Development of a Safety-Aware Intrinsically Passive Controller for Multi-DOF Manipulator

C.A. (Carlos) Cardenas

MSc Report

Committee:
Dr.ir. J.F. Broenink
Dr.ir. T.J.A. de Vries
Dr.ir. T.S. Tadele
Dr.ir. D. Dresscher
Dr.ir. J. van Dijk

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Robotics and Mechatronics
EE-Math-CS
University of Twente
P.O. Box 217
7500 AE Enschede
The Netherlands
Development of a Safety-Aware Intrinsically Passive Controller for a Multi-DOF Manipulator

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C.A. Cardenas Villa

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Assessment: Dr. ir. J. F. Broenink, University of Twente
Committee: Dr. ir. T. J. A. de Vries University of Twente
Dr. T. S. Tadele, Philips N.V.
Dr. D. Dresscher, University of Twente
Dr. ir. J. van Dijk, University of Twente

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A mis padres: Jorge y Karina;
y a mis hermanos: Jorge, Karina, y Pablo.
Los amo.
Abstract

In contexts where robots share their workspace with humans, safety is of supreme importance. Consequently, in recent years, a big impulse has been given to the design of human friendly robots by involving both mechanical and control design aspects. When it comes to controller design, this often involves introducing compliance and ensuring asymptotic stability using an interaction control scheme and passivity theory. Moreover, when human operators physically interact with the robot during work, strict safety measures become necessary with some of these including power and force limitations.

In this thesis, a novel impedance control technique for collaborative robots is presented. The featured controller allows safe human-robot interaction by defining energy and power based safety metrics. An energy tank system is incorporated to this controller, in order to assure passivity of the overall system. The effectiveness of the proposed controller is evaluated with a KUKA robot arm in a co-manipulation environment.
Preface

This report marks the final step in my life as Systems and Control student at the University of Twente, in The Netherlands. It has been a long and tough journey, full of life-changing experiences that will remain written in the book of memories.

I have met so many different people along the way, and I hope they have learned from me as much as I have from them. One is always thankful after finishing a journey, and accomplishing a goal. I would like use this section to specifically thank people who played an important role in this story.

First of all, I would like to give thanks to Consejo Estatal de Ciencia y Tecnologia (COECYT) from the state of Sonora, Mexico, for the scholarship granted to fulfill one of my goals in life.

Theo de Vries, thank you for being a supervisor and mentor during this time; thank you for your time and guidance. Keep inspiring your students, and be always an example of everything that is good.

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Gennaro Raiola, ringraziamenti infiniti, thank you for helping me setting up the KUKA for the experiments. My best wishes for you in Genoa, hope they can provide you with enough chewing gum.

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To the Fellowship of the Ring, my group of international friends, thanks for the memories and time we spent together.

Shamel Fahmi, Adel Abdelhady, Samer Abdelmoeti, my Egyptian brothers, thank you for your friendship throughout these 2 years; you will always be in my heart. You are always welcome to Hermosillo, paradise city, the promised land, world’s capital of genuine tacos, where real men are born. Then, we will have a real Carne Asada.

Muchas gracias a mis padres, por todo el apoyo que siempre me han dado. Todo lo que hago siempre es para honrarlos.

Education is a beautiful gift we should always pursue. When I graduated from Elementary school I received a special acknowledgment for being a dedicated students. It was a framed piece of paper that contained four life advices I have always kept in my heart and mind. I share them now; with the reader, hoping that they will influence on you as much as they did on me:

• Remember your Creator.
• Honor your parents.
• Be humble and grateful.
• Be always an example in everything that is good.

To God be the glory, today and forever.
Carlos Cardenas
Enschede, September 2017

Robotics and Mechatronics C.A. Cardenas Villa
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<tbody>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DLR</td>
<td>German Aerospace Center</td>
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<tr>
<td>EJS</td>
<td>Eulerian Junction Structure</td>
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<tr>
<td>FRI</td>
<td>Fast Research Interface</td>
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<tr>
<td>HIC</td>
<td>Head Injury Criteria</td>
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<td>HIP</td>
<td>Head Impact Power</td>
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<td>HRI</td>
<td>Human-Robot Interaction</td>
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<td>KRC</td>
<td>KUKA Robot Controller</td>
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<td>KUKA Robot Language</td>
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<td>LWR</td>
<td>Lightweight Robot</td>
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<td>PBC</td>
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<td>Port-Controlled Hamiltonian</td>
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<td>PD</td>
<td>Proportional-Derivative</td>
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<tr>
<td>pHRI</td>
<td>Physical Human-Robot Interaction</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
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<tr>
<td>POE</td>
<td>Product of Exponentials</td>
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<tr>
<td>RAM</td>
<td>Robotics and Mechatronics</td>
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<tr>
<td>ROS</td>
<td>Robot Operating System</td>
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<tr>
<td>SAIP</td>
<td>Safety-Aware Intrinsically Passive</td>
</tr>
<tr>
<td>SE</td>
<td>Special Euclidean Group</td>
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<tr>
<td>SO</td>
<td>Special Orthogonal Group</td>
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<tr>
<td>URDF</td>
<td>Universal Robotic Description Format</td>
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Introduction

1.1 Context
Recent advances in the robotics field have accelerated the development of robots that can operate close to humans. To empathize this feature, these new type of robots are often called Cobots, from the neologism of the words collaborative and robot [8].

Since these robots are expected to share their workspace with a human operator, physical contact may occur for two reasons mainly: accidentally, in case of collisions; or deliberately, if the human operator is supposed to physically interact with the robot during work. In both cases, it is important to guarantee a safe physical interaction for injury prevention [62, 8].

Collaborative robots are expected to operate in unstructured human environments and interact with unknown objects [3]. Thus, controller design should incorporate safety issues such as: introducing compliance to minimize injury in case of uncontrolled impact, ensuring asymptotic stability even during interaction and provide limitations in terms of velocities or forces exerted by the robot [8, 54].

A compliant behavior for a robotic manipulator can be obtained at the control level by using techniques such as the impedance control [28]. In this way, the joint torques can be reduced in case of an impact, protecting the robot itself and the individual. To achieve implicit safety, robotic arms in a domestic environment are designed to be lightweight and compliant [27, 3, 10, 31], such that any possible injury is prevented in case of an uncontrolled collision with a human.

1.2 Research Proposal
One of the research topics of the Robotics and Mechatronics (RAM) laboratory at the University of Twente, is to design personal robots that operate safely in home environments by using standardized architectures. Recently, a novel intrinsically passive control design was proposed that is suitable for human-friendly robotic manipulators because of the compliance and passivity it introduces into the system. Nevertheless, this novel algorithm was only tested with a 1-Degree of Freedom (DOF) actuator, and needs to be extended to a multi-DOF instance and evaluated experimentally.

The passivity concept will be adapted to a multi-DOF manipulator, and enhanced by including energy and power based safety norms in a Cartesian impedance controller design, ensuring safe interaction in a domestic environment. In this direction, the main research objective is:

"Develop a Cartesian impedance controller which guarantees a safe environment for human-robot interaction, ensuring passivity of the overall system."
1.3 Related Work

Concerning stability, different authors have exploited passivity theories to design controllers which ensure asymptotically stability of a robotic manipulator [37, 4, 65]. By definition, passive systems are stable dynamic systems whose total energy is less than or equal to the sum of its initial energy and any external energy supplied to it by interaction [38, 6]. As proven in [51], if the robot is not strictly passive, it is always possible that a passive environment destabilizes the robot’s motion and extracts infinite energy from it.

Because most tasks can be defined in terms of energy, different authors introduced energy tank methods [39, 19], to preserve passivity. By doing so, the robot can use a certain amount of energy to perform a task but no more than that.

While addressing the issue of human friendly robots, different safety criteria can be defined based on risk assessments and performance requirements [62, 8]. In [7], the Head Injury Criteria (HIC) is used as a safety criteria to identify a performance limit given in terms of maximum allowed link velocity. Hence, one of the control objectives is to guarantee that the desired trajectories satisfy a safety velocity limitation. The trade-off between safety and performance is evaluated for different actuation mechanisms and possible controller schemes [66].

In [26], the maximum impact force that can be exerted by a multi-DOF robot is used to define a force based safety metric called impact potential. This metric is used to define a hierarchical controller composed by a low level motion controller and a high level safety protection layer. The motion controller generates required torque outputs to achieve a desired motion trajectory and the safety layer checks the impact potential that will be achieved due to the motion controller torque. If the impact potential is within the safe limit it is passed to the robot’s actuators and if not the torque is cut to an appropriate value to avoid unsafe collisions.

Regarding energy limitations as safety measures, [33] uses energy based metrics to design an energy regulation controller that limits the total energy of a manipulator within the required safety limit by modulating the desired trajectory reference. A power based safety metric is proposed in [36], where the minimum power that can cause a concussion from an impact on a human head is investigated, defining a power limit metric experimentally. Energy related safety indicators are proposed in [34], where the kinetic and potential energies of the system are used as constraints to establish a safe environment around a manipulator’s workspace.

The work done in [55], introduces an impedance controller with safety limitations which is defined through a risk based safety analysis performed, following the guideline defined in [25]. In particular, power and total energy of the system are considered as safety metrics for this analysis. The result is a variable impedance controller for a 1-DOF actuator which guarantees safety, through energy and power limitation and establishes passivity and energy consistency of the system, by decoupling the controller from the robot with an energy-tank [48, 39].

Similarly to [33] and [34], and considering the results presented in [36], a safety-aware impedance controller is developed in this thesis. This controller limits the energy exerted by the robot, and the power flowing from the controller to the manipulator, such that safe Human-Robot Interaction (HRI) is guaranteed.

In particular, the control scheme begins with a basic impedance controller which can be tuned according to certain performance requirements [32, 53], and then modifies the controller parameters (i.e. stiffness and damping) such that safety limits defined in a combined energy and power based metrics are met. This variable impedance controller is connected to an energy tanks system to guarantee the passivity and energy consistency of the system.
1.4 Report Outline

The rest of this thesis report is organized as follows:

- **Chapter 2** is introduced as background material for the mathematics notation employed within the context of the project. Relevant equations used for modelling are presented using screw theory notation.

- **Chapter 3** focuses on the principle behind impedance control. This concept is presented for a 1-DOF case, and then extended to a multi-DOF. The resulting Cartesian impedance controller is then shaped using energy and power based safety metrics to come up with a safety-aware controller.

- **Chapter 4** introduces port-Hamiltonian systems as the fundamental starting point of passive controllers. An energy-tank based approach is presented for a 1-DOF case, and the extended for a multi-DOF general case. The main contribution of this project is presented at the end of this chapter, in a multi-layered structure control scheme.

- **Chapter 5** briefly describes the multi-DOF platform employed for simulations and experiments. A port-based modelling of this manipulator is described, and presented as a standardized architecture for reusability and/or extendability.

- **Chapter 6** displays the evaluation of the controller proposed in Chapter 4. Simulation and experimental results are presented here.

- **Chapter 7** summarizes the main contributions of this project. The results from the experiments presented in Chapter 6 are discussed, and an overview of the future work and recommendations is provided.

"It takes a lot of bad writing to get to a little good writing."

- Truman Capote
Mathematics of Robotic Manipulators

This chapter is included as background material to grasp the terminology and main mathematical concepts in robotics, used within the context of this project. The adopted notation to mathematically express the kinematics and dynamics of the analyzed manipulator is screw theory, which was early developed in [5] and later employed by different authors in the field of robotics [35, 50, 46, 45].

2.1 Kinematics of Serial Manipulators

Any point in a coordinate frame $\Psi_i$, can be expressed relatively to a frame $\Psi_j$, in vector form as follows:

$$ p^j = R^j_i p^i + p^j_i $$  \hspace{1cm} (2.1)

which can be rewritten in matrix form as:

$$ \begin{pmatrix} p^j \\ 1 \end{pmatrix} = R^j_i \begin{pmatrix} p^i \\ 1 \end{pmatrix} + \begin{pmatrix} p^j_i \\ 1 \end{pmatrix} $$ \hspace{1cm} (2.2)

The matrix $H^j_i \in \mathbb{R}^{4 \times 4}$, is known as the homogeneous transformation matrix, used to transform vectors from a coordinate frame $i$, to a coordinate frame $j$, and belongs to the Special Euclidean Group ($SE(3)$):

$$ SE(3) := \left\{ \begin{pmatrix} R & p \\ 0_3 & 1 \end{pmatrix} : R \in SO(3), \ p \in \mathbb{R}^3 \right\} $$ \hspace{1cm} (2.3)

where $p$ and $R$, are the position and orientation of a rigid body, respectively. The rotation matrix $R$, is a square orthogonal matrix that belongs to the Special Orthogonal Group ($SO(3)$):

$$ SO(3) = \left\{ R \in \mathbb{R}^{3 \times 3} : RR^\top = I, \ \text{det} \ R = +1 \right\} $$ \hspace{1cm} (2.4)

The position and orientation of a point with respect to any coordinate frame, can be expressed as products of transformation matrices, following the so-called chain rule:

$$ H^0_n = H^0_1 H^1_2 H^2_3 \cdots H^{n-1}_n $$ \hspace{1cm} (2.5)

According to Chasles’ Theorem, any rigid body motion can be accomplished by means of a rotation about a unique geometrical line in space, followed by a translation along that line. This line is called screw axis [5, 61]. In screw theory, generalized velocities for rigid bodies are described by means of a twist [35, 52]. A twist can be expressed in vector and matrix form as:

$$ T = \begin{pmatrix} \omega \\ v \end{pmatrix} \hspace{2cm} \tilde{T} = \begin{pmatrix} \tilde{\omega} & v \\ 0 & 0 \end{pmatrix} $$ \hspace{1cm} (2.6)
where \( \omega \) and \( \upsilon \) stand for the rotational and translational velocities respectively, of the \( x \), \( y \), and \( z \) axes. Note that the tilde mark in \( \tilde{\omega} \) is known as the tilde operator, which associates in a unique way a vector with its matrix form as follows:

\[
\omega = \begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z 
\end{pmatrix} \quad \tilde{\omega} = \begin{pmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{pmatrix}
\] (2.7)

The matrix in (2.7) is a skew-symmetric matrix, which belongs to the vector space \( so(3) \):

\[
s o(3) = \{ \tilde{\omega} \in \mathbb{R}^3 : \tilde{\omega}^T = -\tilde{\omega} \} 
\] (2.8)

Analogous to points in a Cartesian space, change of coordinates can also be achieved on twists by pre- and post-multiplying \( \tilde{T} \) with an homogeneous matrix:

\[
\tilde{T}^{j,i}_k = H^j_i \tilde{T}^{i,j}_k 
\] (2.9)

Nevertheless, in most cases, it is easier to work with the six-dimensional vector form of a twist. Thus, the change of coordinates for twists in vector form is given by:

\[
T^{j,i}_k = Ad_{H^j_i} T^{i,j}_k 
\] (2.10)

With this in mind, given the joint angles it is easy to find the end-effector position and orientation of serial kinematic chains, as in Figure 2.1.

