University of Twente

TNO

Graduation Project

Extending the PowerMatcher using Dynamic Programming

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Abstract

The electricity grid did not change much over the last century. A few power plants are used to supply the electricity demand, whereby the power plants have to ensure that demand and supply of electra are always balanced. The transport of the electricity is realized via a layered network, and the flow of electricity is always from the highest layer, where also the power plants are connected, to the lower layers, where the customers are connected. Three important trends change the electricity network. Because of the increasing demand and supply of electricity and the decentralization of the electricity production, it becomes more difficult to balance electricity demand and supply. In the new situation electricity demand should follow the supply where possible. It is commonly believed that the current electricity network will no longer suffice and an intelligent electricity grid is needed to deal with the changes.

One of the technologies to support the intelligent grid is the PowerMatcher, which is introduced by a group of international partners. It is designed to match current demand and current supply and uses a lot of renewable energy in doing so. The current version of the PowerMatcher has some weaknesses and can therefore be improved. One way to do this is to not only use the current demand and current supply but also predictions about future demand and future supply. An attempt to integrate these predictions has led to the two-time-scale PowerMatcher, an extension of the PowerMatcher.

However, the change from the PowerMatcher to the two-time-scale PowerMatcher does not (yet) lead to the desired improvement. This report presents a different strategy of using predictions in the two-time-scale PowerMatcher. Using dynamic programming, the new strategy tries to optimally use the available supply of renewable energy. This new method turns out to improve the two-time-scale PowerMatcher significantly under certain conditions. Computational results show that the predictions used in the new strategy need to be of good quality, since bad predictions lead to bad decisions and a bad performance of the two-time-scale PowerMatcher. Predictions of good quality lead to an improvement with little room left for further improvements.

Contents

1 Introduction

Energy plays an increasingly important role in many aspects of our life. Electricity is used for lighting and cooling while fuel is used for transportation and heating. Our production and use of energy contributes for a large part to the climate change. Because of the thinning ozone layer, global warming and sea-level rise, environmentally friendly forms of power generation are asked for. The last years a lot of effort and research is put into different environmentally friendly power generation types. Wind and solar power are two examples of these power generation types, which are used more and more in the last years. The number of households that have a solar panel on their rooftop and the number of wind parks offshore increases. In the Netherlands, the aim is to provide at least 1 million households with energy from these wind farms, [17].

Because of these new types of power generation and the ability of households to produce small amounts of energy, the power grids are changing. Instead of power flowing from a few large power plants to a lot of small consumers, power will now flow both ways. Also, energy from renewable resources fluctuate in availability. Keeping the power supply and demand balanced is becoming a challenge through this change. Because of the flexibility on the supply side, also more flexibility on the demand side is needed. The current power grids are not able to manage these new kinds of energy generation and its flexibility. An intelligent system is needed to deal with the new kinds of energy generation and to prevent the grid from overloading, a smart power grid is needed.

An important component of this smart grid is demand-side management. Demand-side management is used to reduce the peak electricity demand. This is where the PowerMatcher comes in. The PowerMatcher is a smart grid technology, developed by TNO together with industry and research partners, that matches energy supply and demand and uses a lot of renewable energy in doing so, without overloading the power grid. The idea behind the PowerMatcher is that every producing and/or consuming device sends a bidding curve which shows the willingness to pay for different electricity prices. These bidding curves are made using a certain bidding strategy and show the electricity demand at each price. This demand is negative when electricity is generated and positive when electricity is consumed. The electricity supply and demand can be balanced by putting all these bidding curves together.

The PowerMatcher uses current demand and current supply, where the generated amount of electricity from renewable energy resources is dependent on the availability of wind/solar radiation. It would be great to know the wind/solar radiation availability in advance, planning could then be used to improve the PowerMatcher. The availability of wind/solar radiation is however not known in advance, forecasts about wind/solar radiation availability are accessible. Planning, based on the forecasts, could still improve the PowerMatcher. Whether this improvement is possible depends for a large part on the quality of the forecasts. Bad forecasts give a wrong idea about the future, which leads to bad decisions. Good forecasts can have an added value to the PowerMatcher and its bidding strategies, depending on how the information is used. The question arises whether an inclusion of the forecasts indeed improves the PowerMatcher and how wind/solar radiation forecasts have to be included into the PowerMatcher. These questions will be studied in this report.

This report starts with some background information and literature about the electricity grid, the need for a smart power grid, the importance of demand-side management and the PowerMatcher, see Chapter 2. Chapter 3 describes the PowerMatcher and introduces an extension of the PowerMatcher, called the two-time-scale PowerMatcher. This two-time-scale PowerMatcher makes it possible to include wind/solar radiation forecasts into the PowerMatcher. With the PowerMatcher described, the main research question of this report is given, see Section 3.4. Chapter 4 explains the algorithm and the used bidding strategy of the two-time-scale PowerMatcher. The two-time-scale PowerMatcher can be improved by using different bidding strategies. Chapter 5 will give a different bidding strategy, one that uses dynamic programming. The different bidding strategies are compared to each other in Chapter 6. The conclusions are given in Chapter 7.

2 Background and literature

Electricity is one of the most widely used forms of energy. It is a secondary energy source and is generated from other sources of primary energy, like coal, natural gas and oil, at a few large power plants. Because electricity is generated from primary energy sources, electricity demand can be seen as energy demand. The terms 'electricity demand' and 'energy demand' will therefore be used interchangeably in this report and have the same meaning.

An electricity grid is an interconnected network where electricity is delivered from producers to consumers. The electricity network did not change much over the last century and depends on large power plants. These few power plants generate electricity which is transported to many consumers all over the country via the electricity grid. The electricity grid is divided into different voltage levels. The electricity generated at the power plants flows into the network at the high voltage level. The voltage level is lowered when the electricity is closer to the customer, this is done using transformers. Finally, the electricity is delivered to the customer via the low voltage distribution grid. Electricity can also flow into the network at lower voltage levels, this electricity is mostly generated from renewable energy sources. This type of inflow has increased in the last years, together with the generation of electricity from renewable sources.

The electricity grid differs from a lot of other networks (e.g. road networks) by the fact that the production and consumption needs to be balanced at all times. Since the network itself has no storage capacity and the consumers do not want to wait for the needed electricity, the generation of electricity is demand driven. As a consequence of the liberalization of the electricity markets, the production, transportation and distribution of electricity is carried out by different independent companies. The production companies own the power plants and generate the electricity, there are multiple production companies that have to compete with each other. The transportation companies own and maintain part of the electricity grid without any competitors. The distribution companies sell the electricity to the customers, these companies have to predict the amount of electricity their customers will use and have to buy this amount from production companies. The distribution companies try to predict the consumption of electricity as accurately as possible to prevent penalties due to imbalance.

There are currently three important trends in the electricity system [1]. These are listed below.

- Demand of electricity increases every year and is expected to keep increasing in the coming years. The electrification of everything leads to this rise in demand. This rise in electricity demand will drive the distribution networks to their capacity limits. Overloading the grid will shorten the life span of the networks, which are already close to their life end.
- Supply of electricity increases and becomes more uncontrollable and unpredictable. More electricity is generated from renewable energy resources, the availability of energy from renewable resources is dependent on the availability of wind/solar radiation. The increased and more unpredictable amount of energy supply makes it harder to balance the supply and demand of electricity.
- The electricity production is becoming more and more decentralised. The number of wind turbines and solar panels has increased and is expected to keep increasing in the coming years. This increase changes the electricity system, the electricity system is now based on a few large power plants and will in the near future be based on many small producers.

These three trends will change the electricity network. In the new network, consumers will become prosumers: sometimes producer and sometimes consumer. This makes the system more complex, instead of coordinating a few large power plants, a huge number of small producers needs to be coordinated. The interactions between users are becoming more important because of this decentralisation. An intelligent electricity grid, generally referred to as 'smart grid' is needed. The smart grid is defined in [4] as an electric system that uses information, two-way, cyber-secure communication technologies, and computational intelligence in an integrated fashion across electricity generation, transmission, substations, distribution and consumption to achieve a system that is clean, safe, secure, reliable, resilient, efficient and sustainable.

Demand-side management will be an essential part in the smart grid. Traditionally, power plants adjust the power generation to meet the rising demand, demand-side management (DSM) will look at the other side, the demand of electricity. DSM encourages consumers to use less energy during peak hours or to move the time of energy use to off-peak hours, see [13]. This encouragement is needed because the peak loads are increasing. The evening peak, for example, will increase because of the increasing amount of plug-in hybrid vehicles that are charged after working hours. The impact and opportunities for the electricity grid with this increasing amount of plug-in hybrid vehicles is studied in the European project: Grid-for-Vehicles (G4V), see [20]. This G4V project has, among others, studied the peak load in the grid using different scenarios, see [14]. Several solutions for DSM of plug-in hybrid vehicles are given in [18]. These solutions do all take advantage of the two-way communications. Another DSM system that takes advantage of the two-way communication infrastructure is given in [12]. In [12] game theory is used and an energy consumption scheduling game is formulated. The players in this game are the electricity users and their strategies are the daily schedules of their household appliances and loads. Game theory is a study of mathematical models of complex interactions between intelligent rational decision makers. In [15] an overview of the potential of applying game theory within smart grid systems is given. Besides [18] and [12], other demand-side management systems are looked at in the last years. A few of these systems are given in [16]. A smart management system is however not enough to create a smart grid, a smart infrastructure and protection system is also needed. A few infrastructure and protection systems are given in [3].

There are many ongoing projects to create a smart grid, see [5]. One of the technologies to support the smart grid is called the PowerMatcher. The PowerMatcher, [10], is a smart grid technology created by TNO in cooperation with industry and research partners. In the PowerMatcher, users determine their bidding curve, which is a curve showing the electricity demand of the user at different electricity prices. These bidding curves are determined using a certain strategy, knowledge about the average price is used to determine this bidding strategy. The bidding curves are used in the PowerMatcher to balance the total supply and total demand. By balancing the total supply and total demand, the PowerMatcher automatically determines the amount of produced/consumed electricity of each user.

The "capability" of the PowerMatcher to support the mass integration of electricity produced by wind energy has been studied in [11]. More precisely, the goal in this research was to adapt flexible household demand and supply to the availability of wind power. The need for fossil fuel based electricity will, in this way, be reduced.

As the real world is different from the testing evnironment, TNO, in cooperation with strategic partners, developed the PowerMatching City, [2]. The PowerMatching City is a demonstration project where a community of homes is connected to a smart grid. The households are connected to each other and each household not only consumes but also generates electricity. The PowerMatcher matches the supply and demand in this smart grid. In May 2013 PowerMatching City II started, in this project the number of households is around 60. The focus of this project is less on the technological side but more on energy services and energy markets. Another project where the PowerMatcher is used in the EcoGrid project, [6]. This EU funded project started in 2011 and coordinates the energy demand of 2.000 households in the island of Bornholm (Denmark) with local production by wind turbines. There are more projects where the PowerMatcher is used/tested, most of them are located in the Netherlands. Results of field deployments and simulation studies are given in [9].

The PowerMatcher can be improved and extended. In the current version of the PowerMatcher the average price is used by the user to determine the bidding strategy. When besides the average price also forecasts about future prices are known, planning can be included in the PowerMatcher. One extension of the standard PowerMatcher is therefore a combination of two instances of the PowerMatcher, the two-time-scale PowerMatcher, see [7]. These two instances operate on different time scales. With these two instances it is intended to make decisions about short term demand using information about the long term, which may lead to more cost-efficient schedules. Whether the two-time-scale PowerMatcher improves the PowerMatcher depends to a large extent on the quality of the long term information.

Why demand-side management is needed and where the two-time-scale PowerMatcher comes in can be shown using a small example network. This network, see Figure 2.1, consists of one windmill, one household with an electric car and flexible energy resources. Energy generated by the windmill is cheaper than the energy produced by flexible energy resources, but wind energy is not always available while energy from flexible energy resources is.

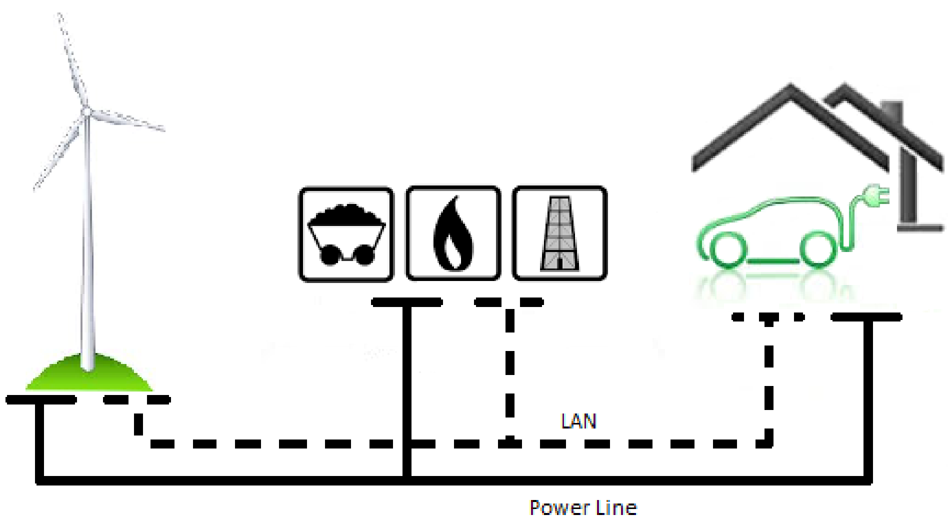


Figure 2.1: Network with one windmill and one household with an electric car

The energy demand of the electric car is shiftable over a long time period. The question arises when the electric car has to be charged to minimize the costs of charging the car. Without any communication and information about future demand, the car is charged as soon as possible. This may lead to high costs and a high amount of energy generated by flexible energy resources. To lower the costs demand-side management is needed, demand-side management is used to match the wind energy and the car's demand. Using communication and information about future demand, the two-time-scale PowerMatcher tries to create a situation where a lot of available renewable energy and a minimal amount of energy from flexible energy resources is used. This report tries to improve the two-time-scale PowerMatcher in order to reduce the total amount of energy used from flexible energy resources and the total costs of satisfying the shiftable demand.

This report explains the PowerMatcher and the two-time-scale PowerMatcher and tries to improve the performance of the two-time-scale PowerMatcher by changing its bidding strategy. A new bidding strategy is introduced in this report, one that uses the available demand forecasts and is supposed to improve the two-time-scale PowerMatcher.

3 The PowerMatcher

This report will look at the bidding strategies used in the PowerMatcher. Before looking at the bidding strategies, first the PowerMatcher has to be explained. The standard PowerMatcher is described in Section 3.1. This PowerMatcher can however be improved. One possible way to do this is to extend the PowerMatcher to the two-time-scale PowerMatcher, this two-time-scale PowerMatcher is introduced in Section 3.2. In both versions of the PowerMatcher bidding strategies are used. Section 3.3 explains how, where and which bidding strategies are used in both versions of the PowerMatcher. With information about the standard PowerMatcher, two-time-scale PowerMatcher and bidding strategies, the research question can be given, the research question is given in Section 3.4.

3.1 The standard PowerMatcher

The electricity network is changing. The electricity demand and supply are increasing and more renewable energy becomes available. Sufficient management is needed to prevent the grid from overloading. The PowerMatcher, [10], is one of the technologies to match energy demand and supply, using available renewable energy and preventing the grid from overloading.

Systems design of the PowerMatcher

Within a PowerMatcher cluster agents are organized into a tree. The agents are the producers and consumers of electricity and are represented by the leafs of the tree, these agents are also called local device agents. One of the leafs could be an objective agent. There is also an auctioneer agent, at the root of the tree. The auctioneer agent is a unique agent that handles the price forming. In order to obtain scalability, concentrator agents can be added to the structure as tree nodes. Figure 3.1 shows an example PowerMatcher agent cluster.

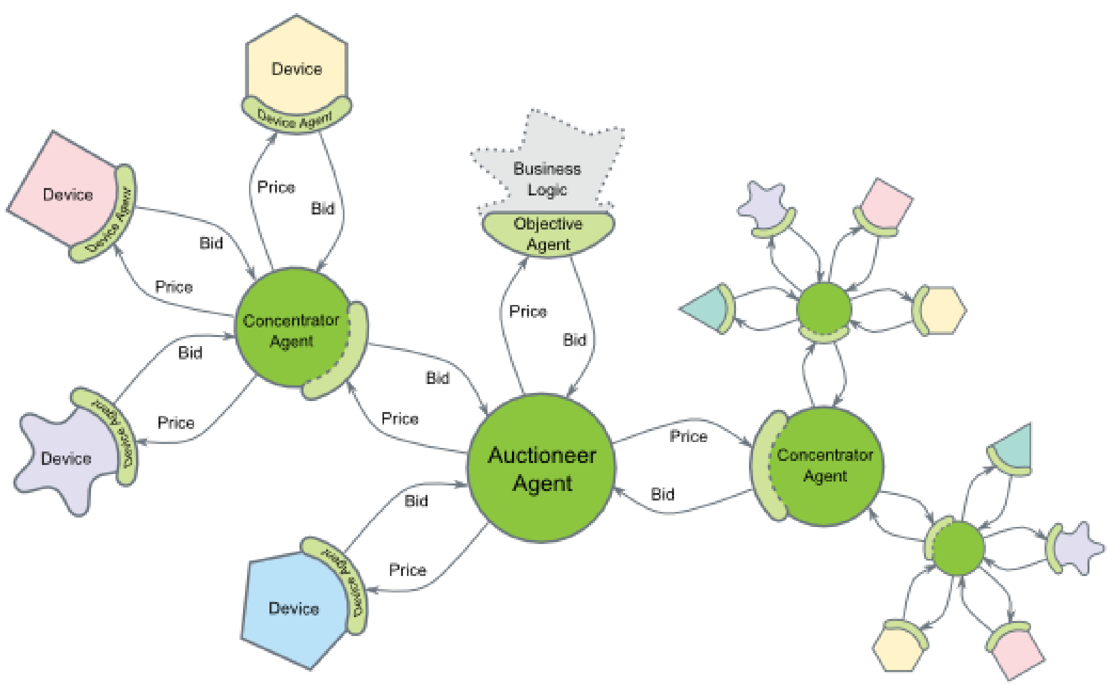


Figure 3.1: Example PowerMatcher agent cluster

The different agent types are described in [10] and are repeated below:

- Local device agent: This agent is the representative of a DER device. The local device agent coordinates its actions with all other agents in the cluster by buying or selling electricity. The agent communicates its latest bidding curve to the auctioneer agent and receives price updates from the auctioneer agent. The latest bidding curve and the current price determine the amount of electricity the local device agent is obligated to produce or consume.

- Auctioneer agent: This agent determines the price. The auctioneer agent receives the bidding curves of all agents and searches for the equilibrium price. The agent communicates the equilibrium price back to all agents.

- Concentrator agent: The concentrator agent represents a sub-cluster of local device agents. The agent concentrates the market bidding curves of the agents in the sub-cluster and aggregates this into one bidding curve. The agent then communicates this curve to the auctioneer agent. The agent also communicates the price, received from the auctioneer agent, to all local device agents in the sub-cluster. The concentrator agents look like the auctioneer agent from the perspective of the local device agents in the sub-cluster.

- Objective agent: The objective agent gives the cluster its purpose. When the objective agent is absent, the goal of the cluster is to balance itself. Depending on the specific application, the goal of the cluster might be different. If the cluster has to operate as a virtual power plant, for example, it needs to follow a certain externally provided setpoint schedule. The externally imposed objective can be realized by implementing an objective agent.

All agents, except the auctioneer agent, send a bidding curve to the higher agent. This bidding curve shows the electricity demand at each price. The bidding curve is explained in more detail in Section 3.3. The auctioneer agent receives all these bidding curves and determines the corresponding price. This price is send back to all agents.

Structure of the PowerMatcher

The structure of the PowerMatcher is given in Figure 3.2. A certain time scale is used in the PowerMatcher, in this figure the hour scale is used. So agents send a bidding curve every hour. This time scale can however be chosen differently, agents could for example sent a bidding curve every 15 minutes.

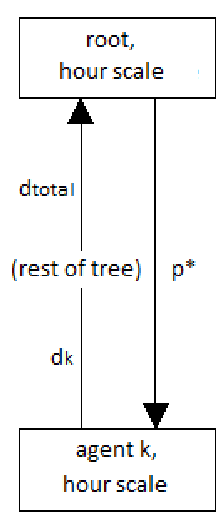


Figure 3.2: Structure of the PowerMatcher

In this figure, agent k represents one of the many agents in the tree. The steps in the PowerMatcher algorithm are given below:

- The PowerMatcher algorithm starts at the bottom, where the agent determines the bidding strategy over the next hour. The bidding curve corresponding to this strategy shows the demand, d_k, the agent is willing to buy or sell at each price, p.
- All agents (leafs in the tree) make such a bidding curve and send this curve to the next higher agent.
- The next-higher agent aggregates the received bidding curves and passes them on. Finally the combined curve, d_{total}, reaches the root.
- The root determines the price, p^*, over the next hour such that d_{total} equals zero (then the supply and demand of electricity are the same).
- This p^* is sent back to all agents and each agent knows the demand it must buy or sell, d_k^*.
- An hour later, t becomes $t+1$ and the whole process is repeated.

