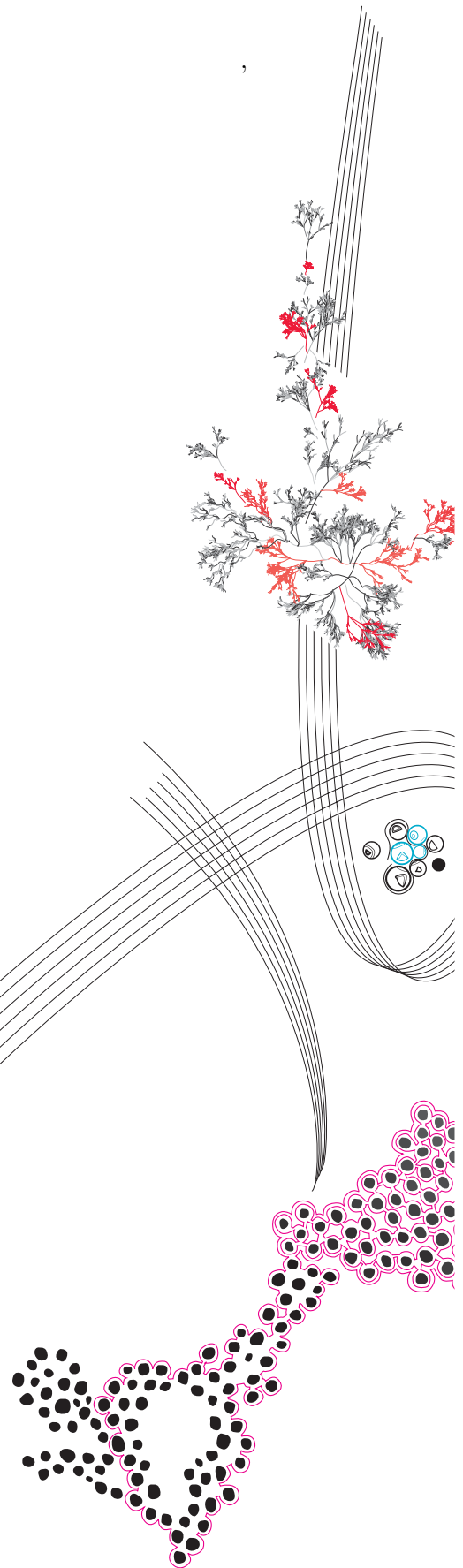




BSc Thesis Applied Mathematics

Immersion cooling of a sphere

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June 23, 2025

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Preface

I want to thank me, Kevin, and my supervisors

Immersion cooling of a sphere.

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This research focuses on cooling a copper sphere immersed in three different dielectric liquids, serving as a foundational study for the future optimization of heat sink shapes in immersion cooling technologies. By examining the cooling process in different liquids, we have determined that the type of liquid has a relatively minor impact on the overall cooling rate. However, it is still possible to tell which one cools the sphere faster due to the usage of an OpenFOAM. Despite this, the simulation of a sphere immersed in a liquid presents significant challenges due to the complex interactions between the sphere and the liquid. These challenges must be addressed to advance the understanding and development of more effective heat sinks.

Keywords: heat transfer, sphere cooling, immersive cooling, OpenFOAM

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1 Introduction

Liquid immersion cooling technology has shown considerable promise for enhancing thermal management in data centre computation and storage systems. (W.Kuncoro et al., 2019). Submerging server systems entirely in dielectric fluid results in superior heat transfer, reduced operating noise, and improved energy efficiency compared to forced-air cooling. (P.R. Naveen et al., 2024) These findings have significant implications for the field of data centre technology and thermal management, offering a more efficient and cost-effective solution for heat dissipation.

