Credit risk modeling and CDS valuation

An analysis of structural models

Master thesis J.A.G. van Beem

April, 2010

Deloitte.

UNIVERSITY OF TWENTE.

Credit risk modeling and CDS valuation

An analysis of structural models

A thesis

submitted in partial fulfillment of the
requirements for the degree of Master of Science
in Industrial Engineering and Management

University of Twente

by

Jeroen van Beem

Thesis Committee

Dr. B. Roorda, University of Twente
Dr. J. Krystul, University of Twente
Dhr. K. Dessens, Deloitte Capital Markets

Abstract

The main objective of this thesis is to determine a credit risk model that could be used to value credit default swaps (CDSs). A literature study identifies that structural models are able to meet this objective and that they could also be used for analysis on default probabilities and recovery rates. Two structural models are selected for further research. In the first model the firm's assets value process is a jump-diffusion process and default is modeled as the first-passage of a constant default barrier. The second model uses the same jump-diffusion process for the firm value, but incorporates a mean reverting leverage process to model default. We implement both models in Matlab using (Brownian Bridge) Monte Carlo simulations to analyze the sensitivity of modeled CDS term structures to value changes in the input parameters. We find that both models can calculate non-zero short-term CDS spreads and that the second model generates upward sloping term structures over a long time horizon. From three cases studies in which we compare modeled CDS spreads to the market spreads of Dutch firms we find that an advanced parameter estimation methodology is necessary to provide consistent jump and diffusion parameter values throughout the economic cycle.

Preface

This is it. Although this preface might be the beginning of this master thesis for you, it is the end for me. It represents the end of long hours of working on this thesis, but also the end of my student time. A great time in which I made lots of friends and learned even more.
This thesis is written to obtain the degree of Master of Science in Industrial Engineering and Management at the University of Twente. The eight months of research that I performed for this thesis at Deloitte Capital Markets were a great experience for me. It was an opportunity to broaden my knowledge and experience working in the financial services industry. Reviewing academic literature on credit risk, programming simulation models in Matlab and performing sensitivity analysis have been an interesting challenge. Now I look back at the project with satisfaction, knowing that I achieved my goals and produced useful results.
I would like to thank dhr. K. Dessens from Deloitte Capital Markets for his great help and critical comments to keep me focussed. Our weekly discussions were crucial for the progress of the project. Furthermore, I would like to thank my colleagues of the Capital Markets department who were always willing to help and open for entertaining small talk. Also my gratitude goes to Dr. B. Roorda and Dr. J. Krystul of the University of Twente. Their insights and feedback helped me to solve difficult parts in this thesis and to focus on the research goals.
Finally, I would like to thank my family and friends. Especially my parents and girlfriend for their valuable advice, ongoing support, and many pleasant moments during my thesis project and before.

Enschede, April 2010.

Management Summary

The main topics in this thesis are credit risk modeling and credit default swap (CDS) valuation. In particular, the study performed in this thesis has the objective to determine a credit risk model that:

1. can be used to value single name cash settled CDS contracts,
2. is able to estimate CDS term structures observed in the market,
3. can evaluate multiple credit risk measures as output,
4. and can be used to analyze the effects of market risks on these measures.

From a literature review of credit risk models we assess several credit risk models along four dimensions that are specified to meet the objectives of this thesis: interpretation of the default event, implementation difficulties, performance, and scope of applications. Structural models score best on these dimensions and are therefore analyzed in more detail.

We describe the first structural model of Merton (1974) and find that this model is unable to estimate values and shapes of term structures of default probabilities and credit spreads observed in the market. Therefore this model is extended in the literature on its simplifying assumptions to improve its performance.
We review the literature on these extensions to the Merton model and select two structural models that are further analyzed. In the first model the firm's assets value process is modeled with a jump-diffusion process and default is modeled as the first-passage of a constant barrier. The second model also assumes a jump-diffusion process for the firm's assets value, but models a mean reverting default barrier.
The models are implemented in Matlab using Monte Carlo (Brownian Bridge) approaches to determine various credit risk measures: probabilities of default, CDS spreads and recovery rates. We focus on modeling CDS term structures by performing a sensitivity analysis that studies the effect of changes in the values of input parameters on these term structures.
From this analysis we find that both model 1 and 2 are able to determine values and shapes of CDS term structures observed in the market. Furthermore, both models can calculate positive short-term CDS spreads. Model 1 results in downward sloping and model 2 in upward sloping term structures over a long time horizon.

Finally, we analyze the performance of the models when they are applied in practice. We perform three case studies in which our structural models estimate the market CDS term structure of Dutch firms. We use a simple parameter estimation methodology that allows us to focus on potential difficulties in parameter estimation from market data. We find that estimation of the jump and diffusion parameters is a challenging task since these parameters are affected by changes in macro economic or firm specific conditions, such as the credit crisis. This makes it difficult to find a single estimation procedure that results in consistent values and shapes for modeled CDS term structures for different firms on different dates.
We finally conclude that the selected structural models can meet the objectives of this thesis. A more advanced parameter estimation method that can provide consistent parameter estimates throughout the economic cycle is an important topic for future research. Furthermore it would be interesting to extend the models to account for non-credit risk factors, such as liquidity, that affect the level of the market CDS spreads.

Notation

B_t	Debt value at time t
$c_{t,T}$	Par CDS spread of a contract between time t and maturity T
D	Promised debt payment or notional amount of a bond
E_t	Equity value at time t
F	Number of defaults in a Monte Carlo simulation
$\mathrm{E}^{\mathbb{Q}}$	Expectations under the risk-neutral measure $\mathbb{Q}$
$\mathbf{1}_{xyz}$	Indicator function
K	Level of the default barrier
l	Right boundary of uniform sampling interval
l_i	Random variable describing (part of) the asset value process in model 2
L_t	Inverse leverage ratio at time t
$\overline{L}$	Target leverage ratio
N	Notional amount of a CDS contract
$N(T)$	Number of jumps in the interval $[0, T]$
P^+	Conditional probability of no barrier crossing regarding the Brownian Bridge approach
P^*	Conditional probability of a barrier crossing regarding the Brownian Bridge approach
P_M	Risk-neutral default probability in the Merton model
q_t	Distribution of the probability of default
Q_t	Cumulative probability of default
r	Risk-free interest rate
R	Recovery rate on which CDS spreads are quoted (input to the model)
RR	Actual recovery rate (output of the model)
s	Random variable regarding uniform sampling
S_t	Cumulative survival probability
t	Time
t_p	Time at which a protection payment is made
Δt	Time between successive protection payments
T	Maturity
V_t	Firm's assets value at time t
dW_t	Wiener process
x_i	Random variable describing the diffusion component of the firm value process
X_t	Log assets value at time t
$X_{\tau_i^-}$	Log assets value the instant before jump i
$X_{\tau_i^+}$	Log assets value the instant after jump i
y	Bond yield
y_i	Random variable describing the occurrence of a jump
dY_t	Poisson process
z	Maximum number of protection payments
γ	Constant to adjust the target leverage ratio
δ	Payout of the firm
λ	Jump intensity
μ_V	Expected log return on firm's assets without jumps
μ_π	Mean of the jump size distribution
ν	Expected value of the jump component

π_i	Random variable describing the jump size
Π	Jump amplitude
σ_V	Volatility of firm's assets
σ_π	Volatility of jump size distribution
τ^*	Default time
τ_i	Time at which jump i occurs
$\Phi(\cdot)$	Cumulative standard normal distribution function
φ	Mean reversion speed

BC	Black & Cox (1976)
bps	basis point
CDG	Collin-Dufresne & Goldstein (2001)
CDS	Credit default swap
CGM	Collin-Dufresne, Goldstein, and Martin (2001)
DAP	Discounted accrual payment
DDP	Discounted default payment
DPP	Discounted protection payments
EAD	Exposure at default
HH	Huang & Huang (2003)
LIBOR	London Interbank Offer Rate
LGD	Loss given default
LS	Longstaff & Schwartz (1995)
LT	Leland & Toft (1996)
PD	Probability of default
ZCB	Zero-coupon bond

Contents

Chapter 1

Research Design

Credit risk[1] has received new and increased attention since the credit crisis. Credit risk appeared to be not just the traditional risk that financial institutions avoid when lending money, but it had become a financial instrument traded around the world.
Many new, exotic credit risk related products were developed by financial institutions and all investors seemed to want a share in it. The dramatic growth of the credit market in the past years illustrates this. For example the market for CDSs almost doubled each year since 2001 to a peak notional outstanding amount of 62 trillion US Dollars in 2007 (ISDA).
Typical examples of firms that took large positions in the new credit instruments and lend too much subprime mortgages are Bear Stearns, Lehman Brothers, AIG, Fannie Mae and Freddie Mac. In the credit crisis we saw the consequences of the high credit risk in their exposures: large capital injections of governments were necessary for the firms to survive or they ended up in bankruptcy.
It is therefore not a surprise that modeling of credit risk has again gained the interest of financial institutions, regulators and academics. They try to improve the pricing methodology of financial instruments and impose new guidelines to manage credit risk exposures. Credit risk modeling is also the topic of this thesis.

1.1 Research case

This thesis is written during an internship at Deloitte Capital Markets. Deloitte Touche Tohmatsu is the brand name of independent partnerships throughout the world that collaborate to provide business services to clients. Deloitte's core business is audit, but the firm is also active in related consulting, financial advisory, risk management and tax services.
In it's audit activities, Deloitte supports small and medium sized enterprizes in setting up their financial statements and verifies the financial statements of large corporates. Since the introduction of the new IFRS accounting standard these audit activities have become more complex. Especially fair value accounting[2] and hedge accounting of financial derivative contracts (IAS 39) is a difficult task, and therefore specialist teams support the audit. The Deloitte Capital Markets (CM) department is one of these supporting teams.
The valuation of financial instruments is one of this teams' specializations. The products range from plain vanilla interest rate derivatives as foreign exchange forwards and interest rate swaps, to more complex instruments as swap contracts, option-type contracts, credit derivatives and CDOs. In this thesis we focus on the valuation of CDS contracts.

Currently, the CM team applies a reduced form approach to value CDS contracts. The advantage of this approach is its speed in use. However, the method is sensitive to the selected input market data and limited in its applications.

[1]The terminology regarding credit risk and CDSs used in this chapter is further explained throughout chapters 2 and 3.

[2]Fair value accounting means that the current, market value of an asset should be disclosed in the financial statements. This is the value at which an asset could be bought or sold in an armslength transaction between willing parties.

Since the reduced form approach does not provide any additional information on variables affecting the valuation of the contract and does not allow for any credit risk analysis of a client, it is valuable for CM to develop a new credit risk model. Such a model needs to be useful for CDS valuation and should be able to provide information for other business opportunities.

1.2 Research objectives and strategy

CM's current practice on CDS valuation and the exploration of new business opportunities related to credit risk and market risk management are the guidelines for this thesis. We define the following research objective:

Determine a credit risk model that:

1. **can be used to value single name cash settled CDS contracts,**
2. **is able to estimate CDS term structures observed in the market,**
3. **can evaluate multiple credit risk measures as output,**
4. **and can be used to analyze the effects of market risks on these measures.**

Since most CDS contracts that Deloitte CM values are single name CDSs that are settled in cash and have a maturity between 3 and 10 years, the model should be able to value at least these types of contracts. Furthermore, the model should be able to calculate CDS term structures that have similar shapes as CDS term structures observed in the market. The estimated CDS spreads should be comparable to the market CDS spreads within an acceptable range that depends on the application. The model should also be able to calculate other credit risk measures than CDS spreads, like default probabilities and recovery rates. In this way the model has more applications than CDS valuation. These measures could for example be used as inputs to the advanced credit risk modeling approach in Basel II (internal ratings based approach). Finally, it should be possible to analyze the effect of market risk, e.g. changes in interest rates and equity volatility, on the calculated credit risk measures.

To design a credit risk model with these objectives we perform our research in three phases: a literature study, a numerical analysis, and an empirical study. Each of these phases has its own goals:

Literature study. The literature study will give us more insight on (a) credit risk and its modeling approaches and (b) credit default swaps and its pricing methods. From this information we determine the type of credit risk model that can be used for the valuation of CDS contracts and is able to perform a sensitivity analysis to additional credit risk measures.
We find that the structural credit risk models can meet the objectives of this study. Literature on these models is further reviewed to specify the structural models that are implemented in the numerical study.

Numerical analysis. Two selected models from the literature review are further analyzed in a numerical study. We focus on modeling CDS term structures and therefore implement different simulation algorithms in Matlab. To investigate the opportunities of the models, we perform a sensitivity analysis in which we study the effect of changes in the values of input parameters to modeled CDS term structures.

Empirical study. In this last phase we test the practical application of the selected models. We perform case studies in which we estimate the input parameters for the models from the market and balance sheet data of Dutch firms. With these input parameters we estimate the firm's market CDS spread to test the performance of our models and identify difficulties when the models are used in practice.

1.3 Outline

The remainder of this thesis is organized as follows. Chapter 2 first introduces credit risk and gives an overview of credit risk models. The second part of chapter 2 focusses on credit derivatives, CDS contracts and derives a pricing methodology to value CDSs. A particular class of credit risk models is described in chapter 3: the structural models. The chapter starts with a description of the first structural model of Merton (1974) and illustrates its shortcomings. Extensions to the Merton (1974) model are identified and an overview of empirical studies that analyze these models is given. From this analysis we determine the structural models that in theory best meet the research objectives. Chapter 4 implements the determined models with various Monte Carlo approaches and performs a sensitivity analysis. Chapter 5 performs three case studies to test the models when they are applied in practice. Chapter 6 concludes this thesis and gives recommendations to further improve the selected models.

Chapter 2

Credit risk and credit default swaps

This chapter introduces credit risk and credit default swaps (CDSs) from a modeling perspective. First, we explain credit risk and credit risk measurement. Then we describe various credit risk models identified in the literature and make a comparison to select the type of credit risk model that best meets the research objectives.
The chapter continuous with an introduction to credit derivatives and further focusses on CDSs. The basics of the CDS are explained and the most important aspects of CDS contracts are specified. Various methods for pricing CDS contracts are briefly discussed and a discounted cash flow method is derived in detail. Finally, we review empirical literature to identify the factors that affect CDS spreads in the marketplace. In this chapter we follow Duffie & Singleton (2003), Hull (2006) and Schönbucher (2003).

2.1 Credit risk

An investor that enters into a financial transactions is faced to various risks. Two important risk types are market risk and credit risk. Market risk is the risk of value changes in a financial asset due to changes in market variables, e.g. interest rates, exchange rates, equity prices, and commodity prices. Credit risk and its measurement are explained below.

2.1.1 Explanation of credit risk

Credit risk can be defined as the risk of a loss due to the inability of a counterparty in a financial contract to fulfill its obligations.
This definition identifies several components associated with credit risk. A simple example of a financial transaction illustrates these items: a bank provides a loan of EUR 1m to a firm and they agree that the firm repays the loan one year from now.
The bank lends the money to the counterparty in the contract, the firm. From this moment, the bank faces the credit risk of the transaction. Usually, the firm repays the outstanding amount of EUR 1m (and interest) to the bank. However, if the firm gets into financial distress and eventually defaults the firm cannot repay the loan.
If such a credit event occurs, a procedure is started to recover funds from the firm's assets to (partially) repay the bank and other lenders. This probably results in a large loss to the bank: for example only 40% of the loan might be recovered resulting in a loss of EUR 0.6m.
The example illustrates that credit risk in a financial transaction can result in large losses. Therefore, lenders will carefully assess this risk of a counterparty before they enter into the contract.
Furthermore, regulators in the financial service industry have developed frameworks for credit risk measurement, such as the Basel II accord of the Bank of International Settlements (BIS). Lenders should use these frameworks to measure and model credit risk as a basis for calculating capital buffers for credit losses. We describe measurement and modeling of credit risk in the next sections.

2.1.2 Credit risk measurement

Since losses due to credit risk can be high, regulatory institutions force investors, especially financial institutions, to actively model and measure the credit risk in their portfolio. Credit risk modeling is discussed in section 2.2. This section describes the measurement of credit risk.
The Basel II framework on capital requirements for credit risk identifies the following most widely used parameters associated with credit risk measurement:

- **Exposure at default (EAD).** The EAD measures the extend to which an investor is exposed to the counterparty in case of a default event at the counterparty. The EAD is the outstanding amount of the contract at default and thus the maximum amount that could be lost. The EAD in the example above is EUR 5m.
- **Loss given default (LGD).** This is the percentage of the EAD that is lost on a contract when the counterparty defaults. One minus the LGD gives the recovery rate RR of an asset. This is the assets value recovered when a default event occurs. In the example, the LGD 60% and RR is 40%.
- **Probability of default (PD).** This is the probability that a default event occurs at the counterparty of a financial contract in a given time period.
- **Effective maturity.** The effective maturity of a financial contract is the longest possible period available to the counterparty to fulfil all of its contractual obligations. In the example, the effective maturity is one year.

Next to CDS spreads, the model that we determine in this study should be able to calculate these credit risk measures. The next section gives an overview of credit risk models that can at least model the PD.

2.2 Credit risk modeling

Credit risk can be modeled with different approaches. The literature distinguishes between methods that use (historical) accounting information to asses or forecast the credit risk of a firm, and methods that use market prices of assets to model credit risk. Figure 2.1 shows a further classification of the models.

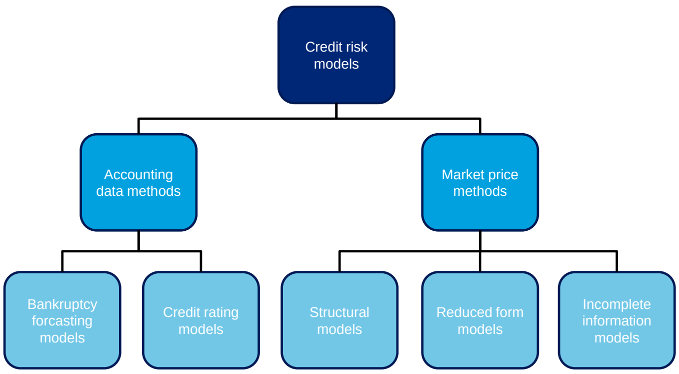


Figure 2.1: Classification of credit risk models.

The models are developed through time to adjust for new market conditions, improve performance and to find new applications. For example Basel II's standardized approach for calculating capital requirements for credit risk uses credit ratings, and Moody's KMV distance to default model uses the structural model framework. The following sections shortly introduce the credit risk models in figure 2.1, such that we can make a comparison.

2.2.1 Bankruptcy forecasting models

This type of models forecasts a firm's bankruptcy risk from its financial statements. A famous example of a bankruptcy forecasting model is Altman's Z-score. Altman (1968) analyzed the historical trends and changes in financial ratios of firm's that ended up in bankruptcy to design the following model:

$$\begin{aligned} Z &= 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5 \\ \text{where} \quad X_1 &= \text{Working capital/Total assets} \\ X_1 &= \text{Retained earnings/Total assets} \\ X_1 &= \text{Earnings before interest and taxes/Total assets} \\ X_1 &= \text{Market value equity/Book value equity} \\ X_1 &= \text{Sales/Total assets} \\ Z &= \text{Overall index.} \end{aligned}$$

Firms with a Z-score higher than 2.99 are considered healthy and thereby not likely to enter into bankruptcy. Firms with a Z-score lower than 1.81 are bankrupt. Firms with scores between 1.81 and 2.99 are in the so called 'grey area', meaning that their future is uncertain.
Altman (1968) shows that the model is accurate in forecasting bankruptcy up to two years between the bankruptcy event. Since 1968, the Z-score model is updated by Altman and other researchers to account for recent market trends by adjusting the model's parameters and ratios.

2.2.2 Credit rating models

Credit rating models summarize credit risk in a credit rating. A credit rating is an opinion about the future credit risk of a firm. Credit rating agencies assign a credit rating to an issuer of debt based on their opinion on the ability and willingness of the issuer to meet its financial obligations. Each credit rating agency uses its own methodology to estimate this credit worthiness of the issuer based on available information on the issuers financial conditions.
The three main credit rating agencies are Standard & Poor's, Moody's and Fitch[1]. Table 2.1 shows the ratings they assign to long-term obligations[2] and their meaning. Standard & Poor's and Fitch

Table 2.1: Credit ratings of long-term obligations and their meaning. AAA to BBB rated obligations are called investment grade. Lower rated obligations are speculative grade.

Standard & Poor's Fitch	**Moody's**	**Meaning**
AAA	Aaa	Highest credit quality, minimal credit risk
AA	Aa	Very high quality
A	A	High quality
BBB	Baa	Good quality, adequate payment capacity
BB	Ba	Speculative, long-term uncertainty
B	B	Speculative, very vulnerable to adverse business
CCC	Caa	High current credit risk
CC	Ca	Currently near default, very high credit risk
C		Very near default (inevitable)
D	C	Default

make a further classification within the categories AA to CCC by the addition of a (+) and (−) sign. Moody's makes this relative classification within the Aa to Caa categories by adding 1, 2, or 3. A (+) and a 1 indicate that the obligation is ranked in the higher end of its class, and a (-) or 3 refers to the lower end of its category.
Credit ratings can apply to firms or countries, and to individual debt issues. When different assets have the same rating, this does not mean that the credit risk of these assets is the same. A credit rating gives an indication of the credit risk of an asset relative to the credit risk of other assets within its class.
Credit ratings can be used to determine a firm's PD. Rating agencies provide probabilities that a

[1] Ratings are not only assigned by commercial rating agencies. Many financial institutions have their own internally developed methodologies, used in for example the internal ratings based approach for their Basel II compliancy for credit risk.
[2] Long-term obligations are defined as obligations with a maturity of one year or more.

firm with a certain rating defaults within a certain time periods. Consider for example figure 2.2 that shows the average cumulative default rates for various credit ratings determined by Moody's.

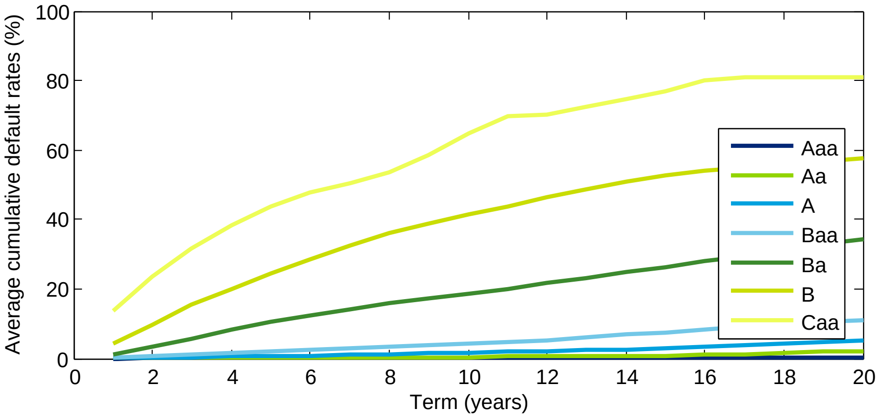


Figure 2.2: Cumulative default rates for various credit ratings over the period 1983-2008 (Moodys Investors Service 2009).

Observe that the default rates largely differ across ratings: the higher the rating, the lower the PD. For investment grade bonds, the PD increases with time. This is because a high rated firm is considered healthy in its first years, but when time passes the probability of a change in its financial conditions increases and thus the PD increases. For lower rated firms the PD strongly increases in its first years, but the marginal PD declines when time passes. Hull (2006) explains that when a firm that is initially considered unhealthy survives its first critical years, the firm is likely to have improved its financial condition, such that the marginal PD declines.
Another observation from figure 2.2 is that default rates are positive even for short maturities. Only AAA-rated firms have PDs of approximately zero over a short horizon. This implies that firms do default on short term debt issues. Chapter 3 further elaborates on these observations.
Credit rating agencies also provide transition matrices that give the credit migration risks of firms. This is the probability that a firm with a certain rating gets an upgrade or downgrade in rating within a certain time period.
Credit ratings are an easily accessible source of information about a firm's credit risk. However, credit ratings do not provide an up-to-date indication of credit risk, since they are not frequently updated. In the recent credit crisis we saw the consequences of this: the sudden changes in financial conditions of firms were not yet reflected in the firm's credit rating. Firms that used credit ratings as the only source of credit risk information were thereby unable to make a correct assessment of their counterparty's credit risk and incurred high losses.

2.2.3 Market price methods

Reduced form models, or intensity models, are developed to take the sudden nature of default events into account. There are several approaches within the reduced form models to determine default probabilities. Central in these models is the default intensity obtained from market prices of defaultable instruments, such as bonds and CDSs. This default intensity is used in an exogenous arrival or jump process to model the default event. Deloitte CM uses a reduced form model to price CDS contracts in which PDs are often determined from market prices of bonds.
Reduced form models are computationally fast. However, since they do not use information from the firm's balance sheet, they provide little economic interpretation for the default event and are not able to provide additional credit risk measures next to the PD.

Structural models assume that a firm defaults when its assets value is insufficient to honor its payment obligations. As we will see in chapter 3, structural models are able to provide more interpretation to default event and can provide more output, but are more difficult to implement.

Incomplete information models try to combine the properties of structural and reduced form models (Elizalde (2005c)). The structural models are used to account for the economic interpretation of the default event, and the reduced form approach has to account for the unexpected nature of default.
In these models, as in the structural approach, a firm defaults when the asset value is too low to serve the obligated payments (the default threshold). However, by assuming that investors do not have complete information on the firm's assets value and default threshold, the default event happens unexpectedly[3] as in the reduced form approach.

2.2.4 Comparison of models

The credit risk models described in the previous sections have both advantages and disadvantages. To determine which types of models are best able to meet the research objectives, we compare them across the following dimensions[4]:

- **Interpretation.** This dimension reflects whether the credit risk model provides an economic interpretation for the occurrence of a credit event and is able to analyze this event in changing market conditions.
- **Implementation.** Implementation concerns the ease of obtaining the credit risk information from the model. This comprises data requirements (availability and amount), parameter estimation procedures, and complexity of the model.
- **Performance.** This indicates whether the credit risk information provided by the model is up-to-date and whether estimate measures of credit risk are comparable to market measures.
- **Application.** This refers to the scope of applications of the models, such as the estimation of several credit risk measures, pricing of securities and ability for market risk analysis.

The models are compared from the perspective of a lender that wants to assess the credit risk of its counterparty in a financial contract. Table 2.2 compares the models by assigning a relative score to each of the models on every dimensions. A (++) refers to the relative highest score and (− −) to the lowest.

Table 2.2: Comparison of credit risk models.

	Interpretation	**Implementation**	**Performance**	**Application**
Bankruptcy forecasting	+/−	+	−	−
Credit rating models	+/−	++	+/−	+
Structural models	++	−	+	++
Reduced form models	−	+/−	+	+
Incomplete information	++	− −	+	++

The bankruptcy forecasting models are easy to implement when the financial data is available. However, the model's parameters and ratios do not always reflect the current economic conditions. Furthermore, these models only forecast bankruptcy and are not able to provide other output.
The advantages of credit ratings are that they are easily available for many asset classes. However, rating agencies mainly provide ratings for large, listed companies and these ratings are not frequently updated.
Structural models provide an economic explanation for the default of a firm, and as we will see in chapter 3 structural models can be designed to estimate several credit risk measures and to calculate prices of bonds, equity and CDS spreads. Major drawback of structural models is parameter estimation of assets value process.
The reduced form models can be used to asses the credit risk of various asset classes of firms with available market data. Implementation of reduced form models takes less effort than structural models, but they do not provide an explanation for the default event and cannot provide additional output.

[3]See section 3.2 for more details on the expectable nature of the default event in structural models.
[4]For a good understanding of this section it is recommendable to first read chapter 3 that elaborates on structural models.

Since the incomplete information models are a combination of the structural and reduced form approach, it takes the advantages of both approaches, but the implementation difficulties rise.

From this comparison we conclude that the structural credit risk models are best able to meet the objectives of this thesis. These models can be designed to determine several credit risk measures, to value single name cash settled CDS contracts and estimate market CDS term structures. Furthermore, the structural modeling approach allows for extensive analysis of the firm's default event.

