The influence of change in reservoir shape on water availability

An assessment of the shape and water availability - in the past and future - of the Pentecoste water reservoir in Ceará, Brazil.

Roy Daggenvoorde
2/24/2015

Universidade Federal do Ceará
November 2014 – January 2015
The influence of change in reservoir shape on water availability

An assessment of the shape and water availability - in the past and future - of the Pentecoste water reservoir in Ceará, Brazil.

Roy Daggenvoorde S1095595
Bachelor Thesis Civil Engineering
University of Twente
26 January 2015

Supervised by
Maarten Krol University of Twente
José Carlos de Araújo Universidade Federal do Ceará
Wellington Lopes Universidade Federal do Ceará
**Preface**

This is my chance to say thanks to everyone who supported me and gave me the opportunity to do my bachelor thesis at the Universidade Federal do Ceará. In the 11 weeks that I spent in Fortaleza and its surrounding I had an amazing time. The things I saw, the people I spoke to and the strange language I heard gave me one of the best times in my life. Travelling abroad is nice, but I could not have imagined that it would be so interesting to perform research in a foreign country as well.

The things that interested me the most were the differences between Brazil and the Netherlands. There are so many differences that I am not going to write them down one by one, instead I only state the two differences that really interested me. The first one is about the speed of life, the first weeks in Brazil I had to get used to the fact that not everything was going to happen directly, it took some effort but in the end I really appreciated this relaxed way of life and work. The second interesting difference is the difference in water management, where the Netherlands are threatened by floods is Brazil the opposite. Water is valuable in Brazil and none can be wasted, because you never know when it will rain again.

In the end I owe my thanks to several people, first Maarten Krol who got me in touch with José Carlos de Araújo, who’s the second person I would like to thank. These are the two people who gave me the opportunity to spend 11 weeks abroad. I would also like to thank Wellington Lopes who arranged the bathymetric survey for me and helped me a lot with the reservoir modelling and remote sensing. The last person I owe special thanks is Luiz Carlos Guerreiro Chaves, he gave me a place to live and I could ask him everything about Fortaleza and Brazil.
Summary

The Pentecoste reservoir is a large water reservoir approximately 85 kilometers west from Fortaleza, Ceará, Brazil. The reservoir is located within the basin of the river Curu and is fed by two rivers: The Rio Canindé and the Rio Capitão Moore. Due to the drought of the last 3 years in the semi-arid region of northeastern Brazil the amount of water in the Pentecoste reservoir is at only 2% of its maximum capacity. It is expected that the drought is not the only reason behind the low water level in the reservoir. The expectation is that the changed shape of the reservoir due to sedimentation also influences the water availability. This is caused by the fact that the regulated outflow of the reservoir is calculated using this shape. The shape which is used in this calculation is outdated since the reservoir will have a different shape now than it had when it was constructed in 1957.

The shape of a reservoir is determined with a bathymetric survey. Bathymetric surveys are normally conducted when the reservoir is full. At that moment the depth is measured at various points in the reservoir. With the corresponding coordinates these depth measurements can be used to model the entire reservoir. This method is very time-consuming; it took 16 days to survey the Pentecoste reservoir in 2009. Another method of bathymetric survey makes use of satellite images. Within this method the bathymetric survey is conducted when the water level is low and satellite images are used to investigate the surface elevation above the water level. This method is less time consuming but it is unknown how accurate the results of this methodology are.

So the water availability is influenced by the shape of the reservoir and the shape of the reservoir can only be determined by time-consuming or inaccurate bathymetric surveys. These facts are the basis for the goals set in this research. The first goal concerns the way how the shape changes and influences the water availability and the second goal concerns the accuracy of the simplified bathymetric methodology. The goals are

Analyse the influence of the change in shape due to sedimentation of the Pentecoste reservoir—in the past and future—on the water availability and analyse and predict how this shape has and will change in the past and future.

Analyse the measurement and propagation errors of the Landsat bathymetric survey methodology in comparison to the real reservoir shape determined by the extensive bathymetric survey in 2009.

If the first goal can be achieved it can make frequent repetition of bathymetric surveys unnecessary because the reservoir shape can be predicted, the second goal will give insight into the accuracy of the simplified bathymetric methodology.

To achieve these goals a research design has been made. This design consists of three parts: the theoretical framework, the methodology and the data collection. The theoretical framework introduces and explains the different theoretical concepts used in this research, such as: reservoir shape, depth-area-volume relations, reliability levels for regulated outflow and water availability. The methodology introduces the methods and tools which will be used to operationalize the concepts and achieve the goals. It introduces the methods which are used to find the relations between the various parameters and time and how the parameters will be used to assess the water availability over time. After these parts which concern the first research goal, the different bathymetric surveys are explained more extensively. The third and last part of the research design shows how the required information to perform the introduced methods is collected, the data collection.

After the research design is presented and the research has been conducted the results are presented. The results are split up in three parts, first the reservoir shape, second the water availability and last the comparison of the different bathymetric surveys.
In the first part of the results the reservoir shape is presented. The four parameters, maximum depth, maximum surface area, maximum volume and the shape-factor are plotted against time. These graphs show different relations between the shape-parameters and time. The maximum depth and maximum volume show a linear decrease over time. The maximum surface area does not show any change over time while it is made plausible that the surface area determined at the construction in 1957 contains a large measurement error. The fourth shape-parameter, the shape-factor, shows an exponential increase over time. The reservoir shape is also presented by the depth-area-volume relations. The equations of these relations are determined for 1957, 2009 and 2064 (50 years after the start of this research). The equations have and will change over various decades. The changes in the depth-area-volume equations show that the Pentecoste reservoir has and will become shallower and that the slopes of the reservoir become more concave. This change can be explained easily by the fact that sediment is transported into the reservoir and settles down within it, particularly onto the lower-lying parts of the bed.

The second part of the results concerns the water availability. The water availability is shown to not change linear over time. This is explained by the increase in evaporation when a reservoir becomes shallower and the increase in spillway loss when the reservoir volume decreases. This effect only becomes visible when the reservoir volume has decreased significantly. Since this is not the case in the first 100 years after construction it is possible to represent this period using a linear function. This linear function represents the yield loss per year, which is 0.14%, 0.13% and 0.12% of the initial volume for respectively the 99%- , 95%- and 85%-reliability levels.

The last part of the results is about the measurement and propagation errors of the simplified bathymetric methodology. The measurement errors are 7.6% and 10.4% for respectively the maximum surface area and the maximum volume. The results of the maximum depth did not show any deviation between both methodologies. The propagation errors, errors in volume and yield reductions per year are larger, this is caused by the propagated error of the maximum volume. The outcomes are still usable to some extend because they have the same order of magnitude. Another conclusion is that the bathymetric survey conducted in 2009 resulted in a more detailed bed elevation map of the entire reservoir, which can be explained by the difference in input data.

All these results add up to the possibility to predict the change of shape of the Pentecoste reservoir. This prediction is possible to make, but it is unknown how accurate it is so it is recommended to do a bathymetric survey in the future, 20 to 50 years from now, to assess the real shape of reservoir and to compare it to the predicted values.

Some remarks have to be made about the validity of this report. In the discussion, several assumptions are pointed out which will need attention in a further report. After these remarks some possibilities for additional research are introduced. Ideas for additional research are for example a bathymetric survey in the future, more comparisons between the two survey methods and a further analysis of the volume decrease under influence of the erosivity factor.
# Table of Contents

Preface ................................................................................................................................. 3  
Summary ................................................................................................................................. 4  
1. Introduction.......................................................................................................................... 7  
   1.1. Background and motive ............................................................................................. 7  
   1.2. Objective ................................................................................................................... 9  
   1.3. Research questions ................................................................................................... 9  
2. Research Design ............................................................................................................... 10  
   2.1. Theoretical framework ............................................................................................. 10  
   2.2. Methodology ........................................................................................................... 14  
   2.3. Data collection ......................................................................................................... 19  
3. Results .............................................................................................................................. 24  
   3.1. Reservoir shape ....................................................................................................... 24  
   3.2. Water availability ..................................................................................................... 31  
   3.3. Different bathymetric methodologies ...................................................................... 34  
4. Conclusion ......................................................................................................................... 40  
5. Discussion ........................................................................................................................ 42  
6. Recommendations ............................................................................................................ 44  
Bibliography ........................................................................................................................... 45  
Appendices ............................................................................................................................ 47  
   Appendix A – The VYELAS-model – ............................................................................. 47  
   Appendix B – Study area, the Pentecoste reservoir – .................................................... 50  
   Appendix C – The BalHidr-model – ................................................................................ 51  
   Appendix D – Equipment – ............................................................................................. 52  
   Appendix E – Bathymetric results – ................................................................................ 54  
   Appendix F – the meaning of the value of shape-factor alpha – .................................... 59
1. Introduction

1.1. Background and motive

The Brazilian state of Ceará is located in the semi-arid region, the Caatinga. The semi-arid region is for 90 percent of its water availability dependent on surface reservoirs. This is caused by the lack of reliable groundwater resources and the intermittency of river flows (de Araújo & Knight, 2005).

Within this semi-arid region the Pentecoste reservoir is one of the biggest reservoirs. The Pentecoste reservoir is located approximately 85 kilometers west of Fortaleza, the fifth largest city of Brazil, and is part of the basin of the river Curu (Table 1 & Figure 1). Two rivers flow into the Pentecoste reservoir; the Rio Canindé and the Rio Capitão Moore. The Pentecoste reservoir is officially called: “Pereira de Miranda”. This reservoir is at this moment at only two percent of its maximum capacity, the maximum capacity is 395 638 000 m$^3$ (de Aragão Araújo, 1982).

<table>
<thead>
<tr>
<th>Characteristics of the basin of river Curu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Origin</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Population</td>
</tr>
<tr>
<td>Number of dams</td>
</tr>
</tbody>
</table>

Table 1 Characteristics of the river Curu (COGERH, 2009)

The low water level in this reservoir is caused by the long term drought, which is afflicting the semi-arid region the last three years. This drought does only explain a part of the low water level in the Pentecoste Reservoir. It is expected that other factors also influence the amount of water in the reservoir. One of these factors is expected to be the shape of the reservoir.

In the current calculations of the volume and yield of the reservoir the operators use the geometry of the reservoir when it was built in 1957. This geometry has been used to determine equations concerning the depth, surface area and volume. These relations require the current depth or current surface area to calculate the amount of water in the reservoir. It is known that sedimentation and reservoir silting influence the shape and volume of reservoirs over time (de Araújo, Günther, & Bronstert, 2006). The impact of using these outdated shape-parameters in the calculation of the outflow is unknown, but it is expected to cause bigger outflows then when actual parameters would be used. To improve the calculation of the water outflow from the Pentecoste reservoir it is important to determine the actual shape-parameters.

Shape-parameters are determined with Bathymetric surveys. In 2009 COGERH, the water management company in Ceará, did a bathymetric survey of the Pentecoste reservoir with almost 100 000 depth measurements. At this moment the reservoir was full and this resulted in a full model of the reservoir shape. This reservoir shape can be used to assess the change since the reservoir was built in 1957. Bathymetric surveys like this one of the Pentecoste reservoir are very time consuming. To avoid this time consuming process, Landsat images are used in many surveys to assess the reservoir shape. But the accuracy of this methodology is unknown. Due to the low amount of water in the Pentecoste reservoir, only two percent of its maximum capacity, this is a good moment to measure the geometry with use of Landsat images. A bathymetric survey will be conducted for shape under the water surface in the reservoir and Landsat images with their corresponding water depth in
various moments in time will be used to assess the shape above the water surface. Due to the drought the last three years there was hardly any inflow into the reservoir, which means there is not transported any sediment into the reservoir. Due to this fact it can be assumed that the reservoir shape did not change since 2009. This results in the possibility to compare the results of the Landsat methodology to the results of a full bathymetric survey.