Sending a bidding curve with a higher/lower electricity demand at each price leads to higher/lower prices which could be advantageous for producers/consumers. It is therefore assumed that agents do not lie about their bidding curve.

3.2 Planning modules for the PowerMatcher

The PowerMatcher algorithm developed in [10] balances the short-term supply and demand in the smart grid. This is done while taking the network capacity constraints into account. The bidding curve corresponding to the agent's bidding strategy is made every fixed time unit, without any forecasts about future prices. Historical data may be available but this is not a good indication of future prices if wind energy is involved. To make more efficient choices, it would for some devices be better to know the long-term price expectations and use this information into their bidding strategy. For devices which are shiftable over a longer time window than the chosen one, it is beneficial to know the lowest prices within that longer time window. One way to predict the long-term price is to use weather forecasts. Including the long-term price predictions into the PowerMatcher may lead to a more cost-efficient schedule for shiftable devices.

To show the potential of including planning modules, a small example is used. Suppose that there are a few electric cars that have to be charged between 18.00 and 08.00 the next day and that an electric car can be charged in five hours if it is charged at full power. Without any planning, all cars start charging at full power at 18.00 . This situation is shown in Figure 3.3. All cars are fully charged at 23.00.

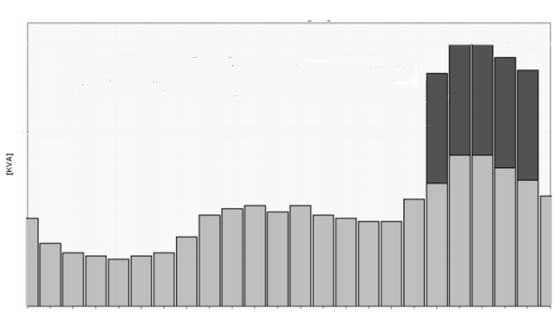


Figure 3.3: Uncontrolled charging

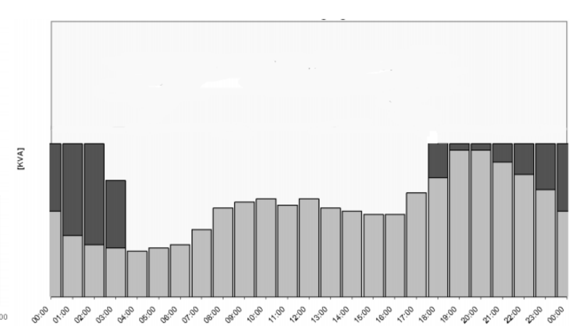


Figure 3.4: Controlled charging

Figure 3.4 shows another way to handle the increasing amount of needed electricity. With controlled charging possible network overloading is avoided. This is also what the standard PowerMatcher does, the standard PowerMatcher tries to control the charging of electric cars to avoid the network from overloading.

There is still room for improvement. The electric cars need to be fully charged at 08.00. As can be seen from Figure 3.4 the electric cars are already fully charged at 04.00. This means that the cars could be charged slower. By taking planning into account the load can be divided more evenly over the night and the peak load of electricity in the network decreases. The network will in that case be far from overloading. This situation is shown in Figure 3.5. The gap between 04.00

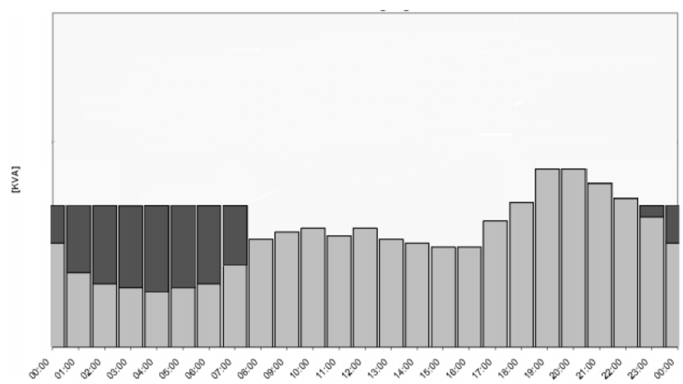


Figure 3.5: Controlled charging using time till 08.00

and 08.00 is filled, it is expected that this leads to lower overall costs. This last case can only be achieved when information about future demands is known. Matching future demands leads to future price expectations.

There are different options for including planning in the PowerMatcher, one of them is the two-time-scale PowerMatcher, [8]. The two-time-scale PowerMatcher charges the electric cars with information about future demands/prices and remaining time. This is done using different time scales. The longer time scale is needed to get information about future demands/prices and the shorter time scale is needed to match demand and supply over the shorter time scale.

In the example above the two time scales used by the two-time-scale PowerMatcher would be the day-ahead scale and the hour scale.

- Day-ahead scale: Agents send their bidding curve over the whole day and an average price is constructed from these curves. This average price can be used to determine the bidding curve for the shorter time scale.

- Hour scale: With the information about the expected average price, agents construct their bidding curve for the next hour. The price for the next hour is determined using the short term bidding curves of all agents.

A moving-horizon approach is used in the two-time-scale PowerMatcher to implement the day-ahead scale, the average price is thus determined every hour instead of once a day. In this way, changes during the day can be taken into account. Different time scales can be used. The day-ahead scale and the hour scale are used here because cars have one night to charge. It can in another example be better to use information about a whole week, then the week-ahead scale can be used in the same way as the day-ahead scale is used here.

Systems design of the two-time-scale PowerMatcher

The systems design of the two-time-scale PowerMatcher is the same as the systems design of the PowerMatcher using one time scale. Within a cluster, agents are organized into a tree. The layout of this cluster is given in Figure 3.1 in Section 3.1, where the different types of agents are also described.

Structure of the two-time-scale PowerMatcher

The structure of the two-time-scale PowerMatcher is given in Figure 3.6. In this figure the day-ahead scale and the hour scale are used.

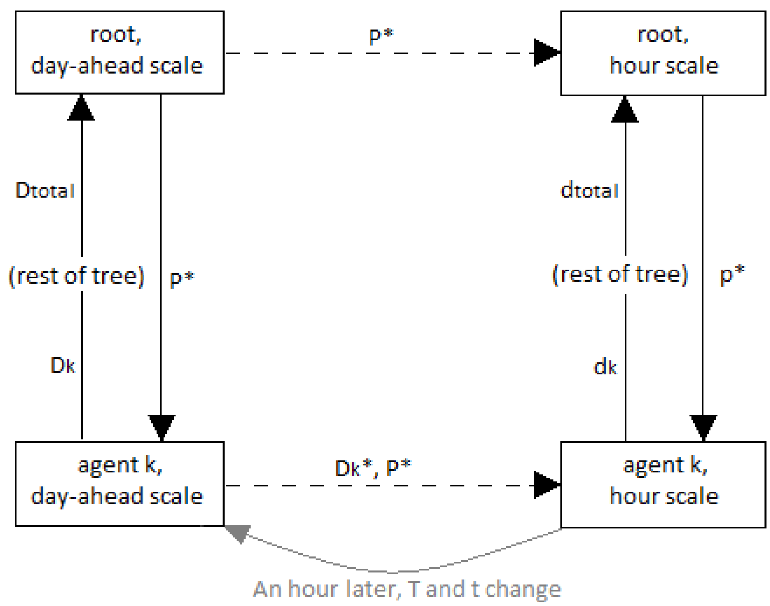


Figure 3.6: Structure two-time-scale PowerMatcher

The right side of Figure 3.6 is equal to Figure 3.2, the two-time-scale PowerMatcher does however start at the bottom left node. Agents determine the day-ahead bidding curve and send this curve to the next-higher agent. By aggregating the different bidding curves, the total bidding curve D_{total} reaches the highest agent/root. This root determines the the day-ahead average price, P^*, and sends this price back to all agents. With P^* known, agents can determine the bidding curve for the shorter time scale. This happens at the right side of Figure 3.6. This right side works exactly the same as explained in Section 3.1, but with P^* as extra information. A more detailed description of the structure of the two-time-scale PowerMatcher is given in Section 4.2.

3.3 Bidding strategies and bidding curves

Above, the PowerMatcher is explained. In the PowerMatcher there are different agents. These agents have to determine their bidding strategy and make the corresponding bidding curve. In the standard PowerMatcher one bidding curve has to be sent at each time epoch, the bidding curve over the next time interval. In the two-time-scale PowerMatcher each agent has to sent two bidding curves, one bidding curve over the longer time scale and one over the shorter time scale.

The agent's bidding strategy is the method by which the bidding curves are made. There are a lot of different bidding strategies possible. Agents can use aggressive and passive bidding strategies. Aggressive bidding strategies have a high demand at very low prices and a very low demand when prices are higher. Passive bidding strategies use more demand at higher prices, compared to the aggressive bidding strategies. Aggressive bidding strategies lead to steeper bidding curves.

There are different kinds of demand, some fixed and some shiftable. Fixed demand has to be satisfied at each price, this is not the case with shiftable demand. The amount of shiftable demand at a certain time depends on the price. At a low price the shiftable demand will be high, the demand can be satisfied at a relatively low price. With the same reasoning, the shiftable demand will be low at a high price. The bidding curve that is send to the higher agent is the summation of the bidding curves per demand type. An example of this is given in Figure 3.7. Suppose that an agent has a fixed and a shiftable demand. The total bidding curve is the summation of the bidding curves of these two different demand types.

(a) shiftable bidding curve (b) fixed bidding curve (c) total bidding curve

Figure 3.7: Agent's fixed and shiftable demand bidding curve is summated to one bidding curve

The fixed demand is the same for each price. The bidding curve for the fixed demand is therefore a straight line. The shiftable demand can be different at each price, the amount of demand at each price depends on the bidding strategy.

Each agent sends a bidding curve at each time epoch. These bidding curves are different for each agent and each time epoch. Each agent wants to fulfill the demand as cheaply as possible, but each bidding strategy leads to different total costs. A few different bidding strategies are looked at in the remainder of this report, these bidding strategies will be compared to each other by looking at the total costs for satisfying a certain amount of demand.

3.4 Research Question

This section summarizes the background and states the research question.

Due to the growing energy needs and larger shares of decentralized and renewable energy, smart control algorithms play an increasing role in the energy field. For the coordinated matching of supply and demand in the electricity network, TNO (together with ECN) developed the PowerMatcher. The PowerMatcher is a coordination mechanism that balances short term demand and short term supply in large clusters of distributed energy resources.

The PowerMatcher uses short time scales. Some types of device agents have a demand which is shiftable over a longer time window. For making locally optimal choices, these shiftable devices need information about the long-term price expectations. These price expectations can be obtained by producing demand forecasts, based on the individual demand predictions of the device agents. This extension of the PowerMatcher is called the two-time-scale PowerMatcher.

The question arises whether this extension to a two-time-scale PowerMatcher provides an added value to the PowerMatcher and how the price expectations have to be included in the bidding strategies of the two-time-scale PowerMatcher. The research question reads therefore:

- Does the performance of the PowerMatcher improve when next to the current demand and current supply also predictions about future demand and future supply are included in the planning and to which extent does the performance improve?
- How should predictions about future demand and future supply be included in the bidding strategies of the two-time-scale PowerMatcher?

3.5 Outline

This report will answer the research question given in Section 3.4. To do this, first the two-time-scale PowerMatcher, introduced in Secton 3.2, needs to be explained in a little more detail. The two-time-scale PowerMatcher uses predictions about future demand and future supply in the planning. How these predictions are included is explained in Chapter 4, where the two-time-scale PowerMatcher algorithm and used bidding strategy are presented. The bidding strategy currently used in the two-time-scale PowerMatcher can be improved. A new bidding strategy that is designed to lead to lower total costs is presented in Chapter 5. This new bidding strategy uses predictions about future prices and dynamic programming to determine which bidding curve to use at a certain time. This bidding strategy works for different levels of information about future prices, how the different levels can be used in the bidding strategy is explained in Chapter 5. The different levels of information about future prices and the different bidding strategies that can be used by the two-time-scale PowerMatcher are compared to each other in Chapter 6. Multiple instances are used to see the difference in the performance of the PowerMatcher using these different bidding strategies. The conclusions of this research are given in Chapter 7, together with options for further research.

4 Two-time-scale PowerMatcher

As stated before, the main goal of this report is to improve the PowerMatcher by not only using current demand and current supply but also predictions about future demand and future supply. Extending the standard PowerMatcher to the two-time-scale PowerMatcher is one of the methods to include planning into the PowerMatcher. This two-time-scale PowerMatcher is already shortly introduced in Chapter 3 and will be explained in more detail in this chapter.

First, in Section 4.1 the assumptions are given under which the two-time-scale PowerMatcher is worked with in the report. Also the most frequently used variables are given in this section. The two-time-scale PowerMatcher algorithm, as developed in [8], will be given in Section 4.2. In this algorithm bidding curves have to be sent by the agents, the agents' bidding strategies and curves in the two-time-scale PowerMatcher are explained in Section 4.3. The bidding strategy used in Section 4.2 is compared to two other demand satisfying strategies in Section 4.4. These two strategies do not have information about future demand and supply.

A new bidding strategy will be introduced in the next chapter. This new bidding strategy will be compared to the bidding strategy explained in Section 4.3, this to see if the current version of the two-time-scale PowerMatcher can be improved.

4.1 Assumptions and frequently used variables

A few assumptions are made throughout this report. These assumptions are listed below.

- It is assumed that agents send a bidding strategy once in every fixed amount of time. This is not what happens in real life. To keep the communication between the agents to a minimum, new bidding strategies are only sent when the local device state changes. Sending new bidding strategies is thus event-based in practice.
- It is possible for agents to lie about their needed demand at a certain price. Producers want to raise the price of energy to earn more, consumers like a low price. By changing the real bidding curve, agents can influence the price to their own benefit. It is assumed in this report that agents play fair.
- It is assumed in this report that there are no storage units. Producers can not store produced energy, to sell it later at a higher price, and consumers can not store bought energy, to use it at another time.
- The information about future prices is in this report based on weather forecasts. Information from the past is not used.
- The prices of two consecutive time periods are independent, the price can thus fluctuate heavily.
- It is assumed that all demand is shiftable, unless stated otherwise.

Some frequently used variables in this section are listed in Table 4.1. A list of all used variables in this report is given in Table 9.1.

4.2 Two-time-scale PowerMatcher algorithm

This section gives the algorithm of the two-time-scale PowerMatcher. The algorithm shows among others when the bidding curves are made and what information is used by the agent to make a bidding curve. The algorithm also shows how the prices are determined.

The outline of the algorithm corresponding to the structure of the two-time-scale PowerMatcher given in Figure 3.6 is given in [8] and will here also be given. This algorithm consists of a few

Variable	Description
t	time (measured in intervals of hours), short time scale
n_t	number of time points
T	time horizon, long time scale
K	number of agents
d	demand at shorter time scale
D	demand at longer time scale
p	unit price of energy at shorter time scale
P	unit price of energy at longer time scale

Table 4.1: Frequently used variables

steps which will all be looked at. The two time scales worked with here will be the hour scale and the day-ahead scale. These two time scales are also mentioned in Figure 3.6.

1. The starting node is the node "agent k, day ahead scale" at the beginning of time t. The day-ahead horizon is denoted by T, $T = [t, \ldots, t + n_t]$.

2. Agent k, $k \in K$, determines the bidding strategy over T and creates the bidding curve $D_k(P;T)$ over T, where P is the price. The demand may depend on the average price, P^* over T but this is only possible when the demand is shiftable over a longer time period than T. When no shiftable demand is available or if the shiftable demand is only shiftable within T, the agent's demand will be constant and therefore independent of P^*. The bidding strategy of the agent is explained in Section 4.3.

3. Agent k sends $D_k(P;T)$ to the next-higher agent in the tree.

4. Each agent in the tree aggregates the received information (bidding curves) and passes it on to the next-higher agent in the tree. Finally, $D_{total}(P;T)$ reaches the root.
$$D_{total}(P;T) \quad = \quad \sum_{k=1}^{K} D_k(P;T).$$

5. The root finds the average price, P^*, such that the total bidding curve at P^* equals zero.
$$D_{total}(P^*;T) = 0.$$
Arguments for the existence and uniqueness of the solution P^* can be found in [10].

6. P^* is the new day-ahead average price. This value is sent to all agents in the tree.

7. By plugging in P^* into $D_k(P;T)$, agent k now knows it's target demand over the day-ahead horizon.
$$D_k^* = D_k(P^*;T).$$

8. Agent k can use P^* and D_k^* as inputs for the bidding curve over the next hour, $d_k(p;t|D_k^*, P^*)$. This $d_k(p;t,d|D_k^*, P^*)$ corresponds to the agent's bidding strategy over the next hour. The bidding curve $d_k(p;t,d|D_k^*, P^*)$ consists of different kinds of demand, see Section 3.3.

9. Agent k sends the bidding curve $d_k(p;t|D_k^*, P^*)$ to the next-higher agent in the tree.

10. Each agent in the tree aggregates the received information (curves) and passes it on to the next-higher agent in the tree. Finally the combined curve $d_{total}(p;t|\{D_k^*\}_{k=1,\ldots,K}, P^*)$ reaches the root. The $d_{total}(p;t|\{D_k^*\}_{k=1,\ldots,K}, P^*)$ equals the sum of all agents' hourly demand.
$$d_{total}(p;t|D_k^*, P^*) \quad = \quad \sum_{k=1}^{K} d_k(p;t|D_k^*, P^*).$$

11. The root finds the price, p^*, such that the total demand at p^* equals zero.

$$d_{total}(p^*; t|D_k^*, P^*) \quad = \quad 0.$$

12. The current price is equal to p^*, this price is sent to all agents in the tree.

13. Agent k can put p^*, into $d_k(p; t|D_k^*, P^*)$. The resulting demand, d_k^* is binding.

$$d_k^* = d_k(p^*; t|D_k^*, P^*).$$

14. An hour later, t becomes $t+1$, the time horizon T gets shifted by one period and the whole algorithm is repeated.

The $d_k(p; t|D_k^*, P^*)$ are made with the knowledge of D_k^* and P^*, this does not mean that the total demand and real average price equal these values determined at the beginning. It turns out that these values differ most of the time. One way by which D_k^* and P^* differ from the total demand and real average price is when producers can't deliver the amount of electricity given in the bidding curve. When the total demand of all agents is positive, electricity has to be bought from expensive but flexible energy resources. When the total demand is negative, energy is thrown away.

4.3 Bidding strategies and curves in the two-time-scale PowerMatcher

Bidding strategies have to be made for the shorter and longer time scale in the two-time-scale PowerMatcher, as explained in Section 4.2. This section explains the bidding strategies for the shorter time scale. The idea behind the bidding strategies for the longer time scale is the same, but without the information about D_t^* and P^*.

The bidding strategy gives the method used to determine the bidding curve and the bidding curve shows the demand at each price. The bidding curve of agent k, $d_k(p; t, d)$, is dependent on price p and can be different for each time epoch t and remaining demand d. The bidding curve currently made in the two-time-scale PowerMatcher is however independent of d. The bidding curve of agent k will in this section therefore be denoted by $d_k(p; t)$. Later on in this report the bidding curve will be dependent of d.

The bidding curve $d_k(p; t)$ would ideally show the demand at each p. The price p is however continuous and it is therefore very hard to determine the demand for each value of p. To deal with this $d_k(p; t)$ will be a piecewise linear function, n possible values of p, $r_1, ..., r_n$ are selected between p_{min} and p_{max} and the corresponding action/demand $a_k(r_1; t), .., a_k(r_n; t)$ is determined such that agent k satisfies the total demand as cheaply as possible. The values of r_1 and r_n are fixed, $r_1 = p_{min}$ and $r_n = p_{max}$. The combinations r_i and $a_k(r_i; t)$ for $i = 1, ..., n$ are the breakpoints of the piecewise linear function. The breakpoints of the function are given in a nx2 matrix.

$$\begin{bmatrix} r_1 & a_k(r_1; t) \\ \vdots & \vdots \\ r_n & a_k(r_n; t). \end{bmatrix}.$$

An agent can have different kinds of demand, fixed and shiftable demand and also demand from wind and flexible energy resources are possible. Each kind of demand has its own bidding strategy. The first two demand types are positive while the last two are negative. The four different demand types and bidding strategies are explained next.
The fixed demand, $d_{k,fixed}(t)$, is an amount which has to be satisfied at time t, so p, D_k^* and P^* do not have any influence on this value, i.e. $d_{k,fixed}(t)$ is a fixed amount, $d_{k,fixed}^{fixed}(t)$.

$$d_{k,fixed}(t) \quad = \quad d_{k,fixed}^{fixed}(t) \quad \forall p.$$

The shiftable demand, $d_{k,shift}(p;t|D_k^*,P^*)$, does depend on p and P^*. It would be beneficial to satisfy a lot of shiftable demand when p is low and to use no shiftable demand when p is very high. If $p = P^*$ for all t, the total shiftable demand at time t, $d_{k,shift\ total}(t)$, would be satisfied equally over t.

$$d_{k,shift}(P^*;t|D_k^*,P^*) \quad = \quad \frac{d_{k,shift\ total}(t)}{T-t+1}.$$

Most of the times $p \neq P^*$, a possible bidding strategy for shiftable demand is given below.

$$d_{k,shift}(p;t|D_k^*,P^*) \quad = \quad \begin{cases} d_{k,shift\ total}(t) \cdot \left(1 + \frac{p}{(p_{min}-P^*)} \cdot \left(1 - \frac{1}{T-t+1}\right)\right) & \text{if p} \leq \text{P}^* \\ \frac{d_{k,shift\ total}(t)}{T-t+1} & \text{if p} = \text{P}^* \\ 0 & \text{if p} \geq \text{P}^* \end{cases}.$$

When $p = P^*$, $d_{k,shift\ total}(t)$ is divided over the remaining times equally and the part assigned to time t is satisfied. When $p \geq P^*$, no shiftable demand will be satisfied. When $p \leq P^*$, the amount of satisfied shiftable demand depends on the value of p. The function $d_{k,shift}(p;t|p \leq P^*)$ is a linear function where $d_{k,shift}(p_{min};t|D_k^*,P^*) = d_{k,shift\ total}(t)$ and $d_{k,shift}(P^*;t|D_k^*,P^*)$ is as given above, one t'th part of the $d_{k,shift\ total}(t)$. The given bidding strategy for shiftable demand is currently used in the two-time-scale PowerMatcher. There are however more bidding strategies possible, agents can choose $d_{k,shift}(p;t|D_k^*,P^*)$ however they want.