![Figure 2.1: Serial kinematic chain.](image)

2.1.1 Brockett’s Product of Exponentials Formula for Direct Kinematics

Considering that:

\[
\tilde{T}^{j,i}_i = H^j_i \tilde{H}^i_j \Rightarrow \dot{H}^j_i = \tilde{T}^{j,i}_i H^i_j 
\]

If \( \tilde{T}^{j,i}_i \) is constant, the solution to the previous differential equation is given by:

\[
H^j_i(q_j) = \exp \left( \tilde{T}^{j,i}_i q_j \right) H^j_i(0) 
\] (2.11)

where \( H^j_i(0) \) is the position for \( q_i = 0 \). In view that the homogeneous transformation matrix goes along with the chain rule:

\[
H^n_j = H^n_0 H^1_0 H^2_0 \cdots H^{n-1}_0 
\] (2.12)
the expression in (2.11) can then be substituted in (2.12) to get:

\[
H_n^0(q_1, q_2, q_3, \ldots, q_n) = e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{n-1}^0(0) e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{n-2}^0(0) \cdots e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{0}^0(0)
\]

Subsequently, by using the identity property, and change of coordinates of the exponential argument:

\[
H_n^0(q_1, q_2, q_3, \ldots, q_n) = \prod_{i=1}^{n} e^{\left(\sum_{j=1}^{i} \tau_j q_j \right)} H_{i-1}^0(0) e^{\left(\sum_{j=1}^{i} \tau_j q_j \right)} H_{i-2}^0(0) \cdots e^{\left(\sum_{j=1}^{i} \tau_j q_j \right)} H_{0}^0(0)
\]

Simplifying, the forward kinematics map for an arbitrary open-chain manipulator with \( n \) degrees of freedom can be expressed as:

\[
H_n^0(q_1, q_2, q_3, \ldots, q_n) = e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{n-1}^0(0) e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{n-2}^0(0) \cdots e^{\left(\sum_{i=1}^{n} \tau_i q_i \right)} H_{0}^0(0)
\]

Equation (2.13) is known as Brockett's Product of Exponentials (POE) formula [16], and its use becomes very advantageous since it only depends on the joint angles \( q_i \). Note that the twists in this expression, are unit twists of the form:

\[
\hat{T} = \begin{pmatrix} \hat{\theta} \\ r \times \hat{\theta} \end{pmatrix}
\]

Furthermore, each exponential product in (2.13) can be determined with the Rodrigues' formula [35]:

\[
e^{\hat{\theta} \theta} = I + \hat{\theta} \sin \theta + \hat{\theta}^2 (1 - \cos \theta)
\]

The theory behind Brockett's POE can be found in [16]. For a well explained procedure, and details behind the intermediate expressions, refer to [35] and [52].

2.1.2 Spatial Manipulator Jacobian

The complete direct kinematics of a manipulator is described by the end-effector position and velocity. It can be proved that given a serial chain manipulator as in Figure 2.1, the end-effector twist can be determined by the following expression:

\[
T_n^{0,0} = T_1^{0,0} + T_2^{0,1} + T_3^{0,2} + \cdots + T_n^{0,(n-1)}
\]

From the definition of kinematic pair [52], the relative motion between two objects for a one degree of freedom joint is constrained by a twist of the form:

\[
T_n^{(n-1),(n-1)} = \hat{T}^{(n-1),(n-1)} \cdot \dot{q}_n
\]

where \( \dot{q}_n \) is the joint velocity, and for a rotational joint \( \hat{T}^{(n-1),(n-1)} \) is a constant unit twist of the form:

\[
\hat{T}^{i,j} = \begin{pmatrix} \hat{\theta} \\ r \times \hat{\theta} \end{pmatrix}
\]
for any \( r \) connecting the origin of the reference frame to the joint axis [52]. Substituting (2.10) and (2.16) in (2.15), and rearranging yields:

\[
T_{n,0}^{0,0} = \left( \hat{T}_{1}^{0,0} \text{Ad}_{H_{1}}^{1,1} \hat{T}_{2}^{2,2} \text{Ad}_{H_{2}}^{2,3} \cdots \text{Ad}_{H_{n-1}}^{n-1,n} \hat{T}_{n}^{(n-1),(n-1)} \right) J(q) \hat{q}
\]

\[
J(q) \dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \cdots & \dot{q}_n \end{bmatrix}^\top
\]

where \( \dot{q} \in \mathbb{R}^n \) is the joint velocity vector; and the matrix \( J(q) \in \mathbb{R}^{6 \times n} \), is known as the manipulator Jacobian, which maps the instantaneous velocities of each joint with the instantaneous end-effector's twist, with respect to the base [18, 46, 61].

## 2.2 Dynamics of Serial Manipulators

The whole screw theory was developed embracing Chasles' and Poinsot's theorems, published in [5]. Poinsot discovered that any system of forces acting on a rigid body, can be expressed as a single force acting along a line, plus a torque around the same line [5, 35]. This force is known as wrench, and is the dual of a twist. Accordingly, wrenches can also be expressed in vector and matrix form as:

\[
W = (m \ f)
\]

\[
\tilde{W} = \begin{bmatrix} \tilde{f} & m^\top \\ 0 & 0 \end{bmatrix}
\]

(2.18)

Note that the vector form of a wrench, is a six-dimensional row vector since:

\[
\text{Power} = F v \quad \Rightarrow \quad \text{Power} = W T
\]

(2.19)

Hence, the change of coordinates for wrenches in vector form is given by:

\[
(W^j)^\top = \text{Ad}_{H_{j}}^T (W^j)^\top \quad \text{where} \quad \text{Ad}_{H_{j}}^T = \begin{bmatrix} R_{j}^T & -R_{j}^T \tilde{p}_{j}^T \\ 0 & 0 \end{bmatrix}
\]

(2.20)

### 2.2.1 Newton-Euler Equation of Motion for a Rigid Body

Considering the principal inertia frame \( \Psi_k \), located in the center of mass of a rigid body, and properly oriented which produces an inertia tensor in its simplest form as:

\[
I = \begin{bmatrix} J & 0 \\ 0 & mI \end{bmatrix}
\]

(2.21)

where \( J \) is a diagonal matrix, the dynamic equations of a rigid body can be expressed by means of screw theory defining a momentum screw [47]:

\[
\begin{bmatrix} P \end{bmatrix}^\top_p = \begin{bmatrix} T \\ m \\ v \end{bmatrix} T
\]

(2.22)

Thus, Newton's law is generalized for rigid bodies as:

\[
\dot{P}^0 = W^0
\]

(2.23)

where \( \dot{P}^0 \) is the rate of change of momentum with respect to time, known as moment; and \( W^0 \) is the wrench acting on the rigid body, expressed in the inertial frame \( \Psi_0 \). If the motion is expressed in the body's coordinate frame \( \Psi_k \), (2.23) becomes:

\[
\dot{P}^k = a d_{r_k}^{T_{x,0}} (\dot{P}^k)^\top + (W^k)^\top
\]

(2.24)
where:

\[
\ddot{a}d_{k,0}^T = \begin{pmatrix}
\ddot{\omega}_{k,0}^T \\
0 \\
\ddot{\omega}_{k,0}^T
\end{pmatrix}
\]  
(2.25)

Equation (2.24) describes the dynamics of a rigid body expressed in the principal inertia frame \(\Psi_k\), and is equivalent to:

\[
\mathcal{I}_k \ddot{T}_{k,0}^k = (-\ddot{\omega}_{k,0}^T - \dddot{\omega}_{k,0}^T) \mathcal{I}_k T_{k,0}^k + (W_k)^T
\]  
(2.26)

\[
\left( J \dot{\omega}_{k,0}^T \right) + \left( \omega_{k,0} \times J \omega_{k,0}^T \right) \left( \omega_{k,0} \times mIv_{k,0}^T \right) = \left( (\tau_{k,0})^T \right) \left( (f_{k,0})^T \right)
\]  
(2.27)

Equation (2.27) is known as the Newton-Euler equation, which is expressed in the body’s principal inertia frame, and describes the motion of a rigid body subject to an external wrench [47, 50]. The relevance of this equation becomes evident in Section 5.2.1, where it is used to model the manipulator’s links to carry out simulation experiments.

### 2.2.2 General Dynamics Equation

A robotic manipulator with \(n\) degrees of freedom is distinguished by a set of \(n\) generalized coordinates \(q^T = [q_1, q_2, \ldots, q_n]\) which define the so-called joint space. These coordinates are defined for serial manipulators with revolute joints, by the angle between two consecutive elements of the kinematic chain [18]. Thus, the general dynamics equation of a generic robotic manipulator can be written in joint space as:

\[
\tau^T = M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q)
\]  
(2.28)

where \(M(q) \in \mathbb{R}^{n \times n}\) is the configuration dependent inertia matrix; \(C(q, \dot{q})\) represents all velocity dependent inertia forces such as Coriolis, centripetal, and gyroscopic effects; \(G(q)\) represents the gravitational forces; and \(\tau^T\) is the equivalent joint torques due to all external and interaction forces [45, 46].

Note that term \(\tau^T\) is transposed, unlike in any other bibliography, since wrenches are defined as row vectors in (2.18), such that (2.19) is satisfied. Therefore, for the rest of the report, wrenches and forces in Cartesian space will be defined as row vectors.
Impedance Control of Robotic Manipulators

Industrial robots commonly use position, or force control when it comes to executing planned motion tasks in a known environment. Nevertheless, control of a vector quantity (e.g. force, velocity, or position) alone is not sufficient [29]. With pure position control, if there is contact with an object, the robot is not expected to go through it. Equivalently, with pure force control tasks and motions without contact become difficult to implement [60].

An alternative control technique is an interaction control scheme, where the manipulator is no longer treated as an isolated system. Thus, the dynamic behavior of the manipulator is regulated as it is interacting with the environment [50]. The manipulation is done through energy exchange, in a bidirectional signal exchange between robot and environment during interaction, where these signals are power conjugated variables. In this way, the interaction with the environment can be controlled by adjusting the dynamics of the robot [42].

One of the most consistently used interaction control schemes is impedance control [28, 29, 30]. This notion is introduced as the groundwork for a proposed novel Cartesian impedance controller in this chapter. The featured controller allows safe HRI through energy and power limitations. The concept behind this controller is first presented for a 1-DOF case, and then extended to a multi-DOF instance. Subsequently, feedforward control is presented as an extension to this, or any other Cartesian impedance controller, in pursuance of performance enhancement tool.

3.1 Impedance Control

The underlying distinctness between impedance control and traditional approaches is that alternatively to controlling a state variable alone, such as position, velocity, or force, a dynamic relationship is established between them [40]. This control strategy offers desirable attributes, such as direct specification of the mechanical interaction with the environment. This interaction is distinguished by the energy exchange, in a bidirectional way, between manipulator and environment [50].

In [28], Hogan states that a complete controlled system can be described with an equivalent physical system. Physical systems can be divided in two categories: admittances, which accept effort inputs and yield flow outputs (e.g. force and velocity, respectively); and impedances, which accept flow inputs and yield effort outputs (e.g. velocity and force, respectively). The physical equivalence of a controlled system, facilitates the understanding of the interacting state variables and exploits passivity concepts that assure stability [49, 38].
3.1.1 Damping Injection Framework

Consider a simplified robotic manipulator modeled as a 1-DOF equivalent mass $m$. If $m$ has to be moved to a desired position $x_d$, the most elementary controller that can achieve this task is a spring connected between the mass and $x_d$. In order to avoid continuous oscillation of the system, a damper is added as shown in Figure 3.1. The resulting controller is a simple impedance controller, which is equivalent to a traditional Proportional-Derivative (PD) controller, that can shape the dynamic behavior of the system.

\[ F_{\text{ext}} - k_c (x_d - x) - b \dot{x} = m \ddot{x} \]  

(3.1)

This expression clearly shows the need to measure the position $x$, and velocity $\dot{x}$. Nevertheless, the latter is roughly never available. Thus, the damping injection framework is proposed as an alternative way to consistently create artificial damping [49]. This notion resolves the need of velocity measurement, and circumvents the existing drawbacks in other velocity estimation solutions such as numerical differentiation, state variable filters, or state observers [49, 53].

The damping injection framework is introduced as an extension of the basic impedance controller from Figure 3.1, introducing a virtual mass $m_c$, and a spring that connects this mass with the simplified plant. See Figure 3.2.

\[ m \ddot{x} + F_{\text{ext}} - k(x_c - x) = 0 \]
\[ m_c \ddot{x}_c - k(x - x_c) + b \dot{x}_c - k_c(x_d - x_c) = 0 \]  

(3.2)

Then, if a very stiff virtual spring $k$ and a small virtual mass $m_c$ are chosen, the mass $m$, which represents the simplified robot plant, will adopt the same motion as the virtual mass $m_c$. Subsequently, a damper can be applied on the virtual mass to insert damping in the plant motion. In this way, the mass velocity is known from the state of the controller. This implementation only requires measurement of the plant position, and ensures a strictly passive behavior with closed loop stability [49, 53].

