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INVESTIGATING THE DYNAMICS OF COALITION FORMING IN INDUSTRIAL SYMBIOSIS SYSTEMS

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Abstract

Implementing industrial symbiosis networks is seen as one of the ways to work towards a circular economy. Traditionally, there are two ways for industrial symbiosis networks to emerge, by planning or by self-organisation. For both of these types of industrial symbiosis networks, a model based on game-theory is given. A cooperative game model is given for planned industrial symbiosis networks as are conditions for finding stable coalitions and a method for dividing the benefits from the cooperation. An algorithm is proposed to find these stable coalitions, which are the industrial symbiosis networks. For self-organised industrial symbiosis networks, a model is proposed based on coalitional bargaining. A non-cooperative game is used for this model and an algorithm is proposed to find the best strategies for each of the companies. Finally, a hypothetical case study about the exchange of waste heat is performed to test the applicability of the planned industrial symbiosis model. In this case study also there is some attention given to the influence of the different operational aspects on the stable coalitions.

Acknowledgements

With this thesis, I conclude my time as a student. It has been six great years in which I ended up doing a lot more than just studying. From spending an entire year as a board member to volunteering almost every summer, it could not have been better. Now, as my student time ends, I feel ready for the next challenge.

I would like to thank a few people who, over the course of the last few months, helped me realise this project. First of all, my supervisors, Devrim and Vahid. Without their ideas and feedback, there would not be much of a thesis now. I would also like to thank Tariq for always supporting me and listening to me when I had run into some issue. Also, a special mention to Sagy, as a fellow graduate you understood all the ups and downs of writing your thesis. Last of all, I would like to thank my sister for always reminding me to keep a positive mindset.

I hope you enjoy reading my thesis.

Femmy Heukers
Enschede, July 5th, 2019

Notation

$\mathbb{R}$	the set of real numbers
$\mathbb{N}$	the set of natural numbers
$\emptyset$	the empty set
(N, v)	a cooperative game with player set N and characteristic function v
N	player set of a cooperative game, also called the grand coalition, $\|N\| = n$
$v(S)$	value of coalition $S \subseteq N$
$(x_i)_{i \in S}$	payoff distribution or payoff vector for coalition $S \subseteq N$, $x_i \in \mathbb{R}$ for all $i \in S$
$C(N, v)$	core of the cooperative game (N, v)
$\Phi_i(N, v)$	Shapley value of player $i \in N$ in the cooperative game (N, v)
m_i^σ	marginal contribution of player i to permutation σ of the grand coalition N in the cooperative game (N, v)
$T(S)$	traditional operating costs of the coalition S
$O(S)$	industrial symbiosis operating costs of coalition S
ISG	industrial symbiosis game
$S_{abc...}$	search space of the coalition generation algorithm, the indices indicate the coalition structure
ISBG	industrial symbiosis bargaining game
$\sigma = (\sigma_1, \ldots, \sigma_n)$	strategy combination for n players
$\sigma_i = (\sigma_i^t)_{t=1}^\infty$	strategy of the i-th player
σ_i^t	t-th round strategy of the i-th player
SSPE	stationary subgame perfect equilibrium
(S_i, y^{S_i})	proposal for an ISBG, S_i is the proposed coalition, y^{S_i} is the proposed payoff distribution
$G^N(\delta)$	ISBG with player set N and discount factor δ
v_j^N	expected payoff of player j in ISBG $G^N(\delta)$
$D(S)$	total demand of coalition S
P_{gas}	price of natural gas per gigawatt hour
$C_{\text{investment}}$	investment costs for pipeline per kilometre
d	distance between two firms
$n_{\text{pipes}}(S)$	number of pipes necessary for coalition S
$D_{\text{supply}}(S)$	supply that the producer companies can deliver

Contents

Chapter 1

Introduction

In 2015, the United Nations formulated the Sustainable Development Goals for 2030 [20]. These goals aim to make our planet a more sustainable place and range from no more poverty to decent work and economic growth. One of these goals, number 12, ensure sustainable consumption and production patterns, can be reached by transitioning towards a more circular economy.

In a circular economy, the value of products, materials, and resources is maintained in the economy for as long as possible, and the generation of waste is minimised [9]. In 2015, the European Commission published an action plan for transitioning to a circular economy [9]. With this action plan, the European Commission hopes to create a sustainable, low carbon, resource efficient and competitive economy. One method that is considered essential to move to a circular economy, is industrial symbiosis [2].

Industrial symbiosis is the "engaging of traditionally separate industries in a collective approach to competitive advantage involving physical exchange of materials, energy, water, and by-products. The keys to industrial symbiosis are collaboration and the synergistic possibilities offered by geographic proximity." [4]. In this thesis, the focus lies on industrial symbiosis cooperation between companies.

1.1 Research Aim and Research Questions

An industrial symbiosis network is a group of companies who are engaging in industrial symbiosis together. The aim of this research is to give insight into the dynamics of cooperation in such an industrial symbiosis network. Specifically, insight into the formation of industrial symbiosis networks. This formation can be planned or self-organised. Both of these approaches will be analysed in this thesis.

The aim of this thesis is to model the emergence of both planned and self-organised industrial symbiosis networks. With these models, we can give decision support to the decision makers for these industrial symbiosis networks. For planned industrial symbiosis networks, decision support will be given to the cluster manager. This decision support will help the cluster manager to make a decision on which firms should cooperate in the industrial symbiosis network. For self-organised industrial symbiosis networks, decision support will be given to the firms themselves. This decision support will help the firms in determining with whom to cooperate and how to divide the economic benefits of this cooperation among them.

For this purpose, the following research questions are formulated.

Planned industrial symbiosis:

- How can we model planned industrial symbiosis as a game?
- What properties do stable coalitions have?
- How can we give decision support to a cluster manager?

Self-organised industrial symbiosis:

- How can we model self-organised industrial symbiosis as a game?
- What properties do stable coalitions have?
- How can we give decision support to the firms?

For both the models, games from the area of game theory will be used. For planned industrial symbiosis, a cooperative game will be used. For self-organised industrial symbiosis, a non-cooperative game will be used. To determine the properties of stable coalitions, these models will be analysed using game-theoretical tools. For the decision support, algorithms will be provided. For planned industrial symbiosis, a hypothetical case study will be used to see the applicability of the proposed algorithm.

1.2 Thesis Outline

In this section, an overview of the chapters of this thesis will be given.

In Chapter 2, the background concepts of this thesis will be given. First, the area of mathematical game theory will be explained. Both cooperative and non-cooperative games will be discussed, as they both play a role in the models later in the thesis. Also, the concept of industrial symbiosis will be explained further. Two different methods of emergence of industrial symbiosis networks will be discussed. Moreover, some real-life examples of companies that have implemented industrial symbiosis will be given.

Chapter 3 gives a model for top-down formation of industrial symbiosis networks. This model is based on cooperative game theory. It will be analysed and a method will be given to form the best possible industrial symbiosis networks out of a cluster of companies. Also, a method for dividing the cooperation benefits will be provided.

In Chapter 4, the focus is on bottom-up emergence of industrial symbiosis networks. For this purpose, another game model will be presented and analysed. Contrary to the model of the previous chapter, this one is based on non-cooperative game theory. Also, a method will be given to help companies make decisions when they are bargaining for an industrial symbiosis network.

In Chapter 5 the model from Chapter 3 will be applied to a hypothetical case about waste heat exchange. With this case, the applicability of this model is tested. Also, some attention is given to the different operational aspects of the case.

Finally, Chapter 6 concludes this thesis. Conclusions will be made and recommendations for further research will be given.

Chapter 2

Background

In this chapter, some background information will be given on the concepts and methods used in this thesis. In Section 2.1, some basic definitions and examples of theoretical games and coalition forming will be given. Section 2.2 explains the concept of industrial symbiosis, planned industrial symbiosis, and self-organised industrial symbiosis.

2.1 Game Theory

In game theory, a specific distinction is made between cooperative and non-cooperative game theory. In *cooperative game theory*, the aim of the game is for the players to find a way to cooperate together. In *non-cooperative game theory*, players try to get the best rewards for themselves by optimising their strategy. Both of these types of games will be explained in this section.

First, a definition of the field of game theory will be given. Peters [16] describes game theory as studying "situations of competition and cooperation between several involved parties by using mathematical methods". This is an extensive definition, but the possible applications of game theory are also extensive. Examples of applications are voting systems, economic competition, and strategic questions in warfare [16].

Some important concepts in a game model are players, rewards, and strategies. The *players* of the game are the decision makers. They make some decision in the game that influences its outcome. The players do this to get some *reward*. This reward can be winning the game or a sum of money, for example. Each player also has a certain way of making decisions. This is called their *strategy*.

Most games are *simultaneous*. This means that the players make their decisions at the same time. A classic example of a simultaneous game is rock-paper-scissors. In an *extensive game* the players play sequentially. In this type of game, players can observe or partially observe the moves of the other players [16]. Think for example of chess.

A special type of games are *bargaining games*. In a bargaining game, the goal of the players is to make an agreement. They can bargain among themselves to come to an agreement.

2.1.1 Non-Cooperative Matrix Games

The most basic non-cooperative games are represented as a bimatrix game.

