Filter Bubbles in Opinion Dynamics

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Abstract

This paper proposes and analyzes a model of opinion dynamics which produces the behavior of filter bubbles. Filter bubbles are feared to seriously narrow the perspectives of individuals in a society, a consequence of the omnipresent online personalization algorithms. This paper produces a full model development description, based upon fundamental assumptions. It is shown that the model presumably asymptotically converges into one or more clusters of opinions. Furthermore, configurations are described under which the model produces filter bubbles.

KEY WORDS: opinion dynamics, continuous opinions, cluster formation, filter bubbles

1. INTRODUCTION

The online world as we know it today is highly tailored to our personal interests. Personalization algorithms determine what you see and more importantly what you do not see, all based on information that was gathered about you. These algorithms lead to a so called personal *filter bubble*. This filter bubble is your own personal unique universe of information that you live in online. More specifically, a filter bubble is the result of a personalized search in which a website algorithm selectively guesses what information its user would like to see, based on information about the user such as location, past click behavior and search history.

The name 'filter bubble' was coined by Eli Pariser. In a TED talk [1] Pariser makes a plea that we take seriously on the (negative) effects of these personalized filters. The vast amount of information that is currently available on the

> " It will be very hard for people to watch or consume something that has not in some sense been tailored for them"
>
> *Eric Schmidt, Google*

web calls for a refined (possibly personalized) selection, which may be found in these selective algorithms. The danger lies in the fact that personalization is now found nearly everywhere on the web. For example, there is no standard Google anymore: for identical search queries, different people might get very different results. Pariser states that the personalization in the Google search engine (without being logged in to a Google-account) is based on 57 input signals. These input signals include anything from the computer that you are using to search history to the time you are spending on certain websites.

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It is not only Google that uses personalization, it is nearly every site on the web, including websites that provide our newsfeed. Pariser names 'The Washington Post' and 'The New York Times' as examples. He states that these personalized filters are moving us very quickly towards a world in which the Internet is showing us a world it thinks we want to see, but not necessarily what we *need* to see. What is in your filter bubble depends on who you are and what you do, but you do not decide on what gets in. More importantly, you do not actually see what gets edited out.

In his TED talk Pariser states, in agreement with the author of this paper, that "if algorithms are going to curate the world for us, and they are going to decide what we get to see and what we do not get to see, then we need to make sure that they are not just keyed to relevance. We need to make sure that they also show us things that are uncomfortable, challenging or important, in other words: other points of view".

With the Internet currently being one of the largest, and maybe *the* largest sources of news, information and communication, the question arises whether a mathematical model can be made that mimics the effect of the personalization algorithms. For this research, a model is sought in the field of *opinion dynamics* that can illustrate the effect of personalization algorithms on opinion development. This will help get a better estimate the costs and risks of these algorithms.. To the best knowledge of the author, a model that studies filter bubbles in opinion dynamics has not yet been developed.

The term 'opinion dynamics' encompasses a wide class of models that describe the evolution of opinions of individuals[1] within a group. Some of these models that will be of use for this paper, are summarized in section 2. In general, in models in opinion dynamics a set of agents is considered where each holds an opinion from a certain opinion space. For continuous opinions this opinion space is typically defined as a certain interval of real numbers. An agent may change his or her opinion when he or she gets aware of the opinion(s) of others in the group.

In this paper a model is proposed that mimics the effect of online personalization algorithms in the field of opinion dynamics, based upon some fundamental assumptions. The goal of this model is to explore the behavior and influences of personalization in opinion dynamics. The resultant model significantly steers and thus modifies the opinions of individuals within a group, thereby creating filter bubbles. Moreover, under certain conditions the model shows how individual opinions may merge into one or more non-communicating clusters of individuals sharing opinions.

This paper is organized as follows: Section 2 gives anoverview of work that is relevant to for this paper. Section 3 reports on the full establishment of the model that is presented. It describes the fundamental assumptions on which the model is based, as well as a step-by-step translation of these assumptions into the model. The behavior and properties of this model are documented in section 4. The overall conclusion of this paper is stated in section 5.

Model Context and Notation

Opinion dynamics can be put in many contexts. Therefore it is important for the reader to be aware of the context relied on when writing this paper. The establishment of the model, as presented in section 3, is for the largest part based on this context. That is decisions and assumptions that are made, and the goals are set with this context in mind.

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[1]The words 'individuals', 'users', 'people' and 'agents' may be used interchangeably throughout the paper.

The context for which the model is developed is an *online social network*. Typical examples are Facebook and Twitter, but one can also include any online *forum*, or anything similar. What characterizes this context is that people in the network (the users) do not meet in person, but instead read each other's posts. The influence on opinions is therefore one-way, that is the 'poster' influences the opinion of the 'reader' (and not vice versa). For generalizing such context, the notation as described here is relied on throughout this paper.

A social network is represented by a directed graph $\mathcal{G}$. This graph consists of $p \geq 0$ nodes which represents the people (or users) in the network. Let $\mathcal{P}$ denote the set of persons in the network such that $|\mathcal{P}| = p$, where $|\mathcal{P}|$ denotes the cardinality of $\mathcal{P}$. Also, let $\mathcal{C}$ denote the set of network *connections* (the direct connection between two people), that is the set of edges of $\mathcal{C}$. Therefore, in short $\mathcal{G}$ can be written as $\mathcal{G} = (\mathcal{P}, \mathcal{C})$. Let $\mathcal{G}$ be such that $(i, i) \notin \mathcal{C}$ for all nodes i, so $\mathcal{G}$ is loop-free. For each node $i \in \mathcal{P}$ the group of friends (or connections) of individual i is denoted by $\mathcal{F}_i = \{j \in \mathcal{P} : (i, j) \in \mathcal{C}\}$. For this node $i \in \mathcal{P}$ the degree of this node (the number of 'friends' of i) is equal to $|\mathcal{F}_i|$.

The opinion or belief of an individual is denoted by a real number between 0 and 100. At time k ($k = 0, 1, 2, ...$) the opinion-state of individual i is denoted as $x_i(k) \in [0, 100]$. The opinion state of all individuals in the network is represented by a *vector of beliefs*, $x(k) \in \mathbb{R}^p$. The vector of initial opinions is denoted as $x(0)$.

2. Literature Review

This section gives an overview of relevant work on opinion dynamics that has been done up to now. It presents a general overview, as well as a more detailed description of some fundamental models.

Models that would nowadays be regarded as opinion dynamics date back to papers by French [2], Harary [3] and DeGroot [4]. In these models opinion updates occur according to a linear, and often convex combination of an individual's current opinion, the opinions of others, and possibly the agent's initial opinion.

For this research only models of continuous opinions are considered. Multiple models on binary opinions have been studied though, for example in [5] and [6]. Notable models in continuous opinion dynamics include models based on bounded confidence, for example models by Deffuant et al. [7] and the Hegselman and Krause model [8], and models that include prejudices where an individual's initial opinion is also taken into account, for instance the model by Friedkin and Johnson [9]. Throughout the years, these models have been extensively studied and extended. One extension one might encounter is that of asynchronous opinion updates, for instance by Frasca et al. [10]. Other studies on for example model convergence are presented in [11], [12] or [13].

The remainder of this section is devoted to briefly summarizing models based on bounded confidence, and a more detailed summary of the model by Friedkin and Johnson [9], and the Friedkin Johnson model with asynchronous updates presented by Frasca et al. [10].

2.1. Models of bounded confidence

Both the Deffuant-Weisbuch model [7] and the Hegselmann-Krause model [8] rely on averaging under bounded confidence. Bounded confidence relates to the model property that an individual

shall only update its opinion with another's if their opinion difference is less than a certain confidence bound denoted by ϵ. The Hegselmann-Krause model yields to a synchronous opinion update, whereas the Deffuant-Weisbuch model uses pairwise encounters (asynchronous). These models with bounded confidence typically restrict an individual's view to only certain other individuals, for whom the absolute opinion difference is less than a certain confidence bound ϵ_i. For example for the Deffuant-Weisbuch an opinion update typically looks as follows:

$$x_i(k+1) = \begin{cases} \frac{x_j(k)+x_i(k)}{2} & \text{if } \left\| x_i - x_j(k) \right\| \leq \epsilon_i , \\ x_i(k) & \text{otherwise.} \end{cases} \tag{1}$$

Figure 2.1 shows a typical simulation of a model of bounded confidence. In figure 2.1, time steps are set out on the horizontal axis, whereas the vertical axis denotes the opinion values (in this case these lie in the interval $[0,1]$). Since models of bounded confidence will not be further considered
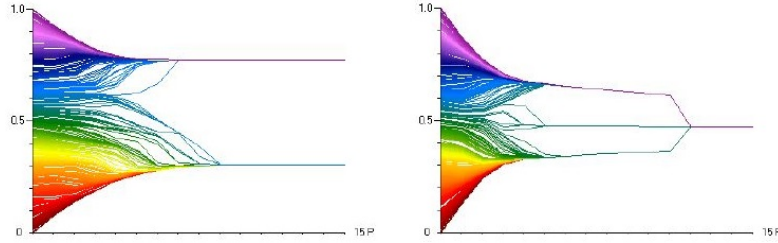

Figure 2.1: Simulation of a model of bounded confidence. Left: confidence bound $\epsilon = 0.15$, right: $\epsilon = 0.25$. [8]

in this paper, the information given above should be sufficient for the reader to get the main idea of these models. For more information the reader is suggested to consult [7] and [8].

2.2. FRIEDKIN AND JOHNSON'S MODEL

The model presented in [9] is defined as follows: Let $W \in \mathbb{R}^{p \times p}$ be a (nonnegative) row-stochastic matrix, the *weight matrix*. The elements w_{ij} denote the weight individual i gives to the opinion of individual j in the network. We let $w_{ij} = 0$ if there is no direct connection between i and j, i.e. $(i,j) \notin C$. Let $\Lambda \in \mathbb{R}^{p \times p}$ be a diagonal matrix which describes the *sensitivity* of each individual to other's opinion. This matrix Λ is defined as $\Lambda = I - \text{diag}(W)$, where $\text{diag}(W)$ denotes the diagonal matrix containing the corresponding diagonal elements of W.

