Optimizing a liquid-based energy conversion system

Master's thesis
by
H.D. Bos
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Graduation committee:
Prof. dr. J.C.T. Eijkel
Prof. dr. ir. A. van den Berg
Dr. A.M. Versluis
Yanbo Xie, M.Sc.
SUMMARY

In this thesis a theoretical multidisciplinary framework to describe and optimize the efficiency of a liquid-based energy conversion system is proposed. The energy conversion system that is described was developed by Yanbo Xie, who developed and improved a system related to the work of Duffin and Saykally in 2008. The system consists of a jetting micropore, that accelerates fluid holding a net charge to high velocity and then converts the velocity to electric energy. We will describe this technique as ‘ballistic electrokinetic energy conversion’ which is derived from streaming potential in microfluidics, but for which can be shown that the mechanism is radically different.

Measurements show conversion efficiencies from pressure mechanical energy to electrical energy of over 30%. This thesis shows that this efficiency is limited mainly by viscous friction in the jetting micropore, surface energy formation in jet and droplets and air friction. Furthermore a significant loss fraction is attributed to the unequally spread velocity of the droplets, causing non-optimal harvesting of the energy.

We show (1) how the working principle of the system can be explained, (2) what loss factors are present and which ones are significant, (3) how measurements can be performed to evaluate the behaviour of the system, (4) how viscous friction in the pore behaves as function of pressure and pore radius, (5) the effects and losses attributed to surface energy formation (6) how air friction on a stream of microdroplets in a large volume of air behaves as function of flow rate, droplet size and electric field (7) how the electric field and loading resistance can be optimally tuned to harvest all energy (8) what limitations are caused by the breakdown characteristics of air, (9) how induction of extra charge can be described and used to lower the required electric field and (10) how the system efficiency is expected to behave when tuning the parameters pore-target distance, pore radius and applied pressure.

Viscous friction in the micropore, surface creation, air friction and velocity dispersion are expected to cause losses of approximately 30%, 20%, 20% and 10% of the original input power for the current system operating at 1.4bar, 15mm distance and with 5µm pore radius. The thus predicted theoretical efficiency of 20% is slightly lower than measured values, most likely caused by inaccuracies in the modelling air friction in the initial millimetres of the air trajectory and of viscous friction.

From the developed model we predict that much higher efficiencies can be obtained by increasing the radius of the pore, where a limiting factor is the charge density that can be induced. Mechanical efficiencies over 80% (excluding the effect of velocity spreading) are predicted for 15µm pore radii, if the device is not limited by the charge density.
ACKNOWLEDGEMENTS

The process of writing this thesis began at the start of academic year 2012-2013. In order to find a challenging, interesting and relevant topic for graduation I turned to the BIOS Lab-on-a-chip group, where professor Albert van den Berg was very helpful. I was joined to work on the project of Ph.D. student Yanbo Xie, with professor Jan Eijkel as supervisor.

The collaboration with Yanbo and Jan was very motivating. The project of Yanbo was already in an advanced stage and Yanbo gladly helped me to work with the setup and quickly answered to any question or remark. We designed the setup for optical measurements together and performing the experiments was a pleasure. For these experiments I also want to thank Mark-Jan van der Meulen, who spent several days to help us assist with the optical measurements at the Physics of Fluids group. For the discussions about the fluidic aspects of the thesis I want to thank also dr. Michel Versluis of the Physics of Fluids group. In several discussions he helped to get a good direction and structure in the work. His feedback on the research process was very valuable.

Throughout this thesis my supervisor Jan Eijkel has been very involved, and together with Jan and Yanbo I spent many hours discussing the newest results from measurements and theory, how the results could be explained and above all, how the system could be improved. I want to thank Jan for the large amounts of scarce time that he enthusiastically spent in supervising this project, and Yanbo for offering this master’s assignment. I was glad that I was allowed to do the work very independently and that I was assigned responsible parts of the project, such as helping with a paper about the project and publishing a theoretical paper myself. This responsibility was very motivating and the process of co-authoring a paper was very good learning experience for me.

For helping in designing and fabricating an excellent chipholder for the experiments I want to thank Hans de Boer. For the very nice working atmosphere I want to thank all the people of the BIOS-group. The atmosphere was always positive, at the coffee corner in coffee and lunch breaks, and everybody was helpful when any help was necessary for the project. I very much enjoyed the time with you in the group.
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1.1 A new energy conversion method

The world is looking for new sources of energy, which are required due to the growing world population and increasing economic wealth in especially China and the Asian world. Between 2008 and 2035 the demand for energy is expected to rise by 36%. Simultaneously the look for renewable and alternative sources of energy is important to reduce the environmental impact of generating electricity. Because of the switch to renewable sources these sources are expected to increase 300% in the same time interval [1]. New sources of energy such as bio-fuels and solar cells need to be developed and improved at a high pace to meet this demand.

One of the new methods to create electric power from water pressure is the use of streaming potential techniques. In streaming potential a separation of electric charges is achieved by flowing water that contains positive ions trough a channel, whilst blocking the transport of negative ions. This is performed in very small channels of a material that contains negatively charged groups, usually glass or silicon. The negatively charged silanol groups attract positive ions in the water, that form an Electric Double Layer (EDL), whilst repelling negative ions. In nanochannels of a radius comparable to the thickness of the EDL, the Poiseuille flow profile overlaps with the EDL, so that the charge will flow with the water, causing a positive current.

This method of electric current generation was first proposed by Morrison and Osterle in 1965 [9]. The theory about streaming currents has developed since their description, and the behaviour is now well known [4, 3]. The most used method of employment of this technique is to employ two reservoirs connected by a nanochannel, of which one is charged by forcing a flow through the channel. The efficiency of converting pressure mechanical to electric energy using this method is limited to less than 5%, which is mainly caused by the high flow resistance of the small channels and the occurrence of back conduction of ions at the surface and in the bulk liquid in the channels. Theory predicts that the flow resistance can be reduced by using boundary slip in the channels, but no experiments are known that used this method successfully.

A new technique in streaming potential was proposed by Saykally and Duffin in 2008 [3]. They eliminate the back conduction in conventional streaming current by employing a liquid water jet that breaks up in droplets, instead of a microchannel. The liquid jet is formed in a thin metal orifice in which the charge separation takes place. By employing this method the
problem of back conduction is eliminated because the droplets are electrically isolated, and the flow resistance is reduced to only the entrance flow of the orifice.

The technique that was proposed by Saykally and Duffin was further analysed and improved at the University of Twente. Saykally and Duffin present their technique as a modification of streaming potential, but Y. Xie shows that liquid water jet power conversion essentially employs a whole new mechanism of energy conversion, which is better described as ‘ballistic conversion’.

This new technique, derived from streaming potential and further referred to as ballistic elektrokinetic conversion, can be extended and optimized in more ways than Saykally and Duffin proposed. They measured a decrease in current when the system was charged to high potentials, which they attributed to leakage current. The more enhanced method do describe the phenomena that is used in this research will show that the decrease in current at high potentials is an inherent limit of this system.

Before the start of this research a system was available that could generate efficiencies of over 30% using ballistic elektrokinetic conversion, that was developed by Y. Xie. The basic device that was used is shown in Figure 1.1. An over-pressure of approximately 1.5bar is applied to the $\text{N}_2$-gas, which pressurizes a water reservoir. The water is jetted through a $10\mu\text{m}$ diameter micropore in a silicon nitride membrane. The water droplets are captured at a metal target at approximately 15mm distance from the pore, creating an electric potential of approximately 15kV. The electric field from the target is shielded by a guard ring at 1.5mm from the pore. Mechanical input power is measured from the $\text{N}_2$ gas pressure and flow rate, and electrical output power is measured from the load resistance and current $I_2$ through the load. Optionally the charge density in the droplets that is generated by the streaming current is amplified by a negative induction potential $U_{\text{ind}}$ at the guard ring.
1.2 Operating principle

The principle of ballistic elektrokinetic conversion as established by the work of Xie is the conversion of external pressure mechanical energy into kinetic energy of charged droplets using a liquid microjet. The kinetic energy is then converted to electrical energy by bringing the charge in the droplets to a higher potential. In this second step the electric field between the target and the guard ring (or micropore, if no guard ring is present) causes a force on the charge in the droplet, which reduces the velocity of the droplet. In this way the electric energy is increased, because charge at a higher potential contains more energy, at the cost of kinetic energy of the droplet. This process is illustrated in Figure 1.2.

The conversion step to kinetic energy is not present in conventional streaming potential systems, where the force of pressure is directly opposed by the electric force. It is therefore that this system provides a radically different operating principle than streaming potential, and requires a completely new analysis of its operation and efficiency. Sources of loss present in streaming potential can be greatly reduced, but new sources of loss, such as the friction of droplets in air, occur. All knowledge domains in this system - streaming potential, a free jet emerging from a micropore, fluidic friction of droplets in air, chemical conversions and electrical fields - can be described by usually well known physics in their respective fields of research. However, this combination and application in this precise environment requires a new study to investigate the full potential of ballistic elektrokinetic conversion.

1.3 Research goals

The purpose of this research is to investigate how the conversion efficiency of a system that employs ballistic elektrokinetic conversion using liquid water jets can be optimized. To achieve
this goal all aspects of the system - electric, fluidic and chemical - are considered, with emphasis on the electric and fluidic (water and air) aspects.

First an analysis will be made of the system based on a selection of available measurements of the system's conversion efficiency. The operating principles and all possible sources of loss will be identified and the most relevant loss factors will be selected for further analysis based on an estimation of the impact of various loss sources. Finally the developed model will be used to give recommendations about optimizing the system, and a prediction of the possibility for efficiency increase.
2.1 Calculations ideal system

In this chapter the behaviour of the theoretical ideal system with no loss factors will be calculated. The conversion from pressure in the reservoir to kinetic energy will be described using an energy balance, equating the power that the pump delivers per volume with the kinetic energy per volume. The surface energy per volume will be subtracted, as this is a fundamental loss factor in this system. Using the kinetic energy per volume we can then calculate the flow speed of the droplets.

Next we can show how the current is related to this velocity and the charge density. The charge density will be left as an unknown parameter, that can be known from measurements. The maximum electrical potential energy that the droplets could obtain will be calculated by again an energy balance. The kinetic energy per volume can then be equated with the potential energy per volume. The maximum voltage multiplied with the current yields the maximum electrical power under no-friction condition.

2.1.1 From pressure to kinetic energy

Gross available energy

In the system water is accelerated using pressure from a pump. At a given pressure the amount of kinetic energy that can be transferred to the water is fixed as well as the resulting jetting velocity. The volume change in the reservoir determines the amount of power delivered by the pump, and when this power is delivered to the volume of water that is expelled, the kinetic energy per volume and the velocity of the water can be calculated:

\[
P_{\text{pump}} = \frac{\delta V}{\delta t} \cdot p \quad [\text{J s}^{-1}] \quad (2.1)
\]

\[
\frac{E_{\text{water, max}}}{V} = p \quad [\text{Pa}] \quad (2.2)
\]

Where \( P \) is power [J s\(^{-1}\)], \( E_{\text{water, max}} \) the maximum kinetic energy that the water can obtain [J], \( V \) is the water volume [m\(^3\)] and \( p \) the pressure difference [Pa]. This energy per unit volume
is converted to the kinetic energy of the water,
\[
\frac{E_{\text{water,kin}}}{V} = \frac{1}{2} \cdot \rho_{\text{water}} \cdot v^2 \quad (\text{For } v << c) \quad [\text{J m}^{-2}] \quad (2.3)
\]
Where \( \rho_{\text{water}} \) is the density of water, for which we will use 1000 [kg m\(^{-3}\)] in the sequel, and \( v \) is the velocity of the expelled water. Equating equations 2.2 and 2.3 results in the velocity of the expelled water
\[
v_{\text{water,\max}} = \sqrt{\frac{2p}{\rho_{\text{water}}}} \quad [\text{m s}^{-1}] \quad (2.4)
\]
This is equal to the relation in the equation of Bernoulli that describes the relation between pressure and kinetic energy of water flow along a streamline, without viscous losses [10, p.99].

Net available energy

Not all energy can go into kinetic energy. The creation of extra surface of the water-droplets will require a part of the energy. We assume that the first part of the jet has a cylindrical shape, and that this part determines the amount of pressure - and thus energy - that goes into surface creation.

We assume the surface of the water in the reservoir to be negligible. The amount of surface that needs to be created per volume depends on the diameter of the water jet after the pore.

The surface-to-volume ratio of the wall of a cylinder is
\[
\frac{A}{V} = \frac{2 \cdot \pi \cdot r_{\text{jet}}}{\pi \cdot r_{\text{jet}}^2} = \frac{2}{r_{\text{jet}}} \quad [\text{m}^{-1}] \quad (2.5)
\]
Where \( A \) is the area of the cylinder [m\(^2\)], \( V \) is the volume of the cylinder [m\(^3\)] and \( r_{\text{jet}} \) is the radius of the cylinder [m], which we assume to be equal to the radius of the pore.

This means that the energy required for the surface per volume of expelled water is
\[
\frac{E_{\text{surface}}}{V} = \gamma \cdot \Delta A_{\text{water-air}} = \gamma \cdot \frac{2}{r_{\text{jet}}} \quad [\text{J m}^{-2}] \quad (2.6)
\]
Here \( \gamma \) is the surface tension of the water [J m\(^{-2}\)] and \( \Delta A_{\text{water-air}} \) is the increase of water-air surface [m\(^2\)].

Taking 0.072 J m\(^{-2}\) for the surface tension of water, 1000 kg m\(^{-3}\) as density of the water and making the approximation that \( r_{\text{jet}} = r_{\text{pore}} \), we can write for the net energy converted to kinetic energy per volume and the resulting velocity:
\[
\frac{E_{\text{water}}}{V} = p - \frac{2 \cdot 0.072}{r_{\text{jet}}} \quad [\text{J m}^{-2}] \quad (2.7)
\]
\[
v_{\text{water}} = \sqrt{(p - \frac{2 \cdot 0.072}{r_{\text{jet}}}) \cdot 0.002} \quad [\text{m s}^{-1}] \quad (2.8)
\]

The energy lost in the surface tension is
\[
\tau_{\text{surface}} = \frac{2 \cdot 0.072}{p} \frac{r_{\text{jet}}}{r_{\text{jet}}} \quad (2.9)
\]
Where \( \tau \) is the loss fraction. For a pressure of 150kPa this yields a loss of 19%
2.1.2 Charge density and resulting current

The current that is carried by the jetting water is determined by the volume flow and the charge density. For a given pore size it is also determined by the charge density and velocity, and thus by the pressure.

\[ I = \rho_{el} \cdot \frac{\delta V}{\delta t} = \rho_{el} \cdot v_{water} \cdot \pi \cdot r_{pore}^2 = \rho_{el} \cdot \sqrt{\left( p - \frac{2 \cdot 0.072}{r_{jet}} \right) \cdot 0.002 \cdot \pi \cdot r_{pore}^2} \quad [\text{A}] \quad (2.10) \]

where \( \rho_{el} \) is the charge density in the droplets [C m\(^{-3}\)].