Definition 2.1.1. A *bimatrix game* is a pair of $m \times n$ matrices (A, B), where m is the number of rows and n is the number of columns [16].

Each of the entries in these matrices represents the rewards for the players. The following example describes a bimatrix game.

Example 2.1.2. Consider the following bimatrix game:

$$\begin{pmatrix} 2,1 & 1,0 \\ 1,3 & 3,2 \end{pmatrix}$$

In this game, there are two players, since the matrices are two-dimensional. These players are the row player and the column player.
The first part of the bimatrix entry represents the payoff of the row player. The second part represents the payoff of the column player.
Each player has two possible actions to play. The row player can choose the first or second row. The column player can choose the first or second column.
If the row player chooses the first row and the column player chooses the second column, the players obtain the following rewards. The row player gets a reward of 1 and the column player gets a reward of 0.

Each player has a way to decide what action to play. This is called their strategy. This strategy can be pure, meaning that they always choose the same action, or mixed, in that case they choose an action based on some probability distribution.

Definition 2.1.3. [16] A *(mixed) strategy* of the row player is a probability distribution $\mathbf{p}$ over the rows of the $m \times n$ matrix A, i.e., an element of the set

$$\Delta^m := \{\mathbf{p} = (p_1, \ldots, p_m) \in \mathbb{R}^m | \sum_{i=1}^{m} p_i = 1, p_i \geq 0 \text{ for all } i = 1, \ldots, m\}.$$

Similarly, a *(mixed) strategy* of the column player is a probability distribution $\mathbf{q}$ over the columns of B, i.e., an element of the set

$$\Delta^n := \{\mathbf{q} = (q_1, \ldots, q_n) \in \mathbb{R}^n | \sum_{j=1}^{n} q_j = 1, q_j \geq 0 \text{ for all } j = 1, \ldots, n\}.$$

To maximise their expected payoff, the players try to play their best reply to each other.

Definition 2.1.4. [16] A strategy $\mathbf{p}$ of the row player is a *best reply* to a strategy $\mathbf{q}$ of the column player in an $m \times n$ bimatrix game (A, B) if

$$\mathbf{p}A\mathbf{q} \geq \mathbf{p}'A\mathbf{q} \text{ for all } \mathbf{p}' \in \Delta^m.$$

Similarly, $\mathbf{q}$ is a *best reply* of the column player to $\mathbf{p}$ if

$$\mathbf{p}B\mathbf{q} \geq \mathbf{p}B\mathbf{q}' \text{ for all } \mathbf{q}' \in \Delta^n.$$

If both players play their best reply to each other, they reach an equilibrium. This is called the Nash equilibrium [16].

Definition 2.1.5. [16] A pair of strategies $(\mathbf{p}^*, \mathbf{q}^*)$ in a bimatrix game (A, B) is a *Nash equilibrium* if $\mathbf{p}^*$ is a best reply of the row player to $\mathbf{q}^*$ and $\mathbf{q}^*$ is a best reply of the column player to $\mathbf{p}^*$. A Nash equilibrium $(\mathbf{p}^*, \mathbf{q}^*)$ is called *pure* if both $\mathbf{p}^*$ and $\mathbf{q}^*$ are pure strategies.
In non-cooperative game theory, the aim is to solve for the equilibrium solution. So, to find the strategies of the players that induce a Nash equilibrium. The following example demonstrates the best reply and the Nash equilibrium of a bimatrix game.

Example 2.1.6. Consider the same matrix game as before:

$$\begin{pmatrix} 2,1 & 1,0 \\ 1,3 & 3,2 \end{pmatrix}$$

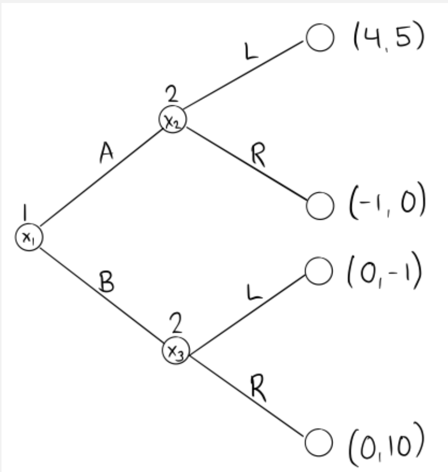
To demonstrate the best reply, assume that the row player plays the first row. In that case, the best reply for the column player is to play the first column. Then the column player receives 1, whereas he would receive 0 if he were to play the second column. So, playing the first column is the best reply when the row player plays the first row.
Actually, the first column is also the best reply if the row player plays the second row, since 3 is greater than 2. So, the column player will always play the first column.
If the column player plays the first column, the best reply for the row player is to play the first row. These two options are best replies to each other. So, the Nash equilibrium in this example is (1,1). Meaning that the row player plays the first row and the column player plays the first column.

2.1.2 Non-Cooperative Extensive Games

In an extensive game, the players take turns in playing an action. Such a game is usually depicted by a decision tree. The final nodes of the tree give the rewards that each player receives if the game ends up in that state.

In this type of game, the equilibrium analysis is focused on finding the subgame perfect equilibrium. This induces a Nash equilibrium in each subgame. The subgame perfect equilibrium is found by working backwards through the tree [16]. In the following example, a sequential game and its subgame perfect equilibrium are given.

Example 2.1.7. Consider the following extensive game:



Player 1 makes the first decision, he can choose action A or action B. Player 2 makes the second decision. If player 1 chose A, player 2 can choose action L or R. If player 1 chose B, player 2 can also choose action L or R, but these result in different payoffs.
To find the subgame perfect equilibrium, we work backwards through the tree. Suppose the game is currently at x_3, so player 1 chose action B. Now player 2 needs to choose. In this case, player 2 will always chose action R, since that will give him a payoff of 10. Now suppose the game is at x_2, so player 1 chose action A. Now player 2 will play action L, since he then receives a payoff of 5.
If the game is currently at x_1, so the first decision moment, player 1 has this information about player 2. If player 1 chooses A, he knows that player 2 will choose action L, so player 1 will receive a payoff of 4. If player 1 chooses B, he knows that player 2 will choose R, so player 1 will get a payoff of 0. So, player 1 will choose action A. So, the subgame perfect equilibrium of this game is {A, L}.

2.1.3 Cooperative Games

In cooperative game theory, the focus is not on strategies, as it is in non-cooperative game theory, but on the payoffs and the coalitions that the players can form. The assumption is that the players can make binding agreements with each other about the coalition and how to divide the payoffs.

A cooperative game is usually denoted by (N, v). Here N is the set of players and v is the characteristic function. The players form coalitions among themselves. These are denoted by $S \subseteq N$. Each coalition S has a value, which is assigned to the coalition by the characteristic function v. This value is denoted as $v(S)$.

In this thesis, only cooperative games with transferable utility are considered. In a *transferable utility* game, the value of each coalition can be expressed by a number, which can be distributed among the coalition member [16].

Definition 2.1.8. [16] A *cooperative game with transferable utility* or *TU-game* is a pair (N, v), where $N = \{1, \ldots, n\}$ with $n \in \mathbb{N}$ is the set of *players*, and v is a function assigning to each *coalition* S, i.e., to each subset $S \subseteq N$a real number $v(S)$, such that $v(\emptyset) = 0$. The function v is called the *characteristic function* and $v(S)$ is called the *worth* of S. The coalition N is called the *grand coalition*. A *payoff distribution* or *payoff vector* for coalition S is a vector of real numbers $(x_i)_{i \in S}$.

To solve a cooperative game, it is usually checked whether the core is nonempty. The core of a cooperative game consists of all payoff distributions for which the coalitions are stable. In a stable coalitions, the coalition members do not want to switch to another coalition.

Definition 2.1.9. [16] For a TU-game (N, v), a payoff distribution $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is

- *efficient* if $x(N) = v(N)$,
- *individually rational* if $x_i \geq v(\{i\})$ for all $i \in N$,
- *coalitionally rational* if $x(S) \geq v(S)$ for all nonempty coalitions S.

The *core* of (N, v) is the set

$$C(N, v) = \{\mathbf{x} \in \mathbb{R}^n | x(N) = v(N) \text{ and } x(S) \geq v(S) \text{ for all } \emptyset \neq S \subseteq N\}.$$

Thus, the core of (N, v) is the set of all efficient and coalitionally rational payoff distributions.

The final solution of a cooperative game is a payoff distribution of the total coalitional value divided between the coalition members. This payoff distribution should obviously be in the core, otherwise coalition members might prefer to be in another coalition and deviate. One very famous and useful way of distributing the payoff between the coalition members is the *Shapley value*. The Shapley value of each player denotes the marginal contribution of the player to the total coalitional value. By using the Shapley values to distribute the payoff, it is ensured that it is done in a fair manner. One issue with the Shapley allocation is that is is not always coalitionally rational. However, there are conditions for games which will ensure that the Shapley allocation is coalitionally rational and therefore also in the core of the game.

Definition 2.1.10. [16] The *Shapley value* of a TU-game (N, v) is denoted by $\Phi(N, v)$. Its i-th coordinate, i.e., the Shapley value payoff to player $i \in N$, is given by

$$\Phi_i(N, v) = \sum_{S \in N : i \notin S} \frac{|S|!(n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)].$$

The following example describes a cooperative game. The core is computed. And also the Shapley values of each player will be calculated.