Bathymetric surveys of both kinds require a lot of time and effort. The one more than the other, but due to this amount of required time it is impossible to use bathymetric surveys to update all the depth-area-volume relations of all the reservoirs in the state of Ceará. If it is possible to predict the change in reservoir shape it will reduce the required amount of bathymetric surveys.

Figure 1 Location of the Pentecoste Reservoir in Curu Basin (França, Wachholz, Neto, & Paulino, 2013)
1.2. Objective
As stated in the previous paragraph water is scarce in the Pentecoste region, the known shape of the Pentecoste reservoir is outdated and the impact of this shape is unknown. This research will focus on the effects of the changing shape and on how the shape has and will change in the past and future. So the objective of this research will be:

*Analyze the influence of the change in shape due to sedimentation of the Pentecoste reservoir—in the past and future—on the water availability and analyze and predict how this shape has and will change in the past and future.*

Next to this main objective a second objective can be formulated. This second objective concerns the accuracy of the Landsat methodology to determine the reservoir shape.

*Analyze the measurement and propagation errors of the Landsat bathymetric survey methodology in comparison to the real reservoir shape determined by the extensive bathymetric survey in 2009.*

The main objective will be translated towards a main question of research in the next paragraph to structure the research process. The second objective will be answered in a sub-question. The research process will consist of answering the main question and its sub-questions to achieve the objectives.

1.3. Research questions
The main question of research is:

*How does the shape of Pentecoste reservoir change due to sedimentation and how does this change in shape influence the water availability in the past and future?*

This main question of research is broken down into three sub-questions. The answers on these sub-questions will add up to an answer on the main question. In addition to this sub-questions which represent the main question of research a fourth sub-question is formulated which concerns the research objective about accuracy of bathymetric methodology. The sub-questions are:

1. **How has shape of the reservoir changed due to sedimentation since construction and how will it change in the next 50 years?**
2. **What are the relations between depth, surface area and volume when constructing the dam, in the present and in the future?**
3. **How has the sedimentation influenced the water availability and how will the future change in shape influence the water availability?**
4. **What are the measurement and propagation errors—in volume, shape and water availability—of the Landsat bathymetric survey methodology?**
2. Research Design
In this chapter the research will be structured, which will be done in three parts. First the necessary theory will be introduced. This theoretical framework will consist out of theories out of the literature which are used in the report. This will help to understand the other parts of the report. The second part is about the methods which will be used. In the paragraph about methodology will be explained how the various steps to achieve the final results are performed. The third and final part will be about the data collection, it will be explained how the data which are required according to the introduced methods are collected.

2.1. Theoretical framework
In this paragraph the necessary theories for the study will be introduced. The introduced theories are about the reservoir shape, depth-area-volume curves, water availability and the VYELAS-model.

2.1.1. Reservoir shape
The main topic of this report is the shape of the reservoir. In the literature there are several parameters used to represent the reservoir shape (Table 2). Table 2 shows that the used parameters are: Depth, surface area, volume and shape-factor alpha. Depth, surface area and volume do not need any extra explanation; they represent the geometry of the reservoir. Shape-factor alpha will need some introduction. Shape factor alpha represents the shape of the reservoir, with the introduction of alpha different shapes are introduced. Reservoirs are no longer round or rectangular shape but can have arms and other varieties in shape. De Araújo et al. (2006) define morphologic parameter alpha as shown in equation 1:

\[ \alpha = \sum \frac{V_i}{\sum h_i^3} \]  

In which \( V_i \) is the reservoir volume at water depth \( h_i \). Parameter ‘\( \alpha \)’ is also known as the shape-factor of reservoirs. With this shape-factor the actual reservoir volume and surface area can be approximated by using the measured depth and formulas 2 and 3.

\[ V(h) = \alpha \times h^3 \]  
\[ A(h) = 3 \times \alpha \times h^2 \]

Formulas 2 and 3 show great similarities to the depth-area-volume equations. These equations will be introduced in the next paragraph. In these equations the values for alpha and the power of ‘\( h \)’ represent the openness and concavity of the reservoir. This will be further explained in the next paragraph. Formulas 2 and 3 schematize the reservoir as with straight slopes with a 45 degree angle. This principle is shown in Appendix F – the meaning of the value of shape-factor alpha –.

<table>
<thead>
<tr>
<th>Article</th>
<th>Used parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(de Araújo, Günther, &amp; Bronstert, 2006)</td>
<td>Depth, area, volume and alpha</td>
</tr>
<tr>
<td>(Liebe, 2002)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Liebe, van de Giesen, &amp; Andreini, 2005)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Annor, van de Giesen, Liebe, van de Zaag, Tilmant, &amp; Odai, 2009)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Rodrigues &amp; Liebe, 2013)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Rodrigues, Sano, Steenhuis, &amp; Passo, 2012)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Grin, 2014)</td>
<td>Depth, area and volume</td>
</tr>
<tr>
<td>(Campos, de Carvalho Studart, Martinz, &amp; Nacimento, 2003)</td>
<td>Depth, area, volume and alpha</td>
</tr>
<tr>
<td>(Campos J. N., 2010)</td>
<td>Depth, area, volume and alpha</td>
</tr>
</tbody>
</table>

Table 2 Shape-parameters in various articles

Within this report the shape of the Pentecoste reservoir will be described by these four presented parameters.
2.1.2. Depth-area-volume relations

In water reservoirs there are relations to be found between depth, area and volume. These relations are power functions. To determine these functions some additional information is needed on the field of power functions in reservoirs, linear regression and goodness of fit. Information on linear regression is needed to be able to determine the values of the formulas and the goodness of fit will be used to check how well the power functions represent the measurements. The information and methods on linear regression and goodness of fit will be given in paragraph 2.2.3 Depth-Area-Volume relations.

Power functions

To describe the depth-area, depth-volume and area-volume relations, power function are most commonly used. Liebe (2002) characterized the volume of reservoirs as the half of a square based pyramid. This approach results in the power function presented in formula 4.

\[ y = k \times x^a \]  

(4)

This function can also be written to represent the relations between depth, area and volume. The most used formulas are those in which volume is expressed as a function of depth and the function in which volume is expressed as a function of the surface area. These two formulas are shown below as formulas 5 and 6.

\[ V = k_1 \times D^{a_1} \]  

(5)

\[ V = k_2 \times A^{a_2} \]  

(6)

Where A is area in m\(^2\); D is depth in m; V is volume in m\(^3\); \(a_n\) is an exponential coefficient and \(k_n\) is a constant.

The exponential coefficient \(a_n\) represents the hillside concavity of the reservoir slopes. This value is in the Area-Volume relation 1.5 when the reservoir is exactly the shape of a half of a square based pyramid, it has straight slopes. If the slopes of the reservoir are more convex \(a_2\) will be smaller otherwise it will be larger (Annor, van de Giesen, Liebe, van de Zaag, Tilmant, & Odai, 2009). In the depth-volume equations it is the other way around; if the reservoir becomes more concave \(a_1\) increase and if it becomes more convex \(a_1\) decreases. The turning point between concave and convex slopes is different in the depth-volume equation where it is 3 instead of 1.5(Figure 2 left).

The constant \(k_n\) represents the openness of the reservoir. An open reservoir is a reservoir with a large surface area and is quite shallow, on the opposite there are reservoirs which are deep with a small surface area. Large, shallow reservoirs are often located in large, flat valleys. The more open the reservoir the larger the value for \(k_n\). \(k_n\) will become smaller when the surface area reduces and the depth increases. The influence of \(k_n\) and \(a_n\) are also shown in the figures below (Grin, 2014).

![Figure 2 The meaning of the different values in the DAV-equations](image-url)
The expressions of volume as a function of depth and area are useful to be able to estimate the reservoir volume. Satellite images and water depth measurements can be used to estimate the volume if these formulas are up to date.

The depth-volume equation (5) has the same shape as equation 2. This means that the shape-factor calculated by equation 1 is a value for $k_1$ with $a_1$ set at 3. This means the reservoir is modelled as a reservoir with straight slopes, neither concave nor convex, when the shape-factor is used.

For smaller reservoirs depth-area-volume relations are often similar to each other. A condition for the phenomena is that the studied small reservoirs need to be in the same area. This is because these reservoirs will have the same conditions (Rodrigues & Liebe, 2013) (Rodrigues, Sano, Steenhuis, & Passo, 2012). For large reservoirs the constants differ too much to find a general expression, the Pentecoste reservoir is considered as a large reservoir so it is needed to find a specific expression for this large Pentecoste reservoir.

2.1.3. Water availability

The goal of all reservoirs in the semi-arid north eastern part of Brazil is water availability throughout the entire year. Water is available from a reservoir when a reservoir can produce controlled outflow, so called yield. This yield can be taken from the reservoir in two ways. First is the obvious controlled outflow through the dam which, in the case of Pentecoste, is the outflow through the turbine. The second form of yield, which is used a lot in the semiarid Brazil, is the water which is withdrawn from the reservoir by water trucks. These trucks pump water out of the reservoir to distribute to the various users in the region (Figure 3).

Yield can be controlled by the dam operators COGERH. They determine whether or not and how much water out of the reservoir may be used. This is done by calculating the amount of water which may be taken out of the reservoir. Since water availability is highly influenced by the weather, rainfall and evaporation, this calculation includes stochastics. These stochastics result in different outcomes for every calculation. To be able to work with these different outcomes reliability levels are introduced. A reliability level of 90% means that the yield corresponding with that reliability level ensures that in 90 of every 100 years the yield is higher than the set minimum water yield.

In the state of Ceará the maximum yield is calculated on three different reliability levels. Which reliability level is used is dependent on the purpose of the reservoir. If the reservoir is used to provide water for human activity the desired reliability level is 99%, if it is used for industrial activity the desired level is 95% percent and when the purpose of the reservoir is to fulfill agricultural water demands a reliability level of 85% is accepted.
These different reliability levels exist because when a lower level is accepted the yearly yield of the reservoir is higher. This higher yield makes it possible to grow more crops and irrigate them better. So if a reservoir is used for agricultural purposes it is economically more desirable to have 85 years in every 100 with a larger water yield and 15 years without yield then to have 99 years with a lower water yield and only one year of failure.

2.1.4. Water availability model – the VYELAS-model

To assess the impact of the shape of the reservoir on the water availability the VYELAS-model will be used. The VYELAS-model calculates the yield of water reservoirs. VYELAS is an abbreviation for ‘Volume-Yield-ELASTicity’. This model uses stochastic modelling techniques to calculate the yield of a reservoir at various reliability levels. The model is developed by José Carlos de Araújo (2004). The working of the model is explained in ‘Appendix A – The VYELAS-model’. The input which is needed for the VYELAS-model is shown in Table 3. How these inputs will be determined will be discussed in paragraph 2.3.5.

<table>
<thead>
<tr>
<th>Required inputs of the VYELAS-model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Inflow in hm$^3$/year</td>
<td>Coefficient of variation of average inflow</td>
</tr>
<tr>
<td>Evaporation in the dry season (m/year)</td>
<td>Maximum Storage capacity (hm$^3$)</td>
</tr>
<tr>
<td>Initial volume in year 1 (hm$^3$)</td>
<td>Number of Yield steps</td>
</tr>
<tr>
<td>Maximum yield (hm$^3$/year)</td>
<td>Minimum operational volume (hm$^3$)</td>
</tr>
<tr>
<td></td>
<td>Minimum yield (hm$^3$/year)</td>
</tr>
</tbody>
</table>

Table 3 Inputs of the VYELAS-model

The output of the VYELAS-model consists of a set number of targeted yield steps and their corresponding reliability levels, mean withdrawal, mean evaporation and mean spillway overflow.
2.2. Methodology

In this paragraph all elements which are needed to answer the sub-questions are discussed. After discussing the methodology it will be clear which data are needed, which will be discussed in the next paragraph.