In this model, the *synchronous* dynamics of opinions is given by:

$$x(k+1) = \Lambda W x(k) + (I - \Lambda)x(0). \tag{2}$$

In (2) the individual's opinion at time step k is (always) influenced by the individual's initial opinion $x(0)$, this aspect is sometimes referred to as *prejudices*. Note that these prejudices are eliminated of $\Lambda = I$ is set. The opinion profile at time step k may be written to be equal to:

$$x(k) = \left((\Lambda W)^k + \sum_{i=0}^{k-1} (\Lambda W)^i (I - \Lambda) \right) x(0).$$

It has been proven in [10] that the limit behavior of the opinions can be described as follows: Assume that from any node $l \in \mathcal{P}$ there exists a path from l to a node m such that $W_{lm} > 0$, then the opinions converge and

$$x^\infty := \lim_{k \to +\infty} x(k) = (I - \Lambda W)^{-1}(I - \Lambda)x(0).$$

In their 2013 paper [10], Frasca et al. extended Friedkin and Johnsen's model to an asynchronous model where agents (asynchronously) interact in pairs, in this paper this model is referred to as the 'Frasca Model'. These pairs are chosen according to a random process, and the 'interaction in pairs' is referred to as *gossiping*. A model that includes asynchronous opinion updates better fits the behavior of individuals in a social network, since they typically read one post at a time, and therefore they may update their opinion at every read.

The Frasca Model [10] is described as follows: Each agent $i \in \mathcal{P}$ starts with an initial belief $x_i(0) \in \mathbb{R}$. At each time k a directed link is randomly sampled from the social network, i.e. randomly sampled from a uniform distribution over $\mathcal{C}$. If link (i, j) is selected at time k, then agent i updates its opinion to a convex combination of its previous opinion, the opinion of j and its initial opinion $x_i(0)$. The update takes place according to (3).

$$x_i(k+1) = h_i \left((1 - \gamma_{ij})x_i(k) + \gamma_{ij}x_j(k) \right) + (1 - h_i)x_i(0),$$
$$x_l(k+1) = x_l(k) \; \forall l \in \mathcal{P} \setminus \{i\}. \tag{3}$$

Here the coefficients h_i are the diagonal elements of the diagonal matrix $H \in \mathbb{R}^{p \times p}$, and γ_{ij} are the elements of matrix $\Gamma \in \mathbb{R}^{p \times p}$. These matrices are determined as follows:

Firstly $D \in \mathbb{R}^{p \times p}$ is defined to be the degree matrix of $\mathcal{G}$, i.e. a diagonal matrix where the diagonal entries d_i are equal to the degree $d_i = |\mathcal{P}_i|$. In social network terms, one could state that d_i equals the number of friends of individual i.

Then

$$h_i = \begin{cases} (d_i - (1 - \lambda_{ii}))/d_i & \text{if } d_i \neq 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\gamma_{ij} = \begin{cases} \frac{d_i(1-h_i)+h_i-(1-\lambda_{ii}w_{ii})}{h_i} & \text{if } i = j, d_i \neq 1, \\ \frac{\lambda_{ii}w_{ij}}{h_i} & \text{if } i \neq j, d_i \neq 1, \\ 1 & \text{if } i = j, d_i = 1, \\ 0 & \text{if } i \neq j, d_i = 1. \end{cases} \tag{4}$$

The asynchronous model that is presented by Frasca et al. [10], may be interpreted as a model for which each individual in the network has an objective view on all other individuals. That is, its feed of information in the network is non-personalized. For this reason this model by Frasca et al. serves as the non-personalized counter-model for the personalized model that is presented in this paper. This means that the analysis of the behavior of the model will include comparisons to the Frasca Model.

3. METHODS

In this section a model is presented here that shows how filter bubbles of opinions may form across a social network. In the remainder of this paper, this model is referred to as the *filtered model*. In section 4 the results of the filtered model will be compared to the Frasca Model [10]. The eventual goal of this filtered model is to personalize the communication of the individuals in the network, and thereby deliberately steer opinions in certain personalized directions and eventually create filter bubbles.

The general structure of this section is as follows: Firstly, the fundamental model assumptions that form a foundation for the filtered model are presented. Secondly, this foundation shall be translated into a mathematical framework, that is, a rough outline for the model is presented. The main bodywork of this section, paragraph 3.2, is formed by the actual establishment of the filtered model. This section is concluded with a summary which recapitulates the full model, see paragraph 3.3.

Recall that in the asynchronous model presented by Frasca et al. [10] an agent couple (i, j), that is an edge from $\mathcal{G}$, is selected uniformly over $\mathcal{C}$. For the filtered model this asynchronous opinion update is adopted, since the author believes that this model sufficiently represents communications within an online network. The main feature of this model is an adapted procedure for selecting these edges. That is, edges will not be selected uniformly over $\mathcal{C}$, but will be drawn from a yet to be determined probability distribution. In this probability distribution $\mathbb{P}^k(i, j)$ denotes the probability of sampling (directed) edge (i, j) at time k. This adapted sampling method shall replicate the personalized feature of the filtered model. For the filtered model, opinions are updated according to equations 3 and 4. Prejudices are not included though, that is $\Lambda = I$.

Assumption 1 and 2 below formulate the fundamental assumptions of the filtered model. Both assumptions are based upon common underlying principles of current online filtering algorithms [14].

> **Assumption 1.** People with alike opinions are more likely to be interested in each others posts.

This assumption is related to the goal of the filtered model in the following way: Given a certain individual i in the network, the goal is to filter out posts of other people that have a very different opinion. In terms of sampling, the probability of sampling people with alike opinions shall be (considerably) larger than people with different opinions.

> **Assumption 2.** People that are interested in each others opinion are more likely to read each others posts, they might even search for them. In terms of edge sampling: the more times edge (i, j) has been sampled, the more likely it is to be re-sampled.

Relating assumption 2 to the model's goal as well, one can say that this assumption relates to the viewing history, that is, the sampling history. Again, the probability of sampling people with a rich history shall be (considerably) larger than people with little sampling history.

Besides these fundamental assumptions, assumptions 3 and 4 are made on the network $\mathcal{G}$, and the activity of the users within the network. When put in context (see Introduction), assumption 3 translates to a network where all posts of all users in principle are public and visible for anyone.

In the same context, assumption 4 indicates that the number of posts a user reads and/or posts is equal for all users, given a certain time span.

Assumption 3. The network graph $\mathcal{G}$ is complete.

Assumption 4. All agents in the network are equally active in the network.

Assumption 4 relates directly to the procedure for edge selection for simulations. By assumption 4 the procedure for selecting edges from $\mathcal{C}$ may be defined as follows:

Definition 1 (Sampling procedure)**.** Selecting edges from $\mathcal{C}$ at time k is conducted according to the following two steps:

(i) The first individual, denoted by N_1, will be drawn uniformly from $\mathcal{P}$. That is, $\mathbb{P}^k(i) = \frac{1}{p}$.

(ii) The second individual, denoted by N_2, will be drawn from $\mathbb{P}^k(i, j|i)$.

In this way the probability of sampling edge (i, j) at time k is denoted by $\mathbb{P}^k(i, j) = \mathbb{P}^k(i)\mathbb{P}^k(i, j|i)$.

In the remainder of this section the focus reduces to finding an expression for $\mathbb{P}^k(i, j|i)$. This expression will be based on assumptions 1 and 2. Paragraph 3.1 presents a mathematisation of the model assumptions that were presented in this section, as well as a rough outline of the model.

3.1. Model outline

Assumptions 1 and 2 in the previous section translate the two main parameters of the filtered model. Namely, the probability distribution $\mathbb{P}^k(i, j|i)$ will depend on *(absolute) difference in opinion* and the *sampling history*. The following two definitions formally define these parameters. Definition 2 follows directly from assumption 1, whereas definition 3 follows from assumption 2.

Definition 2. Let $N_1 = i$ be fixed. The absolute difference in opinion between agent i and (other) agents $j \in \mathcal{P}$ at time k is defined as $\Delta_{ij}^k := |x_i(k) - x_j(k)|$.

Definition 3. The matrix $C^{(k)} \in \mathbb{N}_0^{p \times p}$ is defined to keep track of the sampling history up until time k. The elements of $C^{(k)}$, denoted by, $c_{ij}^k \in \mathbb{N}_0$, indicate the number of times (directed) edge (i, j) has been sampled.

A general expression for $\mathbb{P}^k(i, j|i)$ in terms of Δ_{ij}^k and c_{ij}^k is presented in (5). This equation states that $\mathbb{P}^k(i, j|i)$ is proportional to some function $f(\Delta_{ij}^k, c_{ij}^k)$, an explicit expression for which is yet to be determined. For the sake of avoiding unnecessary complexities, the author deliberately chooses to separate the influences of Δ_{ij}^k and c_{ij}^k into some functions $g(\Delta_{ij}^k)$ and $h(c_{ij}^k)$. These functions are 'combined' by some operator '$\star$'. Note also that α and β are scaling factors for g and h respectively, these may depend on i and/or on k, and possibly on other factors. It is required, though, that α

and β are both nonnegative in any case.

$$\mathbb{P}^k(i, j|i) \propto f(\Delta_{ij}^k, c_{ij}^k) = \alpha g(\Delta_{ij}^k) \star \beta h(c_{ij}^k). \tag{5}$$

For finding an explicit expression for $f(\Delta_{ij}^k, c_{ij}^k)$ in (5), a list of requirements will be set up. Most of these requirements follow from the assumptions made in the previous section.

List of requirements:

(i) Making sure that sampling individuals with alike opinion is favored over individuals with a different opinion requires that f is descending in Δ_{ij}^k, meaning $\frac{\partial f}{\partial \Delta} < 0$. To amplify this effect, the following requirement is set as well: $\frac{\partial^2 g}{\partial \Delta^2} > 0$.

(ii) Also, individuals with a rich sample history are favored over those with little. Therefore the requirements that follows is: f is increasing in c, meaning $\frac{\partial f}{\partial c} > 0$.

(iii) To prevent problems in scaling f to a probability mass function later on, the following requirements follow for g and h: Firstly, $g(\Delta_{ij}^k) > 0$ for all $\Delta_{ij}^k \geq 0$ (note that $\Delta_{ij}^k \geq 0$ is satisfied by the definition of Δ_{ij}^k). Secondly $h(c) > 0$, $\forall c \geq 0$, in particular for $c = 0$. One may set $h(0) = \xi$, for certain $\xi > 0$.

(iv) In combining the influences of g and h in (5), it has to be made sure that the range of g matches that of h (and vice versa).

(v) The influence of sampling history on $\mathbb{P}^k(i, j|i)$ may not be too high if hardly any history has been built up yet.

Implementing these requirements into an expression for the different elements of (5) is described in paragraph 3.2. The reader could decide on leaping to section 3.3, where the finalized model is presented.