Example 2.1.11. Consider a cooperative game with set of players $N = (1,2,3)$ and the following characteristic function:

S	$\emptyset$	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
$v(S)$	0	0	0	0	2	4	4	7

Now we can compute the core. Using Definition 2.1.9, the core of this game is:

$$C(N,v) = \{(x_1,x_2,x_3) \mid x_1 \geq 0,\, x_2 \geq 0,\, x_3 \geq 0,\, x_1+x_2 \geq 2,\, x_1+x_3 \geq 4,\, x_2+x_3 \geq 4, \\ x_1+x_2+x_3 = 7\}.$$

To compute the Shapley values, first we determine the marginal contribution m_i^σ per player i for each permutation σ of N. For each player, we sum these marginal contributions and divide them by $n! = 3! = 6$.

σ	m_1^σ	m_2^σ	m_3^σ
(1,2,3)	0	2	5
(1,3,2)	0	3	4
(2,1,3)	2	0	5
(2,3,1)	3	0	4
(3,1,2)	4	3	0
(3,2,1)	3	4	0
sum	12	12	18

Now we determine the payoff allocation for the players:

Company	Allocation
1	$12/6 = 2$
2	$12/6 = 2$
3	$18/6 = 3$

Note that that this payoff coalition is in the core of the game.

2.2 Industrial Symbiosis

As mentioned in Chapter 1, industrial symbiosis is the "engaging of traditionally separate industries in a collective approach to competitive advantage involving physical exchange of materials, energy, water, and by-products. The keys to industrial symbiosis are collaboration and the synergistic possibilities offered by geographic proximity." [4].

One very famous instance of an industrial symbiosis network is the one in Kalundborg, Denmark [4]. This industrial symbiosis network was unexpectedly discovered in 1989 and is largely self-organised, meaning that the companies cooperated on their own initiative. The companies in this cluster were from different industries, but were intensively cooperating by sharing resources [4]. More details of this eco-industrial park can be found in Section 2.2.2.

When considering industrial symbiotic relations, there are three main ways of sharing resources. These are by-product reuse, utility and infrastructure sharing, and joint provisions of services [5]. In by-product reuse, by-products from one firm are used as a substitute for raw materials in another firm. In utility and infrastructure sharing, firms use and manage common resources, such as water or heat, together. In joint provision of services, firms set up services for common needs, such as transportation, together.

2.2.1 Top-Down Industrial Symbiosis

As mentioned before, there are two ways for an industrial symbiosis network to emerge. In this section, top-down or planned industrial symbiosis will be explained. For planned industrial symbiosis, as the name implies, the industrial symbiotic relations are planned beforehand.

Companies are told, for example by a cluster manager of an industrial park, to exchange waste streams or by-products with other companies.

The best example for top-down industrial symbiosis is a planned eco-industrial park. An eco-industrial park is an industrial park where the business cooperate by sharing resources [11] to, for example, be more sustainable or increase profits. However, most planned eco-industrial parks do not work out so well. In 1996, the President's Council for Sustainable Development in the United States named sixteen project of planned eco-industrial parks. Most of these projects were planned but never put into action [4].

In this thesis, top-down emergence of industrial symbiosis networks is modelled as a cooperative game in Chapter 3.

2.2.2 Bottom-Up Industrial Symbiosis

Bottom-up industrial symbiosis is also called self-organised industrial symbiosis. The companies themselves take the initiative to start an industrial symbiotic relation together. In practice, it is found that industrial symbiosis networks that emerge bottom-up, are more likely to succeed [4].

A good example of a self-organised industrial symbiosis network is again Kalundborg, Denmark. In this eco-industrial park, businesses were first collaborating in 1961 [19]. However, these efforts went unnoticed until, in 1989, some local high school students made a science project which depicted all of the pipelines and connections in scale [4].

In the industrial symbiosis network in Kalundborg, energy, water, and materials are exchanged. There are 25 different resource streams and the resources are exchanged between six private and three public partners [8]. The main players in this network are Anaes Power Station, which is a 1350 megawatt power plant, an oil-refinery of Statoil A/S, Novo Nordisk Novozymes A/S, which is a Danish pharmaceatical and biotechnology firm, plasterboard manufacturer Gyproc Nordic East, soil remedian company A/S Bioteknisk Jordrens, and the municipality of Kalundborg [7]. The Anaes Power Station is the largest power plant in Denmark and has reduced their fraction of directly discarded available energy by 80 percent by exporting part of this energy [7]. Also, with a network of underground pipes, the municipality of Kalundborg distributes heat from the power plant to the residential area [7].

The main motivation for the companies to start cooperating in this way is scarcity of groundwater in Kalundborg [7]. This started in the 1960s, when a project of Statoil required surface water and piped it from Lake Tissø, about 50 kilometres away from Kalundborg [7]. Later on, Asneas and Novo-Nordisk also participated in this project. Since 1987, Statoil has piped cooling water to Asneas, where this is used as raw boiler water [7]. These are just a few little examples of the resource exchanges in the industrial symbiosis network at Kalundborg.

In Chapter 4, the bottom-up emergence of industrial symbiosis networks will be modelled as a non-cooperative sequential bargaining game.

Chapter 3

Top-Down Coalition Forming in Industrial Symbiosis

In this chapter, the focus is on industrial symbiosis networks that are formed in a top-down approach, for example by a cluster manager of an industrial park. So, the decisions on the formation of industrial symbiosis networks are made in a top-down manner. The aim of this chapter is to give decision support to a cluster manager about which industrial symbiosis networks to implement.

To answer this question, a cooperative game model will be used. This model will be explained in Section 3.1. In Section 3.2, this model will be analysed. In Section 3.3, an algorithm for generation of industrial symbiosis networks will be proposed. With this algorithm, the cluster manager will be able to form industrial symbiosis networks from his/her cluster. Also, a method is presented for the actual implementation to divide the economic benefits of the industrial symbiosis relation. This will be done in Section 3.4. Finally, in Section 3.5, the methods from the previous sections will be interpreted for the decision maker, for example, a cluster manager.

3.1 Model and Assumptions

To model top-down coalition forming in industrial symbiosis networks, a cooperative game model is used. In a cooperative game, the aim of the players is to form a coalition that obtains the highest value and to distribute this value among the coalition members. The value of a coalition is usually expressed as a number and can, for example, represent money. In an industrial symbiosis network, firms can reduce their costs or gain benefits by collaborating. This is similar to players in a cooperative game who form a coalition with a higher value. So, cooperative game theory can provide analytical solution concepts for top-down coalition forming in industrial symbiosis networks.

A typical cooperative game in game theory has a set of players N and a characteristic function v [3]. This characteristic function assigns a value $v(S)$ to each coalition $S \subseteq N$ [3]. The aim of a cooperative game is for the players to form coalitions and to distribute the value of the formed coalition among the coalition members.

In [22], forming an industrial symbiosis network between two players is already modelled as a two-player cooperative cost game. In a cost game, the players have to divide some costs in their coalition. In the model of [22], the set of players consists of two companies, A and B, and the worth of each coalition is equal to the costs of the coalition. This means that for the grand coalition, consisting of both the companies, the costs are the total operating cost of the industrial symbiosis. For a coalition consisting of a single company, the costs are the traditional cost for the company. For the empty coalition, which has no members, the costs are defined as 0.

The basis of this two-player cost game, the set of players consisting of the two companies, can be used to formulate a cooperative game for industrial symbiosis networks with n players. Here, $n \in \mathbb{N}$. So, the set of players, N, consists of all companies that can possibly form industrial symbiotic relations with the other players. In total, there are n companies, so $|N| = n$. Each $i \in N$ represents one company. Each subset $S \subseteq N$ represents a possible industrial symbiosis network.

Now that the set of players is defined, it is time to look at the next part of the cooperative game, the characteristic function v. This function assigns a value to each coalition $S \subseteq N$ which the coalition members must distribute among themselves. Before this characteristic function is defined, the costs associated with the exchange of resources in an industrial symbiosis network will be explained.

In an industrial symbiotic relation, there are usually two sides, the waste producing companies and the waste receiving companies. Companies can be a waste producer and a waste receiver at the same time. If there is no industrial symbiotic relation between the companies, the waste producing companies need to discharge their waste and the waste receiving companies need to purchase raw materials. In both cases, there are costs involved. For the waste producers, these costs are waste discharge costs and for the waste receivers, these are the purchasing costs for the raw materials. These costs are called the traditional operating costs. If the companies are in an industrial symbiotic relation, there are other costs involved, since the waste then has to be treated to be used as a substitute for a raw material and needs to be transported from one firm to another. These costs that result from the industrial symbiotic relation are called the industrial symbiosis operating costs.

For the characteristic function in an industrial symbiosis network setting, the definition from [21] is used. Here, the value of each coalition is expressed as $v(S) = T(S) - O(S)$, where $T(S)$ represents the total traditional costs of the coalition if there is no industrial symbiotic relation and $O(S)$ represents the total operating costs of the coalition if there is an industrial symbiotic relation. All coalitions $S \subseteq N$ with $|S| \leq 1$ have $v(S) = 0$. Notice that for $T(S)$ we have that $T(S) = \sum_{i \in S} T(\{i\})$, where $T(\{i\})$ are the operating costs for a single company i.

