Adjusted Present Value

A study on the properties, functioning and applicability of the adjusted present value company valuation model

KPMG

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Foreword

This research report is the result of five months of research into the adjusted present value company valuation model. This master thesis serves as a final assignment to complete the Master Industrial Engineering & Management (Financial Engineering & Management Track).

The research project was performed at KPMG Corporate Finance, located in Amstelveen, from May 21, 2007 up until September 28, 2007, under supervision of Jeroen Weimer (Partner KPMG Corporate Finance), Frank Siblesz (Manager KPMG Corporate Finance), Jan Bilderbeek (University of Twente) and Henk Kroon (University of Twente).

The subject of this master assignment was chosen after deliberation with the supervisors at KPMG Corporate Finance on the research needs of KPMG Corporate Finance.

It is implicitly assumed in this research report that the reader has been educated or is active in the field of corporate finance. It is also assumed that the reader is aware of existence of (company) valuation as part of the corporate finance working field.

Any comments, questions or remarks that come forth from reading this research report can be directed to me through the contact information given below.

The only thing remaining is to wish the reader a pleasant time reading this report and to hope that this report provides the reader with a clear insight in the adjusted present value model.

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Executive summary

The research objective of this research project is to formulate a theory on the differences between the enterprise DCF model and the APV model, and the effects of these differences on the valuation outcome.

There are a number of differences between the enterprise DCF model and the APV model. These differences are as shown in the table below.

	Enterprise DCF model	APV model
Cash flow	FCFF	FCFF
Discount rate	WACC	Ku or Kd
Cost of equity	Levered	Unlevered
Cost of debt	Credit rating based	Credit rating based
Capital structure	Constant leverage ratio	Constant leverage ratio or fixed debt
Probability of default	Not explicitly taken into account	Separate term in the valuation
Costs of financial distress	Not explicitly taken into account	Separate term in the valuation

There are five scenarios in which the APV model can be used to determine the correct value of the company, if the capital structure and/or the probability of default assumption of the enterprise DCF model are violated. The validated differences in valuation outcomes are shown in the table below.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

The APV model is theoretically more correct and also gives a significantly different valuation outcome than the enterprise DCF model in cases where the company under valuation suffers from financial distress.
In other cases, the APV model is theoretically more correct but the difference in valuation outcome with regard to the enterprise DCF model is negligible. The practical situations that relate to the cases in which the difference becomes insignificant are those of a management or leverage buyout and project finance. As a result of the nonsignificant difference, the APV model and the enterprise DCF model can both be used to value the company even though the APV model gives a theoretically more correct company value.
In the remaining cases, either both the enterprise DCF model and the APV model can be used to determine the company value or none of the two models is fit for the determination of the company value.

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Chapter 1: Research design

1.1 Research context

Valuation, for the purpose of this research project, is defined as the process of determining the current worth of an asset or company. Over the years, several valuation models have been developed. One of these models is the enterprise discounted cash flow (DCF) model. For the purpose of this research project, the enterprise DCF model is defined as a valuation model that is used to estimate the value of an asset or a company by using free cash flow projections and discounting them using the weighted average cost of capital (WACC) to arrive at a present value. Another valuation model is the adjusted present value (APV) model. For the purpose of this research project, APV is defined as the net present value of an asset or company if financed solely by equity plus the present value of any financing benefits minus the expected costs of financial distress. Net present value is the difference between the present value of cash inflows and the present value of cash outflows. Present value is the amount that a future sum of money is worth today given a specified rate of return.

The enterprise DCF model is a main valuation model of practitioners. The enterprise DCF model comprises two assumptions, namely that the capital structure of company remains constant over time and that the costs of financial distress are zero. There are scenarios in which the actual capital structure developments and the costs of financial distress differ from the assumptions made by the enterprise DCF model, which causes an error in the valuation. In such scenarios, the APV model is a more appropriate valuation model since it does not contain the two enterprise DCF model assumptions.

The APV model is thus a substitute for the enterprise DCF model in scenarios where the enterprise DCF model assumptions are violated. In order to decide when to use which model a number of aspects have to be analyzed. First, there are circumstances under which either the APV model or the enterprise DCF model should be used. Second, the differences between the valuation outcomes of the two models under the different circumstances have to be analyzed to determine the valuation error as a result of the choice for the inappropriate model. Third, the effect of the nonconstant capital structure and the nonzero costs of financial distress on the APV model valuation outcome need to be analyzed since these are the two aspects on which the APV model differs from the enterprise DCF model. And as a last point of analysis, these two factors need to be modeled in order to obtain the most accurate valuation outcome under different circumstances.

The APV model and, especially, the enterprise DCF model are extensively discussed in the corporate finance literature. However, the amount of corporate finance literature that focuses on the four aspects discussed in the previous subsection is not that substantial. In different textbooks and articles[1] is defined that the APV model is more appropriate in case of a non-constant capital structure. However, the term ‘non-constant’ has not been specified. It is also left unclear whether the nonconstant leverage ratio is the only reason for switching to the APV model. There is also a lack of studies that compare the valuation outcomes of the two models for specific scenarios. Therefore, there are no clear statements on the different outcomes considering certain circumstances. There are also very few authors that discuss the effects of the costs of financial distress on the valuation outcome under the APV model. At last, there are multiple theories for both the modeling of the development of capital structure as well as for the modeling of the probability of default. There is, however, no clear embedment of these modeling approaches into the application of the APV model.

So, the APV model is an alternative for the enterprise DCF model when the assumptions underlying the enterprise DCF model are violated by the valuation situation at hand. There are, however, a number of aspects of the APV model that need to be studied and specified to support a founded choice for either the enterprise DCF model or the APV model under particular circumstances.

[1] For instance in Koller *et al.* (2005) or Kruschwitz & Löffler (1998)

1.2 Research objective, research framework and research issues

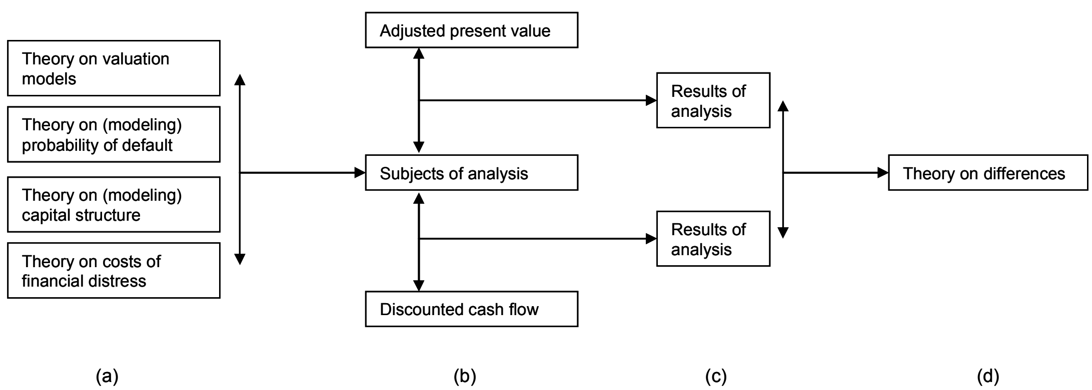
The objective of this research project is based on the research context described in the previous paragraph and functions as a guide for the formulation of the research framework.

The research objective is *to formulate a theory* on the differences between the enterprise DCF model and the APV model, and the effects of these differences on the valuation outcome *by analyzing* the basic assumptions of both models, the circumstances in which either one should be used, the impact of the nonconstant capital structure and the nonzero costs of financial distress on the valuation outcome under the APV model, and the way in which these two factors can be modeled to obtain the most accurate valuation outcome.

In order to arrive at the intended result, a theory on the differences between the two valuation models, two objects have to be studied. These two *research objects* are the enterprise DCF model and the APV model itself. The theory on the differences between the two valuation models will be based on the analysis of each valuation model. To ensure that the analyses of the valuation models can be compared, each model is studied through the *research perspective*. This research perspective consists of a number of *subjects of analysis*. The subjects of analysis are chosen based on their ability to clearly and distinctively study the two valuation models and are retrieved from relevant literature. There are four key concepts that are relevant for studying the valuation models:

1. Basic assumptions of valuation models
2. Probability of default
3. Costs of financial distress
4. Influence of capital structure

The relations in the research framework can be illustrated as follows.



To summarize the research approach, the steps to be taken in the course of the research project are as follows:
a) An analysis of the various aspects of valuation methods that are relevant for the enterprise DCF model and the APV model provides the subjects of analysis b) used to evaluate the enterprise DCF and the APV valuation models. c) A comparison of both evaluations results in d) a theory on the differences between the enterprise DCF model and the APV model and the effect of these differences on the valuation outcome.

The research objective is a formulation of the intended result of the research project; the research framework provides the steps to be taken to arrive at the intended result. The research issues serve as a means to determine the knowledge required for realizing the objective. The research issues are divided into central questions and sub-questions. The three central questions are:

1. What subjects are relevant for analyzing the enterprise DCF and the APV valuation model?
2. How are the enterprise DCF model and the APV model specified in the light of these subjects?
3. What are the differences between the two models and what is the effect of these differences on the valuation outcome under which circumstances?

The sub-questions for the first central question are formulated so that they provide the answer to the central question by combining the answers of the sub-questions. The sub-questions are as follows:

1.1 What subjects can be derived from theories on valuation models?
1.2 What subjects can be derived from probability of default theories?
1.3 What subjects can be derived from theories on capital structure?
1.4 What subjects can be derived from financial distress costs theories?

The sub-questions of the second central question are as follows:

2.1 What are the basic assumptions underlying the enterprise DCF model?
2.2 What are the basic assumptions underlying the APV model?
2.3 What are the specifications of the enterprise DCF model studied in the light of the subjects of analysis?
2.4 What are the specifications of the APV model studied in the light of the subjects of analysis?

An answer to the second central question is provided by the combined answers of the sub-questions. Sub-questions 2.1 and 2.2 describe the basics of each valuation model. This information is mainly relevant as background information for answering sub-questions 2.3 and 2.4. The result of this process is an overview of each valuation model.
The third central question can be subdivided into the following sub-questions:

3.1 What are the differences between the enterprise DCF model and the APV model?
3.2 What is the effect of the differences on the valuation outcome under which circumstances?

1.3 Research strategy

This paragraph focuses on the research strategy followed to arrive at the research objective. First, the literature survey research strategy is described. Second, the decision to use the literature survey research strategy is justified.

1.3.1 Desk research

Desk research is a research strategy whereby the researchers use material produced by others.

A desk research project is characterized by:

1. The use of *existing material*
2. The *absence of direct contact* with the research object
3. Looking at the material being used from *a different* perspective than at the time of its production

In desk research by far the main characteristic is that the material used has been produced entirely by others. Three categories of existing material can be used for carrying out a desk research project: literature, secondary data and official statistical material. Literature is understood to mean books, articles, conference proceedings and such that contain the knowledge products of scientists. Secondary data is empirical data compiled by other researchers or the researcher self during previous research projects. Official statistical material is understood to be data gathered periodically or continuously for a broader public.

Two main variants of desk research can be distinguished, namely literature survey and secondary research. Parallel to this distinction between knowledge sources and data sources, in the first type of research one would use knowledge produced by others, and in the second type of research empirical data produced by others.

1.3.2 Justification

The objective of the research project is to formulate a theory on the differences between the enterprise DCF model and the APV model and the effects of those differences on the valuation outcome. To determine these differences, the focus of the research will be on depth rather than on breadth (which would be the case when the research objective would be to give a complete overview of all existing valuation models).
In order to acquire an in-depth comparison of the two valuation models, a qualitative approach in which multiple aspects of the two models are analyzed is required. The research

project has a non-empirical nature, since the two research objects are theories, which are studied on qualitative aspects.

The above gives a clear indication that the appropriate research strategy is that of a desk research. The research consists of using existing material to compare two theories in an in-depth manner without having direct contact with the research object.

A literature survey is characterized by the fact that knowledge produced by others is used, compared to the secondary search that uses empirical data produced by others. Since the research project compares different theories, a literature survey is the most appropriate form of desk research.

1.4 Structure of the research report

The remainder of this report is divided in chapters that are linked to the research issues. Chapter 2 focuses on the theoretical background of the research project, which is the answer to central question one. In chapter 2, the subjects of analysis are identified through an analysis of available literature on the four aspects. Chapter 3 focuses on the second and third research issue and is the most important chapter of this research project as it contains the analysis which leads to the formulation of the theory on the differences between the two valuation models and the effect of these differences on the valuation outcome, the research objective.

Chapter 4, the validation chapter, aims to validate the results from chapter 3. The theory stated in chapter 3 is tested through the implementation of different scenarios into the valuation model described in appendix II. This leads to an overview of results on the validity of the statements from chapter 3.

Chapter 5, the final chapter of this report, concludes on the valid differences between the two valuation models and gives an overview of the circumstances under which certain effects on the valuation outcomes of each valuation model are realized. As a last contribution, chapter 5 discusses further research suggestions that come forth from this research project.

In appendix II the theoretical results of the analysis in chapter 3 are translated to a valuation model in the shape of a Microsoft Excel workbook. This workbook can be used to determine the value of a company under different circumstances. Appendix II also contains an example of a valuation through the APV model.

Chapter 2: Theoretical background

Introduction

The objective of this research project is to identify the differences between the enterprise DCF model and the APV model. In order to compare the two models in a distinctive manner, they both need to be compared on the same set of subjects.
This chapter discusses the areas of research that are related to the aspects of the two valuation models, as discussed in the research context section of chapter 1. The four research areas are general valuation theory, probability of default estimation, capital structure theory, and the costs of financial distress.
In each paragraph, dedicated to a particular research field, an overview is given on the current state of that research area. The main theories and views are discussed to provide an overview of the particular area and to (implicitly) give an indication of the robustness of the theories and views that currently exist. The discussion of the theories also serves as a foundation for the modeling choices that are made in chapter 3 of this research report.
Each paragraph ends with a conclusion on the relevant subjects of analysis that come forth from the particular research area. These subjects form the basis of the analysis of the two valuation models in chapter 3 of this research report.

2.1 Introduction to valuation theory

Valuation is an important tool for many reasons. It is used in multiple situations for different purposes. Valuation models can be divided in three categories that each have a different approach of determining what the value of an asset or company is. However, in practice, one type of approach, the income approach, is most often applied. Also, the valuation models that are compared in this research project are models in this category. Therefore, this paragraph discusses a number of different income approach valuation models and the different parameters that are used in most of them.
The purpose of this paragraph is to come up with a number of subjects on which the two valuation models can be analyzed.

2.1.1 Definitions in and rationale of valuation

Valuation, for the purpose of this research project, is defined as the process of determining the current worth of an asset or company. In general, valuation is the process of estimating the market value of a financial asset (e.g. investments in marketable securities or intangible assets), liability (e.g. bonds issued by a company or loans), or a company[2].

Common terms for the value of an asset or liability are fair market value, fair value, and intrinsic value. The meanings of these terms differ.
A common term is fair market value defined as the price, expressed in term of cash equivalents, at which property would change hands between a hypothetical willing and able buyer and a hypothetical willing and able seller, acting at arms length in an open and unrestricted market, when neither is under compulsion to buy or sell and when both have reasonable knowledge of the relevant facts.
Fair value is used in different contexts and has multiple meanings. The term is sometimes used to mean the same thing as fair market value. Fair value is also a term used in law and accounting. It is used in the Generally Accepted Accounting Principles (GAAP) for financial reporting and in law in shareholder rights legal statutes. In these cases, fair value is defined in the accounting literature or the law, respectively. Fair value may deviate from fair market value in the account and legal contexts.
Intrinsic value is an asset's true value regardless of the market price. It is the actual value of a company or an asset based on an underlying perception of its true value including all aspects of the company, in terms of both tangible and intangible factors.

[2] Damodaran (2002)

Enterprise valuation is a process applied to determine the fair market value of a company or an owner's interest therein. According to Duffhues (1997), in our western, on market principles based economy, different reasons can be thought of on why a proclamation on the value of a company could be necessary.
In the first place, one can think of the valuation of the shareholder's equity of going concerns in relation to current or future stock transactions, in which the explicit goal of the transaction is to acquire control over the company. Examples of such valuation issues are:

1. The determination of the price against which stocks will be introduced at an exchange (introduction price).
2. The determination of the issue price of securities that are issued.
3. The determination of the conversion price of convertible securities.
4. The determination of the exercise price of warrants.
5. The determination of the exercise price of employee stock options.

Second, one can think of the valuation of companies by tax authorities. In that case, the determination of the value of the tax equity is one of the objectives.
Third, valuation issues are present in the case of the trade of marketable securities; public or private, incidental or continuous. In that case, one should not only think of the sale or purchase of complete companies through mergers or acquisitions, but also of the valuation of securities of companies without any specific relation to mergers and acquisition, that take place on a daily basis on the stock exchanges.

2.1.2 Valuation approaches

There are three different approaches that are used in enterprise valuation: the income approach, the asset-based approach, and the market approach[3]. Within each of these approaches, there are various techniques for determining the fair market value of a company. Generally, the income approach models determine value by calculating the net present value of the benefit stream generated by the company. The asset-based approach models determine the enterprise value by adding the sum of the parts of the company and the market approach models determine the enterprise value by comparing the subject company to other companies in the same industry, of the same size, and/or within the same region. Each model has its own advantages and drawbacks. These should be considered when applying those models to a particular subject company.

The *income approach models* determine fair market value by multiplying the benefit stream generated by the subject company times a discount or capitalization rate. The discount or capitalization rate converts the stream of benefits into present value. There are several different income approaches, including capitalization of earnings or cash flows, discounted future cash flows (DCF), and the excess earnings method (which is a hybrid of assets and income approaches). Most of the income approach models consider the subject company's historical financial data; only the DCF models require the subject company to provide projected financial data. Most of the income approach models use the company's adjusted historical financial data from a single time period; only the DCF models require data from multiple future time periods. The single time period data is often based on normalized data over three historical years. The discount or capitalization rate must be matched to the type of benefits streams to which it is applied. The result of a valuation under the income approach is generally the fair market value of a controlling, marketable interest in the subject company, since the entire benefit stream of the subject company is most often valued, and the capitalization and discount rates are derived from statistics concerning public companies.

The *asset-based approach models* are based on the principle that the value of a company is equal to the sum of its part. This principle is called *value additivity* and defines that in perfect capital markets the present value of two assets combined is equal to the sum of their present values considered separately[4].
In contrast to the income approach models, which require subjective judgments about capitalization or discount rates, the adjusted net book value method is relatively objective. In accordance with accounting conventions, most assets are reported in the books of the subject

[3] Damodaran (2002)
[4] Brealey & Myers (2003)

company at their acquisition value, net of depreciation where applicable. These values must be adjusted to fair market value wherever possible.
The value of a company's intangible assets, such as goodwill, is generally impossible to determine apart from the company's overall enterprise value. For this reason, the asset-based approach models are not the most appropriate models of determining the value of going concern companies.
Adjusted net book value may be the most relevant standard of value where liquidation is imminent or ongoing; where a company's earnings or cash flows are nominal, negative or worth less than its assets; or where net book value is standard in the industry in which the company operates. If these situations do not apply to the company that is being valued, then the adjusted net book value may be used as a 'sanity check' when compared to other methods of valuation, such as the income and market approaches.

The *market approach* to enterprise valuation is based on the economic principle of substitution: buyers will be unwilling to pay more for an item than the price at which they can obtain an equally desirable substitute. The market price of the stocks of publicly traded companies engaged in the same or a similar line of business, whose shares are actively traded in a free and open market, can be a valid indicator of value when the transactions in which stocks are traded are sufficiently similar to permit meaningful comparison. The difficulty lies in identifying public companies that are sufficiently comparable to the subject company for this purpose.

Of the three used approaches in enterprise valuation, the income approach is most often applied. However, since the income approach models rely partially on forecasts, a plausibility check of the forecasts could be used to improve the valuation accuracy.
Koller *et al.* (2005, p. 361) identify these aspects and argue that "the income approach applied through various discounted cash flow methods is the most accurate and flexible method for valuing projects, divisions and companies. Any analysis, however, is only as accurate as the forecasts it relies on. Errors in estimating the key ingredients of corporate value can lead to mistakes in valuation and, ultimately to strategic errors.
A careful multiple analysis (which is an application of the market approach to valuation) – comparing a company's multiples versus those of comparable companies – can be useful in making such forecasts and the discounted cash flow valuations they inform more accurate. Properly executed, such an analysis can help test the plausibility of cash flow forecasts, explain mismatches between a company's performance and that of its competitors, and support useful discussions about whether the company is strategically positioned to create more value than other industry players."

Because both valuation models that are studied in this research project are income approach models, the following sections will go deeper into the aspects of discounted cash flow techniques. Before continuing on those aspects, it is important to note that valuation is more an art than a science, mainly because it requires a significant degree of judgment:

1. There are very different situations and purposes in which one values a company (e.g., a company in distress, for tax purposes, in relation to mergers & acquisitions). In turn this requires different models or a different interpretation of the same models each time.
2. All valuation models have their limitations (e.g., mathematical, complexity, comparability) and could be widely criticized. As a general rule the valuation models are most useful when the same valuation model as the 'partner' you are interacting with is used.
3. The quality of some input data may vary widely.

2.1.3 Parameters normally used in valuation models

The different discounted cash flow techniques all determine value by discounting certain cash flows at a certain discount rate. This section discusses the parameters that are used in almost all discounted cash flow techniques. These parameters are the cash flows, the discount rate, the cost of debt, the cost of equity, and the tax rate. The cash flows and the discount rate form the basis of most discounted cash flow techniques. The cost of debt, the cost of equity, and the tax rate in some cases directly influence the enterprise value, in other cases they serve as an input for the discount rate or cash flows.

Most of the descriptions of the parameters are based on Brealey & Myers (2003) and Koller *et al* (2005). The definitions of the parameters that are derived from Brealey & Myers (2003) and Koller *et al.* (2005) will be explicitly linked to their source through a reference.

Cash flows
A cash flow is the difference between the amount of cash received and the amount of cash paid out over a given period of time.[5] Cash in-flows usually arise from one of three activities: operations, financing or investing. Cash out-flows result from expenses or investments.

The main two types of cash flows used in enterprise valuation are the free cash flow (FCF) and equity cash flow (ECF).
Free cash flow is the after-tax cash flow available to *all* investors: debt holders and equity holders. Unlike "cash flow from operations" reported in a company's financial statement, free cash flow is independent of financing and nonoperating items. It can be thought of as the after-tax cash flow – as if the company held only core operating assets and financed the business entirely with equity. Free cash flow is defined by Koller *et al.* (2005) as:

FCF = NOPLAT + Noncash Operating Expenses – Investments in Invested Capital

Cash flow to equity is a measure of how much cash can be paid to the equity shareholders of the company after all expenses, reinvestment and debt repayment. According to Koller *et al* (2005) it is calculated as:

ECF = Net Income + Noncash Expenses – Net Capital Expenditures – Change in Working Capital + New Debt – Debt Repayment

Or as:

ECF = Dividends + Share Repurchases – New Equity Issues

Discount rate
The first basic principle of finance is that *a Euro today is worth more than a Euro tomorrow*, because the Euro today can be invested to start earning interest immediately. Thus, the present value of a delayed payoff may be found by multiplying the payoff by a discount factor which is less than 1. If C_1 denotes the expected payoff at period 1 (one year hence), then

Present value (PV) = discount factor * C_1

The discount factor is the value today of €1 received in the future. It is usually expressed as the reciprocal of 1 plus a rate of return:

Discount factor = 1 / (1+r)

The rate of return r is the reward that investors demand for accepting delayed payment.
To calculate present value, one discounts expected payoffs by the rate of return offered by equivalent investment alternatives in the capital market. This rate of return is often referred to as the *discount rate, hurdle rate,* or *opportunity cost of capital.* It is called the *opportunity cost* because it is the return foregone by investing in the investment opportunity at hand rather than investing in alternative investment opportunities, such as securities.
The *company cost of capital* is defined as the expected return on a portfolio of all the company's existing securities. It is used to discount the cash flows on investment opportunities that have similar risk to that of the company as a whole.

Cost of debt
The cost of debt is the borrowing rate at which the company is expected to be able to acquire debt, based on the company's current (credit) risk position. The appropriate method of

[5] Brealey & Myers (2003, p. 119)

determining the cost of debt should be selected based on the type of debt that the company has outstanding.
Koller *et al.* (2005) give the following options for estimating the cost of debt:
"To estimate the cost of debt, use the yield to maturity of the company's long-term, option-free bonds. Technically speaking, yield to maturity is only a proxy for expected return, because the yield is actually a *promised* rate of return on a company's debt. For estimating the cost of debt for a company with investment-grade debt (debt rated at BBB or better), yield to maturity is a suitable proxy. When calculating yield to maturity, use *long-term* bonds.
For companies with only short-term bonds or bonds that rarely trade, determine yield to maturity by using an indirect method. First, determine the company's credit rating on unsecured long-term debt. Next, examine the average yield to maturity on a portfolio of long-term bonds with the same credit rating. Use this yield as a proxy for the company's implied yield on long-term debt.
For debt below investment grade, using the yield to maturity as a proxy for the cost of debt can cause significant error. Three factors drive yield to maturity: the cost of debt, the probability of default, and the recovery rate. When the probability of default is high and the recovery rate low, the yield to maturity will deviate significantly from the cost of debt. Thus, for companies with high default risk and low ratings, the yield to maturity is a poor proxy for the cost of debt. To estimate the cost of high-yield debt, we rely on the CAPM (a general pricing model, applicable to any security)."

Cost of equity

The cost of equity is estimated by determining the expected rate of return of the company's stock. Since expected rates of return are unobservable, asset-pricing models that translate risk into expected return are used.
The most common asset-pricing model is the SLB (Sharpe – Lintner – Black) Capital Asset Pricing Model (CAPM). Other models include the Fama-French three-factor model and the arbitrage pricing theory (APT).

The CAPM puts forward that the expected rate of return of any security equals the risk-free rate plus the security's beta times the market risk premium:

$$E(R_i) = r_f + \beta_i [E(R_m) - r_f]$$

where $E(R_i)$ is the security *i*'s expected return, r_f the risk-free rate, β_i the stock's sensitivity to the market and $E(R_m)$ the expected return of the market.
In the CAPM, the risk-free rate and market risk premium (defined as the difference between $E(R_m)$ and r_f) are common to all companies; only beta varies across companies. Beta represents a stock's incremental risk to a diversified investor, where risk is defined by how much the stock covaries with the aggregate stock market.

In 1992, Eugene Fama and Kenneth French stated that equity returns are inversely related to the size of a company (as measured by market capitalization) and positively related to the ratio of a company's book value to its market value of equity. In their model commonly known as the Fama-French three-factor model, a stock's excess returns are regressed on excess market returns (similar to the CAPM), the excess returns of small stocks over big stocks (SMB), and the excess returns of high book-to-market stocks over low book-to-market stocks (HML).
The SMB and HML portfolios are meant to replicate unobservable risk factors, factors that caused small companies with high book-to-market values to outperform their CAPM expected returns. The expected rate of return according to the Fama-French three-factor model is calculated through:

$$E(R_i) = r_f + \beta_1 (E(R_m) - r_f) + \beta_2 E(R_S - R_B) + \beta_3 E(R_H - R_L)$$

The company's three betas are determined through a regression of the stocks returns against the excess market portfolio, SMB, and HML.

Another alternative to the CAPM, the arbitrage pricing theory (APT), resembles a generalized version of the Fama-French three-factor model. In the APT, a security's actual returns are *fully* specified by *k* factors and random noise:

$$\widetilde{R}_i = \alpha + \beta_1 \widetilde{F}_1 + \beta_2 \widetilde{F}_2 + ... + \beta_k \widetilde{F}_k + \varepsilon$$

By creating well-diversified factor portfolios, it can be shown that a security's expected return must equal the risk-free rate plus the cumulative sum of its exposure to each factor times the factor's risk premium (λ):

$$E[R_i] = r_f + \beta_1 \lambda_1 + \beta_2 \lambda_2 + ... + \beta_k \lambda_k$$

Otherwise, arbitrage is possible (positive return with zero risk).
On paper, the theory is extremely powerful. In practice, implementation of the model has been difficult, as there is little agreement about how many factors there are, what the factors represent, or how to measure the factors. For this reason, use of the APT resides primarily in the classroom.

Tax
Companies are obliged by law to pay corporate taxes on the profits that they realize. This corporate tax thus causes a cash outflow. However, companies are allowed to deduct certain costs from their profit (tax-deductible) before the taxes that have to be paid are determined. This leads to a formal cash inflow of the amount deducted times the tax rate. This reduction in corporate taxes that results from taking an allowable deduction from taxable profits is called a *tax shield*.
Debt financing also has this important advantage under the corporate income tax system in the United States, the Netherlands, and in multiple other countries. The interest that the company pays is a tax-deductible expense. Dividends and retained earnings are not. Thus the return to bondholders escapes taxation at the corporate level.

There are two tax rates that are used in valuation: the marginal corporate tax T_c and the net tax saving per dollar of interest paid by the firm T*.
Brealey & Myers (2003) state that one should always use T_c, the marginal corporate tax rate, (1) when calculating the WACC as a weighted average of the costs of debt and equity and (2) when discounting safe, nominal cash flows. In each case the discount rate is adjusted *only* for corporate taxes.
The APV model in principle calls for T*, the net tax saving per dollar of interest paid by the company. This depends on the effective personal tax rates on debt and equity income. T* is almost surely less than T_c, but it is very difficult to pin down the numerical difference. Therefore in practice T_c is almost always used as an approximation.

2.1.4 Five discounted cash flow valuation models

As a final part of this paragraph, five different cash flow valuation models are discussed to show the difference in assumptions and parameters that are used. This should give some insight in the aspects in which valuation models can differ from each other. The five methods discussed are:

- Equity cash flows discounted at the required return to equity.
- Capital cash flows discounted at the WACC before tax.
- Residual Income discounted at the required return to equity.
- EVA discounted at the WACC.
- The risk-free-adjusted equity cash flow discounted at the risk-free rate.

There are four basic discounted cash flow valuation models, two of them being the enterprise DCF model and the APV model. The other two are (a) the model in which the equity cash flows (ECF) are discounted at the required return to equity (K_e) and (b) the model where capital cash flows (CCF) are discounted at the WACC before tax.
For (a), equation (2.1) indicates that the value of equity (E) is the present value of the expected equity cash flow (ECF) discounted at the required return to equity (K_e).

$$E_0 = PV_0[Ke_t; ECF_t] \quad (2.1)$$

The expected equity cash flow is the sum of all expected cash payments to shareholders, mainly dividends and share repurchases.
Equation (2.2) indicates that the value of the debt (D) is the present value of the expected debt cash flows (CFd) discounted at the required return to debt (K_d).

$$D_0 = PV_0[Kd_t; CFd_t] \quad (2.2)$$

The expected debt cash flow in a given period is given by equation (2.3)

$$CFd_t = N_{t-1}r_t - (N_t - N_{t-1}) \quad (2.3)$$

where N is the book value of the financial debt and r is the cost of debt. $N_{t-1}r_t$ is the interest paid by the company in period t. (N_t - N_{t-1}) is the increase in the book value of debt in period t.

With (b), the capital cash flows are the cash flows available for all holders of the company's securities, whether these are debt or shares. They are equivalent to the expected equity cash flow (ECF) plus the expected debt cash flows (CFd).
Equation (2.4) indicates that the value of the debt today (D) plus that of the shareholders' equity (E) is equal to the capital cash flow (CCF) discounted at the WACC before tax ($WACC_{BT}$).

$$E_0 + D_0 = PV[WACC_{BTt}; CCF_t] \quad (2.4)$$

The expression that relates the CCF with the ECF and the FCF is (2.5):

$$CCF_t = ECF_t + CFd_t = ECF_t - (N_t - N_{t-1}) + N_{t-1}r_t = FCF_t + N_{t-1}r_tT_t \quad (2.5)$$

The other three discounted cash flow models are used less often. For the model that uses the residual income (also called economic profit) and K_e (required return to equity), equation (2.6) indicates that the value of the equity (E) is the equity's book value (Ebv) plus the present value of the expected residual income (RI) discounted at the required return to equity (K_e).

$$E_0 = Ebv_0 + PV_0[Ke_t; RI_t] \quad (2.6)$$

The term residual income (RI) is used to define the accounting net income or profit after tax (PAT) minus the equity's book value (Ebv_{t-1}) multiplied by the required return to equity.

$$RI_t = PAT_t - Ke_tEbv_{t-1} \quad (2.7)$$

For the model using the EVA (economic value added) and the WACC, equation (2.8) indicates that of the shareholders' equity (E) is the book value of the shareholders' equity and the debt (Ebv_0 + N_0) plus the present value of the expected EVA, discounted at the WACC:

$$E_0 + D_0 = (Ebv_0 + N_0) + PV_0[WACC_t; EVA_t] \quad (2.8)$$

The EVA (economic value added) is the NOPAT (Net Operating Profit After Tax) minus the company's book value (N_{t-1} + Evc_{t-1}) multiplied by the weighted average cost of capital (WACC). The NOPAT is the profit of the unlevered (debt-free) company.

$$EVA_t = NOPAT_t - (N_{t-1} + Ebv_{t-1})WACC_t \quad (2.9)$$

Last, for the model using the risk-free-adjusted equity cash flows discounted at the risk-free rate, equation (2.10) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected risk-free-adjusted free cash flows ($FCF\backslash\backslash r_f$) that will be generated by the company, discounted at the risk-free rate (r_f):

$$E_0 + D_0 = PV_0[Ku_t; FCF_t \backslash\backslash r_f] \tag{2.10}$$

The definition of the risk-free-adjusted free cash flows is:

$$FCF_t \backslash\backslash r_f = FCF_t - (E_{t-1} + D_{t-1})(WACC_t - R_{Ft}) \tag{2.11}$$

2.1.5 Subjects of analysis regarding the general assumptions of a valuation model

There are multiple approaches for determining the enterprise value of a company. The two models that are studied in this research project both belong to the income approach and have the form of a discounted cash flow model. Each discounted cash flow model consists of four subjects of which the specification determines the functioning of the particular discounted cash flow model. These four subjects are:

1. The way in which the cash flows are modeled.
2. The discount rate that is used.
3. The cost of equity and the cost of debt used.
4. The incorporation of taxes into the valuation.

As the five discounted cash flow models showed, discounted cash flow models often differ on their specifications in relation to these four subjects. Therefore, to compare the enterprise DCF model and the APV model, both models have to be analyzed on these four *general subjects of analysis*.

2.2 Probability of default

The goal of this paragraph is to give an overview of the main theories on estimating the probability of default. Three types of models are commonly used for estimating the probability of default, each with a different type of inputs for the estimation. This paragraph discusses the main models of each type and the comments on these models. The total overview of the models is used to determine the probability of default subjects of analysis.

Default is defined to be the condition that occurs when a company has a delayed or missing contractual debt payment. Unfortunately, data on defaults is not readily available. For this reason, instead of defaults, most studies use bankruptcies as there object of analysis, where a bankruptcy is defined to occur when a company makes either a Chapter 7 (liquidation) or Chapter 11 (reorganization) filing (these titles refer to chapters of the US Bankruptcy Code).

In general, the default risk is a function, very broadly, of two variables: its capacity to generate cash flows from operations and its financial obligations – including interest and principal payments. Damodaran (2002) states that, keeping everything equal:

- Companies which generate high cash flows relative to their financial obligation have lower default risk than firms which generate low cash flows relative to their obligations. Thus, companies with significant assets in place, which generate high cash flows, will have lower default risk than firms that do not.
- The more stability there is in cash flows, the lower is the default risk in the company. Companies which operate in predictable and stable businesses will have lower default risk than otherwise similar companies which operate in cyclicate and/or volatile businesses.

There are three broad sources of information about the creditworthiness of companies, from which one can determine what the probability is that a company will default. These are the views of a specialist credit analyst, information embedded in the company's security prices, or the use of the company's financial statements to make an assessment.

A way to assess a company’s credit standing is to seek the views of a specialist in credit assessment. For example, bond rating agencies, such as Moody’s and Standard and Poor’s, provide a useful guide to the riskiness of the company’s bonds.
Bond ratings are usually available only for relatively large companies. However, information can be obtained on many smaller companies from a credit agency. Dun and Bradstreet is by far the largest of these agencies and its database contains reports on more than 10 million companies.[6]

In addition to checking with a credit agency or a bank, it may make sense to check what the financial community thinks about the company’s credit standing by looking at the yield on the company’s bonds and/or stock price.
Information on security prices can be used to put a figure on the chances of default. Companies have an incentive to exercise their option to default when the value of their assets is less than the amount of their debt. So, if it is known how much the value of the company’s assets may fluctuate, the probability that the asset value will fall below the default point can be estimated.

Security price data may not be available for many companies, and in these cases one will need to rely on the company’s financial statements to make an own assessment of the company’s credit position.

2.2.1 Ratings

The relative quality of most traded bonds can be judged from bond ratings given by Moody’s and Standard and Poor’s. For example, the highest quality bonds are rated triple-A (Aaa) by Moody’s, then come double-A (Aa) bonds, and so on. Bonds rated triple-B (Baa) or above are known as investment-grade bonds.

Brealey & Myers (2003, p. 685) state that: “Bond ratings do reflect the probability of default. Since 1971 no bond that was initially rated triple-A by Standard and Poor’s has defaulted in the year after issue and fewer than one in a thousand has defaulted within ten years of issue. At the other extreme, over two percent of CCC bonds have defaulted in their first year and by year 10 almost half have done so. Of course, bonds rarely fall suddenly form grace. As time passes, and the company becomes progressively more unstable, the agencies revise downward the bond’s rating to reflect the increasing probability of default.”

The following table shows the default probabilities that are linked to the various credit ratings.