Heat sinks play a crucial role in effectively dissipating heat from electronic devices into the surrounding dielectric fluid within an immersion cooling system. The configuration of heat sink fins is critical in enhancing heat transfer by increasing the convective heat transfer surface area. The optimization of heat sinks for single-phase immersion cooling is an area of growing research interest. A well-designed heat sink that leverages natural convection inlet conditions can lead to more efficient and cost-effective cooling solutions, significantly reducing energy consumption and operating costs, enhancing system reliability, and minimizing energy usage. The potential of natural convection flow is a promising avenue for future research, offering a new dimension to heat sink optimization. (G. Gautam et al., 2023)

This paper uses natural conduction to simulate the cooling of a sphere without fluid flow in OpenFOAM in three types of dielectric liquids that are suitable for immersion cooling systems. Those are silicon and mineral oils and polyalphaolefin (PAO). Furthermore, we come to the conclusion that PAO is the best dielectric liquid for a spherical shape, as it cools the shape faster than other liquids due to its highest thermal conductivity κ . That result gives the basis for further research that could begin with optimizing the shape by simulating two spheres or a deformed version of the sphere by changing the parameters in the OpenFOAM simulation files.

To give a full description of the obtained results, this research first describes the problem in Section 2: Problem Statement, which also introduces the analytical approach and the equations that are used for solving the problem utilizing the OpenFOAM tool. Further section - OpenFOAM Simulating describes how the tool works and why it is used for this problem specifically. Section 4: Simulation introduces the simulation itself and gives out its results both by graph and word description. Those results are then confirmed using grid cell refinement in Section 5: Confirming the results. Lastly, the final two sections, Discussion and Conclusion, give the approach for further research and present the final results.

2 Problem statement

The initial problem was about the optimization of the shape that is being cooled in a liquid; however, further work has shown that the time was too limited for the whole research. That is why the problem transformed into the cooling of the sphere immersed in a liquid, and the rest remains for future research that will consider this work as the basis and answer the initial research question: "What is the optimal shape for the heat sink immersed in a dielectric liquid?"

The problem that is viewed here is simpler but not less important than the main one, as it lays out the beginning of the whole research. Here, the single sphere immersed in a liquid will be simulated without fluid flow to show the cooling process, and the refinement will be presented to confirm the correctness of the received result.

To simulate the situation, it is necessary to look at the mathematical part of the problem. The setup that will be used is the sphere immersed in a liquid with the heat transfer process in between the liquid and the sphere, where the temperature of the sphere is uniformly spread, and the liquid has no flow. The goal is to solve the heat conduction equation with appropriate boundary conditions using the principles of heat transfer. An example of such a problem can be seen in the research by Virag et al. (2011)

Before producing the simulation and the following solution, some assumptions need to be made: the thermal properties of the sphere and the liquid are constant, and the heat transfer between the sphere and the liquid follows Newton's law of cooling. Let the radius of the sphere be R , and the initial temperature of the sphere before it is immersed in the liquid be T_{in} . Moreover, the temperature of the liquid is T_w and does not depend on the environment around the setup. Additional constants that will be used are:

- Heat conductivity: k
- Convective heat transfer coefficient: h
- Thermal diffusivity of the sphere's material: $\alpha = \frac{k}{\rho c}$
- Density of sphere's material: ρ
- Specific heat capacity: c

The initial condition is

$$T(r, 0) = T_{in}, 0 \leq r \leq R \quad (1)$$

Where r is a radial coordinate and time t is equal to 0. The boundary conditions are:

$$\frac{\partial T}{\partial r}(0, t) = 0 \quad (2)$$

And at the surface of the sphere, Newton's law of cooling applies, so for $r = R$:

$$k \frac{\partial T}{\partial r} = h(T_w - T_{in}) \quad (3)$$

The spherical coordinates are used for convenience. The following heat conduction equation has to be solved by finite volume method using OpenFOAM:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) \quad (4)$$

