THE IMPACT OF CONSTRAINTS ON EQUITY PORTFOLIOS

A quantitative analysis of the restrictiveness of constraints for developed and emerging market equity strategies

Master thesis
Tim Posthuma

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GENERAL INFORMATION

Author	Tim Posthuma
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Date	21-3-2014
Institute	University of Twente
Faculty	School of Management and Governance Drienerlolaan 5 7522 NB Enschede
Program	Industrial Engineering & Management
Track	Financial Engineering & Management
Company	MN services N.V. Prinses Beatrixlaan 15 2595 AK The Hague

Supervisory committee:

First supervisor	Dr. B. Roorda *University of Twente*
Second supervisor	Dr. R. Joosten *University of Twente*
External supervisor	Drs. A. Van Wieren *MN*

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Management summary

MN is a fiduciary manager and as such entrusted with investment management of several Dutch pension funds. In order to remain ‘in control’ and limit risks, investment constraints are imposed on portfolio managers. The goal of this thesis is to quantify the effects of the constraints. As required by MN the impact of a single asset constraint, a sector constraint and a country constraint (all benchmark relative constraints) should be quantified, using data of the MSCI Emerging Market equity index and the MSCI World equity index. The research question is formulated as follows:

“What are the effects of constraints imposed by MN on an emerging market and developed market equity portfolio?”

A literature review yielded two conceptual approaches to quantify the impact of constraints. The first branch follows the Modern Portfolio Theory proposed by Markowitz (1952) and assesses the impact of constraints in terms of risk and reward. In the reviewed literature this is accomplished by comparing the mean-variance efficient frontier and the resulting frontier of an excess return optimization (mean-Tracking Error Volatility frontier and alpha-Tracking Error Volatility frontier). Furthermore, shrinkage of the Information Ratio and the Sharpe Ratio is used to assess the sub-optimality of the constrained portfolio as opposed to the market portfolio.
The second branch follows the ‘fundamental law of active management’ proposed by Grinold (1989). This law determines the added value of a manager (Information Ratio, IR) by its ability to forecast excess returns (Information Coefficient, IC), the ability to implement his alpha view (Transfer Coefficient, TC) and the number of independent bets (breadth, N):

$$IR \approx TC * IC * \sqrt{N}$$

The Transfer Coefficient is the performance indicator of interest for the purpose of this thesis, it is defined as the correlation between the alpha forecasts of an asset manager and the active weights of the portfolio he manages. Any discrepancies between the alpha forecasts and the positioning of the portfolio lead to a Transfer Coefficient less than one. Since those discrepancies can be a result of implied constraints, the Transfer Coefficient can be used to measure the impact of constraints.

Based on this literature review we propose three methods which should meet the requirements of MN in principle. The first method is an ex-post analysis of a large sample of portfolio values from randomly constructed portfolios. The rationale behind this method is that the underlying assumptions of the efficient frontier do not hold (otherwise active management would not make sense because structural outperformance over the market portfolio would be impossible). The randomly constructed portfolio should provide an overview of the possibilities of the managers. The first two moments of the large sample of portfolio returns should be used to determine the Sharpe ratio, comparing this to the Sharpe ratio of the constrained large sample of portfolio returns leads to Sharpe ratio shrinkage as indicator for the restrictiveness of a particular constraint. In practice it turned out that the procedure to construct random portfolios converged to equally weighted portfolios for benchmarks with a large number of constituents (which is the case with the MSCI Emerging markets and the MSCI World indices). Developing a new procedure to construct true random portfolios is beyond the scope of this thesis.
The second method consisted of an ex-post analysis of the deformation of the efficient frontier by imposing stricter values on a constraint. This procedure provided results about the impact of constraint. It yields a graphical representation of the efficient frontier and deformation of the efficient frontier under constraints, but lacked the ability to quantify the impact of constraints. More important, it led to the notion that assessing constraints in terms of risk and reward leads to biased conclusions with respect to the impact of constraints. An example will be given to clarify the foregoing statement:

If the performance of a benchmark is driven by a specific sector, and historical return data is used as input to analyse a constraint in terms of loss in return and mitigation of risk, then a bias occurs with respect to the conclusions. The logical conclusion is that the sector constraint is very restrictive since the loss in performance exceeds the mitigation of risk.

The third method uses the Transfer Coefficient to overcome this bias. The Transfer Coefficient determines the ability of a manager to transfer his alpha skills into actual portfolio positions. A priori, one cannot say whether a low TC is good or bad. This depends on the context. Since a very skilled manager would see his value added shrink because of a low TC (he is not able to exploit has alpha forecast) whereas a medium or low skilled manager could achieve outperformance given a low TC as the unwanted bets which are a result of the low TC could deliver outperformance (since his alpha forecasts are most of the time wrong).
The flow of the model applied in this study is depicted in Figure 1.

Figure 1: Flow of the Transfer Coefficient model.

The expected return and expected covariance matrix of the MSCI Emerging Market and MSCI World index are determined to calculate the security weights for the MN required strategies. The unconstrained active weights (difference between strategy and benchmark weights) are determined in the second phase as proxy for the alpha view. Third step is to calculate the constrained active weights. The unconstrained and constrained active weights are used to determine the Transfer Coefficient for different levels of the particular constraint. The cut-off point and the slope of the Transfer Coefficient for the part where the constraint is binding are then used to determine how restrictive a constraint is.

Applying the Transfer Coefficient method leads to the conclusion that the country constraint is the most restrictive constraint in the evaluated scenarios. It is binding for all values of the constraint which are currently applied. Furthermore, the restrictiveness of the single asset constraint is most sensitive for changes in the value of the constraint given that it is binding. This is partly due to the fact that it is binding for restrictive values of the constraint and because it is the only constraint which could force the active portfolio to replicate the benchmark.

Preface

Monday morning, 7 a.m., sitting in the train, destination The Hague. A new week as Product Analyst at MN lies ahead of me. Who would have of taught of that?

One year ago I was expected to decide where I would carry out my Master thesis, the finish of an exciting journey. What do I exactly want? Where should I go? I decided to use this opportunity to explore myself. I contacted MN, an institutional investor at heart of the Dutch pension system. After a good meeting they offered me a great assignment. Wait what? Carry out the Master thesis at MN? Leaving friends, family, my girlfriend? Just seize the opportunity, it might even be the last chance to discover things in such a drastic way.

What a ride. I've once climbed Half Dome with friends, they compared it with the process of writing the Thesis. I couldn't think of a better comparison. The promised reward is enormous, but what a struggle to get there. Literally everything is in it, an enormous mountain you have to climb, picking the wrong turn on your way up, seeing the finish but still have to climb for a few hours, the unexplainable satisfaction once you reached the top, the amazing views, but also the long climb down to finish the project. Now here we are, in the train to The Hague, the final version of the thesis will be handed in today and the colloquium is coming Friday. Being aware that I already found a great job, but more important, knowing that I always can fall back on the support of my friends, family and girlfriend.

At this point I am very grateful to a few people I want to mention explicitly, at first I want to thank Arjan van Wieren and his Manager Selection & Monitoring team for giving me the opportunity and freedom to explore the world of Finance and carrying out my Thesis. Especially Abhishek, Wouter and Frederik helped me out at difficult times. Furthermore, I could never finish this thesis without the critical notes and in-depth discussions with my supervisors Berend Roorda and Reinoud Joosten. Ultimately, I could never achieve all this without the continuous support of my parents and my girlfriend, Mirthe.
Enjoy reading the Thesis!

Cheers,

Tim

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Content

Glossary

AIR	Adjusted Information Ratio
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
EPS	Earnings Per Share
EMH	Efficient Market Hypothesis
ERC	Equal Risk Contribution
ETF	Exchange Traded Fund
EW	Equal Weighted
FAF	Financial Assessment Framework
IR	Information Ratio
IC	Information Coefficient
Min Vol	Minimum Volatility
MPT	Modern Portfolio Theory
MVO	Mean Variance Optimization
RE	Risk Efficient
r.v.	Random Variable
SEC	Securities and Exchange Commission
SLLN	Strong Law of Large Numbers
SML	Security Market Line
SVD	Singular Value Decomposition
TC	Transfer Coefficient
TEV	Tracking Error Volatility
VaR	Value at Risk

1 Introduction

"MN is one of the largest pension administrators and asset managers in the Netherlands. With over 60 years of experience in these fields, our clients find in us a partner that can assist them with extensive knowledge of the Dutch pension system. Our services are highly valued: we manage assets worth more than EUR 90 billion for a wide variety of pension funds in the Netherlands and in the United Kingdom.[1]"

This was a small introduction of MN from its website. MN's headquarter is located in The Hague, but it also holds office in Amsterdam and London. MN is a fiduciary manager, this means that it is delegated with the "fiduciary responsibility for investment management and risk management" according to Clark and Urwin (2010).

The difference between partially outsourcing of activities and fiduciary management can be determined by the type of outsourced activities. According to van Nunen (2007) the outsourced activities should at least concern:

- Advice on ALM studies
- Translate ALM studies into a portfolio, an asset mix and a balance between passive and active risk
- Selecting asset managers
- Monitor asset manager
- Report performance

In order to speak of fiduciary management, a more extended list of activities is pointed out by Shackleton (2011). These activities are all covered by the foregoing bullets.

In practice, however, the tasks outsourced from the pension fund to the fiduciary manager differ per client-manager relationship and are specified in a contract (mandate). MN is responsible for the activities as depicted in Figure 2.

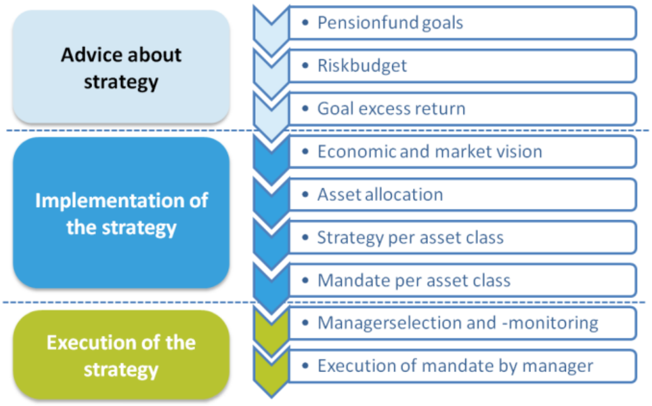

Figure 2: Responsibilities and tasks of MN[2].

One of the responsibilities of the fiduciary manager is advising the client about their strategy and implementing the strategy. This includes the operationalisation of risk management into investment constraints the fiduciary manager is subjected to. This could lead to contradictory objectives for the fiduciary manager, because the fiduciary manager should advise the pension fund. On the other hand it does not want to limit itself too much in order to be able to achieve the performance goals. A situation to which scientific articles usually refer as a principal-agent problem (Eisenhardt, 1989).

[1] From: http://www.mn.nl/portal/page?_pageid=3716,6664201&_dad=portal&_schema=PORTAL
[2] From: MN corporate presentation 2013

Besides advising the client and working within the scope of the mandate, the fiduciary manager has responsibilities with regards to the performance of the assets under management. To optimally fulfil this responsibility, asset management is partially outsourced. Since the fiduciary manager is responsible for the performance it wants to insource risk management. This is done using an Investment Management Agreement (IMA). An IMA usually covers the regulatory aspects concerning the outsourcing, operational agreements, investment guidelines and objectives, investment restrictions and management fees. We focus on investment restrictions in this thesis.

At this point, one can state that the impact of constraints has two dimensions. At first the fiduciary manager should consult the client in the process of operationalisation of risk management. Secondly the fiduciary manager outsources some of the investment activities to external managers, but remains responsible for performance and therefore wants to insource risk management by imposing constraints by means of an IMA. To successfully perform these activities, quantitative insight in the effects of constraints is necessary.

Pension funds, thus MN, are subjected to the 'Pensioenwet' (Pw), therefore pension funds are supervised by the Dutch National Bank (DNB). Part of the 'Pensioenwet' is the Financial Assessment Framework (FAF), the FAF covers the regulatory financial requirements of a pension fund. The indicator which is used most frequently to express a pension fund's health is the coverage ratio. This number expresses the ratio between the available assets of a pension fund and the liabilities. The minimum coverage ratio, dictated by law, is 105%. Besides, pension funds are required to keep a safety buffer for financial setbacks. Because this coverage ratio is determined based on market values, the recent economic turmoil caused declining coverage ratios. Partly due to decreasing interest rates resulting in increasing liabilities, but also due to declining market values of the assets. (see Figure 3)

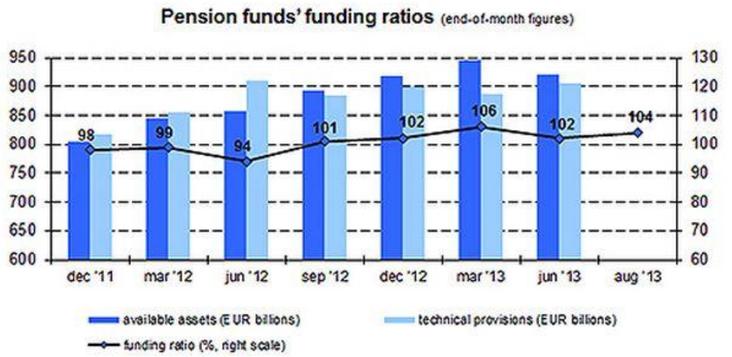

Figure 3: Average coverage ratio of Dutch pension funds[3].

Because of these declining coverage ratios the DNB puts more pressure and emphasis on risk management of the pension funds.

In order to improve risk management and to cope with the demands of the DNB, quantitative insight in the effect of constraints is needed. MN wants to get insight in the following constraints:

- Sector constraint: sector weights in the portfolio might differ X% from the sector weights in the benchmark.
- Country constraint: country weights in the portfolio might differ X% from the country weights in the benchmark.
- Single asset constraint: The minimal amount and maximum amount of a single security in a portfolio.

Investment managers are tempting to outperform the benchmark by using alternative strategies (more on this in Section 3.1). The strategies under which the constraints should be analysed are:

[3] From: http://www.dnb.nl/en/news/news-and-archive/statistisch-nieuwsbericht/dnb296820.jsp

- Equal Risk Contribution.
- Minimum volatility.
- Risk efficient.
- Value strategy.
- Growth strategy.
- Equal Weighted.

The aforementioned constraints are all benchmark relative, the benchmarks required by MN for the analysis are:

- MSCI World equity index.
- MSCI Emerging Markets equity index.

These indices are sufficiently large (resp. 822 and 1610 constituents) and are benchmarks for several MN products, a more extended description of the dataset in Section 4.1.

2 Research design

To formulate the research question, the approach suggested by Verschuren & Doorewaard (2007) will be used. The method consists out of 3 phases:

- Stating the research goals (Section 2.1).
- Design the research model (Section 2.2).
- Formulate the research question (Section 2.3).

Ultimate goal of this approach is to make sure that a research question will be formulated which will lead to a solution of the core problem. Besides, a top level resolution strategy will be constructed to make sure that the problem will be solved in a scientifically sound way.

2.1 Research goal

As pointed out in Chapter 1, the ultimate goal is to see how the investment constraints in an IMA should be structured to ensure a downside performance limit relative to a benchmark. This is however beyond the scope of this master thesis. Goal of this master thesis is:

- *Quantify the effects of constraints on portfolios of external investment managers.*

Result of this should be insight in the restrictiveness of constraints and sensitivity for different values of the constraints.

2.2 Research model

The research model is developed starting from its goal, working its way back to the initial steps. A suitable performance measure should be chosen in order to assess the effects of constraints. Furthermore a content analysis of the IMAs should provide insight in the constraints which are currently applied and it should yield an overview of the prevailing values to which the investment managers are restricted. In order to chose a suitable performance indicators, a literature review should be executed in the field of assessing the effects of constraints. Preliminary research yields that two main branches attempt to assess the effects of constraints in academia. The first branch uses the efficient frontier and assesses the effects of constraints in terms of sub-optimality as compared to the efficient portfolio, whereas the second branch uses the 'Transfer Coefficient' as a measure to assess implementation inefficiency of a portfolio. The research model is depicted in Figure 4.

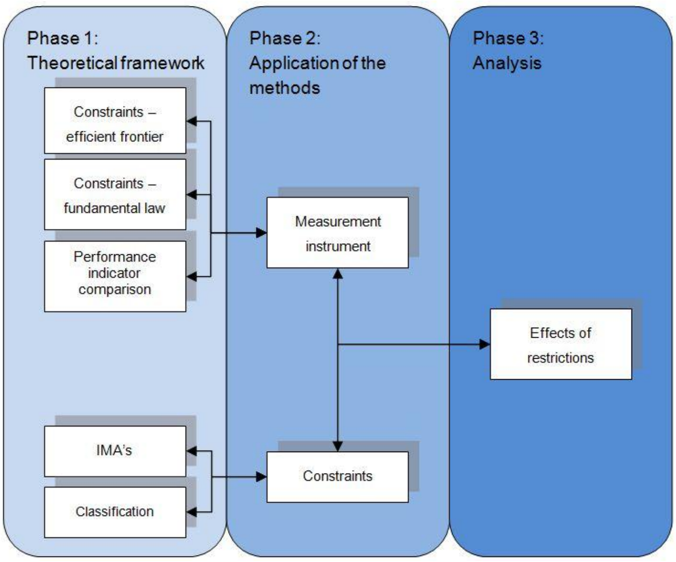

Figure 4: Research model.

This research model can be stated as follows:

A study in the field of constrained portfolio modelling from an efficient frontier perspective as well as a fundamental law of active management perspective should result in a suitable performance indicator, capable of quantifying the effects of IMA investment constraints in terms of the desired performance measures by the problem owner. With this, the ultimate goal of analysing the effects of constraints in terms of restrictiveness and sensitivity can be achieved.

Phase 1: Theoretical framework
The theoretical framework should render an overview of performance indicators suggested in literature. The two main branches in academia which will be reviewed are the branch using the efficient frontier and the branch following the fundamental law of active management. Besides the performance indicators, the main extensions and characteristics should be documented in order to choose a suitable performance indicator.

Phase 2: Application of the methods
The main findings from the theoretical framework will be applied in this phase. After exploring the proposed methods, the practical drawbacks be determined. From that the best suitable model in terms of meeting the MN requirements can be chosen.

Phase 3: Analysis
The constraints will be analysed as soon as an appropriate model has been chosen and implemented. The analysis should provide insight into the restrictiveness and sensitivity of the constraints.

2.3 Research questions

The research goal and the research model of Section 2.1 and 2.2 provide a clear purpose and approach for the assignment but also determine the scope in which the assignment will be carried out. From this, the research question is formulated as follows:

"What are the effects of constraints imposed by MN on an Emerging Market and on aDeveloped Market equity portfolio?"

The research question is split up in sub-questions to answer it in a structured way. The sub-questions are defined as follows:

1. Which performance indicators are proposed in scientific literature to assess the impact of constraints?
2. Which approaches will, in principle, enable the analysis of the MN required constraints within the context of MN?
3. What are the effects of the imposed constraints in terms of restrictiveness and sensitivity for the MN required strategies and benchmarks?

The order and the subjects addressed in the sub-questions are aligned with the research model proposed in Figure 4. The remainder of this thesis is organized as follows, a review of the suggested performance indicators in scientific literature is given in Chapter 3. The performance indicators which suit the requirements of MN will be modelled in Chapter 4. The resulting model from Chapter 4 will be used to analyze the constraints in Chapter 5 and the conclusions and recommendations will be stated in Chapter 6 and 7.

3 Theoretical framework

Imposing constraints on investment companies is not much of a novelty. Almazan *et al.* (2004) state that the first restrictions were imposed by the Securities and Exchange Commission (SEC) to investment companies back in 1940. From this, the amount of legislation and restrictions increased over time. Almazan *et al.* (2004) confirm the situation at MN that restrictions are commonly found in contracts between investors and investment managers and is a monitoring tool to mitigate the agency problems which can occur in an investor – investment manager relationship.
Basically two branches can be distinguished in academia trying to assess the effects of constraints. One branch determines the impact of constraints on the efficient frontier (as proposed by Markowitz (1952)). Developments in this field are heading towards redefining the playing field of the efficient portfolios, for example assessing the efficient portfolio in a mean-variance, mean-tracking error volatility (TEV) or alpha-TEV plane.
The second branch, mainly driven by the work of Grinold (1989), assesses the impact of constraints by shrinkage of the value added of a manager. The Transfer Coefficient (TC) is introduced, a scalar which quantifies the ability of a manager to implement his alpha view.
Appendix A provides an overview of prerequisite knowledge in the field of portfolio theory and active management. Most important definitions for the remainder of this chapter are the definitions of TEV, active weights and alpha. Alpha is defined by Jensen (1968) in the CAPM framework as:

$$\alpha = \left[E(r_p) - r_f\right] - \left[E(r_m) - r_f\right] x\, \beta_{r_p,r_m}$$

With:

α	Beta corrected excess return
$E(r_p)$	Expected portfolio return
r_f	Risk-free interest rate
$E(r_m)$	Expected market return
β_{r_p,r_m}	Beta of the portfolio with the market

Alpha is at best depicted in return-beta space as an offset from the SML (see Figure 5).

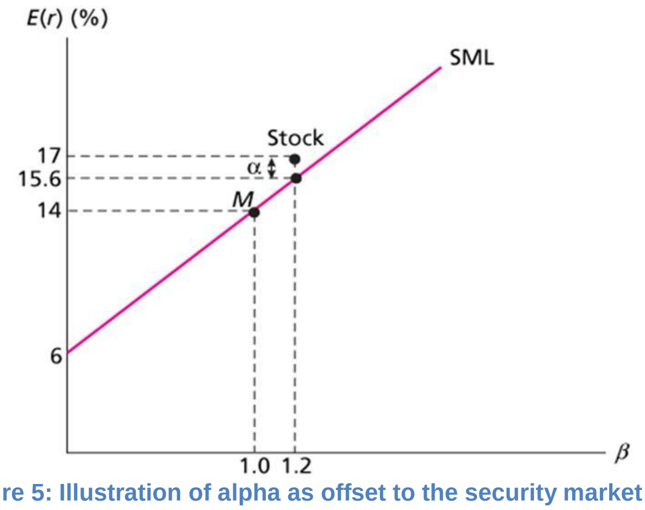

Figure 5: Illustration of alpha as offset to the security market line.

Alpha indicates the amount of excess return over the expected return given the systematic risk of a security or portfolio as compared to a benchmark.

In line with active return, Roll (1992) and Grinold (1989) defined Tracking Error Volatility (TEV) as a measure of the active risk. It is mathematically expressed as:

$$TEV = Std(r_p - r_b)$$

Here r_p is the return series of the active portfolio and r_b the return series of the benchmark portfolio. The TEV is called Tracking Error Volatility because it basically explains how well the active portfolio tracks the benchmark portfolio. Since the returns of the portfolio and the benchmark are not known on forehand this measure is only suitable for ex-post analysis of the TEV. An ex-ante TEV estimate can be made using the active weights.

Since active management is about obtaining alpha, deviating from the benchmark is a given. Cremers & Petajisto (2009) propose a measure to determine the possible alpha an active manager can obtain, called active share. Active share is derived from active weights, which is defined as the difference of the security weight in the active portfolio and the security weight in the benchmark.

$$\Delta w = w_{pf} - w_{bm}$$

Since the relative weights of the portfolio and the relative weights of the benchmark both should sum to 1, the active weights should sum to 0. The active weights are a good proxy for the forecasted alpha on security level, since overweighting or underweighting of a security results from an alpha view. The active weights can also be used to estimate the ex-ante TEV, which is defined by Grinold (1989) as:

$$TEV = \Delta w' * \Sigma * \Delta w$$

With:

n the number of constituents
Δw a nx1 matrix with active weights
Σ the nxn covariance matrix

Different approaches are suggested to obtain alpha, the next section will provide an overview of strategies which are a result of criticism on market capitalization weighted benchmarks.

3.1 α – Strategies

Following Markowitz (1952), investing in the ‘market portfolio’ should be the optimal choice from a MPT perspective. In practice this results in investments in the market portfolio which are usually market capitalization weighted portfolios. Critics of market capitalization weighting, like Hsu (2004) and Treynor (2005), argue that market cap weighted portfolios are not the best representation. Their main argument is that overvalued stocks will be given additional weight in market cap portfolios whereas undervalued stocks will be given less weight. That is why Arnott *et al.* (2005) propose fundamental equity market indices. They have constructed indices using: gross revenue, equity book value, gross sales, gross dividends, cash flow and total employments as weights and show that better performance is achieved on a 43-year time horizon as compared to market cap weighted indices. This line of reasoning has led to a wealth of alternative weighting schemes. As pointed out in Chapter 1, MN requires to assess the impact of constraints for different weighting schemes (strategies) given a benchmark. The required strategies are:

- Value strategy.
- Growth strategy.
- Minimum volatility strategy.
- Equal Weighted strategy.
- Equal Risk Contribution strategy.
- Risk efficient strategy.

A short description of each strategy and the optimization routine to construct each strategy will be given.

Value and Growth strategy

Fama & French (1993) assess whether common risk factors are correlated with stock and bond returns. Five common risk factors are found to be useful to determine if a stock is a value or growth stock. Result of this is their famous three-factor model. Based on the distinction between value and growth stocks, value and growth strategies are developed. MSCI provides a value and a growth index, the methodology they apply uses three parameters to determine whether a stock is a value stock. The three parameters are: Book value/Price, 12-month forward earning/Price and Dividend/Price. As opposed to five parameters to determine if a stock is a growth stock, being: long-term forward earnings per share (EPS) growth rate, short-term forward EPS growth rate, current internal growth rate, long-term historical EPS growth trend and long-term historical sales per share growth trend. These parameters will be used to determine a style score per stock and allocate the full value of the active portfolio to the upper 50% of the ranked style scores (value or growth). Result of this is the fact that value and growth strategies are rather concentrated when applied against a benchmark, since the weight of the constituents of the active portfolio is concentrated on only 50% of the total market value of the benchmark.

The construction of the constrained value and growth strategies starts from unconstrained value or growth portfolio. The constrained portfolios are a result of an optimisation procedure. The objective function of the optimiser has the purpose to minimize the total distance from the unconstrained strategy:

$$\min_{x} \sum_{i=1}^{N} (w_i - x_i)^2$$

With:

w_i unconstrained weight for security i

x_i resulting weight for security i from optimisation

Minimum volatility

A minimum volatility (Min Vol) portfolio is a strategy yielding from Markowitz's (1952) efficient frontier. In absence of a risk free asset, the minimum volatility portfolio is the portfolio on the efficient frontier with the lowest volatility. Moreover, Haugen & Baker (1991) and Clarke, De Silva & Thorley (2006) provide empirical evidence that minimum volatility portfolios add value as compared to market capitalization weighted benchmarks. The minimum volatility portfolio mainly relies on the single stock with the lowest volatility and consists of additional stocks to utilize diversification benefits, its only input requirement is therefore the covariance matrix of the stocks which should be used to construct the portfolio. Because the minimum volatility portfolio largely depends on the stock with the lowest volatility, minimum volatility portfolios are also rather concentrated. Since the purpose of the strategy is to minimize portfolio volatility, the strategy weights are a result of an optimization procedure with the purpose the minimize total portfolio volatility on an ex-ante basis. The objective function of the optimizer is:

$$\min_{x} \sqrt{x'\Sigma x}$$

With:

x the nx1 vector of portfolio weights

Σ the nxn covariance matrix

Equally weighted

Most straightforward asset allocation approach would be the equal weighted (EW) approach. As expected, this weighting scheme allocates the weights evenly over the constituents. It therefore is the least concentrated strategy. Benartzi & Thaler (2001) and Windcliff & Boyle (2004) point out that this strategy is applied in e.g. defined contribution pension plans in which participants have to decide over the allocation of their contribution. Equal weighted or "1/n" portfolios are most beneficial in the case that the constituents are uncorrelated. The calculation of the unconstrained portfolio is rather straightforward, assign $\frac{1}{N}$ to each weight. The constrained portfolio has an objective function which attempts to minimize the total distance between the unconstrained and the constrained portfolio:

$$\min_{x} \sum_{i=1}^{N} (w_i - x_i)^2$$

With:

w_i unconstrained weight for security i
x_i weight for security i from optimizer

Equal Risk Contribution
Where minimum volatility portfolios are rather concentrated as opposed to equal weighted portfolios, both with their own advantages and disadvantages. Equal Risk Contribution (ERC) portfolios are regarded as a compromise. As proposed by Maillard *et al.* (2010), ERC portfolios are minimum volatility portfolios subject to a diversification constraint on the constituent weights. An ERC portfolio is constructed by setting the components risk contributions equal, in which the risk contribution is determined as the first order derivative of the portfolio risk over the portfolio weight of the particular stock. It therefore only requires the covariance matrix of the stocks which are part of the universe from which the portfolio will be constructed. Since the goal is to equalize the risk contribution of each constituent, the portfolio weights are a result of an optimization procedure:
At first, the portfolio volatility should be calculated:

$$\sigma = \sqrt{w'\Sigma w}$$

From that the marginal risk contribution:

$$\partial_{x_i}\sigma = \frac{\partial \sigma}{\partial x_i} = \frac{x_i\sigma_i^2 + \sum_{j\neq i} x_j\sigma_{ij}}{\sigma}$$

As a result the risk contribution for security i:

$$\sigma_i = x_i * \partial_{x_i}\sigma$$

And ultimately, the objective function in order to equal the risk contributions:

$$\min_x \sum_{i=1}^{n}\sum_{j=1}^{n}(x_i(\Sigma x)_i - x_j(\Sigma x)_j)^2$$

Risk efficiency
Amenc *et al.* (2010) propose the risk efficient portfolio (RE). The risk efficient portfolio aims to maximize the Sharpe ratio, which is defined by Sharpe (1966) as:

$$S = \frac{E(r - r_f)}{\sigma}$$

With:
r, the expected return of the portfolio
r_f, the risk-free rate
σ the portfolio volatility

The Sharpe ratio determines the slope of the capital market line. The expected portfolio return and the expected portfolio volatility are functions of the portfolio weights, therefore the optimal portfolio weights are a result of optimization procedure with the objective function for the optimiser of:

$$\min_x - \frac{xr}{\sqrt{x\,\Sigma\,x'}}$$

With:
x the 1xn matrix with constituent weights
r the nx1 vector with expected returns
Σ the nxn covariance matrix

The r_f can be neglected in the optimiser since it is a constant.