S&P's	*Default Probability* [1]	*Moody's*	*Default Probability* [2]	
AAA	0.12	Aaa	0.12	
AA	0.33	Aa	0.24	Investment grade
A	0.75	A	0.54	
BBB	3.84	Baa	2.16	
BB	14.45	Ba	11.17	
B	33.02	B	31.99	
CCC	61.35	Caa	60.83	Subinvestment grade
CC		Ca		
C		C		

[1]Percentage defaulting within 5 years based on default rates between 1981-2003
[2]Percentage defaulting within 5 years based on default rates between 1970-2003

To capture the statements given above in a more formal fashion, the following expression can be used.

A rating of a company can be defined as the mapping of the PoD, the expected probability of default, into a discrete number of quality classes, or rating categories. The PoD is a continuous variable, bounded by zero from below and by one from above.

[6] Brealey & Myers (2003)

$$PoD : \_Companies \rightarrow [0,1]$$

A PoD is the expected relative frequency of a credit event, where the latter is defined as a non-payment of principal or interest due (over a period of at least 30 days, say). The PoD is one component of a lenders' expected loss, as in:

$$E(L) = PoD * E(LGD)$$

Here, E(L) is expected loss, and E(LGD) is the expected loss given default. The expectations are taken over a common time interval, usually one year in the future. Expected loss is thus the average amount a lender is expecting to loose over the next twelve months.
Krahnen & Weber (2001) state that: "Though in theory, PoDs are mapped in rating classes, in practice it is the other way around. Rating classes are mapped into PoDs on the basis of historical data. The established agencies, notably S&P and Moody's, use historical default rates to calibrate their model. The default rate is the percentage of all bond issues outstanding at *t* that will have a credit event between *t* and *t + 1,* e.g., a 12 months period."

The bond ratings assigned by rating agencies are based primarily upon publicly available information, although private information conveyed by the company to the rating agency does play a role. The rating is assigned to a company's bonds will depend in large part on financial ratios that measure the capacity of the company to meet debt payments and generate stable and predictable cash flows. While a multitude of financial ratios exist, the table below summarizes some of the key ratios that are used to measure default risk.

Financial ratios used to measure default risk

Ratio	*Description*
Pretax Interest Coverage	=(Pretax Income from Continuing Operations + Interest Expense)/Gross Interest
EBITDA Interest Coverage	=EBITDA/Gross Interest
Funds from Operations/Total Debt	=(Net Income from Continuing Operations + Depreciation)/Total Debt
Free Operating Cash Flow/Total Debt	=(Funds from Operations - Capital Expeditures - Change in Working Capital)/Total Debt
Pretax Return on Permanent Capital	=(Pretax Income from Continuing Operations + Interest Expense)/ (Average of Beginning of the year and End of the year of long and short term debt, minority interest and Shareholders equity)
Operating Income/Sales	=(Sales - COGS (before depreciation) - Selling Expenses - Administrative Expenses - R&D Expenses)/Sales
Long Term Debt/Capital	=Long Term Debt/(Long Term Debt + Equity)
Total Debt/Capitalization	=Total Debt/(Total Debt + Equity)

There is a strong relationship between the bond rating a company receives and its performance on these financial ratios. Companies that generate income and cash flows that are significantly higher than debt payments, that are profitable, and that have low debt ratios are more likely to be highly rated than are companies that do not have these characteristics. There will be individual companies whose ratings are not consistent with their financial ratios, because the ratings agency does bring subjective judgments into the final mix. For most companies, however, the financial ratios should provide a reasonable basis for estimating the bond rating.

Two remarks can be made with regard to bond ratings. First, based on the distribution of credit ratings for all U.S. and European companies with a market capitalization over $1 billion according to Standard & Poor's, is becomes clear that the vast majority of the companies (72%) are in the rating category of A+ to BBB-.[7]
Second, Gray *et al* (2005) find that interest coverage and leverage ratios have the most pronounced effect on credit ratings, and that profitability variables and industry concentration measures are also important. They also document a consistent trend towards lower ratings – the standard required to achieve a particular rating is increasing over time.

The previous indicates that ratings provide an indication of the probability of default for rated companies. These ratings are based on financial ratios and judgment of the rating agencies. Through the usage of the credit rating of company, one can directly acquire an estimate of the probability of default.

[7] Koller *et al.* (2005)

2.2.1 Altman's Z-score

The second source of information on the creditworthiness of companies is there financial statement. While the usage of a rating to estimate the probability of default is restricted to (mainly) larger companies, the scoring methods based on the financial statements of a company can be used for rating (practically) any company.

In recent decades, a number of objective, quantitative systems for scoring credits have been developed. One of the classic studies of ratio analysis and bankruptcy is Beaver (1967).

Beaver (1967) defines "failure" as the inability of a company to pay its financial obligations as they mature. He finds that financial ratios analysis can be useful to classify failed and nonfailed companies for at least five years before failure and that the ability to predict failure is strongest in the cash flow to total debt ratio.
Another conclusion is that the ratio distributions of nonfailed companies are quite stable throughout the five years before failure. The ratio distributions of the failed companies exhibit a marked deterioration as failure approaches. The result is a widening gap between the failed and nonfailed companies. The gap produces persistent differences in the mean ratios of failed and nonfailed companies, and the difference increases as failure approaches.
However, the ratio analysis cannot be used indiscriminately, because not all ratios predict equally well and ratios do not correctly predict failed and nonfailed companies with the same degree of success. Nonfailed companies can be correctly classified to a greater extent than the failed companies.

Studies, like Beaver's, preceding the Altman (1968) study imply a definite potential of ratios as predictors of bankruptcy. In general, ratios measuring profitability, liquidity, and solvency prevailed as the most significant indicators. Altman (1968) aimed at determining which ratios are most important in detecting bankruptcy potential, what weights should be attached to those selected ratios, and how these weights should objectively be established. The resulting Z-score model is a multivariate approach built on the values of both ratio-level and categorical univariate measures. Caouette *et al* (1998) comment that, the basic Z-score model has endured to this day and has also been applied to private companies, nonmanufacturing companies and emerging markets.

Multiple Discriminant Analysis (MDA) is a statistical technique used to classify an observation into one of several *a priori* groupings dependent upon the observation's individual characteristics. It is used primarily to classify and/or make predictions in problems where the dependent variable appears in qualitative form, e.g., male or female, bankrupt or non-bankrupt. Altman developed his model on the basis of the MDA technique.

From his original list of twenty-two potentially helpful variables, Altman selected five variables as doing the best overall job together in the prediction of corporate bankruptcy. The original 1968 discriminate function is as follows:

$$Z = 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5$$

where X_1 = Working capital/Total assets
X_2 = Retained earnings/Total assets
X_3 = Earnings before interest and taxes/Total assets
X_4 = Market value equity/Book value of total debt
X_5 = Sales/Total assets
Z = Overall Index

The greater a company's bankruptcy potential, the lower its discriminant score. All companies having a Z-score of greater than 2.99 fall into the "non-bankrupt" sector, while those companies having a Z-score below 1.81 are all bankrupt. The area between 1.81 and 2.99 is defined as the "zone of ignorance" or "gray area" because of the susceptibility to error classification.
Altman (1968) concluded, based on his results, that the bankruptcy prediction model is an accurate forecaster of failure up to two years prior to bankruptcy and that the accuracy

diminishes substantially as the lead time increases. He also concluded (1) that all of the observed ratios show a deteriorating trend as bankruptcy approaches, and (2) that the most serious change in the majority of these ratios occurred between the third and the second years prior to bankruptcy.

In Altman (2000), where he revisits the original Z-score model and discusses the developments over the years, he formulates an adjustment to the Z-score model through which the specification of the model is more convenient:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

Using this formula, one inserts the more commonly written percentage, for example, 0.10 for 10%, for the first four variables (X_1-X_4) and rounds the last coefficient off to equal 1.0 (from 0.99). The last variable continuous to be written in terms of number of times.
He also adapted the model for the application to private companies, by substituting the book values of equity value for the market value in X_4. The result of this revision process with a new X_4 variable was:

$$Z' = 0.717(X_1) + 0.847(X_2) + 3.107(X_3) + 0.420(X_4) + 0.998(X_5)$$

The gray area for this model is wider: the lower boundary is 1.23 instead of 1.81 and the upper boundary is 2.90 instead of 2.99.
The next modification of the Z-score model was used to analyze the characteristics and accuracy of a model without X_5. This was done to minimize the potential industry effect, which is more likely to occur when an industry-sensitive variable such as asset turnover is included. The book value of equity was used for X_4 in this case.
The classification results are identical to the revised five-variable model. The new Z'-score model was:

$$Z'' = 6.56(X_1) + 3.26(X_2) + 6.72(X_3) + 1.05(X_4)$$

This particular model is useful in industries where companies finance their assets in very different ways and where adjustments such as lease capitalization are not made.

In 1977, Altman, Haldeman and Narayanan constructed a second generation model with several enhancements to the original Z-score approach. The purpose of this study was to construct, analyze and test a new bankruptcy classification model which explicitly considers recent developments with respect to company failures. The new study also incorporated refinements in the utilization of discriminant statistical techniques. The new model, called ZETA, was effective in classifying bankrupt companies up to five years prior to failure on a sample of companies consisting of manufacturers and retailers. However, since the ZETA model is a proprietary effort, the parameters of the market were not disclosed in 1977 and are still unavailable today.
The variables used in the ZETA model are: X_1 Return on assets, X_2 Stability of earnings, X_3 Debt service, X_4 Cumulative profitability, X_5 Liquidity, X_6 Capitalization, and X_7 Size, measured by the company's total assets.
The ZETA model for assessing bankruptcy risk of companies demonstrates improved accuracy over existing failure classification model (Z-score), as is shown in the table below, and, perhaps more importantly, is based on data more relevant to current conditions and to a larger number of industrial companies.

Classification accuracy between the ZETA model and various forms of the Z-score model

	ZETA model		Altman's 1968 model		1968 model, ZETA sample	
Years prior to bankruptcy	Bankrupt	Non-Bankrupt	Bankrupt	Non-Bankrupt	Bankrupt	Non-Bankrupt
1	96,2%	89,7%	93,9%	97,0%	86,8%	82,4%
2	84,9%	93,1%	71,9%	93,9%	83,0%	89,3%
3	74,5%	91,4%	48,3%	n.a.	70,6%	91,4%
4	68,1%	89,5%	28,6%	n.a.	61,7%	86,0%
5	69,8%	82,1%	36,0%	n.a.	55,8%	86,2%

When the Z-score and ZETA models were developed, a great deal of attention was given to choosing the ratios to be used. This may account for the continued stability of their predictive power and the models' robustness.

One of the primary uses of credit-scoring models is to assign a bond rating equivalent to each score. This enables the analyst to assess the default probability of a company by observing the historical experience of each bond rating.
The following table[8] shows the average Z-score for bonds of different bond classes based on the model $Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$ in which the ratios X_1 through X_4 are expressed as decimals and not percentages.

Average Z-score by Standard & Poor's Bond Rating S&P 500 (1991-1995)

		1995	1994	1993	1992	1991
Rating	No. of Firms	Avg.	Avg.	Avg.	Avg.	Avg.
AAA	11	5.020	4.376	4.506	5.263	6.357
AA	46	4.296	4.047	4.032	4.226	4.386
A	131	3.613	3.472	3.607	3.923	3.736
BBB	107	2.776	2.701	2.839	2.601	2.550
BB	30	2.449	2.276	2.185	2.102	2.219
B	8	1.673	1.876	1.964	1.962	1.887

Note that the average Z-score in 1995 ranged from about 5.0 for AAA bonds down to 1.67 for B bonds. The average score for B-rated companies actually falls in the Z-score's distress zone.

Altman's Z-score is not undisputed. Blums (2003), for instance, states that "the debate on the appropriate values for the scoring model of Altman until this date has latently stalled at this point with most researchers searching among theoretically appropriate variables", and that "the general conclusion from previous research is that on the one hand each study by itself seems to provide a reasonable degree of differentiation between failed and non-failed companies, while on the other hand the various studies hardly show any agreement on what factors are important for failure prediction. More then 30 years of research have failed to produce agreement on which variables are good predictors and why."

2.2.3 Ohlson's O-score

Opposite to the MDA that Altman applied to arrive at a scoring formula, multiple authors use a conditional logit analysis approach to do so. Ohlson (1980) also introduced an alternative scoring method. However, as for most logit approach models, the creators do not provide a scale to convert the score into a probability of default.
Ohlson's model is briefly discussed because he states some clear comments on using MDA and his model gives an indication of ratios that are used in logit approach models.

First, Ohlson (1980) identified four basic factors as being statistically significant in affecting the probability of failure (within one year). These are: (i) the size of the company; (ii) a

[8] Caouette *et al.* (1998)

measure(s) of the financial structure; (iii) a measure(s) of performance; (iv) a measure(s) of current liquidity. Second, he argues that previous studies appear to have overstated the predictive power of models developed and tested.
Ohlson (1980) argues that the econometric methodology of conditional logit analysis was chosen to avoid some fairly well known problems associated with MDA: (i) There are certain statistical requirements imposed on the distributional properties of the predictors. (ii) The output of the application of an MDA model is a score which has little intuitive interpretation, since it is basically an ordinal ranking (discriminatory) device. (iii) There are also certain problems related to the "matching" procedures which have typically been used in MDA.
He also argues that his results again support the contention that size is an important predictor of bankruptcy.

The variable O-score is defined as:

$$O-score = -1.32 - 0.407\log(total\_assets) + 6.03(\frac{total\_liabil.}{total\_assets}) - 1.43(\frac{working\_cap}{total\_assets})$$

$$+0.076(\frac{current\_liabil.}{current\_assets}) - 1.72(1\_if\_total\_liabilities\_>\_total\_assets,\_0\_if\_otherwise)$$

$$-2.37(\frac{net\_income}{total\_assets}) - 1.83(\frac{funds\_from\_operations}{total\_liabilities})$$

$$+0.285(1\_if\_a\_net\_loss\_for\_the\_last\_two\_years,\_0\_otherwise)$$

$$-0.521(\frac{net\_income_t - net\_income_{t-1}}{|net\_income_t| + |net\_income_{t-1}|})$$

2.2.4 Contingent-claims models

The third category of models uses the stock price of a company as an input for the estimation of the probability of default. The general principle of these contingent-claim models is the following.
Holding a corporate bond is equivalent to lending money with no chance of default but at the same time giving stockholders a put option on the company's assets. When a company defaults, its stockholders are in effect exercising their put. The put's value is the value of the limited liability – the value of stockholder's right to walk away from their company's debt in exchange for handing over the company's assets to its creditors. Thus, valuing bonds should be a two-step process:

Bond value = bond value assuming no chance of default – value of put option

Owning a corporate bond is also equivalent to owning the company's assets but giving a call option on these assets to the company's stockholders:

Bond value = asset value – value of call option on assets

To calculate the probability that a company will default, the expected growth in the market value of the assets, the face value and maturity of the debt and the variability of future asset values need to be known. However, the market value of assets for industrial companies is unobservable. This lack can be filled by employing contingent-claims methodology to infer the market value of assets, asset volatility, and default probability from stock price history and the book value of debt[9].

Crouhy *et al.* (2000) discus the main four contingent-claim models which are the credit migration approach, the structural approach, CreditRisk+, and CreditPortfolioView.
The credit migration approach, as proposed by JP Morgan with CreditMetrics, is based on credit migration analysis, i.e. the probability of moving from one credit quality to another,

[9] Fang & Zong (2004)

including default, within a given time horizon, which is often taken arbitrarily as 1 year. The option pricing, or structural approach, as initiated by KMV and which is based on the asset value model originally proposed by Merton (1974) differs somewhat from CreditMetrics as it relies upon the “Expected Default Frequency”, or EDF, for each issuer, rather than upon the average historical transition frequencies produced by the rating agencies, for each credit class.
At the end of 1997, Credit Suisse Financial Products (CSFP) released a new approach, CreditRisk+, which only focuses on default. CreditRisk+ assumes that default for individual bonds, or loans, follows a Poisson process.
McKinsey’s CreditPortfolioView is a discrete time multi-period model, where default probabilities are a function of macro-variables such as unemployment, the level of interest rates, the growth rate in the economy, government expenses, foreign exchange rates, which also drive, to a large extent, credit cycles.

CreditMetrics/CreditVaR I are methodologies based on the estimation of the forward distribution of the changes in value of a portfolio of loan and bond type products at a given time horizon, usually 1 year.
CreditMetrics/CreditVaR I determines the probability of default through the use of a rating system, with rating categories, together with the probabilities of migrating from one credit quality to another over the credit risk horizon. The transition matrix of credit ratings is as follows.

Probabilities of credit rating migrating from one rating quality to another, within one year

Initial rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90,81	8,33	0,68	0,06	0,12	0,00	0,00	0,00
AA	0,70	90,65	7,79	0,64	0,06	0,14	0,02	0,00
A	0,09	2,27	91,05	5,52	0,74	0,26	0,01	0,06
BBB	0,02	0,33	5,95	86,93	5,30	1,17	1,12	0,18
BB	0,03	0,14	0,67	7,73	80,53	8,84	1,00	1,06
B	0,00	0,11	0,24	0,43	6,48	83,46	4,07	5,20
CCC	0,22	0,00	0,22	1,30	2,38	11,24	64,86	19,79

Source: Standard & Poor's CreditWeek (April 15, 1996)

To acquire a cumulative default rate, one should multiply the migration probabilities.

The weakness of CreditMetrics/CreditVaR I is not the methodology, but the reliance on transition probabilities based on average historical frequencies of defaults and credit migration. The accuracy of CreditMetrics/CreditVaR I calculations relies upon two critical assumptions: first, all companies within the same rating class have the same default rate, and second, the actual default rate is equal to the historical average default rate. The same assumptions also apply to the other transition probabilities. In other words, credit rating changes and credit quality changes are identical, and credit rating and default rates are synonymous, i.e. the rating changes when the default rate is adjusted, and vice versa.

This view has been strongly challenged by KMV. KMV has shown through a simulation exercise that the historical average default rate and transition probabilities can deviate significantly from the actual rates. In addition, KMV has demonstrated that substantial differences in default rates may exist within the same bond rating class, and the overlap in default probability ranges may be quite large with, for instance, some BBB and AA-rated bonds having the same probability of default. KMV also showed that the average historical default probability overstates the default rate for a typical obligor.
KMV derives the actual probability of default, the Expected Default Frequency (EDF), for each obligor based on a Merton (1974)’s type model of the company. The probability of default is thus a function of the company’s capital structure, the volatility of the asset returns and the current asset value. The EDF is company-specific, and can be mapped into any rating system to derive the equivalent rating of the obligor.

KMV best applies to publicly traded companies for which the value of equity is market determined. The information contained in the company's stock price and balance sheet can then be translated into an implied risk of default.
The derivation of the probabilities of default proceeds in three stages: estimation of the market value and volatility of the company's assets; calculation of the distance-to-default, which is an index measure of default risk; and scaling of the distance-to-default to actual probabilities of default using a default database. The market value of equity is used to estimate the market value and volatility of the company's assets since the assets value can not directly be observed.
KMV has constructed a transaction matrix based upon default rates rather than rating classes, which is shown below.

KMV 1-year transition matrix based on non-overlapping EDF ranges

Initial rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	66,26	22,22	7,37	2,45	0,86	0,67	0,14	0,02
AA	21,66	43,04	25,83	6,56	1,99	0,68	0,20	0,04
A	2,76	20,34	44,19	22,94	7,42	1,97	0,28	0,10
BBB	0,30	2,80	22,63	42,54	23,52	6,95	1,00	0,26
BB	0,08	0,24	3,69	22,93	44,41	24,53	3,41	0,71
B	0,01	0,05	0,39	3,48	20,47	53,00	20,58	2,01
CCC	0,00	0,01	0,09	0,26	1,79	17,77	69,94	10,13

Source: KMV Corporation

The difference between the various probabilities between the two tables is striking. According to KMV, except for AAA, the probability of staying in the same rating class is between half and one-third of historical rates produced by the rating agencies. KMV's probabilities of default are also lower, especially for the low grade quality. Migration probabilities are also much higher for KMV, especially for the grade above and below the current rating class.

CreditRisk+ applies an actuarial science framework to the derivation of the loss distribution of a bond/loan portfolio. Only default risk is modeled, not downgrade risk. Contrary to KMV, default risk is not related to the capital structure of the company. In CreditRisk+ no assumption is made about the causes of default: an obligor A is either in default with probability P_A, or it is not in default with probability $1 – P_A$. It is assumed that:

- For a loan, the probability of default in any given period, say 1 month, is the same for any other month.
- For a large number of obligors, the probability of default by any particular obligor is small, and the numbers of defaults that occur in any given period is independent of the number of defaults that occur in any other period.

CreditPortfolioView is a multi-factor model which is used to simulate the joint conditional distribution of default and migration probabilities for various rating groups in different industries, for each country, conditional on the value of macroeconomic factors like the unemployment rate, the rate of growth in GDP, the level of long-term interest rates, foreign exchange rates, government expenditures and the aggregate savings rate.

2.2.5 Subjects of analysis regarding the probability of default

There are multiple approaches for estimating the probability of default of a company. Each approach requires a particular type of input, thereby restricting its usefulness to the situations in which the needed input is available.
It is to be expected, from a practical point of view, that the potential for default has an impact on the value of a company. Each of the two valuation models is therefore also expected to incorporate a certain measure or factor of the risk of default to represent that impact.
The first subject of analysis, resulting from this, is whether the valuation models use a measure of the probability of default. The second subject of analysis, in case of a positive answer on the first subject, is what model/approach is used to determine the probability of default. This subject is relevant because the approach implied by the model could impact the

valuation outcome or even the fact whether the probability of default can be determined at all based on the available information.
So, the two subjects are:

1. The usage of the probability of default in the valuation model.
2. The model/approach that is used (in case of actual usage of the probability of default) to determine the probability of default. Which model to use could dependent on the type of company that is being valued.

2.3 Capital structure

The purpose of this paragraph is to give an overview of the theories on capital structure decisions of a company. The effects of a number of factors on leverage are compared to the inferences that the different theories on capital structure decisions make to see which theory describes reality in the most accurate way.
Most of the literature on capital structure is focused on the discussion whether the tax/bankruptcy tradeoff theory or the pecking order theory is the theory that best describes reality. Frank & Goyal (2003a) provide an overview of the factors that determine the amount of leverage that a company adopts. They compare five different theories on the basis of these factors. Because of their extensive and clear review, their article is used as one of the main sources of this paragraph.

2.3.1 Definition of capital structure

According to Duffhues (1997), capital structure is defined as the composition of the total amount of capital that is visible at the right-hand side of a balance sheet.
The concept of capital in the definition of capital structure has the characteristic of total capital. The meaning of total capital is not unambiguously defined in theory or in practice. It has multiple definitions. Three of them are: a balance sheet definition, an active capital definition, and a (middle of the road) stand-alone capital definition. To add insult to injury, these definitions are also expressed in two different valuation principles: book value and market value.
In the first definition, total capital equals the sum of the liabilities according to the regular disposition used in published annual accounts; that is including the induced short-term debt. Induced short-term debt is the total amount of capital that organically grows with the size of the business activities, like accounts payable and deferred wages.
The second definition of capital structure plays an increasingly larger part in annual reporting. In this case, total capital is equal to active capital. All short-term debt is ignored.
The third definition stays in between the first two definitions. Total capital is in this case equal to the sum of all liabilities minus the induced short-term debt.

Duffhues (1997) also states that multiple criteria are available to classify the different components of the total capital. First, the capital structure can be classified in two components that are related to the absence or presence of a debt relation between the company and the providers of capital. If the relation is absent, then the capital component is referred to as equity. Capital components that have a debt relation are named debt. A second criterion is based on the time to maturity of the individual financing transaction. Permanent capital has been disposed to the company for an indefinite period of time. Long-term capital comprises capital with a finite time-to-maturity larger than one year. Short-term capital comprises capital with a time-to-maturity of at most one year. Permanent capital coincides with equity, temporary capital coincides with debt.

If with every choice of the capital structure (stand-alone definition) the market value of the total capital – of which maximization is pursued – would be the same, then the choice of capital structure would be irrelevant. This would be the case in the following two extreme situations:

- There is no need for risk bearing capital.
- There is perfect substitution between the market value of the components of total capital.

When the future cash flows of a company would be certain, then the difference between debt and equity would disappear. All capital providers would receive a return in this situation that is

equal to the risk-free return. The capital structure could than, without any objections, be composed of solely debt.
When future cash flows of a company are uncertain, a need exist for equity to cover potential future losses. However, this does not mean that thinking of the exact choice of the capital structure becomes relevant. This situation creates a fake problem when as a result of exchanging equity for debt (or the other way around) the market value of the one type of capital changes in the same amount as the market value of the other type of capital in the opposite direction thus leaving the sum at an equal value.

2.3.2 Financial deficit

Companies invest in long-term assets (mainly property, plant, and equipment) and net working capital. From the table below, it can be seen that by far the greater part of the money is generated internally. In other words, it comes from cash that the company has set aside as depreciation and from retained earnings.

	1994	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**
Capital Expenditure	83,2%	77,6%	87,6%	81,0%	89,1%	80,4%	86,6%
Investment in net working capital and other uses	16,8%	22,4%	12,4%	19,0%	10,9%	19,6%	13,4%
Total Investment	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%
Total Investment, billions	754	789	755	880	872	1116	1162
Internally generated cash	87,7%	78,6%	89,5%	82,7%	85,7%	72,1%	76,7%
Financial deficit	12,3%	21,4%	10,5%	17,3%	14,3%	27,9%	23,3%
Financial deficit covered by:							
Net stock issues	-6,9%	-7,4%	-9,2%	-13,0%	-30,6%	-12,9%	-14,3%
Net increase in debt	19,3%	28,8%	19,7%	30,3%	45,0%	40,8%	37,6%

Source: Board of Governors of the Federal Reserve System, Division of Research and Statistics, at www.federalreserve.gov/releases/z1/current/data.htm

In most years there is a gap between the cash that companies need and the cash that they generate internally. This gap is the *financial deficit*. Frank & Goyal (2003b) give the following formula for calculating the financial deficit:

$$DEF_t = DIV_t + I_t + \Delta W_t - C_t = \Delta D_t + \Delta E_t$$

With DIV_t as cash dividends in year t, I_t the net investment in year t (i.e., I_t = capital expenditures + increase investments + acquisitions + other use of funds – sale of PPE – sale of investment), ΔW_t the change in working capital in year t, C_t the cash flow after interest and taxes, ΔD_t the net debt issued in year t, and ΔE_t the net equity issued in year t.

To make up the deficit, companies must either sell new equity or borrow. So companies face two basic financing decisions: How much profit should be plowed back into the business rather than paid out as dividends? And, what proportion of the deficit should be financed by borrowing rather than by an issue of equity?
Companies in the United States are not alone in their heavy reliance on internal funds. Internal funds make up more than two-thirds of corporate financing in Germany, Japan, and the United Kingdom.

2.3.3 Effects of capital structure

Although academic researchers have investigated the issue for decades, there is still no clear model for a company's optimal leverage ratio, the leverage that would create the most value for shareholders. But there is evidence that leverage delivers key benefits as well as giving rise to certain costs[10].

1. Tax savings: the most obvious benefit of debt versus equity is the reduction of taxes. However, this tax reduction does not make full debt funding optimal. For example,

[10] Koller *et al.* (2005)

more debt funding may reduce corporate taxes but could actually lead to higher taxes for investors.

2. Reduction of corporate overinvestment: the so-called *free cash flow hypothesis* argues that debt can help impose investment discipline on managers. Debt curbs overinvestment by forcing the company to pay out free cash flow according to scheduled interest and principal obligations.
3. Costs of business erosion and bankruptcy: most notably, bankruptcy costs are the legal and administrative costs of liquidating or restructuring the company for the debt holders after it has defaulted on its debt. Academic research indicates that these costs are relatively small, around three percent of a company's market value. However the costs of business erosion are probably much higher. Highly leveraged companies are more likely to forgo investment opportunities or reduce budgets for research and development and other costs for which the payoffs are further in the future. As a result, these companies may lose significant value creation opportunities. Furthermore, as the risk of financial distress increases, companies become more likely to lose customers, employees, and suppliers. The risk of losing business is high, particularly when the products require long-term service and maintenance.
4. Costs of investor conflicts: higher leverage may cause additional loss of value as a result of conflicts of interest among debt holders, shareholders, and managers.

2.3.4 Capital structure development theories

Frank & Goyal (2003a) compare the empirical support for certain factors that influence capital structure decisions of a company to predictions of five different theories of capital structure choices. These are the following.

1. The pecking order theory: Due to adverse selection, companies prefer to finance their activities using retained earnings if possible. If retained earnings are inadequate, then they turn to the use of debt. Equity financing is only used as a last resort. The pecking order theory is therefore a theory of leverage in which there is no notion of an optimal leverage ratio. Observed leverage is simply the sum of past events.
2. The market timing theory: The basic idea behind the market timing theory is that managers look at current conditions in both debt markets and equity markets. If they need financing, then they will use whichever market looks more favorable at the time.
3. The tax/bankruptcy tradeoff theory: Companies determine an interior leverage optimum by a balancing of the corporate tax savings advantage of debt against the deadweight costs of bankruptcy.
4. The agency theory: Company managers may be tempted to overspend their free cash flow, so high debt is useful to control this overspending impulse. Of course, this increase in leverage does increase the chance of paying deadweight bankruptcy costs. There are agency conflicts between debt holders and equity holders.
5. Stakeholder co-investment theory: In order to insure the willingness of stakeholders, such as employees and business partners to make valuable co-investments, some companies prefer to use little debt when compared to other companies.

2.3.5 Factors that influence capital structure

Frank & Goyal (2003a) find empirical support for fourteen factors that influence capital structure decisions of a company. Factors that have the most statistically robust and economically large effects are classified as Tier 1. Tier 2 factors are less robust, but are still generally supported by the evidence of their study.

In Tier 1, leverage is positively related to median industry leverage, company size as measured by log of sales, intangible assets, and collateral. Leverage is negatively related to firm risk as measured by Altman's Z-Score, a dummy for dividend paying companies, and the market-to-book ratio.

In Tier 2, leverage is positively related to company growth as measured by the change in total assets, the top corporate tax rate, and the Treasury bill rate. Leverage is negatively related to the volatility of a company's own stock returns, its net operating loss carry forwards (NOLCF), corporate profits, and to being financially constrained as measured by Korajczyk & Levy's (2003) financial constraint dummy variable.

Korajczyk & Levy (2003) theoretically define financially constrained companies as the set of companies that do not have sufficient cash to undertake investment opportunities and that face severe agency costs when accessing financial markets.

2.3.6 Support for the different theories by the fourteen factors

Frank & Goyal (2003a) compare the found influence of the different factors on leverage to the predictions of the different theories. The following is an overview of this comparison for each factor.

In every case, companies in a high leverage industry have higher leverage. This is quite natural within a tradeoff model since companies in the same industry must face many common forces. Under a pure pecking order perspective, the industry should only matter to the degree that it serves as a proxy for the company's financing deficit – a rather indirect link. Under the market timing theory, this result is not predicted.

Leverage is positively related to company size as measured by log of sales and positively related to intangible assets. An intangible is defined to be "assets that have no physical existence in themselves, but represent the right to enjoy some privilege". It is easy to imagine that intangible assets, using the above definition, could be used as collateral to support debt. Under this interpretation, the sign is what is as predicted by tradeoff theory. It is difficult to see how this fits under market timing theory. Under the pecking order one might expect that increased intangibles would be associated with increased leverage since such assets are hard to value and thus insiders might know more than outsiders regarding their true value.

Leverage is positively related to collateral. From a tradeoff perspective, a company with more assets can pledge them in support of debt. Under the pecking order theory, a company with more assets has a greater worry about the adverse selection on those assets. Accordingly, one might predict that leverage is positively related to assets. Under the pecking order theory, one might predict a negative relation to debt. This ambiguity stems from the fact that collateral can be viewed as a proxy for different economic forces.

Leverage is negatively related to company risk as measured by modified Altman's Z-Score. Within the tradeoff theory, this makes sense. When there is a greater risk of bankruptcy costs, the company will take offsetting action by reducing leverage. Similarly, in the stakeholder co-investment version of tradeoff theory, even without direct bankruptcy costs, downsizing or other disruptions in normal business impose costs. Companies take actions to avoid these costs by reducing leverage. From the pecking order perspective, it is unclear why risk should matter.

Perhaps dividend-paying companies can avoid paying transaction costs to underwriters involved in accessing the public financial markets. If so, then under the tradeoff theory, dividend payers should have less leverage. This is what was found by Frank & Goyal (2003). Since the financing of dividend is by debt, the implication of the pecking order theory is that dividend-paying companies should have greater leverage. This is not what Frank & Goyal (2003a) found.

The market to book ratio is negatively related to leverage. It is usually interpreted as reflecting a need to retain growth options. This interpretation is consistent with the tradeoff theory. Under the pecking order theory, more profitable companies use less debt. More profitable companies should also have a higher market value. Thus one might expect that a high market to book company would have low leverage. This is consistent with the findings of Frank & Goyal (2003a).

Leverage is positively related to company growth as measured by the change in total assets. Under the tradeoff theory this reflects the fact that assets can be pledged as collateral. Under the pecking order theory, this reflects the fact that debt is used to cover the financing deficit. Leverage is positively related to the top corporate tax rate. This is directly predicted by the tax-based versions of the tradeoff theory. This is not predicted by the market timing theory, pecking order theory, or non-tax based versions of the tradeoff theory.

Leverage is positively related to the interest rate. This is not predicted by the pecking order theory, because higher interest rates lower the debt capacity, and thus lower leverage. The tradeoff theory predicts that the negative effect of a higher interest rate is stronger for equity than for debt, thus increasing leverage. This is consistent with the Frank & Goyal (2003) results.

Leverage is negatively related to the volatility of a company's own stock returns – a simple measure of risk. In the tradeoff theory companies react to risk by reducing leverage. Under the pecking order theory, risk matters to the degree that it is asymmetric.
Leverage is negatively related to net operating loss carry forwards. This is a direct implication of the tradeoff theory.

It is well known that leverage is negatively related to corporate profits. This is inconsistent with static versions of the tradeoff theory, in which the company constantly adapts its financing mix to stay at their target leverage ratio. It is consistent with some dynamic versions of the tradeoff theory, in which the company only changes its financing mix if the leverage ratio passes the endpoints of the optimal leverage ratio interval. It is also consistent with the pecking order theory.

Consistent with the previous literature, Frank & Goyal (2003) find that leverage increases with the average leverage in an industry, with company size, and with the presence of collateral. Also consistent with the literature, riskier companies and high market-to-book companies have lower leverage.
They state that over time the sign on profit is moving in the direction of the predictions of the tradeoff theory.

Most of the evidence is easy to understand within the tradeoff class of theories. Frank & Goyal (2003a) consider three versions of the tradeoff theory: taxes versus bankruptcy costs, agency costs, and stakeholder co-investment. Since tax effects appear to be real, versions of the tradeoff theory that allow for tax effects are preferred.

It is well understood that company circumstances may be important for leverage decisions. For instance, the level of sales is particularly important for non-dividend paying companies, young companies and small companies. Large companies seem more concerned about tax factors than do small companies. However, the major factors have reliable effects across company circumstances.

2.3.7 Debt capacity

Lemmon & Zender (2004) present evidence that companies follow a pecking order in incremental financing choice and offer substantial support for a dynamic version of the pecking order theory articulated in Myers (1984) by explicitly recognizing the role of debt capacity in the theory.

Companies unconstrained by concerns over debt capacity primarily use debt to fill their financing deficit while constrained companies exhibit a heavy reliance on external equity financing. Lemmon & Zender (2004) show that companies appear to "stockpile" debt capacity. When possible, internally generated funds are used to finance new investment and to reduce debt levels. Directly contrary to the tradeoff theory we see that companies with low leverage expecting high profits and having low external financing requirements exhibit this behavior.

Debt capacity was defined by Myers (1977) as the point at which an increase in the use of debt (fixed commitments) actually reduces the total market value of the company's debt. More recently, Shyam-Sunder & Myers (1999) and Chirinko & Singha (2000) define it in terms of "sufficiently high debt ratios" implying that costs of financial distress curtail further debt issues. Adding debt capacity to the pecking order theory suggests that costs of adverse selection are dominant for moderate capital structures but that tradeoff-theory-like forces become primary motivators of financing decisions at extremely high leverage making it more difficult to distinguish the theories.
Lemmon & Zender's primary indicator of whether debt capacity concerns constrain a company's choices is whether the company has rated debt outstanding in a given year.

The study of Lemmon & Zender (2004) shows a striking contrast: for those companies that should face tighter debt capacity constraints, net issues of equity are highly correlated with the financial deficit (as opposed to a high correlation between the financial deficit and net debt issues as predicted by the static pecking order theory).
Further, compared to companies without bond ratings, companies with rated debt outstanding are on average more highly levered. All of these findings are consistent with the idea that companies with rated debt have higher levels of borrowing capacity.
The financing behavior of companies without rated debt is far from that predicted by the static pecking order. For the companies with rated debt outstanding the results are quite different and support the predictions of the static pecking order.

The dynamic pecking order theory predicts that financing behavior will be dependent upon both a company's distance to its current debt capacity and the speed at which the company expects to approach its debt capacity given its current and future financing needs.

Once consideration of debt capacity is taken into account in the pecking order it becomes more difficult to distinguish it from a dynamic version of the tradeoff theory with issuance costs.
The two theories provide contrasting hypotheses, however, for highly profitable companies that are far below their debt capacities. The pecking order theory, both static and dynamic, predicts that profitable companies with low leverage have no incentive to increase their leverage.
Conversely, the dynamic tradeoff theory predicts that in such situations new debt financing would be preferred to an increased use of (internal) equity.
The results provide evidence that internally generated funds are the preferred source of financing, regardless of existing leverage and expected profitability.
Moreover, even companies with low initial leverage use excess profits to reduce their leverage ratios over time. This finding is directly contrary to the tradeoff theory.

