Interest Rate Risk in the Banking Book: The trade-off between delta EVE and delta NII

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Abstract

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-Financial Engineering

Master of Science

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by Philip J.F. ENGBERSEN BSc.

The low interest rate environment has made Interest Rate in the Banking Book (IRRBB) an interesting topic. The Basel Committee on Banking Supervision (BCBS) made new guidelines for regulations available in April 2016. It stated that both delta EVE and delta NII as economic value and earnings perspectives to IRRBB should be taken into account when assessing IRRBB. Several studies have shown that these perspectives relate to each other and can even cause a trade-off when minimising both. Present study is performed to investigate this trade-off and the use of delta EVE and delta NII measures is investigated. By means of a case study and balance sheet data made available by a Dutch bank this trade-off was assessed. A Linear Programming (LP) model was constructed to optimise the hedging portfolio based on IRRBB constraints. The results indicate that for this specific data set the delta EVE and delta NII had very little impact on each other. It also can be derived that plain vanilla swaps could effectively hedge the delta EVE and delta NII.
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<tr>
<td>BCBS</td>
<td>Basel Committee on Banking Supervision</td>
</tr>
<tr>
<td>BIS</td>
<td>Bank of International Settlements</td>
</tr>
<tr>
<td>bp</td>
<td>basis points</td>
</tr>
<tr>
<td>DCF</td>
<td>Discount Cash Flow</td>
</tr>
<tr>
<td>DF</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>DoE</td>
<td>Duration of Equity</td>
</tr>
<tr>
<td>EBA</td>
<td>European Banking Authority</td>
</tr>
<tr>
<td>EONIA</td>
<td>European Overnight Indexed Aggregated rate</td>
</tr>
<tr>
<td>Euribor</td>
<td>Euro InterBank Offered Rate</td>
</tr>
<tr>
<td>EV</td>
<td>Economic Value</td>
</tr>
<tr>
<td>EVE</td>
<td>Economic Value of Equity</td>
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<tr>
<td>FRA</td>
<td>Forward Rate Agreement</td>
</tr>
<tr>
<td>FRM</td>
<td>Financial Risk Management</td>
</tr>
<tr>
<td>FSI</td>
<td>Financial Services Industry</td>
</tr>
<tr>
<td>IRB</td>
<td>Internal Ratings Based approach</td>
</tr>
<tr>
<td>IRR</td>
<td>Interest Rate Risk</td>
</tr>
<tr>
<td>IRRBB</td>
<td>Interest Rate Risk in the Banking Book</td>
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<tr>
<td>KRD</td>
<td>Key Rate Duration</td>
</tr>
<tr>
<td>LIBOR</td>
<td>London InterBank Offered Rate</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>NII</td>
<td>Net Interest Income</td>
</tr>
<tr>
<td>NIM</td>
<td>Net Interest Margin</td>
</tr>
<tr>
<td>NPV</td>
<td>Net Present Value</td>
</tr>
<tr>
<td>OBS</td>
<td>Off Balance Sheet</td>
</tr>
<tr>
<td>OIS</td>
<td>Overnight Indexed Swap</td>
</tr>
<tr>
<td>PV</td>
<td>Present Value</td>
</tr>
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</table>
Chapter 1

Introduction

1.1 Background

1.1.1 Basic banking structure and activities

Traditionally banks have a mediating role between groups of people and businesses that have money, and groups that need it (Asmundson, 2011). This redistribution is done via deposit and savings accounts where clients can store money they don’t need and earn interest on it. Money acquired on these accounts is loaned to customers in need of money via loans and mortgages. This process is called transformation (SVV, 2013). Generally the margin between the interest paid on deposits and earned on loans result in a profit for the bank. This margin is often called the Net Interest Margin (NIM).

These activities of lending and taking deposits is typical for commercial banks (Hull, 2012a). Commercial banks can be divided into two sub categories: retail and wholesale. The retail banks tend to lend relatively little amounts and take deposits of private customers and small businesses, where wholesale banks provide banking services to larger corporate clients, fund managers and other financial institutions (Hull, 2012a). Besides commercial banks, investment banking is a second category of banks. Investment banks assist companies in raising debt and equity, providing advice on mergers and acquisitions, restructurings and other corporate finance decisions (Hull, 2012a). These banks are often also more involved in the trading of securities (Hull, 2012a).

In Table 1.1 a simplified example of a typical retail bank’s balance sheet is given. For retail banks loans are the largest part of their assets, deposits the largest liability. This is represented by the percentages of 72% and 70.5% respectively for loans and mortgages and deposits. The composition of assets and liabilities given in Table 1.1 is based on aggregated public data of the balance sheets of 4 of the largest retail banks in the Netherlands in fiscal year 2015.
The contractual maturity between the assets and liabilities of a bank differ. Loans and mortgages to customers often have a contractual maturity of more than 10 years, while the contractual maturity of deposits is much shorter with contractual maturities of one year on average (Memmel, 2014). This creates a mismatch in maturity between assets and liabilities. This mismatch, where long term loans are financed with short term deposits and savings is also known as maturity transformation (Memmel, 2011; Memmel, 2014; Entrop et al., 2015; SVV, 2013). For many banks maturity transformation forms a large part of their interest income as interest earned on long term assets is often higher than interest on short term assets (Memmel, 2011; Entrop et al., 2015). According to Maes et al. (2004) and Entrop et al. (2015) the existence of a risk premium and expected excess return on long term assets is the main driver of this maturity mismatch at banks. However, with the interest rates changing, this maturity mismatch can result in a decrease of the bank’s earnings (Entrop et al., 2015). Since short term liabilities reprice faster than assets, rising interest rates will have a decreasing effect on the NIM. This since assets keep the original interest rate longer than the liabilities. When interest rates rise, the bank has to pay more interest on liabilities while still earning the same rate on assets. This can lead to a decline in net interest income (Memmel, 2011). This risk is called Interest Rate Risk in the Banking Book (IRRBB). Currently the historically low interest rate environment poses a serious threat to banks as their NIM might decrease as a result of both increasing as decreasing interest rates.

### 1.1.2 Overview of IRRBB practices at banks

A preliminary study on a group of banks across Europe during this research has proven that interest rate risk management is approached differently across banks. An overview of the common methods used by the banks, and their interest rate exposure is given in Table 1.2. It becomes clear that banks use different methods and approaches in measuring IRRBB. In Table 1.2 below, delta EVE is the change in Economic Value of Equity due to changes in interest rates of +100 basis points on the yield curve. Commerzbank forms an exception as they report their delta EVE figure as the result of a 200 basis points shock on the yield curve. Delta NII is the change in

---

**Table 1.1: Typical balance sheet retail bank**

<table>
<thead>
<tr>
<th>Balance Sheet</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans &amp; Mortgages</td>
<td>72</td>
<td>Deposits</td>
</tr>
<tr>
<td>Investments</td>
<td>0.4</td>
<td>Debt</td>
</tr>
<tr>
<td>Trading securities</td>
<td>16</td>
<td>Trading liabilities</td>
</tr>
<tr>
<td>Cash</td>
<td>11.6</td>
<td>Equity</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td><strong>Total</strong></td>
</tr>
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Net Interest Income (NII) due to a change of 100 basis points on the yield curve. In Chapter 2 of this thesis these methods to measure IRRBB will be further elaborated. What can be seen from this table is that many banks are heavily relying on interest income with shares of NII over their total income of above 60 percent. What also becomes clear is that values for IRRBB measures differ per bank. The values of Delta NII for Dutch banks ING and Rabobank are relatively low with 0.17 and 0.2 percent respectively, while BBVA and Lloyds report higher figures with 2.3 and 3.2 percent respectively. This has partly to do with the methodology used in calculating these figures, but also with the banking business model and risk management targets. Also interesting is the relatively low figure of delta EVE of Société Generale. This low figure can be explained by a difference in banking business model compared to the other banks and a better distribution of assets and liabilities over maturity. The mismatch in maturity between assets and liabilities is far lower than for other banks, possibly resulting in a lower delta EVE. The delta EVE figure for Commerzbank is relatively high. This is hard to compare however to the other banks since it is based on a 200 basis points shock, where the rest of the banks use a 100 basis point shock. This indicates the fact that the methodology in using these methods differs between these banks.

Some banks do not report both delta NII and delta EVE figures, Lloyds and Commerzbank are examples. Besides delta EVE and delta NII, ING is the only bank reporting their Present Value (PV) figure. The PV01 represents the change in economic value of the equity of a bank under a shock of 1 basis point on the yield curve. What becomes clear in this overview is that not all banks report the same methods, that banks use a different methodology underlying these methods and that the results differ per method per bank.

<table>
<thead>
<tr>
<th>Name</th>
<th>Equity (bn)</th>
<th>Net interest income</th>
<th>Delta NII/ NII (mln)</th>
<th>Delta EVE/ Equity</th>
<th>PV01</th>
<th>Interest income/ Total income</th>
</tr>
</thead>
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<td>ING</td>
<td>41,5</td>
<td>12.744</td>
<td>0,17%</td>
<td>4,8%</td>
<td>15,7 mln</td>
<td>75%</td>
</tr>
<tr>
<td>Rabobank</td>
<td>41,3</td>
<td>9.139</td>
<td>0,2%</td>
<td>2,4%</td>
<td>NR</td>
<td>69%</td>
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<tr>
<td>ABN Amro</td>
<td>17,6</td>
<td>6.076</td>
<td>1,3%</td>
<td>NR</td>
<td>NR</td>
<td>72%</td>
</tr>
<tr>
<td>Société Generale</td>
<td>62,7</td>
<td>9.306</td>
<td>1,55%</td>
<td>0,07%</td>
<td>NR</td>
<td>35,7%</td>
</tr>
<tr>
<td>Banco Santander</td>
<td>98,7</td>
<td>32.812</td>
<td>1,49%</td>
<td>5,89%</td>
<td>NR</td>
<td>71,1%</td>
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<tr>
<td>Commerzbank</td>
<td>21,1</td>
<td>4.037</td>
<td>NR</td>
<td>8,45%</td>
<td>NR</td>
<td>58,5%</td>
</tr>
<tr>
<td>Lloyds Bank</td>
<td>63,8</td>
<td>15.361</td>
<td>3,2%</td>
<td>NR</td>
<td>NR</td>
<td>64,9%</td>
</tr>
<tr>
<td>BBVA</td>
<td>55,4</td>
<td>16.426</td>
<td>2,3%</td>
<td>2,41%</td>
<td>NR</td>
<td>68,6%</td>
</tr>
</tbody>
</table>
Chapter 1. Introduction

1.1.3 Changing regulatory landscape on IRRBB

In April 2016 the Basel Committee on Banking Supervision (BCBS) published a paper with new guidelines and standards for interest rate risk management called: "Standards on Interest Rate Risk in the Banking Book" (BCBS, 2016a). This paper is a response of the committee on the current low interest rate environment and heterogeneous practices on the measurement and management of interest rate risk.

In these new guidelines, aimed at more standardisation and comparison of management of IRRBB between banks, the use of two methods in measuring interest rate risk is advocated: an earnings based method and an economic value based method (BCBS, 2016a). The earnings based method measures the change in NII due to changes in the interest rates, where the economic value based measures focus on the change in net present value of the bank’s balance sheet. Banks have to comply with these new regulations from the 1st of January 2018.

1.2 Research purposes

The two methods for measuring IRRBB are the main subject of this research. Earnings and economic value based methods to measure IRRBB are different in some aspects. The most important aspect is the time horizon over which IRRBB is assessed. The earnings based methods have a short term focus of 1 to 3 years (BCBS, 2016a; Memmel, 2014; EBA, 2013a). The economic value measures often have an extended focus of more than 5 years. The few literature that is published on these topics claim that a trade-off between both methods occurs when minimising the interest rate risk measured with both these methods. It is not possible to minimise the interest rate risk measured by both of the methods at the same time. In this research the relationship between these two methods is investigated, is determined how this trade-off occurs and how banks should manage their assets and liabilities based on these results.

1.3 Research questions

In order to aid the purposes mentioned in the previous section, and to guide the research, the following research questions is used:

How can banks best change their levels of exposure to interest rate risk, measured from an economic value and earnings perspective, given their risk appetite and business model?

This research is structured through the use of three sub research questions. These three sub research questions will each form a different phase in this research, leading
to the answer to the main research question. The three sub research questions are stated as follows:

1. **What are important aspects and differences between delta NII as earnings and delta EVE as economic value based methods to measure IRRBB?**

2. **What are important aspects to take into account in the calculation of delta EVE and delta NII?**
   - (a) *What is the effect of the yield curve used on the exposure to IRRBB measured from both perspectives?*
   - (b) *What is the impact of changes in duration of assets and liabilities on the exposure to interest rate risk measured from both perspectives?*
   - (c) *What is the impact of the business model of the bank on the calculation of delta EVE and delta NII?*

3. **When is the exposure to interest rate risk measured from both perspectives considered optimal?**
   - (a) *How can a bank best change its exposure to interest rate risk?*

### 1.4 Methodology

In this thesis the main research question will be answered by the answers obtained from the research questions defined in the previous section. The methods and approach used to gather answers to these questions will be described in this section.

The first research question, aimed at getting more understanding of the concepts of IRRBB, economic value and earnings based methods will be answered by extensive literature research in Chapter 2.

The understanding of the basic concepts regarding IRRBB, delta EVE and delta NII will be used during the second step of this research: determining the relevant aspects in this research and their impact on interest rate risk measured. By conducting quantitative analyses on the levels of delta NII as earnings, and delta EVE as economic value perspectives on IRRBB, the second research question will be answered. Relevant aspects that will be dealt with are the yield curves used in computing the levels of delta NII and delta EVE, the duration of assets and liabilities and banking business models. The process of finding these answers and the results will be described in Chapter 3.

The third research question is aimed at answering the main research question: “*How can a bank best change its exposure to IRRBB measured through both delta EVE and delta NII?*” This question will be answered through the use of a case study. A Dutch bank provided data that is used for this purpose. In cooperation with this bank a
model is developed that optimally distributes payer and receiver swaps based on
delta EVE and delta NII measures in order to hedge IRRBB. The bank has provided
an extensive data set containing detailed information on the positions of assets and
liabilities, IRRBB appetite and risk limits set by the management of the bank. This
data, information and expert input of the bank is used to construct the LP model and
to determine the optimal allocation of payer and receiver swaps given a certain risk
appetite. This model is described in Chapter 4.

In order to examine the best direction for banks to change their exposure to IRRBB,
linear programming theory is used to optimise the balance sheet given certain expo-
sures to IRRBB. Knowledge gathered from research question 2 is used to construct
this model. In this research the simplex method will be used to optimise the linear
problem. More on the methodology of this optimisation problem is found in Chap-
ter 4. The results of this model and the answers to research question 3 and the main
research question are discussed in Chapter 5.
Chapter 2

Theoretical Framework and Literature Research

2.1 Introduction

In order to obtain a proper understanding of the concepts relating to IRRBB and both earnings and economic value based methods, in this chapter literature available on these topics is reviewed. The purpose of this chapter is to lay a theoretical foundation from which the research will be built. At the end of this chapter the following research question will be answered:

1. *What are important aspects and differences between delta NII as earnings and delta EVE as economic value based methods to measure IRRBB?*

This is achieved by elaborating on the general literature that is reviewed. First IRRBB in general will be described, further on the focus will be on delta NII and delta EVE as earnings and economic value perspectives on IRRBB.

2.2 Interest Rate Risk in the Banking Book

2.2.1 Definition of Interest Rate Risk

The BCBS defines in its "Principles for the Management and Supervision of Interest Rate Risk", Interest Rate Risk as "the current or prospective risk to the bank’s capital and earnings arising from adverse movements in interest rates that affect the bank’s banking book positions" (BCBS, 2016a). Changes in interest rates affect a bank’s earnings. As stated in Section 1.1 a mismatch between the maturity of assets and liabilities may cause changes in a bank’s earnings when interest rates increase or decrease. Also changes in interest rates may cause the clients of a bank to withdraw their money, or prepay their loans earlier, affecting its NII (BCBS, 2015).
Besides affecting a bank’s earnings, changes in interest rates also have an impact on the underlying value of the bank’s assets, liabilities and off-balance-sheet (OBS) positions. The interest rate is an input variable in the net present value calculation of cash flows. Changing interest rates have therefore an impact on the net present value calculations of assets and liabilities (BCBS, 2004; Memmel, 2014). When interest rates increase, both the value of assets and liabilities decrease. However, since the maturity of the assets is often longer than the maturity of the liabilities, the losses on the assets side are higher than on the liability side. The economic value of equity, the difference between the present value of assets and liabilities, then decreases (Memmel, 2014).

A distinction is made between interest rate risk resulting from trading activities and banking activities. The focus of this thesis lies on Interest Rate Risk in the Banking Book (IRRBB): interest rate risk resulting from other activities than trading activities and market risk (EBA, 2015; BCBS, 2016a). In the next section the difference between these interest rate risks will be elaborated.

### 2.2.2 Banking Book versus Trading Book

After the financial crisis in 2008 a distinction has been made between interest rate risk in the trading and the banking book. Banking book instruments are generally intended to be held to maturity. Changes in market value are therefore not necessarily reflected in profit and loss accounts (BCBS, 2015). Instruments held in the trading book are often not meant to be held to maturity, and changes in the fair value impact profit and loss accounts. Before and during the crisis banks could designate instruments with observable market prices to the trading book by claiming trading intent. During the crisis many of these positions became illiquid. To avoid the impact of these instruments on profit and loss accounts, many banks transferred instruments to the banking book subjecting them to minimum capital requirements (BCBS, 2015). After the crisis the BCBS started a fundamental review of the trading book where different capital charges for the same types of products in the trading and banking book were addressed. In addition, more strict boundaries were set to prevent transferring trading instruments to the banking book and vice versa (BCBS, 2015; BCBS, 2016b). Both trading and banking book products are subject to interest rate risk. In this thesis however, the focus will be on interest rate risk on banking book products.

### 2.2.3 Sources of Interest Rate Risk

In general four sources of interest rate risk in the banking book are defined (Charumathi, 2008; Seetanah and Thakoor, 2013; BCBS, 2004; BCBS, 2015):
2.3. Common practices to measure IRRBB

2.3.1 Introduction

In this section the practices and methods often used by banks to measure IRRBB will be described. In literature many methods are found. One of the first methods to measure interest rate risk was the approach suggested by Flannery and James (1984). In this method the interest rate risk is measured by the sensitivity of a bank's stock prices to changes in interest rates. A negative coefficient means that the value of bank's equity would decrease as a result of changing interest rates. The focus of the
method is primarily on the short term since the changing stock price of the bank is taken as an indicator of risk (Esposito, Nobili, and Ropele, 2015).

As stated in section 2.2 changes in interest rates can affect both a bank’s earnings and economic value. Literature proposes methods to measure IRRBB that fall into these two categories (Seetanah and Thakoor, 2013; Memmel, 2011; Memmel, 2014; Drehmann, Sorensen, and Stringa, 2008; Abdymomunov and Gerlach, 2014; Maes et al., 2004). Also from a regulatory perspective the use of both perspectives is advocated (BCBS, 2004; BCBS, 2015; BCBS, 2016a; CEBS, 2006; EBA, 2013b). It is stated that an interest rate transaction cannot minimise both earnings risk and economic value risk. The longer the duration of an interest rate transaction, the lesser the earnings risk. However, the long duration of the transaction causes more economic value risk. More cash flows take place in the future where the impact of discounting is larger (EBA, 2015). When a bank only minimises its economic value at risk by engaging mostly in short term interest rate transactions, it could run the risk of short term earnings volatility (BCBS, 2016a).

In the following subsections of this chapter the two perspectives on measuring interest rate risk in the banking book will elaborated further. Here the focus will lie on delta NII as earnings perspective, and delta EVE as economic value perspective on IRRBB.

### 2.3.2 Earnings perspective to IRRBB

NII is still one of the most important sources of income for a bank (Racic, Stanisic, and Racic, 2014). Research in interest rate risk on the Belgian sector have proven that interest income still contributes over 60 percent of the total income for banks (Maes et al., 2004). The short analysis of the European banks in Chapter 1 Table 1.2 indicates that this is not only true for Belgian banks. The earnings perspective on IRRBB is in line with the internal management of assets and liability objectives since it represents the ability of the bank to generate stable earnings over a short to medium horizon. These stable earnings provide the bank a stable profit generation and allows the bank to pay a stable level of dividend (BCBS, 2016a).