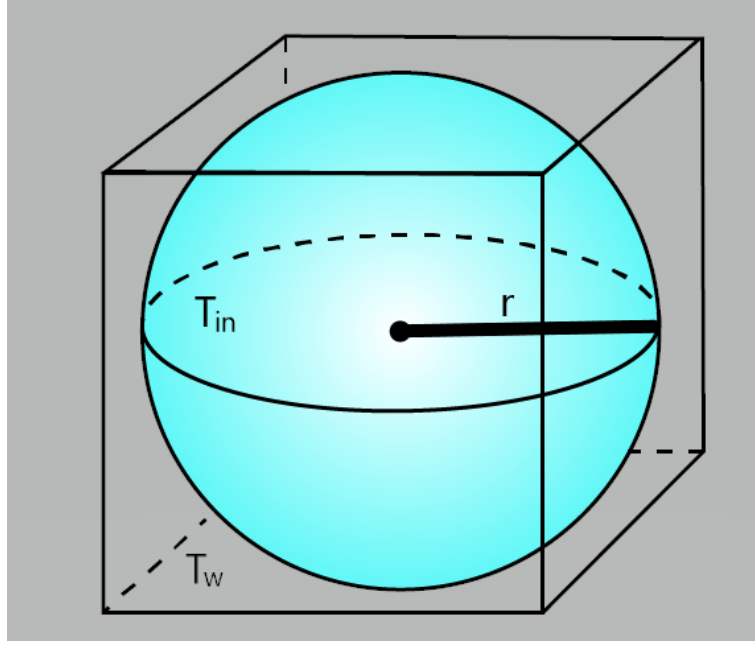


FIGURE 1: Sphere in a cube domain

A similar mathematical approach is utilized inside the OpenFOAM tool to create the simulation that is given further in Section 4 and solves the equation for the temperature in different dielectric liquids.

3 OpenFoam Simulating

For further simulations, the OpenFOAM (Open Field Operation and Manipulation), an open-source Computational Fluid Dynamics (CFD) toolbox that enables the simulation of fluid flow, heat transfer, and associated phenomena, was chosen. It is developed primarily in *C++* and provides a highly customizable and flexible platform for solving complex CFD problems through a collection of solvers, utilities, and libraries. The simulations can be further visualized in ParaView and can be turned into clips to represent the heat transfer problems in a more straightforward, more easily understood form.

OpenFOAM operates by discretizing the governing equations of fluid dynamics and solving them numerically over a defined computational domain. The general workflow in OpenFOAM involves several key steps. First, the geometry of the computational domain is created using a mesh, which divides the domain into discrete cells. Tools like ‘blockMesh’ and ‘snappyHexMesh’ can be utilized for this purpose. Next, boundary and initial conditions are defined to set the physical boundaries of the problem and the initial conditions for the simulation, including specifying the properties of the fluid, thermal conditions, and any external forces.

After preparing the case, an appropriate solver is chosen from OpenFOAM’s extensive library. Solvers in OpenFOAM are specialized for various types of simulations, such as incompressible flow, compressible flow, multiphase flow, and heat transfer. For example, ‘chtMultiRegionFoam’ is used for conjugate heat transfer problems. Once the solver is selected, the simulation is executed. OpenFOAM performs numerical integration over the computational domain to solve the governing equations, often using iterative methods to converge to a solution.

The simulation of an immersed sphere cooling in a dielectric liquid involves analyzing the heat transfer and fluid flow around a heated sphere (which does not apply to this specific case with no fluid flow) submerged in a dielectric fluid. This scenario is relevant in various industrial applications, including electronic cooling and material processing. OpenFOAM was chosen for this simulation due to several key reasons. Its open-source nature allows users to modify and extend its capabilities, which is crucial for tailoring the simulation to specific needs, such as custom boundary conditions, material properties, or solver adjustments.

OpenFOAM includes solvers specifically designed for conjugate heat transfer problems, like ‘chtMultiRegionFoam’, which can handle complex interactions between solid and fluid regions, making them ideal for simulating the cooling of a solid sphere in a fluid. Additionally, the ability to simulate multiple physical phenomena concurrently, such as heat transfer, is essential for accurately capturing the cooling process. OpenFOAM’s multi-physics capabilities ensure that all relevant aspects of the problem are modelled.

In setting up the simulation, the sphere and surrounding fluid domain are modelled using a structured mesh, which is defined in OpenFOAM Userguide by a mesh with implicit connectivity, whose structure allows for easy identification of elements, with a higher mesh density near the sphere, to capture detailed thermal gradients. Thermal boundary conditions are applied to the sphere’s surface to simulate heating by applying Newton’s Law of cooling (3). The ‘chtMultiRegionFoam’ solver solves the conjugate heat transfer problem,

capturing the interaction between the solid sphere and the dielectric fluid. The solver iteratively solves the coupled heat transfer equations using the finite volume method, with parameters such as temperature distribution monitored to ensure convergence. This approach ensures accurate and stable numerical solutions for complex geometries that would be relevant in further research and interactions between solid and fluid regions. Results are then analyzed using ParaView to visualize temperature fields and heat transfer rates, providing insights into the cooling efficiency and identifying any hotspots or flow irregularities.