These results suggest that those companies who issue the majority of external equity can be classified as constrained by concerns over debt capacity, and provide a reconciliation of the results in Fama & French (2002) and Frank & Goyal (2003a) with the pecking order.
The results are consistent with three interpretations. The first is that although the small, high-growth companies in the low predicted rating group may face more asymmetric information concerning the value of their assets in place, they also face relatively more valuable investment opportunity sets. The second is that the market realizes that, due to the constraint imposed by debt capacity, the company has little or no flexibility in its choice of financing instruments and so the announcement of an equity issue is less of a bad signal than it would be for a similar company that could also choose to issue low risk debt. Last, if small, high-growth companies are better at "timing" their equity issues we would expect to see this pattern.
Nonetheless, Lemmon & Zender's results provide a rationale within the pecking order framework for the frequent equity issues by small, high-growth companies, which others have posed as a challenge to the theory.

The results from the research of Frank & Goyal (2003a) identify a number of factors that influence the capital structure decisions of companies. After comparing the effects of these factors to the different theories of capital structure they conclude that the tradeoff theories, especially the taxes/bankruptcy tradeoff theory, are supported most.
Lemmon & Zender (2004) respond to these results by integrating the concept of 'debt capacity' in their analysis of capital structure decision. This leads to the following overview.

Factors	Pecking Order	Market Timing	Tax / Bankruptcy	Agency / Bankruptcy	Stakeholder Co-Investment	*Found relation*	
Industry Leverage			+	+	+	+	
Z-score			-	-	-	-	
Firm size			+	+	+	+	
Dividend-paying	+		-	-	-	-	Tier 1
Intangibles	+		+	+	+	+	
Market-to-book ratio			-	-	-	-	
Collateral			+	+	+	+	
Stock variance	+		-	-	-	-	
Net Operating Loss Carryforwards			-			-	
Financially constrained	- (dc)		+			-	Tier 2
Profitability	-		+	+	+	-	
Change in total assets			+	+		+	
Tax Rate			+			+	
Treasury Bill Rate	-	-	+	+	+	+	

The table show the factors determined by Frank & Goyal (2003a) and their effect on leverage as predicted by the different theories, including the correction for the effect of debt capacity.

2.3.8 Views of other authors

Other authors have also studied the effect of different factors on the capital structure development within a company. When comparing these studies, one can conclude that there is not a specific complete theory to describe capital structure decisions, but a number of theories that apply under certain conditions.

Hovakimian *et al.* (2001) conclude that "although the pecking order considerations affect corporate debt ratios in the short run, companies tend to make financing choices that move them toward target debt ratios that are consistent with tradeoff models of capital structure choice".

Myers (2001) comments that the tradeoff theory cannot account for the correlation between high profitability and low debt ratios. On the other hand, he also comments that potential conflicts of interest between lenders and stockholders significantly increase the potential cost of financial distress through suboptimal investment and operating decisions. This contributes to the tradeoff theory of which financial distress cost are an important factor.

Fama & French (2002) conclude that the tradeoff and pecking order model share many predictions about dividends and leverage. They also comment that the tradeoff model is scarred by the negative relation between profitability and leverage, that the pecking order is wounded by the large use of equity by small low-leverage growth companies and that the issue of the mean reversion of leverage is undecided since mean reversion seems to happen but at a very slow pace.

Korajczyk & Levy (2003) conclude that their results are consistent with elements of both tradeoff and pecking order theories. They also conclude that constrained companies fit the pecking order theory less well than unconstrained companies.

2.3.9 Subjects of analysis regarding capital structure

There are multiple theories on the prediction of capital structure choices. The most well-known are the (tax/bankruptcy) tradeoff theory and the pecking order theory. Although many authors have given arguments over the past years to support one of these or both theories, neither of the two has come to be the 'ultimate' theory of capital structure.

In the analysis of the enterprise DCF model and the APV model, the following subjects with regard to capital structure need to be considered:

1. What are the assumptions on capital structure of the valuation model?
2. What are the conditions of capital structure required for the model to be used?
3. If the model can be used, how can the capital structure decisions be modeled?

The first subject is simply a means to identify the influence of capital structure on the valuation model. The second subject needs to be analyzed to determine whether there are certain capital structure situations in which one of the two valuation models cannot be used. The last subject relates to the theories described above: if the model can be used, how are capital structure decisions modeled?

2.4 Costs of financial distress

The purpose of this paragraph is to give an overview of the theories on the costs of financial distress. Most of the literature on the costs of financial distress focuses on estimating these costs. This paragraph discusses these estimates. It also describes some comments on the costs of financial distress. The paragraph closes with the (final) subjects of analysis, those regarding the costs of financial distress.

2.4.1 Definition of financial distress

Financial distress occurs when promises to creditors are broken or honored with difficulty. Sometimes financial distress leads to bankruptcy. Sometimes it only means skating on thin ice. Financial distress generally leads to negotiations with at least one of the company's creditors. Financial distress is resolved in an environment of imperfect information and conflicts of interest. Yet evidence on the frequency distribution of outcomes for companies in distress proves that it is not synonymous with corporate death.

Corporate bankruptcies occur when stockholders exercise their right to default. That right is valuable; when a company gets into trouble, limited liability allows stockholders simply to walk away from it, leaving all its troubles to its creditors. The former creditors become the new stockholders, and the old stockholders are left with nothing.
The decline in the value of assets is what the mourning is really about. That has no necessary connection with financing. The bankruptcy is merely a legal mechanism for allowing creditors to take over when the decline in the value of assets triggers a default. Bankruptcy is not the cause of the decline in value. It is the result.

Two types of bankruptcy filings are available to corporations: Chapter *7* and Chapter 11 (these titles refer to chapters of the US Bankruptcy Code). Chapter 7 provides for the orderly liquidation of a company's assets by a court-appointed trustee, and payment to claimants in order of priority is always maintained. Chapter 11 provides for reorganization of a company. Participants in a Chapter 11 filing must approve a plan of reorganization, leaving room for negotiations among the various parties and for violation of priority of claims.

Not every company that gets into trouble goes bankrupt. When a company is in trouble, both bondholders and stockholders want it to recover, but in other respects their interests may be in conflict.
According to Brealey & Myers (2003), financial distress is costly when these conflicts of interest get in the way of proper operating, investment, and financing decisions. Stockholders are tempted to forsake the usual objective of maximizing the overall market value of the company and to pursue narrower self-interest instead. They are tempted to play games at the expense of their creditors.
The first game that can be played is *risk shifting*. This game illustrates that, stockholders of levered firms gain when business risk increases. Financial managers who act strictly in their shareholders' interest will favor risky projects over safe ones. They may even take risky projects with negative net present values (NPVs).
This distorted strategy for capital budgeting clearly is costly to the company and to the economy as a whole. The temptation to play this game is strongest when the odds of default are high.

The second game is refusing to contribute equity capital. The game shows that, if one holds business risk constant, any increase in company value is shared among bondholders and stockholders. The value of any investment opportunity to the company's stockholder is reduced because project benefits must be shared with bondholders. Thus it may not be in the stockholders' self-interest to contribute fresh equity capital even if that means forgoing positive-NPV investment opportunities.
As with other games, the temptation to play the next three games is particularly strong in financial distress:

1. Cash in and run
2. Playing for time
3. Bait and switch

The results of these games are poor decisions about investments and operations. These poor decisions are agency costs of borrowing.

Some assets, like good commercial real estate, can pass through bankruptcy and reorganization largely unscathed; the values of other assets are likely to be considered diminished. The losses are greatest for the intangible assets that are linked to the health of the company as a going concern – for example, technology, human capital, and brand image.

2.4.2 Cost effects of financial distress

Altman's (1984) study assumes that the expected bankruptcy cost issue is relevant and that firms do recognize the probability of bankruptcy as an important ingredient when making operating and financial decisions.
Altman argues that, in addition to the costs of liquidation, there are other relevant costs including those which arise due to the process of bankruptcy (either liquidation or reorganization) and those due to the perceived significant potential of bankruptcy (lost opportunities and abnormal loss of sales and profits).
Indirect bankruptcy costs are not limited to companies which actually do fail. Companies which have high probabilities of bankruptcy, whether they eventually fail or not, still can incur these costs.

Wruck (1990) describes the costs of financial distress as:

1. Direct costs. The out-of-pocket or direct costs of financial distress are the easiest to measure. They include legal, administrative, and advisory fees paid by the company. Comparing the direct costs of private workouts with direct costs of bankruptcy suggests that direct costs are almost ten times less when the company is able to restructure debt privately.
2. Indirect costs. Indirect costs are opportunity costs imposed on the company because financial distress affects its ability to conduct business as usual. A distressed company is hampered on three fronts. First, it loses the right to make certain decisions without legal approval. Second, financial distress can reduce demand for the company's product and increase its production costs. Third, management spends considerable time resolving financial distress.

Andrade & Kaplan, in their 1998 study on highly leveraged transactions (HLT) that became financially distressed, also consider qualitative measures of financial distress costs in their analysis. The companies in their sample appear to incur three such costs most frequently. First, a number of companies are forced to curtail capital expenditures, sometimes substantially. Second, a number of companies appear to sell assets at depressed prices. Third, a number of companies delay restructuring or filing for Chapter 11 in a way that appears to be costly. In contrast, they find no evidence that the distressed companies engage in risk shifting/asset substitution of any kind. In addition to costs of financial distress, they also find benefits: many companies cut costs and replace management.
To the extent they occur, the costs of financial distress that they identify are heavily concentrated in the period after the firms become distressed, but before they enter Chapter 11. They find little evidence that Chapter 11 is inefficient or costly.

Finally, Andrade & Kaplan (1998) estimate the cross-sectional determinants of the costs of financial distress. They find that these costs decline with HLT value and the fraction of total debt owed to banks, but are not related to capital structure complexity, the presence of junk bonds, the presence of buyout sponsors, time in distress, or industry performance. These results are not consistent with increased complexity increasing the costs of financial distress. They also suggest that costs of financial distress have a fixed component.

2.4.3 Estimation of the costs of financial distress

According to Andrade & Kaplan (1998), financial economists have found it difficult to measure the costs of financial distress. They state that the difficulty is driven by an inability to distinguish whether poor performance by a company in financial distress is caused by the financial distress itself or is caused by the same factors that pushed the company into financial distress in the first place. Those companies are not only financially distressed, but also economically distressed, making it difficult to identify whether the papers of these financial economist measure costs of financial distress, economic distress, or an interaction of them

Altman (1984) concludes that, based on his sample of eighteen companies, the average total costs of financial distress are 12.4% of the value of the companies three years before bankruptcy to 16.7% of the value of the company at bankruptcy. The table below shows the relation between the direct and indirect costs of financial distress found by Altman.

Average bankruptcy costs relative to company value for the combined corporate sample (N = 18 firms)

	Years prior to bankruptcy			
	3	2	1	0
Direct bankruptcy costs / Value	4.3%	4.6%	4.6%	6.2%
Indirect bankruptcy costs / Value	8.1%	7.1%	6.6%	10.5%
Total bankruptcy costs / Value	12.4%	11.7%	11.2%	16.7%

Altman uses the term bankruptcy costs, but states that “Indirect bankruptcy costs are not limited to firms which actually do fail. Firms which have high probabilities of bankruptcy, whether they eventually fail or not, still can incur these costs.” This means that his bankruptcy costs are a synonym for the costs of financial distress used by other authors and as it is used in this research project.

Weiss (1990) uses three measures to assess the magnitude of the direct costs of bankruptcy: (1) market value of equity, (2) book value of debt plus the market value of equity, and (3) book value of total assets, all measured at the fiscal year-end prior to the bankruptcy filing. On average, the direct costs of bankruptcy are 20.6% of the market value of equity (ranging from 2.0% to 63.6%), 3.1% of the book value of debt plus the market value of equity (ranging from 1.0% to 6.6%), and 2.8% of the book value of total assets (ranging from 0.9% to 7.0%).

Wruck (1990) concludes that direct costs of financial distress average 3.5% of market value. Estimates of indirect costs are less reliable, but evidence to date indicates they lie in the range of 9% to 15% of market value.

Andrade & Kaplan (1998) provide several estimates of the magnitude of the net costs of financial distress. For their entire sample, they estimate that these costs are 10 to 20% of company value one year prior to the financial distress. Their most conservative estimates do not exceed 25% of company value one year prior to the financial distress.
However, they state: “Overall, then, we cannot conclude that our results would hold for companies in high R&D or, possibly, high growth businesses. (In fact, we believe the results are unlikely to hold for such companies.) However, among companies in more mature businesses, it seems likely that the results for our sample Highly Leveraged Transactions would hold.”

Chen & Merville (1999) conclude that it has been accepted that the direct costs of financial failure are on the order of 5% of company value at the time of occurrence of the financial distress (Warner (1977)) and indirect costs around 8-10% of company value at the time of occurrence of the financial distress (Altman (1984)).

Branch (2002) concludes that on the average, the effect of the financial distress on the predistress company value is allocated as follows:
1. The loss, which caused the bankruptcy, consumes about 28%;
2. The cost of dealing with distress consumes about 16%;
3. The net value available to distribute to the claimholders amounts to about 56%.

2.4.4 Comments on the costs of financial distress

Pindado & Rodrigues (2005) find that financial distress costs are positively related to the probability of financial distress, and negatively to leverage and the holding of liquid assets. They also find that the underinvestment problem is more relevant than the overinvestment one. Their results support the relevance of institutional differences across countries for the analysis of financial distress costs.

Chen & Merville (1999) rely on the Z-score model of Altman (1968) as a proxy for the probability of financial distress for two reasons: because its parameters are publicly available and because the interpretations of the scores are widely known.

Wruck (1990) also states that previous studies of financial distress focus on the costs and ignore the possibility that distress can result in beneficial outcomes. These benefits of financial distress are, according to Wruck (1990), as follows:
1. Changes in management and governance. Wruck (1990)'s results are consistent with the idea that leverage acts as a catalyst for organizational change. Poor stock-price performance is not enough to remove incumbent managers, but financial distress provides a mechanism to initiate top-management changes.
2. Changes in organizational strategy and structure. Financial structure interacts with investment decisions; financial distress forces a change in the company's economic activities and the way these activities are organized. Financial distress can, therefore, force managers to undertake value-increasing organizational changes they would not have otherwise undertaken. When company value deteriorates as a result of poor management or when company value is highest in liquidation and management refuses to liquidate, financial distress creates value.
3. Benefits of chapter 11. In some special situations, the ability to enter Chapter 11 is a valuable alternative for security holders. For example, trade creditors and claimants in product-liability suits, are numerous and have heterogeneous claims. Reaching a private agreement with all of them is very difficult. Under Chapter 11, diffuse creditors can be dealt with as a single class, making negotiation manageable and settling protracted disputes once and for all.

Wruck (1990) also comments that: "Although the benefits of distress have not yet been quantified, turnover in top management and changes in governance indicate that corporate insiders are disciplined for poor performance. Evidence from clinical studies and case studies documents changes in strategy and organizational structure following financial distress that are consistent with a process of corporate revitalization."

2.4.5 Subjects of analysis regarding the costs of financial distress

Financial distress is caused by the company's inability to meet the promises that were made to creditors. The costs of financial distress can be split up in direct and indirect costs. The probability of default can be used as a proxy for the probability of financial distress and thus as the probability that the expected cost of financial distress will occur. The direct costs of distress are estimated at 3-5% of the book value of debt plus the market value of equity at the year prior to the occurrence of the financial distress and the indirect cost are estimated at 7-15% cumulating to total costs of financial distress of 10-20% of the book value of debt plus the market value of equity at the year prior to the occurrence of the financial distress.

Based on the elaboration on the costs of financial distress above, the following subjects of analysis can be analyzed:
1. Whether the valuation method incorporates potential costs of financial distress.
2. The way in which the financial distress costs are integrated in the valuation (if the method does in fact use financial distress costs).

Conclusion

The objective of this research project is to identify the differences between the enterprise DCF model and the APV model. In order to compare the two models in a distinctive manner, they both need to be compared on the same set of subjects.
This chapter discussed four research areas that are related to the aspects of the two valuation models. A number of subjects came forth from each research area, which can be used to analyze the two valuation models.

General subjects of analysis

There are multiple approaches for determining the enterprise value of a company. The two models that are studied in this research project both belong to the income approach and have the form of a discounted cash flow model. Each discounted cash flow model consists of four subjects of which the specification determines the functioning of the particular discounted cash flow model. These four subjects are:

1. The way in which the cash flows are modeled.
2. The discount rate that is used.
3. The cost of equity and the cost of debt used.
4. The incorporation of taxes into the valuation.

Subjects of analysis regarding the probability of default

There are multiple approaches for estimating the probability of default of a company. Each approach requires a particular type of input, thereby restricting its usefulness to the situations in which the needed input is available.
It is to be expected, from a practical point of view, that the potential for default has an impact on the value of a company. Each of the two valuation models is therefore also expected to incorporate a certain measure or factor of the risk of default to represent that impact.
The first subject of analysis, resulting from this, is whether the valuation models use a measure of probability of default. The second subject of analysis, in case of a positive answer on the first subject, is what model/approach is used to determine the probability of default. This subject is relevant because the approach implied by the model could impact the valuation outcome or even the fact whether the probability of default can be determined at all based on the available information.
So, the two subjects are:

1. The usage of the probability of default in the valuation model.
2. The model/approach that is used (in case of actual usage of the probability of default) to determine the probability of default. Which model to use could dependent on the type of company that is being valued.

Subjects of analysis regarding the capital structure

There are multiple theories on the prediction of capital structure choices. The most known are the (tax/bankruptcy) tradeoff theory and the pecking order theory. Although many authors have given arguments over the past years to support one of these or both theories, neither of the two has come to be the ‘ultimate’ theory of capital structure.

In the analysis of the enterprise DCF model and the APV model, the following subjects with regard to capital structure need to be considered:

1. What are the assumptions on capital structure of the valuation model?
2. What are the conditions of capital structure required for the model to be used?
3. If the model can be used, how can the capital structure decisions be modeled?

The first subject is simply a means to identify the influence of capital structure on the valuation model. The second subject needs to be analyzed to determine whether there are certain capital structure situations in which one of the two valuation models cannot be used. The last subject relates to the theories described above: if the model can be used, how are capital structure decisions modeled?

Subjects of analysis regarding the costs of financial distress

Financial distress is caused by the company's inability to meet the promises that were made to creditors. The costs of financial distress can be split up in direct and indirect costs. The probability of default can be used as a proxy for the probability of financial distress and thus as the probability that the expected cost of financial distress will occur. The direct costs of distress are estimated at 3-5% of the book value of debt plus the market value of equity at the year prior to the occurrence of the financial distress and the indirect cost are estimated at 7-15% cumulating to total costs of financial distress of 10-20% of the book value of debt plus the market value of equity at the year prior to the occurrence of the financial distress.

Based on the elaboration on the costs of financial distress above, the following subjects of analysis can be analyzed:

1. Whether the valuation method incorporates potential costs of financial distress.
2. The way in which the financial distress costs are integrated in the valuation (if the method does in fact use financial distress costs).

Application of the subjects of analysis

This chapter provides the theoretical framework for the analysis of the two valuation models. The two valuation models are the so-called *research objects* on which the collection of subjects of analysis, the so-called *research perspective*, is applied. The research perspective is therefore like a pair of glasses used to look at the two valuation models.

The next chapter discusses the approach which is taken to analyze the research objects. It also enrolls a justification for the approach taken based on the research context.

In chapter 3, the two valuation models are actually assessed at the subjects of analysis and than compared to each other.

Chapter 3: The two models compared

Introduction

This chapter is the main chapter of this research report. In this chapter each of the two valuation models is analyzed. In the first two paragraphs the valuation models are assessed at the general subjects of analysis. In the third and fourth respectively the enterprise DCF model and the APV model are assessed at the other three subjects of analysis categories. This provides an overview on the assumptions and the functioning of each model. The two valuation models are compared in the fifth paragraph with regard to their theoretical differences and these differences are translated to (expected) differences in the valuation outcome in the sixth paragraph.

The first paragraph discusses the basic assumptions of the enterprise DCF model rather extensive. The rationale behind this is that a number of methods for determining the value of the input variables of the enterprise DCF model are also valid for the APV model. Relevant methods are for instance the determination of the free cash flows and the cost of debt.

3.1 Basic assumptions of the enterprise DCF model

The purpose of this paragraph is to assess the enterprise DCF model on the basis of the general subjects of analysis. These are, as discussed in chapter 2:

1. The way in which the cash flows are modeled.
2. The discount rate that is used.
3. The cost of equity and the cost of debt used.
4. The incorporation of taxes into the valuation.

The first three subjects are explicitly discussed. The fourth subject, the tax rate, returns in multiple sections (for instance in the discussion of the discount rate) and is therefore only discussed implicitly.

Koller *et al.* (2005) discuss the enterprise DCF model extensively. This book is therefore one of the main sources of this paragraph. Damodaran (2002) also discusses the enterprise DCF model in great length. His views are combined with those of Koller *et al*. (2005) to arrive at a comprehensive overview of the enterprise DCF model. Comments from some other authors are used to refine the descriptions from Damodaran (2002) and Koller *et al.* (2005).

3.1.1 Definition of the enterprise DCF model

The enterprise DCF model discounts future cash flows at the weighted average cost of capital.

The enterprise DCF model is the favorite valuation models of many practitioners and academics, according to Koller *et al.* (2005), because it relies solely on the flow of cash in and out of the company, rather than on accounting-based earnings (which can be misleading).

In the 1950s, Franco Modigliani and Merton Miller postulated that the value of a company's economic assets must equal the value of the claims against those assets.[11] Thus, in order to value the equity of a company, two options arise. The first is to value the company's operations and subtract the value of all nonequity financial claims; the second is to value the equity cash flows directly.
Koller *et al.* (2005, p. 103) state that: "Although both methods lead to identical results when applied correctly, the equity method is difficult to implement in practice. Consequently, to value a company's equity, we recommend valuing the enterprise first and then subtracting the value of any nonequity financial claims."

[11] Miller & Modigliani (1958)

3.1.2 Steps in the enterprise DCF model

To value a company's common stock using the enterprise DCF model, the following steps are to be taken:

1. Value the company's operations by discounting free cash flow from operations at the weighted average cost of capital.
2. Value nonoperating assets, such as excess marketable securities, nonconsolidated subsidiaries, and other equity investments. Combining the value of operating assets and nonoperating assets leads to enterprise value.
3. Identify and value all nonequity financial claims against the company's assets. Nonequity financial claims include (among others) fixed- and floating-rate debt, pension shortfalls, employee options, and preferred stock.
4. Subtract the value of nonequity financial claims from enterprise value to determine the value of common stock. To determine share price, divide equity value by the number of shares outstanding.

The value of operations equals the discounted value of future free cash flow. Free cash flow equals the cash flow generated by the company's operations, less any reinvestment back into the business. Free cash flow is the cash flow available to *all* investors, and is *independent* of leverage. Consistent with this definition, free cash flow must be discounted using the weighted average cost of capital. The WACC is the company's opportunity cost of capital and represents a blended required return by the company's debt and equity holders

3.1.3 Modeling of the cash flows

Damodaran (2002, p. 382) uses the term free cash flow to the firm instead of free cash flow. He states that: "The free cash flow to the firm (FCFF) is the sum of the cash flows to all claimholders in the company, including stockholders, bondholders, and preferred stockholders. There are two ways of measuring the free cash flow to the firm.
One is to add up the cash flows to the claimholders, which would include cash flows to equity (defined either as free cash flow to equity or dividends), cash flow to lenders (which would include principal payments, interest expenses, and new debt issues), and cash flows to preferred stockholders (usually preferred dividends):

FCFF = Free cash flow to equity + Interest expense (1 – Tax rate) + principal repayments – new debt issues + preferred dividends

A simpler way of getting to free cash flow to the firm is to estimate the cash flows prior to any of these claims. Thus one could begin with the earnings before interest and taxes, net out taxes and reinvestment needs, and arrive at an estimate of the free cash flow to the firm:

FCFF = EBIT(1 – Tax rate) + Depreciation – Capital expenditure – Δworking capital

where EBIT = Earnings before interest and taxes
Δ = Difference in

Since this cash flow is prior to debt repayments, it is often referred to as an unlevered cash flow."

The Damodaran (2002) FCFF is almost equal to the Koller *et al.* (2005) free cash flow. The difference is that Damodaran (2002) uses EBIT(1-T) and Koller *et al.* (2005) uses NOPLAT. NOPLAT is slightly modified version of EBIT(1-T) and it removes any non-operating items that might affect the reported EBIT. The second definition of Damodaran (2002) is used through the remainder of this research report because the FCFF uses terms that are directly retrievable from the financial statements, and is thus easier to use in practice. This free cash flow to the firm will be called *free cash flow* or *FCFF*.

3.1.4 Structure of the enterprise DCF model

The value of the company is obtained by discounting the FCFF at the weighted average cost of capital. Embedded in this value are the tax benefits of debt (in the use of the after-tax cost of debt in the cost of capital) and expected additional risk associated with debt (in the form of higher costs of equity and debt at higher debt ratios). The version of the model used will depend on assumptions made about future growth.

The value of the company, in the most general case, can be written as the present value of expected free cash flows:

$$Value\_of\_company = \sum_{t=1}^{t=\infty} \frac{FCFF_t}{(1+WACC)^t}$$

where $FCFF_t$ = Free cash flow to the firm in year t
WACC = Weighted average cost of capital
If the company reaches steady state after n years and starts growing at a stable growth rate g_n after that, the value of the company can be written as:

$$Value\_of\_company = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+WACC_{hg})^t} + \frac{[FCFF_{n+1}/(WACC_{st} - g_n)]}{(1+WACC_{hg})^n}$$

where WACC = Cost of capital (hg: high growth; st: stable growth)

3.1.5 Terminal value

As a company grows, it becomes more difficult for it to maintain high growth and it eventually will grow at a rate less than or equal to the growth rate of the economy in which it operates. This growth rate, labeled stable growth, can be sustained in perpetuity, allowing for the estimation of the value of all cash flows beyond that point as a terminal value for a going concern.
Since cash flows cannot be estimated forever, closure in discounted cash flow valuation is generally imposed by stopping the estimation of cash flow sometime in the future an then computing a terminal value that reflects the value of the company at that point:

$$Value\_of\_a\_company = \sum_{t=1}^{t=n} \frac{CF_t}{(1+k_c)^t} + \frac{Termi\_nal\_value_n}{(1+k_c)^n}$$

The terminal value can be found in three ways. One is to assume a liquidation of the company's assets in the terminal year and estimate what others would pay for the assets that the company has accumulated at that point. The other two approaches value the company as a going concern at the time of the terminal value estimation. One applies a multiple to earnings, revenues, or book value to estimate the value in the terminal year. The other assumes that the cash flows of the company will grow at a constant rate forever – a stable growth rate.

Damodaran (2002) states that there are two ways in which the liquidation value can be estimated. One is to base it on the book value of the assets, adjusted for any inflation during the period.

Expected liquidation value = Book value of $\text{assets}_{\text{term year}}$ * $(1 + \text{Inflation rate})^{\text{average life of assets}}$

The limitation to this approach is that it is based on accounting book value and does not reflect the earning power of the assets.
The alternative approach is to estimate the value based on the earning power of the assets. To make this estimate, the expected cash flows first have to be estimated from the assets and then these cash flows have to be discounted back to the present, using an appropriate discount rate.

In the multiple approach, the value of a company in a future year is estimated by applying a multiple to the company's earnings or revenues in that year.

While this approach has the virtue of simplicity, the multiple has a huge effect on the final value and where it is obtained can be critical.
Damodaran (2002) therefore states that: "Using multiples to estimate terminal value, when those multiples are estimated from comparable firms, results in a dangerous mix of relative and discounted cash flow valuation. The only consistent way of estimating terminal value in a discounted cash flow model is to use either liquidation value or to use a stable growth model."

In the stable growth model, which assumes that cash flows, beyond the terminal year, will grow at a constant rate forever, the terminal value can be estimated as follows:

$\text{Terminal value}_t = \text{Cash flow}_{t+1} / (r - \text{stable growth rate})$

If valuing a company, the terminal value can be written as:

$\text{Terminal value}_n = \text{Free cash flow to firm}_{n+1} / (\text{Cost of capital}_{n+1} - g_n)$

where the cost of capital and the growth rate in the model are sustainable forever.
Of all the inputs into a discounted cash flow valuation model, none can affect the value more than the stable growth rate.
The fact that a stable growth rate is constant forever, however, puts strong constraints on how high it can be. Since no company can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth cannot be greater than the overall growth rate of the economy. In making a judgment on what the limits on stable growth rate are, the following three questions have to be considered:

1. Is the company constrained to operate as a domestic company, or does it operate (or have the capacity to operate) internationally? If a company is a purely domestic company, the growth rate in the domestic economy will be the limiting value. If the company is a multinational or has aspirations to be one, the growth rate in the global economy will be the limiting value.
2. Is the valuation being done in nominal or real terms?
3. What currency is being used to estimate cash flows and discount rates in the valuation?

While the stable growth rate cannot exceed the growth rate of the economy in which a company operates, it can be lower. Setting the stable growth rate to be less than or equal to the growth rate of the economy is not only the consistent thing to do, according to Damodaran (2002), but it also ensures that the growth rate will be less than the discount rate.

In every discounted cash flow valuation, there are three critical assumptions that need to be made on stable growth. The first relates to when the company under valuation will become a stable growth company, if it is not already. The second relates to what the characteristics of the company will be in stable growth, in terms of return on investments and cost of equity and capital. The final assumption relates to how the company under valuation will make the transition from high growth to stable growth.

According to Damodaran (2002), three factors should be looked at when considering how long a company will be able to maintain high growth.

1. Size of the companies. Smaller companies are much more likely to earn excess returns and maintain these excess returns than otherwise similar larger companies.
2. Existing growth rate and excess returns. Companies that have been reporting rapidly growing revenues are more likely to see revenues grow rapidly at least in the near future.
3. Magnitude and sustainability of competitive advantage. This is perhaps the most critical determinant of the length of the high growth period. If there are significant barriers to entry and sustainable competitive advantages, companies can maintain high growth for longer periods.

As companies move from high growth to stable growth, they need to be given the characteristics of stable growth companies. A company in stable growth is different from that same company in high growth on a number of dimensions. In general, stable growth

companies are expected to have average risk, use more debt, have lower (or no) excess returns, and reinvest less than high growth companies.
When looking at the cost of equity, high growth companies tend to be more exposed to market risk (and have higher betas) than stable growth companies. As these companies mature, they are expected to have less exposure to market risk and betas that are closer to 1 – the average for the market. Damodaran (2002) recommend that, as a rule of thumb, stable period betas do not exceed 1.2.
Since entire industries often earn excess returns over long periods, assuming a company's returns on equity and capital will move toward industry averages will yield more reasonable estimates of value.
High growth companies tend to use less debt than stable growth companies. As companies mature, their debt capacity increases. When valuing companies, this will change the debt ratio that is used to compute the cost of capital.
Stable growth companies tend to reinvest less than high-growth companies, and it is critical that the effects of lower growth on reinvestment are captured and that it is ensured that the company reinvests enough to sustain its stable growth rate in the terminal phase. Looking at free cash flow, the expected growth in operating income is estimated as a function of the return on capital and the reinvestment rate:

Expected growth rate = Reinvestment rate * Return on capital

Algebraic manipulation yields the following measure of the reinvestment rate in stable growth:

Reinvestment rate in stable growth = Stable growth rate / ROC_n

where the ROC_n is the return on capital that the company can sustain in stable growth. This reinvestment rate can then be used to generate the free cash flow to the firm in the first year of stable growth. Linking the reinvestment rate retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about the stable growth rate. *If the return on capital is equal to the stable growth rate, increasing the stable growth rate will have no effect on value*. In this case, according to Damodaran (2002):

$$\text{Terminal value}_{ROC=WACC} = EBIT_{n+1}(1 - t) / \text{Cost of capital}_n$$

There are three distinct scenarios regarding the transition to stable growth. In the first scenario, the company will maintain its high growth rate for a period of time and then become a stable growth company abruptly. In the second, the company will maintain its high growth rate for a period and then have a transition period where its characteristics change gradually toward stable growth levels. In the third, the company characteristics change each year from the initial period to the stable growth period.

3.1.6 The weighted average cost of capital

To value a company using the enterprise DCF model, the free cash flows are discounted by the weighted average cost of capital (WACC). The weighted average cost of capital represents the opportunity cost that investors face for investing their funds in one particular business instead of others with similar risk.
The most important principle underlying successful implementation of the cost of capital is consistency between the components of WACC and the free cash flows. To assure consistency, according to Koller *et al.* (2005), the cost of capital must meet several criteria:
- It must include the opportunity costs from all sources of capital – debt, equity, and so on – since free cash flow is available to all investors, who expect compensation for the risks they take.
- It must weigh each security's required return by its target market-based weight, not by its historical book value.
- It must be computed after corporate taxes (since free cash flow is calculated in after-tax terms). Any financing-related tax shields not included in the free cash flows must be incorporated into the cost of capital or valued separately (as done in the adjusted present value).
- It must be denominated in the same currency as the free cash flows.
- It must be denominated in nominal terms when cash flows are stated in nominal terms.

To determine the weighted average cost of capital, its three components must be calculated: the cost of equity, the after-tax cost of debt, and the company's target capital structure. Since none of the variables are directly observable, various models, assumptions, and approximations are employed to estimate each component.

In its simplest form, the weighted average cost of capital is the market-based weighted average of the after-tax cost of debt and cost of equity:

$$WACC = \frac{D}{D+E} k_d (1 - T_m) + \frac{E}{D+E} k_e$$

where D/V = Target level of debt to enterprise value using market-based (not book) values
E/V = Target level of equity to enterprise value using market-based values
k_d = Cost of debt
k_e = Cost of equity
T_m = Company's marginal income tax rate

For companies with other securities, such as preferred stock, additional terms must be added to the cost of capital, representing each security's expected rate of return and percentage of total enterprise value.

To estimate the cost of equity, the expected rate of return of the company's stock must be determined. Since expected rates of return are unobservable, one can rely on asset-pricing models that translate risk into expected return. The most common asset-pricing model is the capital asset pricing model (CAPM).
Because the CAPM is discussed at length in modern finance textbooks, its background will not be discussed any further here. Instead, the focus is on best practices for implementation. The CAPM postulates that the expected rate of return on any security equals the risk-free rate plus the security's beta times the market risk premium:

$$E(R_i) = r_f + \beta_i [E(R_m) - r_f]$$

where $E(R_i)$ = Security *i*'s expected return
r_f = Risk-free rate
β_i = Stock's sensitivity to the market
$E(R_m)$ = Expected return of the market

In the CAPM, the risk-free rate and market risk premium (defined as the difference between $E(R_m)$ and r_f) are common to all companies; only beta varies across companies. Beta represents a stock's incremental risk to a diversified investor, where risk is defined by how much the stock covaries with the aggregate stock market.
Koller *et al.* (2005)'s general conclusions with regard to the implementation of CAPM are as follows.

- To estimate the risk-free rate in developed economies, use highly liquid, long-term government securities, such as the 10-year zero-coupon strip.
- Based on historical average and forward-looking estimates, the appropriate market risk premium is currently between 4,5 and 5,5%.
- To estimate a company's beta, use an industry-derived unlevered beta levered to the company's target capital structure.

To estimate the risk-free rate, look to the government default-free bonds. When valuing a company or long-term project, a short-term Treasury bill should not be used to determine the risk-free rate. The short-term bond rate misestimates the opportunity cost of investment for longer-term projects.

Methods to estimate the market risk premium fall in three general categories:

1. Estimating the future risk premium by measuring and extrapolating historical excess returns.
2. Using regression analysis to link current market variables, such as the aggregate dividend to price ratio, to project the expected market risk premium.

3. Using DCF valuation, along with estimates of return on investment and growth, to reverse engineer the market's cost of capital.

Although many in the finance profession disagree about how to measure the market risk premium, Koller *et al.* (2005) believe 4,5 to 5,5% is an appropriate range. (Brealey & Myers (2003) arrive at the conclusion that over a period of 75 years the market risk premium has averaged about 9% per year and Kaplan & Ruback (1995) find that the median implied market equity risk premium equals 7,78% and they state that this value is comparable to the historic arithmetic average market equity risk premium.)

In order to estimate beta, first a raw beta has to be measured through the use of a regression and then the estimate should be improved by using industry comparables and smoothing techniques. The most common regression used to estimate a company's raw beta is the market model:

$$R_i = \alpha + \beta R_m + \varepsilon$$

In the market model, the stock's return is regressed against the market's return.
To improve the precision of beta estimation, one should use industry, rather than company-specific betas. Companies in the same industry face similar *operating* risks, so they could have similar operating betas. The effect of leverage must first be stripped out to compare companies with similar operating risks. Only then can the beta be compared across an industry.
So,

$$\beta_u = \beta_e / \left(1 + \frac{D}{E}\right)$$

where β_e = Levered beta of the company
β_u = Unlevered beta of the company
D/E = Current debt/equity ratio

Since unlevered beta focus solely on operating risk, they can be averaged across an industry (assuming industry competitors have similar operating characteristics).
Koller *et al.* (2005) describe the following four-step process to estimate an industry-adjusted company beta. First, regress each company's stock returns against the market index to determine raw beta. Next, to unlever each beta, calculate each company's market debt to equity ratio. Applying the last given equation leads to unlevered company betas. In step three, determine the industry unlevered beta by calculating the median. In the final step, relever the industry unlevered beta to each company's *target* debt-to-equity ratio (using current market values as proxies) according to the following formula:

$$\beta_e = \beta_u \left(1 + \frac{D}{E}\right)$$

For estimating the cost of debt for a company with investment-grade debt (debt rated at BBB or better), yield to maturity is a suitable proxy. Using the company's bond ratings to determine the yield to maturity is a good alternative to calculating the yield to maturity directly. However, Koller *et al.* (2005) comment to “never approximate the yield to maturity using a bond's coupon rate.”