2.3 Credit derivatives

Credit derivatives are securities with a payoff depending on the credit risk of a reference entity, i.e. one or more financial instruments of firms or countries. An investor exposed to the credit risk of such a reference entity, the protection buyer, enters into a credit derivative contract to (partially) transfer the credit risk to another investor, the protection seller.
The amount of credit risk transferred and the payoff of the contract depend on the type of credit derivative. According to their payoff, we can distinguish between the following credit derivatives (Bielecki & Rutkowski (2001))[5]:

- **Credit event instruments.** These are contracts in which the payoffs are conditional on a credit event at the reference entity. Examples of these type of credit derivatives are CDSs, CDS forwards and options, credit linked notes and first-to-default baskets.
- **Spread instruments.** In these contracts the payoffs depend on changes in the credit quality of the reference entity (e.g. changes in the credit rating or credit spread). Examples are credit spread swaps and credit spread options.
- **Total return instruments.** These securities transfer both the credit risk and market risk of an asset from the protection buyer to the protection seller. Examples of this type are the total return swap and loan swap.

Next section focusses on a popular type of credit derivative: the CDS.

2.4 Credit default swaps

This section introduces CDSs. First, we give a general overview of CDSs. This is followed by an explanation of the terminology associated with CDSs from the elements of a typical CDS contract. The section ends with a description of the relationship between CDS spreads and bond yields.

2.4.1 Overview of a CDS

A CDS is a contractual agreement to transfer the credit risk on one or more reference entities from the buyer of the CDS contract to the seller, see figure 2.3.

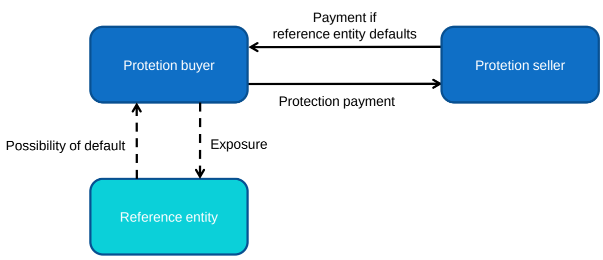


Figure 2.3: Schematic representation of a CDS.

The protection buyer has an exposure to one or more assets of a reference entity. He takes a long

[5]See Bielecki & Rutkowski (2001), Hull (2006), or Schönbucher (2003) for a detailed description of the given examples of credit derivatives.

position in a CDS contract to protect for a loss in case of default of the reference entity. Another investor, the protection seller, takes a short position in this contract and agrees to compensate for this loss when such a credit event occurs. In exchange for the protection the buyer of the contract pays a premium to the protection seller, the CDS spread.
In the standard CDS contracts this premium is a periodic payment until default or maturity of the contract, whichever comes first. This type of contract is also called a running CDS, since the protection payments run throughout the life of the contract. Another type of contract considers an up-front premium. In these contracts the protection buyers pay only an up-front payment at initiation of the contract in return for protection until maturity.
CDSs are traded in the over-the-counter market and can be written on single-name reference entities, indices and tranches of structured credit products. The contracts are quoted on reference entities in the market with a CDS spread and recovery rate. Bid quotes refers to the protection buyer and an offer quote to the protection seller. We study CDS contracts in more detail in the next section.
According to the International Swaps and Derivatives Association (ISDA) CDSs strengthen the financial system, because:

- Banks can use CDSs to transfer credit risk to other investors, such that they can provide more debt to the market.
- CDSs distribute credit risk throughout the financial market to prevent for credit risk concentration.
- CDSs can be used to extract timely information on the credit quality of firms and therefore help in supervisory activities.

However, the recent credit crisis showed that there are also many risk associated with CDS contracts. First, the valuation of CDSs and other more complex credit derivatives requires understanding of advanced financial models, which make the pricing of products less transparent. Second, the CDS market is highly unregulated. For example this made it possible for AIG to trade a huge number of CDS contracts resulting in large off-balance sheet exposures towards credit risk. If the US government did not intervene with billions of support, AIG would have probably been defaulted in the credit crisis. And more recently, CDS investors were blamed to speculate on a default of Greece. The spread that the Greek government needs to pay for borrowing new money strongly increased, such that the country incurred payment difficulties.
Thus CDS contracts have advantages and disadvantages. Therefore regulatory bodies as the BIS and ISDA impose new requirements on CDS trading as a trade off between strengthening the financial system and the risks that CDSs may cause.

2.4.2 Elements of a CDS contract

This section introduces the information specified in CDS contracts, such that we can include this in the CDS pricing framework that we develop in section 2.5.
The majority of single name CDS contracts are specified according to the standards of the ISDA and comprise at least the following information:

- **The reference entity.** The buyer of the contract has an exposure to the reference entity (e.g. a firm) and agrees with the CDS contract to transfer the credit risk of this reference entity to the protection seller.
- **The reference asset.** The assets of the reference entity for which the protection buyer wants to transfer the credit risk. There is no requirement that the protection buyer owns the reference asset. Then the CDS contract is used for speculation.
- **The credit event.** The CDS contract is initiated to protect the buyer of the contract for a credit event at the reference entity. The CDS contracts specifies which events trigger the default payment of the protection seller to the buyer. The default events for CDSs are specified by the ISDA[6]:
    - Failure to pay
    - Bankruptcy, filing for protection

[6] See for a detailed explanation of the credit events the 2003 ISDA Credit Definitions, article IV.

  - Restructuring (e.g. coupon reduction or maturity extension)
  - Repudiation/moratorium
  - Obligation acceleration, obligation default (e.g. violations of bond covenants)

- **The notional value of the CDS.** This is the value of the reference assets that is protected by the contract.
- **The starting date of the CDS.** The date at which the default protection starts.
- **The maturity date.** The end date of the contract conditional on no credit event.
- **The CDS spread.** The CDS spread is the price for the default protection, measured in basis points[7] of the notional value. Multiplication of the CDS spread with the notional value and day count convention, gives the periodic premium payment of the protection buyer to the protection seller.
- **Frequency and day count convention for premium payments**. The contract specifies the period between successive premium payments, typically quarterly or semi-annually. The first payment is usually made at the end of the first period. When a credit event occurs between two payment dates, the protection buyer needs to make a final accrual payment to the protection seller. This is the payment for the protection between the last payment date and default date. The usual day count convention for CDSs is actual/360.
- **Settlement terms at a credit event.** If a credit event occurs before maturity, a CDS contract can be settled either physically or in cash. In a physical settlement, the protection seller buys the reference assets from the protection buyer for their notional value in exchange for the defaulted asset. In a cash settlement, the protection seller pays the difference between the notional value and the post-default market value[8] of the assets to the protection buyer.

Most CDS contracts that Deloitte CM values are single name contracts with a financial institution as reference entity, settlement in cash and with maturities between 3 and 10 years. Before we develop the pricing framework for these type of CDS contracts, we first elaborate on the relation between CDS spreads and bond yields that is used in the structural model framework.

2.4.3 CDS spreads and bond yield

The CDS spread that we described in the last section is an example of a credit spread. In general a credit spread is the premium that an investor requires as a compensation for the credit risk of a financial instrument. The larger the credit risk of an investment, the larger the credit spread an investor demands.

The credit spread is measured as the difference in returns between a risky investment and an equivalent risk-free investment. For example the credit spread is the difference between the return on a corporate bond (the bond yield), and the return of a similar risk-free bond. Also, the credit spread is equal to the CDS spread, since the CDS spread is the compensation for transforming a risky investment into a risk-free investment.

From these observations we infer the following relationship between the CDS spread, c, the bond yield, y, and the risk-free rate, r, of investments with the same maturity and notional value of the (underlying) bond: $c = y - r$. This relationship should hold, otherwise an investor can arbitrage to lock in an immediate profit[9].

However, Hull et al. (2004) impose numerous restrictions on this relation. The choice of the risk-free interest rate is an important point of interest. Bond traders often use Treasury zero curves for the risk-free rate and they measure the corporate bond yield spread as the spread of the corporate bond yield over similar treasury bonds. In this way it is assumed that the yield only reflects the credit risk of the corporate bond.

As we will see in section 2.6 however, there are many other factors that affect the yield on a bond, e.g. liquidity, such that this measure of the yield spread is incorrect. An alternative for the risk-free

[7] One basis point (bps) is 0.01%.

[8] The post-default market value of the reference assets depends on broker quotes from which the recovery rate is determined.

[9] For example if the CDS spread is higher than $y - r$ it is profitable to simultaneously sell the CDS, short the corporate bond and buy a risk-free bond. If the CDS spread is less than $y - r$, buy the CDS and corporate bond, and sell the risk-free bond.

rate is the LIBOR/swap rate often used by derivative traders. This rate is not totally risk-free and therefore captures some of the deficiencies of the Treasury zero rates.
CDS spreads are assumed to be a purer measure of credit risk (see also section 2.6) and thereby less affected by other factors than credit risk. We will use the Euro/swap curve for the risk-free interest rate in our modeling framework and leave the effects of liquidity and other variables for future research.

2.5 CDS valuation

This section develops the framework for valuing CDS contracts. We first give a short introduction to the hedge-based and bond yield-based pricing methods. Then we describe in more detail the discounted cash flow method that will be applied to our credit risk model.

2.5.1 Hedge-based valuation

The main points in hedge-based pricing of CDS contracts are (Schönbucher (2003)):

- The method is based on the assumption that contracts with exactly the same cash flows occurring at exactly the same time should have the same price, otherwise an arbitrage opportunity exists.
- The price of a CDS can thus be determined from constructing and pricing a portfolio of assets with exactly the same payoff.
- The method is popular because it provides hedge strategies and does not require complex pricing models.
- Results of the method are often imprecise and it cannot be used when prices of the required assets for the replication portfolio are not available.

2.5.2 Bond yield-based pricing

In the bond yield-based pricing methods we identify the following main points (Hull et al. (2000a)):

- This method assumes that the market's assessment of a firm's credit risk is reflected in the difference in market price (or yield) of a defaultable bond and a risk-free bonds with the same payoffs and maturity.
- From the comparison of these securities, the probability of default of the issuer of the defaultable bond can be derived. This comparison method is also used in the reduced form modeling approach.
- Next the price of the CDS is calculated with the discounted cash flow method method described in the next section.
- A main concern in this approach is the choice of the risk-free interest rate and the impact of other non-credit risk factors on the yield spread of defaultable bonds, see also sections 2.4.3 and 2.6.

2.5.3 Discounted cash flow method

The discounted cash flow method uses the risk-neutral probability of default calculated with a credit risk model as an input to determine the expected cash flows of the CDS and thereby its value.
We follow Hull (2006) and Yu (2005) to derive the formulas to calculate the expected present values of these cash flows. First we make assumptions and introduce terminology. Then we discuss the firm's survival probability that determines the default time and thereby the expected cash flows from the CDS. Finally we present the formulas to price a CDS contract. The following chapters describe how to determine the default time required for this valuation framework with a structural model.

Assumptions and terminology

The framework is developed to value single name, cash settled CDS contracts of various maturities to meet the research objectives. We make the following assumptions for this framework:

- The recovery rate used in the framework is the recovery rate on which the CDS is quoted.
- There is no correlation between the risk-free interest rate and credit risk.
- Only default of the reference entity is considered, counterparty credit risk is ignored.
- The CDS spread specified in the contract is constant and upfront spread payments are not allowed.

The value of the CDS is evaluated using risk-neutral valuation. This means that the present value of the CDS is equal to the expected value of its future cash flows discounted at the risk-free rate. By applying risk-neutral valuation we assume that the market price of risk is zero and therefore we work under the risk-neutral or $\mathbb{Q}$ probability measure. Expectations under this $\mathbb{Q}$-measure are expressed as $\mathrm{E}^{\mathbb{Q}}$.
The pricing formulas will be derived for a CDS contract that is initiated a time $t_0 = 0$ with maturity T. The protection buyer makes periodic payments at days t_p with $p = 1, 2, \ldots, z$ until maturity or default at time τ^*, whichever comes first. The number of default payments is z. Conditional on no default, the maximum number of protection payments is $z = T/\Delta t$, where Δt is the time period between successive payments, $\Delta t = t_i - t_{i-1}$, is specified in the contract. Each periodic payment provides protection against default of the reference entity for the period $[t_{i-1}, t_i]$.

Survival probability

The survival probability is used to determine the expected cash flows of the CDS contract. Until maturity or default of the contract, the reference entity survives and the protection buyer makes periodic payments to the seller. When a default event occurs the protection payments stop and the protection seller makes a default payment to the buyer.
Since we apply the risk-neutral valuation framework, expected losses from default are discounted at the risk-free interest rate. This is a valid procedure if the expected losses are calculated in a risk-neutral world and this implies that PDs in this valuation framework are risk-neutral.
Default probabilities determined from historical data are real-world PDs. Differences between the risk-neutral and real-world PD are explained from the excess return on defaultable bonds. According to Hull (2006) this excess return could have different reasons, as for example illiquidity and default correlation of defaultable bonds. When there is no excess return the risk-neutral and real-world PD would be the same.

The distribution of the firm's risk-neutral probability of default q_t is determined with the structural model. Integration of q_t gives the cumulative probability of default Q_t. Then, the risk-neutral probability that the firm will not default until time t is

$$S_t = 1 - Q_t = 1 - \int_0^T q_t dt.$$

We use this survival probability to define an indicator function $I(t)$:

$$\mathbf{1}_{\{\tau^* > t\}} = \begin{cases} 1 & \text{if } \tau^* > t \\ 0 & \text{if } \tau^* \leq t \end{cases}$$

The indicator function has a value of one with probability $S(t)$ conditional of no default event up to time t. If a default event occurs at time τ^*, the indicator function has a value of zero. We use the indicator in the pricing formulas for the CDS that we introduce in the next section[10].

[10] In general

$$\mathbf{1}_{xyz} = \begin{cases} 1 & \text{if } xyz \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

.

Pricing formulas

The value of a CDS contract equals the present value of the expected cash flows from the contract. Below, we first determine the present value of the cash flows from the protection buyer and protection seller. With these expressions we then construct the pricing formula for the CDS.
The present value of the cash flows from the protection buyer to the seller consists of two parts:

- The expected present value of the periodic payments until maturity or default. If a credit event occurs before time t_i, the i^{th} and following payments are not paid. The premium payment at time t_i is $Nc_{0,T}\Delta t_i \mathbf{1}_{\{\tau^*>t\}}$, where $c_{0,T}$ is the par CDS spread of a CDS contract issued at time $t_0 = 0$ with maturity T.
- The expected present value of the accrual payment that needs to be paid by the protection buyer when a default event occurs between two payment dates. This accrual payment is $Nc_{0,T}(\tau^* - t_{i-1})$.

From this the expected present value of the total payment made by the protection buyer equals:

$$\mathrm{E}^{\mathbb{Q}}\left[\sum_{i=1}^{z}\left(Nc_{0,T}\Delta t_i e^{-rt_i}\mathbf{1}_{\{\tau^*>t_i\}} + Nc_{0,T}(\tau^* - t_{i-1})e^{-r\tau^*}\mathbf{1}_{\{t_{i-1}<\tau^*<t_i\}}\right)\right]. \tag{2.1}$$

If the reference entity defaults at time τ^*, the protection seller makes a default payment to the protection buyer. We assumed that this settlement is made in cash, such that the default payment equals the expected present value of the LGD of the contract and is written as

$$\mathrm{E}^{\mathbb{Q}}\left[N(1-R)e^{-r\tau^*}\mathbf{1}_{\{\tau^*\leq T\}}\right]. \tag{2.2}$$

Now the market value of a long position in a CDS is the present value of the expected default payment (2.2) minus the present value of the expected premium payments (2.1):

$$\begin{aligned} &\mathrm{E}^{\mathbb{Q}}\left[N(1-R)e^{-r\tau^*}\mathbf{1}_{\{\tau^*\leq T\}}\right] - \\ &\mathrm{E}^{\mathbb{Q}}\left[\sum_{i=1}^{z}\left(Nc_{0,T}\Delta t_i e^{-rt_i}\mathbf{1}_{\{\tau^*>t_i\}} + Nc_{0,T}(\tau^* - t_{i-1})e^{-r\tau^*}\mathbf{1}_{\{t_{i-1}<\tau^*<t_i\}}\right)\right]. \end{aligned} \tag{2.3}$$

A CDS contract is constructed such that the value of the contract is zero at initiation. This is done by specifying a CDS spread that equates the the present value of the protection and default payments. Setting equation 2.3 equal to zero and solving for the credit spread yields:

$$c_{0,T} = \frac{\mathrm{E}^{\mathbb{Q}}\left[N(1-R)e^{-r\tau^*}\mathbf{1}_{\{\tau^*\leq T\}}\right]}{\mathrm{E}^{\mathbb{Q}}\left[\sum_{i=1}^{z}\left(N\Delta t_i e^{-rt_i}\mathbf{1}_{\{\tau^*>t_i\}} + N(\tau^* - t_{i-1})e^{-r\tau^*}\mathbf{1}_{\{t_{i-1}<\tau^*<t_i\}}\right)\right]}. \tag{2.4}$$

Equation 2.4 gives the par CDS spread of a CDS contract issued at time $t_0 = 0$ with maturity T.
The expression in equation 2.3 can be used to determine the value of a single name, cash settled CDS contract when the default time τ^* is determined with a credit risk model. However, in the following chapters of this study we focus on the calculation of CDS spreads with equation 2.4[11]. We evaluate this expression for various maturities to obtain a CDS term structure. Modeled CDS term structures are further analyzed in chapter 4.

2.6 Credit spread puzzle

Equation 2.4 shows that CDS spreads depends on the survival probability and thus the credit risk of the reference entity. However, market CDS spreads are often found to be higher than the spreads obtained from credit risk models. This implies that credit risk is not the only factor that determines market spreads.

[11] Note that a model that calculates CDS spreads with equation 2.4 can be easily transformed to value CDS contracts using equation 2.3.

This section gives an overview of empirical studies that analyze this ‘credit spread puzzle’[12]. In this overview we first present the main findings of studies that analyze bond yield spreads, since this market exists longer than the CDS market, and then the main findings on CDS spreads. We identify determinants besides credit risk that affect the credit spread, such that we could account for these factors in the development of the structural models in chapter 4.

2.6.1 Bond spreads

Collin-Dufresne, Goldstein, and Martin (2001) (CGM) analyze market spreads of corporate bonds. They test whether the following factors affect spread changes:

- **Changes in the risk-free interest rate.** A higher risk-neutral interest rate, reduces the credit spread.
- **Changes in the slope of the yield curve.** An increase in the Treasury yield curve[13], increases the expected interest rate, and thus reduces the credit spread.
- **Changes in leverage**[14]. An increase in the firm’s leverage, increases the credit spread.
- **Changes in volatility.** Intuitively, when volatility increases, the probability of default increases.
- **Changes in the probability or magnitude of a downward jump in firm value.** In section 3.3.4 we will see that this increases the credit spread.
- **Changes in the business climate.** A better business climate[15] lowers the credit spread.

CGM find that these factors explain approximately a quarter of the variation in the market spreads. The remaining variation could be explained by a single unidentified factor, which CGM describe as a result of supply and demand shocks.
Campbell & Taksler (2003) conduct a similar analysis as CGM and conclude that firm specific equity volatility is an important determinant of the corporate bond spread.
Cremers et al. (2006) find that option implied volatility could account for almost one third of the total variation in credit spreads. Furthermore they show that a structural model is able to explain the variation in credit spreads and that there is no evidence of a large unidentified factor affecting spreads as reported by CGM.
Huang & Huang (2003) conduct an empirical analysis of the credit spreads modeled by various structural models. They find that credit risk accounts for only a small fraction (20-30%) of the observed corporate spread for investment grade bonds of all maturities. For junk bonds, however, credit risk accounts for a much larger fraction.

2.6.2 CDS spreads

Skinner & Townend (2002) are the first to analyze the difference between model and market CDS spreads. They claim that CDS can be viewed as a put options. Therefore they analyze whether variables that impact option pricing also impact CDS spreads. They show that the risk-neutral interest rate, the yield of the underlying asset, the maturity and the volatility, are also important in pricing credit default swaps.
Ericsson et al. (2005) conduct a similar research as CGM on CDS spreads. They show that the leverage of the firm, the volatility of the firm’s assets and the risk-neutral interest rate explain approximately 60% of the market CDS spread. They also asses the impact of changes in these variable to changes in the CDS spread. They find that a 1% increase in annual equity volatility increases the CDS spread by 1 to 2bps and that a 1% change in the leverage of the firm raises the CDS spread by approximately 5 to 10bps. Furthermore Ericsson et al. find only weak evidence for a common unidentified factor as CGM describe.

[12] The presented empirical literature uses the term ‘credit spread puzzle’ for the observation that credit spreads are too high to be explained by only credit risk.
[13] The Treasury yield curve describes the relation between the interest rate on a Treasury bond (assumed to be risk-free) and its time to maturity.
[14] CGM define leverage as the book value of debt divided by the sum of market value of equity and book value of debt.
[15] Measured by the return on the S&P 500 index.

Zhang et al. (2005) test with a structural model the impact of the firm's equity volatility and jump risk on the credit spread. They are able to explain 77% of the variation in CDS spread by using this approach.
Alexander and Kaeck (2008) find that interest rates, stock returns and implied volatility have a significant impact on CDS spreads. They again find evidence for a systemic factor as CGM. Furthermore, they show that the impact of the variables differs with the overall market conditions. They show that CDS spreads are sensitive to implied volatility in turbulent times and more sensitive to stock returns in ordinary market conditions.
Bongaerts et al. (2008) study the effects of expected liquidity and liquidity risk on CDS spreads. They find evidence for a systematic liquidity factor in the CDS market and conclude that liquidity factors need to be considered in a CDS pricing model.
Das & Hanouna (2008) observe that CDS spreads are less affected by liquidity and other non-credit risk related factors than bond spreads. However, they show via a hedge relationship between the credit and equity market that CDS spreads are negatively related to the equity liquidity of the reference entity.

We emphasize that this literature review is not exhaustive, but we can observe the following:

- Credit spreads modeled with a structural framework show high correlation with market spreads. However, the modeled spreads are lower than the corresponding market spreads due to non-credit risk related factors. Bond spreads seem to be more affected by these factors than CDS spreads. This suggest that structural models are better able to estimate CDS spreads than bond spreads. This is further investigated in section 3.4.

- The literature identifies several (unidentified) factors that explain part of the credit spread besides credit risk. Most studies agree on the sign of the relations between the credit spread and factors. However, the magnitude of the impact differs amongst studies due to differences in data sets.

This study focusses on modeling CDS term structures and therefore we need to take these factors into account in our modeling approach. We will do this when we the select the components of the structural model that we will implement at the end of chapter 3. And we assess the impact of several (market risk) factors on CDS term structures in chapter 4. As stated before, liquidity factors are left for future research.

2.7 Summary

This chapter provided the basics on credit risk and credit derivatives necessary to understand the following parts of this thesis. We have specified the components of CDS contracts and derived a discounted cash flow pricing method to value single name, cash settled CDS contract of various maturities. Furthermore we studied several credit risk models and determined that structural models are able to meet the research objectives of this study. These models are therefore further investigated in the next chapters.

Chapter 3

Structural models

Chapter 2 determined that structural models of credit risk are best able to meet the objectives of this thesis. This chapter further investigates these models and starts with a description of the first structural model developed by Merton (1974). We compare PDs and CDS spreads evaluated with this model to market data and identify the model's shortcomings. Then we examine extensions to the Merton model that address the Merton model's shortcomings, and describe the implications of these extensions for credit risk modeling. Finally, we review empirical literature to assess the performance of structural models in modeling credit spreads. Based on this literature review, we select the components of the structural models that we further implement and analyze in chapters 4 and 5.

3.1 The Merton model

The literature on structural models for credit risk starts with the papers of Black & Scholes (1973) and Merton (1974). Merton develops a framework that relates the firm's assets value to its credit risk and subsequently uses the Black & Scholes option pricing formulas to price defaultable bonds and equity of the firm.
This section describes the Merton (1974) model in more detail. We first summarize the assumptions underlying the model and analyze the conditions of default. Then we present the formulas to price equity and debt and to calculate PDs and credit spreads. The shortcomings of the Merton model are discussed in section 3.2.

3.1.1 Assumptions and default conditions

Merton (1974) makes the following assumptions to develop his model[1]:

1. There are no transaction costs, bankruptcy costs or taxes, such that the Modigliani-Miller theorem holds[2]. Assets are divisible and trading takes place continuously in time with no restrictions on short selling of all assets. Borrowing and lending is possible at the same, constant interest.
2. There are sufficient investors in the market place with comparable wealth levels, such that each investor can buy as much of an asset he wants at the market price
3. The risk-free interest rate r is constant and known with certainty.
4. The evolution of the firm's assets value V_t follows a stochastic diffusion process:

$$\frac{dV_t}{V_t} = (\mu_V - \delta)dt + \sigma_V dW_t \tag{3.1}$$

[1]Since Merton uses the Black & Scholes (1973) methodology to price securities, Merton makes these assumptions along with some of the Black & Scholes assumptions.

[2]The Modigliani-Miller theorem states that in the absence of bankruptcy costs and taxes the value of a firm is invariant to changes in its capital structure.

where μ_V is the expected return on the firm's assets per unit time, δ is the payout of the firm per unit time[3], σ_V is the volatility of the firms assets per unit time, and dW_t is a Wiener process.

According to Merton (1974) not all of these assumptions are necessary to obtain the model, but are made for convenience. The critical assumptions are continuous time trading and assumption 4.
Furthermore, the model assumes a simplified capital structure for the firm. Total debt consists of only one zero-coupon bond (ZCB) and there are no additional debt issues before maturity of the ZCB. The firm's equity consists of ordinary shares. Both debt and equity are contingent claims on the assets of the firm and the value of total assets (or the firm value) equals the value of total debt B_t and equity E_t: $V_t = B_t + E_t$.

The ZCB has a notional amount D, which has to be paid at maturity T. When the value of the firm's assets at maturity exceeds D, the bondholders receive the full notional amount and the shareholders receive the residual asset value $V_T - D$. When the asset value at maturity is less than D the firm cannot make the promised debt payment and defaults. The bondholders take over the firm and receive the firm value V_T, while the shareholders receive nothing. Shareholder never have to compensate for the bondholders' loss in case of default, which means that E_T cannot be negative.

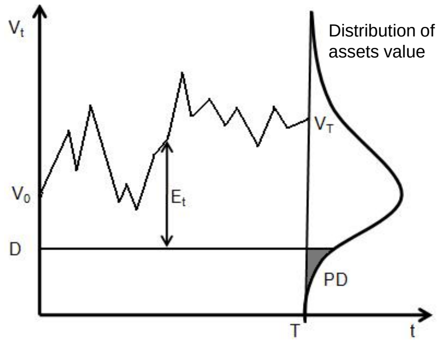


Figure 3.1: Schematic representation of the Merton model, Duffie & Singleton (2003).

Figure 3.1 illustrates the dynamics in the Merton model. Observe that total debt D is constant over time and that the value of equity fluctuates with the value of the firm's assets. Default occurs only when the firm value drops below the default barrier at maturity, such that $V_T < D$. By simulating various paths for the asset value process a distribution of the asset value at maturity is modeled. The shaded area of this distribution is the probability of default.

3.1.2 Security pricing and PD calculation

Based on the assumptions and default conditions described above we can derive the formulas for pricing debt and equity in the Merton framework. Since we need the Black & Scholes option pricing theory for this objective we will work under the risk-neutral probability measure $\mathbb{Q}$[4]. We follow Hull et al. (2004) to derive the formulas.
The payoffs of debt and equity at maturity can be expressed as European options written on the firm's assets with exercise price D and maturity T. We saw in section 3.1.1 that the payoff to the bondholders at maturity equals $B_T = \min(V_T, D)$. We can replicate this payoff with the following portfolio[5]:

$$B_T = V_T - \max(V_T - D, 0) \tag{3.3}$$

[3]A positive δ is a payout to shareholders or liabilities-holders (dividends or interest respectively) and a negative δ is the net amount received from new equity financing.

[4]Under this measure we set the expected return of the firms assets μ_V equal to the risk-free interest rate r to model the firm's asset value process (assuming zero payout, i.e. $\delta = 0$) as a geometric Brownian motion:

$$\frac{dV_t}{V_t} = rdt + \sigma dW_t \tag{3.2}$$

[5]Note that we could also replicate the debt payoff with the portfolio $D - \max(D - V_T, 0)$ that is constructed from a risk-free ZCB with notional value D and a short position in a European put option on the firm's assets with exercise price D.