2.1.3 From kinetic to potential energy

The net available energy per volume of equation 2.7 can be equated with the potential energy of charge in the same volume at plate voltage, to find the maximum value of the plate voltage if there are no losses.

\[ \frac{E_{pot}}{V} = \frac{E_{water}}{V} \rho_{el} \cdot U_{max} = p - \frac{2 \cdot 0.072}{r_{jet}} \]

\[ U_{max} = \frac{p - \frac{2 \cdot 0.072}{r_{jet}}}{\rho_{el}} \quad [\text{V}] \quad (2.11) \]

Combining equations 2.10 and 2.11 we calculate the maximum achievable power when this voltage is applied to the plate.

\[ P_{electric,max} = U_{max} \cdot I = \frac{p - \frac{2 \cdot 0.072}{r_{jet}}}{\rho_{el}} \cdot \rho_{el} \cdot \sqrt{\left( p - \frac{2 \cdot 0.072}{r_{jet}} \right) \cdot 0.002 \cdot \pi \cdot r_{pore}^2} = \left( p - \frac{2 \cdot 0.072}{r_{jet}} \right)^{1.5} \cdot \sqrt{0.002 \cdot \pi \cdot r_{pore}^2} \quad [\text{J s}^{-1}] \quad (2.12) \]

We can see that the term for the charge density drops out in the last equation. This means that the maximum achievable power is in principle independent of the charge density in the water. However, the voltage that needs to be applied to reach maximum power would go to infinity when the charge density goes to zero. Low charge densities occur for example in systems where the pore radius is large, so that the flow rate increases and the streaming current decreases.

2.1.4 Size estimation of variables

From the equations in the previous section we can make an estimation for the range that the variables in our system will take. This will be relevant to be able to make correct assumptions.
To make these rough estimations we assume that the system parameters are $P = 150,000\text{Pa}$ and $r_{pore} = 5\mu\text{m}$ such as in the system of Xie. Measurements by Xie yield an approximate value for the current: $I_{U=0} = 2.6\text{nA}$. From a direct measurement where both current and flow rate were measured we can estimate the charge density. This measurement showed a current of 2.6nA at a flow rate of $0.78\mu\text{l s}^{-1}$, so that $\rho_{el} = 3.4\text{C m}^{-3}$. From these parameters and the mentioned equations we can now estimate the no-loss velocity, flow rate, voltage and electrical power

- $v = 15.6\text{m s}^{-1}$ (from equation 2.8)
- $\frac{4V}{M} = 1.2\mu\text{l s}^{-1}$ (velocity multiplied by $A$)
- $U = 36\text{kV}$ (from equation 2.11)
- $P_{electric} = 150\mu\text{W}$ (from equation 2.12)

All these values are the maximum theoretical values if all losses, except the fundamental loss to surface energy, are ignored. They are not a measure for real system behaviour, but they can be used when a scaling estimation is needed for the variables.

### 2.2 Loss factors estimation

#### 2.2.1 Definitions

The overall conversion efficiency of the system is defined as

$$Eff = \frac{R_{load} \cdot I^2}{P \cdot Q}$$  \hspace{1cm} (2.13)

Where $R_{load}$, $P$, $Q$ are load resistance, pressure and flow rate, respectively.

To be able to indicate where losses occur we define loss factors for the conversion from pressure to kinetic energy and a loss factor of the loss of kinetic energy:

$$L_{pore} = 1 - \frac{\rho_{water}v_0^2}{2 \cdot p}$$  \hspace{1cm} (2.14)

$$L_{air} = 1 - \left(\frac{0.5 \cdot v_{final}^2 + U \cdot \rho_{el}}{0.5 \cdot \rho_{water} \cdot v_0^2}\right)$$  \hspace{1cm} (2.15)

where $L_{pore}$ is the conversion efficiency from pressure to kinetic energy, $v_0$ the initial velocity of the droplets after breakup, excluding air friction losses, $L_{air}$ the loss in the droplet air trajectory, which is the loss in kinetic energy minus the fraction that is converted to electrical energy and $v_{final}$ the remaining velocity (which is not considered an air friction loss).

The conversion efficiencies are thus strictly separated in this definition. Case should be taken that the measured velocity of the droplets after breakup of the jet might incorporate not only a loss factor from the pore, but also a part of the air friction, although the name does not suggest this.
2.2.2 Size estimation of losses

Apart from the fundamental loss of energy to surface energy, we need to consider several loss sources for the optimization of the system. A complete list of the possible loss sources is

- Viscous and turbulent losses in the entrance flow and droplet formation
- Losses to surface energy in the jet and formation of droplets
- Losses of kinetic energy in the droplets due to air friction
- Losses of kinetic energy in inelastic collisions between droplets
- Losses of kinetic energy due to remaining velocity when hitting the target
- Losses of kinetic energy due to droplets not hitting the target
- Losses of kinetic energy due to evaporation of water from droplets
- Losses of electrical energy in the charge conversion at electrodes
- Losses of electrical energy because of back conduction paths

The losses of the measuring system, such as the pump and the electrical circuit, are not considered as losses of the system. In the next section an estimation of the size of the losses will be made, and the significant losses will be considered in this work.

We also want to make an estimation of the size of these loss sources. From this estimation we can choose which sources of loss need further study.

Table 2.1: Scaling estimations of loss sources

<table>
<thead>
<tr>
<th>Loss source</th>
<th>Size estimation</th>
<th>Estimation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrance flow</td>
<td>4-50%</td>
<td>Literature: head loss for contraction as a function of rounding of the inlet edge, [10, p. 418]</td>
</tr>
<tr>
<td>Surface energy</td>
<td>14%</td>
<td>Calculations section 2.1.1</td>
</tr>
<tr>
<td>Air friction</td>
<td>0-100%</td>
<td>Calculation below</td>
</tr>
<tr>
<td>Droplet collisions</td>
<td>&lt;1%</td>
<td>Calculation below</td>
</tr>
<tr>
<td>Surplus velocity and target misses</td>
<td>0-20%</td>
<td>Estimation below</td>
</tr>
<tr>
<td>Evaporation</td>
<td>&lt;5%</td>
<td>Estimation below</td>
</tr>
<tr>
<td>Electrochemical conversion</td>
<td>&lt;1%</td>
<td>Calculation below</td>
</tr>
<tr>
<td>Back conduction</td>
<td>&lt;1%</td>
<td>Estimation below</td>
</tr>
</tbody>
</table>

The losses due to air friction will be an important part of this thesis. A crude estimation is not easily possible, due to the effects of the air velocity that is dragged along with the water droplets.

Air friction estimation We model the air friction of the droplets as the air friction of round spheres in still-standing air. Especially the latter is not accurate because of the slipstream from other droplets. The general expression for force of air friction is

\[ F_d = \frac{1}{2} \cdot \rho_{\text{air}} \cdot v^2 \cdot C_d \cdot A \quad \text{[N]} \]  

(2.16)

where \( F_d \) is the force of drag, \( \rho_{\text{air}} \) is the air density [kg m\(^{-3}\)], \( v \) is the velocity [m s\(^{-1}\)], \( C_d \) is the drag coefficient and \( A \) the reference area [m\(^2\)]. For a sphere the reference area is the
cross-sectional area, that is \( \pi \cdot r^2 \).

The drag coefficient \( C_d \) of a sphere depends on the Reynolds number of the flow. This dependence is shown in Figure 2.1.

At 5m s\(^{-1} \) \( C_d \) is 5.1 and at 10m s\(^{-1} \) \( C_d \) is 3.1. We will take the average velocity at 5m s\(^{-1} \) for the value of \( C_d \) and use the approximation of the frictional force:

\[
F_d = \frac{1}{2} \cdot \rho_{\text{air}} \cdot v \cdot 5 \cdot 5.1 \cdot A
\]

where we have left out the squaring of the velocity to account for the negative slope in \( C_d \) and multiplied by the fixed value of 5m s\(^{-1} \) to simplify the calculation. In the final model the air friction will be calculated more accurately.

To get an estimation of the maximum range of the droplet with this amount of friction, we can solve the simple differential equation 2.18. We assume that \( r = 10 \mu m \) and that \( v(0) = 10 \text{m s}^{-1} \) and solve for the distance that the velocity becomes zero

\[
\begin{align*}
- \frac{\delta v}{\delta x} &= - \frac{\delta v}{\delta t} \cdot \frac{\delta t}{\delta x} = - \frac{\delta v}{\delta t} \cdot \frac{1}{v} \\
&= \frac{F_d}{m \cdot v} \\
&= \frac{1}{2} \cdot \rho_{\text{air}} \cdot 5 \cdot 5.1 \cdot \pi \cdot r^2 \\
&= \frac{4}{3} \cdot \pi \cdot r^3 \cdot \rho_{\text{water}} \\
&\approx 10^3 \\
\end{align*}
\]

\[
v(x) = 10 - 10^3 \cdot x \\
x_{v=0} = 10^{-2} \quad \text{[m]} (2.19)
\]

From this solution we can see that the travelled distance is only 10mm using these assumptions. In this case the loss to air friction would obviously be 100% for pore-target distances over
10mm. This means that it is necessary to take the movement of the air into account, which will be done in later sections. Therefore the estimation of the air friction is undetermined as 0-100%. Because of the low Reynolds number of the system in the liquid flow no turbulent losses will be considered.

**Collisions loss estimation** When two droplets have a different speed, they could collide in-flight. In such a collision the droplets would merge due to the surface tension. A collision where objects that have a different speed collide and stick together is called a perfectly inelastic collision. In such a collision the impulse before and after the collision should be equated. From this we can find the loss of kinetic energy. Two equal mass droplets with a velocity difference of \( \Delta v \) will continue their path with the average velocity:

\[
m \cdot (\bar{v} - 0.5 \cdot \Delta v) + m \cdot (\bar{v} - 0.5 \cdot \Delta v) = (2 \cdot m) \cdot \bar{v}
\]

Where \( m \) is the mass of a droplet, \( \bar{v} \) the average velocity and \( \Delta v \) the difference in velocity.

The kinetic energy before the collision was

\[
E_{\text{kin, before}} = \frac{1}{2} \cdot m \cdot (\bar{v} - 0.5 \cdot \Delta v)^2 + \frac{1}{2} \cdot m \cdot (\bar{v} + 0.5 \cdot \Delta v)^2
\]

\[
= \frac{1}{2} \cdot m \cdot 2 \cdot (\bar{v}^2 + 0.25 \cdot \Delta v^2)
\]

\[
E_{\text{kin, after}} = \frac{E_{\text{kin, after}} - E_{\text{kin, before}}}{E_{\text{kin, before}}}
\]

\[
L = \frac{1}{2} \cdot m \cdot (\bar{v})^2
\]

\[
\approx 0.25 \cdot \left( \frac{\Delta v}{\bar{v}} \right)^2
\]

Where \( E_{\text{kin, before}} \) is the total kinetic energy before the collision, \( E_{\text{kin, after}} \) the total kinetic energy after the collision and \( L \) the fraction of loss in kinetic energy. The approximation in the final step is valid for \( \Delta v \ll \bar{v} \).

We can assume that the speed variation of the droplets after breakup is less than 20% of the average speed. This yields that the loss of kinetic energy is less than 1%.

**Surplus velocity and target misses estimation** The losses that are caused by having a surplus velocity at the target can be avoided by increasing the electric field. However, this increases the chance of droplets with a lower velocity missing the target. Therefore either loss factor can be eliminated by system tuning, but not both factors simultaneously. We estimate that a maximum of 20% is lost if the system is configured optimally. This assumption is evaluated in section 4.3.1.

**Evaporation loss estimation** In literature a high temperature jet evaporation system with a droplet radius of (initially) 20\( \mu \)m turbulent flows is described that [13]. The study measures evaporation of less than 10% of the liquid mass after 25mm. In our system we have no turbulent flow and no high temperature. The measurements of appendix A show that the current \( I_2 \) can be equal to \( I_1 \), indicating no evaporation of charged molecules. We will assume that for our system the evaporation of uncharged molecules is also negligible, and the loss is estimated to be less than 5%.
Electrochemical loss estimation  The charges in droplets are stored in ions, generally $H_3O^+$ ions. These ions need to be produced at the electrode in the reservoir using the oxidation reaction:

$$6H_2O \rightarrow O_2 + 4H_3O^+ + 4e^- \quad (2.22)$$

The produced $O_2$ is an oxidant in water. However, there is nothing to oxidize because no electrons are available. At the target the $H_3O^+$ needs to be reduced to generate a positive current. This is done in the reduction reaction:

$$2H_3O^+ + 2e^- \rightarrow 2H_2O + H_2 \quad (2.23)$$

The produced $H_2$ can escape as a gas. The combined reduction and oxidation reactions form an electrolysis reaction. The potential required to achieve electrolysis of water is 1.48V. This voltage is subtracted from the voltage that the ballistic energy conversion system generates, because energy is required to perform the electrolysis. The working voltages of the ballistic energy conversion system are in the order of many kilovolts, as was calculated from equation 2.11. Therefore the loss of power due to the electrochemical processes is less than 1%. The appearance of dissolved protons in the reservoir might influence the pH of the solution. However, this does not contribute to a loss factor.

Back conduction estimation  In conventional streaming potential systems the back conduction of current is a serious problem. However, in ballistic electrokinetic conversion the electrodes are isolated by air. Therefore during normal operation when no electric breakdown of air or corona discharges occur, the back conduction is zero. The conditions required to achieve this situation will be evaluated in section 4.3.2.
3.1 Measuring overall conversion efficiency

3.1.1 Measurement setup

During operation of the system there are 4 variables crucially important to calculate the overall system efficiency

- Liquid pressure
- Liquid flow rate
- Generated current in target
- Generated voltage or loading resistance

The values of these variables need to be measured to calculate the system efficiency as defined in equation 2.13.

Additional variables are not required to calculate the overall efficiency, but can be measured to gain knowledge about the internal processes of the conversion, and the loss factors.

- Current drawn from top reservoir
- Generated current in target
- Current flowing to other parts of the system

The setup used to measure the electricity conversion efficiency is drawn schematically in Figure 3.1. The system consists of micropore that jets pressurized water towards a metal target, that is connected to TeraOhm resistors. The membrane is a 0.8µm thick SiN membrane with a 10µm diameter circular micropore. The membrane is incorporated in a Si chip, that is mounted in a plastic chipholder. Micro-filtered demineralized water is supplied to the chipholder from a N2-gas pressurized reservoir. The water is jetted through a hole in the guard ring to a metal target at a variable distance. The target is a metal cup with no sharp edges to prevent electric losses when operating at high voltages. The metal target is connected using a high-voltage cable to TeraOhm resistors that are immersed in a bath of dielectric oil. The bath of oil prevents discharges or leakages of high voltages through the air.
The water-filled part of the system is electrically isolated from ground, and a platinum wire contacting the water is mounted in the chipholder. The current into this wire can be measured and is named $I_1$, as well as the current trough the TeraOhm resistors named $I_2$. The guard ring between the pore opening and the target is a metal plate to prevent any electric field to reach the pore. The guard ring has a 1.5mm diameter hole trough which the water is jetted and is mounted at a distance of 1.5µm from the membrane and micropore. A (leakage) current flowing into the guard ring can also be measured and is named $I_3$. In measurements where induction was used, a negative voltage source was inserted between guard ring and ground, in other measurements the guard ring was grounded.