For debt below investment grade, using yield to maturity as a proxy for the cost of debt can cause significant error. Three factors drive the yield to maturity: the cost of debt, the probability of default, and the recovery rate. When the probability of default is high and the recovery rate is low, the yield to maturity will deviate significantly from the cost of debt. Thus, for companies with high default risk and low ratings, the yield to maturity is a poor proxy for the cost of debt. To estimate the cost of high-yield debt, the following method from Damodaran (2002) can be used. First, estimate a company's dollar debt and interest expenses. Second, compute a financial ratio or ratios that measures default risk and use the

ratio(s) to estimate a rating for the company. Third, a default spread, based on the estimated rating, is added to the risk-free rate to arrive at the pretax of debt.
Damodaran (2002) assume that bond ratings are determined solely by interest coverage ratio, which is defined as:

Interest coverage ratio = Earnings before interest and taxes / Interest expense

The interest coverage ratio is chosen for three reasons. First, it is a ratio used by both Standard & Poor's and Moody's to determine ratings. Second, there is significant correlation not only between the interest coverage ratio and bond ratings, but also between interest coverage ratio and other ratios used in analysis, such as the debt coverage ratio and the capital flow ratios. Third, the interest coverage ratio changes as a company changes its financing mix and decreases as the debt ratio increases. The rating agencies would argue, however, that subjective factors, such as the perceived quality of management, are part of the ratings process. One way to build these factors into the analysis would be to modify the rating obtained from the financial ratio analysis across the board to reflect the ratings agencies' subjective concerns.
The data in the following tables were obtained from an analysis of the interest coverage ratios of three types of companies in different ratings classes. The tables also show the interest spread/rating relationship. The risk-free interest rate has to be added to the spread to acquire the cost of debt.

For large manufacturing firms

Interest Coverage Ratio	Rating	Spread
>8.5	AAA	0.35%
6.50-8.50	AA	0.50%
5.50-6.50	A+	0.70%
4.25-5.50	A	0.85%
3.00-4.25	A-	1.00%
2.50-3.00	BBB	1.50%
2.25-2.50	BB+	2.00%
2.00-2.25	BB	2.50%
1.75-2.00	B+	3.25%
1.50-1.75	B	4.00%
1.25-1.50	B-	6.00%
0.80-1.25	CCC	8.00%
0.65-0.80	CC	10.00%
0.20-0.65	C	12.00%
<0.20	D	20.00%

www.bondsonline.com (February 2004)

For financial service firms

Interest Coverage Ratio	Rating	Spread
>3	AAA	0.70%
2.50-3	AA	0.90%
2.00-2.50	A+	1.25%
1.50-2.00	A	1.40%
1.20-1.50	A-	1.50%
0.90-1.20	BBB	2.00%
0.75-0.90	BB+	4.25%
0.60-0.75	BB	4.75%
0.50-0.60	B+	5.75%
0.40-0.50	B	6.00%
0.30-0.40	B-	6.25%
0.20-0.30	CCC	10.50%
0.10-0.20	CC	12.50%
0.05-0.10	C	14.00%
<0.05	D	16.00%

www.bondsonline.com (February 2004)

For smaller and riskier firms

Interest Coverage Ratio	Rating	Spread
>12.5	AAA	0.35%
9.50-12.50	AA	0.50%
7.50-9.50	A+	0.70%
6.00-7.50	A	0.85%
4.50-6.00	A-	1.00%
4.00-4.50	BBB	1.50%
3.50-4.00	BB+	2.00%
3.00-3.50	BB	2.50%
2.50-3.00	B+	3.25%
2.00-2.50	B	4.00%
1.50-2.00	B-	6.00%
1.25-1.50	CCC	8.00%
0.80-1.25	CC	10.00%
0.50-0.80	C	12.00%
<0.50	D	20.00%

www.bondsonline.com (February 2004)

To calculate free cash flow, taxes are computed as if the company were entirely financed by equity. By using all-equity taxes, comparisons can be made across companies and over time, without regard to capital structure. Yet, since the tax shield has value, it must be accounted for. In the enterprise DCF model using the WACC, *the tax shield is valued as part of the cost of capital*. To value the tax shield, the cost of debt is reduced by the marginal tax rate:

After-tax cost of debt = Cost of debt * (1-T_m)

According to research by John Graham (1996), the statutory marginal tax rate overstates the *future* marginal tax rate because of rules related to tax-loss carryforwards, tax-loss carrybacks, investment tax credits, and alternative minimum taxes.
Graham finds that for investment-grade companies, one should use the statutory rate. For a typical company that does not always fully use its tax shields, Graham (1996) estimates that the marginal tax rate is on average 5 percentage points below the statutory rate.

Target weights should be used to determine the cost of capital. Using market values to weight expected returns in the cost of capital follows directly from the formula's derivation. But a more intuitive explanation can also be considered: the WACC represents the expected return on an *alternative* investment with identical risk. Rather than reinvest in the company, management could return capital to investors, who could reinvest elsewhere. To return capital without changing the capital structure, management can repay debt and repurchase shares, but must do so at their *market* value. Conversely, book value represents a sunk cost, so it is no longer relevant.

The cost of capital should rely on target weights, rather than current weights, because at any point, a company's current capital structure may not reflect the level expected to prevail over the life of the business. The current capital structure may merely reflect a short-term swing in the company's stock price, a swing that has yet to be rebalanced by management. Thus, using today's capital structure may cause overestimating (or underestimating) of the value of tax shields for companies whose leverage is expected to drop (or rise).

According to Koller *et al.* (2005), many companies are already near their target capital structure. If the company under valuation is not, then there has to be decided how quickly the company will achieve the target. In the simplest scenario, the company will rebalance immediately and maintain the new capital structure. In this case, using the target weights and a constant WACC (for all future years) will lead to a reasonable valuation. If the rebalancing is expected to happen over a significant period of time, then a different cost of capital should be used each year, reflecting the capital structure at the time. In practice, this procedure is complex; not only have the weights to be modeled correctly, but also the changes in the cost of debt and equity (because of increased default risk and higher betas). For extreme changes in capital structure, modeling enterprise DCF using a constant WACC can lead to significant error.

Koller *et al.* (2005) recommend the use a combination of three approaches to develop a target capital structure for a company:

1. Estimate the company's current market-value-based capital structure.
2. Review the capital structure of comparable companies.
3. Review management's implicit or explicit approach to financing the business and its implications for the target capital structure.

The company's current capital structure can be determined by measuring the market value of all claims against enterprise value. For the most companies, the claims will consist primarily of debt and equity. If a company's debt and equity are publicly traded, simply multiply the quantity of each security by its most recent price. Most difficulties arise when securities are not traded such that prices can be readily observed.
If an observable market value is not readily available, the debt securities can be valued at book value or through the use of discounted cash flow. In most cases, book value reasonably approximates the current market value. This will not be the case, however, if interest rates have changed dramatically since the time of issuance or the company is in financial distress. In these situations, each bond should be valued separately by discounting promised cash flows at the appropriate yield to maturity. Determine the appropriate yield to maturity by examining the yields from comparably rated debt with similar maturities.
If common stock is publicly traded, multiply the market price by the number of shares *outstanding*.

For privately held companies, no market-based values are available. In this case, the equity value (for the cost of capital) has to be determined by either using a multiples approach or through DCF iteratively. To perform an iterative valuation, assume a reasonable capital structure, and value the enterprise using DCF. Using the estimate of debt to enterprise value ratio, repeat the valuation. Continue this process until the valuation no longer materially changes.

To place the company's current capital structure in the proper context, compare its capital structure with those similar companies.

As a final step, management's historical financing philosophy should be reviewed.

3.1.7 Nonoperating assets and nonequity claims

Nonoperating assets can be segmented into two groups, marketable securities and illiquid investments.

International Accounting Standards require companies to report liquid debt and equity investments (e.g., excess cash and marketable securities) at a fair market value on the company's balance sheet. Therefore, when valuing *liquid* nonoperating assets, use their most recent reported balance sheet value, rather than to discount future nonoperating flows.

When valuing a company from the inside, illiquid investments should be valued by using enterprise DCF. If the company is valued from the outside, according to Koller *et al.* (2005), valuation of these assets is rough at best. Companies disclose very little information about illiquid investments, such as discontinued operations, excess real estate, nonconsolidated subsidiaries, and other equity investments.
For nonconsolidated subsidiaries, information disclosure depends on the level of ownership. When a company has some influence but not a controlling interest in another company, it records its portion of the subsidiary's profits on its own income statement and the original investment plus its portion of reinvested profits on its balance sheet. This information can be used to create a simple cash flow statement. To discount the cash flow, a cost of capital, commensurate with the risk of the investment should be used, not the parent company's cost of capital.
When ownership is less than 20%, investments are reported at historical cost, and the company's portion of profits is recorded *only* when paid out to the parent. In most situations, nothing more than the investment's original costs are visible. In this case, use a multiple of the book value or a tracking portfolio to value the investment.

The value of nonoperating assets has to be added to the value of operations to determine enterprise value. To estimate equity value, subtract any nonequity claims, such as debt, unfunded retirement liabilities, capitalized operating leases, and outstanding employee options. The most common nonequity claims are:

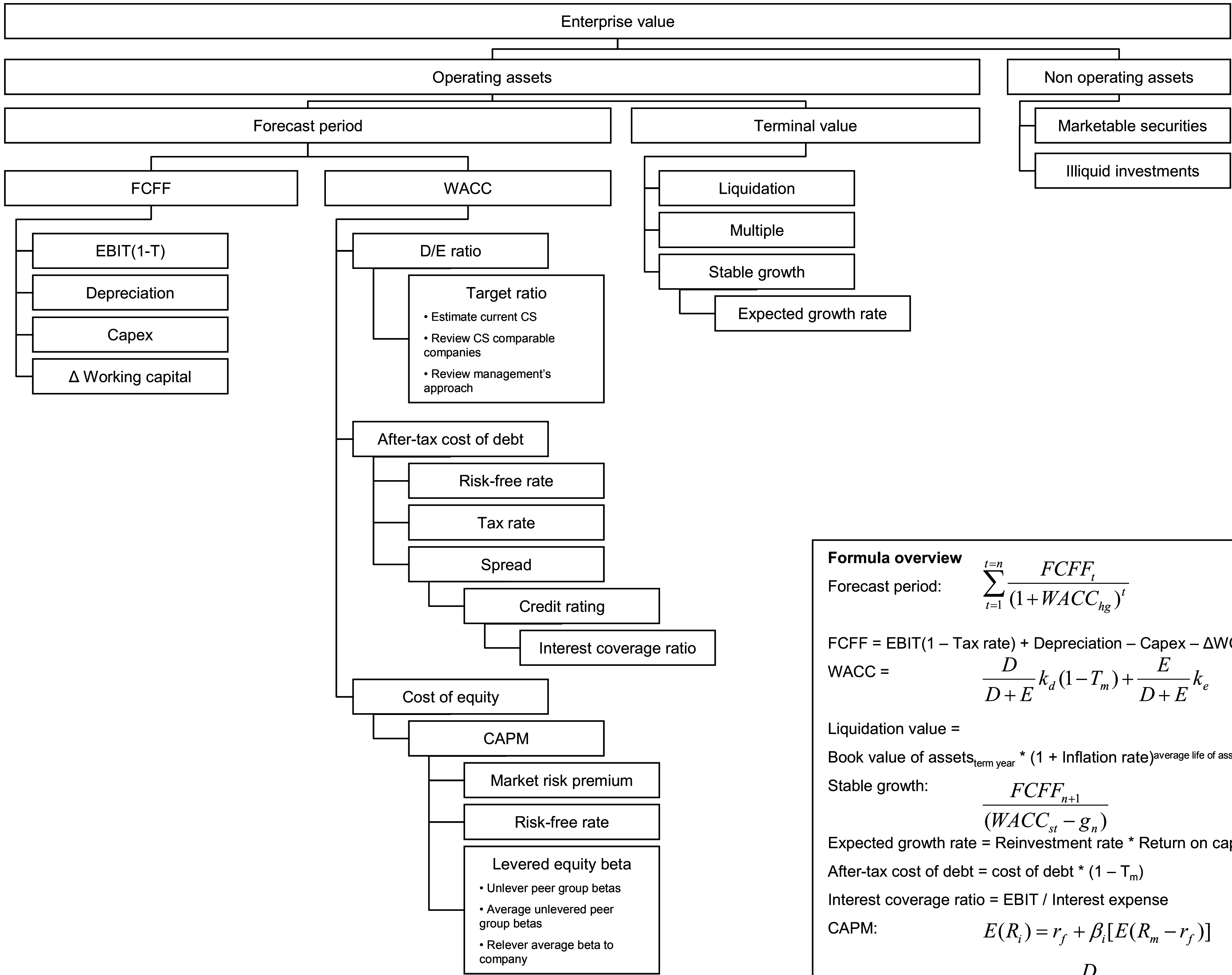
1. Debt
2. Unfunded retirement liabilities
3. Operating leases
4. Contingent liabilities
5. Preferred stock
6. Employee options
7. Minority interest

Once all the nonequity claims have been identified and valued, they can be subtracted from enterprise value to determine equity value.

3.1.8 Conclusion

After analyzing the enterprise DCF model on the general subjects of analysis, it can be concluded that the enterprise value equals the sum of the value of the operational and nonoperational assets of the company. The operational assets are valued by discounting the free cash flows at the weighted average cost of capital. The effect of the tax-deductibility of interest is integrated in the weighted average cost of capital. The nonoperational assets are valued in a different way depending on the type of asset. To arrive at the value of equity, the nonequity claims have to be subtracted from the enterprise value.

Figure 3.1 gives a schematic overview of the building blocks of the enterprise DCF model.



Formula overview

Forecast period: $\sum_{t=1}^{t=n} \frac{FCFF_t}{(1+WACC_{hg})^t}$

FCFF = EBIT(1 – Tax rate) + Depreciation – Capex – ΔWC

WACC = $\frac{D}{D+E} k_d (1-T_m) + \frac{E}{D+E} k_e$

Liquidation value =

Book value of assets$_{\text{term year}}$ * (1 + Inflation rate)$^{\text{average life of assets}}$

Stable growth: $\frac{FCFF_{n+1}}{(WACC_{st} - g_n)}$

Expected growth rate = Reinvestment rate * Return on capital

After-tax cost of debt = cost of debt * (1 – T_m)

Interest coverage ratio = EBIT / Interest expense

CAPM: $E(R_i) = r_f + \beta_i [E(R_m - r_f)]$

Levered equity beta: $\beta_e = \beta_u (1 + \frac{D}{E})$

Figure 3.1: Schematic overview of enterprise DCF model building blocks

3.2 Basic assumptions of the APV model

The purpose of this paragraph is to assess the APV model at the general subjects of analysis. However, the paragraph is structured in such a way that it does not explicitly assess these subjects. The paragraph discusses the three components of the APV model: the unlevered company value, the value of the interest tax shields and the costs of financial distress.

Koller *et al.* (2005), Brealey & Myers (2003) and Damodaran (2002) each give a description of the APV model. Their descriptions are used to illustrate the first and the third component. The valuation of the interest tax shields is a much-debated topic and is therefore discussed more extensively through the use of multiple scientific articles.

3.2.1 Definition of the APV model

Several authors have discussed the APV model. In general, they agree on the fundamentals of the APV model, which are as follows.

The adjusted present value (APV) model separates the value of operations into two components: the value of operations as if the company were all-equity financed and the value of tax shields that arises from debt financing:

Adjusted present value = Enterprise value as if the company was all-equity financed + present value of tax shields

The APV valuation model follows directly from the teachings of Miller & Modigliani, who proposed that in a market with no taxes (among other things), a company's choice of financial structure will not affect the value of its economic assets. Only market imperfections, such as taxes and distress costs, affect enterprise value.
Rather than model the effect of capital structure changes in the weighted average cost of capital, the APV model explicitly measures and values the cash flow effects of financing separately.
Most authors build an APV-based valuation by valuing the company as if it were all-equity financed. They do this by discounting free cash flow by the unlevered cost of equity (what the cost of equity would be if the company had no debt). They then add any value created by the company's use of debt to this value.

3.2.2 Value of the unlevered company

The first step in the APV model is the estimation of the value of the unlevered company. This can be accomplished by valuing the company as if it has no debt:

$$Value\_unlevered\_company = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t}$$

where $FCFF_t$ = Free cash flow to the firm at time t
k_u = Unlevered cost of equity

The free cash flows to the firm are calculated in the same manner as in the enterprise DCF model.
In the special case where cash flows grow at a constant rate in perpetuity,

$$Value\_unlevered\_company = \frac{FCFF_1}{(k_u - g)}$$

where $FCFF_1$ = Expected after-tax operation cash flow to the company
g = Growth rate

The inputs needed for this valuation are the expected cash flows, growth rates, and the unlevered cost of equity. The unlevered cost of equity can be derived by means of the CAPM framework (as discussed in the previous paragraph) with the unlevered beta of the company as in input, instead of the levered beta of the company. Damodaran (2002) gives the following formula for the unlevering of the beta. This formula, however, is tied to the assumption that *the company maintains a fixed amount over time*. The formula is as follows.

$$\beta_{unlevered} = \beta_{levered} /(1+(1-T)D/E)$$

where $\beta_{unlevered}$ = Unlevered beta of the company
$\beta_{levered}$ = Levered beta of the company
T = Tax rate for the company
D/E = Current debt-equity ratio

In case of the assumption that the company maintains a constant leverage ratio, the unlevering formula, as stated by Koller *et al.* (2005), becomes:

$$\beta_u = \beta_e / \left(1+\frac{D}{E}\right)$$

where β_e = Levered beta of the company
β_u = Unlevered beta of the company
D/E = Current debt/equity ratio

So as a general basis, the value of an enterprise is based on the value of the unlevered company and the present value of the interest tax shields (which will be discussed further on in this paragraph). Damodaran, however, extends the basic APV model with a term that accounts for the expected costs of bankruptcy. Other authors acknowledge the theoretical existence of the component, but ignore it most of the times. The ignorance of the expected costs of bankruptcy, according Damodaran (2002), leads these other authors to the conclusion that enterprise value increases as companies borrow money and that it will even yield the conclusion that the optimal debt ratio for a company is 100% debt. Since this is an unrealistic conclusion, the inclusion of a component for the expected costs of bankruptcy is supported.

The Damodaran (2002) APV model consists of three steps to determine the value of the company:

1. Estimating the value of the company with no leverage.
2. Considering the present value of the interest tax savings generated by borrowing a given amount of money.
3. Evaluating the effect of borrowing the amount on the probability that the company will go bankrupt, and the expected cost of bankruptcy.

3.2.3 The expected bankruptcy cost

The third component of the APV model is the evaluation of the effect of debt on the default risk of the company and the expected costs of bankruptcy. Damodaran states that: In theory, at least, this requires the estimation of the probability of default and the direct and indirect costs of bankruptcy. If π is the probability of default and BC is the present value of the bankruptcy cost, the present value (PV) of expected bankruptcy cost can be estimated:

PV of expected bankruptcy cost = Probability of bankruptcy * PV of bankruptcy cost
= πBC

Damodaran (2002, p. 401) comments that: “This component of the adjusted present value approach poses the most significant estimation problems, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.”

Further details on the effect and calculation of the bankruptcy costs component in the adjusted present value model will be discussed in a later paragraph of chapter 4.

3.2.4 Interest tax shields

Although a general basis of the APV model can be formulated based on the approaches of the three authors, differences arise in the determination of the value of the interest tax shields.

Damodaran (2002, p. 401) states that: "The second step in the APV is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate and interest payments of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

Value of tax benefits = (Tax rate * Cost of debt * Debt) / Cost of Debt
= Tax rate * Debt = T_mD
The tax rate used here is the firm's marginal tax rate, and it is assumed to stay constant over time."

Koller *et al.* (2005, p. 124) state that: "If you believe the company will manage its debt-to-value to a target level, then the value of the tax shields will track the value of operating assets. Thus, the risk of tax shields will equal the risk of operating assets (k_{txa} equals k_u). The majority of companies have relatively stable capital structures (as a percentage of expected value), so we favor this method.
If you believe the debt-to-equity ratio will not remain constant, then the value of interest tax shields will be more closely tied to the value of forecasted debt, rather than operating assets. In this case the risk of tax shields is equivalent to the risk of debt (when a company is unprofitable, it cannot use interest tax shields, the risk of default rises, and the value of debt drops). In this case, the following equation gives the value of the interest tax shields in perpetuity:

$$V_{txa} = \frac{(D * k_d) * T_m}{k_d} = D * T_m$$

where T_m = Marginal tax rate
k_d = Cost of debt
This situation occurs frequently in periods of high debt such financial distress and leveraged buyouts."
The cost of debt used in the valuation of the interest tax shields is determined in the same way as in the enterprise DCF model.

Brealey & Myers (2003, p. 541) begin their evaluation of the interest tax shields with the stating of two financing rules. The focus on the valuation of projects, but their reasoning is also valid for enterprise valuation. They state the following:
"What are tax shields worth? It depends on the *financing rule* the company follows. There are two common rules:
- Financing rule 1: debt fixed. Borrow a fraction of initial project value and make any debt repayments on a predetermined schedule.
- Financing rule 2: debt rebalanced. Adjust the debt in each future period to keep it at a constant fraction of future project value."

They illustrate these two rules by means of example based on which they conclude that the calculation of APV (debt rebalanced) gets the implications of Financing Rule 2 only approximately right. Brealey & Myers (2003, p. 542) comment the following on the error in the valuation:
"Even when debt is rebalanced, next year's interest tax shields are fixes. Year 1's interest tax shield is fixed by the amount of debt at date 0, the start of the project. Therefore, year 1's interest tax shield should have been discounted at k_d, not at k_u.
Year 2's interest tax shield is not known at the start of the project, since debt is rebalanced at date 1, depending on the first year's performance. But once date 1's debt level is set, the interest tax shield is known. Therefore the forecasted interest tax shield at date 2 should be discounted for one year at 12% and one year at 8%.
The reasoning repeats. Every year, once debt is rebalanced, next year's interest tax shield is fixed. So the procedure for calculating the exact value of tax shields under Financing Rule 2 is as follows:

1. Discount at the unlevered cost of equity, because future tax shields are tied to actual cash flows.

2. Multiply the resulting present value by $(1+k_u)/(1+k_d)$, because the tax shields are fixed one period before receipt. The k_u stands for the unlevered cost of equity and k_d is the cost of debt."

Brealey & Myers (2003, p. 539) also pose two points of interest regarding the value of the interest tax shields. The first regards the true present value of the tax shields compared to the value that results from the perpetuity formula for the interest tax shields in case of Financing Rule 1 (T_m*interest):
"The true present value of the tax shields is almost surely less than T_m * interest:
- You can't use tax shields unless you pay taxes, and you don't pay taxes unless you make money. Few firms can be sure that future profitability will be sufficient to use up the interest tax shields.
- The government takes two bites out of corporate income: the corporate tax and the tax on bondholders' and stockholders' personal income. The corporate tax favors debt; the personal tax favors equity.
- A project's debt capacity depends on how well it does. When profits exceed expectations, the firm can borrow more; if the project fails, it won't support any debt. If the future amount of debt is tied to future project value, then the interest tax shields are estimates, not fixed amounts."

Their second point is that of the relevance of the accuracy of the valuation of the interest tax shields: "In practice it rarely pays to worry whether interest tax shields are valued approximately or exactly. Your worrying time will be much better spent in refining forecasts of operating cash flows and thinking through what-if scenarios.
But which financing rule is better – debt fixed, or debt rebalanced? As a general rule we vote for the assumption of rebalancing, that is, for Financing Rule 2."

As shown by the above, Koller *et al.* (2005) and Brealey & Myers (2003) consider two scenarios for valuing the interest tax shields: a fixed (dollar) amount of debt or a constant leverage ratio (debt grows in line with the company value). Damodaran (2002) only discusses the situation of the fixed amount of debt.
These two scenarios for the determination of the interest tax shields are based on either the 1963 study of Miller & Modigliani or on the 1980 and 1985 studies of Miles & Ezzell. The remainder of this paragraph will focus on these two scenarios and the discussion in the corporate finance literature on their correctness.

Miller & Modigliani (1963) describe a valuation model for levered companies in which they assume that the company maintains a fixed amount of debt. They state:
"We would expect the value of a levered firm of size X, with a permanent level of debt D_L in its capital structure, to be given by":

$$V_L = \frac{(1-\tau)X}{\rho\tau} + \frac{\tau R}{r} = V_U + \tau D_L$$

where V_L = levered company value
V_U = unlevered company value
τ = marginal tax rate

The formula does not include a component for the potential costs of financial distress but Miller & Modigliani give the following argument with regard to the value maximizing amount of debt: "It may be useful to remind readers once again that the existence of a tax advantage for debt financing – even the larger advantage of the corrected version – does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt in their capital structures.
There are additional considerations, which are typically grouped under the rubric of "the need for preserving flexibility", which will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power. The tax advantage of debt may well tend to lower the optimal size of that reserve, but it is hard to believe that advantages of the size contemplated under our model could justify any substantial reduction, let alone their complete elimination."

Miller & Modigliani (1963) also assume that cash flows have a growth rate of zero.

In Miles & Ezzell (1980) the authors comment on the Miller-Modigliani APV model with regard to the riskiness of the interest tax shields. They state that: “When management acts to maintain a constant debt to total value ratio, in terms of realized market values, the investment decision impacts upon the riskiness as well as the magnitude of future tax shields created by debt financing. Even though the firm might issue riskless debt, if financing policy is targeted to realized market values, the amount of debt outstanding in future periods is not known with certainty (unless the investment is riskless) and consequently, the magnitude of the tax shields cannot be known with certainty.”

The purpose of the Miles & Ezzell (1985) study was to examine the implications for tax shield valuation of maintaining a constant market value leverage ratio instead of a constant debt level. In this paper Miles & Ezzell show that when the company’s financing strategy is to maintain a constant market value leverage ratio, the marginal value of a change in debt level resulting from a change in this leverage ratio is much lower than the corporate tax rate. They also derive the relationship between the company’s equity beta and its unlevered beta under the assumption of a constant leverage ratio.

Miles & Ezzell start their reasoning with the assumption that companies maintain a constant leverage ratio. They develop an equation for the market value of a levered cash flow stream $(\widetilde{X}_1 + \widetilde{TS}_1)$, $(\widetilde{X}_2 + \widetilde{TS}_2)$, …, $(\widetilde{X}_T + \widetilde{TS}_T)$ where $\widetilde{X}_j$ are random unlevered cash flows and $\widetilde{TS}_j$ are random debt-related tax shields in period j, respectively, for j = 1, …, T.

At time T – 1, they express the value of $(\widetilde{X}_T + \widetilde{TS}_T)$ by the CAPM as

$$V_{T-1} = \frac{E_{T-1}(\widetilde{X}_T + \widetilde{TS}_T) - \phi Cov(\widetilde{X}_T + \widetilde{TS}_T, \widetilde{r}_{mT})}{1+r} \tag{3.1}$$

where $E_i(\ )$ = Time i expectation operator
Cov( ) = Covariance operator
Φ = Market price of risk assumed constant across time
$\widetilde{r}_{mj}$ = Time j market rate of return
r = Riskless rate of interest

As a result of the constant leverage ratio assumption, the value of the interest tax shields at time T can be expressed as:

$$\widetilde{TS}_T = \tau r L V_{T-1} \tag{3.2}$$

where τ = Marginal corporate tax rate
L = Leverage ratio (Debt/(Debt+Equity))

At time T - 1, V_{T-1} is known with certainty. Hence, Equation (3.2) shows that $\widetilde{TS}_T$ seen from time T - 1, is also known with certainty. However, at time T – 2, the time T interest tax shield, $TS_T = \tau r L V_{T-1}$ is uncertain due to V_{T-1} being a random variable.
After a number of rearrangements and assumptions, Miles & Ezzell arrive at the following expression:

$$\pi_{T-2}(\tilde{V}_{T-1}) = \frac{E_{T-2}(\tilde{X}_T)}{(1+k_u)^2} + \frac{\tau r L[E_{T-2}(V_{T-1})]}{(1+k_u)(1+r)} \tag{3.3}$$

From this equation (3.3), it is apparent that to obtain the time T – 2 value of the time T interest tax shield, the appropriate discount rate for period T is the risk-free rate, r, and the unlevered cost of capital, k_u, is the appropriate discount rate for period T – 1. Thus, at time T – 1, the market knows the time T tax shield with certainty. However, viewed from time T – 2, the time T – 1 expectation about the time T tax shield is subject to uncertainty.

The next step is iterating formula (3.3) to arrive at the value V_0:

$$V_0 = \sum_{j=1}^{T} \frac{E_0(\tilde{X}_j)}{(1+k_u)^j} + \sum_{j=1}^{T} \frac{E_0(T\tilde{S}_j)}{(1+k_u)^{j-1}(1+r)} \tag{3.4}$$

where

$$E_0(T\widetilde{S}_j) = \tau r L[E_0(\widetilde{V}_{j-1})]$$

Equation (3.4) shows that when the market value leverage ratio is held constant, it is not generally correct to discount interest tax shields at the riskless rate. The tax shield expected at time j is discounted back one period at the riskless rate and the remaining j – 1 periods at the unlevered cost of capital.

This result has a number of implications for the valuation of interest tax shields. The standard analysis of the present value of tax shields in a Miller & Modigliani tax world assumes that the expected unlevered cash income stream, $\widetilde{X}_j$ is a level perpetuity and derives the following expression for levered market value:

$$V_0 = \frac{E_0(\tilde{X})}{k_u} + \frac{\tau r D_0}{r} = \frac{E_0(\tilde{X})}{k_u} + \tau D_0 \tag{3.5}$$

where D_0 is the current level of debt. Implicit in equation (3.5) is that D_0 is permanent debt, and that all future tax shields are nonstochastic. Only in this case will each dollar of additional debt add τ dollars to total levered value. The result, however, is not consistent with equation (3.4).

Compared to the Miller & Modigliani analysis that implies $\frac{dV_o}{dD_o} = \tau$, Miles & Ezzell find that:

$$\frac{dV_0}{dD_0} = \left[\frac{1+k_u}{1+r}\right]\left[\frac{r}{k_u}\right]\tau \tag{3.6}$$

They state, based on this expression, that two testable predictions can be inferred from equation (3.6). First, dV_0 / dD_0 can be much less than τ even in the absence of personal tax biases. Second, dV_0 / dD_0 is a decreasing function of the company's business risk (as measured by either k_u or β_u, the unlevered coefficient). If the company has no business risk, then k_u = r, and equation (3.6) implies dV_0 / dD_0 = τ which is the Miller & Modigliani result. However, if k_u > r, then dV_0 / dD_0 < τ.

Another implication of equation (3.5) relates to the unlevering of the beta. The reasoning of Miles & Ezzell is as follows.
Hamada (1972) assumed that equation (3.5) was correct and developed the following equation for unlevering the equity beta:

$$\beta_u^* = \left(\frac{1-L}{1-\tau L}\right)\beta_e \tag{3.7}$$

where β_u^* and β_e are, respectively, the unlevered beta and the equity beta. The asterisk indicates that Hamada's unlevering procedure is used. To derive (3.7), it must be assumed that future interest tax write-offs are certain and this holds only if the company's future debt

levels are certain. However, if the company maintains a constant leverage ratio, then the correct unlevering procedure is quite different from the one specified above since tax shields occurring behind time 1 are uncertain.
Miles & Ezzell begin by writing levered value from equation (3.3) as

$$V_0 = \frac{E_0(\tilde{X}_1 + \tilde{V}_1^u)}{(1+k_u)} + \frac{\tau r L V_0}{(1+r)} + \frac{E_0(\tilde{V}_1^\tau)}{(1+k_u)} \tag{3.8}$$

where $\tilde{V}_1^u$ is the time 1 value of the unlevered cash flows beyond time 1 and $\tilde{V}_1^\tau$ is the time 1 value of all interest tax write-offs beyond time 1. The first component is simply the unlevered value, the second is the present value of the time 1 tax shield, and the third is the value of all interest tax write-offs beyond time 1. The levered beta coefficients, β_L, can be specified as a weighted average of the betas of these three components.
The beta coefficient of the first value component of equation (3.8) is β_u, the unlevered beta coefficient. Since the third component has the same expected rate of return as the first component, its beta coefficient is also β_u. Since the rate of return on the second value component is riskless, its beta coefficient is zero.
This results in,

$$\beta_L = \left[1 - \frac{\tau r L}{1+r}\right]\beta_u \tag{3.9}$$

Since it is also known that

$$\beta_L = L\beta_d + (1-L)\beta_e \tag{3.10}$$

and following the Hamada assumption that β_d, the debt beta, is zero, the following relationship between the β_e and β_u can be obtained from equations (3.9) and (3.10):

$$\beta_u = \beta_e \left[\frac{1-L}{1-\frac{\tau r L}{1+r}}\right]$$

This is the Miles & Ezzell (1985) beta-unlevering formula under the assumption of a constant leverage ratio.

3.2.5 Comments on the Miller-Modigliani and the Miles-Ezzell framework

Multiple authors of corporate finance and valuation textbooks, as shown earlier in this paragraph, have adopted the theories of Miller & Modigliani and Miles & Ezzell. However, Fernández (2004) takes a different point of view, concluding to an overall incorrectness of both the Miller-Modigliani and the Miles-Ezzell framework.
Fernández (2004) argues that the present value effect of the tax saving on debt cannot be calculated as simply the present value of the tax shields associated with interest. Instead, he claims that the only way to obtain the correct value of the tax shields from debt is to do two present value calculations, one for the unleveraged firm and the other for the leveraged firm, and then subtract the former from the latter. These results are potentially important, because they contradict standard results in the literature. In addition, his results imply that, even though the capital market is complete, value additivity is violated because the value of a stream of cash flows is not independent of adding to another set of cash flows.

Cooper & Nyborg (2006) correct the reasoning from Fernández (2004) and give an overview of the Miller-Modigliani and Miles-Ezzell framework.

In their paper, Cooper & Nyborg reconcile the Fernández results with standard valuation formulae for the tax saving from debt. They show that, as one would expect in a complete market, the value of the debt tax saving is the present value of the tax savings from interest.

Their review is as follows. The two main approaches to leverage policy are the Miller & Modigliani (1963) and the Miles & Ezzell (1980). The difference is that Miles & Ezzell assume that the amount of debt is adjusted to maintain a fixed market value leverage ratio, whereas Miller & Modigliani assume that the amount of debt in each future period is set initially and not revised in light of subsequent developments.
Because the level of risk in the tax savings is different, relations between key parameters are different for the two assumed leverage policies. For the Miller & Modigliani policy, the relation between the cost of equity k_e and k_u is given by

$$k_e = k_u + \frac{D}{E}(k_u - k_d)(1-T)$$

where T is the tax rate and k_d is the cost of debt. For the Miles & Ezzell leverage policy with continuous rebalancing, it is given by

$$k_e = k_u + \frac{D}{E}(k_u - k_d)$$

The value of the tax savings when the Miller & Modigliani policy is followed is

$$VTS^{MM}(g=0) = \frac{Dk_d T}{k_d} = DT$$

The value of tax savings when the Miles & Ezzell policy is followed is:

$$VTS^{ME} = \frac{Dk_d T}{(k_u - g)}$$

The first period tax saving is equal to the interest charge, DK_D, multiplied by the tax rate. With the Miles & Ezzell constant debt to value leverage policy, the tax savings changes at the same rate as the unleveraged cash flows, and the risk of the tax saving is the same as the risk of the company.
Miller & Modigliani does not represent simply the Miles & Ezzell assumption with zero growth. It is a completely different financing strategy.

To calculate an adjusted present value, the amount of the interest tax shields should be added to the unlevered value of the company, which can be calculated using a discount rate set with the unlevered beta. The unlevered beta cannot be observed directly. Assuming riskless debt, it can be estimated from the observable equity beta by

$$\beta_u^{MM} = \beta_e / (1 + (1-T)(D/E))$$

In case of the usage of the Miles & Ezzell assumption of a constant leverage ratio the formula for the unlevered equity beta is

$$\beta_u^{ME} = \beta_e (E/(D+E))$$

Cooper & Nyborg (2006) conclude with the comment that “no consensus exists as to which set of assumptions to use”.

In Cooper & Nyborg (2007), the authors give their view on the application of one of the two frameworks to a valuation situation. In their opinion, the key to valuing tax shields is

consistency. “First, the method used should be consistent with the actual debt policy of the company being valued. Second, the relevering formula should be consistent with the debt policies of the companies whose equity betas are used to estimate discount rates. Third, different methods that assume different assumptions should not be mixed in the same valuation.
The choice between the two possible relevering formulas is important. Both approaches persist in part because the formulas are often used to unlever a discount rate and then relever it back to a leverage ratio similar to where it started. In that application, it doesn’t matter much which approach one uses as long as one uses the same approach to unlever as to relever the rate. In other cases, where the unlevered rate itself is being used in a valuation, it does matter which approach one uses, and the difference is significant.” They also note that the Miles & Ezzell formulas for relevering are usually more likely to be accurate than the Miller & Modigliani formulas.

