Master Thesis

Applied Mathematics - University of Twente

Turning Profile Steering into an auction game

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Abstract

The amount of electricity transported through the electricity grid is increasing. The main causes are the increasing use of renewable energy sources and electrification of our energy use. This causes increasing stress on the network and fluctuating voltage levels. As a solution to this problem, the Profile Steering algorithm was developed. Profile Steering aims to steer houses to spread the electricity consumption across the day to avoid problems within the grid. In profile steering a central controller sends steering signals to each house in a neighborhood. This steering signal can be used by each house to schedule electricity consumption. However, houses know too much about the goal of the central controller due to these steering signals. This goal can be used to create problems for the central controller. Furthermore, the proposed approach has no incentives in place to ensure that houses participate. Therefore, we propose a modification to the profile steering approach, where the steering signals are modified and each house tries to change its electricity consumption according to these modified signals. The houses for which the change in electricity consumption fits best the overall system goals are chosen to carry out their revised plan. These houses are compensated for their change in electricity consumption to incentivize them to carry out their profile. This leads to a game theoretic problem where the participating houses compete to get the compensation. In this thesis, the mentioned game is modeled and methods to optimize the other signals are given that lead to a flattening of the total electricity consumption.

Contents

1 Introduction

Most countries agree that actions are needed with respect to global warming and the non-sustainability of the energy supply chain. The treaty of Kyoto (1997) and the agreement of Paris (2015) about the reduction of greenhouse gas emissions were signed by a lot of countries. The Paris agreement was even signed by all but one. It is generally accepted in the scientific community that CO_2 and other greenhouse gasses produced by humans are for the most part causing global warming [2]. Therefore, targets for the reduction of greenhouse gas emissions were set to be met by a specific date.

Considering the energy supply chain we observe that in the past mainly fossils fuels were used and for a large part still are used for the production of energy. This way of energy generation produces CO_2 and uses resources that are available in limited supply. So these fossils fuels will deplete at some point in time and are not sustainable. As such, governments and industries developed alternatives for fossil fuels to produce energy. Bio-mass, wind energy and solar energy are some prominent examples. They are called renewable energy sources and these sources have been used more and more in the last years. In the following we give an overview of the increasing integration of renewable energy sources worldwide.

1.1 Energy transition

The share of renewable energy in the total energy production is growing. This is needed to meet the goals of the aforementioned reduction targets. This means that for example more and more solar panels are being installed and that the number of wind-turbines is growing (photovoltaics or PV is the term for the whole system of collecting solar energy including solar panels). In 2016 76600 MW (Megawatt) of PV capacity was installed globally compared to only 2524 MW in 2007 [4] (see Figure 1). Wind energy capacity has increased from 18.039 MW globally in 2000 to 486.661 MW in 2016 [12] (see Figure 2). The given figures indicate that the exploitation of these two renewable energy sources is growing rapidly. Thereby, wind energy has a higher installed capacity worldwide which is probably because of it has been around already for a longer time.

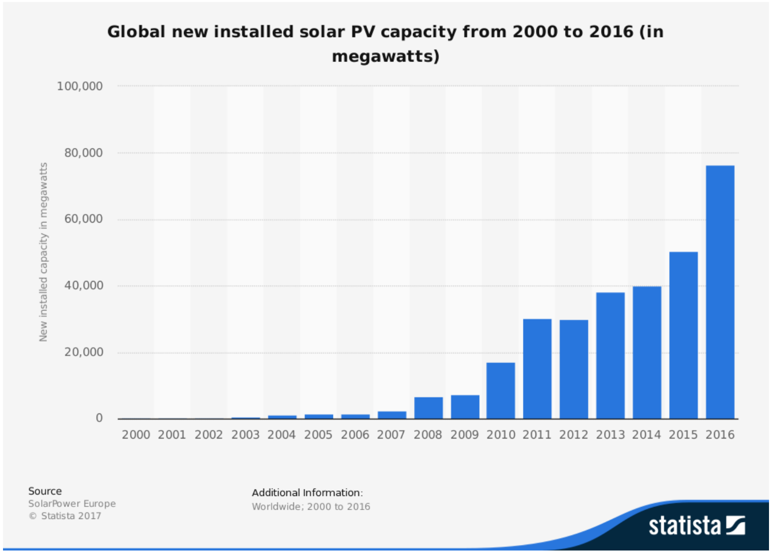

Figure 1: Global new installed PV capacity from 2000 to 2016 [4].

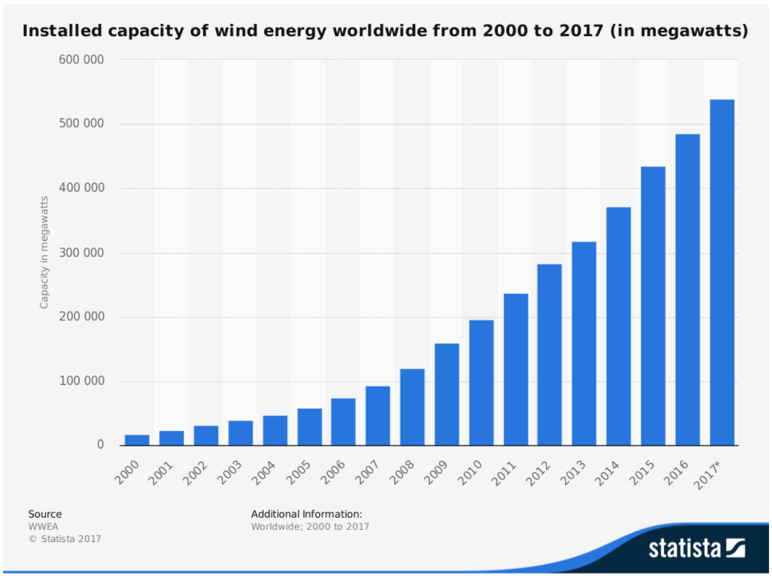

Figure 2: Installed capacity of wind energy worldwide from 2000 to 2017 [12].

In the future it is expected that renewable energy production will continue to grow. In this thesis we look at the consequences of the growing amount of electricity that is produced by renewable and non-renewable energy sources. All this electricity has to be transported from the generation location to the industries and residential houses which require electric power. In the past, the electricity production used to be centralized at large production plants connected to the high voltage grid. Furthermore, this production could be steered to guarantee that exactly the amount of electricity required by consumers (industries and houses) was produced. With the rise of renewable energy sources, electricity is often also produced in other layers of the electricity grid. Furthermore, renewable generation can not be steered. This causes problems inside the grid which are outlined in the next section.

1.2 Electricity grid

In this section we give an overview of the Dutch electricity grid and the effects of the energy transition in the grid. Other countries have different systems but we use the Dutch electricity grid as an example. Although in general an electricity grid is made up of electricity cables which transport electricity from the power plants to the consumers. Different cables transport electricity with a different voltage. Our electricity grid consists of a transmission grid and a number of smaller distribution grids. This is depicted in Figure 3. The transmission grid is used to transport electricity over longer distances from the central power plant (production plant) to the distribution grids. It is divided into two parts, the HV (high voltage) national grid and HV regional grid. The voltage of the HV national grid is 380 or 220 kV (kilovolt) and 150, 110 or 50 kV is used for the HV regional grid. Electricity is produced at the power plant then transported by the HV national grid and after that transported by the HV regional grid. The HV regional grid not only transports electricity but also delivers it to large industries. In the distribution grids the electricity is further transported to all the different customers. The distribution grid is made up of two parts, the medium and the low voltage distribution grid. Electricity from the HV regional grid is first transported by the medium voltage grid and after that by the low voltage grid. The medium voltage distribution grid delivers electricity to local industries and large buildings and has a voltage between 3 and 20 kV. The low voltage distribution grid delivers electricity to houses and small buildings and has a voltage between 230 and 400 V. Between these grids transformers are used to change the voltage of the electricity from the incoming line to the outgoing line.

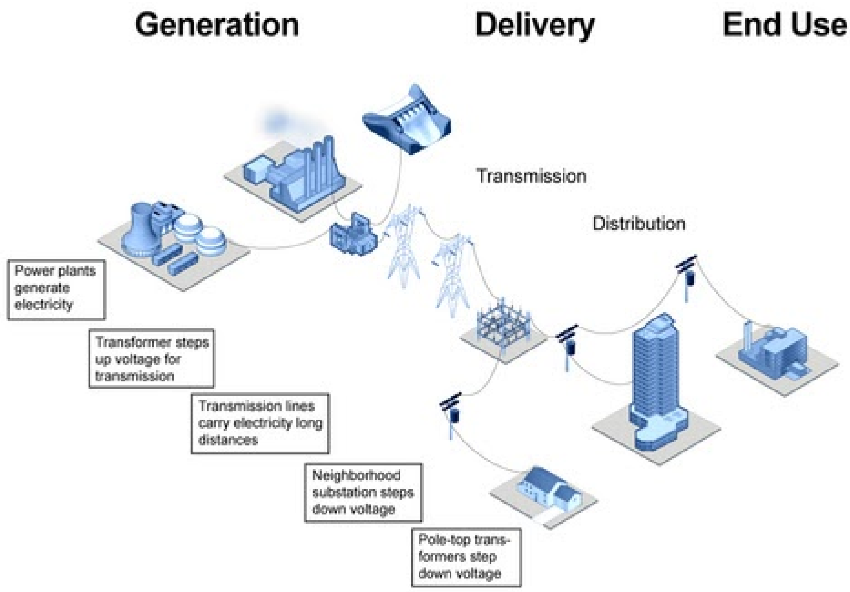

Figure 3: Overview of electricity grid [13].

The electricity running in each line is managed by the system operaters. There are different system operaters for transmission and distribution grids. TSO's (transmission grid operator) own and operate the transmission grids and DSO's (distribution system operator) own and operate the distribution grids. Their job is the safe and efficient operation of the different grids. This job entails ensuring that the electricity running through each line does not violate the capacity of the line. Furthermore, they have to ensure that voltages are kept within their limits and that installations are replaced or upgraded where needed. Finally, DSO's and TSO's are responsible for balancing the power generation and demand at all times, meaning that the electricity running through each line is exactly the amount needed by industries and (residential) buildings. Next to system operators, retailers job is to buy the electricity from the production plants and thereby give the order to produce that amount. Electricity can't be stored in the cables so the demand of electricity must match the generated amount of electricity. So the retailers predict the demand for the next day for each 15 minute interval. They buy the amount of electricity from the generating companies needed to match this predicted demand. Then the power plants must generate this amount of electricity during the next day. The system operators take care of the transportation of electricity from producers to consumers.

With the rise of renewable energy sources the electricity grid is changing. In particular, the unidirectional flow of electricity from producers to consumers is no longer guaranteed. Power plants are no longer the only producer of electricity in the grid. Wind turbines and PV panels are connected to the transmission and distribution grid. On the consumer side more and more devices and appliances require electricity. Examples are electric vehicles, heat pumps and electric stoves. All of this causes an increase in the amount of electricity transported through the grid. Especially when PV panels on a house produce more than needed by the local consumer it can happen that the flow of electricity is "backwards" i.e., up through the to distribution grid towards other areas and customers. The traditional production plants have to adjust their production because part of the demand is produced by these renewable energy sources. This becomes increasingly difficult and costly with a higher share of production from renewables.