Now, the industrial symbiosis game can be formally defined:

Definition 3.1.1. The tuple (N, v) is an *industrial symbiosis game (ISG)*, where N is the set of players $\{1, 2, \ldots, n\}$ and v is the function $v(S) = T(S) - O(S)$ for coalition $S \subseteq N$. Here, $T(S)$ are the traditional costs of the coalition S and $O(S)$ are the operating costs of an industrial symbiosis relation between the coalition S. If $|S| \leq 1$, $v(S) = 0$.

All the players of the industrial symbiosis game are assumed to be rational, meaning that their game strategy is to gain as much benefit as possible. In Example 3.1.2, an example of an industrial symbiosis game will be given.

Example 3.1.2. Let (N, v) be an industrial symbiosis game. $N = \{A, B, C\}$ and v is defined by the following table:

S	$\emptyset$	{A}	{B}	{C}	{AB}	{AC}	{BC}	{ABC}
$v(S)$	0	0	0	0	10	8	4	12

So, N consists of all the companies that can form industrial symbiosis networks. In this case, the companies are A, B, and C. When they operate individually, in coalitions {A}, {B}, and {C}, they obtain no benefit from industrial symbiosis, so the values of these coalitions is zero. When they start cooperating, in coalitions {AB}, {AC}, {BC}, and {ABC}, the benefits from industrial symbiosis are larger than zero, meaning that the companies can gain an economic advantage by cooperating in an industrial symbiosis network.

3.2 Model Analysis

In this section, the industrial symbiosis game from Definition 3.1.1 will be analysed using concepts from cooperative game theory. The aim is to look at properties of this game which ensure that there are coalitions that can be formed in a way that gives each player more benefits than operating alone. These properties can help in answering the questions about which industrial symbiosis networks to form.

For this section, it is assumed that the grand coalition N of all players will form. This assumption is usually made in cooperative game theory. In the case that the grand coalition does not form, the industrial symbiosis game can be split into disjoint subgames for which the grand coalition will form. Later on, in Section 3.3, a solution method for finding these subgames will be proposed.

The value of each coalition in the industrial symbiosis game, assigned by the characteristic function, represents the total obtainable economic benefit of each coalition, if the companies started to cooperate instead of working individually. Each coalition also needs to divide these benefits among its members. This is an important aspect of cooperative game theory.

One well-known solution concept for dividing the benefits is the Shapley value [18]. With the Shapley value and the corresponding Shapley allocation, each coalition member gets his average marginal contribution as a reward. So, in a sense, a Shapley allocation is fair to all of the players. This makes the Shapley allocation an appealing solution concept. For this section, the focus is to derive properties such that the Shapley allocation can be used to divide the benefits of the industrial symbiosis network. When the industrial symbiosis game has these properties, the Shapley allocation can be computed. An example of how to compute the Shapley allocation in an industrial symbiosis game can be found in Section 3.3.

The benefit allocation among the companies in the industrial symbiosis network should be fair. Therefore, the Shapley allocation is used to divide the benefits. The benefit allocation should also be efficient. This means that all the benefits of the coalition are divided among the members. The last property that the benefit allocation should have is coalitional rationality. This ensures that each subcoalition prefers to be in the grand coalition over being in the subcoalition.

Definition 3.2.1. For a given cooperative game (N, v), a benefit allocation $x = (x_1, x_2, \dots x_{|N|}) \in \mathbb{R}^{|N|}$ is *efficient* if $\sum_{i \in N} x_i = v(N)$.

Definition 3.2.2. For a given cooperative game (N, v), a benefit allocation $x = (x_1, x_2, \dots x_{|N|}) \in \mathbb{R}^{|N|}$ is *coalitionally rational* if $x(N) = \sum_{i \in N} x_i \geq v(S)$ for all coalitions $S \subseteq N$.

There is a set of all the benefit allocations that are efficient and coalitionally rational. This is called the core of the game.

Definition 3.2.3. The *core* of a game (N, v) is the set of all efficient and coalitionally rational payoff vectors $C(N, v) = \{x \in \mathbb{R}^{|N|} \mid x(N) = v(N),\ x(S) \geq v(S)\ \forall S \subseteq N\}$.

The Shapley allocation is always efficient [18]. However, it is not always coalitionally rational. So, it is not always in the core. If the Shapley allocation is not in the core, it means that some players or sets of players would prefer not to be in the grand coalition with this benefit allocation. In that case, they have an incentive to deviate from the coalition and the cooperation may fail. So, we need to look for industrial symbiosis games for which the Shapley allocation is in the core.

One issue with the core is that it can be empty. If the core is empty, it is not possible to divide the benefits of the grand coalition in an efficient and coalitionally rational manner. So, we also need to look for games in which the core is non-empty.

Fortunately, these two issues can be solved together. It is known that if a game is convex, the core is non-empty and the Shapley allocation is always in the core [6]. Moreover, if the game is convex, the Shapley allocation is at the centre of gravity of the core. For a game to be convex, its characteristic function v should be supermodular.

Definition 3.2.4. The cooperative game (N, v) is called *convex* if v is supermodular.

Definition 3.2.5. A set function $v : 2^N \to \mathbb{R}$ is called *supermodular* if $v(S) + v(K) \leq v(S \cup K) + v(S \cap K)\ \forall S, K \subseteq N$.

Now the next question arises: when is the characteristic function supermodular? There is a specific subclass of the industrial symbiosis game of Definition 3.1.1 for which the characteristic function is supermodular. This is the class of industrial symbiosis games for which the average benefit per player, $v(S)/|S|$, increases as the coalition S becomes larger. This does not necessarily mean that the benefits are divided equally in this manner, since we want to use the Shapley allocation to divide the benefits.

Proposition 3.2.6. *The industrial symbiosis game (N, v) is convex if the average benefit per player, $v(S)/|S|$, increases as the coalition S becomes larger.*

Proof. To show this, we need to check whether v is supermodular, by checking the following cases: $S = \emptyset$, $S = N$, $S \cap K = \emptyset$, and $S \cap K \neq \emptyset$.

- If $S = \emptyset$, the supermodularity condition becomes:

$$\begin{aligned} v(S) + v(K) &\leq v(S \cup K) + v(S \cap K) \\ v(K) &\leq v(K) \end{aligned}$$

  It is trivial that this always holds.

- If $S = N$, the supermodularity condition becomes:

$$\begin{aligned} v(S) + v(K) &\leq v(S \cup K) + v(S \cap K) \\ v(N) + v(K) &\leq v(N) + v(K) \end{aligned}$$

  Again, this is trivial.

- If $S \cap K = \emptyset$, we get the following condition for supermodularity:

$$\begin{aligned} v(S) + v(K) &\leq v(S \cup K) + v(S \cap K) \\ v(S) + v(K) &\leq v(S \cup K) + 0 \\ T(S) - O(S) + T(K) - O(K) &\leq T(S \cup K) - O(S \cup K) \\ \sum_{i \in S} T(\{i\}) - O(S) + \sum_{i \in K} T(\{i\}) - O(K) &\leq \sum_{i \in S \cup K} T(\{i\}) - O(S \cup K) \\ \sum_{i \in S \cup K} T(\{i\}) - O(S) - O(K) &\leq \sum_{i \in S \cup K} T(\{i\}) - O(S \cup K) \\ -O(S) - O(K) &\leq -O(S \cup K) \end{aligned}$$

  This condition only holds if the industrial symbiosis operating costs are, on average, less for a bigger coalition.

- If $S \cap K \neq \emptyset$, we can use similar computations as for the previous case:

$$\begin{aligned} v(S) + v(K) &\leq v(S \cup K) + v(S \cap K) \\ -O(S) - O(K) &\leq -O(S \cup K) - O(S \cap K) \end{aligned}$$

  Again, this condition holds if the coalition $S \cup K$ and the coalition $S \cap K$ together have less industrial symbiosis operating costs than the coalition S and the coalition K together. This also holds if a bigger coalition has, on average, less industrial symbiosis operating costs.

If the average industrial symbiosis costs, $O(S)/|S|$, decrease if the coalition S grows, it implies that the average benefit per player, $v(S)/|S|$, increases. Therefore, the industrial symbiosis game (N, v) is convex if the average benefit per player, $v(S)/|S|$, increases as the coalition S becomes larger. □

Now, we have a subclass for which we can solve the question of dividing the benefits of cooperation. This subclass of industrial symbiosis games is convex, meaning that we can form the grand coalition of all players and divide the benefits using the Shapley allocation. No player will diverge from this grand coalition, so the coalition is stable. In the next section, industrial symbiosis games will be considered for which the characteristic function is not supermodular and the industrial symbiosis game is not convex. This section will be concluded with two examples, one of a convex industrial symbiosis game and one of a non-convex industrial symbiosis game.

