



Developing an Energy Observer for Energy Bookkeeping

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MSc Report

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Summary

Robots designed to physically interact with humans, such as the SEP, must be guaranteed to be stable. Proper control is necessary as an unstable robot is a dangerous robot. Given that passive systems are inherently stable, an active system such as the SEP could be made stable through passivity control. Passivity control requires proper energy bookkeeping to function correctly, but due to the lack of required sensors, energy bookkeeping is currently not possible with the SEP.

An energy observer was developed to enable energy bookkeeping for the SEP. By taking the available measurements and a model of the SEP, the energy observer is able to give an estimation of the exchanged energy. Any estimation will be affected by the uncertainty of the model parameters and the EO accounts for this by transforming the probability density functions of the parameters to the energy estimation, giving a new probability density function for the estimated energy. How parameter uncertainty affects the distribution of the estimated energy was assessed with simulations and with experiments with the physical SEP. The results show that for different parameter uncertainty ranges, the probability density function for the estimated energy also changed in shape and range. In the simulations the energy observer was validated using the real exchanged energy, but measurement errors meant that this was not possible for the experiments.

1 Introduction

The Shoulder-Elbow Perturbator (SEP, fig. 1.1) is a 2-degrees-of-freedom (DOF) robotic system designed by Hankamp Rehabⁱ to investigate elbow properties of individuals with hemiparetic stroke (Van Hirtum, 2014). One DOF translates along the z-axis (i.e. normal to the horizontal plane) and is passive, designed to support the weight of the arm through a system of pulleys and springs. The second DOF rotates about the z-axis (i.e. follows the rotation of the elbow) and is active, powered by direct-drive DC motor.

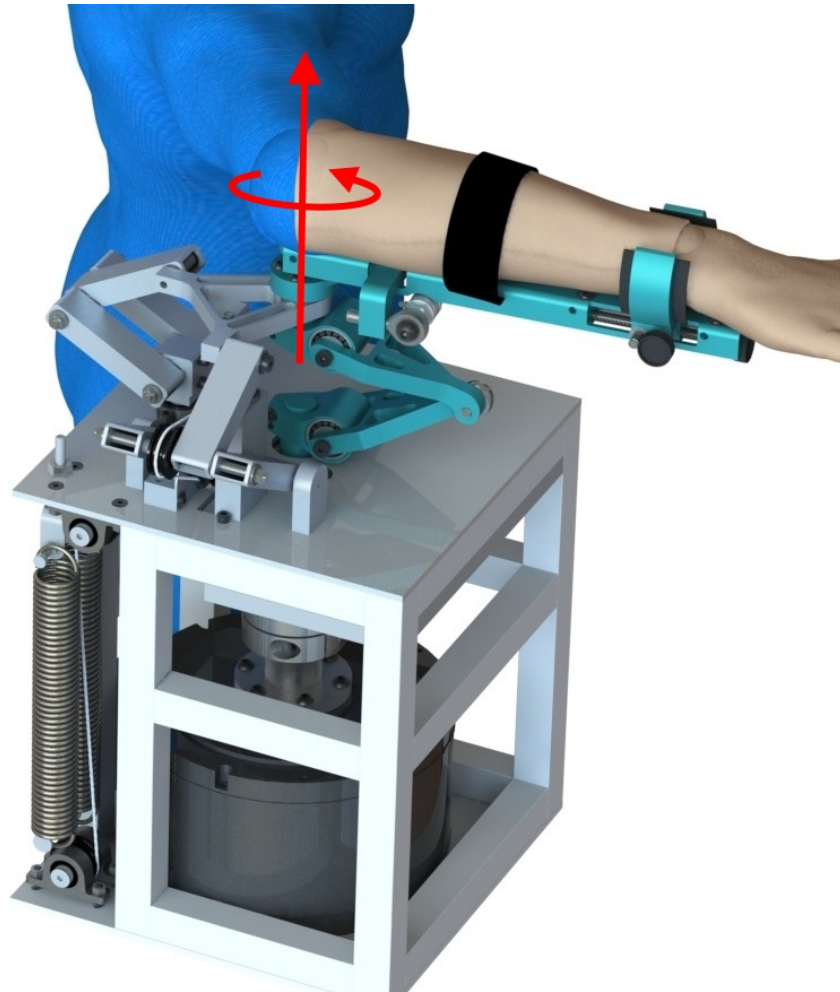


Figure 1.1: The SEP developed by Hankamp Rehab, with one translational DOF and one rotational DOF.

A free-floating stable robotic system that physically interacts with its environment, such as the SEP, can become unstable if not properly controlled. This is unwanted, especially when humans are physically interacting with the robot and can get injured as a result. This makes control stability essential as an unstable robot is an unsafe robot (Tadele et al. (2014)). The field of physical Human-Robot Interaction (pHRI) aims to make robots coupled with humans safe while also meeting their required performance (Alami et al. (2006)). This interconnection of the robot and the human can be considered an active system (the robot) coupled with a passive system, i.e. the human (Hogan, 1989). If the environment is known it is possible to make the interconnected system stable with classic control methods. However, when the environ-

ⁱHankamp Rehab, Enschede, Netherlands, <http://www.hankamprehab.nl>

ment is unknown – which can be the case when the variability of the environment is too large – we cannot assume that the interconnection of systems remains stable (Stramigioli, 2015).

The active system (i.e. the robot) is the reason the interconnected system can become unstable. Conversely, a passive system – which is called passive because it cannot supply more energy than was initially stored into it, see section 2.1 – can never become unstable. A solution to the coupled stability problems could therefore be to make the active robot *behave* passively. If done properly, the robot could be considered a passive system and would no longer become unstable. A method to achieve this is *passivity control*.

Passivity control aims to prevent a system from becoming active, usually by introducing an external controller between the system and the environment, or by including the controller as part of the system. Examples include the PO/PC, which Hannaford and Ryu (2002) designed as a control layer between the robot and environment to dissipate the excess energy coming from the robot. Another approach was proposed by Folkertsma (Folkertsma, 2017, appx. C) with the introduction of embedded, energy-aware actuators (E^2A^2). Rather than trying to make the whole robotic system passive with one PO/PC – which can be a challenging task for large, complex systems – he proposes a decentralized approach where each actuator has a low-level passivity controller: E^2A^2 . This introduces a control layer between the robot actuators and the robot mechanics, designed to make the actuators behave passively. The robot mechanics, consisting of inertial, compliant or dissipative elements, are passive already. A centrally managed energy tank (Schindlbeck and Haddadin, 2015) designates energy budget to each actuator, allowing them to perform tasks using that budget. Once the budget is used up, the actuator is forced to put power output to zero, thereby ceasing energy exchange. Passivity control itself is beyond the scope of this thesis, but it does lead to the problem tackled by this thesis: *energy bookkeeping*.

This thesis considers energy bookkeeping to be the ability to keep track of the energy that is, and has been, exchanged between two systems. This is not a trivial thing and any passivity controller relies on the tracked exchanged energy to properly control the behavior of the system. Incorrect bookkeeping of the exchanged energy makes the passivity control unreliable and can lead to unwanted, and active, behavior of the robotic system.

1.1 Problem Statement

For the aforementioned approaches to passivity control, measuring the energy exchange between systems is necessary. Hannaford and Ryu (2002), for example, assume that correct energy bookkeeping is possible by assuming that the sampling rate of the passivity controller is significantly faster than the dynamics of the system. If this assumption does not hold then correct energy bookkeeping may not be possible. Other causes for incorrect energy bookkeeping are, for example, (variable) time delays between the systems or signal noise affecting the measurements. Even under ideal circumstances where the previous issues do not arise, it may not be possible to measure the required quantities at the locations where the robotic system interacts with the environment. The system might lack sensors to measure the energy exchange, for example. Such is the case with the SEP: it is a multi-part system without rotation measurements at the mass interacting with the human.

When the SEP was originally designed, it was expected that a single mass would be a suitable model representation of the whole system. In practice this representation might not suffice: the first mode of the system might have a relatively low frequency, significantly low enough that it needs to be considered in control design. The stiffness between the rotor mass (henceforth called ‘actuator mass’) and the mass interacting with the human (henceforth called the ‘robot mass’) might therefore have to be considered. For this thesis it will be assumed that this is the case and that a mass-spring-mass model with dissipation is a better representation. In this thesis a linear-time-invariant (LTI) mass-spring-mass system (fig. 1.2) will be used to represent

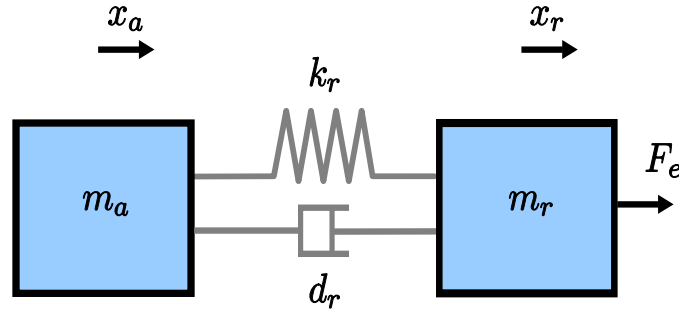


Figure 1.2: The robotic dynamics of the SEP are represented by actuator mass m_a , robot stiffness k_r and robot mass m_r , with dissipation d_r . Two sensors are available: an encoder for position x_a and a force sensor for interaction force F_e .

the SEP. An encoder measures the position x_a of the actuator mass m_a and a force sensor measures the physical interaction force F_e between the robot mass m_r and the human. Without a known velocity v_r of robot mass m_r , energy bookkeeping of the interaction between the SEP and the environment cannot be done. For safety reasons (Tadele et al., 2014) or for performance reasons, such bookkeeping can be necessary. With correct energy bookkeeping of the physical interaction between the SEP and the environment (i.e. the user), it could be possible to control the energy exchange there. The introduction of energy bookkeeping will allow passivity control to be used to guarantee a stable SEP.

The goal of this thesis is to develop an algorithm able to perform energy bookkeeping for the SEP at the mass interacting with the human. This algorithm will be called the energy observer (EO).

1.2 Solution Proposal

Given knowledge about the SEP's robot dynamics, it is possible to estimate the velocity v_r of the robot mass. By combining the measurements of interaction force F_e with that velocity estimation the EO will be able to give an estimation of the interaction energy exchanged between the SEP and environment.

However, the robot dynamics of SEP might not be fully known and there might be uncertainty about parameters m_a , k_r , d_r and m_r . These uncertainties would propagate into the estimation of v_r and, by extension, into the energy estimation. If despite proper system identification there are still parameter uncertainties, these uncertainties will have to be considered. The design of the EO incorporates this by giving energy estimation a known uncertainty. This leads to the first research question:

Research Question 1. *How do uncertainties in the system parameters affect the energy estimation of the Energy Observer?*

By incorporating the system uncertainties into the energy estimation, the true exchanged energy is expected to be within the uncertainty bounds. To validate this expectation, the energy estimations will be compared to an externally measured 'true exchanged energy' through experiments both virtual and with the SEP itself. Thus the second research question is:

Research Question 2. *Does the true exchanged energy lie within the uncertainty range of the estimated exchanged energy?*

1.3 Thesis overview

Following this Introduction comes a Background chapter that gives further detail to concepts that will be used in this thesis. In the Method chapter the design of the EO is given and a

description of the simulations and experiments that will be done with the EO and SEP. The Results chapter gives the results of those simulations and experiments. Answers to the research questions are summarized in the Conclusions & Discussion chapter, which are then followed by the appendices and bibliography.

2 Background

2.1 Passivity

The concept of passivity was described by Colgate (1988) where he relates the passivity of a system to the amount of useful work that could be extracted from it. The first and second law of thermodynamics dictate that the total amount of power P that can be extracted from a physical system must be positive and finite. Following Folkertsma (2017), consider the following system

$$\dot{x} = f(x, u), \quad z = g(x, u) \quad (2.1)$$

where x is the state of the system, u the input and z the output, which take their values in their respective manifolds X , W and Z . The *supply function* is a mapping

$$s : W \times Z \rightarrow \mathbb{R}. \quad (2.2)$$

Supplied power $P = \langle F, v \rangle$ is an example of a supply function. The system eq. (2.1) is dissipative if there exists a storage function

$$E : X \rightarrow \mathbb{R}. \quad (2.3)$$

such that

$$E(x(t_1)) \leq E(x(t_0)) + \int_{t_0}^{t_1} s(w(t), z(t)) dt. \quad (2.4)$$

If the inequality in eq. (2.4) holds, the system can never contain more energy than initially present and consequently the energy is bounded. In physical systems the total energy in the system is often the choice for the storage function.

An active system does not have a lower bounded energy function E and can act as an energy source: it can become unstable and in his chapter on Energy-Aware Robotics, Stramigioli (2015) proved his theorem that for any active system, there can be found a passive system that can make the coupled system unstable.

Theorem 2.1. *Given a non-passive system Σ , there always exists a passive system Υ which connected to Σ gives rise to an unstable behavior of the interconnection of Σ and Υ .*

Theorem 2.1 states that an active system can never be guaranteed stable when coupled to a passive system. Therefore it is useful to make the robot behave passively through passivity control. This moves the requirements for coupled stability away from the now-passive robot to the environment, which can potentially still be an active system. In the case of SEP this is not a problem, as Hogan (1989) showed that the human elbow can be considered passive.