The need arises to plan and predict the demanded, generated and transported electricity. This requires monitoring and controlling the electricity flows. With the current grid architecture this is not possible. That's why the grid needs to be transformed into a smart grid. What a smart grid implies is described in the next section.

1.3 Smart grids

In literature there are many visions on smart grids. Various technologies are mentioned with respect to smart grids. We focus on the following vision (which is comparable to current practice). This vision of a smart grid consists of multiple aspects. One aspect is that in a smart grid there is a two-way communication channel between the consumers and a central controller [9]. The central controller coordinates the electricity use of all the houses within the grid. In this way it can also be the system operator but this does not always have to be the case. The two-way communication channel can be used to manage electricity consumption by communicating problems occuring in the grid to the consumers. Our way to detect such problems is by the use of smart meters. Smart meters record electricity consumption inside a house and communicates it with the controller. Unlike the traditional meters, smart meters can communicate directly with the controller and report the collected data. Smart meters can communicate over wired lines (e.g. power lines) or via wireless connections (e.g. Wi-Fi).

Part of the problems occuring in the grid can be solved by the consumers by changing their demand together to match the generation. Coordination of the controller is required to balance (match) total demand and generation. This requires the abililty of the controller to communicate with consumers directly. Smart meters can be used to improve balance between demand and (renewable) generation and decrease investments in the grid structure. This balancing is done as follows: First, smart meters inside the house record the electricity usage and send it to the controller. The controller gathers this data and analyses it. If he sees problems occuring, he can communicate this to the consumers in order to ask them to adapt their usage of electric devices in such a way, that all consumers together solve some problems in the grid. The consumers react on these requests by changing their electricity usage and communicating these changes in demand back to the controller.

The communication channel of the smart grid must be secure. Privacy sensitive information is transported over this channel meaning that only few people should have access to data of consumer electricity usage for example. Based on this it seems to be a good decision that the controller only get accumulated data of all consumers in the entire grid and not for the households individually. The channel should also be protected from outside attacks on the soft- and hardware.

Another aspect is the automation of the electricity consumption within a household (called home automation). Smart appliances and devices can be monitored and controlled via a channel, which is usually Wi-Fi. These include lighting, heating, air conditioning, washers and many more. A house controller (intermediate controller) controls the electricity consumption of the devices and appliances. So if the house controller gets a signal from the central controller it can determine and execute the best electricity consumption for all devices simultaniously regarding that signal. The devices can communicate with each other to determine a joint consumption. Every device/appliance has different limitations when considering changes in electricity usage, e.g. a heat pump must produce enough heat to keep to temperature stable at a certain temperature, for example 20 °C. Furthermore, most devices/appliances only have a finite number of options for when to turn on the device and for the amount of electricity to use. These options are called the flexibility of the device/appliance, because it states how flexible the device is when it needs to change its consumption. Likewise, the flexibility of a household is the sum of all the flexibility of its devices. So smart devices can change the demand for electricity at a certain time to help to balance demand and generation. Storage is another envisioned aspect of a smart grid. Excess (generated) electricity may be stored in for example a large battery. This electricity could be used to supply the (peak) demand at another time. Storage can shift demand and generation in time to create a better balance between these two. The goal of a smart grid is to cause more reliability and efficiency by improving the balance between generation and demand.

1.4 Thesis overview

The rest of this thesis is outlined as follows. In the problem statement we further discuss the problems occuring in the grid due to the energy transition Furthermore, we specify which problem we want to solve in particular. In the next section (model) we give a possible solution to our problem and we describe the resulting game. After that we describe which optimization problem we have to solve to calculate the possible solution. The fifth section describes the implementation of a simulation and we present the results of the solution in the practical simulation study. Finally, we discuss the results, conclude the thesis, and give recommendations for future research.

2 Problem statement

In this section we discuss some of the problems related to the changing electricity flows caused by the energy transition.

Firstly, most renewable energy sources get their power from the wind or sun. Wind speed and solar radiation fluctuate a lot causing the produced electricity to vary over time. Therefore, it can happen that even when the installed capacity of renewable energy is in principle enough to provide for the demand, the production by renewable energy sources is not enough in practice. These fluctuations happen on different time scales, e.g. minute-to-minute, day-to-day and even on a seasonal scale. Day-to-day fluctuations mean that during some days, the installed capacity of renewable generation cannot supply the daily energy demand and traditional power plants must be turned on to fill the gap which is an expensive and inefficient process. To account for short-term (minute-to-minute) fluctuations, traditional power plants must be kept ready at all times, which is also expensive and ineffecient. The opposite can also happen, meaning that there is more electricity from renewable energy sources available than needed by the consumers. In this case not all electricity can be used so the output of the renewable energy sources is curtailed as storing possibilities of excess electricity are virtually non-existent in the grid. This contributes to a potential increase in losses and decrease in efficiency of the entire electricity system.

Secondly, the energy transition leads to an increase of the demand of electricity by consumers and to a synchronization of consumer demand. This means that many consumers use their devices at the same time. For instance, in the afternoon people come home from work and start using electric devices such as for cooking, heating their house and charging electric vehicles. This causes peaks in the demand for electricity. During such peaks much more electricity is consumed so the needed network capacity has to be adapted to this peak loads and also less demand can be fulfilled by local generation implying that a high amount has to be transported over a long distance. If the capacity of the network is not adapted this can overload and damage the transformers and cables and can cause the voltage and current level to exceed normal grid boundaries (grid boundaries specify for each cable in the grid the minimum and maximum acceptable voltage and current). Often voltage problems occur before the current begins to cause problems. Note that the voltage level fluctuates all the time but within the grid boundaries this is acceptable. Voltage levels exceeding the grid boundaries are not desired when powering (industrial) devices and appliances. Traditionally, such problems would be solved via grid reinforcements, which are costly. Alternatively, (some of) these issues mentioned above may be resolved in another, more efficient, way, namely by introducing the concept of a smart grid.

Smart devices with flexibility in using electricity can change (peak) electricity demand for the consumer. Communicating devices can take into account the electricity usage of the other devices within the same household. Looking at the low voltage grid residential devices and appliances can make a significant difference if they all work together to solve the problems within the grid. (Before, this was only economically and technically feasible for consumers in the medium and large voltage grid [11].) Residential devices can coordinate their electricity usage such that the demand is spread more equally across the day. Moreover, excess electricity of one house can be used locally to compensate demand peaks of other houses in the same LV grid. Because of this local matching the electricity does not have to be transported over a long distance and degradation of cables and transfomers gets reduced. Excess electricity can also be stored in batteries to be used locally at a later point in time. Note that a house can only have excess electricity if it stores or produces electricity in some way. The mentioned solutions decrease demand and production peaks implying that less electricity is transported. This means that the mentioned fluctuations of the demand and production are solved locally as much as possible.

Balancing demand and generation in a grid is done by the central controller. In a smart grid the central controller can communicate with the residential devices, so it can tell the devices how much electricity to consume at each time. However, coordinating all the residential devices in an efficient and reliable way involves a lot of planning from the controller. The devices of each house must be coordinated to compensate for demand and generation peaks. Moreover, all of these devices also have to meet constraints set by the consumer through the day. On top of that devices are not always available to use electricity (e.g. electric vehicles) and devices use different amounts of electricity, resulting in different flexibility for each device. Optimizing the planning of electricity use by all the devices for a whole grid becomes too complex for the central controller. To solve this, intermediate controllers can be established which control a smaller part of the grid. The intermediate controllers are coordinated by the central controller. But even in this situation the problem to determine the electricity usage of residential devices together is too complex for an intermediate controller.

To avoid issues of centralized planning there are approaches that do the calculations decentralized, meaning that a large part of the calculations is done on household level. The central controller communicates to consumers what the desired electricity usage is and consumers calculate a planning for their devices accordingly. A planning includes when to turn on/off devices and how much electricity they consume when turned on. The controller determines whether the individual plannings of the consumer solve the problems occuring in the grid, by considering the planning of all consumers together. This approach is called decentralized energy management (DEM).

2.1 Profile Steering

We eleborate about a decentralized energy management approach formulated by T. van der Klauw [11] which is called Profile Steering. The Profile Steering approach involves letting smart devices make their own planning. The devices make their planning using steering signals send by a controller to the devices via a two-way communication channel. These steering signals indicate to the devices what the devices should do to solve the problems in the grid. In other words steering signals describe the objective the device controller (household) uses to optimize its own planning. This planning is called a schedule or an energy profile in Profile Steering. The possibilities of a device to change its schedule is the flexibility of that device.

Profile Steering is a heuristic that consists of two phases: In the first phase the controller requests an initial schedule for the use of electricity from all the devices in a neighborhood under consideration. All the initial schedules together form the preliminary energy profile. After the initial schedules are made and collected by the controller the second phase starts. The second phase, called the iterative phase, begins with the controller sending steering signals to all the devices. Then, the devices calculate their optimal schedule for the next day according to the steering signals. The calculated schedules usually differ from the initial schedules and are called candidate schedules. The devices send their candidate schedules to the controller, which collects them. All the candidate schedules are compared to the total of the initial schedules that were previously received by the controller. The schedule with the best improvement is chosen, in other words the schedule which complies the most with the steering signals. The corresponding device is asked to update its initial schedule to match the candidate schedule. Next to this, the controller updates the schedule of this device in the preliminary energy profile to the candidate schedule.

The second phase is repeated as long as sufficient progress is made in solving problems occuring in the grid. After both the phases of the Profile Steering approach are executed, the preliminary energy plan/profile generally gives a combined schedule for the devices that is good for the grid. To ensure that this energy plan is executed the devices are expected to closely follow their own schedules. However, nothing in the approach itself prevents households from executing a different schedule than the one chosen by the Profile Steering approach. It is possible to make a legal agreement to ensure the agreed schedule between the controller and the household is executed. However, a household is unlikely to agree to this for free. This means that there has to be some kind of compensation for the household in exchange for executing a different schedule. In other words, there needs to be a compensation for participating in Profile Steering. Such a compensation should incentivize households to sign an agreement stating they will closely follow the schedule agreed with the controller (or possibly pay a fine). Other incentives for participation in Profile Steering are social incentives, such as technological advancement, sustainability or social pressure. In this thesis we consider only the monetary incentives for participation as an extension to Profile Steering. We discuss these incentives is in the next section.

3 Model

In this section we introduce a framework that provides incentives for consumers who participate in profile steering and describe the resulting game. We extend profile steering in two ways. First, we use different steering signals because previously consumers got the objective of the controller as a steering signal. This objective could be used to intentionally damage the grid or cause problems for the controller. The second extension is that consumers are given some reward for offering flexibility. Offering flexibility means that the consumers are changing their energy profile, proposing this to the contoller, getting selected and then sticking to that energy profile. Hereby the reward is chosen such that it is proportional to the overall improvement made by the consumer to the energy profile from the controller's perspective.