Measures under the earnings perspective to IRRBB differ to the extend of the complexity of the calculations of expected income. In general, literature identifies two types of measures (EBA, 2015):

1. GAP Analysis
2. Delta NII (Earnings at risk)

To carry out a GAP analysis, a gap report is constructed by classifying all interest sensitive cash flows of assets and liabilities in time buckets according to their repricing or maturity date. Per time bucket cash flows of interest rates earned and paid
over assets and liabilities are netted to give an exposure per time bucket. GAP analysis is a static measure for interest rate risk in the banking book as it does not adjust the assumptions in the model and calculations under different interest rate scenarios (Charumathi, 2008; EBA, 2015).

The delta NII calculates the change in NII as the difference between the expected net interest income of a base scenario and an alternative scenario (EBA, 2015; BCBS, 2016a). Here the base case scenario reflects the bank’s current corporate plan in projecting volume, pricing and repricing dates of future business transactions. Again the method requires allocation of all the relevant assets and liabilities to maturity buckets by maturity or repricing date. In order to construct a NII forecast, the bank has to make assumptions on the future earnings under both the base and shocked scenario. This suggests that the delta NII measure often assumes a going-concern of the balance sheet, meaning that items that reprice or mature within the assessment horizon are replaced with items that have identical features.

The going concern assumption implies that more assumptions have to be made on the future earnings of the bank. These assumptions range from simple scenarios to more complex dynamic models reflecting changes in volumes and types of business under different interest rate scenarios (EBA, 2013b). With stress test scenarios the method is a more comprehensive and dynamic measure for earnings than the GAP analysis (EBA, 2015).

The delta NII measure normally has a time horizon that focuses on the short term, typically one to three years. This is due to the fact that longer time horizons increase the complexity of calculations. Also the quality of the assumptions underlying the calculations decreases when the time horizon is extended since it is harder to predict the future business production of the bank (BCBS, 2016a; EBA, 2013b).

### 2.3.3 Economic value perspective on IRRBB

The second perspective on IRRBB is the economic value perspective. The economic value of an instrument represents the assessment of the present value of its expected net cash flows. The economic value of a bank can be viewed as the present value of its expected net cash flows defined as the expected cash flows on assets minus liabilities plus the cash flows on Off Balance Sheet (OBS) positions. In this sense it reflects the sensitivity of the net worth of the bank to changes in interest rates (BCBS, 2016a). The Economic Value of Equity (EVE) is also viewed as the amount of future earnings capacity residing in the bank’s balance sheet (Payant, 2007). In contrast to the earnings perspective that has a short term focus on IRRBB, the economic value perspective has a longer time horizon. The economic value perspective evaluates the net worth of a bank’s exposure to changes in all interest rate sensitive portfolios across the full maturity spectrum (Maes et al., 2004).
In literature the following measures are used to determine the sensitivity of a bank’s economic value to changes in interest rates:

- **Duration of equity**: Method that measures the change in value of bank’s equity due to small parallel changes in the yield curve (EBA, 2015).

- **Partial Duration of equity (key rate duration)**: Method that measures the change in value of equity due to small parallel changes in interest rates at specific maturities (EBA, 2015).

- **Delta EVE (capital at risk)**: Method that measures the change in value of equity due to changes in interest rates under several scenarios (EBA, 2015).

- **Value at Risk**: Method that measures the maximum loss of capital under normal market situations and given a specific confidence level (EBA, 2015).

The calculations of delta EVE can either be done with or without the inclusion of equity. In the earnings adjusted economic value calculation, equity is included in the calculation at the same duration as the assets which it is financing. In the standard delta EVE calculation equity is left out of the computations and the outcome is viewed as the theoretical change in economic value of equity (BCBS, 2016a). In this research, the main focus of economic value perspective on IRRBB will be on the standard delta EVE method.

Where the use of EVE is advocated by literature and regulators, there are certain disadvantages in the method. Instruments that are truly meant to be held to maturity are not affected by swings or changes in the market value of the instrument since they will pull back to their original value. The presence or absence of higher/lower accounting values for instruments held at amortising cost is therefore ignored. The economic value perspective can then be misleading. Also, it may be difficult to find a reliable measure since markets for some instruments are highly illiquid or non-existent (Maes et al., 2004).

Additionally, the heterogeneous margins on loans and embedded optionality on assets and liabilities make the determination of delta EVE rather complex. In order to overcome these problems, banks typically determine the delta EVE through the net present value of relevant balance sheet items and contractual or existing cash flows (BCBS, 2016a).

### 2.3.4 Commonalities and differences

In the sections above it became clear that there are certain differences between delta NII and delta EVE as earnings and economic value perspective on IRRBB. In this section the most important commonalities and differences will be summarised in order to give an answer to the first research question: *What are important aspects and
differences between delta NII as earnings and delta EVE as economic value based methods to measure IRRBB?

Commonalities

The most important commonality between the two methods is that both methods measure the effect of changes in interest rates through the use of several scenarios. The original amount of income and EVE in a base scenario is compared with the amount of earnings and EVE as a result of several stressed scenarios.

An other commonality is that both measures can be calculated as the result of dynamic of static assumptions on the balance sheet and future business production.

Differences

Next to the commonalities there are also several differences between the two methods. The most important is the difference in outcome of the measure. As stated in the sections above, the delta NII method measures the change in net interest income over a certain horizon due to changes in interest rates. The delta EVE method measures the change in net present value of the balance sheet due to changes in interest rates. So value on the one hand and future profitability on the other.

Another difference is the time horizon used by the two methods. The nature of the delta EVE method calculating the net present value of all interest sensitive assets and liabilities implies a run-off scenario where all cash flows are incorporated across the full maturity spectrum. The delta NII method on the other hand uses a more shorter time horizon due to increasing complexity of calculations with longer maturities and more uncertainty over future cash flows and business production (EBA, 2015; BCBS, 2016a).

The third and last difference between delta NII and delta EVE methods is the assumption on the future business production. The delta EVE method focuses only on cash flows of products that are already on the balance sheet. Therefore implying a run-off balance sheet. The delta NII method uses a continuous balance sheet where items are replaced with items with identical features after reaching their maturity or repricing date. Here a distinction made between a constant balance sheet and a dynamic balance sheet:

1. **Constant Balance sheet**: The assets and liabilities that are on the balance sheet are maintained assuming like-for-like replacements as assets and liabilities run off.

2. **Dynamic Balance sheet**: Incorporating future business expectations on the basis of specific economic scenarios.
Chapter 3

Analysis of relevant aspects in calculating delta EVE and delta NII

3.1 Introduction

In the previous chapter an introduction to IRRBB and the earnings and economic value perspectives on IRRBB is given. Differences and commonalities between delta NII and delta EVE came forward. In this chapter the purpose is to make the next step in this research by investigating the impact of some of these aspects. The second research question will be answered in this chapter:

2. What are important aspects to take into account in the calculation of delta EVE and delta NII?

In order to determine how levels of delta NII and delta EVE relate to changes in relevant aspects, the delta EVE and delta NII levels are computed on an aggregated balance sheet under changing assumptions on: the yield curve, the maturity profile of assets and liabilities and the banking business model. This is reflected in the subsidiary research questions of research question 2:

a. What is the effect of the yield curve used on the impact of both methods on interest rate exposure?

b. What is the impact of changes in duration of assets and liabilities on the exposure to interest rate risk from both perspectives?

c. How does the banking business model affects the exposure to IRRBB measured from the economic value and earnings perspective?

Previous research has pointed out that the exposure to interest rate risk from both an economic value and earnings perspective differs with the composition of assets and liabilities of a bank (Memmel, 2011). Besides the composition of the portfolio of assets and liabilities, also the maturity profile appears to impact economic value computations to IRRBB (Memmel, 2011). Changing assumptions on these three aspects are made in order to determine the impact on delta EVE and delta NII.
In this chapter first the methodology of this analysis will be described. The assumptions regarding the three aspects will be elaborated, and the methods to measure the delta NII and delta EVE risk measures will be addressed. After the methodology, the results of the analysis will be described. In the end, preliminary conclusions that can be drawn will be covered.

3.2 Methodology

To determine the impact of the aspects mentioned in the previous section, the levels of delta NII and delta EVE are calculated under different situations and assumptions on the yield curve used, the maturity profile of assets and liabilities and the business model of the bank. In total there will be three aspects:

a Aspect 1: Type of yield curve used in delta NII and delta EVE calculations
b Aspect 2: Maturity profile of assets and liabilities
c Aspect 3: Type of banking business model

These three aspects can have different settings as will be described in the sections below. By changing one of these three aspects while keeping the others fixed, the impact of the aspect on interest rate risk exposure will be determined. In this section, the different assumptions on each of the three aspects are addressed. Besides the three aspects, also assumptions underlying the computations of the delta EVE and delta NII is given, as is the data used for this analysis.

3.2.1 Aspect 1: Yield curve used for delta EVE and delta NII calculations

In order to determine the impact of the yield curve on the levels of delta EVE and NII to assess IRRBB, four yield curves are used in this analysis. General literature states that LIBOR and Overnight rates often are used as a proxy for the risk free rate. Treasury rates are viewed too low as a proxy for the risk free rate since regulatory capital requirements on treasury bonds is far lower and tax advantages reduce the cost of treasury bills (Hull, 2012a). However, to create a complete view of the impact of the yield curve used, treasury rates are incorporated in this analysis besides the European Overnight Index (EONIA). Also curves from both European and US currencies are taken into account as they differ in their current level. Hence, in total the following curves are incorporated in this analysis:

a European Overnight Indexed Rate Average (EONIA)
b German Treasury rate
c US treasury rate
d Federal Funds rate
3.2. Methodology

The curves are extracted from Bloomberg at date: 01-07-2016.

3.2.2 Aspect 2: Maturity profile of assets and liabilities

Previous research pointed out that the maturity of an asset or liability has an effect on the level of delta EVE calculated (Memmel, 2011). It is stated that the longer the maturity, the greater the impact on the present value calculation (Memmel, 2011). In order to test this and to determine the effect on both delta EVE and delta NII calculations, the following four situations with changing assumptions on the maturity of assets and liabilities is examined in this analysis:

a. Shorter duration of both assets and liabilities: In this scenario the maximum maturity of both assets and liabilities is assumed to be short. The maximum maturity will be fixed at 6 years.

b. Longer durations for both assets and liabilities: In this scenario the opposite of the previous scenario is assumed. Instead of a maximum duration of 6 years, now all the assets and liabilities that in the previous assumption were assumed to have a maturity of 6 years, are here estimated to have a maturity of 25 years.

c. Duration of assets matched with liabilities: In this scenario the distribution of assets across the maturity ladder is matched with the distribution of the liabilities across the maturity ladder. This means that the distribution of the assets across the maturity buckets is matched with the profile of the distribution of the liabilities.

d. Duration of liabilities matched with assets: In this scenario the distribution of the liabilities across the maturity buckets is matched with the existing profile of the distribution of the assets across the maturity ladder.

The first two assumptions are to determine the maturity effect of assets and liabilities. The second two assumptions test the statement by literature that when assets and liabilities are perfectly matched, a change in economic value can still be present (BCBS, 2016a).

3.2.3 Aspect 3: Type of banking business model

In previous research the relative importance of maturity of assets and liabilities came forward, but the impact could be different depending on the banking business model (Memmel, 2014). Therefore in this research, the type of banking business model will be incorporated, since this may affect the the impact of the aspects described above. In general many banking business models are recognised, in this analysis however, the three business models mentioned in Chapter 1 are examined. These three business models are also in line with recent research on this topic, where on
the basis of balance sheet data and statistical analysis of 222 banks (Roengpitya, Tarashev, and Tsatsaronis, 2014), the following three banking business models are recognised as the main banking business models:

a  **Retail Banking:** This class of banks is characterised by its high share of loans on the balance sheet and its stable funding characteristics as deposits (Roengpitya, Tarashev, and Tsatsaronis, 2014).

b  **Wholesale Banking:** The funding profile of this class of banks strongly resembles the profile of the retail banks in that it also depends on deposits and loans. However, the share of inter-bank liabilities and wholesale debt is much higher than in the previous class (Roengpitya, Tarashev, and Tsatsaronis, 2014).

c  **Investment Banking:** This last class labelled as investment or investment bank holds most of its assets in the form of tradable securities and is predominately funded in the wholesale markets (Roengpitya, Tarashev, and Tsatsaronis, 2014).

When calculating these scenarios, the structure of a bank’s balance sheet is based upon data from the SNL database that contains aggregated financial data of corporations and financial institutions. More on this data can be found in Section 3.3.1.

### 3.2.4 Measuring exposure to interest rate risk

The calculation of the exposures to IRRBB in the analysis described above will be done through the use of six interest rate scenarios. These six interest rate scenarios are described in the “Standards on IRRBB” by the BCBS (BCBS, 2016a). In research published by the BCBS it proved that these six interest rate scenarios are most likely to occur and have the largest impact on the interest rate risk measured by banks (BCBS, 2015; BCBS, 2016a). Hence these interest rate scenarios will be used in this research:

a  **Parallel up shock:** A parallel shock on the yield curve of +200 basis points.

b  **Parallel down shock:** A parallel shock on the yield curve of -200 basis points.

c  **Steepener:** A shock where the yield curve is rotated to get a steeper version of the base curve.

d  **Flattener:** A shock where the yield curve is rotated to get a flattener version of the base curve.

e  **Short rate up:** The short rates of the yield curve up to 1,5 year are increased. The rest of the curve is held equal to the base curve.

f  **Short rate down:** The short rates of the yield curve up to 1,5 year are decreased. The rest of the curve is held equal to the base curve.
3.2. Methodology

3.2.5 Calculation of delta EVE and delta NII figures

The delta EVE and delta NII is calculated over the six interest rate scenarios mentioned in section 3.2.4. In this section the computations for delta EVE and delta NII is described.

Calculation of delta EVE

The delta EVE interest rate risk figure will be calculated according to the method outlined in the paper published by the BCBS (BCBS, 2016a).

The only difference in the approach by the BCBS and this research is that in this research the EVE of the base scenario is subtracted from the EVE of the shocked scenario. In the paper by the BCBS delta EVE is calculated by subtracting the shocked EVE from the base EVE. In the latter case positive values have a negative meaning since the amount that remains is the amount with what the value of equity is decreased. For instance the EVE of the base scenario is 10, and the EVE of the shocked scenario is 2. In the approach by the BCBS then the delta EVE is 8, a decrease of 8. In this approach, it would be -8. To keep the figures and outcomes of the analysis intuitive therefore in this research the EVE of the base scenario is subtracted from the EVE of the shocked scenario.

The first step in the process of calculating the delta EVE is by slotting the repricing cash flows of the assets and liabilities in 19 time buckets in which they belong according to their contractual maturity. The 19 time buckets are given in table 3.1. In order to be consistent with the method used in the "Standards on IRRBB" (BCBS, 2016a) 19 time buckets are chosen, identically defined as in the paper by the BCBS. To allocate the cash flows to these buckets this research makes use of the EDTF 20 publications by Dutch retail banks. This EDTF 20 table contains all notional and coupon cash flows resulting from assets and liabilities and off balance sheet positions organised on their contractual maturity date.

<table>
<thead>
<tr>
<th>Short term rates</th>
<th>Medium term rates</th>
<th>Long term rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overnight</td>
<td>2Y to ≤3Y</td>
<td>7Y to ≤8Y</td>
</tr>
<tr>
<td>≤1M</td>
<td>3Y to ≤4Y</td>
<td>8Y to ≤9Y</td>
</tr>
<tr>
<td>1M to ≤3M</td>
<td>4Y to ≤5Y</td>
<td>9Y to ≤10Y</td>
</tr>
<tr>
<td>3M to ≤6M</td>
<td>5Y to ≤6Y</td>
<td>10Y to ≤15Y</td>
</tr>
<tr>
<td>6M to ≤9M</td>
<td>6Y to ≤7Y</td>
<td>15Y to ≤20Y</td>
</tr>
<tr>
<td>9M to ≤1Y</td>
<td></td>
<td>&gt;20Y</td>
</tr>
<tr>
<td>1Y to ≤1.5Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5Y to ≤2Y</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: 19 time buckets as defined by the BCBS

For the calculation of delta EVE all interest sensitive cash flows that are slotted into the 19 time buckets are discounted using a risk free rate. With $DF(t)$ being the
discount factor of time bucket $t$ and $R_{k,c}$ the interest rate for a certain interest rate scenario $k$ and currency $c$. Since in this research all calculations are done in the euro currency the $c$ is left out of the equations:

$$DF(t) = e^{-R_k \times t} \quad (3.1)$$

Multiplying these discount factors with the net cash flows of assets and liabilities per time bucket midpoint $t$ gives the economic value of equity for interest rate scenario $k$ and currency $c$:

$$EVE_k = \sum_{t=1}^{19} CF_k(t) \times DF_k(t) \quad (3.2)$$

In order to determine the change in economic value of equity, the same process as above is repeated with each interest rate scenario. The change in economic value is the difference between the base scenario and one of the six interest rate scenarios:

$$\Delta EVE = \sum_{t=1}^{19} CF_k(t) \times DF_k(t) - \sum_{t=1}^{19} CF_0(t) \times DF_0(t) \quad (3.3)$$

For example: take a zero coupon bond with a maturity of 3 years and a principal of 100 euros. To get the original value the bond is discounted at a rate of 0.5%. The Discount factor is then according to equation 3.1: $e^{-0.005 \times 3} = 0.985$. Following equation 3.2, this gives an original economic value for the base scenario of: $100 \times 0.985 = 98.5$. Suppose that in the parallel up scenario the interest rate is increased with two percent to 2.5%. This gives an economic value of: $100 \times e^{-0.25 \times 3} = 100 \times 0.927 = 92.7$. This gives a change in economic value of: $92.7 - 98.5 = -5.8$ euro.

**Calculation of delta NII**

For the calculation of delta NII the same data on the distribution of cash flows is used as in the calculation of delta EVE. In the paper published by the BCBS, the calculation of delta NII is made over a time horizon of one year (BCBS, 2016a). In this research delta NII will be assessed on a time horizon of 3 years in order to get a better view of the measure over time. Therefore the assets and liabilities are categorised in the following six maturities:

After the assets and liabilities are allocated to these six buckets, the cash flows per bucket per month over a time horizon of three years are determined. $t$ represents the month ranging from 1 to 36. The cash flows are determined through the notional of assets and liabilities per maturity bucket $j$ and the interest rate earned and paid over assets and liabilities. This interest rate consists of the risk free rate in interest rate scenario $k$: $R_{k,c}$ and the commercial margin for assets per asset class $i$: $M_{assets,i}$. For liabilities it exists out of the risk free rate and the cost of funds represented as a
3.2. Methodology

<table>
<thead>
<tr>
<th>Category j</th>
<th>Contractual Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>More than 3 years</td>
</tr>
<tr>
<td>2</td>
<td>2-3 years</td>
</tr>
<tr>
<td>3</td>
<td>1-2 years</td>
</tr>
<tr>
<td>4</td>
<td>3-12 months</td>
</tr>
<tr>
<td>5</td>
<td>1-3 months</td>
</tr>
<tr>
<td>6</td>
<td>less than a month</td>
</tr>
</tbody>
</table>

Table 3.2: Asset and liability categorisation to maturity

margin per liability class $i$: $M_{liabilities,i}$. In the data used the assets and liabilities are classified into 5 asset and liability classes. The currency in the calculations is just like in the delta EVE calculation in Section 3.2.5 assumed to be the euro. Therefore the $c$ is left out of the equations. This gives the following equation for interest earned on assets:

$$R_{assets,k,i}(t) = R_k(t) + M_{assets,i}$$

(3.4)

The calculation of the cash flows for the liabilities follows the same logic, giving the following formula for the interest paid over liabilities:

$$R_{liabilities,k,i}(t) = R_k(t) + M_{liabilities,i}$$

(3.5)

For the assets the Interest Received (IR) per month $IR_{assets,i}(t)$ are then determined by multiplying the amount of assets per asset class in maturity bucket $j$: $A_{i,j}$ by the interest rate:

$$IR_{assets,k,j}(t) = \sum_{i=1}^{5} (A_{i,j} \times R_{assets,k,i}(t))$$

(3.6)

For the liabilities the same holds but then the notional per maturity bucket $j$ is given by: $L_{i,j}$. This gives the following formula for the Interest Paid (IP) of liabilities:

$$IP_{liabilities,k,j}(t) = \sum_{i=1}^{5} (L_{i,j} \times R_{liabilities,k,i}(t))$$

(3.7)

NII in the base scenario is defined by the summation of the difference in cash flows of assets and liabilities per maturity bucket per month:

$$NII_0 = \sum_{t=1}^{36} \left( \sum_{j=1}^{6} IR_{assets,0,j}(t) - \sum_{j=1}^{6} IP_{liabilities,0,j}(t) \right)$$

(3.8)

To determine the net interest income for the different interest rate scenarios $k$ the same procedure is repeated. However at each of the first repricing dates of the different $j$ categories of assets and liabilities the shocked interest rate of scenario $k$ is taken in stead of the interest rate of the base scenario. The delta NII can then be
calculated by taking the difference of the NII of the base scenario and the interest rate scenario $k$:

$$\Delta NII_k = NII_k - NII_0$$  \hfill (3.9)

In order to clarify the computations the following example is examined. Suppose a loan is on the balance sheet with a notional of 100 euros and a contractual maturity of four years. The risk free rate at one year is -0.05%, and the commercial margin is 2.5%. The total rate earned on the loan per year is then 2.45%. The loan is funded with a deposit that has the same notional but has a contractual maturity of one year. The deposit’s margin is 1%. This gives a total interest paid over the deposit per year of 0.95%

The net interest income per year is thus: $2.45\% \times 100 - 0.95\% \times 100 = 1.50$ euros. Since the NII is assessed over a horizon of three years, this gives $4.50$ euros over three years.