2.2 Port-based modeling

Port-based modeling is a unifying framework for the modeling of systems of different physical domains (mechanical, electrical, hydraulic, thermal, etc.) that uses energy as its *lingua franca*. It was pioneered by Henry Paynter in the 1950s (Paynter (1961)) and expanded by Breedveld (1984, 2009). The location where energy exchange occurs with a system is called a *port*. For most mechanical systems, for example, this is where forces act upon the system. This location, henceforth described as a ‘port’ of that system, represents an interface where energy flows in and out of the system. The power at a port is often described by conjugate variables such as the force-velocity pair. These pairs can be generalized and called *effort* e (e.g. force, potential or pressure) and *flow* f (e.g. velocity, current or volume flow rate) variables. The power conjugate of effort and flow is shown in eq. (2.5):

$$P = \langle f, e \rangle. \quad (2.5)$$

A useful graphical notation of these energy exchanges between systems comes from the field of chemistry in the form of *bonds*. The analogy of chemical bonds connecting molecules was used to create *power bonds* that connected systems' ports. The graph depicting such interconnected systems is therefore called a *bond graph* (fig. 2.1 and section 2.3).

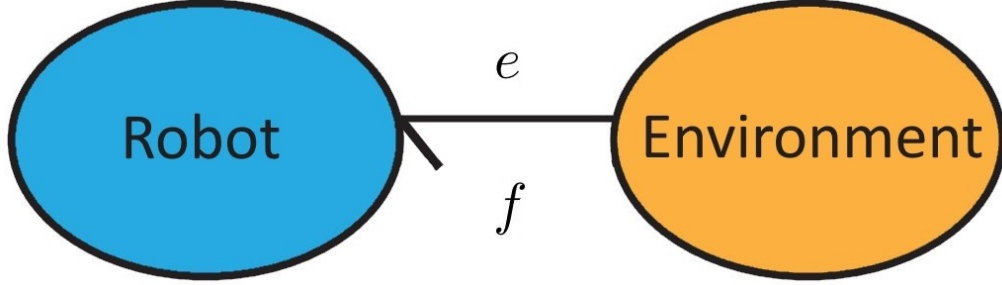


Figure 2.1: The two systems robot and environment are connected through a bond with effort e and flow f , creating the robot-environment interface. The half arrow gives the direction of positive power, meaning energy flows from the environment into the robot.

Port-based modeling gives a concise description of the energy $E(t)$ in the system with relation to the power exchange $P(t)$ at its ports. For a system with N ports, eq. (2.6) gives the total energy $E_t(t)$ in the system.

$$\begin{aligned} E(t) &= \left(\sum_{n=1}^N E_n(t) \right) - E_d, \\ &= \left(\sum_{n=1}^N \int_{-\infty}^t P_n(\tau) d\tau \right) - E_d, \end{aligned} \quad (2.6)$$

where $P_n(t)$ represents the power at time t , E_n is the total energy exchanged through port n until then. E_d is the total dissipated energy. The system is passive iff $E(t) \geq 0, \forall t \geq 0$ – see section 2.1 and eq. (2.4).

By linking systems through their ports the framework creates clarity about the flows of energy through the total interconnected system. The location where two systems are connected can also be called an interface. In the example of fig. 2.1 we can describe the power bond being the robot-environment interface.

If all the ports of a system are known and the power flow is measured, it is possible to discover whether that system behaves passively or actively.

2.3 Bond Graph

The basics of bond graphs are described here, following Broenink (1999).

2.3.1 Elements

A bond graph consists of bonds connecting different systems together. These systems can be divided into abstract subsystems called *elements*. These elements can each be considered as ideal and are the following:

- C** Generalized potential energy storage that is power conservative. Examples are springs (mechanical) or capacitors (electrical).
- I** Generalized kinetic energy storage that is power conservative. Examples are mass (mechanical) or inductors (electrical).
- R** Generalized dissipation element. Examples are dampers (mechanical) or resistors (electrical).

- Se** Effort source, e.g. gravity (force) or electric mains (voltage). Can also be written as a **MSe** when the source is modulated by an external input signal that is part of the bond graph. For example a residual-current device might ground the electric mains, changing its source voltage.
- Sf** Flow source, e.g. pump (water flow) or electric current source. Similarly to **MSe**, it can also be written as **MSf**.
- TF** Transformer, e.g. gears, levers. These elements describe an (ideal) effort-to-effort and flow-to-flow relationship, and is power conservative.
- GY** Gytrators, e.g. piezoelectric sensors, electric motors. Similar to the transformer, the gyrator element describes an ideal effort-to-flow and flow-to-effort relationship, and is power conservative.
- 0** The so-called 0-junction links multiple elements in a power-continuous manner and is analogous to Kirchhoff's current law. All the connected bonds have an equal effort, while the sum of the flows of the connecting bonds is zero. This element is power conservative.
- 1** The so-called 1-junction links multiple elements in a power-continuous manner and is analogous to Kirchhoff's voltage law. All flows of the connecting bonds are equal while the sum of efforts of the connecting bonds is zero. This element is power conservative.

2.3.2 Mass-spring-damper system

Consider the example of a mass hanging from a fixed-world ceiling by a spring-damper combination, shown in fig. 2.2a. The three parts of the system, i.e. the mass, spring and damper, can be represented by the **I**, **C** and **R** elements respectively. The constant force due to gravity can be considered a source element **Se**. Figure 2.2b shows the bond graph of the system described in fig. 2.2a.

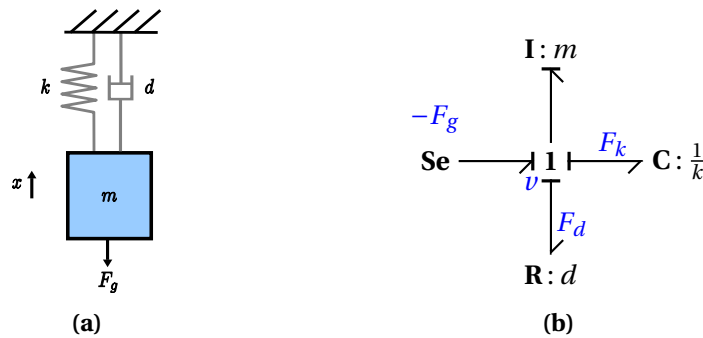


Figure 2.2: (a) A mass m hanging from the ceiling, affected by spring force F_k , damper force F_d and gravitational force F_g . (b) The bond graph of the system in (a), represented by the **I** (mass), **C** (spring), **R** (damper) and **Se** (gravity) elements.

In terms of power flows, the half arrows show the direction of positive energy flow, indicating that **Se** adds to the energy of the system, while the other three elements (**I**, **C** and **R**) only store or dissipate energy. In this specific case, gravity becomes a negative power flow due to the convention of a negative force F_g . The state of system can be described through its total energy

$$E_t = \int_{-\infty}^t P_t(\tau) d\tau, \quad (2.7)$$

with

$$\begin{aligned} P_t &= P_g - P_m - P_k - P_d, \\ P_g &= -F_g v, \\ P_m &= F_m v, \\ P_k &= F_k v, \\ P_d &= F_d v. \end{aligned}$$

Equation (2.7) gives a concise, but complete, description of the system.

$$\mathcal{J} \longrightarrow \text{I} : m$$

(a) Element **I** : m with a effort-in causality.

$$\mathcal{J} \longleftarrow \text{C} : \frac{1}{k}$$

(b) Element **C** : $\frac{1}{k}$ with an flow-in causality.

Figure 2.3: Two examples of possible causalities for bonds connected to arbitrary system $\mathcal{J}$.

2.3.3 Causality

In addition to half arrows, a short vertical line is drawn at one end of a bond to show the *preferred causality* of an element (fig. 2.3). This preferred causality is a result of the physical system represented by the element. For example, consider an electrical capacitor. If a voltage V was imposed on it, the resulting current I would be

$$I = C \frac{dV}{dt}. \quad (2.8)$$

This is a flow-out causality, but represents a problem: physical systems cannot be pure differentiators as shown in eq. (2.8) because a sudden jump in V would require the system to provide infinite energy. The opposite causality (i.e. effort-out causality), when a current is imposed on the capacitor, is therefore preferred:

$$V = \frac{1}{C} \int I dt. \quad (2.9)$$

In general, preferred causality is an intuitive result of the physical systems that are being represented, but for a more systematic approach see (Broenink, 1999; Breedveld, 1984, 2009).

2.3.4 Signal

Sometimes the flow or effort of a bond or element is used as an input signal for another part of the bond graph. This is represented as an arrow, similar to block diagram signals. The concept is shown in fig. 2.4. The choice of using signal lines instead of bonds is very dependent on the situation.

$$\mathcal{J} \longrightarrow \text{I} : L \xrightarrow{i} \text{MSf} \longleftarrow \text{C} : C$$

Figure 2.4: An example of a signal line used in bond graphs: the current i in the inductor element **I**, connected to arbitrary system $\mathcal{J}$, is used as input signal for capacitor element **C**.

3 Method

The goal of this thesis is to create an energy observer (EO) that will be validated. The EO will be assessed for the effects of parameter uncertainty on the energy estimation at the interaction port. Different parameter uncertainties will be compared and for each case, the expected value of the energy estimation will be calculated. This expected value will then be compared to the true exchanged energy.

The EO will be designed in a number of steps. The mass-spring-mass system that represents the SEP will be used as a base for the EO. The system has a limited number of state measurements, similar to the SEP itself, which will be used to calculate the full state of the system. If the full state of the system is known it becomes possible to calculate the exchanged energy.

First the EO will be designed with a model of the SEP and all tests will be done in simulations. The initial design will be ideal and then expanded to include parameter uncertainties. This final version of the EO will be evaluated for multiple parameter uncertainty values and validated against the true exchanged energy.

Second the EO will be adapted to work on the SEP. The SEP's two inputs (x_a and F_a) will be the state measurements that the EO uses to estimate the full state of the system (i.e. the SEP). The parameter values will be based on the SEP with a chosen uncertainty. A set-up will be made to find an external reference measure of the true exchanged energy to validate the EO.

3.1 Dynamic model of the SEP with its energy ports

The mass-spring-mass system that represents the SEP is shown in the bond graph of fig. 3.1. The controller $\mathcal{C}$ applies force F_a on actuator mass m_a , which is connected to robot mass m_r by spring k_r and damper d_r . Robot mass m_r is coupled to the environment $\mathcal{Z}$ regenerating a reaction force F_e .

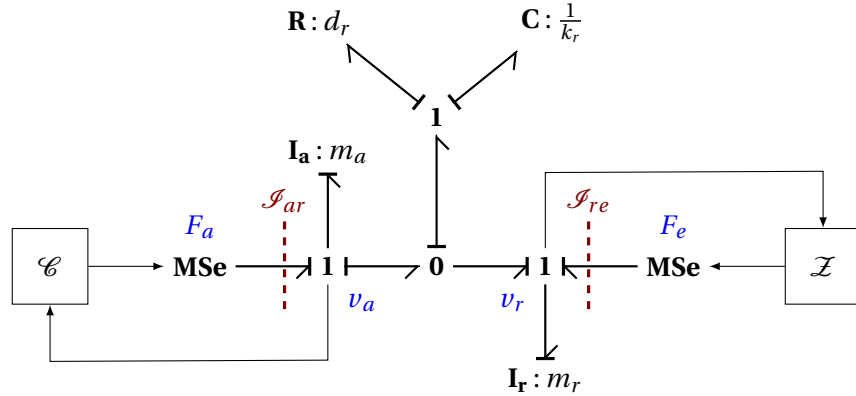


Figure 3.1: The bond graph represents the passive robot system attached to the actuator on the left and the environment on the right. It consists of actuator mass m_a and robot mass m_r with internal dynamics represented by spring k_r and damping d_r . Controller $\mathcal{C}$ applies a force F_a on mass m_a and the environment $\mathcal{Z}$ applies force F_e on mass m_r . The $\mathcal{J}_{ar}$ and $\mathcal{J}_{re}$ ports are the two locations where energy can be exchanged with the system.

The robot system interacts with the world around it through the two $\mathcal{J}_{ar}$ and $\mathcal{J}_{re}$ ports. All energy exchanged with the system goes through those ports, meaning that

$$E_{total} = E_{ar} + E_{re} + E_{diss}, \quad (3.1)$$

$$E_{ar} = \int_0^t P_{ar}(\tau) d\tau, \quad (3.2)$$

$$E_{re} = \int_0^t P_{re}(\tau) d\tau, \quad (3.3)$$

$$E_{diss} = E_{d_r} = \int_0^t d_r (v_r(\tau) - v_a(\tau))^2 d\tau \quad (3.4)$$

where power P_{ar} and P_{re} are

$$P_{ar} = \langle F_a, v_a \rangle, \quad (3.5)$$

$$P_{re} = \langle F_e, v_r \rangle, \quad (3.6)$$

and E_{diss} represents all dissipated energy. The (t) is omitted for brevity. Using the measurements that the SEP provides, position x_a and interaction force F_e are considered known. Velocity v_a can be derived directly from x_a . The controller $\mathcal{C}$ sets F_a and thus it is also known. This means that keeping track of E_{ar} and doing passivity control at this port is possible. The environment $\mathcal{Z}$ and the robot system are both passive systems – masses, spring and dampers are passive elements – but the actuator is not. If the $\mathcal{J}_{ar}$ port is made passive through passivity control, then the full interconnected system is also passive (Folkertsma, 2017).