The demand of a wind turbine, $d_{k,wind}(p;t)$ does not depend on p, it only depends on the available amount of wind. The wind demand is therefore fixed.

$$d_{k,wind}(t) \quad = \quad -d_{k,wind}^{fixed}(t).$$

An agent with storage options can choose to store part of the $d_{k,wind}^{fixed}(t)$. This part could then become available at another time. It is assumed in the beginning of this chapter that this is not possible in this report.

Another type of demand is the demand from a flexible energy resource, like a diesel generator. The bidding curve of a diesel generator, $d_{k,diesel}(p;t)$, is independent of P^*, the bidding curve depends entirely on the price of power generation. $d_{k,diesel}(p;t)$ is given below. Here, the unit cost of generation is denoted by π_g. So, for $p>\pi_g$ the agent with the diesel generator can make profit and will therefore produce as much as possible, $d_{k,diesel}^{max}$. When $p<\pi_g$, the diesel generator will not produce anything.

$$d_{k,diesel}(p;t) \quad = \quad \begin{cases} 0 & \text{if p(t)} \leq \pi_g \\ -d_{k,diesel}^{max} & \text{if p(t)} > \pi_g \end{cases}.$$

When agent k represents a household where the total demand consists of fixed and shiftable demand, the bidding curves of the fixed and shiftable demand, $d_{k,fixed}(t)$ and $d_{k,shift}(p;t|D_k^*,P^*)$, are added, as explained in Section 3.3.

$$d_k(p;t|D_k^*,P^*) \quad = \quad d_{k,fixed}(t) + d_{k,shift}(p;t|D_k^*,P^*).$$

Agent k wants to get the total demand as cheaply as possible and therefore wants to choose $d_k(p;t)$ such that the costs are minimized. The $d_{k,fixed}$ is fixed and there is therefore not much to choose, $d_{k,shift}$ is not fixed and therefore can be chosen by the agent. The goal is now to choose the best one, the one that minimizes the total costs.

The $d_{k,shift}(p;t)$ currently used in the two-time-scale PowerMatcher is given below.

$$\begin{bmatrix} p_{min} & d_{k,shift\ total}(t) \\ P^* & \frac{d_{k,shift\ total}(t)}{T-t+1} \\ P^*+\epsilon & 0 \\ p_{max} & 0 \end{bmatrix}.$$

This $d_{k,shift}(p;t)$ shows the breakpoints of the bidding curve. The total bidding curve can be derived from these breakpoints. The $d_{k,fixed}(t), d_{k,wind}(t)$ and $d_{k,diesel}(t)$ currently used in the two-time-scale PowerMatcher have the same form. These different demands are all fixed. The demand is therefore the same at each price. As an example, the $d_{k,fixed}(t)$ is given by:

$$\begin{bmatrix} p_{min} & d_{k,fixed}^{fixed}(t) \\ \\ p_{max} & d_{k,fixed}^{fixed}(t) \end{bmatrix}.$$

The idea behind $d_{k,wind}(t)$ and $d_{k,diesel}(t)$ is the same, these will therefore not be shown here. Because there is only one bidding strategy possible for $d_{k,fixed}(t), d_{k,wind}(t)$ and $d_{k,diesel}(t)$, the focus of the remainder of this report will be on $d_{k,shift}(p;t)$.

The idea behind the larger time scale PowerMatcher is the same as described above for the shorter time scale. When demand is shiftable inside the larger time scale, the demand will be shiftable for the shorter time scale and fixed for the larger time scale. When the demand is shiftable over a longer time period, the demand is shiftable for the shorter and longer time scale.

4.4 Advantage two-time-scale PowerMatcher over 'simple' strategies

Section 4.2 showed the algorithm of the two-time-scale PowerMatcher. Agents send bidding curves to higher agents/the root. The bidding strategies and corresponding bidding curves are made with information about future demand. This section shows the effect of this information. The bidding strategy of the two-time-scale PowerMatcher is compared to two other demand satisfying strategies. These other two strategies do not have information about future demand. In these strategies the demand is satisfied as fast and as cheaply as possible. The comparison shows whether the information about future demand leads to lower costs of satisfying the demand.

The different strategies are compared using a small network with one windmill and one household which has an electric car. The situation is shown in Figure 4.1.

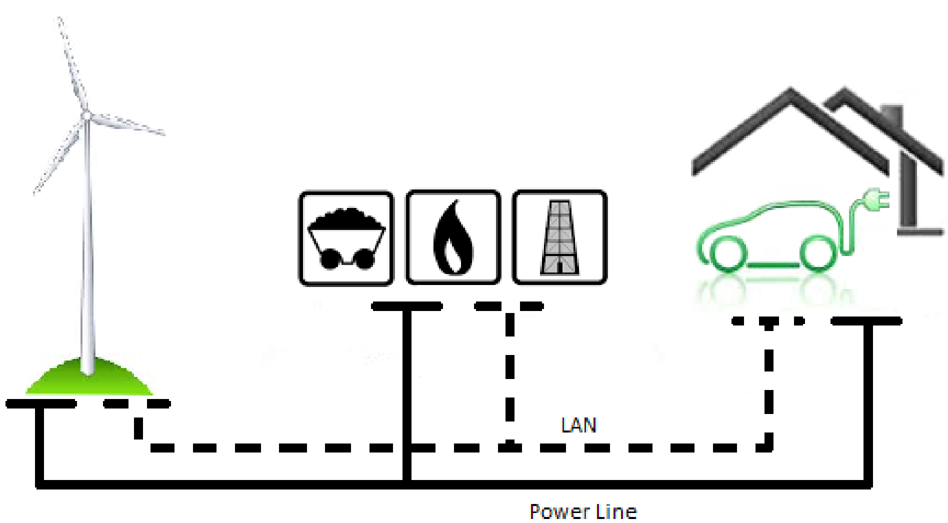


Figure 4.1: Network with one windmill and one electric car

The dashed line is the local area communication network. All communications between the utility and the customers are done through the LAN. The solid line is the power line. The demand of the car is shiftable, the remaining energy demand of the household is fixed. The network will be looked at for 24 hours, where at each hour the household has to decide what amount of energy to use. The windmill in this network can either produce zero, one or two units of wind energy per hour and the household's fixed demand will be between zero and two at each time. The energy produced by the windmill is relatively cheap energy. When not enough wind energy is available, energy from flexible energy resources has to be bought, this is relatively expensive energy. Energy from flexible energy resources is more expensive than wind energy because fuel has to be bought to produce the energy. Wind is freely available and therefore cheap. The total shiftable demand is set to three units and this shiftable demand has to be satisfied between time 10 and 24. The household's capacity is set equal to two units of energy per hour. The decision about the amount of energy demand to use at each hour is made for different strategies. These strategies are listed below.

Strategy 1
In this strategy the household will charge the car as soon and fast as possible. When the fixed demand is met and the household's capacity is not yet reached, the electric car can be charged till the household's capacity is reached. The amount of produced wind energy is not taken into account.

Strategy 2
In this strategy the amount of available wind energy is taken into account. At each time it is checked whether there is wind energy available. When this is still the case after the fixed demand is satisfied, the electric car will be charged. In absence of wind energy, energy from flexible energy resources will be used for the fixed demand, the electric car will not be charged at that specific time. When the car is not fully charged at the end of the day, flexible energy resources are used to charge the car.

Strategy 3
In the third situation not only the current availability of wind energy but the fixed demand and the wind energy for the next 24 hours are taken into account. With this information the average price is determined, which is used to determine the demand at a specific time and price. This is a variant of the two-time-scale PowerMatcher, which is explained in Section 3.2 and Section 4.2.

The strategies will be compared to each other in two different situations. In the first situation it is assumed that the household's fixed demand and the amount of produced wind energy are known for each of the next 24 hours. In the second situation the amount of future wind energy is not known exactly, this information can change over time.

Wind production known

In this situation the household's fixed demand (FD) and the produced wind energy (WD) over the next 24 hours are known. There are 100 instances used to compare the different strategies. The FD and WD of one instance are given in Figure 4.2. This instance is used to see the difference of the strategies more closely. The values of FD and WD are given as a vector of length 24. Each element of the vector represents respectively the fixed demand and wind demand for the corresponding time. As can be seen from the vectors, the values in the fixed demand vector are positive while the vector for wind energy contains negative values. Wind energy is produced, because supply can be seen as negative demand, the second vector contains negative values.

The wind energy is relatively cheap, when two wind energy units are available, the unit costs will be equal to one, $c_{w2} = 1$, when only one unit of wind energy is available the costs for that unit will be equal to three, $c_{w1} = 3$. When not enough wind energy is available, energy has to be bought from flexible energy resources . This is relatively expensive energy, the unit costs of this kind of energy is set to 10, $c_f = 10$. The household wants to satisfy the total demand as cheaply as possible.

$$\mathrm{FD} = \begin{bmatrix} 1,4187 \\ 1,5094 \\ 0,5521 \\ 1,3594 \\ 1,3102 \\ 0,3252 \\ 0,2380 \\ 0,9967 \\ 1,9195 \\ 0,6808 \\ 1,1705 \\ 0,4476 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ 1,5025 \\ 0,5102 \\ 1,0119 \\ 1,3982 \\ 1,7818 \\ 1,9186 \\ 1,0944 \\ 0,2772 \\ 0,2986 \\ 0,5150 \\ 1,6814 \\ 0,5086 \end{bmatrix}, \qquad \mathrm{WD} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \\ 0 \\ -2 \\ -1 \\ -1 \\ 0 \\ -1 \\ -1 \\ -1 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -2 \\ 0 \\ -2 \\ -1 \\ -2 \\ -1 \\ -2 \end{bmatrix}.$$

Figure 4.2: Fixed demand and wind demand vectors used in the network with known wind production

With $A_w(t)$ the amount of wind energy used at time t and $A_f(t)$ the amount of energy used from flexible energy resources at time t, the total costs TC of the household becomes:

$$TC = \sum_t \left(A_w(t) \cdot c_w + A_f(t) \cdot c_f \right).$$

The costs of satisfying all demand in this example network are calculated for each of the strategies, the results are given in Section 6.1.

Wind forecasts

Availability of wind energy in the future can not be known exactly. Wind forecasts are known but change over time. In this part the different strategies are compared to each other when the available information about wind energy changes over time. This comparison is made using 100 instances in the example network given above. The fixed demand vector and wind demand vector of one of these instances are given in Figure 4.3. The fixed demand vector is the same as the fixed demand vector given in Figure 4.2.

$$\mathrm{FD} = \begin{bmatrix} 1,4187 \\ 1,5094 \\ 0,5521 \\ 1,3594 \\ 1,3102 \\ 0,3252 \\ 0,2380 \\ 0,9967 \\ 1,9195 \\ 0,6808 \\ 1,1705 \\ 0,4476 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ 1,5025 \\ 0,5102 \\ 1,0119 \\ 1,3982 \\ 1,7818 \\ 1,9186 \\ 1,0944 \\ 0,2772 \\ 0,2986 \\ 0,5150 \\ 1,6814 \\ 0,5086 \end{bmatrix}, \qquad \mathrm{WD} = \begin{bmatrix} -1,9602 \\ -1,0883 \\ -0,7313 \\ -1,3855 \\ -1,6341 \\ -0,9686 \\ -1,8725 \\ -1,1081 \\ -1,8799 \\ -1,8684 \\ -0,9176 \\ -0,4441 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ -1,6782 \\ -1,7621 \\ -1,3225 \\ -1,1182 \\ -0,0668 \\ -1,4268 \\ -1,9537 \\ -1,2481 \\ -0,8068 \\ -1,7736 \\ -1,6504 \\ -0,9810 \end{bmatrix}.$$

Figure 4.3: Fixed demand and wind demand vectors used in the network with wind forecasts

In this situation, the costs per unit wind energy will be equal to one, $c_w = 1$. Energy from flexible energy resources is relatively expensive, the costs of this kind of energy is set to 10, $c_f = 10$. The wind demand vector will change a little bit at each time t, but stays between zero and minus two. Produced energy can never be positive and can't exceed the capacity. With $n \in N$ and $N \sim \mathcal{N}(0,1)$:

$$WD_{new}(s) \quad = \quad \max(\ \min(\ WD_{old}(s) + (0{,}05 \cdot n), 0), -2) \quad \forall s \geq t.$$

At time t the vector changes for entries $t+1, ..., T$, entries $1, ..., t-1$ do not change because the amount of wind in the past is known for certain. The final wind demand vector (FWD), the wind demand vector at time $t = 24$ of this instance is given in Figure 4.4.

$$\text{FWD} = \begin{bmatrix} -1{,}9602 \\ -1{,}1058 \\ -0{,}6552 \\ -1{,}5516 \\ -1{,}6983 \\ -1{,}1712 \\ -1{,}6523 \\ -1{,}2550 \\ -1{,}7402 \\ -1{,}7562 \\ -0{,}8834 \\ -0{,}5610 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ -1{,}6365 \\ -1{,}9942 \\ -0{,}8771 \\ -0{,}9787 \\ -0{,}1510 \\ -1{,}3234 \\ -2{,}0000 \\ -1{,}1069 \\ -0{,}5251 \\ -1{,}5955 \\ -1{,}5263 \\ -1{,}0626 \end{bmatrix}.$$

Figure 4.4: Final wind demand vector

The three different strategies will be compared to each other to see how the strategies handle changes in the wind forecasts. The third strategy, where the average price P^* is assumed to be known, will be divided into two versions. These two versions of the third strategy are now given.

Strategy 3(1)
In this version of strategy 3, the average price P^* will only be determined at the first time epoch. The real average price will change in time because of the changed wind demand. The value of P^* will not be updated in this strategy.

Strategy 3(24)
In this version of strategy 3, the average price P^* will be updated at every time epoch. This strategy thus includes the changes in wind demand.

There are now four different strategies. The costs of satisfying all demand in the example network are calculated for each of the four strategies, the results are given in Section 6.1.

5 Bidding strategies using Dynamic Programming

The previous chapter, Chapter 4, explained the two-time-scale PowerMatcher and the currently used bidding strategy in the two-time-scale PowerMatcher. Based on this information, new bidding strategies can be introduced. The bidding strategy introduced in this chapter is intended to be used for the shorter time scale and is designed to lead to lower total costs. However, this new bidding strategy could also be used for the larger time scale, in the same way as is explained in this chapter for the shorter time scale.

It is assumed that information about the average price P^* and the target demand D_k^* is already known when the bidding curve for the shorter time scale needs to be calculated, the P^* and D_k^* are used in the bidding strategy. Some of the other variables that are used in this chapter are listed below.

- p: The variable p represents the unit price of electricity. The value of p can have any value between p_{min} and p_{max}, the minimum and maximum price.
- r_i: The variables r_i with $i = 1, ..., n$ give the n possible values of p that are worked with in the new bidding strategy. Price p is continuous, the bidding strategy introduced in this chapter can only be used when the price is discrete, the variable p therefore needs to be discretized. The values of r_i are dependent on the price distribution used.
- q_i: The value of q_i shows the probability that the price equals r_i. The probabilities are used in the dynamic programming part of the new bidding strategy and the values of q_i depend on the price distribution.
- $a_k(r_i; t, d)$: The value of a_k shows the amount of electricity that can best be buyed/sold in the situation with price r_i, time t and remaining demand d.

Using the new bidding strategy, the value of $a_k(r_i; t, d)$ becomes dependent on the remaining demand, d. The form of the bidding curve will therefore be given by:

$$\begin{bmatrix} r_1 & a_k(r_1; t, d) \\ \vdots & \vdots \\ r_n & a_k(r_n; t, d) \end{bmatrix}.$$

Two steps are needed to determine the bidding curve.

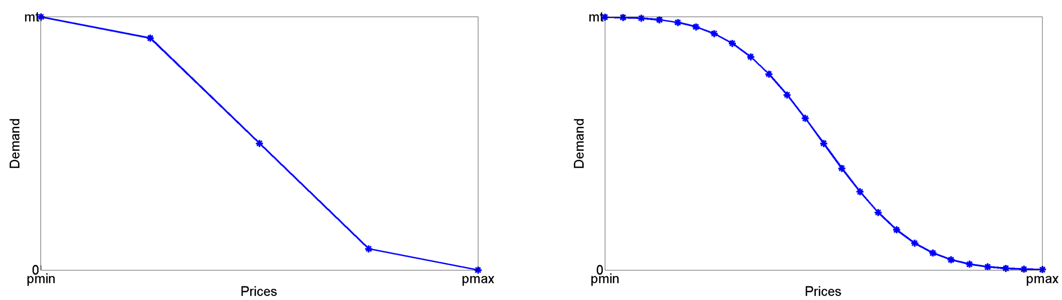
1. The possible values of the price, r_1, ..., r_n, and the corresponding probabilities q_1, ..., q_n, are determined.
2. With step 1 completed, the $a_k(r_i; t, d)$ are calculated.

When r_i, q_i and $a_k(r_i; t, d)$ are determined for all i, the bidding curve is known.

Section 5.1 shows how the possible values of the price, r_1, ..., r_n, and the corresponding probabilities, q_1, ..., q_n are determined for the case with known prices and for the case where prices are stochastic. Different levels of information about the prices are used in this second case. With the r_i and q_i values known, the values of $a_k(r_i; t, d)$ can be determined. Section 5.2 shows how the values of a can be determined when the prices are known. This is done by rewriting the problem as a Knapsack problem and solving this Knapsack problem using dynamic programming. Section 5.3 shows how the values of a can be determined when the prices are stochastic. Sections 5.2 and 5.3 are explained for the situation where an electric car has to be charged before the end of time T. The methods explained in these sections slightly change when other home appliances need electricity. Section 5.4 shows how the method can be applied for different appliances. By putting sections 5.2 and 5.3 together, the current bidding curve is known. A new bidding curve is needed for the next time epoch, in this new time epoch the time horizon can be different. Depending on the appliance, the time horizon is either fixed or moving. Section 5.5 shows what changes when prices are updated.

5.1 Determine price distribution

The first step in determining the bidding curve is to determine the possible values of the price p, $r_1, ..., r_n$, and the corresponding probabilities, q_1, ..., q_n for a given value of n. This value of n determines the number of breakpoints in the bidding curve, where a high value of n means that the demand $a_k(r_i; t, d)$ is known for more possible values of the price. This leads to more detailed information about the bidding strategy, see Figure 5.1. Different values of n will be used throughout the report.



(a) n = 5

(b) n = 25

Figure 5.1: Two examples of a bidding curve

The values of r_1, ..., r_n and corresponding probabilities q_1, ..., q_n are in this section determined for different levels of information about p_t. Section 5.1.1 determines r_i when the prices p are known in advance. In sections 5.1.2 till 5.1.4 p_t is assumed to be a random variable that can take any of n values, where the value of n is handpicked and assumed to be odd. The n possible values of the price, r_1, ..., r_n, can be determined using different distributions. The discrete uniform distribution is used in Section 5.1.2, where the average price P^* is assumed to be known. The normal distribution is used in Section 5.1.3, where besides P^* also the standard deviation of p, σ_p, is assumed to be known. The normal distribution is also used in Section 5.1.4, where the average price and standard deviation at each t, p_t^* and $\sigma_{p,t}$, are known.
The values of r_1, ..., r_n and the corresponding probabilities q_1, ..., q_n determined in this section are used in Section 5.3, where $a_k(r_i; t, d)$ is determined for all r_i, t and d using dynamic programming.

5.1.1 Known p_t

In this section the prices p_1, ..., p_T are assumed to be known. With known prices, the number of possible values for the price is equal to one, $n = 1$. The values of r_1 and q_1 thus need to be determined. The value of r_1 at time t shows the only possible value of p_t, so $r_1 = p_t$. The price p_t is known, therefore $q_1 = 1$. With known prices, the bidding curves have the following form:

$$\begin{bmatrix} p_t & a_k(p_t; t, d) \end{bmatrix}$$

Section 5.2 shows how the value of $a_k(p_t; t, d)$ is determined.