OpenFOAM's comprehensive toolkit makes it an ideal choice for simulating the cooling of an immersed sphere in a dielectric liquid. Its advanced solvers, customizability, and strong community support enable detailed and accurate simulations, providing valuable insights into the heat transfer processes involved. This capability is essential for optimizing cooling strategies in various industrial applications, ensuring effective thermal management and material performance.

4 Simulation

Figure 2 shows a simulation of a sphere immersed in a liquid without flow. Here, we use ‘snappyHexMesh’ to generate an unstructured, primarily hexahedral mesh with local refinement around a spherical object within a domain. To be more specific, it starts with a structured base mesh but then moves to an unstructured format through refinement and layer addition processes. This tool can handle complex geometries, offering flexible refinement capabilities and the ability to create high-quality meshes necessary for accurate simulations. This is useful for making the shapes more complex, which would come in the future, and for the refinement that will be further presented in this research, which is necessary to confirm the result.

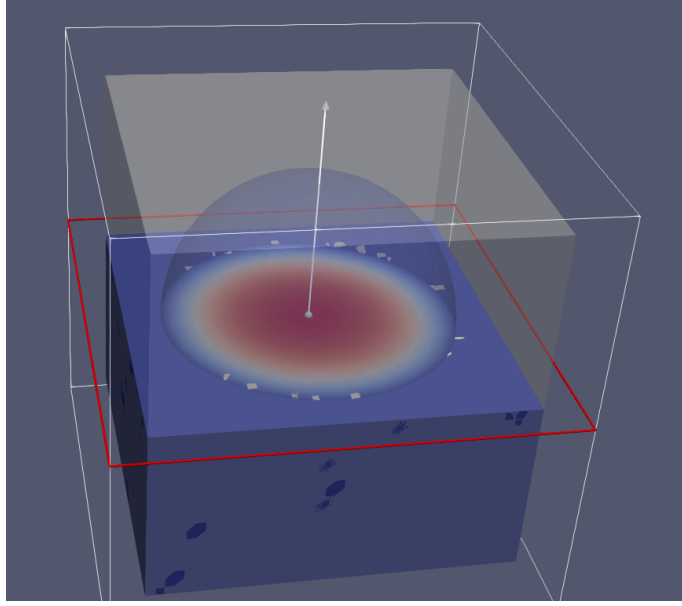


FIGURE 2: Sphere immersed in liquid

The grid type employed is an unstructured hex-dominant mesh with specific refinements around the sphere. Starting from a background structured hexahedral mesh, ‘snappyHexMesh’ refines the mesh locally around specified geometries where higher accuracy is required. The refinement here is similar to the refinement that is discussed further in Section 5, as here it also increases the resolution for better results. The mesh resolution is controlled by defining refinement levels in the ‘snappyHexMeshDict’ file.

Different types of liquids that were simulated are further presented in Figure 3, where it is also visible how fast the sphere is cooling down. Those liquids are silicon and mineral oils, and polyalphaolefin. The sphere is made of copper, which is best suited for being a heatsink (Sushma Chandrashekar, 2021) in the future. The mentioned liquids were specifically chosen to match the requirements of immersion cooling systems, which necessarily include only dielectrics.(Pambudi et al., 2022) The sphere’s radius was initially chosen to be not as big, to be 16 mm, to make the simulation work faster, and to make the temperature change more visible, but the heatsink shape or size can be modified further during the simulations that consider more complicated shapes.