In the following we outline the steps that are involved in our adapted approach. The controller uses historical data of the energy profiles of the households not participating in profile steering to predict the energy profile of those households for the next day. The households that are participating in profile steering are asked to send a first preliminary energy profile. Households are asked what their energy profile would be if they could choose it without any steering signals from the controller. The sum of these predictions and preliminary profiles of all households leads to a preliminary aggregated energy profile $\tilde{x}$ or, in other words, the baseline for the approach.

The controller now analyses this profile to see if there are problems in the grid caused by too much or too little electricity demand in one or more time intervals. If this is the case, he sends a signal for every time interval of the next day to the household indicating if it should consume more, less or the same amount of electricity during that interval. Formally, this signal is either a "$+$", a "$-$" or a "0" indicating whether the household is desired to consume more, less, or the same amount of electricity, respectively. The steering signal of all time intervals form a vector SV of the signals for the next day which is send to every household. Based on this vector SV, household i determines a candidate energy profile $\hat{x}_i$ and sends it back to controller. The controller now chooses the household with the candidate energy profile which solves the largest portion of the problems in the grid. He replaces the household's preliminary energy profile $\tilde{x}_i$ of this household by the candidate energy profile $\hat{x}_i$ and updates the preliminary aggregated energy profile.

After this a new iteration of profile steering starts. For this, the controller adapts the steering signals send to the households because the preliminary aggregated profile is changed and remaining problems need to be solved. These steering signals are send to the households and again each household sends back a candidate energy profile. The operator chooses again the household which can solve the largest portion of the problems and so on. This is done iteratively until no further improvement can be made. Now each household which has a profile $\hat{x}_i$ which has been selected is expected to execute its last energy profile $\hat{x}_i$ profile in $\tilde{x}$. In other words, selected households switch on and off their devices such that their energy profile matches their preliminary energy profile $\tilde{x}_i$. We assume that the not selected households execute their preliminary energy profile approximately or otherwise that the number of not selected households is large enough to even out the deviations from the aggregated preliminary energy profile.

The selected households are compensated for the amount of flexibility they offer with their candidate profile. To push the households to execute their profile, we assume that if consumers diverge too much from their selected energy profile $\hat{x}$ the controller will impose a fine on them.

If a consumer offers a new candidate profile to the controller, this profile deviates somehow from his most prefered profile and thus, the consumer has certain costs assiocated with offering this flexibility to the network operator. These costs are e.g. the loss of comfort of the consumer or the change in degradation of the devices that are used differently in the new profile. To specify these costs of a customer we need to define a function which describes the costs of offered flexibility. We assume that the costs can be split up per device. Thus, for each device in the household we need to define such a function and the function will be different. In this thesis, we define these functions for 6 device categories: baseload, PV (photo-pholtaics), time-shiftable devices, heat pumps, EV's (electric vehicles) and electrical batteries.

The functions should depend on the utility function of the consumers and/or on the characteristics of the device. Thereby, the utility function is the change in the comfort level of the consumer regarding the new candidate profile. Based on the chosen concept, the costs for loss of comfort should always be positive. Furthermore, we assume that comfort is maximized in their baseline profile. This means that the costs are minimal when the energy consumption does not change at all and the consumer can execute their baseline profile. We model the cost functions as convex functions as we assume that with an increasing deviation from the baseline profile the change in the discomfort increases.

We now look at the presented method from the consumers perspective. First of all we assume that every consumer has some incentive to be chosen by the network operator otherwise this approach may not have the desirable effect. As in every iteration only one consumer gets compensation, each consumer has to decide what the best profile is for him in order to be chosen. Note that each candidate energy profile has a probabililty to be chosen, which depends on the steering signals and on the candidate energy profiles of the other consumers. However, these candidate energy profiles of the other consumers are not known in advance. Consumers can only predict candidate profiles of the other users based on historic data about chosen candidate energy profiles. This implies that the consumer's choice for a candidate energy profile is not straightforward. Every choice has different costs, rewards and a different probability to be selected. There is a tradeoff between the probability and reward, so that is a problem that needs to be solved.

To determine the effectiveness of this approach we want to know the choice in energy profile for all consumers in the grid given the used steering signals. We can only predict what consumers are going to choose based on the choices of other consumers. So we need to determine the best choices for all consumers simultaniously. In order to do that we first model the strategic competition between the consumers in a strategic game . The basis of a strategic game is that it has multiple people with more than one choice and these choices influence the reward (payoff). So we model this problem as a game between all the households in a grid where the goal of each household is to achieve as much compensation as possible. We formulate this game and determine the nash equilibria in the following section.

3.1 Auction Game

In this section we formulate a game where households compete for "selling" their flexibility to the controller. Flexibility is the ability of the consumer to change the amount of consumed electricity for time intervals. We model this game as an auction, where the bid is the offered flexibility and the auction prize is the compensation for the offered flexibiliy. Specifically this game is a first-price sealed-bid auction, first-price because the winner gets the compensation of his own bid and sealed-bid because every household bids once per round and the bids are not known by the other households. Furthermore, there are multiple bidding rounds and one round of the auction is equal to one iteration of profile steering. More precisely, the game is as follows:

We have a set of players $N = \{1, 2, ..., n\}$ corresponding to the households and there is one auctioneer corresponding to the central controller of the grid. A round of the auction starts with all players simultaniously sending a bid to the auctioneer. These bids are unknown to the other players. The auctioneer collects all bids and chooses one player as the winner of the auction round. Then, the auctioneer determines the reward of the auction and pays it to winner. This ends the current round of the auction and a new round is started as long as the auctioneer wishes to continue.

Thus, first players send a bid to the auctioneer. A bid b_i for player i consists of an candidate energy profile $\hat{x}_i$ and a minimum reward P_i:

$$b_i = \{\hat{x}_i, P_i\} \quad i \in N$$

The candidate energy profile $\hat{x}_i$ of player i shows the player's consumed electricity for each time interval. Formally it is defined as $\hat{x}_i = \{\hat{x}_{i,1}, \hat{x}_{i,2}, \ldots, \hat{x}_{i,T}\}$ where $t = 1, ..., T$ are the time intervals of a day. In this thesis we assume that the day is split up in 15-minute intervals, so $T = 4 \cdot 24 = 96$. We choose this length of an interval because this is one of the most used interval lenghts within the electricity market. Furthermore, data on minute intervals or less is harder to acquire than 15, 10 or 5 minute data on energy consumption. Smart meters can report this data, but the density of smart meters within a neighbourhood is still quite low.

The players send two pieces of information to the auctioneer. The candidate energy profile $\hat{x}_i$ shows the auctioneer which problems occuring in the grid player i can solve. The minimum reward P_i is used to indicate to the auctioneer that player i will not agree with a reward lower than P_i. With this information the auctioneer must determine the winning player and the corresponding reward.

The auctioneer determines the winner of an auction round based on an objective function f, used to evaluate the received bids. The value of bid b_i, given by $f(b_i)$, puts a value on how much the bid of player i solves problems in the grid and on the minimum reward that is asked for the bid. In other words, this function $f(b_i)$ captures the goals of the auctioneer and the deviation from these goals determines the objective value of a bid. A bid consists of two parts, the candidate energy profile and the minimum reward and the auctioneer values these differently. Therefore the auctioneer has an objective value for both, which added up is the auctioneer's objective value for the bid. The objective function for the candidate energy profile is given by g and the objective function of the minimum reward is h. The auctioneers objective function therefore is $f(b_i) = g(\hat{x}_i) + h(P_i)$. In other words, the better an energy profile achieves the auctioneer's goals, the higher the objective value is. The function h is non-increasing in the minimum reward P_i. The reason is that the more the auctioneer has to reward the player, the less it is worth. There is a preffered reward for the auctioneer captured in the function h, which states what the auctioneer is willing to pay for the offered bid. A player can ask a very high minimum reward which is not proportionate to the objective value and in this case he should not be selected. The player with the highest objective value is chosen, thus a player with a higher objective value has a higher probability to win.

So, the auctioneer has to find a way to optimize his objective value. There are a number of ways for him to do this, it depends what the auctioneer is willing to pay for the offered bid. Firstly, the auctioneer considers only the minimum reward of a bid, so $g(\hat{x}_i) = 0$. In this case the goal is to minimize $h(P_i)$, so the player's bid with the lowest minimum reward is selected. In addition to this a minimum level for the objective value of the candidate energy profile $g(\hat{x}_i)$ can be set such that at least the selected bid does not worsen the problems in the grid. Secondly, the auctioneer considers only the candidate energy profile, so $h(P_i) = 0$. The goal in this case is to maximize $g(\hat{x}_i)$ and therefore the player with the energy profile corresponding to the highest objective value wins. A maximum $h(P_i)$ can be set in the form of a budget for the auctioneer. Players with minimum reward demands more than the budget are not selected. Thirdly, the auctioneer can optimize the difference between $g(\hat{x}_i)$ and $h(P_i)$. In this case, the relative improvement by the candidate energy profile compared to the value for the minimum reward determines the winning player. Another way is to optimize the objective function $f(b_i)$, where the player's bid with the highest objective value wins the auction round. This one is the most logical, because the auctioneer values this bid the most so that player should win. Again, a maximum for $h(P_i)$ and/or a minimum for $g(\hat{x}_i)$ can be used to improve the choice.

The players do not know the objective function of the auctioneer, but nevertheless information about the winning probability can be extracted from historic data on the bid of the winning player. The result is a winning probability for each possible bid. We assume that the probability is non-decreasing in the reward for the player, so if the reward corresponding to a candidate energy profile increases, then so does its winning probability. The reward for player i is determined by the reward function $R(\hat{x}_i)$, which is the possible reward of the candidate profile if the player should win. We assume that the reward function is just the change in electricity consumption beneficial to the auctioneer. This is measured by how much of the problems in the grid are solved by the player switching his base profile to his candidate profile. The problems in the grid are communicated with the players via steering signals and we assume that the reward function depends on these signals. The more the candidate profile follows the steering signals, the higher the reward is. In principle, steering signals indicate how players should change their profile, so the difference between the candidate profile and the preliminary (base) profile should be used to indicate how compliant the candidate profile is to these signals.

As mentioned before, there are three steering signals possible for each time interval t, a "$+$" indicating that the player should consume more electricity, a "$-$" shows that the player should consume less electricity and a "0" indicates that the electricity consumption should stay the same. There are 96 time intervals, one for every 15 minutes of the day. If the difference between the candidate and preliminary profile for one time interval is conform the steering signal then the reward increases. This works as follows:

- The steering signal is a " + " for time interval t: if the difference between the candidate and preliminary profile is positive the reward increases because this was required by the auctioneer. Otherwise, if the difference is negative the reward decreases because it is not beneficial to the auctioneer. If there is no difference, the reward does not change.
- The steering signal is a " − " for time interval t: Here it is the other way around. If the difference is positive the reward decreases because it is the opposite of what the auctioneer wanted. If the difference is negative the reward increases, this is beneficial to the auctioneer. Again, if there's no difference the reward does not change.
- The steering signal is "0" for time interval t: the reward does not change because the steering signal indicates that the electricity consumption should stay the same. The player is not rewarded and not penalized for the difference with the preliminary energy profile in this time interval.