Example 3.2.7. Let (N, v) be an industrial symbiosis game. $N = \{A, B, C\}$ and v is defined by the following table:

S	$\emptyset$	{A}	{B}	{C}	{AB}	{AC}	{BC}	{ABC}
$v(S)$	0	0	0	0	2	3	1	5

For each coalition, average benefit per player, $v(S)/|S|$, can be calculated. This results in:

S	$\emptyset$	{A}	{B}	{C}	{AB}	{AC}	{BC}	{ABC}
$v(S)/\|S\|$	0	0	0	0	1	1.5	0.5	1.67

It can be seen in this table, that for this industrial symbiosis game, the average benefits per player increase as there are more players in the industrial symbiosis network. Therefore, this industrial symbiosis game is convex and all the players prefer to be in the grand coalition {ABC} over any other coalition.

Example 3.2.8. Let (N, v) be an industrial symbiosis game. $N = \{A, B, C\}$ and v is defined by the following table:

S	$\emptyset$	{A}	{B}	{C}	{AB}	{AC}	{BC}	{ABC}
$v(S)$	0	0	0	0	2	3	1	4

For each coalition, average benefit per player, $v(S)/|S|$, can be calculated. This results in:

S	$\emptyset$	{A}	{B}	{C}	{AB}	{AC}	{BC}	{ABC}
$v(S)/\|S\|$	0	0	0	0	1	1.5	0.5	1.33

For the coalition {AC}, the average benefit per player is 1.5. For the grand coalition {ABC}, this is 1.33. So, this industrial symbiosis game is not convex. This means that companies A and C would prefer to be in coalition {AC} over coalition {ABC}. So, the grand coalition is not stable for this industrial symbiosis game.

3.3 A General Characteristic Function

Up until now, it has been assumed that the aim of the industrial symbiosis game is to find a way to divide the benefits from cooperation among the grand coalition. If the grand coalition is formed in a cooperative game, all the players are in one big coalition. However, in realistic industrial symbiosis practices, there are cases in which the game is not convex, so the formation of the grand coalition is not feasible. For convex games, the best coalition is the grand coalition, since no subgroup has an incentive to deviate. For games that are not convex, we can search for a way to split up the players, such that the resulting subgame is convex.

Definition 3.3.1. A *subgame* of the game (N, v) is any game of the form (S, v), where $S \subseteq N$.

The goal here is to look for a way to split the grand industrial symbiosis game into disjoint subgames which maximise the total industrial symbiosis benefit. For this purpose, the algorithm from [17] can be used. This algorithm efficiently finds the best possible coalitions in a coalition formation problem. Another plus of this method is that it is an anytime algorithm, meaning that if the algorithm is terminated before it is finished, the resulting coalitions are within a certain bound from the optimal coalition structure.

In this section, the algorithm from [17] will be tailored for the industrial symbiosis game and an example will be provided.

First of all, for the algorithm of [17], the values of each possible coalition must be known. Also, for each possible coalition size, the average value for that size is computed. The possible coalition sizes of the general cooperative game (N, v) are $1, 2, \ldots, n$, since $|N| = n$. With this information, the search space is then partitioned based on coalition structures. A coalition structure is a way to split up the players into coalitions. For example, a coalition of two players and a coalition of one player is a coalition structure for a three player cooperative game. For an example of the search space partitioning, consider the cooperative game (N, v) with $|N| = 4$. Here, the search space is partitioned in the following subspaces: S_{1111}, S_{112}, S_{13}, S_{22}, and S_4. Here, the index of the subspace indicates the coalition structure. So in S_{1111}, the coalition structure is four coalitions of size 1 and in S_{13} the coalition structure is one coalition of size 1 and one coalition of size 3.

For each of these subspaces, a lower and upper bound are determined using the coalition values and the computed average values per coalition size. If there are subspaces in which the upper bound is lower than the lower bound of some other subspace, these subspaces can be pruned and no longer need to be searched. The algorithm then searches the most promising subspace. This information is used to prune more subspaces or search other promising subspaces. When there are no more subspaces to be searched, the algorithm returns the optimal solution. The following algorithm gives an overview in pseudo code:

Input: industrial symbiosis game (N, v)
Output: subgames of (N, v)
use algorithm 1 from [17];
    for *all possible coalition sizes* **do**
        compute average coalition value;
        compute maximum coalition value;
    end
    partition search space into subspaces of same coalitional structure;
    compute upper and lower bounds for each subspace;
    set initial solution;
use algorithm 2 from [17];
    prune subspaces;
use algorithm 3 from [17];
    while *not all subspaces searched* **do**
        search the most promising subspace using algorithm 4 [17];
        update best solution;
        prune subspaces using algorithm 2 [17];
    end
return *optimal coalition structure*

In Example 3.3.2, it is demonstrated how this algorithm can be applied to an industrial symbiosis game.

Example 3.3.2. The companies A, B, C, and D are looking into forming industrial symbiosis networks. The following information about their possible collaboration is known:

Coalition	Value	Coalition	Value	Coalition	Value	Coalition	Value
{A}	0	{AB}	60	{ABC}	80	{ABCD}	100
{B}	0	{AC}	40	{ABD}	120		
{C}	0	{AD}	80	{ACD}	150	Average	100
{D}	0	{BC}	10	{BCD}	30		
		{BD}	0				
Average	0	{CD}	40	Average	95		
		Average	38.33				

With this information, the search space, consisting of all possible solutions, is partitioned in the following subspaces:

Subspace	Lower bound	Upper bound	Optimal solution
S_{1111}	0	0	?
S_{112}	38.33	80	?
S_{13}	95	150	?
S_{22}	76.66	160	?
S_4	100	100	?

Notice that the optimal solution per subspace is still empty right now, as no subspace has been searched. Immediately, the subspaces S_{1111} and S_{112} can be pruned, since S_{13} will always give a better solution than can be found in S_{1111} or S_{112}. So, now we work with the following subspaces:

Subspace	Lower bound	Upper bound	Optimal solution
S_{13}	95	150	?
S_{22}	76.66	160	?
S_4	100	100	?

We now search S_{22}, as this is the most promising. This results in the coalition structure {{AB},{CD}} with a value of 100. With this information, S_4 can also be pruned, since this subspace does not contain a better solution.

Subspace	Lower bound	Upper bound	Optimal solution
S_{13}	95	150	?
S_{22}	76.66	160	100

Now, only the subspace S_{13} needs to be searched, as this can contain a solution better than 100. In this subspace, we find the solution of {{ACD},{B}} with a value of 150.

Subspace	Lower bound	Upper bound	Optimal solution
S_{13}	95	150	150
S_{22}	76.66	160	100

So, the optimal coalition structure is an industrial symbiosis network with A, C, and D, and B just operating on its own.

3.4 Solving the Industrial Symbiosis Game

We now know when an industrial symbiosis game is convex and how to split up an industrial symbiosis game into subgames if it is not convex. We also know that in a convex game, the benefits can be allocated using the Shapley allocation. This will ensure that the benefits are allocated efficiently, rationally, and fair. In this section, we will continue with the example of

Section 3.3 and show how the benefits of the coalition {A,C,D} can be divided using the Shapley allocation.

Example 3.4.1. Suppose we have an industrial symbiosis game with the companies A, C, and D from Example 3.3.2. The following information about the values of the coalitions is known:

Coalition	Value	Coalition	Value	Coalition	Value
{A}	0	{AC}	40	{ACD}	150
{C}	0	{AD}	80		
{D}	0	{CD}	40		

It is immediately clear that this game is convex. The next step is to compute the Shapley allocation. For this, we check each permutation σ of the players and determine for each permutation the contribution m_i^σ of the players, i =A,C,D. This is then aggregated to compute the marginal contribution for each player.

σ	m_A^σ	m_C^σ	m_D^σ
(A,C,D)	0	40	110
(A,D,C)	0	70	80
(C,A,D)	40	0	110
(C,D,A)	110	0	40
(D,A,C)	80	70	0
(D,C,A)	110	40	0
sum	340	220	340

For each company, the Shapley allocation is now determined by taking the sum of its marginal contributions and dividing it by 3! = 6. This gives the following result:

Company	Allocation
A	$340/6 = 56.67$
C	$220/6 = 36.67$
D	$340/6 = 56.67$

Adding these allocation gives $56.67 + 36.67 + 56.67 = 150$, which is of course the total benefit to be divided by this coalition.

In the last example of this chapter, the coalition finding algorithm and the calculation of the cost allocation will be applied to an industrial symbiosis game with five participating companies.

Example 3.4.2. The companies A, B, C, D, and E are looking into forming industrial symbiosis networks. The following information about their possible collaboration is known:

S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	0	{AB}	16	{ABC}	19	{ABCD}	30	{ABCDE}	32
{B}	0	{AC}	14	{ABD}	22	{ABCE}	26		
{C}	0	{AD}	11	{ABE}	22	{ABDE}	25	Avg	32
{D}	0	{AE}	17	{ACD}	18	{ACDE}	22		
{E}	0	{BC}	18	{ACE}	20	{BCDE}	20		
		{BD}	10	{ADE}	16				
Avg	0	{BE}	11	{BCD}	18	Avg	25.8		
		{CD}	16	{BCE}	18				
		{CE}	15	{BDE}	21				
		{DE}	18	{CDE}	19				
		Avg	14.6	Avg	19.3				

With this information, the search space, consisting of all possible solutions, is partitioned in the following subspaces:

Subspace	Lower bound	Upper bound	Optimal solution
S_{11111}	0	0	?
S_{1112}	14.6	18	?
S_{113}	19.3	22	?
S_{14}	25.8	30	?
S_{23}	33.9	40	?
S_{221}	29.2	36	?
S_5	32	32	?