3.2 The effects of constraints from a risk-reward perspective

The effect of imposing TEV constraints is assessed in a mean-variance framework by Jorion (2003). Motivation to do so was the fact that Roll (1992) pointed out that excess-return optimization in a portfolio construction process leads to higher portfolio risk than the benchmark and is therefore not optimal.
Jorion (2003) determines the efficient frontier under constant TEV and concludes that it is inefficient as compared to the unconstrained mean-variance frontier by assessing the constrained and unconstrained mean-variance efficient frontier. The methodology applied by Jorion (2003) consists of a 'standard' mean-variance optimization using the expected returns and expected covariance to determine the benchmark. Next step is an excess-return optimization with a TEV constraint, using the expected excess-return formula:

q	Vector of index weights
x	Vector of active weights
E	Vector of expected returns
V	Expected covariance matrix
$q_p = q + x$	Vector of portfolio weights
$\mu_B = q'E$	Expected return on the index
$\sigma_B^2 = q'Vq$	Expected variance of index return
$\mu_E = x'E$	Expected excess return over the benchmark returns
$\sigma_E^2 = T = x'Vx$	Tracking error variance
$\mu_P = (q+x)'E$	Active portfolio expected return
$\sigma_P^2 = (q+x)'V(q+x)$	Active portfolio expected variance

The resulting excess-return-variance frontier is calculated for different levels of TEV. As one might expect, the TEV constrained frontier should be pulled to the efficient frontier. Instead Jorion (2003) shows that TEV constrained frontier moves up and to the right in mean-variance plane, which leads to higher levels of portfolio volatility. From that, the possibilities of implying additional constraints to mitigate total portfolio risk are explored. Concluding that an additional constraint on total portfolio volatility improves the performance of the active portfolio.
Alexander & Baptista (2008) executed a similar analysis attempting to use VaR to control total portfolio risk. Their main findings where that adding a VaR constraint mitigates the problem of selecting inefficient portfolios while seeking outperformance. Furthermore, they point out that a long-only constraint reduces the optimal portfolios efficiency loss. Whereas Roll (1992), Jorion (2003) and Alexander and Baptista (2008) use, respectively, beta, variance and VaR to mitigate overall portfolio risk.
Alexander & Baptista (2010) construct an alpha-TEV frontier instead of the mean-TEV frontier used in the foregoing 3 articles. Evaluating the resulting frontiers in a mean-variance plane leads to the initial conclusion that this frontier is more efficient than the mean-TEV frontier if alpha is well chosen (e.g. the intersecting point of the mean-variance and the alpha-TEV frontiers). Overall the comparison leads to a trade off between absolute and relative risk and reward which overall leads to less risky portfolios.
The effect of both a TEV constraint and a weight constraint is analyzed by Bajeux-Besnainou *et al.* (2011). Weight constraints could be imposed to different specific types of securities, for example a constrained sector exposure or country exposure. Important note is the use of Information Ratio (IR) as performance measure:

$$IR = \frac{\alpha}{TEV}$$

In which α (expected excess return) and TEV (ex-ante) are defined as

$$\alpha = x'E$$
$$TEV = x'Vx$$

With:
x Vector of active weights
E Vector of expected returns
V Expected covariance matrix

As pointed out by Bajeux-Besnainou *et al.* (2011), in a TEV constrained setting IR remains constant while relaxing TEV because α will increase. In a TEV and weight constrained setting, IR can be affected by the TEV constraint as wel as the weight constraint. IR will decrease while relaxing TEV because α will not increase (which is expected) due to the weight constraint. Therefore IR is not a coherent risk measure and the Adjusted Information Ratio (AIR) is proposed as alternative. AIR is defined as the standard IR calculated against an adjusted benchmark. Results from their study is that the optimal IR increases when the weight constraint is relaxed as a result, AIR is a more suitable performance measure than IR.
In extend of Alexander & Baptista (2008), Palomba & Riccetti (2012) focussed on portfolio construction under a TEV (relative to the benchmark) and a VaR constraint (absolute). They point out that that TEV and VaR limits are not compatible, at most one of the two constraints can be satisfied. In the case that both VaR and TEV limits are satisfied the portfolio is generally inefficient.

Overall, extensive research is done in the field of portfolio construction under constraints. From an MPT perspective, deviating from the market portfolio should always lead to sub-optimality. Since active managers are attempting to achieve alpha by deviating from the benchmark (excess return optimization), challenge is to control the distance from the benchmark (level of sub-optimality) with imposing constraints. Most straightforward approach would be to impose a TEV constraint, a negative side effect is higher portfolio risk. Literature proposes to control the absolute portfolio risk by constraining beta, portfolio variance or VaR. This will lead to the paradox that it is impossible to satisfy both the benchmark relative constraint (e.g, TEV) and the absolute constraint (beta, portfolio variance, or VaR). Another approach to cope with this challenge is to develop a mean-variance efficient frontier, a mean-TEV efficient frontier and a alpha-TEV efficient frontier. A comparison of the frontiers in mean-variance plane could lead to intersections of the two frontiers that satisfy all efficiency requirements. From this, the effect of constraints can be assessed in two ways. Shrinkage of the IR or AIR can be used as a performance indicator for inefficiency due to constraints. Secondly, deformation of the efficient frontier can be used to assess the impact of constraints.

3.3 The effects of constraints from an implementation perspective

Preliminary work with respect to the implementation perspective dates back to Grinold (1989) who proposes “The fundamental law of active management”. Purpose of this law is to assess the capabilities of an investment manager. This law basically consists of a two variable equation which expresses the ability of a manager to add value in excess of a benchmark.

$$IR = IC * \sqrt{N}$$

With:
IR the Information Ratio
IC the Information Coefficient
N the breadth of the portfolio

The Information Coefficient (IC) is a measure for the skill of a manager to forecast future returns. Where N, the breadth of the portfolio, is the number of available independent ‘bets’ in the universe. In other words the opportunity set. The law is rather intuitive and states that the added value of a manager depends on his ability to forecast stock returns and the number of opportunities where he can apply his skill.

In practice the IR, calculated according Grinold’s (1989) fundamental law, turned out to be lower than the theoretical IR, therefore Clarke *et al.* (2002) propose an additional parameter, the Transfer Coefficient (TC), and define it as:

The transfer coefficient is the cross-sectional correlation coefficient between risk-adjusted active weights and risk-adjusted forecasted residual returns.

As pointed out in Chapter 1, a manager is limited in its ability to implement his alpha vision due to constraints. The Transfer Coefficient determines the ability of a manager to transfer its alpha vision into portfolio positions, if a high IC manager is not able to fully implement its alpha vision it will drag the value added of the manager (IR). Result of this is that Clarke *et al.* (2002) adjusted the fundamental law to:

$$IR \approx TC * IC * \sqrt{N}$$

The equation states that TC is a scalar of the value added of a manager. Their underpinning is that in the generalized version of Grinold (1989) the TC is assumed to be 1, in practice however the TC could reduce due to constraints. The work of Grinold (1989) and Clarke *et al.* (2002) is at best depicted in Figure 6.

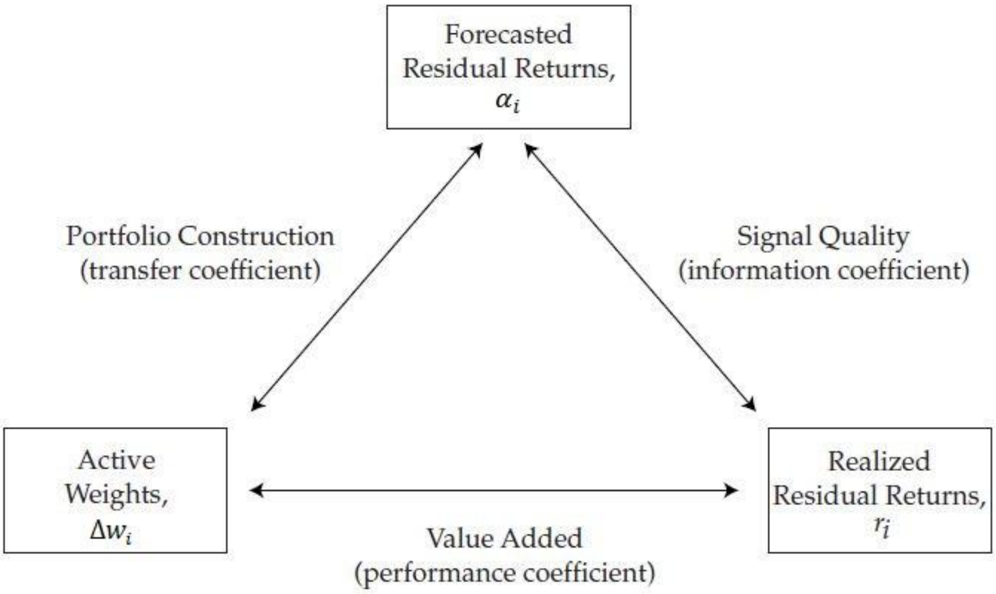

Figure 6: Fundamental law of active management triangle, the relationship between value added of a manager, the forecasting skill of a manager and the ability to implement its alpha vision.

The relation in Figure 6 can be stated as:

$Transfer\ Coefficient = corr(\alpha_i; \Delta w_i)$, the correlation between the alpha forecasts and the active weights.
$Information\ Coefficient = corr(\alpha_i; r_i)$, the correlation between the alpha forecasts and the realized returns.

Whereas the literature in Section 3.2 attempts to construct a constrained optimal portfolio, the impact of constraints is determined in terms of risk and return. The approach explained in the foregoing can be used to assess the impact of constraints without focussing on risk-return, but solely on the effect of constraints while implementing a managers view (implementation inefficiency). The remainder of this section provides an overview of extensions on the adjusted fundamental law of active management by Grinold (1989) and Clarke, de Silva and Thorley (2002).

Where the fundamental law is an ex-ante relationship, Clarke *et al.* (2002) defines the ex-post fundamental law by:

$$R_A \approx (\rho_{\alpha,r} TC + \rho_{c,r}\sqrt{1 - TC^2})\sqrt{N}\sigma_A D$$

With:

$\rho_{\alpha,r}$ the realized IC
$\rho_{c,r}$ noise associated with portfolio constraints
N Breadth of the portfolio
σ_A TEV
D Standard deviation of risk adjusted realized residual returns (return dispersion)
R_A Realized active return

Clarke, de Silva and Thorley (2005) gave the fundamental law more practical significance. They point out that lower TC's results in active returns stemming from the alpha forecasting process and active returns from noise imposed by constraints. As pointed out by Clarke *et al.* (2005):

"Managers frequently experience periods when the forecasting process or signal works but the active performance of the portfolio is poor and, conversely, periods when the signal performs poorly but the active performance is good."

The ex-post relationship, developed by Clarke *et al.* (2002), is used to attribute active performance to noise and quality of the signal:

$$signal \;\approx\; \rho_{\alpha,r} TC\sqrt{N}\sigma_A D$$

$$noise \;\approx \rho_{c,r}\sqrt{1-TC^2}\sqrt{N}\sigma_A D$$

From this, the relative magnitudes of the signal and the noise contributions are scaled by TC^2 and $1 - TC^2$ of the variance in the R_A, assuming independence:

$$var(R_A) = var\left(\rho_{\alpha,r} TC\sqrt{N}\sigma_A D\right) + var(\rho_{c,r}\sqrt{1-TC^2}\sqrt{N}\sigma_A D)$$
$$var(R_A) = TC^2 var\left(\rho_{\alpha,r}\sqrt{N}\sigma_A D\right) + (1-TC^2) var(\rho_{c,r}\sqrt{N}\sigma_A D)$$

Thus, TC^2 percent of the variation in realized performance is attributable to the signal quality and $1 - TC^2$ is due to constraint-induced noise. They compared performance attributions measured according to the fundamental law approach and performance attributions stemming from a factor model. Four portfolios benchmarked against the S&P 500 led to differences of only 1 bps for the contributions from the factor model and the attribution according to the fundamental law, underpinning the validity.

Furthermore, Grinold (2005) focuses on 'implementation efficiency'. Implementation efficiency can be divided in opportunity costs (being the benefits an investor would anticipate in an unconstrained setting) and the implementation costs (being cost of trading, anticipated market impact and the estimated losses associated with attempted trades). The used methodology consists of a mean-variance expected utility framework is expressed by portfolio alpha with penalties for transaction costs and TEV. The difference of the unconstrained and 0 costs utility against the constrained and non-0 costs are the implementation losses. From that the opportunity costs is attributed to different sources. Assessing the different sources as hypotheses leads to insight which enables the user to improve implementation.

The behaviour and characterization of the 3 variables in the fundamental law: IC, N and TC is the starting point for Kroll *et al.* (2005). Calculation of the realized returns is at heart of the IC. A sector oriented manager should measure excess returns based on the sector performance, noise in the active return could be the result otherwise. This confirms the work of Clarke *et al.* (2005) who point out that active returns contain noise for lower values of TC. They conclude the introduction stating that TC and IC are time independent since both are correlations between 2 data sets. From that the focus is on the dynamics of the TC. Looking at behaviour of the TC under different long/short divisions yields insight in the perfect division, conclusion is that shorting improves the TC of portfolios. Moreover, a 130/30 portfolio already improves the TC by 2/3 of the difference between a long-only and a 100/100 portfolio. Kroll *et al*.(2005) stress that the initial model has an oversimplified approach to account for turnover and transaction costs although they can have a significant impact on performance. They attempted to improve the model by adding multi-period turnover and transaction costs. Where the resulting multi-period turnover leads to increasing transaction costs, the ability of the portfolio to generate alpha improves due to implementation of new alpha signals. New insight is the fact that the declining TC, due to increased turnover, stabilizes to a steady state over time.

The approach used to determine the TC is extended by Grinold (2006) to attribute TC and realized IC to different risk sources, sources of outperformance and implementation. Traditionally, attribution is determined by return regression models. But as Grinold (2006) states *"Framing the relevant question in portfolio terms"* enables one to attribute results to the source by determining the correlation between the ideal portfolio and the portfolio under investigation. Important notion is the introduction of the *backlog portfolio*, the basket of trades needed to move the active portfolio to the desired portfolio. This enables one to attribute alpha to multiple sources of risk, whereas traditional return regression models evaluate one source of risk at a time.

An alternative approach is suggested by Scherer & Xu (2007). Where the TC, or IR shrinkage as they call it, is more a headline number, the method developed by Scherer & Xu (2007) enables the user to determine the impact of an individual constraint on security holding level. The closed form solution of the constrained optimization is a function of the Lagrange multipliers which are incurred per constraint. From that they propose the *shadow price* of a constraint as the units of objective function won when relaxing the constraints. As a result, the impact of constraints can be expressed as loss in utility due to constraints and the attribution of the loss in utility to the particular constraints (*shadow costs*).

Building on Clarke *et al.* (2002) and Scherer & Xu (2007), a vector decomposition is suggested by Bender *et al.* (2009). The goal of their paper is to analyze them impact of constraints on risk and return. They start-off with a vector relationship between a constrained portfolio which equals an unconstrained portfolio minus a constraint portfolio (see Figure 7, $h_u - h_c = h_x$). This approach is in line with the notion of a backlog portfolio as defined by Grinold (2006). Next step is to decompose the constrained portfolio in a part which is aligned with the unconstrained portfolio (affects both risk and return therefore constant IR) and a part which is orthogonal to the unconstrained portfolio (only increasing risk, so decreasing IR).

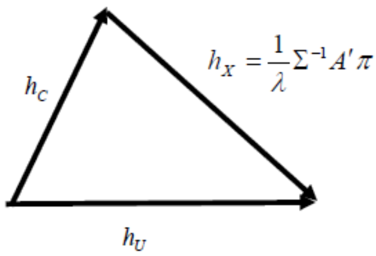

Figure 7: Vector decomposition of the unconstrained active weights (h_u) and the constrained active weights (h_c) in order to determine the backlog portfolio (h_x).

The orthogonal factor represent the unwanted bet part for the manager. This decomposition leads to vectors which can be used to decompose the risk and return of the portfolio in a part which affects both risk and return and a part which only adds risk. Focussing on the amount of risk which is added by a constraint with no return compensation, enables the user to evaluate constraints.

As the amount of literature in the field of constraints and portfolio construction increased, Stubbs & Vandenbussche (2010) shortly summarize the different methodologies. Part of their conclusion is that TC is a good measure of implementation inefficiency on aggregated level. From that, the aim of this article is to develop a method to allocate implementation efficiency to individual constraints. In line with Scherer & Xu (2007) the costs of constraints are measured by loss in utility (*shadow price*). They provided mathematical proof why this method is suitable for all kinds of constraint classes, including constraints that are not differentiable, nonlinear or both.

3.4 Conclusion

MPT is fundamental work in the field of portfolio construction. Together with CAPM it has led to the notion of market efficiency, portfolio optimality and the distinction between systematic and idiosyncratic risk. Portfolio optimality can be determined with the efficient frontier in mean-variance space. The fact that not all underlying assumptions hold, leads to active management i.e. deviating from the market portfolio in order to capture excess return over market return for a given risk level.
Following the initial line of reasoning for active management, deviating from a benchmark is a given in order to obtain alpha. The portfolio construction starts off with developing an alpha view (or alpha forecast per individual security) of the manager. From this, positions will be overweighed or underweighted which results in active weights.
Active weights can be used to reverse engineer the implied alpha per security by looking at the active weights ($\Delta w = w_{pf} - w_{bm}$). Such a reverse-engineering process is suggested by Scherer & Xu (2007) and leads to the notion that active weights can be used as proxy for alpha.

With regards to the impact of constraints, one branch in academia uses alternative efficient frontiers to determine optimal portfolios in a constrained active management setting. Main goal is to construct portfolios which are as efficient as a MVO portfolio. Different spaces like mean-TEV and alpha-TEV are used to determine efficient constrained portfolios. Next to that, the possibility of imposing absolute constraints (like beta, portfolio variance or VaR) next to benchmark relative constraints (like TEV) is explored in order to control efficiency of the active portfolio. Ultimately the mutual effect of constraints on benchmark relative and absolute constraints are assessed. In general, this branch tries to assess the impact of constraints in terms of risk and reward and is therefore more from a portfolio construction perspective. In general, the approach to determine the impact of constraints is:

- Deformation of the efficient frontier.

The second branch, the fundamental law of active management, uses a different approach. The initial work is a rather straightforward relationship which determines the value added of a manager (IR). The effect of constraints is determined in three ways:

- The Transfer Coefficient: this performance measure determines on portfolio level the ability of the manager to implement his alpha forecasts, TC shrinkage can be used to assess the impact of constraints.
- Shadow prices: Lagrangian multipliers in an expected utility optimization are used to determine loss in utility due to imposed constraints on portfolio level. The shadow price is determined by the loss in utility over delta in the constraints.
- Shadow costs: The restrictiveness is assessed by the loss in investor utility attributed to specific constraints.

The Transfer Coefficient is a rather straightforward measure, as opposed to the more sophisticated shadow prices and shadow costs. The latter have more attractive features like e.g, loss attribution to individual constraints, but seem more suitable in a theoretical setting due to their complexity.

Main difference with the approaches from a risk/reward perspective is the level of abstraction. The risk/reward approaches quantify the impact of constraints in terms of loss in return and mitigation of risk, which makes them more suitable from a portfolio construction perspective. The methods stemming from the fundamental law of active management are useful for a constraint assessment perspective. The methods lead to assessment of 'the ideal position not taken' without stating if that is desirable or not. In principle, both branches could work for MN. Therefore 3 methods will be applied in Chapter 4 to determine the practical drawbacks.

4 Application of the methods

The theoretical framework suggests 2 approaches to assess the impact of constraints, the first approach assesses the impact of constraints by sub-optimality (deformation of the efficient frontier) of the constrained portfolio versus the unconstrained portfolio. The second branch uses the TC as a measure to determine the implementation inefficiency of the portfolio. In principle both approaches would meet the requirements of MN, the practical drawbacks are discussed in this Chapter. 2 Methods will be applied using the proposed performance indicators from a risk-reward perspective, at first the impact of constraints will be assessed on random constructed portfolios (method A). From that the deformation of the efficient frontier under constraints will be examined (method B). Method C has the purpose to assess the impact of constraints from an implementation efficiency perspective.
As can be seen in Figure 8, the model is cut in three stages.

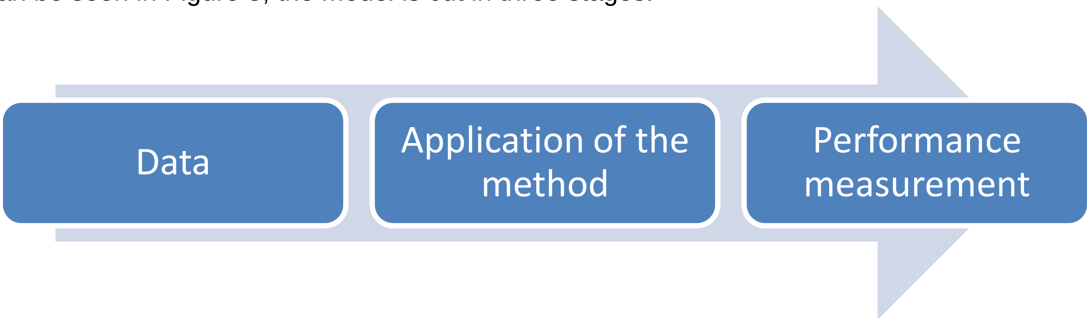

Figure 8: Overview of the stages in the model.

Stage 1 is the data stage, this part is generic for all three methods which will be applied. It consists of a description of the input data and the estimates which will be used. Besides, more in depth insight in the constraints will be provided. The second stage is the application of the model, this part differs per approach. The third stage will elaborate on the performance indicator and how the theory of Chapter 3 will be used in practice.

4.1 Data

As required by MN, MSCI World and the MSCI Emerging Market equity indices will be used to provide sample data, because of their size (resp. 1610 and 822 constituents) the AEX index (25 constituents) will also be used. Due to the limited size of the AEX index, the methods are easier to check and it will be used for illustrative purposes.
The data stage has the purpose of preparing the data for the analysis as depicted in Figure 9.

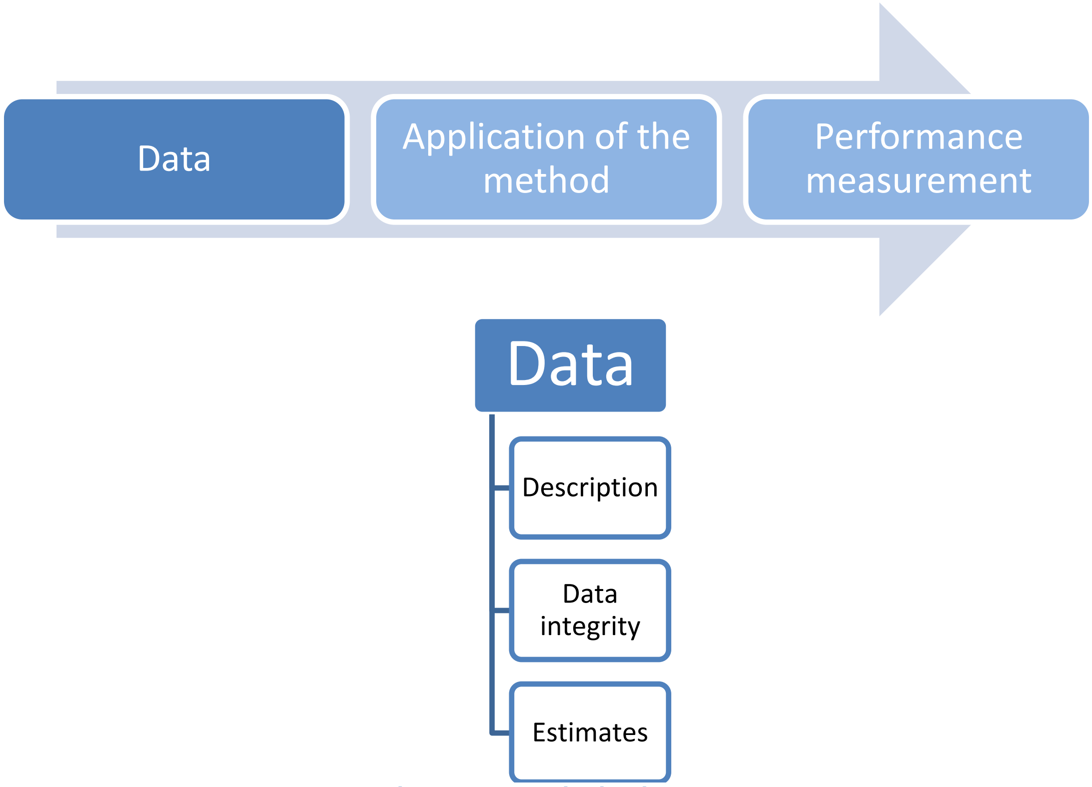

Figure 9: Steps in the data stage.

Especially the data integrity and the estimates are of interest, large datasets will be used as inputs (1 year data for MSCI Emerging Market equals 250 (days) x 822 (constituents) = 205500 data points). As a result the data sets will contain missing values and introduce practical problems in the estimation process.

4.1.1 Description

MSCI World
The MSCI World equity index is an index consisting out of large and mid cap stocks from 23 developed countries. With 1610 constituents it covers a significant large part of the 23 developed countries. The MSCI world gives a good overview of the performance of the equities in the developed markets.

MSCI Emerging Market
The MSCI Emerging Markets index provides an overview of the equity performance in the 21 Emerging Markets countries. The large and mid cap stocks are included which leads to approximately 85% of the market capitalization per country. The MSCI Emerging markets index consists of 822 constituents.

AEX index
The AEX index consists out of the 25 biggest companies listed on the Dutch index. It is a market capitalization weighted index and consists mainly of Dutch companies.

The constituent data is obtained from datastream and split in a static and a dynamic part before it is stored in a local database, the structure of the database is depicted in Table 1, Table 2 and Figure 10.

Table 1: Overview of constituents static data.

Column names in the constituents table	
Column name	**Description**
ID	Unique database identifier
constituent_code	Unique security code provided by Datastream
constituent_name	Name of the constituents
index_code	Index code of the index in which the security constitutes
ISIN	International Securities Identification Number, ISO format global unique security identifier
GICS_sector	Global Industry Classification Standard, an industry taxonomy developed by MSCI and S&P. Sector level classification
GICS_industry_group	GICS classification on industry group level
GICS_industry	GICS classification on industry level
GICS_sub_industry	GICS classification on sub-industry level
country_iso	ISO country code of the company

Table 2: Overview of constituents dynamic data.

Column names in the prices table	
Column name	**Description**
ID	Unique database identifier
security_code	Unique security code provided by Datastream
price_HC	Total return data hedged back to euro's
price_LC	Total return data in the local currency
z_bv_p	MSCI z-score for book value to price
z_fwd_etp	MSCI z-score earnings to price forward
z_div_y	MSCI z-score dividend yield
z_lt_fwd_eps_g	MSCI z-score long term forward earnings per share growth rate
z_st_fwd_eps_g	MSCI z-score short term forward earnings per share growth rate
z_cigr	MSCI z-score current internal growth rate
z_lt_hist_eps_g	MSCI z-score long term historical earnings per share growth trend
z_lt_hist_sps_g	MSCI z-score long term historical sales per share growth trend
time_stamp	Time stamp for which date the foregoing data holds

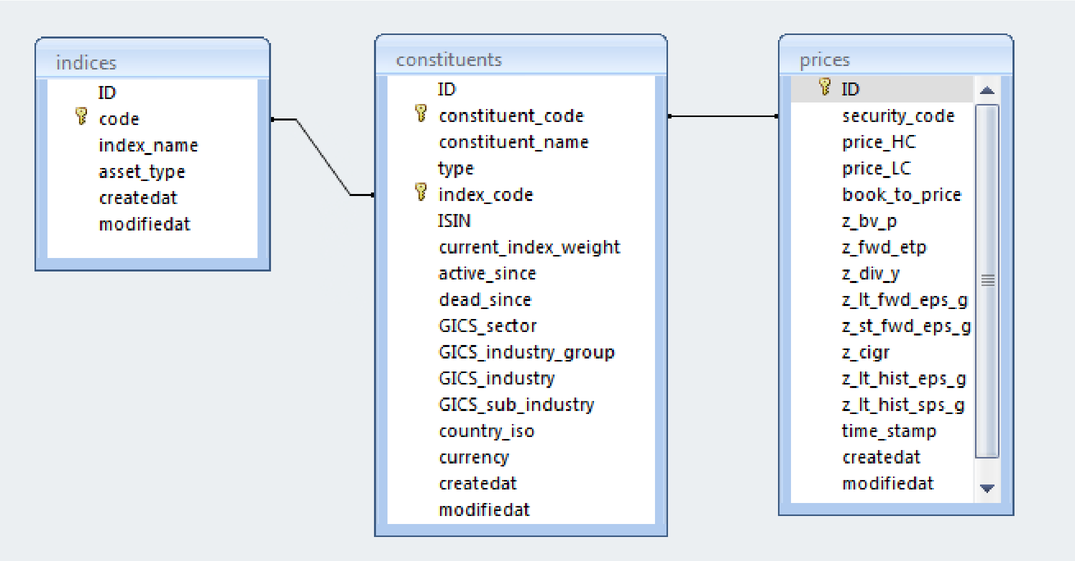

Figure 10: Structure of the database.

As pointed out in Table 2, total return data instead of prices are used for the analysis. The conceptual difference between the two is the fact that dividends are assumed to be re-invested in the stock in the total return case. It is calculated as follows:

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}} * (1 + \frac{DY_t}{100} * \frac{1}{N})$$

With:

RI_t	return index on day t
RI_{t-1}	return index on day t-1
PI_t	price index on day t
PI_{t-1}	price index on day t-1
DY_t	dividend yield % on day t
N	number of working days in the year

Total return data is used to provide better return estimates since dividend payments differ per stock and are not accounted for when solely using price data.

The constituents of the AEX index are quoted in Euros, this is however not the case for all constituents in the MSCI World and the MSCI Emerging Market indices. Unhedged (local currency) total return data is used to estimate expected covariance and expected return, more on this in Section 4.1.3. One year's raw price data is obtained for all securities which constituted the indices in the period 1-1-2013 until 1-1-2014. The raw data is stored in the database, the integrity of the data will be checked when they are extracted from the database.

4.1.2 Data integrity

Due to the large amount of data, missing values can occur in the dataset in three forms:

- Randomly missing values
- Missing values due to the fact that the security was not listed at the start data
- Missing values due to the fact that the security was not listed at the end data

An overview of the missing data is given in Table 3.