The pressure supplied to the system using N2-gas is measured using a pressure meter. The flow rate of water through the micropore is measured by measuring the propagation of an air bubble in the 5mm diameter tubing over a time lapse, and calculating the flow rate from the propagation of the liquid-gas interface. In a later improved setup a flow meter was included to measure the flow rate more accurately. The flow meter causes a small pressure drop. This pressure drop was corrected for afterwards by measuring the pressure drop in the whole flow-line as function of the flow rate, and subtracting this from the measured pressure.

3.1.2 Measurement results

The measurements on the overall conversion efficiency were partially obtained by measurements by Y. Xie before the start of this thesis. These measurements are used here because they are important to illustrate and understand the behaviour of this system. The electric conversion efficiencies illustrated in Figure 3.3(a) and Figure 3.3(b) were obtained by Y. Xie and the author. The full measurement details for the measurement series that are used here can be found in Appendix A.
In a typical measurement the target starts in a discharged state and is connected to a certain resistance. When the jetting reservoir is connected electrically, the current $I_1$ increases immediately, implying that charge is carried in the droplets. The current trough the load resistance $I_2$ then starts to increase with an exponentially decaying slope, slowly reaching the value of $I_2$. An example of this current increase is shown in Figure 3.2(a). This is the result of the charging effect of the target. The target behaves like a capacitor that stores electrical energy in the electric field around it. Although the capacitance of an object at large distance from the ground (in this setup approximately 15mm to the guard ring) is very small, the resistance is very large, so that the RC-time of the system is in the order of minutes.

In some measurements the current $I_2$ does not reach the value of $I_1$, especially when high load resistance are used. In this case the currents show a peak and $I_2$ is suddenly decreased, as is shown in Figure 3.2(b). These drops are caused by electric breakdowns in the setup. These breakdowns could be heard as an audible ‘tick’. In the measurements to find the electrical conversion efficiency the load resistance was typically increases from zero to a value were the current $I_2$ is much smaller than $I_1$. In measurements where the distance between guard ring and target was small and the voltage on the target (calculated by $I_2 \cdot R_{load}$) was large, these breakdowns are one of the efficiency-limiting factors.

Five measurement series were selected from the measurements database that give a representation of the overall system behaviour. The measured $I_2$ currents are shown as function of load resistance in Figure 3.3(a). The efficiencies corresponding to these data are shown in Figure 3.3(b). This overall conversion efficiency is calculated using equation 2.13.

Figure 3.3(b) shows a maximum obtained conversion efficiency of 36%. Chapter 2 showed that there are no fundamental limits limiting the efficiency to this value. Several loss factors have the potential to cause large losses. To distinguish what loss factors are involved in the maximum obtained efficiency of 36% a new measurement was build to measure the fluidic behaviour.
(a): Measured $I_2$ currents with varying load resistance for several measurement series. The indicated current per series is the (average) $I_1$. Full measurement data are in Appendix A.

(b): Efficiencies with varying load resistance for several measurement series. The indicated current per series is the (average) $I_1$. Full measurement data are in Appendix A.

Figure 3.3: Measurement results for electrical conversion efficiency
3.2 Measuring fluidic behaviour

3.2.1 Measurement setup

Three additional variables that are not required to calculate the overall efficiency can be measured to gain knowledge about the internal processes of the conversion, and the loss factors.

- Droplet velocity
- Droplet size
- Generated current in target

A new measurement setup was developed that is capable of measuring the droplet velocity and droplet size using a microscope and dual-illumination laser light source, measuring pressure and flow rate into the reservoir and measuring the current into the reservoir. In the measurement setup the pressure and the distance form the pore could be varied between measurement series, as well as the electric field applied between reservoir and guard ring and between guard ring and target. The setup is shown in Figure 3.4.

The CCD camera in the setup captures images of the droplets through a 10x objective with long focal distance. During the $60\mu$s capturing time of the camera, the triggered laser sources fires two times, with a fixed time delay of $2\mu$s (in some measurements $1\mu$s). Using this way a high resolution camera and a small inter-frame time can be combined in a relatively simple setup [15]. The laser pulses have a wavelength of 532nm and have a pulse width (Full Width at Half Maximum) of $7\text{ns}$.

The double-illuminated images are analysed in Matlab. A script was developed that detects the droplet edges and determines how much the droplet image is displaced between the laser pulses. The detection results needed to be accepted manually by the user, so that false detections could be rejected. Detections of droplets where the shape of the droplet was so much disformed that the centre was not clearly distinguishable were also rejected. Droplets with an oval shape were accepted, because the centre of gravity could still be determined. In edge detection there is an uncertainty in the determination of the precise location of an edge. This does not influence the velocity measurements, because the middle of a droplet
edge is unaffected. The radius measurements are influenced by this uncertainty. The uncertainty of the droplet edge is approximately 1 pixel, which is 6% of the radius of an 8µm droplet.

The size of the CCD pixels was 4.65µm and the objective amplifies 10x (in some measurements 5x), so that the droplet velocity could be determined from:

\[ v_{\text{drop}} = \frac{l_{\text{pixel}}}{A \cdot t_{\text{delay}}} \]  

(3.1)

Where \( l_{\text{pixel}} \) is the length of a pixel, \( A \) is the amplification of the objective and \( t_{\text{delay}} \) the delay between laser pulses. All droplets in the 1392 pixel window are measured, but in some measurement series many droplets are rejected because of the criteria that were mentioned before. For every measurement series the results were averaged over 10-50 images. The statistical average and standard deviation were calculated over the droplet velocity and droplet radius.

3.2.2 Measurement results

First the behaviour of the breakup and the length of the typical jet at 143kPa pressure were determined using the 5x objective. Figure 3.5(a) shows an image in which the whole jet can be seen. The vertical shadow at the left side of the image is the SiN-membrane, having a different color than the surrounding (thicker) silicon. The jet emerges from the pore in the middle of the membrane, at the left side of the membrane we can see a reflection image of the jet in the silicon. The length of the jet averages at approximately 520µm for a series of measurements, with a standard deviation of 20µm.

The Figures 3.5(b) and 3.5(c) show typical images of the droplet stream. Figure 3.5(b) was taken close to the breakup region, where some small droplets are seen that have not merged since breakup, and some larger droplets that must have merged to obtain the larger size. Figure 3.5(c) is taken far from the breakup, where most droplets are much larger than the original size.

A typical histogram of the droplet radius is shown in Figure 3.6, which shows a Gaussian-like distribution. The droplet volumes are expected to be a multiple of the volume of an 8µm droplet, because of merging, but this cannot be distinguished in the histogram at distances further from the pore. The correlation between the droplet velocity and droplet radius was calculated, showing a small but minor positive correlation.

Figure 3.7 shows the flow rate as function of applied pressure that was measured in the flow meter during the measurements. Figures 3.8(a), 3.8(b), 3.8(c) and 3.8(d) show the result of the three main measurement series, in which the droplet size and velocities were determined as a function of applied pressure, distance from the pore and applied induction voltage. These measurements are used in the remaining chapters to develop a model about the droplet kinetics.

The Figures 3.8(a), 3.8(b) and 3.8(c) show that the droplet velocity obtained just after breakup of the droplets is approximately 11m s\(^{-1}\). From equation 2.14 we can calculate that for a pressure of 143kPa this is a loss of 58% or the originally available pressure energy. Figure 3.8(b) shows a decrease in velocity from 11m s\(^{-1}\) to 7.3m s\(^{-1}\) when no electric field is applied, which is of a loss of 56% of the kinetic energy. These loss factors should be reduced, and the measurements of Figures 3.8(a), 3.8(b), 3.8(c) and 3.8(d) provide information about how the losses are influenced by pore-target distance, electric field and pressure. In the next chapter we
(a): Image of the jet and breakup. Taken using 5x objective and single pulse illumination.

(b): Typical droplets image taken near the breakup, at $x = 2.3\text{mm}$. Taken using 10x objective and $t_{\text{delay}} = 2\mu\text{s}$.

(c): Typical droplets image taken far from the breakup, at $x = 13\text{mm}$. Taken using 10x objective and $t_{\text{delay}} = 2\mu\text{s}$.

Figure 3.5: Microscopic images of jet and droplets

Figure 3.6: Histogram of the droplet radius for a measurement taken at $x = 5.9\text{mm}$ and at 7kV.
will develop a model to estimate the losses can be influenced by all relevant system variables, in order to find an optimized system.

Figure 3.7: Flow rate through the pore as function of the applied pressure
(a): Droplet velocity and radius close to the breakup location measured as a function of applied water pressure. No electric fields were applied. Every data point consists of detections from 10 images.

(b): Droplet velocity and radius measured as a function of distance from the pore opening. No electric fields were applied. The pressure was kept constant at 1.41 bar. Every data point consists of detections from 10 images.

(c): Droplet velocity as function of distance from the pore opening for several applied fields. The fields were applied between guard ring at $x = 1.5$ mm and the target at $x = 13$ mm. The applied pressure is 1.46 bar.

(d): Droplet velocity and radius close to the breakup location measured as a function of applied gating voltage. The pressure was kept constant at 1.43 bar.

**Figure 3.8:** Velocity measurement data
This chapter will seek to develop models that accurately describe the behaviour of those elements that critically influence the efficiency of the device. The main loss sources from section 2.2 and relevant effects can be separated by research domain:

- Fluid mechanics domain
  - Viscous losses in orifice and jet
  - Air friction
  - Surface energy
- Electric fields domain
  - Behaviour of the system in different field strengths
  - Deformation of the field
  - Electric breakdown
  - Induction of extra charge in the jet
- (Electro)chemical domain
  - Electric double layer
  - Charge conversion at electrodes
  - Evaporation from droplets

A multidisciplinary approach is required: jetting behaviour is influenced by the electric field and the electric field is influenced by the presence of charged and conducting water. In this thesis the focus is on the electric and fluidic parts, indicated in Figure 4.1, because section 2.2 showed that the chemistry domain is not expected to cause significant losses. First the fluidic behaviour is modelled using assumptions about the electric field behaviour. Secondly the electric field behaviour is modelled. The results form the jet modelling will be used, such as droplet size, shape and distance.

The final section of this chapter will combine the models of all domains and evaluate the behaviour and tuning of the complete system.

4.1 Fluid mechanics - Loss in jet formation

Water is jetted from the reservoir at some pressure through an orifice to a free jet in air. In this process the pressure is converted to kinetic energy of the water, some surface energy, and
kinetic energy. Losses in the jet formation are defined as the ratio of input pressure mechanical energy to the kinetic energy that the droplets have after breakup of the liquid jet. Losses due to air friction will be considered separately. In the liquid two main factors of loss are distinguished. Viscous losses due to the shear rate inside the water, and surface energy losses due to the generation of water-air surface.

The velocity that can be obtained in water when no losses are occurring was described in equation 2.4. However, losses do occur due to surface tension and due to viscous losses, which will be treated by replacing the pressure $p$ by the effective pressure $p_{eff}$

4.1.1 Surface tension losses

Losses due to surface tension were briefly treated in section 2.1.1. In this model we will separate the effect of surface tension in two parts of the jet: in the jet formation region and in the section where the jet breaks up into droplets. In the jet formation section the axial forces at the liquid-air interface are balanced: there is a pulling force in the direction of the membrane and a pulling force in the direction of the jet, as illustrated in Figure 4.2. However, the forces in the azimuthal direction cause Laplace pressure inside the jet. This Laplace pressure for a cylinder is:

$$p_{laplace} = \frac{\gamma}{r_{jet}}$$

(4.1)

Where $\gamma$ is the surface tension. This pressure will decrease the flow rate.

In the breakup section of the jet the Rayleigh-Plateau instability causes the jet to break up into droplets. The axial surface forces are no longer balanced, because the ‘jet side’ of the droplet is pulled by the surface forces, but on the ‘droplet side’ there is no balancing force. However, in this case the Laplace pressure inside the jet causes a positive force on the droplet. This effect is illustrated in 4.2 and was described and measured by Schneider [11, 12]. The velocity was described in a momentum balance:
Figure 4.2: Illustration of surface forces and Laplace pressure effects

Figure 4.3: Velocity of the jet and efficiency of pressure to kinetic energy conversion as function of pressure. The measurements and theory are shown for r=5\(\mu\)m. In the theoretical line viscous friction and surface energy losses are included, air friction losses in the jet are not included.

\[
\frac{dy_{\text{jet}}}{dt} = \frac{dy_{\text{drop}}}{dt} + \pi r_{\text{jet}}^2 \cdot \frac{\gamma}{r_{\text{jet}}} - 2\pi r_{\text{jet}} \cdot \gamma
\]  

(4.2)

Where \(y_{\text{jet}}\) and \(y_{\text{drop}}\) are the momentum of the liquid in the jet and in the droplets, respectively. Working out the equation yields [11]:

\[
v_{\text{drop}} = v_{\text{jet}} \cdot \left(1 - \frac{\gamma}{\rho_{\text{water}} r_{\text{jet}} v_{\text{jet}}^2}\right)
\]

(4.3)

By substituting Equation 4.11 from the next section, which describes the jet velocity by accounting for viscous losses, in Equation 4.3 we can calculate the droplet velocity as function of the pressure. Dividing the kinetic energy by the input energy \(p \cdot Q\) yields the efficiency of pressure to kinetic energy conversion. The resulting theory combined with measured data is shown in Figure 4.3.

The measured data shows a lower efficiency and velocity than the theoretical lines. This is contributed to the fact that no air friction losses are involved in these calculations, but the droplet velocity was measured after the breakup point of the jet, which is approximately 500\(\mu\)m from the pore. Some air friction might slow down the jet before the point of measurement.
4.1.2 Viscous losses in the orifice

Equation 2.4 refers to systems where viscous losses are not considered. To incorporate the effect of viscous losses we need to know what pressure can be effectively used. Although complex additions to the Bernouilli equation to include viscous losses are known, we can only apply those in a simple way when the flow pattern is known. We will first try to determine the viscous loss from literature on empirical studies.

Literature

In [10, p. 419] the loss coefficient is defined as the fraction of pressure loss to the gain in dynamic pressure at some fluidic component:

$$K_L = \frac{\Delta p}{\frac{1}{2}\rho_{\text{water}} \cdot v^2}$$

(4.4)

Where $K_L$ is the loss coefficient, $\Delta p$ the (additional) pressure drop over the element, $\rho_{\text{water}}$ the density of water and $v$ the gain in velocity of the water. Thus for $K_L = 1$ the (additional) pressure drop is equal to the dynamic pressure. The book of Munson [10] lists the pressure drop in a sharp-edged entrance from a reservoir to a channel as $K_L = 0.5$. In the situation that all pressure is converted to dynamic pressure this means that the pressure loss (and thus the energy loss) is 33%. However, this number assumes that there is a ‘vena contracta’ entrance effect, where the water separates from the channel walls, creating a vacuum and being pulled back to the channel wall without regaining full energy. In our system we do not have a channel, so there is no ‘vena contracta’ effect, and the loss can be lower.