Suppose a parallel shock takes place which increases the risk free rate with 2%. The loan reprices after four years meaning that the interest on the loan within the three year horizon is left unchanged at 2.45%. The interest on the deposit in the first year is 0.95%. After the first year it matures and is replaced for a deposit with the same characteristics, except for the fact that the risk free rate has increased with 2%. The new interest on the deposit is calculated at: $-0.05\% + 2\% + 1\% = 2.95\%$. This means that for the second and the third year the net interest income over the loan and deposit is: $2.45\% \times 100 - 2.95\% \times 100 = -0.50$ euros. So the NII over the three year horizon is $100 \times 1.5\% + 2 \times (-0.5\% \times 100) = 1$ euro. Delta NII is then calculated as: $1 - 4.5 = -3.5$, meaning the NII has decreased with $3.50$ euros.

In the determination of the cash flows a time horizon of three years is assumed. Cash flows are examined per month. In the calculations of changes in net interest income a constant balance sheet is assumed, meaning that assets and liabilities are replaced at the features as before the repricing moment.

### 3.3 Data

During the scenario analysis several possible situations have been examined in order to map the effect and the relation between delta EVE and delta NII measures to IRRBB. This scenario analysis has made use of data of typical balance sheets of banks. In this section the data that has been used in this research is described in more detail.

**Data source**

The data that is used in this research has three sources:
a SNL financial database: This database contains industry-specific financial data of companies over the world. The data base includes sector-specific performance metrics, financial reports and statements, and other archived documents.

b Annual reports of financial institutions: In addition to the SNL database, information and data from public annual reports of several banks has been used to construct the data needed to perform the computations for the scenario analysis.

c Bloomberg: Bloomberg terminal has mainly been used in gathering (historical) data on the movements of specific curves of interest rates.

In the following sections the specific data used and the construction of the balance sheet will be discussed.

3.3.1 Balance sheet data

As has been written in previous sections, the scenario analysis is based on three business models in the banking industry:

a Retail Banking

b Wholesale Banking

c Investment Banking

For each of these three models the balance sheet is constructed through the use of data on existing retail, wholesale or investment banks. The data used for the retail business model is described in the section below, for the other two business models this procedure is repeated and the data can be found in Appendix D

3.3.2 Balance sheet retail banking

As written in Section 3.2.3, the retail banking business model is defined by the large share of loans and deposits on its balance sheet. In this analysis the balance sheet of the retail banking business model is primarily based on the combined balance sheet of the four Dutch largest banks. For this data the largest asset and liability categories are given in Table 3.3 relative to the total amount of assets or liabilities. For the construction of this table financial balance sheet data of these four financial institutions from fiscal year 2015 has been used, gathered from the SNL data base. What becomes clear is that the largest asset and liability categories for these four institutions are similar. On the asset side: loans to customers and companies, investments, investment securities and cash. On the liability side: deposits from customers, senior and subordinated debt and trading liabilities. In Table 3.3 it becomes clear that these
eight components together sum up to approximately 98 percent of the assets and liabilities. So when focusing on these components the balance sheet is a representative substitute of a retail bank’s balance sheet.

<table>
<thead>
<tr>
<th>Assets</th>
<th>ABN Amro</th>
<th>ING</th>
<th>De Volksbank</th>
<th>Rabobank</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td>0.677</td>
<td>0.762</td>
<td>0.785</td>
<td>0.686</td>
<td>0.728</td>
</tr>
<tr>
<td>Investments</td>
<td>0.008</td>
<td>0.003</td>
<td>0.002</td>
<td>0.006</td>
<td>0.005</td>
</tr>
<tr>
<td>Trading securities</td>
<td>0.201</td>
<td>0.167</td>
<td>0.133</td>
<td>0.136</td>
<td>0.159</td>
</tr>
<tr>
<td>Cash</td>
<td>0.103</td>
<td>0.051</td>
<td>0.069</td>
<td>0.143</td>
<td>0.092</td>
</tr>
<tr>
<td>Total</td>
<td>0.989</td>
<td>0.983</td>
<td>0.990</td>
<td>0.972</td>
<td>0.983</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits</td>
<td>0.671</td>
<td>0.771</td>
<td>0.816</td>
<td>0.574</td>
<td>0.708</td>
</tr>
<tr>
<td>Debt</td>
<td>0.251</td>
<td>0.162</td>
<td>0.125</td>
<td>0.323</td>
<td>0.215</td>
</tr>
<tr>
<td>Trading liabilities</td>
<td>0.059</td>
<td>0.051</td>
<td>0.037</td>
<td>0.088</td>
<td>0.059</td>
</tr>
<tr>
<td>Total</td>
<td>0.980</td>
<td>0.984</td>
<td>0.978</td>
<td>0.984</td>
<td>0.982</td>
</tr>
<tr>
<td>Equity</td>
<td>0.043</td>
<td>0.048</td>
<td>0.053</td>
<td>0.062</td>
<td>0.051</td>
</tr>
</tbody>
</table>

Table 3.3: Largest assets and liabilities of the 4 largest Dutch banks in percentage

The data in Table 3.3 is combined to form one balance sheet that matches the characteristics of the four banks mentioned above. This balance sheet is given in Table 3.4.

<table>
<thead>
<tr>
<th>Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Loans</td>
</tr>
<tr>
<td>Investments</td>
</tr>
<tr>
<td>Trading securities</td>
</tr>
<tr>
<td>Cash</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

Table 3.4: Typical balance sheet retail bank

The balance sheet in Table 3.4 is taken as a basis for the calculations of delta EVE and delta NII. Since these levels are measured over a certain time horizon, information and data is needed on the distribution of these assets and liabilities over time. This data for loans could be found in the SNL database. For the other asset and liability components annual reports of the banks mentioned in Table 3.3 were used. In these reports banks use to disclose EDTF 20 tables. EDTF 20 tables give information on consolidated assets, liabilities and off balance sheet commitments by remaining contractual maturity at the balance sheet date (Carney, 2014). The EDTF 20 tables of the four largest Dutch banks in combination with the SNL data on the distribution of loans and deposits over time gives a clear but aggregated view on the distribution of assets and liabilities across the maturity ladder. This data is used to construct Table 3.5.
3.4. Results

The data in Table 3.5 describes the distribution of assets and liabilities across the maturity ladder. In the calculations of delta EVE this data is still too aggregated. Therefore, it is assumed that the assets and liabilities are equally distributed over time within the time buckets given in Table 3.5. However in the situation where the aspect maturity profile is changed, other assumptions are made as can be found in Section 3.2.2.

3.4 Results

In this section the results of the analysis is described. The results are dealt with per aspect. First the results of the different yield curves used in the calculations of delta EVE and delta NII will be discussed, then the maturity profile and last the business model.

3.4.1 Results of the yield curve used in calculating delta EVE and delta NII

In this section the results of the analysis of the impact of the first aspect on the delta EVE and delta NII levels measured is described. In this analysis four yield curves, set out in Section 3.2.1, are used to compute the levels of delta EVE and delta NII. This is done for the three business models in Section 3.2.3. First the results on delta EVE are described, thereafter the delta NII results will be elaborated.

Delta EVE

In Figures 3.1, 3.2 and 3.3 the results for the delta EVE figures for respectively the retail, trade and wholesale business models are given. Here the values on the Y axis

<table>
<thead>
<tr>
<th>Assets</th>
<th>Loans</th>
<th>Securities</th>
<th>Investments</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1 M</td>
<td>0,05</td>
<td>0,405</td>
<td>0,014</td>
<td>1</td>
</tr>
<tr>
<td>1-3M</td>
<td>0,1</td>
<td>0,145</td>
<td>0,019</td>
<td></td>
</tr>
<tr>
<td>3-12M</td>
<td>0,05</td>
<td>0,131</td>
<td>0,092</td>
<td></td>
</tr>
<tr>
<td>1-5Y</td>
<td>0,2</td>
<td>0,151</td>
<td>0,456</td>
<td></td>
</tr>
<tr>
<td>&gt;5Y</td>
<td>0,6</td>
<td>0,169</td>
<td>0,419</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Deposits</th>
<th>Debt</th>
<th>Trading Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1 M</td>
<td>0,070</td>
<td>0,391</td>
<td></td>
</tr>
<tr>
<td>1-3M</td>
<td>0,94</td>
<td>0,035</td>
<td>0,098</td>
</tr>
<tr>
<td>3-12M</td>
<td>0,03</td>
<td>0,105</td>
<td>0,099</td>
</tr>
<tr>
<td>1-5Y</td>
<td>0,02</td>
<td>0,070</td>
<td>0,207</td>
</tr>
<tr>
<td>&gt;5Y</td>
<td>0,01</td>
<td>0,720</td>
<td>0,206</td>
</tr>
</tbody>
</table>

Table 3.5: Relative distribution of assets and liabilities over time
are the change in EVE relative to the base scenario. What can be seen from these figures is that for the retail and wholesale business models the parallel shocks have the largest impact. For the investment business model this contrast is less visible. For the investment business model the delta EVE figures are also overall much lower. However, the differences between the business models will be discussed in Section 3.4.3.

Looking at the differences between the yield curves used, it is clear that for parallel shocks the first two yield curves: EONIA and German Treasury curve have a slightly higher delta EVE than the other two US curves. For the non parallel shocks the opposite is true, there the US curves have a slightly higher impact than the EONIA and German Treasury curve.

This can be explained by the fact that the two European curves have lower values in general than the two US curves. A parallel shock of 200 basis points on a curve with
3.4. Results

**Figure 3.3:** Delta EVE per yield curve for investment business model

Lower values in general is therefore relatively bigger. And therefore has relatively more impact on the two European curves than the two US curves.

**Figure 3.4:** Delta NII per yield curve for retail business model

### Delta NII

The impact of the yield curve used on the delta NII is given in Figures: 3.4, 3.5 and 3.6. The delta NII is different per business model which makes the figures more difficult to read. However, it can be seen that there is a slight difference in the yield curve used and the delta NII measured. It becomes clear that the two European curves result in often more negative values than the US curves used. This can be explained by the values of the European curves relative to the two US curves. The values of the two European curves are lower than the two US curves. This causes the floor of 0 percent for interest on liabilities to be reached earlier in the case of
3.4.2 Results of the maturity profile used in the calculation of delta EVE and delta NII

The results for the second aspect: the maturity profile of the assets and liabilities will be discussed in this section. In Figures: 3.7 and 3.8 these results are given. The results for the wholesale business model are omitted in this section for the delta EVE since it is so similar to the results of the retail business model.
3.4. Results

Delta EVE

When looking at Figure 3.7, the distinction between the results per assumption is clearly visible. The figure indicates that the highest delta EVE is measured in the second assumption, where maturity of assets and liabilities is the longest. The delta EVE is the smallest on the two assumptions where the assets and liabilities are matched on their maturity profile. It can therefore be concluded that for retail and wholesale business models the effect of the maturity on delta EVE measured is large. A way to mitigate this effect could be more closely matching the assets and liabilities, since both the retail and wholesale business models are characterised by a great mismatch in maturity between assets and liabilities. The findings in this section are also in line with previous research stating that the effect of maturity on the net present value of assets and liabilities becomes larger as the maturity becomes longer.

For the investment business model there is not such a clear distinction between the results as was the case in the retail and wholesale business models. This is partly due to the nature of the investment business model where the gap in maturity between assets and liabilities is smaller than for the other two business models. This ensures that the change in economic value of the balance sheet is smaller than the other two business models.

Delta NII

For the investment business model there is not really any difference in sensitivity of delta NII to the 4 assumptions made as can be viewed in Figure 3.9. This is expected due to the close matching of the assets and liabilities on the balance sheet. The duration assumptions do not differ much from the original distribution profile of the assets and liabilities.
Chapter 3. Analysis of relevant aspects in calculating delta EVE and delta NII

Figure 3.8: Delta EVE of the duration analysis for investment business model, reference rate: EONIA

Figure 3.9: Delta NII of the duration analysis for investment business model, reference rate: EONIA

For retail and wholesale this difference is bigger. From Figure 3.10 it can be derived that at the retail banking business model the delta NII is more negative for the first two maturity profile assumptions when interest rates increase. The opposite is true for the interest rate scenarios where interest rates decrease. There the second two maturity profile assumptions report the most negative delta NII.

In Figure 3.11 the delta NII sensitivity for the four maturity profile assumptions for the wholesale banking business model is given. It becomes clear that the first two maturity profile assumptions do not have a large impact. The second two assumptions where the assets are matched with the liabilities and the other way around have a bigger impact. Here the fourth assumption where the liabilities are matched with the assets has the lowest negative delta NII.
3.4. Results

As can be seen in the previous two sections, the results on the assumptions on yield curve and maturity profile for delta EVE and delta NII differ per business model. In this section the results of the previous two sections are aggregated per business model. It becomes clear that there is a distinction between the results and therefore appears to exist quite a difference between the business models used.

In order to analyse the results per banking business model, all the figures on delta EVE and delta NII calculated in the previous two Sections 3.4.1 and 3.4.2 are aggregated in order to compare the results per business model. As in the previous Sections 3.4.1 and 3.4.2 the results will be discussed per delta EVE and delta NII.
Chapter 3. Analysis of relevant aspects in calculating delta EVE and delta NII

Delta EVE

The delta EVE figures per shock per banking business model show that the banking business model does have an impact on the delta EVE figures reported. This is mainly due to the asset and liability profile of the business models differing from each other. The investment business model has for instance as the data shows, a smaller gap between maturity of assets and liabilities than the other two business models. Also, it has a shorter maturity profile for both its assets and liabilities than for the retail and wholesale business models. These characteristics show their effect in the results of the analysis.

![Average delta EVE per business model](image)

**Figure 3.12:** Delta EVE per shock per business model, reference rate: EONIA

In Figure 3.12 the results of the delta EVE per business model is shown. It can be seen that the delta EVE for the investment business model is almost zero compared to the other two business models. As discussed above this is mainly due to the asset and liability profile of the business model, and the short maturity profile of its assets and liabilities. The retail business model shows the largest delta EVE as it has the largest maturity mismatch.

Besides the business models, from the figure it can be derived that the parallel shocks have the greatest impact on the delta EVE figures. The last two interest rate shocks where the short rates are moved have the least impact on the delta EVE figures. This is in line with the results in Section 3.4.2 and literature where it is stated that the longer the maturity the larger the impact on the delta EVE.

Delta NII

Just as with the delta EVE results, the delta NII results of Sections 3.4.1 and 3.4.2 are aggregated in order to compare the results per business model. The average delta
NII figures per business model are given in Figure 3.13. The figure indicates differing results per business model. Where the investment business model showed the least delta EVE sensitivity to the six interest rate shocks, for the delta NII it shows the highest negative delta in shock 6: "Short interest rates down".

The retail business model also shows high negative delta NII’s for almost all the interest rate shocks, except for the "Steepener" interest rate shock. The high negative delta for the investment business model can be explained by the great part of assets and liabilities with a short maturity. Since the delta NII focuses on a three year horizon, only the assets and liabilities with a maturity of less than these three years will reprice. Whereas the investment business model has such a large part of short term assets and liabilities, the impact of the delta NII is larger for the investment business model than the other business models. For the retail business model the same reasoning applies. Except here it is only the case for liabilities, that mainly consists of short term maturities. The maturity mismatch amplifies this effect for multiple shocks.

3.5 Concluding remarks

Looking at the results in Section 3.4 can be concluded that all the three aspects tested in this chapter have an impact on the delta EVE and delta NII figures measured. The most important findings are the following:

a The duration or maturity has the largest impact on the delta EVE: as the maturity becomes longer this impact becomes larger. This is in line with previous research.
b The banking business model has an impact on the delta EVE measured. This is largely because of the difference in maturity profile between the business models, and can thus be related to the duration effect. Retail and wholesale business models are characterised by a larger mismatch in maturity of assets and liabilities, and their economic value of equity is therefore more sensitive to changes in interest rates.

c The delta NII is bigger for European curves, or curves that have lower values, than for curves that have higher values, since the floor on interest on liabilities is reached earlier. This means that the margin between assets and liabilities will decrease faster than with curves that have higher values such as the American curves used in this analysis.

These findings are based on the aggregated data of a small group of large systemic banks. What has to be incorporated in the understanding of these findings is that the results are calculated without incorporation of hedging effects. The results of the calculations can therefore be more extreme than they would be when hedging would be incorporated. Also the results are calculated on a static balance sheet assumption. Including optionality assumptions in the different interest rate scenarios might exacerbate the results discussed in the previous sections. More on these two topics can be found in Chapter 7.
Chapter 4

Constructing a model to change exposure to IRRBB

4.1 Introduction

The next step in this research is to determine how banks can change their levels of interest rate risk exposure. In Chapter 4 and 5 this step is discussed. Ultimately, in these two chapters an answer to the third research question and the main research question is given:

3. *When is the exposure to interest rate risk measured from both earnings and economic value perspectives considered optimal?*

And the main research question:

*How can banks best change their levels of exposure to interest rate risk, measured from an economic value and earnings perspective, given their risk appetite and business model?*

As stated in Section 1.4, the answer to these research questions will be found by constructing a Linear Programming (LP). The construction of the LP model will be discussed in this chapter. The results of the optimisation will be described in Chapter 5.

An LP model is an optimisation algorithm that maximises or minimises a target function while being subject to certain constraints (Winston, 2004). Th LP model in this research will optimise the balance sheet via constraints on interest rate risk measured through delta EVE and delta NII.

Literature defines two sets of LP models that are often used for optimising the balance sheet and the management of assets and liabilities:

- *Deterministic Models:* Models that use Linear Programming, assume specific realisations for random events and are computationally tractable for large problems (Zopounidis, Doumpos, and Pardalos, 2010).
• **Stochastic Models**: The use of probability constrained Linear Programming, dynamic programming and Linear Programming under uncertainty (Zopounidis, Doumpos, and Pardalos, 2010).

George B. Dantzig was in 1947 the first to lay a foundation for the simplex method that is mostly used in solving LP problems (Dantzig, 1990). Chambers and Charnes were pioneers in applying deterministic Linear Programming in the management of assets and liabilities with their model on the analysis and optimisation of bank portfolios (Zopounidis, Doumpos, and Pardalos, 2010; Kusy and Ziemba, 1986). All deterministic and many of the stochastic models applied to banking portfolios follow the approach of Chambers and Charnes’ Linear Programming model. Discounted net returns are maximised, subject to budget and liquidity constraints (Kusy and Ziemba, 1986). Deterministic models are often criticised because of their incapability to incorporate uncertainty in their models. Probability distributions can be obtained for different economic scenarios and Linear Programming formulations can be applied to each scenario to determine optimal solutions. However research has pointed out that these approaches will not generate optimal solutions to the total problem (Kusy and Ziemba, 1986).