C.A. Cardenas Villa  University of Twente
3.1.2 Multidimensional Instance

The multidimensional case is based on the same reasoning. Considering the Lagrangian dynamic equation of a general multi-DOF serial manipulator:

$$\tau^T = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)$$  \hspace{1cm} (3.3)

where $M(q) \in \mathbb{R}^{n \times n}$ is the configuration dependent inertia matrix; $C(q, \dot{q})$ represents all velocity dependent inertia forces such as Coriolis, centripetal, and gyroscopic effects; and $G(q)$ represents the gravitational forces [18]. Certainly, an equivalent physical controller must be designed in order to compensate for the mentioned terms. The analogous equivalence of the damping injection framework, from Figure 3.2, in Cartesian space is depicted in Figure 3.3.

Figure 3.3: Impedance control of a multi-DOF manipulator in Cartesian space. A multidimensional virtual spring with stiffness matrix $K \in \mathbb{R}^{6 \times 6}$ is connected between the current and desired end-effector’s configuration, defined by $H^0_t$ and $H^0_d$ respectively. Damping can be either introduced through a spatial damper with damping matrix $B \in \mathbb{R}^{6 \times 6}$, or in joint space via a damper $b_n$ on each DOF.

The Cartesian impedance controller is composed of a spatial geometric spring with a symmetric stiffness matrix $K \in \mathbb{R}^{6 \times 6}$, virtually connected between the current and desired end-effector’s configuration; damping can be either added via a multidimensional damper with damping matrix $B \in \mathbb{R}^{6 \times 6}$, or in joint space via a damper $b_n$ on each DOF, as indicated in Figure 3.3. Matrices $H^0_t$ and $H^0_d$ are homogeneous matrices, as defined in (2.3), which describe the current and desired configurations of the end-effector, respectively.

The spatial geometric spring produces a wrench acting on the manipulator’s end-effector, which is defined as [50, 60]:

$$\left( \begin{array}{c} (m^T_k) \vspace{2pt} \\
(f^T_k) \vspace{2pt} \\
(w^T_k) \end{array} \right) = \kappa \left( \begin{array}{ccc} K_0 & K_c \vspace{2pt} \\
K^T_c & K_r \end{array} \right) \left( \begin{array}{c} \delta \theta^d_t \vspace{2pt} \\
\delta p^d_t \vspace{2pt} \\
\delta \chi \end{array} \right)$$  \hspace{1cm} (3.4)

where $\delta \chi = [(\delta \theta^d_t)^T \ (\delta p^d_t)^T]^T$ is an infinitesimal twist displacement in vector form, and $K_r$, $K_0$, and $K_c$, are the translational, rotational, and coupling symmetric stiffness matrices, respec-
If a set of co-stiffness matrices $G_t$, $G_o$, and $G_c$ are defined as:

$$G_x = \frac{1}{2} \text{tr}(K_x) I - K_x \quad \text{for} \quad x = t, o, c$$  \hspace{1cm} (3.5)

where $\text{tr}()$, is the tensor trace operator, the expression in (3.4) can be rewritten as:

$$\begin{bmatrix} \bar{m}_f \\\ \bar{f}_f \end{bmatrix} = -2\text{as}\left(G_o R_d^t \right) - \text{as}\left( G_t R_d^t \right) p_d^t R_d^t \quad \text{as}\left( G_c R_d^t \right)$$  \hspace{1cm} (3.6)

where the operator $\text{as}()$ gives the antisymmetric part of a square matrix; $R_d^t$ and $p_d^t$ are the rotation matrix and position vector, respectively, of the relative configuration between $H_0^t$ and $H_0^d$, which is defined as:

$$H_d^t = H_0^t \cdot \left( H_0^d \right)^{-1} = \begin{bmatrix} R_d^t & p_d^t \\ 0_3 & 1 \end{bmatrix}$$  \hspace{1cm} (3.7)

For a detailed procedure on how to rewrite (3.4), into its equivalent form in (3.6), refer to [50].

In case damping is injected in Cartesian space instead of joint space, the wrench $W_d^t$ acting on the end-effector due to this damper can be similarly determined as presented in [22]. Thus, the combined wrench acting on the end-effector, expressed in the end-effector’s configuration, is given as:

$$W_t^f = W_t^k + W_d^f$$  \hspace{1cm} (3.8)

Having the total virtual wrench applied on the end-effector, it is possible to calculate the joint torques required to emulate this equivalent wrench. Due to the duality nature of force and velocity, the Jacobian expression from (2.17) can be used to compute the equivalent joint torques as [52]:

$$\tau^T = J^T(q) (W^0)^T \quad \text{where} \quad (W^0)^T = Ad_{H_0^t}(W_f)^T$$  \hspace{1cm} (3.9)

The latter expression can be further extended to define a control law as:

$$\tau^T = J^T(q)(W^0)^T + \hat{C}(q, \dot{q}) \dot{q} + \hat{G}(q)$$  \hspace{1cm} (3.10)

where $\hat{C}(q, \dot{q}) \dot{q}$ is the compensation term for Coriolis and centrifugal forces, and $\hat{G}(q)$ compensates for gravity.

One of the advantages of using a Cartesian impedance controller resides in its clear and simple physical interpretation, as depicted in Figure 3.3. This design choice allows a simple analysis of the controller’s energy and power, exchanged with the manipulator [50]. This impedance control scheme can be employed as base groundwork for the implementation of different controllers that fulfill certain specifications in terms of performance [32, 53] (See Appendix A), or other requirements. In the context of this prokect, a safety-aware impedance controller is proposed in the following sections.

### 3.2 Safety-Aware Robot Control

Recent evolution in the field of robotics have allowed the development of robots that can operate close to humans. Thus, new robots designs must consider coexistence and cooperation with humans in different applications such as assisted industrial manipulation, collaborative assembly, domestic work, entertainment, rehabilitation, or medical applications [8, 25]. In this context, safety becomes a topic of supreme importance and hazardous circumstances must be...
considered. Nevertheless, supervision of risk for humans sharing workspace with robots involves very broad considerations in general, varying from potential electrical hazard, damaged pressurized fluid hose, pinching of limbs, dropping parts, etc.

This means that one of the most challenging features for the new generation of robots will be Physical Human-Robot Interaction (pHRI) [8], where humans and robots share the same workspace in an interaction environment. Robots in a human-friendly environment may physically interact with a human for two main reasons: accidentally, in case of unexpected collisions; or deliberately, if the human operator is supposed to physically interact with the robot during work. In both cases, it is important to guarantee a safe physical interaction for injury prevention [3, 62].

A power based safety metric called Head Impact Power (HIP) is proposed in [36]. In this work, the probability of concussion from an impact on a human head is investigated. The results of the presented experiments identify the minimum power that can cause injury to a person. The power limits are then defined as:

\[
P_{\text{limit}} = \begin{cases} 
12\text{KW} & \text{Frontal Impacts} \\
10\text{KW} & \text{Non-Frontal Impacts}
\end{cases}
\]

The works presented in [63] and [64], identify the maximum allowed energy that can cause neck fracture and failure to cranial bones, respectively. Thus, the energy limits defined for this types of injuries are:

\[
E_{\text{limit}} = \begin{cases} 
517\text{J} & \text{Adult cranium bone failure} \\
127\text{J} & \text{Infant cranium bone failure} \\
35\text{J} & \text{Neck fracture}
\end{cases}
\]

One of the most widely used safety norms regarding robotic safety is the HIC, which was first proposed within the automotive industry by Versace in [59]. This metric, along with the previously mentioned, have constituted a useful foundation for the development and evaluation of safety based robot controllers [26, 66, 7, 55].

In conjunction with power limitation as safety metric [36] and limitation of the manipulator’s total energy similarly to [33], a safety-aware impedance controller is defined in Cartesian space. This controller limits the energy exerted by the robot and the power flowing from the controller to the manipulator, such that safe HRI is guaranteed.

Particularly, the proposed controller begins with a basic impedance controller which can be tuned according to certain performance requirements [32, 53], and then modifies the controller parameters (i.e. stiffness and damping) so that established safety limits are met.

A 1-DOF safety-aware impedance controller, which combines energy and power based safety metrics, was first developed by Tadele and presented in [55]. The core ideas behind the proposed controller are summarized and disclosed in the following section, and then used to extend this notion to a multi-DOF instance.

### 3.2.1 Safety-Aware Impedance Control for a 1-DOF General Case

The interaction scheme proposed by impedance control design provides a favorable tool for human friendly robots, as it enhances safety by introducing compliance and guarantees stability due to its passive essence [38]. Its clear and simple physical interpretation, allows the analysis of its energy content and exchanged power with the manipulator.

Considering the dynamic system from Figure 3.2, if a very stiff binding spring is chosen such that \(k >> k_c\), and a very small virtual mass is defined as \(m_c << m\), the dynamics of the system can be assumed to be the same as in Figure 3.1. Thus, the control implementation of this
elemental impedance controller can be written as:

\[ F_c = k(x_d - x) - b \dot{x} \]  

(3.13)

The parameters of this control law can be designed by choosing metrics which establish safe pHRI [62, 25]. Consider the total energy of the system in Figure 3.1, which is defined by the sum of the kinetic energy of the plant, and the potential energy of the spring:

\[ E_{tot} = \frac{1}{2} k x_e^2 + \frac{1}{2} m \dot{x}^2 \]  

(3.14)

where \( x_e = x_d - x \) is the position error, and \( \dot{x} \) is the velocity of the simplified plant, represented by a single mass. If an energy threshold is defined, based on the amount of energy that a human can tolerate without sustaining injury [63, 64], the parameter \( k \) can be determined in such a way that the total energy of the system does not exceed the maximum allowed value \( E_{max} \). In this way, the stiffness parameter is dynamically modified as:

\[
k = \begin{cases}  
k_0, & E_{tot} \leq E_{max} \\ 2E_{max} - m \dot{x}^2 \over (x_d - x)^2, & E_{tot} > E_{max} \end{cases}
\]  

(3.15)

where \( m \) is the simplified plant mass, and \( k_0 \) is the controller stiffness initially chosen based on performance requirements [53, 58]. In this way, parameter \( k \) in (3.13) will be scaled according to (3.15), if the total energy of the system exceeds \( E_{max} \) due to an uncontrolled situation or accidental collision.

In addition to the energy limitation as a safety metric, a power based metric can also be enabled. On account that a manipulator is driven by controlled actuators, the amount of power transferred to a human during an uncontrolled collision can be limited similarly. Considering the impedance control law (3.13), the power that flows from the controller to the manipulator is given by:

\[
P_c = \frac{k(x_d - x) - b \dot{x}}{F} \dot{x} \]  

(3.16)

In this way, after a stiffness value \( k \) is determined as in (3.15), the power flow between controller and plant can be similarly limited to a maximum value \( P_{cmax} \). The latter is attained by computing the damping parameter \( b \) as follows:

\[
b = \begin{cases}  
b_0, & P_c \leq P_{max} \\ k(x_d - x) \dot{x} - P_{max} \over \dot{x}^2, & P_c > P_{max} \end{cases}
\]  

(3.17)

where \( k \) is the stiffness parameter computed with (3.15), and \( b_0 \) is an initial damping value determined with performance specifications [53]. The result is an impedance controller, with variable stiffness and damping parameters, that adheres to established safety metrics.

### 3.2.2 Safety-Aware Cartesian Impedance Control for a Multi-DOF General Case

Considering the configuration of Figure 3.3, extension of the safety-aware controller to a multi-DOF instance is effected by employing the impedance control law from (3.10), whereas for this case damping is injected in joint space for each DOF:

\[
\tau^T = J^T(q)(W_k^0)^T - \ddot{q} + \hat{C}(q, \dot{q}) \dot{q} + \hat{G}(q)
\]  

(3.18)

where \( (W_k^0)^T \) is the wrench acting on the end-effector produced by the multidimensional spatial spring, expressed in the inertial reference frame; \( \hat{C}(q, \dot{q}) \) and \( \hat{G}(q) \), are the Coriolis and
gravity compensation respectively; and \( \mathbf{B} \in \mathbb{R}^{n \times n} = \text{diag}([b_1 \cdots b_n]) \) is a diagonal matrix with damping parameters for each joint.

The adoption of the safety-awareness concept to Cartesian space requires the evaluation of the total energy of the system, and power transferred from controller to manipulator. The total energy of the system is the sum of the kinetic energy of the manipulator, and the potential energy due to the spatial spring. In this direction, the kinetic energy \( T_k \) of the manipulator is defined as:

\[
T_k(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \mathbf{q}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}
\]

where \( \mathbf{M}(\mathbf{q}) \) is the inertia matrix, and \( \dot{\mathbf{q}} \) is the joint velocity vector. Furthermore, the potential energy \( V_c \) stored in the spatial spring, acting on the end-effector, is given by:

\[
V_c(\mathbf{R}_d^i, \mathbf{p}_d^i) = V_t(\mathbf{R}_t^i, \mathbf{p}_t^i) + V_o(\mathbf{R}_o^i) + V_c(\mathbf{R}_c^i, \mathbf{p}_c^i)
\]

where \( V_t, V_o, \) and \( V_c \), are the translational, rotational, and coupling components, respectively, of the total potential energy of the multidimensional spring. These components are defined in terms of \( \mathbf{p}_d^i \) and \( \mathbf{R}_d^i \), which define the position and orientation of the relative configuration (3.7). The elements of (3.20) are computed as defined in [21, 50]:

\[
\begin{align*}
V_t(\mathbf{R}_t^i, \mathbf{p}_t^i) &= -\frac{1}{4} \text{tr} \left( \mathbf{p}_t^i \mathbf{G}_t \mathbf{p}_t^i \right) - \frac{1}{4} \text{tr} \left( \mathbf{p}_d^i \mathbf{R}_d^i \mathbf{G}_t \mathbf{R}_d^i \mathbf{p}_d^i \right) \\
V_o(\mathbf{R}_o^i) &= -\text{tr} \left( \mathbf{G}_o \mathbf{R}_o^i \right) \\
V_c(\mathbf{R}_c^i, \mathbf{p}_c^i) &= \text{tr} \left( \mathbf{G}_c \mathbf{R}_d^i \mathbf{p}_c^i \right)
\end{align*}
\]

where \( \text{tr}() \) is the tensor trace operator, and \( \mathbf{G}_t, \mathbf{G}_o, \) and \( \mathbf{G}_c \) are the translational, rotational, and coupling co-stiffness matrices respectively, defined in (3.5).