Another option to determine the viscous loss from empirical data is to look at experimental discharge coefficients for ‘nozzles’. The discharge coefficient is defined as [6]:

$$C_D = \frac{Q}{A_{or} \cdot \sqrt{2 \cdot g \cdot h}}$$

(4.5)

Where $Q$ is the volume flow rate, $A_{or}$ the cross-sectional area of the orifice and $\sqrt{2 \cdot g \cdot h}$ is a representation of the pressure due to gravity. We will use the relation $\sqrt{2 \cdot g \cdot h} = \sqrt{\frac{2}{1000}} \cdot p$ to convert this to a term including the pressure, which is not mainly caused by gravity in this study. (The new term is equivalent to the maximum velocity in equation 2.4). Now we have:

$$C_D = \frac{Q}{A_{or} \cdot \sqrt{\frac{2}{1000}} \cdot p}$$

(4.6)

A part of the discharge coefficient being less than 1 is caused by the contraction of the jet after the orifice. This does not cause a loss in energy, thus we want to exclude that effect by defining the coefficient of velocity:

$$C_v = \frac{C_D}{C_C} = \frac{v_j}{\sqrt{\frac{2}{1000}} \cdot p}$$

(4.7)

Where $C_v$ is the coefficient of velocity (def.), $C_C = \frac{A_j}{A_{or}}$ is the coefficient of contraction, $v_j$ is the velocity of the liquid in the jet and $A_j$ is the cross-sectional area of the jet. Because
the energy is proportional to the square of the velocity, the squared coefficient of velocity is a measure for the loss of energy:

\[ L_{or} = 1 - C_v^2 \]
\[ = 1 - \left( \frac{C_D}{C_C} \right)^2 \]  
(4.8)

Where \( L_{or} \) is the loss coefficient of the orifice. To find an estimate of the loss, we need to find values for \( C_D \) and \( C_C \) from literature. Literature delivers values \( C_D = 0.753 \) and \( C_C = 0.943 \) for macro-sized nozzles at \( Re = 100 \) [7], yielding a loss of 36%. The discharge coefficient could also be calculated from equation 4.6 when the pressure and flow rate are measured, and determining \( C_C \) by visual inspection using microscopy. For this thesis no setup was available that could accurately measure the width of the jet visually.

The above methods to determine the loss factor using empirical data both have a major disadvantage. The inflow behaviour of a pipe is different from a free jet because the losses in that situation are related to the ‘vena contracta’ effect, which means that the contraction of the flow in a pipe of fixed diameter causes a loss. This is not the case in a free jet, because the jet can flow freely and assume the optimal shape. The flow profile is not forced into a channel shape. The method using discharge coefficients from nozzles relies on the similarity of the used nozzles. The measurements that are referred to are taken in macro-sized nozzles, not in 10\( \mu \)m orifices. The Reynolds number is the same, but the Ohnesorge number, that takes surface forces into account, is different.

**Simulations**

Because we cannot find a representative study for the viscous friction, a model that applies the laminar flow jetting from a micro sized pore has to be developed. We choose to include the effect of viscosity by modifying the effective pressure in Equation 2.4. We want to find a model for the viscous friction as function of the pore size and the pressure, because these are important variables in which the system could be optimized:

\[ p_{eff} = K_{viscous} \left( r_{pore}, (p - p_{laplace}) \right) \]  
(4.9)

Where \( K_{viscous}(r_{pore}, (p - p_{laplace})) \) denotes a viscous loss function of the variables \( r_{pore} \) and the net pressure \( (p - p_{laplace}) \).

A two-phase fluid flow simulation of a pore system was developed in Comsol. Figure 4.4 shows the boundary conditions and a sample of the results. The viscous power dissipation in Newtonian fluids can be calculated as:

\[ p_{visc} = \dot{\gamma} \cdot \mu \]  
(4.10)

Where \( p_{visc} \), \( \dot{\gamma} \), \( \mu \) are viscous power per unit volume \( [J s^{-1} m^{-3}] \), the shear rate \( [s^{-1}] \) and the dynamic viscosity \( [Pa s] \). The total viscous power in the simulation is calculated as volume integral over equation 4.10 in the volume surrounding the pore. To estimate the power loss this value is divided by the pressure input power, which is calculated by multiplying the measured flow rate with the input pressure.

First we analyse analytically how \( K_{viscous}() \) should relate to \( p \). The viscous power scales with the squared shear rate and thus with the squared flow velocity if the flow pattern is similar, because \( \dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \). Therefore viscous power scales linearly with the pressure, thus
Figure 4.4: Properties of the axisymmetric laminar two phase fluid flow simulation. (Left) The conditions of the simulations. Standard air and water properties are used, but the surface tension is reduced to 5% of the normal value. The phase boundary is simulated using the phase field mode. Boundary conditions: A) has a fixed pressure inlet boundary condition B) has the ‘wetted wall’ condition with contact angle 0.16π C) has fixed normal flow rate inlet condition at a very low flow rate of air and D) is the laminar outflow boundary condition with exit pressure 0kPa and exit length 65μm 1) is the location at which the outflow velocity and flow rate are measured. (Middle) typical velocity flow profile from a simulation with 5μm pore radius and 143kPa pressure. The black line indicates the water-air separation. the jet radius is 4.5μm (Right) Rotated 3D view on the result of the simulation including flow lines.
The relation of \( K_{\text{viscous}} \) with \( r_{\text{jet}} \) is more indirectly derived using 4.10 combined with simulation results. The shear stress and losses are mainly focussed close to the perimeter of the pore, as can be seen in Figure 4.5, and the shear rate near the perimeter is assumed approximately independent of the pore radius, when the fluid velocity in the center of the pore remains constant, as in Figure 4.4(middle). With increasing radius the viscous power loss therefore increases linearly with \( r_{\text{jet}} \), whereas the input power scales with \( r^2 \). Therefore the loss factor is expected to scale inversely with \( r \), so that equation 4.9 can be worked out as:

\[
p_{\text{eff}} = p \cdot (1 - \frac{K_{\text{viscous}}}{r_{\text{jet}}}) - \frac{\gamma}{r_{\text{jet}}}
\]  

(4.11)

Where \( K_{\text{viscous}} \) represents the fraction of viscous loss and \( \gamma, r_{\text{jet}} \) are the surface tension and the radius of the jet, respectively. The term \( \frac{K_{\text{viscous}}}{r_{\text{jet}}} \) denotes the viscous loss.

The loss factor \( K_{\text{viscous}} \) was calculated using equation 4.10 and simulations, performed in Comsol using a two-phase flow model. To separate the effect of surface creation, which is treated analytically, we reduced the surface tension by a factor 20, so that only viscous friction played a significant role. Simulations yield a value for \( K \) of \( 1.30 \cdot 10^{-6} \), which was determined as an average over simulations with several jet radii, and is not valid for very small values of \( a \) where the flow rate is significantly influenced. We also determined that \( r_{\text{jet}} = 0.9 \cdot r_{\text{pore}} \), as can be observed from Figure 4.4, but this value could vary with the pore radius and should be measured to confirm the theory. For these values of \( K \) and \( r_{\text{jet}} \) the fraction of viscous loss is 29%, which is lower than the values of 33% for pipe entrance flow and 36% for macroscopic jets that were found in literature.

Using the velocity of the fluid through the pore we can calculate the flow rate:

\[
Q = \pi r_{\text{jet}}^2 \cdot v_{\text{jet}}
\]  

(4.12)

By substituting Equation 2.4 and 4.11 in Equation 4.12 we find the theoretical flow rate. This theoretical flow rate is in good accordance with measurements over a range of pressures 4.6
as is shown in Figure 4.6, for a pore radius of 5µm. The measured values are slightly below the predicted values, which can indicate that the actual viscous friction factor is higher than $K = 1.30 \cdot 10^{-6}$.

4.2 Fluid mechanics - Air friction

Although ideally all kinetic energy would be converted to potential energy, in reality the friction of the droplets with the air will reduce the energy of the droplets. Because the air friction typically depends on the velocity of an object, we need to describe the motion of the droplets over the whole trajectory from pore to target. This can be done using Newton's second law:

$$F = m \cdot \frac{dv}{dt}$$

$$\frac{\delta v(x, t)}{\delta x} \cdot v(x, t) = -\frac{F_{el}}{m} + \frac{F_{drag}}{m} \quad (4.13)$$

The motion of the droplets can thus be calculated by the electric force $F_{el}$ and the drag force $F_{dr}$ as functions of the position $x$. To simplify notation we will use the force per mass ratios $\frac{F}{m}$ in the calculations. This is advantageous, because we observed in Figure 3.8(b) that the droplet size increased with distance due to the occurrence of merging of droplets in-flight, from a radius of approximately 8µm after jet breakup to 20µm at 13mm from the pore.

The description of the electric force is simple if we assume that the electric field is homogeneous between the target and the guard ring. In this case the force on the droplets is:

$$\frac{F_{el}}{m} = \rho_{el} \cdot \frac{U}{l} \quad (4.14)$$

Where $\rho_{el}$ is the charge density, $U$ is the potential difference between target and guard ring and $l$ is the distance between target and guard ring. Because $\rho_{el}$ is a constant, even when droplets merge, and we assume the electric field to homogeneous, the electric force is a constant in equation 4.13.
In section 2.2.2 we showed that equation 2.16 can be used to calculate the drag force on a sphere. Using rough assumption travelling distance of a single 10µm droplets with an initial velocity of 10 m s\(^{-1}\) was estimated at 10mm. Measurements of the velocity of droplets in Figure 3.8(b) show that the real velocity decrease is smaller, especially at larger distances from the pore. Therefore we cannot treat the droplets as individual objects.

The significantly lower air friction at a distance from the pore is expected, because a wake of air will develop around the stream of droplets. This movement of air decreases the speed of the droplets relative to air. The motion of air is not bound to a nearby stationary wall, and therefore the wake that is formed by the droplets can extend almost unlimited compared to the dimensions of the jet, as illustrated in Figure 4.7. The influence of this wake of air can be modelled as a decrease of the air friction factor \( \frac{F_{\text{drag}}}{m} \). Using the same method as in section 4.1.2 a finite element model was built in Comsol in which the dependency of this parameter on important parameters was studied. The important system parameters are the velocity of the droplets \( v \), the size of the droplets \( r_{\text{drop}} \), the distance from the pore \( x \) and the flow rate \( Q \), which by itself relates to the pore radius \( r_{\text{pore}} \) through equation 4.12.

\[
\frac{F_{\text{drag}}}{m} = C_{\text{drag}}(v, r_{\text{drop}}, x, r_{\text{pore}})
\]  

Where \( C_{\text{drag}}() \) is a function representing the drag force per mass as a function of \( v \), \( r_{\text{drop}} \), \( x \) and \( r_{\text{pore}} \).

### 4.2.1 Air friction relations with system variables

A fluid flow model was developed in Comsol, as is shown in Figure 4.8. To estimate the effect of a stream of droplets in air a FEM-model with 100 droplets of equal size and at a fixed distance was used. The droplets are at fixed locations with the air flowing at a constant velocity from the bottom of the simulation. The distance from the first droplet imitates the effect of the distance of droplets from a pore, because in both situations the difference in velocity between droplet and air is fixed at the pore location (the bottom of the simulation), yielding the maximum drag force, and decreases when the velocity of the air is influenced by the velocity of the droplets. Therefore ‘distance from the pore’ in a jetting system is approximately equivalent to ‘distance from the first droplet’ in the simulations. The simulation is different from reality in two details. First the simulation cannot take the effect of simultaneous decreasing droplet velocity and increasing of air velocity into account. Second, circulating air effects, as illustrated in Figure 4.7 are not accounted for in the simulations.

In the simulations the relation between drag force and parameters such as droplet size or velocity of the droplets cannot be simply studied by keeping all other parameters constant.
Figure 4.8: Properties of the axisymmetric fluid flow simulations. The turbulent flow $k - \epsilon$ physics description was used. (Left) Simulation domain with indicated boundary conditions. There are a total of 100 droplets at a variable radius and distance. The boundary conditions are A) fixed velocity inflow normal to the boundary B) No slip boundary at droplet edges C) open boundary with 0 stress normal to the boundary D) outlet with 0 stress normal to the boundary. (Right) Rotated 3D view of the flow profile. The wake of air around the droplets can be observed to grow larger after the passing of more droplets.
Instead, the flow rate should be kept constant at all distances, to ensure mathematically that no volume or mass is lost. Physically this means that the distance between droplets should become larger when droplets merge and smaller when droplets slow down. This is ensured by using the following relation

\[ d = v \frac{4 \pi r_{\text{drop}}^3}{Q} \]  

(4.16)

Where \( d \) is the distance between droplets. The FEM simulations are validated by comparing the simulation outcomes for a single droplet with the analytical solution using equation 2.16. Figure 4.9(a) shows a non-perfect match, but the results of these simulations will be usable to estimate the behaviour of the system.

Figure 4.9(b) show that for distances larger than 2.5mm, so that a representative wake of air is generated, \( C_{\text{drag}}() \) is not influence by the size of the droplets, so that the function is constant in \( r_{\text{drop}} \). Therefore we conclude that the observed merging of droplets to larger-sized droplets does not influence the effect of air friction after some distance. Figure 4.9(c) shows that \( C_{\text{drag}}() \) scales approximately with the square of the velocity when \( x \) is kept constant at 2.5mm. This is remarkable, because Figure 4.9(a) showed a more linear relation. This can be explained by the additional \( v \)-dependence in Equation 4.16.

The relation between \( C_{\text{drag}}() \) and flow rate \( Q \) is shown in Figure 4.9(d). In the droplet stream simulation the flow rate was used as input variable, but this variable was converted to system parameter \( r_{\text{jet}} \) using Equation 4.12. The first data point of Figure 4.9(d) is the theoretical value for zero flow rate using equation 2.16, with zero flow corresponding to \( r_{\text{pore}} = 2.15 \mu m \) in equation 4.12. Because the flow rate is a constant over the integration variables in differential equation 2.4 it is no problem to fit Figure 4.9(d) with a more complex function:

\[ F_{\text{drag}} = C_{\text{drag}}(v, r_{\text{drop}}, x) \cdot e^{-\frac{r_{\text{pore}}}{5.5 \times 10^{-6}}} \]  

(4.17)

Where \( C_{\text{drag}}(v, r_{\text{drop}}, x) \) is the drag force per mass as function of the remaining parameters.

Finally the relation between \( C_{\text{drag}}() \) and the distance from the pore is studied. It is expected that the drag decreases when the wake of air becomes wider and the shear rate in the air becomes smaller. However, the relation is complex and demands an advanced study because the velocity profile of the air depends on the friction with the droplets along the whole length of the jet, and the velocity profile of the jet depends on the air friction. When the measurement data such as the precise droplet shape is not available, as in this study, a simple relation should be used. Figure 4.9(e) shows the force per mass as function of the distance and a possible fit. This fit is a division by the 6th root of \( x \). However, the velocity measurement data of Figure 3.8(b) requires a larger difference in drag force to account for the change in slope from the first to the last part of the graph. Therefore the relation that the simulations show were not used, and a division by a square root of \( x \) was used instead. The mismatch can clearly be seen in the figure.

The mismatch can be explained by the simplifications that are used in the simulations. The simulations assume that all droplets are equal-sized and perfectly spherical. Figure 3.5(a) shows that this is not the case, especially close to breakup of the jet. The droplets are strongly malformed in at least the plane of sight, which is expected when the droplets have just broken from the jet and the surface tension release causes vibrations. Also the droplets are very unequal in size. Both factors are likely to increase the air drag of the droplets in this region.
(a): Validation result. Total drag force on a 14µm droplet is shown as function of the velocity.

(b): Force per mass as function of droplet size for several velocities measured at 2.5mm from the start of the droplet stream.