5.1.2 Known P^*

It is in this section assumed that the price p is a random variable that can take any of n given values, where n is handpicked and assumed to be an odd number. These n possible values of the price, r_1, ..., r_n, need to be determined, together with the corresponding probabilities, q_1, ..., q_n. With the average price P^* known, the discrete uniform distribution is used to determine r_i and q_i for all i. The values of r_i are equidistantly distributed around P^*, the probability of r_i, q_i, is the same for each i, $q_i = \frac{1}{n}$ for $i = 1, ..., n$. The following equation thus holds.

$$\sum_{i=1}^{n} q_i \cdot r_i \quad = \quad P^*.$$

With stepsize $s = \frac{p_{max} - p_{min}}{n}$, the resulting bidding curve is specified by:

$$\begin{bmatrix} p_{min} & a_k(p_{min}; t, d) \\ p_{min} + s & a_k(p_{min} + s; t, d) \\ \vdots & \vdots \\ P^* & a_k(P^*; t, d) \\ \vdots & \vdots \\ p_{max} - s & a_k(p_{max} - s; t, d) \\ p_{max} & a_k(p_{max}; t, d) \end{bmatrix}.$$

The values of r_i and q_i are used as inputs for Section 5.3, where $a_k(r_i; t, d)$ is determined for all r_i, t and d using dynamic programming.

5.1.3 Known P^* and σ_p

In Section 5.1.2 the values for r_i and q_i are determined using the discrete uniform distribution, using the average price P^*. In this section it is assumed that the prices are normally distributed with a given average price P^* and standard deviation σ_p, $X \sim \mathcal{N}(P^*, \sigma_p)$. This is discretized to a random variable p_t that can take on any of n given values, where n is again assumed to be an odd number. Now, the values of r_i and q_i, for $i = 1, ..., n$ have to be determined such that these values correspond to the given normal distribution.

As the interval $[P^* - 3\sigma_p, P^* + 3\sigma_p]$ covers 99,7 % and thus "almost all" of the range of x, the values r_i for $i = 1, ..., n$ are all chosen in the interval $[P^* - 3\sigma_p, P^* + 3\sigma_p]$, more precisely they are determined as follows: First, the interval $[P^* - 3\sigma_p, P^* + 3\sigma_p]$ is decomposed into n equally sized intervals. Figure 5.2 shows these intervals for n=5.

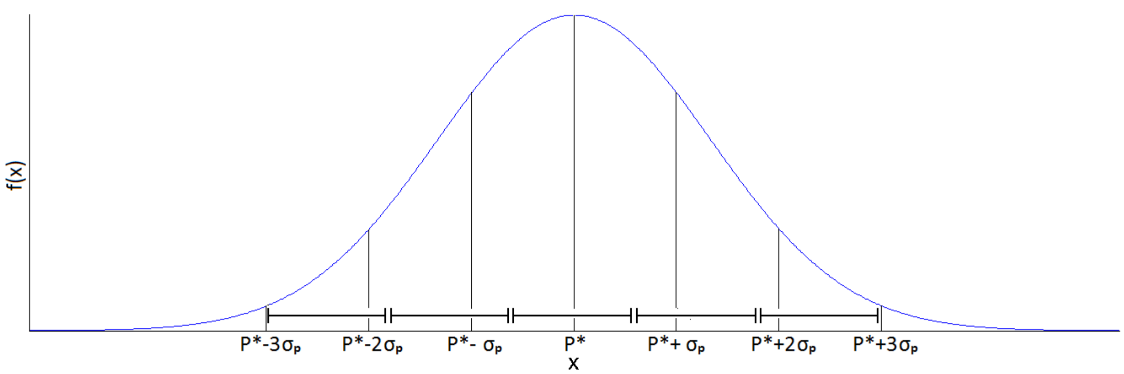


Figure 5.2: Intervals when n=5

With $x_i = P^* - (3 - \frac{6}{n} \cdot i)\sigma_p$, interval i is denoted by $[x_{i-1}, x_i]$. Next, the area left of $P^* - 3\sigma_p$ is added to the first interval and the area right of $P^* + 3\sigma_p$ is added to the n'th interval, so $x_0 = -\infty$ and $x_n = \infty$. Then, for all i, the value r_i is calculated in such a way that r_i is the average price of interval i. At interval $\frac{n+1}{2}$ the curve is symmetric around P^*, so:

$$r_{\frac{n+1}{2}} = P^*.$$

The values r_i for all $i \neq \frac{n+1}{2}$ are more difficult to determine, because the curve is not symmetric in these intervals. At interval i with bounds $[x_{i-1}, x_i]$, r_i is the expected value of x given that x is in interval $[x_{i-1}, x_i]$.

$$r_i = E[x|x \in [x_{i-1},\ x_i]].$$

The value r_i is such that:

$$P(x_{i-1}<X<r_i) \quad = \quad \frac{1}{2}P(x_{i-1}<X<x_i). \tag{5.1.1}$$

The normal distribution probabilities can be calculated through the standard normal distribution.

$$P\left(x_{i-1}<X<x_i\right) \quad \approx \quad \Phi\left(\frac{x_i - P^*}{\sigma_p}\right) - \Phi\left(\frac{x_{i-1} - P^*}{\sigma_p}\right).$$

Now that the values of r_i, $i = 1, ..., n$ are known, the values of q_i, $i = 1, ..., n$ have to be calculated. The value q_i corresponding to r_i is the probability to be in interval i.

$$q_i \quad \approx \quad \Phi\left(\frac{x_i - P^*}{\sigma_p}\right) - \Phi\left(\frac{x_{i-1} - P^*}{\sigma_p}\right). \tag{5.1.2}$$

When the values of r_i and q_i are known for $i = 1, ..., n$, the bidding curve:

$$\begin{bmatrix} r_1 & a_k(r_1; t, d) \\ \vdots & \vdots \\ P^* & a_k(P^*; t, d) \\ \vdots & \vdots \\ r_n & a_k(r_n; t, d) \end{bmatrix}$$

can be determined using dynamic programming, explained in Section 5.3.

5.1.4 Known p_t^* and $\sigma_{p,t}$

In this section the values r_i and q_i for $i = 1, ..., n$ are again determined using the normal distribution. Where in the previous section the values r_i and q_i were determined using P^* and σ_p, in this section the values r_i and q_i are determined for each t seperately using information about the average price and standard deviation at that specific time, denoted by p_t^* and $\sigma_{p,t}$. The average price p_t^* is thus the average price at only time t, where P^* at time t represents the expected average price from time t until T. The same holds for $\sigma_{p,t}$ and σ_p. The price at time t is normally distributed, at each time t let $X \sim \mathcal{N}\left(p_t^*, \sigma_{p,t}\right)$. Then for each t, this is discretized to a random variable p_t, as explained in Section 5.1.3. Based on the values p_t^* and $\sigma_{p,t}$, n intervals are made and the value r_i in each interval $[x_{i-1}, x_i]$ for $i = 1, ..., n$ is determined such that Equation (5.1.1) holds. The values of q_i, for all $i = 1, ..., n$, are determined using Equation (5.1.2). These values of r_i and q_i are determined for each t seperately. The values of r_i and q_i can be different for each t. These values are therefore denoted by $r_{i,t}$ and $q_{i,t}$. The bidding curve at t is now given by

$$\begin{bmatrix} r_{1,t} & a_k(r_{1,t}; t, d) \\ \vdots & \vdots \\ p_t^* & a_k(p_t^*; t, d) \\ \vdots & \vdots \\ r_{n,t} & a_k(r_n; t, d) \end{bmatrix}$$

and can be determined in the same way as in Section 5.1.2 and 5.1.3, this will be explained in Section 5.3.

5.2 DP using known prices

This section shows how to find $a_k(r_i; t, d)$ for all r_i, t and d when all prices p_1, ..., p_T are known. Note that $n = 1$ when p_t is known, the values of $a_k(p_1; t, d)$, ..., $a_k(p_T; t, d)$ need to be calculated. The problem of calculating these values can be rewritten as a Knapsack problem, see [19], and this Knapsack problem can be solved using dynamic programming.

The problem of finding $a_k(p_1; t, d)$, ..., $a_k(p_T; t, d)$ is first transformed to the Knapsack problem.
There is a number of time epochs $t = 1, ..., T$ and each time has a corresponding price p_t. A total of W units of shiftable demand need to be satisfied at the end of time t and only a maximum of m_t units of demand can be satisfied at time t. The decision variable x_t is the amount of demand satisfied at time t. The problem is to minimize the total costs of satisfying W units of demand.

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{T} p_t \cdot a_t && (5.2.1) \\ \text{subject to} \quad & \sum_{t=1}^{T} a_t \geq W, \\ & x_t \in \{0, 1, ..., m_t\}, \ t = 1, ..., T. \end{aligned}$$

This Knapsack problem can be solved using dynamic programming.

How the problem is solved by dynamic programming is shown using a numerical example. In this example $T = 24$, $W = 8$, $m_t = 2$ and p_t for $t = 1$, ..., T is shown in the vector below.

$$\begin{aligned} p \quad = \quad & \begin{bmatrix} 5 & 5 & 4 & 4 & 3 & 4 & 4 & 5 & 5 & 6 & 6 & 6 & \ldots \end{bmatrix} \\ & \begin{bmatrix} \ldots & 5 & 4 & 4 & 5 & 5 & 6 & 6 & 7 & 6 & 5 & 5 & 4 \end{bmatrix}. \end{aligned}$$

Now a decision tree can be made. The tree corresponding to the numerical example is given in Figure 5.3. The nodes in the tree are of the form (t, d), where t specifies the time and d the remaining demand.

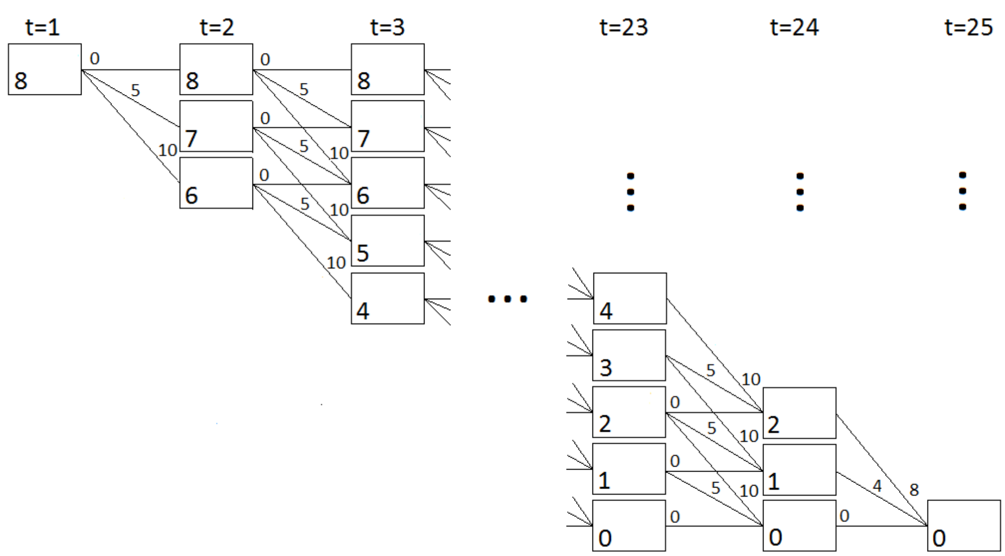


Figure 5.3: Tree with $T = 24$, $W = 8$ and $m_t = 2$

An edge is put between nodes (t, d) and $(t+1, d-x_t)$ if the value of a_t is less then m_t. The weight of an edge represents the costs at time t when a_t is satisfied at t.

$$e((t,d),(t+1,d-a_t)) \quad = \quad p_t \cdot a_t \qquad \forall t,\ d,\ a.$$

Each path in the tree from node (t,d) to $(T+1,0)$ now corresponds to a feasible solution for satisfying demand d in time epochs t, ..., T.

For solving problem (5.2.1) the minimum costs from node (t,d) to node $(T+1,0)$ is needed for each t and d, the minimum costs from node (t,d) to node $(T+1,0)$ is also called the value of node (t,d), denoted by $V_t(d)$. Especially, the value of node $(1,W)$ is interesting. The $V_1(W)$ shows the minimum costs for satisfying all demand over all times. Also, the minimum costs path from node $(1,W)$ to node $(T+1,0)$ shows the best action at each t. Before $V_1(W)$ can be determined, the $V_t(d)$ need to be determined for all t>1, starting with $V_{T+1}(d)$. The value of nodes $(T+1,d)$ for all d are given in equations 5.2.2 and 5.2.3. At time $T+1$ the remaining demand d needs to be exactly zero, $V_{T+1}(d)$ is therefore equal to zero when $d=0$ and set to infinity when the node is not reachable, i.e., $d \neq 0$.

$$V_{T+1}(d) \quad = \quad 0 \qquad d=0, \tag{5.2.2}$$
$$V_{T+1}(d) \quad = \quad \infty \qquad d \neq 0. \tag{5.2.3}$$

For all other t, the value of node (t,d) is given by equation 5.2.4. In $V_t(d)$, $p_t \cdot a_t$ is the costs for using a_t at t and $V_{t+1}(d-a_t)$ is the costs for using $d-a_t$ from $t+1$ to T.

$$V_t(d) \quad = \quad min_{a_t}\left[p_t \cdot a_t + V_{t+1}(d-a_t)\right] \qquad t=T,...,1. \tag{5.2.4}$$

Working backwards from T to 1, $V_t(d)$ and corresponding a_t can be determined for all nodes (t,d), including the desired $V_1(W)$. At time T all remaining demand d needs to be satisfied. There is therefore only one action: satisfy all d. The values of the nodes at time $t=24$ in the numerical example are given below. Because $m_t = 2$, the nodes $(24,8)$, ..., $(24,3)$ can not reach $(25,0)$ and the values of these nodes are therefore set to infinity.

$$\begin{aligned}
V_{24}(8) &= \infty, \\
\vdots \quad & \quad \vdots \\
V_{24}(3) &= \infty, \\
V_{24}(2) &= p_{24} \cdot 2 + V_{25}(0) \\
&= 8, \\
V_{24}(1) &= p_{24} \cdot 1 + V_{25}(0) \\
&= 4, \\
V_{24}(0) &= p_{24} \cdot 0 + V_{25}(0) \\
&= 0.
\end{aligned}$$

At time $t \leq T-1$ there are more choices. The values of nodes $V_{23}(8)$, ..., $V_{23}(0)$ in the numerical example are given next.

$$\begin{aligned}
V_{23}(8) &= \infty, \\
\vdots \quad & \quad \vdots \\
V_{23}(5) &= \infty, \\
V_{23}(4) &= p_{23} \cdot 2 + V_{24}(2) \\
&= 10 + 8 = 18,
\end{aligned}$$

$$\begin{aligned}
V_{23}(3) &= min\left[p_{23}\cdot 2+V_{24}(1), p_{23}\cdot 1+V_{24}(2)\right] \\
&= min\left[10+4, 5+8\right] = 13, \\
V_{23}(2) &= min\left[p_{23}\cdot 2+V_{24}(0), p_{23}\cdot 1+V_{24}(1), p_{23}\cdot 0+V_{24}(2)\right] \\
&= min\left[10+0, 5+4, 0+8\right] = 8, \\
V_{23}(1) &= min\left[p_{23}\cdot 1+V_{24}(0), p_{23}\cdot 0+V_{24}(1)\right] \\
&= min\left[5+0, 0+4\right] = 4, \\
V_{23}(0) &= p_{23}\cdot 0+v_{24}(0) \\
&= 0+0=0.
\end{aligned}$$

The values of all other nodes are determined in the same way. The values of the nodes are given in the upper right corner of the nodes in Figure 5.4. It is assumed that if the value of a certain node is obtained by multiple a_t the highest value of a_t is chosen. The edge (value of a_t) to choose at each of the nodes is shown in the figure as a wider edge. The wider edge is coloured red when the edge is part of the minimum costs path from node $(1, W)$ to node $(T+1, 0)$, the path that is most interesting.

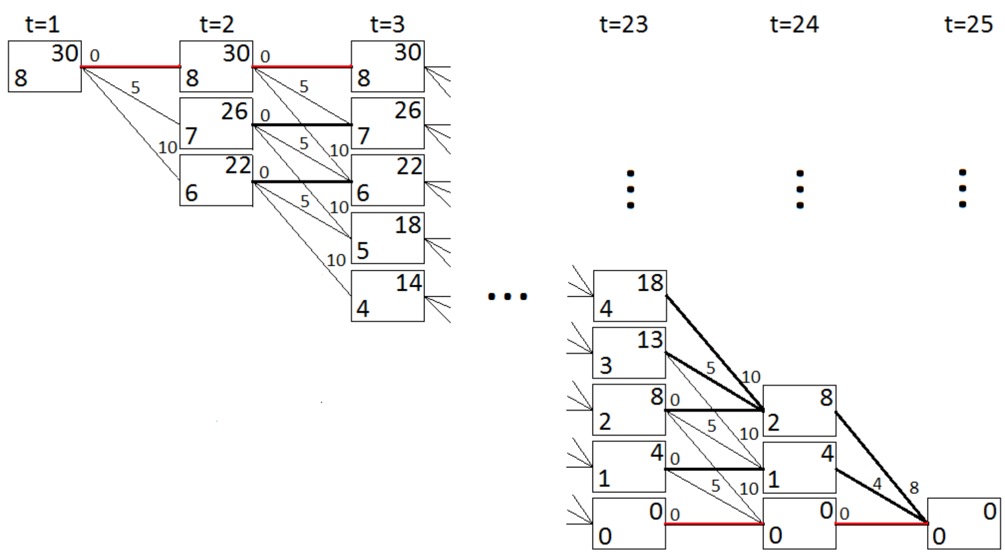


Figure 5.4: Node values and best actions with $T = 24$, $W = 8$ and $m_t = 2$

Now, at $t = 1$ the value of a_t is known for each t. With these values of a_t known, the bidding curve can already be made for each t. The bidding curve at time t with remaining demand d is given by:

$$\begin{bmatrix} p_t & a_k(p_t; t, d) \end{bmatrix}.$$

Finding the value of a_t and the bidding curve at time t becomes more difficult when the prices are forecasted instead of known. Section 5.3 shows how these values are determined when price distributions are used.

5.3 DP using price distributions

This section shows how to find $a_k(r_i; t, d)$ for all r_i, t and d with a given distribution of the prices. Section 5.1 determined r_1, ..., r_n and q_1, ..., q_n, used as input in this section. How the values of $a_k(r_i; t, d)$ for all r_i, t and d are determined using dynamic programming is shown using a numerical example. In this example $T = 24$, $W = 8$, $m_t = 2$, $r_1 = 4$, $r_2 = 5$, $r_3 = 6$ and $q_i = \frac{1}{3}$

for $i = 1, 2, 3$. With this information a decision tree can be made. The tree corresponding to the given example is given in Figure 5.5. This tree consists of two kinds of nodes and has therefore a different form than the tree given in Figure 5.3. At each t, first p_t is determined, this is done at the chance node (shown in the figure as a circle). The p_t can take on values r_1, ..., r_n. With p_t known, the decision about the value of a_t can be made, this is done at the decision node (rectangular node). The rectangular nodes are of the form (t, d) and show the current state, $[d, p_t]$. Edges from a chance node to a decision node show the value of r_i and the probability q_i of r_i. The weight of the edges from decision nodes to chance nodes represent the costs at time t, when a_t is satisfied.

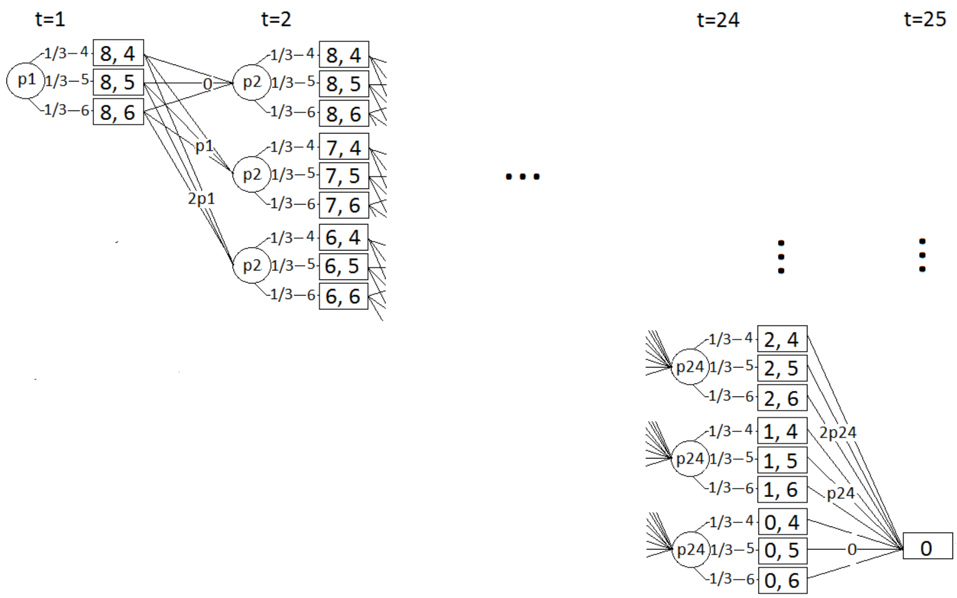


Figure 5.5: Tree with $T = 24$, $W = 8$ and $m_t = 2$

Each path in the tree now corresponds to a feasible solution for satisfying demand d in times t, ..., T.