3.2.6 Which author, which beta?

As shown above, there are two beta unlevering formulas. One based on the Miller & Modigliani leverage policy and the other based on the Miles & Ezzell policy. Damodaran (2002), as described in the beginning of this paragraph, uses the Modigliani & Miller framework. He unlevers the beta through

$$\beta_u = \beta_e / (1 + (1 - T)(D / E))$$

and determines the value of the interest tax shields through

$$VTS = D * T$$

Cooper & Nyborg (2006) give

$$\beta_u^{ME} = \beta_e (E / (D + E))$$

as the formula for unlevering the beta under the Miles & Ezzell framework. However, the original Miles & Ezzell formula for the unlevering of the beta formulated in 1985 was:

$$\beta_u = \beta_e \left[\frac{1 - L}{1 - \frac{\tau r L}{1 + r}} \right]$$

The Cooper & Nyborg (2006) formula is a proxy for the original Miles & Ezzell (1985) formula, because the multiplication of $\tau r L$ becomes very small under normal circumstances, which results in the fact that the denominator approaches one. These normal circumstances would be a tax rate smaller than 50%, an interest rate around 5% and a leverage ratio under 50%. These multiplied gives a value for $\tau r L$ of 0,0125. In order to illustrate the difference between the original Miles & Ezzell (1985) and the Cooper & Nyborg (2006) proxy, consider the following situations and the error in the found proxy for the β_u:

Most likely cases:	Error
35% tax, leverage 30%, interest 5%	0,007
35% tax, leverage 20%, interest 5%	0,005
35% tax, leverage 30%, interest 4%	0,006
35% tax, leverage 20%, interest 4%	0,004
30% tax, leverage 30%, interest 5%	0,006
30% tax, leverage 20%, interest 5%	0,005
30% tax, leverage 30%, interest 4%	0,005
30% tax, leverage 20%, interest 4%	0,004

The table shows the maximum difference between the official Miles & Ezzell formula and Cooper & Nyborg's proxy.

Since the differences are very small, the Cooper & Nyborg (2006) formula will be used in the remainder of this research project.

3.2.7 Comments on the correctness of the Miller-Modigliani framework

The APV model has two versions, each based on a different leverage policy. This means that the choice for one of the two versions should be based on the leverage policy that the company under valuation applies in practice.

The issue addressed by Ehrhardt & Daves (1999) is whether the Miller-Modigliani (MM) version of the APV model is correct. They state that “An estimate of the unlevered cost of equity is necessary for many applications of corporate valuation. However, the widely used MM/Hamada formula is based on two assumptions that may not be true. First, the MM formula assumes that the firm will not grow. This assumption is almost certainly violated in practice, since almost all firms are expected to grow. A second issue is the choice of discount rate for the tax shield. The MM APV model assumes that the discount rate for the tax shield should be the cost of debt. However, there are sound arguments for using a larger rate, perhaps even as high as the unlevered cost of equity. This implies that despite its widespread use by practitioners, researchers, and textbook writers, it is almost always inappropriate to use the MM/Hamada model, since virtually all firms are expected to grow. Given that growth should be incorporated explicitly into the model, the only remaining decision is the choice of discount rate for the tax shield. Unfortunately, there are no empirical tests to provide an answer to this question, and two major econometric problems will confound future research in this area. First, the unlevered beta and unlevered cost of equity for a firm are unobservable. Second, there are large measurement errors in the other required variables.”

Ehrhardt & Daves (1999) show that using the cost of debt as the discount for the tax shield can lead to a levered cost of equity that is less than the unlevered cost of equity. Since this contradicts both intuition and casual observations of levered companies, they believe that a rate higher than the cost of debt should be used.

Ehrhardt & Daves (1999) also show that if a rate less than the unlevered cost of equity is used to discount the tax shield, then the partial derivative of the cost of capital with respect to growth is negative, and it becomes more negative for higher degrees of leverage. Taken alone, this would imply that a high growth company could substantially reduce its cost of capital, and hence increase its value, if it had a high degree of leverage. But this is inconsistent with the observed capital structures of high growth companies, which typically have low levels of debt. They state that: “Although there are other explanations for the phenomenon, such as agency costs and asymmetric information, this suggests once again that a relatively high rate, perhaps even the unlevered cost of equity itself, should be used to discount the tax shield of debt.”

3.2.8 Conclusion

Based on the works of Koller *et al.* (2005), Brealey & Myers (2003) and Damodaran (2002) it can be concluded that the adjusted present value model is based on three steps:

1. Determining the value of the unlevered company
2. Valuing the tax shields
3. Determining the cost of potential distress

This can be represented by the following general APV model formula:

$$EV = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + \sum_{t=1}^{t=n} \frac{ITS_t}{(1+k_{ts})^t} + \pi_a * DC$$

where $FCFF_t$ = Free cash flow to the firm at time t
k_u = Unlevered cost of equity
ITS_t = Interest tax shield at time t
k_{ts} = Appropriate tax shield discount rate
π = Probability of default
DC = Distress costs

There are two frameworks for the valuing of the tax shields. The first is based on the assumption of Miller & Modigliani (1963) that a company maintains a fixed amount of debt. This leads to a tax shield value in perpetuity of $D*T_m$ (debt*tax).
The second is based on the assumption of Miles & Ezzell (1985) that a company maintains a constant leverage ratio. This leads to a tax shield value in perpetuity of $\frac{Dk_d T_m}{(k_u - g)}$.
There is no clear consensus in the literature on which assumption (and approach) is the correct one.
Although different authors (Cooper & Nyborg (2006, 2007); Ehrhardt & Daves (1999)) have pointed out that the Miller & Modigliani assumption is unrealistic or even incorrect, the works all of the three authors that were discussed in this paragraph still hold on to Miller & Modigliani's assumption, with Damodaran (2002) discussing only the Miller-Modigliani framework.

The value of the unlevered firm is determined by discounted free cash flow to the firm (FCFFs) at the unlevered cost of equity. This unlevered cost of equity is derived by using the CAPM framework and the unlevered equity beta as an input. This unlevered equity beta has to be derived from the levered equity beta, since the latter can be observed in the market in contrast to the former which is unobservable.
Depending on the assumption taken (fixed debt amount or a constant ratio), the equation for determining the unlevered equity beta changes. Assuming a constant ratio results in the formula found by Miles & Ezzell (1985) which can be approximated by the unlevering formula given by Cooper & Nyborg (2006):

$$\beta_u^{ME} = \beta_e (E/(D/E))$$

Assuming a fixed debt amount gives the formula stated by Damodaran (2002):

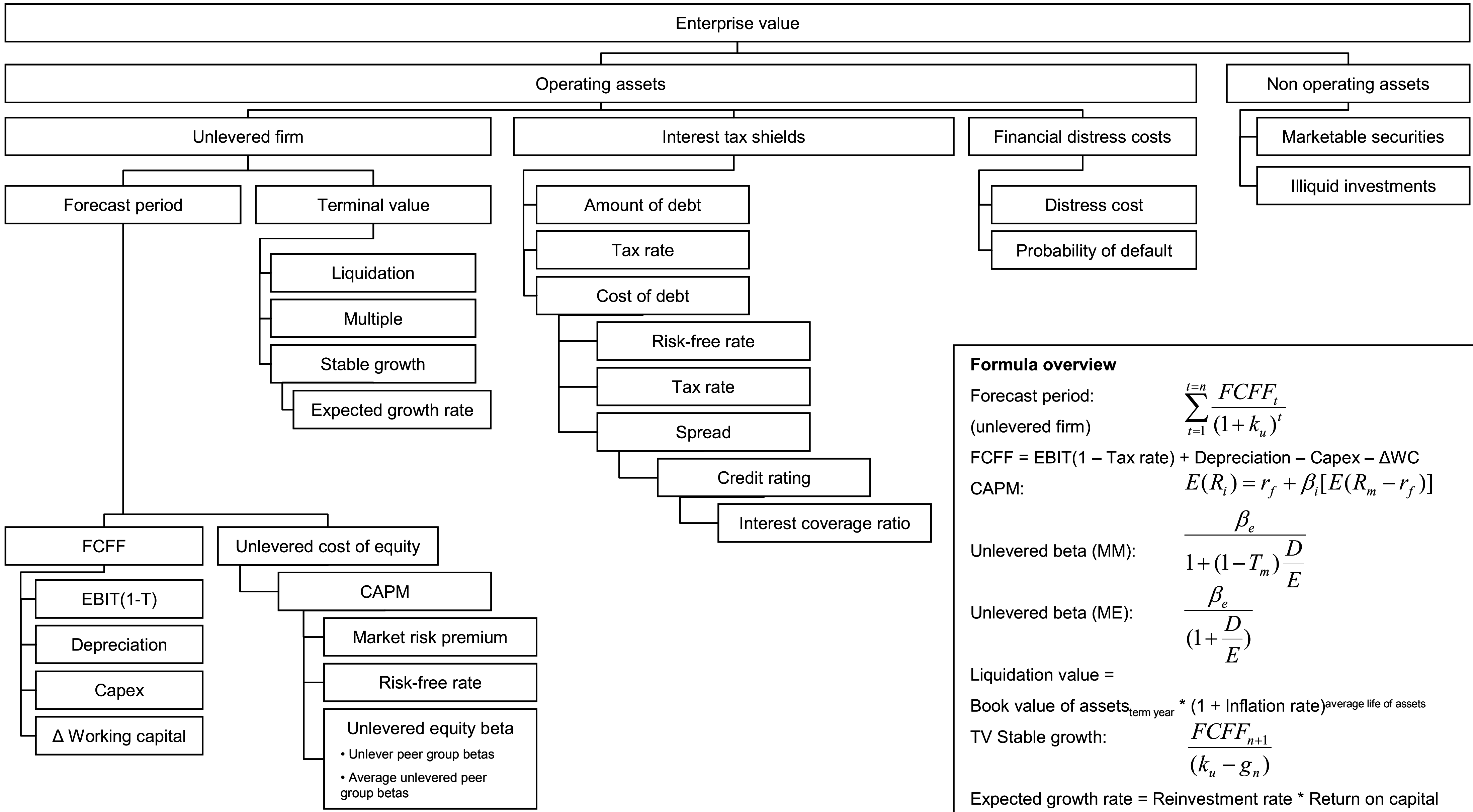
$$\beta_u = \beta_e /(1+(1-T)(D/E))$$

The value of the potential cost of financial distress depends on the cost of financial distress and the probability of the occurrence of these costs.

3.2.9 Comment on this paragraph

The goal of this paragraph is to apply the general subjects of analysis to the APV model. It seems as if there is no yet a consensus on the correct shape of the APV model in the literature: the newest publications on the different related topics are from 2007. In the following paragraphs, the other subject of analysis will be applied to the APV model and an overview will be given on the applicability of the APV model under different circumstances. That will also lead to a comparison with the enterprise DCF model. Therefore this paragraph does not conclude indefinitely on any "right" or "wrong" assumptions.

Figure 3.2 gives a schematic overview of the building blocks of the APV model.



Formula overview

Forecast period: (unlevered firm) $\sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t}$

FCFF = EBIT(1 – Tax rate) + Depreciation – Capex – ΔWC

CAPM: $E(R_i) = r_f + \beta_i[E(R_m - r_f)]$

Unlevered beta (MM): $\frac{\beta_e}{1+(1-T_m)\frac{D}{E}}$

Unlevered beta (ME): $\frac{\beta_e}{(1+\frac{D}{E})}$

Liquidation value =

Book value of assets$_{\text{term year}}$ * (1 + Inflation rate)$^{\text{average life of assets}}$

TV Stable growth: $\frac{FCFF_{n+1}}{(k_u - g_n)}$

Expected growth rate = Reinvestment rate * Return on capital

Interest coverage ratio = EBIT / Interest expense

Interest tax shields (MM): $D * T_m$ for (g = 0)

Interest tax shield (ME): $\frac{T_m D k_d}{(k_u - g_n)} * \left[\frac{1+k_u}{1+k_d}\right]$

Distress cost (DC) = unlevered firm value * 15%

PV financial distress costs = $DC \; * PoD$

Figure 3.2: Schematic overview of APV model building blocks

3.3 The enterprise DCF model assessed at the nongeneral subjects of analysis

Paragraph 3.1 discussed the basic assumptions of the enterprise DCF model. The cash flows are modeled as free cash flows to the firm (FCFFs), which are discounted at the weighted average cost of capital. The cost of debt is determined based on bond prices or credit ratings and the cost of equity can be estimated by using the CAPM framework.
In this paragraph, the enterprise DCF model is analyzed on the basis of the other three subjects of analysis (capital structure, probability of default, costs of financial distress).

3.3.1 Capital structure

In the capital structure paragraph of Chapter 2, two theories were identified as the main theories on capital structure decision making within companies. These were the tradeoff theory and the pecking order theory. Neither of the two theories came forth as the 'ultimate' theory that could explain all the events regarding capital structure development of different companies.
The tradeoff theory states that companies manage their capital structure to an optimal target leverage ratio. This optimal leverage is the outcome of the tradeoff between the potential cost of financial distress caused by the increased leverage, and the tax benefits of debt. A factor that needs to be taken into account is the fact that companies may take on a target leverage ratio that is lower than the optimal point based on the tradeoff between distress cost and tax savings because of the need for access debt capacity.
In this paragraph and the next it is assumed that the tradeoff theory is the right theory. The reasons for this assumption are as follows.

1. Practicality. If companies would not use a target leverage ratio, then no inferences could be made on the expected capital structure. The leverage ratio would depend only on the leverage ratio of the previous time period and this would result in a Markov process with an infinite number of results based on an unknown distribution of the likelihood of the company either issuing debt, equity or using retained earnings.
2. Long-term view. Hovakimian *et al.* (2001) stated that although the pecking order considerations affect corporate debt ratios in the short run, companies tend to make financing choices that move them toward target debt ratios that are consistent with tradeoff models of capital structure choice. Since a large portion of the value of a company is derived from its terminal value, the main focus in the valuation should be on the long term, thus leading to a preference for the tradeoff theory.
3. Interdependence of the valuation inputs. If the pecking order theory is assumed to be correct, then the estimation of the capital structure decisions has to be based on the financial deficit. If there is a financial deficit, then the company will have to acquire new capital and its decision for the type of capital will depend on the amount of retained earnings. This means that changes in capital structure will depend on the estimated cash flows. This makes the WACC and the cash flows are interdependent which creates room for possible errors in both of them when one of the two is incorrect. This problem does not occur when the tradeoff theory is assumed to be correct, since the leverage ratio is independent of the cash flows in this theory.

<u>Assumptions</u>
The enterprise DCF model assumes a constant market leverage ratio. That is, the market value of debt divided by the market value of total capital is constant over time.
This assumption is in line with the tradeoff theory of capital structure discussed in the capital structure paragraph of Chapter 2. This means that the company is expected to correct its leverage at the end of its time period in a way so that it matches it target leverage ratio again. The speed of reversion to this target is however a point of debate. Certain authors state that a company reverses to the target leverage in a very slow manner. The enterprise DCF model, however, implicitly assumes that the speed of reversion is high enough to result in a reversion to the optimal point at the end of each time step, so that the leverage ratio can be considered constant.
The other theory of capital structure discussed in the capital structure chapter, the pecking order theory, states that a company bases its decisions of capital structuring on the adverse selection effects of the different possibilities of acquiring capital. The pecking order theory

assumes that a firm does not have a target leverage ratio. The capital structure of a company is simply the result of all previous financing decisions. It is clear that this theory does not coincide with the assumption of the constant leverage ratio, which is adopted by the enterprise DCF model.

Conditions
In order for the enterprise DCF model to be used, the leverage ratio has to be (at least approximately) constant. If the leverage ratio is not constant, the used WACC will differ from the actual/correct WACC. This is because the WACC has the leverage ratio as an input variable. Also the used cost of equity is dependent on the leverage ratio, therefore it gives an error when the leverage ratio changes.
Translating this condition to capital structure events in which the enterprise DCF model cannot be used gives the following problem scenarios:

1. The company has a large portion of debt on its balance sheet, which it will repay in the coming time periods. This means that the leverage ratio will decline each period. An example of this situation is a management buyout or leveraged buyout.
2. The company is financially constrained in a way that prevents them from further acquiring of debt. This inhibits the company in acquiring new capital in the desired ratio.
3. The company is a special purpose vehicle created to finance a certain project. In this so-called project finance situation, the company first builds up a large debt (up to a leverage ratio of 90%) and then repays this debt over time. This causes a non-constant leverage ratio.

Another situation in which a nonconstant leverage ratio can occur is when the company decides to finance its financial deficit in a certain time period by only one financial instrument (debt or equity). We assume however, based on the tradeoff theory of capital structure, that the company will correct this difference between the current leverage ratio and the target leverage ratio at the end of that period.

Modeling
If the enterprise DCF method can be used because of a constant leverage ratio, then the target leverage ratio has to be determined. The method for determining the target capital ratio has already been discussed on the chapter of the basic assumptions of the enterprise DCF model. In short, to develop a target capital structure for a company, use a combination of three approaches:

1. Estimate the company's current market value-based capital structure.
2. Review the capital structure of comparable companies.
3. Review management's implicit or explicit approach to financing the business and its implications for the target capital structure.

The results of these three approaches should be combined to determine a target leverage ratio. This target ratio is the input for the WACC.

3.3.2 Probability of default

Usage
The enterprise DCF model does not explicitly use the probability of default as an input for the valuation of a firm. The WACC is based on the cost of debt, cost of equity, the marginal tax rate and the leverage ratio. The free cash flows do not incorporate any factor that corrects for the possibility of default.
However, the presence of a probability of default might be implied in the cost of debt and the cost of equity. The providers of debt of the company are assumed to make an estimation of the risk of default for the company and will incorporate this in the interest rate, which they demand. Also, the cost of equity could incorporate the probability of default. This could be through a higher equity beta. However, this beta is derived by taken the average of the unlevered industry-betas and relevering it to the company's leverage ratio. Therefore it can only represent the average probability of default of the companies in the industry in which the company operates. Nonetheless, these possible implications of the probability of default are not enough to state that the probability of default is incorporated in the enterprise DCF model.

As a result of the absence of the probability of default in the enterprise DCF model, error may occur in the valuation of companies that have a significant probability of default. The company

is valued based on the expectation that it will remain a going concern and that its cash flows exist in the future. When the probability of default is significant, the cash flows are no longer certain and the terminal value that is calculated will overstate the correct terminal value. Therefore, the enterprise DCF model gives an incorrect estimation of the value of a company that has a significant probability of default.
There is one exception to this flaw, and that is when a liquidation of the company is foreseen directly after the forecast period. This causes the terminal value to be the liquidation value. However, this exception only occurs when it is stated with certainty that the company will be liquidated at the assumed point in time. If the company becomes distressed or goes bankrupt but does not liquidate or ends up in a stable growth pattern and is liquidated a number of periods later, then the error will still remain in the valuation.

Modeling
Since the probability of default is not incorporated in the enterprise DCF model, the method for determining the probability of default is not relevant here.

3.3.3 Costs of financial distress

Incorporation
The costs of financial distress are not incorporated in the enterprise DCF model. The enterprise DCF model assumes that the company ends up in a stable growth state or that the company is liquidated. In the last case, the liquidation value becomes the terminal value. There are, however, two intermediate states between stable growth and liquidation: financial distress and bankruptcy. With financial distress the company has problems meeting its debt obligations. This causes indirect cost of financial distress to occur. As a result of a period of financial distress, a company can either manage to overcome the situation and return to being a healthy company or the company can go bankrupt. When the company goes bankrupt, direct bankruptcy costs occur. Bankruptcy, however, does not mean that the company is liquidated. The company can be reorganized and can eventually become a healthy company. Therefore, the enterprise DCF model does not incorporate the (full) costs of financial distress.

Manner of usage
Since the enterprise DCF model does not incorporate the cost of financial distress, the method of integrating these in the valuation is not relevant.

3.3.4 Conclusion

The tradeoff theory of capital structure is assumed to be the correct theory for the prediction of capital structure decisions within companies. The enterprise DCF model assumes a constant leverage ratio. In situations where this assumption is violated, the model will give an incorrect value of the company. The enterprise DCF model also does not incorporate a measure for the probability of default and the costs of financial distress. The valuation of a company with a significant probability of financial distress through the enterprise DCF model will therefore also result in an incorrect valuation.

3.4 The APV model assessed at the nongeneral subjects of analysis

Paragraph 3.2 discussed the basic assumptions of the APV model. The cash flows are modeled as free cash flows to the firm (FCFFs), which are discounted at the unlevered cost of equity to determine the value of the unlevered company. The interest tax shields are discounted at either the cost of debt or the unlevered cost of equity depending on the leverage policy adopted by the company. The cost of debt is determined based on bond prices or credit ratings and the unlevered cost of equity can be estimated by using the CAPM framework with the unlevered beta as an input. The costs of financial distress are the third component of the APV model of which the three components combined give the value of the operating assets of the company.
In this paragraph, the APV model is analyzed on the basis of the other three subjects of analysis (capital structure, the probability of default, the costs of financial distress).

3.4.1 Capital Structure

Assumptions

There are two versions of the APV model, each based on a different assumption of capital structure. The Modigliani-Miller version is based on the assumption that the leverage ratio of a company is not constant but that the amount of debt is fixed. So the company has a certain amount of debt at time 0 and this debt will remain in place forever. Linked to the assumption of the fixed amount of debt is that the risk of the interest tax shields is equal to the risk of debt. Therefore, the tax shields are discounted at the cost of debt.
The Miles-Ezzell version assumes that the leverage ratio is constant over time. The risk of interest tax shields is therefore linked to the risk of the company's operations. Miles & Ezzell therefore assume that the interest tax shields should be discounted at the unlevered cost of equity, except for the first period's tax shield, which is discounted at the cost of debt.

Conditions

There are three conditions under which the APV model can be used. The first is the same as for the enterprise DCF model, namely a constant leverage ratio. If the leverage ratio is constant, then certain inferences can be made on the amount of debt at certain points in time enabling the valuer to calculate a terminal value based on a perpetuity formula.
The second possible condition is the fixed amount of debt, which also allows for determining the continuing value by means of a perpetuity formula.
As the first two conditions show, the difficulty in valuing a company by means of the APV model is the fact that some closure is need on the value after the forecast period. As long as some inference can be made on the amount of debt, the value of the interest tax shields can be determined and the company can be valued. If there is no fixed amount of debt and also no constant leverage ratio, then the company that is valued has to have a fixed termination date. This way, no continuing value has to be determined and no inferences on the debt have to be made. This is thus the third condition: a company has to have a known termination date if it does not have a fixed amount of debt and no constant leverage ratio. A note to the third condition is that the amount of debt has to be known for each year in the forecast period. Otherwise, the interest tax shields cannot be valued. In this case the general APV model applies:

$$EV = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + \sum_{t=1}^{t=n} \frac{ITS_t}{(1+k_{ts})^t} + \pi_a * DC$$

where $FCFF_t$ = Free cash flow to the firm at time t
k_u = Unlevered cost of equity
ITS_t = Interest tax shield at time t
k_{ts} = Appropriate tax shield discount rate
π = Probability of default
DC = Distress costs

Because the APV model can be used under any one of the three conditions, most of the scenarios of capital structure decisions are covered. The only scenario under which the APV model cannot be used is that of a company that is expected to stay a going concern, that has a variable amount of debt and this amount of debt is not based on a constant leverage ratio. In this case, no inferences can be made on the amount of debt and the company cannot be valued through the APV model.
However, since we assume (as discussed in the previous paragraph) that companies follow the tradeoff theory of capital structure decision, companies are assumed strive to stay at their target leverage ratio if they are not a situation where they have a fixed amount of debt.

Modeling

In case of the Miles-Ezzell version of the adjusted present value, where the company is assumed to maintain a constant leverage ratio, the situation becomes comparable to the modeling of the capital structure development for the enterprise DCF model: the target leverage ratio of the company has to be determined. This ratio can then be used as an input for the valuation.
In case of the Miller-Modigliani version of the APV model, where the company is assumed to maintain a fixed amount of debt, no modeling of the capital structure is needed. Since only the

amount of debt (and not the amount of equity) is used as an input in the APV model, any other capital structure related elements are not used and thus irrelevant.

3.4.2 Probability of default

Usage

The APV model uses the probability of default as a proxy for the probability of financial distress. The 1-year probability of financial distress is multiplied by the present value of the costs of financial distress to determine the expected costs of financial distress.

Modeling

To determine the probability of default for a company, we have a couple of possible scenarios. The company can have a credit rating, in which case the probability of default can directly be determined, or the company does not have a credit rating, in which case the next step is to determine whether the company is a public or private company.

The rating agencies Moody's and Standard & Poor's track the performances of companies and rate their credibility on the base of a number of ratios. The rating agencies also determine the probability of default for companies in the different rating classes based on historical default data. Therefore, when a company has a credit rating, the probability of default can be determined. Table 3.1 shows the probabilities of default for the different rating classes.

The Altman Z-score formula is based on five financial ratios. The Z-score can be converted to a credit rating. This conversion is based on historical data on the Z-scores of companies that have a certain credit rating. From this credit rating, the probability of default can be determined.
For public companies without a credit rating, the following Altman Z-score formula is to be used:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

where X_1 = Working capital / Total assets
X_2 = Retained earnings / Total assets
X_3 = Earnings before interest and taxes / Total assets
X_4 = Market value equity / Book value of total debt
X_5 = Sales / Total assets
Z = Overall Index

Using this formula, one inserts the more commonly written percentage, for example, 0.10 for 10%, for the first four variables (X_1-X_4). The last variable is to be written in terms of number of times.

For private companies without a credit rating, the following adjusted Altman Z-score formula is to be used, in which the market value of equity is replaced by the book value of equity in the X_4 variable:

$$Z' = 0.717(X_1) + 0.847(X_2) + 3.107(X_3) + 0.420(X_4) + 0.998(X_5)$$

The value of the input variables should be based on the accounting information retrieved from the latest financial report of the company.

After determining the Z-score of a company, the following step is to translate this score to a credit rating from which the probability of default can be determined. The values in conversion are created as follows:

1. The basic inputs are the default probabilities that belong to certain ratings according to Standard & Poor's and Moody's. These probabilities are given in the paragraph on the probability of default in Chapter 2. These probabilities are percentages of companies defaulting within five years. Recalculating these values to 1-year values and taking the average of the two different rating agencies gives the following result.

Credit rating	Average 1-year default probability
AAA	0,02
AA	0,06
A	0,13
BBB	0,61
BB	2,71
B	7,56
CCC	17,20

Table 3.1: Probability of default for different rating classes

2. The Z-scores can be linked to a credit rating on the basis of the following intervals:

Interval Z-score per rating

	Lower	Upper	PoD %
AAA	4,641	infinity	0,02
AA	3,967	4,641	0,06
A	3,117	3,967	0,13
BBB	2,470	3,117	0,61
BB	2,075	2,470	2,71
B	1,617	2,075	7,56
CCC	-infinity	1,617	17,20

There are multiple ways of determining the probability of default, as ways shown in the paragraph on the probability of default. The method described above is a based on a combination of the probabilities of default that are linked to different credit ratings and the Altman Z-score. The reasons for this combination are as follows.

1. The KMV model requires the value of the company as an input for the determination. However, the value is precisely what the complete valuation is supposed to determine. So company value is an output not an input. Therefore, the KMV model is not useful.
2. The Ohlson score can be used to estimate the likelihood, were it not for the fact that there is no scale available to convert the Ohlson score to an actual probability of default.
3. The ZETA model of Altman gives more accurate predictions of the actual probability of default, as Altman (2000) showed. However, the coefficients of the variables are not publicly available. Therefore, this model cannot be used.
4. The CreditMetrics model bases the probability of default on the rating migration probabilities. The model discussed is thus a simple version of this model since the probabilities of default for certain rating classes are the probabilities of migrating from that rating class to default within one period. For simplicity reasons, the model described above assumes that companies either default or remain in their rating class.
5. The Altman Z-score can be used for private companies since it only requires accounting information as an input instead of a market value of publicly traded equity.

3.4.3 Cost of financial distress

Incorporation

The costs of financial distress are incorporated in the adjusted present value model through a separate term. The 1-year probability of default is multiplied by the present value of the costs of financial distress.

Integration

As discussed in the chapter on the cost of default, the total costs of financial distress are estimated at 10-20% of the book value of debt plus the market value of equity. When calculating the cost of financial distress for a company, 15% (the middle of 10-20%) is taken as a basic estimate. And instead of taking this 15% from the book value of debt plus the market value it is taken from the value of the unlevered company. The reason for this approach is the fact that the value of the unlevered company can be determined, but the book value of debt and the market value of equity cannot, since it is the end result of the valuation. Using the unlevered company value as the basis for the cost of financial distress is clearly an approximation. This approximation was introduced by Damodaran (2002).

3.4.4 Conclusion

There are two version of the APV model: one that assumes a constant leverage ratio another that assumes a fixed amount of debt. The general APV model, which makes no assumption on the development of the capital structure, can be used in case of a finite life of the company. The APV model also contains a separate term to incorporate the potential costs of financial distress. These potential costs are based on the distress costs and the probability of default. The probability of default can be estimated based on a method that combines the credit rating of a company and its Z-score.

3.5 Theoretical differences between the enterprise DCF model and the APV model

Based on the previous chapters, the differences between the enterprise DCF model and the APV model can be divided into two categories: the differences in basic assumptions and the differences regarding capital structure, the probability of default and the costs of financial distress.

3.5.1 The differences and similarities in basic assumptions

The enterprise DCF model and the APV model have two basic assumptions in common. They both use free cash flows to the firm (FCFFs) and use the same cost of debt.
They differ in the structure of the valuation model, in the cost of equity used and in the discount rate(s) used. The enterprise DCF model discounts the free cash flows to the firm of each time period at the weighted average cost of capital. The sum of these discounted cash flows is the value of the operating assets of the company.
The APV model splits the valuation in three separate parts: the valuation of the unlevered company, the valuation of the tax benefits of debt and the valuation of the costs of financial distress. The free cash flows are discounted at the unlevered cost of equity to arrive at the value of the unlevered company. The interest tax shields are discounted at either the unlevered cost of equity or the cost of debt. The costs of financial distress are based on the value of the unlevered company.
So the enterprise DCF model uses one discount rate, the APV model uses two different discount rates.
The enterprise DCF model uses the levered cost of equity as in input for the WACC calculation. The APV model uses the unlevered cost of equity as mentioned. Since the cost of equity is derived through the CAPM theorem with an equal risk-free rate and an equal market risk premium for both the APV model and enterprise DCF model, this implies that the enterprise DCF model uses the levered equity beta and the APV the unlevered equity beta.

3.5.2 The differences regarding capital structure, the probability of default and the costs of financial distress

Although the APV model and the enterprise DCF model make different assumptions on the capital structure of the company, companies in both models are assumed to follow a tradeoff theory of capital structure development. The reasons for this assumption were explained in the chapter on the application of the subjects of analysis on the enterprise DCF model. This means that the company will in general strive to maintain a target leverage ratio.

The enterprise DCF model assumes that companies do maintain a constant leverage ratio that remains at a target value over time. The APV model has two different versions, each based on a different assumption of the capital structure. The first version assumes that the company maintains a fixed amount of debt over time. So each occurring capital requirement is met by either retained earnings or by newly issued equity. The second version assumes (like the enterprise DCF model) a constant leverage ratio. The APV model could also be used in a third situation in which no clear capital structure is maintained but where all the developments of capital structure are known at each time period.

The enterprise DCF model and the APV model can thus have a different or an equal assumption of capital structure depending on the situation. However, in case of an equal assumption, namely both assume a constant leverage ratio; the function of that constant leverage ratio is different in each method. The enterprise DCF model uses the leverage ratio to determine the WACC (discount rate effect); the APV model uses the leverage ratio to determine the amount of debt and the resulting interest tax shield (cash flow effect).

The enterprise DCF model does not explicitly take the cost of financial distress and the probability of default into account. The only aspects in the enterprise DCF model that could be related to a probability of default are the cost of equity and the cost of debt. Since the cost of debt is derived from the yield to maturity of the company's bonds (in which the providers of debt are assumed to take into account any default risk) or from the credit rating of the company (in which a the spread over the risk-free rate is related to the probability of default), one can say that some measure of the probability of default is incorporated in it. The cost of equity could incorporate a measure of the probability of default through the use of the beta in its determination. This beta is a measure of the riskiness of the company, but the beta is derived by taking an average industry-beta and thus does not relate to the company's risk of default. Another argument on why the cost of equity and the cost of debt do not add an explicit measure of the probability of default is the fact that the cost of debt and the cost of equity are also used in the APV model. This means that even though the enterprise DCF model might incorporate the probability of default into the valuation, it does not add any extra detail in relation to the APV model. As a last notice, the enterprise DCF model does not adjust the cash flows for the probability of default and it also does not use a cash flow component for the cost of financial distress.

The APV model consists of a separate part that incorporates the potential cost of financial distress. This part is a multiplication of the 1-year probability of default and the cost of financial distress (which is based on the value of the unlevered company). The probability of default is a proxy for the probability of financial distress. If financial distress occurs, both direct and indirect costs of financial distress are realized. These costs are proportional to the value of the unlevered company. The APV model thus takes into account that a company might incur costs when it becomes distress.

3.5.3 Overview

The table below shows an overview of the differences. The effect of these differences on the valuation outcome will be discussed in the next paragraph.

	Enterprise DCF model	**APV model**
Cash flow	FCFF	FCFF
Discount rate	WACC	Ku or Kd
Cost of equity	Levered	Unlevered
Cost of debt	Credit rating based	Credit rating based
Capital structure	Constant leverage ratio	Constant leverage ratio or fixed debt
Probability of default	Not explicitly taken into account	Separate term in the valuation
Costs of financial distress	Not explicitly taken into account	Separate term in the valuation

3.5.4 Method choice

As discussed in the application chapters of the enterprise DCF model and the APV model, each model is applicable under certain conditions. The enterprise DCF model is only applicable when a constant leverage ratio is assumed. This leads to a number of situations in which the usage of the enterprise DCF model is problematic:

- Project finance, in which the leverage ratio increases in the beginning of the project and declines later on.
- A management or leveraged buyout, where the company has a large amount of debt on its balance sheet, which it repays over time.
- A financially-constrained company, where the company is unable to acquire additional capital through debt.
- The company under valuation has a large probability of default.

The APV model can be used in practically every situation. Only in the case of a company with a non-finite life where the amount of debt outstanding at each point in time is unknown can the APV model not be applied.

In order to decide which model can best be used to value a certain company, one can follow the decision tree in Appendix I.

In the situation where both the APV model and the enterprise DCF model can be used (constant leverage ratio, non-significant probability of default), the choice between the two should be based on the priorities of the valuer. The enterprise DCF model gives a valuation based on the WACC. This WACC has a communicational value, since it, as Miles & Ezzell (1980) state, facilitates decentralized capital expenditure analyses and choices where the financing and investment decisions are organizationally separated. Thus, lower level managers are provided with a single discount rate which is intended to reflect not only the company’s operating risk, but also the firm’s financing policies and which is to be used to evaluate, at a decentralized level, the company’s investment opportunities. The enterprise DCF model also does not need to estimate the probability of default, which can be difficult and which can contain errors, therefore being the more efficient valuation model of the two. If the valuer, however, wants to know the distinct effects of the financing activities and the potential financial distress cost on the value of the company, then the APV model provides the desired information.

3.5.5 Adjustment to the enterprise DCF to include distress

This research project aims to identify the differences between the enterprise DCF model and the APV model for the valuation of companies. The theoretical differences that were discussed above are the differences that exist between the generally accepted forms of the enterprise DCF model and the APV model today. The impact of the differences on the valuation outcome will be discussed in the next chapter based on the state of the models as discussed in the previous chapters. However, Damodaran (2006) discusses possible alterations to the enterprise DCF model to include the potential of financial distress. To give a complete overview of the developments of the enterprise DCF model, these alterations will now be discussed.

Damodaran (2006) introduces a number of adjustments that can be made to the enterprise DCF model in order to include the effects of potential financial distress. He states that “distressed firms, i.e., firms with negative earnings that are exposed to substantial likelihood of failure, present a challenge to analysts valuing them because so much of conventional valuation is built on the presumption that firms are going concerns.”

He suggests four ways in which one can incorporate distress into value – simulations that allow for the possibility that a company will have to be liquidated, modified discounted cash flow models, where the expected cash flows and discount rates are adjusted to reflect the likelihood of default, separate valuations of the company as a going concern and in distress and adjusted present value models.”

Damodaran (2006) further states that: “The failure to explicitly consider distress in discounted cash flow valuation will not have a material impact in value if any of the following conditions hold:

1. There is no possibility of bankruptcy, either because of the firm’s size and standing or because of a government guarantee.
2. Easy access to capital markets allow firms with good investments to raise debt or equity to sustain themselves through bad times, thus ensuring that these firms will never be forced into a distress sale.
3. We use expected cash flows that incorporate the likelihood of distress and a discount rate that is adjusted for the higher risk associated with distress. In addition, we have

to assume that the firm will receive sale proceeds that are equal to the present value of expected future cash flows as a going concern in the event of a distress sale.

If these conditions do not hold, and it is easy to make an argument that they will not for some firms at some points in time, discounted cash flow valuation will overstate firm value."

Damodaran (2006) proposes to adopt a form of modified discounted cash flow valuation. He states that "we can adapt discounted cash flow valuation to reflect some or most of the effects of distress on value. To do this, we will have to bring in the effects of distress into both expected cash flows and discount rates.

1. Estimating expected cash flows. To consider the effects of distress into a discounted cash flow valuation, we have to incorporate the probability that a firm will not survive into the expected cash flows.

$$Expected\_cash\_flow = \sum_{j=1}^{j=n} \pi_{jt}(Cashflow_{jt})$$

where π_{jt} is the probability of scenario j in period t and $Cashflow_{jt}$ is the cash flow under that scenario and in that period. These inputs have to be estimated each year, since the probabilities and the cash flows are likely to change from year to year. A shortcut, albeit an approximate one, would require estimated for only two scenarios – the going concern and the distress scenario.

$$Expecte\_cash\_flow_t = (Cashflow_{Goingconcern,t})*\pi_{Goingconcern,t}(Cashflow_{Distress,t})*(1-\pi_{Goingconcern,t})$$

where $\pi_{\text{Going concern}, t}$ is the cumulative probability that the firm will continue as a going concern through period t. The probabilities of distress will have to be estimated for each year and the cumulative probability of surviving as a going concern can then be written as follows:

$$Cumulative\_probability\_of\_survival - period\_t = \pi_t = \prod_{n=1}^{n=t}(1-\pi_{distress,n})$$

where $\pi_{distress,t}$ is the probability that the firm will become distressed in period t.

