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MASTER THESIS

MODEL DEVELOPMENT FOR SUPPORTING RISK-BASED APPROACHES

CENSORED

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Summary

Occurrences are reported and investigated by Air Traffic Control the Netherlands (LVNL) as part of the safety management system. The purpose of this study is to develop a practical mathematical model which is able to analyse statistical characteristics of the available occurrence data. The intention is to use the results as part of more advanced risk-based approaches.

Model

The model developed uses the statistical characteristics of the data to support advanced risk-based approaches in safety assessment of LVNL. Supporting risk-based approaches is done by determining the practicability of reference values as suggested by FABEC, by identification of factors which contribute to risk, and by identifying relations between severity classes to give an indication of risk.

Risk-based approaches

Two reference values suggested by FABEC are studied on practicability, where each reference value is determined for major and serious incidents separately. The first reference value is practicable with exceedance probability $8.2 \cdot 10^{-10}$ for major incidents, and with exceedance probability $3.0 \cdot 10^{-1}$ for serious incidents. For the second reference value the exceedance probability is $9.7 \cdot 10^{-1}$ for major incidents, and the exceedance probability is $9.9 \cdot 10^{-1}$ for serious incidents.
Two case studies are performed in identifying factors which contribute to risk. For the type of occurrence 'deviation taxi' the type of carrier, the type of peak and the type of aircraft are shown to contribute to risk. The states of the factors which contribute to risk are non-home carriers, inbound peaks, and light aircraft. The combination of states of factors which contribute to risk are: {non-home carriers during inbound peaks with light aircraft}, {non-home carriers during outbound peaks with light aircraft}, {non-home carriers during off-peaks with light aircraft}, {home carriers during inbound peaks with light aircraft}, and {home carriers during inbound peaks with medium aircraft}. Where the first has an extraordinary contribution to risk.
For the severity class major the type of carrier and the type of aircraft are shown to be factors which contribute to risk. The states of the factors which contribute to risk are home carriers and light aircraft. The combination of states of factors which contribute to risk are: {non-home carriers with light aircraft} and {home carriers with light aircraft}.
Relations between severity classes have been determined. The correlation between occurrences with no safety effect and occurrences with significant safety effect is strong, and thus the number of aircraft involved with occurrences of these severity classes have a dependency. The correlation between the other severity classes is weak or negligible, and thus independence is assumed. With this information on dependency, distribution functions are made for the ratio of severity classes. When severity classes are independent, the ratio is determined by using the Poisson distribution function. When severity classes are dependent on each other, the distribution is determined empirically by determining the mean and variance of the ratio.

Sensitivity analysis

Sensitivity analysis is performed on the results of the model by using confidence intervals and by adding data. The confidence intervals are considered to be sufficiently small. With adding data the numerical results change slightly, and thus conclusions on risk-assessment change slightly.
Looking at the reference values: both have a decreased exceedance probability for each of the severity classes viewed.

For risk-contributing factors: the same factors contribute to risk, and the combination of states which contribute (extraordinary) to risk remain the same for major incidents. For deviation taxi the same factors contribute to risk, but the combination {home carriers, during outbound peaks, with light aircraft} is now determined to be risk-contributing, while {non-home carriers, during inbound peaks, with medium aircraft} is not anymore. Also, non of the combinations contribute to extraordinary risk anymore.
When looking at the correlation between the number of occurrences in the severity classes, same conclusions are drawn concerning the dependency of the severity classes.

Methods

The first method used to determine statistical characteristics of the data is the test of Kolmogorov-Smirnov to verify the presence of the Poisson distribution.
The second method used is Poisson regression, which identifies whether the influence of factors on the number of occurrences is observable. It also determines the degree of influence the factors have.
Finally, Pearson's correlation coefficient is used to determine the correlation between severity classes.

Recommendations

Further research is recommended on the identification of factors contributing to risk. New factors are suggested and described in detail in section 11. Furthermore, extra research is recommended on the relationship between severity classes and the way this relation can be used in risk-assessment. It is also suggested to seek for a grouping of the types of occurrences such that the groups are independent. Analysis can than be performed on the relation between the type of occurrences and their severity. The last recommendation is to use the model, in adjusted setting, for risk-analysis on occurrence data of other airports.

Contents

List of abbreviations

Abbreviations	
AAA	Amsterdam Advanced ATC-system
ACAS	Airborne Collision Avoidance System
ACC	Area Control Center
ANSP	Air Navigation Service Provider
APP	Approach
ATC	Air Traffic Control
CTA	Control Area
FABEC	Functional Airspace Block Europe Central
IATA	International Air Transport Association
IFR	Instrument Flight Rules
LVNL	Air Traffic Control the Netherlands (Luchtverkeersleiding Nederland)
NLR	National Aerospace Laboratory
R/T	Radio/ Telephony
RIASS	Runway Incursion Alerting System Schiphol
SID	Standard Instrument Departure
SPSS	Statistical program which is used for (part of) execution statistical methods
SSE	Safety Significance Events; scheme which is used to classify severity of occurrences
SSR	Secondary Surveillance Radar
STAR	Standard Arrival Routes
STCA	Short Term Conflict Alert System
TCAS	Traffic Collision Avoidance System
UDP	Uniform Period Daylight
VFR	Visual Flight Rules

Chapter 1

Preface

This is the report for the graduation project at Air Traffic Control the Netherlands (LVNL) as part of the Masters-program Applied Mathematics at the University of Twente.
Appendices C and D contain confidential data/ output. The content of these appendices are left out in the public version of the report.

The research questions regarding the factors which play a role in the number of occurrences emerged whilst finishing my internship-project at LVNL this spring. It was clear that a model could be made to support LVNL's risk-based approach. The research started out by focusing on the Kalman filter and its properties. Soon it became clear that another model was needed to achieve the set goals.
In the conversations with my first supervisor (Judith) about the structure of the data and the goals which were set, it became clear that the methods/ models studied in the courses of my study did not apply. Thus, we searched for models outside our courses and came to Poisson models.
Poisson models were not known widely by me, but when we first came across the term 'Poisson regression' it was clear that the method had potential for the goals which were to be achieved.
I looked deeper into the mechanism of Poisson regression and it indeed appeared to be very suitable. Also, as the Poisson distribution was required, we obviously looked deeper into the possibilities of this aspect.
Finding a distribution which was suitable for the number of occurrences for the different types led to the interest of studying the reference values. Being able not only to see whether the reference values were achieved in previous years, but also learning what the probability distribution looks like and determining the probability of exceeding the reference values.
Learning a distribution for the number of occurrences in the severity classes led to the thought of finding a (stochastic) ratio which indicates the relation between the severity classes when looking at the number of occurrences. Generally a fixed ratio is used. When the number of occurrences is stochastic, it seems logic that the ratio between these numbers is stochastic. Thus, the relation between the numbers is investigated on both dependency and value/ distribution.

Performing the study was a challenge in several ways. First because a steady mathematical model has to be built, whilst not loosing touch of the practical use. Second because the model requested did not exist yet, and the methods used were not yet known by me. There were many challenges on the way, but discovering new methods and combining them to a new model is what I liked best. A model is now at hand which used proper mathematical models, and is directly usable for LVNL in their risk assessment. It feels like an honor that I had the chance to create a model and than learn others at LVNL to use it.

I thank Judith for her support as first supervisor. Even when we did not agree on whether a method was suitable or not, the discussions were good and even pleasant. You have been supportive all the way through. You listen patiently and have a positive way of giving advise and feedback. When I was looking at new methods you always put effort in looking into them as well. That made the discussions well-funded and effective, and also enjoyable.
Further I thank my Roy and Adriaan, as supervisors at LVNL, for all the help they gave during my projects. You have been a great support with constructive feedback, the space to have a good discussion, asking the right questions at the right time, giving space for new ideas, and most of all: investing time to understand the mathematics and all the jumps I have made in creating a model. Thank you for the great working environment!

Also, I would like to thank Richard for starting as second reader and becoming first reader at the end of this project. The clear instructions on how to finish the project helped in completing it without too much delay.
I would also like to thank Klaas for the support he gave during both my internship and my graduation project with my questions regarding statistics. I appreciate your patience in answering questions about both SPSS and the theory in statistics.
Last but not least, I thank all my colleagues at LVNL for a great time. Thank you for the nice conversations, input, feedback, and everything else. You make going to work nice!

Chapter 2

Background, introduction problem, goals

2.1 Background

Air Traffic Control the Netherlands (LVNL) is responsible for providing air traffic services within the flight information region Amsterdam, and provides communication-, navigation- and positioning-services. Other responsibilities lie in the provision of aeronautical information services and publishing aeronautical publications and maps. Also providing training for air traffic safety is an important responsibility, as well as advising both the Minister of Infrastructure and Environment, and the Minister of Defense on matters in the field of civil air traffic management. The responsibility also lies in carrying out other duties assigned by the aviation law [1]. A part of this responsibility thus lies in directing all aircraft at and around the mainport Schiphol.
Mainport Schiphol handles on average 1400 flights a day. For all arriving, departing and transiting aircraft is a schedule, these possible activities are called movements. Deviating events which take place are reported and analyzed, these events are called *occurrences*. An event is an occurrence if it fulfills one or more of the following descriptions [2]:

1. Loss of separation between an aircraft and one or more other aircraft and/ or ground-vehicles.
2. An aircraft or ground-vehicle which deviates from an ATC instruction or procedure.
3. An aircraft or ground-vehicle which follows wrong instructions given by an ATC or to which no instruction is given.
4. Inability or decreased ability to supply ATC services and/ or failure of technical functions.

Every occurrence which lies in the responsibility of LVNL is reported and undergoes an analysis, in which five steps are taken to classify the severity of the occurrence. The occurrences are added to the database, which is used to perform research; e.g., based on the data, research is performed to check whether there is a need to adjust the ATC system (e.g., procedures, equipment, training, etc.).

2.2 Introduction to problem

The current occurrence database is filled with occurrence data starting on January 1st 2010 and is a rich source of information. LVNL wants to learn as much as possible from this database, the question is how to learn from it without losing grip on the complete picture.
The occurrences taken into account in this study are those which took place at ACC, APP and at Schiphol airport. The reason for this is that the responsibility for these airports lie (completely) at LVNL, at other airports in the Netherlands more parties have influence (for example hobby-flights which arrive and depart). Moreover, the influence factors considered in this study are aimed at the infrastructure and procedures of mainport Schiphol.
In [11] a basis was formed to start the statistical analysis on occurrence data. For several intersections of the data it identified whether the normal (Gaussian) distribution behind the number of occurrences is applicable. Also, it searched for a methodological basis to analyse influence of factors, such as: season, number of runway-combination changes, usage of the fourth runway, number of movements on small fields, the standard working protocol, and awareness meetings.

This study focuses on factors which are specified per flight; e.g. type of carrier, type of peak in which the flight took place, and type of aircraft used. This new focus requires a new model, where the aim has been set on Poisson models. The benefits of Poisson models is that they are developed to analyze the statistics concerning events which do not happen with a high average.
The goal of this study is to support risk-based approaches. This includes investigating the practicability of reference values, identifying factors which contribute to risk, searching for relations between types of occurrences and their severity, and finding relations between the number of occurrences in the severity classes.

Research questions stated by LVNL are stated in section 2.3.

2.3 Goals

The main goal of this study is:

Model development and analysis for occurrence data in air traffic management to support advanced risk-based approaches

The following research questions are posed:

- How can LVNL's risk-based approach be supported by the available data?
    1. How can statistical characteristics be used to test reference values on practicability?
    2. How can circumstances which contribute to risk be identified?
        - Which circumstances influence the number of occurrences of a certain type.
        - Which circumstances influence the number of occurrences of a certain severity.
    3. How do the number of occurrences in the severity classes relate to each other?
- Which statistical characteristics can be found concerning the occurrence data?
    1. The distribution for the number of occurrences which take place per month; for all types of occurrences and for all severity classes.
    2. The relation between the type of occurrences and their severity classifications.
    3. The relation between the type/ severity of occurrences and the circumstances concerning occurrences.
    4. The (statistical) relation between the number of occurrences in severity classes.
- How sensitive are the conclusions drawn from the model?

Chapter 3

Literature review

This study analyzes occurrence data in air traffic management where support of a risk-based approach is one of the major issues. The literature investigated can be divided in three categories, namely: relevant research in adjacent fields, methods for investigating statistical characteristic of the data, and difficulties in (supporting) risk-based approaches.

Adjacent fields

There are several adjacent fields to look for occurrence data analysis, some fields which are considered are in road traffic accidents, ship accidents, and other studies for air traffic accidents.

In 2009 LVNL and the NLR published a paper [3] which presents safety criteria concerning ATC-related risk, this article relates to this study in the sense that it searches for accident probabilities, which is of interest to this study since statistical characteristics of the data are sought. The article finds a way to express the "accident probability related to separation between aircraft and other aircraft, their wake vortices and vehicles". However, the model of the study is not used in this study since the search is for a model which shows statistic characteristics and influences for occurrences in more detail. Note: an occurrence is not always an accident.

The same year the NLR published the report called the "Causal model for Air Transport Safety", where a causal model is built to get a "thorough understanding of the causal factors underlying the risks of air transport and their relation to the different possible consequences so that efforts to improve safety can be made as effective as possible"[4].This study is of interest since it looks at factors which have a possible influence on the number of occurrences, as is required in this study. However, a model is sought for finding statistical characteristics of the data, the model proposed is a causal model and thus lies outside the scope of this study.

Time-series analysis of road risk is performed in [5] where it discusses several time-series analysis models. These types of models are of interest due to the fact that they provide insight on the development of accidents through time and the underlying reasons of how they originate. However, these models are used "as a tool to describe, explain and predict changes in the trends of the road safety level", where this study focuses on the statistical characteristics of the data rather then on the predictions for the future. Some of the (time-series) methods have been studied to apply nontheless, but eventually turned out to be inappropriate for the goals of this study.

The usage of Poisson regression is introduced for ship accidents in [6], where the regression searches for the influence of ship-conditions with accidents. This study searches for a model which identifies the influences of factors for occurrences, the type of conditions are similar when looking at them in an abstract way; the studies are thus closely related. Also in [7] and [8] Poisson regression is used to analyze highway and traffic safety with factors of a similar type as the ones in this study. In [9] a link between the financial health of airlines and their safety is analyzed with Poisson regression, here also factors like type of aircraft are taken into account, which is of great interest in this study. The Poisson regression model is appropriate for this study since it gives the desired statistical characteristics on the data, namely: the influence which factors have on the number of occurrences.

Methods for investigating statistical characteristics

Several statistical characteristics are of interest, the subjects of interest are:

- (Tests for) the distribution of the number of occurrences.
- Factors which influence the number of occurrences (of certain types).
- Relation between the type and the severity of occurrences.

In [11] the normal distribution has been investigated for the number of occurrences in the separate severity classes and seasons. This distribution was not applicable for every case, and thus the search for fitting distributions is ongoing. Beside the interest for the distributions for the number of occurrences in the severity classes, there is a growing interest for the distribution on the number of occurrences for the different types of occurrences. Since occurrences can be divided over dozens of types, the distribution which is searched for should be able to handle countable events which do not appear very often. A distribution that comes to mind is the Poisson distribution, as it is a distribution which counts the number of occurrences (with low averages) in a certain time-frame.
Determining whether the occurrences take place according to a Poisson distribution can be done by the statistical test called the Kolmogorov-Smirnov test [10]; this test compares the empirically found distribution to the assumed distribution.

Once distributions are found, regression analysis can be used when looking at factors which influence the number of occurrences. This has also been done in [11]. The focus is on factors which influence occurrences of specific types or severity, a different type of regression than in [11] is necessary to get the desired results. Poisson regression is in place when looking for influence factors for the type and severity of occurrences when the average number of occurrences is low. In contrast to many other types of regression, Poisson regression can handle integer valued output variables without problems (in this case the number of occurrences). In Poisson regression the Poisson distribution is assumed for the number of events, which is verified with the Kolmogorov-Smirnov test.[1]

Once distributions and influence factors are found, the focus shifts to the severity classes. Each occurrence is of a certain type (sometimes multiple types) and a severity class. Knowing the distributions and the influence factors leads to investigation of the division of occurrences over the severity classes. Poisson processes and their characteristics (as described in [12]) are investigated to determine which type of occurrences leads to the different severity classes and with which rate. This way not only the distribution of the severity classes are found, but also the way in which they are built up from the different type of occurrences. The difficulty lies in the requirement of the Poisson processes to be independent for each other to use the desired properties, which is not the case in the organization of the database as shaped by LVNL; an occurrence can be several types at once, where certain combinations are strongly correlated.

Difficulties in (supporting) risk-based approaches

One of the goals of this study is to support an advanced risk-based approach. A swift look has been given to risk analysis in other fields, but this did not give an appropriate method. Many risk-based approaches are aimed at the (possible) damage incurred by occurrences; this can be expressed as either money, lives, etc.. This is not applicable for the situation of this study, since in most cases there is no measurable amount of damage; this is done in [13], also [14] discusses multiple models. This makes it hard to quantify risk, and is thus the reason why statistical characteristics on occurrence data is studied and not the quantification of risk.

[1] Both the Kolmogorov-Smirnov test and Poisson regression are performed in SPSS.

Chapter 4

Available data and its structure

This section gives a description of the structure of the data which is used as input for this study, starting by the types of occurrences and the dependencies therein. Followed by a description of the severity classification which takes place for each occurrence.

4.1 Types of occurrences

A part of the occurrence database is the classification of the 'types' of occurrences. The types focus on the description of the sort of occurrence; also known as 'what-classification'. It is important to realize that one occurrence may exist of multiple types (often two or three), as more than one description is needed to describe the entire occurrence. Due to this characteristic of the occurrences (and thus of the data), the conclusion is drawn that there is a dependency between the possible types, this is discussed and shown in appendix C.2.
Also, some new types of occurrences have been added and some old types of occurrences have been split up through time. This requires some fitting of the data to obtain meaningful results and is discussed below.

Dependencies in types of occurrences

Many mathematical models require independence of variables. Models immediately lose their usability when this fundamental requirement is not fulfilled. In this study a relationship between the type of occurrence and the severity class of an occurrences is sought, the problem with this is that the types of occurrences are sometimes closely related. Many models are viewed to quantify the relation but all assume independence of the variables (and thus independence of the different types of occurrences).
The dependency of the different types of occurrences lies in a few things. First of all, an occurrence (especially a severe one) is often an assembly of events and thus multiple things can be appointed to the occurrence in regard to the things which did not follow procedure. For example: aircraft a accidentally 'enters the runway without clearance', which causes a 'missed approach' for landing aircraft b which thus makes a 'go around' to try landing again later. During this go around an 'airborne-separation' occurs between aircraft b and another approaching aircraft c. This is all denoted as one occurrence, this example shows that an occurrence can be a gathering of events.
Also, some types of occurrences are strongly related since one often cannot appear as an occurrence without the other. For example: a serious incident in the air always includes multiple aircraft which are too close to each other ('airborne separation'), in principle a cause for the lack of separation can be indicated; which is thus filed as another type.
Another dependency lies in the connection with the severity classes, when an 'airspace infringement' is involved in a serious incident, then there has also been an 'airborne separation'.

As demonstrated by the previous examples: there is a strong connection between the different types of occurrences. An overview is made which indicates how often each set of occurrence types have been counted in the data from 01-01-2010 until 31-12-2012, this overview is shown in appendix C.2. Note here: when more than two types are selected in one occurrence then all combinations of those types count as one. E.g., an occurrence includes a 'Runway incursion', a 'Go around' and an 'Airborne separation', then this counts as one for the combination 'Runway incursion' and 'Go around', it counts as one for the combination 'Runway incursion' and 'Airborne separation', and it counts as one for the combination 'Go around' and 'Airborne separation'.

When analysis is desired for the relationship between the types of occurrences and the severity classes it is important that the variables taken into account are independent of each other. One way to accomplish this is by dividing the types into groups which are independent of each other. To illustrate how the type of occurrences can be grouped, three suggestions are given in appendix C.2.
The first suggestion is selecting occurrences on the location of the occurrence, in other words: in which part of the organization the occurrence took place. The parts of the operation are subdivided in ACC, APP, ground-control, or on the runway.

The second suggestion is subdividing the types over cause and effect: is the type of occurrence a cause of the occurrence, or is it the effect of something else.
The third suggestion is subdividing types over flight-phase. A flight at Schiphol airport has several phases, a departing flight has the phases start-up and push-back, line-up, taxiing, take-off, departure, and CTA outbound. An arriving flight has the flight phases CTA inbound, initial and intermediate approach, final approach, landing, and taxiing. A transit flight only has flight-phase CTA transit.

The difficulty in grouping the types is that it is not a one-on-one transformation, some types belong in multiple classes of the groups. Moreover, expert judgment is used to subdivide the different types over the classes, which is thus not defined exactly but somewhat based on a personal view. Where most types have one clear class, some are in a gray area between the classes and thus belong in multiple classes.