The total costs need to be minimized, equations 5.2.2 till 5.2.4 can't be used for this, the p_{t+1}, ..., p_T are unknown at time t and therefore $V_{t+1}(d - a_t)$ is also unknown. The price distribution however is known and therefore the expected values of the nodes can be calculated. The expected value of node (t, d), $EV_t(d)$, shows the expected minimum costs for satisfying d at times $t + 1$, ..., T with a given price distribution. At time $T + 1$ the remaining demand d needs to be exactly zero, $EV_{T+1}(d)$ is therefore equal to zero when $d = 0$ and set to infinity when the node is not reachable, i.e., $d \neq 0$.

$$EV_{T+1}(d) = 0 \quad d = 0, \tag{5.3.1}$$
$$EV_{T+1}(d) = \infty \quad d \neq 0. \tag{5.3.2}$$

For all other t, the expected value of node (t, d) is given by equation 5.3.3. In $EV_t(d)$, $r_i \cdot a_t$ is the costs for using a_t at t with price r_i, $EV_{t+1}(d - a_t)$ gives the expected costs for using $d - a_t$ from time $t + 1$ to T.

$$EV_t(d) = \sum_i q_i \; min_{a_t} \; [r_i \cdot a_t + EV_{t+1}(d - a_t)] \quad t = T, ..., 1. \tag{5.3.3}$$

Working backwards, the $EV_t(d)$ of all nodes can be calculated. In the numerical example the expected value at $t = 1$ with $d = 8$ becomes:

$$
\begin{aligned}
EV_1(8) &= q_1 \cdot min\ [0 + EV_2(8), r_1 + EV_2(7), 2 \cdot r_1 + EV_2(6)] \\
&\quad + q_2 \cdot min\ [0 + EV_2(8), r_2 + EV_2(7), 2 \cdot r_2 + EV_2(6)] \\
&\quad + q_3 \cdot min\ [0 + EV_2(8), r_3 + EV_2(7), 2 \cdot r_3 + EV_2(6)] \\
\\
&= \frac{1}{3} \cdot min\ [0 + 32,10; 4 + 28,08; 8 + 24,02] \\
&\quad + \frac{1}{3} \cdot min\ [0 + 32,10; 5 + 28,08; 10 + 24,02] \\
&\quad + \frac{1}{3} \cdot min\ [0 + 32,10; 6 + 28,08; 12 + 24,02] \\
&= \frac{1}{3} \cdot 32,02 + \frac{1}{3} \cdot 32,10 + \frac{1}{3} \cdot 32,10 \\
&= 32,07.
\end{aligned}
$$

With $EV_t(d)$ known for each node (t, d), the value of a_t can be determined for each t, d and r_i. This value is determined using $V_t(d, r_i)$, the minimum costs at node (t, d) with p_t equal to r_i and p_{t+1}, ..., p_T unknown. The minimum costs at time $T + 1$ are set equal to zero when $d = 0$ and set to infinity when $d \neq 0$.

$$
\begin{aligned}
V_{T+1}(d, r_i) &= 0 \ \ d = 0, & (5.3.4) \\
V_{T+1}(d, r_i) &= \infty \ \ d \neq 0. & (5.3.5)
\end{aligned}
$$

For all other t, the $V_t(d, r_i)$ is given by the following equation:

$$
V_t(d, r_i) = min_{a_t}\ [r_i \cdot a_t + EV_{t+1}(d - a_t)] \ \ t = T, ..., 1. \tag{5.3.6}
$$

At $t = 1$ and $p_1 = 4$ in the numerical example:

$$
\begin{aligned}
V_1(8, 4) &= min\ [0 + EV_2(8), 4 + EV_2(7), 8 + EV_2(6)] \\
&= min\ [32,10; 32,08, 32,02] \\
&= 32,02.
\end{aligned}
$$

The values of all other nodes are determined in the same way. The values of the nodes are given in the nodes of Figure 5.6, together with a_t. The edge corresponding to a_t is shown in the figures as a wider edge. It is again assumed that if the value of a certain node is obtained by multiple a_t the highest value of a_t is chosen. At time t, the best action a_t for each possible price r_i is now known and the bidding curve at time t with remaining demand d can be made.

$$
\begin{bmatrix}
r_1 & a_k(r_1; t, d) \\
\vdots & \vdots \\
r_n & a_k(r_n; t, d)
\end{bmatrix}.
$$

The bidding curve at future t is not yet known. The values of r_i and q_i for all i can change at each t, this could lead to different values of a_t and a different bidding curve.

5.4 Extension to different appliances

In the previous section an electric car has to be charged before the end of time T. A tree is made and the best action for a certain t and d is looked for. This $a_k(r_i; t, d)$ for all r_i can has a value between 0 and m_t and is independent of $a_k(r_i; t - 1, d)$. Not all appliances work like this. For each different kind of appliance, a different kind of tree is needed.

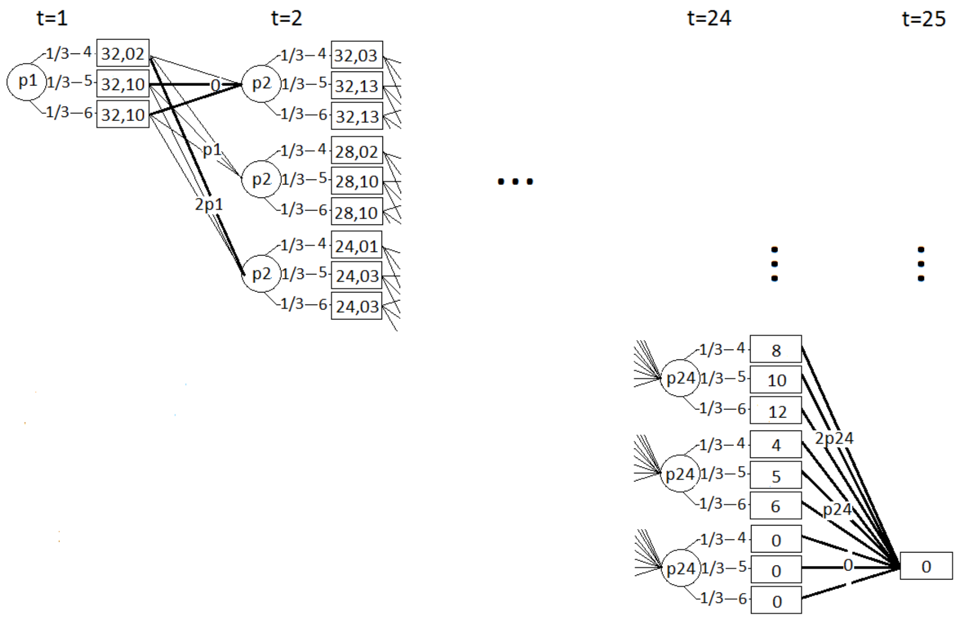


Figure 5.6: Node values with $T = 24$, $W = 8$ and $m_t = 2$

Here, the tree for a washing machine will be given. A washing machine has to be turned on for multiple consecutive time epochs. When at time t the washing machine is turned on, it has to be on for the next few time epochs, i.e., the length of the washing program. Also, the washing machine is either on or off, there is no such thing as half on. So:

$$\begin{aligned} m_t &= 1, \\ W &= \text{length washing program.} \end{aligned}$$

The decision needs to be made when to start the washing program. The tree corresponding to this problem is given in Figure 5.7 for the numerical example where $T = 24$, $W = 3$, $m_t = 1$, $r_1 = 4$, $r_2 = 5$, $r_3 = 6$ and $q_i = \frac{1}{3}$ for $i = 1, 2, 3$. When the remaining demand d equals W there are two choices: start the washing program or wait for at least one time epoch. When $0<d<W$ there is only one choice: keep the washing machine on. When $d = 0$ the washing program is finished.

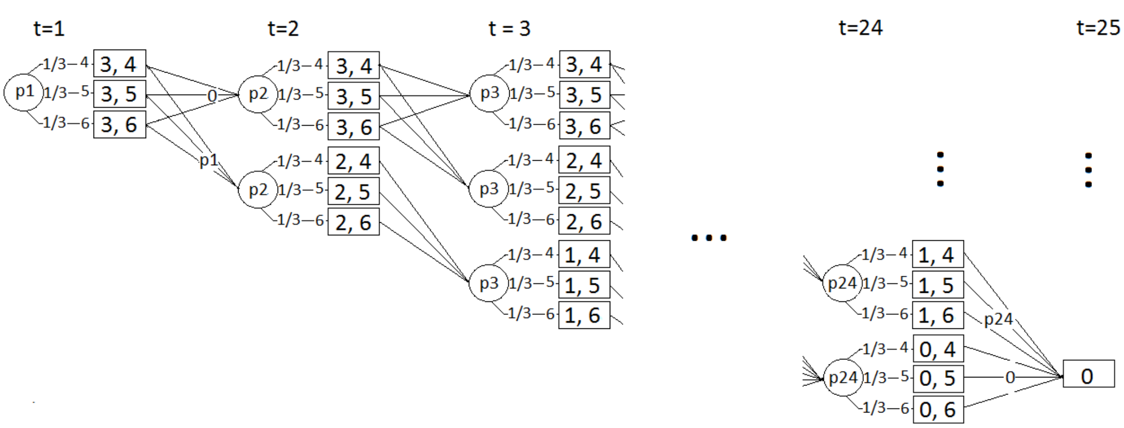


Figure 5.7: Tree for a washing machine with $T = 24$, $W = 3$ and $m_t = 1$

The tree for a battery is also slightly different from the tree for an electric vehicle. A battery can be charged and discharged, the best action at each moment can therefore be a positive or negative value.

5.5 Updating prices

It is assumed in this report that an electric car needs to be charged before the end of time T. Section 5.2 shows how the bidding curve is build when the prices p_t for all t are known in advance. With p_t known for $t = 1, ..., T$, the $a_t(p_t; t, d)$ is also known for all t and the bidding curves can be build for $t = 1, .., T$ in advance. The tree in Section 5.2 needs to be build only once. The situation is different in Section 5.3, in this section the forecasted prices are updated each time. At time t, only the bidding curve for time t can be made. At time $t+1$, a new tree is build, using the updated forecasts, and $a_{t+1}(r_i; t, d)$ is determined to make the bidding curve for time $t+1$.

Starting at time $t = 1$, there are T decision moments. Assuming that the prices are forecasted, the tree changes at each of the decision moments. At time t, decision moments $1, ..., t-1$ are not used and can be removed, the tree then only consists of decision moments $t, ..., T$. So, when t gets closer to T the tree shrinks, see the top part of Figure 5.8.
As said before, the assumption is made that an electric car needs to be charged before some end time. When no end time exists or when the end time exceeds the length of the interval over which prices are forecasted, the tree doesn't shrink. In these cases there is a moving horizon, see the bottom part of Figure 5.8. In this situation also a new tree has to be build when p_t is known for $t = 1, ..., T$.

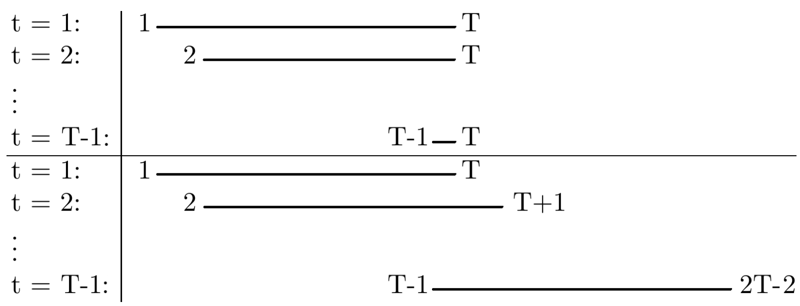


Figure 5.8: Top: Fixed horizon. Bottom: Rolling horizon.

5.6 Conclusion

Agents have to send bidding curves to higher agents. Two steps are needed to determine the bidding curve. In the first step the possible values of the price, r_1, ..., r_n, and the corresponding probabilities q_1, ..., q_n, need to be determined. These values can be determined for different levels of information about p_t, how this is done is shown in Section 5.1. In the second step the values of $a_k(r_i; t, d)$ are determined. Sections 5.2 and 5.3 show how these values are determined when r_1, ..., r_n and q_1, ..., q_n are already determined. It is in these sections important to know the kind of appliance for which $a_k(r_i; t, d)$ is determined. Appliances all work differently, this difference leads to slight changes in the methods of Section 5.2 and Section 5.3.

This new bidding strategy can be used in the two-time-scale PowerMatcher. Chapter 6 analyzes the performance of the two-time-scale PowerMatcher including this new bidding strategy. The new bidding strategy will in Chapter 6 be compared to the currently used bidding strategy in the two-time-scale PowerMatcher.

6 Results

Chapter 3 described the standard PowerMatcher and Chapter 4 the two-time-scale PowerMatcher, in the two-time-scale PowerMatcher planning can be included. This inclusion is supposed to lead to lower total costs for satisfying demand. Chapter 5 introduces some new bidding strategies for the two-time-scale PowerMatcher. These new bidding strategies are intended to further reduce the total costs. A few scenarios are used to see how the different bidding strategies perform. The outcomes of the different strategies are compared to each other to see how and when the new bidding strategies perform better than the currently used bidding strategies.

Section 6.1 shows the advantage of using planning in the bidding strategies. The bidding strategy used in the two-time-scale PowerMatcher is compared to two strategies where no information about future prices is known, these strategies satisfy demand either as fast or as cheaply as possible. The currently used bidding strategy in the two-time-scale PowerMatcher can possibly be improved. New bidding strategies are introduced in Chapter 5, Section 6.2 shows how the bidding curves are determined using these new bidding strategies. The different bidding strategies are compared to each other in Section 6.3. This is done for the case where the available information is perfect and for the case where the available information are forecasts that change over time. Also the computation time of the different strategies are compared to each other in this section.

6.1 Advantage two-time-scale PowerMatcher over 'simple' strategies

In this section the different strategies that are given in Section 4.4 are compared to each other. This is done for two different situations, the situation where the wind production is known and the situation where wind forecasts are worked with.

Wind production known

Using the example network given in Figure 4.1, the three strategies are compared to each other in the situation where the fixed demand and wind demand over the next 24 hours are known. Strategy 1 satisfies the three units of shiftable demand as fast as possible. Strategy 2 tries to satisfy the energy demand using only wind energy, this with the knowledge of the current fixed demand and the wind energy available. The third strategy also tries to satisfy the shiftable demand as cheaply as possible, but in this strategy the fixed demand and wind demand over the next 24 hours are used.

The average total costs for satisfying the fixed and shiftable demand of the 100 instances are given in Table 6.1 for each of the strategies.

	Total costs with known WD
Strategy 1	167,98
Strategy 2	138,75
Strategy 3	138,06

Table 6.1: Total costs for the strategies with wind demand known

Strategy 1 leads to the highest costs. This strategy satisfies the shiftable demand as fast as possible. In this way an unnecessary large amount of energy from flexible energy resources is used. This kind of energy has high costs because it uses finite resources, the use of these energy resources by strategy 1 explains the high costs of this strategy. The costs of strategies 2 and 3 are much lower and close to each other. With strategy 2, the car is charged as soon as wind energy is available. No distinction is made between the two kinds of wind energy. With this strategy it is possible that parts of the cheap wind energy is not used while the more expensive wind energy is used to satisfy shiftable demand. Strategy 3 first uses the cheapest energy to satisfy the shiftable demand. When part of the shiftable demand still needs to be satisfied, the more expensive wind energy is used. The real expensive energy from flexible energy resources is only used for satisfying demand when when there is not enough wind energy available to satisfy all shiftable demand.

The total costs are not the only output. Each strategy divides the shiftable demand over the times in a different way. For the instance given in Figure 4.2, the used demand at each time is shown in figures 6.1 til 6.3 for each of the strategies. The figures show three differently coloured bars.
Blue bar: This bar given the summation of the fixed and wind demand, FD + WD. A positive value means that energy from flexible energy resources has to be bought while a negative value shows the amount of energy that is left. This amount can for example be used to satisfy shiftable demand.
Green bar: The green bar shows the amount of shiftable demand used at each time. At the first 9 times, the green bar will be zero. This because the shiftable demand is only shiftable between times 10 and 24.
Red bar: The red bar shows the summations of the above mentioned bars. Thus the summation of the fixed demand, wind demand and shiftable demand. The excess or shortage of demand can be seen from this bar. Positive values show a shortage of wind energy. In this case energy from flexible energy resources has to be bought. Negative values show an excess of energy, the supply is greater than the demand. In this case energy is thrown away. In the ideal strategy, the amount of discarded energy is minimal.

Figure 6.1 shows the demand at each t using strategy 1. Strategy 1 satisfies the demand as fast as possible, where the shiftable demand can be satisfied from time $t = 10$ onward. The green bar in the figure, which shows the shiftable demand, is zero till $t = 10$. From time 10 onward, the shiftable demand is satisfied as fast as possible. The shiftable demand is fully satisfied at the beginning of time $t = 16$. During the period where shiftable demand is satisfied, the shiftable demand is higher than the available wind demand for shiftable demand, the blue bar. Energy from flexible energy resources is needed, which leads to the high costs given in Table 6.1.

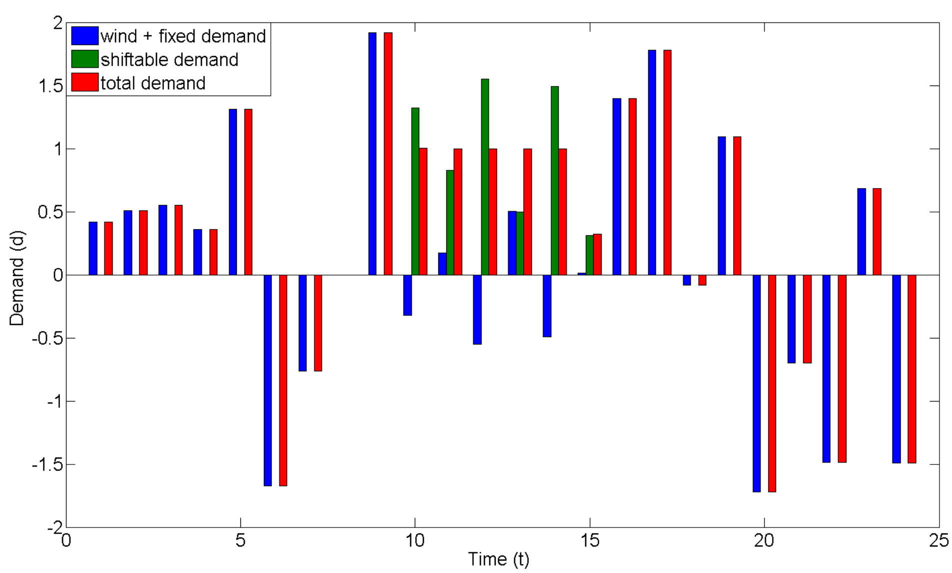


Figure 6.1: Demand of strategy 1 with wind production known

Figure 6.2 shows the different demands using strategy 2. The shiftable demand is satisfied when wind energy is available, the green bar and blue bar cancel each other out. No distinction is made between the cheaper and more expensive wind energy. Part of the cheap wind energy remains unused, this method is therefore not optimal. Strategy 3 does take the cheap and expensive wind energy into account. Figure 6.3 shows the different demands using strategy 3. The shiftable demand is satisfied in a slightly different way.