3.2.   Model establishment

The following paragraphs describe a step-by-step translation of the requirements presented in the previous section into an explicit expression for (5).

3.2.1.   Difference in opinion

The requirements described in paragraph 3.1 lead to a function corresponding to the sketch in figure 3.1. A function that describes the required sketch is of the form $g(\Delta_{ij}^k) = \frac{1}{p(\Delta_{ij}^k)}$, for some function $p(\Delta_{ij}^k)$.

For the actual choice of $g(\Delta_{ij}^k) = \frac{1}{p(\Delta_{ij}^k)}$ it is wise to first examine the domain of $g(\Delta_{ij}^k)$. As opinions are represented by a real number in the interval $[0, 100]$, we have that also $\Delta_{ij}^k \in [0, 100]$, which is the domain of $g(\Delta_{ij}^k)$. Therefore it has to be made sure that admissible Δ_{ij}^k should yield that $p(\Delta_{ij}^k) \neq 0$.
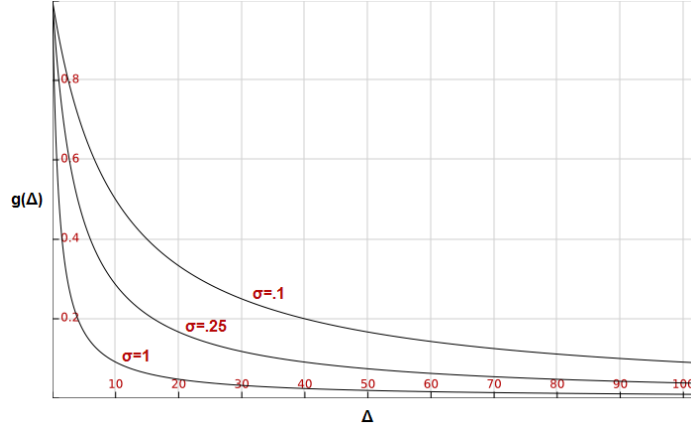
Figure 3.1: Three plots of functions for $g(\Delta_{ij}^k)$, different levels of sensitivity.

For the range of $g(\Delta_{ij}^k)$ it is desired that it is bounded, as to fit the range of $h(c)$ to it later on. The range if $g(\Delta_{ij}^k)$ is therefore set to $[\gamma, 1]$, where $0 < \gamma < 1$, $g(0) = 1$, and $g(100) = \gamma$.

A function that fulfills all requirements is the following:

$$g(\Delta_{ij}^k) = \frac{1}{\sigma \Delta_{ij}^k + 1} \text{ , where } \sigma \text{ is a measure for the vigor of descent.} \tag{6}$$

Suitable choices for σ are $\sigma_W = 0.1$ for weak, $\sigma_M = 0.25$ for mediocre, and $\sigma_V = 1$ for vigorous descend. Please see figure 3.1 for the plots of $g(\Delta_{ij}^k)$ for these values of σ. In section 4 the influence of this parameter σ on the model behavior will be examined.

3.2.2. Sample counter

For the range of h to match that of g, it is set to the interval $(0, 1)$. The function of the following form fits all the requirements:

$$h(c_{ij}^k) = \left(\frac{c_{ij}^k}{c_{ij,\max}^k + 1} \right)^2 \text{ , where } \quad c_{ij,\max}^k := \max_{j|i} \{ c_{ij}^k \}. \tag{7}$$

Note though that the range of h is typically equal to $\big(0, h(c_{ij,\max}) \big]$. It is worth noticing that the upper bound of this interval quickly approaches 1 as $c_{ij,\max}^k$ increases.

3.2.3. Other parameters

The influences of (6) and (7) depend on the choice of α and β in (5) respectively. The influence of opinion difference, i.e. α, may be chosen equal over time. Therefore α is set equal to 1. When it comes to the choice of β, more caution is required.

Basically, β determines the influence of 'history' on the edge sampling. Therefore it is not desired for this influence to be significant when no history has been recorded yet. In other words, the

requirement is set that $0 \leq \beta \ll \alpha = 1$ when $c^k_{i,\text{TOT}} := \sum_{j|i} c^k_{ij}$ is still low. Then, as soon as $c^k_{i,\text{TOT}}$ increases, also β increases asymptotically to $\alpha = 1$. This desired behavior of α and β is demonstrated in figure 3.2.
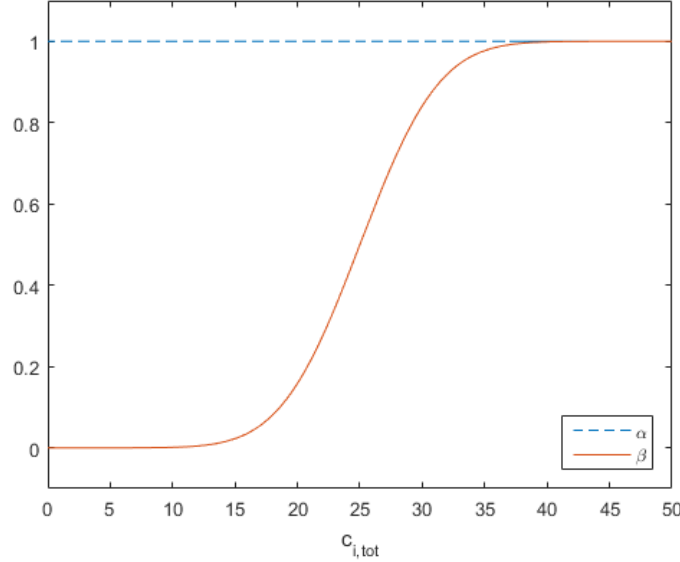

Figure 3.2: Behavior of α and β.

The relation of β to $c^k_{i,\text{TOT}}$ can be described by a function of the same form as the cumulative distribution function of a normal distribution, i.e. a function of the form

$$\beta(c^k_{i,\text{TOT}}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{c^k_{i,\text{TOT}} - p_1}{p_2\sqrt{2}}\right)\right]. \tag{8}$$

Where parameters p_1 and p_2 determine the location of the inflection point and the steepness of the function, respectively. If these parameters are chosen as $p_1 = 25$ and $p_2 = 10$, then the plot in figure 3.2 is acquired. Therefore we determine the value of β as follows:

$$\beta^k_i := \beta(c^k_{i,\text{TOT}}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{c^k_{i,\text{TOT}} - 25}{10\sqrt{2}}\right)\right]. \tag{9}$$

It is assumed here that enough 'history' has been build up after approximately 20 samplings, that is when β^k_i starts increasing to 1 (see figure 3.2).

Lastly, the operator '$\star$' in (5) that combines the influences of Δ^k_{ij} and c^k_{ij} needs to be defined. In order not to over complicate the relation between these two variables, the additive operator ('$+$') is chosen. This operator does the job since it ensures that $f(\Delta^k_{ij}, c^k_{ij}) > 0$ in all cases. Moreover it preserves the influence of β^k_i, where for instance the multiplication operator would not.

3.2.4. Intermediate recapitulation

The previous paragraphs have led to an expression for $f(\Delta_{ij}^k, c_{ij}^k)$ in (5). The final expression of $f(\Delta_{ij}^k, c_{ij}^k)$ is given below for the benefit of clarity.

$$f(\Delta_{ij}^k, c_{ij}^k) = \alpha g(\Delta_{ij}^k) + \beta h(c_{ij}^k) = \frac{1}{\sigma \Delta_{ij}^k + 1} + \beta_i^k \left(\frac{c_{ij}^k}{c_{ij,\max}^k + 1} \right)^2 \text{, in which} \tag{10}$$

(i)　σ is a measure for the intensity, see figure 3.1. Initial choice $\sigma = 0.5$;

(ii)　$\beta_i^k = \beta(c_{i,\text{TOT}}^k) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{c_{i,\text{TOT}}^k - 25}{10\sqrt{2}} \right) \right]$, and $c_{i,\text{TOT}}^k = \sum_{j|i} c_{ij}$;

(iii)　$c_{ij,\max}^k = \max_{j|i} \{ c_{ij}^k \}$.

3.3. FINALIZING THE MODEL

In this paragraph the final expression for $\mathbb{P}^k(i, j|i)$ is presented. Before that, one final assumption is made.

Assumption 5. The attention and view of an individual is limited to a certain number of people, where this number is denoted by n. Especially in large networks, individuals are assumed to only read a certain portion of new posts instead of all new posts. Besides, the model is asked to make a specific selection of the vast amount of information.

Assumption 5 leads to an important aspect of the filtered model, starting with the following definition:

Definition 4. Let i be fixed and let τ be a positive integer, such that $\tau < p$. At time k define

$$\mathcal{T}_i^k(\tau) := \{ j_1, j_2, ..., j_\tau | \, 1 \leq n \leq \tau : \, j_n \in \mathcal{F}_i \},$$

such that $f(\Delta_{ij}^k, c_{ij}^k) \geq f(\Delta_{il}^k, c_{il}^k)$ for all $j \in \mathcal{T}_i^k(\tau)$ and $l \notin \mathcal{T}_i^k(\tau)$.

In definition 4, $\mathcal{T}_i^k(\tau)$ is defined as the collection of τ individuals for which the value of f is the highest. Recall that $i \notin \mathcal{F}_i$. Now define

$$\tilde{f}(\Delta_{ij}^k, c_{ij}^k) := \begin{cases} f(\Delta_{ij}^k, c_{ij}^k) & \text{if } j \in \mathcal{T}_i^k(\tau), \\ 0 & \text{if } j \notin \mathcal{T}_i^k(\tau), \end{cases}$$

$$\mathbb{P}^k(i, j|i) := \frac{\tilde{f}(\Delta_{ij}^k, c_{ij}^k)}{\sum_{m|i} \tilde{f}(\Delta_{im}^k, c_{im}^k)}, \tag{11}$$

such that $\mathbb{P}^k(i, j|i)$ is a probability mass function (PMF). Obviously, at any time k, we have that

$\sum_{j|i} \mathbb{P}^k(i, j|i) = 1$. Moreover one finds that

$$
\begin{aligned}
\sum_i \sum_j \mathbb{P}^k(N_1 = i, N_2 = j) &= \sum_i \sum_j \left(\mathbb{P}^k(N_1 = i) \mathbb{P}^k(N_2 = j | N_1 = i) \right) \\
&= \sum_i \mathbb{P}^k(N_1 = i) \sum_j \mathbb{P}^k(N_2 = j | N_1 = i) = 1.
\end{aligned}
$$

4. RESULTS

In this section the characteristics and the properties of the filtered model, as is presented in section 3, are described. It contains statements on the behavior of the model one would typically see in simulations, as well as an examination of the influences of the different components of (10). Based upon observations of the model behavior several definitions and claims are stated. This section is concluded with a statistical analysis which states under which conditions the presented filtered model actually alters the opinions of individuals within a group, and when the model leads to the creation of filter bubbles.