Table 3: Overview of the data integrity.

From	Till		AEX	MSCI Emerging Markets	MSCI World
1-7-2013	31-1-2013	# Series containing NaNs	0	2	6
		# NaNs	0	183	231
1-1-2013	31-1-2013	# Series containing NaNs	0	6	10
		#NaNs	0	885	1290

Maximum proportion of missing data is 0.7 % (10 out of 1611 series). In order to overcome survivorship bias (Elton, Gruber & Blake, 1996) the data series will not be removed from the dataset if the security is delisted during the period under investigation. When the security was added to the index during the analysis period, the security will be removed from the dataset in order to improve the estimates. Ultimately, randomly missing values will be replaced:

Nearest neighbour interpolation

Since the data in the columns: z_bv_p, z_fwd_etp, z_div_y, z_lt_fwd_eps_g, z_st_fwd_eps_g, z_cigr, z_lt_hist_eps_g and z_lt_hist_sps_g is determined on a quarterly basis, the 'nearest neighbor' interpolation can be applied. A missing value at time point t will be replaced with the value at t+1 if the values at t-1 and t+1 are equal.

Replacement
Randomly missing (return) data will be replaced with the median of the return series.

4.1.3 General estimators

Two estimators should be determined from the data, expected returns and the expected covariance matrix are needed to construct the strategies and as a result determine the alpha views. Next to that, the estimates are needed to construct the unconstrained and constrained efficient frontier. The remainder of this section will elaborate on the estimation procedures which are used.
Starting point for the estimation procedures is the total return data of the constituents.
The total return data of the constituents is converted to daily percentage returns as follows:

$$r_t = \frac{RI_t - RI_{t-1}}{RI_{t-1}}$$

With:
RI_t Return index at day t
RI_{t-1} Return index at day t-1
r_t Return at day t

From which the expected return is calculated as:

$$E(r) = \frac{1}{T}\sum_{t=1}^{N} r_t$$

With:
r_t Return at observation t
T Number of observations
$E(r)$ Estimator of expected return

The use of total return data instead of price data will lead to improved return estimates since dividend payments are also taken into account.
The estimation of the covariance matrix is somewhat more complex, most straightforward approach would be to calculate the sample covariance matrix, the covariance between asset i and j can be calculated as:

$$\sigma_{ij} = \frac{1}{N-1}\sum_{k=1}^{N}(r_{ki} - \overline{r_i})(r_{kj} - \overline{r_j})$$

With:
N Number of observations
r_{ki} k[th] return observation of asset i
r_{kj} k[th] return observation of asset j
$\overline{r_i}$ Average return of asset i
$\overline{r_i}$ Average return of asset j
σ_{ij} Estimator of the covariance between asset i and j

The number of observations used for the calculation is in line with the rule of thumb which is pointed out in Hull (2012):

"A compromise that seems to work reasonably well is to use closing prices from daily data over the most recent 90 to 180 days. Alternatively, as a rule of thumb, n can be set equal to the number of days to which the volatility is to be applied."

The downside of using this rule of thumb is that there are fewer observations than constituents which could harm the covariance estimates. Condition for the matrix to be a covariance matrix is that it should be internally consistent (Hull, 2012), being:

$$w' * \Sigma * w \geq 0$$

With:
w the nx1 vector of asset weights
Σ the nxn covariance matrix

Since the foregoing expression is the matrix notation of the portfolio variance, the condition states that the portfolio variance should be ≥ 0. A matrix that satisfies this condition is known as positive-semi definite. As pointed out by Ledoit & Wolf (2004)

"when the number of stocks under consideration is large, especially relative to the number of historical return observations available (which is the usual case), the sample covariance matrix is estimated with a lot of error."

Furthermore missing data can also harm the consistency of the covariance matrix (Arbuckle, 1996). The consistency of a covariance matrix can be determined, in absence of portfolio weights, by the eigenvalues of the covariance matrix. The eigenvalues should all be larger than or equal to zero.

Inconsistency of the covariance matrix can be solved in two ways, fix the sample covariance matrix (Scherer, 2007) or manipulate the data matrix before the covariance matrix is determined. A manipulation technique is to estimate the covariance matrix from a matrix decomposition of the data matrix (Golub & Reinsch, 1970), this is however only suitable if there are more observations then assets (m>n). Another manipulation technique is to apply matrix shrinkage (Ledoit & Wolf, 2004). All 4 methods will be explained and tested against the available sample data to determine which method is most suitable.

Fixing the covariance matrix

As pointed out by Jobson & Korkie (1980), Ledoit & Wolf (2004) and Scherer (2007), the sample covariance matrix being not positive-semi definite is due to the fact that the estimation of small numbers leads to errors. As a result, very small eigenvalues can become negative (which is the case for inconsistent covariance matrices). Scherer (2007) proposes to fix the sample covariance matrix by adjusting the diagonal of the covariance matrix for the errors:

$$\Sigma_{fixed} = \frac{1}{1 - \min(eig)} (\Sigma - \min(eig)\, I)$$

With:
Σ nxn sample covariance matrix
$\min(eig)$ minimum eigenvalue of Σ
I nxn matrix with 1's on the diagonal
Σ_{fixed} nxn 'fixed' covariance matrix

Ledoit & Wolf (2004) propose a different technique, since the sample covariance contains estimation errors and has no structure they propose to shrink the sample covariance matrix with a structured estimator as e.g. a factor model. Since the extremely positive coefficients in the sample covariance matrix tend to contain positive estimation error, the extremely negative coefficients have negative estimation errors embedded. The result of the compromise is that the extreme values are *shrunk* towards the structured estimator:

$$\Sigma_{shrink} = \delta F + (1 - \delta)\Sigma$$

With:
δ the shrinkage constant, a number between 0 and 1
F Single factor, structured estimator
Σ Sample covariance matrix, unstructured estimator

The single factor, F, proposed in the work by Ledoit & Wolf (2004) is the constant correlation model. This model is a matrix with the average of the coefficients in the sample covariance matrix as the constant correlation. Together with the vector of sample variances this matrix is applied as shrinkage target. The optimal shrinkage factor, δ, is a number between 0 and 1. Ideally this number is a constant that minimizes the expected distance between the shrinkage estimator and the true covariance matrix, which is also provided in Ledoit & Wolf (2004).

Since the covariance matrix of data matrix M can be calculated as:

$$C = M^T M$$

With:
M the data matrix
C the covariance matrix

A decomposition of the raw data is applied to obtain the sample covariance matrix from the roots of the data matrix. A Singular Value Decomposition (SVD) produces a diagonal matrix S of the same dimension as the data matrix with non-negative diagonal elements in decreasing order, an unitary matrix U and V in order to reconstruct the data matrix M as:

$$M = U * S * V^T$$

With:
M the mxn data matrix
U the mxn unitary matrix if m>n and mxm if m<n
S the nxn matrix with singular values if m>n and mxn if m<n
V the nxn unitary matrix

The decomposition can be used to calculate C as follows:

$$C = V * S * U^T * U * S^T * V^T$$
$$C = (V * S) * (S^T * V^T)$$
$$C = V * S^2 * V^T$$

Since the size of U and S depends on the size of the data matrix, the decomposition can only be applied when m>n.

The covariance matrices will be calculated from the data according to the four approaches, from that the consistency will be determined by testing if all eigenvalues > 0. An overview of the results is given in Table 4.

Table 4: Application of the 3 covariance estimation procedures on the sample data.

	Dataset start date:	1-7-2013	1-1-2013
AEX	Sample	Yes	Yes
	Fix	No	No
	Shrinkage	Yes	Yes
	SVD	No	No
MSCI Emerging Markets	Sample	No	No
	Fix	Yes	Yes
	Shrinkage	Yes	Yes
	SVD	Yes	Yes
MSCI World	Sample	No	No
	Fix	Yes	Yes
	Shrinkage	Yes	Yes
	SVD	Yes	Yes

The sample covariance does not meet the consistency condition if the amount of data increases. The covariance matrix ‘fix’ did not meet the consistency condition for the case with limited assets as well as the Singular Value Decomposition. The method of choice is therefore the sample covariance shrinkage technique proposed by Ledoit & Wolf (2004).

4.2 Constraints

A content analysis of the current IMAs yielded an overview of the frequency of which the different constraints are applied as well as the quantities which are used. This section will elaborate on the MN required constraints.

4.2.1 Long only constraint

Individual assets weights are limited in the IMAs. Most important observation with respect to the individual asset weights is the fact that shorting is disallowed.

The asset weight constraint is typically formulated in the IMAs as:

"The maximum percentage of the value of the portfolio invested in any one stock will be benchmark percentage invested in the same stock plus 5% but not to fall below zero percent."

The values which are currently applicable in the IMAs range between 0.1% and 6%. In order to provide a good overview the values which will be analyzed range between 0% and 10% to obtain a good overview of the restrictiveness of the constraint.

4.2.2 Sector constraint

A sector constraint is a benchmark relative constraint. It states the degree of freedom for the manager in terms of deviation from the sector weights in the benchmark. The constraint is typically defined as a percentage maximum deviation from the benchmark with a minimum of 0.

The sector constraint is typically formulated in the IMAs as:

"The manager may not invest in securities of any one company if after such investment, the value of the securities in the sector, as defined by MSCI, GICS tier 1 (Sector Group-level), where the company belongs to, would exceed the following percentages of the value of the portfolio:

Max(0, benchmark weight – 10%) =< portfolio weight =< benchmark weight + 10%"

The values in the IMAs range between 5% and 15% under/overweight of an industry with a minimum absolute exposure of 0%. In order to provide thorough insight in the constraint, the range evaluated in the analysis will be 0% - 20%, although the maximum freedom given in the current mandates is 15% and the minimal value is 5%.

4.2.3 Country constraint

A country constraint is practically the same as the foregoing sector constraint, it only differs in the sense that it determines the degree of freedom with respect country weight in the benchmark.

The country constraint is typically formulated in the IMAs as:

"The manager may not invest in securities of any one company if after such investment, the value of the securities in the country where the company belongs to, according to the MSCI classification of that security and where it exercising the predominant part (meaning > 50%) of their economic activities (in terms of sales), would exceed the following percentages of the value of the portfolio:

Max(0, benchmark weight – 10%) =< portfolio weight =< benchmark weight + 10%"

The country exposures imposed by the mandates range between 3 – 15%, the range evaluated in the analysis will therefore be 0% - 20%.

4.2.4 Constrained optimization

The aforementioned constraints should be transformed to a matrix structure in order to use them in the strategy optimizers. Besides, the mathematical representation of the constraints provides useful insights for the modelling process. The linear inequalities needed as input for the optimization are of the form:

$$\begin{aligned} w_1 * x_1 + w_2 * x_2 + \ldots + w_n * x_n &\leq b_1 \\ w_1 * x_1 + w_2 * x_2 + \ldots + w_n * x_n &\leq b_2 \end{aligned}$$

in matrix notation:

$$\begin{bmatrix} w_1 \; w_2 & \cdots & w_n \\ w_1 \; w_2 & \cdots & w_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

and in short matrix notation:

$$\begin{bmatrix} w_1 \; w_2 \cdots w_n & b_1 \\ w_1 \; w_2 \cdots w_n & b_2 \end{bmatrix}$$

For illustrative purposes, an example of 3 stocks which should have a minimum weight in the portfolio of 0 (long-only) and a maximum weight in the portfolio of resp. 0.3, 0.6 and 0.5. The resulting set of linear equations for the upper bound looks like:

$$\begin{aligned} 1 * x_1 + 0 * x_2 + 0 * x_3 &\leq 0.3 \\ 0 * x_1 + 1 * x_2 + 0 * x_3 &\leq 0.6 \\ 0 * x_1 + 0 * x_2 + 1 * x_3 &\leq 0.5 \end{aligned}$$

and the set of linear equations for the lower bound:

$$\begin{aligned} -1 * x_1 - 0 * x_2 - 0 * x_3 &\leq 0 \\ 0 * x_1 - 1 * x_2 - 0 * x_3 &\leq 0 \\ 0 * x_1 - 0 * x_2 - 1 * x_3 &\leq 0 \end{aligned}$$

the resulting matrices would look like:

$$\begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & 1 & 0 & 0.6 \\ 0 & 0 & 1 & 0.5 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

The same approach applies for a sector constraint, in this example stock 1 and 2 are of the same sector. This sector has a minimum weight in the portfolio of 0.1 and a maximum weight of 0.9. Stock 3 belongs to a different sector which should cover a minimum weight in the portfolio of 0.4 and a maximum weight of 0.8, the resulting set of linear equations:

$$\begin{aligned} 1 * x_1 + 1 * x_2 + 0 * x_3 &\leq 0.9 \\ 0 * x_1 + 0 * x_2 + 1 * x_3 &\leq 0.8 \end{aligned}$$

and the set of linear equations for the lower bound:

$$\begin{aligned} -1 * x_1 - 1 * x_2 - 0 * x_3 &\leq -0.1 \\ 0 * x_1 - 0 * x_2 - 1 * x_3 &\leq -0.4 \end{aligned}$$

the resulting matrices for the sector constraints would look like:

$$\begin{bmatrix} 1 & 1 & 0 & 0.9 \\ 0 & 0 & 1 & 0.8 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -1 & 0 & -0.1 \\ 0 & 0 & -1 & -0.4 \end{bmatrix}$$

The same approach would apply for the country constraint.

4.3 Method A: Risk-reward and randomly constructed portfolios

Following the branch in academia which attempts to assess the impact of constraint in terms of risk/return, a model dependent on randomly constructed portfolios is developed. Since external managers are designated the task to manage a portfolio in order to obtain alpha, the presumption is that the underlying EMH does not hold. Presuming that the EMH does not hold results in the assumption that the benchmark is not efficient, it is therefore feasible to determine the effect of constraints on 'all' possible portfolio's.

The idea behind this model is to do an ex-post comparison of all possible portfolio returns and the likelihood of occurrence in an unconstrained setting and a constrained setting (see Figure 11) the differences can be analysed stage 3 (see Figure 8) in order to assess the impact of the applied constraint.

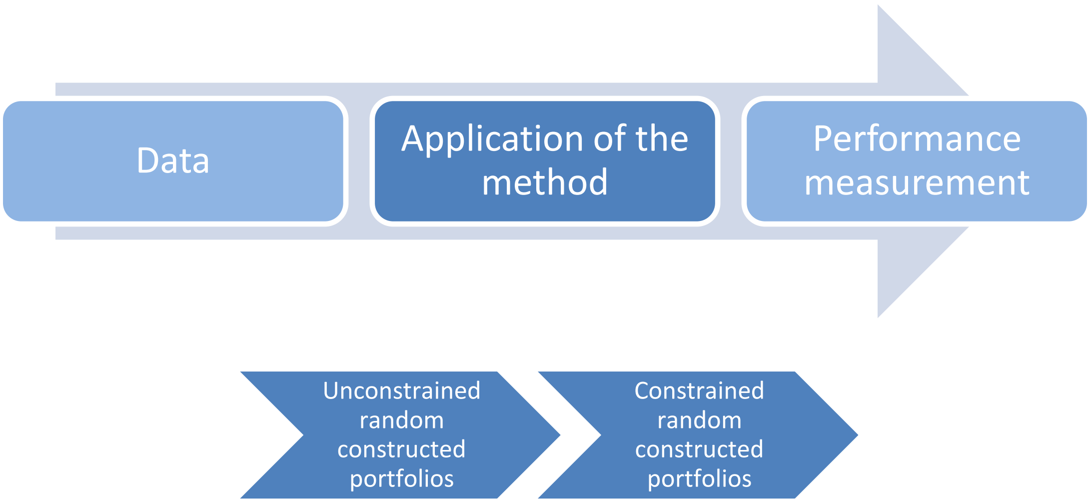

Figure 11: Implementation of the second stage of the model.

A probability density function assigns a probability to each measurable subset of a complete set of outcomes and is therefore a suitable representation of the results (see Figure 13). The resulting probability distribution of portfolio returns for the unconstrained setting and the constrained setting can be compared in order to induce conclusions with regards to the constraints. In order to do so, the first 2 moments of the distribution will be calculated and compared. The first moment of the distribution is the mean, expressing the average realized return. The second moment of the probability distribution is the variance of the distribution.
Result of this analysis is a comparison of the return distribution in an unconstrained setting and in a constrained setting in terms of expected return and variance. The effect of the constraint can therefore be assessed in a change of the Sharpe ratio which is in line with the reviewed literature in Chapter 3.

In order to construct random portfolio returns, random weights are drawn from an uniform (U~(0,1)) distribution. Assuming an initial wealth of 1, the random weights will be rescaled to make sure they add up to 1. The portfolio value can be assessed at each point in time (since it is an ex-post analysis) by performing a matrix multiplication of the random weights with the security returns.

$$\boldsymbol{P} = \boldsymbol{A} * \boldsymbol{V}$$

With:
$\boldsymbol{A}$ the mxn matrix with m the number of time points (prices) and n the number of constituents
$\boldsymbol{V}$ the nx1 matrix with portfolio weights per constituent
$\boldsymbol{P}$ the mx1 matrix with portfolio values per point in time

The Strong Law of Large Numbers (SLLN) proves that for a sufficiently large sample of the portfolio value (a random variable; r.v.), the mean of the sample is the mean of the r.v.

In order to obtain a sufficiently large sample, multiple portfolio should be randomly constructed, this leads to a matrix with portfolio values for each portfolio over time:

$$\boldsymbol{P} = \boldsymbol{A} * \boldsymbol{V}$$

With:
$\boldsymbol{A}$ the mxn matrix with m the number of time-points (prices) and n the number of constituents
$\boldsymbol{V}$ the nxs matrix with portfolio weights per constituent for s portfolios
$\boldsymbol{P}$ the mxs matrix with portfolio values per point in time

The matrix $\boldsymbol{P}$ with portfolio values for each point in time for all portfolios allows a cross-sectional evaluation of the mean and variance of the sample portfolio values ($\boldsymbol{P}$) per point in time.

The constrained portfolios should be constructed likewise, there should only be an additional constraint violation check in place. The portfolio weights will be redrawn as long as the constraints are violated. The same linear algebra will be applied as in the unconstrained scenario in order to obtain portfolio values from the random constructed portfolios per point in time.

An impression of the portfolio values over time is given in Figure 12, this example is from data of the MSCI Emerging Market index in the period 1-Sept-2013 till 17-Oct-2013

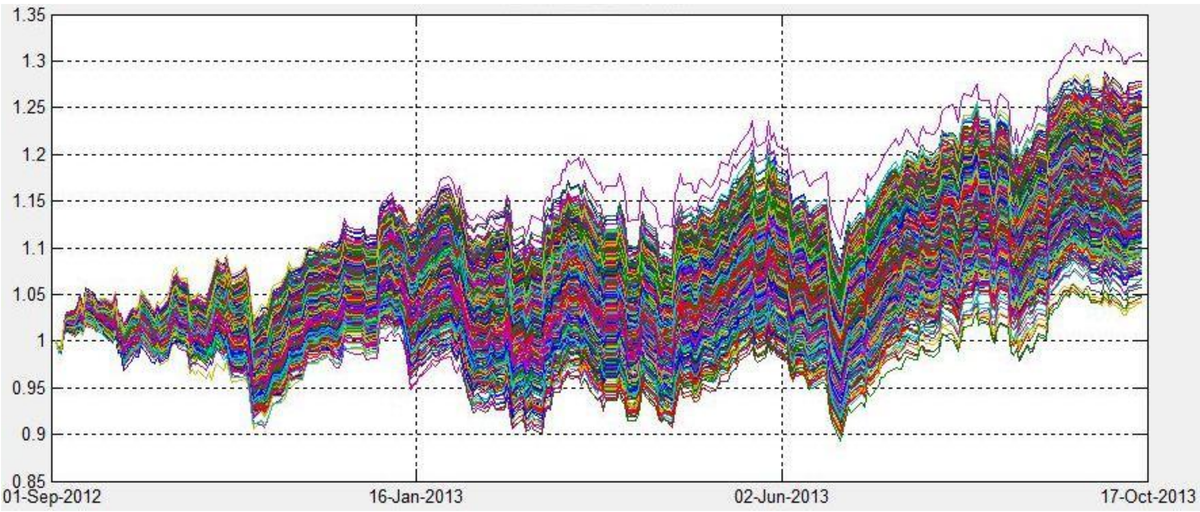

Figure 12: Portfolio values for random constructed portfolio over time.

A cross sectional distribution of the portfolio values can be determined at each point in time. An example of the resulting distribution is given in Figure 13.

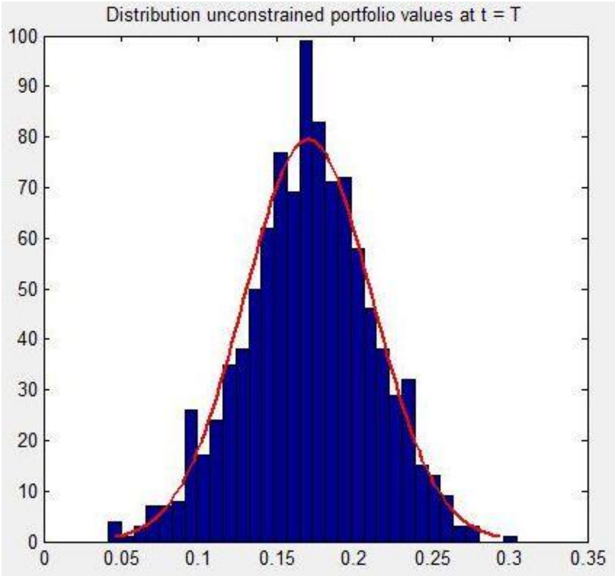

Figure 13: Distribution of portfolio returns at t=T.

A normal distribution can be fitted to the histogram and the mean and variance of the resulting distribution can be determined.

The model did not yield significant differences in the Sharpe ratios, therefore the setup of the model is reviewed.

One of the diagnostics was a check to determine if the current approach to construct random portfolios provides a good overview of all possible portfolio values. The efficient frontier is a useful tool since it is bounded by the single most risky asset and the combination of assets which yield the minimum portfolio volatility (with the long-only constraint imposed). From its definition, the efficient frontier is the upper half of the feasible set of portfolios from a set of assets. In this case however, the complete set of feasible solutions is useful in order to check if the random constructed portfolio comprise a significant part of feasible set.

The feasible set of portfolios will be calculated with the default matlab procedure and drawn in the risk-return plane. All random constructed portfolios will be plotted in the same risk-return plane together with the assets. The results are provided in Figure 14, Figure 15 and Figure 16.

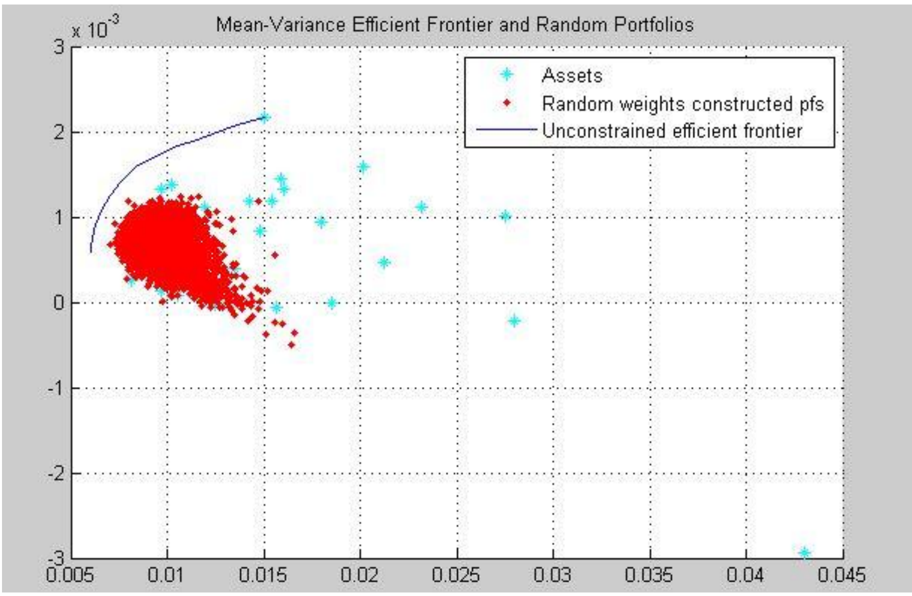

Figure 14: Efficient frontier, AEX constituents and 10000 randomly constructed portfolios.

The AEX index consists out of 25 constituents. The random constructed portfolios, where a random weight is assigned to each constituents, cover a part of the feasible set. Not one of the portfolios is on the efficient frontier.

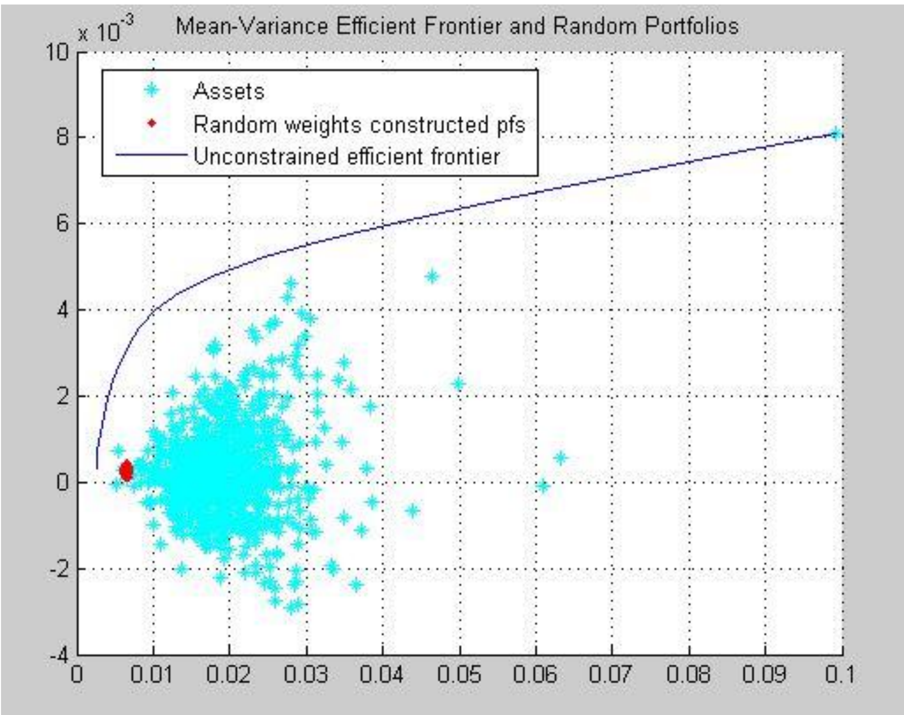

Figure 15: Efficient frontier, MSCI Emerging Markets constituents and 10000 randomly constructed portfolios.

The MSCI Emerging Markets equity index consists out of 822 constituents. The random constructed portfolios, where a random weight is assigned to each constituent, cover a significant smaller part of the feasible set as compared to the random constructed portfolios from the AEX constituents. Besides, the 10000 portfolios are very concentrated.

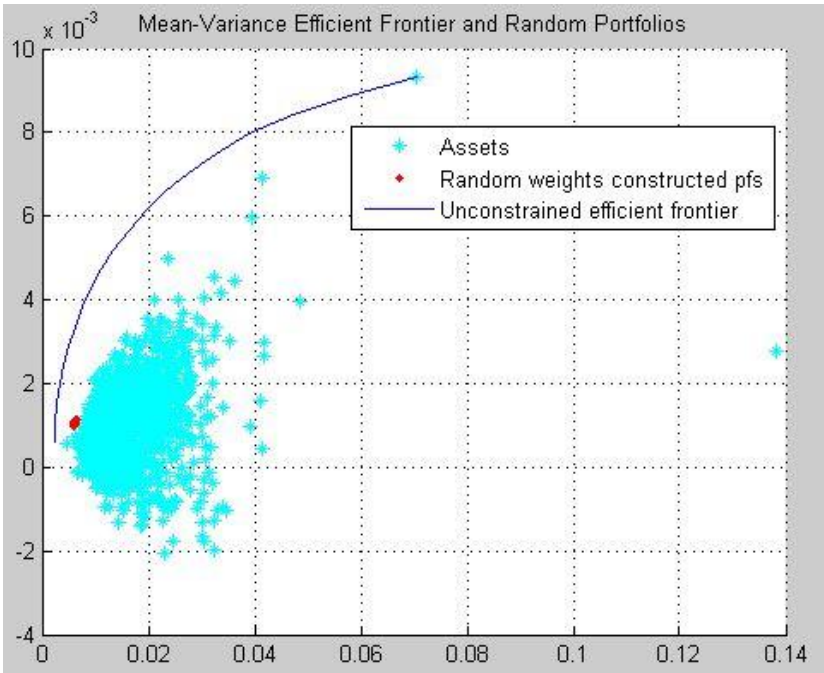

Figure 16: Efficient frontier, MSCI World constituents and 10000 randomly constructed portfolios.

The MSCI World equity index consists out of 1610 constituents. The 10000 random constructed portfolios, where a random weight is assigned to each constituent, cover an even smaller part of the feasible set. As well as in the MSCI Emerging Market case, the portfolios are very concentrated in one point which seems to be a low volatility portfolio.

As can be seen the method to generate random constructed portfolios leads to a concentration of portfolios around a specific point when the number of constituents increases. The method is therefore not suitable to give an overview of all possible portfolios in the current setup.

The SLLN states that the for a large sample of a random variable the average of the sample should be close to the expected value. The random variable X is drawn from a uniform distribution:

$$X \sim U(0,1)$$

By definition:

$$E(X) = \frac{1}{2}(a+b) = \frac{1}{2}(0+1) = \frac{1}{2}$$

Therefore the sum of a portfolio, consisting of 1000 securities, will be:

$$Portfolio\ weight \approx \frac{1}{2} * 1000 = 500$$

The average constituent weight will therefore be:

$$avg\ constituent\ weight \approx \frac{\frac{1}{2}}{500} = 0.001$$

And the maximum constituent weight in the portfolio can not exceed:

$$max\ constituent\ weight \approx \frac{1}{500} = 0.002$$

Since the maximum constituent weight is significantly small, the method leads to portfolio’s with equal weighting scheme characteristics as the number of constituents increases. Therefore the current method to construct ‘random’ portfolios isn’t suitable.