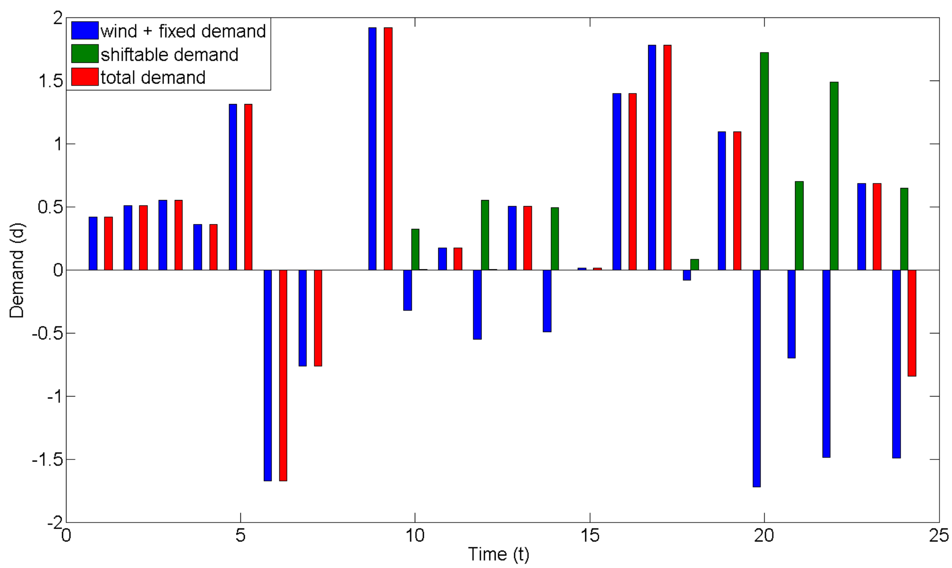


Figure 6.2: Demand of strategy 2 with wind production known

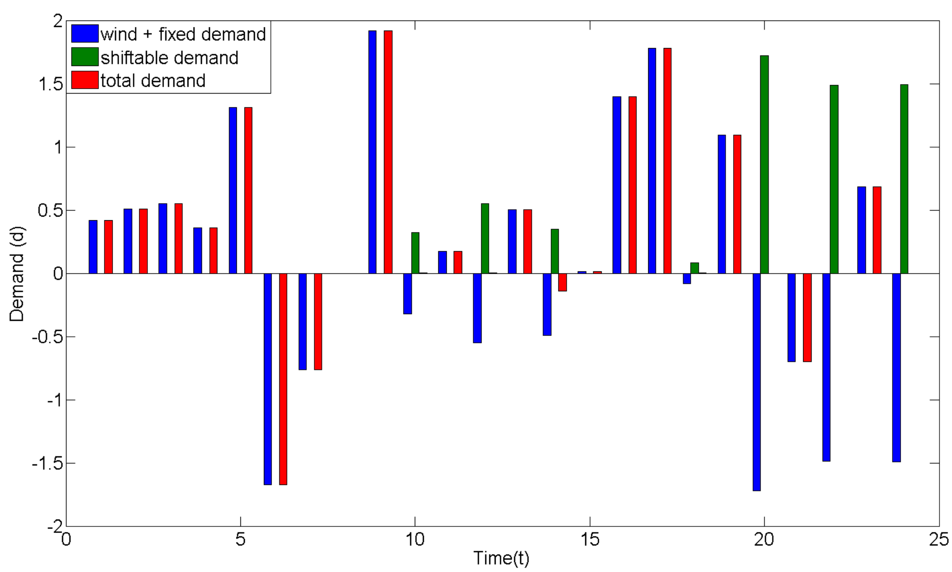


Figure 6.3: Demand of strategy 3 with wind production known

The difference between strategy 2 and 3 is small and difficult to see from the figures. The difference becomes more apparent with Table 6.2, where the satisfied shiftable demand per time unit is given for strategies 2 and 3. The satisfied shiftable demand of strategies 2 and 3 differ at three times, $t = 14$, $t = 21$ and $t = 24$. At times $t = 14$ and $t = 21$ one unit of wind energy is available (at a cost of 3 per unit) while at time $t = 24$ two units are available (at a cost of 1 per unit). Compared to strategy 3, with strategy 2 more shiftable demand is satisfied at $t = 14$ and $t = 21$ and less at $t = 24$. This difference leads to the different total costs.

t = 10	t = 11	t = 12	t = 13
0,3192	0	0,5524	0
t = 14	t = 15	t = 16	t = 17
0,4898	0	0	0
t = 18	t = 19	t = 20	t = 21
0,0814	0	1,7228	0,7014
t = 22	t = 23	t = 24	
1,4850	0	0,6480	

t = 10	t = 11	t = 12	t = 13
0,3192	0	0,5524	0
t = 14	t = 15	t = 16	t = 17
0,3478	0	0	0
t = 18	t = 19	t = 20	t = 21
0,0814	0	1,7228	0
t = 22	t = 23	t = 24	
1,4850	0	1,4914	

Table 6.2: Used shiftable demand for each time (t) greater or equal to 10 for the second strategy (left) and third strategy (right)

Wind forecasts

The strategies will now be compared to each other in the situation where only forecasts about future wind energy are available. This is done for 100 instances in the example network, given in Figure 4.1. The average total costs for satisfying all demand are given in Table 6.3 for each of the strategies.

	Total costs with forecasted WD
Strategy 1	138,26
Strategy 2	114,26
Strategy 3(1)	114,57
Strategy 3(24)	115,50

Table 6.3: Total costs of the strategies with the forecasted wind energy demand

Using the first strategy leads to high costs, compared to the other strategies. An unnecessary amount of energy from flexible energy resources is used. The costs of the other three strategies are close to each other. Strategy 2 leads to the lowest costs, this strategy satisfies shiftable demand as soon as wind energy is available. Strategy 3(1) and 3(24) can lead to higher costs compared to strategy 2 when more wind energy is forecasted than actually available. More energy from flexible energy resources is then needed.

The total costs are not the only output. The strategies divide the shiftable demand differently over the times. Again, strategy 1 doesn't take the future into account, at a certain time only the fixed demand and wind demand at that time are known. Changes in the available wind demand will therefore not look like changes. Figure 6.4 shows the fixed+wind demand and the shiftable demand used at each time. The darker bars show the different demand in the situation where the vector 'WD' changes a bit at each time t. The lighter bars show the different demands in the situation where 'WD' does not change over time. As can be seen from Figure 6.4, the shiftable demand is the same for both situations, this because the fixed demand is the same in both situations. Changes in the wind demand vector do not have any effect on the used shiftable demand at each t.

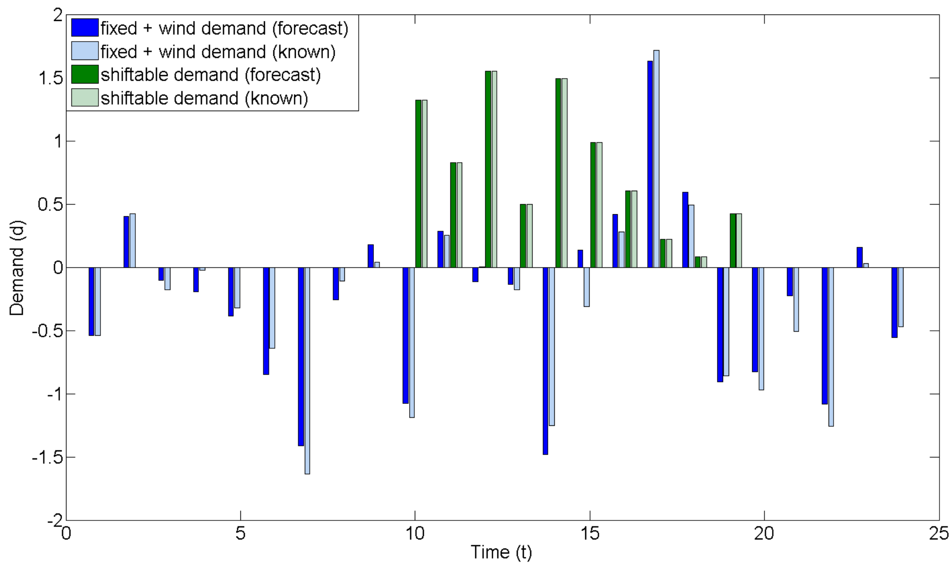


Figure 6.4: Demands of strategy 1 with known and forecasted wind production

Strategy 2 handles changes in the wind demand vector in the same way as strategy 1. Only the fixed and wind demand at a specific time are known and therefore changes in the demand vectors do not seem like changes for strategy 2. The different demands at each t using strategy 2 are given in Figure 6.5. Changes in the wind demand vector does lead to changes in the shiftable demand distribution using this strategy. Strategy 2 satisfies part of the shiftable demand when wind energy is available. An increase in available wind demand leads to a larger amount of satisfied shiftable demand.

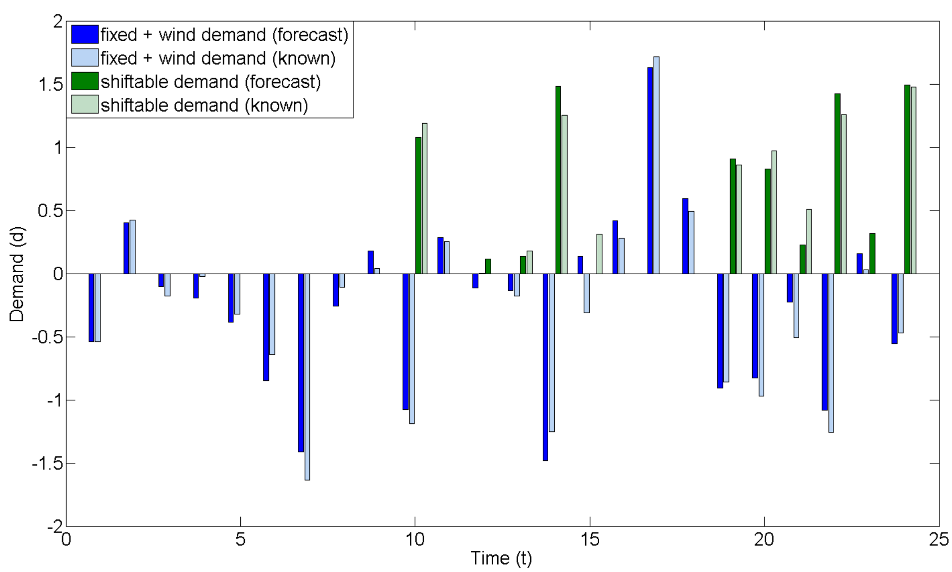


Figure 6.5: Demands of strategy 2 with known and forecasted wind production

Strategy 3(1) does take the future into account, the average price, P^*, is determined at the beginning of the time horizon. Because of changes in the wind demand forecasts, the actual average price also changes. The difference between the determined average price and the actual average price may lead to a non optimal distribution of the shiftable demand. The different demands at each t are given in Figure 6.6. The distribution of the shiftable demand over T does not differ much from strategy 2, the costs of these two strategies are also almost the same, see Table 6.3.

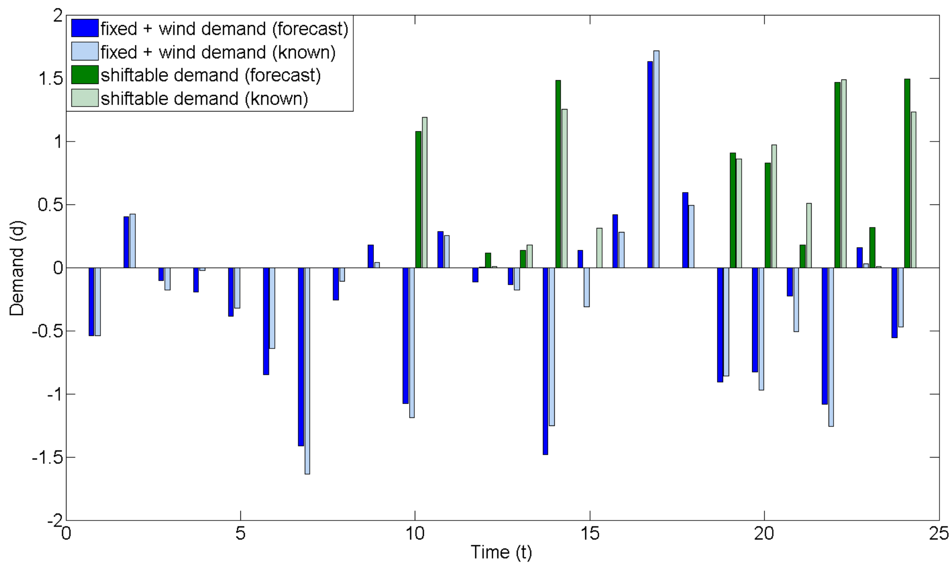


Figure 6.6: Demands of strategy 3(1) with known and forecasted wind production

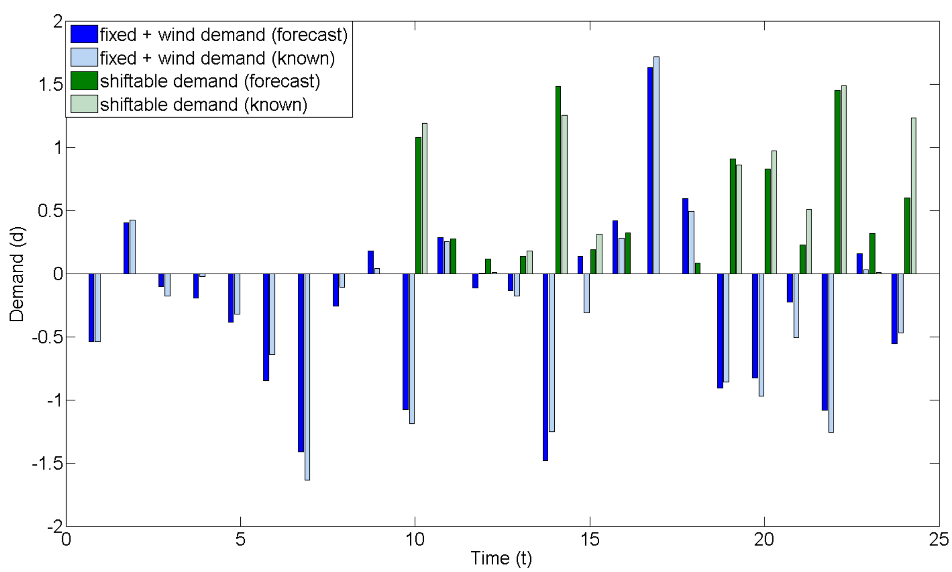


Figure 6.7: Demands of strategy 3(24) with known and forecasted wind production

In strategy 3(24) the average price, P^*, is determined at each time t. Figure 6.7 shows the different demands at each t using strategy 3(24). When the wind demand vector stays the same, strategy 3(24) satisfies the shiftable demand in the same way as strategy 3(1). When the wind demand vector changes, strategies 3(1) and 3(24) differ from each other.

Using the given example network and instances, strategy 2 performs better than strategy 3 and strategy 3(1) performs better than strategy 3(24). It turns out that strategy 3 does not handle changes in the wind demand well. Taking the forecasts into account can lead to lower costs but the forecasts must be used properly.

6.2 Bidding strategies using dynamic programming

Chapter 5 showed how bidding strategies are determined using dynamic programming. Two steps are needed to determine the bidding curve.

1. The price distribution, r_1, ..., r_n and q_1, ..., q_n, is determined.
2. Using dynamic programming, the values of $a_k(r_i; t, d)$ are calculated.

In this section an example is used to show how the bidding curves follow from these two steps. In the example there is only shiftable demand. Futhermore $T = 24$, $W = 8$, $m_t = 2$ and $n = 5$, so there are 8 units of shiftable demand that have to be satisfied at the end of time 24, the maximum satisfied amount per time period is equal to 2 and the number of different r_i and q_i is equal to 5. The available information about future prices is different for each bidding strategy. This information will be given at the beginning of each section.

6.2.1 Known P^*

In this section it is known that $P^* = 5$, no other information about the prices is known. With $n = 5$, five values of r_i have to be handpicked around P^*. The stepsize by which this will be done is set equal to one, $s = 1$.

First, the r_1, ..., r_n and q_1, ..., q_n have to be determined. The values of r_i are picked around P^* with $s = 1$. The corresponding probabilities ar equal to $\frac{1}{n}$, $q_i = \frac{1}{n}$.

$$\begin{aligned} r_1 &= 3 \qquad & q_1 &= \frac{1}{5}, \\ r_2 &= 4 \qquad & q_2 &= \frac{1}{5}, \\ r_3 &= 5 \qquad & q_3 &= \frac{1}{5}, \\ r_4 &= 6 \qquad & q_4 &= \frac{1}{5}, \\ r_5 &= 7 \qquad & q_5 &= \frac{1}{5}. \end{aligned}$$

These values of r_i and q_i for $i = 1, ..., n$ are used in step 2.

In step 2 the values of $a_k(r_i; t, d)$ are calculated for each r_i, t and d. The dynamic programming method used to do this is explained in Section 5.3. At r_1 and r_5 the best actions are simple, respectively $a_k(r_1; t, d)$ as high as possible and $a_k(r_5; t, d) = 0$. The actions for r_2, r_3 and r_4 are more interesting and are shown in Table 9.2. The tables show the best action at each of the nodes (t, d) for r_2, r_3 and r_4. At r_4 the actions look a lot like the ones at r_5. The only difference is at $t = 20$, When the price equals r_4 some demand is used when d is high while no demand is used when the price equals r_5. In general, $a_k(r_i; t, d)$ is higher when i decreases. At the beginning, when t is low, $a_k(r_i; t, d) = 0$ unless $i = 1$, the other possible values of p look too expensive. When

t gets closer to T, other r_i don't look so expensive anymore and $a_k(r_i;t,d) \neq 0$. The t at which this change happens is different for each i and is earlier when i is lower.

With the values of r_i and $a_k(r_i;t,d)$ known, the bidding curve can be made. At $t = 17$ and $d = 7$ the bidding curve is given by:

$$\begin{bmatrix} 3 & 2 \\ 4 & 2 \\ 5 & 1 \\ 6 & 0 \\ 7 & 0 \end{bmatrix}.$$

Using the values of r_i and q_i for all future time periods leads to different values of $a_k(r_i;t,d)$ and different total costs than when future prices are set equal to P^* with probability one. How this difference leads to different costs is shown next. Also, different distributions q_i lead to different bidding curves. This is also shortly looked at, at the end of this section.

Difference with $p = P^*$ and $q = 1$ for future t

The strategy explained in Section 5.3 uses $EV_t(d)$, shown in equations (5.3.1) till (5.3.3). The expected price, $EV_t(d)$, is used to determine $V_t(d)$. This $V_t(d)$ value and corresponding a value is different when future prices p_{t+1}, ..., p_T are set equal to P^*. The difference is shown here.

In the situation where $p_{t+1} = P^*$, ..., $p_T = P^*$, the $EV_t(d)$ becomes:

$$EV_t(d) \quad = \quad \sum_i q_i \, min_{0 \le x_t \le m_t} \ \{r_i \cdot x + P^* \cdot (d - x_t)\} \ \forall t \le T.$$

The future demand $(d - a_t)$ is satisfied with unit cost P^*.

The $EV_t(d)$ formula used in Section 5.3 is given in equation (5.3.3). In this bidding strategy, $d - a_t$ is satisfied using $EV_{t+1}(d - a_t)$. The value of a will be higher when $p \le P^*$ than when $p \ge P^*$. The average price at which d is satisfied in Section 5.3 will therefore be lower than P^*.

The different average price for which the demand is satisfied leads to different actions and different total costs.

Different distributions

Changing the distribution of the values r_i also changes the bidding strategy. With the first distribution given above, below two different distributions are given, these distribution are still symmetric around the mean.
Distribution 2:

$$\begin{aligned} r_1 &= 3 & q_1 &= \frac{1}{6}, \\ r_2 &= 4 & q_2 &= \frac{1}{6}, \\ r_3 &= 5 & q_3 &= \frac{1}{3}, \\ r_4 &= 6 & q_4 &= \frac{1}{6}, \\ r_5 &= 7 & q_5 &= \frac{1}{6}. \end{aligned}$$

Distribution 3:

$$\begin{aligned} r_1 &= 3 & q_1 &= \frac{1}{12}, \\ r_2 &= 4 & q_2 &= \frac{1}{6}, \\ r_3 &= 5 & q_3 &= \frac{1}{2}, \\ r_4 &= 6 & q_4 &= \frac{1}{6}, \\ r_5 &= 7 & q_5 &= \frac{1}{12}. \end{aligned}$$

The values of $a_k(r_i; t, d)$ corresponding to the two distributions are given in tables 9.3 and 9.4 for r_2, r_3 and r_4. For r_1 and r_5, $a_k(r_i; t, d)$ is already known. From the tables can be seen that the values of $a_k(r_3; t, d)$ do not change when the distribution changes, the actions at $r_2 = 4$ and $r_4 = 6$ do change. Because the chance at a very low price decreases with each distribution, $r_2 = 4$ is seen more and more as a low value of p. This leads to different values of $a_k(r_i; t, d)$. With the distribution change, more demand is used at earlier stages. At r_4 the reverse reasoning holds. In the third situation, $a_k(r_4; t, d) = a_k(r_5; t, d)$ for all t, d.

6.2.2 Known P^* and σ_p

In this section it is known that $P^* = 5$ and $\sigma_p = \frac{1}{2}$. The standard deviation is the extra information, compared to Section 6.2.1.

First, the r_1, ..., r_n and q_1, ..., q_n have to be determined. To do this, n intervals have to be made. The total interval is $[P^* - 3\sigma_p, P^* + 3\sigma_p]$, so the length of the total interval is $6\sigma_p$. The n smaller intervals have a length of $\frac{6}{n}\sigma_p = \frac{6}{5}\sigma_p$:

$$\begin{aligned} &\text{interval 1:} && \left[P^* - 3\sigma, P^* - \frac{9}{5}\sigma\right], \\ &\text{interval 2:} && \left[P^* - \frac{9}{5}\sigma, P^* - \frac{3}{5}\sigma\right], \\ &\text{interval 3:} && \left[P^* - \frac{3}{5}\sigma, P^* + \frac{3}{5}\sigma\right], \\ &\text{interval 4:} && \left[P^* + \frac{3}{5}\sigma, P^* + \frac{9}{5}\sigma\right], \\ &\text{interval 5:} && \left[P^* + \frac{9}{5}\sigma, P^* + 3\sigma\right]. \end{aligned}$$

To cover all possible values of p, the first and last interval are modified to:

$$\begin{aligned} &\text{interval 1:} && \left[-\infty, P^* - \frac{9}{5}\sigma\right], \\ &\text{interval 5:} && \left[P^* + \frac{9}{5}\sigma, \infty\right]. \end{aligned}$$

This is done to cover all possible values of p. The values of r_i and q_i for $i = 1, ..., 5$ can now be determined using these intervals, how this is done is explained in Section 5.1.3. The values of r_i

and q_i in this example are:

$$\begin{aligned} r_1 &= 3,9511 & q_1 &= 0,0359,\\ r_2 &= 4,4926 & q_2 &= 0,2383,\\ r_3 &= 5,0000 & q_3 &= 0,4515,\\ r_4 &= 5,5074 & q_4 &= 0,2383,\\ r_5 &= 6,0489 & q_5 &= 0,0359. \end{aligned}$$

The value of P^* can change at each t and therefore also the values of r_i and q_i can change at each t.