3.2 Design of the Energy Observer

For reasons of safety and performance it is also necessary to keep track of E_{re} . This is not trivial because v_r is not known. While the force sensor measures F_e , an estimation of the dynamic behaviour of the robot mass needs to be done to estimate the exchanged energy at port $\mathcal{J}_{re}$. An energy observer (EO) will be designed for this. It is added to the total system as shown in fig. 3.2.

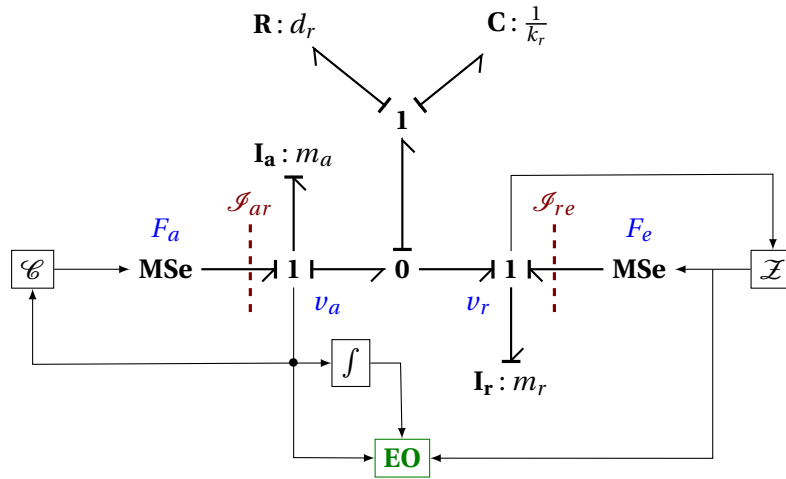


Figure 3.2: Building on fig. 3.1 this figure includes the EO in green. The inputs x_a , v_a and F_e are shown in three signal lines entering the EO. The integration of v_a to x_a is shown by an integration block, which represent the physical relation between the two quantities. The integration block is not part of the EO.

The equations of motion for the SEP (figs. 3.1 and 3.2) are

$$m_a \ddot{x}_a = F_a + k_r (x_r - x_a) + d_r (v_r - v_a) \quad (3.7)$$

$$m_r \ddot{x}_r = F_e - k_r (x_r - x_a) - d_r (v_r - v_a). \quad (3.8)$$

Equations (3.7) and (3.8) can also be written in the state-space form. The state equations for this system are

$$\dot{\mathbf{x}} = \mathbf{A}^* \mathbf{x} + \mathbf{B} \mathbf{u}, \quad (3.9)$$

$$\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u}, \quad (3.10)$$

with $\mathbf{x} = [x_a, v_a, x_r, v_r]^\top$, $\mathbf{u} = [F_a, F_e]^\top$ and $\mathbf{y} = \mathbf{x}$. Matrix $\mathbf{A}^*$ includes an asterisk to differentiate it from matrix $\mathbf{A}$, first shown in eq. (3.17).

Solving eqs. (3.7) and (3.8) gives the full state of the SEP. The EO will be designed to solve eq. (3.8) with x_a , v_a and F_e as inputs (fig. 3.2). From eq. (3.8) it follows that the EO will need information about three parameters: k_r , d_r and m_r . The estimations of these parameters will be denoted as $\hat{m}_r$, $\hat{k}_r$ and $\hat{d}_r$. The state equations for the EO are similar to those described in eqs. (3.9) and (3.10), with the following changes: $\mathbf{u} = [x_a, v_a, F_e]^\top$ and $\mathbf{x} = [x_r, v_r]^\top$, with state-space matrices for the EO becoming

$$\mathbf{A}^* = \begin{bmatrix} 0 & 1 \\ -\frac{\hat{k}_r}{\hat{m}_r} & -\frac{\hat{d}_r}{\hat{m}_r} \end{bmatrix} \quad (3.11)$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\hat{k}_r}{\hat{m}_r} & \frac{\hat{d}_r}{\hat{m}_r} & \frac{1}{\hat{m}_r} \end{bmatrix} \quad (3.12)$$

$$\mathbf{C} = \mathbf{I}_{2 \times 1} \quad (3.13)$$

$$\mathbf{D} = \mathbf{0}_{3 \times 1} \quad (3.14)$$

3.3 Simulations with the model of the SEP

To simulate the interconnected system the model shown in fig. 3.2 will be used. While working in simulations it is possible to merge the SEP state-space equations and the EO state-space equations into one set of equations, with some alternations. These alternations are listed here:

1. There is no input vector $\mathbf{u}$, given that $F_a = \mathcal{C}(\mathbf{x})$ and $F_e = \mathcal{Z}(\mathbf{x})$, each behaving as LTI systems.
2. A new state-space matrix $\mathbf{E}$ is introduced to represent the controller and environment.
3. State-vector $\mathbf{x}$ is expanded to include both the state of the SEP and the state of EO, taking the form $\mathbf{x} = [x_a, v_a, x_r, v_r, \hat{x}_r, \hat{v}_r]^\top$

By taking these alternations into account, the new state-space equations are

$$\dot{\mathbf{x}} = \mathbf{A}^* \mathbf{x} + \mathbf{E} \mathbf{x} \quad (3.15)$$

$$\mathbf{y} = \mathbf{C} \mathbf{x}. \quad (3.16)$$

From eq. (3.15) is only a small step to the set of homogeneous equations

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} \quad (3.17)$$

with

$$\mathbf{A} = \mathbf{A}^* + \mathbf{E}. \quad (3.18)$$

The elegance of writing the whole system in this form is the homogeneous solution

$$\mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}_0 \quad (3.19)$$

where $\mathbf{x}_0$ is the initial state of the SEP and EO. The matrix exponential $e^{\mathbf{A}t}$ is defined as the power series

$$e^{\mathbf{A}t} = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \dots + \frac{\mathbf{A}^k t^k}{k!} + \dots \quad (3.20)$$

and is the same size as matrix $\mathbf{A}$ – see appendix A.

Using eq. (3.19) it is possible to solve eqs. (3.3) and (3.6) to find $\hat{E}_{re}$ and E_{re} . There will be no difference between the true and estimated energy because the parameters of the SEP model and the EO were the same: $\hat{m}_r = m_r$, $\hat{k}_r = k_r$ and $\hat{d}_r = d_r$.

3.3.1 Adding parameter uncertainty to the simulations

In practice the parameters will have uncertainties and cause the estimated parameters to not match the true parameters: $\hat{m}_r \neq m_r$, $\hat{k}_r \neq k_r$ and $\hat{d}_r \neq d_r$. As a result, the estimated $\hat{x}_r$ and $\hat{v}_r$ will deviate from the true x_r and v_r . By extension, the estimated energy $\hat{E}_{re}$, will deviate from the true exchanged energy E_{re} . The uncertainty of $\hat{E}_{re}$ will come directly from the parameter uncertainties, as other uncertainties caused by non-ideal sensors, discretization of signals or time-delays are still ignored.

The uncertain parameters $\tilde{k}_r$, $\tilde{d}_r$ and $\tilde{m}_r$ will be based on the estimated parameters of the SEP but due to measurement errors the parameters will have an uncertainty. In this thesis these uncertainties are chosen to be uniform:

$$\tilde{k}_r = \hat{k}_r \pm \Delta k_r \quad (3.21)$$

$$\tilde{d}_r = \hat{d}_r \pm \Delta d_r \quad (3.22)$$

$$\tilde{m}_r = \hat{m}_r \pm \Delta m_r \quad (3.23)$$

To find the uncertainty of a derived variable (i.e. $\hat{E}_{re}$) it can be possible to analytically transform the uncertainty of the input variable – or parameter, in this case.

The uncertainty of a variable can be described by its probability density function. For example, if $f(\tilde{m}_r)$ is the probability density function of $\tilde{m}_r$ and $g(\hat{E}_{re})$ is the probability density function of $\hat{E}_{re}$, then there might be found a transformation function T such that

$$g(\hat{E}_{re}) = T(f(\tilde{m}_r)). \quad (3.24)$$

This requires knowledge about how $\hat{E}_{re}$ is derived from $\tilde{m}_r$. If there is a function y such that

$$\hat{E}_{re} = y(\tilde{m}_r), \quad (3.25)$$

with inverse $\tilde{m}_r(y)$ then

$$g(\hat{E}_{re}) = f(\tilde{m}_r(y)) \frac{d\tilde{m}_r}{dy}, \quad (3.26)$$

if two constraints on function y are met:

1. Function y is invertible and single-valued.
2. Inverse function $\tilde{m}_r(y)$ is also single-valued.

This means that for every value of $\tilde{m}_r$ there must be a unique value $\hat{E}_{re}$. If the constraints on function y are not met, then there is no analytical result for eq. (3.26) and it is not possible to transform the probability density functions analytically. The derivation of the result in eq. (3.26) and the constraints on function y are fully described in appendix B.

Considering the complex nature of the system in fig. 3.1 and the result of eq. (3.19), it is reasonable to conclude that for the values of $\tilde{m}_r$ not all values of $\hat{E}_{re}$ will be unique. Oscillatory behaviour of the system can cause equal values of $\hat{E}_{re}$ to occur at different values of $\tilde{m}_r$. It is therefore not possible to find a probability density function for $\hat{E}_{re}$ in an analytical manner.

Given that an analytical approach to find an uncertainty for $\hat{E}_{re}$ is not feasible, a numerical method is implemented instead. This means that an N number of simulations will be run per parameter uncertainty. For each simulation a new set of parameters will be sampled from the possible parameters (eqs. (3.21) to (3.23)). This means that for each time t there will be an N number of $\hat{E}_{re}$. The expected value $\bar{E}_{re}$ will then give a value for the estimated energy, given uncertain parameters. At a specific time t the expected value

$$\bar{E}_{re} = \mathbb{E}[\hat{E}_{re}], \quad (3.27)$$

is the average of all estimations in this case:

$$\bar{E}_{re} = \frac{\sum_{n=1}^N \hat{E}_{re}[n]}{N}. \quad (3.28)$$

It is also possible to get the uncertainty distribution around $\bar{E}_{re}$ using histograms of $\hat{E}_{re}$, at each time t . These distributions will be discretized by the number of bins used in the histograms. Using these distributions a confidence intervals (C.I.) can be found for the estimated $\hat{E}_{re}$, resulting in energy bookkeeping that takes parameter uncertainties into account.

To assess the effects of the uncertainty on the energy estimation it is necessary to test different levels of uncertainty. For each parameter three such levels will be chosen: 5%, 10% and 50%. The latter is chosen to discover what extreme uncertainty does to the estimations. Each parameter will be evaluated separately so that it is clear what parameters affects which result.

Each assessment will have the following protocol:

1. Choose a parameter and an uncertainty
2. Define initial conditions $\mathbf{x}_0$
3. For $n = 1 : N$
 - (a) Randomly choose a set $\{\tilde{k}_r, \tilde{d}_r \text{ or } \tilde{m}_r\}$, depending on which parameter is being assessed
 - (b) Calculate $\mathbf{x}(n, t)$ for all t using eq. (3.19)
 - (c) Calculate $\mathbf{y}(n, t)$ for all t using eq. (3.16)
 - (d) Calculate $\hat{E}_{re}(n, t)$ for all t using eq. (3.6) and eq. (3.3)
 - (e) Calculate $E_{re}(t)$ for all t using eq. (3.6) and eq. (3.3), to validate the result
4. For each t
 - (a) Find histogram of $\hat{E}_{re}(n, t)$ over all n
 - (b) Find expected value $\bar{E}_{re} = E[\hat{E}_{re}]$ using eq. (3.28)
5. Plot expected value and uncertainty distribution of $\hat{E}_{re}(n, t)$ together with $E_{re}(t)$

For every parameter the different uncertainties will be compared and discussed.

3.4 Experiments with the physical SEP

To do experiments with the EO and the physical SEP, the SEP will generate input (i.e. x_a , v_a and F_e) for the EO. Position x_a and force F_e are direct measurements from the SEP and the v_a is calculated by using the backward Euler method. The input will be discrete due to the nature of the used sensors, but it will be assumed that the sampling frequency, which is 1 kHz, is significantly higher than the fundamental frequency of the robot dynamics. This allows us to assume that the inputs remain constant over a sample period. In addition, the sensors are assumed ideal, i.e. noiseless and without time-delay.

The EO, described in section 3.2 and eqs. (3.11) to (3.14), will work offline, i.e. outside of the real-time control loop of the SEP, negating the need for it to be discretized. Instead, the EO will be approximated by an ordinary differential equation (ODE) solver such as `ode45` in MATLAB. The EO's energy estimations will also be assessed at three levels of uncertainty for each parameter. Finally, the EO will be validated using an experimental set-up of the SEP. An external reference measurement for the true exchange energy is needed to validate the EO. The experimental set-up, shown in fig. 3.3, will produce this measurement. The SEP will be used as the robot system and attached to it will be a mass m_e representing the environment. The position and velocity of the mass m_e will be the measure used to calculate the true exchange that will be compared to the estimated value of the exchanged energy.