This portfolio consists of a long position in the firm's assets and a short position in a European call option on the firm's assets with exercise price D.
Once the debt has been paid at maturity the remaining assets value belongs to the shareholders. This payoff equals the payoff of a European call option written on the firm's assets with exercise price D:

$$E_T = \max(V_T - D, 0). \tag{3.4}$$

The debt payoff as a function of the assets value is shown in the upper part of figure 3.2. The right graph shows the payoff to the shareholders, the payoff of a European call option.

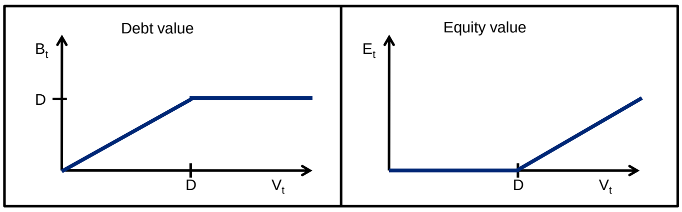


Figure 3.2: Equity and debt values as a function of the assets value in the Merton Model.

We can now apply the Black & Scholes option pricing formulas to determine the value of the firm's debt and equity at time t $(0 \leq t \leq T)$ as

$$B_t = V_t\Phi(-d_1) + De^{-r(T-t)}\Phi(d_2) \tag{3.5}$$

$$E_t = V_t\Phi(d_1) - De^{-r(T-t)}\Phi(d_2) \tag{3.6}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function and d_1 and d_2 are given by

$$d_1 = \frac{\ln(V_t/D) + (r + \frac{\sigma_V^2}{2})(T-t)}{\sigma_V\sqrt{T-t}} \qquad \text{and} \qquad d_2 = d_1 - \sigma_V\sqrt{T-t}.$$

Figure 3.1 showed that the probability of default in the Merton model is given by the probability that the firm's assets value is lower than the obligated debt payment D at maturity. In the Merton framework the risk-neutral probability P_M of default at time T can be calculated as:

$$P_M = \Phi(-d_2). \tag{3.7}$$

From this equation we infer that the probability of default depends on the inverse leverage of the firm V_t/D, the volatility of the firm's assets σ_V, and the time to maturity T. We further analyze these relationships in chapter 4.

3.1.3 Credit spread in the Merton model

Finally we derive the credit spread in the Merton framework using the relationship between the credit spread, risk-free interest rate, and bond yield described in section 2.4.3: $c = y - r$[6]. The yield to maturity y of the ZCB is implicitly given by

$$B_t = De^{-y(T-t)}$$

Solving this equation for y and then substituting $B_{t,T}$ with equation 3.5 gives

$$y_{t,T} = -\frac{\ln\left[(V_t/D)\Phi(-d_1) + e^{-r(T-t)}\Phi(d_2)\right]}{T-t} \tag{3.8}$$

Now we can use $c = y - r$ to write the CDS spread in the Merton model as

$$c_{t,T} = -\frac{\ln\left[(V_t/D)\Phi(-d_1)e^{r(T-t)} + \Phi(d_2)\right]}{T-t}. \tag{3.9}$$

The credit spread also depends on the firm's leverage, the volatility of the firm's assets and the time to maturity.
Equation 3.9 is specific for the Merton model. We will use equation 2.4 with an algorithm to calculate the default time to determine CDS spreads in chapter 4.

[6]See appendix A.1 for a more detailed derivation of the credit spread in the Merton model.

3.2 Shortcomings of the Merton model

Merton's model provides an insightful approach to assess a firm's credit risk and to calculate debt and equity values. However, this section shows that the practical applicability of the model is restricted. We compare modeled PDs and CDS spreads to market data to identify the most important shortcomings of the Merton model. These shortcomings are addressed in subsequent sections.
Figure 3.3 shows par CDS term structures for a number of Dutch firms with various credit ratings (Moody's). Note that firms with a different credit rating can have approximately similar CDS spreads as for example Shell and Philips. Possible explanations are different industry risks and prospects or an 'old' credit rating that does not incorporate the current conditions of the firm that are already reflected in the CDS spread.

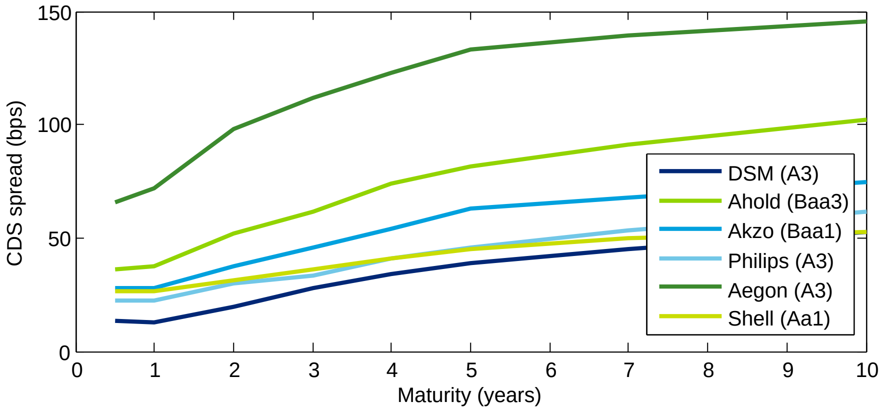
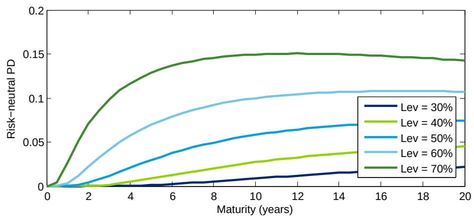
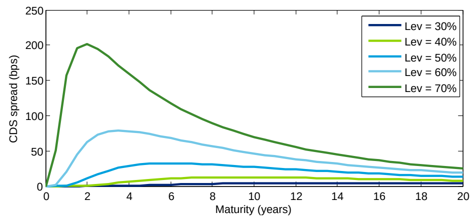


Figure 3.3: CDS term structures of selected Dutch firms on 31-12-2009 (Bloomberg, CMAN).

Figure 3.4 plots term structures of the risk-neutral PD and CDS spread calculated with the Merton model for various levels of the promised debt payment at maturity. These term structures are obtained with a Monte Carlo simulation approach given in appendix A.2.



(a) Probability of default



(b) CDS term structure

Figure 3.4: Probability of default (a) and CDS term structures (b) determined with the Merton Model for various leverage ratios. Note that this leverage ratio is defined as the ratio of the promised debt payment D to the initial firma value. Input parameters are: $r = 0.05$, $\sigma_V = 0.2$, and $V_0 = 100$.

Comparing figure 3.4a with the cumulative default rates in figure 2.2 and figure 3.4b with 3.3 results in the following shortcomings of the Merton model in estimating market PDs and CDS spreads.
The figure illustrates that modeled PDs and CDS spreads are zero for short maturities. This is intuitive, since solvent companies will not default within a short time period. In the market however, we observe positive short-term spread and default probabilities. An explanation for this discrepancy is the firm's asset value process considered in the Merton model. This diffusion type process cannot reach the default barrier within a short time period, such that the modeled firm does not default and the PD and CDS spread will be zero. Due to this slow evolution of the assets value process, the default event never happens unexpectedly in the model. Section 3.3.4 describes an extension to the Merton model that can increases short term maturities and makes the default event unexpected.

Over a long horizon, marginal PDs and especially CDS spreads decrease in the Merton model, while these are increasing functions in the market. This is because firms cannot issue additional debt between initiation and maturity in the Merton model. Due to a positive drift in the assets value process the distance between this process and the constant promised debt payment increases such that the firm is less likely to default. Section 3.3.1 offers a solution to the model's shortcoming of downward sloping term structures.

From the assumptions underlying the Merton model, we derive four more shortcomings. First, the debt structure of a firm is often more complicated than the Merton model assumes. Common features of a firm's debt like coupons, covenants and embedded options cannot be modeled. In section 3.3.1 we describe structural models that allow for a more extend capital structure.
Second, in the Merton model default can only occur at the maturity of debt. This implies that a firm's asset value can drop to almost zero and subsequently recover to D or more before maturity, without going bankrupt. In practice, such a firm would have been defaulted before default. Structural models that introduce more advanced default barriers to deal with this shortcoming are discussed in section 3.3.2
Furthermore, the Merton model assumes a constant risk-free interest rate that is known with certainty. However, the term structure of interest rates observed in the market is not flat and stochastic in time. Merton makes the assumption to omit the correlation between credit risk and interest rates, that occurs in practice. Extensions to the model that relax Merton's interest rate assumptions are described in section 3.3.3.
Finally, another major drawback applying to all structural models is the unobservability of the firm's asset value process. The parameters to model this process therefore need to be estimated from other observable processes, such as equity prices. We address this topic in chapter 5.

3.3 Extensions to the Merton model

The last section identified the shortcomings of the Merton model and their impact on the model's implementation and results. In this section we discuss extensions to the Merton model that address these shortcomings by adjusting the model's assumptions. Extensions with respect to the capital structure of the firm, the default conditions, the interest rate process and the firm value process are presented from a qualitative perspective. In section 3.4 we analyze the performance of these model extensions in approximating credit spreads and PDs. The overview of extensions presented is certainly not exhaustive, but the selected models are often referred and considered as the basic extensions to the Merton model in the literature.

3.3.1 Capital structure

In the Merton model (1974) the firm's debt consists of a single ZCB. To account for coupon paying bonds Merton proposes to consider them as a portfolio of ZCBs, each of which can be priced using his model. Geske (1977) extends this idea and develops a framework that can deal with more complicated debt structures in structural models like coupons, debt subordination, and payout restrictions.
In the Geske model the debt structure of the firm is modeled as several coupon bonds. On each payment date the shareholders have the option to pay the coupon and continue their control of the firm until the next coupon date[7]. The shareholders make the coupon payments from issuing new equity. When they are unable to refinance, the shareholders decide not to pay, and the firm defaults. Then the bondholders take over the firm and receive V_t.
Geske offers a structural model for dealing with complex capital structures. However, in this model the default time is still expected since the firm can only default at the debt payment dates. Furthermore, a firm's default does not only occur due to the failure of debt payments. Firms also default when covenants are breached or their cash flow is insufficient to meet any financial obligation, e.g. accounts payable and taxes.
Collin-Dufresne & Goldstein (2001) (CDG) incorporate a dynamic capital structure in their model to overcome the decreasing default probabilities and credit spreads for long maturities. Their approach

[7]Geske models the shareholders' coupon payments as a compound option, since the option whether or not to pay the coupon is an option on the firm's assets.

allows firms to adjust their capital structure towards a target leverage ratio: when firm value increases the firm issues more debt and when the firm value decreases debt is not replaced at maturity. Now that the leverage of the firm is approximately constant the firm value will drift less far away from the barrier, such that default is more likely to occur.

3.3.2 First-passage models

In the Merton model (1974) a firm can only default when the firm's assets value is below the notional debt value at maturity. In practice firms can default at any time and due to any financial obligation. In this section we describe so called first-passage models that alter the default conditions of the Merton model and also accommodate more advanced debt structures.
In these first-passage models a firm defaults when the firm's assets value drops for the first time below a certain default barrier. Since default is now possible at times $0 \leq t \leq T$, the PD and credit spread obtained from the first-passage models are higher than those implied by the Merton model, resulting in a better approximation of market spreads.
Furthermore, since default can happen before maturity the probability of a positive payoff to the shareholders is lower, thus the value of the firm's equity is lower than in the Merton model. From the balance sheet equation, $V_t = B_t - E_t$, we derive that the debt value should thus be higher in the first-passage time models. This is intuitive because investors have to pay for the protection that the firm value can never drop below the barrier level.
The literature gives various specifications of the default barrier. Below we give a selection of studies in which we distinguish between exogenous barriers that are defined outside the model, and endogenous barriers defined within the model.

Exogenous default barrier

The level of an exogenous default barrier is defined outside the model. Black & Cox (1976) introduced the first-passage models with an exogenous default barrier. They model safety covenants on the debt as a time dependent exponential default barrier. When the firm's assets value drops below the specified covenant level, the firm defaults on all outstanding obligations and the debt holders obtain control over the firm's assets, while the shareholders receive nothing.
Bryis & de Varenne (1997) adjust the exponential threshold of BC to obtain a default barrier that equals the principal debt payment discounted at the risk-free interest rate. By considering a stochastic interest rate process[8], this default threshold becomes stochastic too. Longstaff & Schwartz (1995) (LS) also consider a stochastic interest rate, but model a constant, exogenous given default barrier. Kim et al. (1993) assume a default level that is a function of the coupon rate.
As described in the previous section, CDG incorporate a dynamic capital structure in their model to account for the downward sloping term structures in the Merton model. This is modeled as a mean reverting default barrier in which the level of the default barrier is set equal to the total amount of outstanding debt to obtain a mean reverting leverage process.[9].
There are many more expressions for the exogenous default barrier that we do not discuss in this thesis. In section 5.1.2 we consider a last variant: the stochastic default barrier of the CreditGrades model.

Endogenous default barrier

An endogenous default barrier is defined within the structural model. The level of the barrier is modeled as a shareholders' decision. At each payment date, the shareholders decide whether the firm's prospects are sufficient, such that paying the debt maximizes their (future) equity value.
As long as the firm's assets value is higher than the endogenous default barrier, the firm does not default. The assets value can drop below the required debt payment without a default to occur. In that case, the endogenous default barrier is below the required debt payment (and the assets value), which means that shareholders have decided that it is in their interest to keep control over the firm. Default in models with an endogenous default barrier typically occurs at asset levels below the outstanding debt level.

[8]See section 3.3.3 for a description of stochastic interest rate processes in structural models.
[9]More details of this model are provided in chapter 4.

Leland (1994) and Leland & Toft (1996) (LT) consider endogenous default barriers in their studies on the optimal capital structure of a firm. The former considers debt with an infinite maturity, while the latter examines the effect of debt maturity on bond prices, credit spreads, and the optimal amount of debt. In LT the optimal capital structure is a trade-off between tax advantages and bankruptcy costs. The level of the default barrier is a decision of the shareholders. Default occurs when they cannot maximize their equity value by holding off bankruptcy.
Anderson et al. (1996), and Fan & Sundaresan (2000) model the endogenous barrier as a strategic debt service. In these models a negotiation process between debt and equity holders is assumed. The negotiation can result in a lower debt payment than promised to prevent the firm from default. However, shareholders can also decide to default strategically to obtain concessions from debt holders. These strategic debt service models result in a higher probability of default than the Merton model.

The modeling of default barriers in the first-passage approach provides improvements to the Merton model. In the models, firms can now default before maturity resulting in a higher PD than calculated with the Merton model. Altering the default barrier does increase the complexity of the models, especially in the case of endogenous default barriers. Furthermore, by only adjusting the default barrier the shortcomings of the Merton model regarding expected default times and zero short-term credit spreads are not solved. We will now analyze the impact of relaxing the interest assumptions of the Merton model.

3.3.3 Interest rate process

The Merton model and many of its extensions assume a known, constant interest rate in their framework for simplification and to omit the correlation between interest rates and credit risk. Since this assumption is not in line with market observations on the interest rate term structure, several structural models are developed that model stochastic interest rate process.
LS use the term structure model of Vasicek (1977) for the risk-free interest rate process in their model. CDG also assumes the Vasicek (1977) process in their model, but distinguish from the framework of LS with their definition of the default barrier. Bryis & de Varenne (1997) use a generalized Vasicek process[10], and Kim et al. (1993) suggest the CIR process described in Cox et al. (1985b) for the risk-free interest rate.
A disadvantage of the Vasicek (1977) interest rate model is that it is theoretically possible that interest rates become negative. However, LS argue that the probability of negative interest rates is small for the parameter values used in their structural model. The parameters in the CIR process can be chosen such that the process does not generate negative interest rates. LS and Kim et al. introduce correlation between the interest rate and credit risk by assuming that the Brownian motions in the processes for the firm value and interest rate are correlated. Bryis & de Varenne assume that these Brownian motions are independent, and incorporate a correlation coefficient between the risk-free interest rate and the firm's assets value.
In section 2.6 we saw that the interest rate level is negatively correlated to credit spreads. LS show that this relationship follows from their model and explain that an increase in r effects the drift of the firm value process such that the probability of default tends to be lower and the credit spread decreases. Furthermore, LS find that for investment grade bonds changes in interest rates have more impact on credit spread variations than the fluctuations in the firm's asset value. This effect appears to be less evident for speculative grade debt. These findings of LS suggests that the inclusion of a stochastic interest rate process adds to the performance of a structural model in approximating credit spreads. In section 3.4 we will see whether there is more empirical evidence supporting this finding of LS.
A drawback of stochastic interest rates is the increasing complexity of the model, often making a numerical approximation necessary. Furthermore, the addition of stochastic interest rates does not solve the issue of zero credit spreads and default maturities for short maturities in the Merton model. This issue is the topic of the next section.

[10] In a generalized Vasicek model, the constants in the Vasicek (1977) model are time-dependent.

3.3.4 Assets value process

The Merton model and its extensions considered so far all assume a diffusion process for the firm's value. Under a diffusion process a firm never defaults unexpectedly, because a sudden drop in firm value is impossible. This implies that firms that are currently not in financial distress have a zero PD and credit spread on short-term debt, which is rejected in market observations.
Zhou (1997) addresses these problems by incorporating random jumps in the asset value process. With this jump-diffusion process a default event can occur either from the marginal changes in the firm's assets value (the diffusion component of V_t) or from unexpected shocks in the firm value process (the jump component of V_t). In the first case the firm value equals the default barrier at default, and in the second case the firm value might be below the barrier at default.
Qualitatively the jumps can be seen as new important information becoming available to the investors that causes changes in the firm value, e.g. unexpected financial results, law suits, and acquisition announcements. Figure 3.5 illustrates that a jump-diffusion and diffusion process are approximately similar except for the jumps.

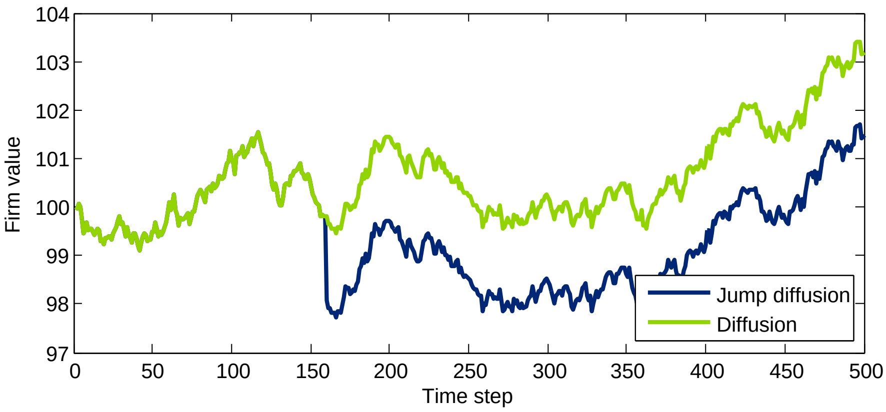


Figure 3.5: Assets value process modeled as a jump-diffusion process and a diffusion process.

According to Zhou (1997), a jump-diffusion type model for the firm value has a number of advantageous implications:

- The jump component enables the term structure of credit spreads to take on the shapes observed in the market: upward sloping, flat, hump-shaped, and downward sloping.
- The model can generate non-zero default probabilities and credit spreads for very short-term bonds of healthy firms.
- The firm value at default is a random variable, implying that recovery rates are stochastic.

A structural model with a jump-diffusion process thus combines the advantage of unexpected default events in the reduced form models with the economic explanation of default in structural models. Furthermore, these models are able to estimate all shapes of CDS term structures, which makes the jump-diffusion process an attractive component of a structural model to reach our study objectives. However, structural models including a jump-diffusion process make parameter estimation more difficult and are therefore less attractive for practical purposes.

3.3.5 Other extensions

We have presented several extensions to the Merton model designed to improve the performance of the model in estimating market prices and spreads. Before we study this performance we briefly discuss two other extensions that might be interesting for the structural model that we determine in this study.
Several structural models, e.g. Merton, Black & Cox (1976) and LS, assume that the recovery rate is a proportion of the remaining assets after default. However, empirical research finds that the recovery rate is correlated with the probability of default, which implies that a constant recovery rate does

not correspond to market observations.
As described above the jump-diffusion approach of Zhou (1997) can account for this. When in this approach the assets value jumps below the default barrier, the firm defaults and the remaining assets value is determined by the jump magnitude. If we now define the recovery rate as a percentage of the remaining assets value the recovery rate is not constant anymore, but stochastic.
Another category of studies claim that the evolution of the firm's assets should not be the only default triggering event in structural models. For example, Davydenko (2005) finds empirical evidence that firms with high assets value, but with a low current cash flow or high financing costs have difficulties in paying their debt obligation and default. Davydenko (2005) therefore concludes that, next to a low assets value level, a firms' liquidity shortages in combination with high external financing costs are also important in explaining the default event.
Based on this observation, Elizalde (2006) claims that structural models should be extended to incorporate defaults due to these liquidity shortages. To do this, she proposes that both the processes for cash flows and external financing, and the correlation between them, should be modeled in the structural framework.
Leland (2005) however, comments on Davydenko (2005) that models with an endogenous default barrier models account for these liquidity-based defaults. Since in these models the shareholders decide on the level of the default barrier to maximize the equity value, they will include future liquidity shortage and high funding costs in their decision.

3.4 Performance analysis of structural models

The previous section introduced several extensions to the Merton model from a qualitative perspective. This section studies the impact of these extensions on the model's performance in approximating market credit spreads. This is done by reviewing empirical studies that compare bond spreads, CDS spreads and PDs estimated with structural models to market observations. We present the main findings of studies that assessed the structural models described in the previous section. From this review we draw ideas for the structural models that will be implemented in chapter 4.

3.4.1 Performance of the Merton model

Jones et al. (1984) present the first empirical study that assesses the performance of the Merton model in practice. They test this model on its ability to price corporate bonds and find that the model prices are far below the corresponding market prices. Eom et al. (2004) confirm this underestimation of bond prices and spreads by the Merton model. Huang & Zhou (2008) find that the Merton model does a poor job in estimating CDS spreads. We will now see whether the extensions to the Merton model can estimate higher bond prices, credit spreads and default probabilities.

3.4.2 Performance of extensions in modeling bond values

Eom et al. (2004) test five structural models for pricing corporate bonds: the Merton, Geske, LS, LT and CDG model. They find that all models show substantial pricing errors and model too low spreads for short maturities. In particular they find that the Merton and Geske models underestimate with a mean absolute errors of 70%. The inclusion of a stochastic interest rate via the LS model leads to a small increase in the model spread on average. However they find that model spreads are still too low and that the results are sensitive to the interest rate volatility estimates of the Vasicek model. The LT model appears to overestimate market spreads and the variation in spreads dependents on the bond's coupon specifications. The model of CDG also overestimates market spreads, but with a lower dispersion than the LT model. Furthermore, they find that the models overestimate the spreads of firms with high leverage and volatility (risky bonds) and underestimate spreads of healthy firms.
Huang & Huang (2003) (HH) assess the performance of structural models in explaining the corporate bond spread. They use as a base case the LS model without stochastic interest rates and compare this model to several extensions. As opposed to Eom et al. (2004) they find that the inclusion of stochastic interest rates, modeled by the total LS model, lowers the calculated credit spread. The LT

model and strategic default models generates higher credit spreads than the base case, especially for investment grade bond. They argue that this is due to the consideration of a perpetual bond in the LT model, and not due to the endogenous default barrier. The model of CDG also produces higher spreads for investment grade debt than the base case. HH also develop their own double exponential jump-diffusion model, that produces higher spreads than the base case.

3.4.3 Performance of extensions in modeling CDSs

Ericsson et al. (2006) assess the endogenous default barrier models of Leland (1994), LT, and Fan & Sundaresan (2000) with respect to bond and CDS spreads. They find that the models underestimate bond spreads but perform better on CDS spreads. Especially the LT model, which overestimates observed CDS spreads with only 8bps on average. They argue that the difference between modeled and market spreads is caused by non-default related factors, such as illiquidity, that influence bond spreads more than CDS spreads.
Huang & Zhou (2008) analyze the performance of five structural models in estimating CDS spreads: the Merton, BC, LS, CDG, and HH model. Their empirical study shows that the Merton, BC, and LS model are inaccurate in estimating CDS spreads. The HH outperforms these three models and the model of CDG performs even better. Huang & Zhou also draw general conclusions from their study that can improve the performance of the models in estimating CDS spreads. First, another interest rate model than the Vasicek (1977) model used in LS may reduce the estimation error. Furthermore, inclusion of a jump process increases the performance on investment grade CDSs, while the inclusion of dynamic leverage improves the results for speculative grade CDSs.

3.4.4 Performance of extensions in modeling PDs

Finally, it is interesting to assess the performance of structural models in modeling default probabilities. This is because the PD is less affected by other factors than credit risk, as described in section 2.6, that cause differences between modeled and market spreads. Tarashev (2005) compares modeled probabilities of default to historical default rates for six structural frameworks: the LS, CDG, HH, Anderson et al. (1996), LT, and Moody's KMV model[11]. The main conclusion of Tarashev (2005) is that in general structural models can provide close forecasts for actual default rates. He finds that models with an endogenous default barrier perform better than models with an exogenous barrier. The LT model performs bests, however the results are sensitive to the dynamics of the firm's leverage in the used dataset. The models with exogenous barriers all underestimate default rates and the Moody's KMV model overestimates default probabilities most.

3.4.5 Findings from the literature review

Although this literature review is not exhaustive we can use the presented findings to draw ideas for the structural models that are further analyzed in the next chapters.
First, we infer that structural models that are used to estimate bond prices and bond yield spreads result in discrepancies with market observations. The main reason for this difference is that the market values are affected by non-default related factors as we also found in section 2.6. The performance of structural models in estimating market CDS spreads seems to be better, because CDS spreads are a more pure measure of credit risk. Probabilities of default can be estimated closely.
Second, we make the following observations that can provide a guideline for the selection of structural models that meet the objectives of this research:

- We need a more advanced structural model than the Merton and Geske model, since these models do not provide accurate credit spreads.
- The first-passage approach improves the results of structural models. In this perspective the CDG and LT model seem to be promising alternatives for estimating CDS spreads. The LT model is also able to account for non-credit risk related factors as liquidity shortages.
- The inclusion of stochastic interest rates raises the credit spread and allows for correlation between interest rates and credit risk. This is important for pricing defaultable bonds and

[11]See section 5.1.1 for a description of the Moody's KMV model

interesting for modeling the interaction of market risk and credit risk. However, the increase in spreads is small and the complexity of the model increases.

- A jump-diffusion process for the firm's assets value increases the model's performance in estimating short-term credit spreads compared to a diffusion process.

3.5 Summary and conclusion

This section presented a literature overview of structural models for credit risk. We started with the first structural model of Merton (1974) and identified its shortcomings from a comparison between modeled and market PDs and CDS spreads. We then gave extensions to the Merton model to overcome its shortcomings regarding capital structure, the conditions for default, the risk-free interest rate process, and the firm value process. The implications of these extensions on the modeled PD and credit spreads were analyzed from a qualitative and quantitative perspective.
The purpose of this study is to implement a credit risk model that can be used to value CDS contracts, estimate market CDS term structures and to perform analysis on several credit risk measures. Based on the analysis in this chapter and the discussion on the credit spread puzzle in section 2.6 we choose the following specifications for the structural models that we implement in the next chapter:

- The firm value process will be a jump-diffusion process, such that default happens unexpected and short-term credit spreads will not be zero. Furthermore, jump-diffusion process can model all CDS term structure shapes observed in the market and result in stochastic recovery rates.
- The specification of the default barrier can provide more economic interpretation to the default event and account for non-credit risk factors in credit spreads. We implement two types of default barriers with a first-passage approach in our model: a constant barrier as in LS, and the mean reverting leverage process as described by CDG. An endogenous default barrier as in LT is an interesting alternative that could account for non-credit risk factors, but since this model is complex this will be left for future research.
- The risk-free interest rate will be constant for two reasons. First, in the literature review we found that a stochastic interest rate has a small impact on modeled results while making the model more complex. Second, this study does not focus on the pricing of defaultable bonds, for which the interaction between credit risk and interest rates is important.

Chapter 4 further determines and analyzes the proposed models and in chapter 5 we compare modeled CDS spreads to market spreads.

