equation 3.1.

$$c_{i,\text{supp}} = \sum_{c=1}^{n} c_{i,c,\text{comp}} \quad (3.1)$$

Another point of view is time as a cost driver. One can argue that there is a double count of work activities when using the summation method. In some industries most costs are generated by waiting for component availability or (machine) redesign. In such a case we can consider the maximum cost component: we then assume the cost to replace a supplier is the cost of replacing the supplier for the component that is most difficult to replace. This is denoted by $c_{i,\text{supp}} = \max_{c=1,...,n} c_{i,c,\text{comp}}$.

Example. For Intier’s example above a resource-driven estimate assuming scale advantages for the components would be €10,000+€100,000+€1,000,000 which is €1,110,000. A time-driven approach assuming design time is the bottleneck is max(€10,000; €100,000; €1,000,000) which is €1,000,000.
3.3 Time, debt, default barriers and interest rates

In this section we consider the inputs that apply to all firms, albeit relative to some firm-specific value. We do so by looking at Moody’s KMV application and analyzing the effects on the models where no clear choices can be found.

3.3.1 Time horizon of the model

The time factor $T$ determines which time horizon is considered for the default probability. A period of one year is most commonly used in industry and we will be doing so here, too. But the fact that it is industry standard is not the only argument. First of all, remember that we are trying to model supply chain risk and not credit risk. When measuring immediate supply risk, we do not care about long-term risk as much as we care about medium length time horizons. Ten years from now we may be using entirely different suppliers altogether. A medium length time interval is more representative of the actual supply chain risk, while a very short time horizon may not provide ample warning.

3.3.2 Debt levels

Debt levels can be taken from the balance sheet. A firm defaults at (or before) time $T$ because its value is below its default point $D$. We must include at least the value of actual debt to be repaid within our horizon of one year. On a balance sheet this is called 'current liabilities' and it is this value that we include. However, such a value largely measures a firm’s liquidity.

Therefore we need to incorporate the long-term view too. Payments will have to be made on a supplier’s debt and the value of the firm should cover at least a part of it. In case the outlook is negative in the long-term due to high levels of long-term liabilities, the probability of a default will definitely be increased since future earnings are unlikely to cover debt. Using such an argument would lead one to conclude that all debt should be included. However, since these liabilities are long-term, firms have some leeway in trading this debt and hence some operate with negative net worth. Moody’s KMV adds part of the long-term debt, as in formula 3.2, and states that the default point $D$ generally lies between short-term and the total liabilities. It successfully tests its model using the value of 0.5, so we choose to use it too.

$$D_{\text{firm}} = D_{\text{current liabilities}} + 0.5 \times D_{\text{long term debt}}$$ (3.2)
3.3.3 Default barriers

The default barrier is used in the first-passage model. Its level should not be too high as debt is partially made up of long-term debt, and it is unreasonable to assume that a bankruptcy can be enforced when the (unobservable) asset value is still comparatively high. Since we have no data to calibrate it, we assume it is a fraction of debt.

It turns out that if the default probabilities are modeled as such, the results are not all that sensitive to the value of the barrier. What we expect is that the first passage model is different from the basic Merton model when asset volatility is low and the relative gap between debt and asset value is small. In such cases, chances are higher that the value dives under the barrier and resurfaces above the strike price, the debt $D$, at time $T$.

We have plotted the results under various conditions in figure 3.2 and see that this is the case, indeed, but differences are not as high as one would expect in commonly encountered situations. In order to have any effect, $B$ should be a fraction of $D$ over approximately 0.8. We tested values ranging from 0.90 to 0.9999, but these only compress the graph and do not discern any more among these. We used a fraction of 0.95 to maximize the chances of it having an effect, while not violating the economic interpretation presented in the previous chapter.

3.3.4 The risk-free interest rate

As for the risk-free interest rate $r$, we use the 1 year rates on US Treasury bills\cite{48} (T-bills) in 2007. This turns out to be 3.5 percent at the end of 2007.
Figure 3.2: On the left side are 3 graphs with a debt/asset ratio of 0.2, the lowest common number in the dataset. On the right side this ratio is 0.5, which is about the average for the set (with a maximum of 0.7). From top to bottom, we have a volatility of 0.25, 0.20 and 0.15, covering almost the entire range of values commonly found for our 42 public companies. For any barrier value under eighty percent of debt, the two models are virtually exactly alike. Differences are largest when there is relatively much debt compared to assets and when volatility is high. Values are low due to the characteristics of the random walk (see results section later this chapter). The blue line is flat since the Merton model is obviously independent of the barrier.
3.4 Firm-specific data for public firms: Asset values and volatilities

Before starting off we again contrast between supplier default risk and credit default risk. In credit default risk you buy a bond of a specific company and know of which company that is, including a credit rating. In supplier default risk, you must stop and ask yourself who is financially and operationally responsible for your orders. So whose balance sheet are we going to use, the parent’s or that of the subsidiary we are doing business with?

Regardless of legal obligations, we assume a holding company of any sorts will discontinue an unprofitable product group. A default is a good reason to discontinue activities. Based on that alone we would therefore advise to take the subsidiary data for the analysis. Rating agency equivalents for these small private enterprises are obtained from ModeFinance (Trieste, Italy). But in many cases and in full contrast with it, customers (especially those in the automotive industry) demand a comfort letter from the supplier’s parent assuring it of its financial backing. This changes the previous conclusion. As the parent is generally assumed to be stronger than the subsidiary, we instead advise to choose the parent company as the unit of analysis in such cases.

In multi layered ownership structures we do not know which parent issues the letter of comfort, so we use the highest unit within the structure that is in the same line of business as the subsidiary under analysis. This parent was found using a snowball search in the Amadeus and Bloomberg databases. The argument of these being in the same line of business was used to prevent extreme distortions of reality such as firms within an unrelated holding. This is not how one would generally tackle such a problem because practitioners know the specifics of their situation. On the plus side, the quality and completeness of data was found to be better at these higher levels. In the CreditRisk+ research by Bode and Wagner choices were not made explicit and the first available dataset was picked for illustration purposes.

3.4.1 Asset values and volatilities using Merton’s and the first passage model

Remember that theoretically the Merton model is based on the notion of a call option on a firm’s assets with the strike price being the firm’s debt. That means the value of the current assets should be the outcome of filling in the European call option pricing formula with maturity of $T$ years. As soon as the asset value drops below the level of debt that has to be repaid, denoted by $D$, a default occurs and our option is worthless.

The problem is that asset values, as well as asset volatilities, are not a directly observable quantity. The reason is that we need market values instead of book values, for as long as a company is still worth more than its debt to be repaid, it is technically not bankrupt. Even for publicly traded companies we still cannot observe asset prices.
and volatilities, but we can observe equity prices and volatilities. We take the standard deviation of the daily return as shown in equation 3.3, after correcting for stock splits, dividends and other class actions. Here, $r$ denotes the return while $p$ denotes the stock price.

$$r_i = \ln(p_i/p_{i-1})$$  \hspace{1cm} (3.3)

We take the standard deviation of these returns and multiply that number by the square root of the number of trading days (often 252, but it depends on the market in which they trade). Yahoo Finance does this for us, but has been found to miss stock splits and, often less problematic, dividend payouts in data used in this thesis on numerous occasions. Such misses may generate severe overestimations of equity volatility, so beware of high volatilities. Common ways to solve this is to manually correct for the class action at hand (which can be hard in the absence of data for an abnormality), or to simply remove these data points from the data set. The latter has been found to have virtually no effect at all.

From Itô’s Lemma which states that

$$\sigma E_0 = \frac{\delta E}{\delta V} \sigma V_0$$  \hspace{1cm} (3.4)

we have that $\delta E/\delta V = N(d_1)$ as can be obtained by taking the derivative of the equation below that we know from option pricing,\(^{39}\) which is

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2).$$  \hspace{1cm} (3.5)

Combining it with equation 3.4 leads to the following system of equations.

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

$$\sigma E_0 = N(d_1)\sigma V_0$$

with:

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma^2_v)T}{\sigma_v \sqrt{T}}$$

$$d_2 = d_1 - \sigma_v \sqrt{T}$$

This nonlinear system of equations can easily be solved using numerical methods. We have implemented this in Matlab as a function, of which the code is made available in
the appendix. A similar method is the implementation in Excel using the solver\(^b\).

A simplified interpretation in place, though, to shed some light on why this works. Debt is intertwined with equity: as soon as equity is taken out of the firm, such a firm is more leveraged, meaning an increased chance it defaults on its debt. This decreases the market value of debt, meaning debt depends on equity. Equity itself depends on the asset value of the firm, namely if it exceeds the value of debt, this is the value of equity. That means both are derivatives on the underlying assets. This lies at the basis of the approach followed above.

When realizing such an influence is present, we should beware of faults in our data. With that in mind we study the sensitivity of an error in inputs for the formulas above. We have Matlab do it for us in figure 3.3. We see that volatility is the most sensitive parameter when D/E is low. In those cases we double check the inputs.

**Example.** Taking Intier again, we have \( D = 3,879,860,000 + 0.5(715,220,000) \) which is €4,237,470,000. Its market capitalization \( E \) is €6,402,421,200 (D/E is not especially low judging from the graphs) and its volatility \( \sigma_E \) over 2007 is 0.2638. After solving the simultaneous equations using the Excel solver we have an estimate for firm value which is \( \sqrt{V_0} = 10,494,000,000 \) and the volatility is \( \sigma_V = 0.1609 \). Using the formulas on the previous page we find \( d_1 \) to be 5.93 and \( d_2 \) to be 5.77. Now the probability of default is \( \mathcal{N}(-d_2) \) which is 3.92591 * 10\(^{-9}\). This seems low but it is the risk-neutral default probability. The first-passage model requires the barrier after which we can apply the formula in chapter two. As implied by the analysis of the barrier done before, this leads to the same default probability.

\(^b\)Simply minimize the sum of the square of the result of both formulas to be zero using the nonlinear solver.
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<td>1,300,000,000</td>
<td>11.82</td>
<td>72,600,000</td>
<td>72,500,000</td>
<td>10,398,300,000</td>
<td>0.86</td>
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<td>0.86</td>
<td>10,398,300,000</td>
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Table 3.1: The input data for 42 public companies in the supplier portfolio.
Figure 3.3: The top two figures show sensitivity of equity volatilities by varying these. In the left top figure, we see that a higher D/E ratio means our input is less sensitive. The more equity there is relative to debt, the more sensitive the asset volatility is to our equity volatility. This makes intuitive sense. On the contrary, asset values hardly change as seen in the figure on the top right.

The bottom two figures vary the D/E ratio and examine the effect on asset volatility for fixed equity volatilities. For higher volatilities, the influence is clearer in the output and the output is more sensitive to it. Such a plot is another way of illustrating the above plot and carries the same reasoning. In the bottom right figure you see that the sensitivity between D/E and asset values is again sensitive when D/E is low, but virtually independent of equity volatility.
3.4.2 Asset values and volatility using KMVs model

The assumption in the Merton model is that we use a European call option formula with a maturity of $T$. This is one way of modeling a firm but certainly not the only. KMV argues that the value of a firm is an ongoing concern\(^{12}\) and hence KMV assumes the value of a firm to be the price of a perpetual option. Such an option is American by nature since their European counterparts would be hard to sell.

The price of such a perpetual call option is given by\(^{37}\) and is

$$E = \frac{D}{h_1 - 1} \left( \left( \frac{h_1 - 1}{h_1} \right) \frac{V}{D} \right)^{h_1} \tag{3.6}$$

with

$$h_1 = 0.5 - \frac{r - q}{\sigma^2} + \sqrt{\left( \frac{r - q}{\sigma^2} - 0.5 \right)^2 + \frac{2r}{\sigma^2}} \tag{3.7}$$

where $q$ represents the dividend yield. We have excluded dividends so far but KMV states it does make a difference.\(^{12}\) From these and using Ito’s Lemma we can produce a new system of nonlinear equations that we can solve using a solver. The above equation, which is the option price that should be equal to the total equity value given $V_0$ and $\sigma_V$ is one of these and the other is given by

$$\sigma_E E_0 = \frac{\delta E}{\delta V} \sigma_V V_0 \tag{3.8}$$

so we take the derivative $\frac{\delta E}{\delta V}$ ($h_1$ serves as a constant since it does not depend on $V$)

$$\frac{\delta E}{\delta V} = \left( \left( \frac{h_1 - 1}{h_1} \right) \frac{V}{D} \right)^{h_1 - 1} \tag{3.9}$$

and insert this into the formula above. We can now solve the nonlinear system using a solver in Excel or Matlab like we did before when we used the Merton model.

Including dividends is important in this model since the value of an American perpetual option without dividends ($q = 0$) is just the asset value ($E_0$), although the result has to be derived differently since $h_1 = 1$ for $q = 0$. Solving the system of nonlinear equations (set $q$ small) would result in $V_0 = E_0$ and therefore $\sigma_V = \sigma_E$ because $\delta E/\delta V = 1$.

We do not use or recommend using this technique in the setting intended here unless the quality of data is exceptional (combining several professional databases) since
its outcome is very sensitive to the dividend yield and its gap with the risk-free interest rate.

3.4.3 Obtaining and handling the input data

For public firms there are many ways to obtain the data. One of the best ways in terms of data quality to do so is by using Bloomberg, a private information terminal. It requires a special keyboard and an account to which we had access. Alternatives are Yahoo Finance, Google Finance, the Google search engine or simply the firm’s website. When using Bloomberg it is important to gather some background information on the firm since there are many listings for some firms. Obtaining data takes a lot of time but having high quality data is well worth the effort. A good example of the value of background research is Norsk Hydro ASA which sold off a division which influenced stock prices a lot. This class action was not listed by Yahoo Finance.

Some remarks are in place about the volatility data. First of all it is industry practice to report stock prices in three digits behind the comma, whereas Yahoo uses only two. For high stock prices, over 10 euro, this is no problem but when taking returns over cents such rounding errors add up. We partially dampened this effect by taking the adjusted stock price unless the raw stock price has a higher resolution (higher prices). In that case we either corrected for the class actions manually or we simply discarded the data points on these days since the changes are small. One of the firms for which this was done is NGK Spark Plug Ltd which listed adjusted prices around 20 cents.

A second remark is regarding the stock market chosen. We have chosen to deal with this luxury problem by using data from the original stock market (in its home currency) on which the stock is listed. The main reason is that the data on class actions such as stock splits and dividends are better. A prime example is BorgWarner Inc which adjusted price looks perfectly normal on the New York Stock Exchange but suffers a massive fifty percent drop on the Frankfurt Stock Exchange. The reason is of course a stock split that was missed. Small differences in the data can occur due to different closing times and exchange rates. Exchange rates themselves of course do not influence the log returns if they remain stable during a day, since we take the log of a fraction and the fraction is unaffected by denoting the amount in a different currency.

Lastly you may encounter A and B stocks. Its definition depends on the firm and the country, but usually A-stocks have (more) voting rights than B-stocks. Since we generally measure the prices of common stock, we take B-stock as opposed to A-stock as our unit of analysis.
3.5 Asset values and volatilities for private companies

Gathering data for private companies is slightly trickier. Whereas websites such as Yahoo remove much of the difficulties of finding the right balance sheet for large public firms, the private firm data was obtained from Amadeus. This database is filled with information and contains data even for local subsidiaries. This means it is important to gather some background information such as product lines, headquarter location and merger and takeover history on the firm to make sure the right data is obtained. In practice this will be easier since most firms know who they are dealing with. We have gathered such information using Google and a lot of elbow grease. Data was originally gathered manually for 150 firms, of which around 45 were deemed public (the mother firm was used) and a further 20 were private and had all the data we were looking for. In practice one could of course simply call the supplier asking for the data.

Problems arise because usually we do not have data on equity prices either. In this case, we have to resort to proxies for the asset value, even for the relatively simple, original Merton model. Before we do that we explore how sensitive our default probabilities are to our inputs.