A more specific algorithm could be developed to provide an overview of the feasible set, this is however beyond the scope of this thesis. Besides, using the efficient frontier to determine the feasible set of portfolios has led to the insight that more specific portfolios are of interest. Therefore the next step is to use the deformation of the efficient frontier to determine the impact of constraints.

4.4 Method B: Risk-reward and the efficient frontier

Although the validity of the underlying assumptions of MPT and CAPM can be questioned, the model and theory (Section 3.2) suggest that deformation of the efficient frontier could be used as a tool to assess the effects of constraints. The efficient frontier provides a better overview of the more extreme portfolios. This method will be used as follows, the efficient frontier will be determined in an unconstrained setting and in a constrained setting for different levels of the constraints. Besides the MSCI Emerging Market index and the MSCI World index, the AEX index will also be part of the analysis since it is significantly smaller than the MSCI Emerging market index and the MSCI World index (25 constituents vs. 822 and 1610 constituents).

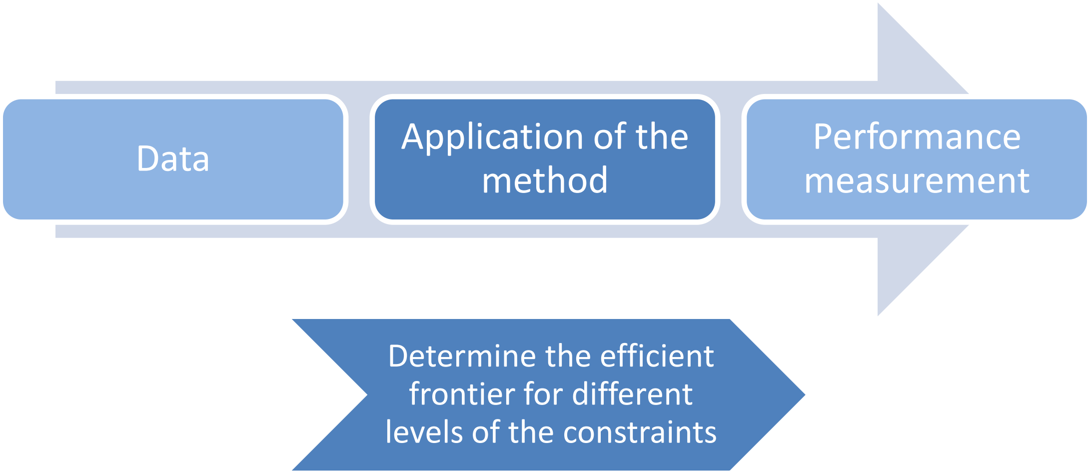

Figure 17: Implementation of the second stage of the model.

As opposed to Section 3.2, where the impact of constraints is assessed by a comparison of the efficient frontier and the resulting frontier from an excess-return optimization (restricted on a fixed value of the constraint). This procedure determines the impact of constraints on the efficient frontier on an ex-post basis. The realized covariance matrix and return vector are used, next to the constraints as described in Section 4.2. The ex-post efficient frontier and the constrained efficient frontiers are depicted in Figure 18 to Figure 23.

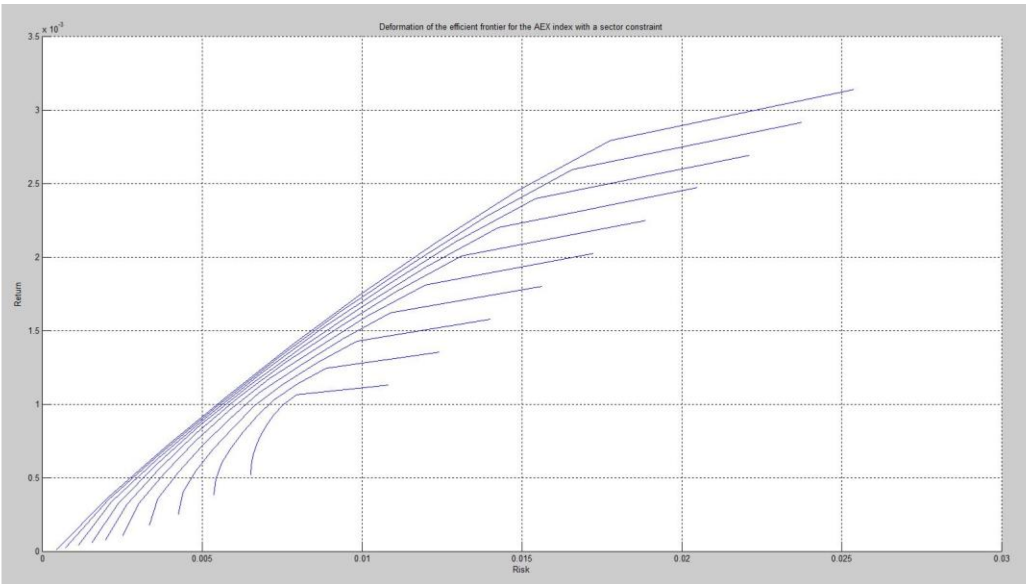

Figure 18: Deformation of the efficient frontier (AEX data) by imposing different levels of the sector constraint.

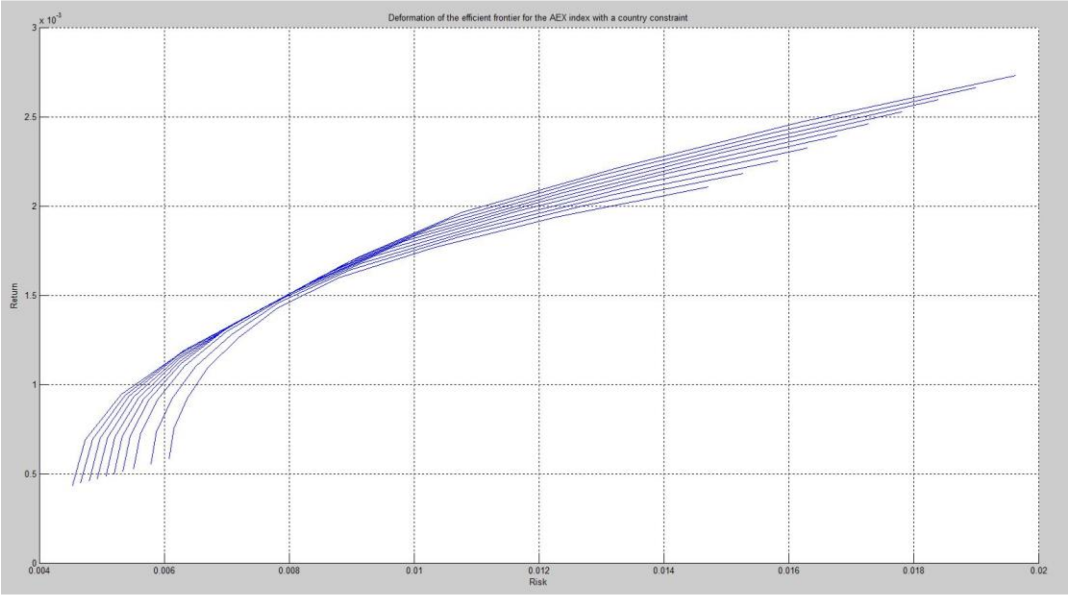

Figure 19: Deformation of the efficient frontier (AEX data) by imposing different levels of the country constraint.

Figure 18 and Figure 19 are the results of a mean-variance optimization under a sector and country constraint. First observation is that in both cases the minimum volatility portfolio shifts to the right (increasing portfolio volatility) and the maximum volatility portfolio shifts to the left (decreasing portfolio volatility). Next to that, the sector constraint seems to have a bigger effect on returns as compared to the country constraint.

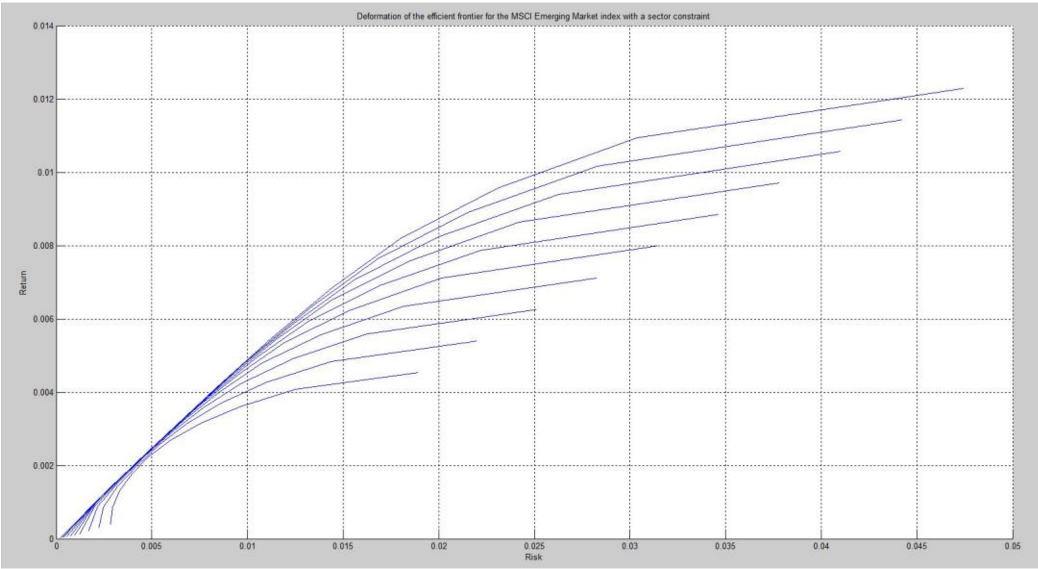

Figure 20: Deformation of the efficient frontier (MSCI EM data) by imposing different levels of the sector constraint.

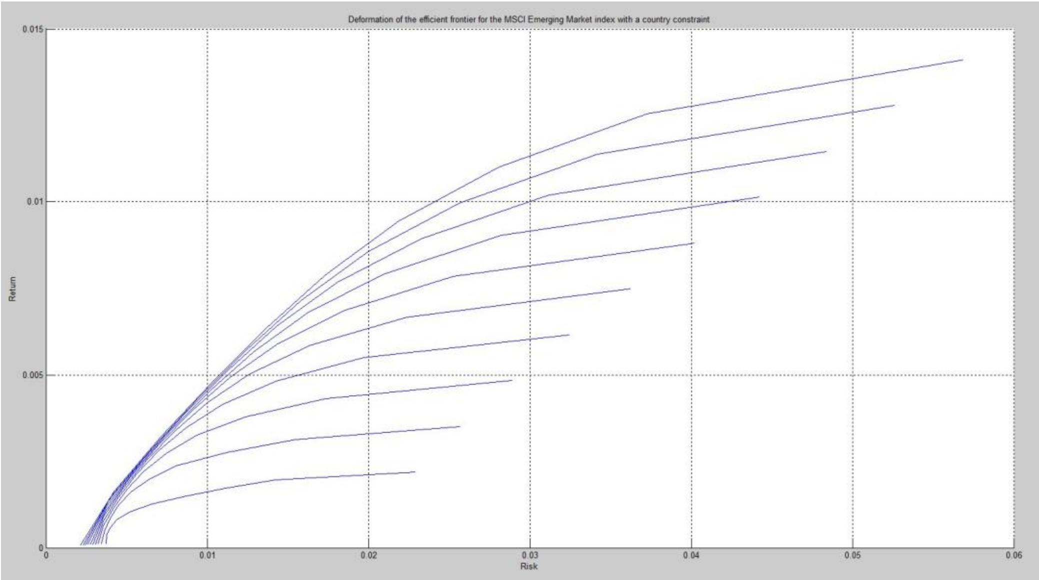

Figure 21: Deformation of the efficient frontier (MSCI EM data) by imposing different levels of the country constraint.

The same behaviour with regards to total portfolio volatility is observed for the MSCI Emerging Market data (see Figure 20 and Figure 21). The movement of the frontier seems to be bigger.

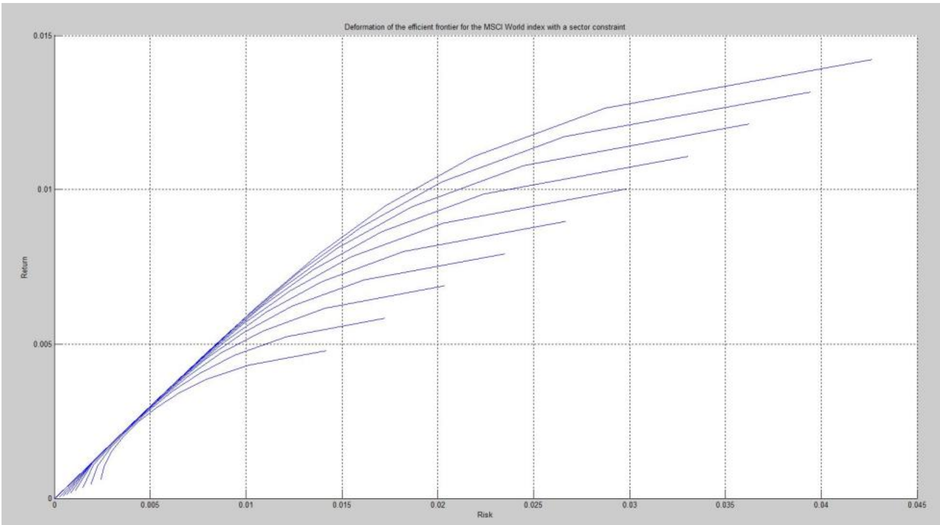

Figure 22: Deformation of the efficient frontier (MSCI World data) by imposing different levels of the sector constraint.

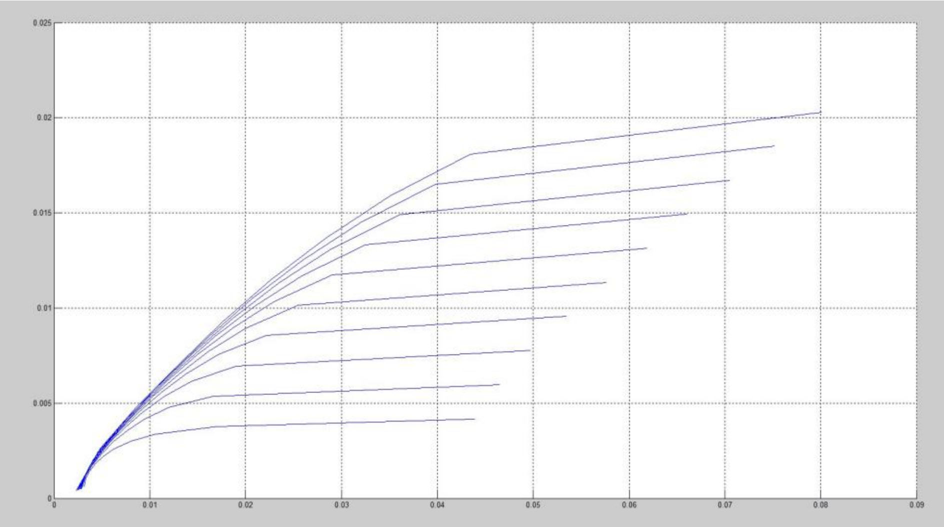

Figure 23: Deformation of the efficient frontier (MSCI World data) by imposing different levels of the country constraint.

The effect of the country and sector constraint on the MSCI World data is depicted in Figure 22 and Figure 23.

An ex-post analysis of the impact of constraints by deformation of the efficient frontier seems to provide graphical insight in the impact of constraints. Nevertheless it is not possible to quantify the effect of the constraints and come up with a good comparison.

Most important insight obtained from the application of the foregoing two methods is the use of risk and reward in order to assess the impact of constraints. Assessing the impact of constraints in terms of limiting the risk (positive consequence of imposing results) and the resulting shrinkage of the reward (negative side effect of imposing constraints) makes the conclusions heavily depend on the used input parameters. An example to clarify the foregoing statement:

If the performance of a benchmark was mainly driven by a specific sector, imposing a strict sector constraint seems to cost performance (more than it limits risk). Resulting conclusion would be that a strict sector constraint is detrimental and that the constraint should be relaxed.

The set-up and conclusions are rather strong although there is no evidence that the markets will behave the same in the future (performance is sector driven). A more abstract perspective on the impact of constraints (like e.g. restrictiveness) would lead to a more sophisticated assessment of the effect of constraints. More on this in Section 4.5.

4.5 Method C: Implementation efficiency and the transfer coefficient

At this point, the random construction of portfolios did not provide a good overview of the possible portfolios, randomly constructed portfolios converge to a ‘1/n’ portfolio if the number of constituents increases in the current implementation. The deformation of the efficient frontier also suffered from several drawbacks. A graphical representation of the impact of constraint is available, but it is not possible to quantify nor compare the effect of constraints.
Because neither methods provided a sufficiently accurate insight in the effect of constraints. The method applied in this section uses the Transfer Coefficient, a performance indicator proposed as extension of the fundamental law of active management.

4.5.1 Application of the method

The fundamental law of active management will be used to assess the effects of constraints on the efficient frontier. As pointed out in Section 3.3, the Transfer Coefficient is proposed as a measure to determine how well a manager can implement his alpha vision. The second stage of the model for this method is depicted in Figure 24.

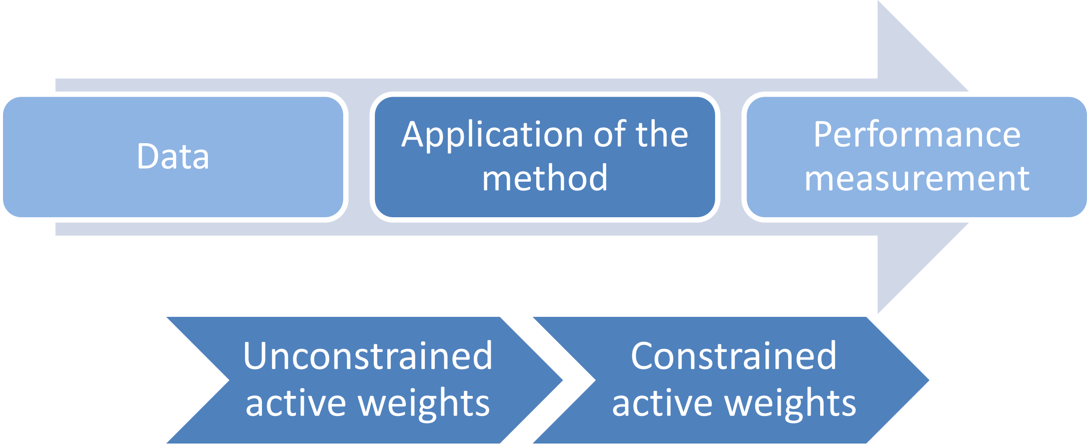

Figure 24: Implementation of the second stage of the model.

The model should assess the restrictiveness of a constraint at a certain point in time for a given benchmark in order to meet the requirements of MN.

First step is to determine the unconstrained active weights, the unconstrained active weights are estimates of the alpha forecasts as pointed out in Section 3.1.

The weights used in this method are all relative weights, as a result the benchmark weigths and the strategy weights sum to 1 and the active weights sum to 0. The active weights can be calculated using the weights of the applied strategy and the benchmark. The weights of the applied strategies are a result of the optimizations described in Section 3.1. An example of a comparison of the weights of the AEX index and the resulting weights of the ERC strategy is provided in Figure 25. The blue bars represent the weights of the constituents of the AEX index, the green bars are the resulting constituent weights when the ERC strategy is applied. The red bars are the active weights, thus the difference between the benchmark weights and the strategy weights.

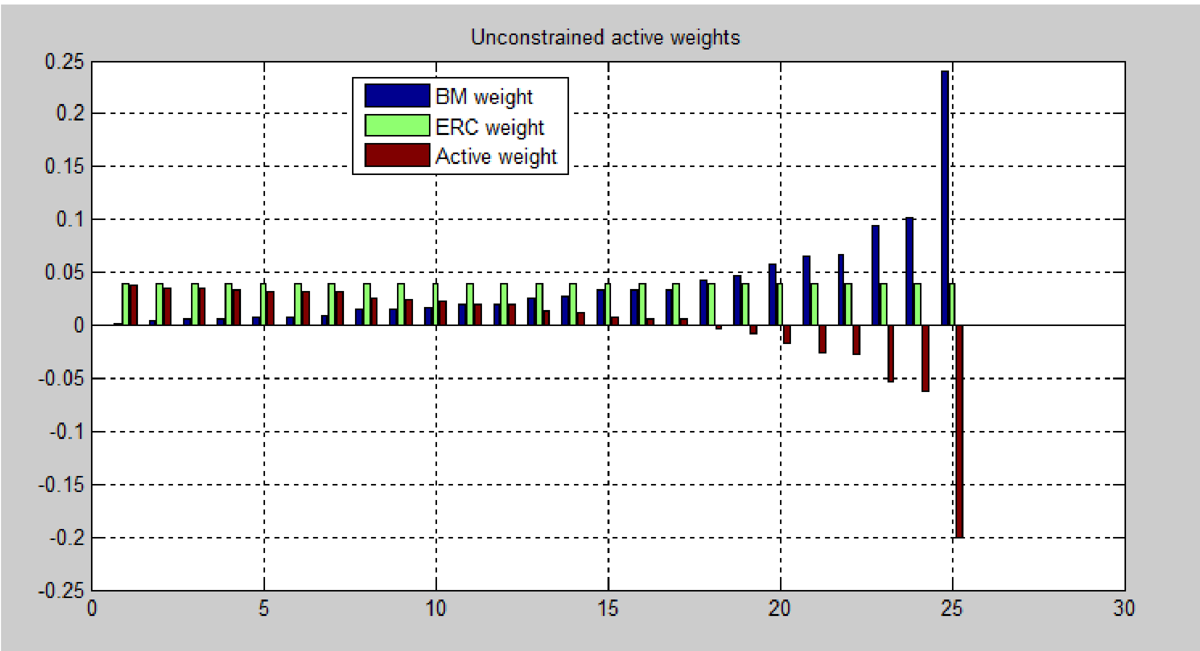

Figure 25: Benchmark weights, unconstrained strategy weights and active weights for the ERC strategy applied to the AEX benchmark; weights are all in relative weight points.

Next step is to determine the constrained strategy weights, the constraints matrix (see Section 4.2) is added as input parameter to the optimization procedure in order to determine the strategy weights. The foregoing example is used with a sector constraint of 5%, the resulting weights are provided in Figure 26.

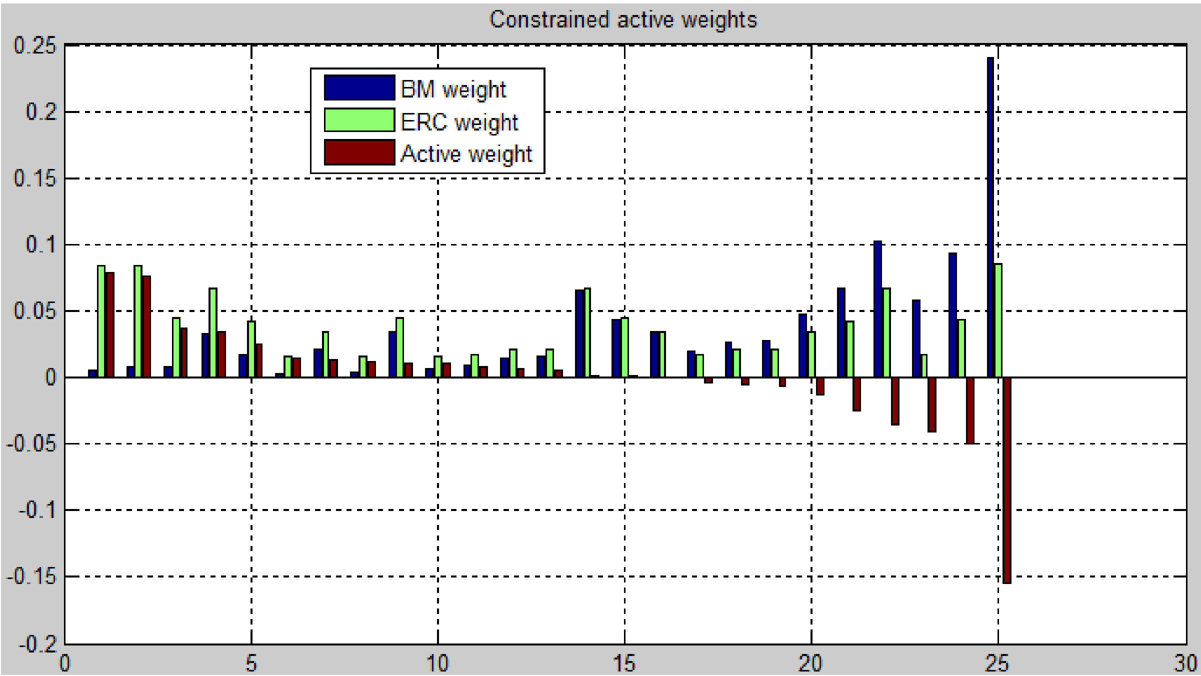

Figure 26: Benchmark weights, constrained strategy weights and active weights for the ERC strategy applied to the AEX benchmark with a 5% sector constraint; weights are all in relative weight points.

As can be observed, the constrained ERC active weights differ from the unconstrained active weights (see Figure 28). The effect of the constraint on sector level is depicted in Figure 27.

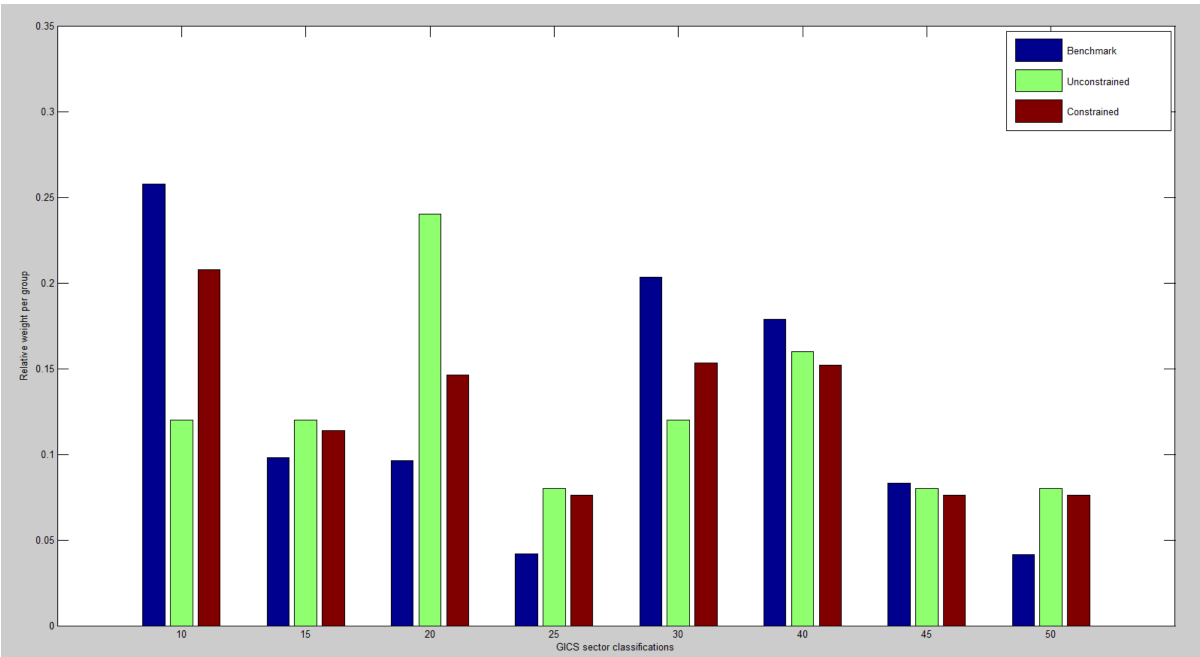

Figure 27: Overview of the sector weights for the benchmark, the unconstrained active portfolio and the 5% sector constrained active portfolio.

The blue bars represent the sector weights in the benchmark, the green bars represent the sector weights if the manager could fully implement his alpha vision and the red bars represent the sector weights for a 5% sector constrained implementation of his alpha vision. Figure 27 clearly shows how the constrained implementation is pulled towards the sector weights of the benchmark. Since each sector consists of multiple stock an alpha view can be implemented although the manager is not allowed to deviate from the benchmark weights. For example the case where a sector consists out of 3 stocks, underweighting one stock with 1% enables to overweight to other stocks within the sector for a total of 1%. This also hold for the country constraint. The single asset constraint on the other hand replicates the benchmark if the restriction is set at 0%.

The unconstrained active weights and the constrained active weights will be used to calculate the TC.

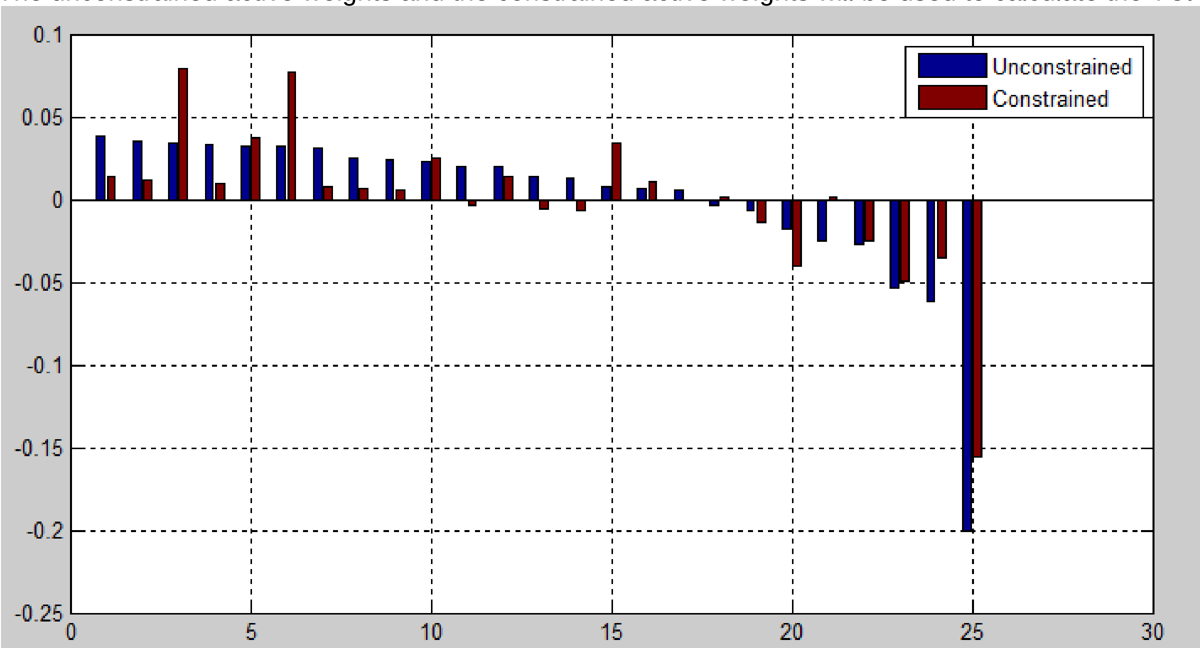

Figure 28: ERC unconstrained and constrained active weights for a 5% sector constraint.