Equivalently to the 1-DOF case, energy limitation is established by regulating the amount of potential energy that the spatial spring supplies to the system. As it can be observed, the potential energy components in (3.21) are all proportional to the co-stiffness matrices \( \mathbf{G}_x \) (for \( x = t, o, c \)). Henceforward, by choosing an array of initial stiffness values \( \mathbf{K}_x \) (for \( x = t, o, c \)), a set of initial co-stiffness matrices \( \mathbf{G}_x \) can be computed with (3.5), defining in this way the energy content of the system as:

\[
E_{\text{tot,i}} = T_k(\mathbf{q}, \dot{\mathbf{q}}) + V_c(\mathbf{R}_d^i, \mathbf{p}_d^i)
\]

where \( V_c \) is the potential energy of the controller spring due to the initial co-stiffness matrices \( \mathbf{G}_x \). Consequently, the total energy of the system can be regulated by scaling the initial co-stiffness matrices with a factor \( \lambda \):

\[
\mathbf{G}_x = \lambda \cdot \mathbf{G}_x \quad \text{for} \quad x = t, o, c
\]

By establishing a threshold value \( E_{\text{max}} \), defined based on safety regulations, the parameter \( \lambda \) can be chosen as follows:

\[
\lambda = \begin{cases} 
1, & E_{\text{tot,i}} \leq E_{\text{max}} \\
\frac{E_{\text{max}} - T_k(\mathbf{q}, \dot{\mathbf{q}})}{V_c(\mathbf{R}_d^i, \mathbf{p}_d^i)}, & \text{otherwise}
\end{cases}
\]

where \( E_{\text{tot,i}} \), and \( V_c \) are the initial total and potential energies respectively, computed with the initial co-stiffness matrices \( \mathbf{G}_x \). Therefore, the total energy of the system can be expressed as:

\[
E_{\text{tot}} = T_k(\mathbf{q}, \dot{\mathbf{q}}) + \lambda \cdot V_c(\mathbf{R}_d^i, \mathbf{p}_d^i)
\]

In this way, the total energy of the system will always be less or equal than the established threshold value \( E_{\text{max}} \), establishing a safe HRI environment.
Analogously to the 1-DOF case, the power transferred from the controller to the plant during motion of the manipulator can be regulated by setting a power based safety metric. If an initial damping matrix \(B_i\) is chosen, the power transferred from the controller is given by:

\[
P_{ci} = \left( J^T(q)(W_0^T - \hat{B}_i \dot{q})^T + \hat{G}(q) \dot{q} \right) \dot{q}^T + \hat{G}(q) \dot{q}
\]

(3.26)

where \((W_0^T)^T\) is the wrench acting on the end-effector due to the spatial Cartesian spring, \(P_{cm}\) is the power that flows from the impedance controller to the manipulator, and \(P_{cg}\) is the power consumed for gravity compensation. In case of an uncontrolled collision with a human, the term \(P_{cm}\) in (3.26) is the power that can be transferred to the individual, and the one which is to be limited below a tolerance value. The latter is attained by adjusting the initial damping with a scaling parameter \(\beta\) as:

\[
\hat{B} = \beta \cdot \hat{B}_i
\]

(3.27)

In this direction, establishing a threshold value \(P_{\text{max}}\), just as with the energy safety metric, the scaling parameter \(\beta\) can be chosen as:

\[
\beta = \begin{cases} 
  1, & P_{cm} \leq P_{\text{max}} \\
  \frac{(J^T(q)(W_0^T)^T \dot{q} - P_{\text{max}})}{\dot{q}^T \hat{B}_i \dot{q}}, & \text{otherwise}
\end{cases}
\]

(3.28)

Hence, the power expression can be rewritten as:

\[
P_c = \left( J^T(q)(W_0^T - \beta \cdot \hat{B}_i \dot{q})^T \dot{q} + \hat{G}(q) \right) 
\]

(3.29)

3.3 Feedforward Control

Feedforward control is introduced as a method to track time varying trajectories and enhance disturbance rejection [46]. Given a Cartesian impedance controller, such as the one presented in Section 3.2.2 or Appendix A, feedforward control can be incorporated as an extension to enhance the transient performance during reference tracking. The implementation of this controller is graphically depicted in Figure 3.4.

![Image](image.png)

**Figure 3.4:** Standard feedforward control system.

For an n-link robot manipulator whose dynamics is described as in (2.28), the minimal-order inverse [43] of the manipulator model is chosen such that the feedforward controller is defined in the frequency domain as:

\[
\tau_{FF} = Q(s) \cdot r(s) = \left[ C + Bs + As^2 \right] \cdot r(s)
\]

(3.30)

or in time domain as:

\[
\tau_{FF} = Cr(t) + B\ddot{r}(t) + A\dot{r}(t)
\]

(3.31)
where $A$, $B$, and $C$ are constant matrices defined as:

\[
A = M(q) \quad B_{ij} = \frac{\partial C_i(q, \dot{q})}{\partial \dot{q}_j} \quad C_{ij} = \frac{\partial G_i(q)}{\partial q_j}
\] (3.32)

The control law (3.31) is realizable as long as the velocity and acceleration of the reference are available [44]. In this way, the output torque command sent to the manipulator is given by:

\[
\tau = \tau_c + \tau_{FF}
\] (3.33)

where $\tau_c$ is the implemented controller computed torque.
4

Ascertaining Passivity Through Energy Tanks

Passive systems are a class of dynamical systems in which energy exchange with the environment plays a central role, since they cannot deliver more energy than what is stored [6]. The connection between the energetic features of a system and its stability, is delineated by means of Passivity-Based Control (PBC) [38].

A straightforward implementation of the safety-aware, and performance-based Cartesian controllers presented in Section 3.2.2 and Appendix A, respectively, contradicts the energetic consistency of impedance control design. The scaling of parameters in the controller allows internal energy production, resulting in the loss of passivity of the overall system.

Thereupon, the energy-tank based controller implementation presented in [55] is adopted as a feasible way to circumvent this issue. After introducing general concepts of passivity and port-Hamiltonian systems, the energy-tank concept is first presented for a 1-DOF case, and then extended for a multi-DOF general case. The latter, together with the controller presented in Section 3.2.2, builds up the so-called Safety-Aware Intrinsically Passive (SAIP) controller, which is the major significance of this study.

4.1 Port-Hamiltonian Systems and Passivity

The essential starting point for PBC design is delineated by port-Hamiltonian systems, as they are a subclass of passive systems [42]. The port-Hamiltonian formulation, defines ports that build up interconnection with other systems. Power-conserving interconnection of port-Hamiltonian structures, allows consistent modeling of complex systems [37]. In this way, different domain subsystems can be interconnected through power conjugated variables. See Table 4.1.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Effort</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanics (translational)</td>
<td>Force $F$</td>
<td>Velocity $v$</td>
</tr>
<tr>
<td>Mechanics (rotational)</td>
<td>Torque $\tau$</td>
<td>Angular velocity $\omega$</td>
</tr>
<tr>
<td>Electric</td>
<td>Voltage $v$</td>
<td>Current $i$</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure $p$</td>
<td>Volume Flow $Q$</td>
</tr>
<tr>
<td>Thermodynamical</td>
<td>Temperature $T$</td>
<td>Entropy flow $\dot{E}$</td>
</tr>
</tbody>
</table>
A passive system satisfies the energy-balance equation [6]:

\[
H[x(t)] - H[x(0)] = \int_0^t u^T(s)y(s)\,ds - d(t) \tag{4.1}
\]

where \(H(x)\) is the total (free) energy function, \(x \in \mathbb{R}^n\) is the state vector, \(u\) and \(y\) are power conjugated variables, and \(d(t)\) is a positive dissipation function that describes (irreversible) energy conversion to the thermal domain.

The port-Hamiltonian formulation allows to describe a physical system in terms of energy exchange. Energy-conserving physical systems can be explicitly modeled in the form of Port-Controlled Hamiltonian (PCH) as:

\[
\begin{align*}
\dot{x} &= J(x) \frac{\partial H}{\partial x} + g(x)u \\
y &= g^T(x) \frac{\partial H}{\partial x}
\end{align*} \tag{4.2}
\]

where \(J\) is a skew symmetric matrix which establishes a power preserving interconnection between internal variables, and \(g\) is a matrix defining how external power is distributed into the system [37]. The latter port-Hamiltonian formalism, provides a framework for modeling physical systems that describe the energy storage and external interaction phenomena.

### 4.2 Energy-Tank Based Controller for a 1-DOF General Case

The nature of a physical system can be outlined in terms of energy flow and energy storage. In this direction, it is essential to define a function representing the total energy of the system. This function must have a Dirac structure [57], which is associated with a skew-symmetric matrix delineating the internal power preserving interconnections.

Consider the mass-spring system in Figure 4.1:

![Elemental mass-spring system.](image)

The total energy of the system is defined by the sum of the kinetic and potential energies as:

\[
E_{tot} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} kx^2 \tag{4.3}
\]

If (4.3) is rewritten as:

\[
H(p, x) = \frac{p^2}{2m} + \frac{1}{2} kx^2 \tag{4.4}
\]

the total energy expression can be displayed in the PCH form as:

\[
\begin{align*}
\frac{d}{dt} \begin{pmatrix} x \\ p \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial x} \\ \frac{\partial H}{\partial p} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} F_{ext} \\
\begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} kx \\ \frac{p}{m} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} F_{ext} \tag{4.5}
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} kx \\ \frac{p}{m} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} F_{ext} \tag{4.6}
\end{align*}
\]
Equation (4.6) is then used as the fundamental starting point of the energy-tank concept presented by Tadele in [55]. The idea is to have a controlled system that satisfies the energy-balance equation (4.1). In this direction, due to its energy-storage nature, a spring is used to model an energy tank, such that a controller force output is applied to the system only when there is available stored energy. The physical interpretation of this notion is depicted in Figure 4.2.

The energy-tank concept of the simplified 1-DOF case incorporates a spring connected to a plant through a transmission. A computational unit \( CU \) is used to determine the variable transmission ratio \( u \), such that the transmission element \( MT \), allows power flow from the controller to the plant as long as there is energy left in the tank \( H(s) \).

Henceforward, if a stiffness constant is chosen as \( k = 1 \), and \( s \) is used to define the spring state of \( H(s) \), the port-Hamiltonian formulation of the system in Figure 4.2 can be expressed as:

\[
\begin{bmatrix}
\dot{s} \\
F_{out}
\end{bmatrix} =
\begin{bmatrix}
0 & u \\
-u & 0
\end{bmatrix}
\begin{bmatrix}
s \\
x
\end{bmatrix}
\]

(4.7)

In this way, the port-Hamiltonian expression in (4.7) can be used to set the desired transmission ratio \( u \) as:

\[
u = -\frac{F_c}{s}
\]

(4.8)

where \( F_c \) is the controller force computed from (3.13). PBC design requires a physically consistent energy coupling. Under these circumstances, the transmission ratio \( u \) should ensure isolation between plant and controller when the energy tank is depleted [55]. The latter is accomplished by the following passivity assuring relation:

\[
u = \begin{cases} 
-\frac{F_c}{s}, & \text{if } (H(s) > \epsilon) \lor (P_c < 0) \\
0, & \text{otherwise}
\end{cases}
\]

(4.9)

where \( F_c \) is the desired controller force; \( H(s) = \frac{1}{2} s^2 \) is the potential energy in the tank model designating \( k = 1 \); \( \epsilon \) is the minimum amount of energy allowed in the tank, before plant and controller isolation; and \( P_c \) is the power flowing from the controller to the plant.

Note that the if condition in (4.9) allows the computation of \( u \), only when there is energy left in the tank model \( (H(s) > \epsilon) \), or when the power flowing from the controller to the plant is negative.
\( P_c < 0 \). The latter condition is satisfied when an external force is acting on the system, which is translated as addition of energy from an external source.

Lastly, once the transmission ratio is settled, the output force command sent to the system plant is computed as:

\[
F_{\text{out}} = -u \cdot s = \begin{cases} 
F_c, & \text{if } (H(s) > \epsilon) \lor (P_c < 0) \\
0, & \text{otherwise}
\end{cases}
\]

(4.10)

where the spring state \( s \) is determined by integrating the following expression:

\[
\dot{s} = u \cdot \dot{x}
\]

(4.11)

### 4.3 Extension to a Multi-DOF General Case

The general 1-DOF simplified case can be extended to a multi-DOF instance, where each joint is examined as a subsystem. Each subsystem is described by the energy storage phenomenon illustrated in Figure 4.2, upholding the energy preserving interconnection of the PCH format in (4.2). The physical perception of the energy-tank based controller for the multi-DOF case is depicted in Figure 4.3.

![Figure 4.3: Physical depiction of the energy-tank based controller for multi-DOF manipulators.](image)

Analogously to the 1-DOF case, the port-Hamiltonian formulation for a joint subsystem \( n \) can be disclosed as:

\[
\begin{bmatrix}
\dot{s}_n \\
\tau_{\text{motor}}
\end{bmatrix} =
\begin{bmatrix}
0 & u_n \\
-u_n & 0
\end{bmatrix}
\begin{bmatrix}
s_n \\
\dot{q}_n
\end{bmatrix}
\]

(4.12)

whereas in this case, the transmission ratio \( u_n \) is determined as:

\[
u_n = \begin{cases} 
-\frac{\tau_c}{s_n} & \text{if } (H_n(s_n) > \epsilon) \\
-\frac{\tau_c}{\gamma^2 s_n}, & \text{otherwise}
\end{cases}
\]

(4.13)
Table 4.2: Energy tank concept comparison between 1-DOF and multi-DOF systems.