We use debt/asset value as a way of measuring how sensitive the output is. The curves in figure 3.4 represent both the simple and the first-passage model due to insensitivity of the barrier. As expected the absolute values increase with the debt/asset value ratio. This is because the increase in debt causes the default percentages to go up by much, which is a property of the bell curve of the Normal distribution that is used in the model. The rightmost graph tells us we should be careful when Debt/asset value is high as we are prone to making large relative errors. Armed with this knowledge we proceed to the

![Figure 3.4: The absolute sensitivity of debt-to-asset-value and asset volatility. The extreme we encountered in our dataset are 0.2 and 0.7. From the shape of the curve we can see sensitivity increases with D/Asset value.](image)
3.5.1 Private firm valuation

The task at hand is to estimate firm value using accounting information and a peer group only. If this does not sound like a daunting task, it should. The valuation of firms using such data is done in value investing and one of its first proponents was Benjamin Graham. The techniques are most commonly used to value stocks and determine which of these are under- or overvalued. Since we have set the goal of risk management, we need not be as exact as an investor and only need a ballpark estimate of what the market thinks of these private firms, since that is the entire premise of the structural models we use. Let it however be said that the accuracy of these models should not be taken at face value: Graham himself introduced the concept 'margin of safety' to illustrate the 'discount' the market is giving. If one would find a method that works perfectly well all the time, he could make a fortune on the stock market and only make the right choices, which is unlikely given the history of even the best performing managers. Academics besides Graham have found that there is truth to value investing such as the three-factor model, while others such as Warren Buffett and Joel Greenblatt have shown that it can be used to make money with. But none of these is believed to work consistently. The best we can hope for is a good estimate.

Where our approach differs a lot from value investing is that we are trying to estimate what the market thinks of our companies as opposed to trying to find undervalued stocks to buy. From theory we borrow the factors we should pay attention to. We estimate how much these contribute to the market capitalization of similar firms in the industry and create a best estimate for the private firms. Using such a method captures some of the market’s view of how important a factor is in this industry, which we then translate into default probabilities.

A good starting point is a formula Graham devised which says that $V = EPS(8.5 + 2G)$. The intrinsic value of a firm is 8.5 plus twice the expected growth $G$ in percentages in the next 7 to 10 years, multiplied by its earnings per share. If we perform a linear least-squares regression analysis on the public firm data (excluding outliers) using market capitalization as independent and EBITDA (earnings before interest, tax, depreciation and amortization) as predictor, we find a reasonably high $R^2 = 0.8587$. Other important ratios mentioned are P/B, which is price-to-book-value ($R^2 = 0.7761$), and P/S, meaning price-to-sales ($R^2 = 0.7605$). The latter is often used to value firms with negative earnings but assumes that D/E is stable, which is why it should be used within an industry. The proxy allegedly used by the KMV Corporation is to compute $EBITDA_{i,j}$, where $i$ is an industry and $j$ a firm. Then we divide industry average market value by the industry average EBITDA (in our case of limited data, the average of 41 comparable companies by EBIT sharing the primary SIC code), thereby obtaining the average equity multiple. Estimates for the market value of equity of company $j$ are
obtained by multiplying its EBITDA by its industry average multiple. In the last step, KMV adds to this the debt listed in the books to obtain asset value.\(^2\) This represents \(V\), or total firm value. Such a valuation is based solely on P/E and immediately creates a problem when encountering negative earnings. While earnings are an important factor, because in theory a stock price should be the discounted sum of all of the company’s future cash flows, both KMV’s method and Graham’s formula would assign negative value to suppliers with negative earnings. At least for the firms for which the model leads to negative equity values we need another valuation method. This would imply the supplying firm has already gone bankrupt while it clearly has not. Earnings can also be very volatile\(^{34}\) and are easier to misreport than sales. Depending on leverage (D/E) a fifty percent drop from one year to the next would mean an equivalent drop in market capitalization (equity) which would dramatically increase default probability. This is not a nice property. Averaging over several years might solve this, but does not solve the negative earnings issue. We need something more robust.

Let us discuss alternatives. Book values might give a good current impression of what it may be worth, but ignores the capacity to earn money and will not be very responsive to changes. Contrary to that, sales shed some light on the earnings capacity of a firm but ignores the current firm value and the costs to realize those sales. Since most valuation methods are based on earnings and sales is closer to that than book values, we are in favor of using sales. Since sales are rarely negative in practice, we expect this to be much more stable. In our dataset its explained variance is also highest for any single variable besides EBITDA.

Since EBITDA is widely used and found to be a good predictor in our regression analyses, we use KMV’s multiplier on EBITDA whenever EBITDA is stable over 3 years and the EBITDA is strictly positive. Our measure of stability is calculated by taking the absolute average relative growth over 3 years of earnings. We do this because such large changes might lead to a high D/Asset value and its corresponding problems shown in

Figure 3.5: Figures plotting EBITDA (left) and sales (right) versus equity value for our public firms. The results seem to be related in a linear fashion.
the sensitivity analysis, as this ratio is highly sensitive to EBITDA in that case. We do not use regression because the intercept would be much too high for the small firms we want to apply it to. In the plots above we show that a line starting in the origin is quite close to the truth. In formula 3.10 $M_{\text{EBITDA}}$ denotes the EBITDA multiplier.

\[ V_{Ei} = M_{\text{EBITDA}} \times \text{EBITDA}_i \tag{3.10} \]

Whenever EBITDA is not stable, we may want to get a second opinion. The next best choice is then to use sales as a predictor, because it is as simple as the EBITDA measure and will always generate positive values for equity. A line from the origin also seems likely and its $R^2$ is among the highest. We use formula 3.11 where $M_{\text{SALES}}$ is the sales multiplier.

\[ V_{Ei} = M_{\text{SALES}} \times \text{SALES}_i \tag{3.11} \]

We can now check which values are candidates for thorough checks using the sensitivity analysis on D/Asset value. We use the proxy $V_{Ai} = V_{Ei} + D$ as used by KMV$^2$ for asset value. Highly leveraged firms might be worth looking into more carefully by finding a peer group of firms in a similar industry, comparable geographical location and turnover. We can for instance take sales as another predictor of income and combine this with a crude measure of the costs we made, for instance in the form of current liabilities and long-term liabilities, and apply a multiple least-squares regression analysis. A regression analysis requires similar turnover values, otherwise its intercept is too high to be useful. For our public firms a regression analysis results in $R^2 = 0.8321$ but we should be careful of the influence of negative values (in our tests we in fact encountered one negative EBITDA value). Adding EBITDA always increases the explained variance but we can only do so when it is positive.

### 3.5.2 Asset volatility of private firms

For asset volatility there is no such procedure. Instead we choose to use the asset volatility of comparable companies again using industry peers, because asset volatility is less volatile and it captures some of the market’s view. Here we used the method employed by KMV$^9$ and look at industry mix, size and geographic region. We do not simply take an average of our public firms because our sensitivity analysis shown before showed that volatility is quite a sensitive parameter.

The firms we analyze are all big and in the automotive industry according to their NAICS codes. We therefore choose to look at geography as the deciding factor. We split each up by continent where we deem the largest firms ‘US’ since that is where most of their headquarters are. Then we take the average of the asset volatilities (which you obtain after solving the nonlinear system for public firms, see Matlab code in the
appendix). This estimate can be improved by finding more similar firms from a region or by creating more specific categories. Doing the latter means running the risk of creating too many and not capturing true volatility.

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<th>Europe</th>
<th>US</th>
<th>Asia</th>
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<tr>
<td>Number of firms</td>
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<td>17</td>
</tr>
<tr>
<td>Asset volatility</td>
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</tbody>
</table>

Table 3.2: Input values we use for the public firms. Note that these are asset volatilities and not equity volatilities, explaining why these are lower than the listing before. Asset volatilities are obtaining by solving the simultaneous nonlinear equations presented before.

Example. Maier Ltd is a C-rated company. We determine the EBITDA multiplier from a set of mid sized European firms in the automotive industry, which turns out to be 5.5988. Maier’s EBITDA is €234,000 and its default point D is €10,473,500 which means we estimate its firm value to be 5.5988* €234,000+€10,473,500. This value of €11,783,630 is directly used in the Merton model. Its volatility of assets is estimated from the same peer firms and is set to 0.1550. Applying the Merton model as before leads to a (very high) default probability of 0.18169. This seems less extreme after we contacted the Italian rating agency modeFinance which informed us of the C-rating it received for 2007.

3.6 Analytical probabilities of default for public firms

Now that we have the input data, we can apply the Merton models and see how well it fits the data supplied by the credit rating agencies. We chose the model variants based on industry practice of the KMV model, the largest commercially applied model. There are closed form formulas available for both variants of the Merton models, which can be found in chapter two. In chapter four we present a Monte Carlo method to provide a more intuitive method.

3.6.1 Benchmarking results with credit ratings

We should not expect to do much better than the rating agencies. Given the fact that this thesis is written with the risk manager in mind, we try to obtain a good result with a bare minimum of data: only that we may reasonably expect the average risk manager to have. Keeping that in mind, let us see how well our model reproduces the credit ratings we already have, namely those of the rating agencies. Rating agencies rate in rating categories instead of default probabilities, so we must take historical default statistics to compute the probability of default within a rating class. The rating data used here comes from Moodys. We use the fine scale, which has more classes to allow for greater accuracy.
Figure 3.6: In the above plots we see 22 mapped credit ratings versus the Merton variants (left basic Merton and first passage) produced here. The Pearson correlation is 0.94131 for the basic Merton model and 0.94132 for the first passage model, but there is a large concentration of measurements in the lower left corner. On the right the data were transformed using the square root to illustrate the relation we mentioned in the text, the correlation there is 0.969. Spearman’s rank correlation is 0.78.

For the sake of the plot we use the observation that default probabilities increase at an exponential scale. From Moodys data, default frequencies of historical defaults are available, but no defaults have been observed for the highest rating classes. This does not mean these are risk-free and one would not argue that they are, so consequently we deal with this by performing a regression analysis on the rating classes. We take historical default probabilities per rating class for each year, give each class a number (Aaa being highest, B3 being lowest) and perform a regression analysis on \( \ln(\text{class mean}) \) and convert these back to ratings. This leads to the result in equation 3.12 where \( x \) is the rating class starting at Aaa.

\[
R_{\text{map}} = e^{-10.5915911 + 0.507587142x} 
\]  

(3.12)

3.6.2 Results for the Merton variants

Results can be seen in figure 3.6 and in table 3.3. Remember that both Merton models produce risk-neutral default probabilities, whereas the credit ratings are real-world probabilities of default. First we notice there is little difference between the two plots. Both seemingly imply a neat relation between the two credit ratings, although such is probably not the case. Risk-neutral probabilities can most likely not be mapped directly, which may make correlation (implying linearity and hence direct mapping) somewhat of a stretch. A direct mapping is extremely unlikely because we are comparing risk-neutral PDs to real-world PDs, more so because in general for large default probabilities we encounter larger differences between real-world and risk-neutral. This is largely due to
greater expected excess returns.\(^\text{39}\)

There are other arguments why a direct mapping may not be possible even if we knew exactly how to, as Moody’s credit ratings are somewhat slower in their updates than our market-based model.\(^\text{52}\) We therefore do not expect a perfect correlation, but do expect a relation between the two if both we and the rating agencies are doing their jobs reasonably well. Since the monotonic transformation between risk-neutral and real-world should hold, a Spearman correlation coefficient \(\rho_s\) is more useful than the standard Pearson correlation. Such a correlation measures rank correlation. Its value is 0.78 for this data and this is significant \((p=0.000021)\).

Another thing that may strike one as odd is that these probabilities are very low. This is due to the nature of the random walk, namely the tail values of the normal distribution that are very unlikely to occur. Since we have low default probabilities we have a low probability of going down. It is known in option pricing that the normal distribution is not a good distribution for the tails,\(^\text{40}\) traders use implied volatility (and the volatility smile, see Hull’s 2010 book) instead. This is part of the reason why Moody’s KMV uses a historical database of defaults and then takes out expected return using the Kealhofer component. As such it implicitly drops the assumption of normality.

The models imply at least one firm that is off the drawn line. That firm is Valeo SA. As said before, this might not be entirely unfounded. In figure 3.7 we see Valeo’s credit situation beyond our credit rating. The question remains if our model was onto something or whether it was a false positive indeed. Looking at the credit history beyond 2007, it was not wrong to implicate Valeo as a potential danger. Of course, we must keep in mind that such reasoning applies to many firms during the credit crunch.

Correlation and perfect mapping are not of principal interest to us. Because of its suggested use as a risk-management system, we only need to know which the rotten apples are since a risk manager is most likely interested in which firms have a much above-average default probability. What we expect of our model is that it indicates the most troubled cases. Since we use it as a warning system upon which we take further action, the worst thing that can happen is a false negative: those instances in which the model does not warn us, but problems are imminent. As seen in the graphs, we have none of those.
Figure 3.7: Valeo SA’s credit situation after our rating. The blue line is Moody’s EDF, unfortunately not available at the time of analysis. The green line is Moody’s equivalent credit rating. The company was downgraded several times within a matter of months. Graph obtained from moodys.com.
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<td>0.558432451</td>
<td>0.017292%</td>
<td>0.087746%</td>
<td>A3</td>
</tr>
<tr>
<td>NGK Spark Plug Co Ltd</td>
<td>3861306772</td>
<td>0.241529657</td>
<td>0.000000%</td>
<td>0.087746%</td>
<td>A3</td>
</tr>
<tr>
<td>Nitto Denko Corp</td>
<td>7480724954</td>
<td>0.228491977</td>
<td>0.000000%</td>
<td>0.087746%</td>
<td>A3</td>
</tr>
<tr>
<td>NSK Ltd.</td>
<td>6085760184</td>
<td>0.212269971</td>
<td>0.000169%</td>
<td>0.145770%</td>
<td>Baa1</td>
</tr>
<tr>
<td>Pinelli &amp; C SpA</td>
<td>8103730073</td>
<td>0.134422484</td>
<td>0.000184%</td>
<td>0.145770%</td>
<td>Baa1</td>
</tr>
<tr>
<td>PPG Industries Inc.</td>
<td>12194537061</td>
<td>0.15014764</td>
<td>0.000000%</td>
<td>0.145770%</td>
<td>Baa1</td>
</tr>
<tr>
<td>Cie de St-Sobain</td>
<td>42404545565</td>
<td>0.144330522</td>
<td>0.000001%</td>
<td>0.145770%</td>
<td>Baa1</td>
</tr>
<tr>
<td>Scapa Group PLC</td>
<td>150522111.4</td>
<td>0.18734031</td>
<td>0.000000%</td>
<td>1.224590%</td>
<td></td>
</tr>
<tr>
<td>SKF AB</td>
<td>7369539451</td>
<td>0.261180954</td>
<td>0.000131%</td>
<td>0.145770%</td>
<td></td>
</tr>
<tr>
<td>Teleflex Inc</td>
<td>2865919846</td>
<td>0.175405029</td>
<td>0.000021%</td>
<td>0.145770%</td>
<td></td>
</tr>
<tr>
<td>Tenneco Inc</td>
<td>23286359702</td>
<td>0.130205239</td>
<td>0.04280%</td>
<td>0.145770%</td>
<td></td>
</tr>
<tr>
<td>ThyssenKrupp AG</td>
<td>43162047825</td>
<td>0.141722327</td>
<td>0.000048%</td>
<td>0.242165%</td>
<td>Baa2</td>
</tr>
<tr>
<td>Trelleborg AB</td>
<td>2816836482</td>
<td>0.160651828</td>
<td>0.008336%</td>
<td>0.242165%</td>
<td></td>
</tr>
<tr>
<td>TRW Automotive Holdings Corp</td>
<td>5639691229</td>
<td>0.078016964</td>
<td>0.008982%</td>
<td>1.110294%</td>
<td>Ba2</td>
</tr>
<tr>
<td>TT electronics PLC</td>
<td>431960818.8</td>
<td>0.206752682</td>
<td>0.008977%</td>
<td>1.110294%</td>
<td>Ba2</td>
</tr>
<tr>
<td>Tupy SA</td>
<td>601846693.3</td>
<td>0.274218111</td>
<td>0.132123%</td>
<td>1.110294%</td>
<td>Ba2</td>
</tr>
<tr>
<td>Valeo SA</td>
<td>6349158195</td>
<td>0.10694172</td>
<td>0.005968%</td>
<td>0.242165%</td>
<td>Baa2</td>
</tr>
<tr>
<td>Varta AG</td>
<td>134738396.4</td>
<td>0.264322481</td>
<td>0.179091%</td>
<td>0.242165%</td>
<td>Baa2</td>
</tr>
<tr>
<td>Volkswagen AG</td>
<td>1.44E+11</td>
<td>0.112735636</td>
<td>0.000039%</td>
<td>0.242165%</td>
<td>Baa2</td>
</tr>
<tr>
<td>WABCO Holdings Inc</td>
<td>2834025262</td>
<td>0.21909385</td>
<td>0.000000%</td>
<td>0.242165%</td>
<td>Baa2</td>
</tr>
</tbody>
</table>

Table 3.3: Results for 42 public companies. Of these, 22 were rated by Moodys. Mapped ratings as described before can be found in the second to last column. Results for the models were aggregated due to their small differences.
3.7 Analytical probabilities of default for private firms

We can compare the default percentages to those of mapped credit ratings just as we did in the case of public firms. Additionally we can apply the Merton model and benchmark it to another accounting-based measure known as Altman’s Z-score (for private firms). The reason to do that here and not before is because the credit rating data is shaky at best due to an unreliable database. Credit ratings here are from modeFinance and were listed in the Amadeus database.