As with the simulations, for the experiments the EO will calculate the $\hat{E}_{re}$ an N number of times per parameter uncertainty, for each time t . Histograms will then be made over all N estimations each time $t = k$, to match the sampled data from the SEP. Using eq. (3.28), the

expected value $\bar{E}_{re}$ can then be calculated at time $t = k$ and histograms of $\hat{E}_{re}$ will give the uncertainty distribution around $\bar{E}_{re}$.

The experimental protocol can be summarized as:

1. Define initial conditions $\mathbf{x}_0$.
2. Set sampling time t_s to match that of input sampling time.
3. For $n = 1 : N$
 - (a) Choose a set $\{\tilde{k}_r, \tilde{d}_r, \tilde{m}_r\}$
 - (b) Calculate $\hat{x}_r(n, t)$ for all t by solving eqs. (3.9) to (3.14)
 - (c) Calculate $\hat{v}_r(n, t)$ for all t by solving eqs. (3.9) to (3.14)
 - (d) Calculate $\hat{E}_{re}(n, t)$ for all t using eq. (3.6) and eq. (3.3)
4. For all k
 - (a) Find histogram of $\hat{E}_{re}(n, t = k)$ over all n
 - (b) Find expected value $\bar{E}_{re} = E[\hat{E}_{re}]$ using eq. (3.28)

3.4.1 Validation

The set-up of fig. 3.3 was used to create inputs x_a , v_a and F_e . Controller $\mathcal{C}$ will be a virtual

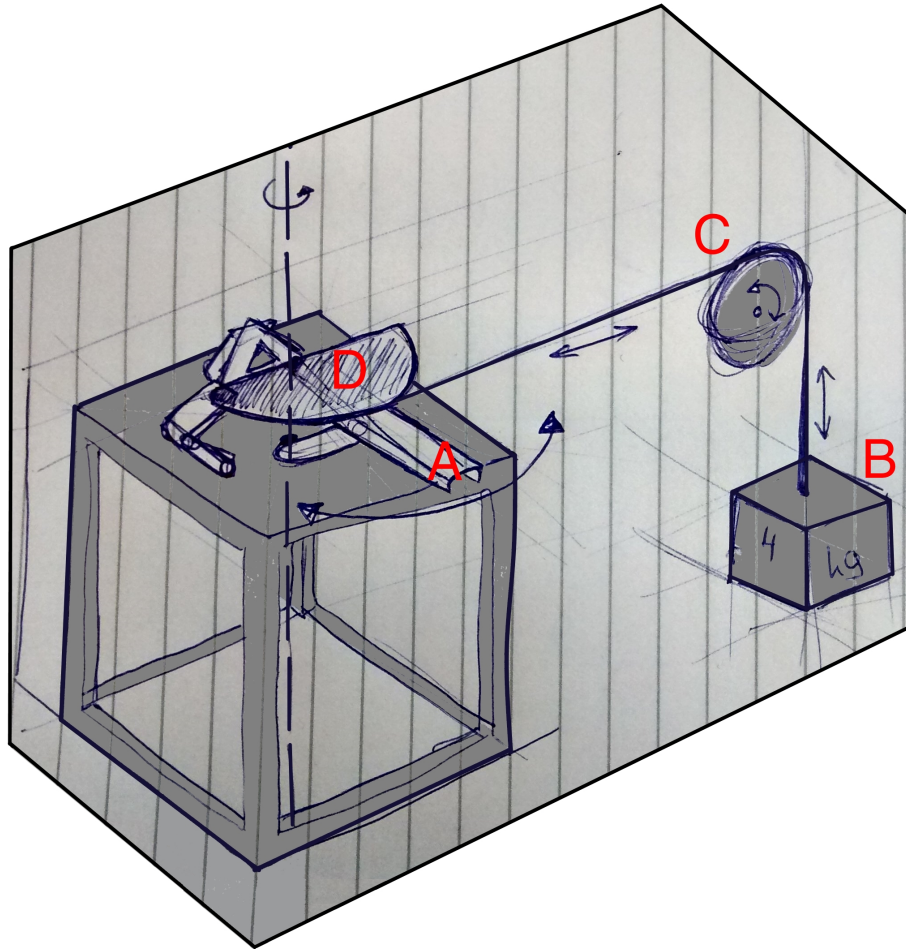


Figure 3.3: The arm of the SEP (A) is connected to a 4 kg mass m_e (B) that will be raised or lowered by means of a non-stretch wire and a pulley (C). The wire connects to a second pulley (D) that keeps the moment created by the mass constant.

spring with proportional gain $-k_v$. A mass m_e (B) will act as the environment and an energy storing element. By lifting or lowering it against the force of gravity an amount of energy will be exchanged between the SEP (A) and the mass, leading to a change in potential and kinetic

energy of the mass. Measuring the difference in height and the rate of change (i.e. the velocity of the mass m_e) will give a measure of energy in the environment. The moment created by the mass m_e is kept constant by the disc D, which acts as a second pulley guiding the wire. A sensor will be mounted on D to provide an external measurement of x_r to validate the EO.

The total energy E_{m_e} in the mass m_e is the sum of the kinetic energy E_{kin} and the potential energy E_{pot} of the mass:

$$E_{m_e} = E_{kin} + E_{pot} \quad (3.29)$$

$$E_{kin} = \frac{1}{2} m_e v_e^2 \quad (3.30)$$

$$E_{pot} = m_e g \Delta x_e. \quad (3.31)$$

The gravitational constant g is assumed to constant throughout the experiment. The initial total energy $E_{m_e}(0)$ is chosen to be zero, i.e. the mass is static. The mass m_e is only attached the SEP, so the interaction port between the SEP and the mass m_e is the only place where energy is exchanged with it. This choice allows us to state

$$E_{re} = -E_{m_e} \quad (3.32)$$

with a negative-sign to match the flow of power. With eq. (3.32) we have an external measure of E_{re} to validate the EO with.

4 Results & Discussion

In this chapter the results of the different simulations and experiments are given and discussed. The chapter is divided into two sections – *simulation results* and *experiment results*. The former handles the simulations done with the EO on the model of the SEP and the latter the experiments done with the EO on the physical SEP. Each section is divided into three subsections, one for each parameter that was assessed: m_r , k_r and d_r . Validation of the EO (i.e. comparison of the expected energy $\bar{E}_{re}$ to the true energy E_{re}) is done simultaneously with the results from the parameter uncertainty.

4.1 Simulation Results

To perform simulations with the model of the SEP, parameters had to be chosen, based on the physical SEP:

$$\begin{aligned} m_a &= 0.014 \text{ kg m}^2 \\ k_r &= 80 \text{ Nm rad}^{-1} \\ d_r &= 10 \text{ Nm rad}^{-1} \text{ s} \\ m_r &= 0.014 \text{ kg m}^2 \end{aligned}$$

For the purpose of the simulations these parameters would be considered the true parameters of the SEP. In addition, the controller $\mathcal{C}$ was chosen to be a virtual spring (i.e. proportional controller) with spring constant $k_v = 3.74 \text{ Nm rad}^{-1}$.ⁱ Finally, an environment $\mathcal{Z}$ had to be chosen. It was chosen to be a spring with spring constant $k_e = 10 \text{ Nm rad}^{-1}$. This spring was considered lossless and would temporarily store energy before returning it to the SEP, and would probably cause oscillatory movements for mass m_r . With the model of the SEP, the controller and the environment set up, it was possible to run the simulations. The simulations followed the steps summarized at the end of section 3.3.

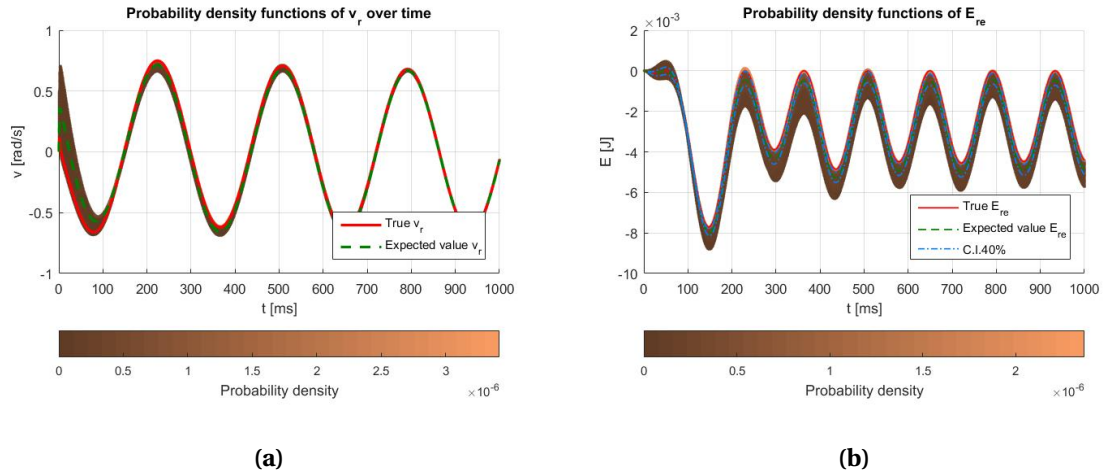


Figure 4.1: An example of the results for $\tilde{k}_r = \hat{k}_r \pm 0.5\hat{k}_r$. **(a)** The EO estimated $\hat{v}_r$, calculated its expected value and gave the uncertainty range around it. **(b)** Taking the results from **(a)**, the EO calculated expected value $\bar{E}_{re}$ based on $N = 2000$ number of estimated $\hat{E}_{re}$. Additionally it is possible to calculate a C.I. range, such as the shown C.I.40%.

ⁱThis value was chosen such that at $x_a = \pm \frac{1}{3}\pi$ rad the moment would be equal to the moment generated by a 3.99 kg mass hanging at 0.1 m from the axis of rotation. See section 4.2.

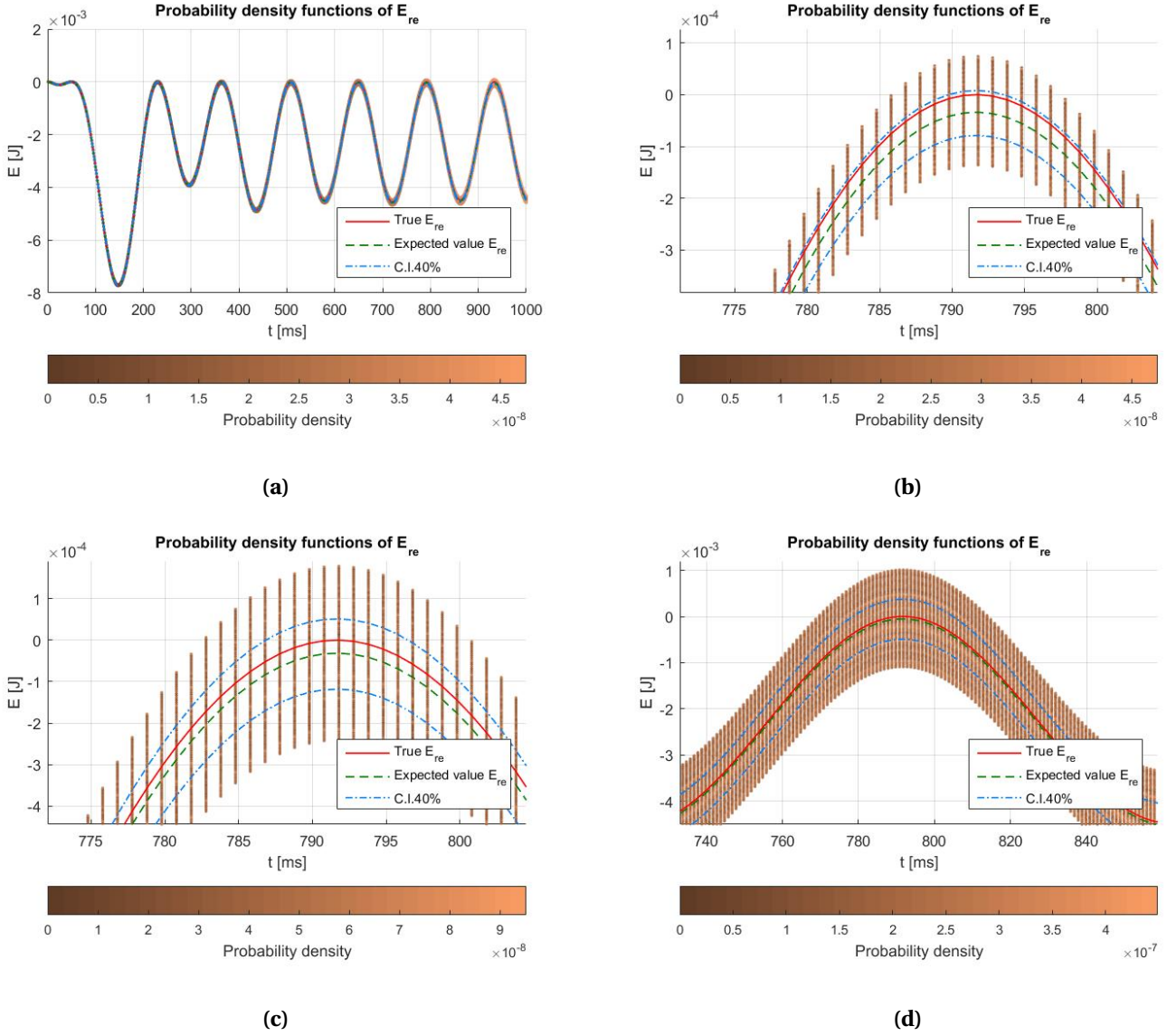


Figure 4.2: The results for the simulations for all three levels of uncertainty: $\Delta m_r(5\%)$, $\Delta m_r(10\%)$ and $\Delta m_r(50\%)$. **(a)** The full simulation results shown for $\Delta m_r(5\%)$, representative for all simulations. **(b)–(d)** An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(5\%)$, $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(10\%)$ and $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(50\%)$, resp.