The reward function is characterized as follows:

$$R(\hat{x}_i) = \sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$$

$$s_t = \begin{cases} +1 & \text{when the steering signal is a " + " for interval } t \\ -1 & \text{when the steering signal is a " − " for interval } t \\ 0 & \text{when the steering signal is "0" for interval } t \end{cases}$$

The reward function is the sum of difference between the candidate and current preliminary profile multiplied with values corresponding to steering signals s_t. The steering value s_t indicates for each steering signal whether positive or negative differences in energy consumption are rewarded.

When the auctioneer determines the reward he must consider the minimum reward P_i of the winning player. If the bid gives a reward lower than P_i, the reward is raised to the minimum reward. Therefore, the reward of the winning player is the minimum of the value of the reward function and P_i. The winning player of round r is denoted by w_r $r = 1, 2, \ldots$ The reward of the winning player R_{w_r} is

$$R_{w_r}(b_i) = \max\{R(\hat{x}_{w_r}), P_{w_r}\}$$

Each player has to send a bid to the auctioneer. The player can choose the bid to send and his goal is to win the auction round with the maximum reward possible. So he has a choice in candidate energy profile and minimum reward. The candidate profile of a house is given by the sum of the candidate profiles for the devices in the house, therefore the player has to choose a candidate profile for every device. Player i has a set of devices $D_i = \{1, ..., M_i\}$. The controllable devices classes are time-shiftables, heat pumps, electric vehicles and batteries.

- Time-shiftables are devices with a fixed energy profile that can be shifted in time. They have a certain starting time for which they are turned on, then they have a fixed load profile for a certain period of time and after that they turn off. Examples are dishwashers and washing machines. The only choice for the player is the starting time of these devices. The player can have restrictions for the starting time of these devices, e.g. that someone has to be home when the washing machine is done.

- The class heat pumps contains devices which consume electricity to generate heat and store this in a buffer. The heat buffer is used to provide the household with heat, e.g. hot water demand and space heating. Each time interval the devices can be on or off, when the device is on the heat in the buffer increases. The heat stored in the buffer must be kept between certain bounds, so the heat in the buffer can fluctuate a bit. Therefore, the flexibility of these devices is that they can switch from on to off or the other way around while keeping the heat in the buffer within the given bounds.

- Electric vehicles are devices that have an internal buffer in the form of a battery. Electric vehicles arrive at a certain time at the house and have to be charged before they leave again. The amount of electricity in the buffer is called the state of charge, which is defined as a percentage of the maximum energy storable in the buffer. When the vehicle arrives it has used some electricity so the state of charge is low. Upon leaving there is a minimum state of charge set by the household. In between the vehicle is charged but the amount does not have to be constant. For the time intervals that the vehicle is present there are a number of charging rates, so the player can vary the charging rate as long as the vehicle's state of charge is higher than the minimum when leaving the house. The choices of the player are how much to charge in each time interval.

- Batteries are devices which can store electricity. Batteries are used to store electricity from (renewable) generation and are discharged when other devices need electricity. The stored electricity can also be send back to the grid to power devices of other households. Batteries can be charged or discharged in each time interval and we assume there are only a finite number of (dis)charging rates. The only restriction of the battery is the capacity. The flexibility of the battery is that it can charge and discharge in each time interval at multiple rates.

We denote by T_i the set of time-shiftable devices, H_i the set of heat pumps, E_i the set of electric vehicles and B_i the set of batteries of player i, i.e. $D_i = \{T_i \cup H_i \cup E_i \cup B_i\}$ and $M_i = |T_i| + |H_i| + |E_i| + |B_i|$.

The set of choices for a player is given in his strategy set. Strategies in this set for the player are combinations of the possible choices for the controllable devices of the player. To define the strategy of player i, we first consider the sets of choices of the different types of controllable devices. There are different choices for each controllable device:

- Time-shiftables can choose their starting time t_s, which are 96 possibilities or less. Users define an interval in which the device can be started. So not all time intervals might be feasible, only the starting times that are within the user defined interval are.
- The devices in the class heat pumps can choose to be on or off for each time interval. These choices can be represented by a boolean vector $hp = \{hp_1, ..., hp_{96}\} \quad hp_j \in \{0, 1\}$. There is a minimum and maximum allowed heat in the buffer for safety reasons, therefore the number of consecutive on states is bounded. The same applies for the number of consecutive off states.
- Electric vehicles can choose the charging rate for each time interval. We assume there are only a finite number of charging rates possible. The rates are denoted by $chev_k$ with $k = 1, ..., n_{EV}$. The strategy of the electric vehicle consists of vectors $EV = \{EV_1, ..., EV_{96}\}$ with $EV_j \in \{0, chev_1, chev_2, ..., chev_{n_{EV}}\}$ for all j. The state of charge stays always within the interval $[0, 1]$, because this is how the state of charge is defined. Furthermore, the vehicle is only parked and plugged in for a limited number of time intervals. Outside of these intervals the vehicle cannot be charged. Also, a minimum state of charge is given that the battery of the vehicle needs to contain at the end of each charging session. This limits the feasible strategies for the EV to those that ensure the vehicle is charged enough when it departs.
- Batteries choose the charging or discharging rates for each time interval, again we assume this is a finite number of states. The discharging amounts are $bdch_l$ with $l = 1, ..., n_{BD}$ and the charging amounts are bch_m with $m = 1, ..., n_{BC}$. The strategy of the battery consists of vectors $BT = \{BT_1, ..., BT_{96}\}$ with $BT_j \in \{-bdch_{n_{BD}}, ..., -bdch_1, 0, bch_1, ..., bch_{n_{BC}}\}$. Again the state of charge must stay within the interval $[0, 1]$, thus limiting the set of feasible charging and discharging rates for consecutive time intervals.

Together the combinations of the strategy sets of the different devices is the strategy set of player i and this is defined as follows

$$S_i = \{t_s^1, ..., t_s^{|T_i|}, hp^1, ..., hp^{|H_i|}, EV^1, ..., EV^{|E_i|}, BT^1, ..., BT^{|B_i|}\}$$

There are $|T_i|$ starting times, one for each time-shiftable of player i, $|H_i|$ boolean vectors for the heat pumps, $|E_i|$ vectors for the electric vehicles and $|B_i|$ vectors for the batteries.

Each of these vectors and starting times results in a candidate energy profile for that device, denoted by $\hat{x}_i^{D_i} = \{\hat{x}_{i,1}^{D_i}, \hat{x}_{i,2}^{D_i}, \ldots, \hat{x}_{i,T}^{D_i}\}$ for device $D_i = \{1, \ldots, M_i\}$. The candidate energy profile is defined as the total energy consumption for each time interval, so the energy consumption of the devices together forms the candidate energy profile of player i. In other words, the candidate profile for player i is the sum of the candidate energy profiles of the devices for player i for each time interval t:

$$\hat{x}_{i,t} = \sum_{D_i=1}^{M_i} \hat{x}_{i,t}^{D_i}$$

Thus, the choices for the devices translate directly into a candidate energy profile and therefore the choices determine the first part of the bid.

Players can choose to offer a candidate energy profile which decreases their comfort, i.e. start a time-shiftable at a less preferable time. Some choices can be good for the network but not for the player. In that case, the player only chooses to do so because he is rewarded for it. The reward should outweigh the loss of comfort caused by their candidate energy profile.

The loss of comfort is reffered to as costs $C_i(\hat{x}_i)$ for player i Each candidate energy profile has certain costs, which are nonnegative. The costs are zero when the player doesn't sacrifice any comfort, otherwise the costs are positive. In the next section we eleborate about the costs of a candidate energy profile.

3.2 Utility functions

Each bid that the player sends to the auctioneer has certain costs for the player. These costs are a quantification of the loss of comfort experienced by the player. Different players have different losses of comfort for the same candidate energy profile, because different players value a change of the candidate energy profile differently. We capture this in utility functions of the player. The player's costs are divided into costs for the devices of the player. Thus, the player has a utility function for each of its devices. We take $U_i^{D_i}(x_i^{D_i})$ to be the utility function of device $D_i = 1, 2, \ldots, M_i$ of player i. The utility function defines the change in costs when changing the use of the flexible device D_i with respect to an *original energy profile*, which is the energy profile the player chooses in absence of steering signals from the auctioneer. All the values of the utility functions for a player's devices together form the costs of that player's candidate energy profile, because the player's device profiles together form the player's candidate energy profile.

Examples of what the utility functions can include are the loss of comfort due to temperature changes, the depreciation of batteries by extra charging or discharging, the change in state of charge (SoC) during the time periods and the loss of comfort due to changes in the start time of time-shiftables. As mentioned, we make the assumption that a player has minimum costs when executing their original energy profile. The reason behind this is, that if households can decide when to turn on/off their devices with no restrictions from the auctioneer the satisfaction of these households should be maximal.

Furthermore, as discussed before, the utility functions are assumed to be convex. For simplicity we assume that all the utility functions are expressed by quadratic functions. The costs are divided into costs per time interval $t = 1, \ldots, T$ and these are divided into costs for different cost factors $j = 1, \ldots, J$. Thus, all utility functions are of the form

$$U_i^{D_i}(y) = \sum_{t=1}^{T} \sum_{j=1}^{J} U_{i,j,t}^{D_i}(y_{i,j,t}) = \sum_{t=1}^{T} \sum_{j=1}^{J} a_{i,j} y_{i,j,t}^2 + b_{i,j} y_{i,j,t} + c_{i,j},$$

where $a_{i,j}, b_{i,j}, c_{i,j}$ are coefficients for player i and cost factor j and $y_i = \{y_{i,1,1}, y_{i,1,2}, \ldots, y_{i,1,J}, y_{i,2,1}, y_{i,2,2}, \ldots, y_{i,2,J}, \ldots, y_{i,T,1}, y_{i,T,2}, \ldots, y_{i,T,J}\}$ are variables that we derive from the energy profile of a device $x_i^{D_i}$. We come back to the specification of y for different devices later. Note, that a quadratic function of the form $ax^2 + bx + c$ is convex iff $a > 0$. The costs are minimal for theoriginal energy profile, therefore for every i, j and t

$$\frac{d}{dy} U_{i,j,t}^{D_i}(y_{i,j,t,or}) = 2a_j y_{i,j,t,or} + b_j = 0,$$

where $y_{i,or} = \{y_{i,1,1,or}, \ldots, y_{i,1,J,or}, \ldots, y_{i,T,1,or}, \ldots, y_{i,T,J,or}\}$ is a variable dependent on the *original energy profile* $x_{i,or}^{D_i}$ of device D_i. From this we obtain a relation for $a_{i,j}$ and $b_{i,j}$:

$$a_{i,j} = \frac{-b_{i,j}}{2y_{i,j,t,or}}$$

Furthermore, the utility function expresses the difference in costs with the original energy profile, so we can assume that $U_{i,j,t}^{D_i}(y_{i,j,t,or}) = 0$ as the difference in costs of the original energy profile with the original energy profile itself is 0. Therefore

$$
\begin{aligned}
0 &= a_{i,j}(y_{i,j,t,or})^2 + b_{i,j}y_{i,j,t,or} + c_{i,j}, \\
&= \frac{-b_{i,j}}{2y_{i,j,t,or}}(y_{i,j,t,or})^2 + b_{i,j}y_{i,j,t,or} + c_{i,j}, \\
&= \frac{b_{i,j}y_{i,j,t,or}}{2} + c_{i,j}.
\end{aligned}
$$