3.7.1 Benchmarking with credit ratings and Altman’s Z-score

For the private firms we do exactly what we did for public firms, but obtaining credit ratings is slightly harder. These ratings were available from previous research but because we are unsure the researchers accurately discerned between a local factory’s accounting details and headquarters we cannot be sure these are fully correct. Data quality and availability of the data obtained from Amadeus, Compustat and Bloomberg is a big issue here. Therefore we decided to also use Altman’s Z-score for private firms, based on discriminant analysis using accounting data. We do not expect the Z-score to perform better since it only uses information on the balance sheet and was originally based on US firms, but consider it to be a second opinion. It should be mentioned that ‘retained earnings’ was not available as such in Amadeus but was found to be a (large) component of another item on the balance sheet. Since no other data could be obtained, we have used this as an approximation.

The formula for the Z-score for private firms is\(^4\)

\[
Z = 0.717T_1 + 0.847T_2 + 3.107T_3 + 0.420T_4 + 0.998T_5
\]  \hspace{1cm} (3.13)

and for public firms\(^3\) it is

\[
Z' = 1.2T_1 + 1.4T_2 + 3.3T_3 + 0.6T_4 + .999T_5
\]  \hspace{1cm} (3.14)

with

\[
T_1 = \text{Working Capital} / \text{Total Assets}
\]

\[
T_2 = \text{Retained Earnings} / \text{Total Assets}
\]

\[
T_3 = \text{Earnings Before Interest and Taxes} / \text{Total Assets}
\]

\[
T_4 = \text{Market Value of Equity} / \text{Total Liabilities}
\]

\[
T_5 = \text{Sales} / \text{Total Assets}.
\]
Table 3.4: Credit ratings, Merton results and Z-score for the private firms under analysis.

Federal-Mogul and First Technology are considered private because their data was incomplete.

These values are not entirely comparable and there are many versions including updates. Altman uses 'safe zone', 'grey zone' and 'distress zone' as the three zones of discrimination that are different for public firms and for private firms. We mainly care about the extreme cases and once again ask if our model manages to pick them out.

The worst case is clearly picked out. Maier gets a CCC rating, has a Z-score of 0.46 and a PD of over 18 percent. The models are unanimous that this is the worst of the bunch. Of the others, CML, First and Wagon are in the distress zones. When comparing credit ratings to Z-scores we see they do not necessarily agree but there is some degree of conformity.

Because of the large differences in credit quality we do not choose for a square root transformation like before, but instead use log-log plots. We can find these in figure 3.8.

Looking at the graphs we see that the Z'-score and the credit ratings sometimes disagree. As mentioned this could be due to data quality, bad performance of the Z'-score, ratings meant for a subdivision or perhaps ratings are simply off. More importantly we see that our Merton model does not do too bad on these plots. In the second plot we expect a negative slope because low Z-scores have high PDs and we see one. In the third plot we expect a positive slope. Partly due to the discretization and low credit rating resolution this is not extremely clear. We also see three large exceptions. One of these is Scherdel, which our Merton model deems as more risky than both credit ratings and bond prices.

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>DP (credit rating)</th>
<th>PD (Merton)</th>
<th>Z-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behr</td>
<td>BB</td>
<td>0.0133</td>
<td>1.96403E-08</td>
</tr>
<tr>
<td>Borgers</td>
<td>BB</td>
<td>0.0133</td>
<td>9.15734E-07</td>
</tr>
<tr>
<td>Bosch</td>
<td>BBB</td>
<td>0.0043</td>
<td>3.54377E-14</td>
</tr>
<tr>
<td>CML Innovative Technologies</td>
<td>BBB</td>
<td>0.0043</td>
<td>0.000422999</td>
</tr>
<tr>
<td>Dräxlmaier</td>
<td>BB</td>
<td>0.0133</td>
<td>8.41318E-14</td>
</tr>
<tr>
<td>Edscha</td>
<td>BB</td>
<td>0.0133</td>
<td>2.51913E-08</td>
</tr>
<tr>
<td>Federal-Mogul</td>
<td>B</td>
<td>0.0429</td>
<td>9.31094E-06</td>
</tr>
<tr>
<td>First Technology</td>
<td>B2</td>
<td>0.0509</td>
<td>3.62892E-06</td>
</tr>
<tr>
<td>Huf Hufeßbeck &amp; Fuerst</td>
<td>BB</td>
<td>0.0133</td>
<td>3.32174E-12</td>
</tr>
<tr>
<td>Kickert</td>
<td>B</td>
<td>0.0429</td>
<td>3.60014E-18</td>
</tr>
<tr>
<td>Kongsberg</td>
<td>BB</td>
<td>0.0133</td>
<td>6.62100E-10</td>
</tr>
<tr>
<td>Maier</td>
<td>CCC</td>
<td>0.1400</td>
<td>0.181692239</td>
</tr>
<tr>
<td>Peguform</td>
<td>BBB</td>
<td>0.0043</td>
<td>1.46279E-16</td>
</tr>
<tr>
<td>Rausch + Pausch</td>
<td>BBB</td>
<td>0.0043</td>
<td>3.77647E-20</td>
</tr>
<tr>
<td>Scherdel</td>
<td>BBB</td>
<td>0.0043</td>
<td>0.033238517</td>
</tr>
<tr>
<td>TI Automotive</td>
<td>B</td>
<td>0.0429</td>
<td>0.001335295</td>
</tr>
<tr>
<td>Wagon</td>
<td>B</td>
<td>0.0429</td>
<td>1.76433E-06</td>
</tr>
</tbody>
</table>
Figure 3.8: From left to right we plot the ratings versus the Z’-scores, Merton versus Z-scores and Merton versus credit ratings. We find some exceptions from linearity.

We have some firms that are B-rated but come out well in our model (Kiekert, Wagon and Federal-Mogul). These firms were values using P/S and not P/E which may indicate that this is the major weakness of our model. On the other hand, the Z-score indicates that Kiekert is very healthy and Federal-Mogul slightly less so (as in our model), but healthy too. Only Wagon seems to perform poorly in both the credit ratings and the Z-score. Unfortunately we do not have the data to investigate if this is truly a weakness or merely a coincidence. Other aspects of the model seem to relate well to the measures presented. Two of the ‘distress zone’ firms in terms of Z-scores have some of the highest PDs. The other two are on the high end of the scale. Really problematic cases are still picked out, so for the purpose at hand we deem the model to be acceptable.

3.8 Mapping the risk-neutral probabilities to real-world probabilities

We have risk-neutral probabilities of default. The only thing we can infer from this is which company is more likely to default, but not its actual default probability. A mapping or transformation would translate its meaning to the real world and serves the purpose of making data directly comparable to credit ratings, as well as allowing us to estimate real-world expected losses.

Unfortunately the KMV mapping (chapter 2) uses historical data which neither we nor risk managers have. That means we must resort to other means. One method is trying to find a mathematical relation between the risk neutral output and some credit ratings, such as a simple linear regression. Such a mapping is plotted in figure 3.6. On the right side in that figure we use the square root function to transform our default probabilities.

Such a method is unlikely to work well. In the real-world, risk preferences are included whereas in the risk-neutral setting they are not. In the case of systematic risk, the return demanded by investors is higher than the risk-free rate of return.52 Such system-
atic preferences are commonly estimated using CAPM, but is nonlinear since the CAPM framework assumes risk-aversion. Such a mapping approach is therefore generally not advisable to use for loss computations as the relation between the two variables is much more complex, depending on industry, company size and geography (among others).

<table>
<thead>
<tr>
<th>Supplier name</th>
<th>Basic Merton Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asahi Glass Company Ltd</td>
<td>-1.0</td>
</tr>
<tr>
<td>Alcoa Inc</td>
<td>11.3</td>
</tr>
<tr>
<td>ArvinMeritor</td>
<td>14035.2</td>
</tr>
<tr>
<td>Basf SE</td>
<td>-1.0</td>
</tr>
<tr>
<td>BorgWarner Inc</td>
<td>-1.0</td>
</tr>
<tr>
<td>Continental AG</td>
<td>28.3</td>
</tr>
<tr>
<td>Johnson Controls Inc</td>
<td>-1.0</td>
</tr>
<tr>
<td>Goodyear Tire &amp; Rubber Co</td>
<td>10544.6</td>
</tr>
<tr>
<td>El du Pont de Nemours &amp; Co</td>
<td>-1.0</td>
</tr>
<tr>
<td>Honeywell International Inc</td>
<td>-1.0</td>
</tr>
<tr>
<td>Hydro Aluminium AS</td>
<td>-1.0</td>
</tr>
<tr>
<td>Textron Inc</td>
<td>-0.9</td>
</tr>
<tr>
<td>Nitto Denko Corp</td>
<td>-1.0</td>
</tr>
<tr>
<td>NSK Ltd</td>
<td>7.4</td>
</tr>
<tr>
<td>PPG Industries Inc</td>
<td>-1.0</td>
</tr>
<tr>
<td>Compagnie de St Gobain</td>
<td>-1.0</td>
</tr>
<tr>
<td>SKF AB</td>
<td>5.5</td>
</tr>
<tr>
<td>Tenneco Inc</td>
<td>2114.2</td>
</tr>
<tr>
<td>ThyssenKrupp AG</td>
<td>1.4</td>
</tr>
<tr>
<td>TRW Automotive Holdings Corp</td>
<td>446.2</td>
</tr>
<tr>
<td>Valeo SA</td>
<td>296.2</td>
</tr>
<tr>
<td>Volkswagen AG</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 3.5: Number of medians dispersed from median. The culprits are easily identified.

### 3.8.1 Relative mapping

An alternative is relative mapping. Simply look at how much more likely a firm is to default compared to the other firms, these are the ones that should be on watch. Under most conditions (crises excluded) one might assume that most operational firms are relatively healthy and that only the most severe should be looked into. This is a compelling argument for such comparative analysis.

Due to the skew of such a distribution of default probabilities, the median is a better reference point than the average. The outcome of the measure is listed in table 3.5. Such a value can be determined using equation 3.15. We use the median as a yardstick (in the divisor), but anything else may be used instead as long as the median is used as the reference point.
Modeling the Merton model for a single supplier

\[ r_i = \frac{\left( \mu_{1/2}(x) - x_i \right)}{\mu_{1/2}(x)} \]  \hspace{1cm} (3.15)

A side note on this type of mapping: when dealing with a large number of very similarly healthy (or unhealthy) companies, it may be wise to include some companies of the same industry whose fortunes are clearer. Including such companies may put some perspective on the standings of the group that is being modeled, perhaps all are doing particularly well. In our model, we have skipped such an exercise because the data is from 2007 and we know Goodyear and ArvinMeritor have not done well since. Such information is sufficient to put the rest into perspective.

### 3.8.2 Absolute mapping

In some cases it is desirable to have actual default probabilities. Professionally this is done using large databases that map default probabilities, often depending on factors such as geography and industry. In our case we have a large pool of suppliers mainly from Europe and mainly within the automotive industry. We therefore present a general example of absolute mapping, but one may choose to split the datasets into groups and perform the procedure on those.

First of all it is important to realize that at such low probabilities of defaults and with small datasets, there is a lot of noise in the data. Moody’s fortunately uses two rating scales from which we have data on the fine rating scale (denoting 1, 2 and 3 within each category). Often the big scale is used and this set of ratings \( R_{big} = (\text{Aaa, Aa, A, Baa, Ba, B, Caa, Ca, C}) \) allows us to aggregate the data. This is necessary and favorable because we have a small sample. We now take the samples for which we have both Moody’s data and a value produced by our structural method and use these to give the unrated samples an unofficial but hopefully a rudimentary Moody’s equivalent credit rating. We do so by again taking the assumption\(^{10}\) of linearity of the logarithms of credit rating values.

\[ \begin{array}{cccccc}
\text{Aaa} & \text{Aa} & \text{A} & \text{Baa} & \text{Ba} & \text{B} \\
\text{Count} & 0 & 1 & 8 & 6 & 4 & 3 \\
\text{Mean} & \text{n/a} & 8.82E-16 & 2.21E-07 & 1.67E-06 & 3.75E-05 & 0.00178 & \text{n/a} \\
\text{St. Dev.} & \text{n/a} & \text{n/a} & 4.79E-07 & 2.31E-06 & 4.47E-05 & 0.00123 & \text{n/a} \\
\ln(\text{Mean}) & \text{n/a} & -34.66 & -15.32 & -13.30 & -10.19 & -6.33 & \text{n/a} \\
\end{array} \]

**Table 3.6:** Using the data to create a mapping. Count describes the number of values in our sample.

Table 3.6 above contains summary statistics for each of the credit rating categories. Even with our low number of samples we see that on average the probabilities of default go up in the higher categories. This confirms what we found in the graph: our procedure does what it promises, at least on average. We can see that the model has trouble discerning
between A and Baa rated firms, or perhaps Moodys ratings are not using the latest information. If this is considered an issue then we may want to increase our sample size. Doing this means finding firms that have a Moodys rating for which key financials are available and at the same time fit in the intended group. The standard deviation can be used to measure the accuracy of the mean.

Now we can assign credit ratings to firms that have none by trying to determine in which group these belong. We perform a regression analysis as before to determine the default probability for each category. For this output we use the big scale data from Moody’s. The outcome of the regression (Aaa=0.046%, Aa=0.116%, A=0.294%, Baa=0.740%, Ba=1.866%, B=4.705%, Caa-C=11.863%) is similar to that of the fine scale with the exception of the higher categories. Using the observation that the natural logarithm of credit ratings is approximately linear we take the natural logarithms of these observations as in the table and assume it holds for our dataset too. Here $x$ represents the category with Aaa being 1 and counting on the big scale. We can apply the result of the regression $e^{-8.605+0.925x}$. Now we propose to use the $x$ from the empirical scale and apply it to Moody’s big scale.

Of course the quality of this procedure depends on the credit ratings data set so at best it should be considered an approximation. Using this method you can also revise credit ratings but we generally do not advise to do so unless the firm has been in the news or Moody’s has not touched it for a long time. On average we may assume Moody’s knows what it does rather well and a simple model should not be able to improve on it consistently.

**Example.** To illustrate, take for instance Continental AG, which has a PD of 0.000587% in our model, or -12.045 when taking the natural logarithm. This is somewhere between category 4 and 5 and we can use linear interpolation ($| -13.3 - -12.045| / | -13.3 - -10.19| \approx 0.4$) and then apply this to the scale with real world probabilities ($e^{-8.605+0.925(4.4)} = 0.0107\%$). Due to the small data set used this is a rough approximation.

### 3.9 Conclusion

In this chapter we have devised a method to estimate the exposure at default. Then we implemented the basic Merton model and its first-passage variant using analysis and information found in literature. Using only publicly available data we showed how a default probability can be obtained in both a public and a private setting. We analyzed model properties and verified results for sixty suppliers. Finally we showed how one might map risk-neutral default probabilities to actual default probabilities and provided ways to interpret the results.
Chapter 4

Modeling alternate scenarios using efficient Monte Carlo techniques

Up until now we have used a simple set of assumptions for individual suppliers and their probability distributions. Because we use a structural model we can change some of these assumptions and obtain default probabilities under alternative scenarios.