(c): Force per mass as function of velocity for several measured for 16µm droplets, \( Q = 0.9\mu\text{ls}^{-1} \), at 2.5mm from the start of the droplet stream.

(d): Force per mass as function of \( r_{\text{pore}} \), which represents flow rate \( Q \) through Equation 4.12. The simulations used \( r_{\text{drop}} = 25\mu\text{m} \) and \( v = 10\text{ms}^{-1} \) measured at 2.5mm from the start of the droplet stream. The first data point is calculated from theory.

(e): Force as function of distance from the first droplet in simulation, as indication of the force as function of \( x \). Simulation was performed for 14µm droplets at 10m s\(^{-1}\). The proper fit of the simulation and the used relation are shown.

**Figure 4.9:** Simulation results compared with theory and measurements
When the air drag is large in the first section of the jet, a wake of air will develop more quickly, so that the air friction is likely to be lower in the part of the jet where the vibrations have damped out. This is one explanation for the mismatch with simulations. Another contributing factor is that although Figure 4.9(b) shows no influence of size, this is only valid when the wake of air has developed. A direct calculation from equation 2.16 learns that the force-to-mass ratio is higher for smaller droplets in absence of a wake of air.

Using the developed relation between the drag force and velocity, position, droplet size and pore radius we can fill in the relations in Equation 4.15:

\[ C_{\text{drag}}(v, r_{\text{drop}}, x, r_{\text{pore}}) = \frac{F_{\text{drag}}}{m} = \frac{C \cdot v^2 \cdot e^{-\frac{r_{\text{pore}}}{5.5 \times 10^{-6}}}}{\sqrt{x}} \]  \hspace{1cm} (4.18)

Where \( C \) is a constant that needs to be determined from simulations. The value for \( C \) was determined to be approximately 5.2. Using this value equation 4.18 gives a good approximation for the simulations as function of the tested variables, except the simulations as function of distance, as was explained before. Limitations to the applicability of this function are that simulations were only performed for the ranges of parameters \( 5 < v < 12 \text{m s}^{-1}, 0 < x < 7.5\text{mm}, 0 < r_{\text{pore}} < 15\text{µm} \) and \( 50 < p < 200\text{kPa} \), and not for all combinations of parameters. We cannot assume that all the relations between these variables are independent, especially for variables that relate to the wake of air.

### 4.2.2 Resulting kinetic behaviour

The velocity profile in the presence of an electrical field is found by solving equation 4.13 for \( v(x) \) with the boundary condition that \( v(0) = v_0 \)

\[ v(x)^2 = -\frac{C_{el}}{C_{\text{drag}}(r_{\text{pore}})} \sqrt{x} + \frac{C_{el}}{4C_{\text{drag}}(r_{\text{pore}})^2} \left( 1 - e^{-4C_{\text{drag}}(r_{\text{pore}})} \right) \]  \hspace{1cm} (4.19)

Where \( C_{el} \) is \( \frac{F_{el}}{m} \), \( C_{\text{drag}}(r_{\text{pore}}) \) equals \( Ce^{-\frac{r_{\text{pore}}}{5.5 \times 10^{-6}}} \) from equation 4.18, and \( v_0 \) is the velocity at \( x = 0 \).

In equation 4.19 we recognize in the first term a decrease in kinetic energy due to energy absorption in the electric field, in the second term an exponential decay because of energy loss in viscous friction, and the third term is a compensation term. The third term adds kinetic energy to the equation, because the electric field slows the droplets down so that the lower velocity of the droplets causes less air friction. From the third term we can observe that increasing the electric field will decrease the losses in air friction because of the lower velocity of droplets.

In practical measurement situations the guard ring blocks the electrical field between the membrane and the guard ring, because this would reduce the induced charge in the droplets. Therefore the differential equation 4.13 should be solved using \( F_{el} = 0 \) for the part of the setup before the guard ring. This yields

\[ v_{\text{init}}(x) = v_0 e^{-2C \cdot e^{-\frac{r_{\text{pore}}}{5.5 \times 10^{-6}}} \sqrt{x}} \]  \hspace{1cm} (4.20)

Where \( v_{\text{init}}(x) \) is the initial velocity equation before the point where the electric field starts and \( v_0 \) is the velocity from equation 4.3. Using equation 4.20 we can calculate the velocity at the position of the guard ring. To combine equation 4.20 with equation 4.19 the velocity at
the position of the guard ring needs to be matched: $v_{\text{init}}(x_{\text{guard}}) = v(x_{\text{guard}})$. To achieve this without modifying equation 4.19 we replace $v_0$ with $v_{0,\text{virt}}$ which is a virtual velocity at $x = 0$ that yields matching velocities at $x_{\text{guard}}$. The velocity development that is predicted using equations 4.19 and 4.20 is shown in Figure 4.10. The prediction shows good resemblance with the droplet velocity that was measured.

4.3 Electric domain

4.3.1 Operating regimes

In the electrical conversion efficiency experiments of section 3.1 we observed that when the load resistance is increased, the efficiency initially increases and the current $I_2$ remains constant. This is expected considering the $I_2^2 \cdot R_{\text{load}}$ term in equation 2.13. In the measurements at some point the efficiency starts to decrease, as well as current $I_2$, whereas current $I_1$ remains constant (Figures 3.3(b) and 3.3(a)). Saykally and Duffin observe the same effect and explain this by the maximum voltage that the target can hold: “the maximum voltage should only be limited by the ability of the receiving vessel to hold unbalanced charge” [3]. However, the description of ballistic energy conversion as a process in which energy is converted via the intermediate step of kinetic energy can show that the voltage on the target is fundamentally limited.

Above a certain value of the load resistance, the target potential reaches a value where the kinetic energy of the droplets becomes insufficient to reach it. No kinetic energy is available to convert to electrical energy. The inability of droplets to reach the target could be observed visually, as droplets were seen to be partly reflected from the target (Figure 4.11) and simultaneously a decrease in $I_2$ was measured. When $R_{\text{load}}$ is too large, the droplet are deflected from the target, so that exactly the maximum voltage is maintained. The efficiency equation

---

**Figure 4.10:** Velocity measurements and predictions as function of distance from the micropore. The pressure is 1.4bar and the distance between pore and target (for dataset 3) is 13mm. In the measurements with applied electrical field, this field was applied between 1.5 and 13mm.
Equation 2.13 can now be rewritten for a constant voltage, instead of a constant current:

\[ P = I_2^2 \cdot R_{\text{load}} = \frac{V_{\text{max}}^2}{R_{\text{load}}} \]  (4.21)

Where \( P \) is the harvested electrical power and \( V_{\text{max}} \) the maximum potential that the kinetic energy of the droplets can overcome. Equation 4.21 shows that increasing \( R_{\text{load}} \) over the optimal value will decrease the efficiency linearly with \( R_{\text{load}} \). Physically this makes sense, because energy is lost when droplets are reflected, so that the maximum efficiency is reached in a regime where no droplets are reflected.

The regime in which the kinetic energy is sufficient to reach the high potential at the target is named the ‘current limited regime’. In this regime the efficiency is lower than the maximum because not all kinetic energy in the droplets is effectively converted to electrical energy, and the remaining velocity in the droplets is lost in the collision with the target. The regime where the voltage of the target no longer increases with \( R_{\text{load}} \) is named the ‘voltage limited regime’. In this regime the efficiency is lower than the maximum because a fraction of the droplets cannot reach the target, and the kinetic energy of these droplets is wasted. This situation is illustrated in Figure 4.12. It shows that the optimum efficiency is reached when the load resistance has an optimal value. This value can be calculated when the maximum voltage that the droplets can reach is known. For a lossless situation this is simply equation 2.11. The optimum voltage for a system involving losses will be calculated in the next section in equation 4.30.

Comparing the measured shape of Figure 3.3(b) with the theoretical shape of Figure 4.12 we note that in the measurements there is no clear peak in the efficiency. This can only partially be explained by changes in \( I_1 \), that can be seen in Appendix A. The flattening of the top efficiency over \( R_{\text{load}} \) is attributed to differences in kinetic energy between droplets. Ideally all droplets would land with exactly zero kinetic energy if the optimum load resistance is used, but if droplets have different kinetic energies, some droplets will be reflected and other droplets will have a remaining velocity. This is the loss factor due to ‘leftover velocity and target misses’ that was described in section 2.2.2. From a comparison of the shape of Figure 4.12 with the measurement series 4 and 5 (that have a stable \( I_1 \) current) in Figure 3.3(b) we conclude the flattening of the peak yields and efficiency decrease of approximately 10%, as is shown in Figure 4.13.
Figure 4.12: Illustration of operation regimes for varying electric field strengths

Figure 4.13: Shape fitting of efficiency measurements with a stable $I_1$ current. The fitting is only done by fitting of the shape, no further theory was used
4.3.2 Electric breakdown

Full breakdown

When the electric field strength in the gap becomes too high the voltage will cause electrical breakdown of the air and the charge that is stored at the target can be lost. An averaged equation for the breakdown voltage at atmospheric conditions was reported in Meek [8] for gap lengths of 0.2mm to 200mm at atmospheric humidity (11g m$^{-3}$). This equation is rewritten for the atmospheric condition at pressure of 1.013bar and 20°C temperature

$$U_{br} = 2.44 \cdot 10^{-6} \cdot d + 6.53 \cdot 10^4 \cdot \sqrt{d}$$

(4.22)

Where $U_{br}$ is the breakdown voltage [V] and $d$ is the sparking gap length [m]. The equation is valid for $0.0001 < d < 0.2$.

The effect of the humidity of air on the breakdown voltage was studied by Kuffel in 1961 [5] and by others reported in [8], showing that a raised humidity can increase the breakdown voltage of air by up to 5% compared. In this project the air will be more humid than atmospheric air due to the presence of the water jet, but the exact humidity is not known and an increase by less than 5% is not significant in this case. Therefore we will use the values for atmospheric air, which is worst-case in preventing electric breakdowns.

The effective pore-target distance (Figure 1.1) in air is smaller because of the presence of water droplets. From the Figures 3.5(b) and 3.5(b) we estimate that 20% of the pore-target distance consists of water.

The typical distance that was used in the experiments was 15mm between the target and the pore. The guard ring however was placed closer to the target, which decreases the distance at the smallest gap to 13.5mm. Subtracting the fraction of water in the total distance an effective distance in air of 10.5mm. Inserting this in equation 4.22 yields a breakdown voltage of 32kV. Applying the same calculation to the shortest pore to target distance that was used in measurements - 7mm - yields a breakdown voltage of 18kV.

At high field strengths the electric force on the water of the target was seen to be strong enough to deform the water droplet at the target plate. In this case the deformation of the droplet led to a smaller plate distance, amplifying the effect. This effect led in some experiments to a breakdown at lower voltages. When this breakdown involved the formation of a water jet, it should be described as electrojetting.

Other breakdown mechanisms

The efficiency at high voltages can be limited by the mechanism of corona discharge. When a sharp-edged conducting particle is present at one of the metal plates in the electric field, such as a dust particle or a defect in the setup, this locally increases the electric field strength. The strongly enhanced field might lead to electric streamers, in which current is conducted into the air, towards a point of lower potential.

Meek [8, p.346-351] summarizes experiments using negative point-to-plane gaps where the onset voltage of corona discharge is measured as a function of the gap length and the point radius. At a gap length of 13.5mm, which is typical for our experiments, onset voltages are
Table 4.1: Overview of voltage limits for a setup with 13mm between the guard ring and metal plate

<table>
<thead>
<tr>
<th>Onset voltage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5kV</td>
<td>Corona discharge from 0.16mm tip</td>
</tr>
<tr>
<td>11.5kV</td>
<td>Corona discharge from 1.08mm tip</td>
</tr>
<tr>
<td>13.8kV</td>
<td>Electrospraying from 49.3°C vertical angle cone of water</td>
</tr>
<tr>
<td>31.3kV</td>
<td>Electric breakdown of air with 22% liquid water</td>
</tr>
</tbody>
</table>

reported from 4.5kV for 0.16mm point radius to 11.5kV for 1.08mm point radius. This is substantially lower than the 41kV from equation 4.22 that is required for full electric breakdown at this distance, and lower than the voltages that are typically achieved in our experiments. The currents that are reported in Meek for a gap length of 13mm and a point radius of 0.86mm show a current increasing almost linearly from 0 to 80µA in the range of 11kV (the onset voltage) to 16kV. These currents are orders of magnitude larger than the currents we measure, indicating that unlimited corona discharges such as reported in Meek need to be prevented in our system.

During measurements we observed loss of current up to several nano ampere related to the presence of dust particles. This was measured as an increase in current from $I_3$ (the guard ring) and a reduction of the current in $I_2$ (the target). These dust particles of approximately 2-4mm would be erected in the direction of the electric field and removal of these particles using a pipet would also eliminate the loss of current. This loss of current from dust particles was associated with corona discharge from the tip of these dust particles.

Electrospraying is another phenomenon that could limit the maximum voltage on the metal plate. The free charges on the surface of water that is on the metal plate are attracted to the ground potential of the guard ring. If this attractive force is larger than the surface tension, water may be jetted back, reducing the current $I_2$. Taylor [14] describes an experiment on the minimum voltage required to sustain electrospraying from a cone with semi-vertical angle of 49.3°C. The measured voltage at which stable electrojetting occurred was

$$U = 1.432 \cdot 10^3 \cdot \sqrt{\gamma \cdot d} \cdot 10^5$$  

(4.23)

Where $d$ is the spraying distance of the electrojet and $\gamma$ the surface tension.

For a guard ring to target distance of 13mm this yields a voltage of 13.8kV. This phenomena thus occurs at voltages in the same order of magnitude as the corona discharge. Whichever phenomenon occurs first will depend on dust particles and disruptions on the surface.

4.3.3 Induction of extra charge

For practical application of the system and to reduce the adverse effects of electric breakdown, it is very advantageous to have a high charge density in the droplets. This can be shown from equation 4.14: a large electric field can be replaced by a low electric field and a high charge density. When the product is equal the effect is the same. Large electric fields cause the electric breakdown effects of section 4.3.2, so we want to avoid them.

In conventional streaming potential systems the charge density is a result of the electric double layer, and can only be changed by adapting the chemistry. In our setup the guard ring below the chip holder can be utilized as charge-attractor to increase the charge density in the droplets. This can be done by, instead of grounding, applying a strong negative potential to
Figure 4.14: Illustration of the jet as a line capacitance

Figure 4.15: Measurement of the current increase when an induction voltage is applied. The used parameters are $Q = 0.9 \mu l s^{-1}$, $R_{jet} = 6.4 \Omega$, $\varepsilon_0 = 8.85 \cdot 10^{-12} F m^{-1}$, $D = 1.5 mm$ $l = 520 \mu m$ and $r_{drop}$ is known from Figure 3.8(b) to be $8 \mu m$.

The jet can be modelled as a line capacitance, as illustrated in Figure 4.14. The breakup section is most relevant, because this determines the charge density after breakup. The capacitances at the sides of the jet are not taken into account, because the current in the jet is modelled as DC current, so that the capacitances have infinite impedance. Therefore the whole jet is modelled as a single resistor. In one of the measurement we measured a conductivity of the DI-water, which was $1.0 mS m^{-1}$. For a jet of $500 \mu m$ long and a radius of $5 \mu m$ this yields a total resistance of

$$R_{jet} = \frac{500 \cdot 10^{-6}}{1 \cdot 10^{-3} \cdot \pi (5 \cdot 10^{-6})^2} = 6.4 \Omega$$ (4.24)

The capacitance of a sphere with respect to ground (at infinity) is

$$C_{sphere} = \frac{q}{U} = 4\pi \varepsilon_0 r_{drop}$$ (4.25)

Where $C_{sphere}$, $q$, $U$, $r_{drop}$ are the sphere capacitance, the attracted charge, the voltage difference and the radius of the sphere.