Immediately, the subspaces S_{11111}, S_{1112}, S_{113}, S_{14}, and S_5 can be pruned. So, now we work with the following subspaces:

Subspace	Lower bound	Upper bound	Optimal solution
S_{23}	33.9	40	?
S_{221}	29.2	36	?

Now, the subspace S_{23}, as it has the highest upper bound. This results in the coalition structure {{ABE},{CD}} with a value of 38. With this information, S_{221} can also be pruned, since this subspace does not contain a better solution.

Subspace	Lower bound	Upper bound	Optimal solution
S_{23}	33.9	40	38

So, the optimal coalition structure is {{ABE},{CD}}. For both of these coalitions, we now compute the Shapley values, so the firms know how to divide their benefits within these coalitions. First, the Shapley values are calculated for the coalition {ABE}.

σ	m_A^σ	m_B^σ	m_E^σ
(A,B,E)	0	16	6
(A,E,B)	0	17	5
(B,A,E)	16	0	6
(B,E,A)	11	0	11
(E,A,B)	17	5	0
(E,B,A)	11	11	0
sum	55	49	28

For each company, the Shapley allocation is now determined by taking the sum of its marginal contributions and dividing it by $3! = 6$. This gives the following result:

Company	Allocation
A	$55/6 = 9.17$
B	$49/6 = 8.17$
E	$28/6 = 4.67$

And finally, the Shapley values are also calculated for the coalition {CD}.

σ	m_C^σ	m_D^σ
(C,D)	0	16
(D,C)	16	0
sum	16	16

Now we divide the sum of the marginal contributions by $2! = 2$ to obtain the Shapley values. This gives the following result:

Company	Allocation
C	$16/2 = 8$
D	$16/2 = 8$

So, now we have solved an industrial symbiosis game with five companies. In this example, the best coalitions are {ABE} and {CD}. Using the Shapley allocation to determine how the firms should divide the benefits of the industrial symbiosis network, we get the following allocations. Company A gets 9.17, company B gets 8.17, company C gets 8, company D gets 8, and company E gets 4.67.

3.5 Interpretation for the Decision Maker

In this chapter, the process of forming industrial symbiosis networks in a top-down manner has been modelled as a cooperative game, with the players being the companies and the characteristic function representing the obtainable benefit by cooperating in an industrial symbiosis network. When the average benefit per company increases as more companies enter the industrial symbiosis network, it was shown that the game is convex. When the game is convex, it can be solved by forming the grand coalition of all players and dividing their benefits using the Shapley allocation. For a convex game, the Shapley allocation is efficient, coalitionally rational, and fair. These are all properties that are desirable for the solution concept.

When the industrial symbiosis game is not convex, it can be solved by splitting the industrial symbiosis game into subgames which maximise the total obtained benefit over all formed coalitions. A method is proposed to do this. When these coalitions are formed and they are convex, the Shapley allocation can again be used to divide the benefits among the industrial symbiosis network.

So, for a cluster manager looking into making the companies in the cluster cooperate in industrial symbiosis networks, this method can help give decision support. A cluster manager usually has some information about all the companies in the cluster. This information can be used to construct an industrial symbiosis game of the companies in the cluster. The cluster manager can use the method of Section 3.3 to form the industrial symbiosis networks and propose to the companies to look into cooperating. By using this model to propose industrial symbiosis networks among the cluster companies, the cluster manager can formally motivate why these companies should cooperate.

Chapter 4

Bottom-Up Coalition Forming in Industrial Symbiosis

For this chapter, we take a new perspective on the formation of industrial symbiosis networks. As explained in Chapter 2, there are two ways for industrial symbiosis networks to form. They can be designed top-down or can emerge bottom-up. In the previous chapter, the focus was on industrial symbiosis networks that are designed top-down. Now, we consider industrial symbiosis networks that emerge bottom-up. When an industrial symbiosis network emerges bottom-up, the companies themselves propose cooperation with other companies to, for example, cut back on waste discharge costs or to be more sustainable. In this case, the decision makers are the companies looking into starting their own industrial symbiosis network with each other.

The aim of this chapter is to describe how companies form industrial symbiosis networks on their own and to provide decision support for companies when they want to start their own industrial symbiosis network with each other. For this purpose, again a game theory approach is used. This time, a non-cooperative bargaining game is used to model the process of companies bargaining for an industrial symbiosis network. Moreover, a method will be provided to generate strategies for the bargaining companies.

In Section 4.1, the model for bottom-up coalition forming will be explained. In Section 4.2, this model will be analysed and a solution method for it will be presented. Section 4.3 gives an algorithm for generating the strategies of the players of the industrial symbiosis bargaining game. In Section 4.4, the implications of this model for the decision makers, in this case the firms themselves, will be explained. Finally, in Section 4.5, the model of this chapter will be compared with the model from Chapter 3, to see where the models differ and what the similarities are.

4.1 Model and Assumptions

In this section, the bottom-up formation of industrial symbiosis networks will be modelled as a non-cooperative coalitional bargaining game. We call this the industrial symbiosis bargaining game (ISBG). For bottom-up industrial symbiosis, the firms themselves take the initiative. There is no higher power telling them to cooperate. Each company decides to partake in an industrial symbiosis network on its own and does so for its own benefit. Because each company is autonomous and plays for its own benefit, the model used for bottom-up coalition forming is based on non-cooperative game theory. In a non-cooperative game, players choose their strategy for their own benefit, and not for the aggregated benefit of all the players. This aligns nicely with an industrial symbiosis network that emerges bottom-up.

For companies to start their own industrial symbiosis network bottom-up, they have to make an offer to other companies to start cooperating. These companies can, in turn, accept or reject this proposal. They are, in essence, bargaining to form a coalition. To model this bargaining behaviour, a coalitional bargaining game is used. In a coalitional bargaining game, players play sequentially. During his/her turn, a player can make an offer for a coalition with a corresponding cost/benefit allocation. Each player of this suggested coalition can then decline or accept it.

If all players of the coalition accept it, the coalition will be formed and the players leave the bargaining game. The players left in the bargaining game will play another round. If there are one or more players who reject the proposal, a new round of the game starts. In Example 4.2.1, the process of the bargaining is explained with an example.

In non-cooperative game theory, the usual solution method is to look for a Nash equilibrium [13]. In a Nash equilibrium, no player can unilaterally change his or her strategy to receive a better payoff, given that the strategies of the other players remain unchanged. This will be the basis for the solution method of the industrial symbiosis bargaining game in Section 4.2.

For this bargaining game, the same player set and the same characteristic function as for the industrial symbiosis game of Chapter 3 can be used. The player set consists of all the companies that want to bargain for an industrial symbiosis network. The characteristic function represents, per possible industrial symbiosis network, the obtainable benefit by cooperating. But, to be able to solve this bargaining game later on, some assumptions need to be made about the characteristic function v. Remember, $v(S)$ of coalition $S \subseteq N$ is expressed as $v(S) = T(S) - O(S)$, where $T(S)$ denotes the traditional operating costs of coalition S and $O(S)$ denotes the costs of maintaining an industrial symbiosis network among coalition S.

To use the same characteristic function v for the bargaining game, the following assumptions about this function need to be made. The characteristic function is assumed to be 0-normalised, essential, and super-additive over the feasible coalitions. In this game, feasible coalitions are coalitions with a value greater than zero. So, only coalitions that can gain from forming an industrial symbiosis network are feasible.

Definition 4.1.1. The characteristic function v of the game (N, v) is 0-*normalised* if $v(\{i\}) = 0$ for all $i \in N$.

Definition 4.1.2. The characteristic function v of the game (N, v) is *essential* if $v(N) > 0$.

Definition 4.1.3. The characteristic function v of the game (N, v) is *super-additive* if $v(S \cup K) \geq v(S) + v(K)$ for all $S, K \subseteq N$ for which $S \cap K = \emptyset$.

With these properties of the characteristic function, we can apply the solution method from [14] to form the coalitions. This will be done in the next section.

4.2 Solving the Industrial Symbiosis Bargaining Game

In this section, the solution method for the industrial symbiosis bargaining game will be explained. The solution method is based on the one presented in [14]. First, the concept of a strategy combination will be presented. The strategy combination contains the strategy of each player. This concept will be used to explain the solution concept of the bargaining game, a stationary subgame perfect equilibrium.

A strategy combination $\sigma = (\sigma_1, \ldots, \sigma_n)$ consists of a strategy σ_i for each player $i = 1, \ldots, n$. This strategy is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$ of mappings where σ_i^t is the t-th round strategy. This gives the proposal that player i will give in round t and a response function replying "yes" or "no" to all possible proposals by other players. This t-th round strategy may depend on the history of the game up until time t [14]. In Example 4.2.2, an example of a strategy and a strategy combination can be found.

Example 4.2.1. The coalitional bargaining game for industrial symbiosis will be explained with the following example. Suppose there are five companies, Red, Purple, Green, Yellow, and Blue. In the following figures, the bargaining process for an industrial symbiosis network will be explained.