In this chapter we do exactly that by using Monte Carlo simulation with importance sampling. We provide some examples based on scenarios possibly encountered in practice. We leave the portfolio model for the next chapter.
4.1 Scenario analysis in a supplier setting

There is a step in between modeling an individual supplier and modeling a portfolio. While in our portfolio model (found in chapter five) we test for the concentration of risk and the improvement as such, in this chapter we provide examples of the influence of alternate scenarios on the individual default probabilities. We do not yet measure the risk stemming from having a geographically concentrated supplier portfolio but instead model assumptions about individual suppliers. Specifically, we focus on only those specific characteristics we care about for some of our important suppliers.

Before we do start making various assumptions, a word of caution. Existing theory tells us it is important to at least distinguish between what is public information and what is truly a scenario analysis. We refer to the efficient market hypothesis. The strong version of the efficient market hypothesis says that stock prices contain all possible information, including insider information. The semi-strong version tells us that these reflect all public information and that prices respond instantly, while the weak version tells us they reflect all past public information (prices). There has been considerable criticism on the efficient market hypothesis mainly from the field of behavioral finance, but also using statistical analysis. Generally there is little support for the strong version. In that study only the weak version is said to hold. We increase the margin of safety in this thesis and assume the semi-weak version holds.

It is important to realize that applying the methods in this chapter to scenarios that are likely to be public information might adjust the probability of default while the stock prices already reflect this information. We therefore recommend only private information is incorporated into the models.

4.2 Probabilities of default using Monte Carlo simulation

Models using stochastic calculus such as in chapter three are fast and useful for standardized methods. Downsides for practitioners are that such models are not intuitive and might seem frightening due to the large amount of complex mathematics involved in deriving them. Errors are hard to track when the model provides a false or counterintuitive result and these models are (partially therefore) inflexible when it comes to modeling particular assumptions. As seen in chapter two, seemingly simple additions such as first passage approaches and jumps lead to complex formulas requiring an above-average understanding of stochastic calculus. Since we would like such flexibility in this section we present an efficient version of Monte Carlo simulation.
4.2.1 Simulating the simple Merton model

The basic Merton model we chose to continue with in chapter three is characterized by the random walk based on a stochastic process. This process is called a Wiener process, or standard Brownian motion. The simplest approach is to simulate the path followed by our total firm assets using random numbers generated by the computer. Should the value $V_T$ of a supplier at end of that path, in our case after one year, be above or at the level of debt (the default point $D$), we characterize the situation as ‘no default’. Should it fall below, we deem the firm to have defaulted. We repeat this artificial path generation process many times over and take the average of all observations at the end of the year as in formula 4.1.

$$ PD = \frac{\# \text{ defaults}}{\# \text{ simulations}}. \quad (4.1) $$

The law of large numbers ensures that our solution converges to the true (risk-neutral) default probability given the assumptions. This also means that in order to be more accurate, we need to simulate a larger number of paths. Measuring how good our estimate is can be done in terms of a confidence interval based on the variance of the estimator.

To simulate the path we use the standard assumptions of the Merton model, namely Brownian motion, constant volatility and constant return. We denote $\Delta t$ as the change in time. Formula 4.2 describes the end result, with $V$ as firm value and $\sigma$ as its volatility of assets. The $\epsilon$ denotes a random value taken from a standard Normal distribution. $\exp(x)$ denotes $e^x$ for readability.

$$ V(t + \Delta t) = V(t) \exp \left[ \left( r - \frac{\sigma^2}{2} \right) \Delta t + \sigma \epsilon \sqrt{\Delta t} \right] \quad (4.2) $$

Simulation of first-passage approach

Simulating the first-passage approach still has its complexities. The basic Merton model defines a default as the asset-value $V_T$ being below a predetermined debt value at time $T$. In the above equation we can set $\Delta t$ to $T$ and therefore need only one random number to simulate an entire path. In the first passage approach we defined a default as being below a barrier $B (< D)$ anytime between 0 and $T$ or below the value $D$ at $T$. This means the accuracy of the first passage approach is not just dependent on the number of simulations, but also heavily depends on the number of time buckets ($\Delta t$) between 0 and $T$ we choose. This is called discretization. When choosing large time buckets, the value may have gone below $D$ and resurfaced without us noticing, meaning we underestimate the probability of default when choosing too few. Simulating the first passage method is therefore hard since it only converges in the limit due to its time bucket size. From
here on forward we do not explicitly derive results for the first passage approach due to the small differences found compared to the basic Merton model in chapter three. For optimization of the discretization readers are referred to the American option valuation method by Tilley\textsuperscript{54} which is based on similar techniques.

4.2.2 Monte Carlo methods for extreme events

Standard Monte Carlo simulation works but converges only slowly. Its error\textsuperscript{30} scales as $1/\sqrt{N}$, $N$ being the number of simulations. As we can see in table 3.3 we have probabilities that are extremely small. Take Asahi Glass Company Ltd, which has a (risk-neutral, where $\mathcal{N}$ denotes the standard normal distribution) default probability of $\mathcal{N}(-5.84)$, meaning we might expect one default every $1/\mathcal{N}(-5.84)$, or once about every 125 million simulated paths (which are referred to as runs).

Why would we even want to know these probabilities? As stated, we mainly care about order-of-magnitude estimates. If we produce a value of zero we have no means of comparing it to other values, not even when adding relatively bad peers. Knowing the order-of-magnitude in this model becomes more important when modeling portfolios where we consider joint defaults, a topic we turn to in the next chapter. In such cases the default dependencies combined with the large number of suppliers can increase such small probabilities substantially. Then we still have the mapped real-world probabilities, which are often much higher as explained at the end of chapter three. The method employed by KMV discerns between factors such as geography and industry and in some industries we may have risk-neutral probabilities that are very low on average, while the real-world probabilities are not.

Now that we have established that we indeed care about these probabilities, let us see if there are better ways of obtaining these. We have knowledge about the distribution we are sampling from since $V, D$ and $\sigma$ all influence the default probabilities. Generally speaking if $D$ is low compared to $V$ and $\sigma$ is low too, we may expect low default probabilities. We are sampling heavily from a region where there are not going to be any defaults and each sample adds little information, or in other words the variance of our estimator is quite high.

There are many variance reduction techniques available to overcome this problem.\textsuperscript{30} In our case it is best to sample from a more favorable distribution, a method known as importance sampling.\textsuperscript{30} Consider the basic problem of estimating the value of an integral (in layman’s terms this is the surface area under a function) as in 4.3.

$\int h(x)f(x)dx$ \hspace{1cm} (4.3)

If we know a density $g(x)$ that is zero when $f(x)$ is zero we can write
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\[ \int h(x)f(x) g(x)dx = E_g \left[ \frac{h(X)f(X)}{g(X)} \right] \]  

where \( E_g \) denotes that we take the expected value with respect to the density of \( g \) instead of \( f \). We now sample not from the distribution \( f \), which would lead to many random numbers going to waste, but from the critical region \( g \) (provided we have chosen \( g \) well). This does however mean we have to correct any result we get from the simulation by \( f(x)/g(x) \) due to the fact that we are using a different distribution to sample from. This ratio is called the likelihood ratio or the Radon-Nikodym derivative. While this sounds impressive it is simply the probability under the original distribution \( f \) divided by the probability under our more favorable distribution \( g \). We measure the importance of each result using this likelihood ratio.

The question remains what the density function \( g \) must be. Theory on this is too vast to recite here but we explain the train of thought. The ideal variance of our estimator is of course zero, which can be expressed mathematically but requires knowledge of the estimated value. Often this is not the case\(^a\). The next best thing is for the variance of the estimate to decrease at maximum speed in terms of the simulation. This is called asymptotic optimality and the maximum attainable result is that the variance decreases by twice the rate of the probability itself. This is what we are aiming to do. We use this fact extensively in the portfolio setting of chapter five.

For our standard option it is not hard to find a decent distribution \( g \). We have for our end value \( V_T \) the formula \( V_0e^{\mu T} \). If we want to increase the probability of a default and decrease the variance of our estimator, we can simply make sure that the value \( V_T \) dwindles around \( D \) for every simulation path. We use the calculus in the equations below to determine what the mean of our distribution must be in this case.

\[ V_0e^{\mu T} = D \]  
\[ \mu = \frac{1}{T}ln(V_0/D) \]

Usually we sample from \( \mathcal{N}(\mu - \frac{\sigma^2}{2}T; \sigma \sqrt{T}) \). We now substitute \( \mu \) for the formula in equation 4.6 above and sample from \( \mathcal{N}(ln(D/V) - \frac{\sigma^2T}{2}; \sigma \sqrt{T}) \). Now we need the likelihood ratio \( f(x)/g(x) \). The easiest way to obtain it is to simply compute the probabilities using Matlab. This is shown in the equation below where \( \phi \) denotes the normal density function.

\(^a\)In this case it is, but only because we use the standard Black-Scholes framework. This is not useful when we start tinkering with the model.
imp\(_p\) = \frac{\phi(ln(D/V) - \frac{\sigma^2}{2}; r - \frac{\sigma^2}{2}T, sig\sqrt{T})}{\phi(r - \frac{\sigma^2}{2}; r - \frac{\sigma^2}{2}T, sig\sqrt{T})} \tag{4.7}

Of course some of the terms cancel out if we use the formulas for the probability density function of the normal distribution because the standard deviation is the same for both distributions. We sample from \(N(\beta; \nu)\) instead of \(N(\alpha; \nu)\). Some calculus leads to the following result.

\[
\frac{1}{\sqrt{2\pi \nu}} e^{-\frac{(Y-\alpha)^2}{2\nu^2}} \quad \frac{1}{\sqrt{2\pi \nu}} e^{-\frac{(Y-\beta)^2}{2\nu^2}} = e^{-\frac{[2(\alpha-\beta)Y-\alpha^2+\beta^2]}{2\nu^2}} \tag{4.8}
\]

All the above may look mathematically complicated but comes down to the following. We adjust the mean of the distribution we sample from, we multiply each default by its likelihood ratio and divide the sum of all weighted defaults by the number of runs. These are our default probabilities. The above approach is also asymptotically efficient for path-dependent options such as the first passage approach.\(^{36}\)

We now consider some examples of suppliers in isolation. Dependencies and portfolios are introduced in the next chapter.

### 4.3 Examples of testing assumptions about an important dispute

A prominent example of a product recall in the automotive industry is the case of Toyota where, among other models, the Prius was said to accelerate due to floor mat entrapment\(^b\). Such a recall may also be a risk for any of our suppliers. This situation may be modeled by adding a jump in the asset value process. Modeling such a jump requires information about the cost of the recall. In section 4.3.2 below we increase the volatility to reflect uncertainty, this is another modeling method. Our goal here is to provide some ideas instead of fully worked-out cases.

#### 4.3.1 A bimodal distribution

Let us start with the bimodal distribution. The supplier of floor mats in our example portfolio is Intier. If Intier’s floor mats were the source of the acceleration problem it may have to carry the costs of replacing these, but we assume here we are not sure yet. Because we are unsure what happens but know that something is about to change, we can also assume that the stock price will either take a hit or it will jump up, but not stay

\(^b\)Information can be found on [http://www.toyota.com/recall/floormat.html](http://www.toyota.com/recall/floormat.html).
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the same. (Of course we may also assume that this is a different scenario and assume the situation will either stay the same or jump upwards.) In this case we encounter a mixture of two lognormal distributions, which is a bimodal distribution.39

Modeling this as a certain thing might be done by simply subtracting the size of the jump with chance $p_{\text{pay}} = 1$. A good proxy for the size of this jump in the latter case may be to subtract the costs of such a recall from the asset value $V_0$ or perhaps do so in only a number of occasions to reflect the uncertainty of the outcome. Starting with the simple case of $p_{\text{pay}} = 1$ (meaning Intier is certainly responsible for the problem) comes down to applying the Merton model.

**Example.** The risk-neutral default probability of Intier is $3.8912 \times 10^{-9}$ in the standard setting. Say we assume they take a big hit and lose one billion for sure. Its analytical default probability is now $1.2975 \times 10^{-7}$. While this is a considerable increase it still does not represent a big threat.

Now we estimate the chance that they lose this amount in value to be $p_{\text{pay}} = 0.5$ (there is a fifty percent chance Intier is the source of the acceleration problem), perhaps from similar cases. Analytically this already becomes harder, and while we may have a clue such as to take the averages of the two situations, we would have to prove this is really the right end result. Fortunately we have an efficient Monte Carlo model in Matlab. We use the following equation

$$V_0 - 1\{P < p_{\text{pay}}\} R e^{\mu_a T}$$

where $R$ represents the recall costs and $\mu_a$ is the adjusted drift generated with its own random number. Note that the importance sampling method is probably not entirely efficient anymore due to the asymmetry, but for reasonable changes (where the modification swings either way, where default is more unlikely or for modest other changes) it outperforms regular Monte Carlo simulation by a wide margin.

**Example.** Using $p_{\text{pay}} = 1$ and $R = 1 \times 10^9$ in 10 simulations with 100,000 runs (0.01s for 100,000 simulations) we get an average of $1.2964 \times 10^{-7}$ with a small standard deviation, versus the exact solution of $1.2975 \times 10^{-7}$. With $p_{\text{pay}} = 0.5$ we get $6.6544 \times 10^{-8}$ and see that Intier is less likely to default. We might of course introduce combinations of simultaneous changes in the asset value to our likings and modify this example as we like.

Even more complicated analytically is to use a distribution of the losses that we perhaps know from previous experience. A brief example is in place. Perhaps the lognormal distribution describes our data or assumptions well. In such cases we can replace $R$ by a realization of a random variable following for example a lognormal distribution with $\ln(500,000,000)$ and 0.25 standard deviation. Applying this, we get an average default

---

*Our random number generator never generates the value 1, so "smaller than" is entirely accurate.
rate of $2.6956 \times 10^{-9}$. We can also choose to use empirical distributions since the default probability is based on Monte Carlo simulation.

### 4.3.2 Increasing volatility

To model uncertainty we can also choose to increase the volatility. Since it is not immediately obvious how much more volatile the process must be, we may want to look to comparable cases (troubled firms with similar leverage ratios) to see what is reasonable. It may even be simplest to determine the increase in equity volatility and solve the simultaneous equations presented in chapter three. As we know from the sensitivity analyses on both the simultaneous equations as well as the model, the volatility is a rather sensitive input value, so we must be careful.

**Example.** Taking the worst cases of our samples (fifty percent equity volatility) we suddenly find a (risk-neutral) default probability of 0.1679 percent for Intier.

A more advanced extension of this might be to introduce stochastic volatility, where the volatility is not stable but has a distribution itself. There are several of such models\(^3\) such as GARCH and Heston’s model in which we would have a more natural effect of the changes. We consider these models too complicated for the purposes stated here due to extensive calibration and the still remaining estimation problem of the hypothetical case.

### 4.4 Examples of modeling market downturns with equity beta

Now we assume we have an important American supplier and we ask ourselves how a downturn of the stock market (called a bear market) would influence its credit quality. This is still for a single, individual supplier, the portfolio problem is considered in the next chapter. As a proxy for the state of the market we use the S&P 500 and the ArvinMeritor stock prices over 2007 to determine a key component from the famous CAPM model, the $\beta$, which measures both direction and magnitude. Note that we can choose the German DAX, the Dutch AEX or the Swiss SMI too if we like, but here we assumed the supplier to be American. All of this data is available on Yahoo Finance. As in the previous section, our goal is to provide some ideas instead of fully worked-out cases.

**Example.** A simple way of estimating past beta is by using linear regression on the returns. For ArvinMeritor we have $\beta_a = 0.1477$, meaning its returns generally follow the S&P 500 index. Another way to interpret this is to say that a 10 percent drop in the market index generally leads to a 1.477 percent drop of ArvinMeritor’s stock price. After solving the simultaneous equations using this value we find that the PD increases very little (from 0.281 percent to 0.282 percent). Such factors can also be incorporated
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in the simulations mentioned in previous paragraphs to test more complex assumptions.

It must be said that betas are not always stable and have been found to mean revert.\textsuperscript{25}
There are several techniques in the literature describing methods of estimating future betas instead of past betas that can be used instead if necessary.

4.4.1 Modeling other risk using betas

Market beta is the most popular beta, but there are other possibilities. Our model is of the form

\[ r_a = \alpha_a + \beta_a x_t + \epsilon_a \]  \hspace{1cm} (4.10)

because we perform regression analysis. The intercept is the \( \alpha_a \), the \( \beta_a \) is the resulting sensitivity, \( x_t \) is the increase in the variable of interest and the \( \epsilon_a \) is the idiosyncratic component. Many models follow such a form.