We will approximate the situation as a sphere that is so small that the external field between the guard ring and the membrane is not significantly influenced. Moreover, we approximate
that the rate of change of this field (over position) is so small that the potential at X distance from the sphere (where \( R << X << D \), \( D \) being the distance to membrane or guard ring) is a constant. We can easily derive the value of the potential at X distance from the sphere from the equations of a parallel plate capacitor. This potential is \( \frac{l_{jet}}{D} \cdot U_{guard} \). Replacing the potential difference in equation 4.25 with a reduced potential difference because of the potential at X we obtain:

\[
C_{sphere} = \frac{l_{jet}}{D} \frac{4 \pi \epsilon_0 r_{drop}}{4 \pi r_{drop}^3} \cdot U = \frac{3 \epsilon_0 Q l_{jet}}{r_{drop}^2 D}
\]

(4.26)

The calculated capacitance yields the charge on a droplet of radius \( R \) that breaks off. Using the volume of the droplet and the flow rate we can calculate the induced current:

\[
I_{ind} = C \cdot U \cdot \frac{Q}{V} = \frac{l_{jet}}{D} \frac{4 \pi \epsilon_0 r_{drop}}{4 \pi r_{drop}^3} \cdot U \cdot \frac{3 \epsilon_0 Q l_{jet}}{r_{drop}^2 D}
\]

(4.27)

where \( Q \) is the flow rate and \( V \) is the volume of a droplet. Now we reduce the charging potential difference by the potential drop over \( R_{jet} \) and solve for \( I_{ind} \):

\[
I_{ind} = \frac{3 \epsilon_0 Q l_{jet}}{R^2 D} (U - R_{jet} \cdot I_{ind})
\]

\[
I_{ind} = \frac{U}{\frac{3 \epsilon_0 Q l_{jet}}{R^2 D} + R_{jet}}
\]

(4.28)

In a measurement to test the effect of the induction (Figure 4.15) we measured a linear increase in current with the induction, as is expected from equation 4.28. The slope of this increase was 35nA kV\(^{-1}\). The equation predicts a slope of 70nA kV\(^{-1}\). The difference is most likely caused by a lower conductivity of the water, which was not measured for the induction measurement, and can change strongly with pH and water contamination. The conductivity of 1.0mS m\(^{-1}\) that we measured in another experiment is higher than expected for micro-filtered demineralized water.

From equation 4.28 we can learn that the induced charge density can be limited by the resistance of the water in the jet. Resistance is not limiting when

\[
R_{jet} << \frac{D R^2}{3 \epsilon_0 Q l_{jet}}
\]

(4.29)

4.4 System efficiency

The model to estimate the droplet velocity at breakup and the model for the air trajectory can be combined with the electric field to yield a equation that describes the velocity at all locations of the system. To find the optimum conversion efficiency from pressure energy to electrical energy we have to calculate the electrical field that can slow the droplets to exactly zero at the position of the target.

Solving equation 4.19 for the electrical field variable \( C_{el} \) that yields \( v(x_{target}) = 0 \) yields for the optimal field strength

\[
C_{el, opt} = \frac{x - x_{guard}}{l} U \rho_{el} = \frac{4 \epsilon_0^2 C_{drag}(r_{pore})^2 e^{-4C_{drag}(r_{pore})} \sqrt{x}}{1 + 4C_{drag}(r_{pore}) \sqrt{x} + e^{-4C_{drag}(r_{pore})} \sqrt{x}}
\]

(4.30)
The electric energy conversion efficiency at the target is simply:

\[ E_{\text{ff}} = \frac{U \cdot \rho_e}{p} \] (4.33)

It can be shown that 4.33 equals \( (1 - L_{\text{pore}}) \cdot (1 - L_{\text{air}}) \), as should be the case. The efficiencies that are calculated using the approximate system parameters of measurement series 1, 4 and 5 of appendix A are shown in Figure 4.16, where the conversion from kinetic energy to electric energy is visualized based on equations 2.14, 4.32 and 4.33.

Figure 4.16: Simulated development of kinetic to electric energy conversion. Guard ring was simulated at 1.5mm from pore, pressure 1.4bar, \( r_{\text{pore}} = 5\mu m \) and \( x_{\text{target}} = 15\text{mm} \)

Where \( U, l, \rho_e \) are the voltage difference between guard ring and target, the distance between guard ring and target and the charge density in the droplets, respectively. The other parameters are defined as in equation 4.19.

The velocity is completely described by substituting equation 4.30 in 4.19 and the losses can be calculated per part of the system. The loss in the pore and breakup section was defined in equation 2.14. The \( v_0 \) in this equation can be calculated from equation 4.3 using equation 2.4.

To calculate the energy that is spent to overcome the drag force, as defined in equation 4.32, we note that

\[
\frac{E_{\text{fric}}}{V} = \frac{1}{V} \int_0^D F_d(x) \, dx
\]

\[ = \frac{1}{V} \cdot Ce^{-\frac{r_{\text{pore}}}{5 \cdot 10^{-6}}} \cdot \int_0^D v(x)^2 \, dx \] (4.31)

\[ L_{\text{air}} = \frac{1}{2v_0^2} Ce^{-\frac{r_{\text{pore}}}{5 \cdot 10^{-6}}} \cdot \int_0^D \frac{v(x)^2}{\sqrt{x}} \, dx \] (4.32)

Where \( E_{\text{fric}} \) is the energy lost in air friction and \( D \) is the distance between the pore and the target.
4.5 Predictions

With the efficiency equations of the liquid losses and air friction losses combined we can determine how the system can be optimized to achieve minimal energy losses. This system efficiency considering these main loss factors is plotted against input pressure, pore-target distance and pore radius in Figure 4.17. In this figure the electric force is optimized such that the final velocity at the target is exactly zero. For real systems where the velocities of droplets show a spread there will be an additional loss because of surplus velocity and target misses. In the current system this is estimated from Figure 4.13 to yield 10% loss. The required field strength is also indicated in the the plots of Figure 4.17, because in practical systems the field strength needs to be limited to prevent breakdown of air and corona discharges. Losses due to these effects are not shown, but the limiting values from section 4.3 can be used. The top value 2.3kV mm\(^{-1}\) of the field-strength-axis is the breakdown voltage of air.

Based on these predictions we can see that a system with a larger pore diameter could potentially yield much higher efficiencies than can be obtained with the current system. At a similar distance to the target this would require electric field strengths above the values that lead to air breakdown, but this can be easily prevented by using a larger distance to the target. From Figure 4.17(b) we can see that this only leads to a minor decrease in efficiency compared to the potential gain from using a larger pore. An additional and more pronounced problem when using a large pore size is that it is more difficult to obtain a sufficient charge density. As was concluded from equation 2.12 the charge density does not directly influence the efficiency, but higher field strengths are required to achieve the optimum efficiency. The effect of streaming current decreases for larger pores, because the velocity profile does not overlap the EDL sufficiently. This means the charge density should be induced by the guard ring for the major part, as in equation 4.28. But the larger droplet radius requires a (linearly) higher voltage to induce the same current, and if we consider that the flow rate increases with the squared radius, we can see that the required induction voltage scales with the third power of the pore radius. An advantage is that the longer jet makes it more easy to place the guard ring close to the point of breakup.

As a hypothetical example we can make predictions for the configuration of a system with 15\(\mu\)m pore size, compared to the current system with 5\(\mu\)m pore size. With this pore size the required field strength is approximately 3 times higher, from Figure 4.17(c). This is solved by increasing the pore-target distance by a factor 3. Because of the increased pore size the flow rate will increase by approximately 9 times, and we will assume that there is no streaming current left. We will apply a gating voltage of -1kV. In section 4.3.3 we calculated a streaming current of 70nA for this voltage. The droplet radius is 3 times higher, thus the streaming current will drop to approximately 23nA according to equation 4.28 (assuming that the electrical resistance of the jet keeps constant). Because of the longer jet we can more easily put the guard ring close to the jet, improving the induction current to 46nA. If the flow rate is 9 times higher (8.1\(\mu\)l s\(^{-1}\)) the charge density will be 5.7C m\(^{-3}\), which is even slightly higher than the charge density in the current systems. The increase in efficiency from Figure 4.17(c) and the decrease from Figure 4.17(b) combined yield a fluidic conversion efficiency of 50%, where the prediction for the original system was 28%. In this example we assumed that induction can be scaled successfully. We did not decrease the pressure in this example, which is an additional option that could be used to yield higher charge densities and a lower required field strength.

The pressure and the target-pore distance are two system parameters that can be tuned to reduce the required field strength. The analysis shows that the effect on the efficiency is small.
Figure 4.17: Predicted system efficiencies for variation of pressure, pore-target distance and pore radius. Indicated with dotted line is the field strength required to operate at this optimum efficiency for a charge density of 3.4C m$^{-3}$. Parameters are $p = 140$ kPa, $d = 15$ mm and $r = 5$ µm, where not stated otherwise.
compared to the gain from a higher pore radius. For pressures below 0.5bar the decrease is more pronounced because the surface energies become significant. For larger pore radii the surface energy is less relevant, so even lower pressures could be used.

Minor improvements that can be made to the system are to perform the experimentation in a dust-free environment to reduce the chance of corona discharge, and to provide a mechanism for removing the jetted water from the target to prevent electro spraying in high electric fields. Another possibility exists because we concluded that 10% in efficiency could be gained when all droplets have exactly the same kinetic energy. An effort could be taken to create droplets of a more uniform size. This is possible by including a piezo-element in the micropore and stimulating the jet breakup by driving the system at the desired breakup wavelength. However, driving a piezo-element costs energy, and the total energy production of the system is small. Therefore the chance of reducing this loss factor using piezo-actuation is small.

Overall we predict that a significant improvement can only be made to the conversion efficiency of the system by increasing the pore radius. To prevent problems with a high field-strength a trade-off should be made with the pressure and pore-target distance. Those system parameters should be tuned to values with a lower system efficiency, but a more usable field strength. Additionally it will be required to use significant induction voltages and an efficient setup to induce voltage in the jet.
DISCUSSION AND CONCLUSIONS

5.1 Discussion

The model that was developed for viscous friction, surface energy losses and air friction predicts mechanical conversion efficiencies around 30%. These predictions match with the measurement series of the droplet velocity. This measurement series show that just after breakup of the droplets (at 520 \( \mu \text{m} \) from the pore) already 58% of the energy is lost (\( v = 11 \text{ m s}^{-1} \)), and at 2.3mm 70%. Additionally, in the electrical measurements a levelling-off of the efficiency versus loading resistance was observed near the theoretical maximum value. Fitting the theoretical curve on the measurements suggests that approximately 10% in efficiency is lost because of differences in kinetic energy between droplets. This can also be seen in the loss of current between the source \( I_1 \) and the target \( I_2 \). This loss of droplets and current is not accounted for in the mechanical efficiency prediction, which should therefore be over 40%. However, electrical efficiency measurements show conversion efficiencies slightly over 30%. There is thus a mismatch of approximately 10% between the droplet velocity measurements, that show a good match to the model, and the highest-efficiency measurements for the electrical conversion efficiency.

The measurement data that is available is insufficient to indicate whether the mismatch in efficiency is located in the viscous pore friction or in the air friction close to the pore and at breakup, or both. We can reason that the theoretical calculations of the surface energies must always yield losses near 20% for a system with a 5\( \mu \text{m} \) pore, this does not depend on measured data, therefore this loss fraction is solid knowledge. The fraction of viscous friction of approximately 30% was derived from simulations, and is therefore less certain, but it is in the same order of magnitude as literature values for macroscopic systems. However, it is possible that the factor for viscous friction is lower than in simulations, possibly due to changes in the contact angle of the material, which was observed to change when a micropore-chip was used for a long time. This could also explain why there can be a difference between measurement series with different micropore-chips. A second uncertainty in the model that could cause the mismatch is the effect of air friction close to the pore. Simulations show a smaller change in air friction over distance to the pore than what we derive from measurements. This was explained by the fact that droplets vibrate after breakup, causing a higher air friction, and by the fact that the model did not incorporate that the simulated air friction was higher for small droplets at distances below 2.5mm from the pore. The relation for air friction as function of distance to the pore that was used in the model is that air friction decreases with \( \frac{1}{\sqrt{x}} \), but
this value was only chosen to fit the measurement data. Using this relation the air friction is very high in the first millimetre, and if this is false it would explain the mismatch between experiments.

Further experiments that measure the droplet velocity in the jet and just after breakup are recommended to verify the results of this theory of the force of air friction. An important measurement to find answers to the above unclarities is to measure the radius of the jet, preferably for multiple chip samples. If the radius of the jet is known we can calculate the true jet velocity from the flow rate, and determine whether the current model over-estimates the loss in the pore, or in the air friction.

The small mismatch in absolute value of the efficiency in the model is no threat to the explanatory power of the model. In section 4.5 we discussed how the system could be optimized. These predictions relate the three main sources of loss (viscous, surface and air friction) to the three main mechanical system parameters (pore-target distance, pressure and pore radius). Of these, only the relation between air friction and distance (for short distances and small droplets) requires more attention. Therefore we can now explain how the system will behave when changing these parameters. Because we have a list of the loss factors and an estimation of their relevance, we can also think of new possibilities to reduce the losses that were not considered in this research. Examples of this are to reduce the viscous friction in the pore by using a slip material, or to reduce the air friction by shaping the air so that an efficient air circulation could form.

5.2 Conclusions

An existing system capable of generating high efficiency ballistic electrokinetic conversion was improved and analysed. First a lossless calculation model was developed and an extensive list of possible loss sources was derived. A short analysis yields that losses due to droplet merging, evaporation, chemical conversion and back conduction are very small and do not need to be studied extensively. Losses due to viscous friction in jet formation, surface creation, air friction and an non optimal velocity of the droplets yield significant losses, and were studied in a more advanced model.

To estimate the losses due to air friction, surface creation and viscous friction a measurement setup was developed to measure the size and kinetic energy of the generated droplets. In the measurement setup microscopic double-illuminated images of the droplets were used to extract the velocity and speed of the droplets under various conditions. Using the size of the droplets the losses in surface creation could be calculated using an analytical model. This model separates the effect of Laplace pressure in the jet, which decreases the flow rate, and the extra loss upon breakup of the liquid jet. The measurements of the velocity as function of the distance travelled in air were used to develop a new model about air friction that is occurring in a stream of droplets under influence of an electrical field. Simulations were used to find the behaviour of air friction in a stream of droplets. For a constant flow rate the force from air friction was found to be independent of droplet size and scaling as square of the velocity. Simulations further suggested that the air friction decreases with a 6th root of the distance from the pore, but measurements show a decrease that fits to a square root of the distance. The loss in air friction depends on the electric field that is applied. It is advantageous to apply a large homogeneous electrical field from directly after the pore, because the decreases velocity of droplets will decrease air friction. This could be observed experimentally. To model
the behaviour of viscous friction in a micro-sized pore literature about existing systems was studied. Because no system was fully comparable with the studied system, a simulations was developed to study the relation between viscous friction, pressure and pore diameter. The results show a viscous friction that decreases with increasing radius of the micropore. Additionally the contraction coefficient of the liquid jet was determined.