4.1. MODEL BEHAVIOR

A description of the general behavior of the model is given with reference to a typical simulation result. Figure 4.1a shows such a result. Included as well is a plot which displays the individual's initial opinion versus its opinion at the end of simulation, see figure 4.1b. An additional simulation outcome is included in appendix B. Figure 4.1 shows the result of a simulation where the number of agents in the network equals 50, and τ equals 8. Figure B.1 shows the result of a simulation where τ equals 25.

Figure 4.2 shows a typical plot for the model with uniform sampling, as presented in [10], this figure is included for the reader to serve as comparison for the results of the filtered model. As has been proven in [10], this model with uniform sampling always converges to group consensus.

What is typically observed in the simulation results of the filtered model is that individual opinions after some time merge into a subgroup of opinions, a so called cluster (formal definition follows later in this section). Also, it seems that all individuals reach such a subgroup eventually, which will also be proven later on in this section. The number of these subgroups depends on the run, but also depends on the choice of τ, as will be shown later on. In cases where a simulation leads to a consensus, as in figure B.1a, its limit value may very well be different than that of the model with uniform sampling. This is illustrated when the reader compares figure B.1a to figure 4.2.

This paragraph is concluded with some formal definitions concerning clusters which will be used to further analyze the model behavior.

Definition 5 (Cluster). The set of individuals $\Omega := \{i_1, ..., i_\omega\}$ form a cluster at time k if $x_i = x_j$ holds for all $i, j \in \Omega$ and also $x_i \neq x_l$ holds for all $i, j \in \Omega$ and for all $l \notin \Omega$. Moreover the size of the cluster is denoted by $|\Omega| = \omega$, that is the number of individuals in the cluster.
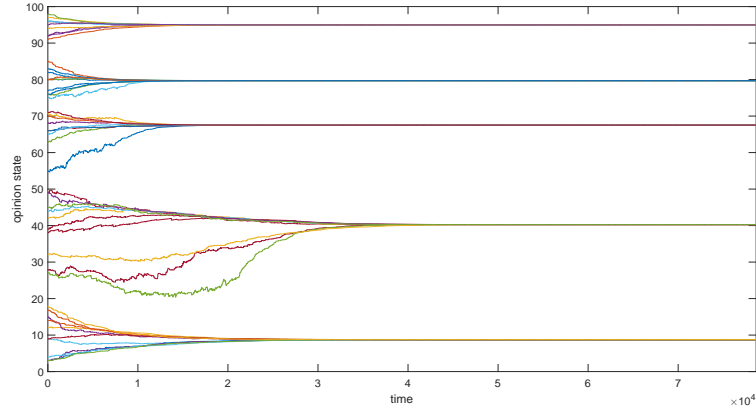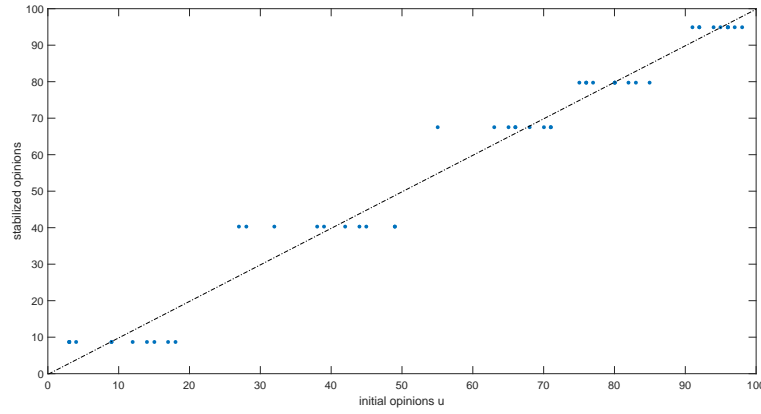
(a) Typical opinion evolution for 50 agents, where $\tau = 8$.

(b) Initial versus final opinions for 50 agents, where $\tau = 8$.

Figure 4.1

Definition 6 (Stable cluster). The set of individuals $\Omega := \{i_1, ..., i_\omega\}$ form a stable cluster if all of the following properties hold for all $i, j \in \Omega$ and for all $l \notin \Omega$:

- $\lim_{k \to \infty} |x_i(k+1) - x_i(k)| = 0$,

- $\lim_{k \to \infty} |x_i(k) - x_j(k)| = 0$, and

- $\lim_{k \to \infty} |x_i(k) - x_l(k)| > 0$.

4.1.1. Model components

In this paragraph the influences of the different components of the model are briefly described. That is, the influences of $g(\Delta_{ij}^k)$ as well as parameter σ, $h(c_{ij}^k)$, and β_i^k. For this section a set of plots is included in appendix C. An intepretation of these plots is given in this section, therefore the author suggests that the reader consults appendix C. Note that this paragraph is meant to give description of the influences of the model components, not a full analysis.
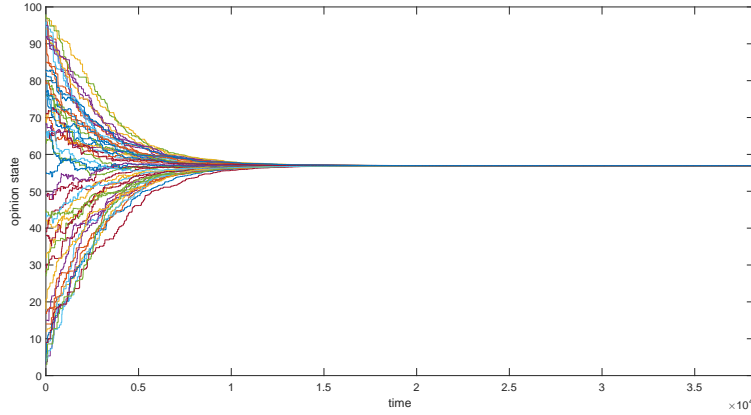
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Figure 4.2: Typical opinion evolution in the Frasca Model.

When only regarding the influence of $g(\Delta_{ij}^k)$, the result is like that of existing bounded confidence models, see figures C.2a and C.2b. A trend that one typically perceives is that different groups of opinions move towards each other slower as σ is increased. Though, due to the random behavior of the model, the simulation time may differ considerably per run. Relating this last finding to figure 3.1, one could say for $\sigma = 1$ and fixing individual i, that i is hardly driven toward individuals j for which $\Delta_{ij}^k > 10$. Whereas this is far more likely for $\sigma = 0.25$. Therefore σ is kept equal to the initial choice of 0.5.

The influence of $h(c_{ij}^k)$ only is shown in figure C.3. Also, $h(c_{ij}^k)$ alone leads to the formation of clusters. What is typical here is that the lines of opinion evolution are far less separated in groups in the early stage of simulation. In figures C.2a to C.2b this separation is far more obvious.

Lastly, if β is set to 1 for all individuals and all time k, a convergent development of opinions is still observed. What is typical for these simulations is that some individuals remain 'in doubt' on which cluster to choose, this can be observed in figure C.4. This 'doubt' for some individuals may be caused by basing the initial sampling on sampling history that is not there, as described earlier.

Overall, one could conclude that the clustering of opinions is caused by $g(\Delta_{ij}^k)$ as well as $h(c_{ij}^k)$, where both components amplify each other. Besides, the choice of including β_i^k (see (9)) pushes 'doubtful individuals' in a certain direction faster.

4.2.   Formation of clusters

Experimental results, as were referred to in the previous section, lead to a set of claims, presumptions and statements on the behavior model. This section is devoted to stating and proving, confirming or motivating these claims, presumptions and statements. One can make a rough distinction between claims on convergence, and claims on the manipulation of opinions.

Claim 1. *A stable cluster contains at least $\tau + 1$ individuals.*

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Proof. Let $\Omega = \{i_1, ..., i_\omega\}$ form a stable cluster at some time k_0, but let $\omega < \tau + 1$. Then for all $i \in \Omega$ there exists an $l \notin \Omega$ such that $\mathbb{P}^k(i, l|i) > 0$. Therefore the probability of sampling edge (i, l) at some time $k > k_0$ is equal to 1.

Then if an edge (i, l) is selected at time k, the opinion $x_i(k)$ is updated with $x_l(k)$. Given that $x_i \neq x_l$ and assuming that $W(i, l) > 0$, we find that $x_i(k) \neq x_i(k+1)$. Therefore $x_i(k+1) \neq x_j(k+1)$, for all $j \in \Omega$. So at time $k + 1$ individual i has left Ω, and therefore Ω is not a stable cluster. $\square$

Claim 1 states the relation that a stable cluster implies that it contains at least $\tau + 1$ individuals. This statement gives rise to a question whether the opposite statement also holds. This opposite claim, including an additional condition, may be found in claim 2 below.

Claim 2. *If $\Omega = \{i_1, ..., i_\omega\}$, $\omega > \tau$, form a cluster at some time k_0, and $c_{ij}^{k_0} > c_{il}^{k_0}$ for all $i, j \in \Omega$ and all $l \notin \Omega$, then the individuals in Ω converge to a stable cluster.*

Proof. Let $\Omega = \{i_1, ..., i_\omega\}$, $\omega > \tau$ form a cluster at time k_0. Also let $c_{ij}^{k_0} > c_{il}^{k_0}$ for all $i, j \in \Omega$ and all $l \notin \Omega$.

It follows that $f(\Delta_{ij}^{k_0}, c_{ij}^{k_0}) > f(\Delta_{il}^{k_0}, c_{il}^{k_0})$, and therefore $\mathbb{P}^{k_0}(i, j|i) > \mathbb{P}^{k_0}(i, l|i)$ holds for all $i, j \in \Omega$ and all $l \notin \Omega$. Since $\omega > \tau$, it holds that $\mathbb{P}^{k_0}(i, l|i) = 0$ for every $i \in \Omega$ and $l \notin \Omega$. Moreover, $\mathbb{P}^k(i, l|i) = 0$ holds for any $k > k_0$.