We can use the beta as described before to measure equity sensitivity to for instance exchange rates\textsuperscript{d}, in a model similar as above by Adler and Dumas.\textsuperscript{1} Over the course of 2007 the US dollar depreciated in value against the euro.

**Example.** In the same spirit as above we can measure the influence on the ArvinMeritor returns, again using regression analysis. We find \( \beta_{	ext{curr}} = 0.1018 \). At the time of writing the Swiss Franc was under strong pressure, dropping from 1.6CHF per euro the year before to 1. Using this alternative beta measures the influence of the exchange rate on the fortunes of a firm.

Other factors can be modeled too. We can model the betas with for instance a raw material price index as well, using a similar model. Again we can combine these with the simulation methods proposed in previous sections.

4.4.2 Using distributions instead of estimates

Here we can use a distribution of the likely prices too. If our beta is unstable over time but we have a sense of direction it might be useful to generate estimates according to a distribution of changes in \( V \), very much like we suggested before in section 4.3. Another idea is that perhaps there are different consequences for large shifts in the exchange rates than for small ones. The Monte Carlo model for which Matlab code can be found in the appendix can easily handle such input without having to do the mathematics.

\textsuperscript{d}A particularly useful website is exchangerate.com where you can find historical daily exchange rates.
4.5 Conclusion

In this chapter we have applied importance sampling to Monte Carlo simulation, which was found to be highly efficient. Then we provided examples of how various assumptions can be modeled in the structural framework. We assumed changes in the value $V$, both independently and linked to factors such as the hypothetical state of the economy. We can use these results in the portfolio version presented in the next chapter.
So far we have considered the case of a single supplier. This is less valuable in practice since in practice such default risks are correlated, due to which the chances of a default and expected losses can rise dramatically.

The basic credit risk models are expandable to multiple counterparties. We do exactly that in this chapter by considering the portfolio problem in an interdependent setting, using Monte Carlo simulation with importance sampling. Then we show examples of how to model some of the specifics of supplier default risk derived from purchasing theory.
5.1 Default correlation

Before we construct and explore models it is important to reflect on some of the differences when considering a portfolio setting. A default correlation between a number of companies means these tend to default at the same time. Since we are working with extremely small probabilities, effects of correlation can be much larger than in the case of high probabilities.

There is a wide array of reasons as to why this may be the case. Sometimes companies are exposed to the same economic conditions, such as the euro crisis sparked by Greece at the time of writing. Such effects may be largely local, perhaps wars or other conflicts, or world-wide, affecting all firms. Business cycles contribute to such situations too and companies using the same inputs (labor, raw materials) sometimes find their faiths intertwined. Credit contagions cause correlations as one default leads to another. These are just some explanations.

In this chapter we use proxies such as asset correlation to model default correlation. It is no secret that neither of these correlations is particularly stable. This becomes even clearer when taking the previous paragraph into account and realizing there are countless variations and combinations. Aspects as mentioned in the previous paragraph can be modeled using for instance a one-factor model instead of using proxies, but such factors are hard to determine and are not obvious at all. We have therefore chosen to use asset correlation because good data is available, markets are liquid and the view of the market is implied by these. It is also successfully used in many commercial applications.18

A special case in supplier default risk is the correlation between defaults of the mother firm and the daughter when credit is backed by the mother. If both go down simultaneously, such a letter may not be worth much. We will not explicitly consider such a situation, but the tools supplier in this chapter are sufficient to make a raw estimate.

5.2 A default distribution for first-tier supplier portfolios

In this section, we start by outlining the theory on supplier portfolios using the Merton model from chapter 2 and the CreditRisk+ model. The Bernoulli model is used as a starting point to exemplify the thought process behind our models.

5.2.1 Bernoulli model

As may be recalled from chapter 2, a Bernoulli process is a binary random variable defined as in equation 5.1. For now we do not consider dependencies.
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\[ Y_i \sim \text{Bernoulli}(p_i) : Y_i = \begin{cases} 
1 \text{ with probability } p_i & \text{if supplier } i \text{ defaults} \\
0 \text{ with probability } 1 - p_i & \text{if supplier } i \text{ survives}
\end{cases} \quad (5.1) \]

Here we assume for a second that we already have the default probability \( p_i \). The number of defaults is then the sum of all of the \( n \) random variables in the portfolio as illustrated in equation 5.2. Using the notation \( L_i \) for a loss given default (LGD) we take the expected value of the losses by multiplying each of the random variables by their corresponding losses (see 5.3).

\[
N = \sum_{i=1}^{n} Y_i \quad (5.2)
\]

\[
L = \sum_{i=1}^{n} L_i Y_i \quad (5.3)
\]

The distribution of such a portfolio of independent suppliers depends on the default probabilities. If these are homogeneous, meaning \( p \) is identical for every firm, we arrive at the binomial distribution in 5.4 by a convolution of Bernoulli variables. The expected value is \( np \) and the variance is \( np(1 - p) \).

\[
P[N = k] = \binom{n}{k} p^k (1 - p)^{n-k}, k \leq n \quad (5.4)
\]

Homogeneous PD’s are not very realistic at all unless a large portfolio is used. Using heterogeneous probabilities, even for a small portfolio, is unfortunately not easily done.\(^{26}\)

To illustrate the difficulties, consider the number of combinations for equation 5.5 below.

\[
P[Y_1 = y_1, ..., Y_m = y_m] = \prod_{i=1}^{m} p_i^{y_i} (1 - p_i)^{1-y_i} \quad (5.5)
\]

Let us consider a simple form of dependency in this model. The so-called Bernoulli mixture model is based on conditional independence.\(^{44}\) Using the notation from McNeil, Frey and Embrechts, this is represented in equation 5.6. Its interpretation is that there is one\(^a\) exogenous risk factor \( \Psi \) upon which realization (given \( \Psi = \psi \) ) the \( p_i \)'s are independent and we obtain the equation below. For every supplier there is a function \( p_i(\psi) \), mapping the realization of \( \psi \) to a probability of default. Note that this is therefore a two-step procedure where we first have to generate factor realizations.

\[
Y_i|_{\psi=\psi} \sim \text{Bernoulli}(p_i(\psi)) \quad (5.6)
\]

\(^a\)This is the one factor model and is commonly used according to McNeil, Frey and Embrechts (2005).
5.2.2 CreditRisk+ model

CreditRisk+ does not explicitly consider the convolutions of the Bernoulli model but rather approximates default probabilities using the Poisson distribution (see 5.7).

\[ f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, k = 0, 1, 2, ..., \lambda \geq 0 \]  

A Poisson distribution is a distribution of intensity, in other words the number of defaults within a time period. Note that this is a reduced form model where the causes of the default are not considered. By considering multiple defaults and modeling these as a mixed Poisson distribution, the sum of these approximately equals the number of suppliers that default (given small Poisson parameters). Instead of a distribution of defaults, we now have a default rate. This makes default itself a random variable, fortunately with the very small risk of having more than one default within a time period: \( P(Y_i \geq 2) \) should be very small.

Such a default rate can be both continuous as well as discrete. In a continuous case we specify the volatility of default probabilities over the years. This is what CreditRisk+ does and rating agencies publish such data. In its discrete analogy we map default rates to credit ratings in the same way we obtained individual credit ratings in chapter 2. Using a transition matrix describing the probability of being downgraded (Aaa to Aa, for instance), we obtain the volatility of these defaults.

Dependencies in CreditRisk+ are introduced using default rate volatilities to represent uncertainty from background factors in default rates, and using sector analysis. Sector analysis is done by specifying several sectors to which suppliers can be partially assigned. Reasons for this approach given by Credit Suisse First Boston are 1) instable and 2) not easy to quantify due to the lack of empirical data. As we shall see later we use equity correlation regardless of this, because we think the factors in the factor model are even harder to quantify. Please note that by introducing the correlations, (pairwise) correlation is still implied, albeit not explicitly.

5.3 A portfolio extension of the Merton model

Now we consider a portfolio in our model. In chapter three we used the Merton model for individual firms and in chapter four we repeated this exercise using Monte Carlo simulation. A natural portfolio extension is by the use of correlated random numbers while simulating several suppliers simultaneously, again using Monte Carlo simulation. Equations for two correlated random numbers are given by the equations below.
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\[ \zeta_1 = x_1 \]  
\[ \zeta_2 = \rho x_1 + x_2 \sqrt{1 - \rho^2} \]  

This is the same as assigning factors \( \alpha_{ij} \) to denote \( \epsilon_2 = \alpha_{21} x_1 + \alpha_{22} x_2 \). We should just make sure that \( \alpha_{21} \alpha_{11} = \rho_{21} \) and that \( \alpha_{21}^2 + \alpha_{22}^2 = 1 \) to make sure the above holds for every pair of equations. This method can be generalized for an arbitrary number of random numbers. Choosing values for \( \alpha \) as described above is known as a Cholesky decomposition and is a standard function in Matlab.

As we have seen in chapter three, the individual PDs are already extremely small in some cases. Simulating these conjointly will then create instances in which probably no defaults are seen at all. We must therefore nudge the Monte Carlo simulation into the right direction to extract such unlikely events using importance sampling as shown in the previous chapter.

5.3.1 Importance sampling in the Merton portfolio model

We have implemented such a method following Han\textsuperscript{36} because we considered this to be the most natural extension of the problem. As in chapter four, we apply a change of drift and sample from a different distribution. Using the same notation as we have throughout this thesis the process is now based on the density \( g \) which is given by

\[ S_T = S_0 e^{\mu - \frac{\sigma^2}{2} - \sigma h_i} \]  

where \( h \) is given by the solution of the sequential equations given by

\[ \sum_{j=1}^{i} \rho_{ij} h_j = \frac{\mu_i}{\sigma_i} - \ln \left( \frac{D_i}{S_{i0}} \right) \frac{\sigma_i T}{\sigma_i} \]  

with \( \rho_{ij} \) being the correlation between \( i \) and \( j \).

If we take the uncorrelated case where the matrix \( \rho \) is a unit diagonal matrix (ones on the diagonal and zeros elsewhere) the density \( g \) reduces to \( \ln \left( D_i / S_{i0} - \frac{\sigma_i^2}{2} \right) \), which was the drift we used in chapter four to set the expected value of the process equal to \( D \).

We have implemented this method (see appendix for code) and tested the uncorrelated case with \( D, \sigma \) and \( V \) equal against the binomial distribution. It appears to be working well (and very well, indeed) only in the tails of the distribution, where all suppliers default simultaneously, but not for the rest of the distribution.
The most likely reason for this is that the distribution is multi-modal and that standard importance sampling is inadequate: we push the supplier values $V$ to unlikely scenarios but oftentimes just below the default barrier (the value is therefore still unlikely to occur, but just as uninformative). The resulting importance will be extremely low since it is the product of several unlikely situations. The probabilities of, for example, the cases ‘no defaults’ and ‘one default’ are commonly encountered, but due to the values $V_I$ of other suppliers being just below the default point $D$, importance is very low. Sometimes a solution does, by chance, have a large importance and as such it characterizes the entire end result. This large variance is exactly what we are trying to avoid.

We have undertaken extensive measures and tests to improve the algorithm using stochastic drift changes (by which the distribution is random) which did lead to significant improvements (results were tested using the binomial distribution as an approximation), but none worked satisfactorily over all scenarios. We found this is due to the multi-modal nature of the portfolio distribution which leads to a large complexity of the problem: adding random changes does work, but only approximates the case where you have to consider all combinations of defaults. This comes close to complete enumeration and leads to problems for larger portfolios. Think for example the number of combinations of defaults possible with ten suppliers for no defaults, one default, two defaults, etcetera.

There are ways to overcome this problem. We have found that Avramidis\textsuperscript{5} has developed an algorithm to find a good density $g$ in such cases that finds the modes of such a distribution, but is much more complex. Due to the shortcomings found here and the complexity of the direct Avramidis alternative, the wish to include techniques presented in chapter four and the wish to use mapped probabilities (as in the end of chapter three), we instead decided to pursue a different approach. We describe it below.

5.4 Estimating the loss distribution using Monte Carlo simulation

The method in the previous section is not our preferred method to estimate the loss distribution, not only due to the fact that it does not perform adequately. Implementing an algorithm such as the one by Avramidis\textsuperscript{5} might resolve the issue of obtaining a distribution but it still does not allow us to obtain the actual loss distribution. Remember that the Merton model provides us with risk neutral probabilities that are fine for ranking the suppliers, but not all that useful for computing expected losses. The mapping procedure at the end of chapter three, using credit ratings, provided a way to deal with this. Sometimes information that is hard or impossible to incorporate into our structural framework might be available and allows one to revise the default probability for a single firm altogether. In the ideal case we use the mapped and revised default probabilities for our portfolio version and use the structural frameworks in chapters three and four to obtain these. By using those we do not have to recompute the default probabilities and maintain maximal flexibility. We have found such a method in literature based on a
Monte Carlo simulation of the mapped default probabilities, with its efficiency improved by importance sampling.

The rest of this section follows the method of \textsuperscript{33} and is somewhat technical. Nearly all of the mathematics below, with the exception of the factor model, are merely used to improve the Monte Carlo simulation using importance sampling and is robust for a wide range\textsuperscript{b} of input probabilities. The non-technical reader can skip it and proceed to the next section immediately. The practically oriented reader can simply input the data and run the Matlab m-file for which the code can be found in the appendix.

5.4.1 The loss distribution for independent defaults

In this section we assume that the probabilities of default are known by applying techniques of chapters three and four including the mapping to real world probabilities. That means we have already applied the Merton model and have a list of mapped default probabilities. Although the mapped probabilities are generally larger than the risk-neutral ones, these are still very small. We use the importance sampling technique introduced in chapter four to make the Monte Carlo simulation more efficient. We introduce the simplified version of independent defaults as a first step but we introduce correlation later, so we cannot solve this analytically in ways presented in the previous sections.

We have a portfolio of \( m \) suppliers where \( p_k \) is defined as the probability of default for supplier \( k \), \( c_k \) its loss given default and \( Y_k \) a binary variable indicating a default (1) or no default (0) of supplier \( k \). Our total losses \( L \) are given by

\[
L = c_1 Y_1 + c_2 Y_2 + ... + c_m Y_m. \tag{5.12}
\]

What we are interested in is the probability of losing more than \( x \) euro, denoted by \( P[L > x] \). To model and compute this we use the approach of Glasserman and Li.\textsuperscript{33} We change the probability of a default depending on a factor \( \theta \) using a so-called exponential twist. Such a twist is a standard method and is done by

\[
p_{k,\theta} = \frac{p_k e^{\theta c_k}}{1 + p_k (e^{\theta c_k} - 1)} \tag{5.13}
\]

where \( \theta \) is the only unknown factor. As before we have to correct any values by multiplying it by the likelihood function or Radon-Nikodym derivative \( dP_\theta/dP \), which is

\[
\frac{dP_\theta}{dP} = exp(\theta L - \psi(\theta)) \tag{5.14}
\]

\textsuperscript{b}Not any set is allowed since computers have limited precision due to their binary nature. In practice this is rarely a problem.
where $\psi(\theta)$ is defined by

$$
\psi(\theta) = \ln E \left[e^{\theta L}\right] = \sum_{k=1}^{m} \ln(1 + p_k(e^{\theta c_k} - 1)).
$$

(5.15)

Glasserman and Li prove the previously discussed asymptotic optimality for

$$
\theta_x = \begin{cases} 
\text{unique solution to } \psi'(\theta) = x & x > \psi'(0) \\
0 & x \leq \psi'(0)
\end{cases}
$$

(5.16)

which is equivalent to $\theta_x = \max(0, \theta)$ where $\theta$ is the unique solution above. What is left to us is to find this solution. First we take the first derivative of $\psi$ to $\theta$ which is

$$
\psi'(\theta) = \sum_{k=1}^{m} \frac{c_k p_k e^{\theta c_k}}{1 + p_k(e^{\theta c_k} - 1)}.
$$

(5.17)

Now we let Matlab find the root of $\psi'(\theta) - x$ to solve this. The steps of the algorithm we use in Matlab is as described below.