The method described in the foregoing has some attractive features. It is likely that the estimators which are used as inputs contain errors (see Section 4.1.3). Since they will only be used to determine the strategy weights for different levels of the constraint, the errors will remain constant. From a practical perspective its attractiveness stems from the fact that additional alpha strategies can be easily added to the model. Furthermore, the model doesn’t rely on any other assumptions.

4.5.2 Performance measurement

Since the procedure described in the foregoing section seems to be applicable, this section will elaborate on the Transfer Coefficient and how to use it to determine the restrictiveness of constraints.

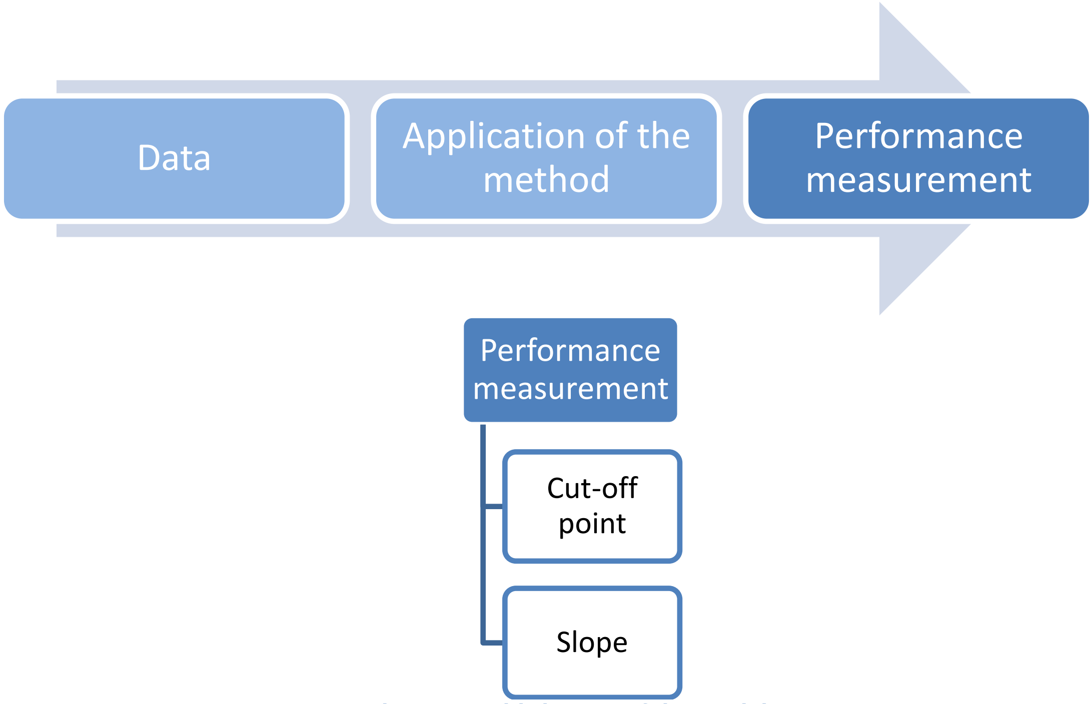

Figure 29: Third stage of the model.

From its definition, the TC will be calculated as the correlation between the alpha view of a manager and the active weights of the portfolio. As pointed out in Section 3.4, unconstrained active weights are a good proxy for the alpha view for a specific strategy and will be used for that purpose in this setting.
The most important conceptual difference with the risk/reward assessment is the fact that the TC is an indicator of the implementation inefficiency of a portfolio, rather than evaluating constraints in terms of good and bad (resp. mitigation of risk and cost of performance). Due to this more abstract approach, the results can be interpreted from multiple perspectives. From a manager selection perspective low values of the TC indicate that the proposed strategy of the manager differs a lot from the benchmark. From an 'impact of constraints' perspective, the TC for different levels of a constraint makes the constraints comparable. Furthermore a value judgement about the impact of constraints depends on the skill of the manager, it could be valuable to apply strict constraints to a manager with less skill.

In order to determine the restrictiveness of a constraint, the TC will be determined for different levels of the constraint. An example of the resulting chart is given in Figure 30.

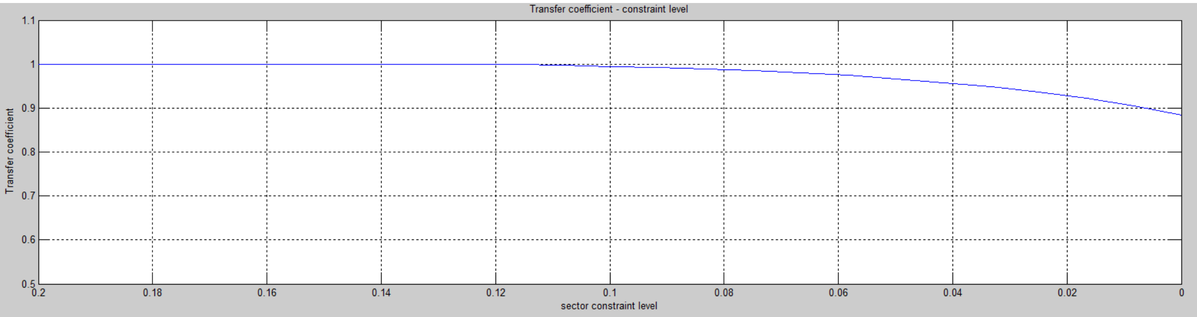

Figure 30: Transfer coefficient (y-axis) for different levels of the sector constraint (x-axis), the value for which the constraint becomes binding is 11% (0.11).

First point of interest in the resulting chart is the cut-off point, for that level of the constraint it becomes binding (see Figure 30; 11% in this particular case). The second property of interest is the slope of the line when the constraint is binding, this will be determined with a linear approximation:

$$slope = \frac{TC_{cutoff} - TC_0}{cutoff(-0)}$$

The method will be applied for a range of 0% - 20% for the country and the sector constraint and 0% - 10% for the single asset constraint, as determined in Section 3.1.

5 Findings

The suggested method in Section 4.5 is applied here. The resulting charts are depicted in Appendix B. The cut-off point and slope for each constraint are determined per strategy.

First observation is the difference in the range of TC per constraint. The TCs for the single asset constraint range from -1 to 1, the TCs for the sector constraint range between 0.45 and 1 and the TCs for the country constraint range from 0.28 to 1. Furthermore the TC's for the value and growth strategy significantly differ from the other strategies for all constraints.

The resulting cut-off points and slopes are presented in Table 5.

Table 5: Overview of the results, the cut-off point is determined for each strategy per constraint.

		Single asset constraint		Sector constraint		Country constraint	
Benchmark	Strategy	Cut-off	Slope	Cut-off	Slope	Cut-off	Slope
AEX	ERC	0.03	65.35	0.085	1.38	0.015	0.12
	Min Vol	0.09	8.78	0.17	1.99	0.04	0.55
	RE	>0.1	6.00	>0.2	2.35	0.035	0.12
	Value	>0.1	6.30	>0.2	1.16	>0.2	0.05
	Growth	>0.1	5.90	>0.2	-0.44	>0.2	0
	EW	0.03	66.33	0.12	1.06	0.015	0.02
MSCI Emerging Markets	ERC	0.01	200.00	0.035	0.20	>0.2	1.07
	Min Vol	0.1	15.20	0.15	0.99	>0.2	1.60
	RE	0.07	23.71	0.12	0.50	>0.2	1.08
	Value	>0.1	4.50	>0.2	-0.04	>0.2	-0.01
	Growth	>0.1	2.71	>0.2	0.13	>0.2	0.046
	EW	0.01	200.00	>0.2	0.11	>0.2	0.92
MSCI World	ERC	0.01	198.00	0.04	0.33	>0.2	0.55
	Min Vol	>0.1	13.60	0.09	0.63	>0.2	1.84
	RE	0.08	17.88	0.09	1.46	>0.2	1.80
	Value	>0.1	1.76	>0.2	-0.24	>0.2	0.14
	Growth	>0.1	1.08	>0.2	0.01	>0.2	0.12
	EW	0.01	200.00	0.02	0.26	>0.2	0.35

The analysis of the restrictiveness is twofold, at first the highest and lowest cut-off points will be compared between the constraints. Higher numbers for the cut-off point indicate a more restrictive constraint. Secondly, the sensitivity of the constraints is assessed by a comparison of the different slope numbers.

Since the evaluated range of the single asset constraint differs from the range for which the sector and country constraints are evaluated, the analysis starts-off with a comparison of the cut-off points for the sector and the country constraint.
The highest values of the cut-off point for the sector and country constraint are marked red. The marked cut-off points are in both cases cut-off points which are beyond the evaluated range. A cut-off point beyond the evaluated range implies that the constraint is already binding for the specific strategy for the least restrictive value of the constraint. As can be seen, the country constraint is binding for all strategies applied in the MSCI Emerging Markets universe and the MSCI World universe. The sector constraint is only binding for the least restrictive value of the constraint in 5 of the 12 scenarios.
The single asset constraint is evaluated on a more restrictive range of values for the constraint (0% - 10%), the cut-off point is beyond the evaluated range for 5 of the 12 scenario's. From this can be concluded that the country constraint is the most restrictive constraint in terms of cut-off point for the evaluated strategies and benchmarks.

Another observation with regards to the cut-off points of the constraints is that the single asset constraint becomes binding in 4 out of 12 scenarios for a constraint level of only 1%. This implies that the single asset constraints is not binding for constraint levels which are currently applicable in the IMAs for the evaluated strategies and benchmarks.

The highest and lowest values for the slope are marked (resp. red and green) in order to compare the constraints. Most important observation is the significant higher levels of the slope for the single asset constraint as compared to the sector and country constraint.

6 Conclusions

The purpose of this master thesis is to quantify the effects of constraints. Since externally managed portfolios are usually subjected to constraints, insight in the impact is needed. Actively managed portfolios have the purpose to outperform a benchmark, fundamental indexation is applied in this thesis to simulate the alpha view of a manager. More specific, the following strategies are applied: Equal Risk Contribution, Minimum Volatility, Risk Efficient, Value, Growth and Equal Weighting.

In order to get an overview of the suggested performance indicators to assess the impact of constraints, a literature review is made. This yielded that basically 2 branches can be distinguished in academia. A branch that assesses the impact of constraints in terms of risk and reward. For example by a comparison of the mean-variance efficient frontier and a constrained excess return optimized frontier or shrinkage of performance indicators like the Sharpe ratio or the Information ratio. The second branch in academia follows the 'fundamental law of active management', the Transfer Coefficient is proposed to measure the inability of a manager to transfer his alpha view to actual portfolio positions.

From this literature review we proposed three methods which should meet the requirements of MN in principle. The first method was an ex-post analysis of a large sample of portfolio values from random constructed portfolios. This method failed in practice due to the fact that the (assumed) random constructed portfolios converged to an equally weighted portfolio for indices with a large number of constituents. Developing a new method to construct real random portfolios is beyond the scope of this thesis. The second method was to evaluate the impact of constraint by deformation of the efficient frontier. Both methods were stemming from the risk/reward literature. This method lacked the ability to provide quantitative insight in the constraints although the resulting frontiers were suitable to provide some graphical results about the impact of constraints.
The third method follows the second branch and attempts to assess the impact of constraints by measuring the Transfer Coefficient.
But more important, this led to the notion that assessing constraints in terms of risk and reward is an ex-post exercise in which the induced conclusions strongly depend on the estimates (most of the time historical data). The Transfer Coefficient in method 3 on the other hand distinguishes the restrictiveness of a constraint and the skill of a manager. An unskilled manager can add value due to constraints (low TC) as a result of undesirable bets, whereas the added value of a skilled manager shrinks due to the same constraints.

Applying the third method leads to the conclusion that the country constraint is the most restrictive one (as compared to a single asset constraint and a sector constraint) in absolute sense, it is binding in most of the evaluated scenario's for the values of the constraint which are currently applicable in the IMAs. Furthermore, the single asset constraint turns out to be most sensitive for small changes in the value of the constraint given that it is binding. This is due to the fact that the single asset constraint becomes binding for low values of the constraint. Second reason is the fact that a single asset constraint of 0% implies that the benchmark is replicated, whereas this is not the case for a 0% sector of country constraint, as a result absolute TC numbers are lower for the single asset constraint.

7 Discussion and further research

The suggested analysis could be improved in multiple ways. In order to provide a more indepth insight in the restrictiveness of constraints, an attribution of the restrictiveness to different risk factors could be executed. This could be valuable to tailor externally managed portfolios to specific risk factors. Another extension could be to do an ex-post analysis of the impact of constraints and the skill of a manager in order to check the validity of the results from this work. Ultimately, relaxing the long only constraint could overcome drawbacks in the constrained active portfolios. As pointed out by Scherer & Xu (2007), the long only constraint leads to a small cap bias in the managed portfolio. Overweighting positions in the active portfolio can only be funded from underweighting of a large cap position, because the underweighting of small caps do not result in sufficient free funds to construct the desired overweight.

The practical significance of the model could also be improved, whereas this work focuses solely on benchmark relative constraints. Absolute constraints like beta, VaR or a max turnover constraint are also applied in practice. But even more, assessing the impact of constraints on different kind of securities (like fixed income) could improve the practical significance. From this it would be interesting to impose multiple constraint and attribute the effects on the portfolio to the particular constraints.

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Appendices

Appendix A: Modern Portfolio Theory and CAPM

Modern portfolio theory: the efficient frontier
Markowitz (1952) formed the fundament for what now-a-days is known as modern portfolio theory (MPT). Dependent on the expected return, expected variance and the expected correlation structure of a universe of assets, a portfolio can be constructed with maximum expected return given an unit of risk or a portfolio with minimum risk given an expected return (mean-variance optimization (MVO)). The efficient frontier is formed by a set of portfolios which are return/risk efficient (see Figure 31). As pointed out in e.g. Luenberger (1998), adding a risk-free investment opportunity results in a tangent line to the efficient frontier that traverses the risk-free rate on the y-axis (expected return axis), this line is called the capital market line (CML).

Following Luenberger's (1998) one-fund theorem:
There is one single fund of risky assets such that any efficient portfolio can be constructed as a combination of the fund and the risk-free asset.

This single fund of risky assets is the tangent point at the efficient frontier and is called 'the market portfolio' (Luenberger, 1998). On the domain $0 < \sigma \leq \sigma_m$, the CML consists of long positions in both the risk-free asset and the market portfolio. On the domain $\sigma_m < \sigma$, the CML consists of a short position in the risk-free asset and a long position in the market portfolio. Figure 31 provides an overview of the efficient frontier, the CML and the market portfolio.

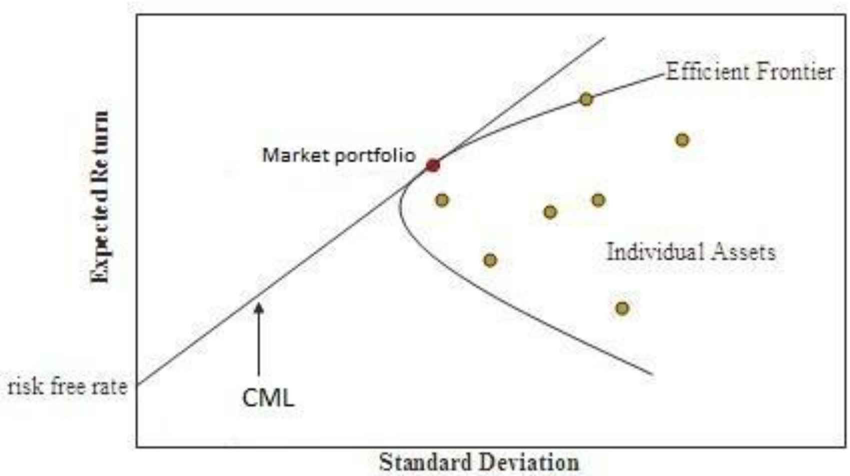


Figure 31: The efficient frontier and the capital market line (CML)

Assuming that the foregoing holds the, investment manager has two possibilities: replicate the benchmark, which is a costly alternative to an index tracker (ETF), or deviate from the benchmark (market portfolio) which would always lead to a suboptimal portfolio from a return/risk perspective. Constraining such portfolios with benchmark relative constraints will pull the portfolio to the benchmark. More on this in paragraph 4.4.

Having defined the market portfolio, Sharpe (1964) and Lintner (1965) built forth on the work of Markowitz and developed the Capital Asset Pricing Model (CAPM). CAPM enables one to determine the required rate of return of an individual security given its level of risk. Main insight provided by the model is the fact that a securities risk consist out of systematic risk and unsystematic risk. As stated by Luenberger (1998):

If the market portfolio M is efficient, the expected return r_i of any asset i satisfies:

$$r_i - r_f = \beta_i * (r_M - r_f)$$

Where

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}$$

Where β_i is the factor which determines the part of the securities risk which is correlated with the market (systematic risk). Unsystematic risk is the difference between the securities volatility and the systematic risk, this part can be cancelled out through diversification. From this a securities expected return can be determined by displaying the CAPM formula in the β-return plane.

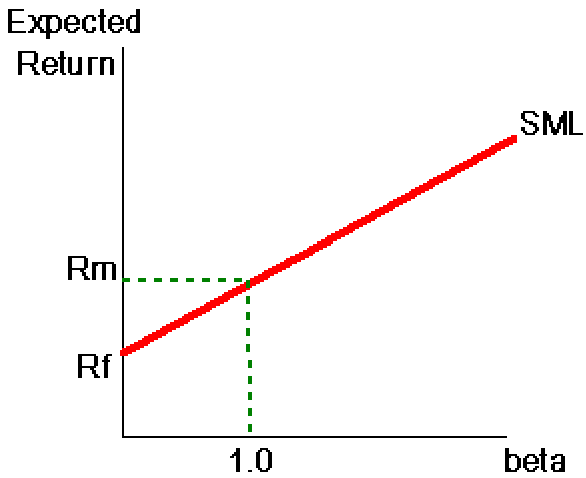


Figure 32: The security market line in the expected return-beta space

Figure 32 gives an overview of the linear relationship, it's a line traversing the y-axis (expected return) through the risk-free rate (r_f) at β = 0, another important point is at β = 1 (x-axis), at this point the expected return equals the market return (r_m). Figure 32 is called the Security Market Line (SML). Following MPT and CAPM, each asset in an efficient market should fall on this SML.
At this point some basic understanding of portfolio construction, risk and risk-return is developed which is necessary for the remainder of this theoretical framework.

Active management: definitions
Following MPT and CAPM, the only portfolio of interest is the market portfolio. In practice however, investment managers are given the task to 'beat' the benchmark. This statement is made by investors who doubt the efficiency of a market and therefore the feasibility of the Efficient Market Hypothesis (EMH). The EMH is one of the underlying assumptions for MPT and CAPM and is formally proposed by Fama (1970) and occurs in 3 versions:

- The *weak* efficient market hypothesis claims that prices on publicly traded assets reflect all available past data.
- The *semi-strong* efficient market hypothesis claims that prices on publicly traded assets reflect all available past data and instantly change when new public data is available.
- The *strong* efficient market hypothesis claims that in addition to the *semi-strong* efficient market hypothesis, the price also reflects inside information.

Malkiel (2003) provides a good overview of the criticism towards the EMH. He analyzes price prediction models stemming from Technical Analysis and Fundamental Analysis. Technical Analysis mainly affects prices through so called behavioural finance. Recognizing patterns in historical data should lead to buy or sell signals, if sufficient investors act on these signals the predicted prices will be realized and the prediction becomes a self fulfilling prophecy. Fundamental Analysis tries to predict stock prices based on fundamental parameters as price/earnings and market value/book value (a so called value stock) or price/earnings (so called growth stocks). Main conclusions are that pricing irregularities exist due to wrong collective judgement of investors or less rational market participants. Although it is very difficult to obtain extraordinary returns without taking excess risk.

Managing a deviating portfolio against a benchmark is called active management, active management is defined by Cremers and Petajisto (2009) as:

> *"Passive management of a portfolio is easy to define: it consists of replicating the return on an index with a strategy of buying and holding all (or almost all) index stocks in the official index proportions.*
> *Active management can then be defined as any deviation from passive management."*

The purpose of active management is to capture alpha (α, excess return or active return), alpha is defined by Jensen (1968) as:

$$\alpha = [E(r_p) - r_f] - [E(r_m) - r_f]\ x\ \beta_{r_p,r_m}$$

Or depicted in the return-beta space (like Figure 32) as can be seen in Figure 33.

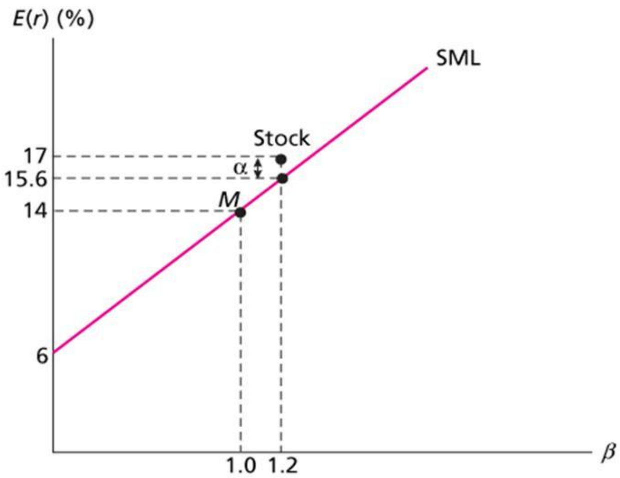


Figure 33: Alpha in the security market line framework

Alpha indicates the amount of excess return over the expected return given the systematic risk of a security or portfolio.

In line with active return, Roll (1992) and Grinold (1989) defined Tracking Error Volatility (TEV) as a measure of the active risk. It is mathematically expressed as:

$$TEV = Std(r_p - r_b)$$

With r_p, the returns of the active portfolio and r_b the returns of the benchmark portfolio. The TEV is called Tracking Error Volatility because it basically explains how well the active portfolio tracks the benchmark portfolio. Since the returns of the portfolio and the benchmark are not known on forehand this measure is only suitable for ex-post analysis of the TEV. An ex-ante TEV estimate can be made using the active weights.

Since active management is about obtaining alpha, deviating from the benchmark is a given. Cremers & Petajisto (2009) propose a measure to determine the possible alpha an active manager can obtain, active share. Active share or active weights are defined as the difference of the stock weight in the active portfolio and the stock weight in the benchmark.

$$\Delta w = w_{pf} - w_{bm}$$

The active weights are a good proxy for the forecasted alpha on security level, since overweighting or underweighting of a security results from an alpha view. The active weights can also be used to estimate the ex-ante TEV, which is defined by Grinold (1989) as:

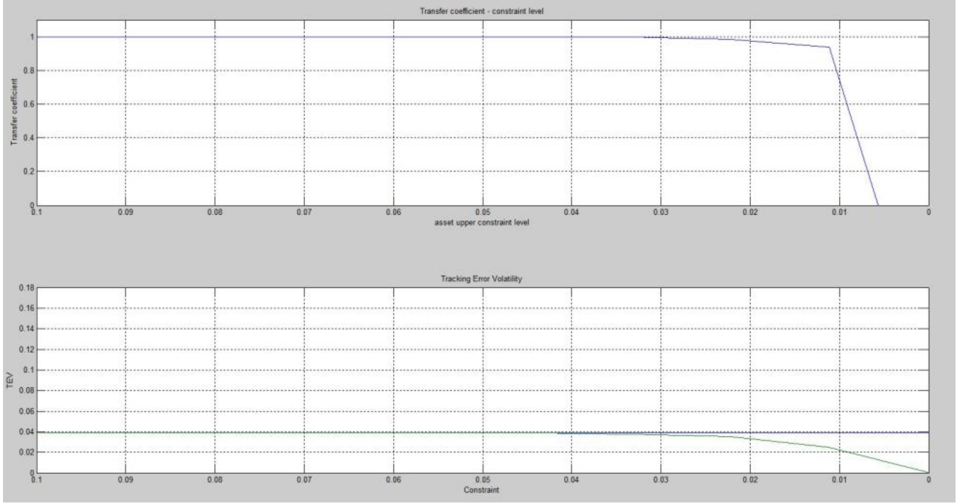
$$TEV = \Delta w' * \Sigma * \Delta w$$

With:
n, the number of constituents
Δw, a nx1 matrix with active weights
Σ, the nxn covariance matrix
Risk, volatility, standard deviation, variance

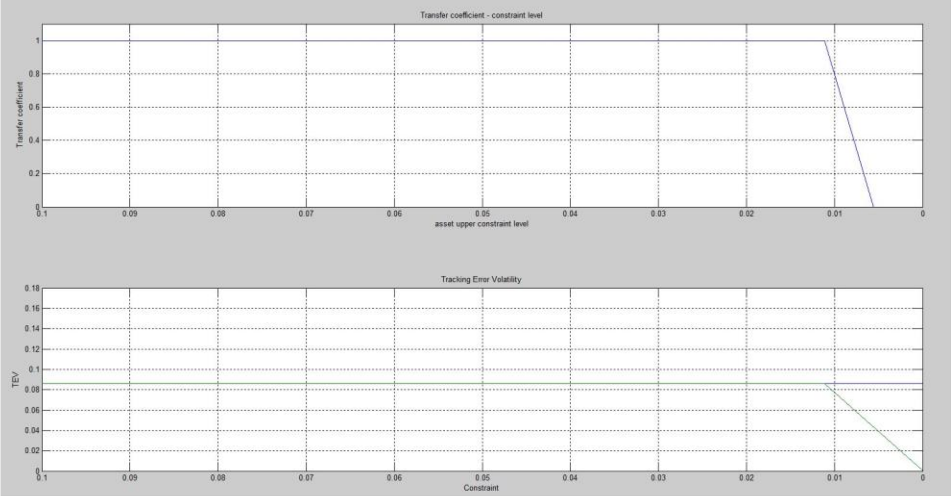
Appendix B: Analysis results

Single asset constraint

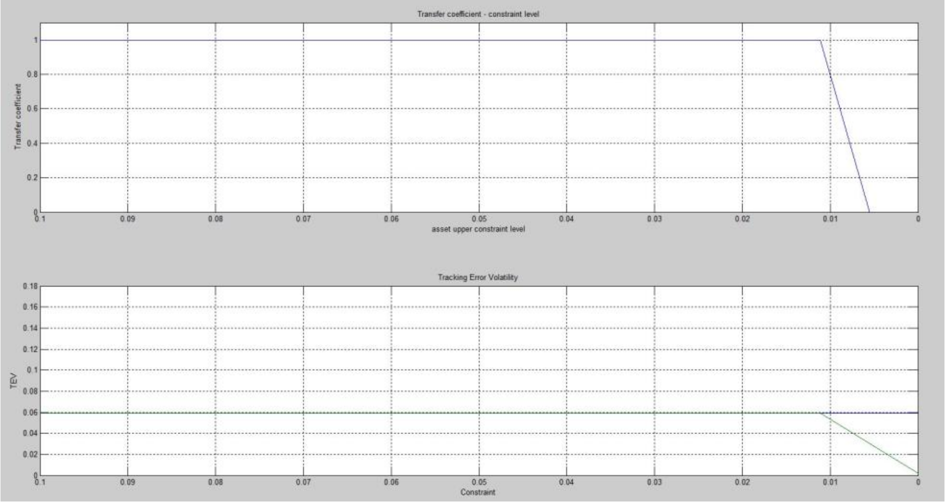
AEX - Equal Risk Contribution



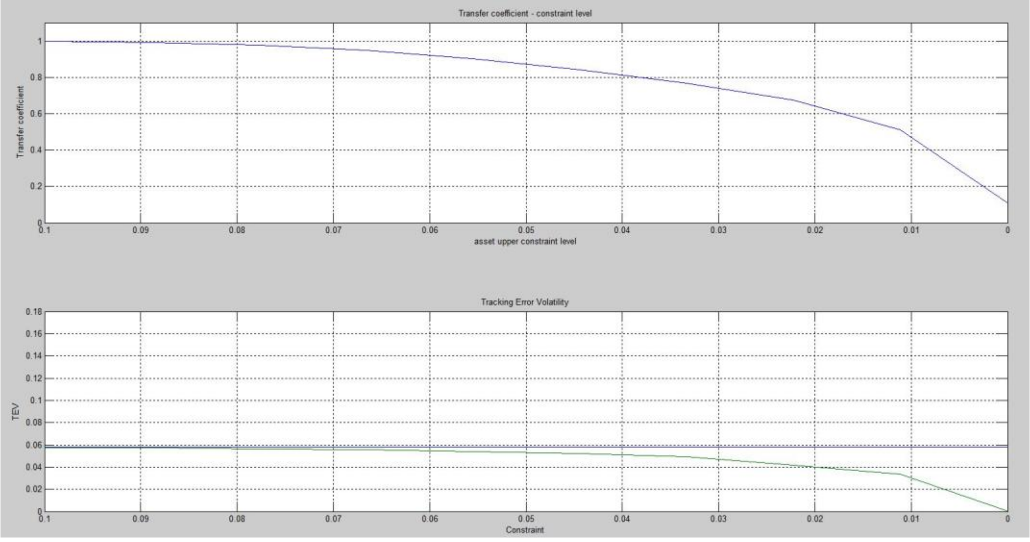
MSCI Emerging Market - Equal Risk Contribution