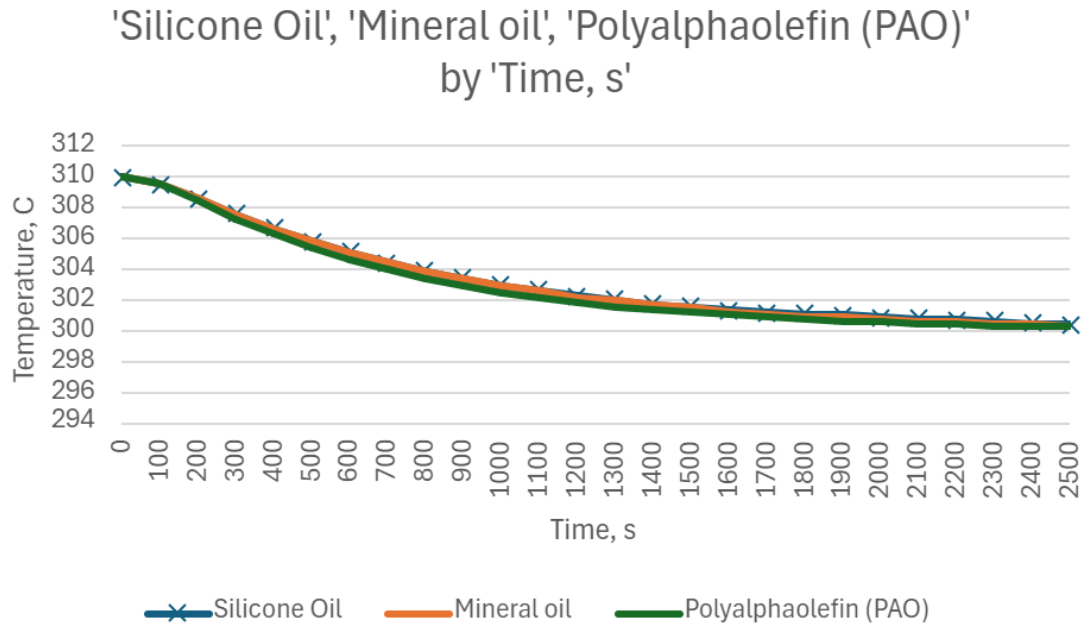


FIGURE 3: Plot of results

The table 1 represents the physical properties of those liquids and shows why the change of the dielectric liquid does not affect the cooling time as much as all of the below properties affect the cooling efficiency.

Property	Silicon Oil	Mineral Oil	Polyalphaolefin (PAO)
Molecular Weight [g/mol]	400	300	500
Thermal Conductivity [W/m/K]	0.14	0.15	0.17
Specific Heat Capacity [J/kg/K]	1500	2000	2300
Density [kg/m ³]	960	870	800

TABLE 1: Physical Properties of Dielectric Fluids

As a result, the liquid with the fastest cooling, as visible in Table 7, is Polyalphaolefin (PAO) due to its largest thermal conductivity and specific heat capacity.

5 Confirming the results

Refinement is needed for every simulation to ensure that the obtained results are correct and have the smallest error. The refinement presented here gives a small error, but the more refinement there would be, the less the error would be. Without refinement, there are a lot of factors that can lead to some mistakes, such as the size of a mesh. For instance, the mesh that is too big (not that many grid cells per axis) may not adequately capture temperature gradients, leading to inaccurate temperature distributions. In this research, we go to 'blockMeshDict' and double the grid cells on every axis to ensure that the temperature solution and graph we obtained are right, and the results can then be confirmed. The number of grid cells for refinement goes from 40 to 80 grid cells per axis. The change in 'blockMeshDict' affected the basic mesh, which further made differences in the unstructured mesh created by 'snappyHexMesh'.

Figure 4 shows the visualization of the sphere after the refinement, where bright red is the hottest temperature and blue is the lowest, and if we compare the two figures, it is visible that the number of grid cells changed, and the contour of the heat waves is presented more accurately.

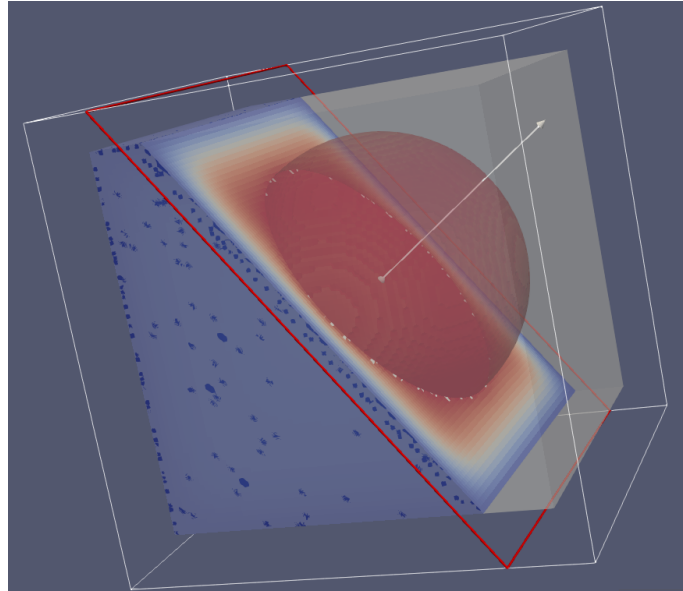


FIGURE 4: Sphere. 80 grid cells.