1. Find the solution of $\psi'(\theta) = x$ for a predefined level of x.
2. Twist the probabilities using formula 5.13.
3. Generate k random numbers of the $U(0, 1)$-distribution for each run.
4. Determine the value of $L_i$ for each run i by setting $Y_k = 1$ when random number k is lower than the twisted probability, using formula 5.12.
5. Compute the value of $\psi(\theta)$ using formula 5.15.
6. Correct the loss per scenario $L_i$ by multiplying it by equation 5.14.
7. Sum all $L_i$ and divide by the number of runs.

All of the above has been implemented in Matlab for which the count including instructions can be found in the appendix.

5.4.2 Capturing dependence through a factor model

Above we have shown how the losses in a portfolio of independent suppliers can be simulated using Monte Carlo simulation using importance sampling.

We now expand the framework to include dependence by the means of a multi factor model. Models like these are used in large commercial applications such as CreditMetrics, we use it for its simplicity but also consider a model used to price securities adequate to
Modeling supplier portfolios

capture dependency for the purpose of risk management. A common way of introducing
correlations is using a factor model\(^c\) of the form

\[
X_k = a_{k1}Z_1 + a_{k2}Z_2 + \ldots + a_{kd}Z_d + b_k\epsilon_k
\]

(5.18)
in which there are \(d\) factors. \(Z_1,\ldots,Z_d\) are risk factors having a \(N(0,1)\)-distribution,
\(\epsilon_k\) is idiosyncratic (meaning unique to this supplier) risk and \(a_{k1},\ldots,a_{kd}\) are so called
factor loadings indicating how much a supplier is affected by this specific factor. These
common risk factors cause the dependency between our suppliers by assigning them
(choosing factor loadings) to the random variables in \(Z\) with weight \(a_{kf}\), where \(f\) denotes
the factor \((f=1,\ldots,d)\). This sounds more complicated than it is. Consider for example a
2-factor model in which geographic region Europe and the general state of the economy
are risk factors. A factor \(a_{11}\) would indicate how much supplier 1 is influenced by the
geographic region and \(a_{12}\) how much this supplier is influenced by the state of the econ-
omy. CreditRisk+ uses a similar structure to introduce correlation.

In order to have nice and simple properties the following conditions apply to these
factor loadings:

\[
a_{k1}^2 + \ldots + a_{kd}^2 \leq 1
\]

(5.19)

\[
b_k = \sqrt{1 - (a_{k1}^2 + \ldots + a_{kd}^2)}
\]

(5.20)

\[
a_{kd} \geq 0
\]

(5.21)

The first two ensure that \(X_k\) is \(N(0,1)\) distributed. The last condition is not strictly
necessary but makes sure that all default indicators are positively correlated, something
much used in practice\(^33\) to ensure positive factor realizations lead to positive correlation.

Now we can determine conditional default probabilities (conditional upon the realization
of the factors, see Glasserman and Li (2005) for a derivation) by

\[
p_k(Z) = \Phi\left(\frac{a_kZ + \Phi^{-1}(p_k)}{b_k}\right)
\]

(5.22)

where \(a_k = (a_{k1},\ldots,a_{kd})\) and thus a vector of factor loadings for supplier \(k\), \(Z = (Z_1,\ldots,Z_d)^T\)
the transposed vector of given factor loadings and \(b_k\) as defined before. Note that \(a_kZ\) is a matrix multiplication.

\(^c\)This is a Gaussian copula and can be combined with any marginal distribution to arrive at more
thick-tailed distributions.
This result is useful when applying importance sampling because, given the realizations of \( Z \), the probabilities are independent, which leads to the procedure we applied before. The question remains how we determine the number of factors and their loadings for each of the suppliers as the correlation between \( X_k \) and \( X_j \) is given by \( a_k a_j^T \) (matrix multiplication) or equivalently \( \sum_{k=1}^{m} a_{ik}a_{jk} \). We present methods to select actual values in section 5.5 and first introduce dependencies.

5.4.3 The loss distribution for weakly dependent defaults

Now that we have defined factor loadings and systematic risk factors we can perform importance sampling between dependent suppliers.\(^{33}\) Because conditional on \( Z = z \) (a realization of the factors) the default indicators are independent, the method is almost the same as before. The exception is that we have to solve \( \frac{\delta}{\delta \theta} \psi_m(\theta, Z) = x \) after every set of random numbers generated because \( p_k \) and therefore \( \psi(\theta) \) now depend on \( Z \).\(^{33}\) The Matlab code to do so can be found in the appendix and can be run without technical knowledge of the procedure.

After determining \( b \) as above (and verifying that the factor loadings meet the criteria we set) we can apply the following procedure for each replication.

1. Generate \( d \) random \( N(0, 1) \) values forming a vector \( Z \).
2. Calculate the conditional default probabilities as in formula 5.22.
3. If \( \sum_{k=1}^{m} p_k(Z)c_k \geq x \) set \( \theta_x(Z) = 0 \), otherwise set it to the solution of \( \frac{\delta}{\delta \theta} \psi_m(\theta, Z) = x \) where \( \psi_m(\theta, z) \) is as in formula 5.15 with \( p_k \) replaced by \( p_k(Z) \).
4. Compute the loss \( L \) as in 5.12 and correct it using the likelihood function.

Then we simply take the average of all of these results as we do in normal Monte Carlo simulation. Because we have only applied importance sampling to the conditional probabilities, this is only efficient for low dependence among suppliers. We address this in the next section.

5.4.4 The loss distribution for generally dependent defaults

The last optimization step done by Glasserman and Li\(^{33}\) is importance sampling over \( Z \), because we have not reduced the variance of the expected value of \( p_k \) fully. This variance is caused by the realizations of \( Z \). All we do is apply importance sampling to the distribution of \( Z \) too. That means we have applied it twice.

As we know from chapter four, the application of importance sampling to a normal distribution is to change the distribution from which we draw random numbers to \( N(\mu, 1) \). The problem here is that in order to normalize we would have to know \( P(L > x) \) and this
is exactly the problem we are trying to solve. In another paper Glasserman suggests solving this using a normal density with the same mode as the optimal density. This is

$$\max_z P(L > x | Z = z) e^{-z^T z / 2}$$  \hspace{1cm} (5.23)$$

for which there are various strategies. Among the ones Glasserman and Li propose they present results for one, but others have tested similar functions and have found the Normal approximation to produce similar results. This approximation is

$$P(L > x | Z = z) \approx 1 - \Phi \left( \frac{1 - E[L|Z = z]}{\sqrt{Var[L|Z = z]}} \right)$$  \hspace{1cm} (5.24)$$

with

$$E[L|Z = z] = \sum_k p_k(z)c_k$$  \hspace{1cm} (5.25)$$

$$Var[L|Z = z] = \sum_k c_k^2 p_k(z) (1 - p_k(z)).$$  \hspace{1cm} (5.26)$$

We can solve this numerically in Matlab which uses an algorithm called the Nelder-Mead Simplex method (it minimizes, so we multiply formula 5.23 by -1). We have found a good starting solution that appears to converge rapidly in the form of $\Phi^{-1}(1 - p_i)$, which is the solution if all $p_i$ are equal and $\rho = 1$.

Equation 4.8 then tells us to multiply our current results (consisting of the result of the simulation multiplied by the likelihood function of the previous section) by the likelihood function because we apply importance sampling to $Z$ in that equation. This would mean our estimators are

$$1[L > x] e^{-\theta_r(Z)L + \psi(\theta_r(Z),Z)} e^{-\mu^T Z + \mu^T \mu / 2}$$  \hspace{1cm} (5.27)$$

because $\alpha = 0$, $\beta = \mu$ and $\nu = 1$. The above can be coded by changing only a few lines in the Matlab code of the previous section. For the sake of convenience the full code can be found in the appendix. We now have an efficient method to simulate losses in supplier portfolios.

### 5.5 Selecting factor loadings in the factor model

In the previous section we defined a multi factor model by
where the matrix $a$ contains the factor loadings for each of the suppliers and the vector $b$ is an idiosyncratic component ensuring nice mathematical properties. By assigning different factor loadings to each of the suppliers we imply different correlations.

Above we mentioned that a similar model for correlation was used in CreditRisk+. In such a model each of the factors gets an economical interpretation, such as the region in which the firm operates or its dependence on the market. We can add as many factors as we like but have to make sure it reflects at least a part of reality.

### 5.5.1 Selecting factors for the factor models

In the simplest factor model, a one factor model, every supplier is affected by a single factor. We can for instance simplify by assuming that since we have suppliers in the same industry, they are only affected by the general economy. We can simplify even more by assuming that all of these are correlated equally.\(^{55}\) In this case we have

\[
a_i = \sqrt{\rho}
\]

where $\rho$ is the correlation. Such an approach seems overly simplistic at first sight, but is often used as an approximation when pricing large portfolios. Even small correlations can already make large differences in the loss distribution.\(^{44}\)

Without large databases of defaults we feel one cannot expect to calibrate a multi factor model using predefined factors. We therefore proceed to another method.

### 5.5.2 Calibration using equity correlation

An alternative to the above is to use equity correlations as a proxy. We choose to do so and gather stock price series over 2007 while making sure the calendar days are matched, since not all exchanges are open on the same days. As discussed before it is important to select high resolution data and verify that no large dividends or stock splits are missing in the data. We then take returns using natural logarithms and now look for linear correlations in stock prices. Now we want a matrix $a$ of factor allocations that matches our correlation matrix $\rho$. We can do this simply with the predefined Cholesky decomposition method as we have described above. This states that (this is linear algebra, $a$ and $\rho$ are matrices)

\[
\rho = aa^T
\]
where $a^T$ is the transposed matrix of $a$ (Matlab requires ‘lower’ as the second parameter here, see appendix for code and explanation). Note that the condition $a_{ij}^2 + \ldots + a_{id}^2 \leq 1$ for every $i$ holds. These are in fact exactly one so our vector $b$ is a zero vector that is unneeded in this case, since the nice mathematical properties are already ensured by the Cholesky decomposition. Our idiosyncratic risks stem from the specific factor loadings.

Combined with the approach above, this allows us to either choose factors ourselves to our own likings, or from any predefined correlation structure we wish to force upon it.

5.6 An illustration of some final results: Producing a loss distribution and risk contributions

Here we provide an example of how the above can be applied. We use some suppliers of Audi’s portfolio, namely the ones with high losses. Let us first obtain a probability distribution, after which we continue with the loss distribution. The input is given in table 5.1 below.

<table>
<thead>
<tr>
<th></th>
<th>ArvinMeritor</th>
<th>Faurecia</th>
<th>Methode</th>
<th>Scapa</th>
<th>Tupy</th>
<th>Varta</th>
<th>PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ArvinMeritor</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td>Faurecia</td>
<td>0.62</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0238</td>
</tr>
<tr>
<td>Methode</td>
<td>0.27</td>
<td>0.51</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>0.0269</td>
</tr>
<tr>
<td>Scapa</td>
<td>-0.77</td>
<td>-0.22</td>
<td>0.01</td>
<td>1</td>
<td></td>
<td></td>
<td>0.0047</td>
</tr>
<tr>
<td>Tupy</td>
<td>0.13</td>
<td>0.33</td>
<td>0.24</td>
<td>0.00</td>
<td>1</td>
<td></td>
<td>0.0438</td>
</tr>
<tr>
<td>Varta</td>
<td>-0.51</td>
<td>-0.08</td>
<td>0.08</td>
<td>0.71</td>
<td>-0.10</td>
<td>1</td>
<td>0.0051</td>
</tr>
</tbody>
</table>

Table 5.1: Input values for our subportfolio that we use as a running example. Asset correlations can be found on the left side, the last column contains mapped (real-world) default probabilities.

We set $c = 1$ (losses are equal for all) and set the vector $b$ to a vector of ones with the same length as $p$. Because our Cholesky decomposition (see above) already ensures the mathematical properties we need, $b$ is a zero-vector and drops out of the equation. Now we can set $x = 0$, solve the model, change $x$ and resolve it until we have results for the area of interest. We can then compute the individual default probabilities using

$$P[L = x] = P[L > x] - \sum_{i=x+1}^{n} P[L > x_i]$$  \hspace{1cm} (5.31)

because it is a discrete distribution (where $P[L > -1] = 1$). Both distributions have been plotted in figure 5.1. The shape of the right distribution can vary and even alternate from low to high for an increasing $x$ when the LGD’s and default probabilities vary wildly.
Chapter 5

Figure 5.1: The cumulative default distribution for the portfolio above. On the right side a density function is formed by subtracting the points for higher $x$. Note that this is actually a stepwise function. We sampled at 0, 1, 2, 3, 4 and 5. The lines in between should not be interpreted as probabilities.

Now we can set $c$ to the estimated losses and repeat this procedure to obtain a loss distribution. The shape of this distribution depends on the aggregation method used in chapter three. If we take the time focused approach and use the maximum for a single suppliers we get very different results than if we take the effort oriented approach which sums up all of the importances. The table below presents us with the loss-given-defaults for both cases. We use the correlation on equity returns computed using CORREL() in Excel (Pearson correlation) presented in table 5.2 below, for which the method is described above.

<table>
<thead>
<tr>
<th></th>
<th>ArvinMeritor</th>
<th>Faurecia</th>
<th>Methode</th>
<th>Scapa</th>
<th>Tupy</th>
<th>Varta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-Oriented</td>
<td>80,000</td>
<td>80,000</td>
<td>20,000</td>
<td>20,000</td>
<td>20,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Effort-Oriented</td>
<td>200,000</td>
<td>80,000</td>
<td>120,000</td>
<td>60,000</td>
<td>60,000</td>
<td>300,000</td>
</tr>
</tbody>
</table>

Table 5.2: Input for the losses. The time-oriented values are the maximum losses while effort-oriented are the sums.

Using the importance sampling technique for portfolios we can sample several points ranging from 0 to the sum of all costs minus one to obtain a loss distribution. Beware of efficiency: for instance picking the value 5,000 given the table above is pointless, since the lowest possible strictly positive loss is 10,000. Instead we pick some informative points and simulate with 5,000 runs. The input we use in Matlab is in thousands since it is easier on the solver.

The results have been plotted below. We see that the distribution is jagged, this is because of the discrete nature of the losses and the differences in default probabilities and dependencies. A density function (non-cumulative) might jump up and down, reflecting this behavior.
Figure 5.2: Left (a) is the time oriented loss distribution, (b) is the effort oriented loss distribution. Naturally, losses are much higher in the rightmost distribution since the inputs are larger. The circles mark points that were evaluated. The lines in between should not be interpolated as it is actually a stepwise function. On the right (c) Lorenz curve of our simplified approximation of risk contributions. The further it is from a straight line from (0,0) to (1,1), the less balanced it is.

5.6.1 Example of estimating risk contributions

To show possible expansions of this framework, we briefly mention risk contributions. This, too, is a simplified example that merely approximates contributions and is unfit for large portfolios due to the large number of simulations that are necessary. Since we are using simulation we cannot take analytical derivatives as one would normally do. We can however approximate it numerically by computing the derivative of $P[L > x]$ to $L_i$, which is the LGD for supplier $i$. If we increase one of these we can monitor the subsequent increase in PD for a given level $x$, upon which we determine PD using $P[L > x]$.

We can then compare the increase in PD for each of these and determine which carries most risk. One way of doing this is by normalizing the changes in PD and plotting them in a curve, for instance such as the Lorenz curve (see Matlab code in appendix) as in figure 5.2 plotted for the effort oriented approach using $x = 500$. This shows how risky each asset is at a level $x$. We see that Varta contributes most of the risk at this level due to its high LGD. Since its high value is almost needed to attain 500 (the sum of the LGDs is 820) or above, this makes sense. It might be sensible to set all LGDs to 1 temporarily for a system reliability perspective.

Such estimations are impractical for large portfolios. We refer the interested reader to Glasserman, $^{31}$ who provides another way of incorporating this in our current framework.
5.7 Ideas of incorporating the unique characteristics of supplier default risk

In the following sections we continue the above example to illustrate some of the possibilities such a framework provides. Above we have modeled a supplier portfolio very much in the sense that a bank would do, for example when analyzing derivatives whose payouts depend on the defaults of a number of counterparties. This is probably a good approximation for suppliers whose defaults have small operational and organizational consequences and merely induce costs. Here we might think of a category of single sourced (see figure above) suppliers for which there are plenty of alternatives, although perhaps at a high cost. In this case the sum of all costs is likely to be a good estimate. But there are cases specific to supply chains that require a different kind of modeling.