A definite conclusion cannot be drawn on how to group the different types of occurrences in such a way that all classes of occurrences are independent of each other. Further research is needed to find a way to cope with the dependencies.

Fitting data - combining types of occurrences

A number of what-categories has been added or changed during the period in which data was gathered, some are new categories and others are old categories which have been split. Categories which have been split are combined to their original category, new categories are gathered in the old category 'other' since these used to be registered there. Also a few categories with very few occurrences and a clear overall category are combined to the overall category (such as Emergency - Mayday, Emergency - Medical, Emergency - Panpan, Emergency - VOS, Emergency / Other; these are combined to Emergency). All combined categories are stated in appendix C.1.

4.2 Severity classification

Occurrences are reported at LVNL, which can be nearly anything: an aircraft which enters the taxiway without clearance, but also an emergency landing. Every reported occurrence undergoes an analysis performed by the department Performance in co-operation with specialized air traffic controllers. The analysis exists of five steps [15]:

1. Gather information
2. Analyze information
3. Categorize according to 'Safety Significant Events'-scheme (SSE)
4. Formulate conclusions
5. Formulate recommendations

Step 2 and 3 of the occurrence analysis are of particular interest in the data analysis. The second step indicates to which type the occurrence belongs; e.g. an aircraft has deviated from the planned route, an emergency call has been made, etc. The third step classifies occurrences on severity based on guidelines, these guidelines take into account the number of aircraft/ vehicles involved, whether or not a loss of separation has occurred, and who detected and/ or solved the problem.
When one aircraft is involved and there are no other aircraft or vehicles close-by, then it is categorized as a 'unilateral occurrence'. Whether or not the separation norm (minimal distance between aircraft/ vehicles) is crossed is checked when other aircraft/ vehicles are nearby. The degree of crossing the separation norm is checked when necessary.

The combination of the above and the one who detects/ solves the occurrence determines the severity.

The severity classes are designed as follows (in increasing order of severity) [15]:

- No safety effect
  - Unilateral occurrence, or
  - Effective ATC solution and no loss of separation
- Significant incident
  - Effective ATC solution and a significant loss of separation, or
  - ATC safety barriers worked and a limited loss of separation, or
  - Less effective ATC solution or airmen solved the event and no loss of separation
- Major
  - ATC safety barrier worked and a major loss of separation, or
  - Less effective ATC solution and a significant loss of separation, or
  - ATC safety barrier did not work and a limited loss of separation, or
  - No ATC safety barriers worked and no loss of separation
- Serious
  - ATC safety barriers did not work and a significant loss of separation

The following definitions are used [15]:

- Major loss of separation: $\leq 50\%$ of needed separation
- Significant loss of separation: $\leq 66\%$ of needed separation
- Limited loss of separation: $> 66\%$ of needed separation

The second and third step of the analysis are important for this study since a link between the severity classes and their distribution is stated as research goal.

Chapter 5

Assumptions

This study has some assumptions regarding the data and the used models. This section discusses the assumptions and illustrates why they are plausible.
The first assumption is that the data used for this research can be considered as reliable. The performance department plays a central role in securing the reliability. Also, all experts are trained and a handbook is available with clear definitions on how to report and analyze occurrences. Also the system supports a consistent working method.
The second assumption regarding the data is that it is uniformly defined: the working methods of LVNL has not changed drastically since the start of the occurrence database on January first 2010. Some small changes have been made in secondary definitions during the four years of filling the database: some types have been split up in multiple types to gain an even more precise insight. These changes can be corrected by changing the new types back to the old types, this is described in section 4.1.
The third assumption is that all occurrences registered are taken into account. This consists of all occurrences reported by LVNL personnel, and reports of others to LVNL such as airlines and Schiphol airport. LVNL has the policy of registering all occurrences, no matter how insignificant it may seem. However, a guarantee cannot be given that some occurrences slip through the system. Also, all occurrences studied are related to the operations of LVNL and are ATC-related
The fourth assumption is that occurrences are independent of each other. When multiple aircraft are part of an occurrence, they are registered as the same occurrence.
Though the occurrences are assumed to be independent of each other, the different types of occurrences are not. This aspect is studied in section 4.1 and arises from the fact that multiple things can deviate from procedures during one occurrence.
Furthermore, LVNL started the database in its current form in January 2010, thus the data can be used from that moment on. The primary data used in this study therefore stems from 01-01-2010 till 31-12-2012, which is exactly three years of data. The data from 01-01-2013 till 31-12-2013 is used for sensitivity analysis on the output of the model.
Finally, the occurrences taken into account are those which occurred at and around Schiphol airport, which includes: Schiphol airport and the airspace which is under control of LVNL.
The occurrences at Rotterdam airport are not taken into account, since it differs from Schiphol airport. Adding occurrences on Rotterdam airport would influence the output of the models incorrectly. E.g., Rotterdam airport has no peak-times due to the fact that there is one runway, also home-carriers are defined as those airlines which have Schiphol airport as home base. The model can be used for occurrences at Rotterdam airport, but a separate analysis is needed.
Another reason for choosing occurrences at and around Schiphol airport is because all movements are known; for some other airports this is not registered precisely. Furthermore, all movements at and around Schiphol airport fall under responsibility of LVNL.

Summarized:

1. The dataset is reliable.
2. The dataset is uniformly defined; new types of occurrences can be changed back to the original types.
3. All occurrences registered by LVNL are taken into account.
4. Occurrences are independent of each other.
5. Initial data used stems from 01-01-2010 till 31-12-2012, data from 2013 is used for the sensitivity analysis.
6. Occurrences taken into account are those which took place at or around Schiphol airport.

Chapter 6

Model

The model is developed to support LVNL in it's risk-based approaches. It is thus made to perform risk-assessment in several ways. The first section (6.1) describes why and how three subjects of risk-assessment are supported.
The subjects in risk-assessment are: finding circumstances which contribute to risk, verifying the practicability of reference values, and finding a relation between the number of occurrences in the different severity classes.
Occurrences and their properties for supporting these subjects in risk-assessment can be described by three elements: the factors describing the circumstances of occurrences, the types of the occurrences, and the severity of the occurrences. Properties of these elements are described (mathematically) in section 6.2.
The relations between the elements describing occurrences are stated in section 6.3. These relations are used to support risk-assessment.
The analysis of the model for risk-assessment is described in section 7.

6.1 Description of the model

The first part of the model for risk-assessment is developed to determine the relation between occurrences and their circumstances. The circumstances are described by factors, where the states of the factors indicate the specific circumstance of that type which is present; e.g., the factor 'type of carrier' has the states 'home carriers' and 'non-home carriers'. These factors are not only determined for the occurrences, but also for each movement which takes place.
The (degree of) influence of the factors is determined with Poisson regression. Poisson regression first establishes if there is a significant influence for each factor, next it determines the degree of influence; Poisson regression and its use is described in appendix A.3. The influence which circumstances have on the number of occurrences is used to find circumstances which contribute to risk. Circumstances in which more than average occurrences take place are viewed as risk-contributing; this analysis is described in more detail in section 7.2.

The second part of the model in risk-assessment concerns practicability of reference values. Governments are developing legislation on the number of occurrences which is acceptable, thus reference values are being developed. This legislation is intended to safeguard air-traffic safety. Thus, there is a need to determine whether these reference values are practicable, which can be done by using a statistical model. The probability that these reference values are (not) exceeded can be determined by using the statistical distribution for the number of occurrences; this detailed analysis for this is described in section 7.1.
Thus, a distribution is sought for the number of occurrences which take place. The Poisson distribution is used as it is known to work well when events occur with a low average. The number of occurrences is counted for each severity and for each type of occurrence, the average number of occurrences is low for the types and severity classes. The presence of the Poisson distribution is verified by using the statistical test of Kolmogorov-Smirnov; which is described in appendix A.2.

The third part in risk-assessment lies in finding a relation between the number of occurrences in the different severity classes. It is presumed that the number of occurrences in the separate severity classes has a strong dependency. In general a fixed number is assumed to describe how the number of occurrences in two severity classes differ. The value of this number is determined by looking at the distribution of the number of occurrences in the severity classes. Thus, the ratio is determined from the number of occurrences in the severity classes. This ratio is assumed to be a stochastic variable and thus the distribution function is sought; this analysis is described in detail in section 7.3.

An effort is made to find a relation between the type of occurrence and its severity. This relation can identify risk and thus where an adjustment in the operational procedure of LVNL should be focused on. When for example a specific type of occurrence leads mostly to a high severity, then one could consider extra effort in preventing this type of occurrence. Details and difficulties of this relation is discussed in section 6.3.

6.2 Main elements of the model

To support risk-based approaches it is necessary to first describe occurrences and their properties. The properties of occurrences are defined in three (main) elements in the model. These elements are described in this section: factors describing the circumstances of movements/ occurrences, types of occurrences, and severity of occurrences.

Factors

Factors (f) describe the circumstances of the movements, these factors have a potential influence on the number of occurrences which take place. The influence can be on a specific type, as well as a specific severity of occurrences. A specific factor f^k consists of several states c, these states describe the alternative options of the factor. The factors are thus categorical. Factor k in state c is denoted by f^k_c, thus:

Definition 6.2.1 (Factors)**.** Factors describe the circumstances of movements and have a potential influence on the number of occurrences.

Definition 6.2.2 (States of factors)**.** Factor k exists of the union of states c, where factor k in state c is denoted by f^k_c:

$$f^k = \bigcup_c f^k_c$$

Examples of factors are the 'type of carrier', 'type of peak', and 'type of aircraft'. The states of these factors are respectively {home carriers, non-home carriers}, {inbound peak, outbound peak, off-peak}, and {light aircraft, medium aircraft, heavy aircraft}.

When looking at movements and occurrences, several factors are needed to describe all circumstances. The entire set of factors to describe the circumstances of movements is given by F^T, which is the union of all available factors f^k. So:

Definition 6.2.3 (Set of factors)**.** The set of factors describing the circumstances of movements is given by F^T and is the union of all factors f^k:

$$F^T = \bigcup_k f^k$$

When making an analysis not all factors are taken into account, only those which are of interest. The set of factors taken into account is denoted by F, which is a subset of F^T:

Definition 6.2.4 (Set of factors for analysis)**.** The set of factors used when making an analysis is given by F and is a subset of all factors F^T:

$$F \subset F^T$$

When a set of selected factors is taken into account and the states of the factors f^k are c, then the combination of states of the factors is denoted by F_C:

Definition 6.2.5 (Combination of states of factors)**.** The combination of factors f^k in specific states c is given by F_C (the combination of the states is given by C), which is the union of f^k_c:

$$F_C = \bigcup_k f^k_c$$

Types of occurrences

As stated in section 2.1: an occurrence is defined as a deviating event. The type of an occurrence indicates which deviation took place. Since multiple deviations can take place at the same time, multiple types can be allocated to one occurrence. The set of types is given by $\mathbb{I}$, and type i is an element of $\mathbb{I}$ ($i \in \mathbb{I}$).
Multiple aircraft can be involved in an occurrence (often two), and thus the number of aircraft involved with occurrences is counted rather than the number of occurrences. Counting the number of aircraft involved with occurrences can either be done *per movement* (denoted by T_i^*), or *per month* (denoted by T_i). Where the first is called the *occurrence rate for type i occurrences*, and the latter has a Poisson distribution with rate λ_i.

Definition 6.2.6 (Occurrence rate for type i occurrences)**.** The number of aircraft involved with occurrences per movement for occurrences of type i is given by T_i^* and is called the **occurrence rate for type i occurrences**.

Assumption 6.2.7 (Distribution for type i occurrences)**.** The number of aircraft involved with occurrences per month of type i is given by T_i and has a Poisson distribution with parameter λ_i:

$$T_i \quad \sim \quad Poisson(\lambda_i)$$

When used, assumption 6.2.7 is verified with the Kolmogorov-Smirnov test; which is described in appendix A.2.

Severity of occurrences

The severity of an occurrence is allocated according to the scheme given in section 4.2, the set of severity classes is given by $\mathbb{J} = \{$no safety effect, significant safety effect, major incident, serious incident$\}$; in contrast with the type of occurrences, only one severity can be allocated to an occurrence. The number of aircraft involved with occurrences per movement of severity j is given by S_j^*; also called the *occurrence rate for occurrences of severity j*.

Definition 6.2.8 (Occurrence rate for occurrences of severity j)**.** The number of aircraft involved with occurrences per movement for occurrences of severity j is given by S_j^* and is called the **occurrence rate for occurrences of severity j**.

The number of aircraft involved with occurrences is also determined per month for each severity class, given by S_j. The number of occurrences per month have a Poisson distribution for each severity class with Poisson parameter λ_j.

Assumption 6.2.9 (Distribution for occurrences of severity j)**.** The number of aircraft involved with occurrences per month of severity j is given by S_j and has a Poisson distribution with parameter λ_j:

$$S_j \quad \sim \quad Poisson(\lambda_j)$$

As before, assumption 6.2.9 is verified with the Kolmogorov-Smirnov test whenever it is used.

6.3 Relations between and in the main elements of the model

To support risk-based approaches it is necessary to determine the relation between the elements of the model. Several relations are established: relations between the factors and the type/ severity of occurrences, relations between the type and the severity of occurrences, and relations between the number of occurrences in severity classes. Each of the relations is described below.

Relation between factors and types / severity of occurrences

A relation is found between factors as described in section 6.2 and the number of (aircraft involved with) occurrences. A relation can be given with respect to the type and with respect to the severity of occurrences.

For the types of occurrences the relation is described as the occurrence rate of type i for the combination of states of factors F_C. This relation is denoted by $T_i^*(F_C)$, where $i \in \mathbb{I}$.
Similarly: for the severity of occurrences the relation is described as the occurrence rate of severity j for the combination of states of factors F_C. This relation is denoted by $S_j^*(F_C)$, where $j \in \mathbb{J}$.

Definition 6.3.1 (Relation between factors and the types of occurrences)**.** The relation between the number of aircraft involved with occurrences per movement of type i and a set of factors in given states F_C is given by $T_i^*(F_C)$.

Definition 6.3.2 (Relation between factors and the severity of occurrences)**.** The relation between the number of aircraft involved with occurrences per movement of severity j and a set of factors in given states F_C is given by $S_j^*(F_C)$.

The relation between the factors and the occurrence rates for the different types and severities is used for risk identification. For example: an occurrence rate which is significantly higher than average for certain circumstances can be used as an identification for circumstances which contribute to risk.
Statements on determination of risk-contributing factors, risk-contributing states of factors, and risk-contributing combinations of states of factors are described in detail in section 7.2.

Relation between types and the severity of occurrences

A relationship exists between the number of aircraft involved with occurrences per month for the types and severity. When the relation between a certain type of occurrence and a severity class is strong one could draw conclusions on the contribution to risk of the type.
For example: when a certain type leads mostly to severe incidents, a strong contribution to risk could be assumed and thus preventing this type of occurrences could get higher priority. The other way round: when a certain type never leads to a high severity, it probably does not need highest priority on preventing it.

The relationship between types of occurrences and their severity is given by $S_j = g(T_i)$. This relationship is not one-to-one for several reasons. First because each type of occurrence can lead to each severity. The second reason is because an occurrence can be of several types at once, but only of one severity. The difficulty of this is explained below.

Definition 6.3.3 (Relation between types and the severity of occurrences)**.** The (not one-to-one) relation between types and the severity of occurrences is given by $S_j = g(T_i)$.

Having an occurrence which is of several types counts as one occurrence for each of these types. Since it is one occurrence, only one occurrence is counted for the corresponding severity class. Simply adding the occurrences from the separate types of occurrences to a corresponding severity class then leads to a higher count of occurrences than actually took place; e.g., an occurrence of types 'airborne separation' and 'airspace infringement' of severity 'major' counts as one for each type and one for the severity class 'major'. Thus, a correction factor is necessary to get the accurate number of occurrences for the severity class.

In determining the relation, one can use the Poisson distribution which is known for the number of aircraft involved with occurrences for each type of occurrence. For each type of occurrence a distribution can be found (empirically) on which part of the occurrences of type i leads to which severity class. Recall, a correction factor has to be included in the relation.

A few aspects should be taken into account when determining the correction factor. The most important are: the frequency in which combinations of types take place in one occurrence, and the (average) number of types allocated to an occurrence of a certain severity.
The first aspect is based on the fact that certain combinations of types take place in the same occurrence often, whereas some combinations never take place in the same occurrence. For example: the types 'airspace infringement' and 'airborne separation' often take place at the same time, whereas 'deviation taxi' and 'airborne separation' never take place in the same occurrence; the latter cannot occur simultaneously as the first is on the ground and the second is a loss of separation between multiple aircraft which are airborne. An indication of the combination of types which take place simultaneously is given in appendix C.2.
The second aspect is based on the knowledge that severe occurrences often have more types attached than occurrences with no safety effect. Thus, the correction factor should not only be based on the types of occurrences, but also on the types of occurrences combined with the severity of the occurrences.

When a correction factor is determined and a direct expression is found for the relation between the type and severity of occurrences, one can compare the determined value of S_j with the corresponding pre-determined Poisson distribution and its parameter (λ_j).
A suitable correction has not been found due to the complexity of the relation.

Relation between the number of aircraft involved with occurrences in severity classes

A relation is expected when looking at the number of (aircraft involved with) occurrences in the severity classes. Generally a fixed number is used to determine the expected number of occurrences for a specific severity class based on the (expected) number of occurrences of a less-severe class of occurrences.

So: assume severity class i is of lower severity than severity class j, with resp. S_i and S_j occurrences. The fixed factor between the number of occurrences of the severity classes is R, thus $S_i = R \cdot S_j$.
The factor can thus be seen as the ratio between the number of (aircraft involved with) occurrences in two severity classes, denoted by $R = \frac{S_i}{S_j}$ where $i \neq j$.

The number of (aircraft involved with) occurrences in the severity classes has a Poisson distribution. This can be used to find a distribution for the ratio R when the number of occurrences in the severity classes are independent of each other. The distribution can be determined empirically when independence cannot be assumed. The distribution functions for the ratio's are given in section 7.3.

Definition 6.3.4 (Relation between the number of aircraft involved with occurrences in severity classes)**.** The ratio for number of aircraft involved with occurrences in severity class i and severity class j ($i \neq j$) is given by $R = \frac{S_i}{S_j}$.

Chapter 7

Analysis

This section describes how the model is analyzed to support the three subjects in risk-assessment as mentioned in section 6.
It first shows how the Poisson distributions are used to determine the probability of (not) exceeding the given reference values in section 7.1. Recall: reference values are developed by governments to safeguard air-traffic safety, the analysed reference values are indicative.
Next, section 7.2 describes how the results of Poisson regression are used to identify if and how factors/ circumstances contribute to risk. Statements are made on how risk is defined.
Last, section 7.3 describes how the relation between the number of occurrences in the severity classes is used in risk-assessment. It describes how a distribution function can be found for the ratio as discussed in section 6.3.

7.1 Practicability of reference values

Testing the practicability of reference values starts by determining the Poisson distribution for all severity classes of occurrences, this is done with the test of Kolmogorov-Smirnov. The Poisson distribution is determined for the number of flights involved with occurrences per month for each severity class:

$$S_j \sim Poisson(\lambda_j)$$

Where:

$$S_j := \text{Number aircraft involved with occurrences of severity } j \text{ per month, } j \in \mathbb{J}$$

Where $\mathbb{J}$ denotes the set of severity classes.

Reference values are given by $FABEC$ and are defined with different units for measuring traffic volume, the unit is thus translated to the number of aircraft involved with occurrences per year:

r := Given reference value, unit as traffic volume v, $r \in \mathbb{R}$, where $\mathbb{R}$ denotes the set of reference values.
$\hat{r}$:= Reference value r transformed to unit 'number of aircraft involved with occurrences per year'.

The Poisson distribution is determined for the number of aircraft involved with occurrences per month. It is transformed to the Poisson distribution with a parameter for the number of aircraft involved with occurrences per year. This is done by summing the Poisson parameter of twelve months, which can be done since the months are assumed to be independent and all have Poisson distribution with equal mean. So:

$$\hat{S}_j \sim Poisson(12 \cdot \lambda_j)$$
$$\hat{S}_j := \text{Number aircraft involved with occurrences of severity } j \text{ per year}$$

Next the practicability of the reference value is verified by observing the exceedance probability, which is done by:

$$\begin{aligned} P(\hat{S}_j > \hat{r}) &= 1 - P(\hat{S}_j \leq \hat{r}) \\ &= 1 - \sum_{k=0}^{\lfloor \hat{r} \rfloor} \frac{(12\lambda_j)^k}{k!} \cdot e^{-12\lambda_j} \end{aligned}$$

Another way of looking at the reference value is by determining which parameter the Poisson distribution should have in order to have an exceedance probability of at most α. So for $X \sim Poisson(\hat{\lambda}_j)$

and given α:

$$\begin{aligned}\alpha &= P(X > \hat{r}) \\ &= 1 - \sum_{k=0}^{\lfloor \hat{r} \rfloor} \frac{(12\hat{\lambda}_j)^k}{k!} \cdot e^{-12\hat{\lambda}_j}\end{aligned}$$

And thus:

$$1 - \alpha = \sum_{k=0}^{\lfloor \hat{r} \rfloor} \frac{(12\hat{\lambda}_j)^k}{k!} \cdot e^{-12\hat{\lambda}_j}$$

Note: α is given and $\hat{\lambda}_j$ is to be determined

Statements for the determination of the practicability of a given reference value are:

Proposition 7.1.1 (Practicability of reference values (1))**.** A reference value r is practicable with probability $1 - \alpha$ if the exceedance probability is equal to α.