This results in

$$
c_{i,j} = \frac{-b_{i,j}y_{i,j,t,or}}{2} = a_{i,j}(y_{i,j,t,or})^2.
$$

Replacing $b_{i,j}$ and $c_{i,j}$ in the utility function with the obtained expression we get

$$
\begin{aligned}
U^{D_i}_{i,j,t}(y_{i,j,t}) &= a_{i,j}(y_{i,j,t})^2 + b_{i,j}y_{i,j,t} + c_{i,j}, \\
&= a_{i,j}(y_{i,j,t})^2 - 2a_{i,j}y_{i,j,t,or}y_{i,j,t} + a_{i,j}(y_{i,j,t,or})^2, \\
&= a_{i,j}((y_{i,j,t})^2 - 2y_{i,j,t,or}y_{i,j,t} + (y_{i,j,t,or})^2), \\
&= a_{i,j}(y_{i,j,t} - y_{i,j,t,or})^2.
\end{aligned}
$$

So the utility functions are given by the sum of the quadratic difference of the variable $y_{i,j,t}$ with the original variable $y_{i,j,t,or}$ multiplied by a constant $a_{i,j}$. In other words, the utility functions are characterized by the variable dependent on the original energy profile $y_{i,or}$, and the constant $a_{i,j}$. Note that, $y_{i,or}$ is the minimum of the utility function and the constant $a_{i,j}$ determines the shape of the utility function. To ensure convexity the constants $a_{i,j}$ are positive. A higher value of mean deviations from $y_{i,j,t,or}$ are penalized more.

Each player has different costs for the changing their energy profile, therefore the constant $a_{i,j}$ differs between players. For modeling purposes we simplify the distribution of the constant $a_{i,j}$. Therefore, we sample $a_{i,j}$ from a gaussian distribution. Thus, $a_{i,j} \sim \mathcal{N}(\mu_{i,j}, \sigma^2_{i,j})$, where $\mu_{i,j}$ is the mean for the constant $a_{i,j}$ and $\sigma^2_{i,j}$ the variance. The $\mu_{i,j}$ and $\sigma^2_{i,j}$ are different for each device to model that each device has different costs. As mentioned before, utility functions of the devices are functions of $y_{i,j,t}$. This variable can be calculated from the candidate energy profile. Examples of $y_{i,j,t}$ are the starting time of a time-shiftable, the energy in the buffer of the heat pump and the state of charge of the battery. Below we define the utility functions for the different devices.

The time-shiftable devices have a fixed energy profile once they are started and are characterized by their starting time. Therefore, the utility function of a time-shiftable is defined based on its starting time. Changing the starting time of time-shiftables may cause discomfort to the player, because players have a preference for the starting time. This implies that $y_{i,j,t} = t^{T_i}_s$

and $y_{i,j,t,or} = t_{s,or}^{T_i}$ where $t_s^{T_i}$ is the starting time of time-shiftable T_i and $t_{s,or}^{T_i}$ the original starting time. Furthermore, $T = 1$ and $J = 1$ for the utility function of a time-shiftable. Therefore, the utility function is of the form:

$$U_i^{T_i}(t_s^{T_i}) = a_i(t_s^{T_i} - t_{s,or}^{T_i})^2.$$

The heat pump is combined with a buffer in which heat can be stored and this is used to provide the house with heat. The buffer is filled with hot water to supply to the house with water for heating and hot tap water. In this case the house itself can also be seen as a buffer. The heat 'stored' inside the house must be enough to ensure the indoor temperature satisfies user constraints. The losses are different than for the water buffer,as the air within the house is heated instead of the water in the buffer. In both cases the heat is measured by the temperature of the substance in the buffer. The temperature in the buffer is controlled by a heat pump, which can be on or off during each time interval. Each player has a preference for a certain indoor temperature for each time interval. Therefore, the utility function for the heat pump is the amount of discomfort caused by a change in temperature. This implies that $y_{i,j,t} = T_t^{H_i}$, where $T_t^{H_i}$ is the average indoor temperature for time interval t. Furthermore, $y_{i,j,t,or} = T_{t,or}^{H_i}$ where $T_{t,or}^{H_i}$ is the original average indoor temperature for time interval t. For the utility function of the heat pump $J = 1$. Therefore, the utility function is defined as follows:

$$U_i^{H_i}(T_t^{H_i}) = \sum_{t=1}^{T} U_{i,t}^{H_i}(T_t^{H_i}) = \sum_{t=1}^{T} a_i(T_t^{H_i} - T_{t,or}^{H_i})^2.$$

To model the indoor temperature of the house, we consider the thermal model used for Profile Steering [11]:

$$T_{t+1}^{H_i} = AT_t^{H_i} + Bx_{i,t}^{H_i} + CO_t + D_t,$$

where A, B, C and D_t are parameters of the model. The parameter D_t is used to model the thermal gains due to for instance human activity and solar radiation. Furthermore, O_t is the outdoor temperature during time interval t. It is assumed that $A \geq 0$ to model that the temperature in the current time interval positively influences the temperature in the next time interval. The parameter B indicates the temperature increase due to the heat production of the heat pump. Lastly, the parameter C is the influence of the outdoor temperature on the indoor temperature.

An electric vehicle arrives at a certain time and has to be charged enough for the next trip before a user-defined time. Between arrival and departure the vehicle is charged and the charging can be done at different rates. For each time interval the vehicle can be charged at certain discrete rates. We assume that EV owners only care about the amount charged at departure.

The state of charge expresses how much energy is stored in the battery as a percentage of the maximum capacity. The SoC (state of charge) can be determined from the energy profile of the electric vehicle and the SoC at arrival. The utility function is dependent on the state of charge at the end of the charging interval $SoC_T^{E_i}$. We assume that EV owners won't experience any discomfort if the state of charge at departure is more than the state of charge at departure in the original energy profile $SoC_{T,or}^{E_i}$. Without including the extra electricity costs. The form of the utility function for the electric vehicle is a bit different than for the other devices. But it is still a convex function. The utility function is :

$$U_i^{E_i}(SoC_T) = \begin{cases} a_i(SoC_T^{E_i} - SoC_{T,or}^{E_i})^2 & \text{if} \quad SoC_T < SoC_{T,or}, \\ 0 & \text{otherwise.} \end{cases}.$$

If the state of charge at departure is lower than in the original energy profile $y_{i,j,t} = SoC_T^{E_i}$, $y_{i,j,t,or} = SoC_{T,or}^{E_i}$ and $J = 1$. Furthermore, the only time interval that is taken into account is the departure of the vehicle. If the SoC at departure is lower than in the original energy profile the costs are zero.

Stand alone electrical batteries can be charged or discharged during every time interval. The utility function for the battery consist of the costs assiocated with the usage of the battery, in other words the depreciation of the battery. These costs are called battery degradation costs. A few factors that cause battery degradation are: high and low SoC, high depth of discharge and high current-rate/high power rate. These are the cost factors, therefore for the utility function of a battery $J = 3$. High depth of discharge occurs when the battery is (dis)charging for many time intervals in a row [6]. high operation temperature is disregarded because we assume there is a climate control system wich takes care of the temperature within the battery. The variable $y_{i,j,t} = \{C_{dd}(t), u(t), C_{SoC}(t))\}$ where $C_{dd}(t)$ is the amount of electricity (dis)charged in a row, $u(t)$ is the (dis)charging current and $C_{SoC}(t)$ is the state of charge of the battery. Likewise, $y_{i,j,t,or} = \{C_{dd,or}(t), u_{or}(t), C_{SoC,ref}(t)\}$ are the factors for the orginal energy profile. The utility function for the battery is based on a method from Koller et al. [6]:

$$\begin{aligned}
U_i^{B_i}(y) &= \sum_{t=1}^{T}\sum_{j=1}^{3} U_{i,j,t}^{B_i}(y_{i,j,t}), \\
&= \sum_{t=1}^{T}\sum_{j=1}^{3} a_{i,j}(y_{i,j,t} - y_{i,j,t,or})^2, \\
&= \sum_{t=1}^{T} Q_{dd}(C_{dd}(t) - C_{dd,or}(t))^2 + Q_u(u(t) - u_{or}(t))^2 \\
&+ Q_{SoC}(C_{SoC}(t) - C_{SoC,ref}(t))^2,
\end{aligned}$$

where $Q_d d$, Q_u and Q_{SoC} are the constants for respectively the factors high depth of discharge, high current-rate/high power rate and high and low SoC. Furtermore, $C_{SoC,ref}(t)$ is the battery SoC which minimizes the degradation. The amount of electricity (dis)charged consecutively is determined by adding the change in the state of charge with the previous interval. When the sign changes, so if the battery goes from charging to discharging or the other way around, then $C_{dd}(t)$ is reset to 0 and again the change in state of charge is added until the sign changes again. The amount of electricity charged or discharged is formulated as follows:

$$C_{dd}(t) = \begin{pmatrix} C_{dd,charge}(t) \\ C_{dd,discharge}(t) \end{pmatrix}$$

Where $C_{dd,\text{charge}}(t)$ is the amount of electricity charged consecutively and $C_{dd,\text{discharge}}(t)$ the amount of electricity discharged consecutively. One of these amounts is 0 for all time intervals t. $C_{dd}(t)$ is determined as follows:

$$C_{dd}(t+1) = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} C_{dd}(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Delta C_{SoC}(t) & \text{if the battery is charging} \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} C_{dd}(t) - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Delta C_{SoC}(t) & \text{if the battery is discharging} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \text{if the battery is idle} \end{cases}$$

If the battery is charging $C_{dd,\text{charge}}$ is increased by the change in state of charge, if the battery is discharging or idle this variable is set to 0. If the battery is discharging $C_{dd,\text{discharged}}$ is decreased by the change in state of charge, if the battery is charging or idle this variable is set to 0. The change in state of charge is negative for discharging, therefore the change in state of charge is substracted from $C_{dd,\text{discharge}}$. The change in state of charge $\Delta C_{SoC}(t)$ consists of the amount of electricity charged in interval t, $u_{\text{charge}}(t)$, multiplied by the charging efficiency η_{charge} and the amount of electricity discharged in interval t, $u_{\text{discharge}}(t)$, multiplied by the inverse of the discharging efficiency $\eta_{\text{discharge}}$. Furthermore, the self-discharging effects of the battery $v(t)$ are taken into account. We assume that a battery can't charge and discharge simultaniously and therefore that $u_{\text{charge}}(t) \cdot u_{\text{discharge}}(t) = 0$.

$$\Delta C_{SoC}(t+1) = \eta_{\text{charge}} u_{\text{charge}}(t) - \eta_{\text{discharge}}^{-1} u_{\text{discharge}}(t) - v(t)$$

The cost functions are given by:

$$
\begin{aligned}
Q_{dd} &= \begin{pmatrix} 1.6 & 0 \\ 0 & 1.6 \end{pmatrix} \\
Q_u &= \begin{pmatrix} 1.2 \times 10^{-7} & 0 \\ 0 & 1.2 \times 10^{-7} \end{pmatrix} \\
Q_{SoC} &= 1
\end{aligned}
$$

These are the utility functions of the players for the different devices. In the next section we formulate the payoff of the players.