<table>
<thead>
<tr>
<th>1-DOF</th>
<th>Multi-DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{pmatrix} \dot{s} \ F_{\text{out}} \end{pmatrix} = \begin{pmatrix} 0 &amp; u \ -u &amp; 0 \end{pmatrix} \begin{pmatrix} s \ \dot{x} \end{pmatrix}$</td>
<td>$\begin{pmatrix} \dot{s}<em>n \ \tau</em>{\text{out}} \end{pmatrix} = \begin{pmatrix} 0 &amp; u_n \ -u_n &amp; 0 \end{pmatrix} \begin{pmatrix} s_n \ \dot{q}_n \end{pmatrix}$</td>
</tr>
</tbody>
</table>

$u = \begin{cases} \frac{-F_c}{s}, & \text{if } (H(s) > \epsilon) \lor (P_c < 0) \\ 0, & \text{otherwise} \end{cases}$

$\begin{cases} \frac{-\tau_{ca}}{s_n}, & \text{if } (H_n(s_n) > \epsilon) \\ \frac{-\tau_{ca}}{s_n} \gamma^2 s_n, & \text{otherwise} \end{cases}$

$F_{\text{out}} = -u \cdot s$

$\dot{s} = u \cdot \dot{x}$

$\dot{s}_n = u_n \cdot \dot{q}_n$

where $\gamma = \sqrt{2 - \epsilon}$, $\tau_{ca}$ is the torque computed from the implemented control law, $s_n$ is the state of the spring in the energy tank $H_n$, and $\epsilon$ is the minimum amount of stored energy before power flow is restrained in joint $n$. Once the transmission ratio is settled according to the energy levels in $H_n(s_n)$ the torque output sent to joint $n$ is computed as:

$$\tau_{\text{out}} = -u_n \cdot s_n$$

(4.14)

The skew-symmetric matrix $J$ from (4.2) delineates the interconnection structure of the power conjugated variables of the physical system in case. Hence, the relevance of $u$ becomes manifest for the energy storage conception described. In this way, attention is brought to the remarkable disparities between the transmission ratio expressions from (4.9) and (4.13), for the 1-DOF and multi-DOF systems, respectively. Both expressions are contrasted with one another in Table 4.2.

The role of the constant value $\epsilon$ for both cases, is to indicate the instant when the tank $H(s)$ has largely used its available stored energy. In this way, power flow from controller to the system plant is restrained by $u$, by changing to the bottom state of the piecewise function as seen in (4.9) and (4.13). This prevents $u$ from becoming an undesirable large value as $s$ approaches zero.

By means of setting $u$ to zero in (4.9), isolation between controller and plant is accomplished, on account that the force output $F_{\text{out}}$ is computed as in (4.10). Consequently, the system can only be re-established if an external source introduces energy into the system, which is the justification of the presence of the power inequality in the upper statement of (4.9).

Unlike (4.9), the piecewise function in (4.13) does not assign a value of zero to the transmission ratio when the energy tank is depleted; moreover, the inequality $(P_c < 0)$ is removed from the if condition. These are obliged adaptations for the multi-DOF instance that induce passivity preservation of the overall system.

Even though each joint is evaluated as a subsystem, they are still correlated between one another as energy exchange takes place among them. The latter can be better discerned by looking back to Figure 4.3. An arbitrary motion that involves multiple joints may use the energy of $H_n$, and then return it back as a consequential motion of a different joint, both being immersed in a spring compression-elongation process triggering energy exchange.

Under these circumstances, when $H_n$ runs out of energy, joint $n$ is virtually shut down by changing the dynamics of $u_n$, instead of making it zero. In this way, the execution of a control action still takes place during the depleted state of the tank. This strategy circumvents the
concern of \( H_n \) being recharged by the compression of the spring in joint \( n \) due to the inertial motion of a different link. The latter can be graphically understood by looking at the results of the simulation experiment from Section 6.3.

By fixing the controller force/torque to a constant value (e.g. \( F_c = -1 \) and \( \tau_{cn} = -1 \)), the behavior of the transmission ratio \( u \) is illustrated in a plot as a function of the spring state \( s \). See Figure 4.4.

![Transmission Ratio as Function of Energy Tank State](image)

**Figure 4.4:** Transmission ratio plotted as a function of the spring state. Transmission \( u \) corresponds to the 1-DOF expression (4.9), and \( u_n \) corresponds to the multi-DOF expression (4.13). The points with coordinates \((\pm \gamma, \pm 1/\gamma)\), where \( \gamma = \sqrt{2/\varepsilon} \), indicate the instant when the energy tank switches to a depleted state \((H(s) \leq \varepsilon)\).

The distinctness from (4.13) with (4.9) analyzed in Table 4.2, becomes evident by looking at Figure 4.4. Note that the transmission ratio \( u_n \), which correspond to the multi-DOF system, adopts a smooth decaying behavior as the tank \( H_n(s_n) \) enters to a depleted state, preserving a control action that gradually shuts down joint \( n \). Contrarily, the transmission \( u \) from the 1-DOF case instantly drops to zero as soon as \( H \) comes to a depleted state, making the tank able to be recharged only when \((P_c < 0)\) is satisfied. This inequality is absent in (4.13), on account that other links may still be in motion when the tank is depleted, which would generate small noise values bringing \( u_n \) to the upper state of the piecewise function. The latter would result in an undesired high frequency turnover between the states of \( u_n \), generating unwanted oscillation behavior in the joint torques values. Therefore, a linear function behavior is chosen for \( u_n \) when \( H_n \leq \varepsilon \), such that \( \tau_{nout} \) drops proportionally to the state \( s_n \). In this way, \((P_c < 0)\) can be removed from the if condition of the upper statement in (4.13), letting \( s_n \) to be purely defined by the integration of \( \dot{s}_n = u_n \dot{q}_n \). Thus, joint \( n \) is virtually shut down when \( H_n \leq \varepsilon \), and is reactivated if energy is added through an external force, modifying \( \dot{s}_n \) which finally defines the state of sprig \( n \).
4.4 Safety-Aware Intrinsically Passive Controller

The energy tank-concept is shaped from the port-Hamiltonian formulation as a subclass of passive systems. This concept, is presented as a tool that can be added to specific task oriented controller, in order to guarantee robust stability in a passive environment. The safety-aware impedance control, presented in Section 3.2.2, is incorporated with the multi-DOF energy-tank concept to build up the so-called SAIP controller. This conception can be summarized in a multi-layered structure control strategy, illustrated in Figure 4.5.

![Figure 4.5: Layered scheme of the proposed controller. The SAIP controller consists on three layers: a safety layer, a motion layer, and a passivity layer. In this way, the manipulator receives joint torques that guarantee a safe environment, establishing passivity of the overall system.](image)

Given a desired configuration described by a matrix $H^0_d$, and the current end-effector’s configuration $H^0_t$, a relative configuration between both matrices is determined as in (3.7). Then, the potential energy of the multidimensional spring is computed with (3.20), whose components are defined in terms of the position $p^d_t$ and orientation $R^d_t$ of the relative configuration. Then, the total energy of the system is inspected in the initial Safety Layer, and adjusted according to (3.24) in case the threshold value $E_{\text{max}}$ is exceeded. In this way, the stiffness parameter of the Cartesian impedance controller is set to a value, such that an energy limitation is assured. Subsequently, the power transferred from the controller is determined with (3.26) and examined in the same layer. If this value exceeds $P_{\text{max}}$, the power is adjusted with the scaling factor $\beta$ using (3.28). This results in a Cartesian impedance controller that ensures a safe environment for pHRI.

Afterwards, the manipulator Jacobian is computed in the Motion Layer as in (2.17). This is used to determine the joint torques equivalent to the virtual wrench acting on the end-effector, in order to define the control law (3.18). Finally, the Passivity Layer incorporates the energy-tank concept from Section 4.3, to ensure passivity of the overall system.

The result is a stable and passive controller which allows HRI in a safe environment. The SAIP controller is tested on a multi-DOF KUKA manipulator [2], which is modeled in Chapter 5. Simulation results and experiments, carried-out in this platform, are presented in Chapter 6.
5

Port-Based Analysis of the Manipulator Dynamics

The port-Hamiltonian formulation enables the analysis of coupled physical systems, using energy as a common language for interconnection. This notion is known as port-based modelling, and allows complex systems, from different domains, to be described by the interconnection of elementary blocks. Port-based modelling uses bi-directional energy ports between interconnected components; whereas classical block diagrams and other signal-based systems deal with data flow in one direction [11]. The graphical representation of this modelling concept is brought by the so-called bond graphs [15].

Followed by an introductory overview of the employed platform, the dynamic model of the multi-DOF manipulator is introduced in a component based manner adopting the bond graph notation. This method ensures extendability, and reusability of the model. A description of each component is presented as an individual element of the overall system, in order to grasp the modelling approach of the unified plant. The kinematic representation of mechanical elements is inspected using screw theory [35, 52].

5.1 KUKA Lightweight Robot 4+ Manipulator

The KUKA Lightweight Robot (LWR) manipulator, developed by the German Aerospace Center (DLR) [1], is one of the earliest generations of manipulators designed for human interaction. This robot features moderate joint compliance, suitable sensing, and control capability [27]. Due to its lightweight essence, dynamic performance is increased by reducing power consumption. The characteristics of the LWR manipulator are decisive for the next generation of robots interacting in a domestic environment.

The manipulator is a 7-axis jointed robot arm, where each joint is equipped with position and torque sensors. Thus, the robot can be operated with position, velocity and torque control. Unlike typical 6-DOF manipulators, the KUKA LWR displays flexibility in scattered workspaces. This is possible due to a redundant DOF, which helps avoiding typical singularities of a 6-axis robot manipulator [10]. General specifications of each joint axis are presented in Table 5.1.

In order to make results easier to compare and to transfer to industry, the KUKA LWR was established as a reference platform for research by the EU-funded research project BRICS [9]. Due to the fitting characteristics displayed by the manipulator within the framework of this project, the KUKA robot arm in case is used as experimental platform. Thus, the foremost step is the development of a port-based dynamic model, so that the SAIP controller concept from Section 4.4 is proven in a simulation environment.

It is intended that the model truthfully behaves egalitarian to the real system. Nevertheless, some idealizations are settled:
Table 5.1: Axis data of the KUKA LWR manipulator [24].

<table>
<thead>
<tr>
<th>Axis</th>
<th>Range</th>
<th>Speed</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 (J1)</td>
<td>±170°</td>
<td>110°/s</td>
<td>176 Nm</td>
</tr>
<tr>
<td>A2 (J2)</td>
<td>±120°</td>
<td>110°/s</td>
<td>176 Nm</td>
</tr>
<tr>
<td>E1 (J3)</td>
<td>±170°</td>
<td>128°/s</td>
<td>100 Nm</td>
</tr>
<tr>
<td>A3 (J4)</td>
<td>±120°</td>
<td>128°/s</td>
<td>100 Nm</td>
</tr>
<tr>
<td>A4 (J5)</td>
<td>±170°</td>
<td>204°/s</td>
<td>100 Nm</td>
</tr>
<tr>
<td>A5 (J6)</td>
<td>±120°</td>
<td>184°/s</td>
<td>38 Nm</td>
</tr>
<tr>
<td>A6 (J7)</td>
<td>±170°</td>
<td>184°/s</td>
<td>38 Nm</td>
</tr>
</tbody>
</table>

- Links are considered as rigid bodies.
- Link mass distribution is considered uniform.
- Effect on link inertias due to weight of low level electronics is neglected.
- Motor dynamics and gear transmissions are not included in joint dynamics models.

The components characterization and relevant dimensions of the manipulator are displayed in Figure 5.1. These values, along with some other pertinent parameters, are taken from the Universal Robotic Description Format (URDF) file provided by the Research Center "E.Piaggio", which is available at their GitHub repository1.

Figure 5.1: Schematic of the KUKA LWR 4+ manipulator with components characterization.

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1https://github.com/CentroEPiaggio
5.2 Bond Graph Dynamic Model

A graphic representation of a dynamic system allows a better understanding of the interacting physical variables involved. Bond graphs are an iconographic description formalism, which simplifies the modeling of engineering processes from different physical domains [13]. Bond graph modelling is founded from the intuitive energy exchange principle outlined by the port-Hamiltonian formulation of physical systems.

The essential feature of this graphic representation, is the interconnection of multidisciplinary engineering systems composed of subsystems, and basic elements that interact by exchanging energy [11]. The work done in [20] introduces a walk-through tutorial on the modelling of a robotic system, in a reusable and extendable object-oriented manner, using bond graphs. In this work, a serial manipulator model is constructed from modeled modular components. This model can be readjusted and enabled to fit to different serial robot manipulators, as in this case study.

The KUKA LWR 4+ manipulator is modeled as a series of rigid bodies, representing the robot links, which are connected through joints. The model for each element of the manipulator is presented individually as a submodel of the entire system. For a more extensive and detailed explanation of each of the elements in the conferred model, refer to [20]. For the interested reader, a meticulous description of bond graph theory and notation can be found in [14, 15].

5.2.1 Bond Graph Model of a Link

In furtherance to impose simpler dynamics, each of the robot’s links is modeled as a rigid body. Subsequently, the Newton-Euler equation (Section 2.2.1) of a rigid body is considered:

\[
\mathbf{I}_k \ddot{\mathbf{r}}^{k,0}_k = \left( -\mathbf{\omega}^{k,0}_k \times \mathbf{I}_k \mathbf{\omega}^{k,0}_k \right) + \left( \mathbf{W}_k \right)^T \tag{5.1}
\]

which can be rewritten as:

\[
\begin{bmatrix}
    J\dot{\omega}^{k,0}_k \\
    m\mathbf{I} \dot{\mathbf{v}}^{k,0}_k \\
end{bmatrix}
+ \begin{bmatrix}
    \mathbf{\omega}^{k,0}_k \\
    \mathbf{\omega}^{k,0}_k \\
end{bmatrix} \times 
\begin{bmatrix}
    J\dot{\omega}^{k,0}_k \\
    m\mathbf{I} \dot{\mathbf{v}}^{k,0}_k \\
end{bmatrix}
= \left( \mathbf{m}_k \right)^T \times \left( \mathbf{f}_k \right)^T \tag{5.2}
\]

Equation (5.2) describes the motion of a rigid body expressed in the body’s principal inertia frame, subject to an external wrench [35, 47]. This expression can be represented in bond graph notation as an effort(force) summation with a 1-junction, as depicted in Figure 5.2.