The main goal of asset and liability management is controlling the net interest margin (Al Shubiri et al., 2010). This is why in the linear (stochastic) programming models this is often the main target function. Other target functions used in models are minimising the penalty cost when achieving a certain goal: "Goal programming" (Zopounidis, Doumpos, and Pardalos, 2010).

In an extensive stochastic Linear Programming problem that has been implemented at the Vancouver City Savings Credit Union (Kusy and Ziemba, 1986), the authors use a maximisation target function that is based on the following components: discounted returns and capital gains on assets, discounted cost of deposits and other liabilities, and expected penalty costs for the violation of constraints (Kusy and Ziemba, 1986; Oguzsoy, Gu, et al., 1997).

This part of the research is executed in cooperation with a Dutch bank. This bank provided data of their asset and liability positions over the fiscal year 2016. These positions were viewed on a time horizon of 30 years. Besides this gapping profile, information on the average coupon earned and paid on their assets and liabilities was incorporated.

In this chapter the main assumptions, data and mathematical formulation of the LP model will be covered in detail.
4.2 LP model constraints and assumptions

The purpose of the LP model is to determine how the bank can best change its levels of delta EVE and delta NII, and how this impacts the return of the bank. This is done by optimising the return of the balance sheet via the target function, while subject to certain constraints, of which delta EVE and delta NII for example. These constraints are used to steer the interest rate risk exposures. This can be done by moving the lower bound for the negative delta EVE or delta NII constraints upwards and thus constraining the IRRBB exposures. Besides the delta EVE and delta NII constraints, other constraints are incorporated in this model as well. In this section the design of the LP model, the data used and the methodology and assumptions underlying the model will be discussed.

4.2.1 LP model design

Decision variables

The bank stated that their interest rate risk position is mainly managed through their hedging portfolio and not through balance sheet restructuring. This is incorporated in the LP model by making the composition of the hedging portfolio the main decision variable in the LP model.

The previous chapter described that one of the most important factors in the determination of delta EVE and delta NII is the maturity profile of assets and liabilities. To make sure that the maturity profile can be adjusted, the number of swaps over time buckets can be changed. Suppose a bank may decide how many swaps with a maturity of one year, two years or three years it needs to obtain a certain interest rate risk profile. The decision variables are the number of swaps per maturity.

The hedging portfolio is assumed to consist out of plain vanilla interest rate swaps. A distinction is made between payer swaps: where the fixed rate is paid and the floating rate is received, and receiver swaps: where the fixed rate is received and floating rate is paid. The decision variables in this model are therefore the number of payer and receiver swaps per maturity.

Both the payer and receiver swaps have the same floating leg: the 3 month Euribor curve extracted from Bloomberg at date 30-06-2016. The swaps have also the same fixed rate leg. The values for this fixed leg per maturity are chosen such that the net present value of the swaps is 0 for all maturities. It is assumed that the swaps can be purchased with maturities ranging from 0.25 up to 30 years, the same time horizon as the balance sheet of the bank. The bank stated that its hedging portfolio is only used for hedging purposes and not for speculation on interest rate scenarios.
Chapter 4. Constructing a model to change exposure to IRRBB

Target function

The Lp model requires a target function that has to be minimised or optimised. In this research the interest is mainly on the exposure to IRRBB measured through delta EVE and delta NII. However, since the measurement of delta EVE and delta NII through the six interest rate shocks is hard to put in one linear format, the delta EVE and delta NII are therefore put as constraints in the model. A third criterion that is used to determine the optimal allocation of the swaps is then the return earned or missed through a specific solution. Therefore the target function of this LP model is to maximise return earned over the hedging portfolio while subject to constraints on interest rate risk and other constraints.

Constraints

This target function will be subject to constraints. Some of these are set due to practical considerations, other such as the interest rate risk constraints are to monitor the relationship between interest rate risk and the return made on the assets and liabilities. The following constraints will apply to the model:

a Balance sheet constraint: This constraint is set to make sure that the asset and liability side of the balance sheet remain equal.

b Decision variables volume constraint: A constraint limiting the maximum volume of the number of swaps.

c Delta EVE constraint: A constraint putting a limit to changes in EVE due to changes of the interest rate.

d Delta NII constraint: A constraint putting a limit to changes in earnings due to changes in interest rates.

e Duration of equity constraint: A constraint putting a limit on the duration of equity as stated in the risk appetite of the bank.

f Key rate or partial duration constraints: A constraint putting a limit on the partial duration or Key Rate Duration per year as stated in the risk appetite of the bank.

g Non-negativity constraint: The LP model requires all variables to be non-negative.

The first constraint ensures that the asset and liability side of the balance sheet are equal to each other. The second constraint puts a limit on the number of swaps a bank can obtain. With maximising the return as target function, the model will take an infinite number of swaps on the balance sheet as there is no limit on the number of swaps and a small positive return can be made per swap. Since the swaps are used for hedging IRRBB and not speculating on the interest rate scenarios a limit on the volume of swaps will be set in terms of notional.
The third and fourth constraint are delta EVE and delta NII constraints on IRRBB. The purpose of the LP model is to determine the optimal distribution of payer and receiver swaps given a certain risk appetite and the impact on the return. The levels of delta EVE and delta NII will be calculated through six interest rate shocks that the bank uses for their IRRBB assessment.

Constraints on the duration of equity and the Key Rate Duration per year complement the interest rate risk constraints mentioned above. The duration of equity is a key economic value measure for IRRBB and is frequently used by the bank to steer its IRRBB profile. The Key Rate Duration constraint (KRD) is set in cooperation with the bank in order to measure interest rate risk per maturity individually. These constraints will prevent the model allocating too many payer and receiver swaps to one or more specific time buckets.

The last constraint is to prevent values to have a value lower than zero. The LP model doesn’t allow variables to become negative. Short selling is however still possible since the pay-off and value of the payer and receiver swaps are the opposite of each other. If a receiver swap with maturity \( x \) has a value of \( y \), the value of the payer swap with the same maturity \( x \) has a value of \(-y\). The decision variable in the LP problem cannot be negative. Its characteristics however can be achieved by multiplying the decision variable with \(-1\).

### 4.2.2 Data used

The data provided by the bank contained information on the asset and liability positions over a time horizon of 30 years considering a run-off balance sheet. The data was available on a quarterly basis, beyond 30 years no data was available. For both assets and liabilities 5 classes are considered in this model that form the main part of the balance sheet of the bank. Due to confidentiality reasons the exact figures will not be disclosed in this thesis. The balance sheet used is given in 4.1:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Mortgages Fixed</td>
<td>1 Term Deposits</td>
</tr>
<tr>
<td>2 Mortgages Floating</td>
<td>2 Sight Deposits</td>
</tr>
<tr>
<td>3 Bonds and Non-commercial loans</td>
<td>3 Wholesale Funding</td>
</tr>
<tr>
<td>4 Receiver Swaps</td>
<td>4 Payer Swaps</td>
</tr>
<tr>
<td>5 Cash</td>
<td>5 Equity</td>
</tr>
</tbody>
</table>

**Table 4.1: Overview asset and liability classes bank’s balance sheet**

For each asset and liability class the following data was available and taken into account:

- Average coupon earned or paid over the asset or liability class;
- Distribution over maturity buckets per asset and liability class;
Chapter 4. Constructing a model to change exposure to IRRBB

- Notional value per asset and liability class;
- Average contract length per asset or liability class.

No information on the average interest rate term per asset or liability category was disclosed. Therefore in the calculations underlying the LP model the average interest rate term was assumed to be equal to the average contract length per asset or liability category.

In the model the allocation of assets and liabilities will be done on a quarterly basis, meaning that in total there are 120 time buckets where the assets and liabilities may be allocated to. This gives 120 decision variables per payer or receiver swap as there are 120 buckets to which a certain number of swaps can be assigned to. Coupons over assets and liabilities are also considered on a quarterly basis.

4.2.3 Methodology and assumptions in the LP model

In the model the delta EVE and delta NII are calculated differently than in Chapter 3. This is due to the fact that the bank uses other interest rate shocks than in Chapter 3 are used. The assumptions that are made in the calculations of delta EVE and delta NII in the LP model will be discussed in this section.

Interest rate shocks

The exposure to interest rate risk is calculated through the use of 6 interest rate shocks that the bank uses in order to determine its interest rate risk exposure. These six shocks are slightly different than the six shocks prescribed by the BCBS. In this section the six interest rate scenarios are discussed:

a. Parallel up shock of 200 basis points on the yield curve.

b. Parallel down shock of 200 basis points in the yield curve.

c. Steepener shock: Plus 200 basis points on the 10 year buckets, and minus 200 basis points on the 1 month bucket, linear interpolation of the shock size in between.

d. Flattener shock: Plus 200 basis points on the 1 month bucket and minus 200 basis points on the 10 years bucket, linear interpolation of the shock size in between.

e. Butterfly up shock: Plus 200 basis points on the yield curve in bucket 10 year and 1 month gradually decreasing to, and increasing from the 1 year point onwards.
4.2. LP model constraints and assumptions

f Butterfly down shock: Minus 200 basis points on the yield curve in buckets 10 year and 1 month, and gradually increasing to, and decreasing from the 1 year point on wards.

These six interest rate shocks are different from the six shocks used in Chapter 3, but are the interest rate scenarios used by the bank. The difference is the way the steepener and flattener shocks are calculated. For the steepener shock the shortest rate is decreased with -200 basis points, and the longest rate is increased with +200 basis points. For the time buckets in between the change in interest rate is calculated via linear interpolation. For the flattener interest rate scenario this is the other way around. This is more simplistic as the way it was determined in Chapter 3 where scalars where used in orde rto determine the decay of the shock over time.

Additionally, where the BCBS uses "short rate up" and "short rate down" shocks, the bank uses "butterfly up" and "butterfly down“ shocks. The butterfly shocks have except for a shock of ±200 bp on the rate with the shortest maturity also the same shock on the longest maturity with linear interpolation for maturities in between. The one-year time bucket is the rotation point of this shock type.

Min-max theory

For each of these six interest rate shocks the delta EVE and delta NII will be calculated. The values of these interest rate exposures will be constrained by a lower bound. This lower bound will be the same for all these six shocks. So the lower bound of the delta EVE by a parallel up shock will be the same as all the other 5 shocks. This enables a min-max theory where the maximum loss in change of eco-nomic value is minimised for all of these six shocks. For example: suppose that the lower bound for delta EVE is set at -15 percent. Results might show a parallel up shock causes the lowest negative delta of -14.5 percent, while the other shocks have higher values. When moving the lower bound of the delta EVE constraint to -13 percent, the LP model optimises the mixture of swaps such that the lowest negative delta EVE falls within this boundary. In the new situation, results might show that delta EVE for the parallel up shock is -12 percent, and the steepener shock has a value -13 percent. This is the min-max theory. By gradually moving the lower bound upwards the maximum negative delta EVE value can be minimised.

The same methodology is used for the delta NII constraint.

Assumptions in calculation of delta NII

For the delta NII calculation, some assumptions are made. In contrast to the delta EVE calculation which assumes a run-off balance sheet, the delta NII assumes a constant balance sheet where assets and liabilities that mature are replaced with
items with identical features. However, there are some assumptions underlying this method. These will be discussed in this section.

**Time horizon:** The time horizon is one of the first assumptions of importance. In the analysis, the delta NII is calculated on a time horizon of one year. This is in line with current practice of the bank and the paper of the BCBS (BCBS, 2016a). Since the delta NII over two-years might be different than the delta NII measured over one year in the model, the delta NII is also calculated over a time-horizon of two years in order to compare the results. **Repricing features:** The assumption on the repricing moments of the assets and liabilities in the delta NII computation is also of importance. As stated in Section 4.2.2, data on the distribution of assets and liabilities over 30 years is used to determine the repricing moments. Here the interest rate term and contractual maturity are assumed to be equal. Therefore the contractual maturity is considered as the repricing moment of the asset or liability.

The new rate at which an asset or liability reprices is determined as follows: the height of the new rate is the normal coupon plus the interest rate shocks. For instance an asset with a coupon of 5%, is assumed to be 5% + 2% = 7% after a parallel up shock.

For non-parallel shocks, as for instance the "Steepener" shock, the average contract length of an asset class is used to determine the repriced rate of the shock. For non-parallel shocks these rates differ with maturity. For instance: the shock size for the "Steepener" shock the maturity of 1 month is -200 bp, but for two years it is +22 bp. The average contract length of an asset or liability class is used to determine the maturity of the new repriced item and the corresponding shock size. Take for instance an asset with average contract length of 2 years, that reprices within one year and has a coupon of 5%. The new coupon is then set at 5% + 0.22% = 5.22%

**Two year delta NII:** For the delta NII with a time horizon of two years the repricing is carried out in the same way as for the delta NII with the horizon of one year, but then two times. Here the asset and liabilities can be categorised in three categories:

a. An asset or liability that reprices in the first year and earns two shocked coupons;
b. An asset or liability that reprices in the second year and earns a normal coupon plus a shocked coupon;
c. An asset or liability that doesn’t reprice within two years and earns two initial coupons.

The shocked coupons are determined in the same way as for the one year delta NII. In order to match market practices the cash flows in the delta NII calculations are not discounted. In order to determine the two year delta NII the change in NII over 2 years is calculated relative to the two year NII forecast of the base scenario. The NII in the base scenario consists of assets and liabilities of the third category only.
4.3 Mathematical formulation of the LP problem

The mathematical formulation of the target function and the constraints is elaborated in this section. The LP model requires all constraints to be in a linear format. Since some computations of these constraints are not linear such as for instance the duration of equity and Key Rate Duration constraints, these have to be rewritten to linear form to fit the format of the LP model. The derivation of the linear constraints as they are stated in this chapter and the following sections can be found in Appendix B.

4.3.1 Target function

The target of the model is, as stated in section 4.2, to maximise the return earned on payer and receiver swaps plus a constant of the return on assets and liabilities of the original balance sheet. Therefore the target function of the LP model can be defined as the interest earned or paid on receiver swaps plus the interest earned or paid on payer swaps. Since the interest rate swaps are the only decision variables in the model, this can be stated as follows:

\[
\text{Max } z = \sum_{t=1}^{120} (x_t R_t) + \sum_{t=1}^{120} (y_t c_t) + C 
\]

(4.1)

Where:

\(x_t\) = The number of receiver swaps in time bucket \(t\)

\(y_t\) = The number of payer swaps in time bucket \(t\)

\(R_t\) = The interest rate earned over receiver swaps allocated to time bucket \(t\)

\(c_t\) = The interest rate paid over the amount of payer swaps allocated to time bucket \(t\)

\(t \in (1, 120)\) = The number of time buckets, divided quarterly over 30 years

\(C\) = A constant representing the interest income earned over the assets and liabilities other than the interest rate swaps

The coupons \(R_t\) and \(c_t\) are the first year cash flows of the receiver and payer swaps. Although the values of the swaps are all 0, the first year cash flows may be non-negative (Hull, 2012b). Therefore, in the first year NII a positive or negative cash flow may result from a position in payer or receiver swaps. These coupons originally are considered to be quarterly since the data of the bank is also on a quarterly basis. In order to display the return over one year these coupons are multiplied by four.
Chapter 4. Constructing a model to change exposure to IRRBB

4.3.2 Balance sheet constraint

The balance sheet constraints are set to keep the two sides of the balance sheet equal. Since only the interest rate swaps are decision variables and the rest of the balance sheet is kept fixed, the total value of the receiver swaps on the asset side must equal the total value of the payer swaps on the liability side of the balance sheet:

\[ \sum_{t=1}^{120} V_t x_t - \sum_{t=1}^{120} V_t y_t = 0 \]  \hspace{1cm} (4.2)  

with: \( V_t \) = the value of the swap in time bucket \( t \).

As the value of the swaps is set at or close to 0 it will be expected that this constraint has a minimum effect on the optimal solution.

4.3.3 Hedging portfolio volume constraints

In order to prevent the LP model to take too many swaps on the balance sheet as a result of the maximisation of the target function, a limit on the total volume of payer and receiver swaps is set. This limit is set on the total volume of the notional of the payer and receiver swaps. With each swap having a notional of 2 million euros, the number of payer and receiver swaps is both limited at 30. With a total balance sheet volume of 100 million the notional for payer and receiver swaps is limited at 60% of the balance sheet. This value is chosen as it represents a large part of the balance sheet and therefore allows the hedging portfolio to be sufficiently large to hedge all risks. But at the same time this limit prevents the balance sheet to become unrealistically large. This gives two constraints. One for the receiver swaps:

\[ \sum_{t=1}^{120} x_t \leq 30 \]  \hspace{1cm} (4.3)  

And one for the payer swaps:

\[ \sum_{t=1}^{120} y_t \leq 30 \]  \hspace{1cm} (4.4)  

4.3.4 Delta EVE constraint

The delta EVE constraint is the fourth constraint that will be discussed. According to literature the change in economic value of equity can be calculated as follows (BCBS, 2015; BCBS, 2016a; EBA, 2015):
\[ \Delta \text{EV}_E^k = (NPV_{\text{Assets},k} - NPV_{\text{Liabilities},k}) - (NPV_{\text{Assets},0} - NPV_{\text{Liabilities},0}) \quad (4.5) \]

With \( k \in (1, 6) \) the indicator of the interest rate shock and 0 the basis interest rate scenario. Equity is left out of the equations as it is the difference between the assets and liabilities. This gives then following constraint:

\[
\begin{align*}
\Delta \text{EV}_E^k = & \sum_{i=1}^{5} \sum_{t=1}^{120} \left( \left( \frac{1}{1 + r_{k,t}} \right)^t + \left( \sum_{t^* = 1}^{t} \frac{1}{1 + r_{k,t^*}} R_{k,t^*} \right) - \left( \frac{1}{1 + r_{0,t}} \right)^t + \left( \sum_{t^* = 1}^{t} \frac{1}{1 + r_{0,t^*}} R_{0,t^*} \right) \right) x_{i,t} - \\
& \sum_{i=1}^{5} \sum_{t=1}^{120} \left( \left( \frac{1}{1 + r_{k,t}} \right)^t + \left( \sum_{t^* = 1}^{t} \frac{1}{1 + r_{k,t^*}} c_{k,t^*} \right) - \left( \frac{1}{1 + r_{0,t}} \right)^t + \left( \sum_{t^* = 1}^{t} \frac{1}{1 + r_{0,t^*}} c_{0,t^*} \right) \right) y_{i,t} \geq -25\% 
\end{align*}
\] (4.6)

The value for the right-hand side of the equation is here chosen at random. In the following chapters the right-hand side of this equation will be used in order to obtain different values for delta EVE.

The derivation of the delta EVE constraint can be found in Appendix B section B.2

### 4.3.5 Delta NII constraint

Besides the delta EVE constraint, the delta NII constraint is an important factor in this model as it constrains the interest rate risk measured from an earnings perspective. According to literature, the change in net interest income can be calculated as follows (BCBS, 2015; BCBS, 2016a; EBA, 2015):

\[ \Delta \text{NII}_k = \text{NII}_k - \text{NII}_0 \quad (4.7) \]

With \( k \in (1, 6) \) the indicator of the interest rate shock applied, and 0 being the base scenario.

Negative values of the delta NII will indicate a decrease in NII due to a change in interest rates, and positive values an increase in NII.

Since the horizon of delta NII is one year, the shock in interest rates will only be applied to the first 4 time buckets, representing the four first quarters. The other
buckets will remain unchanged since assets and liabilities with a maturity of more than one year will not reprice within the one year at which the delta NII is assessed and therefore will keep their original coupon.

Since the LP model requires the constraints to be in linear format, rewriting gives the following result:

$$\Delta NII_k = \sum_{i=1}^{5} \sum_{t=1}^{4} \left( (R_{k,t} - R_{0,t})x_{i,t} - (c_{k,t} - c_{0,t})y_{i,t} \right) \geq -10\%$$  (4.8)

Again the right-hand side of this equation is chosen at random. In the following chapters this right-hand side will be varied in order to obtain different results for delta NII.