Proposition 7.1.2 (Practicability of reference values (2))**.** A reference value $\hat{r}$ is practicable with (given) probability $1 - \alpha$ if the number of flights involved with occurrences is on average $\hat{X}$

$$P(\hat{X} > \hat{r}) = \alpha$$

7.2 Determination of risk-contributing factors

The model determines whether factors and their states contribute to risk by studying the occurrence rate for the different circumstances. A situation which contributes to risk is seen as those (combinations of) circumstances which lead to more than average occurrences of a certain type or severity. Thus, the occurrence rates are determined for each combination of (states of) factors and are compared with each other.

It is first determined whether factors contribute to risk. Next it is determined which states of the factors contribute most to risk, in other words: which state of the factor has the highest occurrence rate when the states of the other factors are kept equal. Following, the occurrence rates for the combinations of states of factors are determined to identify whether combinations lead to a higher than average occurrence rate, and thus which combinations contribute to risk. Last, it is determined which combinations of states of factors lead to an extraordinary contribution of risk, which is seen as combinations with an occurrence rate which is above average and which have more than average movements.

Starting by the contribution of risk of factors: a factor f^k is said to contribute to risk when the occurrence rate for the states c of the factor differ significant.
Poisson regression determines whether the factors have a significant influence on the number of occurrences. When it determines a significant influence it indicates that the states of the factors lead to a significant difference in the number of occurrences. So: when Poisson regression determines a significant influence for a factor, it is directly stated that the factor contributes to risk.

Definition 7.2.1 (Risk-contributing factors)**.** A factor f^k contributes to risk if the occurrence rate for the states c of the factor differ significant.
Factor f^k contributes to risk for occurrences of type i when:

$$T_i^*(f_c^k) \neq T_i^*(f_{c^*}^k)\ \exists c \neq c^*$$

Factor f^k contributes to risk for occurrences of severity j when:

$$S_j^*(f_c^k) \neq S_j^*(f_{c^*}^k)\ \exists c \neq c^*$$

Where $T_i^*(f_c^k)$ denotes the occurrence rate for occurrences of type i with state c of factor f^k, and $S_j^*(f_c^k)$ denotes the occurrence rate for occurrences of severity j with state c of factor f^k.

Next it is determined which states of the factors contribute to risk, given that the factor is risk-contributing. The occurrence rate is determined for each state of the factor, keeping the states of other factors equal. The state of the factor with the highest occurrence rate is defined as the *strongest risk-contributing state of the factor*.

Definition 7.2.2 (Strongest risk-contributing state of a factor)**.** A state $\bar{c}$ of risk-contributing factor f^k contributes strongest to risk for occurrences when factor f^k contributes to risk, and state $\bar{c}$ leads to a higher occurrence rate than other states c of factor f^k.
State $\bar{c}$ of risk-contributing factor f^k contributes strongest to risk for occurrences of type i when:

$$\begin{aligned} T_i^*(f_c^k) &\neq T_i^*(f_{c^*}^k)\ \exists c \neq c^* \\ and & \\ T_i^*(f_{\bar{c}}^k) &> T_i^*(f_c^k)\ \forall c \neq \bar{c} \end{aligned}$$

State $\bar{c}$ of risk-contributing factor f^k contributes strongest to risk for occurrences of severity j when:

$$\begin{aligned} S_j^*(f_c^k) &\neq S_j^*(f_{c^*}^k)\ \exists c \neq c^* \\ and & \\ S_j^*(f_{\bar{c}}^k) &> S_j^*(f_c^k)\ \forall c \neq \bar{c} \end{aligned}$$

Next, the combinations of states of factors are viewed on the contribution they have on risk. This is done with those factors which are already determined to contribute to risk. For each combination of states of factors the occurrence rate is determined. When the occurrence rate for a combination of states is a factor β above the average occurrence rate there is said to be a contribution to risk. So:

Definition 7.2.3 (Risk-contributing combinations of states of factors)**.** A combination of states C from the set of risk-contributing factors F is said to be risk-contributing when the occurrence rate of the combination is a factor β higher than the average occurrence rate.
The combination of states C from the set of risk-contributing factors F for occurrences of type i are contributing to risk when:

$$T_i^*(F_C) > \bar{T}_i^* \cdot \beta$$

The combination of states C from the set of risk-contributing factors F for occurrences of severity j are contributing to risk when:

$$S_j^*(F_C) > \bar{S}_j^* \cdot \beta$$

Where $\bar{T}_i^*$ denotes the average occurrence rate for occurrences of type i, and $\bar{S}_j^*$ denotes the average occurrence rate for occurrences of severity j.

For the case studies performed in section 8.2 the value of β is chosen to be 1, as it gives the desired illustration and avoids a value judgment on where the boundary should lie.

Last, the combinations of states which contribute to risk are reviewed to identify if they have an extraordinary contribution to risk. An extraordinary contribution to risk is defined as those circumstances which have an occurrence rate which is a factor β above the average occurrence rate, and also the number of movements which take place under these circumstances is a factor γ above the average number of movements.

Definition 7.2.4 (Extraordinary risk-contributing combinations of states of factors)**.** A combination of states C from the set of risk-contributing factors F is said to be extraordinary risk-contributing when the occurrence rate of the combination is a factor β higher than the average occurrence rate and the combination has a number of movements which is γ above the average number of movements for an arbitrary combination of states of factors.
The combination of states C from the set of risk-contributing factors F for occurrences of type i contributes extraordinary to risk when:

$$\begin{aligned} T_i^*(F_C) &> \bar{T}_i^* \cdot \beta \\ M_i^*(F_C) &> \bar{M}_i^* \cdot \gamma \end{aligned}$$

The combination of states C from the set of risk-contributing factors F for occurrences of severity j contributes extraordinary to risk when:

$$\begin{aligned} S_j^*(F_C) &> \bar{S}_j^* \cdot \beta \\ M_j^*(F_C) &> \bar{M}_j^* \cdot \gamma \end{aligned}$$

Where $M_i^*(F_C)$ and $M_j^*(F_C)$ denote the number of movements which took place for the given combination of states of the factors for resp. types i and severity j occurrences.
Furthermore, $\bar{M}_i^*$and $\bar{M}_j^*$ denote the average number of movements for an arbitrary combination of states of factors corresponding to resp. type i and severity j occurrences.

Again, for the case studies performed in section 8.2 the values of β and γ are chosen to be 1, as it gives the desired illustration and avoids a value judgment on where the boundary should lie.

7.3 Relation severity classes

To support risk-based approaches, this section analyses the relation between the number of occurrences in severity classes. Generally a fixed number is used to determine the number of occurrences of a certain severity based on the number of occurrences of a lower severity.

A probability function is sought for the number which indicates how the number of occurrences in separate severity classes are related each other. This number is given as the ratio of the number of occurrences in two severity classes. In order to find a distribution for this ratio it is necessary to determine whether the number of aircraft involved with occurrences in severity classes have a dependency. Thus, the correlation between the severity classes is determined with Pearson's correlation coefficient.

The strength of correlation between the number of occurrences in severity classes is determined by the value of the correlation-coefficient ρ:

Definition 7.3.1 (Strength of correlation)**.** The strength of the correlation is based on the correlation-coefficient ρ and is determined as follows:

[0.0 - 0.2] Negligible correlation

[0.2 - 0.4] Weak correlation

[0.4 - 0.6] Reasonable correlation

[0.6 - 0.8] Strong correlation

[0.8 - 1.0] Very strong correlation

When ρ is negative, there is a negative correlation.

The strength of a correlation between variables indicates (statistical) (in)dependency. Weak or negligible correlation suggests independence, where stronger correlations suggest dependence.

Assumption 7.3.2 (Dependency based on correlation coefficients)**.** Variables are assumed (statistically) independent when the correlation is negligible or weak.

The distribution for the desired ratio is determined analytically by using the Poisson distribution function when independence is assumed, and the number of aircraft involved with occurrences per month has a Poisson distribution.

Proposition 7.3.3 (Distribution for the ratio of two Poisson variables)**.** Given $X \sim Poisson(\lambda_x)$ and $Y \sim Poisson(\lambda_y)$, where X and Y are independent. The probability distribution for the ratio $R = \frac{X}{Y}$ is:

$$P\Big\{\frac{X}{Y} \leq t\Big\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda_y}\lambda_y^n}{n!} e^{-\lambda_x} \sum_{k=0}^{\lfloor tn \rfloor} \frac{\lambda_x^k}{k!}$$

Proof:

$$\begin{aligned}
P\Big\{\frac{X}{Y} \leq t\Big\} &= P\{X \leq Yt\} \\
&= P\{X \leq Yt | Y = n\} \cdot P\{Y = n\} \\
&= P\{X \leq tn\} \cdot P\{Y = n\} \text{ (use independence)} \\
&= \sum_{n=0}^{\infty} \sum_{k=0}^{\lfloor tn \rfloor} \frac{e^{-\lambda_x}\lambda_x^k}{k!} \frac{e^{-\lambda_y}\lambda_y^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{e^{-\lambda_y}\lambda_y^n}{n!} e^{-\lambda_x} \sum_{k=0}^{\lfloor tn \rfloor} \frac{\lambda_x^k}{k!}
\end{aligned}$$

□

An analytic derivation of the distribution function for the ratio of two dependent Poisson distributed variables by using the Poisson distribution function is not possible. Thus, the distribution for the ratio is determined empirically with its mean and variance.
This is done by determining the ratio of the number of aircraft involved with occurrences between severity classes for each month, followed by determination of the average value of the ratio and its variance. Note: as division by zero is impossible, the empirical distribution cannot be determined if one or more months of the severity class which is used as denominator has zero occurrences. Thus:

Definition 7.3.4 (Empirical distribution for the ratio of dependent variables)**.** The empirical distribution of the ratio $R = \frac{X}{Y}$ of two variables X and Y is determined by its mean and variance:

$$E\Big[\frac{X}{Y}\Big] = \frac{1}{n}\sum_{i=0}^{n} \frac{x_i}{y_i}$$

$$Var\Big(\frac{X}{Y}\Big) = \frac{1}{n}\sum_{i=0}^{n} \Big(\frac{x_i}{y_i}\Big)^2 - \frac{1}{n^2}\Big(\sum_{i=0}^{n} \frac{x_i}{y_i}\Big)^2$$

Where $y_i \neq 0$.

Chapter 8

Numerical results

The numerical analysis of the model exists of the numerical results from the mathematical methods used. This starts by using the test of Kolmogorov-Smirnov to determine if the Poisson distribution is applicable, this is performed in section 8.1. Next two case studies are presented in which Poisson regression is carried out to determine estimators for the average number of aircraft involved with occurrences for the combinations of states of the factors. This is first done for the type of occurrence called 'deviation taxi' in section 8.2, which is chosen because LVNL has particular interest in this type. Next section 8.2 looks at major incidents, which is chosen because it is an important severity class for risk-analysis at LVNL. Last section 8.3 determines the correlation coefficients for the severity classes, which helps to determine the distribution for the earlier described ratio for the number of occurrences in the severity classes.

8.1 Performing statistical tests

This section gives the distribution found for each type of occurrence and for each severity class for the number of aircraft involved with occurrences by using the test of Kolmogorov-Smirnov. The tested Poisson parameter is the sample average for each case. The Poisson distribution is assumed when the significance value α is larger than the confidence level α_0, the confidence level used is 5%; thus reject the Poisson distribution when $\alpha < 0.05$.

Poisson distributions for the types of occurrences

For performing the Kolmogorov-Smirnov test the data is collected per month, as shown in table B.1 in appendix B.1. In table D.1 the results are shown for the type of occurrences $i \in \mathbb{I}$ (column 1), where the second column gives the Poisson parameter and the third column gives the significance value. The Poisson distribution is assumed to be correct if the significance value is greater than or equal to α.

The test-results show that all types of occurrences i have a Poisson distribution, except for 'Airspace infringement', 'Apron incursion incident including pushback incident', 'Deviation Startup pushback', and 'Deviation Vehicle - airport traffic'. These categories are studied more closely.

Looking at the latter three types, it is known that there were only few reports before February 2011, and that there are more occurrences reported after. This can be explained by the fact that an awareness meeting has been held to emphasize the importance of reporting occurrences, the increase on the number of occurrences is visible from this moment on. Looking for the distribution of these types of occurrences after the awareness meeting, the test indicates that a Poisson distribution is applicable. The results are displayed in table D.2.

Looking at 'Airspace Infringements', a clear seasonal pattern is visible in the number of occurrences per month. In [11] it is already shown that the number of occurrences heavily depends on the IATA seasons, which is defined as the dates which have 'summertime' and those which have 'wintertime'. This is explained by the fact that uncontrolled traffic from the smaller fields which are close to Schiphol airport and are not under the responsibility of LVNL cause airspace infringements. The amount of this type of traffic is considerably larger during the summer months. Thus, the number of occurrences are regarded during the summer- and winter-months are tested separately. The split seasons have a Poisson distribution, with their corresponding sample mean as Poisson parameter. The results are displayed in table D.2.

Poisson distributions for the severity classes

It is known from [11] that the occurrences in several severity classes are normally distributed. This study is interested in the Poisson distribution. Thus, the Kolmogorov-Smirnov test is performed for both the Poisson and the normal distribution. Since they resemble each other when the means increases, the results of the tests are compared. The Poisson distribution is more likely to be applicable when the average number of aircraft involved with occurrences is low and the mean is equal to the variance. The normal distribution is more likely to be applicable when the average number of aircraft involved with occurrences is high and there is overdispersion; which means that the mean and variance are not equal.

Both the Poisson and the normal distribution are tested. The results of the tests are shown in table 8.1, now only the significance level (α) is shown for sake of clarity; again when the significance level is equal to or greater than $\alpha_0 (= 0,05)$ the tested distribution is accepted. The results show that the severity class 'no safety effect' has a normal distribution (with mean 40) and not a Poisson distribution. An explanation for this is that the average number of occurrences per month is relatively high and there is overdispersion.
The severity class 'serious incident' has a Poisson distribution (with mean 0.47) and not a normal distribution, this can be explained by the fact that the average number of occurrences per month is low. The severity classes 'significant safety effect' and 'major incident' (with resp. mean 11 and 3) can be assumed to have both a normal and a Poisson distribution. This result is explained by the fact that the Poisson and normal distribution resemble each other as the mean number of occurrences per month increases, and there is no overdispersion.

Type of severity	**Significance value - Poisson distribution (α)**	**Significance value - Normal distribution (α)**
No safety effect	0.002	0.516
Significant safety effect	0.352	0.845
Major incident	0.709	0.251
Serious incident	0.875	0.000

Table 8.1: Output Kolmogorov-Smirnov - Severity classes

8.2 Performing Poisson regression: case studies

This section illustrates two case studies regarding the determination of estimators for the average number of aircraft involved with occurrences per movement for combinations of states of factors by Poisson regression. Poisson regression is first performed for type of occurrence 'deviation taxi', next it is performed for severity class 'major'. The results on the estimated rates are shown here, the use of these results for identifying factors which contribute to risk is shown in section 9.2.
The progress of obtaining results of Poisson regression in SPSS is stated in appendix B.2. For sake of clarity only the final results of Poisson regression are shown, detailed results can be found in appendix mentioned.

Case study - Factors influencing types of occurrences: Deviation taxi

This case study describes the results of Poisson regression for the type of occurrence called 'deviation taxi'; this type is defined as 'deviation of the instructed or according to the regulations applicable procedure for taxiing'.

Three independent variables are taken into account: the type of carrier (home carriers or non-home carriers), the type of peak in which the occurrence/ flight took place (inbound-, outbound-, or off-peak), and type of aircraft (light, medium, or heavy); taxi-movements are made by all type of carriers, in all kinds of peaks, and by all type of aircraft.

In determining the estimators for the parameters, a selection of the states of the factors is chosen as a 'reference setting'. In this case: non-home carriers ($X_{0,1}$) during the off-peak ($X_{0,2}$) with light aircraft ($X_{0,3}$). The estimators for these parameters are set zero. The remaining variables are defined as:

- Type of carrier: $X_1 = 1$ in case of home carriers, else $X_1 = 0$
- Type of peak: $X_2 = 1$ in case of inbound peak, else $X_2 = 0$
- Type of peak: $X_3 = 1$ in case of outbound peak, else $X_3 = 0$
- Type of aircraft: $X_4 = 1$ in case of heavy aircraft, else $X_1 = 0$
- Type of aircraft: $X_5 = 1$ in case of medium aircraft, else $X_1 = 0$

The choice of these factors results in 18 combinations with in total 163 occurrences for Deviation Taxi. To get a correct view on the effects of (the combinations of) factors, the occurrences corresponding to the chosen circumstances are counted. The number of occurrences and the number of movements is given for each combinations of states of factors in table 8.2.

The initial model equation of Poisson regression is:

$$\begin{aligned} ln\Big(\frac{\mu_i}{V_i}\Big) &= \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2 + \beta_3 \cdot X_3 + \beta_4 \cdot X_4 + \beta_5 \cdot X_5 \\ &+\gamma_{1,2} \cdot X_1 \bullet X_2 + \gamma_{1,3} \cdot X_1 \bullet X_3 + \gamma_{1,4} \cdot X_1 \bullet X_4 + \gamma_{1,5} \cdot X_1 \bullet X_5 + \gamma_{2,3} \cdot X_2 \bullet X_3 \\ &+\gamma_{2,4} \cdot X_2 \bullet X_4 + \gamma_{2,5} \cdot X_2 \bullet X_5 + \gamma_{3,4} \cdot X_3 \bullet X_4 + \gamma_{3,5} \cdot X_3 \bullet X_5 + \gamma_{4,5} \cdot X_4 \bullet X_5 \end{aligned}$$

The test performed than has the null-hypothesis $\beta_i = 0$ and $\gamma_{j,l} = 0$, and the alternative hypothesis $\beta_i \neq 0$ and $\gamma_{j,l} \neq 0$, so:

$$H_0 : \beta_i = 0, \gamma_{j,l} = 0, \forall i, j, l; \qquad H_1 : \beta_i \neq 0, \gamma_{j,l} \neq 0, \exists i, j, l$$

Now for each factor it is first determined if the effect they have on the number of aircraft is observable, which is indicated by the significance value. The significance value is determined by Wald's test, as described in appendix A.3; again significance level $\alpha_0 = 0,05$ is assumed.

Following the progress as stated in appendix B.2: the null-hypothesis is not rejected for those β and γ which have a significance value which is bigger than α_0. Thus, these have value zero and are deleted from the model equation one-by-one; their influence is not observable.

Deleting the non-significant variables one-by one leads to the following results:
First: the significance value is 0.000 for the factors 'type of carrier' and the 'type of aircraft', it is 0.036 for 'type of peak'. This means all factors have an observable influence on the number of occurrences of type 'deviation taxi' and thus remain in the model equation.
Second: looking at the interaction-terms, only the interaction between home carriers with heavy aircraft is significant and thus remains in the model equation.
The model equation becomes:

$$ln\Big(\frac{\mu_i}{V_i}\Big) = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2 + \beta_3 \cdot X_3 + \beta_4 \cdot X_4 + \beta_5 \cdot X_5 + \gamma_{1,3} \cdot X_1 \bullet X_3$$

The estimators of β_i and $\gamma_{j,l}$ are determined as:

$$\beta_\mathbf{0} = -7.953$$

$$\beta_\mathbf{1} = -1.323$$

$$\beta_\mathbf{2} = 0.923$$

$$\beta_\mathbf{3} = 0.565$$

$$\beta_\mathbf{4} = -1.984$$

$$\beta_\mathbf{5} = -1.537$$

$$\gamma_\mathbf{1,3} = 1.018$$

Where β_0 represents the estimator for the reference setting (non-home carriers during the off-peak with light aircraft), β_1 represents the estimator for home carriers, β_2 and β_3 the estimators for resp. the inbound- and outbound-peak, β_4 and β_5 for resp. heavy and medium aircraft. Further, $\gamma_{1,3}$ represents the estimator for the interaction between home carriers with heavy aircraft (during any type of peak).