It follows that Ω meets the properties in definition 6, and therefore the individuals in Ω converge to a stable cluster. $\square$

More experimental results suggest further claims on the convergence of opinions can be made. For any simulations, convergence is supposed satisfied as soon as the following criterion is satisfied:

$$\|x(k) - x(k - 1000)\| < 10^{-5}. \tag{12}$$

Note that by the asynchronous opinion update, a criterion such as $\|x(k) - x(k-1)\| < 10^{-5}$ does not suffice.

All 4900 simulations for appendix D show such a convergent simulation result in finite time. That is, all opinions $x_i(k)$ converge to a cluster as k tends to infinity. By the means of asynchronous opinion update (3), one finds that this convergence is asymptotic. The following claim proposes a statement on convergence.

Definition 7 (Almost sure convergence). $x(k) \in \mathbb{R}^p$ is convergent a.s. to $x(\infty)$ if $\mathbb{P}(\lim_{k \to \infty} x(k) = x(\infty)) = 1$.

Presumption 1. *The filtered model almost sure converges to some $x(\infty)$.*

It is presumed that the filtered model converges almost surely based upon the results of a large amount simulations. As stated before, 4900 simulations for $p = 50$ and for different values of τ show a convergent result for every single simulation. A formal proof for this presumption is left for future work.

Motivation for the verity of presumption 1 may be sought in papers by Zhang and Hong [15] and another paper by Zhang [16]. These papers analyze convergence and specifically clusterization for an asymmetric variant of the Deffuant-Weisbuch model (bounded confidence). The model presented in these papers uses an opinion update protocol identical to that used in the filtered model, that is equation (3) for $\Lambda = I$. The almost sure convergence of such a model of bounded confidence is proven in these papers.

It is believed that the convergence properties presented in [15] and [16] also hold for the filtered model. Reason for this belief is that the filtered model behaves similar to a bounded confidence model, especially at the start of simulation due to the choice of β_i^k (9). Once again, this believe is confirmed by the simulations, though a formal proof remains a subject for future work.

For the claims that follow, it is assumed that the model converges almost surely.

4.2.1. Number of clusters

Claim 1 implies an upper bound for the number of stable clusters that may emerge, which is $\lfloor \frac{p}{\tau+1} \rfloor$. Though, except for the trivial result where $\lfloor \frac{p}{\tau+1} \rfloor = 1$, a result where $\lfloor \frac{p}{\tau+1} \rfloor$ stable clusters emerge, is hardly ever observed. Moreover, it is often observed that the number of stable clusters that emerge is more or less equal to $\frac{1}{2} \lfloor \frac{p}{\tau} \rfloor$. This observation on the model behavior leads to the following claim.

Claim 3. *On average, a number of stable clusters less than or equal to $\frac{1}{2} \lfloor \frac{p}{\tau} \rfloor$ is observed.*

This claim is confirmed by running multiple simulations. That is, for each $\tau \in [1, 49]$, 100 simulations were run until the convergence property was met (12). Among other data, information about the emergence of clusters was stored. Table E.1 in appendix E shows the outcome of these simulations. The table entries represent intensities on the number of clusters that appeared, that is , for instance for $\tau = 5$ a result of 4 stable clusters was observed 30 times. Besides, columns 4 to 6 of table D.1 in appendix D show the average number of clusters, the standard deviation in the number of clusters that emerged, and the highest number of clusters that were observed respectively.

Clearly, the results of the simulations confirm the statement in claim 3. That is, for any value of τ the average number of stable clusters that emerged was less than or equal to $\frac{1}{2} \lfloor \frac{p}{\tau} \rfloor$. One could note that the number of clusters that may emerge for small τ is rather broad, therefore no meaningful statements can be made on this. For larger values of τ one could make a clearer statement on the number of clusters that might emerge.

Figure 4.3 shows plots for the number of clusters that emerged in the simulations. Each line represents a certain number of clusters, see the legend. Values of τ are presented on the horizontal axis, and emergence intensities are shown on the vertical axis. For clarification, for $\tau = 20$ all simulations resulted in the emergence of 1 cluster occurred 53 times, whereas the emergence of 2 clusters occurred 47 times.
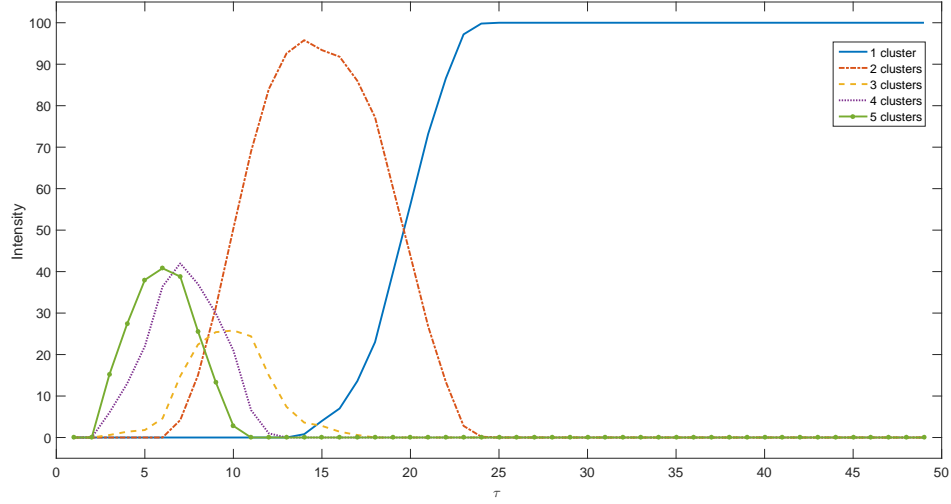
Figure 4.3: Statistics of the number of emerged clusters.

For the figure in which the plots of all emerged number of clusters is displayed, can be found in appendix F. In figure F.1, possibly combined with table E.1, one clearly observes that the spectrum of number of clusters that may emerge for $\tau < 8$ is broad. That is, for $p = 50$ one may observe any of the emerged cluster numbers greater than 2. Whereas, for $\tau = 20$ the number of clusters that will emerge is easier to predict, either 1 or 2.

It is also true that under certain conditions the filtered model produces multiple clusters, and under other conditions only one cluster. This result will be examined in section 4.4. A clear question that arises now is whether the converged opinions in the filtered model are significantly different from those in the model with uniform sampling, and possibly under which conditions. The following paragraph is devoted to answering this question.

4.3. MODIFICATION OF OPINIONS

Before any statement on opinion modificative properties of the filtered model can be made, a definition of 'significant difference in opinion' shall be stated.

Definition 8. Two opinions x and y are defined as being different if $|x - y| > \epsilon$. Here ϵ is defined as being equal to 10% of the length of the total opinion interval. For opinions in the interval $[0, 100]$ it follows that $\epsilon = 10$.

When comparing the converged opinions of the same group, but modeled by different models (for example uniform sampling and filtered sampling), a different definition needs to be stated to define difference in opinion of the group as a whole. Such a definition is required since we may find that the opinion for a certain small number of individuals is not significantly changed, whereas for the rest of the group it is. In such a case one will still mark the opinion of the group as a whole as 'significantly different'.

Definition 9. Let $x(\infty)$ and $y(\infty)$ be the converged outcomes of two models on the same social network. Outcomes $x(\infty)$ and $y(\infty)$ are defined as being significantly different if on average, the individual opinions are significantly different. That is when $\frac{\sum_{i \in \mathcal{P}} |x_i(\infty) - y_i(\infty)|}{p} > \epsilon$, for the same ϵ as in definition 8.

Claim 4. *The resultant opinion vector $x(\infty)$ of the filtered model is significantly different from that of the uniformly-sampled model.*

Claim 4 has been proven/confirmed by executing the Wilcoxon Signed-rank test using MATLAB, for each value of τ between 1 and 49. For each of the tests the number of agents was fixed as $p = 50$, the initial opinions $x(0)$ as well as the weight matrix W were randomly generated before the simulations and kept fixed for all runs. Besides, the order in which all the 'first individuals' are selected, was randomly determined before the simulations, and these as well were kept fixed for all runs. The testing procedure took place as follows:

For each $\tau \in [1, 49]$, 100 simulations were ran (for a sufficient number of time steps to satisfy 12). For every simulation the average opinion difference of the group (definition 9) was calculated. For every τ the Wilcoxon Signed-rank test tests whether the result differs more than 10 or not. The following Wilcoxon Signed-rank test was performed by MATLAB:

H_0: $\frac{\sum_{i \in \mathcal{P}} |x_i(\infty) - y_i(\infty)|}{p} \leq 10$, the average difference is not significant.

H_1: $\frac{\sum_{i \in \mathcal{P}} |x_i(\infty) - y_i(\infty)|}{p} > 10$, the average difference is indeed significant.

The MATLAB output is as follows: $h = 1$ indicates a rejection of the null hypothesis, and $h = 0$ indicates a failure to reject the null hypothesis at the 5% significance level. The outcome of the tests presented in columns 2 and 3 in table D.1 in appendix D.

The statistics show a clear confirmation of claim 4. That is, the filtered model results in a significant change in group opinion, whenever τ is chosen smaller or equal to 33. From these results one can also conclude that for $\tau > 33$ all individual opinions converge to the same limit, where this limit is not significantly different from the opinion limit in the model with uniform sampling.

The previously shown statistical tests were preformed for a fixed number of group size, that is for $p = 50$. One might ask how the model behaves for larger group sizes, for example for $p = 100$, $p = 200$ or any other $p > 50$. Based upon experimenting with the group sizes, the following claim is made.

Presumption 2. *Upscaling p and τ (by the same scale) does not change the outcome of the model.*

This presumption states that the result (number of stable clusters) for $p = 50$ and $\tau = 10$ is the same as for $p = 100$ and $\tau = 20$. We may reasonably assume that this is the case, for the values of $x(0)$ are chosen uniformly over the opinion space. A consequence is that the simulation time until all agents have reached a stable cluster is larger. To support this claim, consider comparing figures G.1a to G.1c in appendix G. One typically sees the same result when upscaling, only the simulation time increases with an increase in group size.

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4.4. RELATING RESULTS TO FILTER BUBBLES

Up until now any relation to filter bubbles has been omitted. In this section the results from the previous sections are related to filter bubbles. The processes that lead to the creation of filter bubbles have several consequences. The consequences that are considered in this section are 'creation of noninteracting subgroups', 'changing opinions' and 'personal filter bubble'.

One property of the filter bubble is that it deliberately changes the opinions of people. In section 4.3 it has been shown that the filtered model does exactly this for $\tau \leq 33$, see table D.1. Even though this steering effect on opinions is a key aspect of filter bubbles, in this model it does not imply that individuals have been split up in several groups of different opinions.