For each simulation a random value for $\tilde{k}_r$, $\tilde{d}_r$ or $\tilde{m}_r$ was taken, depending on which parameter was being assessed. This value remained within the uncertainty range of the parameter and the other two parameters were kept constant. Using input x_a , v_a and f_e , the EO would estimate $\hat{v}_r$. Figure 4.1a shows an example of $\hat{v}_r$ for $\tilde{k}_r = \hat{k}_r \pm 0.5\hat{k}_r$, i.e. $\Delta k_r(50\%)$. Using $\hat{v}_r$ and F_e the estimated energy $\hat{E}_{re}$ was calculated. This was done for $N = 2000$ simulations, after which the EO could calculate the expected value $\bar{E}_{re}$ and the uncertainty around that expected value. The uncertainty of $\hat{E}_{re}$ is shown using probability density functions around expected value $\bar{E}_{re}$ – fig. 4.1b.

The results of the EO can also be used to give a confidence interval (C.I.), such as the arbitrarily chosen C.I.40% shown in fig. 4.1b. The figures can be used to inspect the effects of parameter uncertainty on the final $\hat{E}_{re}$ estimations, but often it was necessary to zoom in on the results. This is done in the following subsections.

The system was excited by giving the system a non-zero initial condition. The initial condition was chosen such that the EO started equal to the true system. For the experiments $\mathbf{x}_0 = [0.1, 0, 0, 0, 0, 0]^T$ was chosen, i.e. $x_a = 0.1$ rad. This means that the initial energy in the system is

$$\begin{aligned} E_t(0) &= E_{k_v}(0) + E_{k_r}(0), \\ &= \frac{1}{2} k_v x_a^2 + \frac{1}{2} k_r (x_r - x_a)^2 \\ &= \frac{1}{2} \times 3.74 \times 0.1^2 + \frac{1}{2} \times 80 \times (-0.1)^2 \\ &= 4.187 \times 10^{-1} \text{ J}. \end{aligned} \tag{4.1}$$

4.1.1 Mass m_r

To assess the effects of uncertainty of the mass m_r , three levels of uncertainty were simulated. The result, an expected $\bar{E}_{re}$ and its uncertainty, was compared between the three levels. When assessing the uncertainty of mass $\tilde{m}_r$, the stiffness $\tilde{k}_r$ and damping $\tilde{d}_r$ were constant and without uncertainty – i.e. $\tilde{k}_r = k_r, \tilde{d}_r = d_r$. The ‘measured’ $\hat{m}_r$ was chosen to be 1% above the ‘real’ value m_r , meaning $\hat{m}_r = 0.01414 \text{ kg m}^2$. This meant that the expected value $\bar{E}_{re}$ and the real value E_{re} would probably not be equal. This was considered more representative of what would happen in practice. For this measured value, the three levels of uncertainty were

$$\Delta m_r(5\%) = 7.07 \times 10^{-4} \text{ kg m}^2, \tag{4.2}$$

$$\Delta m_r(10\%) = 1.414 \times 10^{-3} \text{ kg m}^2, \tag{4.3}$$

$$\Delta m_r(50\%) = 7.07 \times 10^{-3} \text{ kg m}^2. \tag{4.4}$$

For each level of uncertainty $N = 2000$ number of simulations were done and fig. 4.2 shows the results of those simulations. Each simulation lasted one second and fig. 4.2a shows the results for the 5%-uncertainty level. The expected value $\bar{E}_{re}$ followed the true E_{re} very well. The true energy E_{re} (red), the expected value $\bar{E}_{re}$ (green), the probability density functions of the estimations (brown colorbar) and a chosen C.I.40% lines (blue) are hard to differentiate. With such small ranges of uncertainty, it is difficult to see the effect of the parameter uncertainty and thus it was chosen to show enlarged sections of the results rather than the whole simulation results. Therefore figs. 4.2b to 4.2d show enlarged sections of the full simulation, approximately around $t = 0.8$ s, that allow for better inspection of the results. The shape of the curves were all similar to the ones shown in fig. 4.2a and thus the enlarged sections give enough information.

Each figure shows the true energy E_{re} (red), the expected value $\bar{E}_{re}$ of the estimated energy $\hat{E}_{re}$ (green), the probability density functions of the estimations (brown colorbar) and a chosen C.I.40% (blue). The C.I.40% was an arbitrary choice to demonstrate the potential to use the EO as a decision making tool. For example, the passivity controller could use the C.I.40% instead of the full uncertainty range if such a level of uncertainty was considered safe enough.

Figure 4.2a shows E_{re} as an oscillating, mostly negative, curve which started at $E_{re}(0) = 0$ J. After some settling effects the system reaches a steady oscillating state which is reflected by the energy curves. The negative values for E_{re} indicate that energy has left the SEP through the $\mathcal{S}_{re}$ port and was given to the environment $\mathcal{Z}$. The environment then stored the energy in the spring element before returning it to the SEP, shown by the curves returning towards zero. While there is no loss in the environment because it was modeled as an ideal spring, the amount of energy that is returned to the SEP fully depends on the interaction between the two masses (m_a and m_r) and the three springs (k_v , k_r and k_e). For the chosen combination of system parameters it appears that the environment is able to return most of the energy to the SEP with every oscillation. Inspection of the values of energy E_{re} show that the SEP is passive as the amount of energy leaving the SEP through $\mathcal{S}_{re}$ is never larger than the total energy that was in

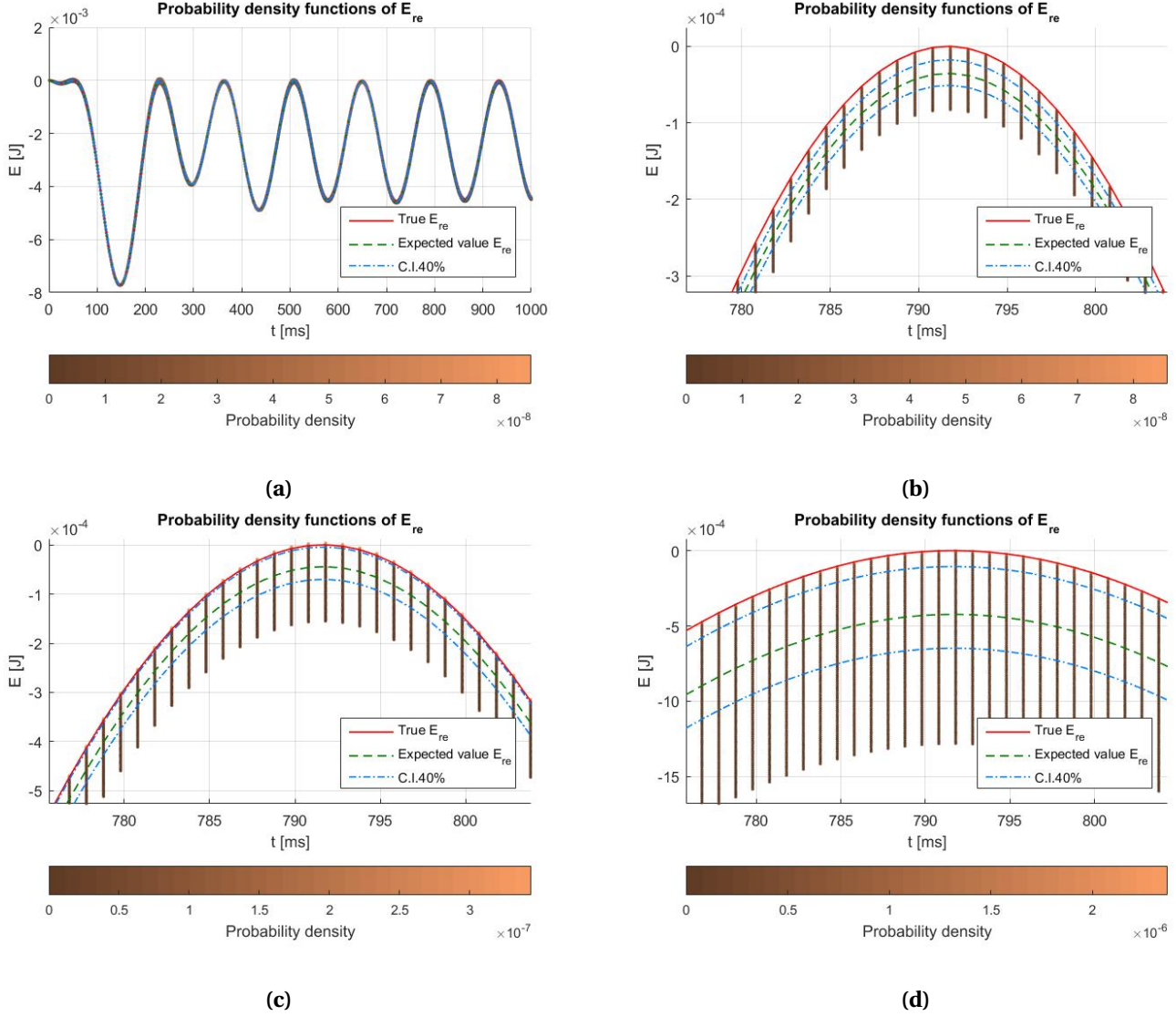


Figure 4.3: The results for the simulations for all three levels of uncertainty: $\Delta k_r(5\%)$, $\Delta k_r(10\%)$ and $\Delta k_r(50\%)$. **(a)** The full simulation results shown for $\Delta k_r(5\%)$, representative for all simulations. **(b)–(d)** An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(5\%)$, $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(10\%)$ and $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(50\%)$, resp.

the SEP-controller combination – i.e. $E_{re}(t) > -E_t(0), \forall t$. This was expected as all elements in the simulation are passive elements (fig. 3.1). Careful inspection shows the uncertainty range increasing over time, as can be expected from a system that is unable to update its state predictions.

Looking at the enlarged sections shown in figs. 4.2b to 4.2d gives a better impression of the effects of parameter uncertainty. First, visual inspection shows that the larger uncertainty Δm_r values give larger uncertainty range for $\hat{E}_{re}$. For the chosen levels of 5%, 10% and 50% the change in $\hat{E}_{re}$ uncertainty appears to change linearly. For example, at $t = 790$ ms the uncertainty range for $\hat{E}_{re}$ resulting from $\Delta m_r(50\%)$ appears approximately ten times larger than the range due to $\Delta m_r(5\%)$. This linear relationship is specific for the chosen system parameters, as different ratios between masses and springs would not only produce shifts in oscillation amplitudes, but also in frequencies. Therefore if a parameter uncertainty, which affects the aforementioned ratios, were to double, the $\hat{E}_{re}$ uncertainty would not just shift spatially, but also temporally.

Second, the EO is validated by the presence of E_{re} within the uncertainty bounds of $\hat{E}_{re}$. For this particular system the true energy lies within the C.I.40% range, which indirectly means that the expected value $\bar{E}_{re}$ is relatively accurate. For a real system, where E_{re} is unknown, the C.I.40% could be used as a ‘worst case scenario’ where the lower bound is used as an indication of the energy that was exchanged with the environment.

4.1.2 Stiffness k_r

As with $\hat{m}_r$, the ‘measured’ $\hat{k}_r$ was chosen to be 1% above the ‘real’ value k_r , meaning $\hat{k}_r = 80.8 \text{ Nm rad}^{-1}$. For this measured value, the three levels of uncertainties were

$$\Delta k_r(5\%) = 4.04 \text{ Nm rad}^{-1}, \quad (4.5)$$

$$\Delta k_r(10\%) = 8.08 \text{ Nm rad}^{-1}, \quad (4.6)$$

$$\Delta k_r(50\%) = 40.4 \text{ Nm rad}^{-1}. \quad (4.7)$$

Figure 4.3 shows the results of the $N = 2000$ simulations, in the same manner the results were shown in fig. 4.2: one representative figure (fig. 4.3a) and three enlarged sections to show the effects of parameter uncertainty (figs. 4.3b to 4.3d). Contrarily, figs. 4.3b to 4.3d show a distinct difference to the simulations done with Δm_r : the true value E_{re} lies very close to the upper bounds of the uncertainty ranges of $\hat{E}_{re}$. As E_{re} is the same in all simulations, this also means that uncertainty of k_r generally gives lower estimations than uncertainty of m_r does. In addition, $\bar{E}_{re}$ and C.I.40% are closer to the upper bounds of the uncertainty range.

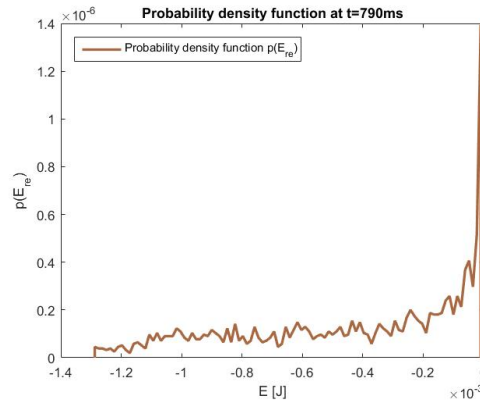


Figure 4.4: A single probability density function $p(E_{re})$ at $t = 790$ ms, taken from the simulation results of $\Delta k_r(50\%)$.