2.2.1. Reservoir modelling

Reservoir modelling is about digitalizing the real reservoir into a model. The current reservoir will be modelled two times. Once using the bathymetric survey out of 2009 and once using the information out of 2014.

2009 – Extensive bathymetric survey of the full reservoir

To find the parameter values and depth-area-volume curves of the current situation in the Pentecoste reservoir a bathymetric survey has been conducted. This survey resulted in data which represent the water depth at various points combined with their coordinates and a GPS-tracked trip around the reservoir’s shores.

These two datasets are loaded into ArcMap 10.0. ArcMap has a function called: ‘Topo to Raster’. This function uses interpolation to calculate the depth of every point within given borders. The function requires two types of input. First it requires an area wherein the interpolation will be conducted. This border is provided by the GPS-tracked reservoir contour. The second type of input which is needed to interpolate is information about the water depth at various points. This information is given by the performed measurements and the reservoir contour. All different points of measurement have their own corresponding depth and the contour, which represents the shoreline, has a water depth of zero. It is important to note that the shoreline is being tracked from inside the boat; obviously the boat cannot follow the exact shoreline. On average the boat was 30 meters out of the shore. This information is used to buffer the area found using the GPS-tracked roundtrip to a shape which represents the actual shoreline.

The interpolation is done based on the ANUDEM program developed by Hutchinson (2008). This program is developed especially for interpolating hydrological areas. This is done by using the information about how water erodes the land. This will result in a connected drainage structure which is expected to show (ESRI, 2012).

This interpolation process results in a colored map which shows the different depths in the various areas of the interpolated waterbody.

2014– Simplified bathymetric survey combined with satellite images

The reservoir modelling in 2014 is conducted in the same way as the reservoir modelling of the 2009 bathymetric survey. The difference is that the reservoir was not full when this bathymetric survey was conducted; the water level was only 43.39 meter above sea level. The maximum level water level of the Pentecoste reservoir is 58 meters (de Aragão Araújo, 1982), so the information about the geometry of the area above the water has to be obtained otherwise. This information is found by using the shorelines found in satellite images. How this information is obtained is explained in paragraph 2.3.2. ‘Remote sensing’.

The shorelines found on the satellite images with their corresponding elevation above sea level are used as extra contours in the ‘Topo to Raster’-function. In this way information about the reservoir above the water level and below the top of the dam is also available and the interpolation of the entire reservoir is possible.
2.2.2. Shape parameters over time

The four parameters, which represent the reservoir shape, will be expressed over time to be able to extrapolate these parameters. To be able to express the values of the parameters over time more information is needed. In this paragraph for each of the four parameters a method will be presented to be able to find these expressions against time.

The maximum reservoir depth will be or is measured in three moments in time, when the dam was built and during the bathymetric surveys. These last two moments will be used as one point in time because it is expected that the reservoir shape has not changed between those two surveys. The change in maximum reservoir depth will plotted as a linear function. This will make it possible to interpolate and extrapolate the maximum reservoir depth.

The maximum surface area is not expected to change over time. This is because the maximum surface area will only change if mayor events happened around the reservoir. Examples for these mayor events are landslides, new large building close to the reservoir and such. Not any of these events is known to have happened around the Pentecoste reservoir. This will be controlled by comparing the maximum surface area in 1957 to the maximum surface area in 2009. If there is no change it is possible to say that the maximum reservoir surface area is constant. When this is not the case Landsat images of moments in the past when the reservoir was full will be used to determine the surface area and the way this area has changed.

The change of reservoir volume over time is harder to determine, because there are only two moments in time in which the maximum volume is known. This is when the dam was constructed and when the bathymetric surveys were conducted. In between these two moments there are no measurements or data available. The surveys of 2009 and 2014 will both be used to determine the volume. This way it is possible to compare their results and to analyze the errors.

Research has been done to be able to determine reservoir volume by using remote sensing. It is shown that is possible to estimate the reservoir volume by measuring the water surface area, but this is only proven for small reservoirs (Liebe, van de Giesen, & Andreini, 2005) & (Costa Lira, Toledo, & Mamede, 2011). Since the Pentecoste reservoir is a large reservoir (volume > 100 000 000 m$^3$ (Costa Lira et al, 2011)) this relation is not valid.

So the volume of the reservoir is only known in two moments in time. If the volume changed by linear function, power function or another function is unknown. So it is needed to find a relation concerning the reservoir volume which can be used to estimate the relation between reservoir volume and time. Lima Neto, Wiegand & de Araújo (2011) showed there is a constant with a relation to reservoir volume, the rate of sediment retention (formula 7). This constant can be used to estimate the volume change if all other factors are known.

$$\xi_m = \frac{\Delta V \rho}{V_0 \sum R} \quad (7)$$

In which $\xi_m$ is the rate of sediment retention (t m$^{-3}$ MJ$^{-1}$ mm$^{-1}$ ha h), $\Delta V$ the change in reservoir volume (hm$^3$), $V_0$ the initial reservoir volume (hm$^3$), $\rho$ the dry bulk density of the sand (t m$^3$) and $\sum R$ is the cumulative rainfall erosivity factor (MJ mm ha$^{-1}$ h$^{-1}$). $V_0$, $\xi_m$ and $\rho$ can be considered as a constant in either the semi-arid region of Ceará or the Pentecoste reservoir and are known. They are respectively 395.638 hm$^3$ (Table 5 page 22), 3.65*10$^{-7}$ (Lima Neto, Wiegand, & de Araújo, 2011) and 1.30 (de Araújo & Knight, 2005). The erosivity factor will be calculated with the use of formulas 8 and 9. These formulas are developed for southern region of Brazil, but are also valid for the semi-arid region (de Araújo, Fernandes, Machado Junior, Lima Oliveira, & Cunha Sousa, 2003).

$$R_m = 67.355 \left( \frac{P_m}{P} \right)^{0.85} \quad (8)$$
\[ R = \sum_{m=1}^{12} R_m \] (9)

In which \( R_m \) and \( R \) are, respectively the monthly and the annual erosivity factors and in which \( P_m \) and \( P \) are respectively the monthly total and annual average rainfall. These rainfall data are known so it is possible to calculate \( R \) and the cumulative rainfall erosivity over a certain period of time. This means all parameters in formula 7 are known except \( \Delta V \), so this formula can be written as (10):

\[ \Delta V = \xi_m \cdot V_0 \cdot \sum R \] (10)

If the time over which the volume change occurs is added to equation 10 this results in equation 11.

\[ \Delta V(t) = \frac{\xi_m \cdot V_0 \cdot \sum_{y=1}^{t} R_y}{\rho} \] (11)

In which \( t \) is the period of time in years and \( R_y \) is the annual rainfall erosivity factor. In this formula \( \Delta V \) and \( t \) are unknown, but \( t \) can be set at a desired time period. So this means \( \Delta V \) can be calculated for different time periods.

With this information the reservoir volume can be estimated between the construction and the bathymetric survey. With these two known volumes and the estimated volume change per unit of time the reservoir volume can be estimated at all moments in time.

The last parameter which will be determined over time is the shape-factor. The formula of the shape-factor alpha is already given in formula 1 and it is shown again below in formula 12.

\[ \alpha = \frac{\sum V_i}{\sum h_i^{3/2}} \] (12)

The formula for alpha shows us that to calculate alpha, various measurements of volume and depth are needed. The maximum volume and maximum water depth are known over time and can be used to find an average value for a longer period. This way 10 values can be determined for a 10 year period and the average value of alpha in that period can be determined. This procedure will result in average values of alpha between the construction and the bathymetric survey. This dataset will suffice to plot parameter alpha over time.

All these parameters plotted against time can be used to estimate future values. This will be done by extrapolation of the found relations. These predictions will be used to estimate and assess water availability in the future, 2064. Relations will be made using both one of the two bathymetric surveys. This will give graphs which represent the same variables calculated with the use of different data.
2.2.3. Depth-Area-Volume relations

The depth-area-volume relations will be power functions as described in paragraph 2.1.2. These power functions will be determined by using linear regression. In this paragraph this methodology will be elaborated. The results will contain depth-area-volume relations at four moments in time; the moment of constructing the dam in 1957, the moment of the bathymetric surveys in 2009 and 2014 and the prediction of the future 50 years from now. The only difference between determining the various relations is the collection of the input data. The relation in 1957 will make use of historical data, the relations in 2009 and 2014 will use the data of the bathymetric survey and the relations in 2064 will be determined with the extrapolated value of alpha and formulas 2 and 3.

Linear regression

To find constant ‘\( a_n \)' and ‘\( k_n \)', linear regression will be used. The goal of the linear regression analysis is to find a linear relation between volume and depth and volume and area. Sawunyama, Senzanje & Mhizha (2006) showed that the log area-log volume and log depth-log volume relations are linear. The formulas of these equations can be transformed to power functions using basic logarithmic principles. The linear equations of the relations between log depth, log area and log volume will have the format shown in formula 13.

\[
Y = b \times X + c
\]  
(13)

The depth-area-volume power function (equations 5 and 6) can be written as formula 14.

\[
\log y = a \times \log x + \log k
\]  
(14)

Combining the four elements of equations 13 and 14 results in the following four equations (equations 15 till 18):

\[
Y = \log y
\]  
(15)

\[
b = a
\]  
(16)

\[
X = \log x
\]  
(17)

\[
c = \log k
\]  
(18)

To express the depth-volume and area-volume relations ‘\( k \)' and ‘\( a \)' are required. The functions 19 and 20 will be used to find these values.

\[
k = 10^c
\]  
(19)

\[
a = b
\]  
(20)

Now ‘\( a_n \)' and ‘\( k_n \)' are known, so the two expressions of volume can be formulated.

Goodness of fit

The goodness of fit will show how well the determined power relation does represent the measured values. This will be done by the coefficient of determination (\( R^2 \)). The coefficient of determination indicates how well the data points fit the model. The value can vary between 0 and 1, the higher the value the more usable the model is. The way how the coefficient of determination is calculated is shown in formulas 21, 22 and 23.

\[
R^2 = 1 - \frac{SE_{\text{Residual}}}{SE_{\text{Total}}}
\]  
(21)
\[ SE_{Residual} = \sum_i (y_i - f_i)^2 \]  
\[ SE_{Total} = \sum_i (y_i - \bar{y}_i)^2 \]

Where \( R^2 \) is the coefficient of determination; \( SE_{residual} \) is the residual sum of squares; \( SE_{Total} \) is the total sum of squares; \( y_i \) is the measured value; \( f_i \) is the corresponding modelled value and \( \bar{y}_i \) is the average value of the observed data.

**Corrected depth-area-volume relations**

The power functions that will be created will not return the maximum volume when the maximum depth or maximum surface area is given as input. This is caused by the fact that the power functions will be fitted as close as possible to the measured points. This results in functions which represent the points as good as possible but do not return the measured maximum. To be able to calculate the maximum volume using depth-area-volume equations, all depth-area-volume equations in this report will be given a so called ‘corrected’ equation as well. This correction is done by dividing the entire equation by the volume found when the maximum depth or surface area is entered and by multiplying it with the measured maximum volume (24). This results in an equation which fits the measured points less well, but this equation does return the maximum volume.

\[ Corrected \ equation = \frac{Equation}{Calculated \ max \ volume} \times measured \ max \ volume \]  

One big remark has to be made when using this method of correcting the depth-area-volume relations. The uncorrected relation is the best possible fit, correcting this relation results in a worse fit. Better fits which represent the maxima would be achieved when the linear regression was conducted with the various maxima as a constraint. The choice is made to not use constrained regression because of the simplicity of the corrected-methodology. The decrease in goodness of fit is not too large; the \( R^2 \)-values are still high, above 0.9.