![Figure 5.2: Bond graph model of a rigid body.](image)

The bond graph model of a rigid body, includes an inertia element \( I \) representing the inertia tensor \( \mathbf{I}_k \) from (5.1); the relation between gyroscopic moments and angular velocities is represented by the modulated gyroscope \( \text{MGY} \) element, in the so-called Eulerian Junction Structure (EJS) [12]; then, addition of external forces is represented by the source of effort \( \text{Se} \) element. Figure 5.2, displays the bond graph equivalence of Equation 5.2. However, this model is later modified so that it can be adopted within the overall robot manipulator model, as a link com-
ponent. Keeping in view that a manipulator’s link is placed between two joints, the reference frames to be considered in the featured submodel are displayed in Figure 5.3.

![Figure 5.3](image)

**Figure 5.3:** Robot manipulator’s link modeled as a rigid body, displaying the reference frames used to describe its configuration. Frame $\Psi_i$ is located at the connection point with the previous joint, frame $\Psi_j$ is located at the connection point with the next adjacent joint, frame $\Psi_k$ is located at the center of mass of the rigid body and is known as the principal inertia frame.

From Figure 5.3, $\Psi_i$ and $\Psi_j$ are reference frames fixed in the interaction points with the preceding and succeeding joints, respectively; $\Psi_k$ is the principal inertia frame; and $\Psi_0$ is the inertial reference frame. In this direction, the bond graph representation corresponding to a robot’s link, is shown in Figure 5.4.

![Figure 5.4](image)

**Figure 5.4:** Manipulator’s link bond graph representation. $\Psi_i$ and $\Psi_j$ are reference frames fixed in the interaction points with the preceding and succeeding links, respectively; $\Psi_k$ is the principal inertia frame. Transformer elements $TF$ are used to express joint reactions, coming from the input/output ports, in the principal inertia frame $\Psi_k$. The force produced by gravity, is modeled with a source of effort $Se$ element, which is then expressed in $\Psi_k$ after going through the modulated transformer element $MTF$. The blocks marked with an $H$ are used to perform transformation of matrices im-
implementing the chain rule as in (2.5). In this way, the bond graph model in Figure 5.4, certainly
serves as a robot link representation, adhering to the dynamics of (5.2).

5.2.2 Bond Graph Model of a Joint

A joint establishes an energetic connection between two links, it imposes a relation between
the wrenches and the twists of the two bodies. This relative motion can be described by screw
theory [35, 52] as:

\[ T_{i,b}^{a} = T_{i,0}^{a} - T_{i,0}^{b} \]  

(5.3)

where \( T_{i,0}^{a} \) and \( T_{i,0}^{b} \) are twists of bodies \( a \) and \( b \) with respect to the inertial frame \( \Psi_0 \), expressed
in frame \( \Psi_i \); and \( T_{i,b}^{a} \) is the relative twist between these twists, expressed in frame \( \Psi_i \).

In light of Newton’s third law of action and reaction, the wrench applied in a joint correspond-
ing to the actuator’s input torque, produces an equal reaction on both adjacent links:

\[ W_{\text{act}} = W_{\text{link}_1} = W_{\text{link}_2} \]  

(5.4)

The relations from (5.3) and (5.4), correspond to a 0-junction in bond graph notation [15, 11].

All joints in the LWR 4+ manipulator are rotational joints. The relation between the actuator’s
torque and the corresponding wrench in the joint is given by:

\[ W = \hat{W} \tau^T \]  

(5.5)

with:

\[ \hat{W} = (\hat{m} \ 0) \]

where \( \hat{\tau} \) is the axis around which the torque is applied. Due to power continuity, a relationship
between the joint velocity and the joint twist is established:

\[ \text{Power} = WT = m\omega + f \nu \]

\[ W T_{i,b}^{a} = \hat{W} \tau^T \Rightarrow \tau = m \]

\[ \omega \Rightarrow \dot{q} = \hat{W} T_{i,b}^{a} \]  

(5.6)

The latter relation is represented with a \( TF \) element in bond graph language. The outright
model of a joint is shown in Figure 5.5.

![Joint Bond-Graph Model](image-url)

Figure 5.5: Joint Bond-Graph Model.
The wrench reactions of the neighboring links are expressed in their corresponding body fixed frame; therefore, change of coordinates is carried out through the $MTF$ element, just as with the link model. A storage element $C$, and a resistive element $R$, are added to the model to constrain the motion only to the joint rotation axis. An additional $R$ element is included to incorporate joint friction. Just as with the link case, the joint submodel preserves a generalized component structure, such that each component can be connected to one another through the input/output ports.

5.2.3 The KUKA LWR 4+ Serial Manipulator Model

Once the submodels of a link and joint are defined, a component-based model for the LWR 4+ serial manipulator is built, as illustrated in Figure 5.6.

![](image)

**Figure 5.6:** Component-based model of the KUKA LWR 4+ manipulator, where each link and joint blocks contain their corresponding submodel as illustrated. The model can be extended and/or adapted to any other serial manipulator as a result of its component-based structure.

Each block labeled as link and/or joint, includes the previously described submodels from Figures 5.4 and 5.5, respectively. Each element is connected to its neighboring component through the input/output ports, present in both illustrated submodels.

The component-based structure of the overall model, discloses extendability and reusability features that facilitate the modelling of any other serial manipulator. The featured model, is then used to evaluate the SAIP controller concept, presented in Section 4.4, through simulation experiments. Pertinent experimental results are reported in Chapter 6.
Experimental Evaluation of the Proposed Controller

The SAIP controller conception, disclosed in Section 4.4, is experimentally evaluated with 20-sim [17] and the Fast Research Interface (FRI) for the KUKA LWR [41]. The concepts considered as most relevant for the understanding of the controller’s essence, are presented first through simulation experiments. Then, a three-phase experiment is enacted in the KUKA multi-DOF platform, exhibiting safe HRI. Note that all simulations and the three-phase experiment are done for the multi-DOF instance, as the 1-DOF notion is already presented in [53] and [55].

6.1 Safety-Aware Robot Control

Given an arbitrary motion in Cartesian space, the total energy of the system $E_{\text{tot}}$ and the power flowing from the controller $P_c$ are limited to $E_{\text{max}} = 1.5$ and $P_{\text{max}} = 0.8$. This is achieved by adjusting the scaling factors $\lambda$ and $\beta$, with (3.24) and (3.28) respectively. Then, energy and power are computed with (3.25) and (3.29). Simulation results are shown in Figure 6.1.

![Figure 6.1: Simulation results - Energy and power limitation. The total energy of the system $E_{\text{tot}}$ is limited by the scaling factor $\lambda$; similarly, the controller power $P_c$ is limited by $\beta$.](image)

It can be observed how parameters $\lambda$ and $\beta$ are scaled to meet the safety specifications for $E_{\text{tot}}$ and $P_c$ plots accordingly. The same simulation was executed for a periodic motion, using the same safety limits. Results are disclosed in Figure 6.2.
Figure 6.2: Simulation results - Energy and power limitation, periodic motion. A periodic motion is given as a reference where $E_{\text{tot}}$ and $P_c$ are limited by adjusting the scaling factors $\lambda$ and $\beta$.

As observed in both simulation results from Figures 6.1 and 6.2, every time the $E_{\text{tot}}$ and $P_c$ plots appear to be clipped the scaling factors $\lambda$ and $\beta$ are adjusted accordingly. Afterwards, an external force was applied on the manipulator’s end-effector to simulate a collision with an individual. The latter case is illustrated in Figure 6.3.

Figure 6.3: Simulation results - Energy and power limitation, periodic motion with collision. An external force is added at $t = 10[s]$, acting on the end-effector for 1 second.

For the given trajectory, the manipulator performs a periodic motion using joint A1 (See Figure 5.1). An external force is applied for 1 second on the manipulator’s end-effector. The force is applied at $t = 10[s]$ in opposite direction of the manipulator’s motion. The same safety metrics are used, which are $E_{\text{max}} = 1.5$ and $P_{\text{max}} = 0.8$. From Figure 6.3, the total energy $E_{\text{tot}}$ is perfectly kept under the established value and the factor $\lambda$ remains close to zero for a long period of time.
This is due to the fact that the potential energy of the virtual spring, which is the one adjusted by $\lambda$, depends on the relative configuration between end-effector and desired position.

In this way, when the external force is applied the manipulator is deviated from the tracked reference, making $p^d_t$ and $R^d_t$ larger, increasing the value of $V_p$ (See Equation 3.20) as a consequence. Eventually, it becomes clear how the $E_{tot}$ and $\lambda$ plots return to its periodic behavior. This is better grasped by looking at the y-axis reference tracking plot of the same simulation, shown in Figure 6.4

![Figure 6.4: Simulation results - Reference tracking with collision. An external force is applied at $t = 10[s]$, acting on the end-effector for 1 second.](image)

From Figure 6.4, it is clear how the manipulator tracks back the reference after being deviated by the applied force. It is assumed that a collision with a person would result in an immediate reaction of the individual of moving apart from the robot. This is why the manipulator continues with its task in a safe way.

An additional remarkable observation can be made from the $P_c$ plot in Figure 6.3. It is noted how $P_c$ is kept at $P_{max} = 0.8$ right after the impact, and how $\beta$ considerably increases its value for a short moment. Afterwards, the controller power goes negative until $t = 11[s]$. The prompt peak of $\beta$ at $t = 10[s]$ appears as the external force tries to stop the manipulator before moving it apart from the reference. Subsequently, $P_c$ becomes negative and power starts flowing from the manipulator to the controller, and not the other way around. This means that the system is being injected with external energy. In other words, the external force first stops the manipulator and then attempts to move it apart from its trajectory, until the force is no longer applied at $t = 11[s]$. Figure 6.5 shows the power and energy tank of joint A1, where it is shown how $H_1(s_1)$ is recharged when $P_{c_1}$ is negative.

![Figure 6.5: Simulation results - Energy tank and joint power. The energy tank $H_1(s_1)$ from joint A1 is recharged when $P_{c_1} < 0$, while the external force is applied.](image)
6.2 Performance/Safety: Trade-off

The cornerstone of pHRI design is intrinsic safety. This means that a robot must be safe to humans at all times, even during failure, malfunctioning, or misuse [33]. Nevertheless, flawless safety is not achievable for processes that must deliver performance in terms of motion, parts positioning, or weight lifting. Thus, a trade-off between performance and safety is inherent to pHRI [8]. This is illustrated in Figure 6.6.

![Figure 6.6: Simulation results - Performance and safety trade-off. A periodic motion is given to joint A1 (See Figure 5.1) such that the reference moves along the y-axis. After the initialization phase, the tracking error is equal to 0.006[m]; when safety limitations are active, the error is equal to 0.047[m].](image)

The trade-off between performance and safety is displayed in Figure 6.6. For this case, safety limitations are not considered after the initialization phase. Afterwards, they are turned on such that the performance during reference tracking clearly decreases. The controller parameters initially chosen are as follows: $K_t = 350I$, $K_p = 350I$, $B = 10I$. Once the safety limits are set as $E_{\text{max}} = 1.5$, and $P_{\text{max}} = 0.8$, stiffness and damping parameters are scaled according to (3.24) and (3.28), respectively. The maximum error when safety limitations are off is 0.006[m], and when safety limits are activated the error is 0.047[m]. The same simulation is run again, but now using two different values as safety limits. Results are shown in Figure 6.7.

![Figure 6.7: Simulation results - Performance and safety trade-off. After initialization, safety limits are set to $E_{\text{max}} = 1.5$, and $P_{\text{max}} = 0.8$, where the maximum error during reference tracking is equal to 0.047[m]. At $t = 20[s]$, safety is increased by setting $E_{\text{max}} = 1.0$, and $P_{\text{max}} = 0.65$. During this phase the maximum error is equal to 0.12[m].](image)

Given the same periodic motion as in simulation from Figure 6.6, safety limits are first set to $E_{\text{max}} = 1.5$ and $P_{\text{max}} = 0.8$, where the maximum error during reference tracking is equal to 0.047[m]. Following, safety is increased by setting $E_{\text{max}} = 1.0$, and $P_{\text{max}} = 0.65$. During this phase the maximum error is equal to 0.12[m].
6.3 Energy Tanks

The energy tanks notion from Section 4.3 is evaluated in the following simulation experiments. An arbitrary motion is given to joint A1, such that the energy tank $H_1(s_1)$ is discharged in order to analyze the dynamics of the transmission ratio disclosed in Figure 4.4. Results are presented in Figure 6.8.

![Image](image_url)

**Figure 6.8**: Simulation results - Depleted energy tank. Energy tank state $s_1$ is initially defined as $s_1 = 1$, such that $H_1(s_1) = 0.5$. The corresponding joint A1 is given an arbitrary motion, until $H_1$ reaches a depleted state $H_1(s_1) \leq \epsilon$. The section of interest is zoomed in at the bottom part of the figure.

Figure 6.8 shows the energy tank, torque, and transmission ratio plots for joint A1. The region of interest is particularly when $H_1(s_1) = \epsilon$, as this points out the instant in which the energy tank is depleted. At this moment, as stated in Section 4.3, the corresponding joint is virtually shut down according to (4.13). The bottom part of Figure 6.8 displays the zoomed section of the region of interest. It is observed that for the 1-DOF case, the transmission ratio $u$ becomes zero when $H \leq \epsilon$, making the torque output equal to zero according to (4.10) (Considering $F_{out} = \tau$). On the other hand, the multi-DOF instance shows how $u_1$ smoothly approaches zero, giving the same behavior to $\tau_1$ as a consequence, according to (4.14).