Figure 4.3a is very similar to fig. 4.2a and does not offer new insights.

A closer inspection of the probability density function of $\hat{E}_{re}$ at $t = 790$ ms, shown in fig. 4.4, gives an explanation: the probability of $\hat{E}_{re}$ is highly asymmetrical and highest at the upper values of $\hat{E}_{re}$. The shape of the probability density function is a direct result of the transformation function (i.e. the interconnected system) between $\hat{k}_r$ and $\hat{E}_{re}$. Larger uncertainty values result in a wider range with a similar asymmetrical shape.

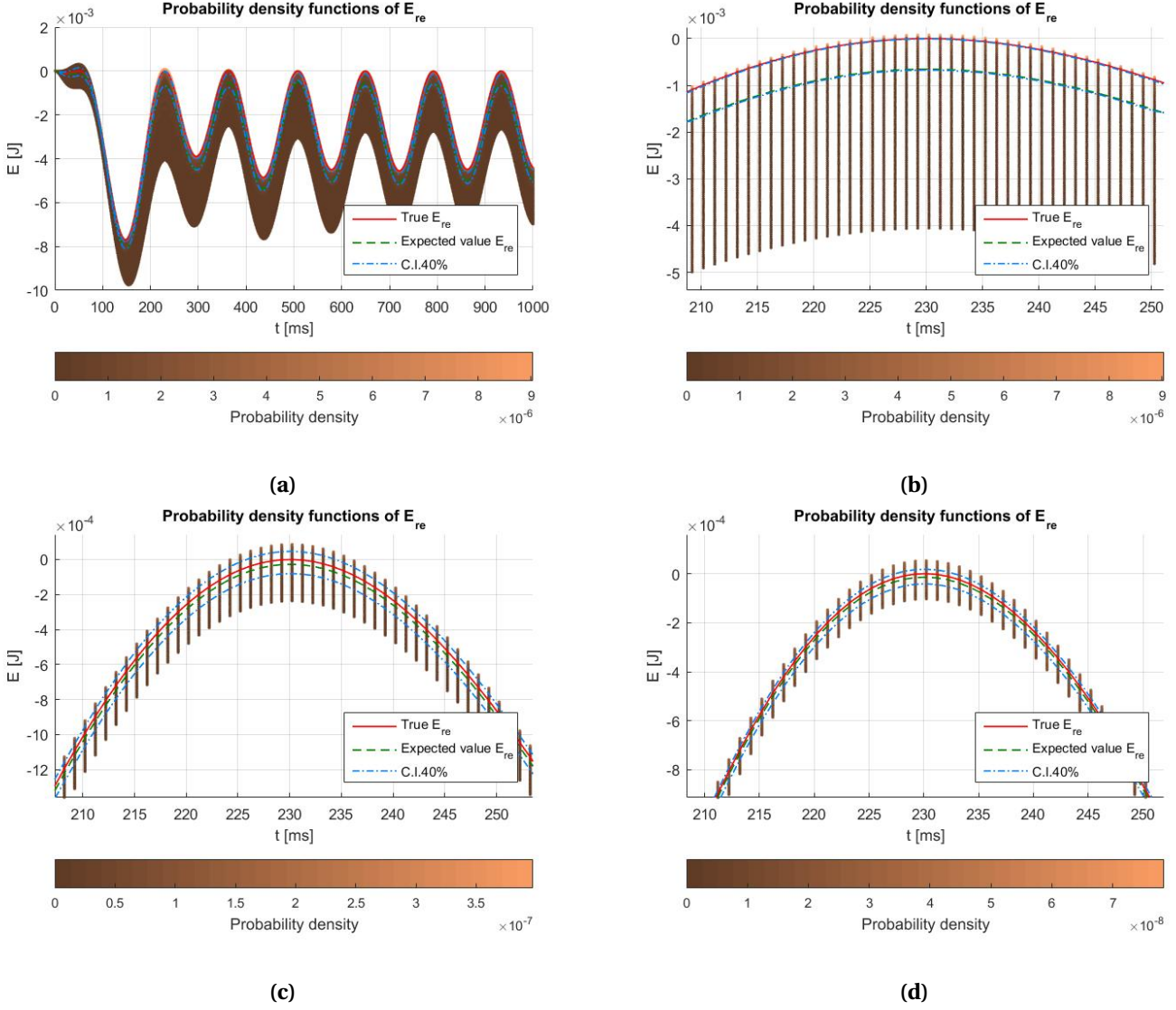


Figure 4.5: The results for the simulations for all three levels of uncertainty: $\Delta d_r(5\%)$, $\Delta d_r(10\%)$ and $\Delta d_r(50\%)$. (a) The full simulation results shown for $\Delta d_r(50\%)$, representative for all simulations. (b)–(d) An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(50\%)$, $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(10\%)$ and $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(5\%)$, resp.

4.1.3 Dissipation d_r

The ‘measured’ $\hat{d}_r$ was chosen to be 1% above the ‘real’ value d_r , meaning $\hat{d}_r = 10.1 \text{ Nm rad}^{-1} \text{ s}$. For this measured value, the three levels of uncertainty were

$$\Delta d_r(5\%) = 0.505 \text{ Nm rad}^{-1} \text{ s}, \quad (4.8)$$

$$\Delta d_r(10\%) = 1.01 \text{ Nm rad}^{-1} \text{ s}, \quad (4.9)$$

$$\Delta d_r(50\%) = 5.05 \text{ Nm rad}^{-1} \text{ s}. \quad (4.10)$$

For each level of uncertainty $N = 2000$ number of simulations were done and fig. 4.5 shows the results of those simulations. Unlike the results shown in figs. 4.2a and 4.3a, which were for $\Delta k_r(5\%)$, fig. 4.5a shows the results for $\Delta d_r(50\%)$. The spread caused by this level of uncertainty is significantly larger than those with $\Delta d_r(5\%)$ and $\Delta d_r(10\%)$ (figs. 4.5b to 4.5d) and its effects are most visible at the beginning of the simulation time. This spread is likely caused by low damping values dissipating less energy from the system and thereby allowing more energy

to flow into the environment. Figure 4.5b indicates that while this spread is large, the probability density functions are still very asymmetrical and focused on the upper bounds of the range. Expected value $\bar{E}_{re}$, true E_{re} and C.I.40% can be found near the top of the range. Note that the expected value $\bar{E}_{re}$ lies close to the lower C.I.40% line and can be hard to distinguish.

4.2 Experiment Results

The set-up of fig. 3.3 is represented by the block diagram of fig. 4.6. The environment mass $m_e = 3.99$ kg was attached to the SEP at with an arm $a = 0.1$ m from the axis of rotation. Combined with the pulley system, the mass applied a constant moment

$$F_e = m_e g a, \quad (4.11)$$

$$= 3.99 \times -9.81 \times 0.1, \quad (4.12)$$

$$= -3.91 \text{ Nm} \quad (4.13)$$

on the robot mass m_r . Gravitational constant g was assumed constant throughout the experiments. The controller $\mathcal{C}$ of the SEP was the same as during the simulations, namely a virtual spring, but in order to excite the system a sinusoidal disturbance was added to the actuator force F_a . The disturbance force is $F_{dist} = A \sin(2\pi f t)$ Nm, with amplitude $A = 3.91$, frequency $f = 0.2$ Hz and time t . In addition, an offset force $\tilde{F}_e \approx F_e$ was added to balance the system against the environment mass m_e .

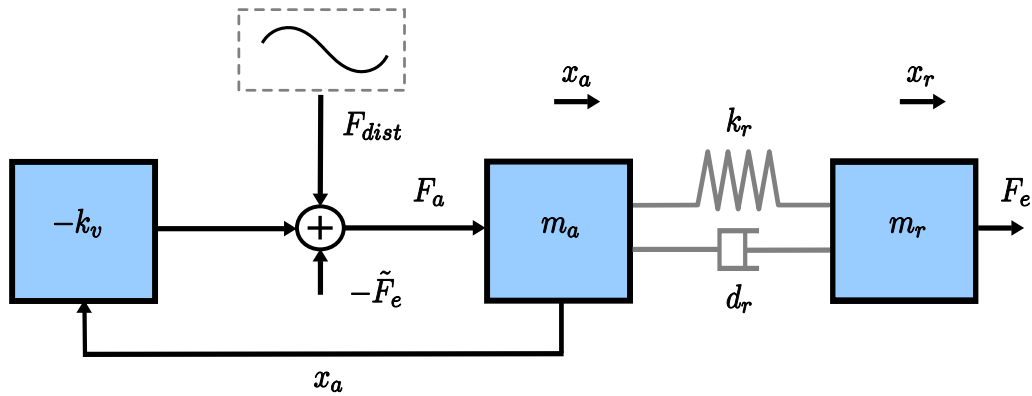


Figure 4.6: The model of the SEP (fig. 1.2) is expanded to include controller $\mathcal{C}$ and environment force F_e can be represented by a mass-spring-mass system with dissipation.

The values for the set-up were chosen such that the actuator mass m_a was expected to remain between $-\frac{1}{3}\pi$ rad and $\frac{1}{3}\pi$ rad. Mechanical constraints of the SEP limited the range of motion for m_a to less than π rad.

At the time of writing this thesis no proper system identification had been done on the SEP. Documentation of the motor gave the moment of inertia for actuator mass $m_a = 0.014 \text{ kg m}^2$, and this value was also chosen for the second mass, the robot mass m_r . The stiffness $\hat{k}_r$ was derived by measuring the difference in x_a and x_r while the set-up was static and $F_e = -3.91$ Nm:

$$\hat{k}_r = \frac{F_e}{x_r - x_a} = \frac{-3.91}{-0.049} = 80 \text{ Nm rad}^{-1} \quad (4.14)$$

Finally, $\hat{d}_r = 10 \text{ Nm rad}^{-1} \text{ s}$ was chosen arbitrarily, as proper identification was not done.

A single measurement run was done, lasting 60 s. The set-up started stationary at position $x_a = 0$ rad and the disturbance force F_{dist} made the system move. Once the measurements of x_r and F_e were done, v_r was derived with the backward Euler method. These three inputs were used as input for the EO and then $N = 2000$ experiments were done per parameter and uncertainty level. The experiment protocol summarized in section 3.4 was used and in the following three

subsections the results are shown for each of the three parameters $\hat{m}_r$, $\hat{k}_r$ and $\hat{d}_r$. However, due to computational limitations and time restraints only 4.5 s of the measurements were used. The EO ran on the MATLAB using the ODE45 solver, with a minimal step-size of 0.2 ms, which made computing the results time-consuming. It was nonetheless expected that the 4.5 s of measurements is enough data to draw conclusions, as the system was expected to oscillate every 5 s. New oscillations would most likely not give new insights.

After the EO estimations had been computed it was to be validated against an externally measured E_{re} . Unfortunately, the measurements of x_r had not been done correctly. The external sensor was an AS5600 12-bit contactless potentiometerⁱⁱ that uses Hall sensors to measure the rotation of a magnet placed on-axis. The sensor had not been mounted correctly on the SEP and the resulting measurements of x_r were non-linear as the magnet had not been aligned correctly with the Hall sensors. This was discovered too late and therefore the measurements could not be redone. This also had the consequence that $\hat{k}_r$ was most likely incorrect – see section 4.2.2.

With the lack of a correct ‘true’ E_{re} , the choice was made to introduce a so-called ‘naive’ E_{re}^n , based on the assumption that the SEP could be represented by a single mass m_a . This would mean that

$$E_{re} = \int_0^t P_{re}(\tau) d\tau \quad (4.15)$$

where

$$P_{re} = \langle F_e, v_a \rangle. \quad (4.16)$$

This naive E_{re} would not properly validate the EO’s estimations, but should still give insight how correctly the EO functioned.

To be complete, the ‘true’ E_{re} will still be included in all the results, despite being incorrect.

4.2.1 Mass m_r

The ‘measured’ $\hat{m}_r$ was 0.014 kg m² and for this measured value, the three levels of uncertainties were

$$\Delta m_r(5\%) = 7 \times 10^{-4} \text{ kg m}^2, \quad (4.17)$$

$$\Delta m_r(10\%) = 1.4 \times 10^{-3} \text{ kg m}^2, \quad (4.18)$$

$$\Delta m_r(50\%) = 7 \times 10^{-3} \text{ kg m}^2. \quad (4.19)$$

For each level of uncertainty $N = 2000$ number of experiments were done and fig. 4.7 shows the results of those simulations. Each experiment lasted 4.5 s second long and fig. 4.7a shows the results for the $\Delta m_r(5\%)$ level. The true energy E_{re} (red), the expected value $\bar{E}_{re}$ (green), the probability density functions of the estimations (brown colorbar) and the naive energy E_{re} (blue) are shown.

Figures 4.7b to 4.7d show that as the uncertainty levels increase, so does the uncertainty range around $\bar{E}_{re}$. However, in none of the figures the true or naive E_{re} fall within the uncertainty range, as expected. In the experiments the larger values for $\hat{E}_{re}$ are exchanged between the SEP and the environment and the uncertainty range match this increase.