These estimators of the independent variables indicate the degree of influence the variables have on the number of occurrences. The estimators for the interaction terms indicate the degree of influence variables have when multiple variables are present simultaneously.
Interaction terms can be interpreted as the effect which a variable has, depending on the value of another variable. So: the interaction between variable X_k and X_l indicates the effect which variable X_k has on the value of the dependent variable given the value of variable X_l. For the case of 'deviation taxi': the effect which home carriers have on the number of occurrences differs for different 'values' of the outbound peak (and vice versa).

The estimated and empirically determined occurrence rates for all combinations of states of factors are given in table 8.2. The empirical and the estimated occurrence rates are close to each other, which gives confidence in the correctness of the estimation.
Further usage of the results for supporting risk-based approaches is given in section 9.2.

Type carrier	**Type peak**	**Type WTC**	**Number of occurrences**	**Number of movements**	**Empirical occurrence rate**	**Estimated occurrence rate**
home carriers	inbound peak	light	0	1	0.00E0	2.32E-4
home carriers	inbound peak	medium	16	297 773	5.37E-5	5.73E-5
home carriers	inbound peak	heavy	5	45 448	1.10E-4	1.24E-4
home carriers	outbound peak	light	0	1	0.00E0	1.67E-4
home carriers	outbound peak	medium	8	251 104	3.19E-5	4.12E-5
home carriers	outbound peak	heavy	3	49 250	6.09E-5	8.90E-5
home carriers	off-peak	light	0	0	0.00E0	1.16E-4
home carriers	off-peak	medium	1	50 198	1.99E-5	2.88E-5
home carriers	off-peak	heavy	0	23 337	0.00E0	6.21E-5
non-home carriers	inbound peak	light	17	16 832	1.01E-3	9.42E-2
non-home carriers	inbound peak	medium	49	217 574	2.25E-4	2.33E-4
non-home carriers	inbound peak	heavy	8	51 958	1.54E-4	1.81E-4
non-home carriers	outbound peak	light	10	14 073	7.11E-4	6.78E-4
non-home carriers	outbound peak	medium	34	194 482	1.75E-4	1.67E-4
non-home carriers	outbound peak	heavy	4	37 887	1.06E-4	1.31E-4
non-home carriers	off-peak	light	0	3 400	0.00E0	4.73E-4
non-home carriers	off-peak	medium	8	33 731	2.37E-4	1.17E-4
non-home carriers	off-peak	heavy	0	16 315	0.00E0	9.11E-5

Table 8.2: Deviation Taxi

Case study - Factors influencing severity class major

This case study describes Poisson regression in detail for severity class major, the method is performed similar as for 'Deviation taxi'. The data is represented in table 8.3.

Type carrier	Type peak	Type WTC	Number of occurrences	Number of movements
home carriers	inbound peak	light	0	2
home carriers	inbound peak	medium	40	297 811
home carriers	inbound peak	heavy	14	45 457
home carriers	outbound peak	light	0	2
home carriers	outbound peak	medium	33	251 142
home carriers	outbound peak	heavy	13	49 259
home carriers	off-peak	light	0	1
home carriers	off-peak	medium	8	50 217
home carriers	off-peak	heavy	0	23 341
non-home carriers	inbound peak	light	8	18 148
non-home carriers	inbound peak	medium	22	276 205
non-home carriers	inbound peak	heavy	8	59 162
non-home carriers	outbound peak	light	8	15 389
non-home carriers	outbound peak	medium	23	253 113
non-home carriers	outbound peak	heavy	6	45 091
non-home carriers	off-peak	light	0	4 058
non-home carriers	off-peak	medium2	3	63 047
non-home carriers	off-peak	heavy	1	19 917

Table 8.3: Data for major incidents

Again the observability of the factors is first determined by examining the significance value, if it is smaller than or equal to 0.05 it is observable. The significance level of 'type of carrier' is 0.001 and that of 'type of aircraft' is 0.000, which indicates that their influence is observable. The significance level of 'type of peak' is 0.104 and thus the influence is not observable. All interactions are not significant and thus left out of the model.
Again a selection of parameters is chosen as a 'reference setting': non-home carriers with light aircraft. The estimators for these parameters are set zero. The type of peaks are left out since their influence is not observable.
The remaining independent variables are:

- Type of carrier: $X_1 = 1$ in case of home carriers, else $X_1 = 0$
- Type of aircraft: $X_2 = 1$ in case of medium aircraft, else $X_2 = 0$
- Type of aircraft: $X_3 = 1$ in case of heavy aircraft, else $X_3 = 0$

The model equation becomes:

$$ln\left(\frac{\mu_i}{V_i}\right) = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2 + \beta_3 \cdot X_3$$

The estimators for the independent variables are determined as:

$$\beta_\mathbf{0} = -7.762$$

$$\beta_\mathbf{1} = 0.543$$

$$\beta_\mathbf{2} = -1.678$$

$$\beta_\mathbf{3} = -1.199$$

Where β_0 represents the estimator for the reference setting, β_1 represents the estimator for home carriers, β_2 and β_3 for resp. medium and heavy aircraft.
The estimated occurrences rates for all combinations of states of factors are given in table 8.4. The empirical and the estimated occurrence rates are again close to each other, which gives confidence in the correctness of the estimation.
Further usage of the results for supporting risk-based approaches is given in section 9.2.

Type carrier	Type WTC	Number of occurrences	Number of movements	Empirical occurrence rate	Estimated occurrence rate
home carriers	light	0	5	0.00E00	7.32E-4
home carriers	medium	81	599170	1.35E-4	1.37E-4
home carriers	heavy	27	118057	2.29E-4	2.21E-4
non-home carriers	light	16	37595	4.26E-4	4.25E-4
non-home carriers	medium	48	592365	8.10E-5	7.95E-5
non-home carriers	heavy	15	124170	1.21E-4	1.28E-4

Table 8.4: Empirical and estimated occurrence rates major incidents

8.3 Determining correlations between severity classes

The dependency between the severity classes is determined by the correlation coefficient for each combination of the severity classes. This correlation is determined by looking at the number of (aircraft involved with) occurrences for each month. The correlation coefficient is determined as explained in appendix A.4, and is between -1 and 1. The coefficients are shown in table 8.5.

Severity class	No safety effect	Significant safety effect	Major incidents	Serious incidents
No safety effect	1,000	0,663	0,211	0,061
Significant safety effect	0,663	1,000	0,226	-0,026
Major incidents	0,211	0,226	1,000	-0,214
Serious incidents	0,061	-0,026	-0,214	1,000

Table 8.5: Correlation severity classes

The correlation coefficients on the diagonal are 1 as it represents the correlation for each severity class and itself.
The correlation coefficient between severity class 'no safety effect' and 'significant safety effect' is 0.663, it is 0.211 between severity class 'no safety effect' and 'major incidents', and it is 0.061 severity class 'no safety effect' and 'serious incidents'. The correlation coefficient is 0.226 between the severity class 'significant safety effect' and 'major incidents', it is -0.026 between severity class 'significant safety effect' and 'serious incidents'. The correlation coefficient is -0.214 between severity class 'major incidents' and 'serious incidents'.

Further usage of the correlation coefficients for supporting risk-based approaches is given in section 9.3.

Chapter 9

Supporting risk-based approaches

This section describes how the results of the methods are used to support risk-based approaches. Starting by studying the practicability of reference values in section 9.1 with the Poisson distributions found. Section 9.2 than describes how the results of Poisson regression are used to identify (combinations of) factors which contribute to risk. Next, section 9.3 describes how the determined correlation of severity classes helps in supporting risk-based approaches.

9.1 Practicability of reference values

Safety-management is important in Air traffic control. Continuous effort is made to improve safety. On national level LVNL is working with the Ministry of Infrastructure and Environment to further search for safety performance indicators. On European level LVNL participates in FABEC, which is an 'initiative driven by civil and military partners of six states existing from high-level officials from the Ministry of Transport and Defense of Belgium, France, Germany, Luxembourg, and the Netherlands', together with 'seven civil air navigation service providers designated in these countries' [16].
Also, legislation concerning the safety in Air traffic control is developing on both national and European level. This is done on global level by for example ICAO.
In this context, it is of interest to investigate what indicative reference values could be concerning the number of occurrences for different severity classes. These reference values can be chosen with respect to several parts of the operation, and with respect to the measuring unit of the reference value. A first search is performed by FABEC on how these reference values could be filled.

This section discusses two possible reference values which are analysed by means of exceedance probabilities. The conceptual reference values are suggested by FABEC:

- The number of aircraft involved with severe incidents in the entire operation, with measuring unit the number of aircraft involved with occurrences per movement; for major and serious incidents separately.
  A reference value is defined for both major and serious incidents, which includes all incidents both in the air- and on the ground-operation of Schiphol airport. The number of incidents are expressed relative to the amount of traffic, thus the total number of aircraft which are under the control of LVNL; the number of incidents per flight.

- The number of aircraft involved with severe incidents in the air (thus, at ACC and APP), with measuring unit the number of aircraft involved with occurrences per flight-hour; for major and serious incidents separately.
  Again a reference value is defined for both major and serious incidents, but now only those incidents which took place when they were under control of the part of the operation at Schiphol airport called ACC and APP. The number of incidents are expressed relative to the amount of time in which aircraft have been at ACC and APP, and thus the total number of flight-hours in which aircraft were handled at ACC and APP; the number of incidents per flight-hour.

The concept reference values are developed by FABEC for all its members. The differences between the airspace/ airports for which the reference values are designed differ in the sense that the number of aircraft handled differs a lot, it also differs in the time an aircraft is under control when it is in the corresponding airspace.

First, the time an aircraft is under control of an ANSP in the upper areas of the airspace depends on the ANSP which is viewed. The German and French ANSP also have an Upper Airspace Control Unit and flights on average are much longer under their control compared to e.g. LVNL. The length of this time-interval has great influence on the analysis outcome of *reference value 2*. This reference value is more likely to be practicable for airports which have inbound, outbound and transit flights under their control for a longer period of time.
Second, the number of movements which airport handle also differs per airport. This explains why the 'accepted' number of aircraft involved with occurrences is high for Schiphol: it handles relatively many aircraft.

The model gives an added value in analyzing the reference values by providing extra insight in the practicability of the indicative reference values. It does not only show whether the reference values are exceeded in the passed years, it also gives the probability that a reference value is (not) exceeded.

The goal of this section is to illustrate how the model gives an indication on the practicability of the proposed reference values based on the data.
The distributions found are in terms of the number of aircraft involved with occurrences per month, thus first a translation is made such that a comparison can be made, followed by an indication for the practicability of the reference value according to propositions 7.1.1 and 7.1.2.

Note here, the reference value is determined as the number of 'aircraft involved with occurrences' rather than 'number of occurrences'. A major or serious incident often includes multiple aircraft, and thus the averages may seem large.

Reference value 1

The first reference value viewed is based on the number of aircraft involved with occurrences per movement which take place at the entire operation of Schiphol for the severity classes major and serious. The Poisson distribution is verified as the number of aircraft involved with occurrences per month. The practicability of the reference value is determined as the number of flights involved with occurrences which take place in one year. Thus, a translation is made to equalize the unit of the reference value and of the Poisson distribution.

The Poisson distribution for the number of aircraft which are involved with incidents per year has the Poisson parameter $\hat{\lambda} = 12 \cdot \lambda$, as it is assumed and verified that the months are independent and identically distributed with same mean λ; summation of Poisson distributions give a Poisson distribution with a parameter which equals the sum of the individual parameters.
The Poisson distribution is determined with data from 2010 to 2012 per month and has the sample mean as parameter λ, which is 5.19 per month for major incidents and is 0.83 per month for serious incidents. This results in parameter $\hat{\lambda}$ of resp. 62.3 and 10.0 aircraft involved with occurrences per year.

Translating the reference value to the desired unit requires some extra data. The average number of movements per year are determined, this is based on data which again stems from 2010 to 2012. Knowing the average number of movements per year is sufficient to determine the number of aircraft involved with occurrences for the given reference value.
The given reference value is given as $2.40 \cdot 10^{-4}$ aircraft involved with occurrences per movement for major incidents, and $2.40 \cdot 10^{-5}$ for serious incidents. Translating the reference value requires data on the number of movements. First the number of movements per month is determined for each year, which is on average 38386 movements per month in 2010 (m_{2010}), 40669 movements per month in 2011 (m_{2011}), 41681 movements per month in 2012 (m_{2012}), which is on average 40245 movements per month (m). Next the reference value is translated to the number of aircraft involved with occurrences per month by multiplying the average number of movements and the reference value. Then the reference value is translated to the number of aircraft involved with occurrences per year by multiplying with 12 (months). So:

$$\hat{r} = r \cdot 12 \cdot m$$

Translated to a yearly base results in a reference value 115.6 aircraft involved with occurrences per year for major incidents, and 11.6 for serious incidents.

Figure 9.1 and 9.2 show the reference value for resp. major and serious incidents including the Poisson distribution as described above. Further, it shows the number of aircraft which were involved with occurrences in 2010, 2011, and 2012.

Figure 9.1 shows that the number of aircraft involved with occurrences for major incidents in 2010, 2011, and 2012 (resp. 51, 69, and 67) are below the reference value (115.6). Furthermore, almost the entire distribution-curve lies on the left of the reference value. The area under the distribution-curve on the right of the reference value is the probability of exceeding the reference value, which is $8.2 \cdot 10^{-10}$.

Figure 9.2 shows that the number of aircraft involved with occurrences for serious incidents in 2011 and 2012 (resp. 8 and 1) are below the reference value (11.6), but 2010 (21) is above the reference value. Also, the probability of exceeding the reference value is larger than it is for the major incidents: $3.0 \cdot 10^{-1}$.

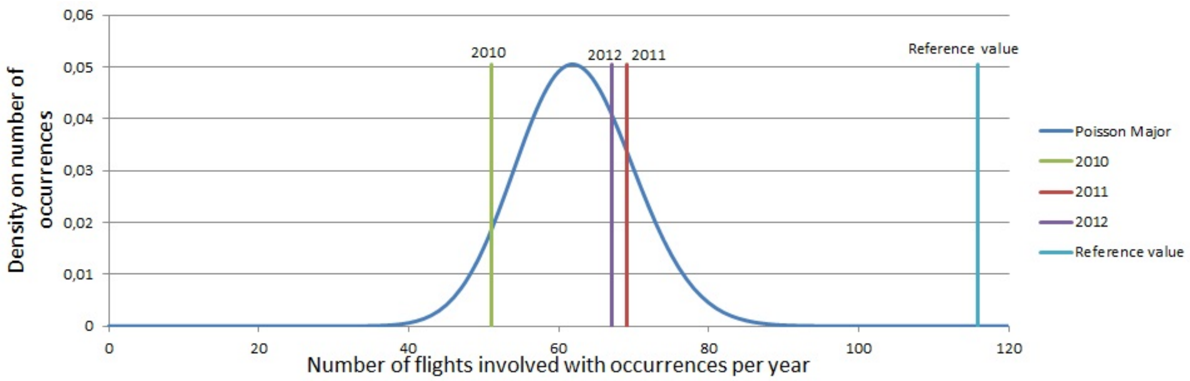

Figure 9.1: Reference value 1: Number of aircraft involved with major incidents per movement for entire operation.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011 and 2012.

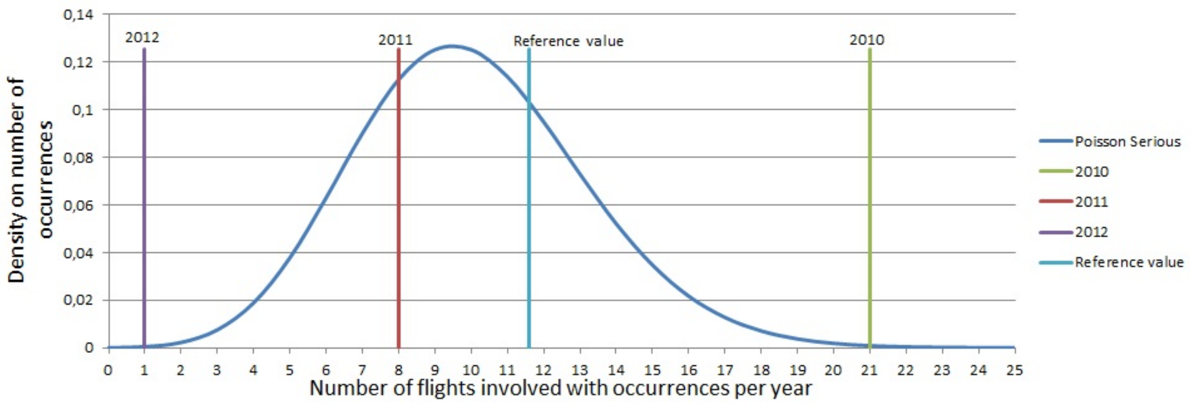

Figure 9.2: Reference value 1: Number of aircraft involved with serious incidents per movement for entire operation.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011 and 2012.

Reference value 2

The second reference value viewed is based on the number of aircraft involved with occurrences per flight-hour which take place at ACC and APP for the severity classes major and serious. The Poisson distribution is again verified as the number of aircraft involved with occurrences per month. Also, the practicability of the reference value is again determined as the number of aircraft involved with occurrences which take place in one year. Thus, a translation is made to equalize the unit of the reference value and of the Poisson distribution.

The translation for the Poisson distribution is done as for *reference value 1*, the Poisson parameters $\hat{\lambda}$ are determined as 15.7 aircraft involved with occurrences per year for major incidents and 4.7 for serious incidents.

The reference value is given as the number of aircraft involved with occurrences per flight-hour, which has to be translated to the number of aircraft involved with occurrences per year. Data is collected to determine the average number of movements per year and the average time an aircraft is at ACC and at APP.
The average time an aircraft is at ACC and APP differs for the different type of movements: inbound flights spend on average 9.9 minutes at ACC and 13.9 minutes at APP, outbound flights spend on average resp. 7.4 and 7.5 minutes at ACC and APP, and transit flights spend on average resp. 15.8 and 6.8 minutes at ACC and APP. The average time of movements at ACC and APP are thus 23.4 for inbound flights, 14.9 for outbound flights, and 22.6 for transits.
Next the number number of movements is determined for ACC and APP separately for each year. Again, separately for inbound, outbound, and transit flights.

From this data the number of flight-minutes per year for type i (V_i) is determined by multiplying the number of movements for flights of type i (m_i) by the average time it spends at ACC and APP (t_i) in hours:

$$V_i = m_i \cdot t_i$$

The total number of flight-hours per year is then determined by summing the number of flight-hours of the different types of flights:

$$\begin{aligned} V &= \sum_i V_i \\ &= \sum_i m_i \cdot t_i \end{aligned}$$

Next the reference value is determined as the number of aircraft involved with occurrences per year by multiplying the given reference value by the number of flight-hours per year:

$$\begin{aligned} \hat{r} &= V \cdot r \\ &= \sum_i r \cdot m_i \cdot t_i \end{aligned}$$

With this data the reference value is translated to 8.00 aircraft involved with occurrences per year for major incidents, and 0.80 for serious incidents.

Figure 9.3 and 9.4 show the reference value for resp. major and serious incidents including the Poisson distribution determined as described above. Further, it shows the number of aircraft which were involved with occurrences in 2010, 2011, and 2012.

Figure 9.3 shows that the number of aircraft which were involved with occurrences of severity major in 2010, 2011 and 2012 (resp. 21, 16, and 8 aircraft) are all above the reference value. Also, the probability of exceeding the reference value is large: $9.7 \cdot 10^{-1}$.

Figure 9.4 shows the same effect: the number of aircraft which were involved with occurrences of severity major in 2010, 2011, and 2012 are resp. 9, 4, and 1. These realizations are all above the reference value and the exceedance probability is: $9.9 \cdot 10^{-1}$.

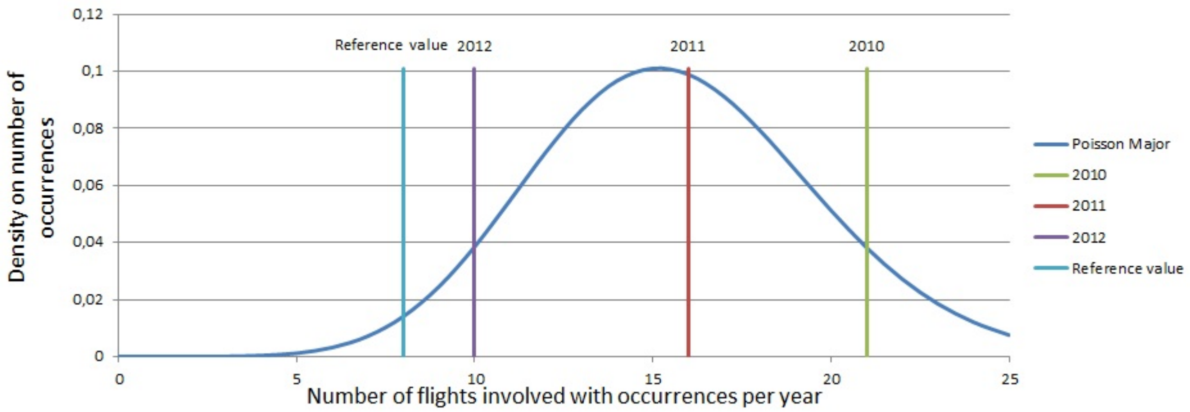

Figure 9.3: Reference value 2: Number of aircraft involved with major incidents at ACC and APP per flighthour.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011 and 2012.