3.3 Payoff

From the perspective of the player his payoff is only positive if he wins the auction round. Then his payoff is the reward given to him by the auctioneer minus the costs corresponding to the bid of the player. If the player does not win his payoff is 0, so the minimum reward is set such that the payoff is nonnegative if he wins the auction round. The payoff function of player i is:

$$
u_i(b_1, \ldots, b_i, \ldots, b_n) = \begin{cases} R_i(b_i) - C_i(\hat{x}_i) & \text{if } f(b_i) \geq f(b_j) \text{ for all } j \in N, \\ 0 & \text{otherwise,} \end{cases}
$$

where $\tilde{x}_{i,t}$ is the preliminary energy profile for time interval t. The costs $C_i(\hat{x}_i)$ are the sum of all the utility functions of the devices for player i. As mentioned before, the bids are defined as $b_i = \{\hat{x}_i, P_i\}$. In other terms the payoff of the winning player is

$$
R_i(b_i) - C_i(\hat{x}_i) = \max\{\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t}), P_i\} - \sum_{D_i=1}^{M_i} U_i^{D_i}(\hat{x}_i^{D_i})
$$

In our formulation, if multiple players have the same objective value all these players win the auction. This situation is called a tie and all of the winning players receive a reward from the auctioneer. We argue that the number of ties is small enough such that the effect to the aggregated energy profile is negligible. The reason is that the number of choices of each consumer is large and therefore the probability of a tie is small. Furthermore, each player has different choices and different costs for each device.

This concludes the formulation for the method that provides incentives for consumers to participate in profile steering. In the next section we study the existance of a Nash equilibrium for this game and their properties.

4 Nash equilibrium

Now that we have characterized the auction game we want to find its Nash equilibrium. In a Nash equilibrium each player bids his best response b_i^* to $(b_1^*, ..., b_{i-1}^*, b_{i+1}^*, ..., b_n^*)$, this means that the player is aiming for a bid b_i^* with

$$E[u_i(b_1^*, ..., b_{i-1}^*, b_i^*, b_{i+1}^*, ..., b_n^*)] \geq E[u_i(b_1^*, ..., b_{i-1}^*, b_i', b_{i+1}^*, ..., b_n^*)] \ \forall \ b_i' \in S_i.$$

The player's expected payoff for the bid $b_i^* = \{\hat{x}_i^*, P_i^*\}$ should be more than his expected payoff for any other possible bid $b_i' = \{\hat{x}_i', P_i'\}$ taking into account all the bids of the other players. The expectation of the payoff is the probability that a bid will win multiplied by the payoff corresponding to the bid. Thus, we get that:

$$E[u_i(b_1^*, ..., b_{i-1}^*, b_i^*, b_{i+1}^*, ..., b_n^*)] = G(b_i^*) \left(R_i(b_i^*) - C_i(\hat{x}_i^*)\right)) + (1 - G(b_i^*)) \cdot 0,$$

where $G(b_i)$ is the winning probability of player i for bid b_i. A bid b_i^* is a best response if

$$G(b_i^*) \left(R_i(b_i^*) - C_i(\hat{x}_i^*)\right)) \geq G(b_i') \left(R_i(b_i') - C_i(\hat{x}_i')\right) \forall \ b_i' \in S_i.$$

We assume that the winning probability can be determined from historical data of bids of previous winners. This implies there have been a number of bidding rounds.

Now we are going to describe a Nash equilibrium of the problem and proof that this is a Nash equilibrium. The goal is to maximize the expected payoff against the strategies of the other players. The expected payoff consists of two parts, the winning probability and the payoff. Both of them should be optimized simultaniously. The winning probabillity is assumed to be increasing with the change in energy profile beneficial to the auctioneer. So we put weights on these objectives (change in energy profile and payoff) which sum to one. The reasoning is that if we optimize both the objectives and therefore also the winning probability, then this also optimizes the multiplication of the winning probability and the payoff. Thus, the Nash equilibrium is $(b_1^*, ..., b_{i-1}^*, b_i^*, b_{i+1}^*, ..., b_n^*)$ such that b_i^* is the solution of the *bid optimization problem*:

$$\begin{aligned}
\max_{b_i, w_1, w_2} \quad & w_1 \sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t}) + w_2 \left(R_i(b_i) - C_i(\hat{x}_i)\right) && (1)\\
s.t. \quad & R_i(b_i) - C_i(\hat{x}_i) > 0 && (2)\\
& w_1 + w_2 = 1 && (3)\\
& b_i \in S_i, \ w_1, w_2 > 0 && (4)
\end{aligned}$$

where w_1 and w_2 are the weights given to the objectives. The objective function (1) represents the weighted sum of the two objectives. Furthermore equation (2) states that the payoff must be positive. The reasoning is that placing a bid that doesn't win produces the same or better payoff, namely 0, than a bid corresponding to a negative payoff. Equation (3) states that the weights must sum to one. Lastly, the bid has to be in the strategy set of player i (equation (4)).

We still have to prove that the solution of the bid optimization problem forms a Nash equilibrium, we do so in the following lemma.

Lemma 1. *The solution to the bid optimization problem given in (1)-(4) for every player i constitutes a Nash Equilibrium for the auction game.*

Proof. We have to proof that

$$E[u_i(b_1^*, ..., b_{i-1}^*, b_i^*, b_{i+1}^*, ..., b_n^*)] \geq E[u_i(b_1^*, ..., b_{i-1}^*, b_i', b_{i+1}^*, ..., b_n^*)] \ \forall \ b_i' \in S_i$$

for all players i=1,...,n. If the solution b_i^* of the *bid optimization problem* (1)-(4) for every player is also a solution of

$$\max_{b_i \in S_i} G(b_i) \left(R_i(b_i) - C_i(b_i)\right)$$

for every player it follows that the solution b_i^* is the best response to the other bids. Then $(b_1^*, .., b_n^*)$ is a Nash equilibrium. So we have to proof that these 2 optimization problems have the same solution. Let's take a bid b_i' which is a solution of he bid optimization problem. Because of (2) the payoff is positive for b_i'. Furthermore, $\sum_{t=1}^{T} s_t(\hat{x}'_{i,t} - \tilde{x}_{i,t})$ is positive. If this is negative, the player's bid will not be chosen and this implies that the payoff is zero. This is not a feasible solution of the problem because of (2) and gives a contradiction. Thus, both the payoff and the change in energy profile beneficial to the auctioneer are positive.

b_i' maximizes the objective value of the bid optimization problem and both $w_1 \sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and $w_2\left(R_i(b_i) - C_i(\hat{x}_i)\right)$ are positive. This implies that b_i' maximizes $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and $\left(R_i(b_i) - C_i(\hat{x}_i)\right)$. Otherwise, if one of these two has a lower value than the maximum for the bid b_i' the weighted sum is not the maximum for b_i'. This gives a contradiction with the fact that b_i' is a solution of the bid optimization problem. We assumed that the winning probabiltity is nondecreasing for the change in energy profile beneficial to the auctioneer $\sum_{t=1}^{T} s_t(\hat{x}'_{i,t} - \tilde{x}_{i,t})$, therefore a maximum $\sum_{t=1}^{T} s_t(\hat{x}'_{i,t} - \tilde{x}_{i,t})$ implies that $G(b_i')$ gives the maximum winning probability.

Both the winning probability and the payoff is maximal for the solution of the bid optimization problem b_i'. This implies that the multiplication of the payoff and winning probability is maximal. In other words $G(b_i')\left(R_i(b_i') - C_i(b_i')\right)$

is maximal for the bid b_i'. Therefore, the bid b_i' is a solution of the optimization problem

$$\max_{b_i \in S_i} G(b_i)\left(R_i(b_i) - C_i(b_i)\right).$$

$\square$

Note that a solution of bid optimization problem (1)-(4) is different for each device and for each household. This is caused by the differences in the strategy set and the bids of other players. In the remainder of this chapter, we look at the difference in Nash equilibria for the device classes.

4.1 Time-shiftables

The energy profie of a time-shiftable device is defined by its starting time. There are at most 96 possible starting times (15 minute intervals), so 96 possibilities to choose from. The potential reward for every starting time depends on the player's starting time for the preliminary and candidate energy profile and the steering signals. Therefore the reward for a certain time interval differs between the players and there is no common best starting time for all players. Recall that the costs increase when the time differences between initial and possible starting time increase. So the problem is to choose a starting time with low costs that is close to the *original starting time* and that has a high reward. An equilibrium between the two objectives must be found according to the winning probability of every starting time.

4.2 Heat pumps

Heat pumps can be on or off during each time interval, so we get 2^{96} possibilities. Each energy profile can be compared to a temperature profile, giving the average temperature for each of the time intervals. Usually the temperature has to stay within a certain deadband predefined by the user to ensure user comfort. However, the reward may outweigh the loss of comfort due to temperature changes. Therefore we allow the temperature to go outside this deadband. We set a minimum and maximum temperature for health and safety issues. This implies that not all possibilities are feasible, but a lot more are than in the case with a relatively small deadband set for user comfort. For these possibilities the difference between on and off state for each time interval and the steering signals determine the reward. The reward increases with more changes in the energy profile beneficial to the auctioneer. On the other hand the costs are determined by the change in temperature. But there is not a one-to-one correspondence between the number of time intervals with a difference in energy profile and the the costs.

This is caused by the fact that the temperature depends also on the temperature in the previous interval. So again there's not a common best choice of on and off states for every player. The winning probability depends on the number of changes in the energy profile beneficial to the controller. The payoff is determined by the changes in the energy profile and the changes in the temperature profile, so both must be optimized.

4.3 Electric vehicles

Electric vehicles can be charged between arrival and departure. We simplify the charging by assuming there is only one charging mode. An on and an off state. The end state of charge now depends on the number of states. There are $2^{t_{EV}}$ possibilities for charging the vehicle, where t_{EV} is the number of time intervals per day the vehicle is available for charging. The reward is again determined by the number of changes in the energy profile beneficial to the controller and so is the winning probability. The payoff depends on the reward and the state of charge at the end of the interval. The state of charge depends on the number of on-states during t_{EV} intervals and is a multiple of the charging rate plus the state of charge at arrival. So on the one hand the objective is to optimize the number of state changes according to the steering signals and on the other hand the goal is to optimize the number of on-states to match the minimal state of charge at the end of the intervals.