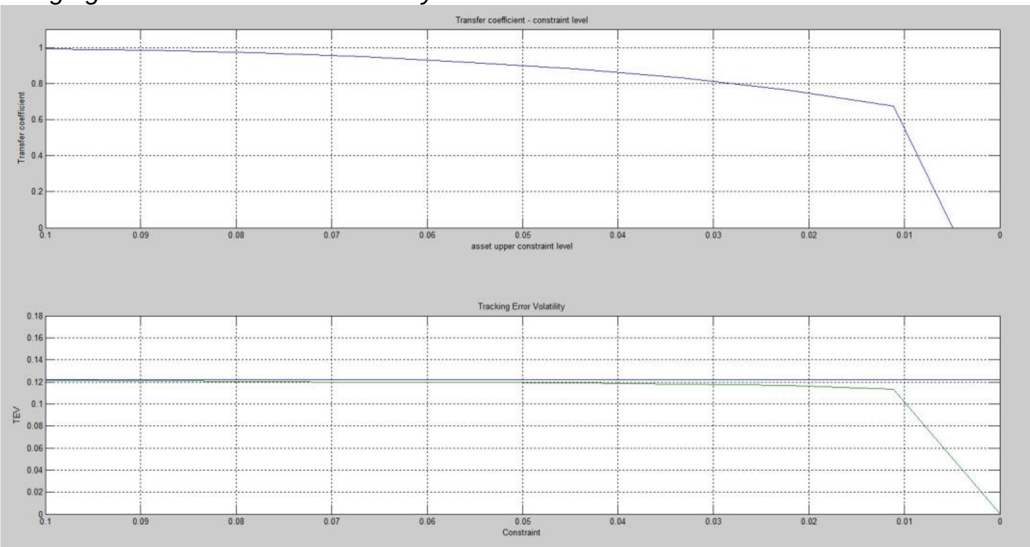
MSCI World - Equal Risk Contribution



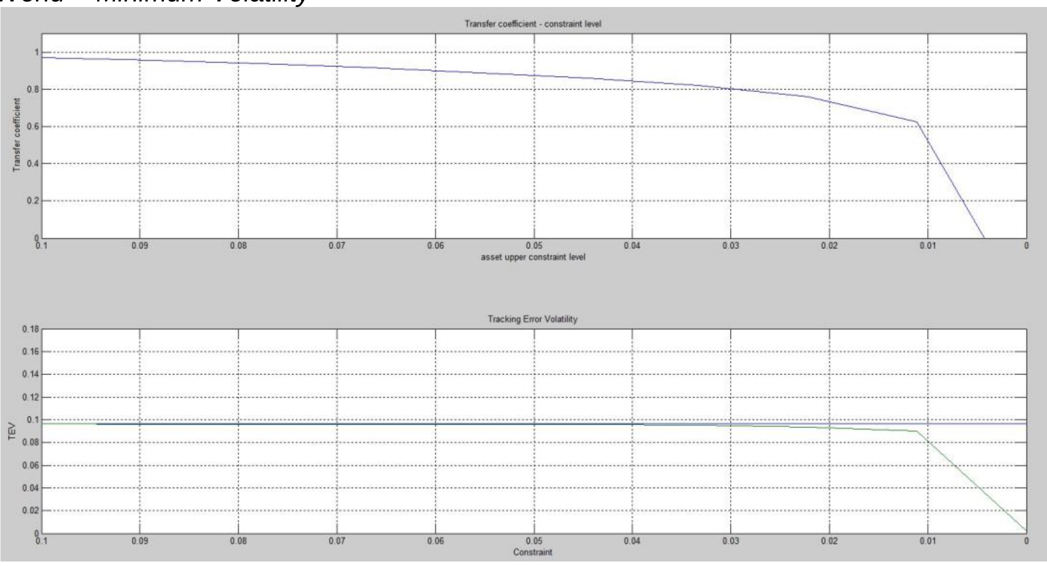
AEX - Minimum Volatility



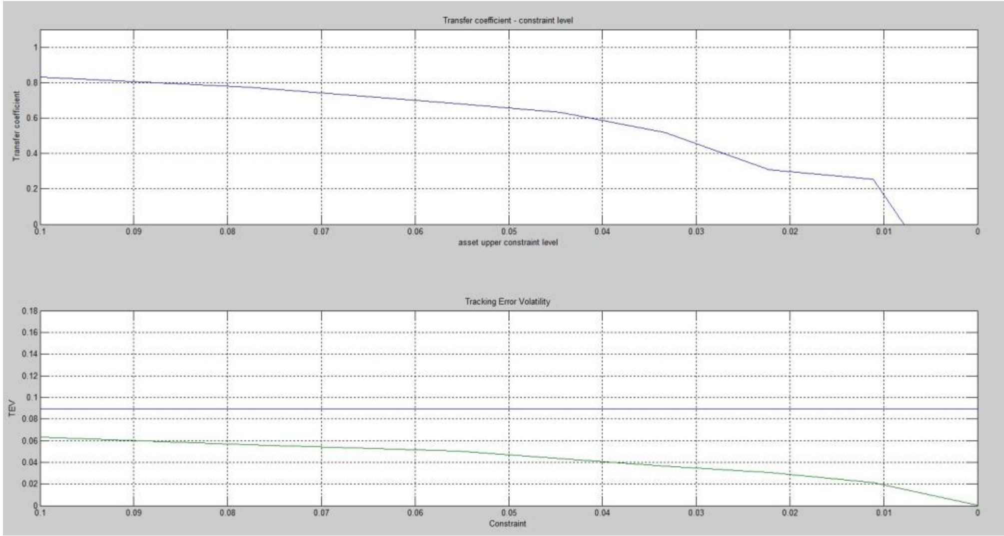
MSCI Emerging Market - Minimum Volatility



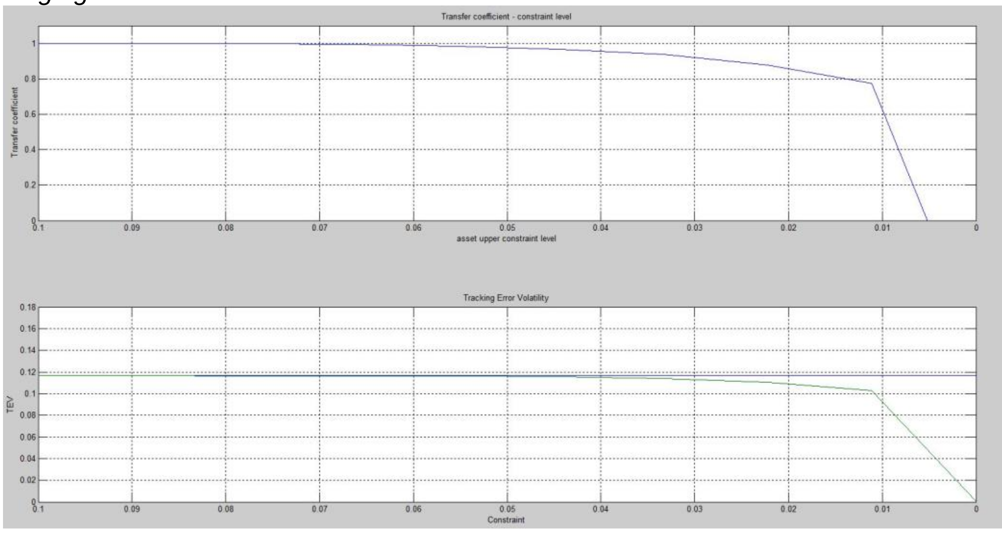
MSCI World - Minimum Volatility



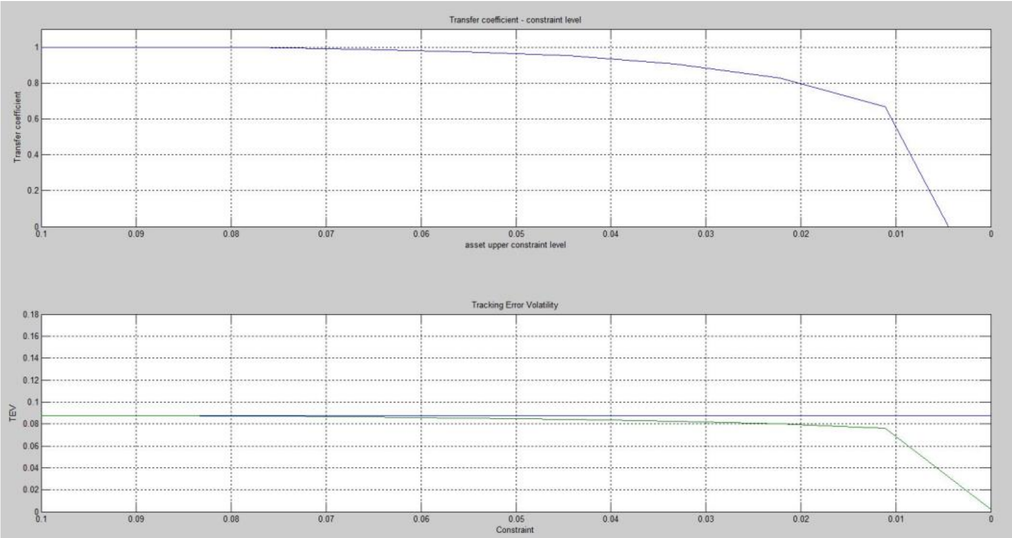
AEX – Risk Efficient



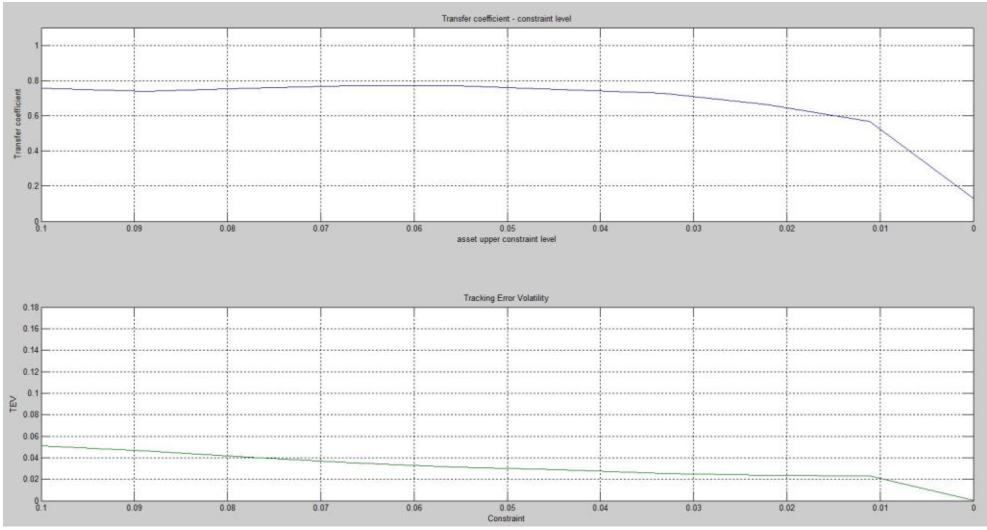
MSCI Emerging Market – Risk Efficient



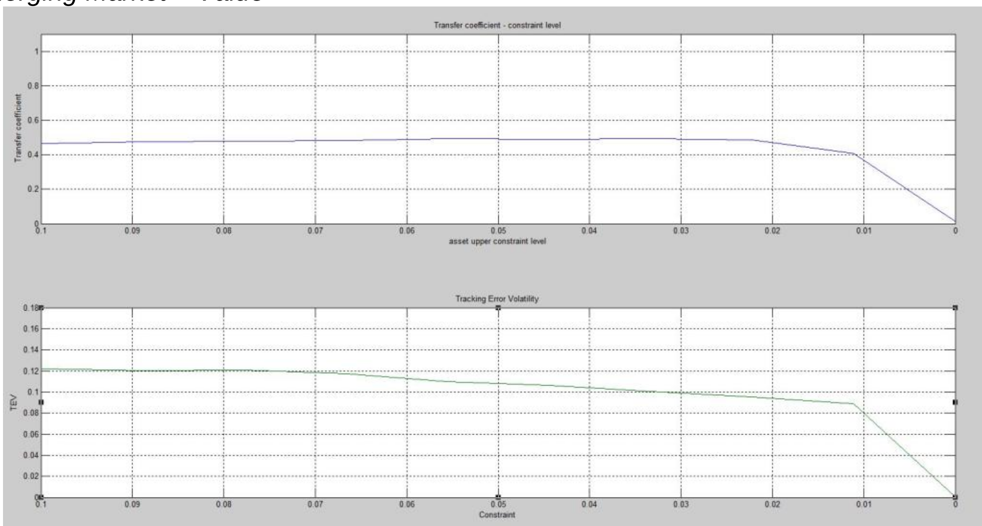
MSCI World – Risk Efficient



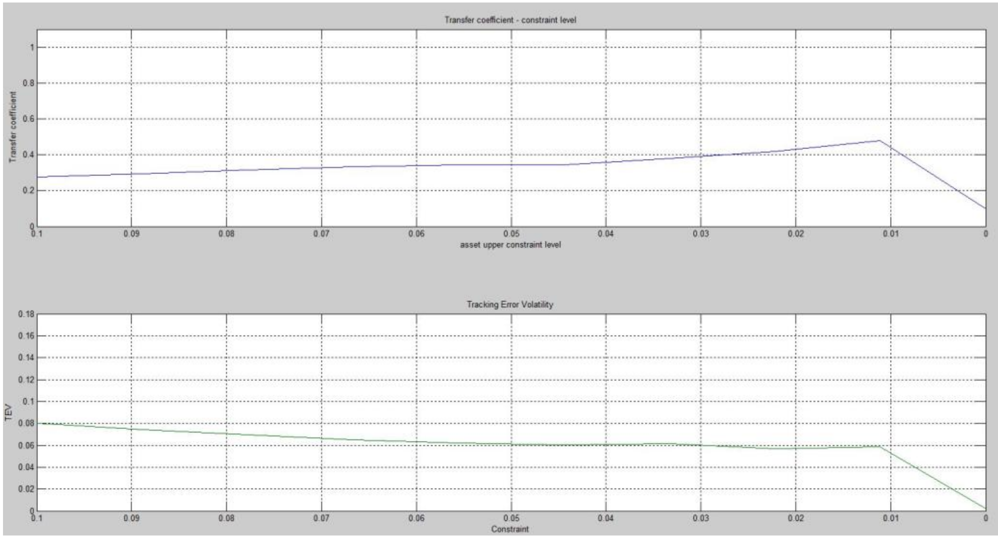
AEX – Value



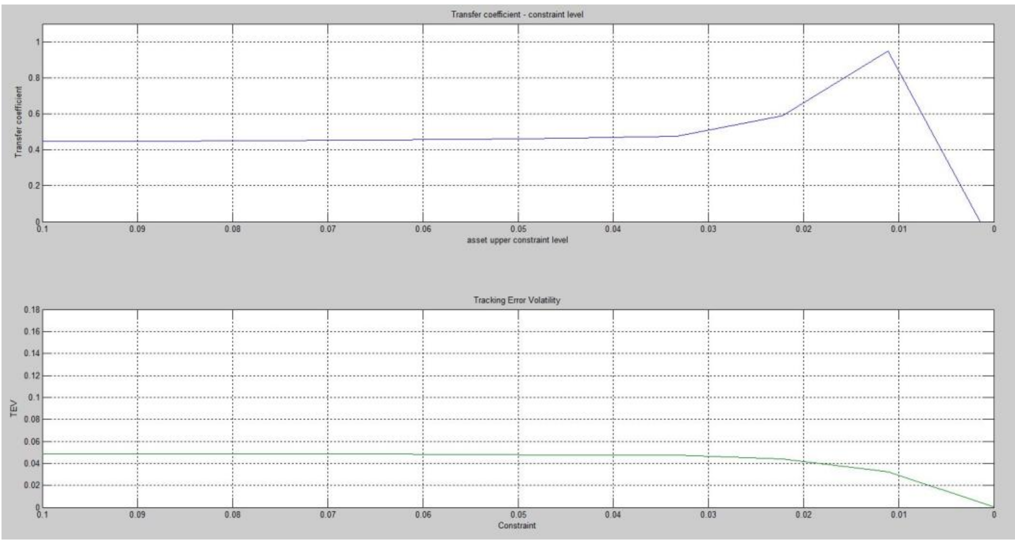
MSCI Emerging Market – Value



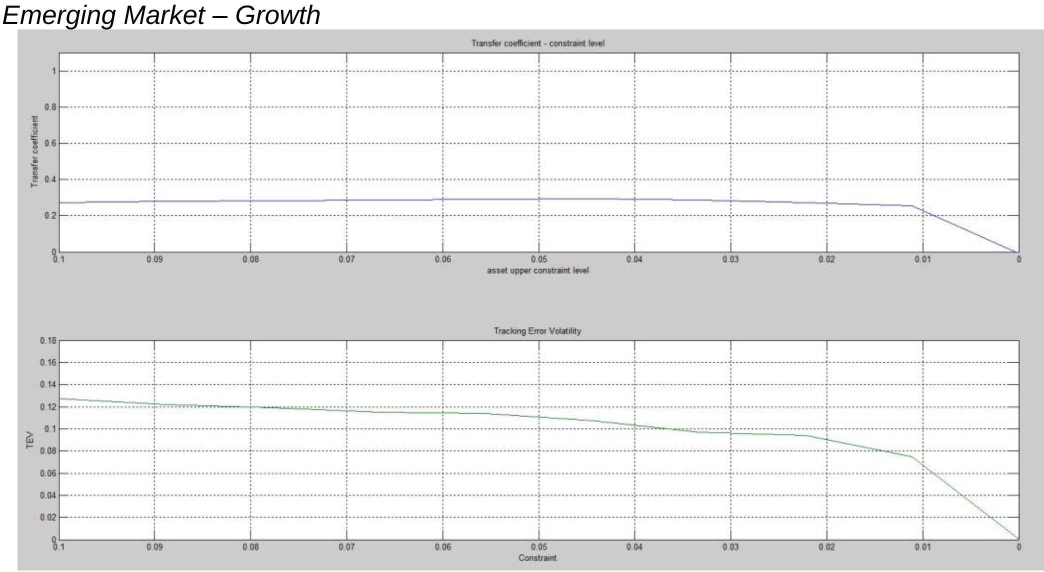
MSCI World – Value



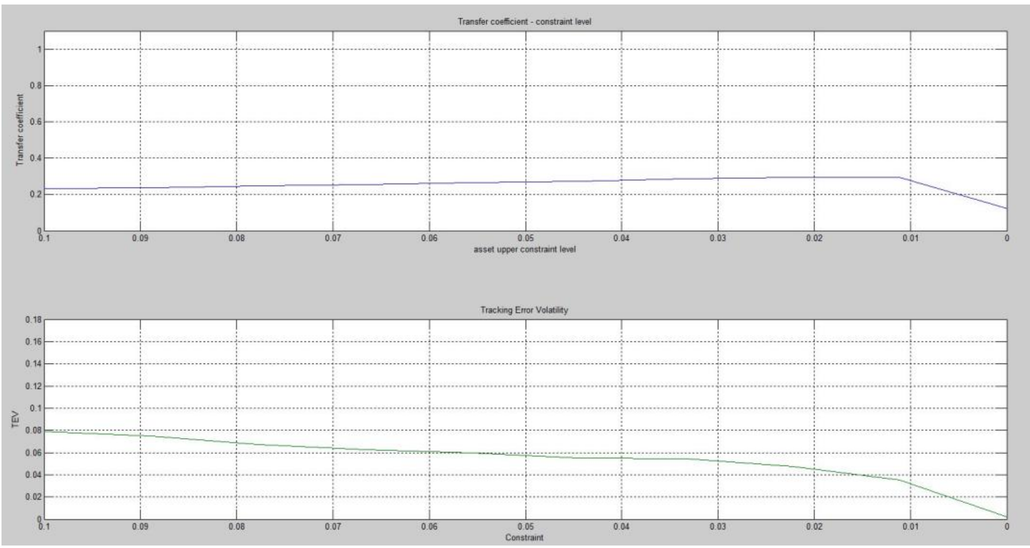
AEX – Growth



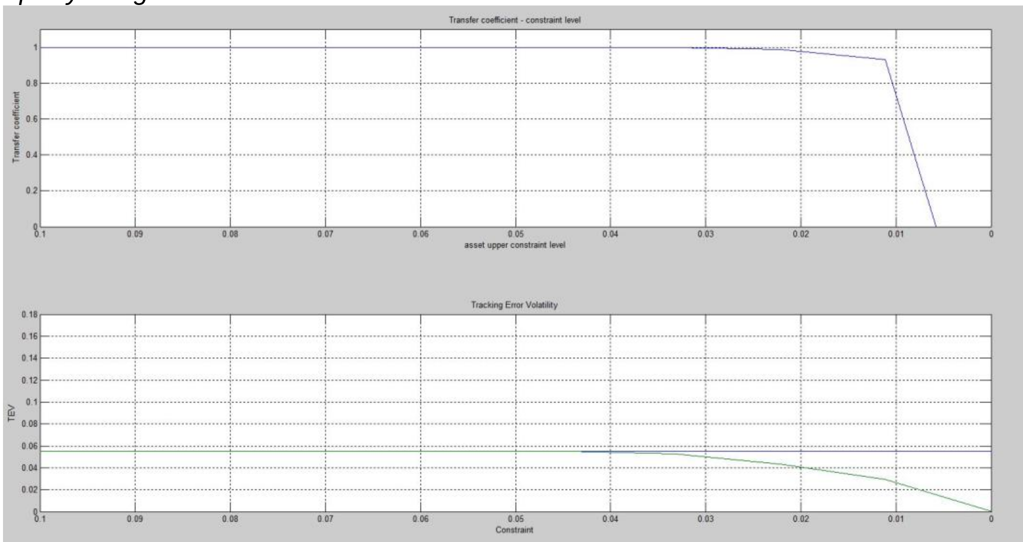
MSCI Emerging Market – Growth



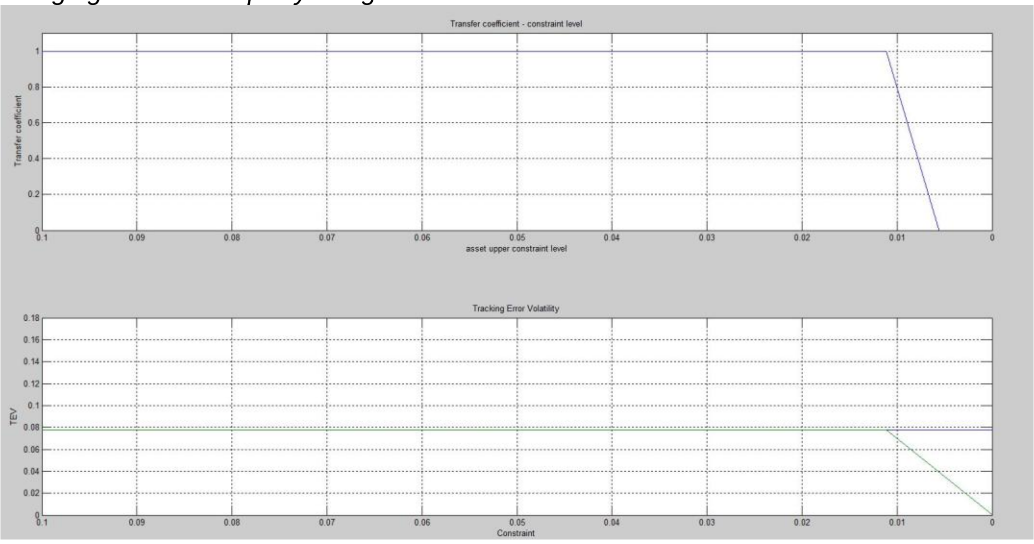
MSCI World – Growth



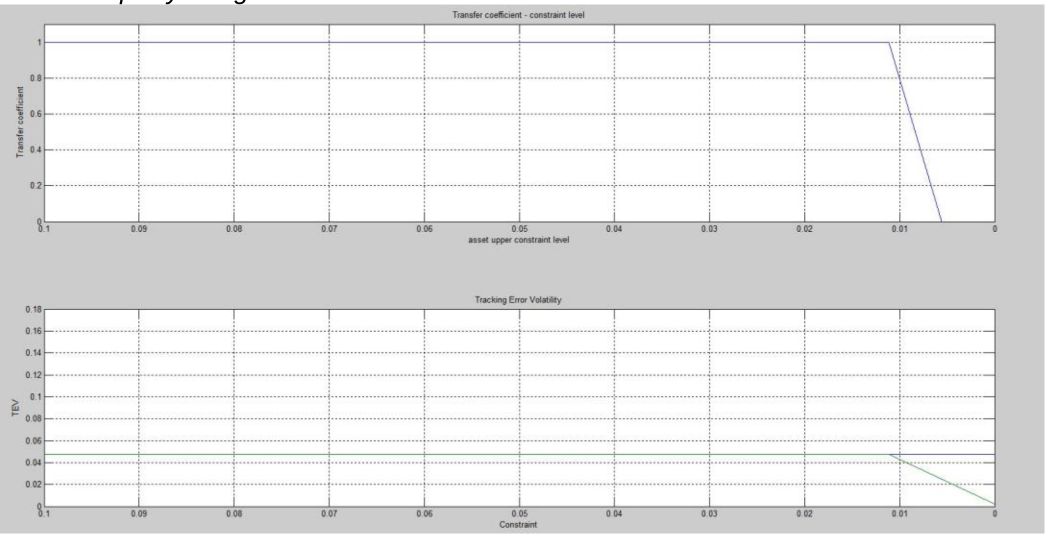
AEX – Equally Weighted



MSCI Emerging Market – Equally Weighted

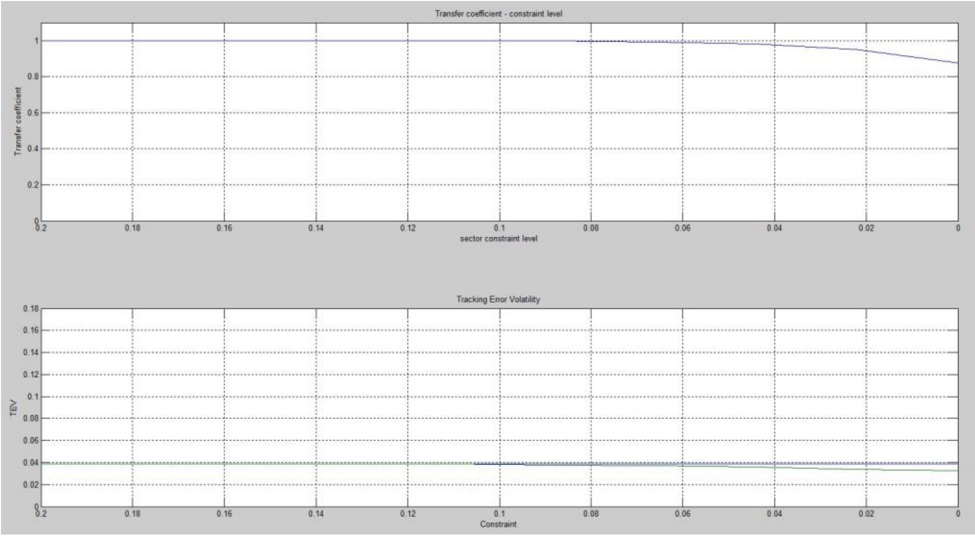


MSCI World – Equally Weighted

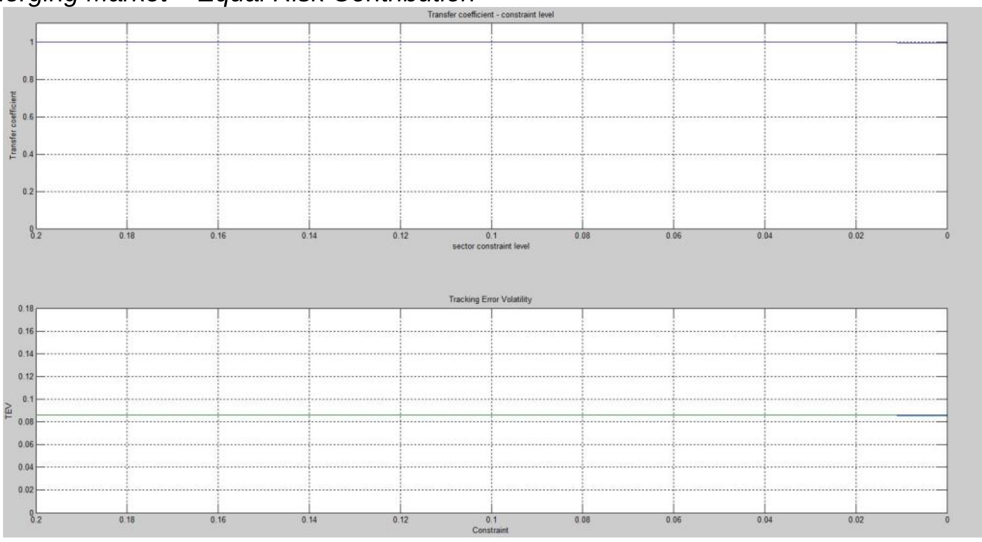


Sector constraint

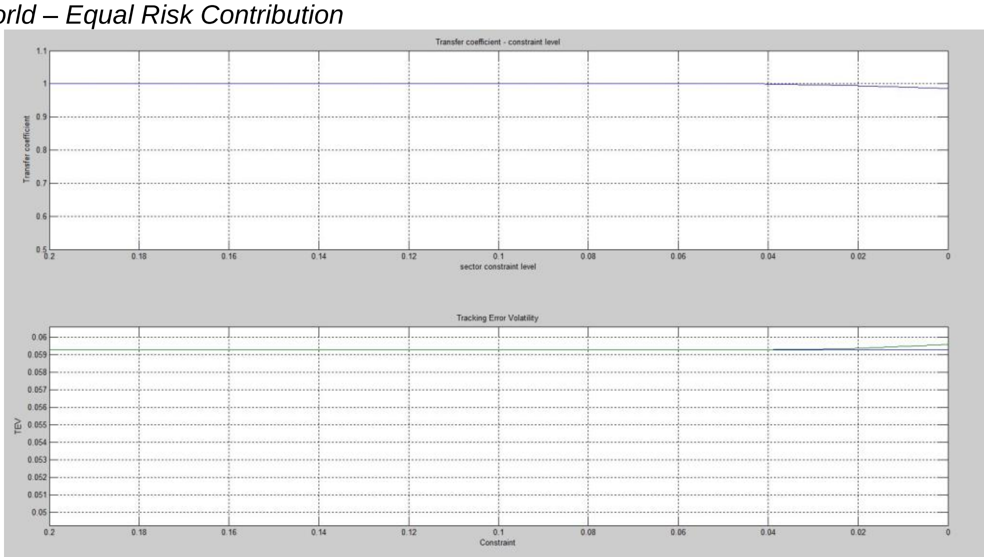
AEX – Equal Risk Contribution



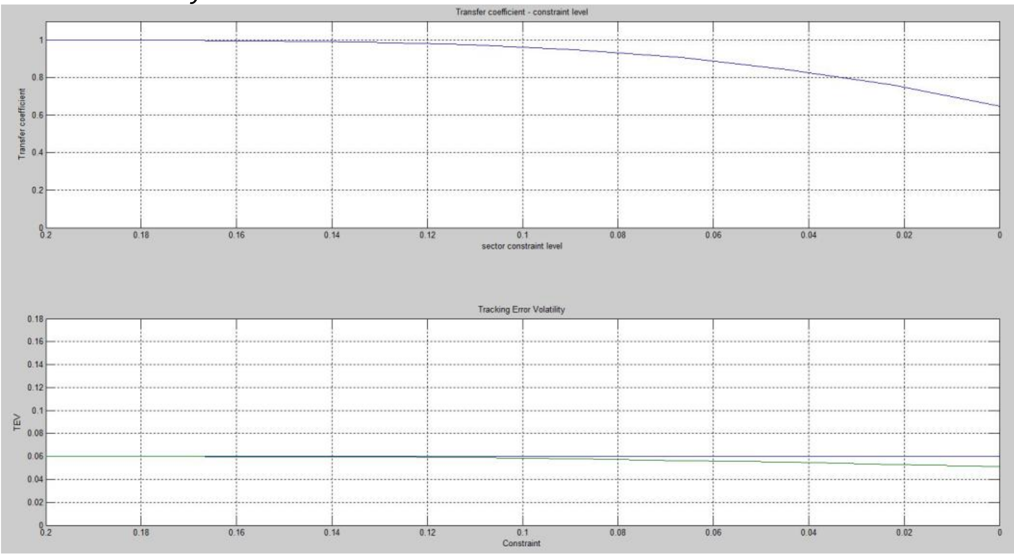
MSCI Emerging Market – Equal Risk Contribution