The following graph in Figure 5 represents the temperature change in different dielectric liquids. Although it is hardly visible how much it changed in the plot, it is visible in the table of temperature differences (Figure6). The change there, on average, is 0.01 degrees Celsius by absolute value. To be more specific:

Liquid	Average Error
Silicon Oil	0.01052
Mineral Oil	-0.0117324
Polyalphaolefin (PAO)	-0.0117248

TABLE 2: Average errors per liquid

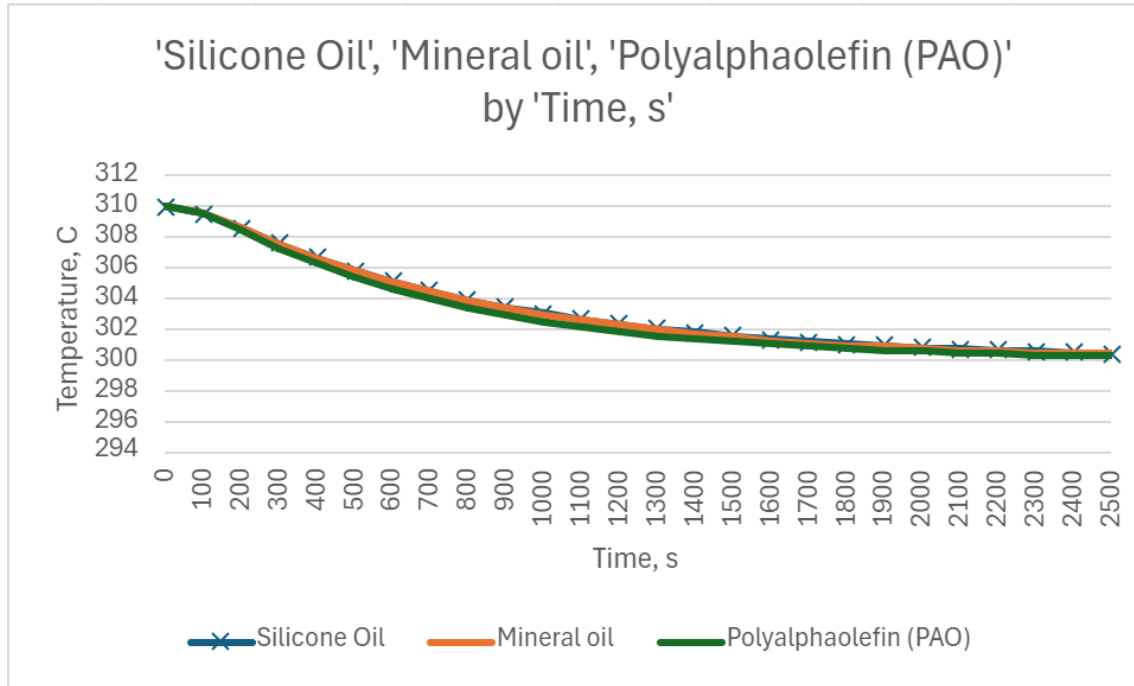


FIGURE 5: Plot of results 80 grid cells

The difference between the temperatures is represented in the following table: the value of the initial grid - the value of the refinement grids. The table shows that Silicon Oil exhibits the largest temperature differences between initial and refined grids, which can be explained by its thermal properties. The lower thermal conductivity and specific heat capacity lead to larger temperature gradients, which in turn make the numerical solution more sensitive to mesh resolution. Thus, Silicon Oil shows greater differences in temperature between different mesh resolutions.