2. Estimating discount rates. In conventional valuation, we often estimate the cost of equity using a regression beta and the cost of debt by looking at the market interest rates on publicly traded bond issued by the firm. For firms with significant probability of distress, these approaches can lead to inconsistent estimates. Consider first the use of regression betas. Since regression betas are based upon past prices over longer periods (two to five years, for instance), and distress occurs over shorter periods, we will find that these betas will understate the true risk in the distressed firm. With the interest rates on corporate bonds, we run into a different problem. The yields to maturity on the corporate bonds of firms that are viewed as distressed reach extremely high levels, largely because the interest rates are computed based upon promised cash flows (coupons and face value) rather than expected cash flows. The presumption in a going concern valuation is that the promised cash flows have to be made for the firm to remain a going concern, and it is thus appropriate to base the cost of debt on promised rather than expected cash flows. For a firm with a significant likelihood of distress, this presumption is clearly unfounded.
   To estimate the cost of equity, we have two options that provide more reasonable estimated than regression betas:
   a. CAPM beta adjusted for distress: Instead of using regression betas, we could use the bottom-up unlevered beta (the weighted average of unlevered betas of the businesses that the firm operates in) and the current market debt-to-equity ratio of the firm.
   b. Distress factor models: In addition to the standard factor for market risk, we could add a separate distress factor for the cost of equity."

Damodaran (2006) recommends using the interest rate based upon the company's bond rating to estimate the cost of debt for a distressed company. To compute the cost of capital, an estimate of the weights on debt and on equity is needed. In the initial year, the current market debt to capital ratio (which may be very high for a distressed company) should be used. As the forecasts for future years are made and build in the expectations of improvements in profitability, the debt ratio should be adjusted towards more reasonable levels. The conventional practice of using target debt ratios for the entire valuation period (which reflect the industry averages or the optimal mix) can lead to misleading estimates of value for companies that are significantly over levered.

Another alternative that Damodaran (2006) proposes is dealing with distress separately. He states that "an alternative to the modified discounted cash flow model is to separate the going concern assumptions and the value that emerges from it from the effects of distress. The value of the firm can then be written as:

Firm value = going concern value * (1 – $\pi_{distress}$) + distress sale value * $\pi_{distress}$

where $\pi_{distress}$ is the cumulative probability of distress over the valuation period."
To value a company as a going concern, only those scenarios are considered where the company survives. When estimating discount rates, the assumption is made that debt ratios will, in fact, decrease over time, if the company is overlevered, and that the company will derive tax benefits from debt as it turns the corner to profitability.
Damodaran (2006) considers, three ways in which the probability of distress can be estimated. The first is a statistical approach, where the probability of distress is related to a company's observable characteristics – company size, leverage and profitability, for instance – by contrasting companies that have gone bankrupt in prior years with companies that did not. The second is a less data intensive approach, where the bond rating for a company is used, and the empirical default rates of companies in that rating class to estimate the probability of distress. The third is to use the prices of corporate bonds issued by the companies to back out the probability of distress, which is based on the following formula:

$$BondP\_rice = \sum_{t=1}^{t=n} \frac{Coupon(1-\pi_{distress})^t}{(1+Riskfree\_Rate)^t} + \frac{Face\_Value\_of\_Bond(1-\pi_{Distress})^n}{(1+Riskfree\_Rate)^n}$$

This equation can now be used, in conjunction with the price on a traded corporate bond to back out the probability of default.
Consequently, a key input that needs to be estimated are the expected proceeds in the event of a distress sale. There are three choices:

1. Estimating the present value of the expected cash flows in a discounted cash flow model, and assume that the distress sale will generate only a percentage (less than 100%) of this value.
2. Estimating the present value of expected cash flows only from existing investment as the distress sale value.
3. The most practical way of estimating distress sale proceeds is to consider the distress sale proceeds as a percent of book value of assets, based upon the experience of other distressed companies.

By incorporating the changes described by Damodaran (2006), one could adjust the enterprise DCF model for the potential financial distress. But, as discussed before, these adjustments are not part of this research project. This research project's objective is to determine the differences between the enterprise DCF model and the APV model as currently accepted in literature.

3.5.6 Conclusion

The enterprise DCF model and the APV model use the same cash flows and the same cost of debt. The differences between the two models are created by the fact that they use a different discount rate and a different cost of equity. The enterprise DCF model has only one version where a constant leverage ratio is assumed. The APV model has two versions, one with a

constant leverage ratio assumption and one with a fixed amount of debt assumption. A general form of the APV model can also be used in special situations.
The choice for one of the two models in different situations is mainly based on the capital structure and the probability of default.
There are a number of adjustments that could be made to the enterprise DCF model to incorporate the costs of financial distress more accurately. These adjustments lie outside the scope of this research project and will not be discussed any further.

3.6 Impact of the differences on the valuation outcome

The differences in assumptions between the enterprise DCF model and the APV model were discussed in the previous paragraph. In this paragraph the impact of these differences on the valuation outcome will be analyzed.

The decision tree in Appendix I shows the different criteria based on which one can choose to use a version of the APV model or the enterprise DCF model to value a company. The decision tree links either an APV model or the enterprise DCF model to a particular condition because the assumptions underlying the chosen model correspond to this condition. The chosen model should give the most accurate valuation of the company.
To determine the effects of the differences between the enterprise DCF model and the APV model on the valuation outcome, each ‘best-chosen’ model is compared with its alternative. Since each ‘best-chosen’ model is an APV model, the ‘best-chosen’ model will be compared to the valuation outcome under the enterprise DCF model. The special case in which both the enterprise DCF model and the APV model can be used will also be addressed.
The following table gives an overview of the different scenarios and the ‘best-chosen’ model. The remainder of this chapter is used to address each scenario.

Scenario		Model	Compare to
1	Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.
2	Constant leverage ratio & significant PoD	APV (ME)	DCF
3	Fixed amount of debt	APV (MM)	DCF
4	Finite life & debt known	APV (general)	DCF
5	Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF

3.6.1 Scenario 1: Constant leverage ratio & no significant probability of default

Differences between valuation outcomes arise because of the fact that one of the two models is based on assumptions of capital structure and/or the probability of default that differ from the state of these elements in the scenario.
Since this first scenario comprises no such differences, the valuation outcomes of both models should be the same. The ME APV model and the enterprise DCF model both assume a constant leverage ratio and since there is no significant probability of default the term of the costs of financial distress in the ME APV model has no significant effect.

3.6.2 Scenario 2: Constant leverage ratio & significant probability of default

This scenario is the same as the previous except for the fact that the probability of default is significant. This means that the difference between the two valuation outcomes is based on the third term of the ME APV model. The ME APV model is stated as:

$$\sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + \frac{T_m D k_d}{(k_u - g_n)} * \left[\frac{1+k_u}{1+k_d}\right] - \pi DC$$

The third term, πDC, represents the costs of financial distress.
Since the present value of the costs of financial distress are estimated at 15% of the value of the unlevered firm, the difference between the enterprise DCF model valuation outcome and the ME APV model valuation outcome will be:

$$\Delta = 0.15 * V_u * \pi$$

where V_u = the value of the unlevered company
π = probability of default

The value of the difference (Δ) will depend on the value of the unlevered company:
Since $0 \le \pi \le 1$ and $V_u \ge 0$, $\Delta < 0$. This means that in this scenario, the valuation outcome of the ME APV model will be lower than the valuation outcome of the enterprise DCF model.
So $V_{enterpriseDCF} > V_{APV/acutal}$ under this scenario.

3.6.3 Scenario 3: Fixed amount of debt

The difference in valuation outcome between the MM APV model and the enterprise DCF model in the scenario of a fixed amount of debt consists of two parts. The difference in valuation outcome based on the difference in the capital structure assumption depends on the development of the capital structure. The MM APV model is formulated as:

$$\sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + DT_m - \pi DC$$

The capital structure has an impact on the value through the second term, DT_m, where D is the amount of debt and T_m is the marginal tax rate. If the company is expected to increase its total amount of capital under the condition of a fixed amount of debt, then this means that the company's leverage ratio will decrease. The enterprise DCF model, however, assumes a constant leverage ratio. This means that the enterprise DCF model overestimates the amount of debt and thus overestimates the benefits of debt. This results in a higher value under the enterprise DCF model than under the MM APV model. If the company is expected to decrease its total amount of capital while keeping its amount of debt fixed, then the opposite occurs: the enterprise DCF model assumes less debt than is actually present thereby underestimating the benefits of debt and thus underestimating the value of the company. This can be summarized in the following way.

For $\frac{d(D+E)}{dt} > 0$, $D_{enterpriseDCF} > D_{APV/actual}$, which leads to $V_{enterpriseDCF} > V_{APV/acutal}$.
And in the opposite development of the capital structure of the company:

for $\frac{d(D+E)}{dt} < 0$, $D_{enterpriseDCF} < D_{APV/actual}$, which leads to $V_{enterpriseDCF} < V_{APV/acutal}$.

These conclusions on the relation of the company value under the two valuation models are only correct if the probability of default is not significant.
In case of a significant probability of default the third term of the MM APV model, the costs of financial distress, also affects the valuation outcome. In the case of an increasing amount of capital the $V_{enterpriseDCF} > V_{APV/acutal}$ relation remains the same, only the difference will be larger since the costs of financial distress always lower the value of the company (as was shown in scenario 2).
In the situation where the amount of capital decreases and the relation between the company values under the two methods based on the difference on the capital structure assumption is

given by $V_{enterpriseDCF} < V_{APV/acutal}$, the effect of the costs of financial distress reduces the company value under the MM APV model and thus reduces the difference between the valuation outcome under the enterprise DCF model and the MM APV model. Whether $V_{enterpriseDCF} < V_{APV/acutal}$ or $V_{enterpriseDCF} > V_{APV/acutal}$ depends on the change in the amount of capital in relation to the probability of default. It might even be the case that $V_{enterpriseDCF} = V_{APV/acutal}$, because of the fact that the two errors in the enterprise DCF model cancel each other out.

3.6.4 Scenario 4: Finite life and a known amount of debt at each point in time

In this scenario, there is neither a fixed amount of debt nor a constant leverage ratio. This results in the fact that the value of the interest tax shields cannot be calculated by means of a perpetuity formula. Instead, each interest tax shield has to be valued separately. The value of the company can be determined by the use of the general APV model:

$$\sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + \sum_{t=1}^{t=n} \frac{ITS_t}{(1+k_{ts})^t} - \pi DC$$

where k_{ts} = the discount rate of the interest tax shields
ITS_t = the interest tax shield at time t

Since the amount of debt is known at each point in time, the appropriate discount rate for the tax shields is the cost of debt: $k_{ts} = k_d$.

The difference in the valuation outcome between the general APV model and the enterprise DCF model depends on the difference in the amount of debt assumed at each point in time. For instance, if the amount of capital increases by the amount of X then the amount of debt assumed by the enterprise DCF model is X * leverage ratio + previous debt. The benefits of debt are related to this amount of debt. If the actual amount of debt in a period is higher than the debt assumed by the enterprise DCF model then the value of the benefits of debt at that point in time are underestimated by the enterprise DCF model. The opposite is true is the actual amount of debt is below the assumed amount of debt of the enterprise DCF model. The final difference between the valuation outcomes of the enterprise DCF model and the general-APV model will depend on the present value of the differences between the assumed interest tax shields. This final difference also has to be corrected for the costs of financial distress if the probability of default is significant. The effect of this correction is the same as discussed under scenario 3.

The difference between the valuation outcomes of the enterprise DCF model and the general-APV model under this scenario thus depends on the situation.

3.6.5 Scenario 5: Debt known up to time t, followed by a constant leverage ratio

This scenario is a combination of scenario 4 and scenario 1 (if the probability of default is not significant) or a combination of scenario 4 and scenario 2 (if the probability of default is significant).

For the first part of the valuation, up to point t where the amount of debt is known, scenario 4 applies. This means that it depends on the situation whether the valuation outcome of the APV (general) model is higher or lower than the valuation outcome of the enterprise DCF model.

For the second part, after point t, the company is assumed to have a constant leverage ratio. In the case of a probability of default that is not significant, both the ME APV model and the enterprise DCF model give the same valuation outcome as discussed in scenario 1. In case of a significant probability of default, the valuation outcome of the ME APV model will give a lower value than the enterprise DCF model as discussed in scenario 2.

The total difference between the general APV model / ME APV model combination and the enterprise DCF model is the sum of the differences of the two parts.

This scenario is not an independent scenario since it is a combination of a number of the previous scenarios. The reason for discussing it briefly is to indicate that the methods can be combined. This particular scenario is also the model of the situation that is created by a management or leveraged buyout. In a buyout situation a company starts with a high leverage ratio as a result of the buyout and, after reaching the desired leverage ratio through debt repayments, the company is assumed to take on a constant leverage ratio.

3.6.6 Conclusion

For each scenario described, an APV model can be used to determine the value of the company. Using an enterprise DCF model in these situations gives an incorrect valuation, except for scenario 1. The difference between the enterprise DCF model valuation outcome and the APV model valuation outcome are given in the table below. In the case of a constant leverage ratio and an insignificant probability of default one can use both the enterprise DCF model and the ME APV model to determine the correct company value.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

Conclusion

In this chapter each of the two valuation models is analyzed. The valuation models are assessed at the subjects of analysis. This provides an overview on the assumptions and the functioning of each model. The two valuation models are compared with regard to their theoretical differences and these differences are translated to (expected) differences in the valuation outcome.

Basic assumptions of the enterprise DCF model

After analyzing the enterprise DCF model on the general subjects of analysis, it can be concluded that the enterprise value equals the sum of the value of the operational and nonoperational assets of the company. The operational assets are valued by discounting the free cash flows at the weighted average cost of capital. The effect of the tax-deductibility of interest is integrated in the weighted average cost of capital. The nonoperational assets are valued in a different way depending on the type of asset. To arrive at the value of equity, the nonequity claims have to be subtracted from the enterprise value.

Basic assumptions of the APV model

Based on the works of Koller *et al.* (2005), Brealey & Myers (2003) and Damodaran (2002) it can be concluded that the adjusted present value model is based on three steps:

1. Determining the value of the unlevered company
2. Valuing the tax shields
3. Determining the cost of potential distress

This can be represented by the following general APV model formula:

$$EV = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+k_u)^t} + \sum_{t=1}^{t=n} \frac{ITS_t}{(1+k_{ts})^t} + \pi DC$$

where $FCFF_t$ = Free cash flow to the firm at time t
k_u = Unlevered cost of equity

ITS_t = Interest tax shield at time t
k_{ts} = Appropriate tax shield discount rate
π = Probability of default
DC = Distress costs

There are two frameworks for the valuing of the tax shields. The first is based on the assumption of Miller & Modigliani (1963) that a company maintains a fixed amount of debt. This leads to a tax shield value in perpetuity of D^*T_m (debt*tax).
The second is based on the assumption of Miles & Ezzell (1985) that a company maintains a constant leverage ratio. This leads to a tax shield value in perpetuity of $\frac{Dk_d T_m}{(k_u - g)}$.
There is no clear consensus in the literature on which assumption (and approach) is the correct one.
Although different authors (Cooper & Nyborg (2006, 2007); Ehrhardt & Daves (1999)) have pointed out that the Miller & Modigliani assumption is unrealistic or even incorrect, the works all of the three authors that were discussed in this paragraph still hold on to Miller & Modigliani's assumption, with Damodaran (2002) discussing only the Miller-Modigliani framework.

The value of the unlevered firm is determined by discounted free cash flow to the firm (FCFFs) at the unlevered cost of equity. This unlevered cost of equity is derived by using the CAPM framework and the unlevered equity beta as an input. This unlevered equity beta has to be derived from the levered equity beta, since the latter can be observed in the market in contrast to the former which is unobservable.
Depending on the assumption taken (fixed debt amount or a constant ratio), the equation for determining the unlevered equity beta changes. Assuming a constant ratio results in the formula found by Miles & Ezzell (1985) which can be approximated by the unlevering formula given by Cooper & Nyborg (2006):

$$\beta_u^{ME} = \beta_e (E/(D/E))$$

Assuming a fixed debt amount gives the formula stated by Damodaran (2002):

$$\beta_u = \beta_e / (1 + (1 - T)(D/E))$$

The value of the potential cost of financial distress depends on the cost of financial distress and the probability of the occurrence of these costs.

The enterprise DCF model assessed at the nongeneral subjects of analysis

The tradeoff theory of capital structure is assumed to be the correct theory for the prediction of capital structure decisions within companies. The enterprise DCF model assumes a constant leverage ratio. In situations where this assumption is violated, the model will give an incorrect value of the company. The enterprise DCF model also does not incorporate a measure for the probability of default and the costs of financial distress. The valuation of a company with a significant probability of financial distress through the enterprise DCF model will therefore also result in an incorrect valuation.

The APV model assessed at the nongeneral subjects of analysis

There are two version of the APV model: one that assumes a constant leverage ratio another that assumes a fixed amount of debt. The general APV model, which makes no assumption on the development of the capital structure, can be used in case of a finite life of the company. The APV model also contains a separate term to incorporate the potential costs of financial distress. These potential costs are based on the distress costs and the probability of default. The probability of default can be estimated based on a method that combines the credit rating of a company and its Z-score.

Differences between the enterprise DCF model and the APV model

The enterprise DCF model and the APV model use the same cash flows and the same cost of debt. The differences between the two models are created by the fact that they use a different discount rate and a different cost of equity. The enterprise DCF model has only one version where a constant leverage ratio is assumed. The APV model has two versions, one with a constant leverage ratio assumption and one with a fixed amount of debt assumption. A general form of the APV model can also be used in special situations.
The choice for one of the two models in different situations is mainly based on the capital structure and the probability of default.
There are a number of adjustments that could be made to the enterprise DCF model to incorporate the costs of financial distress more accurately. These adjustments lie outside the scope of this research project and are not discussed any further.

Impact of the differences on the valuation outcome

For each scenario described, an APV model can be used to determine the value of the company. Using an enterprise DCF model in these situations gives an incorrect valuation, except for scenario 1. The difference between the enterprise DCF model valuation outcome and the APV model valuation outcome are given in the table below. In the case of a constant leverage ratio and an insignificant probability of default one can use both the enterprise DCF model and the ME APV model to determine the correct company value.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

Practice

The second, third and fourth part of the research framework discussed in chapter 1 have been implemented in this chapter. The result of these three steps is an overview of the differences between the enterprise DCF model and the APV model and the effects of these differences on the valuation outcome.

The found differences are purely based on the theories underlying the enterprise DCF model and the APV model. In order to determine the significance of the differences, these theories have to be translated to a practical tool for the valuation of companies. Appendix II focuses on the formulation and demonstration of a practical APV model.

Chapter 4 serves as a testing chapter. The purpose of this chapter is to test the theories stated in this chapter on different scenarios. The result of chapter 4 is a tested set of theories on the differences between the enterprise DCF model and the APV model.

Chapter 4: Validation

Introduction

The purpose of this chapter is to test the theories stated in chapter 3 through of the valuation tool developed and discussed in appendix II. The theories are tested through the valuation of a fictitious company. The first paragraph of this chapter discusses the key financials of the fictitious company (Company X). The second paragraph focuses on the validation of the five scenarios discussed in chapter 3. The third paragraph is dedicated to the identification of the factors that cause the valuation outcomes to deviate from the values predicted by the chapter 3 theories. These factors form the conditional framework under which the theories hold. The last paragraph concludes on the retrieved validation results and links to the final chapter of the research report: the conclusions.
All the Euro amounts mentioned in this chapter are in thousands of Euros.

4.1 Validation approach

Company X, the company that serves as the means to test the theories from chapter 3, is a stable growth company. The company is assumed to have 15.000 EUR of sales in 2003 and the expected annual sales growth is 2,0%. The direct costs comprise 50% of sales; the operating costs are fixed at 4.000 EUR per year. The corporate tax rate is based on the actual rate in the Netherlands and the long-term corporate tax rate is assumed to be 25,5%.
Company X is assumed to own tangible fixed assets with a value of 9.000 EUR in 2003, 60% of sales, and is assumed to have depreciation costs of 1.000 EUR per year. The investments in the tangible fixed assets are such that the value of the fixed assets remains at 60% of sales in the corresponding year. The intangible fixed assets are assumed to have a value of 3.000 EUR in 2003, 20% of sales in the corresponding year. The yearly tax-deductible amortization is assumed to be 250 EUR, the non-tax deductible amortization is assumed to be 200 EUR. The yearly investments in the intangible fixed assets are such that the value of the intangible fixed assets remains at 20% of sales in the corresponding year.
The inventory (stock) is assumed to be 2,5% of sales and the debt outstanding with creditors is assumed to be 1,5% of direct costs. Company X is assumed to pay out two-thirds of its net profit to its shareholders in the form of dividends.
The assumed risk-free rate is 4,0%, the market risk premium 5,0%, the additional risk premium 3,0% and the assumed spread is also 3,0%. The company is assumed to have a levered beta of 1,0 and the D/E ratio used for the calculation of the cost of capital is set at 20,4% (which translates to a debt to value ratio of 17,0%). The development of the capital structure depends on the scenario that is studied.

The validation outcomes resulting from this validation approach will be discussed for each basic scenario in the next paragraph.

4.2 Basic scenarios

This paragraph discusses the five (basic) scenarios discussed in chapter 3. The five scenarios and their results, as predicted in chapter 3, are shown in the table below.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

4.2.1 Scenario 1: Constant leverage ratio & no significant probability of default

The first scenario, which is based on a constant leverage ratio and no significant probability of default assumption, should result in a valuation outcome from the enterprise DCF model that is equal to the valuation outcome of the APV model.

The inputs for this scenario are based upon the valuation approach described in the first paragraph of this chapter. The following tables show the inputs in the different sections of the valuation tool.
The first inputs (shown in table 4.1) are the sales and (direct and operational) costs. These add up to the EBITDA, which, after deduction of the depreciation costs, results in the EBITA. The other blue-colored cells are also input cells for respectively the amortization costs and the dividends paid out.
The assets are modeled as shown in table 4.2, 4.3 and 4.4. The amount of excess cash for each year is determined through the calculation of the cash flows. This cash flow calculation is shown in table 4.5.
The debt-equity distribution that results from maintaining a fixed market leverage ratio of 17% is shown in table 4.6. The value of the assets in 2003 is 12.375 EUR. For the first four years, a 30% book leverage ratio is used. This input is chosen arbitrarily, and is irrelevant for the valuation outcome since the valuation is based on the years 2007 and onward.
The market leverage ratio is determined through an iteration of the DCF analysis. A starting debt to value ratio is inserted in the WACC calculation leading to a company value. This value is realized with a certain amount of debt at time t is zero. The amount of debt outstanding divided by the enterprise value (from the DCF analysis) gives the implied leverage ratio. This implied leverage ratio is inserted into the WACC calculation to arrive at a new company value and a new implied leverage ratio. This procedure is repeated until the implied debt to value ratio is equal to the leverage ratio inserted in the WACC calculation. This is the final implied market leverage ratio, which is used as an input for the unlevered cost of equity calculation.
The next step is to determine the amount of debt at each point in time. These amounts are determined by calculating the value of the company at each point in time and multiplying this value with the market leverage ratio. Since the value of the company depends on the amount of excess cash, the amount of excess cash on the amount of debt, and the amount of debt on the value of the company, iteration has to take place to find the right values.
The probability of default for Company X is (automatically) determined through the Altman Z-score formula and is estimated at 0.06% (equal to a AA-rating).
The last (relevant) inputs are inserted in the cost of capital calculation. These inputs are shown in table 4.7 and 4.8. The relevant unlevered cost of equity is the ME unlevered cost of equity, since the main assumption in this scenario is that of a constant leverage ratio.

Company X - profit and loss statement												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Net sales	**15.000**	**15.300**	**15.606**	**15.918**	**16.236**	**16.561**	**16.892**	**17.230**	**17.575**	**17.926**	**18.285**	**18.650**
% growth		*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*
Direct cost 1	(7.500)	(7.650)	(7.803)	(7.959)	(8.118)	(8.281)	(8.446)	(8.615)	(8.787)	(8.963)	(9.142)	(9.325)
Total direct costs	(7.500)	(7.650)	(7.803)	(7.959)	(8.118)	(8.281)	(8.446)	(8.615)	(8.787)	(8.963)	(9.142)	(9.325)
Gross profit	**7.500**	**7.650**	**7.803**	**7.959**	**8.118**	**8.281**	**8.446**	**8.615**	**8.787**	**8.963**	**9.142**	**9.325**
% of sales	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*
Operating cost 1	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.080)
Other operating costs					-	-	-	-	-	-	-	-
Total operational costs	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.080)
% of sales	*-26,7%*	*-26,1%*	*-25,6%*	*-25,1%*	*-24,6%*	*-24,2%*	*-23,7%*	*-23,2%*	*-22,8%*	*-22,3%*	*-21,9%*	*-21,9%*
EBITDA	**3.500**	**3.650**	**3.803**	**3.959**	**4.118**	**4.281**	**4.446**	**4.615**	**4.787**	**4.963**	**5.142**	**5.245**
% of sales	*23,3%*	*23,9%*	*24,4%*	*24,9%*	*25,4%*	*25,8%*	*26,3%*	*26,8%*	*27,2%*	*27,7%*	*28,1%*	*28,1%*
Depreciation	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.020)
EBITA	**2.500**	**2.650**	**2.803**	**2.959**	**3.118**	**3.281**	**3.446**	**3.615**	**3.787**	**3.963**	**4.142**	**4.225**
% of sales	*16,7%*	*17,3%*	*18,0%*	*18,6%*	*19,2%*	*19,8%*	*20,4%*	*21,0%*	*21,6%*	*22,1%*	*22,7%*	*22,7%*
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
EBIT	**2.050**	**2.200**	**2.353**	**2.509**	**2.668**	**2.831**	**2.996**	**3.165**	**3.337**	**3.513**	**3.692**	**3.766**
% of sales	*13,7%*	*14,4%*	*15,1%*	*15,8%*	*16,4%*	*17,1%*	*17,7%*	*18,4%*	*19,0%*	*19,6%*	*20,2%*	*20,2%*
Interest on debt	(245)	(264)	(275)	(286)	(325)	(367)	(411)	(457)	(504)	(554)	(605)	(631)
Interest on cash	-	-	-	-	-	-	-	-	-	-	-	-
Profit before tax	**1.805**	**1.936**	**2.078**	**2.223**	**2.343**	**2.463**	**2.585**	**2.708**	**2.833**	**2.959**	**3.088**	**3.136**
Income tax	(692)	(737)	(718)	(717)	(648)	(679)	(710)	(742)	(773)	(806)	(838)	(852)
Net profit	**1.113**	**1.199**	**1.360**	**1.506**	**1.695**	**1.784**	**1.875**	**1.967**	**2.060**	**2.154**	**2.249**	**2.284**
% of sales	*7,4%*	*7,8%*	*8,7%*	*9,5%*	*10,4%*	*10,8%*	*11,1%*	*11,4%*	*11,7%*	*12,0%*	*12,3%*	*12,2%*
Dividend payments	(742)	(799)	(907)	(1.004)	(1.130)	(1.189)	(1.250)	(1.311)	(1.373)	(1.436)	(1.500)	(1.523)
Retained earnings	**371**	**400**	**453**	**502**	**565**	**595**	**625**	**656**	**687**	**718**	**750**	**761**

Table 4.1: Profit & loss statement of Company X

Company X - tangible fixed assets												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Tangible fixed assets begin of period		9.000	9.180	9.364	9.551	9.738	9.938	10.138	10.338	10.548	10.758	10.968
Gross investments tangible fixed assets		1.180	1.184	1.187	1.187	1.200	1.200	1.200	1.210	1.210	1.210	1.020
Book value of disposal of fixed assets	-	-	-	-	-	-	-	-	-	-	-	
Depreciation	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.020)
Tangible fixed assets end of period	**9.000**	**9.180**	**9.364**	**9.551**	**9.738**	**9.938**	**10.138**	**10.338**	**10.548**	**10.758**	**10.968**	**10.968**
% of sales	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	60,0%	58,8%

Company X -intangible fixed assets												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Intangible fixed assets begin of period		3.000	3.060	3.121	3.184	3.247	3.310	3.373	3.438	3.507	3.577	3.648
Gross investments intangible fixed assets		510	511	513	513	513	513	515	519	520	521	523
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
Intangible fixed assets end of period	**3.000**	**3.060**	**3.121**	**3.184**	**3.247**	**3.310**	**3.373**	**3.438**	**3.507**	**3.577**	**3.648**	**3.712**
% of sales	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	20,0%	19,9%

Table 4.2 & 4.3: Tangible fixed assets and intangible fixed assets of Company X

Company X - Balance sheet - Assets												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Tangible fixed assets	**9.000**	**9.180**	**9.364**	**9.551**	**9.738**	**9.938**	**10.138**	**10.338**	**10.548**	**10.758**	**10.968**	**10.968**
Intangible fixed assets	**3.000**	**3.060**	**3.121**	**3.184**	**3.247**	**3.310**	**3.373**	**3.438**	**3.507**	**3.577**	**3.648**	**3.712**
Non-consolidated subsidiaries					-	-	-	-	-	-	-	-
Deferred tax assets					-	-	-	-	-	-	-	-
Other financial fixed assets 1		-	-	-	-	-	-	-	-	-	-	-
Other financial fixed assets 2					-	-	-	-	-	-	-	-
Financial fixed assets	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Stock (Inventory)	375	383	390	398	406	414	422	431	439	448	457	466
Debtors	-	-	-	-	-	-	-	-	-	-	-	-
Other receivables	-	-	-	-	-	-	-	-	-	-	-	-
Prepayments and accrued income	-	-	-	-	-	-	-	-	-	-	-	-
Operational cash	-	-	-	-	-	-	-	-	-	-	-	-
Current assets	**375**	**383**	**390**	**398**	**406**	**414**	**422**	**431**	**439**	**448**	**457**	**466**
Excess cash	**-**	**707**	**679**	**1.466**	**2.036**	**2.633**	**3.270**	**3.951**	**4.643**	**5.380**	**6.155**	**6.895**
ASSETS	**12.375**	**13.329**	**13.554**	**14.599**	**15.427**	**16.295**	**17.203**	**18.158**	**19.137**	**20.163**	**21.229**	**22.042**

Table 4.4: Assets overview of Company X

Company X - Cash flow statement												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
EBITA	**2.500**	**2.650**	**2.803**	**2.959**	**3.118**	**3.281**	**3.446**	**3.615**	**3.787**	**3.963**	**4.142**	**4.225**
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
EBIT	**2.050**	**2.200**	**2.353**	**2.509**	**2.668**	**2.831**	**2.996**	**3.165**	**3.337**	**3.513**	**3.692**	**3.766**
Operational taxes	(776)	(828)	(804)	(802)	(731)	(773)	(815)	(858)	(902)	(947)	(993)	(1.012)
NOPLAT	**1.274**	**1.372**	**1.549**	**1.707**	**1.937**	**2.058**	**2.181**	**2.307**	**2.435**	**2.566**	**2.700**	**2.754**
% growth		*7,7%*	*12,9%*	*10,2%*	*13,4%*	*6,2%*	*6,0%*	*5,8%*	*5,6%*	*5,4%*	*5,2%*	*2,0%*
Change in provisions	-	2	2	2	-	-	-	-	-	-	-	-
Depreciation	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.020
Amortisation	450	450	450	450	450	450	450	450	450	450	450	459
Gross cash flow	**2.724**	**2.824**	**3.001**	**3.159**	**3.387**	**3.508**	**3.631**	**3.757**	**3.885**	**4.016**	**4.150**	**4.233**
Book value of disposal of fixed assets		-	-	-	-	-	-	-	-	-	-	-
Capital gain on disposal of fixed assets (net of tax)		-	-	-	-	-	-	-	-	-	-	-
Investments tangible fixed assets		(1.180)	(1.184)	(1.187)	(1.187)	(1.200)	(1.200)	(1.200)	(1.210)	(1.210)	(1.210)	(1.020)
Investment intangible fixed assets		(510)	(511)	(513)	(513)	(513)	(513)	(515)	(519)	(520)	(521)	(523)
Investments net working capital		(5)	(5)	(5)	(6)	(6)	(6)	(6)	(6)	(6)	(6)	(6)
Operational free cash flow		**1.129**	**1.301**	**1.454**	**1.681**	**1.789**	**1.912**	**2.036**	**2.150**	**2.280**	**2.413**	**2.683**
Extraordinary result (net of tax)		-	-	-	-	-	-	-	-	-	-	-
Result non-consolidated subsidiaries (net of tax)		-	-	-	-	-	-	-	-	-	-	-
Interest on debt (net of tax)		(160)	(194)	(188)	(227)	(240)	(254)	(267)	(282)	(295)	(309)	(323)
Interest on cash (net of tax)		-	-	-	-	-	-	-	-	-	-	-
Cash flow from change in fixed assets		-	-	-	-	-	-	-	-	-	-	-
Change in equity		-	-	-	-	-	-	-	-	-	-	-
Change in debt		546	(231)	534	256	260	263	272	259	266	266	-
Change in debt equivalents		-	-	-	-	-	-	-	-	-	-	-
Financing cash flow		**386**	**(425)**	**346**	**29**	**20**	**9**	**5**	**(23)**	**(29)**	**(43)**	**(323)**
Generated cash		**1.515**	**876**	**1.800**	**1.710**	**1.809**	**1.922**	**2.041**	**2.128**	**2.251**	**2.370**	**2.361**
Dividend payments		(808)	(903)	(1.013)	(1.140)	(1.212)	(1.285)	(1.360)	(1.436)	(1.514)	(1.594)	(1.621)
Change in cash		**707**	**(28)**	**787**	**570**	**597**	**637**	**681**	**692**	**737**	**776**	**740**

Table 4.5: Cash flow statement of Company X

KPMG

Company X - Balance sheet - Equity & Liabilities												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Share capital	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291
Minority interests					-	-	-	-	-	-	-	-
Cumulative retained earnings	-	371	775	1.227	1.733	2.303	2.909	3.551	4.231	4.949	5.706	6.503
Exchange rate differences					-	-	-	-	-	-	-	-
Equity from change financial fixed assets	-	-	-	-	-	-	-	-	-	-	-	-
Change in equity	-	-	-	-	-	-	-	-	-	-	-	-
Retained earnings current year	371	404	452	506	570	606	642	680	718	757	797	810
Equity	**8.662**	**9.066**	**9.518**	**10.024**	**10.594**	**11.200**	**11.842**	**12.522**	**13.240**	**13.997**	**14.794**	**15.604**
Provision 1	100	102	104	106	106	106	106	106	106	106	106	106
Provision 2	-	-	-	-	-	-	-	-	-	-	-	-
Provision 3	-	-	-	-	-	-	-	-	-	-	-	-
Operating provisions	**100**	**102**	**104**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**
Bank loans	3.500	4.046	3.815	4.349	4.605	4.865	5.128	5.400	5.659	5.925	6.191	6.191
Bonds	-	-	-	-	-	-	-	-	-	-	-	-
Other long-term interest bearing debt	-	-	-	-	-	-	-	-	-	-	-	-
Long-term liabilities	**3.500**	**4.046**	**3.815**	**4.349**	**4.605**	**4.865**	**5.128**	**5.400**	**5.659**	**5.925**	**6.191**	**6.191**
Preferred equity	-	-	-	-	-	-	-	-	-	-	-	-
Long-term operating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Nonoperating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Debt equivalents	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Credit institutions					-	-	-	-	-	-	-	-
Creditors	113	115	118	120	122	124	127	129	132	134	137	140
Taxation & social security	-	-	-	-	-	-	-	-	-	-	-	-
Other liabilities	-	-	-	-	-	-	-	-	-	-	-	-
Accruals and deferred income	-	-	-	-	-	-	-	-	-	-	-	-
Current liabilities	**113**	**115**	**118**	**120**	**122**	**124**	**127**	**129**	**132**	**134**	**137**	**140**
EQUITY & LIABILITIES	**12.375**	**13.329**	**13.554**	**14.599**	**15.427**	**16.295**	**17.203**	**18.157**	**19.137**	**20.162**	**21.228**	**22.041**

Table 4.6: Balance sheet of Company X

Company X - Weighted Average Cost of Capital			
Risk free rate	4,0%	Risk free rate	4,0%
Market risk premium	5,0%	Spread	3,0%
Additional risk premium	3,0%	Tax rate	25,5%
Unlevered beta peer group	0,83		
D/E	20%		
Levered beta	1,00		
Cost of equity	**12,0%**	**After-tax cost of debt**	**5,2%**
E/(E+D)	83%	D/(E+D)	17%
WACC			**10,8%**

Table 4.7: WACC calculation of Company X

Company X - Unlevered cost of equity (ME)			
Risk free rate	4,0%	Risk free rate	4,0%
Market risk premium	5,0%	Spread	3,0%
Additional risk premium	3,0%	Tax rate	25,5%
Unlevered beta peer group	0,83		
Unlevered cost of equity	**11,2%**	**Cost of debt**	**7,0%**
		After-tax cost of debt	**5,2%**
D/E	20%		
E/(E+D)	83%		

Table 4.8: Unlevered cost of equity calculation of Company X

These inputs result in the following valuation outcomes:

Scenario 1 **in EUR x1000**	DCF analysis	APV analysis
Enterprise value	25.626	25.767
Shareholder value	21.277	21.418
Abs. difference in EV	141	
Rel. difference in EV	0,55%	

The valuation outcome of the APV analysis can be divided into the following four components:
- Unlevered company value: 23.303 EUR
- Value of interest tax shields: 999 EUR
- Costs of financial distress: 2 EUR
- Excess cash: 1.466 EUR

These results show that the theory on scenario 1, stated in chapter 3, holds.