This last phenomenon may be observed for $\tau \leq 22$, again see table D.1. If the model leads to the formation of stable clusters, then these are non-communicating. This means that each cluster forms an isolated group, one could call this group a 'bubble'. Due to the design of the filtered model, in such a bubble people only get to communicate with others that have alike opinions.

As described in the Introduction of this paper, Eli Pariser defines the filter bubble as a personal thing. That is, the bubble is seen from the point of view of an individual, and not as a birds-eye view upon a group. The following definition describes a *personal* filter bubble, as Pariser describes it.

> **Definition 10** (Personal filter bubble). The personal filter bubble of individual i is formed by a fixed set of individuals $\mathcal{B}_i(\tau) := \{j_1, j_2, ..., j_\tau | \ 1 \leq n \leq \tau : \ j_n \in \mathcal{F}_i\}$, such that only posts from individuals from this set are shown.

The following claim (and its proof) states the relation between the filtered model and the emergence of personal filter bubbles.

> **Claim 5.** *Any individual i in a stable cluster develops a fixed set of individuals $\mathcal{B}_i(\tau)$ for which for some $k > K$ ($K \in \mathbb{N}$) it holds that $\mathbb{P}^k(i, j|i) \neq 0$ for all $j \in \mathcal{B}_i(\tau)$, and $\mathbb{P}^k(i, l|i) = 0$ for all $l \notin \mathcal{B}_i(\tau)$.*

Proof. For an individual i to converge to a stable cluster, its probability of sampling an individual from that cluster should converge to 1. Therefore individual i only samples individuals from that cluster. For all j in that cluster Δ_{ij}^k is equal, therefore the sampling probability depends only on c_{ij}^k.

Given a time K, there exists a set of individuals $\mathcal{B}_i(\tau) := \{j_1, j_2, ..., j_\tau | \ 1 \leq n \leq \tau : \ j_n \in \mathcal{F}_i\}$, such that $c_{ij}^K > c_{il}^K$ holds for all $j \in \mathcal{B}_i$ and $l \notin \mathcal{B}_i$. Therefore $\mathbb{P}^K(i, j|i) > \mathbb{P}^K(i, l|i) = 0$. For any $k \geq K$ only individuals from $\mathcal{B}_i$ are sampled, and therefore $\mathbb{P}^k(i, j|i) > \mathbb{P}^k(i, l|i) = 0$ holds for any $k \geq K$. Hence $\mathcal{B}_i$ forms a fixed set. $\qquad\square$

4.5. SUMMARY OF RESULTS

The overall results suggest that the model that was proposed in section 3 leads to the creation of filter bubbles. Under certain model settings (referring to τ) the filtered model leads to a significant opinion change compared to the Frasca Model [10]. Under more strict model settings

the model may lead to the clusterization of opinions into non-communicating or isolated groups of individuals. Moreover, the developed model leads to the generation of personal filter bubbles, where a fixed group of individuals form the entire feed for a certain person.

Furthermore, the model has been proven to asymptotically converge to a stable opinion profile. Multiple claims were also made and proven on the number of clusters that may emerge.

5. Conclusion

In this paper a model was presented that produces the behavior of filter bubbles in opinion dynamics. This model was built upon fundamental assumptions, based on the principles of current online personalization algorithms. The model was shown to presumably converge into stable clusters of opinions. Besides, the model has also been proven to create personal filter bubbles for each of the individuals in the social network. Furthermore, some interesting claims and statements were made on the model's properties.

The full filtered model was based on assumptions 1 and 2, together with the requirements presented in section 3.1. Even though these assumptions and requirements are fundamental, the implementation is not unambiguous. In this research a model was established for it that did the job. No research was done to other implementations of these assumptions and requirements. This may be a point for further research. Besides, the current model only contains two main inputs. As briefly mentioned in the introductory section, Google's personalization algorithms use 57 input signals. Further research may be focused on extending the number of input signals for the model. Lastly, in the current model all sampling history is considered for sampling new edges. One could reasonably assume that more recent history is of more value, than history from longer ago. This feature of weighting or deleting history has not been included, and also this might be a feature for future models.

6. Acknowledgements

My most sincere thanks go to my mentor, Dr. Paolo Frasca. I thank him for his useful advice, fruitful discussions and his overall supervision throughout this project. Besides, I thank him for introducing me to the field of opinion dynamics, and that of filter bubbles. I also wish to thank Dr. Miles Macleod and fellow students David Doppenberg and Mike Wendels for their valuable comments and suggestions for the improvement of this work. I wish to thank fellow student Sander Dijkstra for his general additions to this work.

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A. LIST OF SYMBOLS

Symbol	Discription
$\mathcal{G}$	The graph that represents the network
$\mathcal{P}$	Set of people/users in the network
p	Number of people/users in the nework
$\mathcal{C}$	Set of network connections
$\mathcal{F}_i$	The group of friends (or connections) of individual i
k	Denotes (discrete) time steps
$x(k)$	Vector of opinion states at time k, also denoted as vector of beliefs
$x(0)$	Vector of initial opinion states
$\mathbb{N}_0$	The set $\mathbb{N} \cup \{0\}$
$\mathbb{P}^k(i,j)$	Probability of sampling edge (i,j) at time k
N_1	When sampling of edges, N_1 denotes the first-sampled node
N_2	When sampling of edges, N_2 denotes the second-sampled node
$\mathbb{P}^k(i,j\|i)$	Probability of sampling $N_2 = j$ when $N_1 = i$ has just been sampled
Δ_{ij}^k	The absolute difference in opinion of agent i and j on time k, that is $\|x_i(k) - x_j(k)\|$.
c_{ij}^k	Counter for the number of times directed edge (i,j) has been sampled up until time k. These c_{ij}^k's form the elements of a matrix matrix $C^{(k)}$.
σ	Parameter for vigor of descent of function $g(\Delta)$ in equation (5)
$c_{ij,\max}^k$	Defined as $\max_{j\|i}\{c_{ij}^k\}$
$c_{i,\text{TOT}}^k$	$c_{i,\text{TOT}}^k := \sum_{j\|i} c_{ij}^k$
$\mathcal{T}_i^k(\tau)$	The collection of τ individuals for individual i at time k for which the initial sampling probability is highest
Ω	Notation for the set of individuals $\{i_1, ..., i_\omega\}$ that form a cluster

B. Typical simulation results
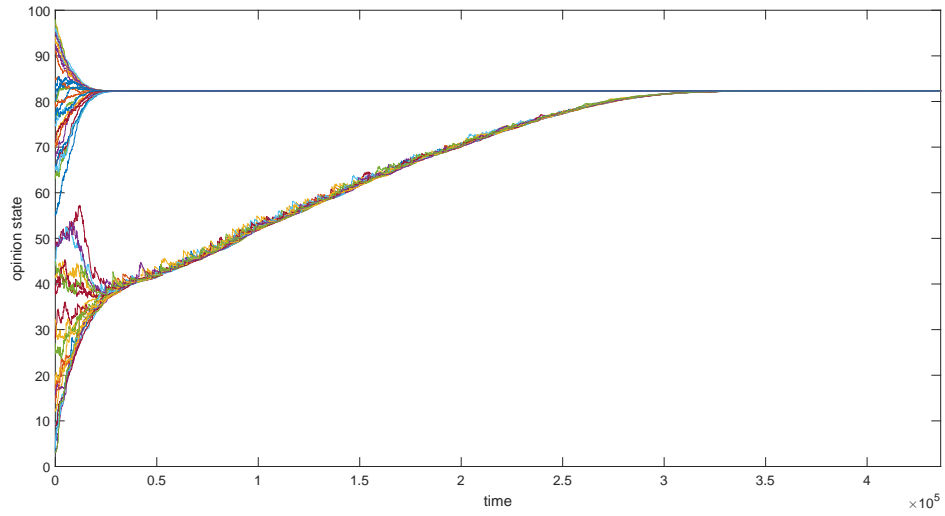
(a) Typical opinion evolution for 50 agents, where $\tau = 25$.
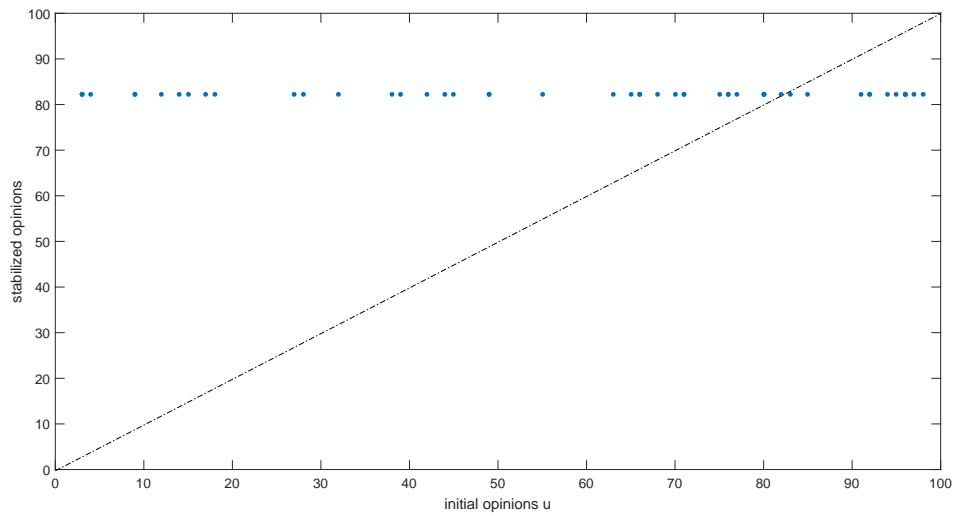
(b) Typical opinion evolution for 50 agents, where $\tau = 25$.