The total mechanical loss factor from combined pore friction and air friction was determined using a model that incorporates the viscous liquid friction, surface creation effects and air friction under influence of an electric field. From the equation of the velocity the optimum field was calculated, for which the droplets would convert all kinetic energy to electrical energy. Energy losses around 50% were predicted in the conversion from pressure energy to kinetic energy in droplets for the current system \((p = 1.4\text{bar} \text{ and } r_{\text{pore}} = 5\mu\text{m})\) of which 30% is due to viscous losses, and 20% due to surface creation. From the remaining energy 40% is lost in friction with air for a 15mm pore-target distance, yielding a total mechanical loss of 70%.

In addition to the mechanical losses there is a loss because of the non-optimal velocity of droplets. The effect of various load resistances was analysed in a theoretical description of the system behaviour. In this description the system has a current-limited operating regime for values below the optimal load resistance and a voltage limited regime for values above this resistance. In the current-limited regime the droplets have more kinetic energy than is necessary to bring the charge to the target potential, and remaining kinetic energy is lost. In the voltage limited regime the target stabilized at the maximum voltage, but some droplets are reflected because of insufficient kinetic energy. If all droplets had the same velocity the measurements would show a sharp peak in between the regimes. Instead the measurements show a flat top on the efficiency-to-resistance graphs. Comparing the ideal shape to the measurements shows that the efficiency is approximately 10% lower because of this effect.

The electric breakdown of air and the possibility to induce a higher charge density in the droplets are factors that do not directly influence the efficiency of the system, but can be a limiting factor. Several causes for electrical breakdown and losses were studied. Corona discharge and a combination of electro jetting and breakdown of air are effects that were observed in some measurements, but should be prevented. Corona discharge can happen from sharp objects, so the system should be kept dust-free. To prevent electrojetting the water surface should be unable to deform. For the prevention of breakdown of air the electric field strength should be kept below 2.3kV mm\(^{-1}\). Increasing the charge density in the droplets is an effective method to decrease required field strength, and thus the adverse effects of high electric fields. The possibility to induce charge in droplets from the (flat) guard ring was studied. The predicted charge induction for the system was 70nA kV\(^{-1}\), and 35nA kV\(^{-1}\) was measured, likely caused by a lower conductivity of the water.

From the equations as function of the system parameters predictions could be derived about the efficiency of the system in other configurations. The relations show that increasing the pressure or decreasing the distance can increase the conversion efficiency by a small amount, but this comes at the cost of a large increase in required field strength. Increasing the radius of the micropore has the potential to increase the mechanical efficiency of the system to values over 80%, assuming that the same pressure and pore-target distance can be used.

The final conclusion is that the best way to optimize the current system is to increase the radius of the jetting micropore, and to employ several methods to avoid problems with low charge densities. Studying the possibility to use larger pores in combination with lower
pressure, larger pore-target distance and an optimally configured charge-induction system is highly recommended, because the mechanical losses can be greatly reduced with promising predictions of mechanical conversion efficiencies over 80%.


MEASUREMENT DATA

A.1 Used equipment

Table A.1: Equipment that was used for conversion efficiency measurements

<table>
<thead>
<tr>
<th>Equipment Type</th>
<th>Specific Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow meter</td>
<td>Coriflow M12-RBD-11-0-S</td>
</tr>
<tr>
<td>Pressure sensor</td>
<td>SensorTechnics CTE8016GY7</td>
</tr>
<tr>
<td>Current meter $I_1/I_2$</td>
<td>Keithley 2410 1100V sourcemeter</td>
</tr>
<tr>
<td>Current meter $I_1/I_2$</td>
<td>Keithley 6485 picoammeter</td>
</tr>
<tr>
<td>Current meter $I_3$</td>
<td>Keithley 6487 picoammeter/voltage source</td>
</tr>
<tr>
<td>HV power supply</td>
<td>FUG HCN 7E-12-500 (negative)</td>
</tr>
<tr>
<td>Water preparation filter</td>
<td>Millipore millipak 0.22µm</td>
</tr>
</tbody>
</table>

A.2 Electric efficiency measurements

Table A.2: Efficiency measurement series 1.

Solution: degassed DI water.
Gating voltage: 0V.
Distance pore-target: 13mm (of which 1.5mm pore-guard ring).
Applied pressure: 148kPa.
Measured flow rate (manual): 0.73µLs⁻¹.

<table>
<thead>
<tr>
<th>$R_{load}$ (TΩ)</th>
<th>0</th>
<th>1.1</th>
<th>2.2</th>
<th>3.3</th>
<th>4.3</th>
<th>5.3</th>
<th>6.4</th>
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<th>8.0</th>
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<td>4.1</td>
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<td>3.62</td>
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<td>3.5</td>
<td>3.7</td>
<td>4.1</td>
<td>4.0</td>
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<tr>
<td>$I_2$ (nA)</td>
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<td>3.5</td>
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<td>2.5</td>
<td>2.1</td>
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<td>0.53</td>
<td>0.63</td>
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<tr>
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<td>0.1</td>
<td>0.5</td>
<td>0.9</td>
<td>1.2</td>
<td>1.3</td>
<td>1.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>0</td>
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<td>24.9</td>
<td>25.7</td>
<td>24.9</td>
<td>21.6</td>
<td>19.2</td>
<td>16.0</td>
<td>5.2</td>
<td>2.3</td>
<td>3.7</td>
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</tbody>
</table>
Table A.3: Efficiency measurement series 2a.
Solution: degassed DI water.
Gating voltage: 0V.
Distance pore-target: 7.5mm (of which 1.5mm pore-guard ring).
Applied pressure: 143kPa (corrected).
Measured flow rate: 0.9µLs⁻¹.

<table>
<thead>
<tr>
<th>$R_{\text{load}}$ (TΩ)</th>
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<td>4.7</td>
<td>4.5</td>
</tr>
<tr>
<td>$I_2$ (nA)</td>
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<td>4.45</td>
<td>4.3</td>
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<tr>
<td>Efficiency (%)</td>
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<td>22.9</td>
<td>30.1</td>
<td>32.3</td>
<td>30.1</td>
<td>29.9</td>
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</table>

Table A.4: Efficiency measurement series 2b.
Solution: degassed DI water.
Gating voltage: 0V.
$R_{\text{load}}$: 2.2TΩ.
Distance pore-target: variable (1.5mm pore-guard ring).
Applied pressure: 143kPa (corrected).
Measured flow rate: 0.9µLs⁻¹.

<table>
<thead>
<tr>
<th>Distance (mm)</th>
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<th>6</th>
<th>4</th>
<th>3</th>
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<td>$I_1$ (nA)</td>
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<tr>
<td>Efficiency (%)</td>
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<td>23.5</td>
<td>30.1</td>
<td>32.3</td>
<td>26.1</td>
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Table A.5: Efficiency measurement series 3a.
Solution: degassed DI water.
Gating voltage: 0V.
Distance pore-target: 11.5mm (of which 1.5mm pore-guard ring).
Applied pressure: 143kPa.
Measured flow rate: 0.83µLs⁻¹.

<table>
<thead>
<tr>
<th>$R_{\text{load}}$ (TΩ)</th>
<th>0</th>
<th>1.1</th>
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<td>18.8</td>
<td>16.7</td>
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</table>

Table A.6: Efficiency measurement series 3b.
Solution: degassed DI water.
Gating voltage: 0V.
$R_{\text{load}}$: 3.3TΩ.
Distance pore-target: variable (1.5mm pore-guard ring).
Applied pressure: 143kPa.
Measured flow rate: 0.83µLs⁻¹.

<table>
<thead>
<tr>
<th>Distance (mm)</th>
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<tr>
<td>Efficiency (%)</td>
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<td>27.1</td>
<td>28.9</td>
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Table A.7: Efficiency measurement series 4.
Solution: degassed DI water.
Gating voltage: -433V.
Distance pore-target: 15mm (of which 1.5mm pore-guard ring).
Applied pressure: 143kPa.
Measured flow rate: 0.90µLs⁻¹.

<table>
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<tr>
<td>Efficiency (%)</td>
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<td>28.8</td>
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<td>27.3</td>
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Table A.8: Efficiency measurement series 5.
Solution: degassed DI water.
Gating voltage: -250V.
Distance pore-target: 15mm (of which 1.5mm pore-guard ring).
Applied pressure: 140kPa.
Measured flow rate (manual): 0.71µLs⁻¹.

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<td>Efficiency (%)</td>
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The following paper was written as result of this research. The paper requires some improvements and the data needs updating.
IDENTIFYING FLUIDIC ENERGY LOSSES IN A STREAMING POTENTIAL SYSTEM

1. INTRODUCTION

The demand for energy has risen enormously in recent years and new supply of energy, especially new energy sources, are strongly desired. Many new energy sources, such as solar cell, bio-fuel cell and so on, have proved that there are still many possibilities to investigate non-traditional energy sources without producing harmful chemicals for environment.

Microfluidic techniques have developed quickly in recent years and can also provide a convincible method to harvest electrical energy, such as reversed electro-wetting and ion diffusion. Electro-kinetic phenomenon, such as streaming current, can also convert liquid kinetic energy into electrical energy, first proposed by Osterle. Near the wall of wetted glass or silicon surfaces charges are accumulated, named electric double layer (EDL), caused by the electrical field that is produced by ionic species at the surface. These net charges (ions) will be moved in a liquid flow, representing electrical current. Efficiencies around 5% were obtained by using EDL overlapping in nanochannels. The energy conversion performance can be improved by boundary slip, predicted by theory. However, it has not been achieved in experiment.

A new technique using two phase flow was introduced in energy conversion from streaming current by Duffin and Saykally. A single micro liquid jet was shown, showing that the efficiency can be easily over 10%. They attribute this increases of conversion efficiency to the lack of long high-resistance microchannels and to elimination of conduction current, since the droplets that break off from the microjet are isolated by air.

Our analysis shows that this method is fundamentally different from traditional energy conversion using streaming potential, and our experiment shows that the efficiency can be over 30%. In this new method pressure pushes liquid from an orifice forming a jet. Due to the Rayleigh-Plateau instability, the liquid jet will break into droplets, containing net charges from the EDL. The charged droplets are collected at a target, creating a current and generating an electrical potential through a high ohmic (several Tera Ohm) resistance. The created potential will slow the charged droplets down, because they move against an electrical field. The kinetic energy of the droplets can overcome this electrical field, so that essentially pressure mechanical energy is converted to electrical energy via kinetic energy. Because the basic system consists of only a membrane with micropore and a target, this is a simple and straightforward method to convert mechanical energy to electrical energy.
Figure 1 A. shows the principle and setup. Droplets, from break of Rayleigh jet, are charged by EDL. Inertia of droplets moves against electrical field and convert (mechanical) kinetic energy to electrical energy. B. shows a typical measurement that efficiency changes as function of load resistance. The maximum efficiency can be over 35%.

Figure 1B shows typical data from an experiment with increasing load resistance. The conversion efficiency increases with load resistance at the beginning, and then drops down. The average maximum efficiency is over 30% and the maximal value is over 35%. However, the sources of energy losses in the system are still unknown, and a study is required to find how much energy efficiency could be achieved theoretically.

In this paper, we will analyze the energy conversion principle and theoretically study the sources of energy loss in this system, and predict how to improve the performance of energy conversion.

2. MODEL

Energy conversion from external pressure to electric energy takes place in two steps, as shown in Figure 2. In the first step external applied pressure is converted to kinetic energy in the acceleration of water in a liquid jet, that breaks up in a stream of droplets. Losses involved in this step are named ‘liquid losses’, and they are further differentiated in viscous losses in the pore and losses during breakup of the jet. The second conversion step is deceleration of the charged droplets in an electrical field achieving kinetic energy to electric energy conversion. This step includes losses due to air friction.
To acquire information about jet formation and the velocity of droplets under the influence of electrical forces and air friction, a custom setup was built as shown in Figure 3. The setup was capable of capturing double-illuminated images of droplet at all desired distances from the micropore whilst applying a 0-7kV potential difference between target, water and guard ring and measuring pressure, flow rate and electrical current. Droplet velocity information was extracted from the double-illuminated images by dividing displacement by the time step between illumination pulses. [REF supplementary]

**Figure 3 Setup for optical velocity measurements. Flow meter and water inlet are electrically isolated from the setup.**

### 2.1. Loss Factors in Liquid

Liquid losses are defined as the ratio of input pressure mechanical energy to the kinetic energy that the droplets have after breakup of the liquid jet. In the liquid two main factors of loss are distinguished. Viscous losses due to the shear rate inside the water, and surface losses due to the generation of water-air surface. Surface effects are treated separately at the location of the pore and near the breakup of the jet, as illustrated in Figure 4. Jet formation takes place in the pore and initial part of the jet, and determines the flow rate. In this section the surface forces are balanced and do not reduce the energy of the jet, but the Laplace pressure in the jet does reduce...
the net pressure exerted on the fluid. In the breakup section the Laplace pressure yields a positive contribution to the kinetic energy of the droplets, but the surface forces are not balanced and they form a negative contribution to the droplet kinetic energy [REF]. We will first treat the forces in the pore and jet formation section, followed by the effect of the breakup section.

![Figure 4 Illustration of surface forces and Laplace pressure effects](image)

2.1.1. **Jet Formation: Viscous Friction**

The viscous losses can be calculated when the stream profile of water through the circular pore is known. Such a profile is known analytically for creeping flow through a circular submerged orifice, known as Sampsons solution [REF]. However, the Reynolds number of our system is close to 100, which is laminar and not creeping flow, and moreover there is a fundamental difference between a submerged orifice and a free jet, because a free jet has a radically different outflow behaviour. Empirical data is available for the head loss or pressure loss in a sudden circular contraction into a circular pipe [REF], but again the outflow behaviour is different from a free jet because most losses in that situation are related to the ‘vena contracta’ effect, which does not cause large losses in free jets. Empirical data of the outflow of micro-sized orifices was not available.

The Bernouilli equation describes the relation between pressure and kinetic energy of water flow, without viscous losses:

\[
\rho \frac{d}{dt} (\frac{1}{2} \rho \mathbf{v}^2) = \rho \mathbf{F} - \nabla \mathbf{p} - \frac{\mathbf{J} \cdot \mathbf{H}}{\mu} + \mathbf{f} - \rho \frac{d}{dt} \mathbf{v}
\]

Where \( \rho \), \( Q \), \( m \), \( v \), \( \eta_{water} \) are the effective pressure, flow rate, liquid mass, liquid exit velocity and water mass density, respectively.