### 4.3.6 Duration of equity constraint

Duration of equity is a widely used risk measure for the exposure of a portfolio to movements in the yield curve. Duration measures the sensitivity of percentage changes in an asset’s or liability’s price to changes in its yield (Hull, 2012a). A duration of equity constraint is incorporated in the LP model since it is part of the risk appetite of the bank and since it is an indication of the economic value sensitivity of the balance sheet. The duration of equity can be viewed as the difference in duration of assets and liabilities (Cummins et al., 2009):

$$DoE = Duration_{Assets} - \frac{assets}{liabilities} \cdot Duration_{Liabilities}$$  (4.9)

The duration of an asset or liability can also be viewed as the weighted average of all the cash flows of assets and liabilities multiplied with the time they occur (Hull, 2012a).

These weights are calculated by dividing the cash payment at time $t$ with the total net present value of the asset or liability (Hull, 2012a). The value of the boundary for the duration of equity constraint is given by 2.1. For confidentiality purposes this figure can not be disclosed but replaced by an indicative figure. The linear format of this equation is given in the following formula:

$$Duration = \sum_{i=1}^{n} t \left( \frac{R_t}{(1+r_{0,t})^t} \right)$$  (4.10)

In the LP model the constraint will be defined as:
4.3. Mathematical formulation of the LP problem

\begin{equation}
D\text{OE} = \left( \sum_{t=1}^{120} \left( \left( R_{i,t} \frac{1}{(1+r_{0,t})^t} \right)(t-2.1) \right) + \frac{1}{(1+r_{0,t})^t}(t-2.1) \right) x_{i,t} - \left( \sum_{t=1}^{120} \left( \left( R_{i,t} \frac{1}{(1+r_{0,t})^t} \right)(t-2.1) \right) + \frac{1}{(1+r_{0,t})^t}(t-2.1) \right) y_{i,t} \leq 0
\end{equation}

(4.11)

The mathematical derivation of equation 4.11 can be found in Appendix B, section B.2.1.

4.3.7 Key Rate Duration constraint

In order to constrain the Key Rate Duration per year, a set of 30 Key Rate Duration constraints is constructed to prevent peaks of allocations of swaps in specific time buckets. Key Rate Duration is defined as the change in value of assets and liabilities due to a shock to the yield curve at a specific maturity (Hull, 2012a). It is given as:

\[ KRD_T = -\frac{1}{P} \frac{\Delta P_T}{\Delta r_T} \]

Where \( P \) is the total price or value of an asset/liability, and \( \Delta P_T \) is the corresponding change in value of the asset. Just like the duration constraint and the calculation of the delta EVE constraint, the \( \Delta P_T \) is calculated as the difference between the original price and the price after the change in value due to the small shock of the interest rate: \( \Delta P_T = P_T - P_0 \). The small change in interest rates \( \Delta r_T \) is determined as 1 basis point, or 0.01 percent. Any change could be chosen here. However, due to convexity with greater changes a small change of 1 basis point is chosen (Hull, 2012a).

This change is applied to a one year period of the yield curve. Since the assets and liabilities can be allocated to a total of 120 time buckets (30 years), there are 30 Key Rate Duration constraints where for each constraint a different period of one year on the yield curve is shocked. Each of these constraints then have to comply with equation 4.12.

In the risk appetite statements of the bank, values for the bounds on the KRD constraints were given. This information is confidential, therefore the following value is chosen to replace it: 1.5. This gives the following Key Rate Duration \( KRD_T \) with \( T \) the year on the yield curve that is shocked to determine the Key Rate Duration:
Chapter 4. Constructing a model to change exposure to IRRBB

\[ KRD_T = -\frac{1}{P} \frac{\Delta P_T}{\Delta r_T} \leq 1.5 \]  \hspace{1cm} (4.12)

Literature considers liabilities as a short position in a bond (Hull, 2012b). In line with the duration of equity calculations, the net Key Rate Duration position per year is defined as:

\[ KRD_T = KRD_{assets,T} - KRD_{liabilities,T} \]  \hspace{1cm} (4.13)

Implementing equations B.30, B.31 and B.2.2 in equation 4.13 for \( T = 1 \) gives the equation for the Key Rate Duration of assets, where:

\[
KRD_{assets,1} = \left( \sum_{i=1}^{5} \left( \sum_{t=1}^{4} \frac{1}{(1 + r_{0,t} + \Delta r_{t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*} + \Delta r_{t^*})^t c_{k,t^*}} \right) \right) + \right.
\sum_{i=1}^{5} \sum_{t=4}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*})^t c_{k,t^*}} \right) \right) \right) \left( \sum_{i=1}^{5} \sum_{t=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*})^t c_{k,t^*}} \right) \right) (1 + \Delta r_{t,XX}) \right) x_{i,t} 
\]  \hspace{1cm} (4.14)

and:

\[
KRD_{liabilities,1} = \left( \sum_{i=1}^{5} \left( \sum_{t=1}^{4} \frac{1}{(1 + r_{0,t} + \Delta r_{t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*} + \Delta r_{t^*})^t c_{k,t^*}} \right) \right) + \right.
\sum_{i=1}^{5} \sum_{t=4}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*})^t c_{k,t^*}} \right) \right) \right) \left( \sum_{i=1}^{5} \sum_{t=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{t^* = 1}^{t} \frac{1}{(1 + r_{0,t^*})^t c_{k,t^*}} \right) \right) (1 + \Delta r_{t,XX}) \right) y_{i,t} 
\]  \hspace{1cm} (4.15)

4.3.8 Hedging portfolio

The decision variables in this model are the number of payer and receiver swaps to buy. As was stated in previous sections, these swaps are used by the bank to manage its interest rate risk exposure. In order to give some more information on the assumptions and calculations underlying these payer and receiver swaps, they are discussed here in this section.
4.3. Mathematical formulation of the LP problem

A swaps has two legs: one floating leg and a fixed leg. The floating leg of the swap is based on the 3 month Euribor forward curve at 30-06-2016. This curve is annually compounded. The fixed leg is chosen such that the value of the swap for each maturity is 0 or close to 0. The both legs of the swaps are discounted using the OIS curve at date 30-06-2016.

Calculating the value of the interest rate swap

In the calculation of delta EVE the value of swaps has a big impact. When interest rates rise, the rates of the floating leg will rise at the same time, while the fixed rates remain fixed. This results in a difference in earnings and thus in a higher value for payer swaps and a lower value for receiver swaps. The opposite holds for decreasing interest rates. The change in value of the swaps will be incorporated in the delta EVE and delta NII computations to model the effect.

The value of an interest rate swap can be calculated according to the following two approaches:

a Valuation in terms of Bond prices: The value of the swap is the difference between a fixed and a floating bond that represents the fixed and floating legs of the swap (Hull, 2012b).

b Valuation in terms of Forward Rate Agreements (FRA’s): The value of the swap is calculated as the value of a portfolio of FRA’s (Hull, 2012b).

In this research the second approach is used: valuation in terms of FRA’s. A Forward Rate Agreement is a contract where the specifics of a certain interest rate, applied to either borrowing or lending a certain principle in the future, are determined (Hull, 2012b). An example of an FRA is the situation where a company agrees that it receives a fixed rate of 4% on a principal of 100 million for a 3-month period starting in three years and in exchange pays LIBOR. If the 3-month LIBOR is 4.5% for the three month period, the cash flow to the lender will be: 100,000,000 \times (0.04-0.045) \times 0.25 = -$125,000 (Hull, 2012b).

A swap can be viewed as a portfolio of FRA’s, since in an interest rate swap a certain interest rate applies to a certain principal at multiple times in the future (Hull, 2012b). Key in the valuation of an FRA is the assumption that forward interest rates are realised. The interest rate swap therefore also makes use of this assumption.

The first step is determining the forward curve that applies to the floating leg. In this case the Euribor curve extracted from Bloomberg is the forward curve that can be applied. The second step is to use the assumption that floating rates equal the Euribor forward rates to calculate the cash flows of the two legs of the swap. In the final step the swaps cash flows are discounted in order to determine the value. This procedure is given in the following formula:
Chapter 4. Constructing a model to change exposure to IRRBB

\[ \text{Value}_{\text{payerswap}}_{k,t} = \sum_{t=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} \text{Euribor}_{k,t} \right) - \left( \frac{1}{(1 + r_{0,t})^t} \text{Fixed}_t \right) \]  (4.16)

Where \( \frac{1}{(1 + r_{0,t})^t} \) is the discount factor that applies, \( \text{Euribor}_{k,t} \) is the realised Euribor forward rate under interest rate scenario \( k \) and maturity \( t \), and \( \text{Fixed}_t \) the value of the coupons of the fixed leg.

Discounting all cumulative cash flows earned and paid for each maturity and determining the net result will lead to the value of the swap. The same holds for the receiver swap, but then the legs have to be switched.

An example of a receiver swap with a maturity of 1 year is given in Table 4.2. The fixed coupon is set at 0.00029, the notional is 1 euro and the floating leg is assumed to equal the 3M Euribor rate.

The Euribor 3M forward rates are -0.00153, -0.00159, -0.00165 and -0.00166 for 0.25, 0.5, 0.75 and 1 year respectively with quarterly compounding. The cash flow for the first quarter is determined by: \(-0.00153 \times 0.25 = -0.000383\) euros.

Discounting these net cash flows gives a total value of 0.00278 for this receiver swap.

Table 4.2: Valuation of a 1 year swap in terms of FRA's

<table>
<thead>
<tr>
<th>Time</th>
<th>Fixed Cash flows</th>
<th>Floating Cash flows</th>
<th>Net Cash flows</th>
<th>Discount Factor</th>
<th>Present Value net Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.00029</td>
<td>-0.000383</td>
<td>0.00067</td>
<td>1.0018</td>
<td>0.00067</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00029</td>
<td>-0.000397</td>
<td>0.00069</td>
<td>1.0028</td>
<td>0.00069</td>
</tr>
<tr>
<td>0.75</td>
<td>0.00029</td>
<td>-0.000412</td>
<td>0.00070</td>
<td>1.0038</td>
<td>0.00071</td>
</tr>
<tr>
<td>1</td>
<td>0.00029</td>
<td>-0.000412</td>
<td>0.00071</td>
<td>1.0049</td>
<td>0.00071</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.00278</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculating the earnings of the swaps

The earnings received on the payer swaps is the difference in the interest earned on the floating leg minus the interest paid on the fixed leg. Since the earnings are only incorporated in the delta NII measure, the horizon of the cash flows is one year. This gives the following equation:

\[ Earnings_{\text{payerswap}}_{k,t} = \text{Euribor}_{k,t} - \text{Fixed}_t \]  (4.17)

The same holds for the receiver swap but then the other way around. This gives subsequently:

\[ Earnings_{\text{receiverswap}}_{k,t} = \text{Fixed}_t - \text{Euribor}_{k,t} \]
4.4 Concluding remarks

In this chapter the construction of the LP model is discussed. The target function of the LP model is to maximise the return earned over payer and receiver swaps. The target function is constrained by delta EVE, delta NII, (key rate) duration and balance sheet constraints.

The design of the LP model gives an answer to the third research question: 3. *When is the exposure to interest rate risk measured from both earnings and economic value perspectives considered optimal?* The model uses boundaries for the levels of delta EVE and delta NII measured. The height of these boundaries differ per bank and their risk appetite. In this research the min max theory will be used together with sensitivity analysis in order to determine how low the boundaries of these constraints can be set. The answer to the third research question is then that the exposure to IRRBB is considered optimal as it its minimum. The target of the LP model is however to maximise the return over the hedging portfolio. In that case the most optimal solution would be the solution that maximises the return given the IRRBB constraints.
Chapter 5

Results LP problem

5.1 Introduction

In this chapter the results of the optimisation of the hedging portfolio through the LP problem will be discussed as they present the key findings of this thesis. When discussing these results, an answer to the main research question will be provided.

Before describing the results of the LP problem outlined in Chapter 4, this chapter will start with a simple example to explain the LP model. This example will show how the LP model selects the optimal solution and how the results should be interpreted. Next, the results of the optimisation of the hedging portfolio will be described. Here the lower boundaries of delta EVE and delta NII are increased simultaneously in order to confine the delta EVE and delta NII exposure in the model. For each combination of values for these lower bounds the return will be plotted in a surface. In the trade-off surface that results from this operation, several points will be compared with each other on levels of delta EVE, delta NII and return achieved. Also these levels will be compared to the values of the original balance sheet. Further it will be described how these levels of IRRBB exposure are achieved by examining the composition of the hedging portfolio.

The LP model has a constraint on the one-year delta NII, meaning that earnings risk is limited by the model over a one year horizon. However, the earnings risk for longer time horizons is not limited which may cause a unilateral view on the delta NII. The last section of this chapter addresses this problem and compares the results of the one-year delta NII with the two-year delta NII.

5.2 Example of LP model and results

In the previous chapter it became clear that the LP model is quite complex and that understanding the results can be difficult. The target function of the model is to maximise the return while subject to multiple constraints on IRRBB. The levels of delta EVE and delta NII are calculated through with six interest rate shocks which
makes the interpretation of the outcome of the levels of delta EVE and delta NII even more complex. This since the six interest rate shocks each have a different impact on the delta EVE and delta NII levels. In this section therefore a simple graphical example of the LP model is given in order to aid the understanding of the LP model and the results in the next sections.

In this example a simple balance sheet is taken consisting out of one loan with a maturity of five years. The loan has a notional of 100 million euros and a yearly coupon of 3.9%. After five years the notional is paid back to the bank. In this example the decision variables of the model are the number of payer or receiver swaps with a maturity of five years to take on the balance sheet. The model is constrained by the maximum level of negative delta EVE and delta NII, measured through the six interest rate shocks described in Section 4.2.3.

In Figure 5.1 the amount of return, delta EVE and delta NII for this simple example is given per number of payer swaps. In this figure a negative amount of payer swaps represents the number of receiver swaps as they have opposite characteristics.

The green line represents the one year return earned over the combination of the loan and the number of payer swaps. This line shows a decreasing trend when the number of payer swaps increases. This is due to the fact that the earnings from payer swaps in the first year are negative, meaning that the more payer swaps are taken on the balance sheet, the more the return decreases. This is the other way around when receiver swaps are taken on the balance sheet.

The blue and yellow lines show the levels of minimum delta EVE and delta NII resulting from the loan and the number of swaps, measured through the six interest
rate shocks. In the figure the delta NII is zero at the moment that there are no swaps taken on the balance sheet. The delta NII decreases when either more payer or more receiver swaps are taken on the balance sheet. Since the loan reprices after five years and the delta NII is measured over a one-year time horizon, the loan has no earnings risk within this one-year time horizon. This means that when there are zero swaps on the balance sheet the delta NII is zero. The payer and receiver swaps do have an impact on the delta NII. Increasing the number of swaps, either payer or receiver swaps, have both a negative impact on the delta NII measured from this simple example.

In contrast to the delta NII, the delta EVE of the loan is not zero. This is due to the fact that a change in interest rates has an impact on the value of the loan when the cash flows are discounted. From Figure 5.1 can be derived that the minimum level of delta EVE is at maximum -2.56 million euro. In order to achieve this level of delta EVE, 85 payer swaps with a notional of 1 million euros have to be taken on the balance sheet. The delta EVE is just as the delta NII measured through the use of six interest rate shocks. It can be seen that the delta EVE increases as the number of swaps move from receiver to payer. Until the point of 85 payer swaps. From this point the delta EVE decreases with the number of payer swaps. This is due to the calculation of the delta EVE as the minimum of the six interest rate shocks. Until the point of 85 swaps the minimum delta EVE was caused by the delta EVE measured from the parallel up shock. After this point the delta EVE measured from the Steepener shock is lower than the delta EVE from all the other shocks, causing the delta EVE to decline again.

In order to show how the LP model determines how many swaps should be taken on the balance sheet, the simple example is given constraints on the level of delta EVE and delta NII. For both delta EVE and delta NII a maximum negative delta EVE or delta NII of -5 million euros is taken. In Figure 5.2 the feasible region of the model is shown as the area between the two black vertical lines. Since the delta EVE in this simple example is far lower than the delta NII, the feasible region is between the two black vertical lines showing the points at which the delta EVE of the portfolio of the loan and the swaps is -5 million euros. Since the delta NII is smaller than the delta EVE it has no impact on the feasible region.

The LP model determines just like in Figure 5.2 what its feasible region is given its constraints. Within this feasible region it wants to maximise its target function, the return earned over the portfolio of the loan and swaps. The model can take a number of payer swaps on the balance sheet ranging from 57.6 to 128 to stay within this feasible region. The model chooses to take 57.6 payer swaps on the balance sheet as this maximises the return and keeps the delta EVE at -5 million exactly.

In Figure 5.3 the same simple example is given, but the constraint for delta NII is changed. Where it previously was constrained at -5 million, in the new situation it
Chapter 5. Results LP problem

Figure 5.2: Return, delta EVE and delta NII per number of payer swaps with feasible region

is constrained at -2.5 million. The delta EVE constraint is left unchanged. The figure indicates that the feasible region has narrowed. The right boundary for the feasible region has moved to the left since the level of delta NII is lower than -2.5 million on the right side of the boundary. The left boundary remained at 57.6 payer swaps since here the delta EVE level is -5 million. The outcome of the model hasn’t changed since the maximum return can still be achieved by taking 57.6 payer swaps on the balance sheet.

Figure 5.3: Return, delta EVE and delta NII per number of payer swaps with adjusted feasible region
5.3 Changing exposure to both delta EVE and delta NII

In this simple example the decision variable was the number of payer or receiver swaps to take on the balance sheet with a maturity of 5 years. In the example the effect of all the number of payer and receiver swaps on the return, delta EVE and delta NII is examined in order to decide what the best option is. This becomes a more difficult exercise when there are more options allowed, for instance swaps for multiple maturities. All the combinations have to be examined in that case, which with 120 maturity options for both payer and receiver swaps is virtually impossible to solve manually. Besides the larger number of decision variables, there are also more constraints in the model that are taken into account. This makes the LP solving methodology a better tool for optimising the balance sheet. The idea of the LP optimisation tool is the same as in the simple example described above, but has better and more efficient ways to compare all possible solutions.

5.3 Changing exposure to both delta EVE and delta NII

After the simple example the next step is to discuss the results of the optimisation problem as outlined in Chapter 4. In this optimisation problem the data of the bank is loaded in order to determine the trade-off between the delta EVE, delta NII and the return. This data is used for two reasons: the first is that it is much more granular than the data that can be obtained from open sources like annual reports. The second reason is that it provides an opportunity for a case study. Before optimising the hedging portfolio first the initial IRRBB exposure of the bank is discussed.

5.3.1 Initial IRRBB exposure

Based on the bank’s balance sheet data, the initial levels of delta EVE and delta NII are calculated for the six interest rate shocks described in Section 4.2.3. As mentioned in Chapter 4 the data provided by the bank is not disclosed in this thesis due to confidentiality reasons. In Table 5.1 the values of delta EVE and delta NII for the initial balance sheet are given. The results indicate that the bank is less sensitive to parallel shocks than to rotating shocks, especially regarding the delta NII. For parallel shocks it denotes values of delta NII of $\pm 0.03\%$ where this is much higher for rotating shocks steepener and flattener: $\pm 6.25\%$. This can be explained by the relatively longer duration for liabilities on the bank’s balance sheet. This means that assets reprice earlier than the liabilities leading to a decrease of the NII in case short rates change.

The delta EVE figures show that for a parallel down shock the EVE decreases with 1.74 percentage points. This is in line with the expectations as the data made available by the bank discloses a duration of equity that is in line with these values.
Table 5.1: Overview of initial delta EVE and delta NII on the bank’s balance sheet

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Delta EVE</th>
<th>Delta NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Up</td>
<td>1.41%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Parallel Down</td>
<td>-1.74%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Butterfly Up</td>
<td>-0.81%</td>
<td>6.25%</td>
</tr>
<tr>
<td>Butterfly Down</td>
<td>0.51%</td>
<td>-6.25%</td>
</tr>
<tr>
<td>Steepener</td>
<td>1.83%</td>
<td>-6.17%</td>
</tr>
<tr>
<td>Flattener</td>
<td>-2.13%</td>
<td>6.17%</td>
</tr>
</tbody>
</table>

Just as for delta NII, for delta EVE the bank is more sensitive to non-parallel shocks than to parallel shocks. The delta EVE measured through the flattener shock is lower than for the parallel down shock meaning more decrease in economic value. This can be explained by the maturity profile of the assets and liabilities on the balance sheet. The longer duration for liabilities leads to a larger decrease in value in the case of rising interest rates for liabilities than assets. So a rotating shock, where the short rates increase and the long rates decrease, leads to a decrease in value of assets and an increase in value of liabilities. Resulting in a lower EVE. In the case of parallel shocks this effect is less strong as the assets increase or decrease in value at the same time as the liabilities increase or decrease in value. This shows the importance of the measurement of IRRBB through the use of multiple shocks since the parallel shocks only don’t give a complete view on the exposure to IRRBB.