### 2.2.4. Water availability graphs

Two types of water availability graphs will be made; water availability against reliability and water availability against time. Of the first there will be three graphs, the situation in 1957, 2019 and 2064. Of the second, the availability against time, there will be made three graphs. One for each of the common used reliability levels, 99\%, 95\% and 85\%.

The water availability-reliability graphs will be outcomes out of the VYELAS-model and can be made easily. The model will be run with the determined input data of each of the four moments in time.

The water availability over time will be made by running the VYELAS model with interpolated and extrapolated data from the period between 1957 and 2064. These runs will be made several times. The water availability at the three reliability levels of each run will be documented. This dataset will be used to plot the water availability at the three reliability levels against time. This process will be conducted with the interpolated and extrapolated data of the 2009 and 2014 bathymetric surveys.

### 2.2.5. Measurement and propagation errors

The last sub-question concerns the measurement and propagation errors of the simplified bathymetric survey methodology. The measurement errors will be expressed as a percentage of deviation from the real value determined by the extensive bathymetric survey of 2009. The measurement and propagation errors will be assessed of all determined and predicted parameters.
2.3. Data collection

In this paragraph the collection of the required datasets will be discussed. General information about the study area is also collected, but is presented in Appendix B – Study area, the Pentecoste reservoir. The collected data that are presented below are all the data which need to be collected to fulfill the methodology as described in 2.2.

2.3.1. Bathymetric survey

2014

The bathymetric survey was conducted on the 3rd of December 2014. This bathymetric survey was conducted with manual devices. Two devices were used; a GPS-tracker to determine the coordinates of the point of measurement and an echo-sounder (Appendix D – Equipment –).

The GPS-tracker has a memory; this is used to store the coordinates of every point where a depth measurement is made. The echo-sounder does not have its own memory, so all measured depths are documented manually. This process resulted in 281 points of measurement. This number of points is enough to assess the reservoir shape. The reservoir surface area was estimated to be around 70 hectares. It was impossible to measure water depths below half a meter, because these areas were not accessible for the boat and the depth meter would not work in such shallow water.

In addition to the depth measurement the reservoir shape was assessed. This was done by making a trip along the shores of the reservoir. While making this trip the GPS-tracker was used to collect the route. This route will result in an approximation of the reservoir shape. Also with these measurements the boat was not able to follow the shoreline everywhere due to shallow water. The points of measurement are shown in Figure 4.

![Figure 4 Points of measurement Bathymetric survey 3 December 2014 (the image on the right is zoomed to the surveyed area)](image-url)
2009

The extensive bathymetric survey took 16 days to perform and was conducted in October 2009. This survey was conducted by COGERH and consisted of 94659 points of depth measurement. When this survey was conducted the water reservoir was completely full, this made the bathymetric survey more time-consuming. 13 days were needed to measure the depth all around the reservoir and it took three days to assess the shape of the surface area. The points of measurement are shown in Figure 5, due to the high density of points of measurement the separate points are not visible but Figure 5 does give an impression of the geographical spread of all points of measurement.

Figure 5 Points of measurement Bathymetric survey October 2009
2.3.2. Remote sensing

To complete the reservoir modelling of the simplified bathymetric survey Landsat 5 images are used. These are images that are collected from the Landsat 5 satellite which was launched in 1984. The images collected from this satellite are used to determine the water surface area in various moments in time. The elevation of the water level in the Pentecoste reservoir is 43.39 meter at the moment of the simplified bathymetric survey. The maximum elevation is 58 meter (Table 5 page 22). Five Landsat 5 images will be used to determine the shoreline at various depths. The most accurate result would be achieved when the images are evenly spread between 43.39 and 58 meter. This is not possible due to the fact that Landsat images are not taken daily of the Pentecoste region and images where clouds cover the Pentecoste reservoir are not usable. It is also good to use recent images, but this is not entirely possible due to the fact that the Landsat 5 satellite is taken out of business in January 2013 (USGS, 2014). The more recent Landsat 7 satellite could also be used but those images are not freely available. The dates of the used Images and their corresponding water levels are shown in Table 4.

<table>
<thead>
<tr>
<th>Date</th>
<th>Water level (meters above sea level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>07-04-11</td>
<td>55.32</td>
</tr>
<tr>
<td>09-07-10</td>
<td>54.58</td>
</tr>
<tr>
<td>05-10-07</td>
<td>52.71</td>
</tr>
<tr>
<td>12-08-99</td>
<td>48.05</td>
</tr>
<tr>
<td>30-10-93</td>
<td>47.41</td>
</tr>
</tbody>
</table>

Table 4 Used Landsat 5 images

The shoreline will be determined out of these images. Landsat 5 images consist out of seven different bands with varying wavelengths. Each band and its corresponding wavelength represent one color. The band combination which gives the clearest view of land-water boundaries is 4, 3 and 2. These bands represent respectively the colors red, green and blue. The first step is to combine these three bands into one image. This is done in with software called ENVI 4.3. After these 3 bands are compiled to one image this image needs to be georeferenced. Georeferenced means that the image will be given coordinates. The coordinate system used UTM WGS 84 zone 24 in the southern hemisphere. This step is important, because it will make it possible to use the results of the image analysis in ArcMap, because these programs now use the same coordinate system. The next step is to tell ENVI which colors in the image are of interest. This is done by using the regions of interest tool. This tool is used to create colored groups of the same surface area as shown in Figure 6 left. The next and last step is to create an image in ENVI, in which the program has filled out the entire defined land surfaces with the same color. ENVI does this by using the maximum likelihood tool.
This georeferenced surface area of the reservoir can be loaded as a raster into ArcMap. ArcMap has a tool called ‘Raster to Polygon’ which can be used to create polygons of every contour in the image. After deleting all polygons except the contour of the entire reservoir this polygon can be converted to a polyline which can be used as a contour in the previously described ‘Topo to Raster’ tool. The polygon can also be used to determine the surface area of the reservoir at the corresponding moment of time.

2.3.3. Depth-area-volume relations

Two types of data need to be collected to be able to construct the depth-area-volume relations as described in the methodology. The first information is the historical information, which is shown in Table 5. The other data are the depth, area and volume data of the modelled reservoirs. This information is obtained using the SurfaceVolume-tool in ArcMap. This tool returns the area and volume for a given elevation. A script is used in the Python extension of ArcMap to obtain these data for every elevation between the minimum and maximum water levels, with an interval between the steps of 0.1 meter.

<table>
<thead>
<tr>
<th>Technical data of the Pentecoste Reservoir</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build between</td>
</tr>
<tr>
<td>Maximum height</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>Mean Annual Rainfall</td>
</tr>
<tr>
<td>Catchment Area</td>
</tr>
<tr>
<td>Maximum Discharge</td>
</tr>
<tr>
<td>Hydraulic Basin</td>
</tr>
<tr>
<td>Annual Inflow</td>
</tr>
</tbody>
</table>

Table 5 Technical Information about the Pentecoste Reservoir (de Aragão Araújo, 1982)

2.3.4. Erosivity factor

The erosivity factor is calculated using rainfall data form the Curu basin. These data are the same data as used by Costa Lira (2012). Formulas 8 and 9 as given in paragraph 2.2.2 will be used to determine this value.

2.3.5. Data collection VYELAS model

As stated in the theoretical framework in paragraph 2.1.4 and in Appendix A – The VYELAS-model – the VYELAS-model has eleven different inputs. As shown in Appendix A, four of these inputs are used to enhance the models accuracy and performance. The other seven parameters represent natural factors which influence the water availability. In the upcoming paragraphs all the different inputs will be determined.

The average inflow and its variance

The mean inflow and its variance will be calculated with another model of de Araújo (2013), the BalHidr-model. This model uses water levels combined with the formula for the relation between volume and depth, medium evaporation, height of the spillway and the relation between outflow and water depth. These data are used to create a simplified water balance. This water balance can be used to determine the inflow over entire period of which depth is known. An extensive elaboration can be found in Appendix C – The BalHidr-model –.

The dataset created with this method will consist of a large range inflow data. These data will be used to determine the mean, the variance and the coefficient of variation by use of formula 25, 26 and 27.

\[
\text{Average inflow} = \frac{\sum \text{inflow}}{n} \quad (25)
\]

\[
S^2 = \frac{\sum (x - \bar{x})^2}{n - 1} \quad (26)
\]
\[ CV = \frac{\text{Average inflow}}{\sqrt{s^2}} \]  

(27)

Where \( n \) is the number of data; \( s^2 \) the variance; \( x \) is the measured value; \( \bar{x} \) is the average value and 
\( CV \) the coefficient of variance.

**Reservoir shape**

The reservoir shape is represented by the shape-factor, how the shape-factor is determined is already discussed in the methodology.

**Evaporation in the dry season**

The dry season in the semi-arid region lasts from June till January (Malheiros Ramos, Rodrigues dos Santos, & Guimarães Fortes, 2009). Malheiros Ramos et al. documented the monthly evaporation in Fortaleza, which is close to Pentecoste, and of which it is expected to have a similar evaporation. This monthly evaporation is shown in Table 6. The months of the dry season are added together this results in the total evaporation in the dry season of 1121 mm.

<table>
<thead>
<tr>
<th>Month</th>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>M</th>
<th>J</th>
<th>J</th>
<th>A</th>
<th>S</th>
<th>O</th>
<th>N</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaporation (mm)</td>
<td>127.7</td>
<td>93.8</td>
<td>72.4</td>
<td>67.5</td>
<td>80.5</td>
<td>93.5</td>
<td>115.2</td>
<td>153.2</td>
<td>159.2</td>
<td>163.9</td>
<td>158.9</td>
<td>149.4</td>
</tr>
</tbody>
</table>

Table 6 Average evaporation per month in Fortaleza (1961-1990)

**Maximum storage capacity**

Similar to the reservoir shape the maximum storage capacity is already discussed extensively in the methodology.

**Minimum operational volume**

The minimal operational volume is 50 000 000 m³. This was the volume which was in the reservoir when the decision was made to put a ration on the amount of discharged water in July 2013. This volume is determined by COGERH using the depth volume relation.

**Initial volume**

The initial volume will be set at half of the maximum storage capacity. This is done because the initial volume is unknown. The initial volume does only influence the results when the timescale is very small. This is caused by the fact that the reservoir will reach in minimum or maximum within a few years of simulation.

**Range of yield**

The range of yield will be set between 50hm³ and 350hm³ with a 1000 steps between them. This will result in reliability levels ranging from 0% to 100%.

**Number of simulations**

50000, this is the default setting of the VYELAS-model. Various tests have been done to check whether this amount of simulations keeps providing the same outputs. This is the case so 50000 simulations gives usable outcomes.
3. Results
The next three paragraphs will present the results. First a paragraph concerning the shape, second a paragraph concerning the water availability and the third paragraph will concern the difference in the results between the different bathymetric surveys.

3.1. Reservoir shape
As described in chapter 2 ‘Research Design’, the shape of the reservoir is represented by four parameters and the depth-area-volume relations. First the change of the four parameters over time will be discussed; afterwards the depth-area-volume relations will be given and analyzed.

3.1.1. Shape over time
The four parameter values will be given for the years 1957, 2009 and 2064. After these parameters are given each of them will be presented over time.

Parameter values
The measured and calculated parameter values of the Pentecoste reservoir in 1957 and 2009 are shown in the table below; the predicted values for 2064 are added to Table 7 as well. These predictions are made by extrapolations of the measured values.