Now we need to calculate what fraction of the pressure is converted effectively. We subtract the losses due to viscous friction. Viscous friction scales with the squared shear rate and thus with the squared flow velocity and for a similar flow pattern. Therefore it scales with the pressure. The shear stress and losses are mainly focussed close to the edge of the pore and the shear rate near the edges is assumed approximately constant. With increasing radius the viscous power increases linearly with \( r \), whereas the input power scales with \( r^2 \). Therefore the loss factor is expected to scale inversely \( r \). Finally we subtract the Laplace pressure in the jet, yielding a term from the pressure

\[
p_{eff} = p \left( 1 - \frac{K}{a} \right) \frac{v}{a} 
\]

where

\[
a = 0.9r
\]

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\[
p_{eff} = p \left( 1 - \frac{K}{a} \right) \frac{v}{a} 
\]

where

\[
a = 0.9r
\]
Where \( p, K, a, r, \gamma \) are the input pressure, viscous loss factor, jet radius, pore radius and surface tension, respectively. The term \(-\frac{K}{a}\) denotes the viscous loss that scales with the pore radius, and the term \(-\frac{\gamma}{a}\) is the opposing Laplace pressure in the jet.

The loss factor \( K \) can only be calculated when the flow profile is known, therefore it was calculated from simulations, performed in Comsol using a two-phase flow model. To separate the effect of surface creation, which we will treat analytically, we reduced the surface tension by a factor 20, so that only viscous friction played a significant role. Simulations yield a value for \( K \) of \( 1.30\times10^{-6} \), which was determined as an average over simulations with several jet radii, and is not valid for very small values of \( a \) where the flow rate is significantly influenced. The relation \( a = 0.9r \) was taken from the same simulations, but should be measured to confirm the theory.

Using the velocity of the fluid through the pore we can calculate the flow rate:

\[
Q = \pi a^2 \cdot v
\]  

(3)

By substituting Equation 1 and 2 in Equation 3 we find the theoretical flow rate. This theoretical flow rate is in good accordance with measurements over a range of pressures (Figure 5), for a pore radius of 5μm.

![Figure 5 Flow rate for several pressures in 5μm radius pore. The theory is based on a jet radius 4.5μm.](image)

### 2.1.2. Breakup: Surface Energies and the Momentum Balance

In the breakup section of the jet the Rayleigh-Plateau instability causes the jet to break up into droplets. The surface forces are no longer balanced, because the ‘jet side’ of the droplet is pulled by the surface forces, but on the ‘droplet side’ there is no balancing force. However, the Laplace pressure inside the jet causes a positive force on the droplet. This effect was described and measured by [Schneider] and the velocity was described in a momentum balance [REF Schneider]:

\[
\frac{dy_{jet}}{dt} = \frac{dy_{drop}}{dt} + \pi a^2 \cdot p - 2\pi a \cdot \gamma
\]  

(4)

Where \( y_{jet}, y_{drop} \) are the momentum of the liquid in the jet and in the droplets, respectively.

Working out the equation yields [REF Schneider]:
Substituting Equation 1 in Equation 5 we can calculate the droplet velocity as function of the pressure. Dividing the kinetic energy by the input energy \( p Q \) yields the efficiency of pressure to kinetic energy conversion. The resulting theory is shown in Figure 6 combined with measured data.

![Figure 6](image)

**Figure 6** Velocity of the jet and efficiency of pressure to kinetic energy conversion as function of pressure. The measurements and theory are shown for \( r=5 \text{[um]} \).

The measured data shows a lower efficiency and velocity than the theoretical lines. This is contributed to the fact that no air friction losses are involved in these calculations, but the droplet velocity was measured after the breakup point of the jet, which is approximately 500\text{um} from the pore. Some air friction might slow down the jet before the point of measurement.

## 2.2. Loss Factors in Air

Air friction losses are defined as the ratio of the kinetic energy of the droplets just after breakup to the generated electric energy plus remaining kinetic energy after travelling to the target.

The motion of droplets during the trajectory from pore to target is described using Newton's second law:

\[
F = m \frac{dv}{dt}
\]

\[
v \cdot \frac{dv}{dx} = \frac{F_{el}}{m} + \frac{F_{dr}(x)}{m}
\]

The motion of the droplets can thus be calculated by the electric force \( F_{el} \) and the drag force \( F_{dr} \) as functions of the position \( x \). To simplify notation we will use the force per mass ratios \( \bar{F}_m \) in the calculations, so that the forces in calculations do not depend on droplet size. The droplet size was observed to increase with distance due to the occurrence of merging of droplets in-flight, from approximately 8\text{um} radius after jet breakup to 20\text{um} radius at 13mm from the pore. [REF Supplementary]

The air friction on an object can be calculated using the drag equation and the coefficient of drag for that object at some Reynolds number.
\[ F_{dr} = \frac{1}{2} \eta_{air} v^2 c_d A \]  

Where \( \eta_{air}, v, c_d, A \) are the mass density of air, the speed of the object relative to air, the coefficient of drag and the reference area, which is \( \pi r^2 \) for a sphere. For perfectly spherical droplets of 10μm radius at a velocity of 10m/s (Re=13.3) this yields \( F_{dr} = 18 \cdot 10^3 \) and for droplets of 20μm radius (Re=26.6) \( F_{dr} = 6 \cdot 10^3 \). This would mean that 10μm radius droplets decelerate approximately 1.8m/in 1mm.

Measurements of the velocity of droplets as function of the distance from the pore are shown in Figure 7. The decrease of velocity between 0.5 and 2.3mm is 1.7m/s, which is smaller than from the drag equation. From 2.3 till 13mm the velocity decrease is still much smaller, at approximately 0.2m/s per mm.

![Figure 7 Velocity measurements and predictions as function of distance from the micropore. The pressure is 1.4bar and the distance between pore and target (for dataset 3) is 13mm. In the measurements with applied electrical field, this field was applied between 3 and 13mm.](image)

The significantly lower air friction at a distance from the pore is expected, because a wake of air will develop around the stream of droplets. This movement of air decreases the speed of the droplets relative to air. The motion of air is not bound to a nearby stationary wall, and therefore the vortex that is formed by the droplets can extend almost unlimited. The influence of this vortex of air can be modelled as a decrease of the air friction factor \( \frac{F_{dr}}{m} \) and was simulated using a FEM fluid flow model in Comsol. [REF supplementary].

Simulations show that for distances larger than 2.5mm, so that a representative wake of air is generated, the \( \frac{F_{dr}}{m} \) coefficient is not influence by the size of the droplets. [REF supp merging] Therefore we conclude that the observed merging of droplets to larger-sized droplets does not influence the effect of air friction after some distance. The \( \frac{F_{air}}{m} \) coefficient can further be influenced by the flow rate and droplet velocity. Both influence the distance between droplets, and therefore reduce the effect of the wake from previous droplets. For a stream of equal-sized, equal-distance perfectly spherical droplets this distance is:
Where, $d$, $v$, $Q$, $r$ are distance between droplets' centres of mass, droplet velocity, flow rate and droplet radius, respectively.

During the trajectory, the air friction can thus decrease for three reasons. First is that the velocity decreases, second that the distance decreases because of this decrease of velocity, and third that the velocity of the air in the wake of other droplets increases. We will model the air friction coefficient $\frac{F_{dr}}{m}$ as

$$
\frac{F_{dr}}{m} = C_{dr} \frac{v^2}{\sqrt{x}}
$$

(9)

Where $C_{dr}$ is a coefficient that should be determined from simulations and $\sqrt{x}$ denotes the assumption that air friction will decrease with the root of distance from the pore, due to development of an air wake. This assumption is not valid for very small values of $x$, where the friction would become infinite. This can be solved by choosing a particular solution of Equation 6 that yields the correct initial velocity at the position of breakup of the jet, and ignoring the implications for very small values of $x$. The assumption of $\frac{1}{\sqrt{x}}$ relation for the influence of air wake is evaluated with simulations in the supplementary information.

From simulations we determined $C_{dr} = 2.11$ for a flow rate of $0.9\mu$L/s and at a distance of $2.5$mm from the pore, with a maximum deviation of $0.18$ when the droplet radius is varied from $8$ to $20$um and the velocity is varied from $5$ to $12$ m/s. The fact that $C_{dr}$ remains constant over this range of velocities shows that the dependency of $v$ in Equation 8 is properly covered in the $v^2$ in Equation 9.

Using $C_{dr} = 2.11$ we can substitute Equation 9 in Equation 6 and solve for $v(x)$. To obtain a particular solution we note that $v(0.5mm) = 11$m/s, as was obtained from measurements and from the predictions about the pore friction. The resulting theory line is shown in Figure 7.

The value for $\frac{F_{el}}{m}$ can be calculated from the charge density in the droplets and the electrical field strength, assuming the electric field is constant:

$$
C_{el} = \frac{F_{el}}{m} = \frac{U}{l} \frac{\rho}{\eta_{water}}
$$

(10)

Where $C_{el}$, $U$, $l$, $\rho$, $\eta_{water}$ are the coefficients for electric force (defined), potential difference, length of the electric field, charge density in the droplets and mass density of water, respectively.

The charge density is calculated from $\rho = \frac{i}{Q}$, where $i$ is the current into the reservoir. The theory line with an applied electrical field is also shown in Figure 7. The charge density was $3.4C/m^3$ ($2.9nA$ at $0.85\mu$L/s). Because of the distance between the micropore and the guard ring in the measurements, the theory line is shown from $x = 3$mm, and $v_0$ was adapted so that $v(3mm)$ was equal to the velocity without applied electrical field. Combining all parameters yields $C_{el} = 2380$ for $7kV$ over $10$mm and $C_{el} = 0$ for $0kV$.

Solving Equation 6 with substituted Equation 9 and Equation 10 and initial condition $v(0) = v_0$ yields
In this equation we recognize in the first term a decrease in kinetic energy due to energy absorption in the electric field, in the second term an exponential decay because of energy loss in viscous friction, and the third term is a compensation term. The third term adds kinetic energy to the equation, because the electric field slows the droplets down, and the lower velocity of the droplets causes less air friction. From the third term we can observe that increasing the electric field will decrease the losses in air friction because of the lower velocity of droplets.

The velocity of Figure 7 is shown as kinetic energy fraction in Figure 8. The effect of the third term of Equation 11 can be observed in the theory lines because the 'Kinetic + electric energy’ line lies above the theory line with no applied electrical field. The electric energy per volume is calculated as:

\[
\frac{E_{el}}{V} = \frac{x - x_0}{l} U \rho \tag{12}
\]

Where \(x_0\) is the location of the guard ring, where the electric field starts. This equation uses the same assumption as Equation 10, that the electric field is homogeneous.

![Figure 8 Energy redistribution in the system as function of distance from the jetting micropore. The lines show the kinetic energy of the water as fraction of the initial kinetic energy. Additionally a line is shown with added electrical energy, to show that air friction losses are smaller when an electrical field is applied. In the measurements with applied electrical field, this field was applied between 3 and 13mm.](image)

### 2.3. SYSTEM EFFICIENCY

We can determine the overall efficiency of the jetting micropore system by substituting Equation 1 and 2 in Equation 5 and substituting Equation 5, 9 and 10 in Equation 11. Substitution Equation 5 in Equation 11 yields:

\[
v^2(x) = \frac{-C_{el}}{C_{dr}} \sqrt{x} + \frac{2}{\eta_{water}} \left( \frac{1 - \frac{K}{a}}{\frac{y}{a}} \right) \left( 1 - \frac{y}{2(p(a - K) - y)} \right)^2 e^{-\frac{C_{el}}{4 C_{dr}^2} \left( 1 - e^{-\frac{C_{el}}{C_{dr}}} \right)} \tag{13}
\]
Equation 13 can be converted to efficiency by dividing by \( \frac{2pQ}{\eta_{water}} \). The efficiency of converting pressure energy to electric energy is calculated by dividing Equation 12 (using \( x_0 = 0 \)) by the input pressure. In Equation 13 the factor \( C_{dr} \) and \( \frac{K}{a} \) were measured for a single value of \( r \) (5ums) and the relation with \( r \) was determined from simulations. The factor \( C_{dr} \) can be approximated with:

\[
C_{dr} = 4.6 e^{-5.5[\mu m]} \tag{14}
\]

Substituting Equation 14 in Equation 13 and using the assumption from Equation 2 that \( a = 0.9r \) we can determine the efficiency of the system as function of the pore radius.

With all the efficiency equations of the liquid losses and air friction losses combined we can determine how the system can be optimized to achieve minimal energy losses in the streaming potential system. This system efficiency is plotted against input pressure, pore-target distance and pore radius in Figure 9. In these system efficiencies the electric force is assumed to be optimized such that the final velocity at the target is exactly zero, by solving Equation 13 at \( v = 0 \) and \( x = x_{target} \) for \( E_{el} \). The required field strength is also indicated in the figure, because in practical systems the field strength needs to be limited to prevent breakdown of air and corona discharges. The breakdown field strength of air is 2.3kV/mm, corona discharge can happen at lower voltages.

![Figure 9 Predicted system efficiencies for variation of pressure, pore-target distance and pore radius. Indicated with dotted line is the field strength required to operate at this optimum efficiency for a charge density of 3.4C/m³. Parameters are p=140kPa, d=15mm and r=5um, where not stated otherwise.](image)

### 3. Discussion

We measured and modelled how energy was lost during the conversion process from pressure energy to electrical energy. Energy losses around 50% were observed in the conversion from pressure energy to kinetic energy in droplets, for 1.4bar pressure and 5um radius pores. These losses were modelled as losses due to viscous friction, which attributed over 30% for a 5um pore radius, and losses in surface creation. Surface creation accounts to the remaining 20% at this pore size.

In the conversion from kinetic energy to electrical energy approximately 40% of the kinetic energy is lost in friction with air for a 15mm pore-target distance. This fraction depends on the
electric field that is applied. It is advantageous to apply a large homogeneous electrical field from directly after the pore, because the decreases velocity of droplets will decrease air friction. This could be observed experimentally. It is also advantageous to reduce the distance between pore and target, but at the cost of a higher required electrical field.

The amount of viscous drag in air decreased more quickly in experiments than simulations of a 'perfect stream' of droplets predict [REF Supplementary info]. This can be explained by the fact that a real jet shows oscillations of droplets in the first millimetres after breakup, where many droplet merges occur. These oscillations will increase the apparent size, and thus the drag of the droplets. The increased drag will accelerate the air more, so that the relative velocity between droplets and air is smaller at larger distance from the pore where droplets do not oscillate anymore. There the friction is lower than in simulations.

The relation between air friction loss and pore-target distance was modelled and could be verified experimentally, as well as 'liquid losses' for variations in pressure. The resulting system efficiency for 5um pore and 1.4bar pressure matches with the maximum electrical efficiencies of around 30% that were measured in other experiments. [REF other paper or intro]

The relation between losses and pore radius was modelled using a combination of predictions from simulation data for viscous losses in the pore, theory and literature data about the surface energy losses and simulation data about the air friction coefficients for various flow rates. The model that was developed based on this data predicts that substantially higher efficiencies can be obtained for larger pore sizes. It is therefore recommended that the predictions about the efficiencies for larger pore sized be tested in an experimental setup.

To operate the streaming potential system at its optimum potential it is recommended that an effort is made to develop a system with a larger micropore size. Increasing the operating pressure or decreasing the pore-target distance can also yield higher efficiencies, but the gain is low and it is not expected that much progress can be made here. Some system limits might be encountered when increasing the micropore size, such as the requirement to use larger pore-target distances that increase the losses to air friction and limits in the possibility to induce large enough charge densities in larger droplets. Further study and confirmation of the simulation data is recommended for the subject of increasing the micropore size, as the predicted 80% efficiencies for larger micropores are very promising for optimization of the streaming potential system.

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