Chapter 4

Modeling

Chapter 3 described that the structural model of Merton (1974) is extended to improve its performance in estimating market credit spreads. We identified that especially the introduction of a jump-diffusion process for the firm's assets value and the first-passage approach with an appropriate definition of the default barrier increase this performance.
This chapter analyzes two structural models that incorporate these model specifications. In the first model the assets value process is a jump-diffusion process and the default barrier is constant. The second model also includes a jump-diffusion process for the firm value, but with a mean reverting default threshold. For both models we derive a modeling framework to calculate CDS term structures using Monte Carlo simulations.
The modeled CDS term structures are compared to market term structures to analyze whether modeled term structure shapes are realistic. Furthermore, we perform a sensitivity analysis to analyze the effects of changes in the values of input variables on on modeled CDS spreads.

4.1 Model 1: Constant default barrier

This section describes the framework for a structural model with a jump-diffusion process and constant default barrier. The framework is based on the studies of Zhou (1997 and 2001) for the jump-diffusion process, BC for the first-passage approach, and LS for the default conditions.
The section starts with an explanation of the assumptions underlying the model. Then we derive the modeling framework to implement the model in Matlab with three different Monte Carlos simulation approaches. Finally, modeled CDS term structures are compared and analyzed for various input parameters.

4.1.1 Model assumptions

The underlying assumptions for the first model follow the assumptions of BC, LS, and Zhou (1997 and 2001) to price defaultable bonds. We have adapted these assumptions to price CDSs.

Assumption 1. Markets are assumed to be perfect and frictionless: there are no transaction costs, bankruptcy costs or taxes[1], assets are divisible, there are no restrictions on short selling of all assets, and borrowing and lending is possible at the same risk-free interest rate. In these markets, securities trade continuous in time and arbitrage opportunities do not exist.

This is a standard assumption made in literature for risk-neutral valuation of derivatives, as for example in Black & Scholes (1973) and Merton (1974).

Assumption 2. The risk-free interest rate r is constant over time.

[1]The absence of bankruptcy costs and taxes implies that the Modigliani-Miller theorem holds. This theorem states that the total value of a firm does not depend on its capital structure. Under this theorem, raising debt has no tax advantage or bankruptcy costs, but is compensated by the amount of equity such that the firm value is not affected.

Although the risk-free interest rate in the marketplace is not constant over time, we concluded from the literature review presented in chapter 3 that the benefit of implementing a stochastic interest rate does not outweigh the increased modeling complexity.

Assumption 3. The dynamics of the firm's assets value are given by the following jump-diffusion process (Zhou (1997)) under the real world probability measure:

$$\frac{dV_t}{V_t} = (\mu_V - \lambda\nu)dt + \sigma_V dW_t + (\Pi - 1)dY_t, \tag{4.1}$$

where

$$\begin{aligned}
V_t &= \text{Total market value of the firm's assets at time } t\\
\mu_V &= \text{Expected log return on the firm's assets excluding jumps}\\
\lambda &= \text{Intensity parameter of a Poisson process}\\
\nu &= \text{Expected value of the jump component}\\
\sigma_V &= \text{Volatility of the firm's assets excluding jumps}\\
dW_t &= \text{A Brownian motion process}\\
\Pi &= \text{Jump amplitude with expected value equal to E}[\Pi] = \nu - 1\\
dY_t &= \text{Poisson process with intensity parameter } \lambda
\end{aligned}$$

dW_t, dY_t, and Π are mutually independent. The term $\lambda\nu$ is added to the drift to account for the jump values.
The Poisson process dY_t with intensity parameter λ determines the number of jumps within a time interval. Since the arrival of jumps is a Poisson process the time intervals between successive jumps are exponential distributed. Furthermore, we can express the probability that the firm survives until t years from now as $S_t = \exp(-\lambda t)$.
At jump time τ, $\Pi < 1$ (downward jump) or $\Pi > 1$ (upward jump), otherwise $\Pi = 1$. The process Π thus represents the jump size and $\Pi - 1$ the change in the firm's asset value due to the jump[2]. This jump amplitude is an i.i.d. log-normally random variable[3], such that

$$\ln(\Pi) \sim \Phi(\mu_\pi, \sigma_\pi^2).$$

And this implies that

$$\nu \equiv \text{E}[\Pi - 1] = e^{(\mu_\pi + \sigma_\pi^2/2)} - 1.$$

The process in equation 4.1 contains a diffusion and a jump component. The diffusion process accounts for the gradual fluctuations in the firm's assets value due to small changes in the economic environment or the arrival of new information with little impact. The jump component describes sudden changes in the assets value due to the arrival of information with a large effect on the firm's market value such as an earnings warning.

Assumption 4. A positive, constant threshold K exists for the value of the firm's assets at which the firm enters in financial distress and at which a credit event occurs. If $V_t > K$, the firm is expected to make its obligated debt payments and to continue its operations. If $V_t \leq K$, the firm immediately defaults on all of its current and future payment obligations.

This assumption introduces the first-passage approach into the model and specifies the constant default barrier. LS interpret the constant barrier as an average of all payment obligations between times $0 \leq t \leq T$. We model the default barrier as the total debt outstanding measured as a percentage of the firm value at $t = 0$.
In the original LS model, the firm's assets value is assumed to follow a diffusion process, as in equation 3.1, such that the firm's assets value equals the default barrier upon default. In the jump-diffusion framework, the assets value can also jump below K resulting in a random recovery rate. This is also

[2] The change in the firm's assets value due to a jump is $(V_a - V_b)/V_b = V_a/V_b - 1$, where V_b and V_a are respectively the asset value before and after the jump. This implies that $\Pi = Va/Vb$, or $V_a = V_b\Pi$.

[3] This means an independent and identically distributed random variable.

observed in practice, where recovery rates vary across time periods and firms, and within firms on different bond issues.
In the model we distinguish two recovery rates. First we have the recovery rate R on which CDS spreads are quoted in the market. This is an input to the model to calculate CDS spreads. Furthermore, the model can evaluate the LGD as the ratio of the firm value at default and the level of the default barrier. Now we can determine an output recovery rate RR using $RR = 1 - \text{LGD}$.
This study focusses on the calculation of CDS spreads for valuation purposes and therefore we only use the constant recovery rate R as input to the model. We do provide the procedures to calculate RR, but do not present results of RR explicitly. However, it would be interesting to keep track of RR since large discrepancies between the modeled and quoted recovery rate indicate over or under insurance that might be reflected in the CDS spread. This topic will be left for future research.
The assumption that the firm defaults on all of its obligations is realistic. If a firm defaults on one debt issue, it often (has to) defaults on other issues as well. For example, the Basel II framework prescribes that when a counterparty defaults on one contract with a financial institution, it must default on all outstanding contracts with that institution.

Assumption 5. We model CDS term structures of a single name cash settled contract with a running CDS spread.

The reference entity of the CDS is a single obligation (a firm). The total outstanding debt of this firm is modeled as a ZCB with notional N. The CDS contract is written on the total debt of this firm. The protection buyer pays a quarterly premium to the protection seller and the protection seller pays $N(1 - R)$ to the protection buyer in case of default.

Finally note that since the model considers a jump-diffusion process for the firm's assets value the market is incomplete, such that the risk-neutral measure $\mathbb{Q}$ is not unique. This implies that perfect hedging of a financial instrument is not possible and that several parameter sets can determine the same value for a CDS contract. Since we focus on a sensitivity analysis to modeled CDS term structures we neglect this non-uniqueness.

4.1.2 Modeling framework

This section describes the framework to evaluate CDS spreads based on the assumptions made in the previous section. We specify the default time and method to calculate the CDS spread and then present three different simulation approaches.
The default time τ^* is specified with assumption 4. It is the first time that V_t crosses default threshold K, or

$$\tau^* = \inf\{t | V_t \leq K, t \geq 0\}.$$

Using this definition of the default time we can calculate CDS spreads, probabilities of default and recovery rates for various input parameters. Since this study focuses on CDS term structures we further determine the modeling framework to calculate this credit risk measure.
Section 2.5.3 developed the discounted cash flow method to value single name, cash settled CDS contracts. Expression 2.4 provided the formula to calculate the par CDS spread of a contract initiated at time $t = 0$ with maturity T:

$$c_{0,T} = \frac{\mathrm{E}^{\mathbb{Q}}\left[(1-R)e^{-r\tau^*}\mathbf{1}_{\{\tau^* \leq T\}}\right]}{\mathrm{E}^{\mathbb{Q}}\left[\sum_{i=1}^{z}\left(\Delta t_i e^{-rt_i}\mathbf{1}_{\{\tau^* > t_i\}} + (\tau^* - t_{i-1})e^{-r\tau^*}\mathbf{1}_{\{t_{i-1} < \tau^* < t_i\}}\right)\right]}. \tag{4.2}$$

Evaluation of equation 4.2 for various maturities results in a term structure of CDS spreads. Next sections present three different Monte Carlo simulation approaches to calculate these CDS term structures.

Approach 1A: discrete Monte Carlo

We use a Monte Carlo simulation approach to model default times and calculate CDS spreads. A Monte Carlo simulation is a numerical approach to solve equations in four steps (Seydel, 2006):

1. Define the distributions of the input variables.
2. For a large number of runs randomly draw values for the input variables from their distributions.
3. For each run, calculate the equations inserting the input variables.
4. Calculate the final answer by taking the average value over all runs.

In simulation approach 1A we follow the Monte Carlo approach outlined in Zhou (1997) to determine the default time.
Define $X_t = \ln(V_t)$, such that the jump-diffusion process of equation 4.1 can be transformed into[4]

$$X_{t_i} - X_{t_{i-1}} = x_i + y_i \cdot \pi_i, \tag{4.3}$$

where x_i, y_i, and π_i are mutually and serially independent random variables with distributions

$$x_i \sim \Phi\left((r - \sigma_V^2/2 - \lambda\nu)\tfrac{T}{n}, \sigma_V^2 \tfrac{T}{n}\right),$$

$$\pi_i \sim \Phi(\mu_\pi, \sigma_\pi^2),$$

$$y_i = \begin{cases} 0 & \text{with probability } 1 - \lambda \cdot \frac{T}{n} \\ 1 & \text{with probability } \lambda \cdot \frac{T}{n} \end{cases}$$

The following Monte Carlo algorithm is implemented in Matlab to model the process X_{t_i} and calculate CDS spreads:[5]

Step 1. Divide the time interval $[0, T]$ into n equally sized time periods.

Step 2. Determine the time points t_p as a fraction of T on which periodic protection payments are made. Assume that the time between successive protection payments is constant, such that the payment times can be expressed recursively as $t_p = t_{p-1} + \Delta t$, for $p = 1, 2, \ldots, z$. Where z is the maximum number of protection payment made conditional on no default and $t_0 = 0$.

Step 3. Perform Monte Carlo simulations by repeating the following procedures $j = 1, 2, \ldots, M$ times:

(a) For each j, generate a series of mutually and serially independent random vectors (x_i, y_i, π_i) for $i = 1, 2, \ldots, n$ according to their distributions specified in equation 4.3.

(b) Let $X_{t_0} = \ln(V_0)$ and calculate X_{t_i} according to equation 4.3, for $i = 1, 2, \ldots, n$.

(c) Find the smallest $i \leq n$ for which $X_{t_i} \leq \ln(K)$. If such an i exists, default has occurred and $i = \tau^*$ as a fraction of T. Otherwise the contract exists until maturity, such that $\tau^* > T$.

Step 4. For each j, perform the following actions:

(a) Evaluate the denominator of equation 4.2 as the sum of

- The discounted protection payments as long as $\tau^* \geq t_p$:

$$DPP_j = \sum_{p=1}^{z} \Delta t e^{-rt_p}.$$

- The discounted accrual payment:

$$DAP_j = \begin{cases} (\tau^* - t_p)e^{-r\tau^*} & \text{if } \tau^* \leq T \\ 0 & \text{if } \tau^* > T \end{cases}$$

(b) Evaluate the numerator of equation 4.2 as

$$DDP_j = \begin{cases} (R-1)e^{-r\tau^*} & \text{if } \tau^* \leq T \\ 0 & \text{if } \tau^* > T \end{cases}$$

[4] See appendix B.1 for a detailed derivation of this equation.

[5] Although we focus on the term structure of CDS spreads, we also provide the procedures to calculate the probability of default and recovery rate with this model.

(c) Calculate the log LGD if default occurs as $\ln(K) - (X_{\tau^*})$

Step 5. Calculate the output:

(a) The CDS spread is

$$c_{0,T} = \frac{\frac{1}{M}\sum_{j=1}^{M} DDP_j}{\frac{1}{M}\sum_{j=1}^{M}(DPP_j + DAP_j)}.$$

(b) The probability of default is

$$PD = \frac{\sum_{j=1}^{M} \mathbf{1}_{\{\tau^* \leq T\}}}{M}$$

(c) The actual recovery rate of the firm is

$$RR = 1 - \frac{\sum_{f=1}^{F} LGD_f}{F}$$

Where F is the number of defaults in simulation runs $j = 1, 2, \ldots, M$.

Running this algorithm for various maturities gives the term structure for the par CDS spread. The input parameters and values of M and n to be used, are described in section 4.1.3. The results are presented and analyzed in section 4.1.4.

Approach 1B: Brownian Bridge

An alternative to the discrete Monte Carlo approach is to use a Brownian Bridge approach. This approach calculates the path of a Brownian motion (diffusion process) between two time points using an explicit formula. Therefore this method should be computational faster and should provide smoother graphs than approach 1A. This section determines the Brownian Bridge simulation approach to model 1, following Metwally & Atiya (2002).

The first step is to determine the jump times τ_i between initiation and maturity. We assume that the number of jumps follows a Poisson process with intensity parameter λ. This implies that the inter jump times $[\tau_{i-1}, \tau_i]$ are independent and follow an exponential distribution with mean $1/\lambda$.
On each jump time τ_i let $X_{\tau_i^-}$ be the log asset value the instant before the jump and $X_{\tau_i^+}$ be the log asset value the instant after τ_i. On the interval between jump times, $t \in [\tau_{i-1}^+, \tau_i^-]$, the firm's assets value follows the Brownian motion:

$$\frac{dV_t}{V_t} = (r - \lambda\nu)dt + \sigma_V dW_t$$

Using Itô's Lemma we obtain the process for $X_t = \ln(V_t)$:

$$dX_t = (r - \frac{\sigma_V^2}{2} - \lambda\nu)dt + \sigma_V dW_t.$$

Applying the result of Hull (2006) in appendix B.1 this can be expressed as

$$X_{\tau_i^-} - X_{\tau_{i-1}^+} \sim \Phi\left((r - \frac{\sigma_V^2}{2} - \lambda\nu)(\tau_i - \tau_{i-1}), \sigma_V^2(\tau_i - \tau_{i-1})\right). \tag{4.4}$$

To evaluate equation 4.4 we need an expression for $X_{\tau_i^+}$, the log asset value the instant after a jump. In assumption 4 of model 1 we identified that Π is the percentage change in the asset value due to a jump, such that $\ln(\Pi) = X_{\tau_i^-} - X_{\tau_i^+}$. Now we can calculate the log asset value after a jump by drawing $\ln(\Pi)$ from its distribution $\Phi(\mu_\pi, \sigma_\pi^2)$.
Default occurs when $X_{\tau_i^-}$ or $X_{\tau_i^+}$ is equal to or smaller than $\ln(K)$. The default time is then equal to the jump time. However, between successive jumps the firm's asset value might have crossed the default barrier. Since we apply the Brownian Bridge methodology we cannot observe this event directly as in approach 1A.

To account for the barrier crossings we calculate the conditional probability P^+ that the minimum of the Brownian Bridge is always above the barrier in the interval $[\tau_{i-1}, \tau_i]$ given $X_{\tau_{i-1}^+}$ and $X_{\tau_i^-}$ with

$$P^+ = \begin{cases} 1 - \exp\left(-\frac{2[\ln(K) - X_{\tau_{i-1}^+}][\ln(K) - X_{\tau_i^-}]}{(\tau_i, \tau_{i-1})\sigma_V^2}\right) & if\ X_{\tau_i^-} > \ln(K) \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

The conditional probability that the log asset value does drop below the default barrier in the interval $[\tau_{i-1}, \tau_i]$ is then $P^* = 1 - P^+$. To transform this probability into a default time, we use a uniform sampling method[6]. We draw a random number s from the uniform distribution, $s \sim \text{UNIFORM}[\tau_i, l]$ distribution, where $l = \tau_{i-1} + (\tau_i - \tau_{i-1})/P^*$. If $s \in [\tau_{i-1}, \tau_i]$ the log assets value has crossed the barrier in the interval and the firm has defaulted with default time $\tau^* = s$.

The Brownian Bridge framework is implemented in Matlab to calculate CDS spreads with the following algorithm:

Step 1. Determine the CDS payment dates with step 2 of simulation approach 1A.

Step 2. Perform Monte Carlo simulations by repeating steps 3 and 5 for $j = 1, 2, \ldots, M$.

Step 3. Determine the Brownian Bridge time points t_{BB}, by performing the following procedures:

(a) Draw the number of jumps $N(T)$ in the interval $[0, T]$ from the Poisson distribution with intensity parameter λ.

(b) Determine the jump times τ_i for $i = 1, 2, \ldots, N(T)$, by drawing the inter jump times, $\tau_i - \tau_{i-1}$, from the exponential distribution with mean $1/\lambda$.

(c) The Browian Bridge time points are now specified as $t_{BB} = 0, \tau_1, \ldots, \tau_{N(T)}, T$ when the last jump occurs before maturity. When a jump occurs at maturity or when jumps $i, \ldots, N(T)$ occur after maturity, the Brownian Bridge time points are $t_{BB} = 0, \tau_1, \ldots, \tau_i, T$.

Step 4. Calculate the log asset value at each Brownian Bridge point:

(a) At initiation, $t_{BB} = 0$, $X_0 = \ln(V_0)$.

(b) The log asset value before a jump, $X_{\tau_i^-}$, is derived with equation 4.4.

(c) The log asset value directly after a jump is $X_{\tau_i^+} = X_{\tau_i^-} + \ln(\Pi_i)$.

Step 5. Evaluate the occurrence of default and the default time for each Brownian Bridge interval with the following procedures:

(a) Determine the conditional probability P^+ according to equation 4.5.

(b) Generate a possible crossing time $s \sim \text{UNIFORM}[\tau_i, l]$

(c) If $s \in [\tau_{i-1}, \tau_i]$, default has occurred and $\tau^* = s$

(d) If $X_{\tau_i^+} \leq \ln(K)$, default has occurred and $\tau^* = \tau_i$.

(e) Otherwise no default occurred and $\tau^* > T$.

Extend this algorithm with steps 4 and 5 of approach 1A to calculate the output of the Brownian Bridge approach.

Approach 1C: Jump to default

Approach 1C models the default time as the first jump in an exogenous process or the time at which the firm's assets value crosses the default barrier.
We consider this approach since it is computational fast and simple compared to approach 1A and 1B. Furthermore, it combines the best elements of structural models and reduced form models: the company can default due to a decreasing assets value, but also due to unexpected shocks in the assets value as a result of new external information.

[6]See appendix B.2 for more explanation on the uniform sampling method.

The assets value process between $t = 0$ and the termination date is modeled with a Brownian Bridge approach. The termination date is the first jump time τ_i in the exogenous process or maturity, whichever comes first. Since no jumps occur in the assets value process until the termination date the drift of the assets value process does not account for jumps. The process for X_t is defined as

$$X_{B_T} - X_0 \sim \Phi\left((r - \frac{\sigma_V^2}{2})B_T, \sigma_V^2 B_T\right),$$

where $B_T = T$ if $\tau_1 > T$ and $B_T = \tau_1$ otherwise.
The default time is equal to the jump time determined by the exponential distribution with mean $1/\lambda$ or the time at which $X_t \leq \ln(K)$. The latter condition is calculated using the conditional probability of a barrier crossing in $[0, B_T]$ and the uniform sampling method as described for approach 1B. Since we assume that a jump in firm value triggers default the jump size is not relevant in this approach. Note that due to this assumption we cannot calculate the recovery rate RR with this model.

The algorithm for this model encompasses the following steps:

Step 1. Determine the CDS payment dates with step 2 of simulation approach 1A.

Step 2. Perform Monte Carlo simulations by repeating steps 3 and 4 for $j = 1, 2, \ldots, M$.

Step 3. Determine the Brownian Bridge points:

(a) Generate the first jump time, $\tau_1 \sim \text{EXP}(1/\lambda)$

(b) The Brownian Bridge interval is $[0, B_T]$, where $B_T = T$ if $\tau_1 > T$ and $B_T = \tau_1$ otherwise.

Step 4. Evaluate the default occurrence and default time in the Brownian Bridge interval:

1. Determine the conditional probability P^+ according to equation 4.5.
2. Generate a possible crossing time $s \sim \text{UNIFORM}[0, l]$
3. The default time is specified as
   - If $s \in [0, B_T]$, default has occurred and $\tau^* = s$
   - If $s \notin [0, B_T]$, but $\tau_1 \leq T$, then $\tau^* = \tau_1$
   - Otherwise no default occurred and $\tau^* > T$.

CDS spreads and default probabilities are calculated by extending this algorithm with steps 4 and 5 of approach 1A.

4.1.3 Parameters and Monte Carlo settings

Before we analyze modeled CDS term structures, this section gives the inputs parameters and Monte Carlo settings for simulation approach 1A, 1B and 1C.
The recovery rate of the CDS is $R = 0.40$. Protection payments are made quarterly and we ignore the day count convention. In practice CDSs are quoted for 6 months, 1 year, 2, 3, 4, 5, 7, and 10 years. In this study we will model CDS term structures with maturities ranging from 6 months to 20 years.
The following parameters are used as a base case in the sensitivity analysis: $V_0 = 100$, $r = 0.05$[7], $\sigma_V = 0.2$, $\lambda = 1$[8], $\mu_\pi = 0$, $\sigma_\pi = 0.1$, and the level of the constant default barrier is $K = 0.5V_0$

Appendix B.3 determines the settings of the Monte Carlo simulations for approach 1A, 1B and 1C. We use graphical procedures in which we determine CDS spreads for increasing levels of the time periods n in which we divide the interval $[0, T]$ and the number of Monte Carlo runs M. We then select the values for n and M as a trade off between computational speed and stability in the calculated CDS spread. With stability defined as the range of basis points between which the CDS spread varies for increasing n or M. Table 4.1 gives an overview of the settings for each simulation approach to model 1.

[7]Note that we use r as drift rate in the assets value process and for discounting the expected CDS cash flows. The empirical study in chapter 5 uses different values for these purposes.

[8]This means that the jump intensity is 1 jump a year. For approach 1A and 1B this is realistic, since a jump could be a default event as well as important news. In approach 1C we take smaller values for λ.

Table 4.1: Monte Carlo settings for the simulation approaches to model 1.

Approach	n	M	**Range**
1A	400	50000	5bps
1B	not applicable	1000000	1bps
1C	not applicable	1000000	1bps

4.1.4 Results of model 1

This section models par CDS spreads of contracts initiated at time $t = 0$ with maturity T with model 1. Specifically, we analyze the sensitivity of evaluated CDS term structures to changes in the values of the input parameters. First the sensitivity to the jump component is analyzed, followed by an analysis of the parameters of the assets value process and the level of the constant default barrier. We use the market CDS spreads of figure 3.3 as a benchmark to compare whether the model can produce similar values and shapes for the term structures.

Sensitivity to the jump parameters

This section analyzes the sensitivity of the CDS term structure to the jump intensity λ and jump size σ_π for approach 1A, 1B and 1C. Next section performs a similar analysis to the jump component in approach 1A, but applies a restriction to the total variance of the firm.
We first analyze and compare the CDS term structures modeled with approach 1A and 1B since these results should be approximately similar. Figure 4.1 shows CDS term structures for different levels of the jump parameters.

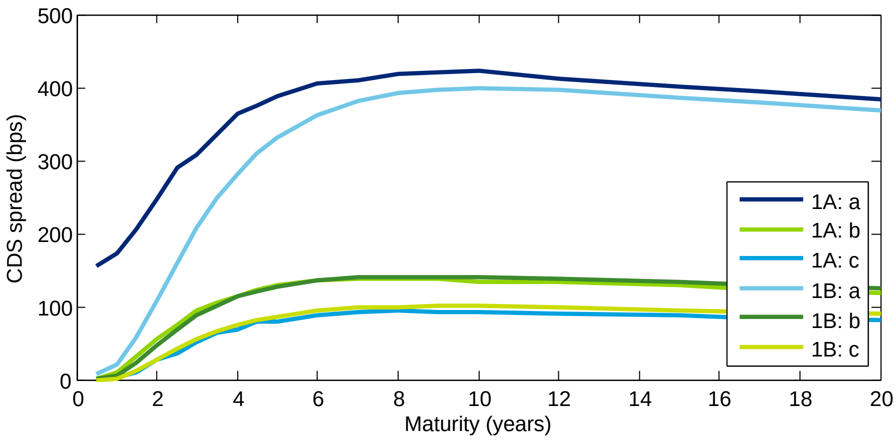


Figure 4.1: Sensitivity of CDS term structures modeled with approach 1A and 1B to changes in the jump parameter values. Case a: $\lambda = 0.1$ and $\sigma_\pi = 1$. Case b: $\lambda = 1$ and $\sigma_\pi = 0.1$. Case c: $\lambda = 10$ and $\sigma_\pi = 0.01$.

As expected it is first notable that approach 1B is computational faster and it also generates smoother term structures than approach 1A.
Intuitively, increasing λ and σ_π should increase the CDS spread. Observe in figure 4.1 that especially the jump size affects the value of the CDS spread[9]. As we will see, increasing λ and decreasing σ_π results in term structures that are more similar to term structures obtained with a pure diffusion process for the firm's assets value.
In comparison to the term structures obtained with the Merton model in figure 3.4b, we observe that both approach 1A and 1B can generate positive short-term CDS spread. These are higher for approach 1A than for 1B, which is caused by the different timing of jumps. In approach 1A a jump can occur in each time period i with the same probability, such that jump times will be equally divided over the interval $[0, T]$. For example when $T = 1$ and $\lambda = 1$ at least one jump will occur on

[9]See figure 4.3a for an alternative illustration of this effect.

the interval $[0, T]$ and the jump time will be $\tau = 0.5$ year on average.
Approach 1B however, draws jump times from the exponential distribution with mean $1/\lambda$. When $T = 1$ and $\lambda = 1$ the average jump time $\tau = 1$ year, such that it is not certain that a jump occurs in $[0, T]$, and if a jump occurs the jump time will be later on average than in approach 1A. This results in lower short-term CDS spreads for approach 1B. As we see in the graph, increasing the jump intensity decreases the difference in short-term CDS spreads between both approaches.
For longer maturities the values of the CDS spreads decrease. In figure 3.3 however, we saw that these are often increasing in the marketplace. In chapter 3 we explained that the decreasing term structure is caused by the the positive drift in the assets value process. Since the default barrier is constant in model 1 the distance between the default barrier and the firm value increases, such that default is less likely to occur.
For approach 1A the long-term decrease in CDS spreads is higher than for approach 1B. We can explain this with the restriction of one jump per time period i in approach 1A. For example when $T = 20$ the interval $[0, T]$ is divided in 400 time periods of 0.05 years. If $\lambda = 10$ the inter jump time is exponentially distributed with a mean of 0.1 years, such that time periods with more than one jump are likely to occur. Since approach 1B models every jump, this approach results in higher CDS spreads.
The observations described above indicate that approach 1A is more useful for modeling short maturities, while approach 1B provides better results over a longer horizon. Furthermore, in approach 1B a single parameter set to model an entire term structure does not seem appropriate. To increase short-term spreads in this approach we could increase σ_π, but this results in too high spreads for long maturities. For example $\lambda = 1$ and $\sigma_\pi = 0.2$ result in a 1 year CDS spread of 41bps, which is 4bps higher than the 1 year spread of Ahold in figure 3.3. However, the 2 year spread is already 141bps and the 5 year spread 266bps, while these are 52bps and 82bps for Ahold respectively.

Figure 4.2 shows the sensitivity of the CDS term structure to the jump intensity in approach 1C.

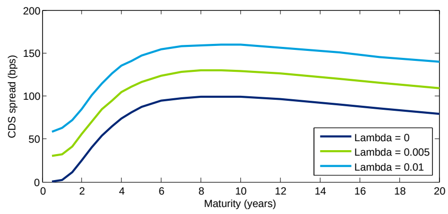


Figure 4.2: Sensitivity of the par CDS term structure to the jump intensity in approach 1C.