Figure B.1

C. Plots for different model parameters
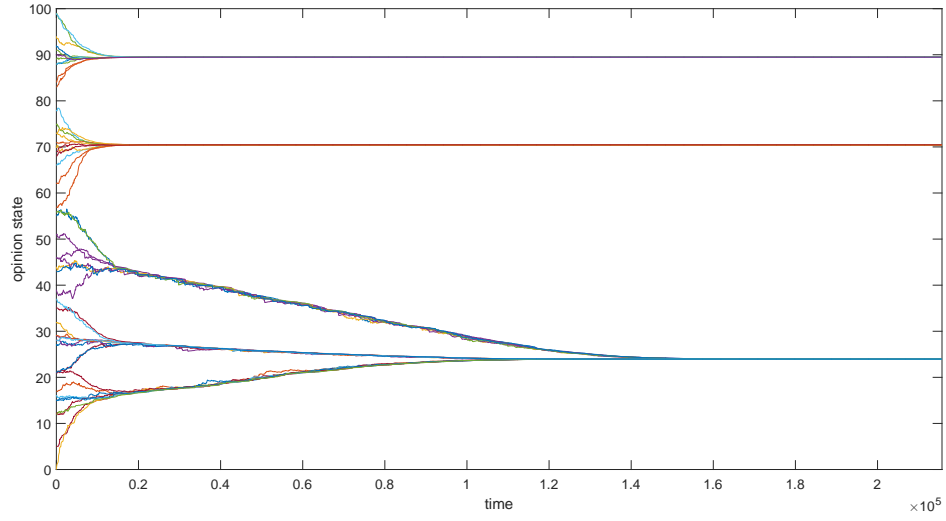
(a) Result of only $g(\Delta_{ij}^k)$, for $p = 50$, $\tau = 10$ and $\sigma = 1$.
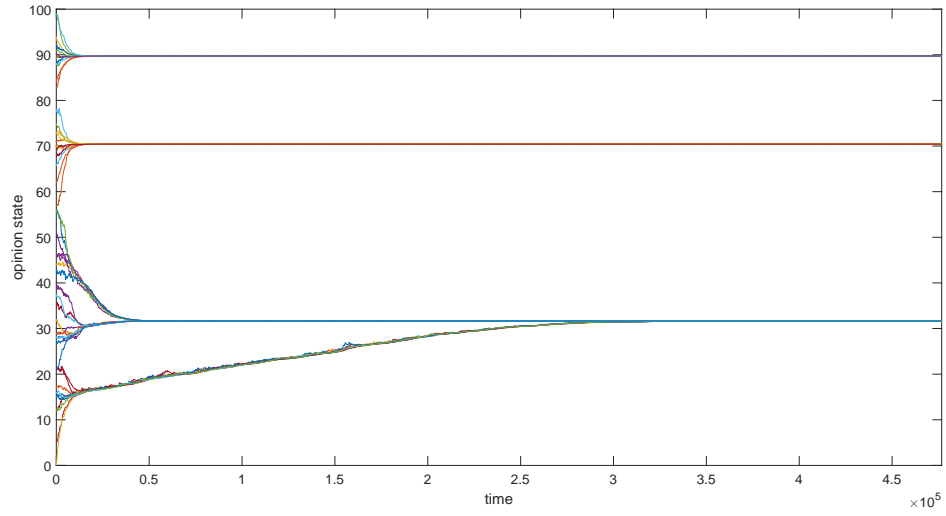
(b) Result of only $g(\Delta_{ij}^k)$, for $p = 50$, $\tau = 10$ and $\sigma = 0.5$.

Figure C.1

C-1

(a) Result of only $g(\Delta_{ij}^k)$, for $p = 50$, $\tau = 10$ and $\sigma = 0.25$.

(b) Result of only $g(\Delta_{ij}^k)$, for $p = 50$, $\tau = 10$ and $\sigma = 0.1$.

Figure C.2

(a) Result of only $h(c_{ij}^k)$, for $p = 50$ and $\tau = 10$.

(b) Initial versus final opinions for $p = 50$ and $\tau = 10$.

Figure C.3

Figure C.4: Result when $\beta = 1$, for $p = 50$ and $\tau = 10$.

D. STATISTICAL RESULTS

Table D.1: Statistical results for $p = 50$.

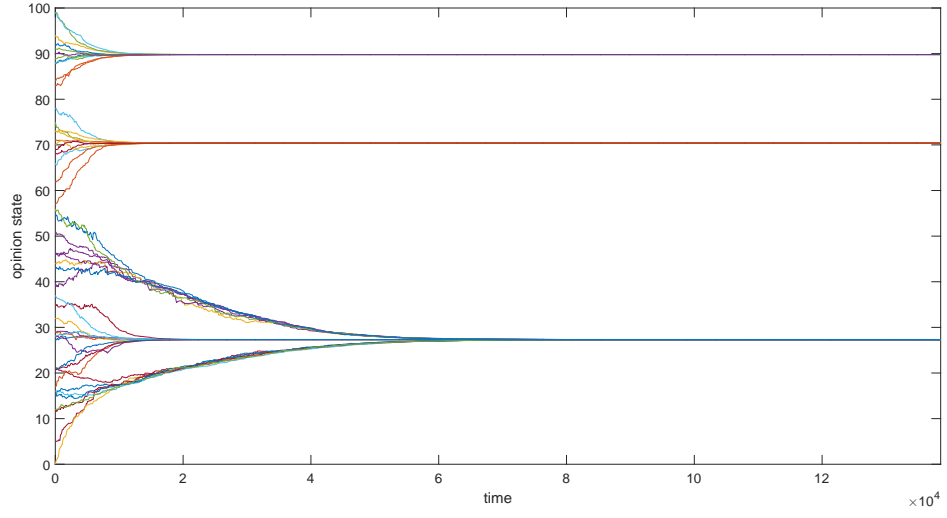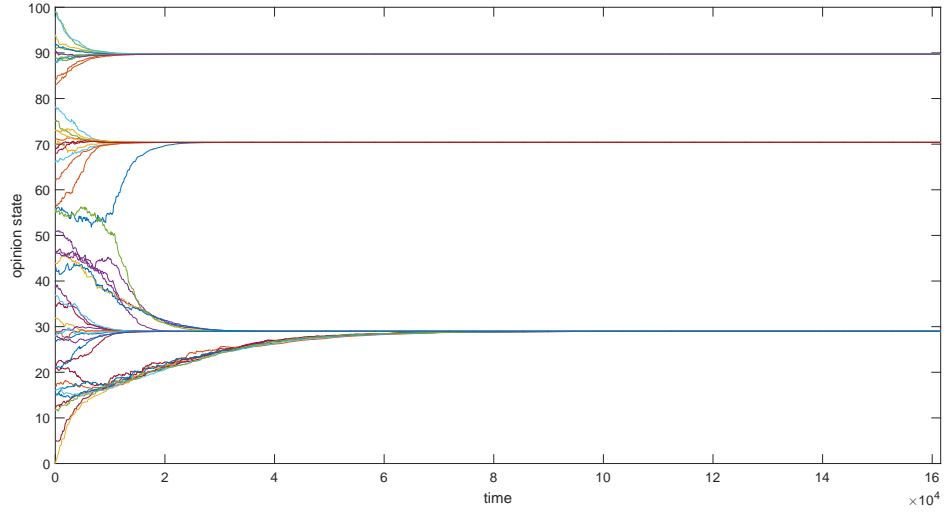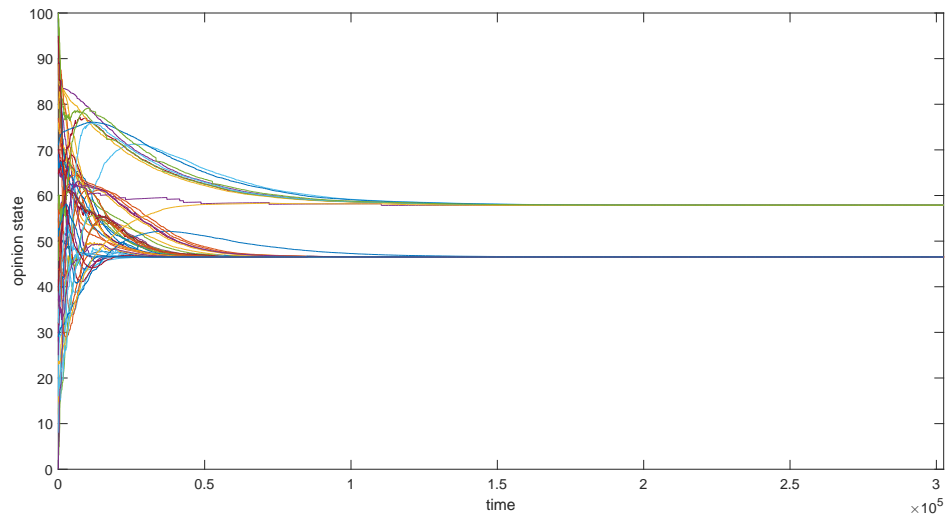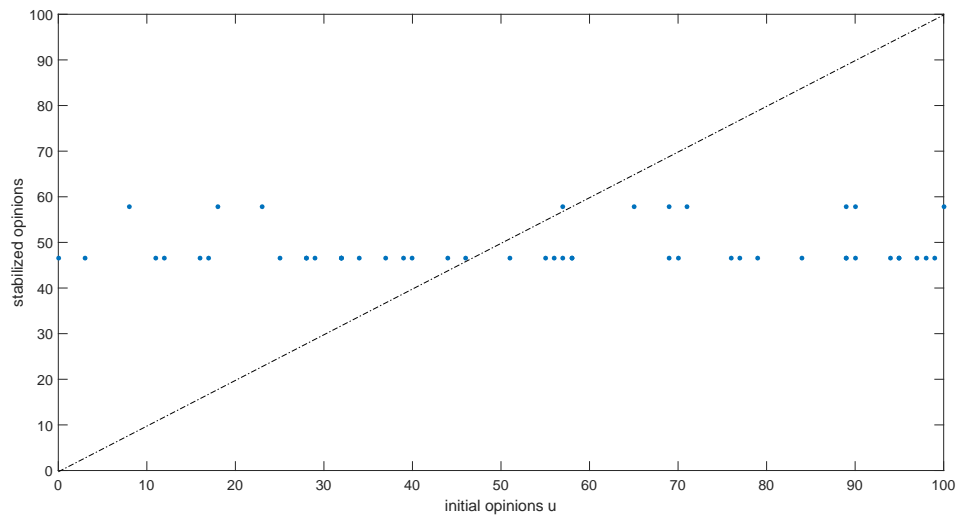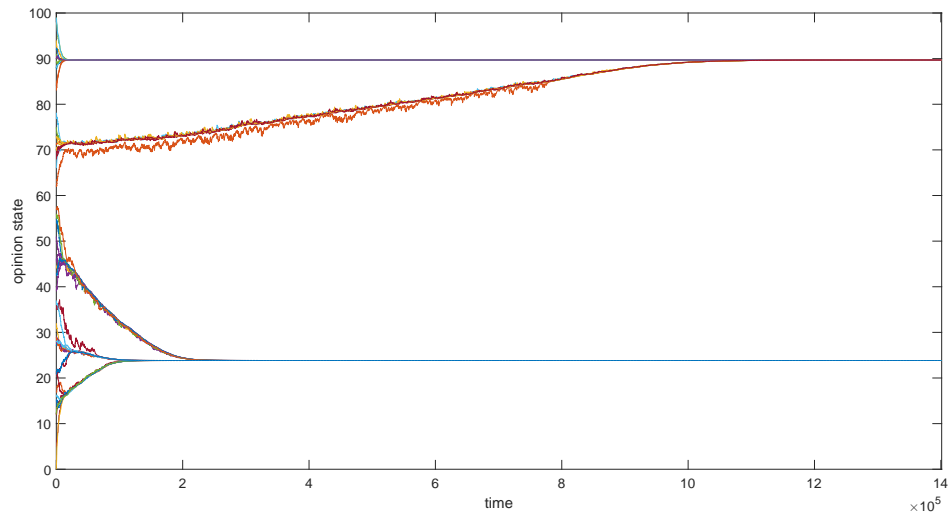| τ	accepted hypothesis	p-value	average number of stable clusters	std number of stable clusters	highest number of stable clusters
1	1	1,9780e-18	16	0	16
2	1	1,9780e-18	9,5400	4,3703	12
3	1	1,9780e-18	7,4600	1,8114	8
4	1	1,9780e-18	5,8600	1,8533	7
5	1	1,9780e-18	4,2100	1,4995	6
6	1	1,9780e-18	3,6600	1,9081	5
7	1	1,9780e-18	4,0700	1,4513	5
8	1	1,9780e-18	3,7200	1,1466	5
9	1	1,9780e-18	2,6900	1,1951	4
10	1	1,9780e-18	2,2900	0,9244	4
11	1	1,9780e-18	2,0500	0,6416	3
12	1	1,9780e-18	1,9100	0,5143	3
13	1	1,9780e-18	1,8500	0,6571	3
14	1	1,9780e-18	1,9000	0,4381	2
15	1	1,9780e-18	1,8500	0,5573	3
16	1	1,9780e-18	1,8900	0,4471	2
17	1	1,9780e-18	1,6500	0,6723	2
18	1	1,9780e-18	1,6600	0,6670	2
19	1	1,9780e-18	1,4500	0,7160	2
20	1	1,9780e-18	1,4000	0,6356	2
21	1	1,9780e-18	1,0900	0,3786	2
22	1	1,9780e-18	1,0100	0,1000	2
23	1	1,9780e-18	1	0	1
24	1	1,9780e-18	1	0	1
25	1	1,9780e-18	1	0	1
26	1	1,9780e-18	1	0	1
27	1	1,9780e-18	1	0	1
28	1	1,9780e-18	1	0	1
29	1	2,4434e-18	1	0	1
30	1	2,1396e-17	1	0	1
31	1	8,9881e-16	1	0	1
32	1	4,6649e-12	1	0	1
33	1	0,0117	1	0	1
34	0	0,9999	1	0	1
35	0	0,9999	1	0	1
36	0	1	1	0	1
37	0	1	1	0	1
38	0	1	1	0	1
39	0	1	1	0	1
40	0	1	1	0	1
41	0	1	1	0	1
42	0	1	1	0	1
43	0	1	1	0	1
44	0	1	1	0	1
45	0	1	1	0	1
46	0	1	1	0	1
47	0	1	1	0	1
48	0	1	1	0	1
49	0	1	1	0	1