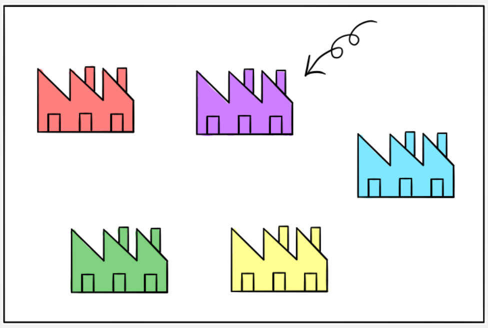


Figure 4.1: Purple is randomly selected to make the first proposal.

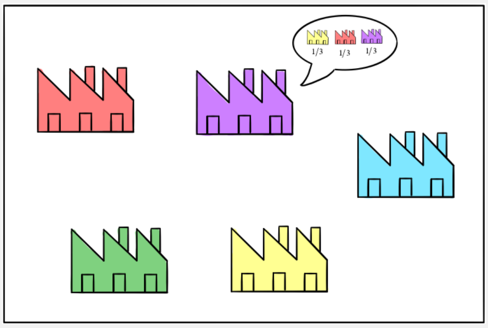


Figure 4.2: Purple proposes a coalition between Red, Yellow, and Purple, with an equal payoff distribution.

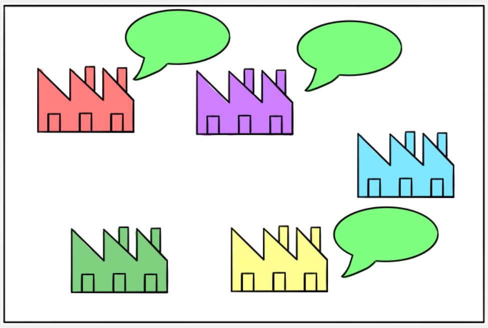


Figure 4.3: One thing that can happen is that all the prospective coalition partners accept the proposal. The coalition is formed and they leave the bargaining game.

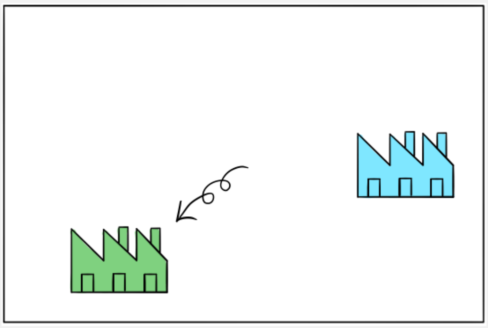


Figure 4.4: Two players are left in the bargaining game. The next round starts and Green is randomly selected to make the next proposal.

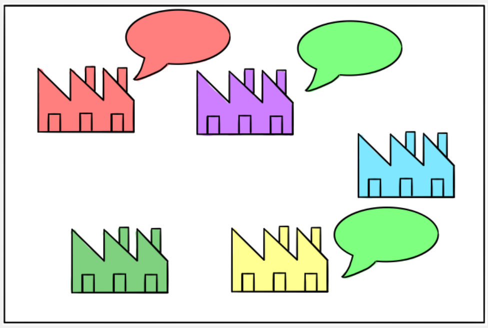


Figure 4.5: Another thing that can happen after Purple makes it proposal is that not all the prospective coalition partners accept the proposal. Here, Red rejects the proposal. No coalition is formed and the bargaining game continues with all players.

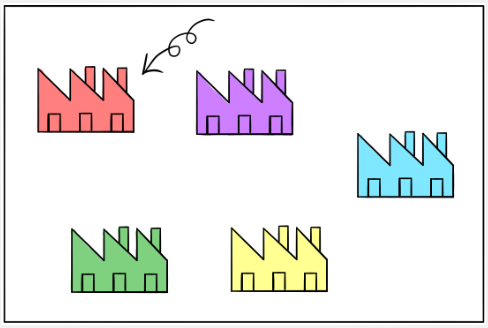


Figure 4.6: Another round of the game starts and Red is randomly selected to make the next proposal.

Example 4.2.2. Consider the following industrial symbiosis bargaining game. The players are the companies A, B, and C. The characteristic function is as follows:

Coalition	Value	Coalition	Value	Coalition	Value
{A}	0	{AB}	2	{ABC}	3
{B}	0	{AC}	1		
{C}	0	{BC}	3		

For company A, his strategy in round 1 may be to accept the proposed coalition only if he gets at least 2 and to propose coalition {ABC}, where he gets 2 and B and C both get 0.5. In round 2, he might accept anything that gives him 1.8 and propose again coalition {ABC}, where he gets 1.8 and B and C both get 0.6. So, his t-th round strategy is to accept any proposal that at least gives him $2 - 0.2 \cdot (t-1)$ (as long as it is greater than zero) and always propose coalition {ABC} where he gets $2 - 0.2 \cdot (t-1)$ and B and C equally divide the rest.
For company B, his strategy might be to accept if he receives at least 1 and to propose coalition {BC}, where they equally divide the benefits.
For company C, his strategy might be to accept if he gets more than zero and B is in the coalition and to propose {ABC} where each company is rewarded 1.
All of these individual strategies together for this game form a strategy combination.

The strategy combination can be seen as a collection of the strategies of all the players in an industrial symbiosis bargaining game. With this, we can work towards the solution concept, a stationary subgame perfect equilibrium point (SSPE), which is defined in the following two definitions.

Definition 4.2.3. A *subgame perfect equilibrium* is a strategy combination that induces a Nash equilibrium in every subgame.

Definition 4.2.4. A strategy combination of a game is a *stationary subgame perfect equilibrium point* if it is a subgame perfect equilibrium with the property that, for all $t = 1, 2, \ldots$, the t-th round strategy of every player depends only on the set N^t of active players at round t [14].

So, to solve the industrial symbiosis bargaining game, we look for a strategy combination that is a stationary subgame perfect equilibrium. Similarly as in [14], the industrial symbiosis bargaining game has random proposers, meaning that in each round of the game, a random player is selected to make the proposal. Because of this property and the fact that we look for a stationary subgame perfect equilibrium point, where the strategy of the players is only dependent on the currently active players, an interesting result can be derived.

In [14] it is shown that in the coalitional bargaining game with random proposers, there is no delay of agreement. This means that, in the first round of the game, an agreement is reached for forming a coalition. For the industrial symbiosis bargaining game this means that there will be an industrial symbiosis network formed in the first round of the game. So, the first player selected to make a proposal will always propose a coalition and payoff distribution that he knows will be accepted by the other players in the proposed coalition. Since the players have perfect information about each other and all players are assumed to be rational, each player knows what the other players will accept. They can build their own strategy on this information.

Example 4.2.5. Consider the following industrial symbiosis bargaining game. The players are the companies A, B, and C. The characteristic function is as follows:

Coalition	Value	Coalition	Value	Coalition	Value
{A}	0	{AB}	2	{ABC}	3
{B}	0	{AC}	1		
{C}	0	{BC}	3		

Assume that company A is selected at first to make a proposal. Company A will not propose the grand coalition, since companies B and C will only accept this proposal if they get all the payoff, so in that case, company A will not gain anything from cooperation. Company A will also not propose a coalition with only company C, since company C will reject that as well. Company A knows that the only proposal to be accepted is a coalition with only company B, where company B will get 1.5 of the payoff and A will take 0.5. A payoff proposal where company A gets more will always be rejected by company B.

Proposition 4.2.6. *In an industrial symbiosis bargaining game, a coalition will be formed with no delay of agreement.*

Proof. Since the industrial symbiosis bargaining game is a modification of the coalitional bargaining game of [14] and fulfils the criteria that the characteristic function is 0-normalised, essential, and super-additive over the feasible coalitions, the results of [14] can immediately be applied. Therefore, in the industrial symbiosis bargaining game, a coalition will be formed with no delay of agreement.

□

So, in the first round of the industrial symbiosis game, a coalition is immediately formed. But, this is not necessary the grand coalition. As shown in [14], the grand coalition is always formed in the first round if and only if the coalitional value per capita, $v(S)/|S|$, increases as the coalition becomes larger.

Proposition 4.2.7. *In an industrial symbiosis bargaining game, the grand coalition will only be formed in the first round if and only the coalitional value per capita, $v(S)/|S|$, increases as the coalition becomes larger.*

Proof. Since the industrial symbiosis bargaining game is a modification of the coalitional bargaining game of [14] and fulfils the criteria that the characteristic function is 0-normalised, essential, and super-additive over the feasible coalitions, the results of [14] can immediately be applied. Therefore, in the industrial symbiosis bargaining game, the grand coalition will only be formed in the first round if and only the coalitional value per capita, $v(S)/|S|$, increases as the coalition becomes larger.

□

With the solution method of a stationary subgame perfect equilibrium point, the strategies for the players can be determined. This will be done in the next section.

4.3 Strategy Generation

In this section, a method will be presented to generate a strategy for the players of the industrial symbiosis bargaining game. This strategy will aid the companies in determining which proposals to accept, which to reject, and which to suggest when it is their turn to propose a coalition. This method for generation strategies is based on Theorem 1 from [14], which gives a maximisation problem that each player uses to determine their proposal.