5.3.2 Changing IRRBB exposure of the bank

The exposure to IRRBB as given in Table 5.1 can be changed by increasing the lower boundaries of the delta EVE and delta NII constraints. In this case the model searches for a combination of payer and receiver swaps to take on the balance sheet such that it satisfies these new constraints. Releasing the constraints of delta EVE and delta NII allows more exposure to IRRBB on the balance sheet.

In the following sections the impact of higher and lower values for the boundaries of the delta EVE and delta NII constraints will be discussed. Here the impact of different values for these boundaries on the composition of the hedging portfolio and the level of return is examined. This will be done in two steps: first the trade-off surface between boundary levels for the delta EVE, delta NII and the return will be constructed. Secondly, different points in the surface will be compared in order to determine the possible strategies for the bank and how to achieve them.

Effect payer and receiver swaps on return

Before going through the results, a change in target function is discussed. During the analysis it appeared that the way the return on the hedging portfolio was calculated
5.3. Changing exposure to both delta EVE and delta NII

did not lead to satisfying results. In the previous chapter the return over new swaps was calculated as the difference in fixed and floating leg cash flows over the first year. When optimising the balance sheet, this leads to very large results regarding the number of swaps in the optimal solution. This is illustrated in the following example.

In Figure 5.4 the number of payer and receiver swaps is given after the LP model has determined the optimal solution given the constraints. The limit on the volume of the swaps is omitted in this example. As one can see the number of payer and receiver swaps is very large. Considering that each swap has a notional of 2 million euros, this number of swaps is virtually impossible as the bank would blow up its balance sheet with just swaps.

![Figure 5.4: Composition of hedging portfolio over maturity with original target function](image)

While the swaps have an initial value of zero, the cash flows in the first year may be non-negative. Especially for swaps with longer maturities this difference may be larger as more cash flows take place in the future. Taking the cash flows of the first year into account in the return is doubtful for two reasons. The first is that it is very short minded. Keeping in mind that the value of all the swaps initially set at zero means that the cash flows for the same swap in the future may be different and even negative. Secondly the purpose of the LP model was to determine the composition of swaps needed to hedge the IRRBB and not to speculate on the interest rate scenarios.

Figure 5.4 indicates that the model finds a way of satisfying the constraints while increasing the number of swaps to a very large number. By offsetting the maturities a small positive difference in return can be made. From the model’s point of view
this is regarded as "free" return, and therefore it keeps taking these swaps on the balance sheet as long as the constraints are not violated.

In order to overcome this problem and to model the impact of swaps on the return in a more realistic way, it is assumed that for new swaps a trading cost is incurred. The previous way of determining the impact of the swaps on the return is omitted. These trading costs are a more realistic approach for the reason that when going to the market for swaps trading costs also have to be paid. The second reason is that in this way the model doesn’t take any more swaps on the balance sheet than necessary. Since the target function of the LP problem is to maximise the return, the model only takes as many swaps on the balance sheet as necessary to satisfy the IRRBB constraints. In this case swaps aren’t taken on the balance sheet anymore from a speculative point of view and the LP model therefore provides a more realistic hedging solution than before.

In order to determine the trading costs that have to be incurred in the analysis, literature was reviewed. In a report published by Greenwich Associates in 2015 a total cost analysis of interest rate swaps is conducted. The costs of making a trade are divided in the following four categories (Greenwich, 2015):

a. Liquidity costs: The bid-ask spread for the given instrument.

b. Initial margin and funding costs.

c. Futures Commission Merchant (FCM) fees: execution, clearing and capital usage fees.

d. Central Counter Party clearing House (CCP) fees: Exchange, execution and clearing fees charged by relevant clearinghouses.

In multiple interviews with over 40 market participants Greenwich Associates estimated the average costs for buying or selling an interest rate derivative. In Table 5.2 the total cost analysis of swaps per duration is given. The costs are given for trading a swap with a notional of 1 million euros (Greenwich, 2015).

<table>
<thead>
<tr>
<th>Duration of contract</th>
<th>2 Years</th>
<th>5 Years</th>
<th>10 Years</th>
<th>30 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margin/funding</td>
<td>€2,631.00</td>
<td>€7,433.00</td>
<td>€15,051.00</td>
<td>€33,434.00</td>
</tr>
<tr>
<td>CCP</td>
<td>€2,900.00</td>
<td>€4,900.00</td>
<td>€8,400.00</td>
<td>€12,900.00</td>
</tr>
<tr>
<td>FCM</td>
<td>€5,592.00</td>
<td>€10,288.00</td>
<td>€17,736.00</td>
<td>€35,711.00</td>
</tr>
<tr>
<td>Liquidity</td>
<td>€10,739.00</td>
<td>€26,688.00</td>
<td>€49,666.00</td>
<td>€113,452.00</td>
</tr>
<tr>
<td>Total</td>
<td><strong>€21,862.00</strong></td>
<td><strong>€49,309.00</strong></td>
<td><strong>€90,853.00</strong></td>
<td><strong>€195,497.00</strong></td>
</tr>
<tr>
<td>BP</td>
<td>2.1862</td>
<td>4.9309</td>
<td>9.0853</td>
<td>19.5497</td>
</tr>
</tbody>
</table>

Table 5.2: Total cost analysis interest rate swap over time

From Table 5.2 can be derived that the costs in trading an interest rate derivative are relatively small. From the table also can be derived that the costs of trading an interest rate swap increase with the duration of the contract. This is in line with the literature of the BCBS stating that the market requires a liquidity and duration
5.3. Changing exposure to both delta EVE and delta NII

The liquidity spread is a premium representing the market appetite of buyers and sellers. The market duration spread is the premium that the market requires for products that have more risk since the cash flows take place further into the future (BCBS, 2016a).

In this research the costs given in Table 5.2 are used to determine the trading costs per trade. The number of basis points is multiplied of the notional of the interest rate swap. In the model outlined in Chapter 4 the notional per swap is 2 million euros. In the report the costs are given for 4 maturities. In order to determine the costs of trading the swaps for the maturities in this research, ranging from 0.25 to 30 years, linear interpolation is used.

General results changing exposure to delta EVE and delta NII

In this section the results of the LP model will be described. In order to determine how the levels of the lower bounds for delta EVE and delta NII affect the levels of delta EVE, delta NII and return, a trade-off surface is created. This surface is given in Figure 5.5. This trade-off surface is created by gradually increasing the lower bounds for delta EVE and delta NII in the LP model from -100% to 0%. For each combination of the lower bound for delta EVE and delta NII the return is plotted. By gradually moving the lower bounds upwards, the model allows less exposure to delta NII and delta EVE.

![Figure 5.5: Trade-off surface delta EVE, delta NII and return](image)

In Figure 5.5 the Y-axis represents the value for the lower bound for delta NII, the X-axis the value for the lower bound for delta EVE and the Z-axis the return in millions. In the figure, the green area represents the maximum return of the feasible solutions. The figure indicates that the lower bound for delta EVE can be increased...
to 0% while still leading to feasible solutions, meaning that there is no delta EVE measured on the balance sheet of the bank. For the delta NII on the contrary the lower bound can only be increased to a maximum value of 5.78%. If the boundary is moved beyond this value, the model will not yield any feasible solutions anymore satisfying all constraints. This is displayed by absence of the green surface at the right-hand side of the steep decline in return in Figure 5.5. This means that although the risk on delta EVE can be minimised, there still remains an earnings risk to some extent.

In order to have a better view on the surface only the feasible solutions are considered. This surface is given in Figure 5.6.

![Figure 5.6: Trade-off surface delta EVE, delta NII and return, feasible solutions only](image)

From Figure 5.6 can be derived that increasing the lower boundaries of delta EVE and delta NII, and allowing less risk has a small impact on the theoretical return of the bank. In fact there is only an effect at the points where the lower bounds for delta EVE and delta NII are already close to 0%. For lower values of these lower bounds there is virtually no impact on the return as the surface is flat. This is in fact a logical result as the balance sheet of the bank already shows low initial values for delta EVE and delta NII as was given in Table 5.1. If the boundaries of delta EVE and delta NII are below these levels, meaning that there is more delta EVE and delta NII allowed, the model would not take any more swaps on the balance sheet. This since the swaps have a negative impact on the target function and therefore would cost the bank return.

In order to determine the impact of different combinations of lower bounds for delta EVE and delta NII, four points in this surface are chosen that represent four different strategies the bank could take. In Figure 5.6 the four points are marked with red
5.3. Changing exposure to both delta EVE and delta NII

dots. The four points are characterised by the following values for the lower bounds of delta EVE and delta NII:

- **Point 1**: At this point the lower bound for delta EVE is 0% and for delta NII is -15%.
- **Point 2**: At this point the lower bound for delta EVE is 0% and for delta NII is -5.93%.
- **Point 3**: At this point the lower bound for delta EVE is -10% and for delta NII is -5.78%.
- **Point 4**: At this point the lower bound for delta EVE is -1.50% and for delta NII is -7.4%

These first three points are specifically chosen as they represent the points at which either the delta EVE, delta NII or both are minimised by putting the lower bound for delta EVE and delta NII at the maximum level possible. By looking at these three points in more detail, insight in the strategy in minimising either delta EVE, delta NII or both can be gained. The fourth point is examined as this point is characterised by a relatively high return and low corresponding levels for delta EVE and delta NII.

With the target function of maximising the return this point is specifically interesting. In the following sections the results of the LP model at the four points on the trade-off surface will be discussed.

**Delta EVE, delta NII and return in point 1**

In Table 5.3 the levels of delta EVE and delta NII for the six interest rate shocks measured in point 1 in the trade-off surface are given. The first point is characterised by a lower bound for delta EVE of 0% and a lower bound for delta NII of -15%. The results in Table 5.3 indeed give a minimum of 0% for the level of delta EVE measured on three shocks. The maximum delta EVE is 2.58% on the butterfly down and steepener shock. The values in the table indicate that it is possible to completely mitigate a negative change in EVE resulting from the six interest rate shocks. Still three shocks result in a positive change in EVE, meaning that the balance sheet isn’t completely insensitive to interest rate shocks measured through delta EVE.

The lower boundary for the delta NII constraint is set at -15%. The results however show that the minimum delta NII is just -7.96% meaning that the lower boundary for delta NII is not constraining the solution of the LP model. The minimum value for delta NII has decreased from -6.25% in the original balance sheet to -7.67%. The return that the bank is able to achieve however has decreased to 1.348 million euros, where previously it was 1.355 million for the original balance sheet.

These levels of delta EVE, delta NII and return are achieved by taking a hedging portfolio on the balance sheet as given in Figure 5.7. In this figure the number of
Table 5.3: Values of delta EVE and delta NII for point 1 on the trade-off surface

<table>
<thead>
<tr>
<th></th>
<th>Delta EVE</th>
<th>Delta NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>1.86%</td>
<td>-6.80%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>0.00%</td>
<td>5.21%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>0.00%</td>
<td>6.10%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>2.58%</td>
<td>-7.75%</td>
</tr>
<tr>
<td>Steepener</td>
<td>2.58%</td>
<td>-7.96%</td>
</tr>
<tr>
<td>Flattener</td>
<td>0.00%</td>
<td>6.32%</td>
</tr>
</tbody>
</table>

receiver swaps per maturity is given. The red negative bars indicate that at that point a negative amount of receiver swaps should be taken on the balance sheet. Since the characteristics of the receiver and payer swaps are the opposite of each other, this negative figure for the number of receiver swaps means a positive amount of payer swaps. For instance: -3 receiver swaps are the same as 3 payer swaps. The maturity is measured in quarters, going up to 120 quarters or 30 years.

The figure indicates that in order to achieve the results on delta EVE and delta NII as given in Table 5.3, the bank needs 5.3 payer swaps in the first quarter, 4 receiver swaps with a maturity of 3 years and few swaps with even longer maturities of both payer and receiver. Here the payer swaps have shorter maturities than the receiver swaps.

The position of the receiver swaps indicate that in this point the bank is hedged mainly against short rates moving up and longer rates moving down. The negative values for delta EVE in Table 5.1 have increased to 0%. These negative values for delta EVE were measured at the shocks where interest rates at short maturities increased and interest rates at longer maturities decreased. The payer swaps at the
5.3. Changing exposure to both delta EVE and delta NII

short maturities in Figure 5.7 increase in value when the rates move up, the receiver swaps in the longer maturities decrease in value for increasing interest rates.

**Delta EVE, delta NII and return in point 2**

The results on delta EVE and delta NII for the second point are given in Table 5.4. The second point is characterised by the highest lower bounds for both delta EVE and delta NII possible. The table indicates that the minimum value for delta EVE is again 0%. The maximum value for the delta EVE is 2.80% on both the butterfly down and Steepener shock. This is slightly higher than the maximum delta EVE in point 1.

<table>
<thead>
<tr>
<th></th>
<th>Delta EVE</th>
<th>Delta NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>1.35%</td>
<td>-5.62%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>0.00%</td>
<td>5.68%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>0.00%</td>
<td>5.78%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>2.80%</td>
<td>-5.78%</td>
</tr>
<tr>
<td>Steepener</td>
<td>2.80%</td>
<td>-5.70%</td>
</tr>
<tr>
<td>Flattener</td>
<td>0.00%</td>
<td>5.70%</td>
</tr>
</tbody>
</table>

**Table 5.4: Values of delta EVE and delta NII for point 2 on the trade-off surface**

As Table 5.4 indicates, the minimum delta NII in point 2 is measured at the butterfly down shock with a value -5.78%. This is 2.18 percentage points higher than in point 1 and 0.47 percentage points higher than in the original balance sheet. The return corresponding to point 2 is with a value of 1.342 million euros lower than the return in point 1: 1.348 million euros and the initial balance sheet: 1.355 million euros. The decrease in delta NII therefore comes at a slight cost, as already was indicated by the decreasing slope of the surface in Figure 5.6.

In Figure 5.8 the number of swaps per maturity is given for point 2. Comparing Figure 5.8 with Figure 5.7, one can see that the number of receiver swaps with a maturity of 3 years has increased, and a few receiver swaps with a maturity of 3.25 years are added. The number of payer swaps has remained the same, however the maturity is increased to 1.25 years. The increase in receiver swaps with a maturity of 3 years is logical when comparing the values for delta NII of point 2 with the values of point 1. The negative delta NII has become smaller for shocks where the short rates are decreased: butterfly down shock and steepener. The effect of the receiver swap on the NII is positive for decreasing rates as the floating rate that is paid decreases while the fixed rate that is received remains the same. This comes forward in the results.
Chapter 5. Results LP problem

Figure 5.8: Number of payer and receiver swaps for point 2 on the trade-off surface

Delta EVE, delta NII and return in point 3

Results on delta EVE and delta NII corresponding to the third point in the trade-off surface in Figure 5.6 are given in Table 5.5. The third point is characterised by the highest lower boundary possible for delta NII of -5.78% and a lower boundary for delta EVE of -10%. Table 5.5 indicates that delta NII has a minimum value of -5.78. The value delta EVE has decreased compared to the delta EVE in point 1 and 2 to -10%.

<table>
<thead>
<tr>
<th></th>
<th>Delta EVE</th>
<th>Delta NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>7.88%</td>
<td>-5.62%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>-10.00%</td>
<td>5.68%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>2.09%</td>
<td>5.78%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>-3.13%</td>
<td>-5.78%</td>
</tr>
<tr>
<td>Steepener</td>
<td>4.89%</td>
<td>-5.70%</td>
</tr>
<tr>
<td>Flattener</td>
<td>-5.93%</td>
<td>5.70%</td>
</tr>
</tbody>
</table>

Table 5.5: Values of delta EVE and delta NII for point 3 on the trade-off surface

In Figure 5.9 the distribution of swaps corresponding to the solution for point 3 is given. The figure indicates that the number of receiver swaps compared to point 2 has decreased. The volume of the payer swaps has also decreased, but less than the receiver swaps compared to the hedging portfolio corresponding to point 2.

The effect is evident in the delta EVE levels measured. Where previously the parallel down shock resulted in a value of delta EVE of 0%, in the case of point three it has a value of -10%. This decrease can be explained by the decrease in number of both payer and receiver swaps.
As a result of the lower number of saps in the portfolio, the return in point 3 is slightly higher than the return in point 2 and slightly lower than in point 1 with 1.349 million euros. So the decrease in payer swaps has a small positive effect on the return.

Delta EVE, delta NII and return in point 4

The fourth and last point examined in this research is characterised by its relatively high return and relatively low levels of delta EVE and delta NII. In Table 5.6 the values for delta EVE and delta NII corresponding to point 4 on the trade-off surface are given. In comparison to the initial balance sheet the minimum delta EVE measured is slightly higher, the delta NII however has decreased.

<table>
<thead>
<tr>
<th></th>
<th>Delta EVE</th>
<th>Delta NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>1.95%</td>
<td>0.90%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>-1.50%</td>
<td>-1.68%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>-1.48%</td>
<td>6.52%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>1.92%</td>
<td>-7.29%</td>
</tr>
<tr>
<td>Steepener</td>
<td>1.02%</td>
<td>-7.35%</td>
</tr>
<tr>
<td>Flattener</td>
<td>-0.57%</td>
<td>6.58%</td>
</tr>
</tbody>
</table>

Table 5.6: Values of delta EVE and delta NII for point 4 on the trade-off surface

In Figure 5.10 the number of swaps belonging to point 4 on the trade-off surface are given. In the figure it appears that these levels of delta EVE, delta NII and return are achieved by taking very little extra swaps on the balance sheet. The swaps that are taken on the balance sheet are mainly payer swaps. This explains the increase in
minimum delta EVE compared to the original balance sheet. However, the amount of payer swaps relative to the amount of receiver swaps in this portfolio make that the minimum delta NII measured decreases. This is in line with the results of the previous point, where payer swaps were mainly cause increasing the minimum level of the delta EVE and receiver swaps the increase in minimum level of delta NII.

![Number of swaps per maturity Point 4](image)

**Figure 5.10:** Number of payer and receiver swaps for point 3 on the trade-off surface

### 5.4 Difference in one and two year time horizon for delta NII

In the LP model the delta NII is measured at a one-year time horizon as described in Chapter 4. The optimisation algorithm therefore only has a constraint on the one-year delta NII. In order to see how this affects the longer term earnings risk, the delta NII over a one-year time horizon is compared with the delta NII measured over a two-year time horizon. This is done at the three points on the trade-off surface.

In tables 5.7, 5.8, 5.9 and 5.10 the one- and two-year delta NII are given for point 1, 2, 3 and 4 respectively. In the left column the delta NII over the one-year time horizon is given, in the right column the delta NII over the two-year time horizon. Note that the model constraints the one-year delta NII according to the specifications of the boundaries corresponding to the four points as discussed in the previous sections. On the two-year delta NII there is no constraint.

The four tables are relatively the same in that they show the same pattern of two-year delta NII. In all the four tables the parallel down shock denotes a minimum delta NII far lower than the rest of the shocks. The tables also indicate that the minimum delta NII measured on the parallel shocks has switched form parallel up to parallel down for the one- and two-year delta NII.
There are two reasons why the parallel shocks denote more extreme values for the two-year delta NII than the other shocks. The first is the way the shocks are calculated. In the rotational shocks the rotating point of the shock is in the one year time bucket. This means that in the second year the effect of the shock is the opposite of the effect in the first year, and less strong since the height of the shock is interpolated over more time buckets than the height of the shock before the rotating point of one year. This means that the effect of the receiver swaps is also the opposite. Parallel shocks don’t have this rotating point and have the same shock over the whole yield curve.

A second reason for the high values of parallel down shocks in the two-year delta NII is the high volume of swaps with a maturity of one year. In the second year all of these swaps reprice their fixed leg resulting in a lower impact on the delta NII in the second year. The swaps that do have a maturity of more than two years are mainly receiver swaps. These swaps still have an impact on the delta NII as they don’t reprice in the first or second year on which the delta NII is assessed. The receiver swaps have a negative effect in the case of decreasing rates as the floating rate that is paid decreases while the fixed rate that is received remains the same. This explains the change in negative delta NII from the parallel up to the parallel down shock for the two-year delta NII.