When $\lambda = 0$ the assets value process is a pure diffusion process with zero short-term CDS spreads. Increasing λ results in an upward shift of the CDS term structure. This is intuitive since a higher λ increases the occurrence of a jump and thus the occurrence of a default event.
Observe that this method only gives hump-shaped CDS spreads since the modeled process is a diffusion process. Furthermore, we see that the term structure modeled with $\lambda = 0.005$ has positive short term CDS spreads, with not too high longer term CDS spreads for the same input parameter sets, which is an advantage compared to approach 1B.

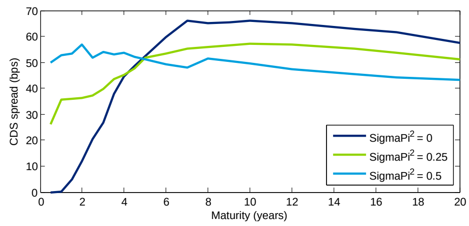
Jump sensitivity with total variance restriction

The variance in the assets value process consists of two components: the variance of the diffusion component and the variance of the jump component. Adjusting σ_V or σ_π increases the total variance of the process, such that the relative effect of a parameter change is less clear. This section follows Zhou (2001) to focus on the relative effect of changing the jump parameters while keeping the total variance of the assets value process σ_X^2 constant.
Zhou (2001) assumes that $\mu_\pi = 0$, such that the total instantaneous variance of the log assets value X_t is

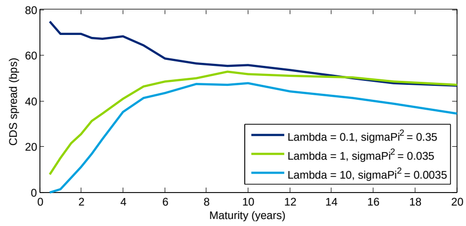
$$\sigma_X^2 = \sigma_V^2 + \lambda\sigma_\pi^2$$

As in Zhou (1997) we set the total variance of the process to the constant level of $\sigma_X^2 = 0.035$, such that we can investigate how the composition of σ_X^2 affects the results. We only apply this analysis to approach 1A, since the variance of the assets value process in approach 1B and 1C only consists of a diffusion component between the jump times.

Figure 4.3 shows the sensitivity of the par CDS term structure to the level of the jump parameters when the variance restriction is applied.



(a) 1A: changing σ_π^2



(b) 1A: changing λ and σ_π^2

Figure 4.3: Sensitivity of the CDS term structure to the jump parameters in approach 1A with variance restriction. Panel (a) shows CDS spreads for various jump sizes with $\lambda = 0.05$ and $\sigma_X^2 = 0.035$. Panel (b) shows the impact of the jump intensity and jumps size, such that in each case $\sigma_V = 0$.

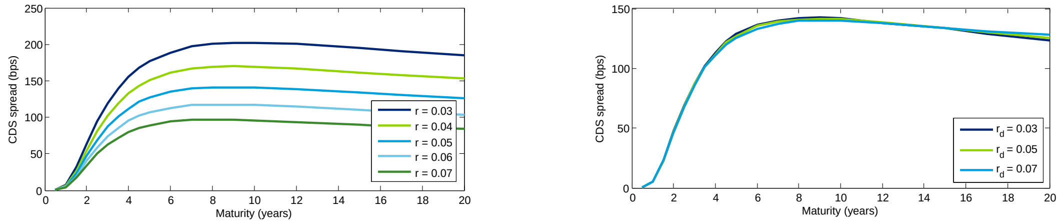
Observe in figure 4.3a that the modeled term structure shapes compare at least to two term structure shapes observed in the market[10]: hump shaped for $\sigma_\pi^2 = 0$ and downward sloping for $\sigma_\pi^2 = 0.5$.
The figure clearly shows that a higher jump size σ_π^2 increases the short-term CDS spread. For $\sigma_\pi^2 = 0$ the log asset value process is a diffusion process and as explained in chapter 3 this results in zero CDS spreads for short maturities and decreasing CDS spreads for long maturities.
For $T > 5$ years we observe the reverse: the effect of the jump decreases and the diffusion process eventually shows a higher CDS spread. We can explain this with the term structures in figure 4.3b. This figure shows the impact of the jump intensity on the par CDS spread. The total variance of the firm is constant, $\sigma_X^2 = 0.035$, such that the variance of the diffusion component is zero and the log asset value process is a pure jump process. A larger λ and a smaller σ_π^2 makes the log assets value process more continuous than the reverse. We observe this in figure 4.3b: for $\lambda = 10$ and $\sigma_\pi^2 = 0.0035$, the CDS term structure has a similar shape as the diffusion process in figure 4.3a. Furthermore, the figure shows that a pure jump process increases the CDS spread for short maturities, but spreads decrease for a longer horizon and even have a lower CDS spread than the pure diffusion process in figure 4.3a. Thus a pure jump process is more likely to default in the short term but less likely to default on long maturities than a diffusion process. Since we limit the variance of the jump-diffusion process, this explains why a firm with a higher jump size is more likely to default on short-term debt, than on long term debt.

Sensitivity to the diffusion parameters

The relation between the diffusion parameters r and σ_V and CDS spreads is intuitive. A higher risk-free interest rate implies a higher drift in the log assets value process. This results in an increasing distance between the firm value and default barrier, such that the risk-neutral PD and CDS spread decrease. A higher assets value volatility decreases the drift and increases uncertainty in the firm value, such that CDS spreads will increase. This section further analyzes these relations.
Figure 4.4 shows modeled CDS term structures for various levels of the risk-free interest rate. In 4.4a both the drift term and the discount rate of the CDS cash flows are altered when r changes. In 4.4b the drift term is constant at $r = 0.05$ while the interest rate used to discount the cash flows is changed.

Results are as expected: a higher level of the drift term results in lower CDS spreads. This effect is more pronounced over long maturities, because the assets value process can farther drift away from the default barrier. We can ignore the change in the discount rate in this observation.
This can be seen in figure 4.4b where a change in the discount rate results in a small spread change. For long maturities we observe that a smaller discount rate seems to results in smaller CDS spreads.

[10]See the comment of Zhou in section 3.3.4 on the shapes of term structures



(a) 1B: changing r

(b) 1B: changing the discount rate

Figure 4.4: Sensitivity of the the par CDS term structure to the asset value drift and risk-free interest rate for approach 1B. Panel (a) shows CDS spreads for different levels of r, where r is both the drift parameter and interest rate to discount the CDS cash flows. Panel (b) shows CDS term structures with a constant drift of $r = 0.05$ and varying discount rates.

However, we need to be careful with this result. The difference in CDS spread for $r_d = 0.03$ and $r_d = 0.07$ is only 5bps, which is slightly higher than the 1bps precision of the applied Monte Carlo settings. An experiment in which we model CDSs with $T = 50$ for $r_d = 0.03$ and $r_d = 0.07$ determines a spread of respectively 93bps and 113bps. We can thus conclude that a higher interest rate indeed increases the CDS spread.
This analysis was a preliminary study on the interaction between market risk and credit risk of a firm. To further extend this analysis, a more advanced interest rate process needs to be incorporated in the model. This is addressed in the topics for future research.

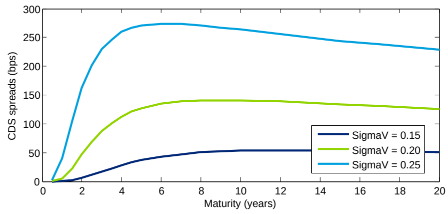


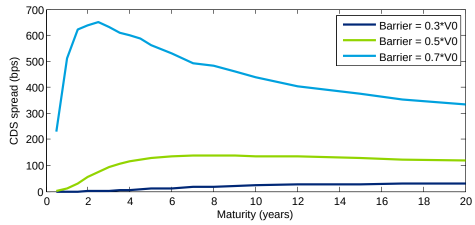
Figure 4.5: Sensitivity of the par CDS term structure to σ_V in approach 1B.

Figure 4.5 shows that CDS spreads strongly increase with a higher σ_V. An increase from $\sigma_V = 0.15$ to $\sigma_V = 0.2$ causes the 5 year CDS spread to increase with 90bps.

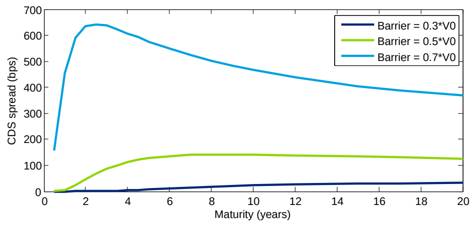
Sensitivity to the level of the default barrier

The last analysis to model 1 regards the sensitivity of the CDS term structure to the level of the default barrier. The default barrier is set as a percentage of the initial assets value and is constant over time. Figure 4.6 shows the results.

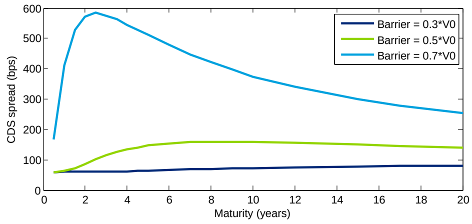
CDS spreads increase with the level of the barrier. Interesting are the various shapes of the term structures. For approach 1A and 1C the term structure modeled with a barrier level of 30% of V_0 is an increasing function. A barrier level of 50% models a decreasing term structure for longer maturities, and a barrier level of 70% results in a strong increase followed by a decrease in spreads.
The shape of the 70% default barrier can be explained as follows. A firm with a high initial leverage has problems with surviving in its first years. When the firm successfully survives these years its probability of surviving increases and therefore the CDS spread decreases. A similar observation is made for speculative grade debt in the market, see 2.2.
We can provide a similar explanation to the shape of the term structure with a barrier of 30% of the initial asset value. The probability of default for investment grade debt tends to increase with time, and thus the CDS spread increases.
In 4.6c we observe the same shapes of CDS term structure as in 4.6a. However, short-term CDS spreads are higher. From the graphs with barrier level 30% and 50% we imply that for short maturities



(a) Approach 1A



(b) Approach 1B



(c) Approach 1C

Figure 4.6: Par CDS spread term structure for different levels of the default barrier. Panel (a) refers to approach 1A. Panel (b) shows the result of approach 1B, and panel (c) shows approach 1C with $\lambda = 0.01$.

default is mainly triggered by a jump. When T increases the log assets value process is also able to reach the barrier, such that CDS spreads with a barrier level of 50% of V_0 increase more.

4.1.5 Conclusions model 1

This section described three Monte Carlo simulation frameworks to a jump-diffusion model with constant default barrier. The CDS term structures obtained with these approaches were compared and analyzed for different input parameters. We observed that the inclusion of a jump component in the asset value process increases short-term CDS spread. An increase in the drift of the assets value process results in lower CDS spreads and a higher assets volatility increases the term structure. Furthermore, a lower default barrier results in lower CDS spreads.
The discrete Monte Carlo modeling approach is computational slow, but is able to generate various shapes of CDS term structure. The Brownian Bridge method appears to be difficult to implement since it might require different input parameter sets to estimate CDS spreads of different maturities. The jump to default approach is computational fast and we saw that this method is able to produces realistic CDS spreads for short and long term maturities with the same input set.
The main drawback in each simulation approach for model 1 is the constant default barrier. Since the jump-diffusion process is an increasing function with time the term structure becomes downward sloping with time. In the market however, CDS spreads tend to be upward sloping. CDG account for this observation in their structural model by modeling the leverage of the firm as a mean reverting process. We will implement this model in the next section.

4.2 Model 2: Stationary leverage ratio

This section extends the framework of model 1 with a mean reverting leverage process that triggers default to account for downward sloping CDS term structures. We follow CDG to adapt the assumptions of model 1 to incorporate this process in a new modeling framework. Two simulation approaches are determined to model CDS term structures with this framework and finally we present the results.

4.2.1 Model assumptions

In the previous section we found that CDS term structures modeled with model 1 are downward sloping for long maturities, while these are increasing in the marketplace. This was explained by the positive drift in the jump-diffusion process that makes the assets value process drift away from the constant default barrier.
CDG introduce a dynamic capital structural for the firm in their structural model to overcome this problem. They incorporated the market observation that firms issue more debt when their leverage ratio B_t/V_t drops below a target level and tend to wait with replacing this debt when the leverage ratio is above target. In this way the firm adjusts its outstanding debt in response to changes in its firm value to obtain a target leverage ratio. Thus, when the firm value increases the firm will issue more debt to keep its leverage ratio on target, such that the term structures will not be downward sloping.
Under these dynamics the leverage ratio of the firm will be mean reverting to a constant level, or otherwise stated the firm's leverage ratio is expected to be stationary over time. We will now extend the assumptions of model 1 to incorporate the mean reverting leverage process in the modeling framework.

Assumptions 1-3 and 5 of model 1 also apply to model 2. The specification of the default threshold in assumption 4 of model 1 is adjusted according to CDG:

New assumption 4. A dynamic default barrier K_t exists for the value of the firm's assets at which the firm enters in financial distress and at which a credit event occurs. The log default threshold $\ln(K_t)$ is modeled as the mean reverting process

$$d\ln(K_t) = \varphi(X_t - \gamma - \ln(K_t))dt, \tag{4.6}$$

where φ is the mean reversion speed, $X_t = \ln(V_t)$ and γ is a constant to adjust the target leverage ratio.

The default barrier is set equal to the value of the firm's debt: $K_t = B_t$. Now this barrier specifies that firms issue debt when their leverage ratio B_t/V_t drops below a target level and do not replace debt when their leverage ratio is above target. Equation 4.6 shows this mean reverting process: when $\ln(K_t) < X_t - \gamma$, $d\ln(K_t)$ is positive, such that the firm's debt value increases and vice versa. When we set $\varphi = 0$, the default barrier is constant and we obtain model 1. Assumption 4 offers more opportunities to model a firm's debt policy. With φ we can model how quick the firm adjusts its debt level to changes in the assets value and with γ we can alter the target leverage of the firm.

4.2.2 Modeling framework

This section determines the modeling framework for model 2. First we define the default time and implement the mean reverting leverage process in the jump-diffusion process. Then we present two simulation approaches to evaluate CDS term structure.
Since we assume that $K_t = B_t$ the inverse leverage ratio can be defined as $L_t = V_t/K_t$ or $\ln(L_t) = \ln(V_t/K_t)$. Now default occurs when $\ln(L_t) \leq 0$ for the first time, such that the default time is specified as

$$\tau = \inf\{t| \ln(L_t) \leq 0, t \geq 0\}.$$

In appendix B.1 we showed that $X_t = \ln(V_t)$ can be written as

$$dX_t = (r - \sigma_V^2/2 - \lambda\nu)dt + \sigma_V dW_t + \Pi dY_t.$$

Using $\ln(L_t) = \ln(V_t/K_t) = X_t - \ln(K_t)$ and equation 4.6 yields the following expression for the log inverse leverage ratio:

$$d\ln(L_t) = (r - \sigma_V^2/2 - \lambda\nu)dt + \sigma_V dW_t + \Pi dY_t - \varphi(X_t - \gamma - \ln(K_t))dt. \tag{4.7}$$

This can be simplified to

$$d\ln(L_t) = \varphi(\ln(\overline{L}) - \ln(L_t))dt + \sigma_V dW_t + \Pi dY_t \tag{4.8}$$

in which the target log inverse leverage ratio $\ln(\overline{L})$ is

$$\ln(\overline{L}) = \frac{(r - \sigma_V^2/2 - \lambda\nu)}{\varphi} + \gamma.$$

Equation 4.8 shows that the process for $\ln(L_t)$ is mean reverting to the target log inverse leverage ratio $\ln(\overline{L})$. This target leverage ratio is assumed to be constant over time and can be set equal to leverage ratio at $t = 0$ or according to the future prospects of the company. We analyze the impact of this decision in section 4.2.4.
The process described in equation 4.8 is modeled with a discrete time framework similar to approach 1A for model 1. We therefore transform the process into

$$\ln(L_{t_i}) - \ln(L_{t_{i-1}}) = l_i + y_i \cdot \pi_i, \tag{4.9}$$

where l_i, y_i, and π_i are mutually and serially independent random variables with distributions

$$l_i \sim \Phi\left(\varphi(\tfrac{(r-\sigma_V^2/2-\lambda\nu)}{\varphi} + \gamma - \ln(L_{t-1}))\tfrac{T}{n}, \sigma^2\tfrac{T}{n}\right),$$

$$\pi_i \sim \Phi(\mu_\pi, \sigma_\pi^2),$$

$$y_i = \begin{cases} 0 & \text{with probability } 1 - \lambda \cdot \frac{T}{n} \\ 1 & \text{with probability } \lambda \cdot \frac{T}{n} \end{cases}$$

With this equation we can calculate the firm's default time and this serves as an input for equation 4.2 to evaluate the par CDS spread with model 2. Two simulation approaches are considered to model 2: the first is similar to approach 1A and the second is a discrete jump to default approach[11].

Approach 2A: discrete Monte Carlo

This Monte Carlo simulation approach is similar to approach 1A. Only step 3 of the algorithm described in section 4.1.2 needs to be adjusted to incorporate the stationary leverage process. The new step 3 is:

Step 3. Perform Monte Carlo simulations by repeating the following procedures for $j = 1, 2, \ldots, M$:

(a) For each j, generate a series of mutually and serially independent random vectors (l_i, y_i, π_i) for $i = 1, 2, \ldots, n$ according to their distributions specified in equation 4.9.

(b) Let $\ln(L_{t_0}) = \ln(V_0/K_0)$ and calculate $\ln(L_{t_i})$ according to equation 4.9, for $i = 1, 2, \ldots, n$.

(c) Find the smallest $i \leq n$ for which $\ln(L_{t_i}) \leq 0$. If such an i exists, default has occurred and $\tau^* = i$ as a fraction of T. Otherwise the contract exists until maturity, such that $\tau^* > T$.

With approach 2A we can evaluate CDS spreads, probability of defaults and recovery rates with the same procedures as for approach 1A.

Approach 2B: Jump to default

The second simulation approach to model 2 is a jump to default approach. Since we cannot apply the Brownian Bridge methodology as in approach 1C to the process of equation 4.8 we use a similar approach as in 2A.
When a jump occurs in time period i the company immediately defaults, such that the default time $\tau^* = i$. Since we do not need to model the jump size, the process in equation 4.9 reduces in approach 2B to:

$$\ln(L_{t_i}) - ln(L_{t_{i-1}}) = l_i, \tag{4.10}$$

where

$$l_i \sim \Phi\left(\varphi(\frac{(r - \sigma_V^2/2)}{\varphi} + \gamma - \ln(L_{t-1}))\frac{T}{n}, \sigma^2\frac{T}{n}\right),$$

The algorithm is straightforward, we only need to adjust step 3 in the algorithm of approach 1A.

[11] Note that we cannot apply the Brownian Bridge methodology to the process in equation 4.8.

Step 3. Perform Monte Carlo simulations by repeating the following procedures for $j = 1, 2, \ldots, M$):

(a) For each j, generate a series of serially independent random vectors l_i for $i = 1, 2, \ldots, n$ according to its distributions specified in equation 4.10.

(b) Let $\ln(L_{t_0}) = \ln(V_0/K_0)$ and calculate $\ln(L_{t_i})$ according to equation 4.10, for $i = 1, 2, \ldots, n$.

(c) For each $i = 1, 2, \ldots, n$, draw the number of jumps from the Poisson process with intensity λ.

(d) Find the smallest $i \leq n$ for which a jump occurs or $\ln(L_{t_i}) \leq 0$. If such an i exists, default has occurred and $\tau^* = i$ as a fraction of T. Otherwise the contract exists until maturity, such that $\tau^* > T$.

Approach 2B is able to evaluate CDS term structures and default probabilities.

4.2.3 Parameters and Monte Carlo settings

This section presents the input parameters for simulation approaches 2A and 2B. To compare with the results of model 1 we apply the same values for the input parameters. In some cases of the sensitivity analysis we use other values for the parameters and these values are given throughout the text.
The values of the parameters introduced in model 2 are selected from CDG: $\varphi = 0.18$ and the target leverage ratio is set to 55%. According to CDG this risk-neutral value for the long run leverage corresponds to a real world ratio of 38%, which is typical for AAA-rated bonds. The target leverage ratio is altered in the simulations by varying γ. Note that approach 2A and 2B differ in their expression for the target leverage ratio (and thus γ) due to the exclusion of the jump component in approach 2B.
The Monte Carlo settings n and M are determined with the same graphical procedures as for the simulation approaches to model 1. The plots of the CDS spread as a function of n and M are given in appendix B.4. Table 4.2 summarizes the simulation settings.

Table 4.2: Monte Carlo settings for the simulation approaches to model 2.

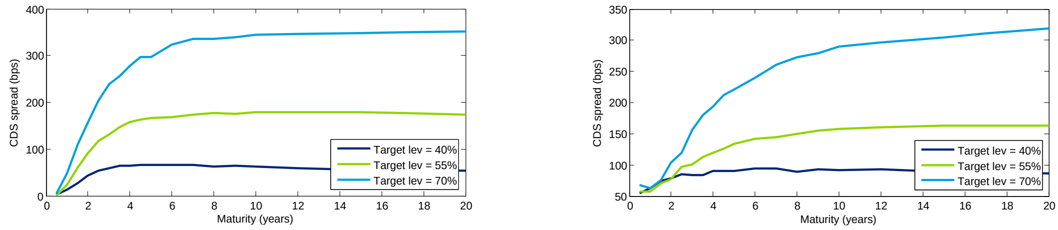
Approach	n	M	**Range**
2A	400	50000	2bps
2B	400	50000	5bps

4.2.4 Results of model 2

This section performs a sensitivity analysis to the CDS term structures evaluated with simulation approaches 2A and 2B. We analyze the sensitivity to the level of the target leverage ratio, the initial leverage ratio, the jump parameters, the assets volatility and the mean reversion speed.

Sensitivity to the target leverage ratio

To analyze the impact of the target leverage ratio on the CDS term structure we take an initial firm leverage of 55% and set the target leverage ratios to 40%, 55%, and 70% by altering γ.
A firm with initial leverage ratio of 55% and long run ratio of 40% can be interpreted as a firm that buys back its debt over time and thereby decreases its leverage. An initial ratio and long run ratio of 55% corresponds to a firm that only adjusts its outstanding debt to changes in the total value of the firm, such that B_t/V_t stays constant. A target leverage ratio of 70% with initial ratio of 55% is a situation in which the company raises more debt and thereby increases its leverage. The latter situation typically results in higher default probabilities and CDS spreads.
Figure 4.7 shows the obtained term structures for both approach 2A and 2B.
For a target leverage ratio of 40% we do not observe the described dynamics. Since the initial leverage is 55% we would expect that CDS spreads first decline and than become approximately constant. This is probably caused by the values for the input parameters: for a smaller value of σ_V we do



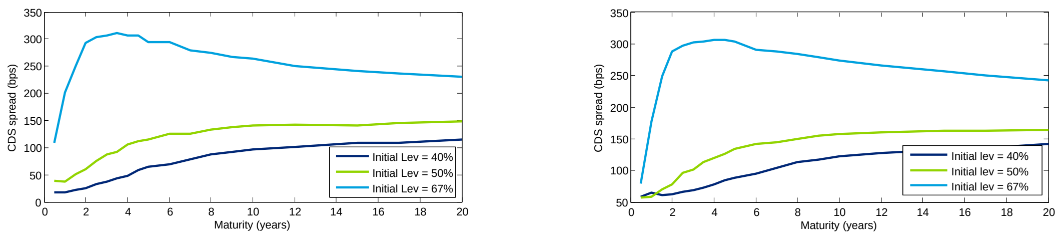
(a) Approach 2A **(b)** Approach 2B

Figure 4.7: Par CDS term structures for varying levels of the long run target leverage of a firm with initial leverage ratio of 55%. Panel (a) shows the effect of increasing the target leverage ratio in approach 2A for respectively $\gamma = 0.777$, $\gamma = 0.459$, and $\gamma = 0.218$ for increasing target leverage ratio. Panel (b) shows varying target leverage ratio in approach 2B for respectively $\gamma = 0.750$, $\gamma = 0.431$, and $\gamma = 0.190$ for increasing target leverage ratio.

observe the described dynamics.
As we saw for model 1 the applied values for λ and σ_π result in underestimation of short-term CDS spreads. We also observe this in 4.7a: short-term CDS spreads strongly increase from zero and this effect is higher for a higher level of the target leverage ratio.
For long maturities we do observe the expected shape of the term structure. As a consequence of the long run leverage ratio CDS spreads are approximately constant or even increasing and the level of the CDS spreads depends on the level of the target leverage ratio. This is an improvement to the downward sloping term structures obtained with model 1.
Approach 2B results for these input parameter values in higher short-term CDS spreads. When we further decrease the target leverage ratio for the same initial leverage ratio, the term structure becomes more flat but not downward sloping. This indicates as in approach 1C that the choice of λ is the main driver of the level of the CDS spread in this approach.

Sensitivity to the initial leverage ratio

Next we analyze the sensitivity of CDS term structure to the leverage ratio of the firm at $t = 0$. For approach 2A we take new values for the jump parameters to increase short-term CDS spreads. Figure 4.8 shows the results for approach 2A and 2B.



(a) Approach 2A **(b)** Approach 2B

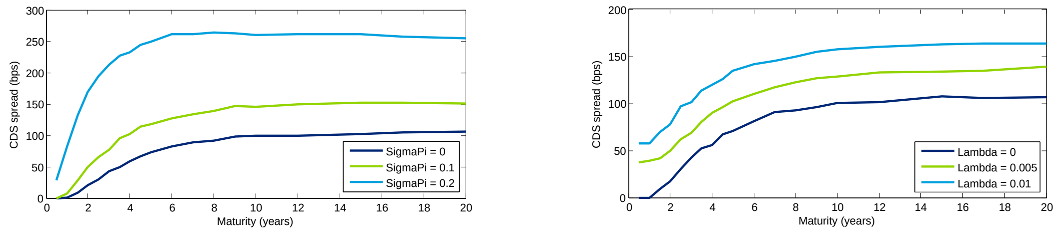
Figure 4.8: Sensitivity of the par CDS term structure towards the initial leverage of a firm with a target leverage ratio of 55%. Panel (a) shows term structures modeled with approach 2A with $\lambda = 0.05$, $\sigma_\pi = 0.5$, and $\gamma = 0.468$. Panel (b) shows the relation for approach 2B with $\lambda = 0.01$, and $\gamma = 0.431$.

A high initial leverage of 70% results in high short-term CDS spreads and these decline when the firm improves its financial conditions. The CDS term structure for firms with a lower initial leverage tends to increase with time. This illustrates the effect of the mean reverting leverage process: the assets value increases with time and the leverage is kept constant. In contrast the term structures in figure 4.6 are downward sloping.
Especially for approach 2B we observe that the graphs tend to converge to each other. The term structures modeled with an initial leverage of 40% and 70% increase respectively decline to the more constant term structure with initial leverage of 50%. This is intuitive since each term structure has

a target leverage ratio of 55% and since an initial leverage of 50% is closer to this target, this term structure is more constant.
When we compare figure 4.8 to figure 3.3 we observe that the term structure with initial leverage of 40% for approach 2A has a similar shape and similar values as the term structure of Ahold on 31-12-2009. Furthermore, the term structure with initial leverage of 50% modeled with approach 2B has similar values to the term structure of Aegon. This indicates that model 2 is able to estimate CDS term structures observed in the marketplace and the parameters used to obtain these modeled spreads provide a benchmark for estimating market CDS term structures in chapter 5.

Sensitivity to the jump parameters

For model 1 we presented an extensive analysis on the sensitivity of the CDS term structure to changing values of the jump parameters. Since the effects are similar in model 2 we limit this analysis. Figure 4.9a presents a sensitivity analysis for approach 2A and 2B.



(a) 2A: changing σ_π^2 **(b)** 2B: changing λ

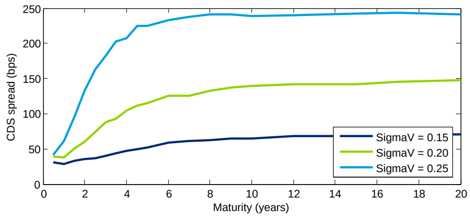
Figure 4.9: Sensitivity of the par CDS term structure to the jump parameters in approach 2A and 2B. Panel (a) shows the effect of changing the jump size on the par CDS term structure for approach 2A. $\lambda = 1$ and the target leverage ratio is 55%, such that $\gamma = 0.431$, $\gamma = 0.459$, $\gamma = 0.543$ for increasing σ_π. Panel (b) shows the effect of increasing the jump intensity in approach 2B. The target leverage ratio is 55%, such that $\gamma = 0.431$.