4.4 Batteries

Batteries can be charged, be discharged or be idle during each time interval. Before we stated that this happens at different rates, now we assume that there is only one rate for charging and discharging. Furthermore, we assume that the energy losses are negligible. The reward for offered flexibility and thereby the winning probability are calculated from the changes for each time interval. The payoff is calculated from the reward and the cost, where the latter is a combination of three factors. As mentioned before, these factors are depth of discharge, power rate and state of charge. This translates into short charge/discharge intervals, a low charging/discharging rate and a state of charge close to the optimum value. These three objectives and the reward determine the bid and the energy profile of each player.

In this section we described the optimization of a bid for the player such that all the bids together form a Nash equilibrium. In the next section we describe the implementation of this problem and the simulations of the auction rounds.

5 Simulations

In this section we describe the implementation of the bid optimization problem and the calculation of a solution for this problem. First we describe the number of households and devices used, then the values needed for the calculation of the payoff. After that we describe the solution method for the optimization of a single bid according to other bids and the steering signals. Lastly, multiple rounds and the auctioneer's choice for the best bid are discussed.

We've modeled a group of 4 households, therefore the game consists of 4 players. Each of these players have a time-shiftable device, a heat pump and an electric vehicle. The values per device are:

- Time-shiftable: All players have a time-shiftable with an on-period of 2 hours. The electricity usage is 0.1125, 0.2125, 0.3125 and 0.0125 kWh per time interval for respectively the time-shiftable of player 1,2,3 and 4. The original starting time is 20:00, 5:00, 10:00 and 15:00 hour for respectively players 1,2,3 and 4. The coefficient $a_{i,j}$ is drawn from a gaussian distribution with $\mu_{i,j} = 1$ and $\sigma^2_{i,j} = 0.25$.
- Heat pump: The electricity usage when the heat pump is on is 2.0, 1.9, 1.8 and 2.1 kWh per time interval for players 1,2,3 and 4, respectively. The original energy profile for player 1 is that the heat pump is on non-stop from 21:00 until 4:00 hour, the heat pump of player 2 is on from 4:00 until 11:30 hour, the heat pump of player 3 is on from 11:30 until 19:00 hour and the heat pump of player 4 is alternating on and off from 9:30 until 0:00 hour. For the thermal model all players have the same parameters. The temperature at the start of the day (0:00 hour) the indoor temperature is 18 °C, the outdoor temperature O_t is taken from a cold day in december and ranges from -1 to 4 °C, the thermal gains D_t are 0.5 at 18:00 hour and diminish over time. The parameter $B = 0.2$, $C = 0.01$ and $A = 1 - C = 0.99$. The coefficient $a_{i,j}$ is drawn from a gaussian distribution with $\mu_{i,j} = 1$ and $\sigma^2_{i,j} = 0.25$.
- Electric vehicle: The charging rate of the electric vehicle is 1.5, 1.4, 1.3 and 1.6 kWh per time interval for players 1,2,3 and 4, respectively. The original energy profile for the electric vehicle of player 1 is charging non-stop from 18:00 until 21:00 hour, for player 2 it is charging from 0:00 until 3:00 hour, for player 3 it is charging from 5:00 until 8:00 hour and the original energy profile for the electric vehicle of player 4 is charging non-stop from 15:00 until 18:00 hour. The vehicle arrives at 18:00 hour for players 1,2 and 3 and for player 4 the vehicle arrives at 15:00 hour. The vehicle departs at 8:00 hour for all the players. The SoC at arrival is 0.2 for all the players and the maximum energy

storable in the battery of the vehicle is 24 kWh. The coefficient $a_{i,j}$ is drawn from a gaussian distribution with $\mu_{i,j} = 1$ and $\sigma^2_{i,j} = 0.25$.

For each of these four players we solve the *bid optimization problem.* The optimization we execute in matlab using the method *intlinprog*, which is the matlab tool for solving mixed-integer linear programming problems. We take (1) as objective and (2)-(4) as constraints. For modeling purposes we take weights: $w_1 = w_2 = 0.5$. The constraint that $b_i \in S_i$ propagates as follows: For every time interval t we have that $15\,°\mathrm{C} \leq T_t^{H_i} \leq 25\,°\mathrm{C}$ and that $0.5 \leq SoC_T^{E_i} \leq 1$. Thus, the safety margins for the indoor temperature are 15 and 25 °C and the minimum SoC of the electric vehicle at departure is 0.5.

Intlinprog uses the branch and Bound algorithm to find a solution for the *bid optimization problem* and for a first guess it uses heuristics. Intlinprog is designed to handle mixed-integer linear programs. The *bid optimization problem* is a mixed-integer problem, but it is not linear. The costs $C_i\hat{x}_i$ in the objective function (1) are quadratic. This implies that intlinprog doesn't work for this problem, but with an adaption it works for quadratic objective functions. This adaption is called a cutting plane method and is mentioned in [5]. This cutting plane method is described as follows: First, we introduce a slack variable z to represent the quadratic term

$$\begin{aligned} \max_{b_i, w_1, w_2} \quad & w_1 \sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t}) + \\ & w_2 \left(R_i(b_i) - z - \sum_{t=1}^{T}\sum_{j=1}^{J} -a_{i,j} 2 y_{i,j,t} y_{i,j,t,or} + a_{i,j}(y_{i,j,t,or})^2 \right), \\ s.t. \quad & \sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_{i,j,t})^2 - z \leq 0, z \geq 0, \end{aligned}$$

so we add a nonlinear constraint to get a linear objective. This problem is solved without taking into account the nonlinear constraint and the solution is called y_0. Then, we approximate this new constraint locally near the current point by using the first-order Taylor approximation in the point $y_{i,j,t} = y_0 + \delta$:

$$\begin{aligned} \sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_{i,j,t})^2 - z &= \sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2 + 2\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j} y_0 \delta - z + \mathcal{O}(|\delta|^2), \\ &= -\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2 + 2\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j} y_0 y_{i,j,t} - z + \mathcal{O}(|y_{i,j,t} - y_0|^2). \end{aligned}$$

Therefore, we add the constraint that

$$-\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2 + 2\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}y_0y_{i,j,t} - z \leq 0,$$

$$2\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}y_0y_{i,j,t} - z \leq \sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2.$$

The resulting optimization problem is solved with Intlinprog. The solution of this new problem becomes y_0 and is used to add yet another approximation of a constraint. This is done iteratively until the difference between the slack variable z and the value of $\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2$ is small enough. For the simulation the algorithm stops if the difference between these two parts is less than 1 %. The last value for y_0 is an approximation for the solution of the *bid optimization problem.*

With the method above we can calculate what the players will bid when receiving steering signals from the auctioneer. In the simulation we use for the first round of the auction as steering signals the sign of a sine function, because during the day there is typically too much production, which gives a " + " steering signal and during the evening/morning there is too much demand for electricity corresponding to a " − ". The steering signals are:

$$\operatorname{sgn}(\sin(\frac{\pi t}{48} + \frac{\pi}{2})),$$

where t is the time interval, $sgn(x) = \frac{x}{|x|}$ for $x \neq 0$ and $sgn(x) = 0$ for $x = 0$. The period of this sine is 96 time intervals and the shift is 24 time intervals, therefore from 6:00 until 18:00 the steering signals are " − " and from 18:00 until 6:00 hour the steering signals are " + ".

When players send their bid to the auctioneer, the auctioneer has to choose the winner. This is done according to his objective function $f(b_i) = g(\hat{x}_i) + h(P_i)$. In the simulation the auctioneer maximizes only $g(\hat{x}_i) = \sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$. Furthermore, the auctioneer has a budget for $h(P_i) = P_i$. In the next section we describe the results of the simulation.

6 Results and discussion

The simulation contains 4 players, which solve the *bid optimization problem* to determine what their best bid is. We use a cutting plane method to determine this bid. The difference between the slack variable and the quadratic part of the objective function $\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2$ should converge to one another. Figure (4) gives an overview of the values for these two parts for every iteration of the cutting plane algorithm.

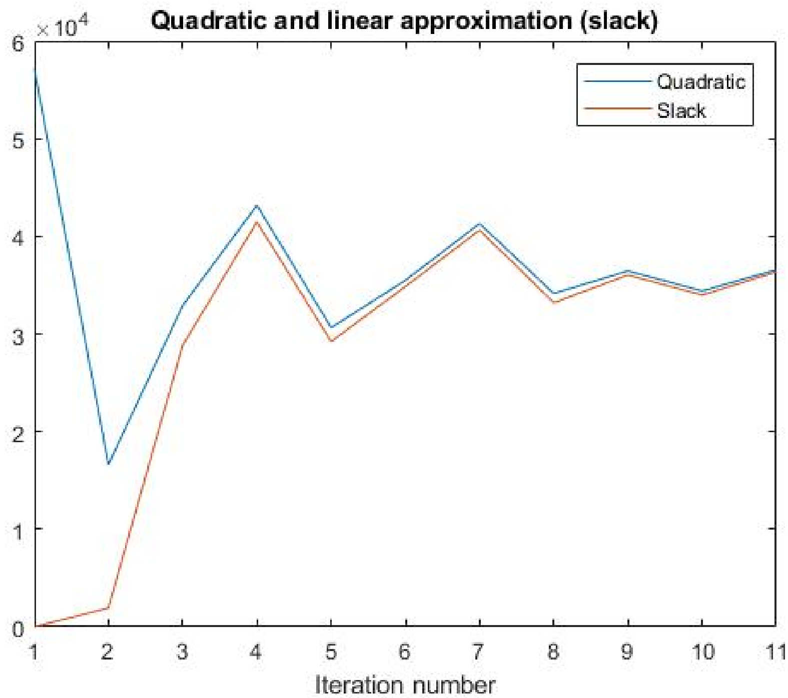

Figure 4: The values for the quadratic part of the objective $\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2$ and the value of the slack variable z for every iteration of the cutting plane algorithm

In this figure we see that the slack variable z converges to the quadratic part of the objective $\sum_{t=1}^{T}\sum_{j=1}^{J} a_{i,j}(y_0)^2$. Therefore, the resulting bid is a good approximation for the solution of the *bid optimization problem.* For each player the optimization of their bid needed about 11 iterations to reach a difference between the two parts of less than 1 %.

This results in 4 bids for the players, which are send to the auctioneer. The auctioneer chooses the bid, which gives him the highest objective value $f(b_i)$. Therefore, we look at how much the candidate energy profile complies to the steering signals of the auctioneer. For player 1 $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and the steering signals are given in figure (5).