4.2.2 Scenario 2: Constant leverage ratio & significant probability of default

This second scenario is identical to scenario 1 accept for the difference in the probability of default. This results in a difference in the costs of financial distress. In scenario 1, Company X received a AA-rating based on its Z-score. In this scenario, the probability of default is manually adjusted to a CCC-rating, which results in a probability of default of 17.20%. The costs of financial distress increase, as a result of the increased probability of default, to 601 EUR (an increase of 599 EUR). The valuation outcomes after this adjustment are as follows.

Scenario 2 **in EUR x1000**	DCF analysis	APV analysis
Enterprise value	25.626	25.168
Shareholder value	21.277	20.819
Abs. difference in EV	-458	
Rel. difference in EV	-1,79%	

This result supports the theory stated in chapter 3 that in case of a significant probability of default, the valuation outcome under the APV model is lower than the valuation outcome under the enterprise DCF model.

4.2.3 Scenario 3: Fixed amount of debt

The main assumption in the third scenario is that the company under valuation maintains a fixed amount of debt. Debt, in this case, refers to interest-bearing debt. The amount of interest-bearing debt has to be fixed so that the interest tax shields are known at each point in time.
To simulate this scenario, the financial data in the equity & liabilities overview of Company X are adjusted. The bank loans are fixed at the 2006 value (which is 4.349 EUR). All the other inputs remain the same as in the previous scenarios. The valuation outcomes for the fixed the amount of debt scenario are as follows.

Scenario 3 in EUR x1000	DCF analysis	APV analysis
Enterprise value	25.626	25.375
Shareholder value	21.277	21.026
Abs. difference in EV	-251	
Rel. difference in EV	-0,98%	

This result supports the theory stated in chapter 3 that, in case of a fixed amount of debt and an increasing amount of capital, the enterprise DCF model overestimates the benefits of debt. The enterprise DCF model assumes a constant leverage ratio, and since the total amount of capital increases, the amount of debt is also assumed to grow. However, this is not correct since the amount of debt is fixed. The enterprise DCF model therefore gives an incorrect valuation.
If the Company X would have a CCC-rating, instead of an AA-rating (based on the Z-score), then the difference between the enterprise DCF model and the APV model would increase to 3.27%. This effect is also as stated in chapter 3.

The theory in chapter 3 on the difference in valuation outcome between the enterprise DCF model and the APV model under scenario 3 also stated that in case of a decreasing amount of total capital, the APV model would give a higher company value than the enterprise DCF model if the probability of default was not significant. This is caused by the fact that the enterprise DCF model underestimates the benefits of debt.
The financial data of Company X have to be adjusted at a number of points to create a scenario in which the amount of debt is fixed and the total amount of capital is decreasing. These adjustments are the following:

- Sales growth rate of -1,0%.
- Depreciation increased to 2.000 EUR.
- Non-tax deductible amortization reduced to 100 EUR for the year 2003-2011 and to zero for 2012 and onward.
- Tax deductible amortization increased to 500 EUR for the years 2003-2011 and to 600 for 2012 and onward.
- Tangible fixed assets value remained constant at 9.000 EUR through investments in these assets.
- Intangible fixed assets value remained constant at 3.000 EUR through investments in these assets.
- Fixed amount of share capital with a value of 7.628 EUR.
- Dividend payout ratio set to 100%.
- Amount of debt fixed at 4.500 EUR.

These changes result in an overall decrease in the amount of total capital of 1,9% over the period 2003 – 2013. The valuation outcomes after these changes are shown in the table below.

Scenario 3b in EUR x1000	DCF analysis	APV analysis
Enterprise value	1.930	2.694
Shareholder value	-2.570	-1.806
Abs. difference in EV	764	
Rel. difference in EV	39,59%	

This shows that the enterprise DCF model underestimates the benefits of debt in case of scenario where the total amount of debt is fixed and the amount of total capital is decreasing. This is in line with the theory stated in chapter 3.

4.2.4 Scenario 4: Finite life & known amount of debt

The financial data of Company X in scenario 4 are identical to those in scenario 3 but with an amount of debt in 2003 of 3.500 EUR. Company X is assumed to pay down its debt in the 10 years after 2003. This means that the amount of debt is each year reduced by 350 EUR. The equity & liabilities overview that results from these adjustments is shown in table 4.9.
After inserting these adjustments to the basic scenario, the enterprise value of the company becomes as follows.

Scenario 4 **in EUR x1000**	DCF analysis	APV analysis
Enterprise value	9.225	9.123
Shareholder value	6.775	6.673
Abs. difference in EV	-102	
Rel. difference in EV	-1,11%	

These findings support the theory stated in chapter 3 on the difference in valuation outcome between the two valuation models under this scenario. The amount of capital is growing, so when using the enterprise DCF model, an increased benefit of debt is assumed. However, the amount of debt outstanding is decreased over time to zero in 2013. This means that the enterprise DCF model overestimates the benefits of debt which causes the valuation outcome under the enterprise DCF model to be higher than the valuation outcome under the APV model. This was predicted in chapter 3.
If the Company X would have a CCC-rating, instead of an A-rating (based on the Z-score), then the difference between the enterprise DCF model and the APV model would increase to 3.72%. This effect is also as stated in chapter 3.

4.2.5 Scenario 5: Debt known up to time t, then a constant leverage ratio

In this scenario, Company X is assumed to start with an amount of debt of 9.000 in 2003 (which gives a book leverage ratio of 65%). This could for instance be the case after a LBO or MBO. Company X is also assumed to have a constant leverage ratio from 2011 and onward. The other input financials are the same as prescribed in the validation approach paragraph. The equity & liabilities overview of this scenario is shown in table 4.10.

Company X - Balance sheet - Equity & Liabilities												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Share capital	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291
Minority interests					-	-	-	-	-	-	-	-
Cumulative retained earnings	-	371	775	1.241	1.764	2.367	3.016	3.713	4.458	5.251	6.094	6.988
Exchange rate differences					-	-	-	-	-	-	-	-
Equity from change financial fixed assets	-	-	-	-	-	-	-	-	-	-	-	-
Change in equity	-	-	-	-	-	-	-	-	-	-	-	-
Retained earnings current year	371	404	466	523	603	649	697	745	794	843	894	918
Equity	**8.662**	**9.066**	**9.532**	**10.055**	**10.658**	**11.307**	**12.004**	**12.749**	**13.542**	**14.385**	**15.279**	**16.197**
Provision 1	100	102	104	106	106	106	106	106	106	106	106	106
Provision 2	-	-	-	-	-	-	-	-	-	-	-	-
Provision 3	-	-	-	-	-	-	-	-	-	-	-	-
Operating provisions	**100**	**102**	**104**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**
Bank loans	3.500	3.150	2.800	2.450	2.100	1.750	1.400	1.050	700	350	-	-
Bonds	-	-	-	-	-	-	-	-	-	-	-	-
Other long-term interest bearing debt	-	-	-	-	-	-	-	-	-	-	-	-
Long-term liabilities	**3.500**	**3.150**	**2.800**	**2.450**	**2.100**	**1.750**	**1.400**	**1.050**	**700**	**350**	**-**	**-**
Preferred equity	-	-	-	-	-	-	-	-	-	-	-	-
Long-term operating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Nonoperating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Debt equivalents	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Credit institutions					-	-	-	-	-	-	-	-
Creditors	113	115	118	120	122	124	127	129	132	134	137	140
Taxation & social security	-	-	-	-	-	-	-	-	-	-	-	-
Other liabilities	-	-	-	-	-	-	-	-	-	-	-	-
Accruals and deferred income	-	-	-	-	-	-	-	-	-	-	-	-
Current liabilities	**113**	**115**	**118**	**120**	**122**	**124**	**127**	**129**	**132**	**134**	**137**	**140**
EQUITY & LIABILITIES	**12.375**	**12.433**	**12.553**	**12.731**	**12.986**	**13.288**	**13.637**	**14.034**	**14.480**	**14.976**	**15.522**	**16.443**

Table 4.9: Scenario 4 Equity & liabilities overview

Company X - Balance sheet - Equity & Liabilities												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Share capital	3.860	3.860	3.860	3.860	3.860	3.860	3.860	3.860	3.860	3.860	3.860	3.860
Minority interests					-	-	-	-	-	-	-	-
Cumulative retained earnings	-	302	637	1.034	1.488	2.020	2.602	3.233	3.916	4.649	5.422	6.234
Exchange rate differences					-	-	-	-	-	-	-	-
Equity from change financial fixed assets	-	-	-	-	-	-	-	-	-	-	-	-
Change in equity	-	-	-	-	-	-	-	-	-	-	-	-
Retained earnings current year	302	335	396	454	533	582	631	682	734	772	812	825
Equity	**4.162**	**4.497**	**4.894**	**5.348**	**5.880**	**6.462**	**7.093**	**7.776**	**8.509**	**9.282**	**10.094**	**10.919**
Provision 1	100	102	104	106	106	106	106	106	106	106	106	106
Provision 2	-	-	-	-	-	-	-	-	-	-	-	-
Provision 3	-	-	-	-	-	-	-	-	-	-	-	-
Operating provisions	**100**	**102**	**104**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**
Bank loans	8.000	7.500	7.000	6.500	6.000	5.500	5.000	4.500	4.774	5.048	5.321	5.321
Bonds	-	-	-	-	-	-	-	-	-	-	-	-
Other long-term interest bearing debt	-	-	-	-	-	-	-	-	-	-	-	-
Long-term liabilities	**8.000**	**7.500**	**7.000**	**6.500**	**6.000**	**5.500**	**5.000**	**4.500**	**4.774**	**5.048**	**5.321**	**5.321**
Preferred equity	-	-	-	-	-	-	-	-	-	-	-	-
Long-term operating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Nonoperating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Debt equivalents	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Credit institutions					-	-	-	-	-	-	-	-
Creditors	113	115	118	120	122	124	127	129	132	134	137	140
Taxation & social security	-	-	-	-	-	-	-	-	-	-	-	-
Other liabilities	-	-	-	-	-	-	-	-	-	-	-	-
Accruals and deferred income	-	-	-	-	-	-	-	-	-	-	-	-
Current liabilities	**113**	**115**	**118**	**120**	**122**	**124**	**127**	**129**	**132**	**134**	**137**	**140**
EQUITY & LIABILITIES	**12.375**	**12.214**	**12.115**	**12.074**	**12.108**	**12.192**	**12.326**	**12.511**	**13.521**	**14.570**	**15.658**	**16.486**

Table 4.10: Scenario 5 Equity & liabilities overview

The calculation of the enterprise value of Company X through the APV model has to be manually adjusted to match the situation at hand. The years 2007-2010 are modeled according to the MM APV model, since the amount of interest-bearing debt outstanding is assumed to be known in these years. For the years 2011 and onward, the ME APV model is used, since a constant leverage ratio is assumed.
As a result of this split in the valuation, the free cash flows for the years 2007-2010 have to be discounted at the MM unlevered cost of equity and the free cash flows for 2011 and onward have to be discounted at the ME unlevered cost of equity.
Also, the interest tax shields for 2007-2010 have to be discounted at the cost of debt and the interest tax shields for 2011 and onward have to be discounted at the ME unlevered cost of equity. This results in the following valuation outcomes.

Scenario 5 **in EUR x1000**	DCF analysis	APV analysis
Enterprise value	23.138	23.248
Shareholder value	16.638	16.748
Abs. difference in EV	110	
Rel. difference in EV	0,48%	

The leverage ratio inserted in the WACC calculation is 17,2%. This is the assumed value of the constant leverage ratio in the period 2011 and onward.
The implied leverage ratio at time t is zero is 28,1%. This means that Company X is currently not at its target leverage ratio. This causes the DCF analysis to underestimate the benefits of debt. This leads to a valuation outcome of the DCF analysis that is lower than the (correct) APV model valuation outcome.

The difference in valuation outcomes that is found is in line with the theory stated in chapter 4 for this scenario. The leverage ratio decreases and causes the enterprise DCF model to underestimate the benefits of debt. However, the leverage ratio is assumed to be constant from 2011 and onward, which causes the error in the valuation outcome to stabilize.

When manually adjusting the credit rating from the A-rating (based on the Z-score) to a CCC-rating, the valuation outcome of the APV analysis is 2,11% lower than the valuation outcome of the DCF analysis. This shows that the effects of underestimating the benefits of debt and underestimating the costs of financial distress by the enterprise DCF model can cancel each other out. If Company X would have a BB-rating, the difference in valuation outcomes would be reduced to 0,06%.

4.3 Sensitivity analysis

The validation of the basic scenarios in the previous paragraph is based on a fictitious company. The financial data that form the basis for the valuation are artificial. Therefore, the validation outcomes might be biased by the validation approach chosen. This paragraph discusses the effects of changing a number of input variables to give some indication of the sensitivity of the validation outcomes to the validation approach that was chosen.

The five variables of which the influence on the difference in valuation outcomes is analyzed are the levered beta, the cost of debt, the market leverage ratio, the amount of sales in 2003 and the tax rate. The influence of these variables is tested for both scenario 1 and 3.

4.3.1 Levered beta

The levered beta has a standard value of 1,0 in the validation approach. Adjusting the levered beta has practically no effect on the relative difference between the enterprise DCF model and the APV model valuation outcome for both scenario 1 & 3, as is shown in the tables below.

S1	Beta	*0,1*	*0,4*	*0,7*	*1*	*1,3*	*1,65*	*2*
	Abs. Diff	256	200	164	141	125	111	102
	Rel. Diff	0,59%	0,57%	0,56%	0,55%	0,55%	0,54%	0,55%
S3	Beta	*0,1*	*0,4*	*0,7*	*1*	*1,3*	*1,65*	*2*
	Abs. Diff	-429	-346	-291	-251	-219	-191	-166
	Rel. Diff	-0,99%	-0,99%	-0,99%	-0,98%	-0,96%	-0,94%	-0,90%

This means that the validation outcomes are independent of the levered beta inserted in the validation approach.

4.3.2 Cost of debt

The validation outcomes are also insensitive to the cost of debt that is chosen in the validation approach (standard value of the cost of debt is 7,0%). The size of the relative difference between the two valuation outcomes is affected by a change in the cost of debt under scenario 1, however, the sign of the difference remains the same and the relative difference stays under 1% as shown in the tables below.

S1	Kd	4%	5%	6%	7%	8%	9%	10%
	Abs. Diff	123	132	138	141	144	146	148
	Rel. Diff	0,34%	0,41%	0,49%	0,55%	0,62%	0,68%	0,74%
S3	Kd	4%	5%	6%	7%	8%	9%	10%
	Abs. Diff	-357	-313	-278	-251	-227	-209	-190
	Rel. Diff	-0,98%	-0,98%	-0,98%	-0,98%	-0,97%	-0,97%	-0,96%

This means that the validation outcomes are independent of the cost of debt inserted in the validation approach.

4.3.3 Tax rate

The value of the tax rate used as an input in the validation approach has no effect on the sign of the relative difference between the two valuation outcomes. It does have an effect on the size of the relative difference. This effect, for both scenario 1 & 3, is shown in the tables below.

S1	Tax rate	*10,0%*	*15,0%*	*20,0%*	*25,5%*	*30,0%*	*35,0%*	*40,0%*
	Abs. Diff	51	80	108	141	170	202	237
	Rel. Diff	0,17%	0,28%	0,40%	0,55%	0,70%	0,89%	1,11%
S3	Tax rate	*10,0%*	*15,0%*	*20,0%*	*25,5%*	*30,0%*	*35,0%*	*40,0%*
	Abs. Diff	-101	-149	-198	-251	-292	-337	-381
	Rel. Diff	-0,33%	-0,52%	-0,73%	-0,98%	-1,20%	-1,48%	-1,79%

This shows that the validation outcomes are independent of the tax rate selected as an input in the validation approach.

4.3.4 Market leverage ratio

The sensitivity of the validation outcomes to the value of the market leverage ratio inserted in the validation approach shows the same pattern as the sensitivity to the tax rate inserted in the validation approach: the sign of the relative difference in valuation outcomes is insensitive to the market leverage ratio chosen, but the size of the relative difference does change when a different market leverage ratio is inserted in the validation approach. The results of the sensitivity analysis are shown below.

S1	D/V Ratio	*4,42%*	*8,58%*	*12,52%*	*16,97%*	*21,62%*	*26,30%*	*34,94%*
	Abs. Diff	23	49	78	141	190	219	356
	Rel. Diff	0,10%	0,21%	0,33%	0,55%	0,72%	0,82%	1,24%

S3	D/V Ratio	*4,42%*	*8,58%*	*12,52%*	*16,97%*	*23,12%*	*27,83%*	*32,20%*
	Abs. Diff	-64	-131	-200	-251	-428	-549	-692
	Rel. Diff	-0,28%	-0,56%	-0,83%	-0,98%	-1,65%	-2,04%	-2,48%

This shows that the validation outcomes are independent of the market leverage ratio selected as an input in the validation approach.

4.3.5 Amount of sales in 2003

For the previous four variables that were discussed, the validation outcome was insensitive to the values of the variables inserted in the validation approach. The last variable discussed in this paragraph, the amount of sales in 2003, has a different effect on the validation outcome. The validation outcome is sensitive to low values of the amount of sales in 2003.

The amount of sales is assumed to grow each year with a fixed percentage, as do the direct costs that are assumed to remain at 50% of sales. The other costs, such as operating costs and depreciation are assumed to remain constant and thus cause the free cash flow to grow. The growth of the free cash flow is what ultimately affects the valuation outcome.

In case of scenario 3, the relative difference in the valuation outcomes of the APV model and the enterprise DCF model increases significantly for lower values of the amount of sales in 2003 (below 12.500). The results of the sensitivity analysis are shown in the table below.

S3	Sales '03	*11.000*	*12.500*	*15.000*	*20.000*	*25.000*	*30.000*
	Abs. Diff	-779	-370	-251	-187	-164	-153
	Rel. Diff	-9,08%	-2,49%	-0,98%	-0,39%	-0,24%	-0,17%

This shows that the chosen value for the amount of sales in 2003 that is inserted in the validation approach under scenario 3 has an effect on the size of the relative difference between the valuation outcomes. However, the sign of the relative difference stays the same.

The situation becomes more peculiar under scenario 1. The costs of financial distress, that lower the valuation outcome under the APV model, increase stepwise. This results in the fact that the relative difference in valuation outcomes is positive (that is the APV model gives a higher valuation outcome than the enterprise DCF model) for an amount of sales in 2003 of 11.100 EUR and negative for an amount of sales in 2003 of 11.000 EUR. These results are shown in the table below.

S1	Sales '03	*11.000*	*11.100*	*11.500*	*12.500*	*15.000*	*20.000*
	Abs. Diff	-59	41	20	116	141	153
	Rel. Diff	-0,69%	0,46%	0,19%	0,78%	0,55%	0,32%
	PoD	17%	8%	8%	1%	0%	0%

“PoD” stands for the probability of default based on the Altman Z-score. These results show that the sign of the validation outcome is sensitive to the value of the amount of sales in 2003.

Conclusion

The purpose of this chapter is to test the theories stated in chapter 3. These theories are formulated as scenarios under which a difference between the APV model valuation outcome and the enterprise DCF model valuation outcome should occur. The table below shows these scenarios and their predicted results.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

Based on the validation approach chosen, scenarios 2 to 5 were validated. Theory 1 was only weakly validated.
A difference of 0,55% between the two valuation outcomes under scenario 1 was found, while the theory in chapter 3 stated that the two models should give the same valuation outcome. One of the causes of the nonzero difference in scenario 1 is the growth rate of both the free cash flows and the interest tax shields inserted in the validation approach. The free cash flows and interest tax shields are assumed to have a yearly growth rate of 2,0% in perpetuity. The difference in valuation outcomes can be reduced by setting the growth rate to the value of the growth implied by the free cash flows and interest tax shields. For instance, the free cash flow is assumed to be 2.413 EUR in 2013 and 2.683 EUR in the time period after 2013 (this is the input free cash flow for the calculation of the terminal value). This change in free cash flow implies a 11,2% growth rate. The difference between the valuation outcomes under scenario 1 of the APV model and the enterprise DCF model is reduced to 0.12% if the growth rate of the free cash flows and interest tax shields is set to 11,2%.
However, assuming an 11,2% growth rate in perpetuity is not very realistic since the change in free cash flow between the last year of the forecast period and the input for the terminal value calculation is caused by a couple of modeling decision that allow for the determination of this terminal value. This is also indicated by the fact that the average growth in free cash flow in the years 2007-2013 is 6%. Therefore, the growth rate of the free cash flows and interest tax shields is assumed to be equal to the growth in sales, which gives a more realistic long-term view of the expected free cash flow and interest tax shield growth.
As a result of this decision, the difference between the valuation outcome of the APV model and the enterprise DCF model is nonzero. However, since the difference is very small (0,55%), theory 1 is assumed to be validated by the test performed in this chapter.

The test of scenario 3b shows the strongest validation result: a difference between the valuation outcomes of the two models of 40%. The other tests show validation results that validate the theories stated in chapter 3, but these results are less significant than the result from the test of scenario 3b.

The validation outcomes are insensitive to the levered beta, the cost of debt, the market leverage ratio and the tax rate inserted in the validation approach for both scenario 1 and 3. The validation outcome of scenario 1 is sensitive to the amount of sales in 2003 inserted in the validation approach. Under scenario 3, inserting a low value for the amount of sales in 2003 results in a larger difference between the valuation outcomes of the two models (difference increases from 0,98% for 15.000 EUR of sales in 2003 to 9,08% for 11.000 EUR of sales in 2003). However, the sign of the difference remains the same which means that theory 3 is validated regardless of the validation approach chosen.
Under scenario 1, the difference between the valuation outcomes of the two models changes sign for an inserted amount of sales in 2003 of around 11.000 EUR (11.000 EUR gives a difference of 0,69% and 11.100 EUR gives a difference of 0,46%). This change in sign is caused by the stepwise increase of the costs of financial distress. However, the difference between valuation outcomes stays within plus or minus 1%. Since theory 1 states that in case

of a constant leverage ratio both valuation models should give the same valuation outcome, a change in sign of the difference is not damaging the validity of theory 1.

The theories on the difference between the valuation outcomes of the APV model and the enterprise DCF model under five different scenarios are found to be valid, based on the results from the tests discussed in this chapter. The implications of these validated differences for the decision on which valuation model to use in which situation will be discussed in chapter 5.

Chapter 5: Conclusion

Introduction

The research objective of this research project, as stated in chapter 1, is:

To formulate a theory on the differences between the enterprise DCF model and the APV model, and the effects of these differences on the valuation outcome *by analyzing* the basic assumptions of both models, the circumstances in which either one should be used, the impact of the non-constant capital structure and the non-zero costs of financial distress on the valuation outcome under the APV model, and the way in which these two factors can be modeled to obtain the most accurate valuation outcome.

The purpose of this chapter is to conclude on the extent that the research objective has been met. This is done by discussing the differences found between the enterprise DCF model and the APV model and by commenting on their validity based on the validation results from chapter 4. The situational conditions of the differences in the valuation outcomes are discussed in the second paragraph. The implications of the validated conditional differences are discussed in the third paragraph.
The chapter ends with a paragraph on the further research suggestions that come forth from the overall results of the analysis of the differences between the enterprise DCF model and the APV model and the theoretical framework that was formulated in chapter 2, which functioned as a means for the analysis.

5.1 Overview of (valid) differences

There are a number of differences between the enterprise DCF model and the APV model. These differences are summarized in the table below.

	Enterprise DCF model	**APV model**
Cash flow	FCFF	FCFF
Discount rate	WACC	Ku or Kd
Cost of equity	Levered	Unlevered
Cost of debt	Credit rating based	Credit rating based
Capital structure	Constant leverage ratio	Constant leverage ratio or fixed debt
Probability of default	Not explicitly taken into account	Separate term in the valuation
Costs of financial distress	Not explicitly taken into account	Separate term in the valuation

The impact of the differences on the valuation outcome is based on the difference in capital structure assumption and the inclusion of the costs of financial distress in the valuation models. The enterprise DCF model is assumed to give a correct valuation in the case of a company that maintains a constant leverage ratio over time and that has no significant probability of default. The enterprise DCF model gives an incorrect company value if one of these assumptions is violated.
There are five scenarios in which the APV model can be used to determine the correct value of the company, if the capital structure and/or the probability of default assumption of the enterprise DCF model are violated. The expected differences in valuation outcomes are shown in the table below.

Scenario	Model	Compare to	Effect on valuation outcome
Constant leverage ratio & **no** significant PoD	DCF or APV (ME)	n.a.	V (APV) = V (DCF)
Constant leverage ratio & significant PoD	APV (ME)	DCF	V (APV) < V (DCF)
Fixed amount of debt	APV (MM)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Finite life & debt known	APV (general)	DCF	V (APV) > V (DCF) or V (APV) = V (DCF) or V (APV) < V (DCF)
Debt known up to t, then constant leverage ratio	APV (general) + APV (ME)	DCF	Combination of scenario 4 and 1 or a combination of scenario 4 and 2

The five scenarios were simulated in chapter 4 to test whether the stated effects on the valuation outcome do indeed occur. The outcome of the tests is that all the predicted effects on the valuation outcome are validated. This means that the following theories can be stated with regard to the effect of the differences between the APV model and the enterprise DCF model on the difference in valuation outcomes.

Theory 1: If the company under valuation is assumed to maintain a constant leverage ratio and if the company has no significant probability of default, then the valuation outcomes of both models are the same.

Theory 2: If the company under valuation is assumed to maintain a constant leverage ratio and if the company has a significant probability of default, then the enterprise DCF model valuation outcome is incorrect and higher than the correct valuation outcome of the APV model.

Theory 3: If the company is assumed to maintain a fixed amount of debt, then the enterprise DCF model overestimates the benefits of debt in case of an increasing total amount of capital and underestimates the benefits of debt in case of a decreasing total amount of capital. This error in the estimation of the benefits of debt is complemented by a potential error in the estimation of the costs of financial distress. If the company under valuation has a significant probability of default, then the APV model correctly includes a separate term for these costs whereas the enterprise DCF model ignores these costs. Combining the errors in the estimation of the benefits of debt and the estimation of the costs of financial distress result in the total error in the company value determined through the enterprise DCF model compared to the correct company value determined by the APV model.

Theory 4: If the company under valuation is assumed to have a finite life and a known amount of debt at each point in time, then error in the valuation outcome of the enterprise DCF model depends on the difference between the amount of debt assumed by the enterprise DCF model and the actual amount of debt at each point in time. The error in the estimation of the benefits of debt has to be added to the error that results from ignoring the costs of financial distress when the company under valuation has a significant probability of default.

Theory 5: If the company under valuation is assumed to have a known amount of debt at each point in time up to time t and if the company is assumed to have a constant leverage ratio from time t and onward, then the difference between the valuation outcomes of both models is based on a combination of theory 4 and 1 or on a combination of theory 4 and 2. The combination of theory 4 and 1 implies that the error in the valuation outcome of the enterprise DCF model is based on the incorrect estimation of the benefits of debt over the time period up to time t since the enterprise DCF model gives a correct value for the period after t in which the company maintains a constant leverage ratio and has no significant probability of default as is stated in theory 1. The combination of theory 4 and 2 implies that the error in the valuation outcome of the enterprise DCF model is based on the incorrect estimation of the benefits of debt over the time period up to time t and on the fact that the costs of financial distress are ignored regardless of the significant probability of default of the company under valuation.

5.2 Situational conditions of valuation outcome differences

The valid differences discussed in the previous paragraph occur under ideal circumstances. This paragraph discusses which factors could cause the valuation outcomes to deviate from the theoretically stated values. This paragraph also addresses the extent of the deviation caused by each factor.

In paragraph 4.3, the sensitivity of the validation outcomes to five different variables was analyzed. These five variables are the tax rate, the cost of debt, the market leverage ratio, the levered beta and the amount of sales. The result of the sensitivity analysis was that the effect on the valuation outcome under each scenario was insensitive to changes in one of the five variables. This means that each of the five theories regarding the effects on the difference between the valuation outcomes of the APV model and the enterprise DCF model holds for any reasonable value of the five variables.

The *extent* of the effect on the valuation outcome is sensitive to some of the five variables. The levered beta and cost of debt have no influence on the difference between the valuation outcomes. However, the tax rate, market leverage ratio and the amount of sales do influence the size of the difference between the valuation outcomes. The difference in valuation outcomes increases for higher values of the tax rate and market leverage ratio and for lower values of the amount of sales in case of an assumed fixed amount of debt (scenario 3).
This can be explained as follows. The difference between the valuation outcomes of the two valuation models under scenario 3 is caused by the fact that the enterprise DCF model assumes a growing amount of debt in case of an increasing amount of total capital. The amount of debt outstanding is however fixed. This causes an overestimation of the benefits of debt by the enterprise DCF model. A higher tax rate and/or a higher market leverage ratio amplifies the overestimation of the amount of debt: if the market leverage ratio is 50% instead of 20%, each increase in total capital is assumed to give 2,5 times more benefits of debt and thus a 2,5 times larger error in the valuation outcome of the enterprise DCF model.
The influence of the amount of sales is a case on its own. Suppose that the company under valuation is first assumed to have a fixed amount of debt of 1.000 EUR and sales in the amount of 5.000 EUR. This amount of sales generates free cash flow and this leads to an enterprise value. The implied market leverage ratio is based on the ratio of the amount of debt to the enterprise value. Now suppose that the amount of sales is reduced to 3.000 EUR. This means that the free cash flow decreases and so does the enterprise value. The implied market leverage ratio increases since the amount of debt is assumed to be fixed. The change in the implied market leverage ratio is larger for low enterprise values than for high enterprise values and thus larger for low amounts of sales. These high implied market leverage ratios are the cause of the error in the valuation outcome of the enterprise DCF model for low amounts of sales.

The enterprise DCF model assumes a constant leverage ratio and no significant probability of default. If one of these two assumptions is violated, then the valuation outcome of the enterprise DCF model is incorrect. The difference in valuation outcomes depends on the capital structure and the probability of financial distress of the company under valuation. These differences where discussed in the previous paragraph. The tests performed in chapter 4 showed that the stated differences are valid. The validity of the theories that predict the difference in valuation outcomes does not depend on the cost of debt, tax rate, levered beta, market leverage ratio and the amount of sales, as was also shown in chapter 4. The size of the difference however does depend on the market leverage ratio, tax rate and (indirect) on the amount of sales. A higher tax rate and market leverage ratio amplifies the difference and so does a lower amount of sales. This means that it becomes more important to use the right model in each situation when the market leverage ratio and tax rate are high and the amount of sales are low. The actual value of the size of the difference in valuation outcomes depends on the characteristics of the company under valuation.

It is important to note that the conclusion on the validity of the differences between the valuation outcomes of the APV model and the enterprise DCF model under different scenarios are based on tests on the valuation of a single, fictitious company. This means that is unrealistic to state that the differences formulated are generally correct. However, the stated theories on the differences between the valuation outcomes are based a multitude of sources and have not be rejected by the tests performed in chapter 4.

The next paragraph discusses the implication of the differences between the two valuation models and respectively the effect of these differences on the valuation outcomes for the decision on the usage of either the enterprise DCF model or the APV model under certain circumstances.

5.3 Implications of the differences between the two valuation models

The first paragraph of this chapter discussed the valid differences between the enterprise DCF model and the APV model and the effect of those differences on the valuation outcome for a number of scenarios. As was shown in chapter 4, the difference between the valuation outcomes under the enterprise DCF model and the APV model is in some cases significant (40% under scenario 3 with a declining amount of total capital) and in other cases negligible.

The function of the APV model for the valuation of companies can therefore be formulated as follows:

- In certain cases the APV model is theoretically more correct and also gives a significantly different valuation outcome than the enterprise DCF model. These cases are (with the difference in value between brackets):
    1. Companies with a fixed amount of debt and a decreasing amount of total capital (40%).
    2. Companies with a fixed amount of debt and low amounts of sales (9%).
    3. Companies that maintain a constant leverage ratio and that have a significant probability of default (2%).

  These three cases have in common that the company under valuation is in some sort of financial distress: the total amount of capital decreases, which can be caused by declining sales, high fixed costs and high dividend payout ratios; the amounts of sales are low, which combined with significant fixed costs result in low profits and low free cash flow; and a significant probability of default, which also indicates financial distress. *Based on this mutual aspect of the three cases, the APV model seems to be especially appropriate for the valuation of a company that suffers from financial distress.*
- In other cases, the APV model is theoretically more correct but the difference in valuation outcome with regard to the enterprise DCF model is negligible. These cases are (with the difference in value between brackets):
    1. Companies with a known amount of debt up to a certain point in time and with a constant leverage ratio from that point onward (0,5%).
    2. Companies with a finite life and known amounts of debt at each point in time (1%).
    3. Companies with a fixed amount of debt and high amounts of sales (1%).

  The small differences between the valuation outcomes in the first two cases are caused by the influence of the terminal value on the enterprise value. In the first case, the assumptions of the enterprise DCF model are not violated after point t, which means that the error in the valuation outcome is only created by the years up to point t. The terminal value, which contributes a major part of the enterprise value, does not contain an error. In the second case, the company is assumed to have a finite life. Since the error in the enterprise DCF model valuation outcome accumulates over the years, the size of the error remains negligible for companies with a short, finite life.
  The difference between the valuation outcomes for the first two cases thus depends on the relation between the forecast period and the terminal value. The longer the forecast period, the larger the difference. This means that it at a certain length of the forecast period, the difference between the valuation outcomes of the APV model and the enterprise DCF model becomes significant.

  As a result of the nonsignificant difference, the APV model and the enterprise DCF model can both be used to value the company even though the APV model gives a theoretically more correct company value. The choice for either the APV model or the enterprise DCF

model should be based on the information requirements of the valuer and the quality of the available input information required by each of the models.
The practical situations that relate to the cases in which the difference becomes insignificant are that of a management or leverage buyout (the first case) and project finance (the second case). For these cases both the enterprise DCF model and the APV model can be used if the period in which the enterprise DCF model assumptions are violated is small.

- In the remaining cases, either both the enterprise DCF model and the APV model can be used to determine the company value or none of the two models is fit for the determination of the company value.

Note however that from a practical point of view that the valuation tool discussed in appendix II automatically calculates the company value through both the enterprise DCF model and the APV model after the insertion of the required (general) input data. This means that the valuer does not have to decide which model he chooses to use. The decision can be postponed up to the point where both valuation outcomes are known.

5.4 Further research suggestions

The suggestions for further research focus on two topics:

1. The estimation of the costs of financial distress.
2. The determination of the exact differences in valuation outcomes under different circumstances.

5.4.1 Costs of financial distress

The costs of financial distress are often ignored in discussion of the adjusted present value in the corporate finance literature. Damodaran (2002) does include and discuss the separate term that is added in the adjusted present value for the expected costs of financial distress. However, Damodaran (2002) does not specify any specific ways for determining the probability of default and the costs of financial distress.
The costs of financial distress are estimated by different authors, as described in chapter 2, but the results are diverse. Further research into the costs of financial distress could improve the accuracy of the estimation of the costs of financial distress.
There are various ways to estimate the probability of default of a company. The Altman Z-score is useful for estimating the probability of default of nonlisted and nonrated companies. Further research into the relation between the Z-score and the probability of default for different types of companies in different industries could provide conversion tables to more accurately estimate the probability of default for these companies.
By combining the improved estimation of both the costs of financial distress and the probability of default and through a discussion on the inclusion of the distress term into the APV model, the accuracy of the APV model ought to be improved.

5.4.2 Exact differences in valuation outcomes

In chapter 4, the differences between the valuation outcomes of the APV model and the enterprise DCF model were shown for different values of input variables. However, these results are based on the valuation of one fictitious company. Further research on different scenarios, companies and values of input variables should provide a *quantitative* overview of the differences in valuation outcome. The results from these cases can be used to conclude on the exact size of the difference between the valuation outcomes of the APV model and the enterprise DCF model in different situations which can be used to make a better founded decision on the type of valuation model to use for the company valuation.

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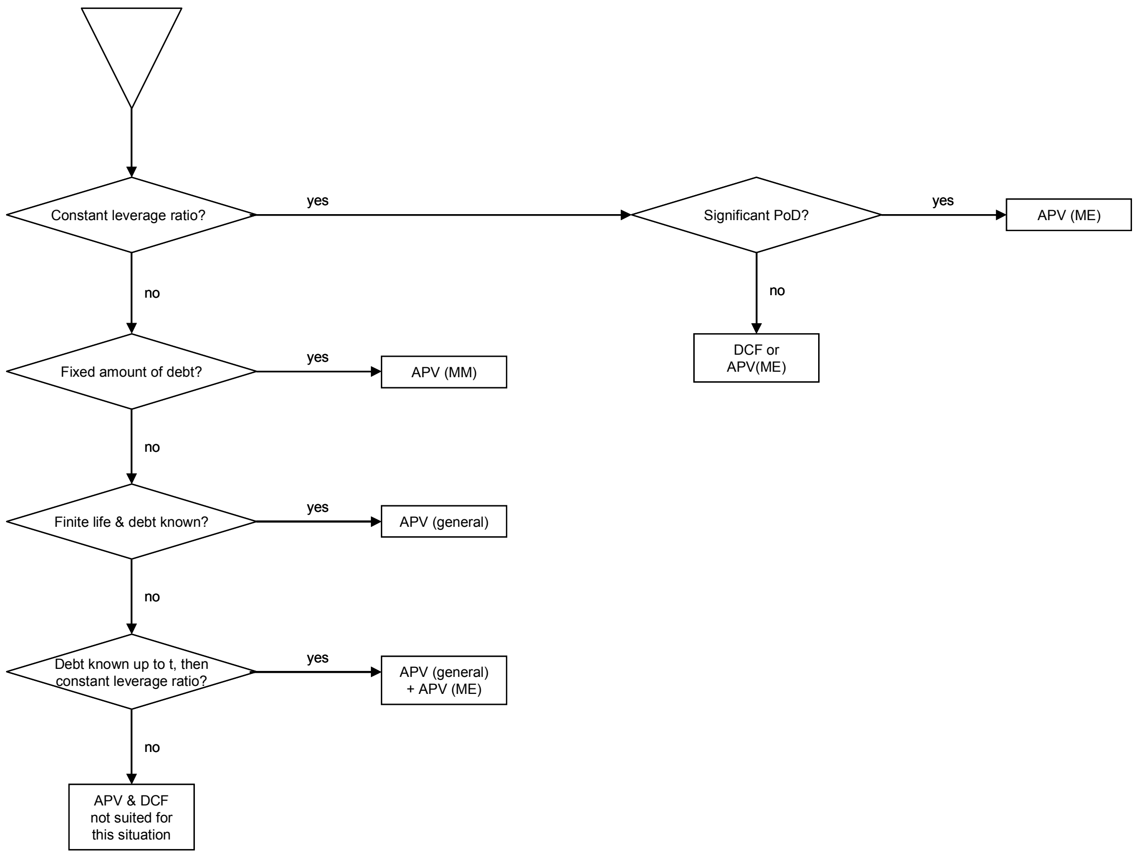
Appendix I: Decision tree



Appendix II: Valuation tool

Introduction

The purpose of this appendix is to translate the theories found in chapter 3 into a practical tool that can be used to value companies on the basis of both the enterprise DCF model and the APV model. The first paragraph will focus on the structure of the tool, which has the format of a Microsoft Excel workbook. The tool was provided by KPMG and already contained the functionality to value a company on the basis of the enterprise DCF model. The second paragraph comments on the specific adjustment and additions that were made to include the functionality of the APV model. The last paragraph discusses an example of a company valuation through the use of the tool.