As in model 1, a larger jump size increase the CDS spread. Comparing the term structure of $\sigma_\pi = 0.1$ in figure 4.9a to the term structure of 1A case b in figure 4.1 finds that 2A generates upward and 1A downward sloping term structures. Applying a total variance restriction in approach 2A gives similar results as in figure 4.3, but again for approach 2A the term structures are upward sloping.
Figure 4.9b plots the CDS term structures for different jump intensities in approach 2B. As in figure 4.2 a higher value of λ results in an upward shift of the term structure.

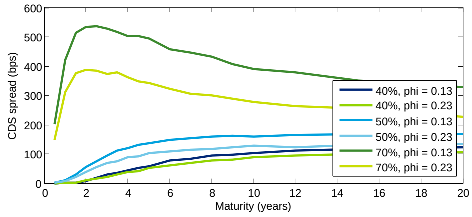
Sensitivity to σ_V and φ

Figure 4.10a shows that a higher assets value volatility increases the CDS spread. Compared to figure 4.5 approach 2A generates upward sloping term structures and with the applied values of $\lambda = 0.05$ and $\sigma_\pi = 0.5$ short-term CDS spreads are positive. The parameters used to calculate the term structure for $\sigma_V = 0.15$ provide a guideline to model the CDS term structure of AkzoNobel as shown in figure 3.3.

Finally, we analyze the sensitivity of the CDS term structure to the mean reversion speed. In figure 4.10b we observe that for an initial leverage of 40% the mean reversion speed of 0.23 models results in higher CDS spreads than for $\varphi = 0.13$ up to $T = 2$ year. This illustrates that with a higher mean reversion speed the firm's leverage faster increases to the target leverage of 55%, such that the CDS spread is higher. The same effect is seen when the initial leverage is 50%, but now up to $T = 0.5$ year. When the initial leverage is 70% a higher φ faster decreases the leverage of the firm, such that the CDS spread will be lower than for a smaller mean reversion speed.
Furthermore, we observe that all term structures converges to the target leverage level of 55%. We see that the term structures modeled with initial leverage of 40% and 50% and a higher mean reversion speed result in lower CDS spreads over a long horizon. A possible explanation might be that a higher



(a) 2A: changing σ_V



(b) 2A: changing φ

Figure 4.10: Panel (a) shows the effect of changing the assets volatility on the par CDS term structure for approach 2A for $\lambda = 0.05$ and $\sigma_\pi = 0.5$. The target leverage ratio is 55%, such that $\gamma = 0.420$, $\gamma = 0.468$ and $\gamma = 0.531$ for increasing σ_V. Panel (b) shows the sensitivity of the CDS term structure to the mean reversion speed for various levels of the initial leverage ratio. The target leverage ratio is 55%, such that $\gamma = 0.406$ and $\gamma = 0.489$ for respectively $\varphi = 0.13$ and $\varphi = 0.23$.

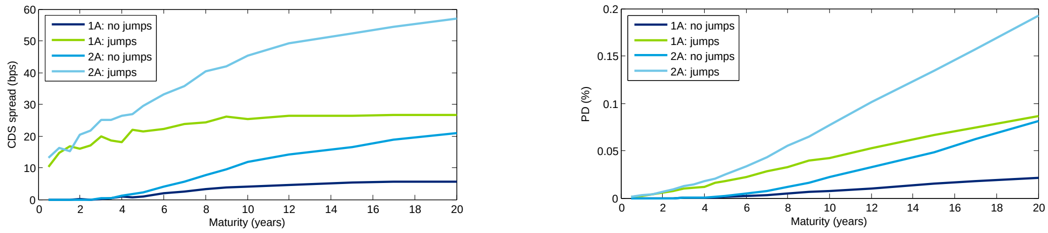
φ makes the leverage process more stationary than with a smaller φ: the firm can quicker adjust its debt level to a change in firm value. Further research for $T > 20$ is necessary to verify this observation.

4.2.5 Conclusions model 2

This section analyzed a structural model with a jump-diffusion process and mean reverting leverage ratio. Two Monte Carlo simulation approaches were developed to evaluate CDS term structures with this model.
We saw that the model is able to generate positive short-term CDS spreads and upward sloping term structures for longer maturities. This latter aspect is an improvement to model 1. Furthermore we compared modeled spreads to market spreads and found that both simulation approaches are able to match shapes and values of market term structures.
The main drawback of simulation approaches 2A and 2B is their long computational time.

4.3 Overview and conclusions

This chapter described two structural model. In the first model the assets value process was modeled with a jump-diffusion process and the default barrier was assumed to be constant. The second model incorporates a jump-diffusion process for the firm's assets value and a mean reverting leverage process.
The models are implemented in Matlab using several Monte Carlo simulation algorithms to calculate three credit risk measures: CDS spreads, probabilities of default and recovery rates. We focussed on the modeled CDS term structures and performed a sensitivity analysis to study the effect of value changes in input parameters on this term structure.
In the sensitivity analysis we found that the models can evaluate CDS term structures shapes and values that are also observed in the market. As a final overview we present in figure 4.11 CDS term structures and default probabilities evaluated with four different model configurations: a diffusion and jump-diffusion process with constant default barrier, and a diffusion and jump-diffusion process with mean reverting leverage ratio. The first two configurations are modeled with approach 1A, the latter two with approach 2A.
The models with a diffusion process for the firm's assets value generate zero CDS spreads and default probabilities for short maturities. Due to the low default barrier of 40% approach 1A without jumps also results in low spreads and PD over a long horizon. Adding the mean reverting leverage process to the diffusion process increases long-term spreads and PDs.
The inclusion of jumps in the assets value process increases short-term values. The mean reverting leverage ratio process further increases spreads and PDs over a longer horizon. We must note however, that the effect of the stationary leverage ratio is most pronounced for maturities of $T > 5$. Since most CDS contracts have a maturity of 5 years, this effect might be less relevant for CDS spread analysis in practice.



(a) CDS spreads

(b) Risk-neutral PD

Figure 4.11: CDS spreads (a) and risk-neutral default probabilities (b) for different model configurations. The initial leverage ratio or default barrier is 40% of V_0, $\sigma_V = 0.15$, and the target leverage ratio is 55%. For the diffusion processes $\lambda = 0$ and $\sigma_\pi = 0$, such that $\gamma = 0.3834$. For the jump-diffusion processes $\lambda = 0.05$ and $\sigma_\pi = 0.5$, such that $\gamma = 0.420$.

In this chapter we further found that the standard Monte Carlo approaches 1A and 2A are most complete: they can generate several credit risk measures, can calculate term structure shapes observed in the market and allow for an extensive sensitivity analysis. Drawbacks of these methods are the one jump per time period restriction and their low computational speed compared to approach 1B and 1C.

Approach 1B seems to be unable to estimate CDS spreads with different maturities using the same input parameters. This will cause difficulties when the model is applied in practice. Jump to default approach 1C is limited in its output possibilities and sensitivity analysis, and the term structure is downward sloping for long term maturities. Approach 2B that is similar to approach 1C, but computational slow can provides upward sloping term structures and has proven to match market CDS spreads.

Concluding, the proposed Monte Carlo algorithms have each its advantages and disadvantages. Therefore we will further analyze all models in the empirical study in the next chapter.

Chapter 5

Parameter estimation and application

Chapter 4 described two structural models and analyzed the sensitivity of modeled CDS term structures to changing input parameters. We found that both models are able to generate shapes and values of CDS term structures that are also observed in the marketplace. Furthermore, both models are able to calculate recovery rates and default probabilities and can be used for market risk analysis. To use the models in practice we need to take another step: estimation of input parameters based on firm data. This chapter gives an introduction to this step by identifying possible problems in estimating parameters from market data. These issues can be applied as guidelines for more advanced parameter estimation techniques that are left for future research.
To identify possible problems we first describe two market applications of structural models: the distance to default and CreditGrades model. Then we collect data and describe a methodology to determine input parameters from the data. With these parameters we perform three case studies in which we compare modeled CDS term structures to market CDS term structures of Dutch firms and focus on observed discrepancies due to the selected input set.

5.1 Two applications of structural models

The first step to get more insight on the implementation of structural models to market data is to study the practice of others. This section therefore describes two well known applications of structural models: Moody's KMV's distance to default model and the CreditGrades model.

5.1.1 Moody's KMV distance to default model

Moody's KMV distance to default model uses the results from the Merton Model to determine a measure for a firm's credit quality. This distance to default is the number of standard deviations by which the firm's assets value must change for a default event to occur T years from now. The distance-to-default is given by d_2 in the Merton model:

$$\text{Distance to default} = \frac{\ln{(V_t/D)} + (r - \frac{\sigma_V^2}{2})(T-t)}{\sigma_V \sqrt{T-t}}$$

When the distance to default decreases, the probability of default increases and the firm is more likely to default. The distance to default of a particular firm is compared to the historical default experience of firms with the same distance to default to obtain an expected default frequency.
Moody's KMV takes for the promised debt payment or default barrier D a combination of current book value of debt, the face value of near term debt and a fraction of long term debt. The firm's assets value is the sum of D and the firm's market capitalization, and the assets volatility is determined from historical equity prices.

5.1.2 CreditGrades model

RiskMetrics, JP Morgan, Goldman Sachs and Deutsche Bank developed the CreditGrades model to create a more simple link between the credit and equity market than most structural models. Due to this relative simplicity, CreditGrades has become an industry benchmark for determining credit spreads in a structural framework.
Creditgrades is different from other applications of structural models for two reasons. First its purpose is not to accurately determine default probabilities, as most structural models. CreditGrades aims to track credit spreads and detect when market spreads deviate from values that the firm's conditions suggest.
Second many applications focus on the modeling of unobservable parameters like the firm's asset value and volatility. CreditGrades is more practical using a specially designed parameter estimation methodology based on a small parameter set that can be determined from market observables. For example the assets volatility is estimated from the equity volatility over a 750-1000 day horizon.
Due to these simplifying assumptions, the model is not useful for pricing purposes. It is more suitable for analyzing trading opportunities, e.g. for capital structure arbitrage as in Yu (2005), or in risk management to monitor changes in the credit worthiness of firms based on information from the equity market. The CreditGrades model is derived from the BC model. To account for the expected default event and low short-term credit spreads in the BC model, a random variable is included in the default barrier to make it stochastic. Since the level of the default barrier is now stochastic, a default event will happen unexpectedly. Furthermore, when at $t = 0$ the default barrier is specified, this barrier might be above the initial firm value V_0, such that the firm defaults at initiation. This results in a positive default probability and credit spread for short maturities at $t = 0$. However, for $t > 0$ CreditGrades produces unrealistic dynamics of short-term credit spreads, which makes the model less applicable for pricing purposes after $t = 0$.

As noted in chapter three and confirmed by these applications, the unobservability of the firm's assets value process is a major difficulty in the implementation of structural models. Academic studies propose advanced estimation methodologies to determine the parameters of this process. However, in applications of structural models simplifying assumptions are made to model the process. This limits the precision of the models but makes them more practical.
Since our models need to be both practical and precise to value CDS contracts a suitable parameter estimation method should be selected. This is left for future research. We apply a simple estimation methodology to identify possible problems in the parameter estimation process when we compare modeled and market spreads in three cases studies.

5.2 Case studies: data and methodology

To identify estimation issues that are typical for the two models considered in chapter 4 we perform three case studies. In these case studies we model CDS term structures of a firm based on various input parameter sets that are estimated from that firm's market and balance sheet data. The modeled CDS spreads are compared to market CDS spreads of the firm and the differences are analyzed. This section describes the data collection and estimation methodology. The next section presents the results.

5.2.1 Data

Since Deloitte Capital Markets is primarily active in the Netherlands we perform the case studies on Dutch firms. To determine the input parameters for the models we use a simple methodology described below. To apply this methodology, we need for each firm approximately 10 years of equity prices and balance sheet data, time series of CDS spreads for various maturities, and a (recent) credit rating. Table 5.1 shows the selected firms based on these requirements.
All data is collected from Bloomberg. For each trading day between 1 January 2010 and 1 January 2000, we take the firm's closing price of equity, the market capitalization, and the outstanding amount of short and long term debt of the firm if available. Furthermore, we acquire dividend payout and dividend yield information. As a proxy for the risk-free interest rate we take the Euro/swap curve

Table 5.1: Selected Dutch firms and their rating and industry (Moody's).

Firm	Rating	Rating date	Industry
Royal Ahold	Baa3	26-07-2009	Retail: food and grocery
Akzo Nobel N.V.	Baa1	16-03-2009	Chemicals: specialty
Aegon N.V.	A3	17-02-2009	Financial services: insurance

for Euro denominated shares on various dates[1].
The CDS spreads are mid quotes for maturities of 6 months, 1, 2, 3, 4, 5, 7, and 10 year with recovery rate $R = 40\%$. To compare term structures of CDS spread we need on a typical date a quote of each maturity. For the selected firms, this data is available from January 2005.

5.2.2 Methodology

This section describes the methodology to determine the input parameters for model 1 and 2 from the acquired data. We use a fast and simple methodology based on Jones et al (1984) such that we can focus on estimation issues and the sensitivity of the CDS spreads to the input parameters. More advanced parameter estimation methods may be recommendable but are left for future research.

Jump parameters

Section 5.1 identified that the unobservability of the firm's assets value process is the main problem in applications of structural models is. It requires that the parameters μ_V, σ_V, μ_π, σ_π, and λ need to be determined from other data. This section derives the jump parameters and the next section determines the diffusion parameters of the assets value process.
We construct a time series of daily assets values using the balance sheet equation. According to this equation the total assets of a firm equals the sum of total debt and total equity. On each trading day we add the firm's market capitalization as total equity and the book value of debt as total debt to construct a time series of daily assets value. From this time series we calculate the daily log returns of the firm's assets value to determine the paramters[2].
In simulation approaches 1A, 1B and 2A we define a jump in the assets value process as a daily log return of at least $4\sigma_V$[3]. As we will see this jump definition results in similar values for λ and σ_π as used in the sensitivity analysis of model 1 in chapter 4. The number of jumps occurring in a year is a measure for the jump intensity λ. The mean and volatility of these jumps are calculated to obtain μ_π and σ_π respectively.

In the jump to default approaches 1C and 2B we determine the jump intensity from cumulative default rates provided by Moody's. This is an intuitive method since the jump intensity is calculated from historical default data. This implies that in these approaches the occurrence of a jump is a real default event and not an arrival of important news like an earnings warning.
We use the following relationship from Hull (2006) to determine λ:

$$Q_t = 1 - e^{-\lambda t} \quad \text{such that} \quad \lambda = -\frac{\ln(1 - Q_t)}{t}, \tag{5.1}$$

where Q_t is the cumulative probability of default by time t. Moody's provides these default probabilities for all credit ratings and maturities up to 20 years, see for example figure 2.2.

Diffusion parameters

The diffusion parameters μ_V and σ_V are calculated from the daily log returns of the firm's assets value. In approaches 1C and 2B μ_V and σ_V are the yearly mean and volatility of this time series.
In simulation approach 1A, 1B and 2A we first remove the jumps from the time series of daily log

[1]Note that in this method the risk-free interest rate is only used to discount the CDS cash flows and not as drift in the firm's assets value process.
[2]Since equity prices have a lognormal distribution we take the log returns of the time series of the firm's assets, such that μ_V, σ_V, μ_π and σ_π are normally distributed
[3]In the case studies we also apply jump criterions of $3\sigma_V$ and $2\sigma_V$, such that λ increases and σ_π decreases.

assets returns. From this jump-free series we take the yearly mean and volatility to obtain μ_V and volatility σ_V[4].
We set the sample horizon for estimating the jump and diffusion parameters equal to the contract's maturity. This means that if we model a 6 month CDS spread on a specified date, we determine the parameters based on the daily log assets returns in the 6 months preceding that date. A CDS spread of a contract with maturity of 5 years is calculated using 5 years of data, etcetera.
Although this methodology for the sample horizon is a common practice we need to be aware of extraordinary events in applying it. For example the credit crisis (mid 2007-2009) and the internet bubble (2003) will have an impact on the firm's assets return and volatility that might not be reflected in each sample horizon.

Other parameters

As initial firm value V_0 we take the sum of total debt and market capitalization on the evaluation date of the contract. So, if we want to determine the CDS term structure on 1 January 2010 we calculate the firm value on this date.
In approach 1A and 1B we specify the default barrier as the total amount of outstanding debt at the evaluation date. It would be more appropriate to take a promised debt payment as in Moody's KMV's distance to default model, but we do not have the required data.
In approach 2A and 2B the initial leverage is specified as the ratio of total debt to total firm value at the evaluation date. For the long run leverage we take the average ratio of total debt to firm value for the sample horizon. We use the literature value of $\varphi = 0.18$ for the mean reversion speed.
The Euro/swap curve is used to determine discount factors. We apply a single interest rate to each maturity. For example all cash flows of a CDS contract with maturity of 5 years are discounted using the 5 year Euro/swap rate.
Finally, the firm's assets value process could be extended to account for dividend payments. Therefore, we subtract the yearly dividend yield from the yearly assets return. In this way the drift of the assets value process decreases, such that CDS spreads increase. This method is easily applied in approach 1C and 2B. However, in the other approaches it must be evaluated whether or not the dividend payments are removed from the time series of daily log assets returns by applying the jump definition.

5.3 Case studies: results

The methodology described in the last section is the basis approach to determine the input parameters of the models in the case studies. Appendix C gives the calculated parameters for each company on two evaluation dates.
The following sections describe the results and issues of implementing models 1 and 2 to market data of Ahold, AkzoNobel and Aegon respectively. For each firm we first review firm specific data to identify possible problems in parameter estimation. Then we model the firm's CDS structure using the basic parameters and compare the result to the the market CDS spreads. Finally we propose changes to the input parameter set to solve observed implementation issues.

5.3.1 Ahold

To identify extraordinary events in the Ahold data, we analyze Ahold's time series of CDS spreads. Figure 5.1 shows the 1, 5, and 10 year CDS spread on every trading day between 1 January 2005 and 1 January 2010. Ahold's CDS spreads are volatile especially during the credit crisis, which makes parameter estimation difficult: an input set that might calculate a close estimate on a certain date might be useless a few days later. In the periods before 1 January 2007 (post credit crisis) and after September 2009 the CDS spreads are more stable. Therefore we will test the models in estimating CDS term structures on 31 December 2009 and 2 January 2007.
Furthermore, we observe a large downward jump in equity prices between 21 February 2003 and 24 February 2003. This jump is less pronounced in the assets returns than in the equity returns, but it

[4]Note that in the Merton (1974) model the diffusion parameters μ_V and σ_V can be directly estimated from equity prices by solving a system of two equations (Hull, 2006). This method is typical for the Merton model and is not applicable to the jump-diffusion type of models that we consider.

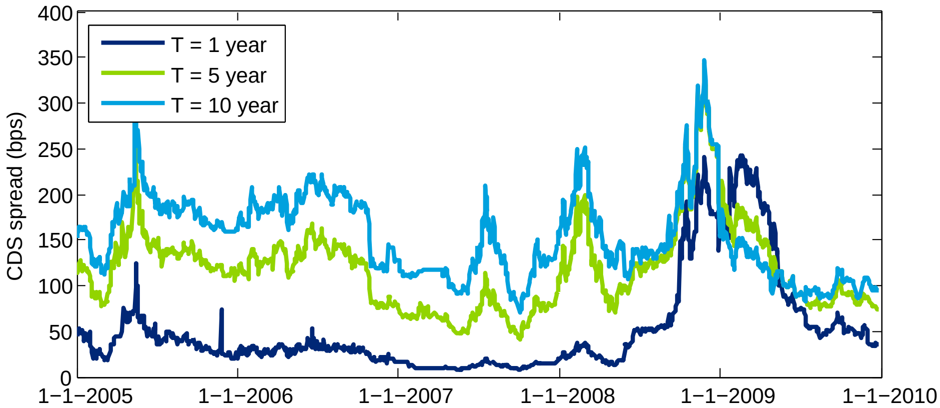
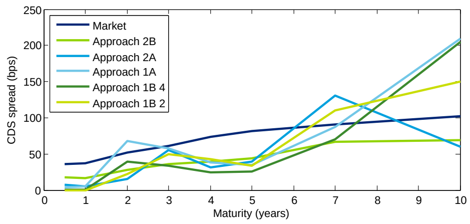
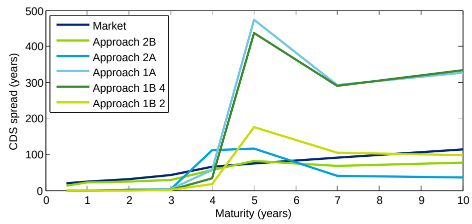


Figure 5.1: 1, 5 and 10 year CDS spreads for Ahold between 1 January 2005 and 1 January 2010.

might be necessary to adjust the data horizon for this jump.



(a) CDS term structure per 31-12-2009



(b) CDS term structure per 2-1-2007

Figure 5.2: Market and modeled CDS term structures of Ahold on 31 December 2009 and 2 January 2007. The 2 and 4 in the legend of approach 1B represent the size of the jump criterion, respectively $2\sigma_V$ and $4\sigma_V$.

Figure 5.5 shows modeled and market CDS term structures on two different dates[5]. Observe that modeled CDS term structures show large discrepancies with the market term structure. The main explanation is the changing sample horizon that we use to determine the input parameters for each maturity. Especially the results of approach 1A and 1B-4 in 5.2b for $T > 4$ are different. The Ahold data per 2-1-2007 in appendix C shows that from a maturity of 4 years, μ_V becomes negative and σ_V increases, which result in higher CDS spreads.

We further observe that on 31 December 2009 short-term CDS spreads are underestimated. From the data we infer that asset returns are high and asset volatilities are low in the post crisis period resulting in low modeled CDS spreads. Another sample horizon is therefore necessary to determine the input parameters for calculating short-term spreads.

In both figures approach 2B underestimates market spreads, but the term structures have approximately the same shape. In the analysis in chapter 4 we found that we can shift the CDS term structure modeled with approach 2B upward by increasing the jump intensity. In our estimation methodology this implies that the cumulative default rates corresponding to Ahold's Baa3 rating should be higher than applied. This might be possible, since our default data is based on the period 1983-2008 and therefore does not include part of the credit crisis in which many firms defaulted. Unfortunately we do not have more recent data. Another possibility to increase the term structure of approach 2B could be to account for non-credit risk related factors (e.g. liquidity) in the model that increase the market spread. This is left for future research.

The term structures of approach 2A and 1B-2 move around the market term structure and seem to provide a good fit on average. This might be obtained by using a single sample horizon to estimate the input parameters instead of the proposed sampling method. For example a horizon of 3 year

[5]Results of approach 1C are not included in the graph, since modeled CDS spreads blow up for large maturities.

might be appropriate to calculate the entire term structure since approach 2A closely estimates the 3 year market spread figure 5.2a.

From the observations in figure 5.5 we infer that the applied parameter estimation methodology is not suitable for Ahold: calculated parameters show large differences over various sample horizons. We will now adjust the input parameter set on 31-12-2009 to see whether this implementation issue can be fixed.
A first attempt would be to adjust approach 2A, since we found in section 4.2.4 that this approach with an initial leverage ratio of 40% provides a close approximation to the Ahold term structure. Unfortunately we cannot match the input of that term structure with the input data for Ahold, especially σ_π is smaller.
Based on the analysis of figure 5.5 we might the following adjustments to input parameters:

- **Trial 1.** For approach 2B we estimate the parameters over a 3 year horizon, between 31-12-2009 and 2-1-2007.
- **Trial 2.** For approach 1B-2 we take the jump and diffusion parameters over the same 3 year horizon as in trial 1.
- **Trial 3.** In approach 2A we set σ_V and the target leverage ratio to the 3 year horizon.
- **Trial 4.** In approach 1C we determine the parameters from the period between 2-1-2007 and 2-1-2004, thus between the credit crisis and internet bubble.

The term structures modeled with these input parameter sets are shown in figure 5.3[6].

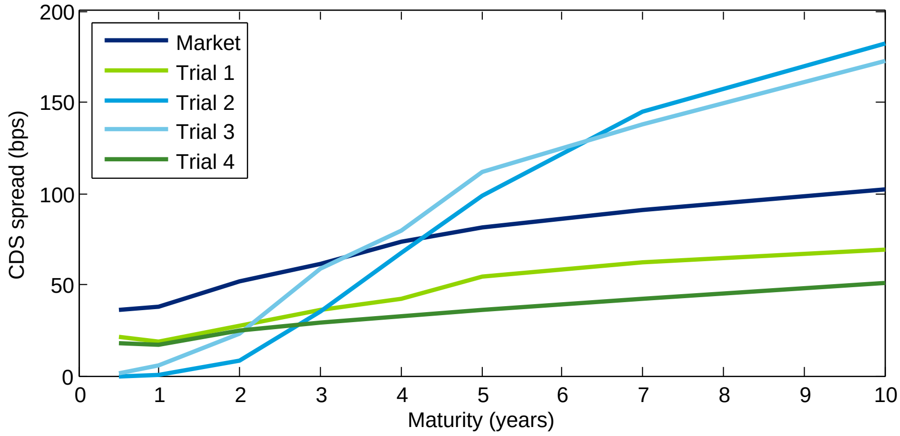


Figure 5.3: Estimation of the CDS term structure of Ahold on 31 December 2009 with various input parameter sets.

As can be seen in the figure, the single data horizon to determine input parameters makes the term structures more stable. We further recognize the hump-shaped term structures also observed in chapter 4.
Trial 1 and 4 have a similar shape but underestimate the market term structure on all maturities. Since trial 1 results in a term structure with similar values as in figure 5.2a, we infer that this approach is mostly depending on the choice of λ.
Trial 2 and 3 underestimate short-term CDS spreads due probably to the low jump size but overestimate CDS spreads with $T > 4$ years. This can be explained by the high σ_V in the input data between 02-01-2007 and 31-12-2009 due to the credit crisis.

5.3.2 AkzoNobel

The second case study analyzes the ability of our models to estimate the CDS term structure of AkzoNobel and identifies the parameter estimation problems for this process.
Figure 5.4 shows that the CDS spreads of AkzoNobel are especially volatile during the credit crisis

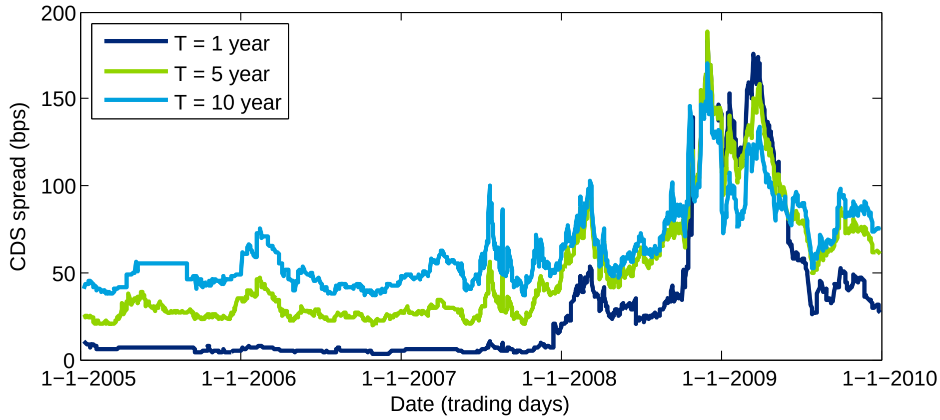
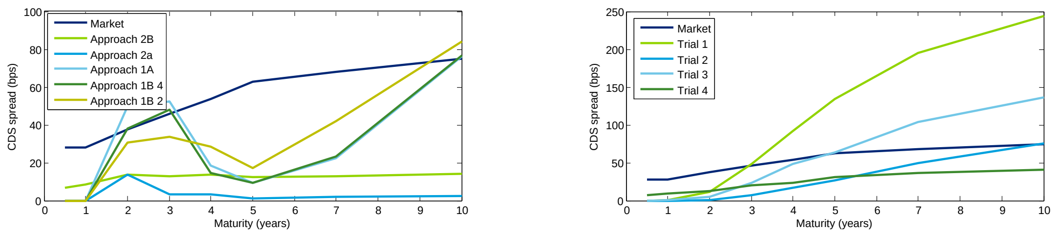


Figure 5.4: 1, 5 and 10 year CDS spreads for AkzoNobel between 1 January 2005 and 1 January 2010.

and relatively stable in the period before 1 January 2007. There are no other extraordinary events in AkzoNobel's data and the parameter values calculated with our standard estimation methodology are given in appendix C. We use these parameters to estimate the CDS term structure of AkzoNobel on 31 December 2009 in figure 5.5a.