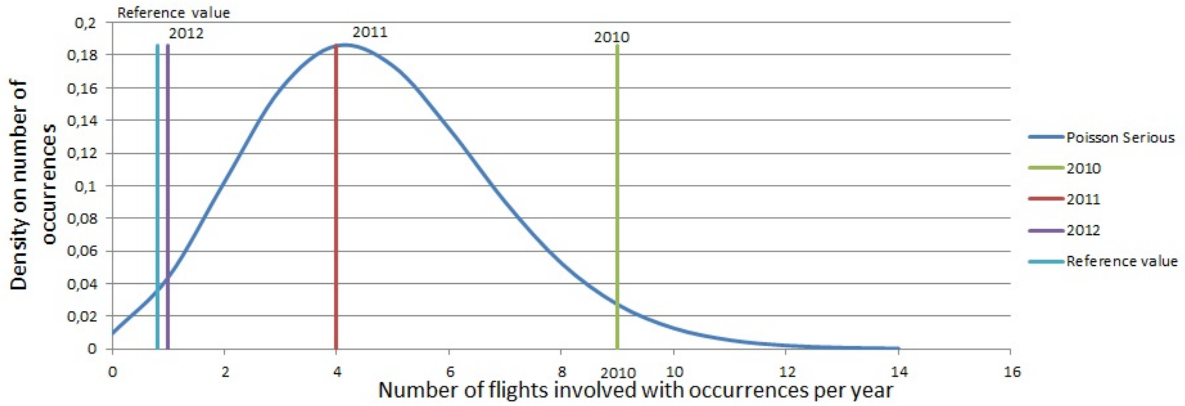

Figure 9.4: Reference value 2: Number of aircraft involved with serious incidents at ACC and APP per flighthour.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011 and 2012.

Conclusions on practicability of the indicative reference values

Following proposition 7.1.1: *reference value 1* is practicable with probability $1 - \alpha$, where $\alpha = 8.2 \cdot 10^{-10}$ is the exceedance probability when looking at major incidents. Looking at serious incidents the exceedance probability is $\alpha = 3.0 \cdot 10^{-1}$; the reference value is thus practicable with probability $1 - \alpha = 7.0 \cdot 10^{-1}$.
Reference value 2 is practicable with probability $1 - \alpha$, where the exceedance probability is $\alpha = 9.7 \cdot 10^{-1}$ for major incidents and the exeedance probability $\alpha = 9.9 \cdot 10^{-1}$ for serious incidents.

Following proposition 7.1.2: an practicability with exceedance probability of at most 5.0% for *Reference value 1* is achieved if the Poisson parameter (and thus mean number of aircraft involved with occurrences per year) for major incidents is 98.9, and is 6.93 for serious incidents.
An practicability with exceedance probability of at most 5.0% for *Reference value 2* is achieved if the Poisson parameter (and thus mean number of aircraft involved with occurrences per year) for major incidents is 4.7, and is 0.4 for serious incidents.

Note on practicability of the reference values

The exceedance probabilities for the indicative reference values are shown to differ, an explanation for this difference is given below.
The number of incidents is expressed relative to the amount of traffic. The definition of the amount of traffic has influence on the applicability of the reference values. If the value of the amount of traffic is large, the exceedance probability is smaller then when this value is small (when the number of occurrences remains equal). Say for example the number of flights are 100, and the time of a flight is small: say half an hour. Than the reference value is smaller and thus the exceedance probability is bigger than when the time of a flight had been two hours.

The following example illustrates which is stated above. This example compares air-traffic to the traffic on highway-routes.

Take two routes for vehicles on highways:

- Route 1: Amsterdam to Utrecht
- Route 2: Amsterdam to Berlin

Now take the two indicative reference values:

- N1: the number of accidents per trip.
- N2: the number of accidents per driving-hour of the trip.

For sake of simplicity: assume the number of vehicles on route 1 is equal to the number of vehicles on route 2. Assume that the time to follow route 1 is a lot shorter than the time to follow route 2 (rough estimation: 30 minutes for route 1, and 6 hours for route 2).

The reference values which FABEC is developing are intended to be the same for all participating ANSP. So: assume the reference values N1 and N2 to be equal for route 1 and route 2.
Reference value N2 is advantageous for route 2 compared to route 1; as more driving-hours are made. However, looking at the reference value N1, one will see that route 1 and route 2 have the same exceedance probability; intuitively there is more time to have something going wrong, this is left aside for sake of clarity.
Thus, reference value N1 is favorable for route 1 as the time needed to follow the route is short. Reference value 2 is less favorable for route 1, due to the short driving-time for each vehicle on the route; reference value 2 can be seen as more strict.

The tested reference values are being developed to be suitable for each ANSP which is joined in the FABEC project. Schiphol airport can be compared to route 1 in the example above: much traffic, but each movement is in Schiphols airspace for a brief period of time. This explains the difference in the probability of exceeding the reference value as shown above.

9.2 Identifying risk-contributing factors

Poisson regression as performed in section 8.2 gives estimates for the regression parameters with which an estimation is made for the number of aircraft involved with occurrences per movement for each combination of states of factors. Both case studies are discussed separately in this section. For each case it is first determined whether factors contribute to risk according to definition 7.2.1. Second, it is determined which state of a risk-contributing factor contributes to risk according to definition 7.2.2. Next it is determined which combination of states of factors contribute to risk as defined in definition 7.2.3. Last it is determined if combinations of states contribute to extraordinary increase of risk as stated in definition 7.2.4.

Factors which contribute to risk for type of occurrence 'deviation taxi'

The factors taken into account for type of occurrence 'deviation taxi' are the type of carrier, the type of peak, and the type of aircraft. These factors are of interest for LVNL and are thus used in this study.
When Poisson regression shows a significant influence of a factor, than the states of the factor show significant differences in the occurrence rates. Poisson regression shows a significant influence for each factor. Thus following definition 7.2.1: all factors contribute to risk.
Definition 7.2.2 shows which states of the factors have strongest contribution to risk. For the factor concerning the type of carrier the state 'non-home carriers' contributes strongest to risk, since the estimated occurrence rate is higher than other states in type of carrier when keeping type of peak and type of aircraft equal. Similar: the state 'inbound peak' in the factor concerning the type of peaks contributes strongest to risk. Also, the state 'aircraft type light' of the factor concerning the type of aircraft contributes strongest to risk.

Type of carrier	**Type of peak**	**Type of aircraft**	**Number of occurrences**	**Number of movements**	**Estimated occurrence rate**
home carriers	inbound peak	light	0	1	2.32E-4
home carriers	inbound peak	medium	16	297 773	5.73E-5
home carriers	inbound peak	heavy	5	45 448	1.24E-4
home carriers	outbound peak	light	0	1	1.67E-4
home carriers	outbound peak	medium	8	251 104	4.12E-5
home carriers	outbound peak	heavy	3	49 250	8.90E-5
home carriers	off-peak	light	0	0	1.16E-4
home carriers	off-peak	medium	1	50 198	2.88E-5
home carriers	off-peak	heavy	0	23 337	6.21E-5
non-home carriers	inbound peak	light	17	16 832	9.42E-2
non-home carriers	inbound peak	medium	49	217 574	2.33E-4
non-home carriers	inbound peak	heavy	8	51 958	1.81E-4
non-home carriers	outbound peak	light	10	14 073	6.78E-4
non-home carriers	outbound peak	medium	34	194 482	1.67E-4
non-home carriers	outbound peak	heavy	4	37 887	1.31E-4
non-home carriers	off-peak	light	0	3 400	4.73E-4
non-home carriers	off-peak	medium	8	33 731	1.17E-4
non-home carriers	off-peak	heavy	0	16 315	9.11E-5

Table 9.1: Deviation Taxi revised

The estimated occurrence rate of each combination of the states of the factors can be found in table 9.1. As the average estimated occurrence rate $\bar{T}_i^*$ is $2.18 \cdot 10^{-4}$, the combination of states of factors which contribute to risk accordingly to proposition 7.2.3 are:

- $c^* =$ {non-home carriers, inbound peak, light aircraft}
- $c^* =$ {non-home carriers, outbound peak, light aircraft}
- $c^* =$ {non-home carriers, off-peak, light aircraft}
- $c^* =$ {home carriers, inbound peak, light aircraft}
- $c^* =$ {non-home carriers, inbound peak, medium aircraft}

Following proposition 7.2.4: given the average number of movements $\bar{M}_i^*$ is 72409 and the average estimated occurrence rate $\bar{T}_i^*$ is $2.18 \cdot 10^{-4}$, the combination $F_C =$ {non-home carriers, inbound peak, WTC medium} contributes to risk extraordinary.

Factors which contribute to risk for severity class major

The factors taken into account for severity class major are the type of carrier, the type of peak, and the type of aircraft. Using the results of Poisson regression: definition 7.2.1 shows that the factor concerning the type of peak does not contribute to risk, the type of carrier and the type of aircraft do.
Definition 7.2.2 shows that the states contributing strongest to risk are the 'home-carrier' and 'aircraft type light'.
The estimated occurrence rates can be found in table 9.2 for each combination of states of the factors 'type of aircraft' and 'type of carrier'.

Type of carrier	Type of aircraft	Number of occurrences	Number of movements	Estimated occurrence rate
home carriers	light	0	5	7.32E-4
home carriers	medium	81	599170	1.37E-4
home carriers	heavy	27	118057	2.21E-4
non-home carriers	light	16	37595	4.25E-4
non-home carriers	medium	48	592365	7.95E-5
non-home carriers	heavy	15	124170	1.28E-4

Table 9.2: Occurrence rates major incidents - revised

As the average estimated occurrence rate $\bar{T}_i^*$ is $2.87 \cdot 10^{-4}$, the combination of states of factors which contribute to risk accordingly to proposition 7.2.3 are:

- $c^* =$ {non-home carriers, light aircraft}
- $c^* =$ {home carriers, light aircraft}

Following proposition 7.2.4: as the average number of movements $\bar{M}_i^*$ is 245227 and the average estimated occurrence rate $\bar{T}_i^*$ is $2.87 \cdot 10^{-4}$. None of the combinations of states of the factors lead to an extraordinary contribution to risk.

Graphic display of estimated occurrence rate for combinations of (states of) factors

Figure 9.5 and figure 9.6 give the estimated occurrence rates of the case studies graphically. On the x-axis the number of movements which correspond to the combination of states of factors is given (based on data of 2010 to 2012), the y-axis gives the estimated number of aircraft involved with occurrences per movement (occurrence rate) for the corresponding combination on a logarithmic scale. The labels attached to the estimates give the corresponding combination of states of factors. For figure 9.5 the labels indicate {type of carrier, type of aircraft}. For figure 9.6 the labels indicate {type of carrier, type of peak, type of aircraft}. The horizontal line indicates the estimated average number of occurrences for the considered type/ severity class. The vertical line indicates the average number of movements for the considered type/ severity class.

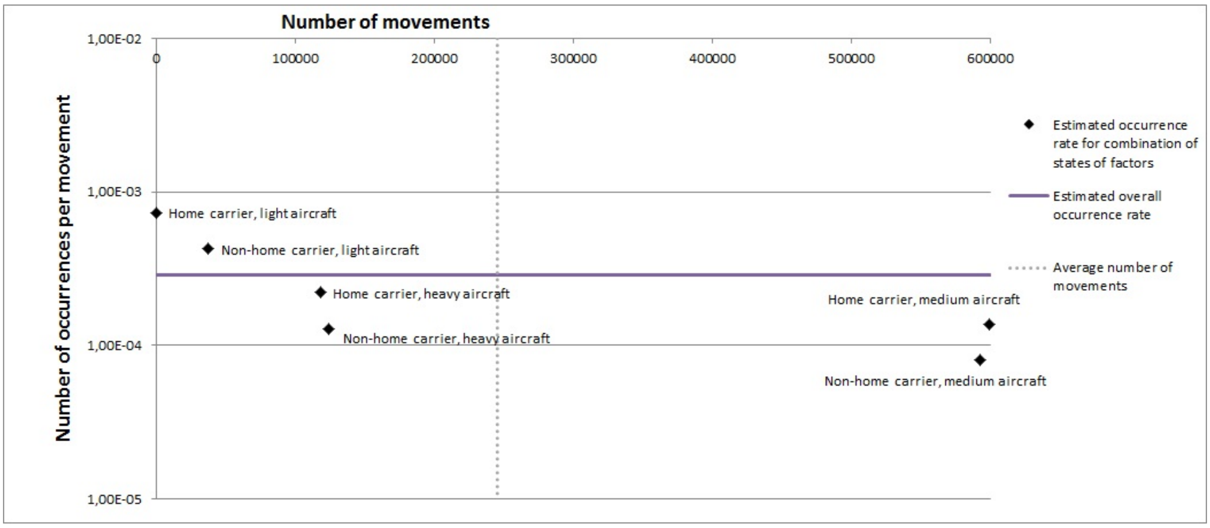

Figure 9.5: Estimated occurrence rate for combinations of states of factors for severity class 'major'. Factors: type of carrier, and type of aircraft.

Figure 9.6: Estimated occurrence rate for combinations of states of factors for occurrences of type 'deviation taxi'. Factors: type of carrier, type of peak, and type of aircraft.

9.3 Risk indication by using correlations between severity classes

The correlation between severity classes is studied because of LVNL's interest regarding the relation between severity classes. The presumption is that the number of occurrences in the separate severity classes is highly dependent on each other. The number of occurrences of low severity could be an indication of a disturbed operation. When the number of these occurrences is above average, it could lead to an increased probability on severe incidents and could thus be seen as an indication of risk.

With correlation coefficients (ρ) one can identify the relation between the number of occurrences in the severity classes. Dependency between the severity classes is strong when the correlation is strong, the strength of correlation is given in definition 7.3.1. Independence between the severity classes is assumed when the correlation is weak or negligible, as stated in assumption 7.3.2.
Knowing the strength of the correlation, and thus the strength of dependence between severity classes, gives insight in identifying the risk on a severe incident. This section looks at the strength of the correlation between severity classes, and how the ratio of the number of aircraft involved with occurrences in different severity classes is distributed.

Section 8.3 determined the correlation coefficient between the severity classes; the coefficients (ρ) are repeated in table 9.3.

Severity class	No safety effect	Significant	Major	Serious
No safety effect	1,000	0,663	0,211	0,061
Significant	0,663	1,000	0,226	-0,026
Major	0,211	0,226	1,000	-0,214
Serious	0,061	-0,026	-0,214	1,000

Table 9.3: Correlation severity classes - revised

Definition 7.3.1 is used to determine the strength of the correlations. A strong correlation is visible between the severity classes 'no safety effect' and 'significant' ($\rho = 0.663$), the number of occurrences in these severity classes is strongly related to each other; thus, an increase in occurrences of one severity class is strongly related to the number of occurrences in the other severity class. The correlation between severity class 'major' and 'no safety effect' ($\rho = 0.211$), and the correlation between severity class 'major' and 'significant' is weak ($\rho = 0.226$), which implies a weak relation between the number of occurrences in these severity classes. Also, there is a weak negative correlation between the number of occurrences in severity class 'major' and 'serious' ($\rho = -0.214$), which indicates that more occurrences in one of these severity classes weakly relates to fewer occurrences in the other severity class. Further, the severity class 'serious' has a negligible correlation with severity classes 'no safety effect' and 'significant' (resp. $\rho = 0.061$ and $\rho = -0.026$), which indicates that the number of occurrences in these classes barely relates to each other. The strength of the correlations is given graphically in figure 9.7.

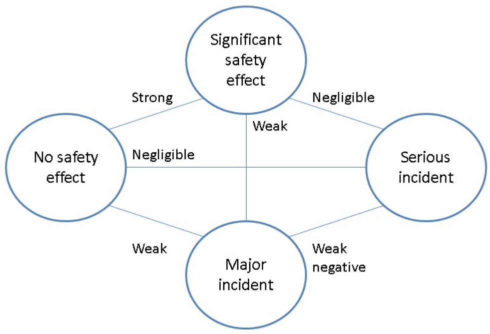

Figure 9.7: Strength correlations between severity classes

Following assumption 7.3.2: independence between severity classes cannot be assumed for each combination of the severity classes. The two severity classes which have a clear dependence in the number of occurrences are 'no safety effect' and 'significant safety effect'. The other combinations of severity classes are assumed to be independent of each other.

The interest in risk-assessment lies mostly in occurrences with a safety effect, so for all severity classes except the class 'no safety effect'. Since the three severity classes with safety effect have a Poisson distribution and can be assumed independent, the distribution function for the ratio between these severity classes can be determined as in proposition 7.3.3. The severity classes 'significant' (S_{sig}), 'major' (S_{maj}), and 'serious' (S_{ser}) are Poisson distributed with corresponding parameter λ_i (i=sig, maj, ser). The Poisson parameter is equal to the average number of aircraft involved with occurrences per month of the corresponding severity class, which are resp. 10.9, 3.0 and 0.47.
Recall: the ratio $R_{i,j}$ of severity classes i and j ($i \neq j$), is given by:

$$R_{i,j} = \frac{S_i}{S_j}$$

Figure 9.8 illustrates which ratio's can be determined by proposition 7.3.3 (indicated by 'use prop. 7.3.3'). The distribution for three ratio's cannot be determined by proposition 7.3.3 (denoted by 'use def. 7.3.4'), there are two reasons for this: first, the number of occurrences in severity class 'no safety effect' has no Poisson distribution. Second, the number of occurrences in severity 'no safety effect' and 'significant safety effect' has a dependence.

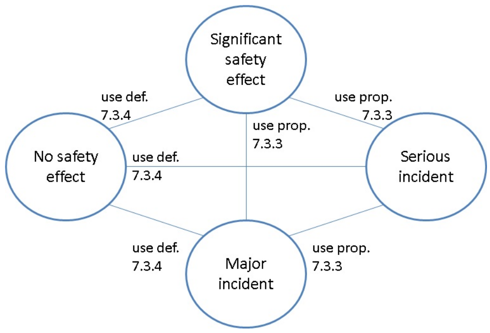

Figure 9.8: Use of propositions/ definitions for ratio between severity classes

The ratio between the severity classes 'significant safety effect', 'major incidents' and 'serious incidents' are determined by proposition 7.3.3. The choice is made to choose the class with highest severity as numerator in the ratio.
Recall the ratio is given by:

$$P\left\{\frac{S_i}{S_j} \leq t\right\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda_j}\lambda_j^n}{n!} e^{-\lambda_i} \sum_{k=0}^{\lfloor tn \rfloor} \frac{\lambda_i^k}{k!}$$

Thus, for 'significant safety effect' with 'major incidents':

$$P\Big\{\frac{S_{\text{major incidents}}}{S_{\text{significant safety effect}}} \leq t\Big\} = \sum_{n=0}^{\infty} \frac{e^{-10.9}10.9^n}{n!} e^{-3.0} \sum_{k=0}^{\lfloor tn \rfloor} \frac{3.0^k}{k!}$$

For 'major incidents' with 'serious incidents':

$$P\Big\{\frac{S_{\text{serious incidents}}}{S_{\text{major incidents}}} \leq t\Big\} = \sum_{n=0}^{\infty} \frac{e^{-3.0}3.0^n}{n!} e^{-0.47} \sum_{k=0}^{\lfloor tn \rfloor} \frac{0.47^k}{k!}$$

And for 'significant safety effect' with 'serious incidents':

$$P\Big\{\frac{S_{\text{serious incidents}}}{S_{\text{significant safety effect}}} \leq t\Big\} = \sum_{n=0}^{\infty} \frac{e^{-10.9}10.9^n}{n!} e^{-0.47} \sum_{k=0}^{\lfloor tn \rfloor} \frac{0.47^k}{k!}$$

When the severity classes do not both have a Poisson distribution or have a dependency, one can determine the empirical distribution for the ratio between $R_{i,j}$ by its variance and mean accordingly to definition 7.3.4.
The difficulty herein lies in the fact that the number of occurrences in severity classes major and serious is occasionally zero: when the denominator is zero, the empirical distribution cannot be given. As the number of occurrences with 'no safety effect' in a month has never been zero, the empirical distribution function can thus be given.
For occurrences with 'no safety effect' and 'significant safety effect' the expected value of the ratio and its variance is:

$$E\Big[\frac{S_{\text{significant safety effect}}}{S_{\text{no safety effect}}}\Big] = 2.8E-1$$

$$Var\Big(\frac{S_{\text{significant safety effect}}}{S_{\text{no safety effect}}}\Big) = 1.1E-2$$

For occurrences with 'no safety effect' and 'major incidents' the expected value of the ratio and its variance is:

$$E\Big[\frac{S_{\text{major incidents}}}{S_{\text{no safety effect}}}\Big] = 8.1E-2$$

$$Var\Big(\frac{S_{\text{major incidents}}}{S_{\text{no safety effect}}}\Big) = 3.9E-3$$

For occurrences with 'no safety effect' and 'serious incidents' the expected value of the ratio and its variance is:

$$E\Big[\frac{S_{\text{serious incidents}}}{S_{\text{no safety effect}}}\Big] = 1.4E-2$$

$$Var\Big(\frac{S_{\text{serious incidents}}}{S_{\text{no safety effect}}}\Big) = 1.1E-3$$

Note on ratio between severity classes

The proportion which is commonly used for the relation of number of occurrences in the severity classes is the following:

- Significant safety effect : major incidents has the proportion 100:1
- Major incidents : serious incidents has the proportion 10:1

When verifying this proportion one can think of using the Chi-squared test. However, there are rules of thumb on when this test gives reliable results, which are all not fulfilled. The most important observations are:

- The sample size is too small (36 months/ samples).
- The average value of the samples is too small for severity classes major and serious (< 5).
- The number of samples which count zero is too large for severity class serious.