MSCI World – Equal Risk Contribution



AEX - Minimum Volatility



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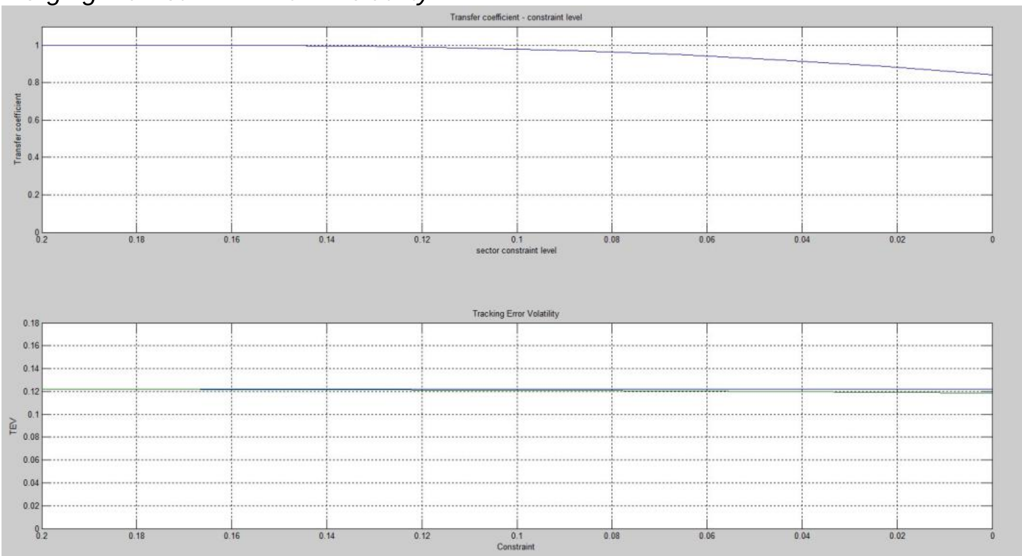
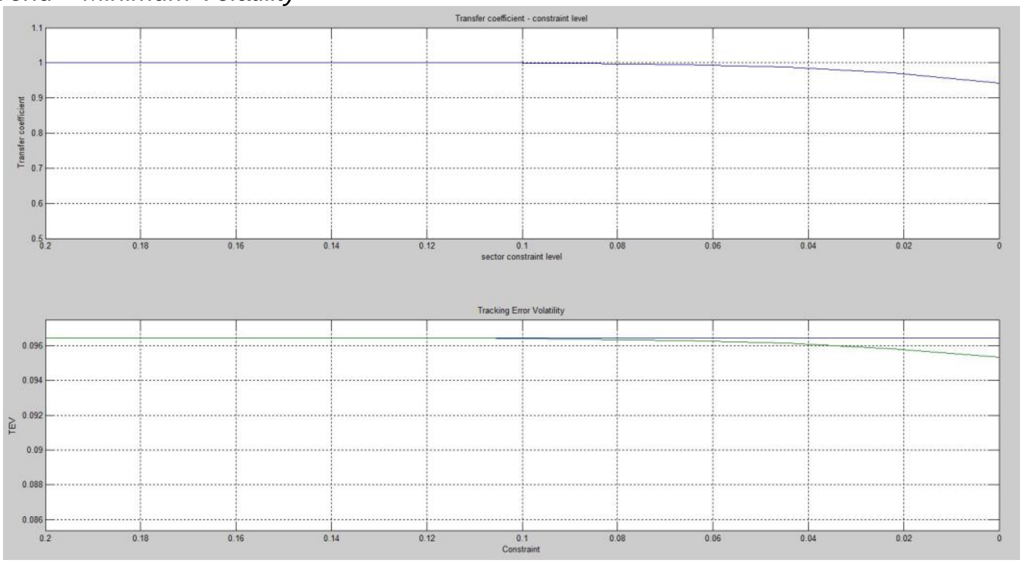
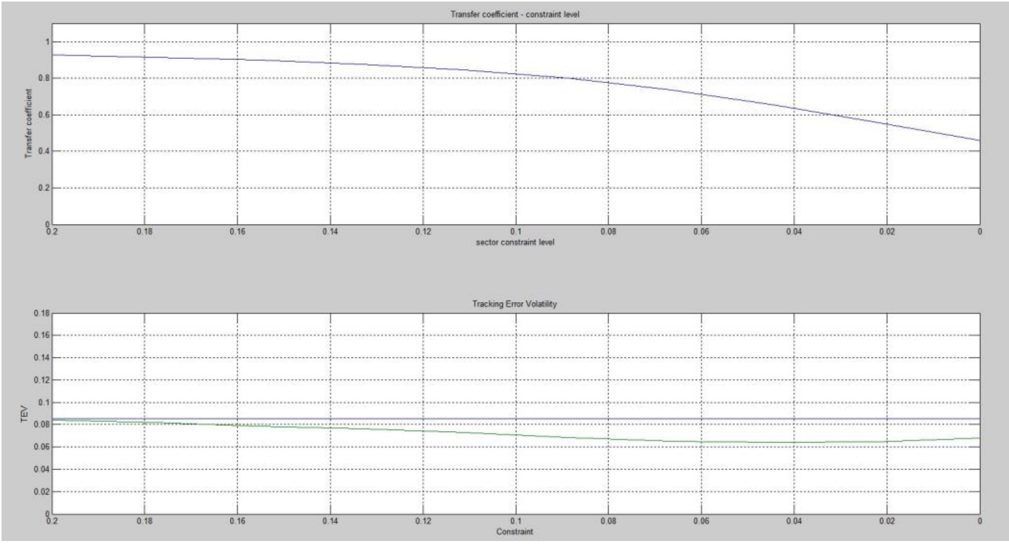


MSCI World – Minimum Volatility



AEX – Risk Efficient



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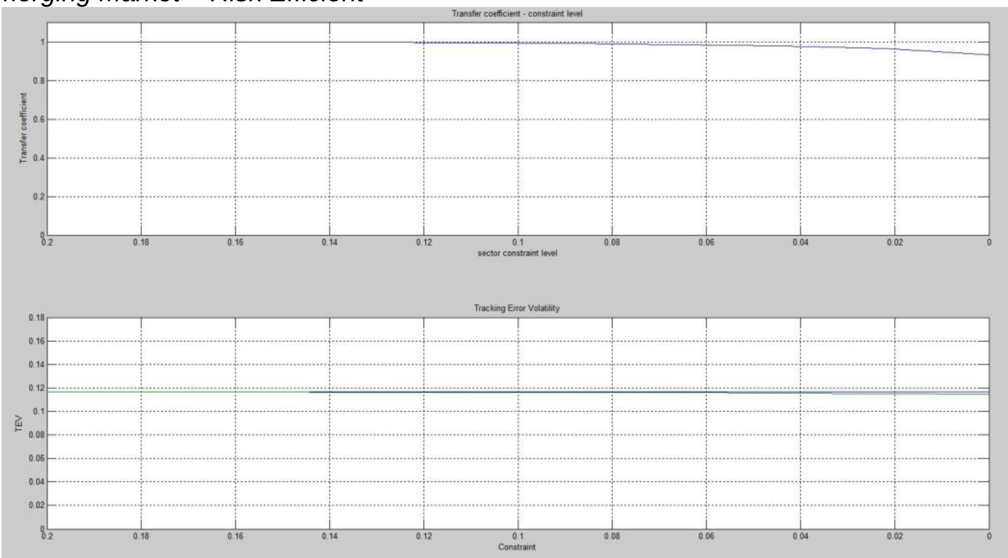

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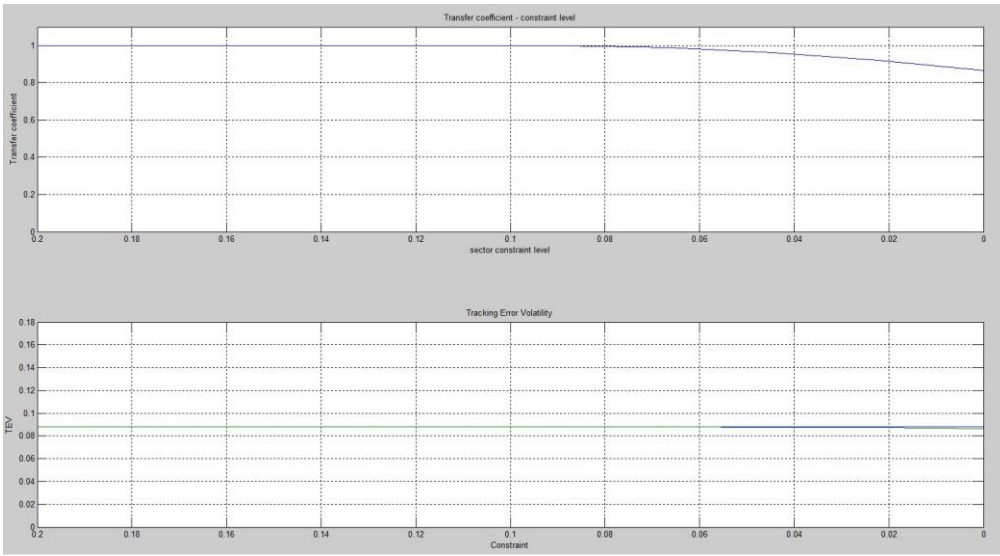
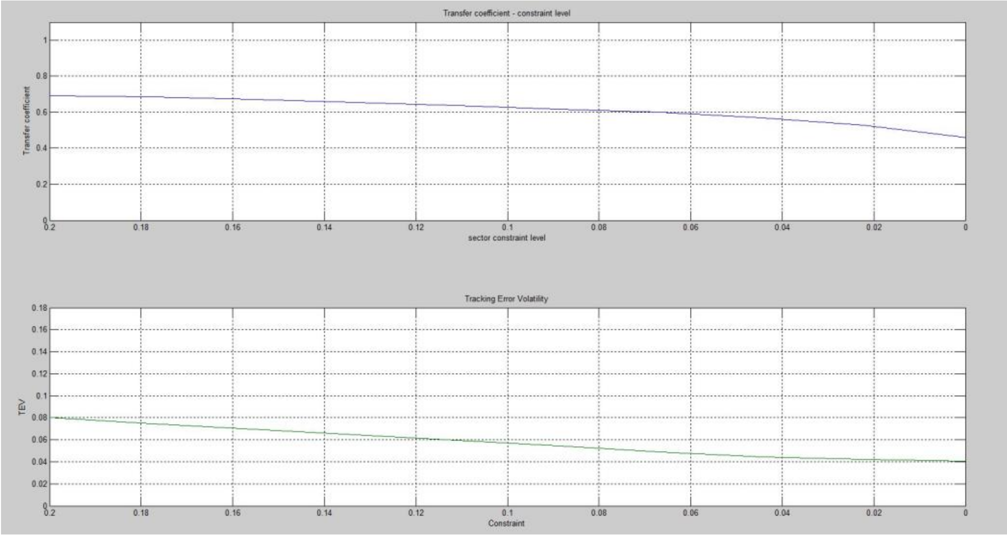
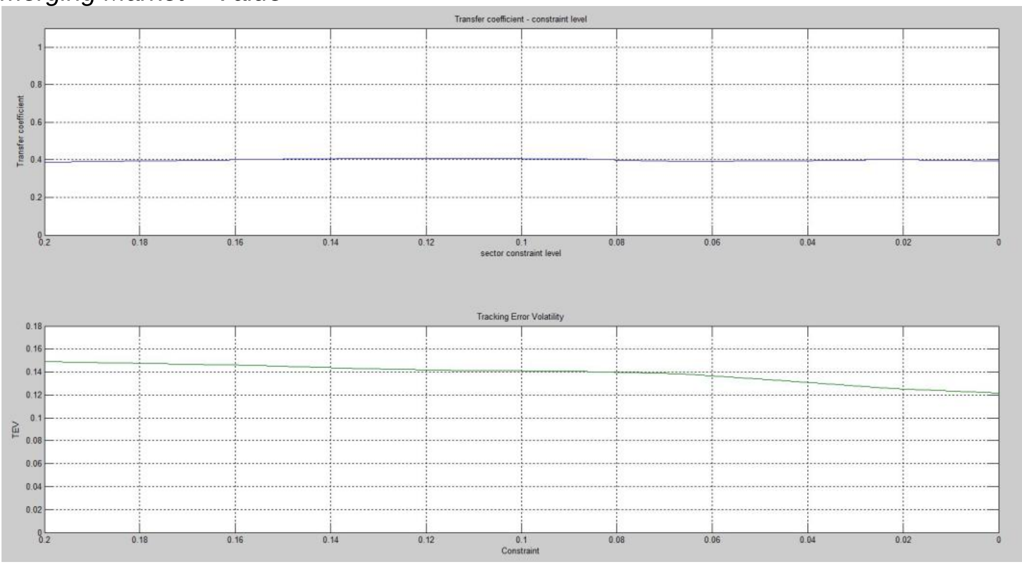


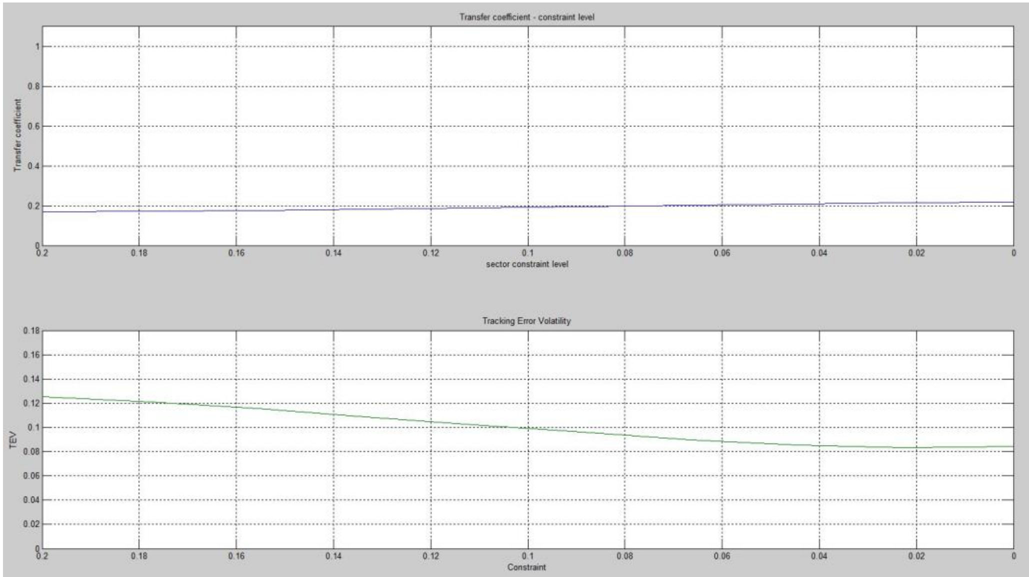
AEX – Value



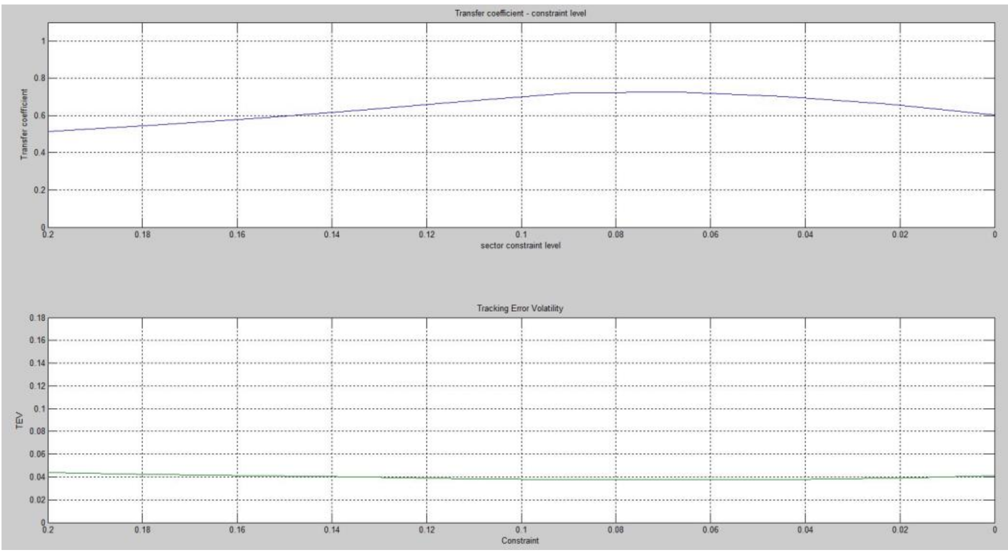
MSCI Emerging Market – Value



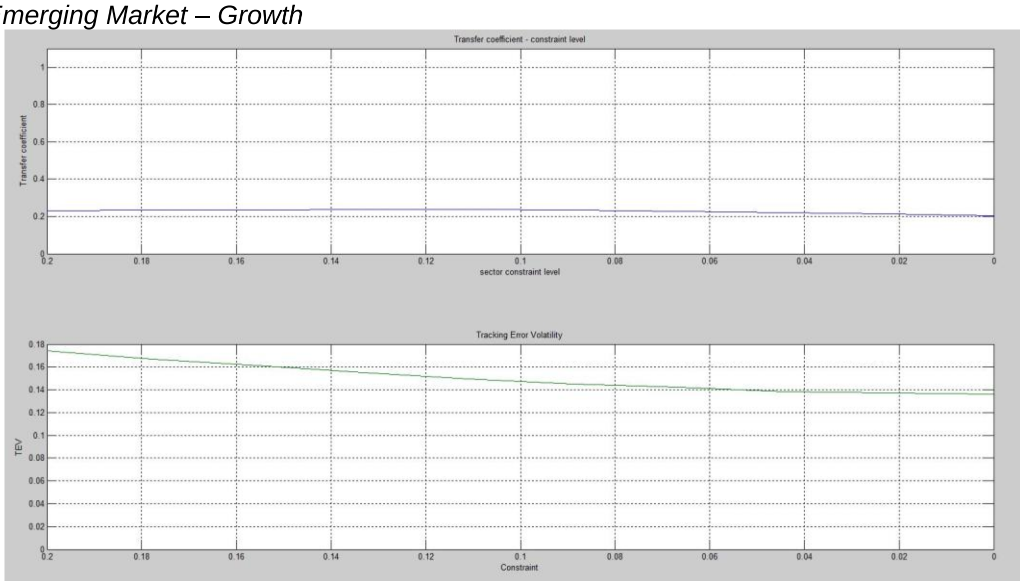
MSCI World – Value



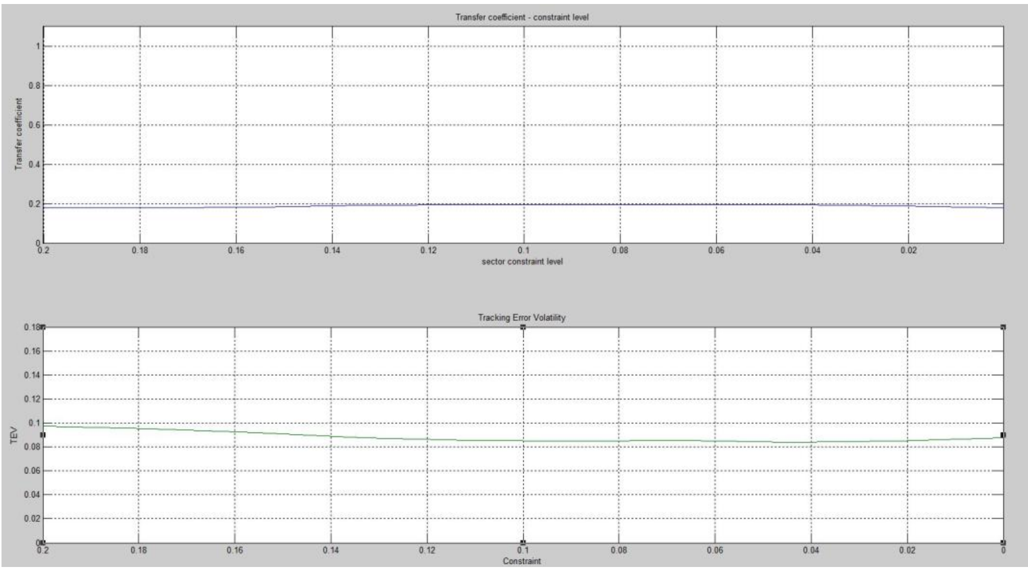
AEX – Growth



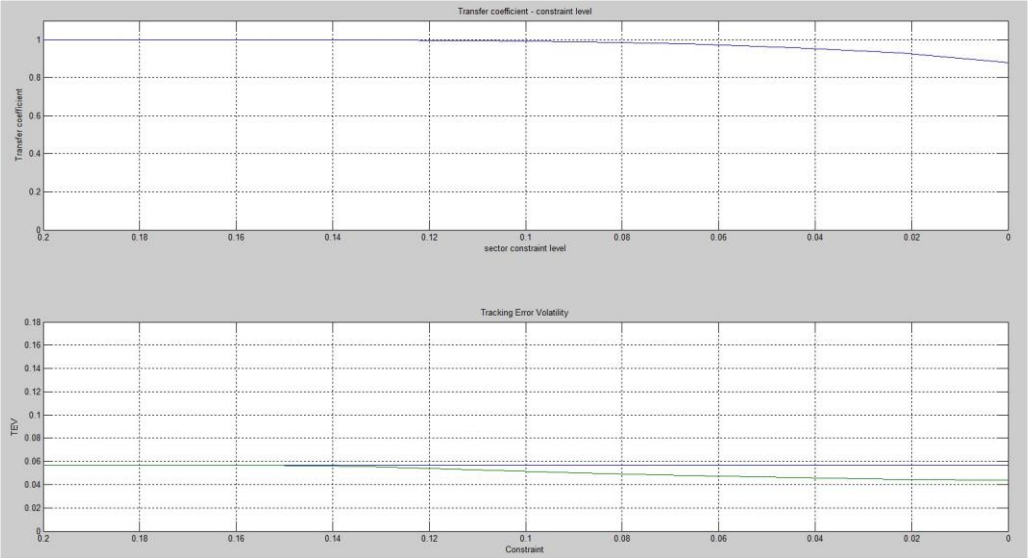
MSCI Emerging Market – Growth



MSCI World – Growth



AEX – Equally Weighted



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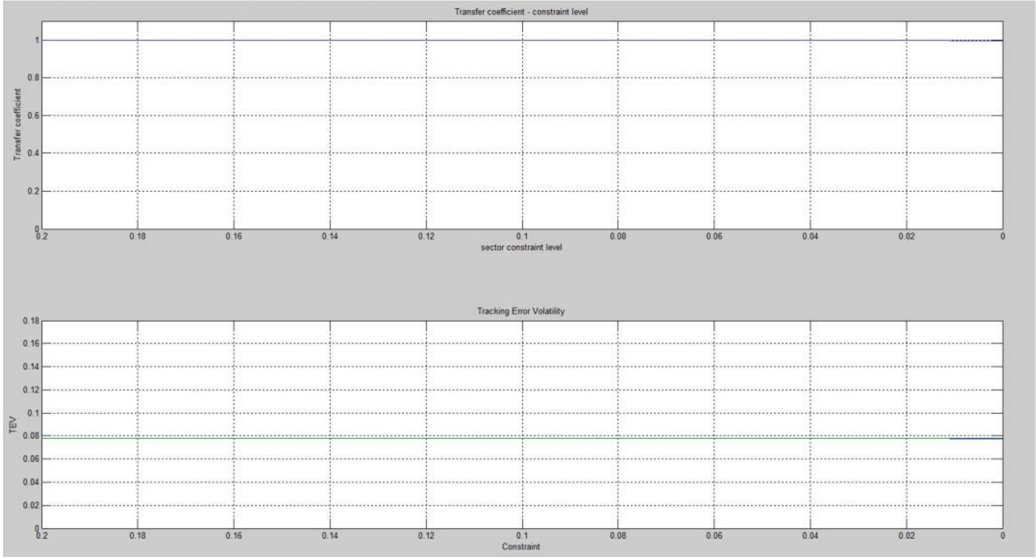
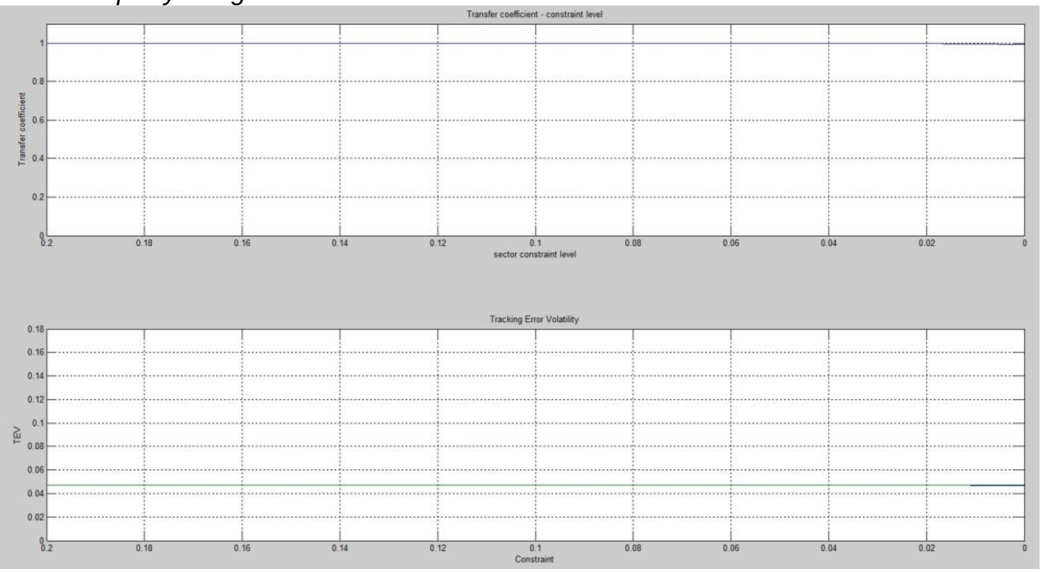
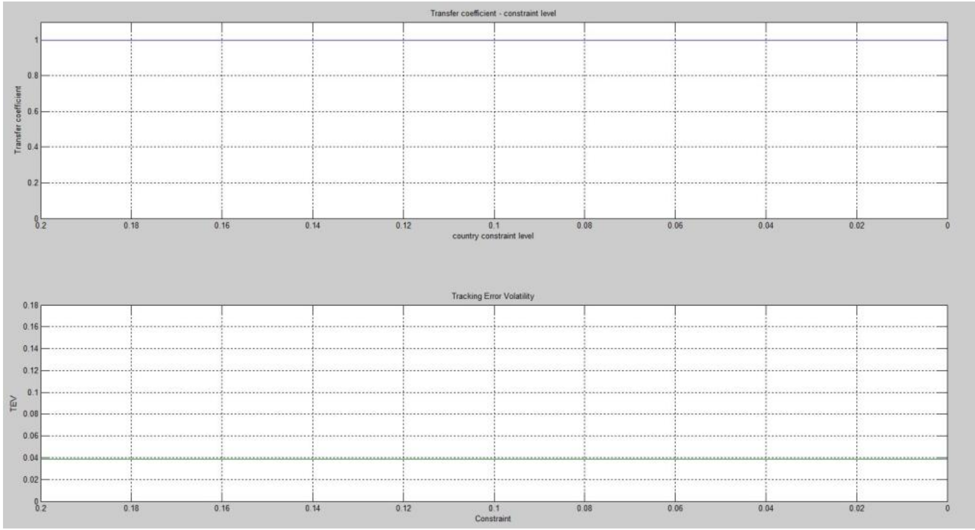