<table>
<thead>
<tr>
<th>Year</th>
<th>Maximum depth (m)</th>
<th>Maximum surface area (m²)</th>
<th>Maximum volume (m³)</th>
<th>Shape-factor (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>22</td>
<td>57 000 000</td>
<td>395 638 000</td>
<td>37.227</td>
</tr>
<tr>
<td>2009</td>
<td>20.00</td>
<td>48 487 669</td>
<td>357 453 679</td>
<td>46.183</td>
</tr>
<tr>
<td>2064</td>
<td>17.88</td>
<td>48 487 669</td>
<td>315 597 789</td>
<td>52.193</td>
</tr>
</tbody>
</table>

Table 7 Shape parameter values

Maximum depth over time

![Maximum water depth over time](image)

In the period between 1957 and 2009 the maximum depth of the reservoir has been reduced with two meters; from 22 meters in 1957 to 20 meters in 2009. As stated in the theoretical framework it is assumed the relation between the maximum water depth and time is linear. This results in equations 28 and 29. When equation 29 is used to calculate the maximum depth in 2064 it gives 17.88 meter.

\[
h = h_0 + (\text{Year} - \text{Year}_0) \cdot \frac{\Delta h}{\text{Year}} \quad (28)
\]

\[
h = 22 - (\text{Year} - 1957) \cdot 0.0385 \quad (29)
\]
In Figure 8 the surface area over the time is visible. The maximum surface area in 2009 was not equal to the maximum surface area in 1957. As stated in the methodology, satellite images of earlier moments in time are used to examine the surface area in the past. Three images are used of moments in time when the water level was around 58 meters above sea level. The dates and surface areas of these three images can be found in Table 8. In Figure 8 the line through the four points which are measured in this research is plotted. This line is almost horizontal; this insinuates that the surface area does not change in time. The equation of the line is shown below as equation 30.

\[
\text{Maximum surface area (km}^2\text{)} = 0.0085 \times \text{Year} + 30.08
\]

This equation shows that the surface area changes with 0.0085 km\(^2\) per year, with a measured surface area of 48.488 km\(^2\) this change is negligible as it is only 0.018% of the total surface area per year (equation 31). Because of the negligibility of the change, the surface area is considered constant.

\[
\frac{0.0085}{48.488} \times 100\% = 0.018\%
\]

The surface area in 1957 is not used to determine the linear function. This is done because it is unknown how and how accurate this measurement is executed in the past. The value of the surface area in 1957 can be explained in two possible ways. The first states that the surface area is a measurement error. The second possibility is that a mayor event has influenced the Pentecoste reservoir in the period between 1957 and 1986. This last possibility is unlikely, because there is not any knowledge about such a mayor event and an event which would influence 10km\(^2\) of reservoir surface would be documented.

<table>
<thead>
<tr>
<th>Date</th>
<th>Surface area (m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-5-2004</td>
<td>45 467 300</td>
</tr>
<tr>
<td>31-5-1996</td>
<td>46 466 100</td>
</tr>
<tr>
<td>20-5-1986</td>
<td>47 479 300</td>
</tr>
</tbody>
</table>

Table 8 Satellite images for examining the surface area over time
Figure 9 shows the maximum volume. The blue points in the graph represent the volume calculated with the change in volume dependent on the erosivity factor. The red points represent the volume measurements of the bathymetric survey in 2009 and the initial volume calculated in 1957. The volume by erosivity factor shows a linear trend through both the red points. Because the red points are actual points of measurement and the blue points are based on the assumption that the rate of sediment retention is constant over time, the red points are used to construct a linear function for the volume over time (equations 32 and 33).

\[
V = V_0 + (Year - Year_0) \cdot \frac{\Delta V}{Year}
\]  

(32)

\[
V = 395\,638\,000 - (Year - 1957) \cdot 734\,314
\]  

(33)

Equation 33 is used to calculate the expected maximum volume in 2064, which was presented previously in Table 7. The R-squared value of this equation related to the expected volume with the use of the erosivity factor is 0.9927.

Equation 33 also shows the amount of volume of the Pentecoste reservoir is yearly filled up with sediment. This is 734,314 m³ per year.
Shape-factor over time

Figure 10 shows the development of the shape-factor over time. The different values of the shape-factor are determined by using the maximum depth and the maximum volume found in the previous paragraphs over a period of ten years. This procedure results in a set of ten depths with their corresponding volume. This set makes it possible to use equation 1 to calculate the shape-factor.

Figure 10 also shows the measured shape-factors in 1957 and 2009. Values calculated with the previously described procedure underestimate both measured values. This is probably caused by the fact that the procedure uses the maximum depth and maximum volume. This method excludes the values with lower water levels and lower volumes. When there is a certain range in depth in the reservoir which is different in geometry than the highest range of depth this could increase or decrease the shape-factor. The shape-factor will increase if the average angle of the slopes of the reservoir is smaller than 45°, it will decrease if the average angle is larger. This principle is explained in Appendix F – the meaning of the value of shape-factor alpha – and is based on the principle that when the numerator is larger than the denominator the outcome increases. So it is expected slopes below the maximum water level of the reservoir have an average angle smaller than 45°. The formula which calculates the shape-factor represents a reservoir with straight slopes with a 45° angle.

The underestimation described above cannot be proven, only expected. The underestimation could also be caused by measurement errors since the difference is not very large. This is why the final formula for the shape-factor over time is not corrected for the underestimation. The equation of the Shape-factor over time is shown below as equation 34.

\[
\alpha = 36271 \times e^{0.0033 \times (\text{year} - 1597)}
\]  

(34)
3.1.2. Depth-area-volume curves

Below the final corrected depth-area-volume curves are presented. How these graphs are constructed can be found in Appendix E – Bathymetric results – on page 54.

Depth-volume relations

![Depth-volume relations](image)

**Figure 11 Depth volume-curves in 1597, 2009 and 2064**

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation ( V = k_1 * D^{a_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>( V = 27291 * D^{3.1089} )</td>
</tr>
<tr>
<td>2009</td>
<td>( V = 7113 * D^{3.6135} )</td>
</tr>
<tr>
<td>2064</td>
<td>( V = 55469 * D^{3} )</td>
</tr>
</tbody>
</table>

**Table 9 Equations of the depth-volume curves**

In Figure 11 and Table 9 the depth-volume curves and their corresponding equations are visible. It is important to notice that the values for \( k_1 \) are large. As told in the theoretical framework a large value for \( k_n \) means there is a large open reservoir. To be able to assess the change of \( k_n \) over time, new equations are made. In these new equations \( a_1 \) is set to 3. This will make \( k_1 \) the only changing variable, which results in equations were that specific variable can be assessed. These values for \( k_1 \) are found by an iterative process in which \( R^2 \) is maximized. The results are shown below in Table 10.

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation ( V = k_1 * D^{a_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>( V = 37000 * D^{3} )</td>
</tr>
<tr>
<td>2009</td>
<td>( V = 46000 * D^{3} )</td>
</tr>
<tr>
<td>2064</td>
<td>( V = 55469 * D^{3} )</td>
</tr>
</tbody>
</table>

**Table 10 Depth-volume equations with \( a_1 \) set to 3**

Table 10 shows that \( k_1 \) has increased over the years. As described in the theoretical framework in paragraph 2.1.2 large values of \( k_1 \) represent reservoirs with a large surface area which are shallow. An increase of \( k_1 \) over time indicates the reservoir has become shallower, this is expected because the maximum depth has decreased and sediment is being deposited into the reservoir over time.

The same method is used to assess the change in \( a_1 \) over time. The value of \( k_1 \) is set equal to the value of 2064, 55469, for each of the equations and the best fitting \( a_1 \) is determined. The results are shown in Table 11.
Table 11: Depth-volume equations with $k_1$ set to 55469

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation ($V = k_1 \times D^{a_1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>$V = 55,469 \times D^{2.865}$</td>
</tr>
<tr>
<td>2009</td>
<td>$V = 55,469 \times D^{2.935}$</td>
</tr>
<tr>
<td>2064</td>
<td>$V = 55,469 \times D^3$</td>
</tr>
</tbody>
</table>

Table 11 shows that $a_1$ has and will increase over time. $a_1$ indicates the whether the reservoir is convex or concave. Larger values for $a_1$ indicate concave reservoirs so an increase in $a_1$ over time means the reservoir has become more concave. This is probably caused by the settling of sediment on the reservoir bottom.

‘$a_1$’ from the 2064 equation is already exactly 3 because this relation is equal to equation 35. The shape-factor in 2064, 52193, is not equal to $k_1$ because the equation is corrected.

\[ V(h) = \alpha \times h^3 \]  

(35)

Area-volume relations

![Area-volume relations](image)

Figure 12: Area-volume curves in 1957, 2009 and 2064

Table 12: Equations of the depth-volume curves ($V$ and $A$ are in m$^3$ and m$^2$ respectively)

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation ($V = k_2 \times A^{a_2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>$V = 0.0008 \times A^{1.504}$</td>
</tr>
<tr>
<td>2009</td>
<td>$V = 0.006 \times A^{1.4016}$</td>
</tr>
<tr>
<td>2064</td>
<td>$V = 0.0009 \times A^{1.5}$</td>
</tr>
</tbody>
</table>

The same method to assess the change of $a_2$ and $k_2$ over time will be used as was used on the previous pages to assess the depth-volume curves. It is expected $a_2$ and $k_2$ show the same direction of change as $a_1$ and $k_1$ did.

First all values of $a_2$ are set to 1.5 to be able to assess $k_2$. The results of this step are shown below in Table 13. Table 13 shows that $k_2$ has increased in the period between 1957 and 2009, this was expected because $k_1$ also increased over time. It is also expected that in the period between 2009 and 2064 $k_2$ will further increase which will result in an even shallower reservoir.
The results of the analysis with \( k_1 \) set equal to the value in 2064, 0.00094, are shown in Table 14.

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation ( (V = k_2 * A^{a_2}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>( V = 0.00090 * A^{1.5} )</td>
</tr>
<tr>
<td>2009</td>
<td>( V = 0.00093 * A^{1.5} )</td>
</tr>
<tr>
<td>2064</td>
<td>( V = 0.00094 * A^{1.5} )</td>
</tr>
</tbody>
</table>

Table 13: Area-volume equations with \( a_2 \) set to 1.5

Table 14: Area-volume equations with \( k_2 \) set to 0.00094

Table 14 shows a small increase in \( a_2 \) between 1957 and 2009. The depth-volume relation showed that the reservoir becomes more concave, this would mean a decrease in \( a_2 \). This is not the case. The unexpected increase in \( a_2 \) is possible caused by the fact that the maximum surface area is different in 1957 than in 2009. This can explain the increase between those years. The formula has not changed between 2009 and 2064. This does not support the expectation that the reservoir has become more concave, says the slopes have not changed. This observation is less reliable than the observation in the depth-volume equations because both the area (equation 35) and volume (equation 36) in 2064 are calculated using the shape-factor. Hence a deviation in the shape-factor has a large influence on the area-volume curve in 2064. Because the observations in the area-volume equations are considered less reliable it is still assumed the reservoir has become more concave.

\[
A(h) = 3 \propto h^2
\]  

(36)
3.2. Water availability

3.2.1. Water availability against reliability

Figure 13 shows the Yield-reliability curves of the years 1957, 2009 and 2064. The most important observation in this graph is that the horizontal distance between 1957 and 2009 smaller is then the distance between 2009 and 2064. If the decrease in yield should be linear over time the distance between 1957 and 2009 should be larger than the distance between 1957 and 2009 because of the difference in time period. So it can be concluded that the decrease is not linear. This can be explained by the change in the reservoir shape. In the previous paragraph the analysis of the depth-area-volume curves showed that the reservoir will become shallower. When the reservoir becomes shallower and less deep in the same process, the surface area relative to the volume will increase. This will cause an increase in evaporation which is one of the uncontrolled ways water can leave the reservoir. So more water will be lost over time due to evaporation and the yield will decrease.