In this figure we observe that all simulation approaches underestimate short-term CDS spreads. This can be explained by the high return on assets in the period after the credit crisis. As we saw before a higher return on assets means a higher drift in the assets value process and thus a lower CDS spread. We further observe that especially model 1 generates unrealistic term structure shapes. As in the Ahold case study this is a result of the changing sample horizon that we use to estimate the jump and diffusion parameters. From the estimated input data set of AkzoNobel we find that especially the daily log assets return without jumps has different values for different maturities: high returns in the post credit crisis period, negative returns during the credit crisis and again positive returns for a sample horizons of 4 years or longer.

The term structures obtained with model 2 are more stable. This is because the main input parameters to this model, the average leverage ratio and assets volatility over de sample horizon, are more stable than the input parameters to model 1. The low target leverage ratio, 22-31%, is the reason for the underestimation of AkzoNobel's whole term structure with approaches 2A and 2B.



(a) Basic input parameters

(b) Adjusted parameter set

Figure 5.5: Panel (a) shows market and modeled CDS term structures of AkzoNobel on 31 December 2009 using the standard estimation method. Panel (b) shows the market CDS term structure and modeled term structures evaluated with other input parameter sets.

The findings in chapter 4 are the starting point to find input parameter values that improve the models' estimations of the CDS term structure of AkzoNobel. In section 4.2.4 we found that the term structure of AkzoNobel could be approximated with approach 2A when the target leverage ratio is 55% (or 38% in the real world) and $\sigma_V = 0.15$. However, the target leverage ratios estimated from the balance sheet data of AkzoNobel are smaller and the assets volatility without jumps higher,

[6]We also applied a 10 year sample horizon to approach 2B since the average leverage over 10 year is 40%, but this resulted in CDS spreads ranging between 200 and 650bps and this trial is therefore left from the figure.

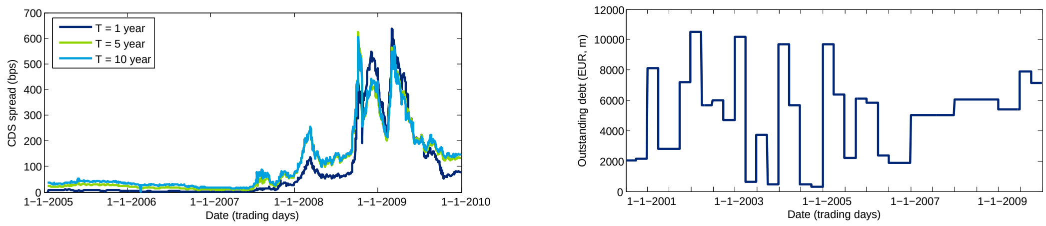
such that we need to select other input parameter sets. We propose the following four input parameter sets to improve the term structure estimates:

- **Trial 1.** For approach 1B-4 we determine the jump and diffusion parameters over a 3 year horizon between 31-12-2009 and 2-1-2007.
- **Trial 2.** For approach 1A-4 we estimate parameters over the data horizon between 31-12-2009 and 1-7-2002.
- **Trial 3.** For approach 1A-2 all parameters are calculated over a 5 year horizon between 31-12-2009 and 2-1-2005 and the dividend yield is subtracted from the assets returns with jumps[7].
- **Trial 4.** For approach 2B we take the input parameters over a 2 year horizon between 31-12-2009 and 2-1-2008. In theory this should increase the term structure, since this sample period has the highest σ_V and a relatively high target leverage ratio.

The term structures modeled with these trials are given in figure 5.5b. The results of approach 1A and 1B (trials 1-3) are now more stable and we observe the diffusion type shape. As expected, the term structure modeled with approach 2B (trial 4) has increased compared to its result in figure 5.5a. All trials are unable to provide close estimates for the short-term CDS spreads of AkzoNobel. An explanation is the low values for the jump size in the input set. We used values of approximately $\sigma_\pi = 0.1$ to calculate CDS spreads for all maturities and in chapter 4 we found that this jump size does not generate high short-term CDS spreads. Another methodology to estimate jumps sizes and jump intensities seems therefore necessary to use the models in practice. This is left for future research.

5.3.3 Aegon

The CDS contracts that Deloitte CM values often have a financial institution as reference entity. This last case study therefore analyzes whether our models can estimate the CDS term structure of Aegon.
During the credit crisis Aegon was one of the financial institutions that suffered great losses and obtained financial support from the government to survive. As can be seen in figure 5.6 this financial turmoil is reflected in Aegons high and volatile CDS spreads.



(a) 1, 5 and 10 year CDS spreads

(b) Outstanding debt

Figure 5.6: Panel (a) shows 1, 5 and 10 year CDS spreads for Aegon between 1 January 2005 and 1 January 2010. Panel (b) shows the outstanding amount of debt of Aegon between 1-7-2000 and 31-12-2009.

Figure 5.6b shows that Aegon's level of outstanding debt frequently changes with a large amount. This will probably result in a relatively high jump intensity calculated from the log assets return time series. Furthermore, these debt changes will make the level of the default barrier in model 1 and initial leverage in model 2 more volatile.

In the case study on AkzoNobel we found that the models have problems in estimating CDS spreads when parameters are calculated over a horizon that includes the credit crisis. Since Aegon's parameters are even more volatile than AkzoNobel during this crisis, we focus in this analysis on Aegon's term structure before the crisis period. We take as estimation date 2 January 2007 just after an

[7]This is a trial and error attempt to decrease the drift in the assets value process, such that the CDS spreads should increase.

increase in debt value (see figure 5.6b), such that the default barrier and initial leverage in the models is higher. Furthermore, in the case studies on Ahold and AkzoNobel we found that the standard sampling procedure to estimate parameters resulted in term structures shapes that do not correspond to the market spreads. In this case study we therefore only analyze the results of alternative input parameter sets.
The input parameters estimated for Aegon on 2-1-2007 using the standard approach are given in appendix C. Aegon's assets volatility with jumps increases to 0.43 and 0.61 for sample horizons of 3 and 4 years respectively. These values will result in high CDS spreads in approach 1C and 2B and therefore we limit the sampling horizon for these models to 3 years or less.
The assets return and volatility without jumps seem to be useful until 1 January 2003: for a sampling horizon longer than 4 years the assets return becomes negative, which will result in too high CDS spreads. For a 4 year sample horizon $\sigma_\pi = 0.38$ which might be an interesting value to increase short-term CDS spreads in approach 1A or 1B.
Based on these observations on the parameter values we model term structures with the following input sets:

- **Trial 1.** For approach 1A we estimate the input parameters over a 2 year horizon between 2-1-2007 and 2-1-2005[8]
- **Trial 2.** For approach 1B we estimate the input parameters over a 2 year horizon between 2-1-2007 and 2-1-2005.
- **Trial 3.** For approach 2A the parameters are estimated over a 2 year horizon between 2-1-2007 and 2-1-2005.
- **Trial 4.** For approach 2B we estimate the parameter over a 2 year horizon between 2-1-2007 and 2-1-2005[9].

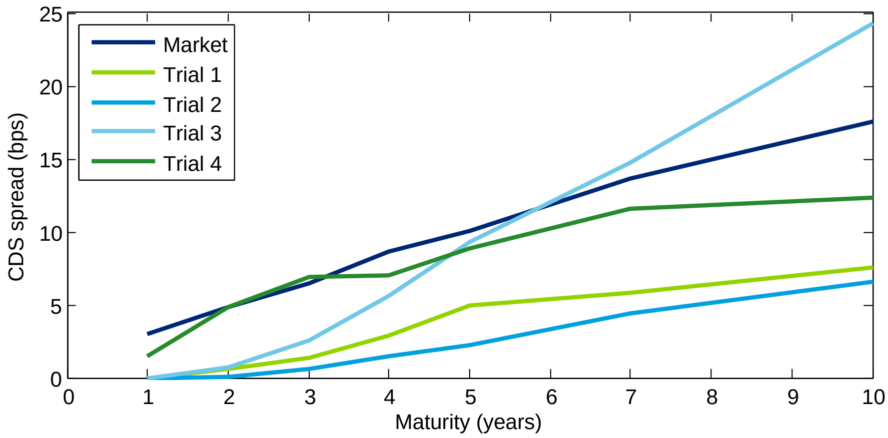


Figure 5.7: Aegon market and modeled CDS term structures on 2 January 2007.

Figure 5.7 presents the term structures modeled with these trials. Trial 4 results in the closest estimate of Aegon's market term structure and it is also the only trial that models a positive 1 year CDS spread. The estimate could be further improved with a higher value for λ, but the difference might also be explained by other factors for which the model does not account, such as liquidity effects.
Trial 1 and 2 result in a similar shape as the market term structure of Aegon but underestimate it due to the low jump size and high assets return without jumps. Trial 3 results in a diffusion shaped term structure that overestimates the 7 and 10 year spreads of Aegon. Compared to the inputs of trial 1 and 2, this should be caused by the effect of the target leverage ratio.

[8] We also used the parameters estimated over a 4 year horizon as input for this approach because of the higher value for σ_π. However, this resulted in a 6 month spread of 31bps, 1 year spread of 75bps and further increasing spreads for higher maturities. Parameters estimated over a 3 year horizon overestimate for $T > 1$ year, therefore we apply a 2 year horizon.

[9] We also modeled term structures with approach 2A and 2B using parameters estimated over a 3 year horizon. This resulted in 10 year CDS spread of respectively 85bps and 43bps, which is too high compared to Aegon's market spreads.

5.4 Summary and conclusions

This section identified possible parameter estimation problems when the structural models are applied to market data. We first analyzed two applications of structural models and found that most difficulties are experienced in estimating the parameters of the firm's assets value process. Then we described a simple methodology to estimate input parameters for the models determined in chapter 4. Finally we performed three cases studies to test the performance of the models in estimating market CDS term structures and we focused on possible difficulties in the input parameter set.
From the case studies we identified the following parameter estimation difficulties:

- Using a sample horizon equal to the maturity of the contract to estimate input parameters results in volatile term structures. This is because the effect of changes in macro economic or firm specific conditions are not reflected in each sample horizon. For example in the AkzoNobel case we estimate high positive values for the drift parameter in the post credit crisis period, while this parameter has a negative value when the sample horizon includes the crisis period.
- There is no consistent methodology that provides close term structure estimates with the applied estimation methodology. For each firm, each simulation approach, and each maturity we often need to apply another estimation horizon to fit the market CDS spread. This is not useful in practice.
- Short-term CDS spreads are often underestimated. This can be explained by the low values of σ_π estimated with the applied methodology. Only the jump to default approaches 1C and 2B, in which the jump intensity is determined from cumulative default rates, seem to provide positive short-term CDS spreads.
- Also long term CDS spreads are often underestimated. This might be explained by other factors than credit risk described in section 2.6 that might increase the market spread.

Since we observed in chapter 4 and also in this chapter that our models can provide similar shapes and values as market term structures for the right parameter choices, we conclude that our structural models are correct. However, to apply the models in practice a more advanced parameter estimation methodology, especially for the jump parameters, is necessary. Furthermore, it would be interesting to extend our structural models to account for non-credit risk factors to increase modeled spread. These two issues are the main topics for future research.

Chapter 6

Conclusions and future research

6.1 Conclusions

The research objective of this thesis is to determine a credit risk model that:

1. can be used to value single name cash settled CDS contracts,
2. is able to estimate CDS term structures observed in the market,
3. can evaluate multiple credit risk measures as output,
4. and can be used to analyze the effects of market risks on these measures.

We reviewed the literature on credit risk modeling and identified that structural credit risk models are able to meet these research objectives. In these models the assets value of a firm is modeled over time and default occurs if the value of the firm's assets drops below a certain default threshold. In this way structural models provide an intuitive, economic explanation for the default process. Furthermore, they can be implemented to estimate and analyze changes in various credit risk measures, such as default probabilities, recovery rates, and CDS spreads. We focused on the modeling of CDS term structures.

From the literature we selected two types of structural models for further research as a trade off between modeling complexity and performance. In the first model, the firm's assets value process is modeled with a jump-diffusion process and the default barrier is assumed to be constant. The second model also assumes a jump-diffusion process for the assets value process, but incorporates a mean reverting leverage default barrier.

The models were implemented in Matlab using different Monte Carlo (Brownian Bridge) simulation algorithms. We focused on modeling CDS spreads calculated with a discounted cash flow method. This method is also used for the valuation of CDS contracts, such that the models can be easily extended to value these contracts.

We performed a sensitivity analysis to analyze the effect of value changes in the input parameters on modeled CDS term structures. In this analysis we found that both model 1 and model 2 can model term structure shapes and values observed in the market. Both models are able to calculate positive short-term CDS spreads for the right values of the jump parameters. Over a longer time horizon model 1 generates downward sloping term structures due to the positive assets value drift and constant default barrier in this model. Model 2 overcomes this shortcoming and generates increasing term structures since in this model the leverage ratio is stationary over time.

Then we performed three case studies to identify difficulties when the models are used in practice. In these case studies we used a simple methodology to estimated input parameters from the market and balance sheet data of Dutch firms. These parameters were used as inputs to our structural models to estimate the market term structure of the firm.

We focused in the case studies on the identification of potential problems in the parameter estimation process using the simple methodology. We found that especially the estimation of the jump and diffusion parameters of the assets value process is challenging. These parameters are affected by changes in macro economic or firm specific conditions, such as the credit crisis, which makes a consistent estimation procedure for these parameters difficult. Furthermore, the jump parameters and especially the jump size should be estimated sufficiently large to model positive short-term CDS spreads. A further complicating factor is that modeled CDS spreads are in general low compared to market spreads. This indicates that the market spreads are also increased by non-credit risk factors as liquidity.

Finally, we conclude that the selected structural models are able to meet the research objectives of this thesis. However, in our study we made several assumptions to limit modeling complexity. With respect to CDS valuation we ignored counterparty credit risk and day count conventions, and we further restricted the framework to single name cash settled contracts with a running spread. In the structural models we assumed a constant risk-free interest rate, simplified the firm's capital structure and did not track recovery rates.
At this moment, Deloitte CM could use the selected structural models for a sensitivity analysis of CDS spreads, default probabilities and recovery rates of firms to changes in market variables. Further research and development is necessary to cope with the simplifying assumptions underlying the models, extend the models' opportunities and make them more practical. The next section introduces the main directions for future research.

6.2 Future research directions

In this study we made various assumptions that limit the opportunities and performance of the selected credit risk models. To extend the models' applications and to account for their shortcoming we propose the following main topics for future research:

- **Parameter estimation methodology.** In the case studies in chapter 5 we found that our simple parameter estimation methodology did not provide appropriate parameter values for estimating market CDS term structures. A more advanced calibration methodology should be used that can cope with high volatile firm data during economic downturns and can estimate jump parameter values that increase short-term spreads. An example of such a more advanced calibration method is maximum likelihood estimation, which is frequently used in the literature to estimate the parameters for structural models.
- **Non-credit risk factors.** In chapter 2 we found that CDS spreads are a more pure measure of credit risk than bond spreads. However, several empirical studies in the literature indicate that CDS spreads are also affected by non-credit risk factors, such as liquidity and also our modeled CDS spreads underestimate the market spreads in chapter 5. It would therefore be interesting to investigate methods that can account for these non-credit risk factors in the proposed models. An alternative is to incorporate an endogenous default barrier in the model. For example Leland (2005) claims that since an endogenous default barrier is chosen to maximize the value of the shareholders it is in their interest to account for factors like liquidity in the default decision.
- **Recovery rates.** Recovery rates are an important factor in the valuation of CDSs. CDS spreads are quoted on a constant recovery rate and we used this rate as input to our models. However, the quote might differ from actual or historical recovery rates, which might be reflected in the CDS spread. Therefore we should determine actual recovery rates with our model and make adjustments to modeled CDS spreads if modeled and quoted spreads differ. Further research on the sensitivity of CDS spreads to recovery rates is therefore necessary.
- **Interest rate modeling.** In our models we assumed the risk-free interest rate to be constant, which does not correspond to interest rates in the marketplace. The models' opportunities and performance can be increased by incorporating a stochastic interest rate process, as for example one of the processes described in section 3.3.3. We found in the literature that stochastic interest rates in a structural model could increase modeled CDS spreads, which is attractive for our models. Furthermore, the inclusion of a stochastic interest rate process will allow for a more advanced analysis on the interaction between market risk and credit risk.

There are many more interesting topics to investigate and thereby improve our models, such as another jump-diffusion process, a time varying default barrier in model 1 or long run leverage ratio in model 2, and time varying parameters in the assets value process. However, the proposed future research directions are the first most important steps to make the models more practical and to better meet the research objectives of this thesis.

Bibliography

[1] Alexander, C. & Kaech, A. (2008). Regime dependent determinants of credit default swap spreads. *Journal of Banking & Finance*, 32, 1008-1021.

[2] Altman, E. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance*, 23, 589-609.

[3] Anderson, R., Sundaresan, S. & Tychon, P. (1996). Strategic analysis of contingent claims. *European Economic Review*, 40, 871-891.

[4] Bajlum, C. & Larsen, P. (2008). Capital Structure Arbitrage: Model Choice and Volatility Calibration. Working paper.

[5] Bielecki, T. & Rutkowski, M. (2001). Credit risk: modelling, valuation, and hedging. Springer Finance.

[6] Black, F. & Cox, J. (1976). Valuing corporate securities: some effects of bond indenture provisions. *Journal of Finance*, 31, 351-367.

[7] Black, F. & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81, 637-654.

[8] Bongaerts, D., de Jong, F. & Driessen, J. (2008). Liquidity and liquidity risk premia in the CDS market. Working paper, University of Amsterdam.

[9] Briys, E. & de Varenne, F. (1997). Valuing risky fixed rate debt: An extension. *Journal of Financial and Quantitative Analysis*, 32, 239-248.

[10] Campbell, J. & Taksler, G. (2003). Equity volatility and corporate bond yields. *Journal of Finance*, 58, 2321-2350.

[11] Collin-Dufresne, P. & Goldstein, R. (2001). Do credit spreads reflect stationary leverage ratios? *Journal of Finance*, 56, 1929-1957.

[12] Collin-Dufresne, P., Goldstein, R. & Martin, S. (2001). The determinants of credit spread changes. *Journal of Finance*, 56, 2177-2207.

[13] Cox, J., Ingersoll, J. & Ross, S. (1985b). A theory of the term structure of interest rates. *Econometrica*, 53, 385-407.

[14] CreditGrades technical document (2002). Available at http://www.creditgrades.com/resources/pdf/CGtechdoc.pdf.

[15] Cremers, M., Driessen, J., Maenhout, P. & Weinbaum, D. (2006). Individual stock-option prices and credit spreads. Working Paper, Yale School of Management.

[16] Das, S. & Hanouna, P. (2008). Hedging credit: Equity liquidity matters. *Journal of Financial Intermediation*, 18, 112-123.

[17] Davydenko, S. (2005). When do firms default? Working paper, University of Toronto.

[18] Duffie, D. & Singleton, K. (2003). Credit risk: Pricing, measurement, and management. Princeton University Press.

[19] Elizalde, A. (2005b). Credit risk models II: Structural models.
Available at www.abelelizalde.com.

[20] Elizalde, A. (2005c). Credit risk models III: Reconciliation structural-reduced form models.
Available at www.abelelizalde.com.

[21] Elizalde, A. (2006). Equity-credit modelling. Where are we. Where should we go.
Available at www.abelelizalde.com.

[22] Eom, Y., Helwege, J. & Huang, J. (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies*, 17, 499-544.

[23] Ericsson, J., Jacobs, K. & Oviedo, R. (2005). The determinants of credit default swap premia. Working paper McGill University.

[24] Ericsson, J., Reneby, J., & Wang, H. (2006). Can structural models price default risk? Evidence from bond and credit derivative markets. Working paper, McGill University.

[25] Fan, H. & Sundaresan, S. (2000). Debt valuation, renegotiation, and optimal dividend policy. *The Review of Financial Studies*, 13, 1057-1099.

[26] Geske, R. (1977). The valuation of corporate liabilities as compound options. *The Journal of Financial and Quantitative Analysis*, 12, 541-552.

[27] Hilberink, B. & Roger, L. (2002). Optimal capital structure and endogenous default. *Finance and Stochastics*, 6, 237-263.

[28] Huang, J. & Huang, M. (2003). How much of the corporate-treasury yield spread is due to credit risk? Working paper, Pennsylvania State University.

[29] Huang, J. & Zhou, H. (2008). Specification analysis of structural credit risk models. Working paper, Pennsylvania State University.

[30] Hull, J. (2006). Option, futures and other derivatives. Prentice Hall, 6th edition.

[31] Hull, J. (2007). Risk management and financial institutions. Pearson, International Edition.

[32] Hull, J., Nelken, I. & White, A. (2004). Merton's model, credit risk and volatility skews. *Journal of Credit Risk*, 1, 1-27.

[33] Hull, J., Predescu, M. & White, A. (2004). The relation between credit default swap spreads, bond yields, and credit rating announcements. *Journal of Banking & Finance*, 28, 2789-2811.

[34] Hull, J. & White, A. (2000a). Valuing credit default swaps 1: No counterparty default risk. *Journal of Derivatives*, 8, 29-40.

[35] Jones, E., Mason, S. & Rosenfeld, E. (1984). Contingent claims analysis of corporate capital structures: An empirical investigation. *The Journal of Finance*, 39, 611-625.

[36] Kim, J., Ramaswamy, K. & Sundaresan, S. (1993). Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model. *Financial Management*, 22, 117-131.

[37] Leland, H. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, 49, 1213-1252.

[38] Leland, H. (2005). Comments on Davydenko (2005). Available at
www.moodyskmv.com/conf05/pdf/presentations/h_lelandSDavydanko.pdf.

[39] Leland, H. & Toft, K. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, 51, 987-1019.

[40] Longstaff, F. & Schwartz, E. (1995). A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance*, 49, 789-819.

[41] Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29, 449-470.

[42] Merton, R. (1976). Option pricing when the underlying stock returns are discontinuous. *Journal of Financial Economics*, 3, 125-144.

[43] Metwally, S. & Atiya, A. (2002). Using Brownian bridge for fast simulation of jump-diffusion processes and barrier options. *The Journal of Derivatives*, 10, 43-54.

[44] Moody's Global Credit Policy, February 2009. Corporate default and recovery rates, 1920-2008. Available at www.moodys.com.

[45] Schönbucher, P. (2003). Credit derivatives pricing models: Models, pricing and implementation. John Wiley & Sons.

[46] Seydel, R. (2006). Tools for computational finance. Springer, 3th edition.

[47] Skinner, F. & Townend, T. (2002). An empirical analysis of credit default swaps. *International Review of Financial Analysis*, 11, 297-309.

[48] Tarashev, N. (2005). An empirical evaluation of structural credit risk models. BIS Working Papers, No 179.

[49] Vasicek, O. (1977). An equilibrium characterisation of the term structure. *Journal of Financial Economics*, 5, 177-188.

[50] Yu, F. (2005). How profitable is capital structure arbitrage? Working paper, University of California.

[51] Zhang, B., Zhou, H. & Zhu, H. (2005). Explaining credit default swap spreads with equity volatility and jump risks of individual firms. BIS Working Papers, No 181.

[52] Zhou, C. (1997). A jump-diffusion approach to modeling credit risk and valuing defaultable securities. Working paper, Federal Reserve Board.

[53] Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking & Finance*, 25, 2015-2040.

Appendix A

Appendix to chapter 3

A.1 Credit spread in the Merton model

To derive the credit spread in the Merton (1974) model, equation 3.9, we first need an expression for the bond yield. The yield y of a zero-coupon bond is implicitly given by

$$B_t = De^{-y(T-t)},$$

such that we can calculate the yield as

$$\begin{aligned} e^{-y(T-t)} &= \frac{B_{t,T}}{D} \\ -y(T-t) &= \ln \frac{B_{t,T}}{D} \\ y &= -\frac{\ln \frac{B_{t,T}}{D}}{T-t}. \end{aligned}$$

Substituting equation 3.5, $B_t = V_t N(-d_1) + De^{-r(T-t)}N(d_2)$, into the expression for y gives equation 3.8:

$$y = -\frac{\ln\left[(V_t/D)N(-d_1) + e^{-r(T-t)}N(d_2)\right]}{T-t}.$$

The relation between the CDS spread and bond yield, $c = y - r$, could now be used to obtain equation 3.9 for the credit spread c in the Merton model:

$$\begin{aligned} c &= y - r \\ &= -\frac{\ln\left[(V_t/D)N(-d_1) + e^{-r(T-t)}N(d_2)\right]}{T-t} - r \\ &= -\frac{\ln\left[(V_t/D)N(-d_1) + e^{-r(T-t)}N(d_2)\right]}{T-t} - \frac{\ln e^{r(T-t)}}{T-t} \\ &= -\frac{\ln\left[(V_t/D)N(-d_1) + e^{-r(T-t)}N(d_2)\right] + \ln e^{r(T-t)}}{T-t} \\ &= -\frac{\ln\left[(V_t/D)N(-d_1)e^{r(T-t)} + N(d_2)\right]}{T-t}. \end{aligned}$$

A.2 Merton model algorithm

The following Monte Carlo algorithm evaluates CDS spreads and default probabilities with the Merton model[1]:

Step 1. Determine the time points t_p as a fraction of T on which periodic protection payments are made. Assume that the time between successive protection payments is constant, such that the payment times can be expressed recursively as $t_p = t_{p-1} + \Delta t$, for $p = 1, 2, \ldots, z$. Where z is the number of protection payments made and $t_0 = 0$.

Step 2. Since in the Merton model default can only occur at maturity, all protection payments are made and there is no accrual payment. Calculate the sum of the discounted protection payments as:

$$DPP = \sum_{p=1}^{z} \Delta t e^{-rt_p}.$$

Step 2. Perform Monte Carlo simulations for $j = 1, 2, \ldots, M$, and perform the following procedures for each j:

(a) Draw a random number ϵ from its distribution $\Phi(0, 1)$.

(b) Calculate the assets value at maturity as:

$$V_T = V_0 \exp\left(r - \frac{\sigma_V^2}{2}T + \sigma_V \epsilon \sqrt{T}\right).$$

(c) Determine whether a default occurs:

- If $V_T \leq D$, default occurs at maturity and calculate the default payment as

$$DDP_j = (R - 1)e^{-rT}.$$

- If $V_T > D$, no default occurred.

Step 5. Calculate output:

(a) The CDS spread is

$$c_{0,T} = \frac{\frac{1}{M}\sum_{j=1}^{M} DDP_j}{DPP}.$$

(b) The probability of default is

$$PD = \frac{\sum_{j=1}^{M} \mathbf{1}_{\{V_T \leq D\}}}{M}.$$

[1] Chapter 4 provides more details on Monte Carlo simulations. It is recommended to first study that chapter before reading this algorithm.