Moreover, when looking at the data swiftly, it is observed that the first proportion does not hold for the dataset given. Looking at occurrences with significant safety effect: the largest count in one month is 26. Whereas the largest count of major incidents is 10. Moreover, looking at the data over three years: there are 392 occurrences with 'significant safety effects' and 108 'major incidents'. Thus, a proportion of 100:1 seems inappropriate.

Further, the largest count of serious incidents in one month is 4, and 26 of the 36 months have no serious incidents.
Also, the number of major incidents per month in 2010, 2011 and 2012 are resp. 2.4, 3.2 and 3.4. Where the number of major incidents per month in 2010, 2011 and 2012 are resp. 1, 0.3 and 0.08. So the number of major incidents shows an increase where serious incidents show a decrease. These averages also show that the proportion 10:1 does not seem realistic.

It is known that LVNL is constantly improving its procedures to prevent occurrences. When occurrences happen with high severity, it is analysed how this could happen and more important: how it can be prevented. A way is sought to prevent occurrences with high severity from happening.
For certain types of occurrences it is known that they appeared in serious incidents, so procedures were adapted and a change appeared: the occurrences still happen, but the severity has decreased. This can be a reason why an increase in major incidents appears, and on the same time a decrease in serious incidents appears.
The shift in the number of occurrences is to be examined further, but lies outside the scope of this study.

Chapter 10

Sensitivity analysis

This section determines the sensitivity of the data for the mathematical methods used. The sensitivity analysis consists of three elements, namely:

- Sensitivity of the Poisson parameter by using confidence intervals.
- Sensitivity of the output of Poisson regression by using confidence intervals.
- Comparing results case studies 2010-2012 and 2010-2013.

The confidence intervals indicate with which probability the estimated mean/ parameter is in the given interval. Using confidence level $\gamma = 0.95$, there is a certainty of 95% that the estimated mean lies in the given interval. The confidence intervals for the mean number of occurrences for the different types are given in section 10.1. Confidence intervals for the estimated parameters of Poisson regression is given in section 10.2.
Section 10.3 performs the (numerical) analysis from sections 8 and 9 with the data of 2013 added. The results of the studies with data from 2010-2012 and 2010-2013 are compared, which give insight in the development of the data and thus insight in the stability of the data.

10.1 Sensitivity Poisson distribution by using confidence intervals

In table D.3 the confidence intervals of the Poisson parameters (which are determined as the sample mean) are given for each type of occurrence. The results show a wide interval for the type 'Airspace infringement', the type of occurrence which is already discussed to have a seasonal pattern and thus the interval is wide. For the other types the confidence intervals are not always very small, but neither very big. Do note: the confidence intervals are determined theoretically and can thus be negative, in reality this will not happen; there is no such thing as a negative number of occurrences.

Table 10.1 shows the confidence intervals for the severity classes, again the confidence intervals are small.

Severity class	**Average**	**Lower bound**	**Upper bound**
No safety effect	40,42	34,99	45,84
Significant	10,89	9,19	12,59
Major	3,00	2,20	3,80
Serious	0,47	0,15	0,79

Table 10.1: Confidence intervals severity classes

10.2 Sensitivity Poisson regression by using confidence intervals

Confidence intervals used for the estimated parameters of Poisson regression are 95% Wald confidence intervals. The parameters chosen as 'reference setting' have no confidence interval: they are set at zero and are used to determine the other parameter's estimators (as discussed in section 8.2). Thus, only the parameters which are not included in the reference setting have a Wald confidence interval. Note: choosing the reference setting has no influence in determining the estimated number of occurrences, the estimates for the combinations of states of factors remain the same.
The intervals for the case study with type of occurrences 'deviation taxi' are given in table 10.2; where still β_0 represents the estimator for the reference setting, β_1 represents the estimator for home carriers, β_2 and β_3 the estimators for resp. the inbound- and outbound-peak, β_4 and β_5 for resp. aircraft type heavy and medium, and $\gamma_{1,3}$ is the estimator for the interaction between home carriers and outbound peaks.
For the case study concerning major incidents the confidence intervals are given in table 10.3. Still: β_0 represents the estimator for the reference setting, β_1 represents the estimator for home-carriers, β_2 and β_3 for resp. aircraft type medium and heavy.

Parameter	Parameter estimate	Lower bound	Upper bound
β_0	-7.953	-8.694	-7.212
β_1	-1.323	-1.766	-0.880
β_2	0.923	0.238	1.608
β_3	0.565	-0.138	1.267
β_4	-1.984	-2.664	-1.303
β_5	-1.537	-1.966	-1.107
$\gamma_{1,3}$	1.018	0.190	2.017

Table 10.2: Confidence Interval: case study - Factors influencing types of occurrences: Deviation taxi

Parameter	Parameter estimate	Lower bound	Upper bound
β_0	-7,762	-8,252	-7,272
β_1	0,543	0,232	0,854
β_2	-1,199	-1,806	-0,592
β_3	-1,678	-2,234	-1,122

Table 10.3: Confidence Interval: case study - Factors influencing Major incidents

10.3 Comparing results case studies 2010-2012 and 2010-2013

Numerical results in section 8 are obtained from data which stems from January 1, 2010 until December 31, 2012. Data of 2013 has become available while performing this study, thus sensitivity analysis is performed on the numerical results by using data of all four years.
It is first viewed if the Poisson distribution is applicable for the types and severity of occurrences. Next is determined if the Poisson parameter based on four years is in the confidence intervals found in section 10.1. Than Poisson regression is performed, the results of the case studies with the extra year of data are compared to the results of the original case studies. Last the correlation coefficients are reviewed with the extra year of data.

Poisson distribution and parameter

The Poisson distribution is tested for each type of occurrence and for each severity of the occurrences. Note here: in section 8.1 it is determined that the data of 2010 should not be taken into account for the types 'Apron incursion incl. pushback incident', 'Deviation Startup pushback', and 'Deviation vehicle - airport traffic', this is also done here. Also, 'Airspace infringement' is again tested with and without seasons separated.

The results for the severity of occurrences is shown in table 10.4, the results of the tests for the type of occurrences are displayed in table D.5. One can see that the significance for the type of occurrences 'Deviation Startup pushback' is too low to assume the Poisson distribution. This can be explained by the high number of occurrences of this type in 2013, the average number per month increased dramatically; on average 19 per month in 2013, and 7 per month in 2011 and 2012.
'Airspace infringement' is again not Poisson distributed when not splitting seasons, but is when seasons are split. This is displayed in table D.4.

The Poisson parameters when determined with four years of data, are viewed for the confidence intervals of the Poisson parameters of the original data used (three years). One sees that each estimated Poisson parameter based on four years of data lies within the confidence interval determined on three years of data; confidence intervals are given in section 10.1.

Type of severity	**Significance value - Poisson distribution**	**Significance value - Normal distribution**
No safety effect	0,002	0,746
Significant safety effect	0,119	0,742
Major incident	0,154	0,172
Serious incident	0,673	0,000

Table 10.4: Adding data 2013: Output Kolmogorov-Smirnov - Severity classes

Practicability reference values

Adding data for finding the practicability of reference values has impact on several elements: the parameter (and thus shape) of the Poisson distribution, the exceedance probability of the reference values, and the values for the Poisson parameter which lead to an exceedance probability of 5%.

Starting by the parameter (and thus the shape) of the Poisson distribution:
Looking at reference value 1 (number of aircraft involved with occurrences in the entire operation), the Poisson parameter decreased for both major and serious incidents; the parameters were resp. 62.3 and 10.0, the parameters are now resp. 57.0 and 8.5.
The exceedance probability of the reference value for major incidents decreased from $8.2 \cdot 10^{-10}$ to 0.0 , for serious incidents this probability decreased from $3.0 \cdot 10^{-1}$ to $3.6 \cdot 10^{-3}$.
The Poisson parameter which is required to obtain an exceedance probability of at most 5% is 136.97 for major incidents and 10.04 for serious incidents.
Figures 10.1 and 10.2 show the Poisson distribution for resp. major and serious incidents, the reference values and the realizations of 2010, 2011, 2012, and 2013.

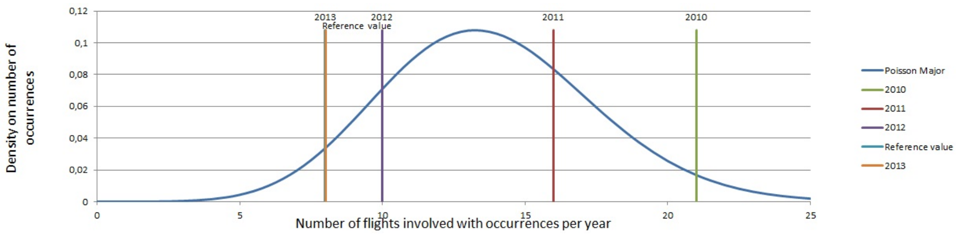

Figure 10.1: Reference value 1: Number of aircraft involved with major incidents per movement for entire operation.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011, 2012 and 2013.

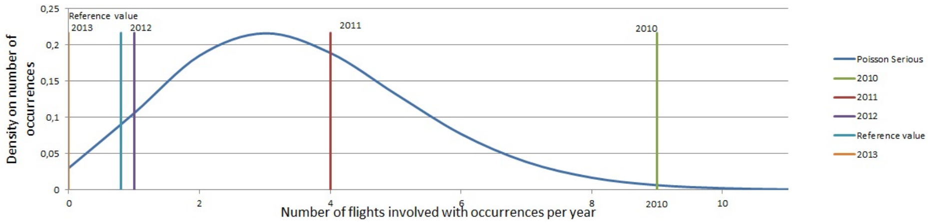

Figure 10.2: Reference value 1: Number of aircraft involved with serious incidents per movement for entire operation.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011, 2012 and 2013.

Looking at reference value 2 (number of aircraft involved with occurrences in the upper airspace), the Poisson parameter decreased for both major and serious incidents; the parameters were resp. 15.7 and 4.7, the parameters are now resp. 13.8 and 3.5.
The exceedance probability of the reference value for major incidents decreased from $9.7 \cdot 10^{-1}$ to $9.3 \cdot 10^{-1}$, for serious incidents this probability decreased from $9.9 \cdot 10^{-1}$ to $9.7 \cdot 10^{-1}$.
The Poisson parameter which is required to obtain an exceedance probability of at most 5% is 6.17 for major incidents and 0.36 for serious incidents.

Figures 10.3 and 10.4 show the Poisson distribution for resp. major and serious incidents, the reference values and the realizations of 2010, 2011, 2012, and 2013. Note here: the number of occurrences in 2013 is slightly above the reference value for major incidents.

Regression parameters and their influence

Adding data of 2013 gives the possibility to observe how the (measure of) influence of the given factors change compared to the data of 2010 to 2012. The comparison made contains the significance of the influence, and whether the value of the estimators lies in the original confidence interval based on the first three years of data.

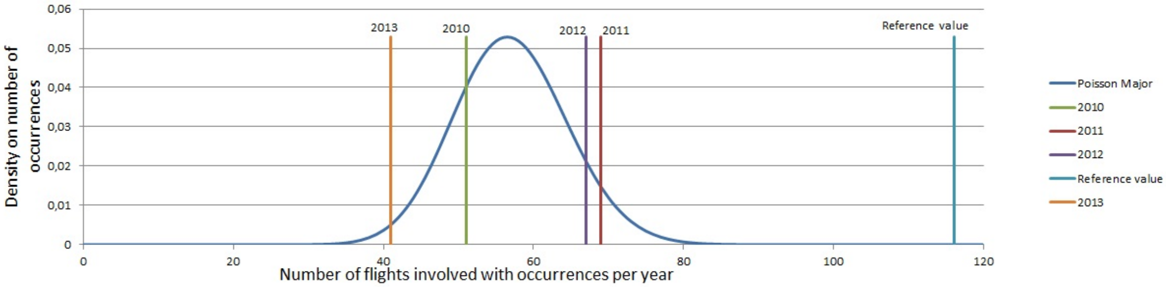

Figure 10.3: Reference value 2: Number of aircraft involved with major incidents at ACC and APP per flighthour.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011, 2012 and 2013.

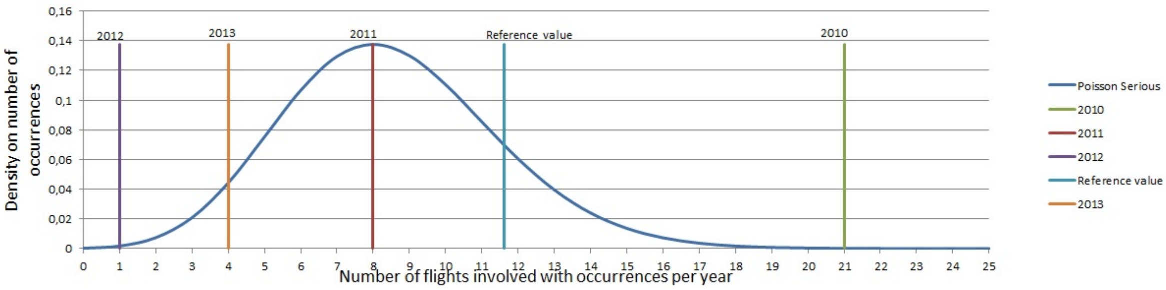

Figure 10.4: Reference value 2: Number of aircraft involved with serious incidents at ACC and APP per flighthour.
Translations made to a yearly base: Poisson distribution, reference value, and realizations of 2010, 2011, 2012 and 2013.

The conclusions upon the influence of factors and interactions remain the same when looking at the type 'deviation taxi'. The conclusions upon the contribution to risk for the states of factors remain the same. The combinations of states of factors which were already contributing to risk remain the same, except for two cases: the combination {home carriers during the outbound peak with light aircraft} is now also determined to be a risk-contribution combination, whereas the combination {non-home carriers during the inbound peak with medium aircraft} is no longer a risk-contributing combination. Further, non of the combinations has an extraordinary contribution to risk anymore.

Table 10.5 gives the estimated occurrence rate and the number of movements for each combination of states of factors for the data of 2010-2012 and that of 2010-2013 for type 'deviation taxi'. The average number of movements for 2010-2012 is 72 409, the average occurrence rate is $2.09 \cdot 10^{-4}$. For the data of 2010-2013 these averages are resp. 97 315 and $2.38 \cdot 10^{-4}$.

Type of carrier	Type of peak	Type of aircraft	Number of movements 2010-2012	Estimated occurrence rate 2010-2012	Number of movements 2010-2013	Estimated occurrence rate 2010-2013
home carriers	inbound peak	light	1	2,3E-04	1	3,1E-04
home carriers	inbound peak	medium	297773	5,7E-05	394640	6,7E-05
home carriers	inbound peak	heavy	45448	4,5E-05	59818	5,5E-05
home carriers	outbound peak	light	1	1,7E-04	1	2,5E-04
home carriers	outbound peak	medium	251104	4,1E-05	338357	5,4E-05
home carriers	outbound peak	heavy	49250	3,2E-05	64434	4,4E-05
home carriers	off-peak	light	0	1,2E-04	0	1,8E-04
home carriers	off-peak	medium	50198	2,9E-05	76229	3,9E-05
home carriers	off-peak	heavy	23337	2,2E-05	32681	3,2E-05
non-home carriers	inbound peak	light	16832	9,4E-04	20942	9,8E-04
non-home carriers	inbound peak	medium	217574	2,3E-04	281594	2,1E-04
non-home carriers	inbound peak	heavy	51958	1,8E-04	69284	1,8E-04
non-home carriers	outbound peak	light	14073	6,8E-04	18159	7,9E-04
non-home carriers	outbound peak	medium	194482	1,7E-04	256159	1,7E-04
non-home carriers	outbound peak	heavy	37887	1,3E-04	50259	1,4E-04
non-home carriers	off-peak	light	3400	4,7E-04	5696	5,6E-04
non-home carriers	off-peak	medium	33731	1,2E-04	58663	1,2E-04
non-home carriers	off-peak	heavy	16315	9,1E-05	24756	1,0E-04

Table 10.5: Deviation Taxi (including 2013)

When looking at major incidents the conclusions remain the same with three years of data as for the analysis with data of 2013 added. This is true for the factors which have a contribution to risk, the states of factors which contribute to risk, and the combinations of states which have an (extraordinary) contribution to risk.

Table 10.6 gives the estimated occurrence rate and the number of movements for each combination of states of factors for the data of 2010-2012 and that of 2010-2013. The average number of movements for 2010-2012 is 245 227, the average occurrence rate is $2.87 \cdot 10^{-4}$. For the data of 2010-2013 these averages are resp. 335 391 and $2.31 \cdot 10^{-4}$.

Correlations between severity classes

The strength of the correlation between the number of aircraft involved with occurrences in severity classes changes slightly after adding data of 2013. The conclusions on the (in)dependence of the severity classes and the distribution function for the ratio of the occurrences in separate severity classes remain as before.
The correlations are given in table 10.7.

Type of carrier	**Type of aircraft**	**Number of movements 2010-2012**	**Estimated occurrence rate 2010-2012**	**Number of movements 2010-2013**	**Estimated occurrence rate 2010-2013**
home carriers	light	5	7,3E-04	285	5,6E-04
home carriers	medium	599170	1,4E-04	823056	1,2E-04
home carriers	heavy	118057	2,2E-04	156964	1,9E-04
non-home carriers	light	37595	4,3E-04	56883	3,3E-04
non-home carriers	medium	592365	7,9E-05	806389	7,0E-05
non-home carriers	heavy	124170	1,3E-04	168770	1,2E-04

Table 10.6: Occurrence rates major incidents

Severity class	No safety effect	Significant safety effect	Major incidents	Serious incidents
No safety effect	1.000	0.548	0.387	-0.005
Significant safety effect	0.548	1.000	0.227	-0.109
Major incidents	0.387	0.227	1.000	-0.216
Serious incidents	-0.005	-0.109	-0.216	1.000

Table 10.7: Correlation severity classes with 2013

Chapter 11

Conclusions and recommendations

This section first describes the conclusions for main goal and research questions as stated in section 2.3, next recommendations on further research are discussed.

Conclusions on the goals

Main goal: Model development and analysis of occurrence data in air traffic management to support advanced risk-based approaches.

The model developed has three subjects in which it supports LVNL's advanced risk-based approach: verifying the practicability of reference values by using exceedance probabilities, finding circumstances which contribute to risk, and finding a relation between the number of occurrences in the severity classes.
To do so the model describes the occurrences with three elements: the circumstances of the occurrences, the type of the occurrences, and the severity of occurrences.
Properties of the elements are determined. For each type of occurrence ($i \in \mathbb{I}$) and for each severity of occurrences ($j \in \mathbb{J}$), the number of occurrences per month (resp. S_i and S_j) has a Poisson distribution. The model describes the relation between the elements of the model. It describes the relation between factors regarding the occurrences and the type/ severity of occurrences ($T_i^*(F_C)$ and $S_j^*(F_C)$). The relation between the types and the severity of occurrences has been studied, but no specific expression is derived due to the complexity of the relation; the relation however is denoted by $S_j = g(T_i)$. Also, a relation between the number of occurrences in the separate severity classes is determined, distributions are found for the ratio of the number of occurrences in the severity classes.

Develop a model which supports LVNL's risk-based approach.