Diff.S.O.	Diff.M.O.	Diff.PAO.
0	0	0
0,00206	0,00178	0,00171
0,00008	0,00017	-0,00104
-0,00418	-0,00411	-0,0061
-0,00827	-0,00832	-0,01067
-0,01158	-0,01178	-0,01415
-0,01406	-0,01437	-0,01656
-0,01577	-0,01615	-0,01803
-0,01685	-0,01727	-0,01877
-0,01742	-0,01784	-0,01893
-0,01758	-0,01798	-0,01867
-0,01745	-0,01779	-0,0181
-0,01706	-0,01736	-0,01731
-0,01651	-0,01675	-0,01639
-0,01583	-0,01601	-0,01536
-0,01506	-0,01518	-0,01431
-0,01424	-0,0143	-0,01324
-0,01338	-0,0134	-0,01218
0,04040	-0,01248	-0,01116
0,11689	-0,01159	-0,01018
0,10167	-0,01072	-0,00925
0,08843	-0,00989	-0,00839
0,07691	-0,00908	-0,00758
0,06689	-0,00833	-0,00682
-0,00782	-0,00761	-0,00614
-0,00716	-0,00695	-0,0055

FIGURE 6: Table of temperature differences

6 Discussion

6.1 Results of the research

This research has demonstrated the cooling of a copper sphere in various dielectric liquids using the OpenFOAM tool. One of the obtained results shows that Polyalphaolefin (PAO) cools the sphere faster than other liquids due to its highest thermal conductivity and specific heat capacity, which suggests that PAO might be superior for applications requiring efficient heat dissipation, at least out of the three liquids that were presented in this paper. Despite PAO's advantage, the difference is relatively small, which can be visible in the result tables presented in the Appendix.

The error of the results can be modified with further refinement. However, the error is small enough to proceed with further research using 80 grid cells per axis.

The shape of the cooling curve as a function of time generally follows an exponential decay, characterized by an initial rapid decrease in temperature followed by a slower approach to the ambient temperature. The graph result was initially expected and then was confirmed by the simulation. The thermal properties of the cooling fluid, such as thermal conductivity and specific heat capacity, influence the specific shape and rate of cooling. These properties explain why different fluids show varying cooling behaviours.

6.2 Suggestions for future research

The initial description of the task and the outcome differ mainly due to the limited time conditions for the research. Moreover, the major limitation of this study was the assumption of a stationary liquid with no fluid flow, which was used because of the research's time limitations. In real-world applications, the fluid flow is necessary to simulate the immersion cooling systems. Future research should include fluid flow to provide a more accurate representation of the cooling process. The flow would change the cooling to be faster in time because it introduces convection, which can be either natural (driven by buoyancy forces) or forced (induced by external means like pumps or fans). Convection enhances heat transfer by moving heated fluid away from the heat source and bringing more excellent fluid into contact.

This research implied knowledge of the Partial Differential Equation course and knowledge of the OpenFOAM tool. A challenge during this research was the complexity of using OpenFOAM as a tool for such simulations, at least because of the programming language that it uses (C++). Despite this, the study successfully utilized the tool, laying a solid foundation simulation for future work.

Further research should also explore the impact of different complex shapes on cooling performance. While this study focused on a single sphere, future research could simulate shapes, such as fins, ridges, or multiple spheres, to optimize the design of heat sinks. Investigating how these shapes influence fluid flow and heat dissipation could provide significant findings. Another possible direction for future research is the investigation of heat sink designs adapted to specific cooling requirements. This involves solving the inverse problem, where the shape and configuration of the heat sink are optimized based on the given cooling characteristics, such as temperature gradients and thermal resistance.

Experimental validation of the simulation results is also essential. Conducting physical experiments to measure the cooling rates of spheres in different dielectric liquids could provide practical data to validate the computational findings. This step is necessary to confirm the accuracy of the simulations in general and identify any differences that need to be addressed. Exploring alternative dielectric fluids with even better thermal properties could further improve cooling performance. Research into nanofluids, which contain nanoparticles suspended in a base fluid, might offer new possibilities for improved heat transfer.

This research's broader implications extend to industries requiring efficient thermal management. In particular, data centres could benefit from immersion cooling technologies research, leading to improved energy efficiency, reduced operating costs and more sustainable performance.

7 Conclusions

In conclusion, this research, which analyzes the conductive cooling performance of a copper sphere in various dielectric liquids, provides crucial findings that pave the way for future studies to optimize heat sink designs. The result that Polyalphaolefin (PAO) is the most efficient dielectric liquid for cooling highlights the importance of thermal properties in immersion cooling. These significant findings can guide the development of more efficient and sustainable thermal management technologies. This study contributes to the academic understanding of immersion cooling and opens up possibilities for further exploration in the field.