In addition to the decrease in yield due to increased evaporation the yield is further decreased by the declining maximum volume. Due to the lower capacity of the reservoir, a lower volume of water can be stored. This results in a lower buffer for dry periods, an increase in increase in spillway loss and an increased chance in decreased yield.

The two phenomena are schematically presented in Figure 14, which shows cross sections of a fictional reservoir. On the left Figure 14 shows the initial situation and on the right the situation after a longer period when sediment has been transported into the reservoir. Both reservoirs contain the same amount of water. ‘X meter’ represents the evaporation, the water level decreases X meters due to evaporation. Clearly visible in Figure 14 is the larger evaporating body of water on the right side of the figure. So the amount of water lost to evaporation has increased. The downward pointing arrow represents an event of rainfall. Due to sedimentation the storage capacity of the reservoir has decreased with 100hm³. This results in an extra 100hm³ water is lost to uncontrolled outflow (horizontal arrows).

These two phenomena add up, resulting in a non-linear decrease of yield over time.
Figure 14 Increase in evaporation and uncontrolled outflow

3.2.2. Water availability against time

Figure 15 Yield reliability over time

<table>
<thead>
<tr>
<th>Reliability level</th>
<th>Equation</th>
<th>R squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>99% – Human activities</td>
<td>$Y_{99} = -0.1163 \times year + 309.73$</td>
<td>$R^2 = 0.9794$</td>
</tr>
<tr>
<td>95% – Industrial activities</td>
<td>$Y_{95} = -0.1336 \times year + 363.15$</td>
<td>$R^2 = 0.9750$</td>
</tr>
<tr>
<td>85% – Agricultural activities</td>
<td>$Y_{85} = -0.1441 \times year + 405.78$</td>
<td>$R^2 = 0.9837$</td>
</tr>
</tbody>
</table>

Table 15 Yield equations for the three reliability levels

The trends shown in Figure 15 seem inconsistent with the conclusions made based on Figure 13 in the previous paragraph. These conclusions showed that the relation between yield and time is not linear and Figure 15 shows the opposite. De Araújo et al. (2006) came upon the same contradiction in their research. They explained that the effects which were described with the evaluation in paragraph 3.2.1 only increase exponentially in magnitude when the loss in volume reaches higher values. The percentage of volume loss in 2064 is only 20.2% of the initial volume. When this volume
loss will reach values above 60% the exponential characteristics of these equations will show more clearly.

To assess this hypothesis, the values for alpha and volume are extrapolated further into the future, the VYELAS-model is run again and the outcomes are plotted in Figure 16. In Figure 16 the best fitting functions are plotted, these are exponential functions. The functions fit the datasets with a $R^2$ value a little above 0.8 which is considered acceptable. A better fit could be achieved by simulating more years. This is not done because of the purpose of Figure 16, Figure 16 shows that there is an exponential increase in volume loss. It also shows that the first ±100 years after reservoir the reservoir is constructed the yield reduction per year hardly increases; hence it is possible to approximate this first part as a linear function, which is confirmed by the high $R^2$-values.

<table>
<thead>
<tr>
<th>Reliability level</th>
<th>Absolute decrease of yield per year (hm³)</th>
<th>Relative decrease of yield per year (%) $(Y_0=1957)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99% – Human activities</td>
<td>0.1163</td>
<td>0.14</td>
</tr>
<tr>
<td>95% – Industrial activities</td>
<td>0.1336</td>
<td>0.13</td>
</tr>
<tr>
<td>85% – Agricultural activities</td>
<td>0.1441</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 16 Decrease in yield due to change in reservoir shape

![Figure 16 Yield reduction per year](image-url)
3.3. Different bathymetric methodologies

In the previous paragraphs the outcomes of the extensive bathymetric survey conducted in 2009 are used to assess shape and water availability. In Appendix E – Bathymetric results – both the bathymetric surveys of 2009 and 2014 are elaborated. The calculations and graphs shown in the previous paragraph are also made with the data from the 2014 bathymetric survey. In Table 17 both the outcomes of the 2009 and 2014 bathymetric survey are presented. Figure 18 and Figure 19 the elevation of the reservoir is shown as result of the conducted surveys.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>2009</th>
<th>2014</th>
<th>%deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max depth</td>
<td>20 meter</td>
<td>20 meter</td>
<td>Equal</td>
</tr>
<tr>
<td>Max surface area</td>
<td>48 487 669 m²</td>
<td>52 213 165 m²</td>
<td>7.6%</td>
</tr>
<tr>
<td>Max volume</td>
<td>357 453 679 m³</td>
<td>319 767 545m³</td>
<td>10.4%</td>
</tr>
<tr>
<td>Shape-factor</td>
<td>46 183</td>
<td>35 521</td>
<td>23.1%</td>
</tr>
<tr>
<td>Depth-volume relation</td>
<td>$V = 7 , 113 \times D^{3.6135}$</td>
<td>$V = 5 , 959.4 \times D^{3.6353}$</td>
<td>-</td>
</tr>
<tr>
<td>Area-volume relation</td>
<td>$V = 0.006 \times A^{1.4016}$</td>
<td>$V = 0.0191 \times A^{1.3247}$</td>
<td>-</td>
</tr>
<tr>
<td>2064 D-V relation</td>
<td>$V = 55 , 469 \times D^{3}$</td>
<td>$V = 44 , 298 \times D^{3}$</td>
<td>-</td>
</tr>
<tr>
<td>2064 A-V relation</td>
<td>$V = 0.0009 \times A^{1.5}$</td>
<td>$V = 0.0007 \times A^{1.5}$</td>
<td>-</td>
</tr>
<tr>
<td>Volume reduction per year</td>
<td>734 314 m³</td>
<td>1 331 061 m³</td>
<td>85.7%</td>
</tr>
<tr>
<td>99% Yield reduction per year</td>
<td>$Y_{99} = -0.1163 \times year + 309.73$</td>
<td>$Y_{99} = -0.1953 \times year + 465.33$</td>
<td>-</td>
</tr>
<tr>
<td>95% Yield reduction per year</td>
<td>$Y_{95} = -0.1336 \times year + 363.15$</td>
<td>$Y_{95} = -0.2146 \times year + 522.38$</td>
<td>-</td>
</tr>
<tr>
<td>85% Yield reduction per year</td>
<td>$Y_{85} = -0.1441 \times year + 405.78$</td>
<td>$Y_{85} = -0.2513 \times year + 616.58$</td>
<td>-</td>
</tr>
<tr>
<td>99% Relative decrease of yield per year ($Y_0=1957$)</td>
<td>0.14%</td>
<td>0.24%</td>
<td>71.4%</td>
</tr>
<tr>
<td>95% Relative decrease of yield per year ($Y_0=1957$)</td>
<td>0.13%</td>
<td>0.21%</td>
<td>61.5%</td>
</tr>
<tr>
<td>85% Relative decrease of yield per year ($Y_0=1957$)</td>
<td>0.12%</td>
<td>0.21%</td>
<td>75.0%</td>
</tr>
</tbody>
</table>

Table 17 Comparison of the results of the 2009 and 2014 bathymetric survey

Table 17 shows the results of the different bathymetric methodologies. The fourth column, %deviation, shows how much the results of the Landsat-methodology deviate from the results of the extensive bathymetric survey.

The best result with the 2014-bathymetric survey is achieved with the measurement of the maximum depth. This caused by the fact that both survey methodologies have made depth measurement using an echo-sounder at the deepest point of the reservoir. The most important consequence of this result is that it supports the assumption that the reservoir shape has not changed in the period between 2009 and 2014.

The maximum surface area and the maximum volume deviation are at an acceptable level, respectively 7.6 and 10.4 percent. The most important parameter is the maximum volume, because to calculate all other results shown in Table 17, the maximum volume is used. When the
measurement error of the maximum volume would be smaller all these other results will have a smaller deviation.

The results of the 2014-bathymetric survey seem to be unusable for further calculations. This is not entirely the case because all results are within the same order of magnitude. The parameters which show large deviations (>25%) are all predictions of the future, except the volume reduction per year. All predictions made in this report are based on several assumptions and simplifications which also causes uncertainty. This means that all results which are used to predict the future should not be taken for granted.

The only parameter which has a large deviation which is not a prediction is the volume reduction per year. The volume reduction per year is the same as the slope of the volume over time graph, the volume over time graph using the 2014-bathymetric survey is shown below in Figure 17. The $R^2$-value for this graph is -1.3; this means the function fits the point of measurements less well than the function which represents the average of all points of measurement.

![Volume over time](image)

**Figure 17** Volume over time with 2014-input

**Depth maps**

On pages 37, 38 and 39 the bed elevation maps and their differences are shown. Figure 18 shows the elevation map which is constructed with input from the bathymetric survey from 2009, Figure 19 shows the same information with input from 2014 and Figure 20 shows the difference in meters between the two maps.

The biggest difference between the two elevation maps is the amount of detail in the parts with higher elevation (44-58 meter above sealevel). This difference is because in these areas the different methodologies are applied. The Landsat-methodology results in less data for these parts of the reservoir which means there are fewer points available for the interpolation, which results in less detail. This difference is especially visible within the range of elevation from 56 till 58 meters above sealevel. This is caused because the information on this range of elevation was the border of the reservoir, which was 58 meters elevated and the satellite image of 07 April 2011 when the water level was 55.31 meters. This low amount of information to use in the interpolation process resulted
in the large parts of the reservoir in which the bed elevation is between 56 and 58 meters above sealevel.

Figure 20 shows that the parts of the reservoir determined with depth measurements show almost no deviation between the two surveys. Most parts of the comparison show no big differences. Big differences are considered as larger areas with more than 6 meters difference, the large areas are required because small areas of high values can be caused by a small difference in coordinates. The big differences all have a negative value; this means the elevation of the 2014 survey was higher. This is probably caused by the absence of depth measurements in this area, this absence results in an unknown maximum depth in this area and makes the interpolation use a different minimum elevation. This different minimum elevation is higher than the measured minimum in 2009.
Figure 18 Map showing the elevation of the bottom of the Pentecoste reservoir in 2009
Figure 19 Map showing the elevation of the bottom of the Pentecoste reservoir in 2014
Figure 20 Comparison in difference in results of bed elevation, 2009 survey minus 2014 survey
4. Conclusion

This report started with the introduction of the goals, the main question of research of this report. All the sub-questions introduced in chapter 1 Introduction have been answered in the previous chapter. In this chapter these answers will be summarized in the form of conclusions and will be combined to form the answer of the main question of research. When the questions are answered, other findings which are not covered in the research questions.

Sub-questions

1. How has shape of the reservoir changed due to sedimentation since construction and how will it change in the next 50 years?

The reservoir shape has been represented by four parameters in this research; maximum water depth, maximum surface area, maximum volume and the shape-factor. The changes of all of these parameters have been researched and the following five conclusions are made. The maximum depth and maximum volume both have a linear relation with the time and decrease 3.85 cm and 734 314 m$^3$ per year respectively. The maximum surface area does not change over time and it is expected that the measurement of the maximum surface area in 1957 has a large measurement error. The last parameter, the shape-factor, shows an exponential relation with the time.

2. What are the relations between depth, surface area and volume when constructing the dam, in the present and in the future?

The depth-area-volume relations have been found. Two important findings have been made based on the depth-area-volume relations. The analysis of the equations of these relations showed that the reservoir has become shallower and that the slopes have become more concave.

3. How has the sedimentation influenced the water availability and how will the future change in shape influence the water availability?