II.1 Model assumptions

The model consists of thirteen worksheets:

1. Main assumptions
2. Abs Prognose
3. Prognose %
4. WACC
5. Unlevered CoE
6. Probability of default
7. CoCo
8. CoTrans
9. Outcomes
10. Graph
11. Sensitivity (WACC)
12. Sensitivity (APV)
13. Summary

The relation between the worksheets is described in Appendix III.

II.1.1 General guidelines

The grey-colored cells are input cells and the user should set their value. The white-colored cells are output cells; these should not be changed since there value is linked to other cells.

II.1.2 The input sheets

In the 'main assumptions'-sheet, the main assumptions that will be used in all the other sheets can be selected. Most of the aspects are obvious and do not need any explanation. The last four might require some explanation, therefore the following comments:

- Financial figure types: the user can choose whether the absolute forecasts or the relative forecasts should be used in the valuation.
- Net working capital: there are three options for the calculation of the net working capital. The decision for either one of them is a matter of the user's preference.
- Capital structure: if the company under valuation is expected to maintain a fixed amount of debt, then the first option should be selected. If the company under valuation is expected to maintain a constant leverage ratio, then the second option should be selected. The choice for either the Miller & Modigliani or the Miles & Ezzell assumption has its impact on the valuation outcome determined by the APV model part of the tool.
- Costs of financial distress: as discussed in chapter 3, the estimate for the costs of financial distress is 15% of the unlevered company value. If the user has a reason to believe that this value is incorrect, then the costs of financial distress can be changed to the desired value.

The ‘CoCo’-sheet is the second input sheet. The user is supposed to insert the data from the peer group of the company under valuation to acquire the target debt/equity ratio and the unlevered beta.

The ‘CoTrans’-sheet is the last input sheet, in which the user can enter any available transaction data from relevant transaction. These data serve as input for the transaction multiples.

II.1.3 The processing sheets

These sheets are the ones that implement the theories from chapter 3. Based on the assumptions made in the first sheets ('main assumptions'), the appropriate valuation model is used to determine the valuation outcome.

The 'Abs Prognose'-sheet and the 'Prognose %-sheet are the sheets that calculate the cash flows based on the financial data of the company. The 'Abs Prognose'-sheet can best be used if the company under valuation has provided forecasts in absolute values. These can be inserted at the assigned locations (grey-colored cells). In case of financial data in the form of percentages, the 'Prognose %'-sheet should be used.
Each of these two sheets has the same structure. Most of the sections of the sheets speak for themselves. Grey-colored cells should be used to insert data from the financial statement(s) of the company.
The DCF analysis derives the free cash flow through the use of NOPLAT. This differs from the definition discussed in chapter 3, but since the APV analysis is based on the same free cash flows and since the NOPLAT definition differs only marginally from the definition in chapter 4, using free cash flow determined through NOPLAT does not significantly affect the valuation outcome.
The discount factor used in the DCF analysis is the WACC. The free cash flows in the APV analysis are discounted at the unlevered cost of equity. The interest tax shields are discounted at the unlevered cost of equity if the Miles & Ezzell assumption is selected and at the cost of debt if the Miller & Modigliani assumption is selected. However, the first interest tax shield is always discounted at the cost of debt.
The costs of financial distress in the APV analysis are determined through the multiplication of the costs of financial distress times the probability of default times the unlevered company value.

The 'Probability of default'-sheet takes input variables from either the 'Abs Prognose'-sheet or from the 'Prognose %'-sheet depending on the assumption selected. The probability of default is determined on the basis of the credit rating of the company. In the absence of a credit rating, the Altman Z-score (as discussed in paragraph 3.4) is used to determine the probability of default.

The 'Unlevered CoE'-sheet determines the unlevered cost of equity based on the chosen capital structure assumption. The difference between the Miller-Modigliani and Miles-Ezzell determination of the unlevered cost of equity occurs through the difference in the unlevered beta. The unlevered beta is retrieved from the 'CoC'-sheet. The user is supposed to insert the desired risk-free rate, market risk premium, additional risk premium and spread.

The 'WACC'-sheet is the last processing sheet. The WACC is calculated on the basis of the cost of equity and the after-tax cost of debt. The user is supposed to insert the required inputs, thereby making sure that these inputs are consistent with the inputs in the 'Unlevered CoE'-sheet.

II.1.4 The output sheets

The first output sheet, the 'Outcomes'-sheet, determines the market multiples and transaction multiples based on the inputs in the 'CoCo'-sheet and the 'CoTrans'-sheet.

The 'Graph'-sheet can be used to acquire a graph of the key financials of the company.

The 'Sensitivity (WACC)'- sheet and the 'Sensitivity (APV)'-sheet uses a duplicated overview of respectively the DCF analysis and the APV analysis to determine the sensitivity with regard to respectively the long term EBIT margin/WACC and the long term EBIT margin/unlevered cost of equity. An interval of 2% is taken for each parameter to test its sensitivity.

The 'Summary'-sheet is the final sheet of the Microsoft Excel tool. The sheet shows the values found through each of the four valuation models. The intervals are based on the sensitivity analyses.

II.2 Comments on the model

The enterprise DCF part of the Microsoft Excel tool was provided by KPMG. The APV model is built into the tool through a number of additional adjustments and assumptions. Appendix V contains a list of all the adjustment that where made to the original Microsoft Excel tool. This appendix can be used to further expand the tool.

II.3 Example

This paragraph discusses an example of the application of the Microsoft Excel tool to illustrate its functioning. All the Euro amounts mentioned in this paragraph are in thousands of Euros. The scenario that is implemented is scenario 1 from paragraph 3.4: the company is assumed to maintain a constant leverage ratio and the probability of default is assumed to be not significant.
The financial data from Company X are not based on a real company. Most of the components of the profit and loss account are modeled in such a way that they represent a yearly growth of 2.0%. Next to that, the amount of debt outstanding is being adjusted at each period to maintain a constant market leverage ratio of 17% (debt / total capital). The long-term tax rate is assumed to be 25,5% and the costs of financial distress are 15% of the unlevered company value. The company is also assumed to have a levered beta of 1,0.

The first step in the valuation process is selecting the peer group. Since the company under valuation is a fictitious company, no existing companies are used in the peer group. To make sure that the beta analysis shows a levered beta of 1,0 and a D/E ratio of 20,4% (which corresponds to the leverage ratio of 17%), the peer group companies are given exactly the same specifics as the company under valuation. This results in the following.

Company X - beta analysis

	Country	D/E	Tax rate	Levered beta	Unlevered beta (MM)	Unlevered beta (ME)
Company A	NL	20,4%	25,5%	1,00	0,87	0,83
Company B	NL	20,4%	25,5%	1,00	0,87	0,83
Company C	NL	20,4%	25,5%	1,00	0,87	0,83
Company D	NL	20,4%	25,5%	1,00	0,87	0,83
Company E	NL	20,4%	25,5%	1,00	0,87	0,83
Average		**20,44%**	**25,50%**	**1,00**	**0,87**	**0,83**
Median		**20,44%**	**25,50%**	**1,00**	**0,87**	**0,83**

The second step is to estimate and forecast the financial data in the ‘Abs Prognose’-sheet and the ‘Prognose %’-sheet. The estimated data are shown in table II.1 (profit & loss account), table II.2 (net working capital overview), table II.3 (balance sheet – assets overview) and table II.4 (balance sheets – liabilities overview). The grey-colored cells are the input cells in which financial data retrieved from the financial statement(s) of the company should be inserted. In this case, the values are chosen in a way that the sales grow at 2,0% a year, the gross margin in stable at 50% and two-thirds of the net profits are paid out as dividends. The amount of bank loans outstanding is chosen at each point in time such that the market leverage ratio remains constant.

Company X - profit and loss statement												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Net sales	**15.000**	**15.300**	**15.606**	**15.918**	**16.236**	**16.561**	**16.892**	**17.230**	**17.575**	**17.926**	**18.285**	**18.650**
% growth		*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*	*2,0%*
Direct cost 1	(7.500)	(7.650)	(7.803)	(7.959)	(8.118)	(8.281)	(8.446)	(8.615)	(8.787)	(8.963)	(9.142)	(9.325)
Total direct costs	(7.500)	(7.650)	(7.803)	(7.959)	(8.118)	(8.281)	(8.446)	(8.615)	(8.787)	(8.963)	(9.142)	(9.325)
Gross profit	**7.500**	**7.650**	**7.803**	**7.959**	**8.118**	**8.281**	**8.446**	**8.615**	**8.787**	**8.963**	**9.142**	**9.325**
% of sales	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*	*50,0%*
Operating cost 1	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.080)
Other operating costs					-	-	-	-	-	-	-	-
Total operational costs	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.000)	(4.080)
% of sales	*-26,7%*	*-26,1%*	*-25,6%*	*-25,1%*	*-24,6%*	*-24,2%*	*-23,7%*	*-23,2%*	*-22,8%*	*-22,3%*	*-21,9%*	*-21,9%*
EBITDA	**3.500**	**3.650**	**3.803**	**3.959**	**4.118**	**4.281**	**4.446**	**4.615**	**4.787**	**4.963**	**5.142**	**5.245**
% of sales	*23,3%*	*23,9%*	*24,4%*	*24,9%*	*25,4%*	*25,8%*	*26,3%*	*26,8%*	*27,2%*	*27,7%*	*28,1%*	*28,1%*
Depreciation	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.020)
EBITA	**2.500**	**2.650**	**2.803**	**2.959**	**3.118**	**3.281**	**3.446**	**3.615**	**3.787**	**3.963**	**4.142**	**4.225**
% of sales	*16,7%*	*17,3%*	*18,0%*	*18,6%*	*19,2%*	*19,8%*	*20,4%*	*21,0%*	*21,6%*	*22,1%*	*22,7%*	*22,7%*
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
EBIT	**2.050**	**2.200**	**2.353**	**2.509**	**2.668**	**2.831**	**2.996**	**3.165**	**3.337**	**3.513**	**3.692**	**3.766**
% of sales	*13,7%*	*14,4%*	*15,1%*	*15,8%*	*16,4%*	*17,1%*	*17,7%*	*18,4%*	*19,0%*	*19,6%*	*20,2%*	*20,2%*
Interest on debt	(245)	(245)	(283)	(267)	(304)	(322)	(341)	(359)	(378)	(396)	(415)	(433)
Interest on cash	-	-	-	-	-	-	-	-	-	-	-	-
Profit before tax	**1.805**	**1.955**	**2.070**	**2.242**	**2.364**	**2.508**	**2.656**	**2.806**	**2.959**	**3.117**	**3.278**	**3.333**
Income tax	(692)	(743)	(715)	(723)	(654)	(691)	(728)	(767)	(806)	(846)	(887)	(902)
Net profit	**1.113**	**1.212**	**1.355**	**1.519**	**1.710**	**1.818**	**1.927**	**2.040**	**2.154**	**2.271**	**2.391**	**2.431**
% of sales	*7,4%*	*7,9%*	*8,7%*	*9,5%*	*10,5%*	*11,0%*	*11,4%*	*11,8%*	*12,3%*	*12,7%*	*13,1%*	*13,0%*
Dividend payments	(742)	(808)	(903)	(1.013)	(1.140)	(1.212)	(1.285)	(1.360)	(1.436)	(1.514)	(1.594)	(1.621)
Retained earnings	**371**	**404**	**452**	**506**	**570**	**606**	**642**	**680**	**718**	**757**	**797**	**810**

Table II.1: Profit & loss statement of Company X

Company X - net working capital												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Stock (Inventory)	375	383	390	398	406	414	422	431	439	448	457	466
Current assets	**375**	**383**	**390**	**398**	**406**	**414**	**422**	**431**	**439**	**448**	**457**	**466**
Creditors	113	115	118	120	122	124	127	129	132	134	137	140
Taxation & social security					-	-	-	-	-	-	-	-
Other liabilities					-	-	-	-	-	-	-	-
Accruals and deferred income					-	-	-	-	-	-	-	-
Total current liabilities	**113**	**115**	**118**	**120**	**122**	**124**	**127**	**129**	**132**	**134**	**137**	**140**
Net working capital	**263**	**268**	**273**	**278**	**284**	**290**	**296**	**302**	**308**	**314**	**320**	**326**
as % of sales	*1,8%*	*1,7%*	*1,7%*	*1,7%*	*1,8%*	*1,8%*	*1,8%*	*1,8%*	*1,8%*	*1,8%*	*1,8%*	*1,8%*
Investments in net working capital		**5**	**5**	**5**	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**

Table II.2: Net working capital overview of Company X

Company X - Balance sheet - Assets												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Tangible fixed assets	**9.000**	**9.180**	**9.364**	**9.551**	**9.738**	**9.938**	**10.138**	**10.338**	**10.548**	**10.758**	**10.968**	**10.968**
Intangible fixed assets	**3.000**	**3.060**	**3.121**	**3.184**	**3.247**	**3.310**	**3.373**	**3.438**	**3.507**	**3.577**	**3.648**	**3.712**
Financial fixed assets	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Stock (Inventory)	375	383	390	398	406	414	422	431	439	448	457	466
Debtors	-	-	-	-	-	-	-	-	-	-	-	-
Other receivables	-	-	-	-	-	-	-	-	-	-	-	-
Prepayments and accrued income	-	-	-	-	-	-	-	-	-	-	-	-
Operational cash	-	-	-	-	-	-	-	-	-	-	-	-
Current assets	**375**	**383**	**390**	**398**	**406**	**414**	**422**	**431**	**439**	**448**	**457**	**466**
Excess cash	**-**	**707**	**679**	**1.466**	**2.036**	**2.633**	**3.270**	**3.951**	**4.643**	**5.380**	**6.155**	**6.895**
ASSETS	**12.375**	**13.329**	**13.554**	**14.599**	**15.427**	**16.295**	**17.203**	**18.158**	**19.137**	**20.163**	**21.229**	**22.042**

Table II.3: Asset overview of Company X

Company X - Balance sheet - Equity & Liabilities												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
Share capital	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291	8.291
Minority interests					-	-	-	-	-	-	-	-
Cumulative retained earnings	-	371	775	1.227	1.733	2.303	2.909	3.551	4.231	4.949	5.706	6.503
Exchange rate differences					-	-	-	-	-	-	-	-
Equity from change financial fixed assets	-	-	-	-	-	-	-	-	-	-	-	-
Change in equity	-	-	-	-	-	-	-	-	-	-	-	-
Retained earnings current year	371	404	452	506	570	606	642	680	718	757	797	810
Equity	**8.662**	**9.066**	**9.518**	**10.024**	**10.594**	**11.200**	**11.842**	**12.522**	**13.240**	**13.997**	**14.794**	**15.604**
Provision 1	100	102	104	106	106	106	106	106	106	106	106	106
Provision 2	-	-	-	-	-	-	-	-	-	-	-	-
Provision 3	-	-	-	-	-	-	-	-	-	-	-	-
Operating provisions	**100**	**102**	**104**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**	**106**
Bank loans	3.500	4.046	3.815	4.349	4.605	4.865	5.128	5.400	5.659	5.925	6.191	6.191
Bonds	-	-	-	-	-	-	-	-	-	-	-	-
Other long-term interest bearing debt	-	-	-	-	-	-	-	-	-	-	-	-
Long-term liabilities	**3.500**	**4.046**	**3.815**	**4.349**	**4.605**	**4.865**	**5.128**	**5.400**	**5.659**	**5.925**	**6.191**	**6.191**
Preferred equity	-	-	-	-	-	-	-	-	-	-	-	-
Long-term operating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Nonoperating provisions	-	-	-	-	-	-	-	-	-	-	-	-
Debt equivalents	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
Credit institutions					-	-	-	-	-	-	-	-
Creditors	113	115	118	120	122	124	127	129	132	134	137	140
Taxation & social security	-	-	-	-	-	-	-	-	-	-	-	-
Other liabilities	-	-	-	-	-	-	-	-	-	-	-	-
Accruals and deferred income	-	-	-	-	-	-	-	-	-	-	-	-
Current liabilities	**113**	**115**	**118**	**120**	**122**	**124**	**127**	**129**	**132**	**134**	**137**	**140**
EQUITY & LIABILITIES	**12.375**	**13.329**	**13.554**	**14.599**	**15.427**	**16.295**	**17.203**	**18.157**	**19.137**	**20.162**	**21.228**	**22.041**

Table II.4: Equity & liabilities overview of Company X

The third step is to determine the probability of default, the unlevered cost of equity and the weighted average cost of capital (WACC). The probability of default can be determined by entering the credit rating of the company. Since the fictitious company does not have a credit rating, choosing this option in the drop-down menu leads to an estimation of the probability of default on the basis of the Altman Z-score. The estimation of the probability of default is shown in the table below.

Company X - Probability of default	
Company type	1 Public
Credit rating	no rating
Z-score	used
Estimated probability of default	**0,06%**
Input variables	**current year**
Working capital	278
Retained earnings	506
EBIT	2.509
Market value of equity	21.418
Book value of equity	10.024
Net Sales	15.918
Book value of debt	4.575
Total assets	14.599
Z-score	**4,54**

The estimation of the unlevered cost of equity is linked to the Miles & Ezzell assumption of the constant leverage ratio. The data from the following table are therefore used in the further calculations of the company value.

Company X - Unlevered cost of equity (ME)			
Risk free rate	4,0%	Risk free rate	4,0%
Market risk premium	5,0%	Spread	3,0%
Additional risk premium	3,0%	Tax rate	25,5%
Unlevered beta peer group	0,83		
Unlevered cost of equity	**11,2%**	**Cost of debt**	**7,0%**
		After-tax cost of debt	**5,2%**
D/E	20%		
E/(E+D)	83%		

The input for the WACC is practically the same as for the unlevered cost of equity. The grey-colored cells are the input cells. The tax rate is retrieved from the 'Main assumptions'-sheet and the beta from the 'CoCo'-sheet. The calculation of the WACC looks as follows.

Company X - Weighted Average Cost of Capital			
Risk free rate	4,0%	Risk free rate	4,0%
Market risk premium	5,0%	Spread	3,0%
Additional risk premium	3,0%	Tax rate	25,5%
Unlevered beta peer group	0,83		
D/E	20%		
Levered beta	1,00		
Cost of equity	**12,0%**	**After-tax cost of debt**	**5,2%**
E/(E+D)	83%	D/(E+D)	17%
WACC			**10,8%**

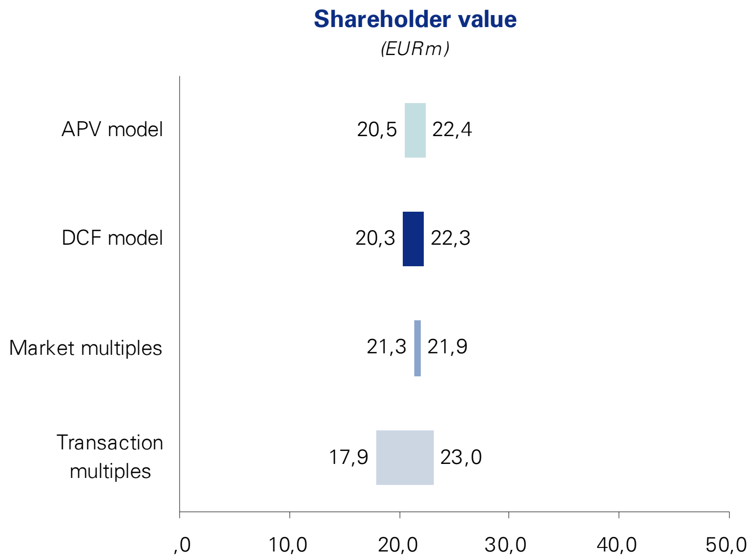
After inserting all the input values in the input sheets, the valuation outcome through both the enterprise DCF model and the APV model is determined. The free cash flows are determined as defined by Koller *et al.* (2005):

FCF = NOPLAT + Noncash Operating Expenses – Investments in Invested Capital

This results in the DCF analysis shown in table II.5 and the APV analysis shown in table II.6.

The company is also valued through the use of the determined market multiples and transaction multiples. A sensitivity analysis is performed for both the DCF analysis and the APV analysis. This is done for the DCF analysis by taking an interval for the WACC of 9,8% - 11,8% and an interval for the long-term EBIT margin of 19,2% - 21,2%. For the APV analysis, an interval for the unlevered cost of equity is taken of 10,2% - 12,2% and an interval for the long-term EBIT margin (the same as for the DCF analysis) of 19,2% - 21,2%.

Combining the results of the sensitivity analysis into an overview of the valuation outcomes under the four different valuation approaches gives the following charts for the shareholder value and the enterprise value.

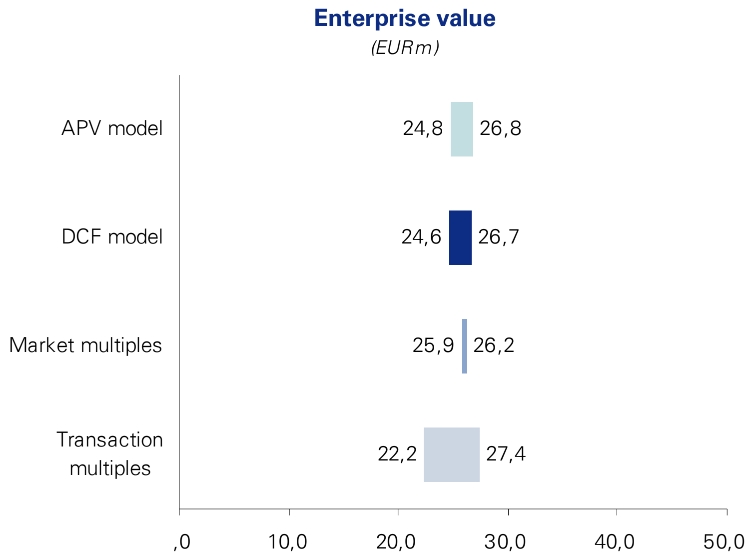


Company X - DCF analysis												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
EBITA	**2.500**	**2.650**	**2.803**	**2.959**	**3.118**	**3.281**	**3.446**	**3.615**	**3.787**	**3.963**	**4.142**	**4.225**
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
EBIT	**2.050**	**2.200**	**2.353**	**2.509**	**2.668**	**2.831**	**2.996**	**3.165**	**3.337**	**3.513**	**3.692**	**3.766**
Operational taxes	(776)	(828)	(804)	(802)	(731)	(773)	(815)	(858)	(902)	(947)	(993)	(1.012)
NOPLAT	**1.274**	**1.372**	**1.549**	**1.707**	**1.937**	**2.058**	**2.181**	**2.307**	**2.435**	**2.566**	**2.700**	**2.754**
% growth				*10,2%*	*13,4%*	*6,2%*	*6,0%*	*5,8%*	*5,6%*	*5,4%*	*5,2%*	*2,0%*
Change in provisions	-	2	2	2	-	-	-	-	-	-	-	-
Amortisation	450	450	450	450	450	450	450	450	450	450	450	459
Depreciation	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.020
Operational cash flow	**2.724**	**2.824**	**3.001**	**3.159**	**3.387**	**3.508**	**3.631**	**3.757**	**3.885**	**4.016**	**4.150**	**4.233**
Book value of disposal of fixed assets					-	-	-	-	-	-	-	-
Capital gain on disposal of fixed assets (net of tax)					-	-	-	-	-	-	-	-
Investments tangible fixed assets					(1.187)	(1.200)	(1.200)	(1.200)	(1.210)	(1.210)	(1.210)	(1.020)
Investment intangible fixed assets					(513)	(513)	(513)	(515)	(519)	(520)	(521)	(523)
Investments in net working capital					(6)	(6)	(6)	(6)	(6)	(6)	(6)	(6)
Free cash flow					**1.681**	**1.789**	**1.912**	**2.036**	**2.150**	**2.280**	**2.413**	**2.683**
Terminal adjustment factor					1,000	1,000	1,000	1,000	1,000	1,000	1,000	11,301
Time to cash flow					1,000	2,000	3,000	4,000	5,000	6,000	7,000	7,000
Discount factor: 10,8%					0,902	0,814	0,734	0,662	0,598	0,539	0,486	0,486
Present value operational free cash flow					**1.516**	**1.456**	**1.404**	**1.349**	**1.285**	**1.229**	**1.173**	**14.747**
Value of operations					**24.159**							
Non-consolidated subsidiaries					-							
Deferred tax assets					-							
Other financial fixed assets 1					-							
Other financial fixed assets 2					-							
Tax losses carry forwards					-							
Excess cash					1.466							
Enterprise value (EV)					**25.626**		**Ratio**	**Value**	**Debt**			
Total nonequity claims					(4.349)		Implied D/E	20,44%	4.349			
Shareholder value (SV)					**21.277**		Implied D/V	16,971%	4.349			
Implied multiples												
EV/Sales					1,6x	1,5x	1,5x	1,5x	1,5x	1,4x	1,4x	
EV/EBITDA					6,2x	6,0x	5,8x	5,6x	5,4x	5,2x	5,0x	
EV/EBITA					8,2x	7,8x	7,4x	7,1x	6,8x	6,5x	6,2x	
SV/Net profit					12,4x	11,7x	11,0x	10,4x	9,9x	9,4x	8,9x	

Table II.5: DCF analysis of Company X

Company X - APV analysis - ME												
EUR '000	**2003-A**	**2004-A**	**2005-A**	**2006-A**	**2007-F**	**2008-F**	**2009-F**	**2010-F**	**2011-F**	**2012-F**	**2013-F**	**RV**
EBITA	**2.500**	**2.650**	**2.803**	**2.959**	**3.118**	**3.281**	**3.446**	**3.615**	**3.787**	**3.963**	**4.142**	**4.225**
Non-tax deductible amortisation	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(200)	(204)
Tax deductible amortisation	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(250)	(255)
EBIT	**2.050**	**2.200**	**2.353**	**2.509**	**2.668**	**2.831**	**2.996**	**3.165**	**3.337**	**3.513**	**3.692**	**3.766**
NOPLAT	**1.274**	**1.372**	**1.549**	**1.707**	**1.937**	**2.058**	**2.181**	**2.307**	**2.435**	**2.566**	**2.700**	**2.754**
% growth				*10,2%*	*13,4%*	*6,2%*	*6,0%*	*5,8%*	*5,6%*	*5,4%*	*5,2%*	*2,0%*
Operational cash flow	**2.724**	**2.824**	**3.001**	**3.159**	**3.387**	**3.508**	**3.631**	**3.757**	**3.885**	**4.016**	**4.150**	**4.233**
Free cash flow					**1.681**	**1.789**	**1.912**	**2.036**	**2.150**	**2.280**	**2.413**	**2.683**
Terminal adjustment factor					1,000	1,000	1,000	1,000	1,000	1,000	1,000	10,927
Time to cash flow					1,000	2,000	3,000	4,000	5,000	6,000	7,000	7,000
Discount factor: 11,2%					0,900	0,809	0,728	0,655	0,589	0,530	0,477	0,477
Present value operational free cash flow					**1.512**	**1.448**	**1.393**	**1.334**	**1.267**	**1.209**	**1.151**	**13.989**
Value of operations					**23.303**							
Interest on debt tax shield					78	82	87	92	96	101	106	111
Interest on cash tax shield					-	-	-	-	-	-	-	-
Interest tax shield					**78**	**82**	**87**	**92**	**96**	**101**	**106**	**111**
Terminal adjustment factor, assumed growth: 2,0%					1,000	1,000	1,000	1,000	1,000	1,000	1,000	10,927
Time to cash flow					1,000	2,000	3,000	4,000	5,000	6,000	7,000	7,000
Discount factor: 11,2%					0,935	0,809	0,728	0,655	0,589	0,530	0,477	0,477
Present value interest tax shield					**73**	**67**	**63**	**60**	**57**	**54**	**50**	**576**
Value of interest tax shields					**999**							
Value of operations					**23.303**							
Costs of financial distress					**(2)**							
Excess cash					1.466							
Enterprise value (EV)					**25.767**		ΔAPV - DCF	0,55%				
Total nonequity claims					(4.349)							
Shareholder value (SV)					**21.418**							
Implied multiples												
EV/Sales					1,6x	1,6x	1,5x	1,5x	1,5x	1,4x	1,4x	
EV/EBITDA					6,3x	6,0x	5,8x	5,6x	5,4x	5,2x	5,0x	
EV/EBITA					8,3x	7,9x	7,5x	7,1x	6,8x	6,5x	6,2x	
SV/Net profit					12,5x	11,8x	11,1x	10,5x	9,9x	9,4x	9,0x	

Table II.6: APV analysis of Company X



The final result of the valuation example is that the enterprise value found under the APV model is 25.767 EUR and 25.626 EUR under the enterprise DCF model.

The difference between the two valuation outcomes is 0,55% of the value found under the enterprise DCF model. According to the theory stated in paragraph 3.4, there should not be a difference between the valuation outcomes of both methods. Considering the assumptions that had to be made regarding, for instance, the probability of default, the relation between the amount of debt outstanding and the free cash flows that are realized, it can be concluded that the APV model, as implemented in the Microsoft Excel tool, correctly represents the theory of the APV model as stated in chapter 3.

Conclusion

In this appendix, the theories formulated in chapter 3 are translated into a practical tool for the valuation of companies in the form of a Microsoft Excel workbook. The example discussed shows the working of the tool and also indicates that the results found through the use of the practical tool match the valuation outcome predicted by the theory in chapter 3 in case of scenario 1.

The next chapter is concerned with the validation of the theories stated in chapter 3 on the effect of the differences between the enterprise DCF model and the APV model through the implementation of different scenarios. Among these scenarios are the scenarios discussed in chapter 3.

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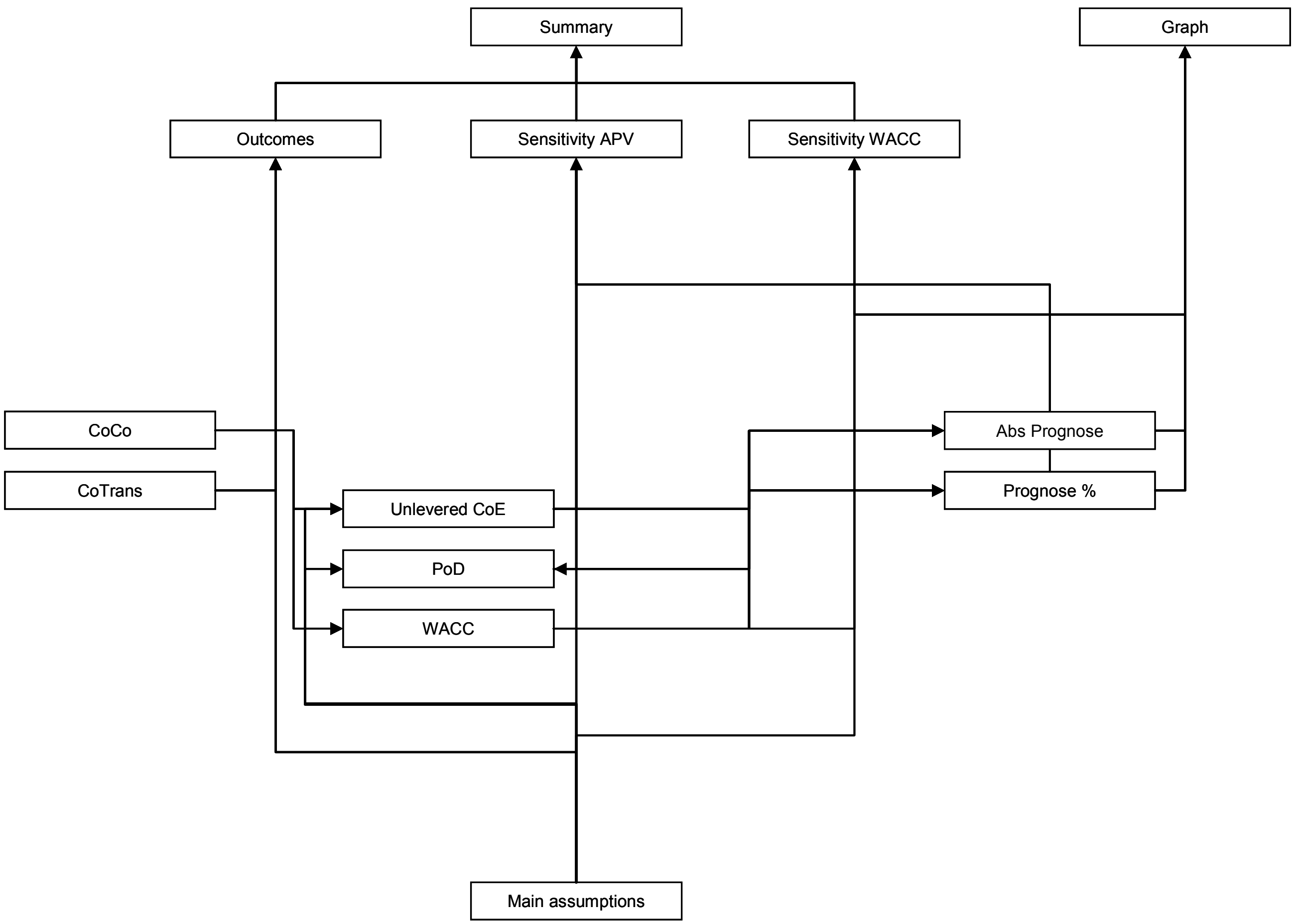
Appendix III: Relations between tool worksheets



Appendix IV: Comments on valuation tool

Sheet(s): Main assumptions
Added:

1. Capital Structure – Drop-down list. Goal: to allow for the choice between the MM and ME assumption
2. Cost of financial distress – Input variables that is used to determine the present value of the costs of financial distress, displayed as the percentage of unlevered company value.

Sheet(s): Abs Prognose & Prognose %
Added:

1. APV analysis
    a. Up to Free Cash Flows the same as DCF analysis
    b. Discount factor (B442) = the correct discount rate based on the capital structure assumption.
    c. Tax shields are based on the interest paid/received and the tax rate (row 447 & 448).
    d. Tax shield growth rate (B451) = the correct growth rate based on the capital structure assumption.
    e. Discount factor (B454) = the correct discount rate based on the capital structure assumption. However, the first interest tax shield is always discounted at the cost of debt, because it is known at time = 0. The perpetuity is case of the ME scenario is not multiplied by the correction factor, since this would create a double effect. This is a deviation from the model described in the theory.
    f. Costs of financial distress = the multiplication of the costs of financial distress and the probability of default.

Sheet(s): Unlevered CoE (Cost of equity) (=new sheet)
Added:

1. Unlevered cost of equity (MM)
    a. Unlevered beta (MM) (C9) is used as an input in CAPM to determine the unlevered cost of equity.
2. Unlevered cost of equity (ME)
    a. Unlevered beta (ME) (I9) is used as an input in CAPM to determine the unlevered cost of equity.
3. Unlevered cost of equity, show the current assumption and the current unlevered cost of equity.

Sheet(s): Probability of default (=new sheet)
Added:

1. Input cells
    a. Company type (C6) to indicate whether the company under valuation is a public or private company.
    b. Credit rating (C7), where one can enter the credit rating of the company that was determined by a credit rating agency.
2. Input variables Z-score (C13-C20). Cells are used to calculate the z-score (C22).
3. Estimated probability of default (C10), based on either the credit rating or, in absence of a rating, on the z-score. Cell C8 shows whether the z-score is used for determining the probability of default.

Sheet(s): CoC
Added:

1. Unlevered beta (MM) (Column BR), based on the unlevering formula of Modigliani & Miller.
2. Unlevered beta (ME) (Column BS), based on the unlevering formula of Miles & Ezzell.

Sheet(s): Sensitivity (APV) (=new sheet)
Added:

1. APV analysis, based on the chosen SELECTOR (which is the type of financial figures, stated in Main Assumptions!.D19.
2. Sensitivity analysis based on changes in the unlevered cost of equity and the long term EBIT margin.

Sheet(s): Summary
Added:

1. An ‘APV method’-series in the charts of the shareholder value and the enterprise value.

Word of thanks

This research project has been realized with the help of several people. Each of them has contributed to the final result and/or other activities that are related to the completion of my master Industrial Engineering & Management.

First of all, I would like to thank Jeroen Weimer & Frank Siblesz for their support and supervision, feedback and questions.

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