Appendix B

Appendix to chapter 4

B.1 Deriving equation 4.3

To obtain equation 4.3 from the jump-diffusion process in equation 4.1 we use Itô's Lemma for semi-martingales that applies to this type of assets value process:

$$dX = \left(\frac{\partial X}{\partial t} + \mu V \frac{\partial X}{\partial V} + \frac{1}{2}\sigma_V^2 \frac{\partial^2 X}{\partial V^2}\right) dt + \sigma_V \frac{\partial X}{\partial V} dW_t + \left[X(V+\Pi, t) - X(V,t)\right] dY_t,$$

where $X(V,t)$ is a function of V_t and t.
Applying $X(V,t) = \ln(V_t)$ yields

$$dX_t = (r - \sigma_V^2/2 - \lambda\nu)dt + \sigma_V dW_t + \ln(\Pi) dY_t. \tag{B.1}$$

Under the assumption that dW_t and dY_t are serially independent we can use the result of Hull (2006). This yields that when r, σ_V, λ, and ν are constants, the change in the diffusion component of X_t is normally distributed:

$$X_{t_i} - X_{t_{i-1}} \sim \Phi\left((r - \sigma^2/2 - \lambda\nu)\frac{T}{n}, \sigma^2\frac{T}{n}\right). \tag{B.2}$$

Assuming that only one jump can occur per time period i, we can extend (B.2) with the jump component to obtain equation 4.3:

$$X_{t_i} - X_{t_{i-1}} = x_i + y_i \cdot \pi_i,$$

where x_i, y_i, and π_i are mutually and serially independent random variables with distributions

$$x_i \sim \Phi\left((r - \sigma^2/2 - \lambda\nu)\tfrac{T}{n}, \sigma^2\tfrac{T}{n}\right),$$

$$\pi_i \sim \Phi(\mu_\pi, \sigma_\pi^2),$$

$$y_i = \begin{cases} 0 & \text{with probability } 1 - \lambda \cdot \frac{T}{n} \\ 1 & \text{with probability } \lambda \cdot \frac{T}{n} \end{cases}$$

B.2 Uniform sampling

Equation 4.5 specifies the probability P^+ that a geometric Brownian motion will always be above the barrier K on the interval $[\tau_{i-1}, \tau_i]$ conditional on the log assets value at the boundaries of this interval. The conditional probability of a crossing on this interval given the boundary values for the log assets value is $P^* = 1 - P^+$.
The probability that the crossing time s lies in the interval $[\tau_{i-1}, \tau_i]$ is given by:

$$P^* = P(\tau_{i-1} \leq s \leq \tau_i) = F_s(\tau_i) - F_s(\tau_{i-1}),$$

where $F_s(s)$ is the cumulative distribution function (cdf) of s.
Take $F_s(s)$ as the cdf of $s \sim \text{UNIFORM}(\tau_{i-1}, l)$, then

$$\begin{aligned} P(\tau_{i-1} \leq s \leq \tau_i) &= F_s(\tau_i) - F_s(\tau_{i-1}) \\ &= \frac{\tau_i - \tau_{i-1}}{l - \tau_{i-1}} - \frac{\tau_{i-1} - \tau_{i-1}}{l - \tau_{i-1}} \\ &= \frac{\tau_i - \tau_{i-1}}{l - \tau_{i-1}} \end{aligned}$$

Now set $l = \tau_{i-1} + \frac{\tau_i - \tau_{i-1}}{P^*}$, such that

$$P(\tau_{i-1} \leq s \leq \tau_i) = P^*.$$

So, drawing a time s from $\text{UNIFORM}(\tau_{i-1}, l)$ results in a default time $\tau^* = s$ if $s \in [\tau_{i-1}, \tau_i]$. When s does not lie in this interval, no default occurred.

B.3 Monte Carlo settings Model 1

B.3.1 Model 1: Approach 1A

To determine the number of time steps n in which we divide the interval $[0, T]$ in approach 1A we evaluate the 1 and 5 year CDS spreads for different values of n. Figure B.1 shows the results.

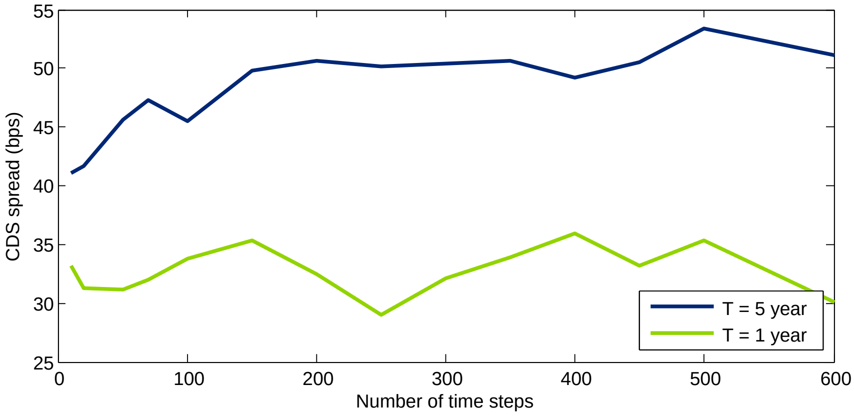


Figure B.1: 5 and 1 year CDS spread as a function of the number of time steps n in which the interval $[0, T]$ is divided in approach 1A.

Observe that convergence starts (the graphs get more stable) from $n = 200$ for a maturity of 5 years. The 1 year par CDS spread varies within 5bps from $n = 300$. We will therefore take $n = 400$ in the simulation settings of approach 1A.
Figure B.2 plots the 5 year CDS spread for increasing number of Monte Carlo runs M to determine the number of runs to be used in approach 1A.

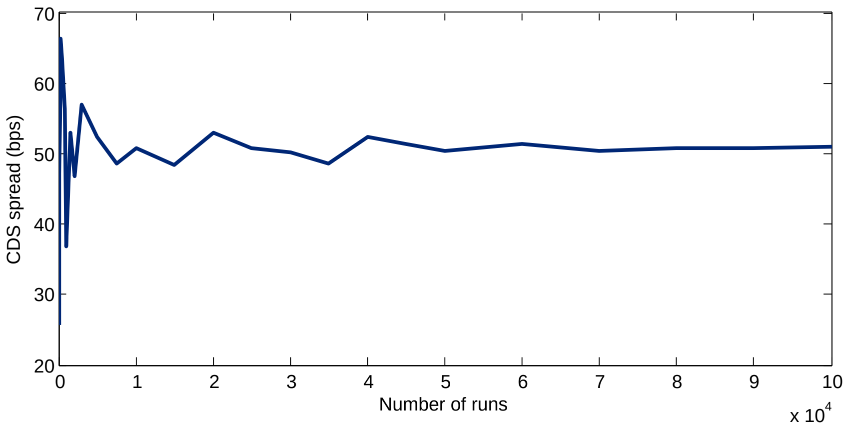


Figure B.2: 5 year CDS spread for different number of Monte Carlo runs M determined with approach 1A.

Convergence starts from $M = 10000$, but we will use $M = 50000$ to limit the range of the CDS spread to 2bps.

B.3.2 Model 1: Approach 1B

The number of Monte Carlo runs M for approach 1B is determined from the graphs in figure B.3.

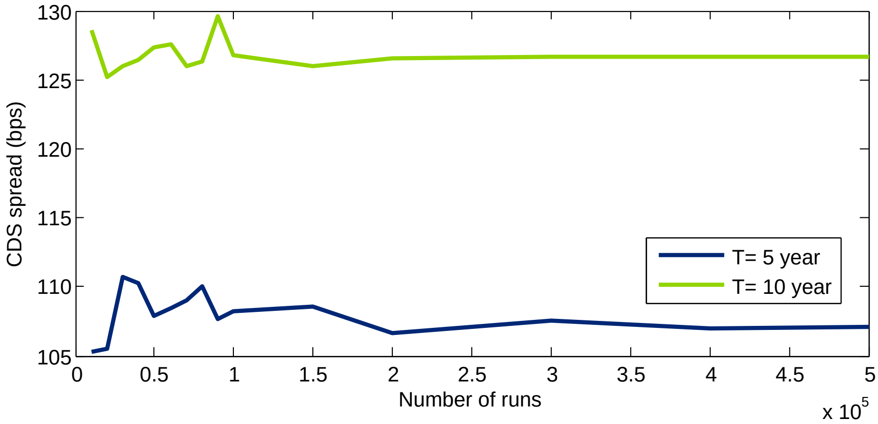


Figure B.3: CDS spread as a function of the number of runs M in the Monte Carlo simulation of approach 1B.

Observe that for both maturities the CDS spread converges to a range of 2bps for a minimum of 200000 runs. To stabilize results within 1bps we will use 1000000 simulations.

B.3.3 Model 1: Approach 1C

Figure B.4 shows the 5 and 10 year CDS spreads to determine the number of Monte Carlo runs for approach 1C.

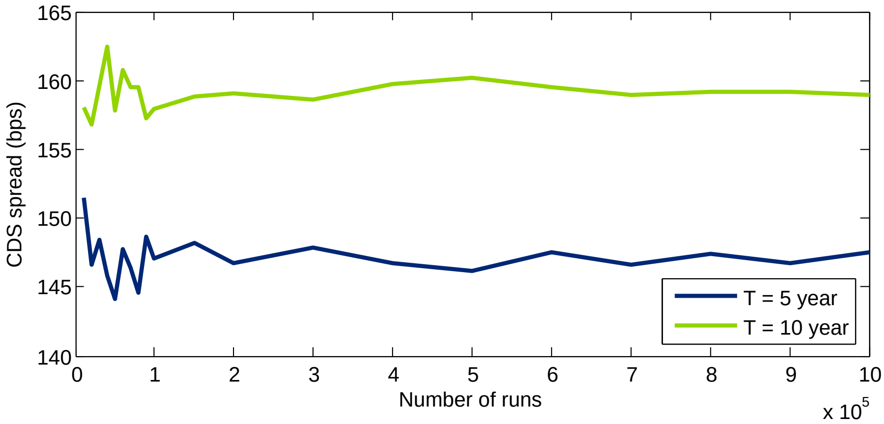


Figure B.4: CDS spread as a function of the number Monte Carlo runs M in approach 1C.

Convergence starts from 200000 runs. We will use 1000000 runs, such that the variance in the CDS spread reduces to 1bps.

B.4 Monte Carlo settings Model 2

B.4.1 Model 2: Approach 2A

To determine the Monte Carlo settings for model 2 we use the same methodology as for approach 1A. Figure B.5 shows the 1 and 5 year CDS spread calculated with approach 2A for an increasing number of time steps n in which we divide the interval $[0, T]$.

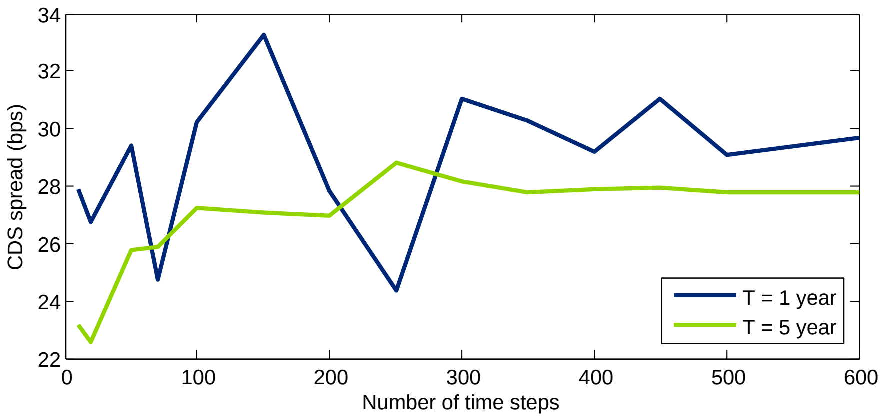


Figure B.5: 1 and 5 year CDS spread evaluated with approach 2A for an increasing number of time steps n.

Observe that convergence start from $n = 300$ for both the 1 and 5 year CDS spread. We take $n = 400$ in the simulations to limit the fluctuation to approximately 2bps.
In figure B.6 we plot the 5 and 10 year CDS spread to determine the number of Monte Carlo runs M for approach 2A.

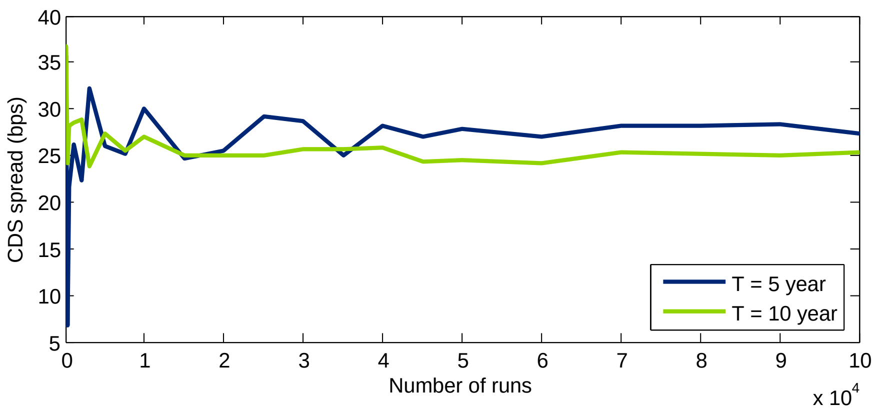


Figure B.6: 5 and 10 year CDS spread as a function of the number of Monte Carlo runs M in approach 2A.

We will use $M = 50000$, such that the CDS spreads vary within a range of 2bps.

B.4.2 Model 2: Approach 2B

Figure B.7 shows 1 and 5 year CDS spread evaluated with approach 2B for an increasing number of time steps n in which we divide the interval $[0, T]$.

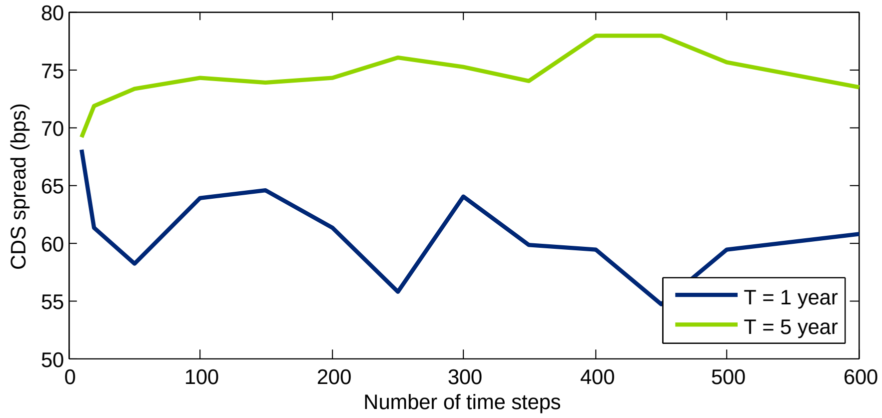


Figure B.7: 1 and 5 year CDS spreads for an increasing number of time steps n calculated with approach 2B.

Observe that for a maturity of 1 year the CDS spread stabilizes to a range of 5bps from $n = 400$. More time steps might be appropriate for this approach, but we will use $n = 400$ to save computational time.
Figure B.8 presents the 5 and 10 year CDS spread for different values of the number of Monte Carlo runs M.

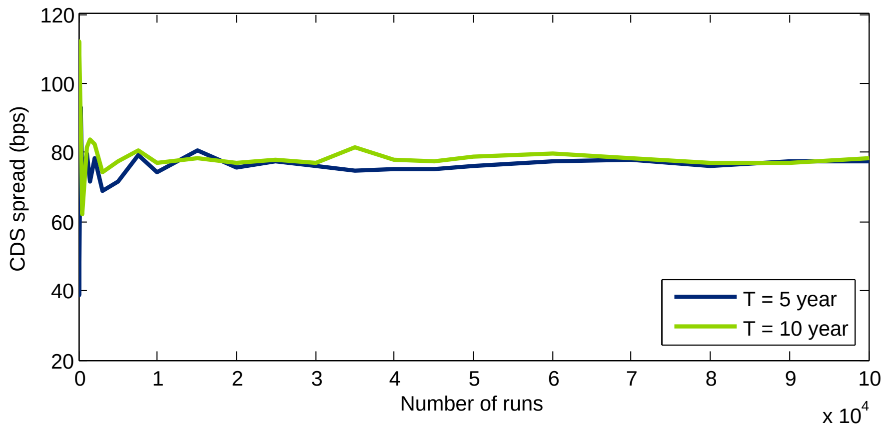


Figure B.8: 5 year CDS spread as a function of the number of runs M in the Monte Carlo simulation of approach 2B.

From $M = 50000$ the CDS spreads fluctuates within 2bps.

Appendix C

Appendix to chapter 5

The following tables present the input data estimated with the simple parameter estimation methodology developed in section 5.2.2 to model the CDS term structures of Ahold, AkzoNobel and Aegon. Note that these parameter values are estimated based on a minimum jump size of $4\sigma_V$.

Ahold input parameters

Input parameters Ahold as per 31-12-2009.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.0399	0.0480	0.0745	0.0781	0.0699	0.0651	0.0674	0.0666
MuV (with jumps)	0.0682	-0.0050	-0.0865	-0.0865	-0.0313	-0.0394	-0.0723	-0.0737
SigmaV (with jumps)	0.1584	0.1907	0.2957	0.3101	0.2774	0.2584	0.2675	0.2641
MuV (without jumps)	0.1728	0.0481	-0.0676	0.0199	0.0309	0.0241	0.0064	-0.0394
SigmaV (without jumps)	0.1398	0.1834	0.2406	0.2264	0.2048	0.1900	0.1882	0.2042
Lambda (counted)	2.0000	1.0000	1.0000	1.6667	1.5000	1.8000	1.7143	1.5000
Lambda (rating)	0.0029	0.0029	0.0041	0.0049	0.0054	0.0059	0.0065	0.0067
MuPi	-0.0537	-0.0537	-0.0195	-0.0648	-0.0420	-0.0357	-0.0467	-0.0224
SigmaPi	0.2444	0.2444	0.2444	0.1708	0.1627	0.1348	0.1455	0.1384
Initial firm value (Euro m)	14904	14904	14904	14904	14904	14904	14904	14904
Default barrier (Euro m)	3968	3968	3968	3968	3968	3968	3968	3968
Initial leverage	0.2662	0.2662	0.2662	0.2662	0.2662	0.2662	0.2662	0.2662
Average leverage	0.2855	0.2994	0.3052	0.2946	0.3217	0.3539	0.4273	0.3869
RiskFree rate	0.0099	0.0130	0.0184	0.0227	0.0256	0.0280	0.0323	0.0363
Dividend yield	0.0097	0.0194	1.8821	0.0125	0.0094	0.0075	0.0054	0.0114

Input parameters Ahold as per 2-1-2007.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.0332	0.0348	0.0380	0.0391	0.0581	0.0631	0.0606	0.0606
MuV (with jumps)	0.1521	0.1430	0.0356	-0.0049	-0.0595	-0.1608	-0.0664	-0.0664
SigmaV (with jumps)	0.1318	0.1383	0.1509	0.1552	0.2306	0.2505	0.2404	0.2404
MuV (without jumps)	0.1521	0.1430	0.0690	0.0172	0.0161	-0.1147	-0.0544	-0.0544
SigmaV (without jumps)	0.1318	0.1383	0.1433	0.1504	0.1599	0.2025	0.1987	0.1987
Lambda (counted)	0.0000	0.0000	0.5000	0.3333	1.5000	1.0000	1.0000	0.7000
Lambda (rating)	0.0029	0.0029	0.0041	0.0049	0.0054	0.0059	0.0065	0.0067
MuPi	0.0000	0.0000	-0.0677	-0.0677	-0.0513	-0.0474	-0.0118	-0.0118
SigmaPi	0.0000	0.0000	0.0000	0.1396	0.1396	0.1584	0.1429	0.1429
Initial firm value (Euro m)	19573	19573	19573	19573	19573	19573	19573	19573
Default barrier (Euro m)	6864	6864	6864	6864	6864	6864	6864	6864
Initial leverage	0.3507	0.3507	0.3507	0.3507	0.3507	0.3507	0.3507	0.3507
Average leverage	0.3726	0.3932	0.4306	0.4614	0.5101	0.4833	0.4133	0.4133
RiskFree rate	0.0382	0.0399	0.0404	0.0404	0.0404	0.0404	0.0407	0.0413
Dividend yield	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0097	0.0108

Bloomberg calculates total debt of industrials as the sum of short and long term debt, where:
Short term debt, including Includes bank overdrafts, short-term debts and borrowings, repurchase agreements (repos) and reverse repos, short-term portion of long-term borrowings, current obligations under capital (finance) leases trust receipts, bills payable, bankers acceptances, and current portion of hire purchase creditors.
Long term debt, including all interest-bearing financial obligations that are not current.
Includes convertible, redeemable, retractable debentures, bonds, loans, mortgage debts, sinking funds, long-term bank overdrafts and capital (finance) lease obligations. Excludes short-term portion of long-term debt, pension obligations, deferred tax liabilities and preferred equity. Includes subordinated capital notes.

AkzoNobel input parameters

Input parameters AkzoNobel as per 31-12-2009.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.0505	0.0628	0.0817	0.0741	0.0682	0.0627	0.0607	0.0621
MuV (with jumps)	0.5235	0.3294	-0.1341	-0.0329	0.0072	0.0409	0.0235	-0.0187
SigmaV (with jumps)	0.2003	0.2494	0.3241	0.2942	0.2708	0.2487	0.2408	0.2465
MuV (without jumps)	0.4031	0.2673	-0.0608	-0.0235	0.0503	0.0628	0.0387	0.0041
SigmaV (without jumps)	0.1813	0.2419	0.2942	0.2635	0.2360	0.2130	0.2111	0.2184
Lambda (counted)	2.0000	1.0000	2.0000	1.6667	2.2500	2.4000	2.0000	1.5000
Lambda (rating)	0.0014	0.0014	0.0018	0.0021	0.0022	0.0022	0.0022	0.0021
MuPi	0.0642	0.0642	-0.0375	-0.0059	-0.0192	-0.0090	-0.0076	-0.0116
SigmaPi	0.1055	0.1055	0.1055	0.1155	0.0937	0.0878	0.0864	0.0847
Initial firm value (Euro m)	14671	14671	14671	14671	14671	14671	14671	14671
Default barrier (Euro m)	3899	3899	3899	3899	3899	3899	3899	3899
Initial leverage	0.2658	0.2658	0.2658	0.2658	0.2658	0.2658	0.2658	0.2658
Average leverage	0.2921	0.3128	0.2721	0.2229	0.2164	0.2208	0.2423	0.2468
RiskFree rate	0.0099	0.0130	0.0184	0.0227	0.0256	0.0280	0.0323	0.0363
Dividend yield	0.0396	0.0499	0.0477	0.0388	0.0358	0.0353	0.0382	0.0359

Input parameters AkzoNobel as per 2-1-2007.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.0452	0.0462	0.0397	0.0390	0.0482	0.0526	0.0526	0.0526
MuV (with jumps)	0.1495	0.1336	0.1543	0.1031	0.0671	-0.0079	-0.0079	-0.0079
SigmaV (with jumps)	0.1795	0.1832	0.1575	0.1549	0.1914	0.2089	0.2089	0.2089
MuV (without jumps)	0.2979	0.2080	0.1914	0.1123	0.0381	-0.0018	-0.0017	-0.0017
SigmaV (without jumps)	0.1453	0.1675	0.1485	0.1466	0.1703	0.1912	0.1941	0.1941
Lambda (counted)	2.0000	1.0000	0.5000	0.6667	2.2500	1.6000	0.8571	0.6000
Lambda (rating)	0.0014	0.0014	0.0018	0.0021	0.0022	0.0022	0.0022	0.0021
MuPi	-0.0748	-0.0748	-0.0748	-0.0137	0.0133	-0.0035	-0.0048	-0.0048
SigmaPi	0.0863	0.0863	0.0863	0.0863	0.0618	0.0693	0.0751	0.0751
Initial firm value (Euro m)	16319	16319	16319	16319	16319	16319	16319	16319
Default barrier (Euro m)	2961	2961	2961	2961	2961	2961	2961	2961
Initial leverage	0.1814	0.1814	0.1814	0.1814	0.1814	0.1814	0.1814	0.1814
Average leverage	0.1929	0.1969	0.2174	0.2317	0.2596	0.2655	0.2655	0.2655
RiskFree rate	0.0382	0.0399	0.0404	0.0404	0.0404	0.0404	0.0407	0.0413
Dividend yield	0.0253	0.0266	0.0301	0.0335	0.0377	0.0365	0.0344	0.0344

Bloomberg calculates total debt of industrials as the sum of short and long term debt, where:
Short term debt, including Includes bank overdrafts, short-term debts and borrowings, repurchase agreements (repos) and reverse repos, short-term portion of long-term borrowings, current obligations under capital (finance) leases trust receipts, bills payable, bankers acceptances, and current portion of hire purchase creditors.
Long term debt, including all interest-bearing financial obligations that are not current.
Includes convertible, redeemable, retractable debentures, bonds, loans, mortgage debts, sinking funds, long-term bank overdrafts and capital (finance) lease obligations. Excludes short-term portion of long-term debt, pension obligations, deferred tax liabilities and preferred equity. Includes subordinated capital notes.

Aegon input parameters

Input parameters Aegon as per 31-12-2009.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.0693	0.1202	0.1233	0.1043	0.0964	0.0934	0.1361	0.1331
MuV (with jumps)	0.0311	0.1884	-0.2666	-0.2097	-0.1516	-0.1025	-0.1554	-0.1644
SigmaV (with jumps)	0.2750	0.4769	0.4895	0.4139	0.3827	0.3709	0.5400	0.5284
MuV (without jumps)	0.0311	0.0050	-0.2788	-0.2960	-0.2406	-0.1431	-0.1042	-0.1356
SigmaV (without jumps)	0.2750	0.4404	0.4583	0.3775	0.3316	0.3021	0.3653	0.3689
Lambda (counted)	0.0000	1.0000	1.0000	1.3333	2.2500	2.4000	2.0000	1.5000
Lambda (rating)	0.0003	0.0003	0.0008	0.0011	0.0012	0.0014	0.0017	0.0018
MuPi	0.0000	0.1864	0.0113	0.0645	0.0392	0.0166	-0.0302	-0.0181
SigmaPi	0.0000	0.2476	0.2476	0.1556	0.1311	0.1463	0.3139	0.3061
Initial firm value (Euro m)	15034	15034	15034	15034	15034	15034	15034	15034
Default barrier (Euro m)	7152	7152	7152	7152	7152	7152	7152	7152
Initial leverage	0.4757	0.4757	0.4757	0.4757	0.4757	0.4757	0.4757	0.4757
Average leverage	0.4654	0.4688	0.3846	0.2896	0.2394	0.2423	0.2244	0.1983
RiskFree rate	0.0099	0.0130	0.0184	0.0227	0.0256	0.0280	0.0323	0.0363
Dividend yield	0.0097	0.0708	0.0830	0.0691	0.0596	0.0551	0.0508	0.0476

Input parameters Aegon as per 2-1-2007.

Parameter\maturity (years)	**0.5**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
Jump of 4*sigmaV	0.06067	0.06746	0.07425	0.10762	0.15376	0.15188	0.1454	0.1454
MuV (with jumps)	0.41303	0.03142	0.06226	0.01435	0.00809	-0.12124	-0.14035	-0.14035
SigmaV (with jumps)	0.24077	0.26774	0.29469	0.42711	0.61022	0.60277	0.57704	0.57704
MuV (without jumps)	0.17988	0.03989	0.14289	0.09094	0.0979	-0.02372	-0.02084	-0.02084
SigmaV (without jumps)	0.17431	0.20366	0.16985	0.18196	0.24896	0.28002	0.28708	0.28708
Lambda (counted)	2	2	2.5	2.66667	2.25	2.2	1.71429	1.2
Lambda (rating)	0.00032	0.00032	0.00076	0.00106	0.00116	0.00143	0.00173	0.00182
MuPi	0.12007	-0.00415	-0.03226	-0.02118	-0.04028	-0.02799	-0.01309	-0.01309
SigmaPi	0.17566	0.17566	0.16813	0.25369	0.37607	0.34705	0.3349	0.3349
Initial firm value (Euro m)	28703.5	28703.5	28703.5	28703.5	28703.5	28703.5	28703.5	28703.5
Default barrier (Euro m)	5025	5025	5025	5025	5025	5025	5025	5025
Initial leverage	0.17507	0.17507	0.17507	0.17507	0.17507	0.17507	0.17507	0.17507
Average leverage	0.07812	0.11881	0.18391	0.18967	0.19155	0.19215	0.16633	0.16633
RiskFree rate	0.03816	0.03993	0.04043	0.04041	0.04041	0.04043	0.04065	0.04125
Dividend yield	0.04585	0.04118	0.03617	0.03679	0.0365	0.03788	0.1065	0.03744

Bloomberg calculates total debt of insurance firms as the sum of short and long term debt, where:
Short term debt, including bank overdrafts, short-term debts and borrowings, repurchase agreements (repos), including short-term portion of long-term borrowings and current obligations under capital (finance) leases.
Long term debt, including all interest bearing financial obligations that have maturities greater than one year. Includes convertible debentures, bonds, loans, mortgage debts, sinking funds, LT bank overdrafts, capital (finance) lease obligations, quarterly or monthly Income Preferred Securities with a fixed maturity, and redeemable preferred stock.