4.2.2 Stiffness k_r

The ‘measured’ $\hat{k}_r$ is 80 Nm rad⁻¹, but this was based on the assumption that x_r was measured correctly. In hindsight this should have been an indication of incorrect measurements, as the real stiffness of the SEP is probably 2 to 3 orders of magnitude higher. The material between

ⁱⁱams Sensors Germany GmbH, Reutlingen, Germany, www.ams.com.

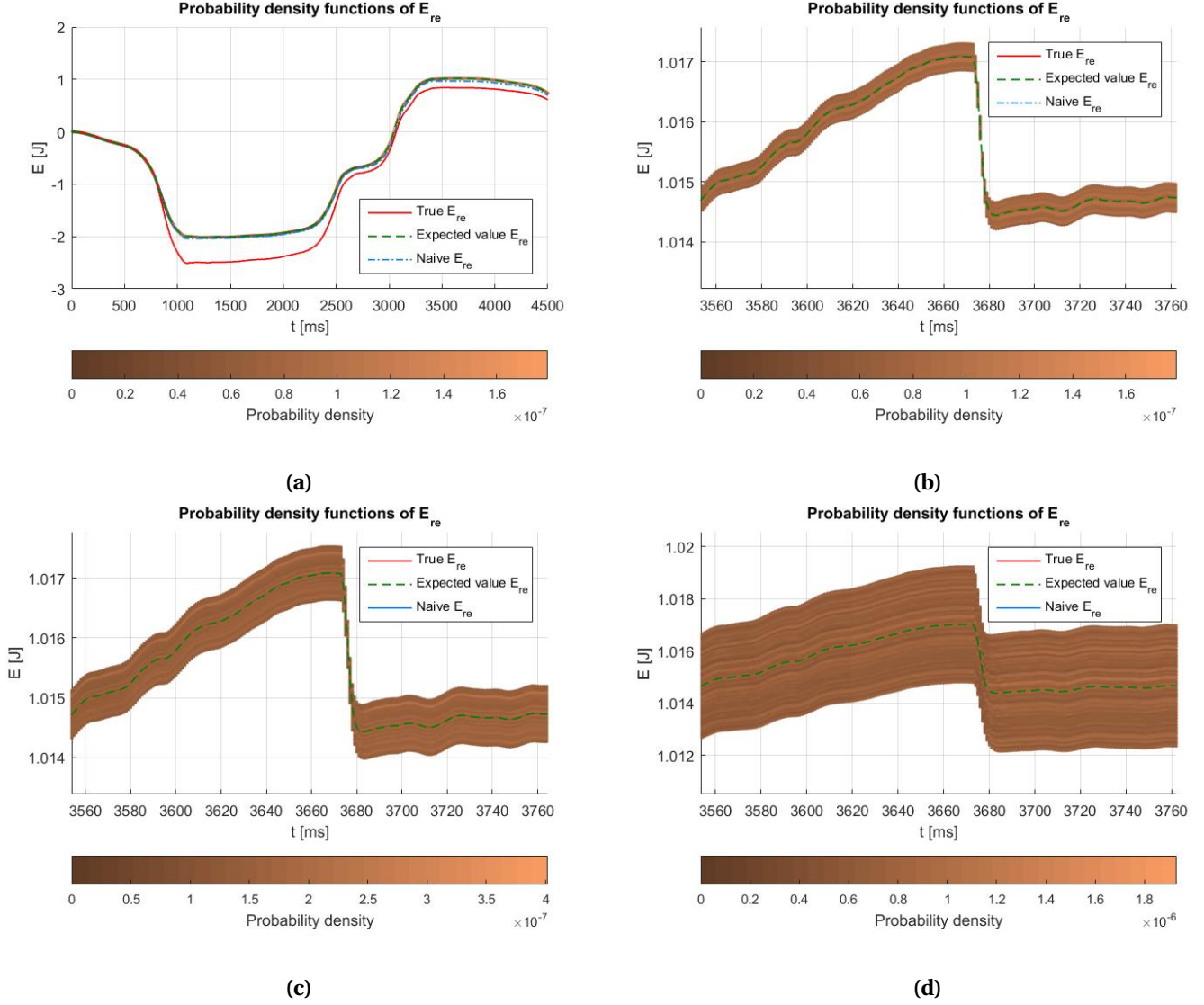


Figure 4.7: The results for the experiments for all three levels of uncertainty: $\Delta m_r(5\%)$, $\Delta m_r(10\%)$ and $\Delta m_r(50\%)$. (a) The full simulation results shown for $\Delta m_r(5\%)$, representative for all experiments. (b)–(d) An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(5\%)$, $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(10\%)$ and $\tilde{m}_r = \hat{m}_r \pm \Delta m_r(50\%)$, resp.

the actuator mass and the robot mass is a combination of mostly the metals aluminum and steel, both very stiff metals. For example, an aluminum rod of 0.15 m with a diameter of 0.04 m would have a torsion stiffness of $41900 \text{ Nm rad}^{-1}$.ⁱⁱⁱ The EO used $\hat{k}_r = 0 \text{ Nm rad}^{-1}$ for the results shown in fig. 4.8.

At $\hat{k}_r = 80 \text{ Nm rad}^{-1}$, the three levels of uncertainty were

$$\Delta k_r(5\%) = 4 \text{ Nm rad}^{-1}, \quad (4.20)$$

$$\Delta k_r(10\%) = 8 \text{ Nm rad}^{-1}, \quad (4.21)$$

$$\Delta k_r(50\%) = 40 \text{ Nm rad}^{-1}. \quad (4.22)$$

On the full scale of fig. 4.8a it is not possible to view the uncertainty ranges around $\bar{E}_{re}$, but it is clear that true E_{re} does not lie within its range. Further inspection shows that for $\Delta k_r(5\%)$

ⁱⁱⁱ The torsion constant J of such a rod is $\frac{0.04^4 \pi}{32} \text{ m}^4$ and the shear modulus G of aluminium is approximately 25 GPa. For a given length of $l = 0.15 \text{ m}$, the torsional stiffness $k_T = \frac{JG}{l} = 41888 \text{ Nm rad}^{-1}$

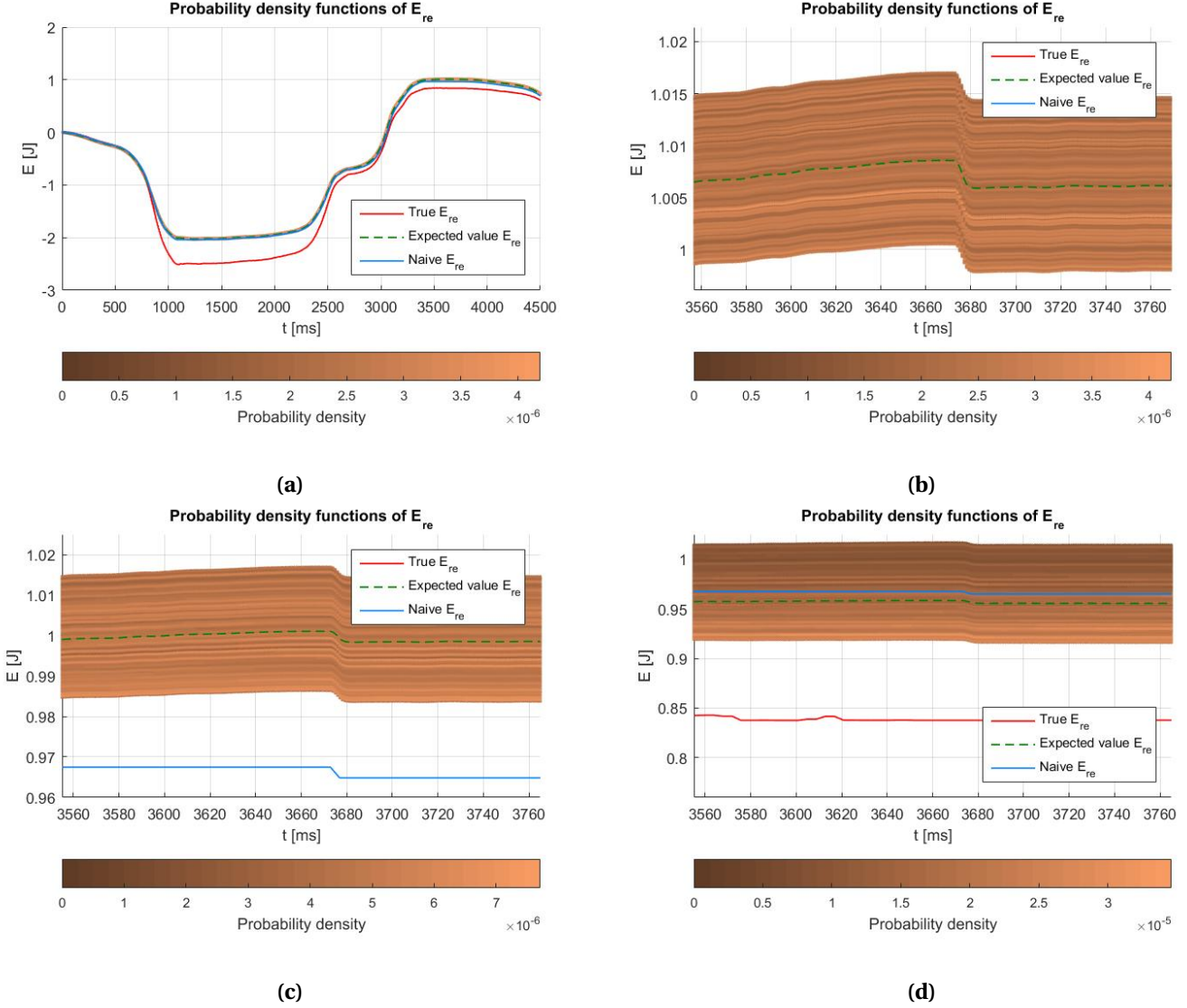


Figure 4.8: The results for the experiments for all three levels of uncertainty: $\Delta k_r(5\%)$, $\Delta k_r(10\%)$ and $\Delta k_r(50\%)$. **(a)** The full simulation results shown for $\Delta k_r(5\%)$, representative for all experiments. **(b)–(d)** An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(5\%)$, $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(10\%)$ and $\tilde{k}_r = \hat{k}_r \pm \Delta k_r(50\%)$, resp.

and $\Delta k_r(10\%)$ (figs. 4.8b and 4.8c) the naive E_{re} is not within the range either. However, for the largest uncertainty $\Delta k_r(50\%)$ (fig. 4.8d) the uncertainty range of $\hat{E}_{re}$ does encompass the naive E_{re} .

4.2.3 Dissipation d_r

Without proper system analysis it was difficult to find a realistic value for the dissipation in the SEP. The chosen $\hat{d}_r = 10 \text{ Nm rad}^{-1} \text{ s}$ was used for the experiments with the EO and thus the three levels of uncertainty were

$$\Delta d_r(5\%) = 0.5 \text{ Nm rad}^{-1} \text{ s}, \quad (4.23)$$

$$\Delta d_r(10\%) = 1 \text{ Nm rad}^{-1} \text{ s}, \quad (4.24)$$

$$\Delta d_r(50\%) = 5 \text{ Nm rad}^{-1} \text{ s}. \quad (4.25)$$

Figure 4.9 shows that for the chosen $\hat{d}_r$ and the three levels of uncertainty, the results are similar to those of section 4.1.3. On the large scale of fig. 4.8a it is not possible to view the uncertainty