As explained in Section 4.3, joints in the multi-DOF instance must be virtually shut down using control. If $u_n$ is set to zero when $H_n(s_n) \leq \epsilon$, the corresponding link will not completely stop because of its inertial motion. This is an undesired situation, since the state of each energy tank is defined by the integration of $\dot{s}_n$, which depends on the transmission $u_n$ ratio and joint velocity $\dot{q}_n$ (See Table 4.2). From the same simulation experiment in Figure 6.8, the position of joint A1 is plotted along with the transmission ratio $u_1$ and joint torque $\tau_1$. Figure 6.9 shows the case in which $u_1 = 0$ when $H_1(s_1) \leq \epsilon$, and Figure 6.10 the case where $u_1 = (-\tau_{c_1}/\gamma^2)s_1$ when $H_1(s_1) \leq \epsilon$. 
By looking at Figures 6.9 and 6.10, it is evident how the transmission ratio plays an important role when an energy tank is depleted. The prime function of $u_n$ is to isolate controller and plant when $H_n(s_n) \leq \epsilon$. Nevertheless, as mentioned in Section 4.3, a control action must take place in the multi-DOF occasion to avoid affecting the remaining joint subsystems. It is clear how the angular position plot in Figure 6.9 increases when the tank has been depleted. In this case $u_1 = 0$ as soon as $H_1(s_1) = \epsilon$, making $\tau_1 = 0$. Consequently, joint A1 does not immediately stop, as this joint holds most of the manipulator’s overall mass, which was in motion before the tank $H_1$ used its available energy. On the other hand, the angular position plot from Figure 6.10 holds still when $H_1(s_1) \leq \epsilon$, due to the smooth dynamics of $u_1$. This makes clear why $u_n$ is modified for the multi-DOF case, as presented in Table 4.2.

One of the advantages of having energy tanks monitoring the energetic consistency of the overall system, is the isolation between plant and controller when $H_1(s_1) = \epsilon$. This not only maintains passivity, but can also intercede as a safety measurement for any kind of controller. Assuming a performance based impedance controller is employed, if the robot goes out of control without exceeding its internal torque and velocity limitations, the manipulator turns its workspace into a highly hazardous area. The introduction of the energy tanks notion in a situation like this, would immediately stop the robot before generating significant damage to its surroundings. In order to show the latter case, a hypothetical circumstance is simulated where the stiffness of the Cartesian virtual spring is changed with random values at a high frequency. This induces the joint torques to dramatically go out of control, as shown in Figure 6.11.
Figure 6.11: Simulation results - Manipulator out of control. The stiffness of the Cartesian controller spring is changed with high frequency random values, such that joint torque outputs go out of control.

Figure 6.11 shows a hypothetical case in which the stiffness of the impedance controller’s spring (See Figure 3.3) is given a random noise generator as input value. By doing so, the virtual wrench acting on the end-effector (3.4) changes its magnitude in the same way, producing high frequency random joint torque outputs with (3.9). For this case safety limitations are disabled allowing the manipulator itself to go out of control.

For a precise analysis, joint E1(J3) is chosen to be inspected. Position, velocity, and torque for joint E1(J3) are shown in Figure 6.12 for the uncontrolled mentioned case.

Figure 6.12: Simulation results - Joint E1(J3) out of control.

Figure 6.12 shows joint E1(J3) data for the case presented in shown in Figure 6.11. Safety limitations are disabled, and the energy tanks notion is not incorporated to the impedance controller. The same simulation is performed including the energy tanks, which in this case help as a safety feature. Results are shown in Figure 6.13.
Figure 6.13: Simulation results - Joint E1(J3) is virtually shut down when $H_3(s_3) = \epsilon$.

As it can be observed in Figure 6.13, as soon as $H_3(s_3) = \epsilon$ the corresponding joint is virtually shut down, isolating controller and manipulator. It is clear how the position $q_3$ remains still, and the joint velocity $\dot{q}_3$ rapidly goes zero. Under these circumstances, the incorporation of the energy tanks concept acts as a safety feature for the Cartesian impedance controller. Nevertheless, since the energy tanks are initially loaded with a certain value it is important to determine how big this value will be. The amount of initial energy in $H_n$ will determine how much time joint $n$ will be allowed to be out of control. Figure 6.14 shows the results of a simulation where $H_3$ is initially loaded with a higher value.

Figure 6.14: Simulation results - Energy tank in joint E1(J3) is initially loaded with a higher value.

Figure 6.14 clearly shows the importance of initially choosing an appropriate value for $H_3$. As it can be observed, $\tau_3$ goes out of control for a longer period, since $H_3$ requires more time to be depleted. Note that the values in the previous simulation results might look unrealistic, but a hypothetical case was simulated in order to highlight the energy tanks incorporation as a safety feature.
6.4 Safe Human-Robot Interaction

To validate the proposed control scheme on a real platform, a three-phase experiment was conducted with the KUKA LWR 4+, shown in Figure 6.15.

Figure 6.15: Experimental set-up.

In order to establish communication with the robot, and be able to use the proposed controller, the FRI [41] and the KUKA LWR Robot Operating System (ROS) package were used along with the resources provided by the Research Center "E. Piaggio" 1. The FRI gives direct low-level real-time access to the KUKA Robot Controller (KRC) [41, 23]. With the FRI all the industrial features of the LWR controller are still available through the KUKA Robot Language (KRL). An overview of the FRI architecture is displayed in Figure 6.16.

Figure 6.16: Overview of the FRI control system architecture.

For this experiment, a Cartesian reference trajectory was defined as a periodic motion along the y-axis of the base frame as:

\[
p^i_0(t) = \begin{bmatrix} x \\ y(t) \\ z \end{bmatrix} = \begin{bmatrix} -0.6 \\ 0.3 \sin\left(\frac{2\pi}{T} t\right) \\ 0.6 \end{bmatrix}
\]

\[R^i_0(t) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}
\]

(6.1)

1Research Center "E.Piaggio" GitHub repository: https://github.com/CentroEPiaggio
Table 6.1: Experimental results - Safe human robot interaction. Mean (M) and standard deviation (SD), of the translation and orientation error for each part of the experiment.

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
<th>Part III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Translation</td>
<td>0.0220</td>
<td>0.0076</td>
</tr>
<tr>
<td>Orientation</td>
<td>0.2790</td>
<td>0.1619</td>
</tr>
</tbody>
</table>

In order to evaluate the performance variations due to the safety limits, the experiment is divided in three parts. In the first part the safety limitations are not activated. In the second and third part, the controller enforces the limitations reducing both energy and power. In the third part, the user safely interacts with the robot.

The tracking error for the translation part $||e_t||$ is computed as the norm of the position vector $p_t^v$ of the relative configuration (3.7), while the orientation error $||e_o||$ is computed from the matrix $R_t^v$ from (3.7) using the angle and axis representation [45].

The controller gains are defined as $K_t = 1000I$, $K_o = 100I$, $B = 50I$, and the safety parameters as $(E_{max} = 1.0, P_{max} = 2.0)$. The experiment was executed using the KUKA LWR 4+ manipulator, shown in Figure 6.15, which is located in the RAM laboratory at the University of Twente. The results of the experiment are shown in Figure 6.17, and reported in Table 6.1.

Note that more attention was given to the translation error $||e_t||$, since the manipulator did not count with an end-effector at that time and was not required for the test in case.

Figure 6.17: Experimental results - Safe human robot interaction. In the first part of the experiment the safety limitations are not applied, the robot is able to better track the reference $||e_t||$: $(M = 0.0220, SD = 0.0076)$, $||e_o||$: $(M = 0.2790, SD = 0.1619)$. In the second part safety limits are enabled, and the energy and the power are limited to the threshold values $(E_{max} = 1.0, P_{max} = 2.0)$ thanks to the scaling stiffness (3.24) and damping (3.28). As a consequence, the tracking error increases $||e_t||$: $(M = 0.0293, SD = 0.084)$, $||e_o||$: $(M = 0.2985, SD = 0.1548)$. In the last part of the experiment, the user interacts with the robot while the safety limits are enabled, where energy and power limitations guarantee a safe interaction $||e_t||$: $(M = 0.0795, SD = 0.1143)$, $||e_o||$: $(M = 0.3216, SD = 0.1645)$. 

C.A. Cardenas Villa University of Twente
Conclusions and Recommendations

7.1 Conclusions

Considering the research objective stated in Section 1.2:

"Develop a Cartesian impedance controller which guarantees a safe environment for human-robot interaction, ensuring passivity of the overall system."

The energy-tank concept introduced by Tadele in [55] has been extended for a multi-DOF, combined with a Cartesian impedance controller, in an energy and power based safety metrics framework. This safety-aware controller has been incorporated with energy tanks, to ensure passivity of the overall system. Both concepts are presented as individual tools, such that they can be combined with different control design choices.

The features of this novel passive impedance controller have been evaluated in a simulation environment, and in an experimental platform. It has been shown Section 6.1 how the total energy of the system, and the controller power can be limited to establish a safe environment where humans and robots can share workspace. Moreover, the role of the energy tanks notion in an uncontrolled scenario is revealed in Section 6.3. Finally, the proposed controlled is tested in a real platform and results are shown in Section 6.4, where safe HRI is achieved.

The major contribution of this project is wrapped in the so-called Safety-Aware Intrinsically Passive (SAIP) controller disclosed in Section 4.4. This concept is presented in a multi-layered structure control scheme, which combines de safety-aware Cartesian controller from Section 3.2.2 with the energy tanks system from Section 4.3.

This contribution has been submitted, under the title "Development of a Safety-Aware Intrinsically Passive Controller for Collaborative Robots", to the International Conference of Robotics and Automation (ICRA) to be celebrated in 21-25th of May 2018, in Brisbane, Australia.

7.2 Future Recommendations

Considering ongoing work in passivity based control design, and safety metrics in domestic environments, the following recommendations are proposed:

- **Feedforward Control:** The idea of feedforward control is introduced, but not tested. Simulations with a performance-based impedance controller were satisfactorily carried out. Nevertheless, due to time constraints and prioritized tasks, the SAIP controller was not extended with this tool in a simulation, nor an experiment.

- **Estimation of Energy:** As shown in the simulation results from Figure 6.14, the amount of energy initially given to the energy tanks is crucial. Therefore, a good study on energy estimation can be developed, such that the amount of required energy to complete a task can be known beforehand. In this way, the energy tanks can be initially loaded with...
this known amount, knowing that the manipulator will freely complete its task unless an uncontrolled situation or collision occurs.

- **Optimization of the Manipulator Configuration:** This task is directly related to the estimation of energy. The configuration of the manipulator can be optimized such that the robot finds the most effective way to achieve the desired configuration.

- **Force Sensor:** A safe controller was designed using energy and power as safety metrics. Nevertheless, these magnitudes were always computed inside the control software with the position and velocity feedback from the manipulator. It is suggested to use a force sensor to externally measure and corroborate that safety limits are accurate.
Appendices
Performance-Based Impedance Control

Given a closed-loop system, as shown in Figure A.1:

\[
G(s)K(s) + \frac{1}{1 + G(s)K(s)} \cdot r(s)
\]  
\[
(A.1)
\]

Figure A.1: Standard feedback control system.

A controller can be designed according to desired performance requirements, such as the maximum allowed motion error [58]:

\[
e(s) = \frac{1}{1 + G(s)K(s)} \cdot r(s)
\]

Motion tracking is a typical low frequency phenomenon. Thus, the maximum allowed motion error \(e(s)_{\text{max}}\) can be defined as:

\[
e(s)_{\text{max}} = \max \left\{ \lim_{s \to 0} e(s) \right\}
\]

The work done in [53], presents an impedance control design approach which maps the traditional Proportional-Integral-Derivative (PID) control tuning rules to the damping injection framework from Section 3.1.1. This notion is later extended to a multi-DOF instance in [56].

The ideas presented in both works, are considered as relevant in the context of this work, since they constitute a clear example on how a controller that satisfies certain specifications can be designed based on the interaction scheme offered by impedance control.

With this in mind, the relevant ideas that constitute the 1 and multi-DOF cases are briefly presented here, suggesting the reader to refer to [53] and [56] for clarified details.

A.1 Performance-Based Impedance Control for 1-DOF Manipulators

The 1-DOF case is developed by initially considering the dynamic equations from (3.2), with which the damping injection framework, shown in Figure 3.2, is represented in a simplified block diagram as displayed in Figure A.2.

From this block diagram, it is observed that the damping injection framework is composed of a setpoint pre-filter \(F(s)\), and a phase-lead compensator \(C(s)\), which has the same frequency response as a PD controller. See Figure A.3.

In light of these observations, classical PID control tuning rules are mapped to the damping injection framework, where a desired maximum phase frequency is obtained based on the
maximum velocity of the motion profile, and the maximum allowed motion error $e_{\text{max}}$. In this way, the parameters from the transfer function block $C(s)$ in Figure A.2 are computed, and the controller is defined. The detailed procedure on how to compute these parameters, is presented in [53].

### A.2 Performance-Based Cartesian Impedance Control

Extension to a multi-DOF case follows a similar approach. Firstly, simpler dynamics are established by ‘masking’ the true nonlinear inertia of the manipulator, with rigid body dynamics. Consequently, a space diagonal inertia tensor $\mathbf{I}_m = m \cdot \mathbf{I}^6 \times 6$, now interacts with the spatial spring and damper of the controller [29]. The latter consideration is portrayed in Figure A.4, where $\Psi_n$ is the end-effector’s coordinate frame, $\Psi_d$ is the desired position’s frame, and $\mathbf{p}_0^d$ and $\mathbf{p}_0^n$ are the position vectors of the end-effector and desired configuration, respectively, expressed in the inertial frame $\Psi_0$.