From the results it appears that the lowest minimum two-year delta NII is achieved in the fourth point examined on the trade-off surface. This makes sense as this points is characterised by its high return, but low amount of swaps. The lack of swaps results in a low minimum value for the two-year delta NII.
Chapter 5. Results LP problem

<table>
<thead>
<tr>
<th></th>
<th>Delta NII one-year</th>
<th>Delta NII two-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>-5.62%</td>
<td>11.65%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>5.68%</td>
<td>-11.63%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>5.78%</td>
<td>6.43%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>-5.78%</td>
<td>-6.43%</td>
</tr>
<tr>
<td>Steepener</td>
<td>-5.70%</td>
<td>-5.76%</td>
</tr>
<tr>
<td>Flattener</td>
<td>5.70%</td>
<td>5.76%</td>
</tr>
</tbody>
</table>

Table 5.9: Values of one- and two-year delta NII for point 3 on the trade-off surface

<table>
<thead>
<tr>
<th></th>
<th>Delta NII one-year</th>
<th>Delta NII two-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel up shock</td>
<td>0.90%</td>
<td>14.05%</td>
</tr>
<tr>
<td>Parallel down shock</td>
<td>-1.68%</td>
<td>-14.00%</td>
</tr>
<tr>
<td>Butterfly up shock</td>
<td>6.52%</td>
<td>6.62%</td>
</tr>
<tr>
<td>Butterfly down shock</td>
<td>-7.29%</td>
<td>-6.57%</td>
</tr>
<tr>
<td>Steepener</td>
<td>-7.35%</td>
<td>-5.90%</td>
</tr>
<tr>
<td>Flattener</td>
<td>6.58%</td>
<td>5.94%</td>
</tr>
</tbody>
</table>

Table 5.10: Values of one- and two-year delta NII for point 4 on the trade-off surface

The results on the two-year delta NII indicate that the high volume of short term receiver swaps compared to the longer term payer swaps is the main cause of the larger delta NII values for the parallel down shocks. The increasing costs for swaps with longer maturities in our target function is the main cause of this result. Since the delta NII is limited for a one-year time horizon only, the model is better off by putting as less swaps as possible in longer maturities. When also a constraint on the two-year delta NII or longer time horizons is set, the model would allocate more swaps to longer maturities at a higher cost meaning that the return would be lower.

5.5 Concluding remarks

In the analysis in this chapter a few points came forward. The first conclusion is that the negative change in delta EVE can be limited to 0% for this data set. For the delta NII the highest minimum level is at -5.78%.

The second conclusion is that the hedging portfolio is composed such that it minimises the delta EVE measured from rotational shocks where the short rates increase and the longer rates on the yield curve decrease. This fits the original distribution of the balance sheet where the liabilities have a longer duration than the assets, meaning that in these rotational shocks the delta EVE is the largest.

Decreasing the delta NII measured is mainly done by increasing the number of receiver swaps with a maturity just over 3 years. This has the effect that the delta NII over a one-year time horizon can be lowered to some extend. However, the fact that many swaps are allocated in the first year has the effect that the delta NII assessed
over a two-year time horizon for all the four points examined in the trade-off surface is larger than the one-year horizon. To overcome this problem the bank would have to take more swaps with longer maturities on the balance sheet which would have a negative impact on the return.

In the introduction of Chapter 4 the aim of the model was described. This chapter was aimed at finding an answer to the third and main research questions by optimising the balance sheet:

*How can banks best change their levels of exposure to interest rate risk, measured from an economic value and earnings perspective, given their risk appetite and business model?*

The IRRBB limits differ per bank, as does the balance sheet. So it depends per bank how they want to change their IRRBB exposure. In this chapter based on the data of a specific bank a trade-off surface was created in order to compare several different combinations of delta EVE, delta NII and return.

The results indicated that the exposure to IRRBB can be changed effectively using plain vanilla swaps. The negative delta EVE could be minimised to a level 0%, where the delta NII could be minimised to -5.78%. The analysis that was carried out in this chapter on the data of a bank indicated that this bank was mainly exposed to IRRBB resulting from rotational shocks. The hedging portfolio was therefore mainly directed to overcome the negative values resulting from these shocks.
Chapter 6

Conclusion

6.1 Introduction

In this research many topics relating to interest rate in the banking book were addressed, especially focusing on the measurement of IRRBB through delta EVE and delta NII as economic value and earnings based perspectives. The aim was to determine the dynamics of these two methods, and their relationship to the steering of assets and liabilities. In the process this research has been partly conducted in cooperation with a Dutch bank. In this chapter the main findings of this research will be presented and concluded. This will be done at the hand of the three research questions that were designed to guide this research and to find the answer to the main research question:

*How can banks best change their levels of exposure to interest rate risk, measured from an economic value and earnings perspective, given their risk appetite and business model?*

In the next three sections the main conclusions to the three research questions will be discussed.

6.2 Conclusions to the first research question

The first research question was designed to get more knowledge on the concepts of interest rate risk in the banking book and the relationship between the two perspectives on IRRBB:

*What are important aspects and differences between delta NII as earnings and delta EVE as economic value based methods to measure IRRBB?*

In the literature review that is conducted in Chapter 2, the basic concepts between IRRBB, economic value and earnings perspectives to IRRBB were elaborated on.

In order to give an answer to the research question, it can be stated that the most important conclusions from this stage of the research are the following:
• Earnings and economic value measures are linked in the fact that earnings affect the net present value of assets and liabilities, and thus the economic value of assets and liabilities;

• Economic value and earnings based perspectives on IRRBB differ in the time horizon at which IRRBB is measured. Delta NII is assessed on a short term horizon where delta EVE has a longer time horizon. The methods also differ in the use of different assumptions on the future business production of the bank.

6.3 Conclusions to the second research question

The second research question was aimed to get more insight in the assumptions underlying the delta NII and delta EVE calculations: What are important aspects to take into account in the calculation of delta EVE and delta NII? The target was to determine how assumptions on three different aspects in the calculation of delta EVE and delta NII would impact the exposure measured. In order to do so the levels of delta NII and delta EVE were calculated under different assumptions on the following three aspects:

• The yield curve used in the calculations of delta EVE and delta NII;

• The impact of maturity profile of assets and liabilities on the calculations of delta EVE and delta NII;

• The impact of the banking business model used in the calculation of delta EVE and delta NII.

Looking at the results in Section 3.4, it can be concluded that all the three aspects tested in that chapter have an impact on the delta EVE and delta NII figures measured:

a The duration or maturity profile of assets and liabilities has the largest impact on the delta EVE measure to IRRBB: as the maturity becomes longer, this impact becomes larger.

b The banking business model has an impact on the delta EVE measured. This is largely because of the difference in gapping profile between assets and liabilities for the business models, and can therefore be related to the maturity profile of assets and liabilities. Retail and wholesale business models are characterised by a much larger maturity mismatch, and their economic value of equity is therefore more sensitive to changes in interest rates.

c The negative delta NII is bigger for European curves, or curves that have lower values, than for curves that have higher values, since the floor is hit earlier. This means that the margin between assets and liabilities will fall earlier than
with curves that have higher values such as the American curves used in this analysis.

6.4 Conclusion to the third research question

The third research question was designed to make the link between interest rate risk in the banking book and the management of assets and liabilities:

*When is the exposure to interest rate risk measured from both perspectives considered optimal?*

a. *How can a bank best change their exposure to interest rate risk?*

In Chapter 4 a linear programming model was constructed that through the use of constraints on delta EVE, delta NII and (key rate) durations determines the optimal distribution of the hedging portfolio in order to maximise the return on assets and liabilities. A real answer to the third research question was not found. The design of the model and the methodology used imply that the less delta EVE and delta NII measured the better. So the optimal exposure would be the solution with the least exposure to IRRBB.

In Chapter 5 a trade-off surface was created where delta EVE, delta NII and the return were displayed. This surface was formed by plotting the return for each combination of lower bound f delta EVE and delta NII. In the surface four points were examined to show the effect of the position of the lower bounds for delta EVE and delta NII on the theoretical maximum return earned on the balance sheet.

In the analysis several conclusions came forward. The first conclusion is that the exposure to negative delta EVE can be mitigated so that the minimum value for delta EVE is 0%. For the delta NII this is possible only to a certain level given this data set.

The second conclusion that can be made is the fact the hedging portfolio is directed such that it minimises the delta EVE measured from rotational shocks where the short rates increase and the longer rates on the yield curve decrease. This fits the original distribution of the balance sheet where the liabilities have a longer duration than the assets, meaning that in these rotational shocks the delta EVE is the largest.

Decreasing the measured delta NII is mainly done by increasing the number of receiver swaps with a maturity just over three years. This has the effect that the delta NII over a one-year time horizon can be lowered to some extend. The delta NII assessed over a two-year time horizon for all the four points examined in the trade-off surface is much larger than the one-year horizon. Therefore it can be concluded that limiting the one-year delta NII can have the opposite effect on longer maturities. To
overcome this problem the bank would have to take more swaps with longer maturities on the balance sheet which would have a negative impact on the return.

6.5 Conclusion

In this research many different conclusions and aspects of earnings and economic value based measures to interest rate in the banking book have come across. It can be concluded that the maturity profile of assets and liabilities plays an important role in the interest rate risk exposure measured. In the optimisation problem this is taken into account in order to address the main research question:

*How can banks best change their levels of exposure to interest rate risk, measured from an economic value and earnings perspective, given their risk appetite and business model?*

The IRRBB risk appetite differs per bank. It depends per bank how they want to change their IRRBB exposure. On the basis of data made available by a Dutch bank a trade-off surface was created in order to compare several different combinations of delta EVE, delta NII and return.

The results indicated that the exposure to IRRBB can be changed effectively using plain vanilla swaps. The minimum Delta EVE measured on the basis of six interest rate shocks amounts 0% where the minimum delta NII measured amounted to -5.78%. The analysis that was carried out on data of the bank indicated that this bank was mainly exposed to IRRBB resulting from rotational shocks. The hedging portfolio was therefore primarily directed to overcome the negative values resulting from these shocks.
Chapter 7

Future Research

7.1 Introduction

The research that is conducted and outlined in this thesis contained many assumptions. These assumptions have a large impact on the results. Some of these results also lead to new questions that were not addressed in this research. In order to address these issues in future research, the most important aspects are mentioned in this chapter.

7.2 Recommendations

In the modelling of the linear programming problem to investigate the effect of the delta EVE and delta NII constraints on management of IRRBB some assumptions are made that influenced the results and may be interesting for future research. The LP model is mainly focused on repricing risk and yield curve risk. Optionality of term deposits or mortgages or other assets and liabilities is not incorporated in the model. It is assumed that these assumptions will have an impact on the exposure to interest rate risk as the term deposits or mortgages reprice earlier and to a different rate than expected. This may have an impact on the composition of the hedging portfolio. It might be interesting for future research to investigate how optionality of the balance sheet affects the composition of the hedging portfolio.

The current research uses linear programming as optimisation method. Research pointed out that stochastic programming models are quite often used in the management of assets and liabilities. The use of stochastic programming models can enable the use of stochastic interest rate scenarios and even more dynamic stress testing of interest rate risk exposure than in the current research. It would be interesting for future research to determine how IRRBB is impacted by more and heavier interest rate scenarios. The use of stochastic programming could be used in order to determine this purpose. By assigning a probability to a certain interest rate scenario, a stress test and Value at Risk (VAR) figure could possibly be generated. This would
be a different point of view on IRRBB measurement as chance is not incorporated in this research.

In this research, during the optimisation of the balance sheet, the hedging portfolio is used as decision variable. This is to represent current practices at the bank. However, from both a scientific as practical point of view it would be interesting to see current constraints on IRRBB and target function to be applied on a complete balance sheet in combination with other constraints on liquidity, solvency and other. In that case it would give a more broader view on the management of assets and liabilities in general instead of only focusing on the hedging portfolio and interest rate risk constraints only. It could also be an interesting comparison in the management of IRRBB on balance sheet via restructuring of assets and liabilities versus off balance sheet via swaps.

Besides this integrated approach, there is also another topic that is left out in this research but forms a major part of interest rate risk: basis risk. It would be interesting to see how interest rate risk measures from an earnings and economic value perspective are impacted by, or have an impact on the basis risk measured.
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Appendix A

Modelling of interest rate scenarios
Chapter 3

A.0.1 Modelling of interest rate scenarios

To model the six different interest rate scenarios mentioned in section 3.2.4, the approach given in the BCBS standards on IRRBB is used (BCBS, 2016a). This approach uses the specified size of 3 types of shocks: parallel, short rate and long rate. The sizes of these shocks are used in order to determine the level of change per interest rate scenario per time bucket (BCBS, 2016a). The sizes of the three types of shocks are given by the BCBS and updated every 5 years. For the Euro and US dollar the sizes can be found in table A.1. The complete table with sizes of the three shocks for all currencies is given in table C.1 in appendix C.

<table>
<thead>
<tr>
<th>Currency</th>
<th>Parallel</th>
<th>Short</th>
<th>Long</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>200</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>USD</td>
<td>200</td>
<td>300</td>
<td>150</td>
</tr>
</tbody>
</table>

The specified size per shocktype per currency is denoted as $R_{\text{shocktype},c}$. To determine the size of the parallel shock per time bucket midpoint ($t_k$) the following formula is applied:

$$\Delta R_{\text{parallel},c}(t_k) = \pm R_{\text{parallel},c}$$

This parallel shock is added or subtracted in case of respectively a parallel up or down shock to the base yield curve.

The steepener and flattener interest rate scenarios are composed by using the specified sizes of the short and long rates as given in table A.1. The size of the short rate is multiplied by a scalar $S_{\text{short}}(t_k) = e^{-x t_k}$ where $x = 4$ as given in the paper by the BCBS (BCBS, 2016a). This scalar indicates the decay of the shock over the time buckets. This means the longer the maturity, the lesser the effect of the short shock becomes on the yield curve. For the long rates the opposite is the case. The scalar...
for the long shock is \( S_{long}(t_k) = 1 - S_{short}(t_k) \). This scalar now indicates the increase in effect of the long rate over the time buckets. This gives the following formulae for the short and long shock (BCBS, 2016b):

\[
\Delta R_{short,c}(t_k) = \pm R_{short,c}(t_k) \times S_{short}(t_k) = \pm R_{short,c}(t_k) \times e^{-\frac{t_k}{\tau}}
\]

\[
\Delta R_{long,c}(t_k) = \pm R_{long,c}(t_k) \times S_{long}(t_k) = \pm R_{long,c}(t_k) \times (1 - e^{-\frac{t_k}{\tau}})
\]

The shift in interest rate at each tenor midpoint for the steepener and flattener interest rate scenario is then determined by the following formula (BCBS, 2016a):

\[
\Delta R_{steepener,c}(t_k) = -0.65 \times |\Delta R_{short,c}(t_k)| + 0.9 \times |\Delta R_{long,c}(t_k)|
\]

For the steepener interest rate scenario and for the flattener interest rate scenario:

\[
\Delta R_{flattener,c}(t_k) = +0.8 \times |\Delta R_{short,c}(t_k)| - 0.6 \times |\Delta R_{long,c}(t_k)|
\]

Applying these formulae to the size of the Euro currency shocks gives the following results:

<table>
<thead>
<tr>
<th>Time Bucket ((t_k))</th>
<th>1(,0028)</th>
<th>0,0417</th>
<th>0,1667</th>
<th>0,375</th>
<th>0,625</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in years ((t_k))</td>
<td>0,024982506</td>
<td>0,024741</td>
<td>0,02398</td>
<td>0,022763</td>
<td>0,021384</td>
</tr>
<tr>
<td>(\Delta R_{short})</td>
<td>0,009937738</td>
<td>0,009938</td>
<td>0,00994</td>
<td>0,009943</td>
<td>0,009947</td>
</tr>
<tr>
<td>(\Delta R_{long})</td>
<td>-0,007294664</td>
<td>-0,00714</td>
<td>-0,00664</td>
<td>-0,00585</td>
<td>-0,00495</td>
</tr>
<tr>
<td>(\Delta R_{steepener})</td>
<td>0,014023362</td>
<td>0,01383</td>
<td>0,013219</td>
<td>0,012244</td>
<td>0,011139</td>
</tr>
<tr>
<td>(\Delta R_{flattener})</td>
<td>0,014023362</td>
<td>0,01383</td>
<td>0,013219</td>
<td>0,012244</td>
<td>0,011139</td>
</tr>
</tbody>
</table>

In table A.2 only the first five time buckets are given. For the complete results please find table C.2 in appendix C. Applying these changes to the base EONIA curve gives the following 6 interest rate risk scenarios that are displayed in figure A.1:
Figure A.1: EONIA Interest rate scenarios
Appendix B

Mathematical derivation of LP optimisation problem

B.1 Introduction

In this appendix the mathematical derivations of the constraints in the LP model are elaborated. Most of the constraints that are used in the LP optimisation problem and are stated in Chapter 4 are of a non-linear form. Since the LP package used in R only recognises constraints of a linear form, these formulae have to be rewritten in order to fit the requirements. The constraints as stated in Chapter 4 are written in linear form. Since the computations are quite complex, the mathematical derivation of these constraints will be dealt with in this appendix. In total the following constraints will be dealt with in this appendix:

- Delta EVE constraint;
- Delta NII constraint;
- Duration of equity constraint;
- Key rate duration constraint.

B.1.1 Delta EVE constraint

The delta EVE constraint is the first constraint that will be dealt with. According to literature the change in economic value of equity can be calculated as follows (BCBS, 2015; BCBS, 2016a; EBA, 2015):

$$\Delta EV_{E_k} = (NPV_{Assets,k} - NPV_{Liabilities,k}) - (NPV_{Assets,0} - NPV_{Liabilities,0}) \quad (B.1)$$

Where $k$ is one of the 6 interest rate scenarios mentioned in Chapter 4 section 4.2 and 0 is the base scenario.
The difference in net present value of assets between a shocked interest rate scenario and the base scenario is the change in economic value of assets. The change in net present value of liabilities is subtracted to get the net change in economic value of equity. In a linear form the delta EVE calculation looks as follows:

\[
\Delta EV_{E_k} = (NPV_{Assets,k} - NPV_{Assets,0}) - (NPV_{Liabilities,k} - NPV_{Liabilities,0}) \quad (B.2)
\]

The net present value of assets and liabilities can be calculated by multiplying all the cash flows and notional repricing cash flows per maturity bucket with a discount factor corresponding to the risk free rate at that time. The sum of these discounted cash flows is then the net present value.

The payer and receiver swaps are the only decision variables in this LP model. However, all asset and liability classes as set in Chapter 4 section 4.2 are taken into account in the calculation of the delta EVE constraint. The notional repricing cash flows in this thesis are based on the gapping information provided by the bank. This notional repricing cash flow is represented in the model by \( x_{i,j} \) where \( i \) is one of the 5 asset classes set in Chapter 4 section 4.2 and \( j \) one of the 120 maturity buckets the asset can be allocated to. The liabilities are represented by \( y_{i,j} \) with class \( i \) in maturity bucket \( j \).

The discount factors are computed via continuously compounding. This is done by the following formula: \( \exp^{-r_{k,j}t_j} \), with \( r_{k,j} \) being the risk free rate under scenario \( k \) at time bucket \( j \), and \( t_j \) the time at time bucket \( j \). The net present value of the notional repricing cash flows are then: \( \exp^{-r_{k,j}t_j} x_{i,j} \) for assets and \( \exp^{-r_{k,j}t_j} y_{i,j} \) for liabilities.

Besides the notional cash flows, the coupon payments that are made on assets and liabilities are also incorporated in this model. An asset in a certain time bucket \( j \), has earned \( jR_{i,j} \) until the moment of repricing at moment \( j \). In order to determine the net present value of these coupon payments, they should be discounted with the rate at which they occur. To do this, the first coupon \( R_{i,1} \) should be discounted with discount factor \( \exp^{-r_{k,1}t_1} \). The second coupon \( R_{i,2} \) should be discounted with the discount factor corresponding with the discount factor at time \( t_2: \exp^{-r_{k,2}t_2} \) etc.