Sourcing comes in four primary structures as seen in figure 5.3: single, multiple, parallel and delegated. The first two speak for themselves. Parallel sourcing sources distinct products from two suppliers where each has the capability to supply the product the other supplies. In delegated sourcing, the responsibility for some suppliers is given to another suppliers, thereby creating a small network. Single and delegated sourcing are the most fragile structures in terms of default: they provide no immediate alternatives.

5.7.1 Explicitly incorporating sourcing strategies

We again use the selection of firms presented in 5.6, which are the ones with the high exposure-at-defaults in Audi’s portfolio. Since we have already obtained a distribution in the previous section, we can now look into the effects of different sourcing strategies. Let us assume that the most important products supplied by Faurecia, exhaust systems, can also be supplied by Valeo (a supplier of exhausts, indeed). We now use a bivariate normal distribution to determine the probability that either of these goes bankrupt. A multivariate distribution, a generalization of the bivariate, takes a vector of $\mu$ and a covariance matrix $\Sigma$ as input.

We now replace Faurecia by a new supplier we call FV (Faurecia-Valeo). First we
normalize the default probabilities to a standard normal distribution by taking the inverse

\[ x_i = \Phi^{-1}(p_i) \]  

(5.32)

from which we have \( x_1 = -1.9816 \) and \( x_2 = -2.0358 \). Our covariance matrix is then simply our correlation matrix,\(^{18}\) which consists of ones along the diagonal and the correlation between these assets in the upper and lower triangle of the matrix. This is \( \rho = 0.0383 \), obtained from equity returns. Such a low value is good news for a firm because it means these suppliers are almost independent. We now compute \( p_{fv} \) as the probability of default for our multi-sourced value using the cumulative distribution function. There is no closed form available, but Matlab provides an efficient algorithm which we denote by the working programming code

\[ p_{fv} = \text{mvncdf}([x_1 \ x_2], 0, [1 0.0383; 0.0383 1]) \]  

(5.33)

where the first are the \( x \)-values, the second is \( \mu \) and the third is \( \Sigma \). Of course we have to recompute the correlated of other assets with this one. We do so by summing the returns of Faurecia and Valeo (representing a pooling effect). Our end result is in table 5.3 below.

<table>
<thead>
<tr>
<th>ArvinMeritor</th>
<th>FV</th>
<th>Methode</th>
<th>Scapa</th>
<th>Tupy</th>
<th>Varta</th>
<th>PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ArvinMeritor</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td>FV</td>
<td>0.19</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>0.0047</td>
</tr>
<tr>
<td>Methode</td>
<td>0.27</td>
<td>0.02</td>
<td>1</td>
<td></td>
<td></td>
<td>0.0269</td>
</tr>
<tr>
<td>Scapa</td>
<td>-0.77</td>
<td>0.01</td>
<td>0.01</td>
<td>1</td>
<td></td>
<td>0.0047</td>
</tr>
<tr>
<td>Tupy</td>
<td>0.13</td>
<td>0.13</td>
<td>0.24</td>
<td>0.00</td>
<td>1</td>
<td>0.0438</td>
</tr>
<tr>
<td>Varta</td>
<td>-0.51</td>
<td>0.11</td>
<td>0.08</td>
<td>0.71</td>
<td>-0.10</td>
<td>0.0051</td>
</tr>
</tbody>
</table>

Table 5.3: Values when applying multi-sourcing to a supplier. FV’s probability of default is significantly lower than Faurecia’s. By repeating the distribution and cost analysis above we can determine the benefit of having such multi-sourcing.

This is quite informative since we can now determine the added benefit to our supplier portfolio from having this specific multi sourcing. Of course we can generalize the above from 2 to \( n \)-sourcing.

Note that the same procedure holds for parallel sourcing. In that case we can simply set the PD’s and correlations the same for two suppliers, thereby replacing both with the same unit and considering it as a sort of ‘double single sourcing’.

\[ 83 \]
5.7.2 An example of modeling the network characteristics of suppliers

The last sourcing strategy we consider is delegated sourcing. In the simple case, we can consider a supplier portfolio such as in figure 5.4. This section is very much an example and application of some of the possibilities and not at all intended as a complete theory.

![Figure 5.4: A small sample supplier network.](image)

Here we move from a flat portfolio to a networked structure. Suppliers four, five and six might be important to us but are actually second-tier suppliers. We are now analyzing a network.

We might feel like our network is still a portfolio and that it is a distinction without a difference. After all, any failing component leads to problems in our supply chain. We argue that this is not the case because second-tier suppliers are tailored to first tiers and often these second-tier suppliers carry responsibility for them. Although a failure of second-tier suppliers would probably cause problems for us indeed, we do not incur the costs of resourcing and fixing it until operational disturbances occur. Then there is the issue of a first-tier bankruptcy. In the case of delegated sourcing we would lose some of our second-tier suppliers that may actually supply key products and will have to renegotiate contracts and arrangements while doing so. The difference is then that the costs of bankruptcy for a first-tier supplier most likely increases (we lose their managerial duties and matching technology).

We can increase the LGD of our first-tier suppliers depending on the links to second-tier suppliers: these are default costs we incur for finding a new in-between supplier and renegotiating with second-tier suppliers until we find a new supplier to take over the delegated sourcing. The question remains by how much these costs should increase. It is probably less than the sum of finding replacements for all but might be a considerable number if rework needs to be done. We leave it up to the practitioner to determine such a cost increase, perhaps from past experience or reference cases. The section below might provide initial directions.
Determining reliability

We can also choose to determine the fragility, or reliability, by finding the probability of a supplier going down using the multivariate normal as discussed before. (We may also choose to find a distribution as shown in previous sections.) The situation in the figure above is more unreliable since 4, 5 and 6 are (partially) dependent on 1 and 2. We can apply reliability theory\textsuperscript{50} to determine the fragility, including the correlations.

The suppliers 1 through 6 represent the suppliers from the example in the beginning of the previous section, with 1 representing ArvinMeritor and 6 representing Varta. To illustrate the reduced reliability of supplier 4, note that it is now the probability that supplier 4 or 1 goes bankrupt. For supplier 5, both 1 and 2 or 5 itself has to go bankrupt. We can approximate this by assuming independence and using techniques from reliability analysis.\textsuperscript{50} Of course we can simply add multi or parallel sourcing by first applying the methods of the previous section.

A brief example. Supplier 4’s reliability depends on 4 itself and 1 too. The default probability for supplier 1 is $PD(1) = 0.0033$, for 4 is it $PD(4) = 0.0047$. The probability that supplier 4 is unavailable to us (assuming independence) is then $1 - (1 - 0.0033)(1 - 0.0047) = 0.00798$. This is much higher than $PD(4)$ and might help us to select an adequate value for the exposure-at-default of supplier 1 now we know how it affects supplier 4.

5.7.3 An example of modeling organizational consequences

For a particular type of supplier, one that cause an exceptional work load in the case of joint defaults, we may want to revise our model somewhat more. Modeling these as a portfolio fails to recognize another large difference between credit risk and supplier default risk: having multiple supplier defaults at once can have organizational consequences, are sourced under various sourcing strategies and are linked in a network. Our basic model only superficially acknowledges these differences in the form of a summation of exposure-at-defaults in the resource based method, and a flat portfolio. We can model the organizational effects of multiple defaults, but emphasize that this section is again merely an illustration of the possibilities of analyses.

In this section we switch from a supplier-centered approach to a product/service centered approach because actually we do not care directly about the supplier itself, but rather about the important product or service that stops being delivered. This is of course not independent of the supplying party.

To assess organizational consequences of joint defaults we need to define a way in which to differentiate between different types of suppliers. The Kraljic matrix\textsuperscript{19} in figure 5.5 is a simple method of doing so by considering products and services instead of suppliers.
Please note that this is only one of the many ways of positioning products and services and that we have chosen it, besides the fact that it is apt, for its simplicity and its frequent use in literature. Kraljic classifies products and services along two dimensions: the impact on the business and the supply market complexity. Business impact is the value or profit a product/service has to the organization. This does not necessarily mean that something is highly profitable, since a high value product can be a relatively cheap but key component that cannot fail for the product to maintain functionality. Such considerations can be unique to organizations since a chip failing in a telephone can have very different consequences from the same chip failing in an airplane. The second dimension is the complexity of the market. Porter gives five factors that may influence such complexity: barriers to new entrants, power of buyers, substitutes, power of suppliers and industrial rivalry. The last refers to the growth and competition within an industry that can make the market complex.

<table>
<thead>
<tr>
<th>Impact on business (internal issues)</th>
<th>Classification of purchase items</th>
<th>Critical: Cooperation (High profit impact, high supply risk)</th>
<th>Bottleneck: Supply continuity (Low profit impact, high supply risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Leverage: Best deal (High profit impact, low supply risk)</td>
<td>• Unit cost management important because of volume usage</td>
<td>• Unique specification</td>
</tr>
<tr>
<td></td>
<td>• Substitution possible</td>
<td>• Supplier technology important</td>
<td>• Production-based scarcity due to low demand and/or few sources of supply</td>
</tr>
<tr>
<td></td>
<td>• Competitive supply market with several capable suppliers</td>
<td>• Changing source of supply difficult or costly</td>
<td>• Usage fluctuation not routinely predictable</td>
</tr>
<tr>
<td>Low</td>
<td>Routine: Efficiency (Low profit impact, low supply risk)</td>
<td>• Standard specification or “Commodity”-type items</td>
<td>• Potential storage risk</td>
</tr>
<tr>
<td></td>
<td>• Substitute products readily available</td>
<td>• Competitive supply market with many suppliers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Commodity-type items</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.5:** The Kraljic matrix, divided into four quadrants. Defaulting suppliers delivering products or services from the strategic and bottleneck quadrant require most resourcing effort due to market complexity. In the leverage quadrant it usually pays off to negotiate good quality or a good price and requires medium effort. The routine quadrant requires least resourcing effort.

In this matrix, characterizations of suppliers that supply products are usually not independent of sourcing strategies. The fragile structure of single sourcing is commonly found in the bottleneck and critical quadrant, while delegated sourcing is commonly found in critical and leverage (an example might be that to truly leverage, you usually buy in bulk). We now argue that the suppliers delivering products and services in the strategic quadrant require a high level of organizational sourcing effort due to its high business impact and market complexity, while a medium effort is required in the bottleneck and leverage quadrants. A low effort is required in the routine quadrant in which
multiple sourcing is very common.

Now we have an example of a framework in which to analyze organizational effort and the extra costs that arise from joint defaults. In a way we have loss dependency. Let us put the suppliers of products for which we use the fragile structures of single and delegated sourcing delivering products into quadrants of the Kraljic matrix. We consider the high and medium effort groups, respectively found in the strategic quadrant and the bottleneck and leverage quadrants. Let $r_h$ and $r_m$ represent resourcing costs for respectively high and medium effort suppliers, $C$ the total additional organizational costs, $p(N_h > x)$ and $p(N_m > x)$ the probabilities of having more than $N$ defaults in respectively the high and medium group and $f_h$ and $f_m$ the factors by which these costs grow. We assume a formula as in 5.34 to describe the additional costs of organizational effort. The exponents are due to the more than linear increasing complexity when multiple firms default: the organization is increasingly more upset when many firms default.

$$C = p(N_h > x)r_h\alpha^{f_h} + p(N_m > y)r_m\alpha^{f_m} \quad (5.34)$$

We can use the models that were previously introduced to estimate the probabilities within each of these subsets to obtain a result of the sub portfolio like in 5.1 and use these to fill in the formula. The chance of two suppliers within this group simultaneously failing is shown to be just over 0.05 in the results section. We (again) leave it to the practitioner to select an appropriate value, in this case for $\alpha$.

5.8 Conclusion

In this chapter we have constructed a portfolio model using a multi factor dependency model and applied importance sampling to its Monte Carlo simulation. We used the probability of default values resulting from chapters three and four as input. After doing so we have shown how to obtain a distribution of the losses. Furthermore we have provided examples of how specific supply chain components such as sourcing strategies, network typology and organizational consequences might be modeled.
Chapter 6

Conclusions and further research

We conclude this thesis by drawing up the results and evaluating whether the goals have been met. In the process we briefly summarize the accomplishments of this thesis.

Consequently we outline some possible directions for related further research.
6.1 Conclusion

The first goal we set was quantifying order-of-magnitude (real world) risks of default and exposure-at-default. We have done so by using Moody’s KMV approach as a basis, explored and selected the components that we considered useful, given data that the average risk manager can reasonably be able to obtain. We provide a way to estimate exposure-at-default based on switching costs. Using a structural framework as a basis we have leveraged the advantages of this approach by providing examples of how various views of the future can be modeled to obtain these order-of-magnitude risks of default. Compared to CreditRisk+ we can produce default probabilities for a larger number of firms, while being able to incorporate specific assumptions regarding the supplier in question.

The second goal was to model the supplier portfolio. Here we have opted for a flexible way of modeling dependency using a multi factor model. Effects of individual differences have been explored not just in terms of probabilities and loss distributions but we have also provided examples using theory from supply chain and purchasing theory. The method we employed is similar to CreditRisk+, but allows for a wider variety of dependencies to be modeled while being much simpler than the relatively complicated CreditRisk+. Especially the latter is expected to make a big difference in practice.

All of this was motivated by finding a way that is accessible for the average risk manager while providing good results, to strike a balance between useful results and the complexity needed to arrive at them. Lastly, the methods of arriving at such results should be intuitive and easily explained even to laymen. Using a running example based on real data from the automotive industry, explaining the intuition behind models combined with readily available implementations we believe we have struck such a balance by providing tested models and robust Matlab implementations which require only knowledge of the input data to produce results.

Shortly summarizing, we have constructed a model able to incorporate various assumptions, using almost only publicly available data as input that can serve as a basis for a wide array of supplier default analyses.

6.2 Further research

During this research we have encountered several points that require a deeper look. Among the many we select some of the more interesting ones.

- The most prominent differences between supplier default risk and credit risk are the loss distributions. In this thesis we have provided extensions of the credit risk models that are specific to supplier default risk on the basis of assumptions.
The cost of losing a supplier is almost surely a more complex thing than we have assumed it to be in this thesis. Finding empirical evidence to determine which factors influence these losses is key to improve on these estimates and would certainly improve the quality of the results.

- In chapter 3 we have performed analysis on the methods which we have supported with some empirical evidence to select a model. Although the Merton model has been tested extensively and proven to be useful, the differences between various modeling aspects and the risk-neutral to real-world mapping have not. We have not done so because we do not have historical default data available, but it would certainly be interesting to use a larger database and see how these methods relate.

- The private firm model was implemented using simple means because we did not have the data to develop more advanced methods. Especially the volatility of assets might be related to other firm properties such as balance sheet items, for instance EBITDA volatility. For firms with negative EBITDA we used a sales multiplier, but there may be better ways to do so such as negative multiplier values.

- We have assumed a simple factor model, but we can use a generalized copula to introduce several forms of dependency with for example thicker tails. Expanding the model will surely be helpful to model an even wider array of dependencies.

- For the expansion of the portfolio model we have simply assumed a particular type of dependency based on purchasing theory and supply chains from which we have made assumptions. A case study investigating how costs truly evolve in the case of joint defaults might provide very different results.

- A link with other areas in operations research is evident. We have presented a link to purchasing management and various sourcing strategies, but it is not unimaginable to integrate the results or even the models presented and developed here in fields such as inventory management.

Naturally there are other directions in which to continue. There is a wide variety of methods available including countless variations on the structural models presented here that may improve the default probabilities. The question is if that is really useful, which raises the issue of how this information is used.
Bibliography


Appendix A

Source code

This chapter contains the source code, including comments. All code is in Matlab unless stated otherwise. The % means the line is a comment.

Please note that there are line breaks so copy/paste may not work.