The water availability does not change linear over time. This is caused by increased evaporation and spillway loss when the reservoir becomes shallower and the slopes become more concave. The non-linear trend in this relation only shows itself when a large part of the reservoir volume has been lost. This gives the possibility to present the loss in water availability as a linear function for the first 100 years. This resulted in yield loss per year of 0.1163 hm$^3$, 0.1336hm$^3$ and 0.1441hm$^3$ for respectively the 99%-95%- and 85%-reliability levels.

4. What are the measurement and propagation errors—in volume, shape and water availability—of the Landsat bathymetric survey methodology?

It is shown that the measurement errors of the bathymetric survey with remote sensing of the maximum depth, surface area and volume, are not bigger than 10.4%. The propagation errors are larger, which is caused by the measurement error in the maximum volume. The propagation errors make the results of the simplified bathymetric survey less useful, but they are still useful to some extent, because the results of both surveys have the same order of magnitude. The last finding which has been made is that the main difference between both surveys is the detail level.

Main question of research

How does the shape of Pentecoste reservoir change due to sedimentation and how does this change in shape influence the water availability in the past and future?
With the conclusions of the sub-questions it can be stated that the shape of the Pentecoste reservoir changes over time by sedimentation, towards a shallower more concave reservoir. This change in shape results in an increase in evaporation and spillway loss. These increasing losses result in a decrease in yield, this decrease in yield increases exponentially over time. When the decrease in yield increases exponentially over time the actual yield decreases exponentially over time, hence the water availability over time can be described as a exponential function of time. However the exponential characteristics become only visible when a large part of the reservoir is filled with sediment. Due to the low amount of sedimentation in the first 100 years of the Pentecoste reservoir this exponential characteristics are not visible. Hence it is possible to describe the yield over time as a linear function from 1957 till 2064, the scope in time of this report.

Other findings
Another observation made within this report which is noteworthy is the correlation between volume over time and the erosivity factor. The volume over time calculated with the rate of sediment retention and the erosivity factor shows great similarity with the linear decrease of volume over time, measured with the bathymetric survey in 2009. This makes it possible to use this relation and/or the measurement of rainfall data to estimate the volume in the future, which reduces the need of performing another costly bathymetric survey.
5. Discussion

As in every research, some remarks have to be made. These remarks represent assumptions and choices made within this research which could have influenced the outcomes of the research. Every chapter of this report has been reflected in this chapter. Each paragraph within this chapter represents one chapter of the report. This way all chapters are reflected upon and all steps of research have been considered once again.

Chapter one introduced the goals and questions within this report. Within this chapter the most arguable choice or assumption is the choice to extrapolate the future values till the year 2064. The choice for 2064 has been made because it was 50 year after the start of the research. The choice to extrapolate 50 years is also subjective, but in the end this choice does not really influence the results. This is because the results mostly consist of relations between parameters and the explanations why these relations have a certain form. These will not change when the scope of this research is changed from 50 year to 100 years or 1000 years. The only effect which the increase in scope will have, is that it will show the moment when the reservoir is completely filled with sediment.

The second chapter consists of descriptions of all theories, methods and the collection of data. Within this chapter the most important assumptions have been made. The most important assumption is the assumption of a linear relation between depth and time. This choice has been made to be able to interpolate and extrapolate the maximum depth of the reservoir. The maximum depth has influence on most parts of the research. The maximum depth influences the predicted value of the shape-factor in the past and future and in this way the maximum depth influences all the results related to water availability. This means when another research will show that the relation is not linear all quantitative result of this result will change except the shapes in 1957, 2009 and 2014, the maximum surface area and the maximum volume, since these are the only parameters within the research which have not used the assumed linear relation between maximum depth and time.

A second point of discussion within chapter two is a part of the data collection. It is about the collection of satellite images especially. The used satellite images are old, which was caused by the ending of the Landsat 5-project. More recent satellite images will probably result in more accurate measurement of the surface area and the shoreline. This will increase the accuracy of the Landsat bathymetric methodology. The measurement and propagation errors could be decreased in this way.

The third point of discussion which has to be stated by chapter two is about the correction of the depth-area-volume relations. This is done by scaling the relations to the desired maxima. A better method to introduce the maxima in the relation is by using constrained regression to determine the relations. In this research this has not been done, because of the simplicity of the correction method and the acceptable $R^2$-values above 0.9. The decrease of the $R^2$-values, when correcting, was only 0.07 or less.

The third chapter consisted of the results. One important remark will be made about this chapter, which is about the amount of input data within this research. The results of this research are in the end based on two moments in time, the year of construction and the year of the bathymetric survey, either 2009 or 2014. It is shown that one of the measurements in 1957 has a large measurement error, the surface area. This means that the measurements out of 1957 are not completely reliable and cause uncertainty within the research. To decrease this uncertainty it would be good to validate with historical data. This is only done to certain extend with the volume over time and the surface area over time. All other parameters are based on two points of measurement.

Chapters four, five and six do not need a lot of reflection. The conclusions are already made in the previous chapters and this way chapter four does not introduce new points of discussion. The only
argument which could be made about chapter five is about the completeness of this discussion. It is tried to cover this by discussing and reflecting every chapter. Chapter six also needs one small remark about completeness. It is impossible to be sure that all recommendation have been made, another reader might find other information within this report which could be used to recommend other actions.
6. Recommendations

In this report various findings, observations and remarks have been made. These gave various ideas for recommended steps which should be taken. These further steps are researches about various parts of this report which can result in useful information.

These recommendations are aimed at both scientific readers as COGERH, because if the recommended follow-ups are carried out the outcomes of this research and the follow-ups can be used to better predict and evaluate the reservoir shape. Hence it is possible for COGERH to adjust the outflow more frequently to the actual situation, which will result in a better water availability and less scarcity in the semi-arid region.

The first research should be a future bathymetric survey to compare the predicted values to the actual values. The survey should be conducted 20 to 50 years from now, after 20 years the reservoir shape will have changed and the predictions are made till 50 years from now. This analysis will result in data which represents the accuracy of the predictions. If the predictions are shown to be within an acceptable range of the actual values it will result in a less urgent need of bathymetric surveys in the Pentecoste reservoir, because the shape can be calculated. If it is shown that the deviations between the actual values and the predictions are very large, it will show that the assumptions in this research are not true and it will give to possibility to assess the relation between shape and time again with more available data.

A second possible aspect to investigate further is the relation between the maximum volume and the erosivity factor. This research can be conducted at any moment; it will consist of bathymetric surveys of various reservoirs and the volume determined with these surveys can be compared to the volume calculated by the methodology within this research. If it is shown that the difference between these two ways to determine the maximum volume result in comparable results in various locations and reservoirs, it will reduce the need for bathymetric surveys. It will reduce the need because the maximum volume can be determined of individual reservoirs without conducting a survey.

The third recommended follow-up would be to assess the change of depth-area-volume relations over time. If it is possible to show that there are relations which represent the change of \( a_n \) and \( k_n \) over time, it will result in the possibility to construct depth-area-volume equations without doing a bathymetric survey.

The fourth idea for further research focusses on the bathymetric survey methodologies. If more comparisons will be made between these two methodologies it will be possible to calculate a standard deviation of this methodology. If this is possible it will result in the opportunity to use the Landsat Methodology and correct its results. This results in reliable less time-consuming method to perform bathymetric surveys.

The fifth and last recommendation is about a research which includes more external factors. One of these factors could be climate change; climate change would change the inflow and the evaporation values. Other external factors can also be introduced within a further research. The introduction of extra factors could increase the possibility to represent the actual water availability over time.
Bibliography


Appendices

Appendix A – The VYELAS-model –

The assumptions and calculations used in the VYELAS model are extensively elaborated by de Araújo et al (2006). In this appendix this will be shortly repeated and explained.

The VYELAS model makes use of the stochastic experimental method developed by Campos (1996) and Campos, de Araújo % Sousa Filho (1997) and the long term probability of success in providing the water yield in one year (McMahon & Mein, 1986). The model consists of calculating a simplified water balance with seasonal time steps. These seasonal time steps consist of a step representing the wet season and one representing the dry season.

The water balance used within the VYELAS-model is shown in equation 37. With the assumptions shown in equation 38 and 39 equation 40 is formulated.

\[
\frac{dV}{dt} = (Q_A + Q_H + Q_{GW}) - (Q_E + Q_S + Q_l + Q_G)
\]

\[
Q_E = Q_{E,w} + Q_{E,d}
\]

\[
Q_H + Q_{GW} \approx Q_{E,w} + Q_l
\]

\[
\frac{dV}{dt} = (Q_A) - (Q_{E,d} + Q_S + Q_G)
\]

Where V is volume in m$^3$; t is time (year); $Q_A$ is inflow from the river network; $Q_H$ is water input by rainfall directly on the reservoir surface; $Q_{GW}$ is groundwater discharge to the reservoir; $Q_E$ is evaporation from the reservoir surface, with $Q_{E,w}$ and $Q_{E,d}$ respectively the evaporation in the wet and dry season; $Q_S$ is reservoir outflow over the spillway; $Q_G$ is outflow due to regulated water yield with a reliability level G. All the values for the different Q's are in m$^3$/year.

Several other assumptions are made to simplify the model. Three assumptions characterize the wet season. The first is that all inflow to the reservoir ($Q_A$) occurs in the wet season. The second assumption is that whenever the maximum reservoir capacity is reached all new inflow will directly become spillway overflow. Thirdly there are no water extractions during the wet season. Thus the reservoir volume at the end of the wet season is the initial volume plus the inflow minus to spillway overflow. This results in equations 41 and 42, where $V_w$ is the volume at the end of the wet season; $V_i$ is the volume at the start of the wet season; $\Delta t_w$ is the length of the wet season and $V_{max}$ is the maximum reservoir capacity.

\[
V_w = \max(V_i + Q_A * \Delta t_w, V_{max})
\]

\[
Q_S = \max(0, V_i + Q_A - V_{max})
\]

The dry season is characterized by volume depletion. The reservoir volume reduces due to evaporation ($Q_{E,d}$) and withdrawal ($Q_G$). The reservoir volume at the end of every year will be calculated by reducing the dry season water depletion from the volume calculated at the end of the wet season. The water balance in the dry season can be written as equation 43, this can be rewritten with the introduction of surface area A this results in equation 44.

\[
\frac{dV}{dt} = -Q_{E,d} - Q_G
\]
Where $A$ is the water surface ($\text{m}^2$) and $h$ the water level (m). In paragraph 2.1.1 parameter alpha is introduced with two equations, these equations are also shown below (equation 45 and 46). Considering equation 46, the surface area can be approximated by equation 47, because the area is the derivative of the volume. Multiplying equation 44 with $dt$, integrating over the length of the dry season and substituting the area with an average area during the dry season, $\bar{A} = \frac{3}{2} \alpha \sum (h_t^2 + h_{t+\Delta t_d}^2)$ results in equation 48.

\[
\alpha = \frac{\sum V_i}{\sum h_i^3} \quad (45)
\]

\[
V(h) = \alpha \cdot h^3 \quad (46)
\]

\[
A = 3 \alpha \cdot h^2 \quad (47)
\]

\[
h_{t+\Delta t_d}^3 + \frac{3}{2} h_{t+\Delta t_d}^2 \cdot E = h_t^3 - \frac{3}{2} h_t^2 \cdot E - \left(\frac{Q_G}{\alpha}\right) \cdot \Delta t_d \quad (48)
\]

In this equation index $t$ refers to the beginning of the dry season where $t+\Delta t_d$ refers to its end. The only remaining unknown, for a given $Q_G$, is the final height $h_{t+\Delta t_d}$, which can be obtained numerically.