The developed model supports LVNL's risk-based approach by investigating the practicability of the reference values given by FABEC in section 9.1. The reference value referring to the number of aircraft per movements involved with occurrence is practicable with exceedance probability $8.2 \cdot 10^{-10}$ for major incidents, and $3.0 \cdot 10^{-1}$ for serious incidents. Whereas the reference value referring to the number of flights per flight-hour has exceedance probability $9.7 \cdot 10^{-1}$ for major incidents and $9.9 \cdot 10^{-1}$ for serious incidents.

The model also supports LVNL's risk-based approach by identifying factors which contribute to risk (section 8.2). For the type of occurrence 'deviation taxi' it is determined that the type of aircraft, the type of carrier, and the type of peak are risk-contributing factors. The states of factors light aircraft, non-home carriers, and inbound peaks contribute strongest to risk. The combination of states of factors which contribute to risk are: {non-home carriers, during inbound peaks with light aircraft}, {non-home carriers, during outbound peaks with aircraft light}, {non-home carriers, during off-peaks with light aircraft}, {home carriers, during inbound peaks, with light aircraft}, and {home carriers, during inbound peaks, with medium aircraft}. Where the last even has an extraordinary contribution to risk.
For the severity class major similar analysis is performed with different outcome. The influence of peaks is not observable and thus do not contribute to risk. The type of carrier and the type of aircraft are factors which do contribute to risk. The states of factors which contribute strongest to risk are home-carriers and light aircraft. The combination of states of factors which contribute to risk are: {non-home carriers with light aircraft} and {home carriers with light aircraft}, these combinations do not contribute to risk extraordinary.

The correlation found in section 8.3 shows a strong correlation between severity classes no safety effect and significant safety effect, and thus dependence. Whereas the other correlations between severity classes are weak or negligible and thus independence is be assumed. The distribution function for the ratio of the number of aircraft involved with occurrences is determined by using the Poisson distribution when independence and the Poisson distribution is assumed. The distribution of the ratio is determined empirically when the Poisson distribution or independence cannot be assumed.

Find statistical characteristics concerning the occurrence data.

The Poisson distribution is verified as a suitable distribution for the number of occurrences for the different types of occurrences and for the number of occurrences in the severity classes, severity class 'no safety effect' is the only severity class in which this is not the case; the latter has a normal distribution. The parameters of the distributions are discussed in section 8.1.

Correlations between the type of occurrence and its severity is studied. Due to the dependence between the different types of occurrences a conclusion on what the relation is between an occurrence its type and its severity is not drawn. Further research is desired and described below.

The relation between factors and the type/ severity of occurrences is investigated by using Poisson regression. For the type of occurrences 'deviation taxi' it is determined that the influence of the type of carrier, type of peak, and type of aircraft is observable. For the severity class major it is determined that the type of carrier, and type of aircraft have an observable influence, and the influence of the type of peak is not observable. The strength of the influence is shown in section 8.2.

The statistical relation between the severity classes is discussed in section 8.3. The correlation between severity class 'no safety effect' and 'significant' is strong. A weak correlation is found between severity class 'major' and 'no safety effect', and between severity class 'major' and 'significant'. A weak negative correlations is found between severity class 'major' and 'serious'. A negligible correlation is found between both severity class 'serious' and 'significant', and between severity class 'serious' and 'no safety effect'.

Identify the sensitivity of the conclusions drawn in the model.

In section 10.1 and section 10.2 the sensitivity of the parameters tested and estimated are tested on sensitivity by using confidence intervals. These confidence intervals are deemed sufficiently small and thus reliable results are acquired.
In section 10.3 the results from the model are verified on sensitivity by adding data. The results as acquired in sections 8 and 9, and thus with data from 2010-2012, are compared to the results of the model when the data of 2013 is added.
Adding data of 2013 does not change conclusions on the (Poisson and normal) distributions for all types and severity classes except for one type: 'Deviation startup-pushback'. An increase on the the average number of occurrences of this type has occurred.
Testing the reference values on applicability shows a decrease in the exceedance probability for each reference value of each severity class.
Also, adding data of 2013 does not change the conclusions in identifying (combinations of) factors which contribute to risk for the type 'deviation taxi' and severity class 'major incidents' dramatically. The estimated occurrence rates change slightly, but the conclusions on contribution to risk remain the same for major incidents. For type 'deviation taxi': the estimated occurrence rates change slightly, conclusions on contribution to risk change slightly. The combination of states {home carriers, during the outbound peak, with light aircraft} is now also determined to be risk-contributing, whereas the combination {non-home carriers, during the inbound peak, with medium aircraft} is no longer risk-contributing. Furthermore, non of the combinations has an extraordinary contribution to risk anymore.
The correlation coefficients for the combinations of the severity classes change slightly. Conclusions on the strength of the correlation changes slightly, but conclusions on the dependency of the severity classes remain as before.

Recommendations

Analysis on occurrence data of other airports.

The model can be used for risk-analysis on other airports, only the factors taken into account should be adjusted to the selected airport. E.g., Rotterdam airport is of interest, but as they do not have peaks as Schiphol airport, one should not take that factor into account.

Identify extra factors which contribute to risk.

The developed model developed is analyzed for a few case studies, the second recommendation is to apply the model for the identification of more factors which potentially contribute to risk. Some factors are discussed below.

The first class of factors which is suggested concerns the location of the occurrences, which can be defined in several ways. One definition is the runway which an aircraft uses, this results in a categorical variable with six states; the runways available are the Kaagbaan, Zwanenburgbaan, Buitenveldertbaan, Aalsmeerbaan, Polderbaan, Schiphol-Oostbaan. The idea behind this is that certain runways lead to a more complicated traffic flow, which might cause an increase in occurrences and thus contribute to an increase of risk. Another definition of location can be the sectors in the air, which results in five states. Again some sectors may cause a more complicated traffic flow leading to more occurrences and thus contribute to risk.
In line with the locations, one can think of a factor which indicates the combination of runways which is used. Some runway combinations are known to be experienced as 'more complicated' for the pilot, which could thus lead to an increase in risk.

The second class of factors could be focused on the properties of the traffic itself. E.g., the origin of the flight. Origin of flights can be either whether its regional or not, or the continent it departed from, or the origin of the carrier which carries out the flight. Also a difference can be made in whether a flight is inbound, outbound, or transit.

A third class of factors can be chosen with respect to situations which do not directly influence them, but do have an indirect influence. One can think of military activity in the adjacent air space. It influences the flow of the air traffic, the combinations of runways which are used, and maybe even increases the risk for occurrences of types like 'airspace infringement'.
In this line, one can also have interest in a factor which indicates if the 'Zwanenburgbaan' is being used, traffic flow is experienced as more complicated when it is in use.

A fourth class of factors which can be investigated is the 'time' in which occurrences/ movements take place. E.g., during or outside UDP, which indicates whether flights take place during daylight or not.

The last class of factors suggested is the positioning of air traffic controllers, e.g., the number of air traffic controllers on a position; single or double. Some sectors can be divided over two controllers in certain situations, which can lead to a decrease or increase of occurrences.

Relation between severity classes.

In section 9.3 the correlation between the severity classes is determined, this led to determination of (in)dependence and the distribution for the ratio of the number of aircraft involved with occurrences. Further research should be performed to find or develop methods which identify risks using this knowledge.

Grouping types of occurrences to create independent intersections of the data.

It is recommended to find a division for the type of occurrences in such a way that independent groups are formed. In section 4.1 the dependency of types of occurrences is shown, and an effort is made in dividing the available types in independent groups. When independent subdivisions are made, an analysis can be done on the relation between the type of occurrences and the severity classes. Which is again a way to identify risk.

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Appendix A

Mathematical methods

This appendix gives information on the mathematical models used. Appendix A.1 first describes the properties of the Poisson distribution, followed by the test of Kolmogorov-Smirnov which is used for verifying the Poisson distribution in appendix A.2. Next the formal setting of (Poisson) regression is stated in appendix A.3, followed by the theory on correlation coefficients of variables in appendix A.4.

A.1 Poisson distribution

The Poisson distribution is a distribution which is suitable for count-data. It has one parameter, μ, which corresponds to the mean number of occurrences. The mean and the variance are equal in the Poisson distribution. The probability of having y events with a given mean μ is given by:

$$P(Y = y; \mu) = \frac{\mu^y}{y!} e^{-\mu}$$

Where $y!$ is the y factorial. The form of the mass probability plot depends heavily on the value of μ, an idea of the distribution function is given in figure A.1. Note here: the form of the distribution function resembles the normal distribution when μ becomes large.

When summing multiple Poisson distributions, the summed distribution is again Poisson distributed with parameter the sum of the individual parameters.

$$\sum_{i=1}^{n} Poisson(\lambda_i) = Poisson(\sum_{i=1}^{n} \lambda_i)$$

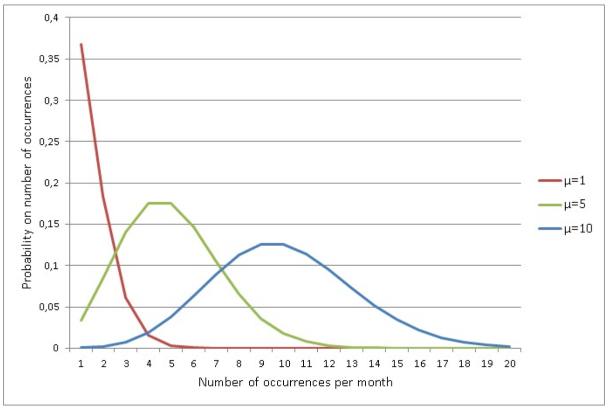

Figure A.1: Poisson distribution

A.2 Testing for Poisson distribution: Kolmogorov-Smirnov

Count-data is available, thus a Poisson distribution with unknown mean is expected. The Kolmogorov-Smirnov test is used to determine if the Poisson distribution is applicable for the number of occurrences, where the tested parameter is the sample mean.

Statistical tests are methods to check whether an assumption (null-hypothesis) should be rejected or not, based on observations (the sample). First the hypothesis of the test is determined, it exists of the null-hypothesis (H_0) and the alternative hypothesis (H_1). This is displayed as follows:

$$H_0 : \theta \in \Theta_0; \qquad H_1 : \theta \in \Theta_1$$

where: $\Theta_0 \bigcup \Theta_1 = \Theta$ and $\Theta_0 \bigcap \Theta_1 = \emptyset$. Also, Θ represents all possible outcomes of the sample.

The null-hypothesis is rejected when the sample gives sufficient proof to assume that H_0 is not correct, the null-hypothesis is accepted when the sample does not give sufficient proof against H_0.

The critical region of a test is the set of possible outcomes which give rise to the rejection of H_0 (the hereby established decision procedure is called a (statistical) test). The sample function $T = t(X_1, X_2, \dots, X_n)$ is called the test statistic and c_i $(i = 1, 2)$ the critical values of a test, the critical region of a test is then of the form:

$$Z = \{(x_1, \dots, x_n) | t(x_1, \dots, x_n) \leq c_1 \text{ or } t(x_1, \dots, x_n) \geq c_2\}$$

The test is called right-sided when $c_1 = -\infty$ and it is called left-sided when $c_2 = \infty$, in all other cases the test is called two-sided. The value of c is determined by the significance level α of the test and the distribution which is tested. The value of α is often chosen to be small, like 0.01, 0.025 or 0.05; the value of α depends on the desired level of (un)certainty [17].

Kolmogorov-Smirnov

The Kolmogorov-Smirnov test can be used to test several distributions [10]. In general it compares samples with a reference probability distribution, where the statistic quantifies the distance between the empirical distribution function and the reference distribution; this is illustrated in figure A.2. The distributions which can be verified are the normal, Poisson, uniform and exponential distribution.

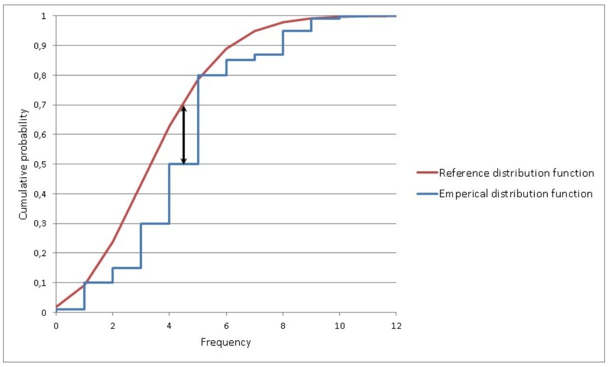

Figure A.2: Kolmogorov-Smirnov

In this study the Kolmogorov-Smirnov test is used to check whether the Poisson and normal distribution is applicable to model occurrence data, so the Poisson distribution is used as reference probability distribution.
The hypothesis are as follows:

$$H_0 : F = F_0; \qquad H_1 : F \neq F_0$$

Where F denotes the empirical distribution function and F_0 the reference distribution function.
The test statistic is defined as [10]:

$$K_n = \sqrt{n} \sup_{-\infty < x < \infty} |F_n(x) - F(x)|$$

Here $F_n(x)$ represents the empirical distribution function and $F(x)$ represents the reference probability distribution. Also, sup denotes the smallest upper bound and n the number of measurements.
In this test H_0 is to be rejected when K_n is sufficiently large, depending on the choice for α.

A.3 Regression: general and Poisson regression (with rates)

Poisson regression is used to estimate the influence which factors have on the number of occurrences for a certain type or severity.

Regression analysis is a statistical technique to estimate the relationship between variables. The relationship involves that the value of a stochastic variable y can be predicted by using other variables x. Variable y is called the dependent variable and x is (are) called the independent variable(s). Also, regression contains a measurement error called ϵ which is assumed to be normally distributed ($N(0, \sigma^2)$) [18]. Another assumption in regression analysis is that the Gauss-Markov property is fulfilled; which states that the expected error is zero, the variation of the error is constant, and the observations are uncorrelated [18].

Multiple regression is used when multiple factors influence the dependent variable y. When k independent variables are taken into account, the regression model equation is formulated as:

$$y_i \quad = \quad \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_k x_{ik} + \epsilon_i \text{ for } i = 1, 2, \ldots, n$$

Here n denotes the number of measurements. There are thus n of these equations, which can be given in matrix form by $y = X\beta + \epsilon$.
Regression determines estimators for the parameters β_i, which is done by using the least squares method.

Poisson regression (with rates) is different with respect to other regression models in several ways, the most important differences are:

- Assumption of Poisson distribution regarding the distribution of μ (instead of the normal distribution), where μ is the expected number of occurrences.
- The value of μ represents a counting variable and is thus always integer valued ($\mu \in \mathbb{N}$)[1].
- Influence factors taken into account can be either continues, integer valued, or categorical[2].

The data should have a Poisson distribution, which is verified in 8.1. Knowing this, the likelihood function is:

$$\begin{aligned} P(data|\mu_1, \ldots, \mu_n) &= \frac{\mu_1^{x_1}}{x_1!} e^{-\mu_1} \frac{\mu_2^{x_2}}{x_2!} e^{-\mu_2} \ldots \frac{\mu_n^{x_n}}{x_n!} e^{-\mu_n} \\ &= \prod \frac{\mu_i^{x_i}}{x_i!} e^{-\mu_i} \end{aligned}$$

When interaction of the independent variables is not taken into account, than μ_i is estimated by:

$$ln(\mu_i) \quad = \quad \beta_0 + \sum_j \beta_j \cdot X_j$$

or equivalent:

$$\mu_i \quad = \quad e^{\beta_0 + \sum_j \beta_j \cdot X_j}$$

Here β_0 is the offset parameter, and β_j is the parameter for the independent variables. X_j is the indicator variable ($X_j \in \{0, 1\}$) which denotes if the variable is taken into account or not.

[1] μ was earlier denoted by y in the model equation, now denoted as μ due to practical reasons.
[2] Categorical: variable which exists from categories. E.g., X_i denotes whether flights take place during an inbound-, outbound-, or off-peak.

When count data is represented as rates ($R_i = \frac{\mu_i}{V_i} = \frac{\text{Number of occurrences}_i}{\text{Number of movements}_i}$) the Poisson regression takes the following form:

$$ln(R_i) = \beta_0 + \sum_j \beta_j \cdot X_j$$

Following the properties of logarithms[3] this is equal to:

$$ln(\mu_i) = ln(V_i) + \beta_0 + \sum_j \beta_j \cdot X_j$$

Usually denoted by:

$$ln(\mu_i) - ln(V_i) = \beta_0 + \sum_j \beta_j \cdot X_j$$

Poisson regression applied on rates assumes the outcome is the count (numerator), while the log of the denominator ($ln(V_i)$) is considered as covariate with regression coefficient fixed to 1. To distinguish it from the other covariates, it is referred to as the offset [20].

An addition to the model is the interaction of the independent variables. The interaction shows whether the influence of variables is conditional on each other. The interaction between for example two categorical variables X_1 and X_2 shows the influence variable X_1 has given the value of X_2 and vice versa. The coefficients of these interaction-terms are denoted by $\gamma_{k,l}$, the model equation becomes:

$$ln\left(\frac{\mu_i}{V_i}\right) = \beta_0 + \sum_j \beta_j \cdot X_j + \sum_{\forall k \neq l} \gamma_{k,l} \cdot X_k \bullet X_l$$

Where $X_k \bullet X_l$ is the product between two indicator variable and is thus an indicator variable itself ($X_k \bullet X_l \in \{0, 1\}$).

The estimators for the coefficients β_i and $\gamma_{k,l}$ are again estimated by the least squares method.

Determining if the variables have a significant influence is done by using Wald's test. Wald's test has the following hypothesis:

$$H_0 : \beta_i = 0 \text{ and } \gamma_{k,l} = 0 \ \forall_{i,k,l}; \qquad H_1 : \beta_i \neq 0 \text{ and } \gamma_{k,l} \neq 0 \ \exists_{i,k,l}$$

The test statistic is compared to the chi-squared distribution, the test statistic is:

$$\frac{(\hat{\theta} - \theta)^2}{var(\hat{\theta})}$$

Where $\hat{\theta}$ is the maximum likelihood estimator of the parameters, and θ is the proposed value of the estimators.

When the significance value of an independent variable is larger than the significance level, than the independent variable has no significant influence and should be left out the model.

[3] $ln(R_i) = ln(\mu_i) - ln(V_i)$

A.4 Correlation of variables

To determine the correlation between the severity classes the correlation coefficient is determined between the severity classes. This correlation coefficient is determined with Pearson's correlation coefficient, which is defined as [19]:

$$\begin{aligned}\rho(X,Y) &= \frac{covariance(X,Y)}{\sigma_X \sigma_Y} \\ &= \frac{E[(X-\mu_X)(Y-\mu_Y)}{\sigma_X \sigma_Y}\end{aligned}$$

Where σ denotes the standard deviation.
The estimation of the correlation with a given dataset is calculated by [19]:

$$\begin{aligned}\rho(x,y) &= \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y} \\ &= \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}}\end{aligned}$$

Where $\bar{x}$, $\bar{y}$ denotes the expected number of occurrences of resp. x and y.

The value of ρ indicates the strength and nature of the correlation, where $-1 \leq \rho \leq 1$. When ρ is close to one there is a positive correlation, when it is close to minus one it indicates a negative correlation, and when it is close to zero there is no (or little) correlation.

Appendix B

End-user of model

This section describes how the model is used in practical sense: the usage of SPSS in determining distributions, and performing Poisson regression. This is described in appendix B.1. Carrying out the model requires knowledge on the practical necessities and limitations of the model, this is described in appendix B.3

B.1 Guide through SPSS

This section gives information on how to gain and interpret output from SPSS (version 21) for the models used in this study.

Selection of data in SPSS

Selection of data in SPSS for performing the Kolmogorov-Smirnov test starts by creating a table which has the form of table B.1.

Where m denotes the number of the months considered and n indicates the number of types consid-

Number occurrences type 1	Number occurrences type 2	...	Number occurrences type n
# month 1	# month 1	...	# month 1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
# month m	# month m	...	# month m

Table B.1: Input Kolmogorov-Smirnov

ered. Note here: the lay-out for the different types of occurrences (what-categories) and the different types of severity classes is identical.

For selecting data to perform Poisson regression, the data set should be of a form similar to table B.2. Note: in SPSS the options of the factors have to be labeled so SPSS sees them as categories and not as integers.

Factor 1	...	Factor i	...	Factor n	Number occurrences	Number of corresponding movements
$j_1 \in \{f_1\}$	...	$j_i \in \{f_i\}$	...	$j_n \in \{f_n\}$	$m_{occurrence} \in N$	$m_{movement}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$k_1 \in \{f_1\}$	...	$k_i \in \{f_i\}$	...	$k_n \in \{f_n\}$	$m_{occurrence} \in N$	$m_{movement}$

Table B.2: Input regression for type of occurrence or severity class

Where n denotes the number of factors taken into account, $m_{occurrence}$ the number of occurrences corresponding to the combination of factors, $m_{movements}$ the number of movements corresponding to the combination of factors. All combinations of the options within the factors must be taken into account, as done in section 8.2.