When we consider asset 2 in time bucket 3, this gives the following computation for the total net present value of the coupon payments: \( \sum_{j=1}^{3} (x_{2,3} R_{2,3} \exp^{-r_{k,j}t_j}) \).

The total net present value for asset 2 in time bucket 3 is thus:

\[
NPV_{\text{notional}} + NPV_{\text{coupon payments}} = (\exp^{-r_{k,3}t_3} x_{2,3}) + \sum_{j=1}^{3} (x_{2,3} R_{2,3} \exp^{-r_{k,j}t_j}) \quad (B.3)
\]
The linear format requires the notional repricing cash flow to be kept outside brackets. This gives:

\[
NPV_{\text{asset},2,3} = \left( \exp^{-r_{k,3}t_3} + \sum_{j=1}^{3} (R_{2,3} \exp^{-r_{k,j}t_j}) \right) x_{2,3} \]  

(B.4)

Repeating this for every asset class \(i\) and all time buckets \(j\) gives the total net present value of assets and liabilities:

\[
NPV_{\text{assets},k} = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{k,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{k,t})^t} R_{k,j'} \right) \right) x_{i,j} \]  

(B.5)

\[
NPV_{\text{liabilities},k} = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{k,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{k,t})^t} c_{k,j'} \right) \right) y_{i,j} \]  

(B.6)

\[
NPV_{\text{assets},0} = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{0,t})^t} R_{0,j'} \right) \right) x_{i,j} \]  

(B.7)

\[
NPV_{\text{liabilities},0} = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{0,t})^t} c_{0,j'} \right) \right) y_{i,j} \]  

(B.8)

Implementing these four in equation B.2 gives the total delta EVE calculation in linear format:

\[
\Delta EVE = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{0,t})^t} R_{0,j'} \right) \right) x_{i,j} - \left( \frac{1}{(1 + r_{k,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{k,t})^t} R_{k,j'} \right) \right) x_{i,j} \right) - \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \left( \frac{1}{(1 + r_{0,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{0,t})^t} c_{0,j'} \right) \right) y_{i,j} - \left( \frac{1}{(1 + r_{k,t})^t} + \left( \sum_{j'=1}^{j} \frac{1}{(1 + r_{k,t})^t} c_{k,j'} \right) \right) y_{i,j} \right) \]  

(B.9)

B.2 Delta NII constraint

As mentioned in Chapter 2, the delta NII constraint is set as the difference in net interest income of a base scenario, and one of the six interest rate shocks as described
in the Chapter 4. According to literature the change in net interest income can be calculated as follows (BCBS, 2015; BCBS, 2016a; EBA, 2015):

$$\Delta NII_k = NII_k - NII_0$$  \hspace{1cm} (B.10)

Here negative values of the delta NII will indicate a decrease in NII due to a change in interest rates, and positive values an increase in NII.

Since the time horizon on which the delta NII is monitored is one year, the shock in interest rates will only be applied to the first 4 time buckets. This since assets and liabilities with a maturity of more than one year will not reprice within that one year at which the delta NII is assessed and thus will keep their original coupon. This then gives the following computations for the components of equation B.10:

$$NII_k = \sum_{i=1}^{5} \sum_{j=1}^{4} \left( (x_{i,j} R_{k,j}) - (y_{i,j} c_{k,j}) \right)$$  \hspace{1cm} (B.11)

and

$$NII_0 = \sum_{i=1}^{5} \sum_{j=1}^{4} \left( (x_{i,j} R_{0,j}) - (y_{i,j} c_{0,j}) \right)$$  \hspace{1cm} (B.12)

Here $R_{k,j}$ is the interest earned on assets in interest rate scenario $k$ and time bucket $j$. $R_{0,j}$ is the original coupon of the base scenario. Note that interest rate earned $R_{k,j}$ is not the same as the risk free rate.

Implementing these components in equation B.10 gives the following equation:

$$\Delta NII_k = \sum_{i=1}^{5} \sum_{j=1}^{4} \left( (x_{i,j} R_{k,j}) - (y_{i,j} c_{k,j}) \right) - \left( (x_{i,j} R_{0,j}) - (y_{i,j} c_{0,j}) \right)$$ \hspace{1cm} (B.13)

Rewriting to linear format gives:

$$\Delta NII_k = \sum_{i=1}^{5} \sum_{j=1}^{4} \left( (R_{k,j} - R_{0,j}) x_{i,j} - (c_{k,j} - c_{0,j}) y_{i,j} \right)$$ \hspace{1cm} (B.14)

### B.2.1 Duration of equity constraint

The duration of equity is the difference that can be viewed as the difference in duration of assets and liabilities (Cummins et al., 2009):

$$DoE = Duration_{Assets} - (assets/liabilities)Duration_{Liabilities}$$  \hspace{1cm} (B.15)
The duration of an asset or liability can also be viewed as the weighted average of all the cash flows of assets and liabilities multiplied with the time they occur. These weights are calculated by dividing the cash payment at time $t_j$ with the total net present value of the asset or liability (Hull, 2012a). This is given in the following formula:

$$\text{Duration} = \frac{1}{\text{NPV}_{\text{asset}}} \sum_{j=1}^{n} t_j \left( \frac{R_j}{(1 + r_j)^t} \right)$$

(B.16)

Here $R_j$ is the coupon at time $j$, and $r_j$ the risk free rate at time $j$.

In the LP model the duration of equity constraint will be calculated in a similar manner. However since the LP model requires a linear format for the constraints, equation B.16 has to be rewritten to fit this purpose. In the next few steps this process will be elaborated. During this process, the duration of assets will be taken as example, for the duration of liabilities this however works exactly the same, but for simplicity it is omitted.

In equation B.16 $R_j$ contains both coupon and principal payments (Hull, 2012a). In our linear format this is not the case, and $R_{i,j}$ only represents the coupon payments. The principal payment is in the case of our LP format denoted with $x_{i,j}$. Just like in the delta EVE computations, the linear format requires us to deal with the coupon payments and the principal payment separately.

Let’s begin with the coupon payments. As in our delta EVE calculation, coupons are earned on a quarterly basis until the moment that the asset reaches its maturity. This means that asset 2 in maturity bucket 3 earns 3 quarterly coupons during that time. These coupons have to be discounted at the rate corresponding to the time they occur. This gives for asset 2 in time bucket 3: $\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t_j})^t} \right)$. In order to determine the weighted average cash flows, first the coupon cash flows have to be multiplied with the time $t_j$ at which they occur. This gives:

$$\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t_j})^t} t_j \right)$$

(B.17)

Besides the coupon cash flows, the notional payment also has to be discounted and multiplied with the time $t_j$ at which it occurs:

$$x_{2,3} \exp^{-r_{0,t_3} t_3}$$

In total the cash flows can be written as:
Appendix B. Mathematical derivation of LP optimisation problem

\[
\text{coupon + notional} = \sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} t_j \right) + \left( x_{2,3} \frac{1}{(1 + r_{0,3})^3} t_3 \right) \tag{B.18}
\]

To determine the weighted average of these cash flows, equation B.18 has to be divided by the total cash flows of asset 2 in time bucket 3:

\[
D_{2,3} = \frac{\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} t_j \right)}{\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,3})^3} \right)} + \left( x_{2,3} \frac{1}{(1 + r_{0,3})^3} t_3 \right) \tag{B.19}
\]

In the risk appetite statement by the bank, the risk limit for duration of assets is determined at XX. This gives the following duration of assets constraint:

\[
D_{2,3} = \frac{\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} t_j \right)}{\sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,3})^3} \right)} + \left( x_{2,3} \frac{1}{(1 + r_{0,3})^3} t_3 \right) \leq XX \tag{B.20}
\]

To rewrite this to a linear format, both the left- and right-hand side of equation B.20 are multiplied with the total discounted cash flows of asset 2 in maturity bucket 3:

\[
D_{2,3} = \sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} t_j \right) + \left( x_{2,3} \frac{1}{(1 + r_{0,3})^3} t_3 \right) \leq XX \left( \sum_{j=1}^{3} \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} \right) + \left( x_{2,3} \frac{1}{(1 + r_{0,3})^3} \right) \right) \tag{B.21}
\]

Subtracting the right hand side of equation B.21 of the left hand side gives the following result:

\[
D_{2,3} = \sum_{j=1}^{3} \left( \left( x_{2,3} R_{2,j} \frac{1}{(1 + r_{0,t})^t} \right) (t_j - XX) \right) + \left( x_{2,3} \left( \frac{1}{(1 + r_{0,3})^3} \right) (t_3 - XX) \right) \leq 0 \tag{B.22}
\]

Rewriting gives:
B.2. Delta NII constraint

\[ D_{2,3} = \left( \sum_{j=1}^{3} \left( \frac{1}{(1 + r_{0,t})^t} \right) (t_j - XX) \right) + \frac{1}{(1 + r_{0,t})^3} (t_3 - XX) \right) x_{2,3} \leq 0 \]  

(B.23)

Assuming the same risk limit for duration of assets and liabilities, equation B.23 can be implemented for all assets and liabilities. Together with equation B.15, this gives the following formulation for the duration of equity constraint:

\[ D_{OE} = \left( \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{0,t})^t} \right) (t_j - XX) \right) + \frac{1}{(1 + r_{0,t})^t} (t_j - XX) \right) x_{i,j} \]

-B.2.2 Key rate duration constraint

In order to determine and constrain the partial duration per year, a set of 30 key rate duration constraints is constructed to prevent peaks of asset and liability allocations in specific time buckets. Key rate duration is defined as the change in value of assets and liabilities due to a partial shock to the yield curve (Hull, 2012a) and is given as:

\[ D_j = -\frac{1}{P} \Delta P_j \]

Where \( P \) is the total price or value of an asset/liability, and \( \Delta P_j \) is the corresponding change in value of the asset. Just like the duration constraint and the calculation of the delta EVE constraint, the \( \Delta P_j \) is calculated as the difference between the original price and the price after the change in value due to the small shock of the interest rate: \( \Delta P_j = P_j - P_0 \). The small change in interest rates \( \Delta r_j \) is determined as 1 basis point, or 0.01 percent. Any change could be chosen here however, due to convexity problems with greater changes here the small change of 1 basis point is chosen (Hull, 2012a).

This change is applied to a one year period of the yield curve. Since the assets and liabilities can be allocated to a total of 120 time buckets (30 years), there are 30 key rate duration constraints where for each constraint a different period of one year
on the yield curve is shocked. Each of these constraints then have to comply with equation B.25.

The risk appetite statements of the large Dutch bank that cooperated to build this model state that these key rate durations can not be larger than XX. This gives the next partial duration $D_j$ with $j$ the time bucket, and the year on the yield curve that is shocked to determine the key rate duration:

$$D_j = -\frac{1}{P} \frac{\Delta P}{\Delta r_j} \leq xx$$

Rewriting this to a linear form gives:

$$D_T = \Delta P_T + P_0 \Delta r_T xx \leq 0$$

Substituting $\Delta P_T$ for $P_T - P_0$ gives:

$$D_T = P_T - P_0 + P_0 \Delta r_T xx \leq 0$$

Rewriting gives the following linear form of the key rate duration constraint:

$$D_T = P_T - P_0 (1 - \Delta r_T xx) \leq 0$$

(B.26)

In the LP model $P_0$ is represented by the net present value of all cash flows of the asset. Just like the delta EVE constraint and the duration constraints this is given by the following formula:

$$P_0 = \sum_{j=1}^{120} \left( \frac{1}{(1 + r_{0,t_j})^{t_j}} + \sum_{j' = 1}^{j} \frac{1}{(1 + r_{0,t_j'), (t_j')^{c_{k,j'}}})} x_{1,j} \right)$$

(B.27)

For every $T$ a period of one year on the yield curve is shocked with $\Delta r_T$. This gives in total 30 key rate duration constraints. For $T = 1$, $P_T$ is given as:

$$P_T = \sum_{j=1}^{4T} \left( \frac{1}{(1 + r_{0,t_j} + \Delta r_j)^{t_j}} + \sum_{j' = 1}^{j} \frac{1}{(1 + r_{0,t_j'}, + \Delta r_{j'})^{t_j'}} x_{1,j'} \right) + \sum_{j=4T}^{120} \left( \frac{1}{(1 + r_{0,t_j})^{t_j}} + \sum_{j' = 1}^{j} \frac{1}{(1 + r_{0,t_j'})^{t_j'}} x_{1,j'} \right) x_{1,j}$$

(B.28)

If $T \in (2, 30)$ than $P_T$ is given as:
B.2. Delta NII constraint

\[
P_T = \sum_{j=1}^{4(T-1)} \left( \frac{1}{1 + r_{0,t_j}} \right)^{T_j} + \left( \sum_{j^* = 1}^{j} \frac{1}{1 + r_{0,t_{j^*}} + \Delta r_{j^*}} \right)^{x_{1,j}} \]

\[
\sum_{j=4(T-1)+1}^{4T} \left( \frac{1}{1 + r_{0,t_j} + \Delta r_j} \right)^{T_j} + \left( \sum_{j^* = 1}^{j} \frac{1}{1 + r_{0,t_{j^*}} + \Delta r_{j^*}} \right)^{x_{1,j}} \]

(B.29)

Repeating this for all asset classes gives the key rate constraint for assets:

\[
P_0 = \sum_{i=1}^{5} \sum_{j=1}^{120} \left( \frac{1}{1 + r_{0,t_j}} \right)^{T_j} + \left( \sum_{j^* = 1}^{j} \frac{1}{1 + r_{0,t_{j^*}} + \Delta r_{j^*}} \right)^{x_{i,j}} \]

(B.30)

For \( T = 1 \):

\[
P_T = \sum_{i=1}^{5} \sum_{j=1}^{4T} \left( \frac{1}{1 + r_{0,t_j} + \Delta r_j} \right)^{T_j} + \left( \sum_{j^* = 1}^{j} \frac{1}{1 + r_{0,t_{j^*}} + \Delta r_{j^*}} \right)^{x_{1,j}} \]

(B.31)

And for \( T \in (2, 30) \):

\[
P_T = \sum_{i=1}^{5} \sum_{j=1}^{4(T-1)} \left( \frac{1}{1 + r_{0,t_j}} \right)^{T_j} + \left( \sum_{j^* = 1}^{j} \frac{1}{1 + r_{0,t_{j^*}} + \Delta r_{j^*}} \right)^{x_{1,j}} \]

(B.32)

Implementing equations B.30, B.31 and in equation B.26 for \( T = 1 \) gives the total equation for the key rate duration of assets.
Appendix B. Mathematical derivation of LP optimisation problem

$$KRD_{assets,1} = \left( \sum_{i=1}^{5} \sum_{j=1}^{4} \left[ \frac{1}{(1 + r_{0 \cdot t_j} + \Delta r_j) t_j} + \frac{1}{(1 + r_{0 \cdot t_{j^*}} + \Delta r_{j^*}) t_{j^*} c_{k,j^*}} \right] \right) + \sum_{i=1}^{5} \sum_{j=1}^{120} \left[ \frac{1}{(1 + r_{0 \cdot t_j}) t_j} + \sum_{j^*=1}^{j} \exp^{-r_{0 \cdot t_j} t_{j^*} c_{k,j^*}} \right] - \left( \sum_{i=1}^{5} \sum_{j=1}^{120} \left[ \frac{1}{(1 + r_{0 \cdot t_j}) t_j} + \sum_{j^*=1}^{j} \exp^{-r_{0 \cdot t_j} t_{j^*} c_{k,j^*}} \right] \right) x_{i,j}$$  \hspace{1cm} (B.33)

Just like the duration constraints the same procedure is repeated for all liability classes. The key rate duration per year is then the difference in key rate duration of assets minus liabilities:

$$KRD_{liabilities,1} = \left( \sum_{i=1}^{5} \sum_{j=1}^{4} \left[ \frac{1}{(1 + r_{0 \cdot t_j} + \Delta r_j) t_j} + \frac{1}{(1 + r_{0 \cdot t_{j^*}} + \Delta r_{j^*}) t_{j^*} c_{k,j^*}} \right] \right) + \sum_{i=1}^{5} \sum_{j=1}^{120} \left[ \frac{1}{(1 + r_{0 \cdot t_j}) t_j} + \sum_{j^*=1}^{j} \exp^{-r_{0 \cdot t_j} t_{j^*} c_{k,j^*}} \right] - \left( \sum_{i=1}^{5} \sum_{j=1}^{120} \left[ \frac{1}{(1 + r_{0 \cdot t_j}) t_j} + \sum_{j^*=1}^{j} \exp^{-r_{0 \cdot t_j} t_{j^*} c_{k,j^*}} \right] \right) y_{i,j}$$ \hspace{1cm} (B.34)

The total key rate duration per maturity bucket is then given by:

$$KRD_j = KRD_{assets,j} - KRD_{liabilities,j}$$ \hspace{1cm} (B.35)
Appendix C

Tables
### Table C.1: Overview shock size per currency

<table>
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<tr>
<th>Currency</th>
<th>Parallel</th>
<th>Short</th>
<th>Long</th>
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</thead>
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<tr>
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<td>0,05</td>
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### Table C.2: Overview changes per rate per maturity

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<th>$\Delta R_{long}$</th>
<th>$\Delta R_{steepener}$</th>
<th>$\Delta R_{flattener}$</th>
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Appendix D

Data Chapter 3

<table>
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<th>Commerz Bank</th>
<th>Belfius</th>
<th>Row Average</th>
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<td>0.057</td>
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<td>0.055</td>
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*Table D.1: Data wholesale business model*
### Balance Sheet

<table>
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<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td>Deposits</td>
</tr>
<tr>
<td>Investments</td>
<td>Debt</td>
</tr>
<tr>
<td>Trading securities</td>
<td>Trading liabilities</td>
</tr>
<tr>
<td>Cash</td>
<td>Equity</td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
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<tr>
<td>Investments</td>
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<td>Trading liabilities</td>
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</tr>
<tr>
<td>Cash</td>
<td>15.5</td>
<td>Equity</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
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**Table D.2: Typical balance sheet wholesale business model**

<table>
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<tr>
<th>Assets</th>
<th>Loans</th>
<th>Securities</th>
<th>Investments</th>
<th>Cash</th>
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<td>0.000</td>
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<td>0.075</td>
<td>0.003</td>
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<tr>
<td>1-5Y</td>
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<td>0.221</td>
<td>0.052</td>
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<tr>
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<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Deposits</th>
<th>Debt</th>
<th>Trading Liabilities</th>
</tr>
</thead>
<tbody>
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<td>&lt;1 m</td>
<td>0.769</td>
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<tr>
<td>1-3M</td>
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<td>0.089</td>
<td>0.745</td>
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<tr>
<td>3-12M</td>
<td>0.004</td>
<td>0.203</td>
<td>0.067</td>
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<tr>
<td>1-5Y</td>
<td>0.011</td>
<td>0.287</td>
<td>0.090</td>
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<tr>
<td>&gt;5Y</td>
<td>0.010</td>
<td>0.218</td>
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**Table D.3: Data balance sheet wholesale business model over time**

<table>
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<th>HSBC</th>
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<td>Cash</td>
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<tr>
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<td>0.052</td>
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<tr>
<td>Equity</td>
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<td>0.058</td>
<td>0.057</td>
<td>0.049</td>
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</table>

**Table D.4: Data investment business model**
### Balance Sheet

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<td>Loans</td>
<td>Deposits</td>
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<tr>
<td>35.3</td>
<td>23.8</td>
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<tr>
<td>Investments</td>
<td>Debt</td>
</tr>
<tr>
<td>0</td>
<td>8.7</td>
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<tr>
<td>Trading securities</td>
<td>Trading liabilities</td>
</tr>
<tr>
<td>55</td>
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<tr>
<td>Cash</td>
<td>Equity</td>
</tr>
<tr>
<td>9.7</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>Total</strong></td>
</tr>
<tr>
<td>100</td>
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</tbody>
</table>

**Table D.5:** Typical balance sheet investment business model

<table>
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<th>Securities</th>
<th>Investments</th>
<th>Cash</th>
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</thead>
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<td>&lt;1 m</td>
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</tbody>
</table>

<table>
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<th>Debt</th>
<th>Trading Liabilities</th>
</tr>
</thead>
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<td>0.090</td>
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<tr>
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**Table D.6:** Data balance sheet investment business model over time