PAO emerged as the fastest cooling agent of the three dielectric liquids studied due to its higher thermal conductivity and specific heat capacity. These properties allow PAO to transfer heat from the copper sphere more efficiently, resulting in faster cooling rates than Silicon Oil and Mineral Oil. The study observed that Silicon Oil exhibited the largest temperature differences between the initial and refined meshes, indicating a higher sensitivity to mesh resolution. This emphasizes the necessity of fine mesh refinement to capture steep temperature gradients accurately and reduce numerical errors. The refinement process significantly improved the accuracy of the simulation results, underscoring its importance in CFD studies. Additionally, OpenFOAM demonstrated its robustness and flexibility in handling complex thermal simulations. The platform's capabilities allowed for detailed analysis and accurate modelling of the cooling processes, paving the way for its use in future research to explore more intricate aspects of immersion cooling and heat sink design.

Building on these findings, future studies have the potential to significantly impact the field by focusing on optimizing heat sink designs. By considering different dielectric fluids' thermal properties and behaviours, such research can lead to developing more effective cooling solutions tailored to specific applications. Further investigations are needed to understand the impact of fluid flow, including both natural and forced convection, on cooling performance. Analyzing how different fluid properties and flow conditions influence heat transfer can provide deeper insights into enhancing immersion cooling systems.

This study contributes significantly to the academic understanding of immersion cooling, emphasizing the critical role of thermal properties in determining cooling efficiency. Addressing current limitations and pursuing the suggested future research directions can lead to substantial advancements in thermal management technologies. They will lead to more efficient and sustainable cooling solutions, crucial for meeting the growing demands of modern data centres and other high-performance computing environments.

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A Appendix

Time, s	Silicone Oil	Mineral oil	Polyalphaolefin (PAO)
0	310	310	310
100	309,55174	309,58123	309,49853
200	308,58410	308,62354	308,41086
300	307,57839	307,60484	307,29135
400	306,65203	306,65717	306,27538
500	305,82748	305,81121	305,387
600	305,10167	305,06718	304,61998
700	304,46520	304,4165	303,96077
800	303,90781	303,8487	303,39517
900	303,41989	303,35367	302,91019
1000	302,99287	302,92223	302,49445
1100	302,61915	302,54627	302,13808
1200	302,29210	302,21866	301,83262
1300	302,00588	301,9332	301,57079
1400	301,75540	301,68447	301,34638
1500	301,53621	301,46774	301,15402
1600	301,34438	301,2789	300,98915
1700	301,17651	301,11435	300,84783
1800	301,08252	300,97098	300,7267
1900	301,02959	300,84605	300,62288
2000	300,90103	300,73719	300,53389
2100	300,78852	300,64234	300,45761
2200	300,69005	300,5597	300,39223
2300	300,60389	300,48768	300,3362
2400	300,46249	300,42494	300,28816
2500	300,40474	300,37026	300,247

FIGURE 7: Table of results 40 grid cells

Time, s	Silicone Oil	Mineral oil	Polyalphaolefin (PAO)
0	310	310	310
100	309,54968	309,57945	309,49682
200	308,58402	308,62337	308,4119
300	307,58257	307,60895	307,29745
400	306,66030	306,66549	306,28605
500	305,83906	305,82299	305,40115
600	305,11573	305,08155	304,63654
700	304,48097	304,43265	303,9788
800	303,92466	303,86597	303,41394
900	303,43731	303,37151	302,92912
1000	303,01045	302,94021	302,51312
1100	302,63660	302,56406	302,15618
1200	302,30916	302,23602	301,84993
1300	302,02239	301,94995	301,58718
1400	301,77123	301,70048	301,36174
1500	301,55127	301,48292	301,16833
1600	301,35862	301,2932	301,00239
1700	301,18989	301,12775	300,86001
1800	301,04212	300,98346	300,73786
1900	300,91270	300,85764	300,63306
2000	300,79936	300,74791	300,54314
2100	300,70009	300,65223	300,466
2200	300,61314	300,56878	300,39981
2300	300,53700	300,49601	300,34302
2400	300,47031	300,43255	300,2943
2500	300,41190	300,37721	300,2525

FIGURE 8: Table of results 80 grid cells