Selection of statistical tests in SPSS

To perform the test for Kolmogorov-Smirnov the following selections are made in SPSS:

1. Go to 'Analyze', 'Nonparametric Tests', 'Legacy Dialogs', and '1-Sample K-S...'
2. Select all categories of which the distribution is to be tested in 'Test Variable List'
3. Select the Poisson distribution in the box 'Test Distribution'
4. Press 'OK'

Selection of Poisson regression in SPSS

When the table similar to that in table 8.3 of section 8.2 is inserted in an SPSS datafile, take the following steps:

1. Go to 'analyze' and select 'generalized linear models' (twice)
2. Choose 'Poisson log-linear' in the tab 'Type of model'
3. In tab 'Response' select the number of occurrences as 'Dependent variable'
4. Select the factors chosen as 'Factors' in the tab 'Predictors'.
   Go to options to select 'Descending' in 'Category Order for factors' .
   Also select 'number of taxi-movements' as 'Offset variable' in the tab 'Predictors'
5. In tab 'Model' select all factors to the box called 'Model'
6. Press 'OK' at the bottom of the screen.

Interaction terms are added to the model in tab 'Model'. This is done as follows:

1. Select the first factor of the interaction term
2. Press the pointing downwards (underneath the box 'Factors and Covariates).
3. Press the button 'By *' underneath the box 'Term'
4. Select the second factor of the interaction term
5. Press the pointing downwards (underneath the box 'Factors and Covariates).
6. The box 'Term' now says 'Factor1*Factor2'
7. Press 'Add to model'

The interaction term for three of more factors can be added similarly by repeating step 1 to 3.

Output tables in SPSS for Poisson regression

SPSS gives the following tables as output after performing Poisson regression:

- **Model information** tells what is chosen as dependent variable, probability distribution, Link function, and Offset variable.
- **Case processing summary** gives the number of cases which are in-/ excluded, make sure that all cases are included; this should be the case automatically.
- **Categorical variable information** gives the number of cases for each (choice within the) factor.
- **Continuous variable information** gives information about the dependent and the offset variable; it gives the number of cases, the minimal and maximal value, the mean and the standard deviation.
- **Goodness of fit** gives several values, where 'Value/ df' is the only one that has to be considered for now. This value indicates how well the Poisson distribution fits. The Poisson distribution is applicable when this value is close to one, when it is bigger than one it implies that an $over-dispersed$ Poisson distribution might be a better fit; there is no formal test to decide on preference for the regular Poisson or the over-dispersed Poisson distribution.
  The Poisson distribution has a mean which is equal to the variance. An over-dispersed distribution means a distribution has a variance larger than the mean.
- **Omnibus test** compares the fitted model against the null-model, when this value is below α it is said that the 'model outperforms the null-model'.

- **Tests of model effects** indicates whether the influence of factors is observable, which is the case when the values of 'Sig.' are smaller then the significance level α.

- **Parameter estimates** gives information on each estimated parameter, including: the values of β_i, the corresponding standard errors, the 95% Wald confidence Interval, and the outcomes of the Hypothesis test.

B.2 Obtaining numerical results Poisson regression in SPSS

This section describes in detail how the numerical results of the case studies in section 8.2 are obtained with SPSS. The process in SPSS is discussed per step, which is done for both case studies.

The procedure is to perform Poisson regression as mentioned before, where all factors and all interaction terms are taken into account. When the output is generated, start looking at the significance of the factors and interaction terms in the table called 'Tests of model effects'.
When there are interaction terms have a significance value which is larger than 0.05 they have to be removed from the model. The interaction-term which has highest significance is removed from the model, than output is generated again. If there is still an interaction term with significance value larger than 0.05 remove it from the model as well. This process is continued until there are no interaction terms with significance level higher than 0.05.
When only significant interaction-term are left, start looking at the significance level of the factors. Remove the insignificant factors by the same process as the interaction-terms.

The occurrence rates are determined when all significant factors and interactions are determined. They can be determined by the model equation for each combination of the states of the factors.

Deviation taxi

The data is collected as mentioned in table 8.2 of section 8.2.
All steps of performing Poisson regression as mentioned above are followed, starting with generating the output when all factors and all interaction terms are taken into account.
The output shown is given in figure B.1:

Tests of Model Effects

	Type III		
Source	Wald Chi-Square	df	Sig.
(Intercept)	3004,031	1	0,000
Type_Carrier	5,393	1	,020
Type_Peak	4,260	2	,119
Type_WTC	57,246	2	,000
Type_Carrier * Type_Peak	,556	2	,757
Type_Carrier * Type_WTC	3,923	1	,048
Type_Peak * Type_WTC	,075	2	,963

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, Type_Carrier * Type_Peak, Type_Carrier * Type_WTC, Type_Peak * Type_WTC, offset = Log_Number_Movements

Figure B.1: Output for all factors and all interactions of factors.

Next the interaction terms (and factors) with no significance are removed one by one, starting by the interaction between the type of peaks and the type of aircraft. The output is than given by figure B.2:

Tests of Model Effects

Source	Type III		
	Wald Chi-Square	df	Sig.
(Intercept)	2384,505	1	0,000
Type_Carrier	6,600	1	,010
Type_Peak	7,981	2	,018
Type_WTC	57,353	2	,000
Type_Carrier * Type_Peak	,790	2	,674
Type_Carrier * Type_WTC	4,311	1	,038

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, Type_Carrier * Type_Peak, Type_Carrier * Type_WTC, offset = Log_Number_Movements

Figure B.2: Output for all factors and all interactions of factors.

Next the interaction between the type of carrier and the type of peak is removed from the model, the output is given in figure B.3:

Tests of Model Effects

Source	Type III		
	Wald Chi-Square	df	Sig.
(Intercept)	3860,460	1	0,000
Type_Carrier	10,217	1	,001
Type_Peak	9,893	2	,007
Type_WTC	56,913	2	,000
Type_Carrier * Type_WTC	3,992	1	,046

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, Type_Carrier * Type_WTC, offset = Log_Number_Movements

Figure B.3: Output for all factors and all interactions of factors.

All factors and interactions in the model are now significant, thus no more factors and interactions are removed from the model.
The parameter estimates are given in figure B.4. The column 'B' gives the estimates for the parameters β and γ.

Parameter Estimates							
			95% Wald Confidence Interval		Hypothesis Test		
Parameter	B	Std. Error	Lower	Upper	Wald Chi-Square	df	Sig.
(Intercept)	-7,953	,3781	-8,694	-7,212	442,468	1	0,000
[Type_Carrier=1]	-1,323	,2259	-1,766	-,880	34,317	1	,000
[Type_Carrier=0]	0[a]						
[Type_Peak=2]	,565	,3585	-,138	1,267	2,482	1	,115
[Type_Peak=1]	,923	,3495	,238	1,608	6,977	1	,008
[Type_Peak=0]	0[a]						
[Type_WTC=3]	-1,984	,3471	-2,664	-1,303	32,662	1	,000
[Type_WTC=2]	-1,537	,2192	-1,966	-1,107	49,160	1	,000
[Type_WTC=1]	0[a]						
[Type_Carrier=1] * [Type_W	1,018	,5096	,019	2,017	3,992	1	,046
[Type_Carrier=1] * [Type_W	0[a]						
[Type_Carrier=0] * [Type_W	0[a]						
[Type_Carrier=0] * [Type_W	0[a]						
[Type_Carrier=0] * [Type_W	0[a]						
(Scale)	1[b]						

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, Type_Carrier * Type_WTC, offset = Log_Number_Movements

a. Set to zero because this parameter is redundant.
b. Fixed at the displayed value.

Figure B.4: Estimates for parameters β for all significant factors and interactions of factors.

Note: a small modification on the data is made to prevent numerical problems. Two states of two factors cause numerical problems when determining influence and estimating parameters, this combination is the home carriers with light aircraft. For the inbound peak there have been no movements, for the other peaks there is a single movement.
These number are so small that they do not cause numerical problems when interaction is not taken into account, but they do when interaction is taken into account. The number of movement for the other combination of states are all well over 10.000, some are well over 100.000.
To prevent numerical problems the combination where home carriers with light aircraft are taken out of the data. The results with interaction (without home carriers using light aircraft) are compared to the results without interaction (with home carriers using light aircraft), one can see a clear improvement of the estimators. This improvement can be seen by looking at the estimated occurrence rates, and how much they differ from the empirical occurrence rates. By modifying the data slightly, the estimated occurrence rates are closer to the empirical occurrence rates and are thus assumed to be an improvement of the model.

Major incidents

The data is collected as mentioned in table 8.3 of section 8.2.
All steps of performing Poisson regression are followed as in the previous example, starting with generating the output when all factors and all interaction terms are taken into account.

The results found are given in figure B.5:

Tests of Model Effects

Source	Type III Wald Chi-Square	df	Sig.
(Intercept)	2272,271	1	0,000
Type_Carrier	7,428	1	,006
Type_Peak	5,037	2	,081
Type_WTC	25,795	2	,000
Type_Carrier * Type_Peak	,611	2	,737
Type_Carrier * Type_WTC	,277	1	,599
Type_Peak * Type_WTC	4,414	3	,220

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, Type_Carrier * Type_Peak, Type_Carrier * Type_WTC, Type_Peak * Type_WTC, offset = Log_Number_Movements

Figure B.5: Output for all factors and all interactions of factors.

Now the interaction terms with no significance are removed one by one, no interaction-terms remain; first the interaction between type of carrier and type of peak is removed, than the interaction between type of carrier and type of aircraft, last the interaction between type of peak and type of aircraft is removed from the model.
This results in a model without interaction terms, but also the factors need to be viewed on significance. The results without interactions are given in figure B.6.

Tests of Model Effects

Source	Type III Wald Chi-Square	df	Sig.
(Intercept)	4413,289	1	0,000
Type_Carrier	11,560	1	,001
Type_Peak	4,528	2	,104
Type_WTC	39,292	2	,000

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_Peak, Type_WTC, offset = Log_Number_Movements

Figure B.6: Output for all factors and all interactions of factors.

The significance value of the type of peak is to high and thus this factor is left out the model, resulting in figure B.7:

Tests of Model Effects			
	Type III		
Source	Wald Chi-Square	df	Sig.
(Intercept)	6724,602	1	0,000
Type_Carrier	11,722	1	,001
Type_WTC	38,201	2	,000

Dependent Variable: Number_Occurrences
Model: (Intercept), Type_Carrier, Type_WTC, offset = Log_Number_Movements

Figure B.7: Output for all factors and all interactions of factors.

The parameter estimates are given in figure B.8. The column 'B' gives the estimates for the parameters β.

Parameter Estimates							
			95% Wald Confidence Interval		Hypothesis Test		
Parameter	B	Std. Error	Lower	Upper	Wald Chi-Square	df	Sig.
(Intercept)	-7,762	,2500	-8,252	-7,272	964,044	1	0,000
[Type_Carrier=1]	,543	,1585	,232	,854	11,722	1	,001
[Type_Carrier=0]	0[a]						
[Type_WTC=3]	-1,199	,3098	-1,806	-,592	14,971	1	,000
[Type_WTC=2]	-1,678	,2835	-2,234	-1,122	35,027	1	,000
[Type_WTC=1]	0[a]						
(Scale)	1[b]						

Dependent Variable: Number_OccurrencesModel: (Intercept), Type_Carrier, Type_WTC, offset = Log_Number_Movements
a. Set to zero because this parameter is redundant.
b. Fixed at the displayed value.

Figure B.8: Estimates for parameters β for all significant factors and interactions of factors.

B.3 Carrying out the model

This section describes how the model can be used properly. The model is developed to support advanced risk based approaches, this section describes the benefits and limitations of the model. This section first describes how finding statistical distributions helps in risk assessment. Next it describes how performance standards are identified by distributions in the model, followed by the description on how to identify factors which contribute to risk. Last, the limitations in using the model are viewed on both theoretical and practical level.

Finding and using Poisson distributions

The model used is built on the fact that the Poisson distribution is applicable for the number of occurrences which take place in a month; for both the number of occurrences for certain types and certain severity classes. The Poisson distribution is a reasonable choice to look for since it counts occurrences which happen with a low average per time unit, but which are not rare; as discussed in section 6.2. Finding statistical distributions helps identifying characteristics of the data, and makes it possible to use certain models for the analysis of the data.
First of all, using the found distribution to study reference values is a simple but insightful way to perform safety analysis on the operation.
Second, the Poisson distribution enables the use of Poisson regression, it neatly connects to the analysis for finding factors which contribute to risk. A major advantage of regression analysis is that it looks at the factors directly connected to occurrences rather then overall circumstances. Furthermore, it is capable to take multiple factors into account and it identifies its individual (potential) influence, and also quantifies it.

Note: as stated in section A.1, there is a close relation between the Poisson and the normal distribution: when the parameter λ of the Poisson distribution increases, then the Poisson distribution resembles the normal distribution. When more data is collected it is not necessarily true that the obtained Poisson distribution converges to the normal distribution. As long as the average number of occurrences per month does not increase, the Poisson distribution is likely to still hold.

Practicability of reference values

The first step in identifying practicability of reference values is the identification of the distribution behind the number of occurrences, where the focus in this study is on the Poisson distribution (as discussed in section 6.2). For LVNL the most interesting performance standard is focused on severe occurrences, since they give an important indication on how safe their operation is. In section 9.1 the practicability of reference values is indicated for two possible choices, there are multiple other definitions which the reference values can have.

Studying reference values starts by determining the distribution behind the number of occurrences, which is done with unit 'number of occurrences per month'. The number of occurrences per time unit is chosen because the Poisson distribution needs integer values (it is a 'counting' distribution). The time unit month is chosen due to practical reasons, enough data-points are needed to determine the distribution, but also the number of occurrences per time unit should not be too small. If the number of occurrences per time unit is too small it is called a 'rare event', and a statistical distribution is probably not found. There is four years of data available for this study, which makes time unit 'years' to gross, and 'days' to refined.

The reference values come in sight once distributions are found. However, reference values are most likely to be chosen with a unit other then 'number of occurrences per time unit', and thus a translation has to be made for making the practicability of the reference value relevant.
The examples discussed in section 9.1 are the translation from flight-hours and movements to years, which is done by determining the average number of movements per year and the average time a flight is in the Schiphol area.

An additional option in this analysis is plotting the average number of occurrences per time unit which took place in the separate years, this identifies how the average changed over time. This value can be plotted as a line similar to the reference value-line.

The benefits of using the Poisson distribution when searching for the practicability of the reference value, is that an exceedance probability can be estimated. Using the average number of occurrences and the reference value gives an indication on whether the number of occurrences is higher or lower than the reference value.

However, when the average number of occurrences is smaller then the reference value, but a considerable part of the probability distribution lies above the reference value, than there is still a (possibly large) probability of exceeding the reference value. As can be seen in section 9.1, the average number of occurrences in 2012 is close to reference value 2. However, the major probability mass lies above reference value 2 and there is thus a large probability of exceeding the reference value; even though the average does not differ much from the reference value.
A similar thing can happen when the average value lies below reference value 1, as can be seen in section 9.1. The average number of occurrences in 2013 is far below the reference value. However, the area under the graph on the right of the reference value indicates that there is still a considerable probability of exceeding the reference value.
Thus, the added value of using the Poisson distribution is that it is not only possible to see where the average number of occurrences lies with respect to the reference value, but also what the probability is of exceeding the reference value.

Identifying factors which contribute to risk

Poisson regression is used for identifying factors which contribute to risk, for this model the Poisson distribution is required. So again the Poisson distribution is tested first for the desired setting of the circumstances.

Once the Poisson distribution is determined factors are chosen to take into account. The factors taken into account must be chosen with great care, there are some strict limitations which must be fulfilled to obtain meaningful results.

The first property of the factors is that the factor is either integer valued[1] or categorical. Where categorical means that the factor is divided into certain categories; e.g., the different type of carriers.
The second property of the factors taken into account is that they have to be independent of each other; e.g., the type of carrier does not depend on the type of peak, but the level of visibility can be dependent on the precipitation (and vice versa). This restriction holds for each type of regression.
A third property of the factors taken into account is that they cannot be a direct cause of occurrences. One cannot say that event A has a potential influence on the number of occurrences when it actually is the occurrence. E.g., Initially it was intended to find how the type of occurrences influence the severity classes, but studying if an airspace infringement (which is an occurrence) influences the number of occurrences is rather silly.

Once the factors are chosen the data is gathered, for each possible combination of the items within the factors the number occurrences and the number of movements is determined. The Poisson regression determines whether influence is observable and quantifies it; this is shown in section 8.2.
Next, the model parameters are used to determine the expected number of occurrences per month for the chosen combination of factors and is made insightful in a graph. This graph shows the expected number of occurrences per movement on the y-axis (which is on a logarithmic scale), and the number of movements on the x-axis.
Looking at the graph one has a quick overview of the influence of the factors. Moreover, one can identify risk swiftly by looking at the graph.

Limitations model

Each part of the model has its limitations, studying reference values for the number of occurrences can still be performed when a distribution other then Poisson is found, the only difference in approach is that the distribution drawn is different. The remaining analysis for the reference values remains the same. The suggested analysis is not applicable as described when no distribution is found.

For Poisson regression the Poisson distribution is required, the model equation of Poisson regression is based on the distribution function of the Poisson distribution. When the Poisson distribution is not applicable, then the (Poisson) distribution function is not valid and thus the model equation of regression is not valid. When other distributions are applicable, other forms of regression can be searched for.

[1] Whole numbers, $n \in \mathbb{N}$

There is no theoretical restriction for Poisson regression with respect to the number of factors which can be taken into account, however a practical restriction is present. The number of factors taken into account is limited: if too many factors are taken into account, than there are too many possibilities in the number of combinations. Notice here: the number of combinations grows exponentially. When only factors are taken into account which all have two options, the number of combinations is doubled when one factor is added. The formula to determine the number of combinations N is:

$$N = f_1 \cdot f_2 \cdot \ldots \cdot f_n$$

Where f_i denotes the number of possible values of factor i, and n denotes the number of factors taken into account.
In deciding the number of combinations that are taken into account one should realize that choosing the number of combinations which is considered acceptable depends on the number of occurrences that have taken place: when there are only a few occurrences (e.g., serious incidents) the number of combinations should be low, since regression cannot extract influence when there are too few observations. Numerical problems can occur. On the other hand: when many occurrences are available (e.g., occurrences with no safety effect), more factors can be taken into account. Always be careful not to overdo the number of factors, this causes loss of overview.

The number of movements linked to the occurrences considered also influences the outcome of the regression analysis. When only few occurrences are available, but there are many movements, the regression analysis may not be able to extract the influence of the factors taken into account; e.g., 10 occurrences have taken place for a certain selection of the data, 16 combinations are possible, and there are 1.5 million movements, regression might have trouble identifying the influence correctly. Moreover, it is likely to indicate that there is no (significant) influence.
The fact that the occurrences are divided over the combinations taken into account does not help regression's cause. It is not the reason regression cannot extract influence, but it does not help either. Most factors can be used for different divisions of the data used (e.g., type of aircraft), but for each division a close look should be given on the presence of the options within the factors, since they are not always all needed. E.g., sometimes the type of aircraft is not known (often with Airspace Infringements), so in principle the type of aircraft can be: light, medium, heavy, and unknown. However, on the ground-operation of Schiphol Airport all types of aircraft are known, so when the occurrences regarding the ground operation are considered one does not need to use the option 'unknown type of aircraft'. Moreover, the regression analysis will not take this option into account as it does not have any movements (and thus also no occurrences).

Appendix C

Data

Contents of this section is only shown in the confidential version of the report.

C.1 Types of occurrences: combined types

C.2 Types of occurrences: dependencies

Combinations of types

Table C.1: Dependencies in type of occurrences - part 1

Table C.2: Dependencies in type of occurrences - part 2

Table C.3: Dependencies in type of occurrences - part 3

Grouping of types

Table C.4: Dependencies in type of occurrences - part 4

Table C.5: Dependencies in type of occurrences - part 5

Table C.6: Dependencies in type of occurrences - part 6

Table C.7: Dependencies in type of occurrences - part 7

Table C.8: Dependencies in type of occurrences - part 8

Table C.9: Dependencies in type of occurrences - part 9

Table C.10: Grouping types of occurrences on location

Table C.11: Grouping types of occurrences on cause/effect

Table C.12: Grouping types of occurrences on flight-phase (1)

Table C.13: Grouping types of occurrences on flight-phase (2)

Appendix D

Confidential output tables

This section contains confidential output tables D.1, D.2, D.3, D.4, and D.5.

Table D.1: Output Kolmogorov-Smirnov - Types of occurrences

Table D.2: Output Kolmogorov-Smirnov - Types with closer look, data from February 2011 until December 2012

Table D.3: Confidence intervals - Types of occurrences

Table D.4: Adding data 2013: Output Kolmogorov-Smirnov - Airspace infringements

Table D.5: Adding data 2013: Output Kolmogorov-Smirnov - Types of occurrences