A.1 Simultaneous equations

Below is the code that solves the simultaneous nonlinear equations in the basic single firm Merton model introduced in chapter 3. See Hull (2009), page 307 and 308.

```matlab
% EXAMPLE USE
defsolve (@(z) eqns(z,D,T,r,ez,se),[1 1], optimset('Display','off'))

function fcns=eqns(z,D,T,r,ez,se)
    x=z(1);
y=z(2);
    fcns(1)=se*ez-(x.*y.*normcdf(((log(x./D))+((r+(y.^2)/2))*T))/(y.*sqrt(T)))
    fcns(2)=ez-(x.*normcdf(((log(x./D))+((r+(y.^2)/2))*T))/(y.*sqrt(T)))
        -D*exp(-r*T)*normcdf(((log(x./D))+((r-(y.^2)/2))*T))/(y.*sqrt(T))
end
```

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A.2 Monte Carlo simulation using importance sampling

The source code below are for various implementations of Monte Carlo (MC) simulation Importance Sampling (IS). Details can be found in Glasserman’s book.\(^\text{30}\)

We generate matrices and manipulate these because Matlab is much faster with matrices. The following hypothetical supplier with default probability \(PD_{bsm}=5.4919\times10^{-11}\) is simulated using 100,000 replications. \(PD_{mc}=5.4833\times10^{-11}\) and the simulation runs in 0.014134 seconds on a MacBook Pro with a 3.06GHz Intel Core 2 Duo processor. Note that the simulation time does not increase linearly with the number of replications since the computer will run into memory problems.

A.2.1 IS MC Simulation for the standard Merton Model

The code below is the implementation of importance sampling (IS) that changes the drift to have an expected value of \(K\) as described in the beginning of chapter 4.

```
clear

% Inputs - Simulation Setup
runs = 1000000; % number of replications

% Inputs - Parameters
r = 0.035; % risk-free interest rate
sig = 0.25; % volatility of assets
V = 500; % current value of assets
D = 100; % default level
T = 1; % maturity
mu = r; % risk-free return

% Variables
driftIto = mu-power(sig,2)/2; % original drift factor
driftAdj = log(D/V)-power(sig,2)/2; % adjusted drift
volIto = sig*sqrt(T);

disp('Running the Monte Carlo Simulation, please hold...')
tic

% Monte Carlo Simulation
randomNums = randn(1,runs);
Component = driftAdj.*T+sig.*sqrt(T).*randomNums;
ResultBrownian = V.*exp(Component); % value given realization
Importances = normpdf(Component,driftIto,volIto)./normpdf(Component,driftAdj,volIto);

toc

defaultRate = sum((ResultBrownian-D<0).*Importances)/runs
```
A.2.2 IS MC simulation in an independent portfolio setting

The code below is an implementation of the IS exponential twist applied in chapter 5. The function \( \psi(\theta) \) is solved with the function below.

\[
\% Inputs – Simulation Setup
\]
\[
x = 5; \quad \% P[L>x]
\]
\[
\text{runs} = 50000; \quad \% \text{replications}
\]
\[
p = [0.003 0.003 0.003 0.003 0.003 0.003]; \quad \% \text{predefined}
\]
\[
c = [1 1 1 1 1 1]; \quad \% \text{LGD}
\]
\[
disp(["Starting the simulation of \( \psi_0 \), \text{num2str}(\text{runs}), \text{\textquotesingleruns\textquotesingle}]);
\]
\[
\% Determine twisted probabilities
\]
\[
\theta = \max(gam(c,p,x),0);
\]
\[
pTwisted = (p.*exp(\theta.*c))/(1+p.*(\exp(\theta.*c)-1));
\]
\[
disp(["Theta is \( \psi_0 \), \text{num2str}(\theta), \text{\textquotesingle}for x=\text{\textquotesingle}, \text{num2str}(x)");
\]
\[
\% Monte Carlo Simulation
\]
\[
\text{randoms} = \text{rand}(\text{runs},\text{length}(p));
\]
\[
\text{LperScenario} = \sum(((\text{randoms}<\text{repmat}(pTwisted,\text{runs},1))).*\text{repmat}(c,\text{runs},1));
\]
\[
\text{gammaFunc} = \sum(\log(1+p.*(\exp(\theta.*c)-1)));
\]
\[
\text{pLX} = \sum((\text{LperScenario}>x).*\exp(-\theta.*\text{LperScenario}+\text{gammaFunc}))/\text{runs};
\]
\[
disp(["P[L>x]=\text{\textquotesingle}, \text{num2str}(\text{pLX})]);
\]

The function \( \text{gam}(c,p,x) \) below is used to determine \( \theta_x \).

\[
\% c is a vector of losses, p a vector of probabilities, x a threshold value
\]
\[
\% solves \( v'(t)=x \) for \( t \) given \( x \)
\]
\[
\text{function} \quad \theta = \text{gam}(c,p,x)
\]
\[
f = @(t) \sum((c.*p.*\exp(t.*c))/(1+p.*(\exp(t.*c)-1)))-x;
\]
\[
\theta = \max(fzero(f,0),0);
\]

end

A.2.3 IS MC simulation in a mildly dependent portfolio setting

Here we have applied importance sampling to the probabilities given \( Z \) as in chapter 5. For high correlations the procedure is not very efficient because large losses depend on large outcomes of \( Z \).

\[
clear
\]
\[
\% Inputs – Simulation Setup
\]

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% Chapter A

\textbf{Example Code:}

```
x = 0;
runs = 10000;
pDep = [0.003 0.003 0.003]; % predefined
c = [1 1 1];
a = [0.2 0.1 0.05; 0.01 0.03 0.1; 0.2 0.1 0];

% Form a correlation matrix, set diagonals to 1
correlations = a*a';
correlations(logical(eye(size(correlations)))) = 1;

% Check matrices to verify \(N(0,1)\)-distribution holds
chk = sum((a.^2)');
if (max(chk)>1)
    disp('Your factor loadings do not meet the second criterion.')
    break;
end;
b = sqrt(1-chk);

% Monte Carlo Simulation
chk = sum((a.*c));
if (sum(chk) >= x)
    theta = 0;
else
    theta = gam(c,p,x);
end;

% generate one extra set of random numbers
subruns = 1;
pTwisted = (pZ.*exp(theta.*c))./(1+pZ.*(exp(theta.*c)-1));
randoms = randn(subruns,length(p));
LperScenario = sum(((randoms<repmat(pTwisted,subruns,1)).*repmat(c,subruns,1)).');
gammaFunc = sum(log(1+pZ.*(exp(theta.*c)-1)));

% store this value since we need it later
pLX(1,i) = sum((LperScenario>x).*exp(-theta.*LperScenario+gammaFunc));
end;
pLXFinal = sum(pLX)/(runs*subruns);
```
A.2.4 IS MC Simulation in a generally dependent Portfolio setting

Here we have applied importance sampling to the probabilities and to Z as in chapter 5. This procedure is more efficient for correlated defaults. The code is almost identical to the one above but is printed in full for convenience.

clear

% Inputs – Simulation Setup
x = 0;
runs = 10000;
pDep = [0.003 0.003 0.003]; % predefined
c = [1 1 1];
a = [0.2 0.1 0.05;
    0.01 0.03 0.1;
    0.2 0.1 0];

% Form a correlation matrix, set diagonals to 1
correlations = a.*a';
correlations(logical(eye(size(correlations)))) = 1;

% Check matrices to verify N(0,1)–distribution holds
chk = sum((a.^2)');
if (max(chk)>1)
    disp('Your factor loadings do not meet the second criterion. ');
    break;
end;

b = sqrt(1-chk);

% using the Normal approximation to determine a vector of mu
mu = fminsearch(@(z)−((1−normcdf((x−sum(normcdf(((a*z)'+
norminv(pDep))./b,0,1).'*c))/sqrt(sum(c.^2.*normcdf(((a*z)'+
norminv(pDep))./b,0,1).*((1−normcdf(((a*z)'+
norminv(pDep))./b,0,1))))),0,1)))*exp(−z'*z/2)),
    [norminv(1−pDep)]');

% Monte Carlo Simulation
disp([ 'Starting the simulation of ', num2str(runs), ' runs.']);
tic
plX = zeros(1,runs);
for i=1:runs
    % gen d N(mu,1) random numbers and compute pZ
    Z = mu+randn(length(a),1);

    disp([ 'P[Z> ', num2str(x), ']= ', num2str(pLXFinal) ]);
\[ pZ = \text{normcdf} \left( \left( a \cdot Z \right) + \text{norminv} \left( p_{\text{Dep}} \right) \right) / b, 0, 1 \];

\[ \text{if } \left( \text{sum} \left( pZ \cdot c \right) \geq x \right) \]
\[ \quad \text{theta} = 0; \]
\[ \text{else} \]
\[ \quad \text{theta} = \text{gam} \left( c, p, x \right); \]
\[ \text{end}; \]

\% generate one extra set of random numbers
\[ \text{subruns} = 1; \]
\[ p\text{Twisted} = \left( pZ \cdot \exp \left( \text{theta} \cdot c \right) \right) / \left( 1 + pZ \cdot \left( \exp \left( \text{theta} \cdot c \right) - 1 \right) \right); \]
\[ \text{randoms} = \text{rand} \left( \text{subruns}, \text{length}(pZ) \right); \]
\[ \text{LperScenario} = \text{sum} \left( \left( \text{randoms} \cdot \text{repmat} \left( p\text{Twisted}, \text{subruns}, 1 \right) \right) \cdot \text{repmat} \left( c, \text{subruns}, 1 \right) \right); \]
\[ \text{gammaFunc} = \text{sum} \left( \log \left( 1 + pZ \cdot \left( \exp \left( \text{theta} \cdot c \right) - 1 \right) \right) \right); \]

\% store this value since we need it later
\[ p\text{LX} \left( 1, i \right) = \text{sum} \left( \left( \text{LperScenario} > x \right) \cdot \exp \left( - \text{theta} \cdot \text{LperScenario} + \text{gammaFunc} \right) \right) \cdot \exp \left( - \mu' \cdot Z + \mu' \cdot \mu / 2 \right); \]
\[ \text{end}; \]

\[ p\text{LXFinal} = \text{sum}(p\text{LX}) / \left( \text{runs} \cdot \text{subruns} \right); \]
\[ \text{toc} \]
\[ \text{disp}(['P[L] > x', \text{num2str}(x), ']' = '', \text{num2str}(p\text{LXFinal})); \]

### A.2.5 Function solving for theta

A function that solves for the value of \( \theta \).

\[ \text{function } \text{theta} = \text{gam} \left( c, p, x \right) \]
\[ \quad f = @(t) \text{sum} \left( \left( c \cdot p \cdot \exp \left( t \cdot c \right) \right) / \left( 1 + p \cdot \left( \exp \left( t \cdot c \right) - 1 \right) \right) \right) - x; \]
\[ \quad \text{theta} = \text{fzero} \left( f, 0, \text{optimset} \left( '\text{TolX}', 1e-12 \right) \right); \]
\[ \text{end} \]

### A.2.6 Estimating tail probabilities using a full structural setting

This is source code based on section 5.2.3. It only estimates tail probabilities well because the modes of the distribution are too far apart anywhere besides the tail. We provide this only for completeness and do not recommend using this model.

\% create a covariance matrix to determine the scenario importance below
\[ \text{for } f = 1: \text{length}(\text{assetCorrelation}) \]
\[ \quad \text{for } g = 1: \text{length}(\text{assetCorrelation}) \]
\[ \quad \quad \text{covarMatr}(f, g) = \text{sig}(f) \cdot \text{sig}(g) \cdot \text{assetCorrelation}(f, g); \]
\[ \quad \text{end} \]
\[ \text{end} \]
%% create a covariance matrix to determine the scenario importance below
for f = 1:length(assetCorrelation)
    for g = 1:length(assetCorrelation)
        covarMatrAdj(f,g) = sigAdj(f)*sigAdj(g)*assetCorrelation(f,g);
    end
end

%% we do some preprocessing to adjust the drift so our Monte Carlo
%% simulation is more favorable (see Han, 2010)

hvals = zeros(1,length(assetCorrelation));
for i = 1:length(assetCorrelation)
    extraTerm = 0;
    if i>1
        for e = 1:(i-1) \% compute the values of the other terms
            extraTerm = assetCorrelation(i,e)*hvals(e)+extraTerm;
        end
        xTerm(1,i) = extraTerm;
    end
    rhs(1,i) = (mu(i) / sig(i) - log(D(i)/V(i))/sig(i));
    hvals(1,i) = (rhs(1,i)-extraTerm)/assetCorrelation(i,i);
end

%% Variables

discountFac = exp(-sig.*T); % the discount factor for a full year
driftIto = mu-power(sig,2)/2; % the drift factor of lnS by Ito’s Lemma: mu-power(sig,2)/2

% driftExtra = log(D./V);
driftAdj = driftIto-sig.*hvals; % adjusted drift log(D./V)-power(sig,2)/2 for tails — driftIto-sig.*hvals | driftIto-0.225*sig.*
volIto = sig.*sqrt(T);

Running the simulation can be done using the code below. The factor lim was used to
experiment with the modes of the distribution. Depending on its setting some reasonably
good results can be obtained but these depend on the problem setup.

%% do the Cholesky decomposition
cholDecomp = chol(assetCorrelation);

%% generate the random numbers for each of the assets and correlate them
randomMatrix = randn(runs,length(assetCorrelation));
randomMatrixCorr = randomMatrix*cholDecomp; % correlate the columns, this is matrix multiplication

lim = 0/1000; % 830 maximizes the probability that 1
% defaults uniquely (out of 6)

%% for each asset, perform the simulation
for i = 1:length(D)
randomNums = randomMatrixCorr(:, i)'; \% we have \#runs numbers that are correlated by column
randomOn = rand(1, length(randomNums));
randomDrift(i,:) = ((randomOn<lim).* driftIto(i)+(randomOn>=lim).* driftAdj(i));
Component(i,:) = randomDrift(i,:).*T(i)+sigAdj(i).*sqrt(T(i)).* randomNums;
ResultBrownian = V(i).*exp(Component(i,:));
Importances(i,:) = normpdf(Component(i,:), driftIto(i), volIto(i))./ normpdf(Component(i,:), driftAdj(i), volIto(i));
defaultResult(i,:) = ResultBrownian-D(i)<0;
end
randomMatrix;
randomMatrixCorr;
clear randomNums;
ResultBrownian;
toc
disp('Simulation is complete. Analyzing the losses ... ')
tic
for l = 1:runs
    importancePerScenario(l) = mvnpdf(Component(:,l), driftIto', covarMatr)/
    mvnpdf(Component(:,l), randomDrift(:,l), covarMatrAdj);
end
probabilityPerScenario = importancePerScenario/runs;
Component;
for j = 1:length(D)
    rateNumDefaults(j,:) = [sum((defaultResult==j).*
    importancePerScenario)/runs binopdf(j,6, defaultAnalytical)];
end
Importances;
importancePerScenario;
rateNumDefaults
structure = [sum(sum(defaultResult==0)); sum(sum(defaultResult==1)); sum(
    sum(defaultResult==2)); sum(sum(defaultResult==3)); sum(sum(
    defaultResult==4)); sum(sum(defaultResult==5)); sum(sum(defaultResult) ==6)];
toc;

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A.3 Additional Matlab code

Here we provide some additional code to generate a Lorenz curve and a distribution plot.

A.3.1 Code to plot a Lorenz curve

The inputs should be a vector of probabilities summing up to 1. If these are not, they can be normalized.

```matlab
function [ output_args ] = plotLorenz( riskContributions )
% A quick way of plotting the risk contributions
    sortedRiskContributions = sort(riskContributions, 2, 'descend');
    cumulativeContributions = [0 cumsum(sortedRiskContributions)];
    populationPercentage = [0 cumsum(repmat(1/length(riskContributions), 1, length(riskContributions)))];
    plot(populationPercentage, cumulativeContributions, 'bo--', 'MarkerFaceColor', 'b');
    axis([0, 1.001, 0, 1.001]);
    xlabel('Cumulative number of suppliers');
    ylabel('Cumulative risk contributions');
    title('Lorenz curve of risk contributions');
    disp('The Lorenz curve has been plotted.');
end
```

A.3.2 Plot for the loss distribution

The code below is useful to quickly plot a loss distribution. The inputs is a 2-by-n matrix of which the first row contains losses and the second corresponding probabilities.

```matlab
function [ output_args ] = plotLossDist( distributionData )
% A quick way of plotting the loss distribution
    plot(distributionData(1,:), distributionData(2,:), 'ko--', 'LineWidth', 1, 'MarkerFaceColor', 'k');
    xlabel('Losses in Euros');
    ylabel('Probability');
    title('The portfolio loss distribution');
    disp('The loss distribution has been plotted.');
end
```

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