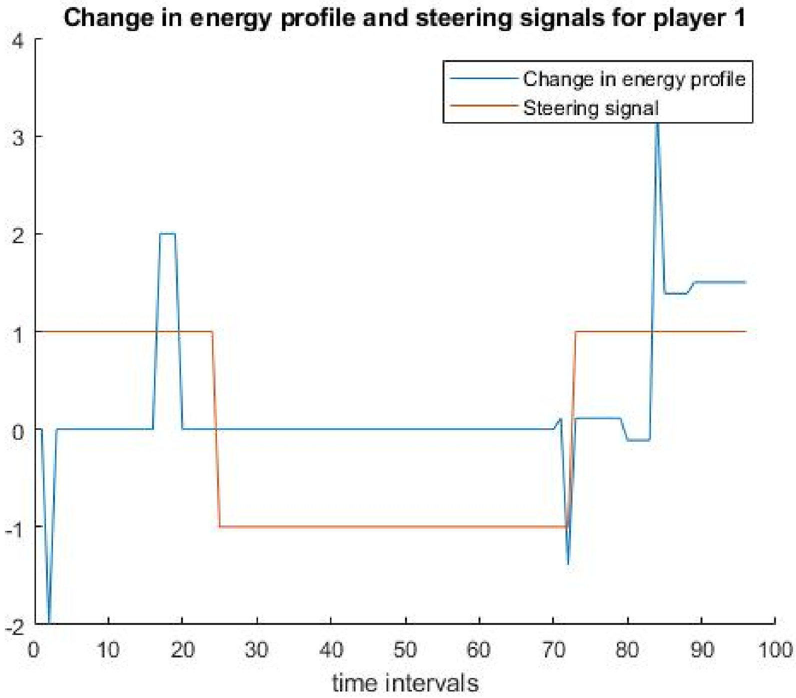

Figure 5: The change in energy profile and the steering signals for player 1

We see that only at a few time intervals it is the change is the opposite of what is preferable for the auctioneer. Most of the time intervals stay the same and at the end of the day there are a lot of preferable time intervals with a preferable change. For player 2 $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and the steering signals are given in figure (6).

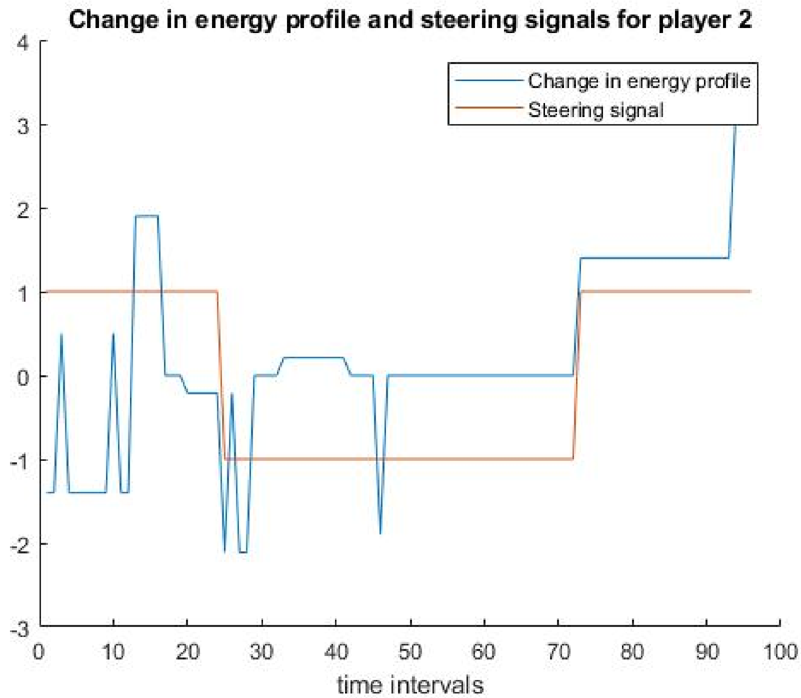

Figure 6: The change in energy profile and the steering signals for player 2

The change in energy profile for player is a bit better than for player 1. After 18:00 hour it follows the steering signals perfectly and there are more time intervals with a change preferable by the auctioneer. But, there are also more time intervals in which the energy changes in the wrong direction. For player 3 $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and the steering signals are given in figure (7).

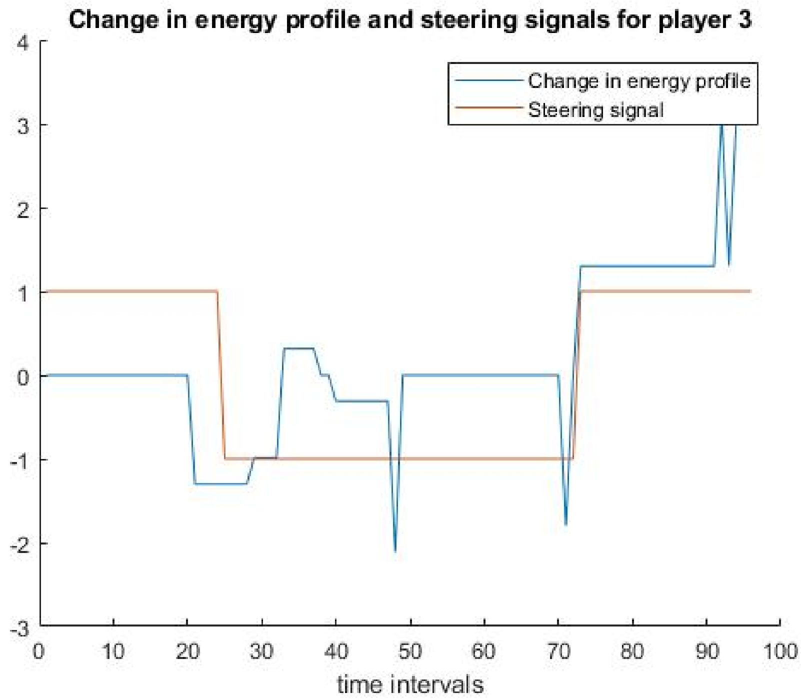

Figure 7: The change in energy profile and the steering signals for player 3

The change in energy profile for player 3 is almost as preferable as the one from player 2. Less time intervals change in the wrong direction, but also less time intervals change for the better. For player 4 $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ and the steering signals are given in figure (8).

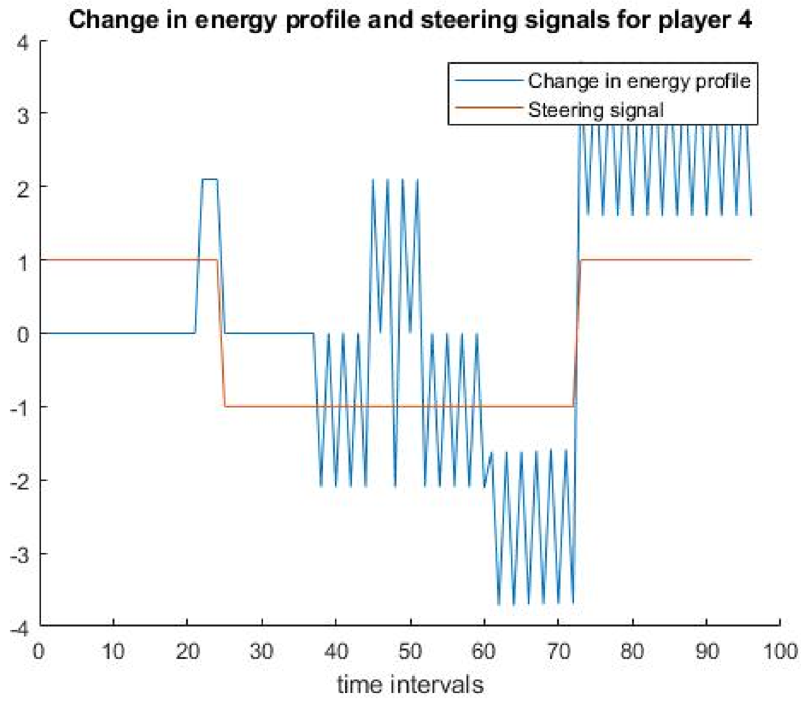

Figure 8: The change in energy profile and the steering signals for player 4

The change in energy profile for player 4 is much better than for the other players. There are only a few intervals with less preferable change of the energy profile. Furthermore, there are a lot more time intervals that comply to a " $-$ " steering signal. This can also be seen from the values of $\sum_{t=1}^{T} s_t(\hat{x}_{i,t} - \tilde{x}_{i,t})$ for all the players. These are 26.550, 39.375, 47.20 and 114.35 for player 1,2,3 and 4, respectively.

The auctioneer will choose player 4 for this auction round if the minimum reward is not higher than the budget. This may cause an "overshoot effect", where "overshooting" means that instead of too much production there is now too much demand for a certain time interval. The reason is that the change of the candidate energy profile with the preliminary energy profile is much higher than for the other players. The change is higher for much more time intervals than for the other player's bids. This is mainly due to the candidate energy profile for the heat pump of player 4. This heat pump has a very different original energy profile than the other heat pumps. Therefore, it can change its energy profile a lot more than the other heat pumps for the same loss in utility.

In some cases it might be better for the auctioneer to chose the bid of player 3, because the bid has a change in energy profile which is more moderate. It only contains a couple of relatively high changes in the energy profile. So, if the auctioneer demands less change in energy profile the bid of player 3 is a better choice. In other situations the auctioneer might require a bid with a change in energy profile somewhere in the middle of the bid of player 3 and 4.

The problem with this is that the players don't know exactly how much they should change their energy profile, because the steering signals are only a "$+$", a "$-$" or a "0". This problem can be improved by introducing more options for the steering signals, such as a "$++$" and a "$--$". But, then the problem arises how to choose the boundaries between the options and if the boundaries should be conveyed to the players.

In the next section we conclude the thesis and give some recommendations for future research.

7 Conclusions and recommendations

In this thesis we set up an approach to incentivize players to participate in profile steering. We also changed the steering signals to provide less information on the objective of the central controller to the households. This involved an auction where households bid for a monetary compensation for their change in energy profile. The strategy set, the payoff and the winning conditions of the players were described and the costs of the players for changing their energy usage were defined. These were described per device of the players. After that the Nash equilibrium of the game was determined by setting up the *bid optimization problem.* The solution of the *bid optimization problem* is the best response to the bids of the other player taking into account the steering signals.

After that a simulation of the auction was executed with 4 households. Each of the households was simulated with 3 devices and each household had different utility functions and different preferences. Thereafter, we described the optimization in matlab using the branch and bound algorithm together with a cutting plane method. The results give inside into the effects of the bids resulting from the *bid optimization problem* and the convergence of the cutting plane method. The results were that players had different bids with different utility functions and different devices. This difference resulted in player 4 winning the auction round, but his bid can cause an "overshoot effect" in the total energy profile.

The recommondations for future research are: First of all what the form of the solution of the *bid optimization problem* is and the effect of this form for the Nash equilibrium. This can give a stronger result for games of this kind. Furthermore, more simulations could have been executed. But, due to time constraints and the computing time of the algorithms this was not done. The simulation of 4 households took 2 days to complete. So my recommondation would be to speed up the calculations of the bid. Other algorithms could be implemented for this purpose. Moreover, introducing more options for the steering signals, such as a "$++$" and a "$--$" could be beneficial for the bids and therefore for the total energy profile.

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