With the known values of r_i and q_i, the corresponding values $a_k(r_i;t,d)$ for all r_i, t, d can be determined using dynamic programming, see Section 5.3. With $a_k(r_i;t,d)$ known, the bidding strategy can be made. This is done in the same way as in Section 6.2.1.

6.2.3 Known P_t^* and $\sigma_{p,t}$

In this section a lot of information is known about forecasted future prices. For each future t, the average price P_t^* and the standard deviation $\sigma_{p,t}$ are known. In this example the values of P_t^* and $\sigma_{p,t}$ are:

$$\begin{aligned} P^* &= [5\ 5\ 4\ 5\ 5\ 6\ 5\ 5\ 4\ 4\ 5\ 5\ 6\ 6\ 5\ 5\ 4\ 4\ 4\ 5\ 5\ 6\ 6\ 6]^T,\\ \sigma_{p,t} &= \frac{1}{2} \quad \forall t. \end{aligned}$$

At time t, n intervals are made for each future t. When these intervals are known, the values of $r_{i,t}$ and $q_{i,t}$ can be determined for each t. At time $t = 1$ the values of $r_{i,t}$ and $q_{i,t}$ are

$$\begin{aligned} r_1 &= 3,9511 & q_1 &= 0,0359,\\ r_2 &= 4,4926 & q_2 &= 0,2383,\\ r_3 &= 5,0000 & q_3 &= 0,4515,\\ r_4 &= 5,5074 & q_4 &= 0,2383,\\ r_5 &= 6,0489 & q_5 &= 0,0359. \end{aligned}$$

The values of $r_{i,t}$ and $q_{i,t}$ are the same for all other times with $P_t^* = 5$. The values of $r_{i,t}$ and $q_{i,t}$ are different when $P_t^* \neq 5$.

When $r_{i,t}$ and $q_{i,t}$ are known, dynamic programming can be used to find the values of $a_k(r_{i,t};t,d)$. This part is explained in Section 5.3. With the known values $r_{i,t}$ and $a_k(r_{i,t};t,d)$, the bidding strategy is made in the same way as in Section 6.2.1.

6.3 Compare different bidding strategies

This section compares the bidding strategies explained earlier in this report with each other. The new bidding strategy can be used for different levels of information about p_t, the new bidding strategy is therefore divided into three different strategies. All different bidding strategies are compared to each other. This is in Section 6.3.1 done with perfect information about p_t. In Section 6.3.2 the bidding strategies are compared to each other in the situation where information about p_t changes every time. Sections 6.3.3 and 6.3.4 compare the bidding strategies with each other for the situation where P^* respectively increases and decreases in time. The computation times of the different bidding strategies are given at the end of this section.

6.3.1 Perfect information

In this section the different strategies are compared to each other in the situation where information about the price is very accurate and does not change over time. In this situation: $T = 24$,

$W = 20$, $m_t = 2$, $s = \frac{p_{max} - p_{min}}{n}$ with $p_{min} = 1$ and $p_{max} = 10$. Furthermore $P^* = 5$ and $\sigma_p = 1$. The average costs of satisfying 20 demand units, $W = 20$, with perfect information is shown in Table 6.4 for each of the strategies.

n	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P^*_t and $\sigma_{p,t}$
3	95,14	95,08	93,01	86,67	82,60
5	95,14	95,08	88,79	84,82	82,66
7	95,14	95,08	86,39	83,90	82,60
9	95,14	95,08	85,54	83,58	82,60
11	95,14	95,08	84,95	83,36	82,58
13	95,14	95,08	85,03	83,27	82,58
15	95,14	95,08	84,88	83,19	82,55
17	95,14	95,08	84,75	83,11	82,55
101	95,14	95,08	84,18	82,86	82,41

Table 6.4: Costs of the different strategies for different n with perfect information

When no information about p_t is known, the demand is satisfied equally over the times t. The total costs of satisfying 20 units of demand will then be $100,16$. The cheapest way of satisfying W has a total cost of $82,28$.

As can be seen from the table, the costs decrease when more information about p is known. Including dynamic programming in the bidding strategy seems to lead to lower costs. Whether and how much the total costs decrease depends on how good P^* and σ_p are. In this section the forecasted and real P^* and σ_p are the same. In the next sections the forecasted P^* and σ_p will differ from the real P^* and σ_p. The information about p will in the next sections not be known in advance and can change over time. The next sections show how well the methods perform when information about p changes.

Different trends

Table 6.4 shows the average total costs of satisfying W demand units, the total costs of each of the instances lies around these average total costs. The trend of the prices has an influence on the total costs. The total costs will be lower when the cheap prices are available in the earlier stages and the high prices at the end. An example of this is shown here, where the total costs of a certain price trend are compared with the total costs of the opposite trend. The following prices are used. In this example: $T = 24$, $W = 20$ and $m_t = 2$. The prices of the next 24 times are:

$$\begin{aligned} p \quad = \quad & [7,9080 \quad 6,3790 \quad 6,1275 \quad 6,0984 \quad 5,8252 \quad 5,7015 \quad 5,5080 \quad 5,3502 \\ & 5,2820 \quad 5,0335 \quad 5,0229 \quad 4,7380 \quad 4,7275 \quad 4,7221 \quad 4,7143 \quad 4,7009 \\ & 4,6462 \quad 4,5314 \quad 4,1764 \quad 3,9418 \quad 3,6663 \quad 3,4229 \quad 3,2498 \quad 2,9482]^T. \end{aligned}$$

These prices are however not yet known by the agents in the PowerMatcher. The trend is clear, prices decrease. The total costs and the used demand at each t corresponding to this trend will be compared to the total costs and used demand of the opposite trend. In the opposite trend the prices are mirrored around P^*, in this case $P^* = 4,9343$. The two trends will be compared to each other using the bidding strategy where besides P^* also σ_p is assumed to be known, furthermore $n = 101$. The total costs and used demand are given next. Using the price trend given above, the total costs of satisfying demand W is equal to $81,2847$. Satisfying the demand is cheaper when the opposite trend is used, the total costs are then equal to $79,6086$. This difference in total costs can also be seen in the used demand table, Table 6.5. In the left part of this table, prices are high at the beginning and decrease with time. At $t = 13$ some demand is already used, future prices are not known by agent k and using some demand at $t = 13$ makes sure that less demand is needed at possible high prices in the future, not knowing that the prices only decrease. By playing safe, the total costs turn out to be higher than needed. This problem does not occur when the opposite trend is used, see right part of Table 6.5, this is why the total costs are lower using the opposite trend.

Total costs: 81,2847

Used demand:

t = 1 0	t = 2 0	t = 3 0	t = 4 0
t = 5 0	t = 6 0	t = 7 0	t = 8 0
t = 9 0	t = 10 0	t = 11 0	t = 12 0
t = 13 0,7002	t = 14 2	t = 15 2	t = 16 0
t = 17 2	t = 18 2	t = 19 2	t = 20 2
t = 21 2	t = 22 2	t = 23 2	t = 24 1,2998

Total costs: 79,6086

Used demand:

t = 1 2	t = 2 2	t = 3 2	t = 4 2
t = 5 2	t = 6 2	t = 7 2	t = 8 2
t = 9 2	t = 10 0	t = 11 0	t = 12 0
t = 13 0	t = 14 0	t = 15 0	t = 16 2
t = 17 0	t = 18 0	t = 19 0	t = 20 0
t = 21 0	t = 22 0	t = 23 0	t = 24 0

Table 6.5: Total costs and used shiftable demand for each time (t) for the price trend (left) and opposite price trend (right)

Using different prices and different strategies lead to the same results, the total costs decrease when low prices are available at the beginning instead of the end.

6.3.2 Average price changes every t

In the previous section the information about future prices was known exactly. In this section the forecasted P^* and σ_p will differ from the real P^* and σ_p. In this section again: $T = 24$, $W = 20$, $m_t = 2$ and $s = \frac{p_{max} - p_{min}}{n}$. This s value leads to n equally sized intervals. The length of these intervals does not change while the value of P^* does change. The $p_t \in X$ for $t = 1, ..., T$ with $X \sim \mathcal{N}(5, 1)$. With P^* and possibly σ_p known, the different strategies can determine the bidding curve at time t. When the bidding curve is made, the price at time t, p_t, is determined. This p_t can be different from the forecasted price. At each t, the forecasted p_t, ..., p_T changes. This change l is normally distributed, $l \sim \mathcal{N}(0, c)$. The value of c will in the remainder of this section be equal to either $0,1$ or 1. The demand corresponding to p_t is known from the bidding curve and p_t. The different strategies are compared to each other in situations where P^* and σ_p change over time.

The following tables show the total costs of satisfying W units of demand for each of the methods. This is done for different values of σ_p, c and n. The σ_p equals $0,0001$, $0,5$, 1 or 2 in respectively tables 6.6, 6.7, 6.8 and 6.9. The c values used are $c = 0,1$ and $c = 1$. Nine different values for n are used, this to see the influence of n on the performance of the bidding strategies. The standard/one-time-scale PowerMatcher (OTS PM) and the current two-time-scale PowerMatcher (TTS PM) are independent of this value of n, these two strategies only use p_{min}, p_{max} and P^*. The tables show the costs using the standard/one-time-scale PowerMatcher (OTS PM), the current two-time-scale PowerMatcher(TTS PM), the DP method with known P^*, the DP method with known P^* and σ_p and the DP method with known P_t^* and $\sigma_{p,t}$. There are 100 different instances used, the tables show the average costs of satisfying W demand units over these instances.

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P^*_t and $\sigma_{p,t}$
3	0,1	99,31	99,31	99,02	94,88	94,21
	1	73,33	73,11	65,11	65,39	60,24
5	0,1	99,31	99,31	98,38	94,46	94,15
	1	73,33	73,11	60,95	61,99	59,35
7	0,1	99,31	99,31	97,80	94,33	94,12
	1	73,33	73,11	59,87	60,57	59,33
9	0,1	99,31	99,31	97,30	94,29	94,10
	1	73,33	73,11	59,31	60,23	58,93
11	0,1	99,31	99,31	96,92	94,25	94,09
	1	73,33	73,11	59,10	59,81	58,95
13	0,1	99,31	99,31	96,73	94,23	94,04
	1	73,33	73,11	59,19	59,50	58,61
15	0,1	99,31	99,31	96,46	94,17	94,02
	1	73,33	73,11	58,89	59,20	58,70
17	0,1	99,31	99,31	96,22	94,18	94,02
	1	73,33	73,11	58,74	59,29	58,55
101	0,1	99,31	99,31	95,96	94,08	93,95
	1	73,33	73,11	58,02	58,23	58,14

Table 6.6: Costs of different strategies for different n with changing P^* and $\sigma_p = 0,0001$

When absolutely no information about p_t is known, the demand is satisfied equally over the times t. In the case where $c = 0,1$ the total costs will then be $99,95$. When p_t is known for all t, the total costs of satisfying 20 units of demand will be $93,63$. When $c = 1$ the costs will be respectively $103,40$ and $55,23$.

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P^*_t and $\sigma_{p,t}$
3	0,1	98,21	98,21	97,29	91,45	89,66
	1	73,38	73,22	65,11	65,67	59,73
5	0,1	98,21	98,21	95,35	90,46	89,54
	1	73,38	73,22	60,73	61,85	59,08
7	0,1	98,21	98,21	93,75	90,06	89,46
	1	73,38	73,22	59,74	60,53	58,74
9	0,1	98,21	98,21	92,83	89,86	89,45
	1	73,38	73,22	59,27	59,88	58,67
11	0,1	98,21	98,21	92,04	89,77	89,45
	1	73,38	73,22	59,13	59,68	58,52
13	0,1	98,21	98,21	92,00	89,73	89,45
	1	73,38	73,22	59,08	59,38	58,39
15	0,1	98,21	98,21	91,74	89,71	89,42
	1	73,38	73,22	58,86	59,16	58,18
17	0,1	98,21	98,21	91,70	89,65	89,39
	1	73,38	73,22	58,74	59,14	58,29
101	0,1	98,21	98,21	91,24	89,53	89,28
	1	73,38	73,22	58,14	58,23	57,79

Table 6.7: Costs of different methods for different n with changing P^* with $\sigma_p = 0,5$

With $\sigma_p = 0,5$ and no information about p_t, the total costs of satisfying 20 demand units will be $100,14$ in the case where $c = 0,1$ and $103,63$ in the case where $c = 1$. With perfect information about p_t these costs will be respectively $89,03$ and $55,25$.

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P_t^* and $\sigma_{p,t}$
3	0,1	94,53	94,56	91,62	85,50	81,71
	1	72,88	72,60	64,05	65,02	59,02
5	0,1	94,53	94,56	87,18	83,56	81,59
	1	72,88	72,60	59,75	60,96	58,33
7	0,1	94,53	94,56	84,85	82,68	81,56
	1	72,88	72,60	59,09	59,70	58,24
9	0,1	94,53	94,56	84,11	82,35	81,48
	1	72,88	72,60	58,66	59,28	58,08
11	0,1	94,53	94,56	83,60	82,19	81,46
	1	72,88	72,60	58,48	58,71	57,68
13	0,1	94,53	94,56	83,56	82,08	81,46
	1	72,88	72,60	58,45	58,74	57,79
15	0,1	94,53	94,56	83,54	81,95	81,40
	1	72,88	72,60	58,23	58,38	57,75
17	0,1	94,53	94,56	83,40	81,94	81,37
	1	72,88	72,60	58,12	58,39	57,46
101	0,1	94,53	94,56	82,65	81,62	81,25
	1	72,88	72,60	57,47	57,47	57,01

Table 6.8: Costs of different methods for different n with changing P^* with $\sigma_p = 1$

Table 6.8 shows the total costs using the different stategies when $\sigma_p = 1$. When the demand is satisfied equally over the times t, the costs will be $100,32$ in the case where $c = 0,1$ and $103,97$ in the case where $c = 1$. When p_t is known for each t, the total costs will be respectively $80,96$ and $54,68$.

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P_t^* and $\sigma_{p,t}$
3	0,1	81,88	81,56	74,45	73,25	65,58
	1	70,32	70,09	61,05	64,31	56,54
5	0,1	81,88	81,56	68,91	69,41	65,49
	1	70,32	70,09	57,29	59,22	56,12
7	0,1	81,88	81,56	67,36	67,81	65,52
	1	70,32	70,09	56,58	57,56	55,83
9	0,1	81,88	81,56	67,01	67,13	65,39
	1	70,32	70,09	56,45	57,00	55,68
11	0,1	81,88	81,56	66,76	66,83	65,29
	1	70,32	70,09	56,10	56,75	55,60
13	0,1	81,88	81,56	66,56	66,53	65,26
	1	70,32	70,09	55,88	56,48	55,70
15	0,1	81,88	81,56	66,47	66,39	65,27
	1	70,32	70,09	55,66	56,39	55,65
17	0,1	81,88	81,56	66,54	66,16	65,13
	1	70,32	70,09	55,76	56,28	55,43
101	0,1	81,88	81,56	65,78	65,54	64,87
	1	70,32	70,09	55,11	55,55	55,23

Table 6.9: Costs of different methods for different n with changing P^* with $\sigma_p = 2$

When $\sigma_p = 2$ and the demand would be satisfied equally over all times t, the total costs would respectively be $100,90$ and $104,90$ in the cases where $c = 0,1$ and $c = 1$. The cheapest way to satisfy 20 demand units are respectively $64,58$ and $52,51$.

First of all, it can be seen from tables 6.6 till 6.9 that the total costs decrease when the value of σ_p increases. With $P^* = 5$ and $\sigma_p = 0,0001$ the probability of a very low p_t is very small. With $\sigma_p = 2$ this probability is much higher. A larger value of σ_p gives the opportunity to have low prices, a good bidding strategy will use these low prices to satisfy the demand. The costs will therefore be lower when the value of σ_p is higher.

The costs are almost the same for the standard PowerMatcher and the current two-time-scale PowerMatcher. Compared to the total costs without any information and the total costs with perfect information, the PowerMatcher can be improved. Especially with a low σ_p and c the total costs are high, almost equal to the total costs when no information about p is known. The DP used bidding strategies perform a lot better than the currently used bidding strategy. These DP used bidding strategies lead to lower total costs than the currently used bidding strategy, no matter the level of information about p worked with and the value of n. Using dynamic programming in the bidding strategies thus seems to have a possitive effect on the total costs.

The level of information about p worked with and the value of n does have influence on the performance of the DP used bidding strategy itself. Increasing the value of n from 3 to 7 really improves the performance of the bidding strategy. Increasing an already high value of n does not have as much of an effect. A higher n leads to lower costs, but the difference of the costs will at a certain point be negligible. Increasing n also brings the performance of the different levels of information closer to each other and closer to the total costs with perfect information. When n is high, the bidding strategy can almost not be improved further, especially for a low c. A low value of c means that the received forecasts are accurate. The highest level of information about p leads in this case to the lowest costs. This is however different for a larger value of c. A larger value of c means that the forecasts are not very accurate and that the forecasts can change a lot. Working with bad forecasts can lead to higher total costs than working with less information. Bad forecasts lead to bad decisions.

Variation in total costs

Tables 6.6, 6.7, 6.8 and 6.9 show the average costs of satisfying W over 100 instances. It is also important to look at the variation of the total costs. Figure 6.8 shows the total costs of each of the 100 instances for each of the bidding strategies, $\sigma_p = 1$ and $c = 1$ in these instances. The OTS PM and TTS PM strategies always lead to higher costs, compared to the other strategies. The variation of the total costs is approximately the same for each of the strategies. The total costs of each of the strategies lies in a range of 50.

More choices for a_t

In the above mentioned situations $a_t \in \{0, 1, ..., m_t\}$ with $m_t = 2$. The bidding curves will be different when more choices for a_t are possible. Table 6.8 shows the total costs for each of the strategies when $a_t \in \{0, 1, ..., m_t\}$ and Table 6.10 shows the total costs for each of the strategies when $a_t \in \{0, \frac{1}{3}, ..., m_t\}$, both with $m_t = 2$ and $\sigma_p = 1$. The total costs when demand is satisfied equally over the times t and when p_t is known for each t are the same in both situations and are given directly under Table 6.8.

With more choices for a_t, the bidding curve can show the best action at each of the possible prices more precise. This increase of choices lead to lower total costs, as can be seen from tables 6.8 and 6.10. The results are the same for other values of σ_p. These tables are omitted here.

In this section the changes in p_t, ..., p_T are normally distributed, $l \sim \mathcal{N}(0, c)$. The prices can either increase or decrease. The P^* does also change, either positive or negative. How the bidding strategies perform when P^* is either only increasing in time or only decreasing in time is shortly looked at in the next two sections.

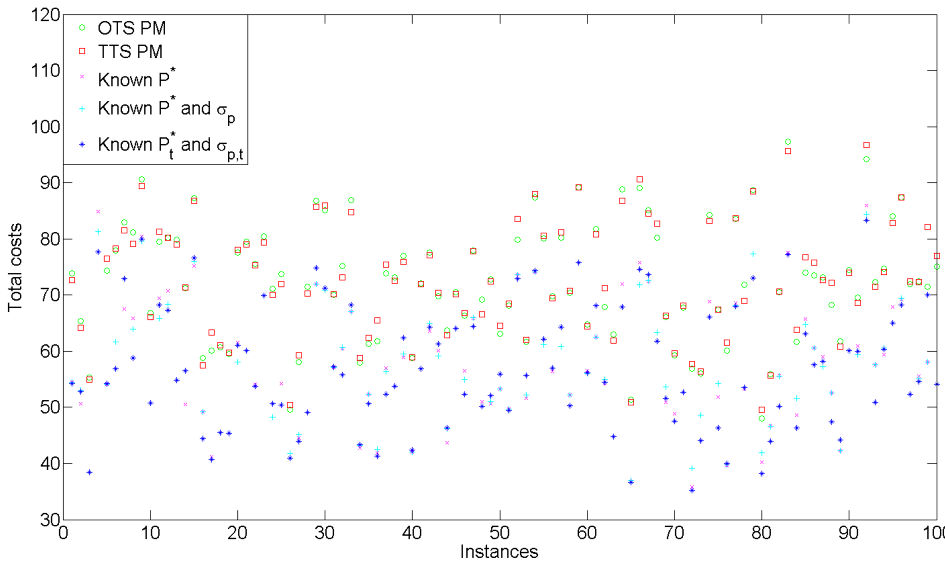


Figure 6.8: Scatterplot of total costs for 100 instances for each of the bidding strategies

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P_t^* and $\sigma_{p,t}$
3	0,1	94,53	94,56	91,45	85,33	81,49
	1	72,88	72,60	63,63	64,69	58,70
5	0,1	94,53	94,56	87,02	83,38	81,42
	1	72,88	72,60	59,50	60,60	57,82
7	0,1	94,53	94,56	84,64	82,55	81,36
	1	72,88	72,60	58,51	59,34	57,80
9	0,1	94,53	94,56	83,96	82,20	81,31
	1	72,88	72,60	58,25	58,90	57,66
11	0,1	94,53	94,56	83,48	82,08	81,31
	1	72,88	72,60	58,00	58,45	57,35
13	0,1	94,53	94,56	83,52	81,95	81,29
	1	72,88	72,60	58,11	58,28	57,42
15	0,1	94,53	94,56	83,37	81,84	81,28
	1	72,88	72,60	58,03	58,08	57,35
17	0,1	94,53	94,56	83,25	81,78	81,27
	1	72,88	72,60	57,82	58,10	57,21
101	0,1	94,53	94,56	82,58	81,53	81,21
	1	72,88	72,60	57,38	57,42	57,00

Table 6.10: Costs of different strategies for different n with changing P^* with $\sigma_p = 1$ and more a_t values

6.3.3 Average price increases

This section will look at the performance of the bidding strategies in the situation where P^* only increases in time. In this situation $T = 24$, $W = 20$ and $m_t = 2$. The stepsize $s = \frac{p_{max} - p_{min}}{n}$, such that there are n equal intervals. This value of s does not change, even though P^* does change. Again, 100 instances and the normal distribtution with $P^* = 5$ and $\sigma_p = 1$ is used. The p_t, ..., p_T change every t, this change l is normally distributed with a positive mean, $l \sim \mathcal{N}(c, 0, 1)$. Different values of n are used, this to see the influence of n on the performance of the bidding strategies. The average total costs of satisfying W units of energy is shown in Table 6.11 for the different bidding strategies and levels of information.