Thus, in consideration of (3.6) and (3.9), the translational component of the wrench exerted on the end-effector, due to the multidimensional spatial spring with diagonal stiffness matrix $\mathbf{K} = \text{diag}([K_0, K_t])$, can be expressed as:

$$\mathbf{f}^0 = k_t (\mathbf{p}_d^0 - \mathbf{p}_n^0) = k_t \cdot \mathbf{p}_{d,n}^0$$  (A.3)

where $k_t$ is the translational stiffness constant, and $\mathbf{p}_{d,n}^0$ is the position vector of the relative configuration between frames $\Psi_d$ and $\Psi_n$, which represents the position error expressed in the inertial frame $\Psi_0$. If a diagonal damping matrix is also selected as $\mathbf{B} = \text{diag}([B_0, B_t])$, the low frequency component of the motion error can be defined for each DOF as:

$$e_i = \frac{b_t}{k_t} \cdot \mathbf{v}_{n,i}^0, \quad i \in x, y, z$$  (A.4)

where $\mathbf{v}_{n,i}^0$ is the velocity component of the twist of the diagonal inertia $\mathbf{I}_m$, and $b_t$ is the translational damping constant. In this direction, Cartesian impedance controller parameters are
defined in terms of the absolute motion error $e_{\text{max}}$ as:

$$k_t = \frac{4m\zeta}{e_{\text{max}}^2} \cdot (\nu_{\text{max}}^0)^2$$

$$b_t = 2\zeta \sqrt{k_t m}$$

This method is extended towards a generalized approach that can be used for a nonlinear manipulator dynamics, such that the inconvenience of invertible matrix requirements is avoided. Thus, a worst-case design approach is proposed, where a controller is designed to achieve a desired maximum allowed motion error on the heaviest possible inertia. In this direction, if a preliminary diagonal inertia matrix is chosen, the following inequality is defined:

$$\lambda \cdot \frac{1}{2} \left( T_n^{0,0} \right)^\top I_p T_n^{0,0} > \frac{1}{2} \dot{q}^\top M(q) \dot{q}$$

where $\lambda$ is a scaling parameter used to define the upper bound inertia as $I_m = \lambda \cdot I_p$, and at the same time scales the Cartesian impedance parameters determined using $I_p$. The detailed procedure to develop the displayed equations and controller parameters is presented in [56].

Figure A.4: Nonlinear effects on manipulator masked with a diagonal inertia $I_m$. 
Simulation Environment 20-sim

Simulation experiments were performed with 20-sim [17]. The components of the simulated controlled system are briefly described in this appendix.

Figure B.1 shows the controlled system model which basically consists of three submodels: the reference submodel, the controller submodel, and the plant submodel. These submodels are briefly described in the following sections.

![Figure B.1: 20-sim controlled system.](image)

B.1 Reference Submodel

Figure B.2 shows the contents of the reference submodel, which is labeled as Joint Angles Set-Points.

![Figure B.2: 20-sim joint angles set-points submodel.](image)

The Motion Profile block takes the step signals inputs and gives as an output a 7 dimension vector with joint angle references. This vector enters the H-matrix Calculation block and the configuration of the desired position is computed and stores in an homogeneous matrix structure. This output is sent to the Controller Submodel.
B.2 Controller Submodel

The Controller Submodel receives as inputs the reference configuration in matrix form, joint positions vector, and joint velocities vector. Figure B.3 shows the blocks corresponding to this submodel.

![Figure B.3: 20-sim controller submodel.](image)

The Kinematics block receives the joint positions vector and computes the current end-effector’s configuration, the manipulator Jacobian, and the robot’s mass matrix. These three elements are introduced in the Controller block which receives the desired configuration and the joint velocities vector. Inside this block, energy and power limitations are computed, with the incorporated energy tanks notion to produce the joint torques output vector that will be sent to the manipulator as described in Section 4.4.

B.3 Plant Submodel

The Plant Submodel basically includes the port-based model of the manipulator, described in Section 5.2.3.

![Figure B.4: 20-sim component based model of the manipulator.](image)

This submodel receives the torques coming from the Controller, and sends the joint positions and velocities vectors in the feedback loop.
Safety-Aware Intrinsically Passive Controller

This appendix includes the pseudo-code of the SAIP controller implementation.

C.1 Parameters

```plaintext
real ko = 250; // Stiffness constant
real kt = 250; // Stiffness constant
real b = 3; // Damping coefficient
real epsilon = 0.001; // Minimum energy in tank
real Emax = 0.6; // Maximum allowed energy
real Pmax = 0.8; // Maximum allowed power
real state_s = 1;
```

C.2 Declaration of Variables

```plaintext
real tau1,tau2,tau3,tau4,tau5,tau6,tau7; // Output joint torques
real qdot1,qdot2,qdot3,qdot4,qdot5,qdot6,qdot7; // Input joint velocities
real sdot1,sdot2,sdot3,sdot4,sdot5,sdot6,sdot7; // sdot = u * qdot
real s1_in,s2_in,s3_in,s4_in,s5_in,s6_in,s7_in; // Initial value of energy tank state
real pc1,pc2,pc3,pc4,pc5,pc6,pc7; // Power of the controller on each joint
real s1,s2,s3,s4,s5,s6,s7; // Energy tanks states
real u1,u2, u3,u4,u5,u6,u7; // Transmission Ratios for each joint
real H1,H2,H3,H4,H5,H6,H7; // Energy content for each joint
real tip_error, tip_x, tip_y, tip_z; // x, y, and z components of the end-effector
real xyz_error, x_error, y_error, z_error; // Error in cartesian space
real Htipref[4,4], Hreftip[4,4]; // H-matrix from end-effector to desired position
real Rtd[3,3]; // Rotation matrix of relative configuration
real Rdt[3,3]; // Inverse of Rtd
real ptd[3]; // Position vector of relative configuration
real Htot; // Total energy from energy tanks
real Bi[7,7], B[7,7]; // Joint damping matrix
real beta; // Scaling parameter for joint damping matrix
real Koi[3,3]; // Initial orientation stiffness matrix
real Kti[3,3]; // Initial translation stiffness matrix
real Kci[3,3]; // Initial coupling stiffness matrix
real Goi[3,3], Go[3,3]; // Orientation co-stiffness matrix
real Gti[3,3], Gt[3,3]; // Translation co-stiffness matrix
real Gci[3,3], Gc[3,3]; // Coupling co-stiffness matrix
real lambda; // Scaling parameter for co-stiffness matrices
real KE; // Kinetic Energy of the system
real Voi, Vo; // Potential energy due to rotational component of spatial compliance
real Vti, Vt; // Potential energy due to translational component of spatial compliance
real Vci, Vc; // Potential energy due to coupling component of spatial compliance
real Vi, V; // Total potential energy V = Vo + Vt + Vc
real Etoti, Etot; // Total energy of the system Etot = KE + V
```
real \(t[3,1]\), \(t_G[3,3]\); // Torques (Rotational part of wrench)
real \(f[3,1]\), \(f_G[3,3]\); // Forces (Translational part of wrench)
real \(tGo[3,3]\), \(tGt[3,3]\), \(tGc[3,3]\); // Torque components
real \(fGt[3,3]\), \(fGt2[3,3]\), \(fGc[3,3]\); // Force components
real \(astGo[3,3]\), \(astGt[3,3]\), \(astGc[3,3]\); // Anti-symmetric part of Torque components
real \(asfGt[3,3]\), \(asfGt2[3,3]\), \(asfGc[3,3]\); // Anti-symmetric part of Force components
real \(Wsn0[6,1]\); // Wrench acting on end-effector
real \(W0[6,1]\); // Wrench acting on end-effector expressed in inertial frame
real \(Fci[7,1]\), \(Fc[7,1]\); // Initial controller force
real \(Fdi[7,1]\), \(Fd[7,1]\); // Force due to joints damping
real \(Fs[7,1]\); // Force due to spatial compliance
real \(Pci\), \(Pc\); // Power of the controller
real \(gama\); // Constant related to epsilon

### C.3 Initial Equations

```plaintext
Bi = b * eye(7);
Koi = ko * eye(3);
Kti = kt * eye(3);
Kci = 0;
Goi = 0.5*trace(Koi)*eye(3)-Koi;
Gti = 0.5*trace(Kti)*eye(3)-Kti;
Gci = 0.5*trace(Kci)*eye(3)-Kci;
gama = sqrt(2 * epsilon);
```

### C.4 Safety Layer

```plaintext
/* Potential Energy */
Vti = -0.25*trace(skew(ptd)*Gti*skew(ptd)) -0.25*trace(skew(ptd)*Rtd*Gti*Rdt*skew(ptd));
Voi = -{trace(Goi*Rtd)};
Vci = trace(Gci*Rdt*skew(ptd));
Vi = Vti + Voi + Vci; // Initial potential energy
/* Energy of the System */
Etoti = KE + Vi;
if Etoti > Emax then
    lambda = (Emax - KE)/Vi;
else
    lambda = 1;
end;
// New co-stiffness matrices using scaling parameter lambda
Go = lambda * Goi;
Gt = lambda * Gti;
Gc = lambda * Gci;
// New potential energy components using new co-stiffness matrices
Vo = -{trace(Goi*Rtd)};
Vc = trace(Gc*Rdt*skew(ptd));
V = Vt + Vo + Vc; // Potential energy
Etot = KE + V; // Total energy of the system
/* Power of the System */
Fs = transpose(Jacobian) * W0; // Force due to spatial compliance
Fdi = Bi * qdot; // Force due to joint damping
Fci = Fs - Fdi; // Initial controller force
Pci = transpose(Fci) * qdot; // Initial power of the controller
if Pci > Pmax then
    beta = ((transpose(Fs)*qdot) - Pmax)/ (transpose(Fdi)*qdot);
else
    beta = 1;
end;
```
APPENDIX C. SAFETY-AWARE INTRINSICALLY PASSIVE CONTROLLER

43 // New joint damping matrix using scaling parameter beta
44 B = beta * Bi;
45
46 // New power of the controller using new joint damping matrix
47 Fd = B * qdot; // Force due to joint damping
48 Fc = Fs - Fd; // Controller force
49
50 Fc = transpose(Fc) * qdot; // Power of the controller

C.5 Motion Layer

1 /* Wrench Acting on End-Effector */
2
3 // Rotational part of the wrench
4 tGo = Go * Rtd;
5 astGo = (tGo - transpose(tGo))./2;
6 tGt = Gt * Rdt * skew(ptd) * skew(ptd) * Rtd;
7 astGt = (tGt - transpose(tGt))./2;
8 tGc = Gc * Rtd;
9 astGc = (tGc - transpose(tGc))./2;
10 tG = (-2*astGo) - (astGt) - (2*astGc);
11 t = [tG[3,2]; tG[1,3]; tG[2,1]];
12
13 // Translational part of wrench
14 fGt = Gt * skew(ptd);
15 asfGt = (fGt - transpose(fGt))./2;
16 fGt2 = Gt * Rdt * skew(ptd) * Rtd;
17 asfGt2 = (fGt2 - transpose(fGt2))./2;
18 fGc = Gc * Rtd;
19 asfGc = (fGc - transpose(Gc))./2;
20 fG = (-Rdt * asfGt * Rtd) - (asfGt2) - (2 * asfGc);
21 f = [fG[3,2]; fG[1,3]; fG[2,1]];
22
23 Wsn0[1:3,1] = t;
24 Wsn0[4:6,1] = f;
25
26 W0 = transpose(Adjoint(inverse(Htip0))) * (Wsn0); // Wrench change of coordinates
27
28 // Controller torques for each joint
29 tau1 = Fc[1];
30 tau2 = Fc[2];
31 tau3 = Fc[3];
32 tau4 = Fc[4];
33 tau5 = Fc[5];
34 tau6 = Fc[6];
35 tau7 = Fc[7];
36
37 // Potential energy in each tank
38 H1 = 0.5*s1*s1;
39 H2 = 0.5*s2*s2;
40 H3 = 0.5*s3*s3;
41 H4 = 0.5*s4*s4;
42 H5 = 0.5*s5*s5;
43 H6 = 0.5*s6*s6;
44 H7 = 0.5*s7*s7;
45 Htot = H1 + H2 + H3 + H4 + H5 + H6 + H7; // Total energy in tanks
46
47 // Power of the controller on each joint
48 pc1 = tau1 * qdot1;
49 pc2 = tau2 * qdot2;
50 pc3 = tau3 * qdot3;
51 pc4 = tau4 * qdot4;
52 pc5 = tau5 * qdot5;
53 pc6 = tau6 * qdot6;
54 pc7 = tau7 * qdot7;

C.6 Passive Layer

1 // Setting transmission ratios
2 if H1 > epsilon then
3     u1 = -tau1/s1;
4 else

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\begin{verbatim}
u1 = ((-tau1/gama)/gama)*s1;
end;

if H2 > epsilon then
u2 = -tau2/s2;
else
u2 = ((-tau2/gama)/gama)*s2;
end;

if H3 > epsilon then
u3 = -tau3/s3;
else
u3 = ((-tau3/gama)/gama)*s3;
end;

if H4 > epsilon then
u4 = -tau4/s4;
else
u4 = ((-tau4/gama)/gama)*s4;
end;

if H5 > epsilon then
u5 = -tau5/s5;
else
u5 = ((-tau5/gama)/gama)*s5;
end;

if H6 > epsilon then
u6 = -tau6/s6;
else
u6 = ((-tau6/gama)/gama)*s6;
end;

if H7 > epsilon then
u7 = -tau7/s7;
else
u7 = ((-tau7/gama)/gama)*s7;
end;

tau[1] = -u1 * s1; sdot1 = u1 * qdot1; s1 = int(sdot1,s1_in);
tau[2] = -u2 * s2; sdot2 = u2 * qdot2; s2 = int(sdot2,s2_in);
tau[3] = -u3 * s3; sdot3 = u3 * qdot3; s3 = int(sdot3,s3_in);
tau[4] = -u4 * s4; sdot4 = u4 * qdot4; s4 = int(sdot4,s4_in);
tau[5] = -u5 * s5; sdot5 = u5 * qdot5; s5 = int(sdot5,s5_in);
tau[6] = -u6 * s6; sdot6 = u6 * qdot6; s6 = int(sdot6,s6_in);
tau[7] = -u7 * s7; sdot7 = u7 * qdot7; s7 = int(sdot7,s7_in);
\end{verbatim}
Bibliography

Development of a SAIP Controller for a Multi-DOF Manipulator


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