E. Statistical results - number of clusters

Table E.1: Results for 100 simulations for each $1 \leq \tau \leq 49$, for $k = 50$. Table entries represent intensities on the number of clusters that appeared, i.e., e.g. for $\tau = 5$ a result of 4 stable clusters was observed 30 times.

		number of clusters →														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
τ	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
↓	2	0	0	0	0	0	0	0	0	0	5	43	52	0	0	0	0
	3	0	0	0	0	0	3	21	76	0	0	0	0	0	0	0	0
	4	0	0	0	0	10	41	49	0	0	0	0	0	0	0	0	0
	5	0	0	3	30	66	1	0	0	0	0	0	0	0	0	0	0
	6	0	0	4	35	61	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	2	45	53	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	14	72	14	0	0	0	0	0	0	0	0	0	0	0
	9	0	21	51	28	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	54	41	5	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	81	19	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	96	4	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	93	7	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	96	4	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	0	97	3	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	4	97	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	16	84	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	18	15	85	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	19	33	67	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	47	53	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	21	87	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	22	99	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	23	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	24	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	25	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	26	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	27	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	28	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	29	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	30	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	31	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	32	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	33	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	34	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	35	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	36	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	37	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	38	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	39	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	40	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	42	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	43	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	44	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	45	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	46	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	47	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	48	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	49	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

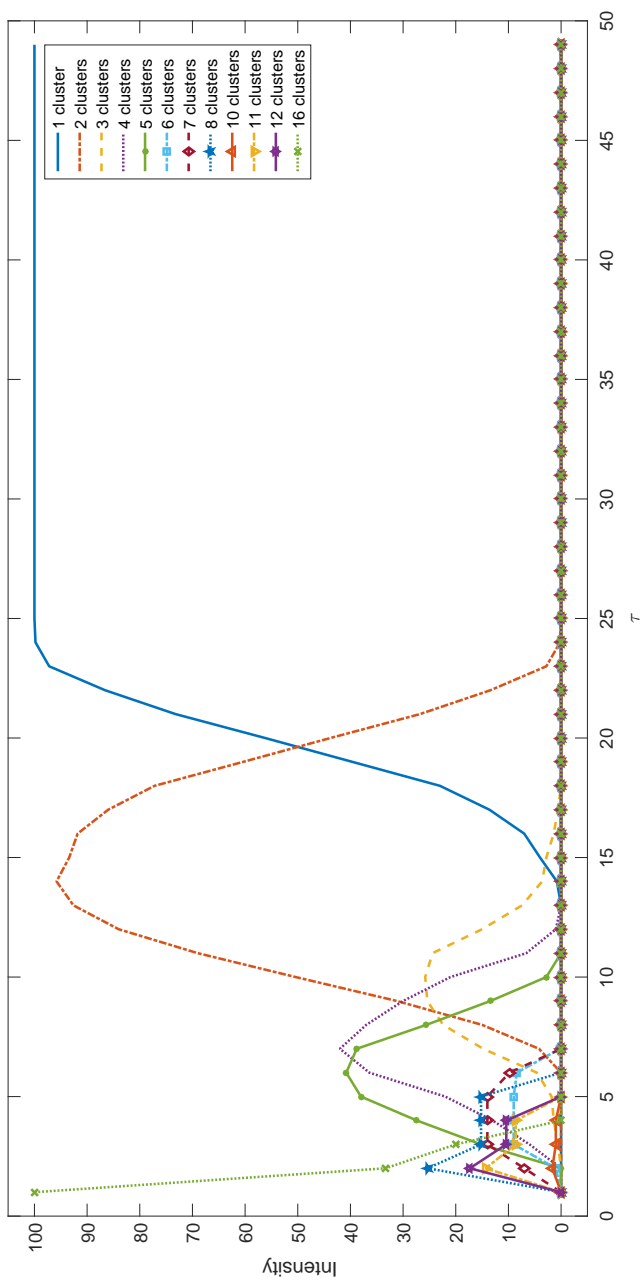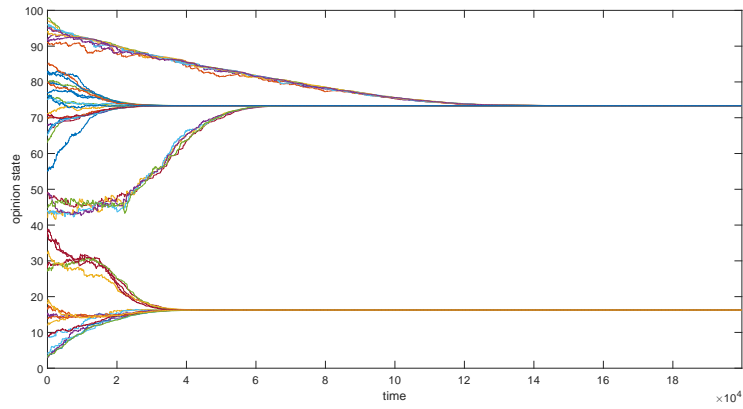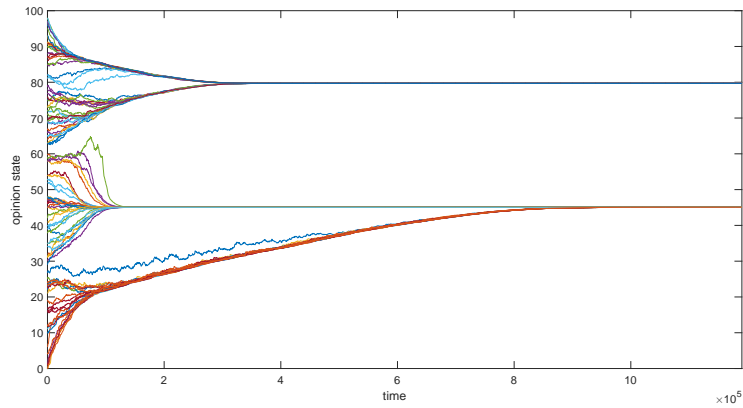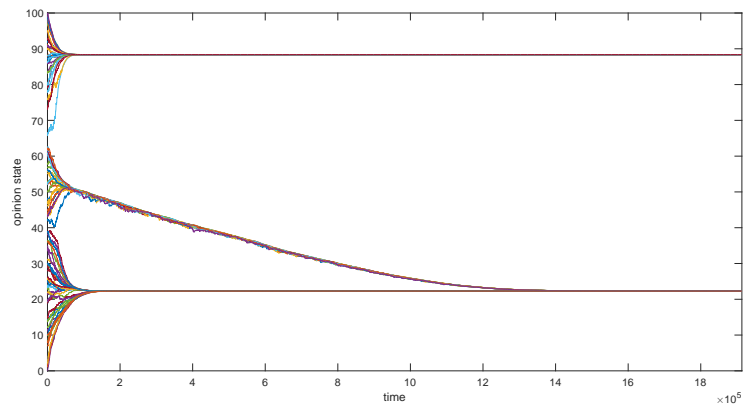## F.   Full plot cluster intensities

Figure F.1: caption

G. Comparison group sized

(a) Typical opinion evolution for 50 agents, where $\tau = 15$.

(b) Typical opinion evolution for 100 agents, where $\tau = 30$.

(c) Typical opinion evolution for 200 agents, where $\tau = 60$.

Figure G.1