With all presented equations it is possible to calculate the volume of the water in the reservoir at the end of the wet season (equation 41), given the initial volume is known. This volume can be transformed to the water level with equation 46. When this water level is known the water level in the reservoir at the end of the dry season can be calculated, to do so $Q_G$ should be given.

Now that the assumptions and relations behind the VYELAS-model are known it is important to understand the operational rules which are applied within the model. This rule consists of defining the annual water withdrawal ($Q_G$), also called the effective reservoir yield. The effective reservoir yield is set equal to the annual water withdrawal, which is defined as the users’ water demand. In years when $Q_G$ would lower the reservoir water level below the minimal operational volume (determined in paragraph 2.3.5) $Q_G$ is adjusted by an iterative procedure. This iterative procedure targets to find an effective reservoir yield which will result in a volume at the end of the dry season between zero and the minimum operational volume. This iterative process is shown in Figure 21. This figure is based upon the figure shown by de Araújo et al (2006).
Figure 21 Process of determining effective yield, where $Q_{eY}$ is the effective yield; $Q_G$ the annual water withdrawal; $V_{t+1}$ is the volume at the end of the dry season; $V_{min}$ is the minimal reservoir volume; $Q_i$ the $i$th iteration of $Q$ and $dQ$ the increment decrement in withdrawal discharge.

Each time the effective yield ($Q_{eY}$) is lower than the annual withdrawal ($Q_G$) the corresponding year is marked as a year of failure. The reliability level can be calculated by equation 49.

$$G = 1 - \left( \frac{N_F}{N} \right)$$  \hspace{1cm} (49)

Where $G$ is the reliability level (%); $N_F$ is the number of years of failure and $N$ is the total number of simulated years.

With this operational rule VYELAS is able to calculate the reliability related to a certain amount of targeted water withdrawal. To do this it is shown VYELAS needs various input: The annual inflow into the reservoir, which is determined stochastically by VYELAS, so it needs the mean and the variance of this inflow; the initial volume in the reservoir; the capacity of the reservoir; the shape-parameter alpha; the evaporation in the dry season and minimal operational volume. Four other inputs are needed but they concern the accuracy of the model. They are the minimum annual withdrawal, the maximum annual withdrawal, the amount of steps which have to be calculated between these minimum and maximum and the total number of simulations.
Appendix B – Study area, the Pentecoste reservoir –

General information of the reservoir and its dams
The Pentecoste reservoir is located 85km west of Fortaleza. The exact location of the reservoir is 3°48'13.6"S 39°15'00.1"W. The reservoir is created in 1957 by building two dams. The largest dam is the main dam. This dam is made of earth and has a height of 29.4 meter measured from the bottom of the dam. At the base the dam has a width of 130 meter which converges to a width of 6 meters at the top. The main dam stretches over a distance of 1 274 meter. These dimensions give the dam a total volume of 1 100 445 m³.

The second dam, called the saddle dam, is the smaller dam which bounds the Pentecoste reservoir. This dam is only 5 meter height and stretches over a distance of 190 meter. The volume of this second dam is 9 650 m³. (de Aragão Araújo, 1982).

Curu Basin
The Pentecoste reservoir has inflow from two different rivers: The Canindé and the Capitão Moore. These rivers are as well as the reservoir located in the Basin of the river Curu.

Climate and rainfall
Figure 22 below shows the average precipitation, evaporation and temperature per month.

![Climatograph Curu basin](image)

Figure 22 Climatograph Curu-basin (Malheiros Ramos, Rodrigues dos Santos, & Guimarães Fortes, 2009)
Appendix C – The BalHidr-model –

The BalHidr-model uses the change in water depth, average daily evaporation, spillway height and the relations between volume, depth and discharge to calculate the outflow. When the model is run it results in a dataset of outflow per year for several years. This dataset can afterwards be used to calculate the mean, standard deviation and coefficient of variation of the inflow. This model has been run and used within the Hidrosed research group of the agricultural engineering department of the Universidade Federal do Ceará and resulted in the following outcomes which are used in this research.

- Mean = 182.66 hm³/year
- Standard deviation = 116.77hm³/year
- Coefficient of variation = 0.639
Appendix D – Equipment –

Echo-sounder
The echo-sounder used for the bathymetric survey on 3 December 2014 is shown below in Figure 23. Further specifications of this echo-sounder are unknown, because the labels are unreadable.

Figure 23 The echo-sounder
**GPS-tracker**

The used GPS-tracker is the Garmin eTrex Legend HCx, the specifications of this device are shown below in Table 18, Table 19 and Table 20 (Garmin, 2014).

<table>
<thead>
<tr>
<th>Physical &amp; Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical dimensions</td>
</tr>
<tr>
<td>Display size, WxH</td>
</tr>
<tr>
<td>Display resolution, WxH</td>
</tr>
<tr>
<td>Display type</td>
</tr>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Battery</td>
</tr>
<tr>
<td>Battery life</td>
</tr>
<tr>
<td>Water rating</td>
</tr>
<tr>
<td>Floats</td>
</tr>
<tr>
<td>High-sensitivity receiver</td>
</tr>
<tr>
<td>Interface</td>
</tr>
</tbody>
</table>

Table 18 Physical & Performance specifications GPS-tracker

<table>
<thead>
<tr>
<th>Maps &amp; Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basemap</td>
</tr>
<tr>
<td>Preloaded maps</td>
</tr>
<tr>
<td>Ability to add maps</td>
</tr>
<tr>
<td>Built-in memory</td>
</tr>
<tr>
<td>Accepts data cards</td>
</tr>
<tr>
<td>Waypoints/favorites/locations</td>
</tr>
<tr>
<td>Routes</td>
</tr>
<tr>
<td>Track log</td>
</tr>
</tbody>
</table>

Table 19 Maps & Memory specifications GPS-tracker

<table>
<thead>
<tr>
<th>Features &amp; Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automatic routing (turn by turn routing on roads)</td>
</tr>
<tr>
<td>Electronic compass</td>
</tr>
<tr>
<td>Touchscreen</td>
</tr>
<tr>
<td>Barometric altimeter</td>
</tr>
<tr>
<td>Camera</td>
</tr>
<tr>
<td>Geocaching-friendly</td>
</tr>
<tr>
<td>Custom maps compatible</td>
</tr>
<tr>
<td>Photo navigation (navigate to geotagged photos)</td>
</tr>
<tr>
<td>Hunt/fish calendar</td>
</tr>
<tr>
<td>Sun and moon information</td>
</tr>
<tr>
<td>Area calculation</td>
</tr>
<tr>
<td>Custom POIs (ability to add additional points of interest)</td>
</tr>
<tr>
<td>Unit-to-unit transfer (shares data wirelessly with similar units)</td>
</tr>
<tr>
<td>Picture viewer</td>
</tr>
<tr>
<td>Garmin Connect™ compatible (online community where you analyze, categorize and share data)</td>
</tr>
</tbody>
</table>

Table 20 Features & benefits specifications GPS-tracker
Appendix E – Bathymetric results –
In this appendix the deduction the depth-volume and area-volume curves is shown. This will be done by graphs which show the separate steps of the deduction. The goodness of fit will also be given of all formulas including the corrected ones. All formulas will be given in meters, square meters and cubic meters.
Figure 24 Depth-area-volume curves of the 2009 bathymetric survey
Log Depth
Log Volume

\[ Y = 3.6135 \times X + 3.9864 \quad R^2 = 0.996 \]  \hspace{1cm} (50)

Log Area
Log Volume

\[ Y = 1.4016 \times X - 2.3718 \quad R^2 = 0.9946 \]  \hspace{1cm} (51)

Depth-volume

\[ V = 9692 \times D^{3.6135} \quad R^2 = 0.9960 \]  \hspace{1cm} (52)

Depth-volume corrected

\[ V = 7113 \times D^{3.6135} \quad R^2 = 0.9235 \]  \hspace{1cm} (53)

Area-volume

\[ V = 0.0042 \times A^{1.4016} \quad R^2 = 0.9946 \]  \hspace{1cm} (54)

Area-volume corrected

\[ V = 0.006 \times A^{1.4016} \quad R^2 = 0.9294 \]  \hspace{1cm} (55)
### Measured Depth-Volume

![Graph showing depth-volume relationship](image)

### Measured Area-volume

![Graph showing area-volume relationship](image)

### Log D Log V

![Graph showing log depth-log volume relationship](image)

- \( y = 3.6353x + 3.8584 \)
- \( R^2 = 0.9896 \)

### Log A Log V

![Graph showing log area-log volume relationship](image)

### Depth-volume

- Depth volume measured
- Depth volume model
- Corrected

![Graph showing depth-volume data](image)

### Area-volume

- Area volume measured
- Area volume model
- Corrected

![Graph showing area-volume data](image)
Log Depth
Log Volume  
\[ Y = 3.6353 \times X + 3.8584 \]  \[ R^2 = 0.9896 \]  \( (56) \)

Log Area
Log Volume  
\[ Y = 1.3247 \times X - 1.8236 \]  \[ R^2 = 0.9791 \]  \( (57) \)

Depth-volume  
\[ V = 7218 \times D^{3.6353} \]  \[ R^2 = 0.9896 \]  \( (58) \)

Depth-volume corrected  
\[ V = 5959.4 \times D^{3.6353} \]  \[ R^2 = 0.9313 \]  \( (59) \)

Area-volume  
\[ V = 0.0150 \times A^{1.3247} \]  \[ R^2 = 0.9791 \]  \( (60) \)

Area-volume corrected  
\[ V = 0.0191 \times A^{1.3247} \]  \[ R^2 = 0.9615 \]  \( (61) \)
Appendix F – the meaning of the value of shape-factor alpha –

The shape-factor is determined by the ratio between the volume and the corresponding height. Within the formula for the shape factor the reservoir is schematized as shown in Figure 25. The reservoir is a square based pyramid with an angle between the sloped walls and the floor of 45 degrees. This is shown by the example calculations with the angle of 45°. These calculations show that the length, width and height are equal within these calculations when the angle is 45° and in this way the volume can be stated as height cubed. The example calculations using 30° and 60° show whether alpha increases or decreased when the reservoir slopes have these angles.

Figure 25 schematic top view of a square reservoir

<table>
<thead>
<tr>
<th>Angle</th>
<th>Tan(angle)</th>
<th>Δheight</th>
<th>ΔArea</th>
<th>ΔVolume</th>
<th>height^3</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.577</td>
<td>2</td>
<td>12.000</td>
<td>24.000</td>
<td>8</td>
<td>ΔV&gt;Δh shape-factor increases</td>
</tr>
<tr>
<td>45°</td>
<td>1.000</td>
<td>2</td>
<td>4.000</td>
<td>8.000</td>
<td>8</td>
<td>ΔV=Δh shape-factor does not change</td>
</tr>
<tr>
<td>60°</td>
<td>1.732</td>
<td>2</td>
<td>1.333</td>
<td>2.667</td>
<td>8</td>
<td>ΔV&lt;Δh shape-factor decreases</td>
</tr>
</tbody>
</table>

Table 21 Influence of slope angle

\[
\Delta \text{Height} = \text{Tan}(\text{angle}) \times \Delta \text{Length} \tag{62}
\]

\[
\Delta \text{Length} = \frac{\Delta \text{Height}}{\text{Tan}(\text{angle})} \tag{63}
\]

\[
\Delta A = \Delta \text{Length} \times \Delta \text{Width} \tag{64}
\]

\[
\text{Length} = \text{Width} \tag{65}
\]

\[
\Delta A = \Delta \text{Length}^2 \tag{66}
\]

\[
\Delta A = \left(\frac{\Delta \text{Height}}{\text{Tan}(\text{angle})}\right)^2 \tag{67}
\]

\[
\Delta V = \Delta A \times \Delta h \tag{68}
\]