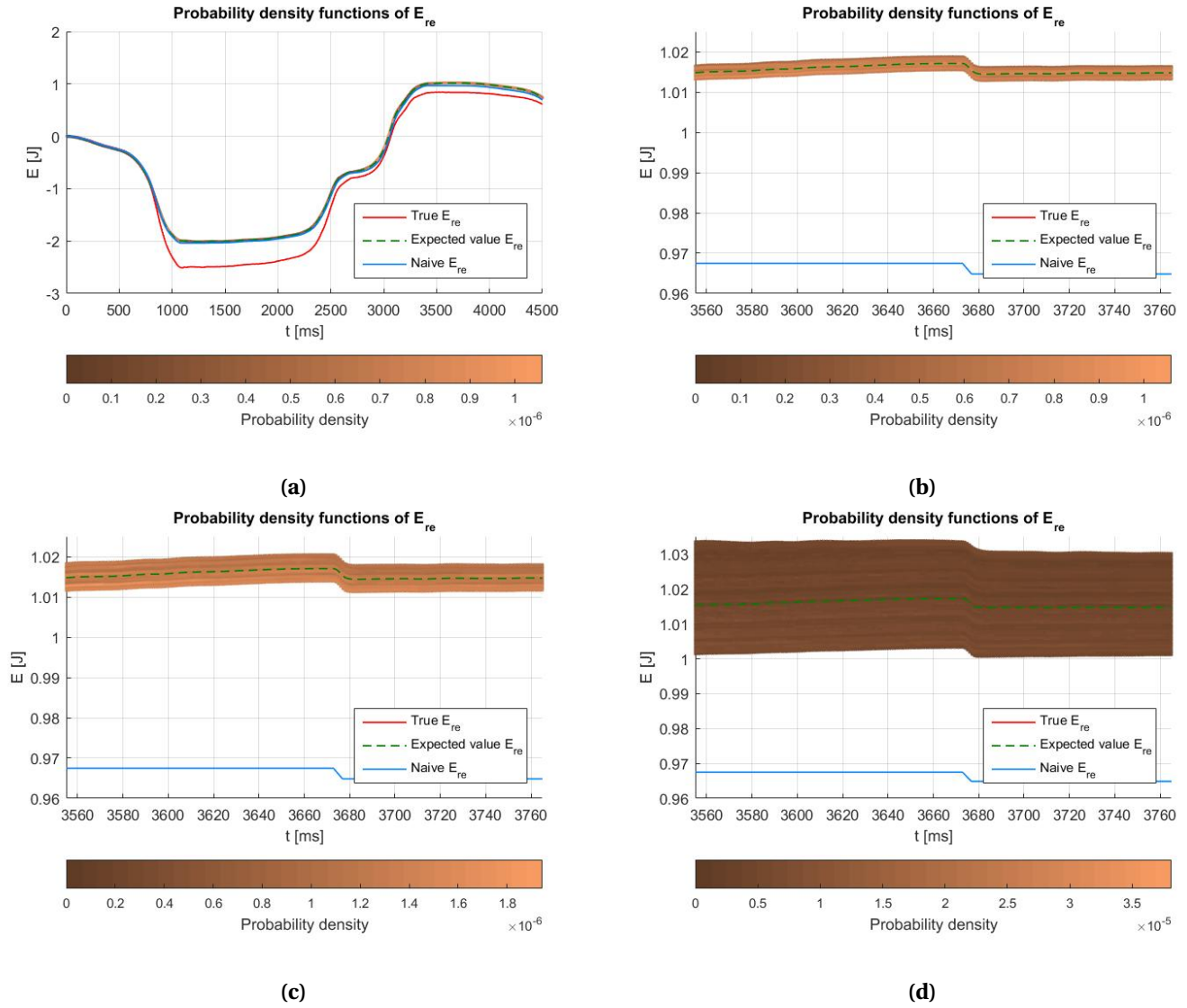


Figure 4.9: The results for the experiments for all three levels of uncertainty: $\Delta d_r(5\%)$, $\Delta d_r(10\%)$ and $\Delta d_r(50\%)$. (a) The full simulation results shown for $\Delta d_r(50\%)$, representative for all experiments. (b)–(d) An enlarged section of the results for E_{re} , for uncertain parameter $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(50\%)$, $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(10\%)$ and $\tilde{d}_r = \hat{d}_r \pm \Delta d_r(5\%)$, resp.

ranges around $\bar{E}_{re}$, but it is clear that the true E_{re} does not lie within the range. Further inspection shows that for $\Delta k_r(5\%)$ and $\Delta k_r(10\%)$ (figs. 4.8b and 4.8c) the naive E_{re} is not within the range either. However, for the largest uncertainty $\Delta k_r(50\%)$ (fig. 4.8d) the uncertainty range of $\hat{E}_{re}$ does encompass the naive E_{re} .

4.3 Summary

The EO works as expected. For a given system with measured parameters and known uncertainties, the EO could be a good tool to get an energy estimation that accounts for uncertainties. It gives the freedom to choose a certain C.I. range that is deemed ‘good enough’ and even with extreme parameter uncertainties the results could be considered useful. The EO was validated virtually with a model of the SEP, but due to measurement errors the EO was not validated using the physical SEP.

5 Conclusion & Discussion

The goal of this thesis was to answer two research questions. Those questions are repeated and answered below.

Research Question 1. *How do uncertainties in the system parameters affect the energy estimation of the Energy Observer?*

Uncertainties in the system parameters cause uncertainty in the energy estimation of the EO. If the uncertainties in the parameters increase, so do the uncertainties in the energy estimation. That said, the shape of the probability density functions of the estimations might also change, giving less expected results.

Research Question 2. *Does the true exchanged energy lie within the uncertainty range of the estimated exchanged energy?*

If the true values for the system parameters lie within the parameter uncertainty ranges, then the true exchanged energy will also lie within the uncertainty range of the estimations done by the EO. This result was validated using simulations but not using a physical set-up of the SEP, due to incorrect measurements of x_r . With proper implementation of the experimental method described in this thesis, it is expected that validation of the EO is still possible. The SEP or a similar set-up as described in section 3.4.1 could be used. As long as there is an external method of finding E_{re} , the set-up would be able to validate the EO.

In this work each parameter was tested separately to discover what the effects of specific parameter uncertainties were. Additionally, each parameter had a uniform distribution. Future work could evaluate the effects of multiple parameter uncertainties simultaneously to assess how the interplay affects the energy estimations. Multiple uncertainty ranges would increase the uncertainty range of the energy estimation, but might also change the shape of the distribution. This could mean that a certain C.I. range could actually decrease instead of increase. Furthermore, different distributions could be evaluated for similar reasons. For example, a gamma distribution could be used to describe uncertainty in the dissipation in a system.

For this thesis the EO was implemented on a linear system, but it is not limited to such a system. The EO requires knowledge of the structure of the system to model an optimal representation - i.e. the order of the system, the types and number of elements that could best represent the system, the initial conditions, etc. If for the given representation of the system there can be found an analytical or numerical solution, the principle of the EO could be applied using that solution. For example, the `ode45` used in this work could also be used for non-linear systems.

However, a drawback of using the `ode45` is its computation time. This is the result of trying to approximate a time-continuous model, and the current iteration of the EO could not simply be implemented inside a real-time control loop. With the purpose of passivity control in mind, the implementation of the EO into the real-time loop is recommended. Switching from time-continuous models to explicitly discrete models might be a method of lowering the computation time of the EO. A discrete model can be relatively easily implemented in an existing real-time loop, which will also be discrete.

It can be noted that once a parameter is chosen, it will not change over time. The value of the parameter, though uncertain, is constant over time – this is a direct result of considering time-invariant systems. Changing to time-variant systems would require a redesign of the algorithm of the EO. Parameters would not only be uncertain, they would change over time. The set of parameters would have to be re-sampled every period. This redesign could be based on particle filters, which also re-sample every period. An added benefit of such a redesign would be the ability of being able to include uncertain measurement variables. Sensor noise for F_e ,

for instance, could then be included in the uncertainty of $\hat{E}_{re}$, as that is also a time-variant uncertainty.

This thesis showed that an Energy Observer has potential as an energy bookkeeping algorithm. In the future it could be used to supply a passivity controller with the required information to guarantee passivity of a robotic system. It would keep track the exchanged energy in a way that explicitly regards uncertainties in the robotic system. The use of the EO could facilitate moving the passivity controller out of the ideal, theoretical realm into the unknown, real world.

Appendices

A Matrix Exponential

The goal of this section is to explain why eq. (3.19) is the homogenous response of the linear, time-invariant model described in eq. (3.9), repeated here for the sake of convenience. The homogenous response of the state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (\text{A.1})$$

is

$$\mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}_0. \quad (\text{A.2})$$

Following Rowell (2002), consider the response of a first-order (scalar) system with state equation

$$\dot{x} = ax + bu \quad (\text{A.3})$$

with initial condition x_0 . The form of eq. (A.3) requires that we find a function that, when differentiated, has the same shape as the original function. The exponential form is such a function:

$$\frac{d}{dt} e^t = e^t \quad (\text{A.4})$$

$$\frac{d}{dt} e^{at} = ae^t \quad (\text{A.5})$$

The homogeneous response $x_h(t)$ (i.e. $u = 0$, $\dot{x} = ax$) of eq. (A.3) is then

$$x_h(t) = e^{at} x_0. \quad (\text{A.6})$$

The exponential term e^{at} in eq. (A.6) may be expanded into a power series, to give:

$$e^{at} = \left(1 + at + \frac{a^2 t^2}{2!} + \frac{a^3 t^3}{3!} + \dots + \frac{a^k t^k}{k!} + \dots \right) \quad (\text{A.7})$$

where the series converges for all finite $t > 0$. Thus

$$x_h(t) = \left(1 + at + \frac{a^2 t^2}{2!} + \frac{a^3 t^3}{3!} + \dots + \frac{a^k t^k}{k!} + \dots \right) x_0. \quad (\text{A.8})$$

Consider next that eq. (A.3) describes a higher order linear time-invariant (LTI) system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}. \quad (\text{A.9})$$

Let us now assume that the homogeneous response $\mathbf{x}_h(t)$ can also be expressed as an infinite power series, similar to eq. (A.8), but now in terms of square matrix $\mathbf{A}$:

$$\mathbf{x}_h(t) = \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \dots + \frac{\mathbf{A}^k t^k}{k!} + \dots \right) \mathbf{x} \quad (\text{A.10})$$

with initial state $\mathbf{x}_0$. Each term in this series is a matrix of size $n \times n$, and the summation of all terms yields another matrix of size $n \times n$.

Next, the series can be differentiated (to time t) term by term. Matrix differentiation is defined on an element by element basis and because the system is LTI, each element in $\mathbf{A}$ is constant. That means

$$\dot{\mathbf{x}}_h(t) = \left(\mathbf{0} + \mathbf{A} + \mathbf{A}^2 t + \frac{\mathbf{A}^3 t^2}{2!} + \cdots + \frac{\mathbf{A}^k t^{k-1}}{(k-1)!} + \cdots \right) \mathbf{x}_0 \quad (\text{A.11})$$

$$= \mathbf{A} \left(\mathbf{I} + \mathbf{A} t + \frac{\mathbf{A}^2 t^2}{2!} + \cdots + \frac{\mathbf{A}^{k-1} t^{k-1}}{(k-1)!} + \cdots \right) \mathbf{x}_0 \quad (\text{A.12})$$

$$= \mathbf{A} \mathbf{x}_h(t) \quad (\text{A.13})$$

which is the same form as eq. (A.1). This means that eq. (A.10) is the homogenous response of eq. (A.1). Due to its similarity to eqs. (A.7) and (A.8), it is convenient to define the *matrix exponential* of square matrix $\mathbf{A}$ as

$$e^{\mathbf{A}t} = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \cdots + \frac{\mathbf{A}^k t^k}{k!} + \cdots \quad (\text{A.14})$$

and the homogenous response for eq. (A.1) as

$$\mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}_0 \quad (\text{A.15})$$

□

B Transforming Probability Density Functions

For a random variable X , with probability density function $f(x)$, $Y = y(X)$ is the derived random variable. The distribution of Y is described by probability density function $g(y)$. Note that a capital letter X is used to describe the unspecified variable with a distribution while the small letter x explicitly represents a value for that variable.

The probability $P(X)$ is defined as

$$P(a \leq X < b) = \int_a^b f(x) dx. \quad (\text{B.1})$$

where $0 \leq P(X) \leq 1$. Using eq. (B.1) it becomes possible to find $g(y)$. It is assumed that if $a \leq X < b$ then $y(a) \leq Y < y(b)$, such that

$$P(y(a) \leq Y < y(b)) = P(a \leq X < b). \quad (\text{B.2})$$

The general equation for substituting a variable in an integral is

$$\int_a^b f(x) dx = \int_{y(a)}^{y(b)} f(x(y)) \frac{dx}{dy} dy \quad (\text{B.3})$$

and is used to calculate $P(Y)$:

$$P(y(a) \leq Y < y(b)) = P(a \leq X < b) \quad (\text{B.4})$$

$$= \int_a^b f(x) dx \quad (\text{B.5})$$

$$= \int_{y(a)}^{y(b)} f(x(y)) \frac{dx}{dy} dy. \quad (\text{B.6})$$

The integrand can be replaced by $g(y)$, thus giving

$$g(y) = f(x(y)) \frac{dx}{dy}, \quad (\text{B.7})$$

$$P(y(a) \leq Y < y(b)) = \int_{y(a)}^{y(b)} g(y) dy. \quad (\text{B.8})$$

For example, if function $y = x^2$ and probability density function $f(x)$ was,

$$f(x) = \begin{cases} \frac{1}{2} & \text{for } 0 \leq x < 2 \\ 0 & \text{otherwise,} \end{cases} \quad (\text{B.9})$$

then probability density function $g(y)$ could be calculated using eq. (B.7):

$$x(y) = \sqrt{y}, \quad (\text{B.10})$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{y}}, \quad (\text{B.11})$$

$$g(y) = f(x(y)) \frac{dx}{dy}, \quad (\text{B.12})$$

$$g(y) = \begin{cases} \frac{1}{4\sqrt{y}} & \text{for } 0 \leq y < 4 \\ 0 & \text{otherwise.} \end{cases} \quad (\text{B.13})$$

The result is verified using eq. (B.8) and taking the integral from $-\infty$ to ∞ and finding that $P(-\infty \leq Y \leq \infty) = 1$.

B.1 Constraints

To analytically transform probability density functions, eq. (B.2) is a crucial step. This step applies constraints on the transformation function $y(x)$:

1. For every value x of random variable X in the useful range a to b , both $y(x)$ and the inverse $x(y)$ must be defined and single-valued.
2. This also means that throughout the range $\frac{dx}{dy}$ is defined and is *either* $\frac{dx}{dy} \leq 0$ *or* $\frac{dx}{dy} \geq 0$.

If $\frac{dx}{dy}$ were to change sign, it means that $y(x)$ would not be single-valued. The consequence of the constraints is that $y(x)$ must increase or decrease monotonically over the useful range of x . In addition, because probability density functions cannot have negative values it is customary to rewrite eq. (B.7) to

$$g(y) = f(x(y)) \left| \frac{dx}{dy} \right|. \quad (\text{B.14})$$

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