MSCI World – Equally Weighted

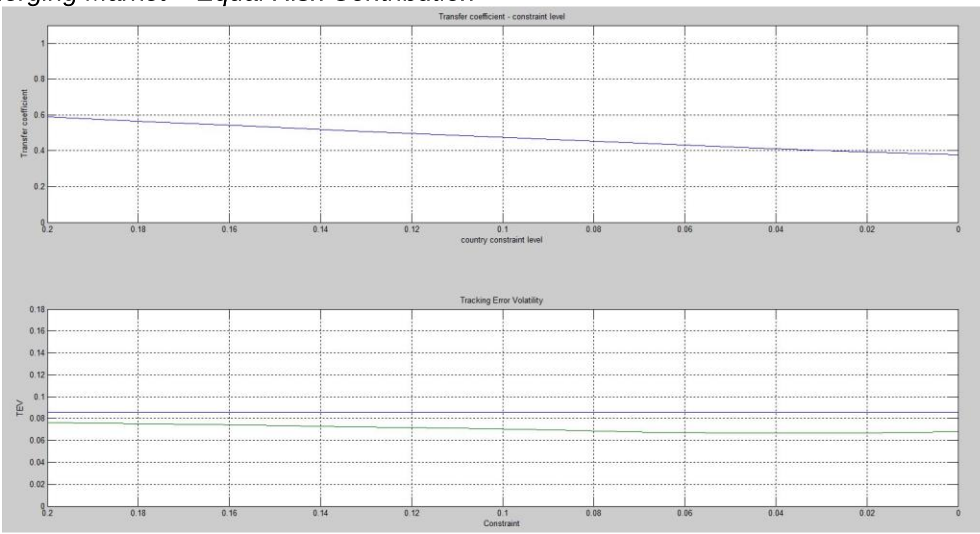


Country constraint

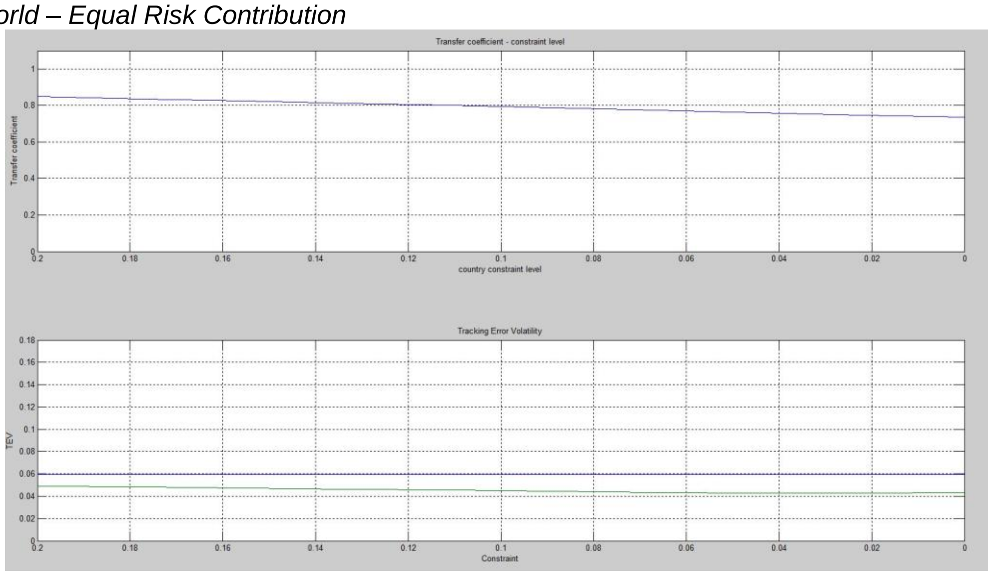
AEX – Equal Risk Contribution



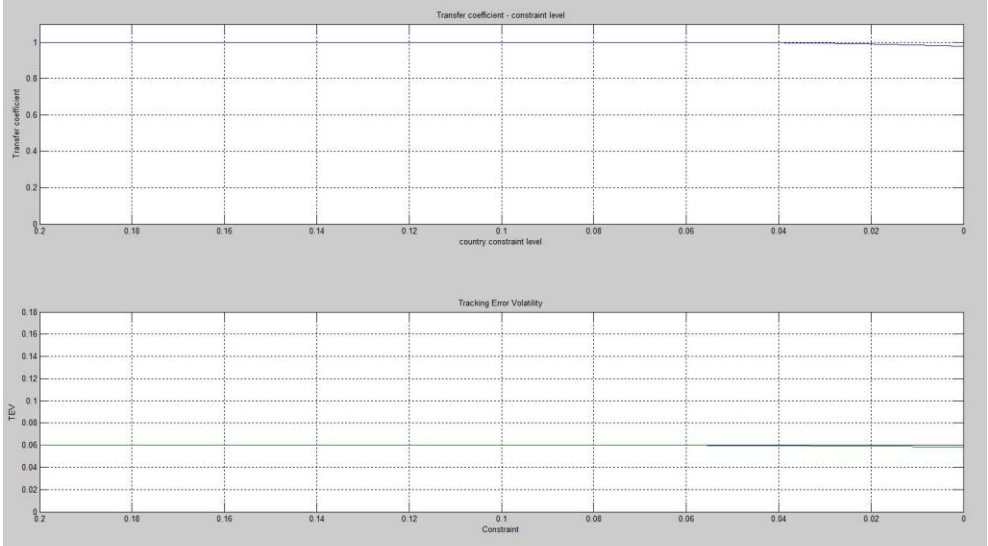
MSCI Emerging Market – Equal Risk Contribution



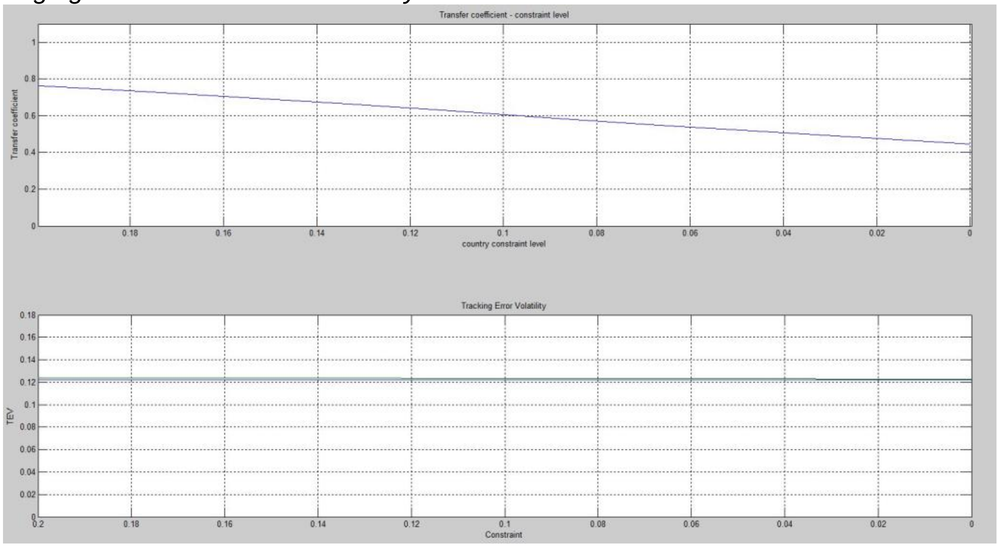
MSCI World – Equal Risk Contribution



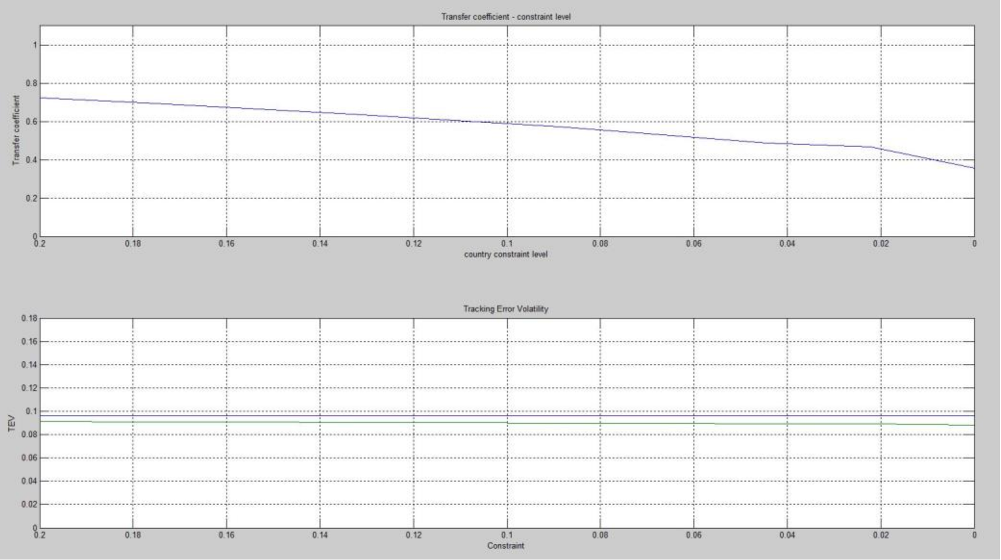
AEX – Minimum Volatility



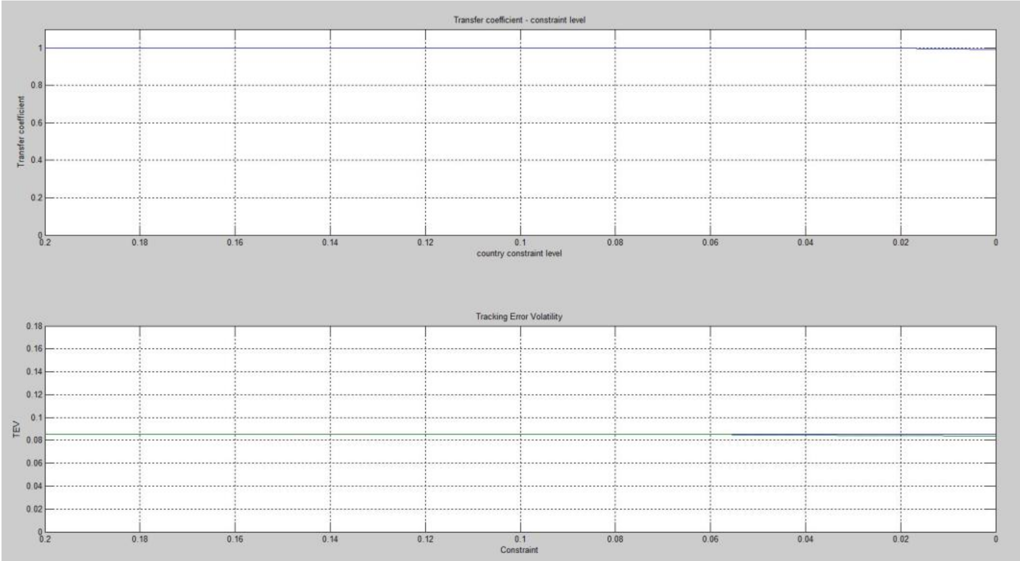
MSCI Emerging Market – Minimum Volatility



MSCI World – Minimum Volatility



AEX – Risk Efficient



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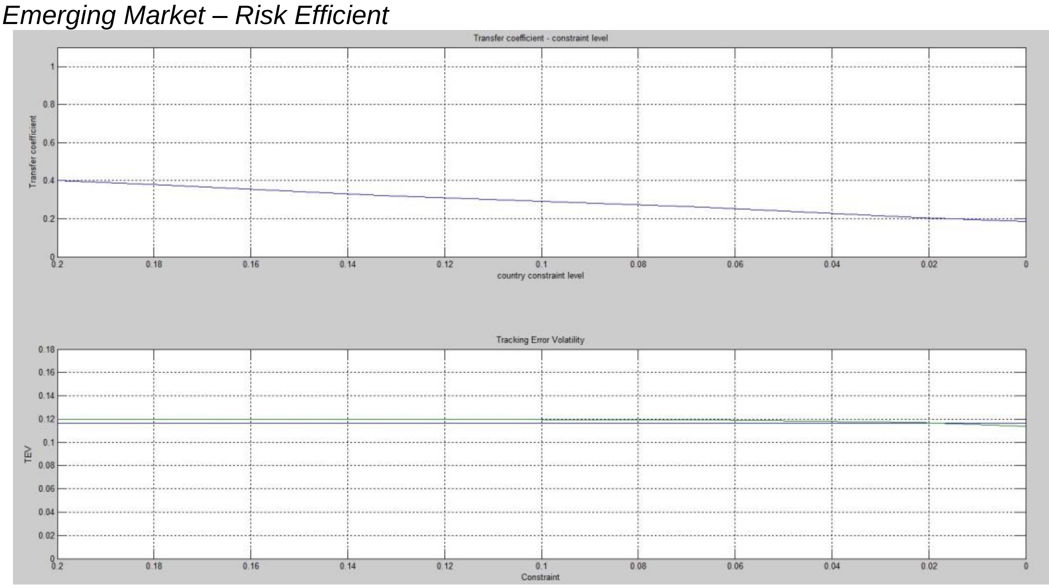
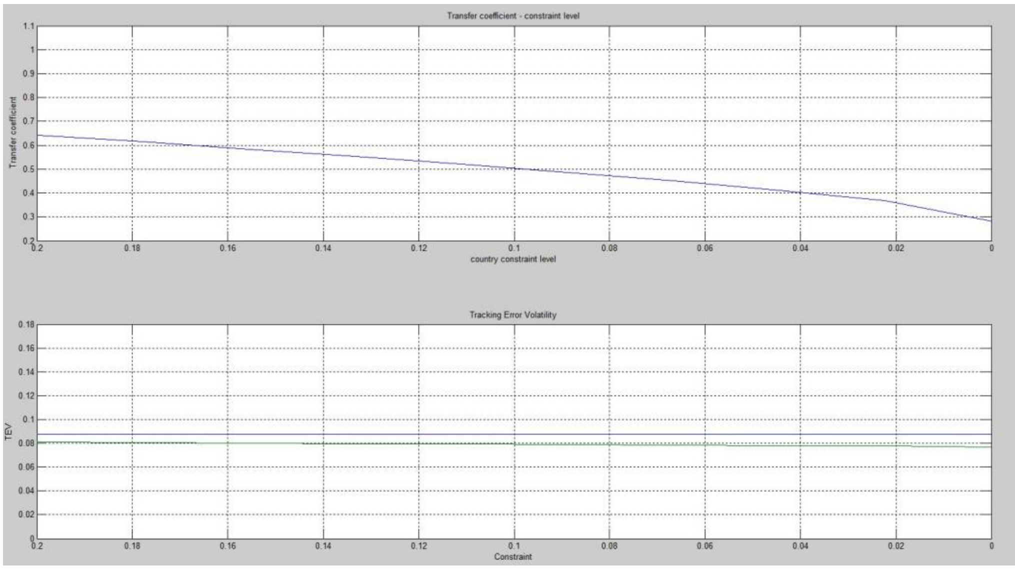
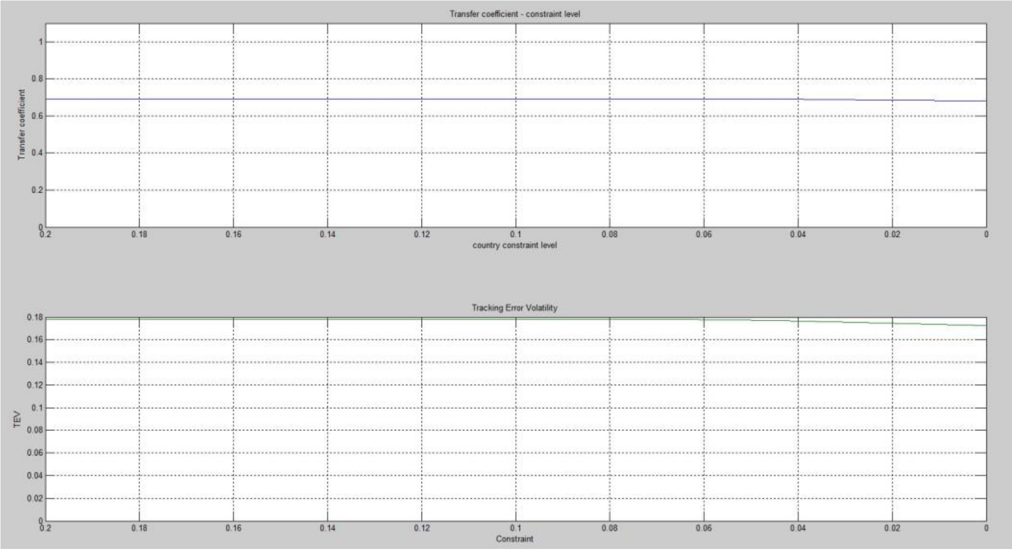


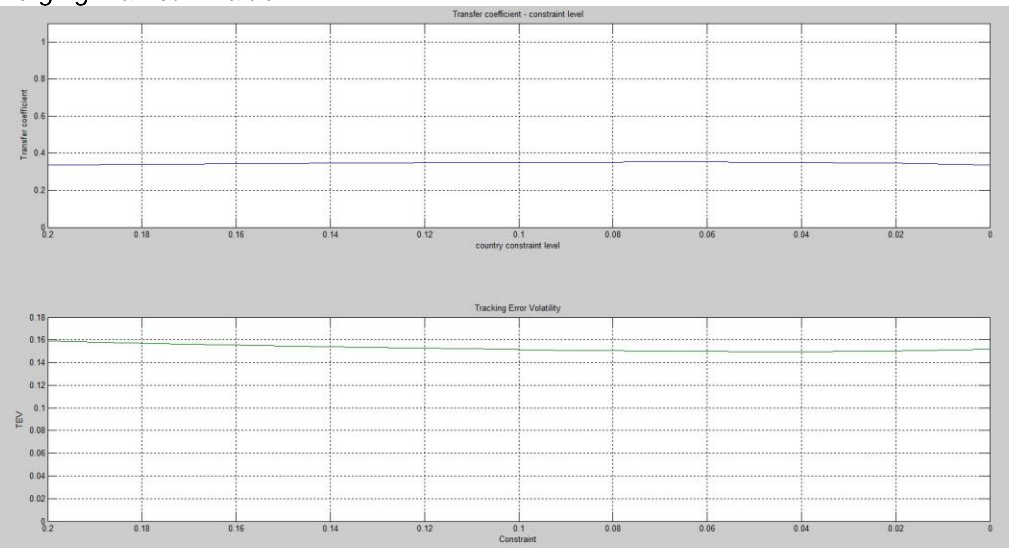
MSCI World – Risk Efficient



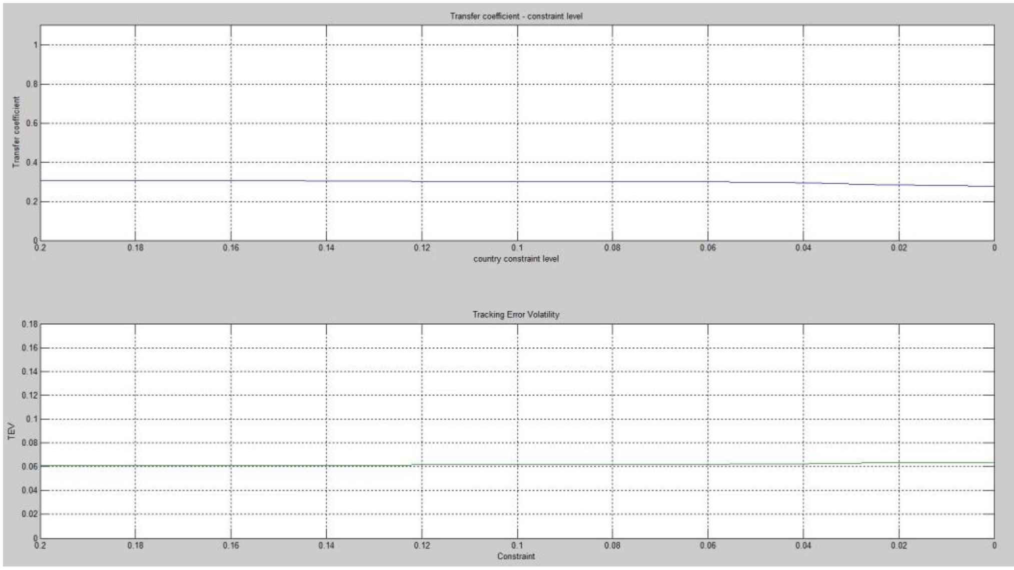
AEX – Value



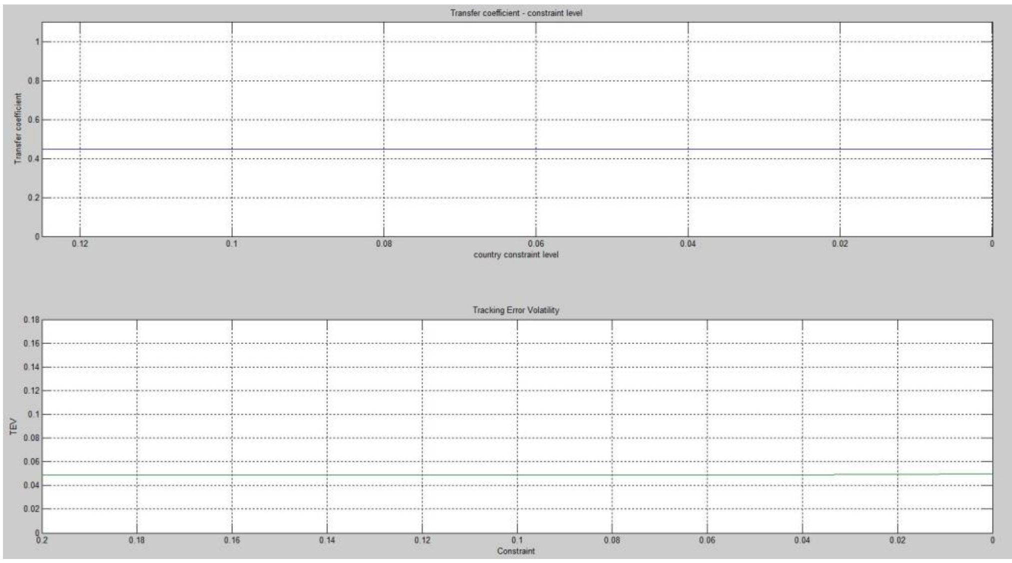
MSCI Emerging Market – Value



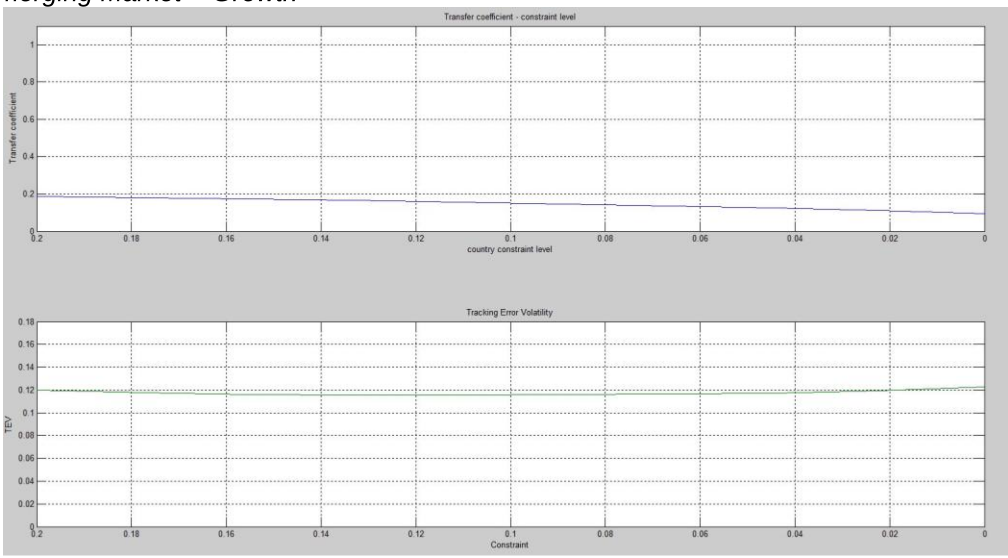
MSCI World – Value



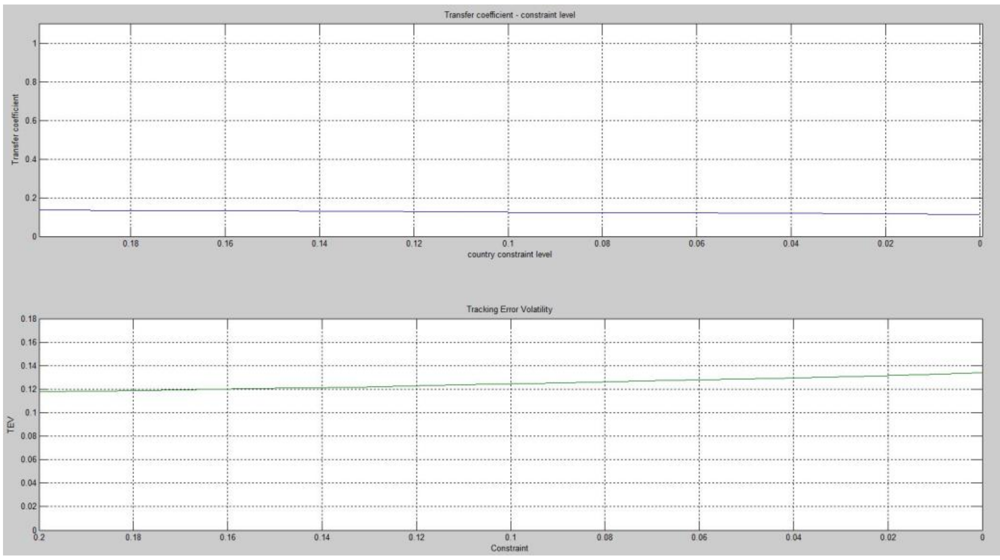
AEX – Growth



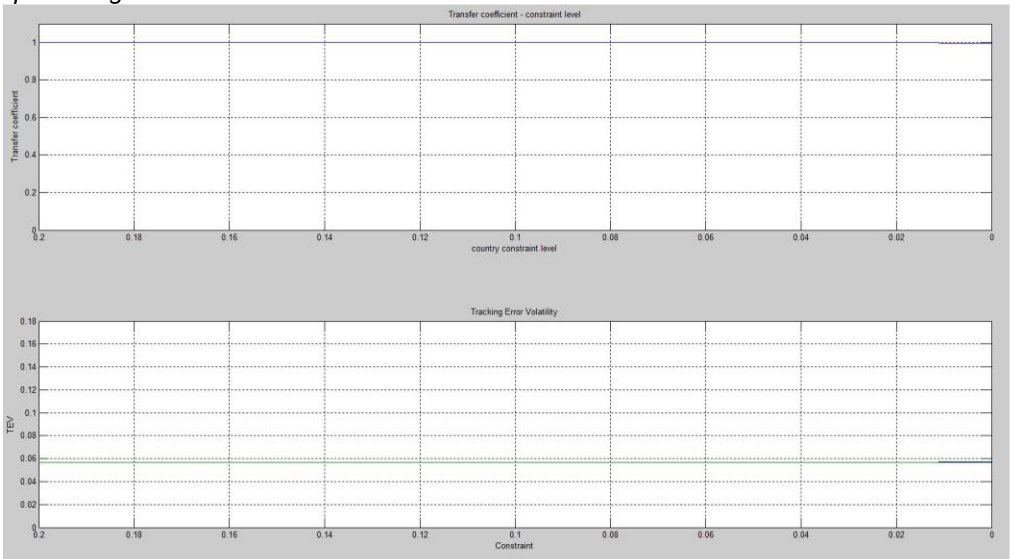
MSCI Emerging Market – Growth



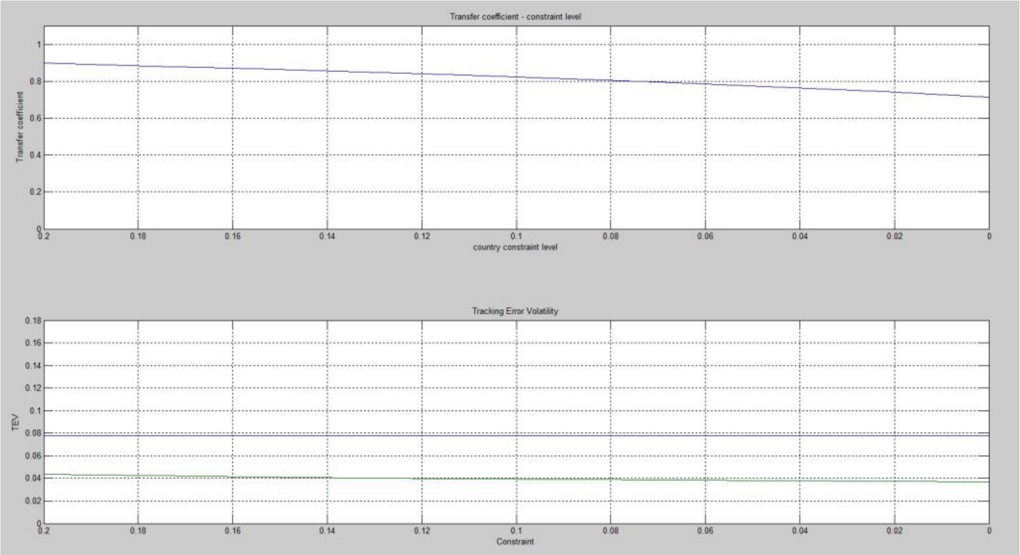
MSCI World – Growth



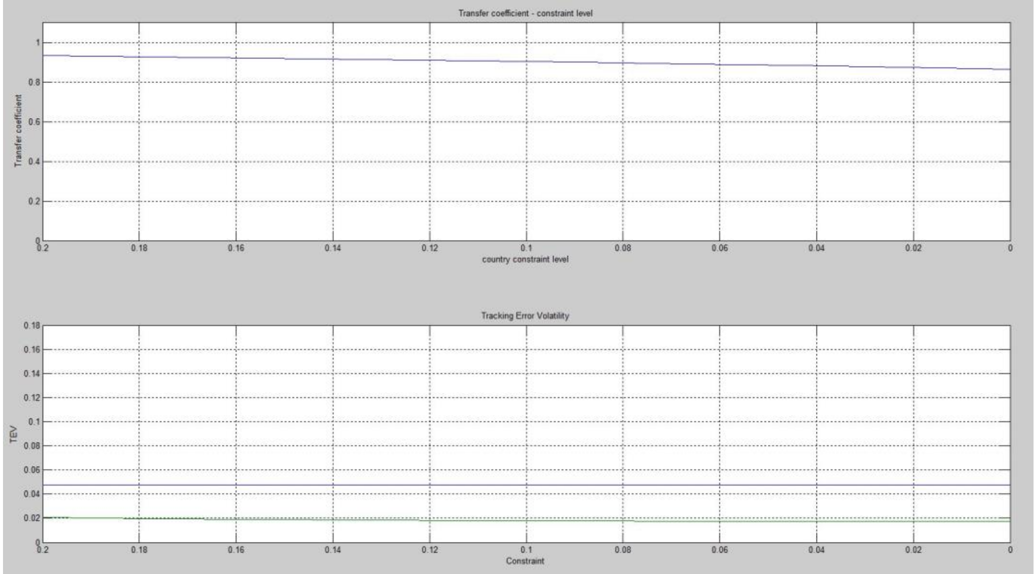
AEX – Equal Weighted



MSCI Emerging Market – Equal Weighted



MSCI World – Equal Weighted



Appendix C: List of figures and tables