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P_t^* and $\sigma_{p,t}$
3	0,1	121,55	118,93	118,65	112,86	107,62
	1	194,21	192,45	198,26	197,48	193,32
5	0,1	121,55	118,93	114,11	110,55	107,51
	1	194,21	192,45	197,71	196,95	195,07
7	0,1	121,55	118,93	111,58	109,23	107,54
	1	194,21	192,45	197,21	196,55	195,03
9	0,1	121,55	118,93	110,75	108,48	107,59
	1	194,21	192,45	196,84	196,23	195,27
11	0,1	121,55	118,93	110,22	107,99	107,46
	1	194,21	192,45	196,57	196,00	195,11
13	0,1	121,55	118,93	110,27	107,70	107,47
	1	194,21	192,45	196,47	195,81	195,13
15	0,1	121,55	118,93	110,52	107,45	107,50
	1	194,21	192,45	196,63	195,66	195,05
17	0,1	121,55	118,93	110,41	107,36	107,50
	1	194,21	192,45	196,53	195,54	195,27
101	0,1	121,55	118,93	110,61	107,68	107,61
	1	194,21	192,45	197,29	195,12	195,03

Table 6.11: Costs of different methods for different n with increasing P^*

With $c = 0,1$, the total costs with respectively no information and perfect information are $125,31$ and $102,02$. With $c = 1$ these values are $191,26$ and $179,03$.

When $c = 0,1$ the total costs of all strategies are between the bounds, the total costs with no information and perfect information. Also, the total costs decrease with more information about p. The costs of the strategies are however a bit higher compared to the costs with perfect information. The updated P^* differs more from the original P^* than in the previous section. In the previous section the prices could increase and decrease, the P^* .. around the same value. The greater difference in this section leads to the respectively higher costs. When $c = 1$ the prices p_t, ..., p_T are badly forecasted and the forecasted p_t, ..., p_T keeps increasing per update. The given information has such a bad quality that it is better to have no information about p_t, ..., p_T. The $p_{max=10}$. As can be seen from Table 6.11, the average price for which W is satisfied is almost equal to p_{max}.

6.3.4 Average price decreases

This section looks at the performance of the different strategies when P^* only decreases in time. Again, $T = 24$, $W = 20$ and $m_t = 2$. The stepsize $s = \frac{p_{max}-p_{min}}{n}$, such that there are n equal intervals. There are 100 instances used. At the beginning of each instance the $p_t \sim \mathcal{N}(5,1)$. The p_t, ..., p_T change every t, this change l is normally distributed with a negative mean, $l \sim \mathcal{N}(-c, 0, 1)$. The average total costs of satisfying W units of energy is shown in Table 6.12 for the different bidding strategies and levels of information.

With $c = 0,1$, the total costs with respectively no information and perfect information are $75,53$ and $52,29$. With $c = 1$ these values are respectively $25,59$ and 20.

The results are comparable to the results of Section 6.3.3. When $c = 0,1$ the total costs are between 20 and $25,59$, the total costs with respectively perfect and no information. The change of

n	c	OTS PM	TTS PM	Known P^*	Known P^* and σ_p	Known P^*_t and $\sigma_{p,t}$
3	0,1	73,78	70,89	65,09	60,33	58,32
	1	28,07	26,88	22,78	25,23	25,77
5	0,1	73,78	70,89	61,17	59,03	57,98
	1	28,07	26,88	24,41	26,10	26,79
7	0,1	73,78	70,89	59,34	59,00	57,94
	1	28,07	26,88	25,47	26,62	26,95
9	0,1	73,78	70,89	58,90	59,12	57,88
	1	28,07	26,88	25,86	27,00	26,96
11	0,1	73,78	70,89	58,55	59,22	57,82
	1	28,07	26,88	25,45	27,25	27,01
13	0,1	73,78	70,89	58,40	59,36	57,76
	1	28,07	26,88	25,42	27,45	27,02
15	0,1	73,78	70,89	57,86	59,24	57,63
	1	28,07	26,88	24,85	27,56	26,88
17	0,1	73,78	70,89	57,83	59,15	57,59
	1	28,07	26,88	25,08	27,67	26,90
101	0,1	73,78	70,89	56,57	58,25	57,32
	1	28,07	26,88	25,01	27,43	26,88

Table 6.12: Costs of different methods for different n with decreasing P^*

P^* over time is greater than in Section 6.3.2, this lead to respectively higher costs in this section. When $c = 1$ the total costs are really high compared to the total costs with no information. The p_t, ..., p_T decrease a lot over time, all demand satisfied at the beginning, at a price higher than p_{min}, is paid too much for. The demand W can be satisfied at a price of p_{min}, $p_{min} = 1$ in this example.

6.3.5 Computation time bidding strategies

The different bidding strategies are compared to each other in the sections above. It turns out that using dynamic programming in the bidding strategy reduces the costs, dynamic programming however also needs more steps to determine the bidding curve. The bidding curve has to be determined once in every time period, therefore the process of determining which bidding curve to use can't take forever. This section will look at the computation time of the different bidding strategies used in this report.

The computation time is different for each bidding strategy and also depends on the number of remaining time epochs, the value of n and the number of possible values of $a_k(p; t, d)$ for the last three bidding strategies, where dynamic programming is used. With more time epochs left, the trees of sections 5.2 and 5.3 are greater. Greater trees lead to a higher computation time. With the same reasoning, more possibilities of $a_k(p; t, d)$ leads to a higher computation time. In Table 6.13 the number of possible values of $a_k(p; t, d)$ is equal to three, $z = 3$. Table 6.14 shows the computation time when there are seven possible actions, $z = 7$. As can be seen from tables 6.13 and 6.14, the bidding curves corresponding to the OTS PM and TTS PM strategy are determined very fast. The bidding curves of the other strategies are determined much slower, compared to the first two strategies. The computation time is however still less than a second, which is fast enough.

As said before in Section 5.2, the problem of satisfying demand can be rewritten as a Knapsack problem when prices are known. The Knapsack problem is an NP-complete problem, but can be solved in pseudo-polynomial time using dynamic programming. The time complexity of this dynamic programming solution is $O(TW)$. When the price can take on multiple values, the tree becomes more complex. The value of the node needs to be determined once more, not only for the case where the current price is known but also once extra for the case where the current price is unknown. Why this extra value is needed is explained in Section 5.3. These extra computations will lead to a higher computation time.

strategy	remaining time							
OTS PM	24	23	22	21	20	19	18	17
n=3	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	16	15	14	13	12	11	10	9
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	8	7	6	5	4	3	2	1
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
TTS PM	24	23	22	21	20	19	18	17
n=3	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	16	15	14	13	12	11	10	9
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	8	7	6	5	4	3	2	1
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Known P^*	24	23	22	21	20	19	18	17
n=3	0,0080	0,0074	0,0069	0,0059	0,0055	0,0044	0,0036	0,0030
n=101	0,0325	0,0292	0,0268	0,0242	0,0220	0,0198	0,0173	0,0153
	16	15	14	13	12	11	10	9
n=3	0,0022	0,0013	0,0012	0,0009	0,0007	0,0006	0,0006	0,0004
n=101	0,0136	0,0119	0,0104	0,0087	0,0076	0,0064	0,0054	0,0044
	8	7	6	5	4	3	2	1
n=3	0,0004	0,0003	0,0003	0,0002	0,0002	0,0002	0,0001	0,0001
n=101	0,0036	0,0025	0,0020	0,0015	0,0009	0,0006	0,0003	0,0002
Known P^* and σ_p	24	23	22	21	20	19	18	17
n=3	0,0076	0,0054	0,0052	0,0049	0,0047	0,0044	0,0040	0,0035
n=101	0,0364	0,0339	0,0321	0,0302	0,0288	0,0272	0,0253	0,0238
	16	15	14	13	12	11	10	9
n=3	0,0029	0,0018	0,0018	0,0014	0,0014	0,0012	0,0010	0,0010
n=101	0,0260	0,0215	0,0206	0,0199	0,0188	0,0180	0,0176	0,0169
	8	7	6	5	4	3	2	1
n=3	0,0009	0,0009	0,0008	0,0007	0,0007	0,0007	0,0006	0,0006
n=101	0,0163	0,0160	0,0160	0,0153	0,0148	0,0150	0,0145	0,0143
Known P_t^* and $\sigma_{p,t}$	24	23	22	21	20	19	18	17
n=3	0,0148	0,0137	0,0127	0,0120	0,0109	0,0104	0,0094	0,0087
n=101	0,3431	0,3275	0,3126	0,2981	0,2827	0,2661	0,2513	0,2366
	16	15	14	13	12	11	10	9
n=3	0,0085	0,0077	0,0071	0,0063	0,0059	0,0054	0,0051	0,0049
n=101	0,2238	0,2073	0,1938	0,1791	0,1661	0,1515	0,1371	0,1235
	8	7	6	5	4	3	2	1
n=3	0,0041	0,0036	0,0030	0,0025	0,0019	0,0016	0,0010	0,0005
n=101	0,1095	0,0962	0,0823	0,0683	0,0546	0,0410	0,0274	0,0139

Table 6.13: Computation time of bidding strategies with $n = 3$ and $n = 101$ where $z = 3$

strategy	remaining time							
OTS PM	24	23	22	21	20	19	18	17
n=3	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	16	15	14	13	12	11	10	9
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	8	7	6	5	4	3	2	1
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
TTS PM	24	23	22	21	20	19	18	17
n=3	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	16	15	14	13	12	11	10	9
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	8	7	6	5	4	3	2	1
n=3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
n=101	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Known P^*	24	23	22	21	20	19	18	17
n=3	0,0498	0,0420	0,0363	0,0311	0,0285	0,0211	0,0185	0,0145
n=101	0,1557	0,1443	0,1217	0,1037	0,0897	0,0788	0,0692	0,0586
	16	15	14	13	12	11	10	9
n=3	0,0120	0,0099	0,0075	0,0060	0,0042	0,0027	0,0020	0,0012
n=101	0,0493	0,0420	0,0367	0,0297	0,0248	0,0208	0,0164	0,0119
	8	7	6	5	4	3	2	1
n=3	0,0009	0,0006	0,0004	0,0003	0,0003	0,0002	0,0001	0,0001
n=101	0,0083	0,0055	0,0037	0,0027	0,0018	0,0010	0,0004	0,0001
Known P^* and σ_p	24	23	22	21	20	19	18	17
n=3	0,0434	0,0362	0,0279	0,0221	0,0186	0,0131	0,0100	0,0070
n=101	0,1522	0,1336	0,1111	0,0920	0,0784	0,0658	0,0551	0,0441
	16	15	14	13	12	11	10	9
n=3	0,0051	0,0038	0,0029	0,0021	0,0016	0,0014	0,0011	0,0009
n=101	0,0396	0,0315	0,0283	0,0248	0,0231	0,0206	0,0193	0,0185
	8	7	6	5	4	3	2	1
n=3	0,0009	0,0008	0,0008	0,0008	0,0008	0,0007	0,0007	0,0007
n=101	0,0176	0,0172	0,0165	0,0162	0,0160	0,0157	0,0156	0,0155
Known P_t^* and $\sigma_{p,t}$	24	23	22	21	20	19	18	17
n=3	0,0518	0,0412	0,0316	0,0272	0,0210	0,0171	0,0141	0,0116
n=101	0,4849	0,4490	0,4096	0,3798	0,3521	0,3257	0,3001	0,2757
	16	15	14	13	12	11	10	9
n=3	0,0101	0,0082	0,0075	0,0069	0,0062	0,0055	0,0051	0,0050
n=101	0,2527	0,2352	0,2177	0,1993	0,1817	0,1650	0,1479	0,1330
	8	7	6	5	4	3	2	1
n=3	0,0044	0,0033	0,0033	0,0027	0,0021	0,0015	0,0012	0,0007
n=101	0,1174	0,1030	0,0874	0,0735	0,0584	0,0445	0,0289	0,0156

Table 6.14: Computation time of bidding strategies with $n = 3$ and $n = 101$ where $z = 7$

7 Conclusions and Further Research

This report presents different bidding strategies that can be used by the two-time-scale PowerMatcher. Section 7.1 gives the main results and conclusions about the performance of the two-time-scale PowerMatcher, using these different strategies. In particular the strategies where dynamic programming is used are looked at. The remarks and the options for further research are given in Section 7.2.

7.1 Conclusions

This report introduces different bidding strategies that can be used by the agents in the PowerMatcher. The new bidding strategies use dynamic programming and can be applied to shiftables where the only restriction is that the demand needs to be satisfied before some end time, electric vehicles for example. The performance of the two-time-scale PowerMatcher, using different bidding strategies, is looked at. The different bidding strategies can not only be compared to each other, but can also be compared to the standard PowerMatcher.

The difference in performance of the standard PowerMatcher and the currently used two-time-scale PowerMatcher shows how much the PowerMatcher is already improved by adding planning. This is not a big improvement. Over the 100 different instances used in this report, the two-time-scale PowerMatcher does perform a little bit better than the standard PowerMatcher but the average total costs decrease with less than 2% of what it could be. Some of the instances even lead to higher costs using the two-time-scale PowerMatcher. Compared to the minimum costs for satisfying demand over the 100 used instances, the costs can decrease up to 25%. A decrease of 2% is therefore not very satisfying, the two-time-scale PowerMatcher has to be improved.

The bidding strategy used in the two-time-scale PowerMatcher does only take the prices and remaining demand into account, it does not include the remaining time. By taking the remaining time into account, it should be able to improve the PowerMatcher further. This is where the new bidding strategy, introduced in this report, comes in. This new strategy looks at the possible prices, the remaining demand and also the remaining time. This bidding strategy uses dynamic programming to determine the bidding curve. Depending on the variation of future prices and the quality of the forecasted average price, the new bidding strategy can really improve the PowerMatcher. The total costs of satisfying demand decreases with 4% till 20%, using the values and instances mentioned in this report. The percentage by which the total costs decrease depends on the quality of the forecasted information and the variation of the prices. Good quality of the forecasted information and a high variation lead to a bigger decrease. In the extreme case where the forecasts are really bad and the forecasted future prices are steeply increasing, the new bidding strategy performs worse than the currently two-time-scale PowerMatcher used bidding strategy. The percentage by which the total costs can maximally decrease also depends on these two values. The total costs of the PowerMatcher can maximally decrease with 6% till 25%. As mentioned above, the improvement of the PowerMatcher using the new bidding strategy is huge when predictions of reasonable quality are available, the total costs then approach the minimum total costs. There is however some room left for further improvement.

The new bidding strategy can perform even better when not only the forecasted average price but also the forecasted standard deviation of future prices is known. Information about the standard deviation leads to a better incentive as to what prices are really cheap. Using this extra information could lead to a improvement of another 2%. The used forecasts do however have to be of good quality. Bad forecasts do lead to slightly higher total costs, compared to the new bidding strategy where only the average price forecasts are used.

Till now, the PowerMatcher works with the forecasted average price and standard deviation over future prices. With the new bidding strategy it is also possible to work with forecasted future prices and standard deviations for each time seperately. With more information about future prices the total costs will decrease. But, again, the forecasts need to be of good quality. With forecasts of good quality, this version of the new bidding strategy leads to slightly lower costs than the previous mentioned versions of the new bidding strategy.

It turns out that the new bidding strategy, introduced in this report, really improves the performance of the PowerMatcher. The total costs for satisfying demand decrease to almost the minimum total costs, where the minimum total costs can only be obtained with perfect information about future prices. The different versions of the bidding strategy leads to different total costs. Which version can best be used depends on the available information about future prices and the computation time the agent finds acceptable. The more information about future prices is available, the lower the costs and the higher the computation time. Using the last mentioned version generally leads to the lowest costs but there is a risk of high costs, these high costs occur when the forecasts are of bad quality. The quality of the forecasts for each seperate time is not yet known. When the bidding strategy is chosen, two other values need to be picked. These two values given the number of possible prices and the number of possible actions in each bidding curve. High values lead to lower costs/ a more precise bidding curve but also to a higher computation time.

The new bidding strategy makes the PowerMatcher stable and also increases the overall power efficiency. The new bidding strategy therefore leads to an improved PowerMatcher. There are different versions of the bidding strategy, working with different levels of information about future prices, the agents have to choose which version to use and what values to use in the bidding strategy.

7.2 Remarks and further research

The introduced bidding strategy seems to improve the two-time-scale PowerMatcher. There are however some remarks and further research options, these are given in this section .

Some assumptions are made in this study. For example, bidding curves are determined periodically in time and prices of two consecutive time periods are independent. It is interesting to see what happens to the performance of the bidding strategy when these assumptions are not made, before implementing the bidding strategy into the two-time-scale PowerMatcher.

This report mainly looked at the situation where a electric car has to be charged in a certain time interval. This problem can be solved with dynamic programming, as is explained in this report. This problem however changes when the electric car turns into a washing machine or battery. An electric car can be charged with interruptions, this is not possible with the washing machine. When a washing machine is turned on, it stays on till the end of the washing program, see Section 5.4. A battery can be charged and discharged, this also leads to a more complex problem. For these different devices, a different version of the bidding strategy is needed. These versions do not yet exist, further research is needed.

No simulations or field tests of the two-time-scale PowerMatcher including the new bidding strategy are carried out yet. These simulations and field tests show how well the new bidding strategies perform in real life and are therefore very useful.

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9 Appendix

Variable	Description
a_k	best action of agent k
d	remaining demand
d_k	demand of agent k at shorter time scale
d_k^*	demand of agent k at shorter time scale with p^*
d_{total}	total demand at shorter time scale
$d_{k,shift}$	shiftable demand of agent k
$d_{k,fixed}$	fixed demand of agent k
$d_{k,wind}$	wind demand of agent k
$d_{k,diesel}$	diesel demand of agent k
D	demand at longer time scale
D_k	demand of agent k at longer time scale
D_k^*	demand of agent k at longer time scale with P^*
D_{total}	total demand at longer time scale
$EV_t(d)$	expected value of node (t, d)/ expected minimum costs of satisfying d
FD	fixed demand
FWD	final wind demand
K	number of agents
m_t	maximum amount of demand to satisfy at time t
n	number of possible values of price p
n_t	number of time points
p	unit price of energy at shorter time scale
p^*	average price at shorter time scale
p_t^*	average price at time t
p_{min}	minimum price
p_{max}	maximum price
P	unit price of energy at longer time scale
P^*	average price over T
q_i	probability of r_i
r_i	i'th possible value of price p
s	stepsize of prices
t	time (measured in intervals of hours), short time scale
T	time horizon, long time scale
TC	total costs
$V_t(d)$	value of node (t, d)/ minimum costs of satisfying demand d
W	total demand to satisfy
WD	wind demand
z	number of possible actions at each t
π_g	unit cost of power generation at diesel generator
σ_p	standard deviation of price p
$\sigma_{p,t}$	standard deviation of price p at time t

Table 9.1: List of all variables used in the report

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	2	2	2	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	2	2	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	2	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.2: best actions with distribution 1, from top till bottom when $r_2 = 4$, $r_3 = 5$ and $r_4 = 6$. The rows represent the remaining demand d and the columns represent time t. A non reachable node is denoted by '-'.

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-	-	-
7	-	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	2	2	2	2	2	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	2	2	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	2	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.3: best actions with distribution 2, from top till bottom when $r_2 = 4$, $r_3 = 5$ and $r_4 = 6$. The rows represent the remaining demand d and the columns represent time t. A non reachable node is denoted by '-'.

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-	-	-
7	-	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-	-	-
6	-	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-	-
5	-	-	0	0	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	2	2	2	2	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	2	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

d t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-	-	-
7	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-	-	-
6	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-	-
5	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-	-
4	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-
3	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-
2	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
1	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.4: best actions with distribution 3, from top till bottom when $r_2 = 4$, $r_3 = 5$ and $r_4 = 6$. The rows represent the remaining demand d and the columns represent time t. A non reachable node is denoted by '-'.