Before diving into this maximisation problem, some notation will be introduced. A proposal made by player $i \in N$ is denoted by (S_i, y^{S_i}). Here, S_i is the proposed coalition and y^{S_i} is the proposed payoff distribution. The set X^S contains all feasible payoff distributions for coalition $S \subseteq N$ and X_+^S contains all payoffs in X^S with non-negative components. $G^N(\delta)$ denotes an industrial symbiosis bargaining game with player set N and discount factor δ. This discount factor is introduced to discount future payoffs. v_j^N is the expected payoff of player j in the game $G^N(\delta)$.

With v_j^N, the first part of the strategy can already be determined. With an expected payoff of the game for each player, a decision rule can be made for accepting or rejecting a proposal. Each player is rational, so they want at least their expected payoff in this game. But, rejecting a proposal means that there will be no payoff in this round. Therefore, this expected payoff should be discounted in the decision rule. This gives the following algorithm for the decision rule of player j:

```
Input: industrial symbiosis bargaining game G^N(δ), proposal y, expected payoff
       v_j^N
Output: accept or reject
if y_j ≥ δv_j^N then
    return accept
else
    return reject
end
```

Algorithm 1: Decision rule for the industrial symbiosis bargaining game.

Now, to determine the proposal to make in the current round, the linear program given in [14] will be used. This linear program can be found in Equation 4.1.

$$\begin{array}{lll} \max\limits_{S,y} & v(S) - \sum\limits_{j \in S, j \neq i} y_j & \\ \text{subject to} & i \in S \subseteq N, & y \in X_+^S \\ & y_j \geq \delta v_j^N & \text{for all } j \in S \text{ with } j \neq i \end{array} \tag{4.1}$$

The linear program 4.1 uses the expected payoff v_j^N of each player j to determine the strategy of a player. If these are know, an individual company can use the maximisation problem given in Equation 4.1 to solve for their own best strategy. However, the strategies of the players in turn influence the expected payoff. To solve this linear program for all players, we thus need to solve it iteratively for all players, since the result of the linear program for one player influences the expected playoff for both himself and the other players. So, by changing the strategy for one player, the strategies for the other players are influenced and might need to be adjusted. Algorithm 2 solves the LP 4.1 iteratively for all players until the difference between their old expected payoffs and their new expected payoffs is very small. The choice for a very small difference was made to make sure that the algorithm terminates in case it starts to cycle.

```
Input: industrial symbiosis bargaining game G^N(δ), convergence bound ϵ
Output: proposals for all players
initialise the proposals y^i for all players i ∈ N;
initialise the expected payoffs v_j^N for all players j ∈ N;
initialise the updated expected payoffs w_j^N for all j ∈ N;
while ||v_j^N − w_j^N||_2^2 > ϵ do
    update v_j^N to w_j^N for all j ∈ N;
    for all players i ∈ N do
        calculate δ · v_j^N for all j ∈ N, j ≠ i;
        for all coalitions S ∈ N, i ∈ S do
            calculate the benefit in S for player i using δ · v_j^N;
            update best possible coalition for i;
        end
        update y^i using best coalition;
    end
    update w_j^N;
end
return y^i for all players i ∈ N
```

Algorithm 2: Strategy generation for the industrial symbiosis bargaining game.

The algorithm will give each player i a proposal (S_i, y^{S_i}) that will be accepted by all players in S. This is an immediate result of Proposition 4.2.6. Player i maximises his own payoff, while still ensuring that the other players in the proposed coalition will accept it.

To end this section, Algorithm 2 is applied to an example with five companies.

Example 4.3.1. Consider an industrial symbiosis bargaining game with five players and discount factor $\delta = 0.9$. The characteristic function is given by the following table:

S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	0	{AB}	16	{ABC}	19	{ABCD}	32	{ABCDE}	38
{B}	0	{AC}	14	{ABD}	22	{ABCE}	35		
{C}	0	{AD}	11	{ABE}	22	{ABDE}	34		
{D}	0	{AE}	16	{ACD}	18	{ACDE}	33		
{E}	0	{BC}	18	{ACE}	20	{BCDE}	36		
		{BD}	10	{ADE}	16				
		{BE}	11	{BCD}	18				
		{CD}	16	{BCE}	18				
		{CE}	15	{BDE}	21				
		{DE}	16	{CDE}	19				

This is very similar to Example 3.4.2, except that the characteristic function is adjusted a little in order to make it superadditive. We now apply Algorithm 2 to this example to see the strategies it generates for each player.
To let the algorithm solve this example, it was implemented in Python. The complete code for this specific example can be found in Appendix A. For the convergence bound a value of 0.4 was chosen. With a lower value, the algorithm got stuck in a loop, since there are multiple possible solutions for the strategies, which influence the expected payoffs, which in turn again influences the strategies.
The algorithm gives the companies the following strategies:

Company	Coalition	Payoff distribution
A	{A, B, C, D}	{13.49, 6.75, 6.24, 5.52}
B	{A, B, D, E}	{5.17, 16.27, 5.61, 6.95}
C	{A, B, C, E}	{5.20, 6.56, 16.33, 6.90}
D	{B, C, D, E}	{8.00, 6.33, 14.80, 6.87}
E	{A, C, D, E}	{5.24, 6.34, 5.69, 15.73}

It is curious that all companies would propose a coalition with four companies. As can be seen in the characteristic function, a coalition of four companies gives the most benefits per company. So, in a coalition with four companies, there is the most to be gained.
This means that this industrial symbiosis bargaining will be done in one round. Since there is no delay of agreement, at round one there will be a proposal made that will be accepted. Since all companies will propose a coalition of size four, there will be only one player left after the first round and the game is over.

4.4 Interpretation for the Decision Makers

At the start of this chapter, the companies who wanted to start their own industrial symbiosis network needed a way to make a well-motivated decision about what coalitions to propose and what coalitions to accept. By modelling this problem as a non-cooperative coalitional bargaining game, it is possible to generate strategies for these companies which will result in the immediate formation of an industrial symbiosis network.

With the strategy generation method presented in Section 4.3, companies are able to calculate which proposals they should accept and which they should propose. An important assumption made here, is that all the companies have perfect information. This means that all companies know exactly what they can gain from cooperating with the other companies. In the more realistic case that the companies do not have perfect information, this approach will not be useful. In that case the expected payoff could, for example, be replaced by the payoff of the companies if they are not in a coalition. More suggestions for the industrial symbiosis bargaining game with imperfect information can be found in Section 6.2.

4.5 Discussion

In this final section, the two different models from Chapter 3 and this chapter will be compared. There are some interesting similarities in these models which will be discueed here.

At first glance, the bargaining game of this chapter is very similar to the industrial symbiosis game of Chapter 3. Both games have the same set of players and the same characteristic function. The biggest difference between these games, is that the industrial symbiosis game is cooperative and the bargaining game is non-cooperative. In a cooperative game, the players form coalitions to get the highest total payoff, thus to maximise the sum of all the coalition values. In a non-cooperative game, each player plays for his own benefit. They want their own best payoff and are not interested in the total payoff of all players. Because of this fundamental difference in the way the game is played, the approach to solving this game is also different. It is still based on game theory, but on the non-cooperative game theory, whereas the previous model was based on cooperative game theory.

Another big difference from the industrial symbiosis game of Chapter 3 is that the bargaining game is played sequentially. The players take turns in proposing a coalition and a payoff distribution. In the industrial symbiosis game of Chapter 3, there is no concept of time in the game. The game is solved by finding the best possible coalitions. They form and distribute their payoffs. In the bargaining game, a proposition is made in each round of the game. This proposition is either accepted or rejected. Since there is no clear ending to this game, it could go on forever.

Some similarity between the models can be found in Proposition 4.2.7, as it lines up with earlier conditions from Proposition 3.2.6 for convexity of the industrial symbiosis game. In a convex game, the grand coalition is always preferred, just as it is here on the same condition. For this result the following proposition can be formulated.

Proposition 4.5.1. *If the grand coalition forms in an industrial symbiosis bargaining game, the resulting cooperative industrial symbiosis game is convex.*

Proof. Assume we have an industrial symbiosis bargaining game in which the grand coalition forms. From Proposition 4.2.7, we know that the grand coalition forms if and only if the coalitional value per capita, $v(S)/|S|$, increases as the coalition becomes larger. Since this holds here, we know from Proposition 3.2.6 that the industrial symbiosis game which underlies the bargaining game is convex.

□

So, there are some similarities between the models, but the main differences can be found in the game theoretical approach and the essence of time in the model.

Chapter 5

Application to a Hypothetical Case

In this chapter, the applicability of the model and algorithm of Chapter 3 is tested. This was the model for top-down industrial symbiosis networks. To do this, a hypothetical case about exchanging waste heat will be used. The data from this case can be found in [12]. In Section 5.1, the case is described. In Section 5.2, the case will be moulded into an industrial symbiosis game. Sections 5.3, 5.4, and 5.5 explain the different scenarios for the case and the solution given by the algorithm for each scenario. Finally, in Section 5.6, the results from the case study will be summarised for the decision maker.

5.1 Case Description

In this case study, we consider six firms of otherwise unrelated industries [12]. These firms are geographically close together. There are two types of firms: firms that generate waste heat and firms that use heat as an input resource. The firms that generate heat are called producer companies, since they produce waste heat. Normally, they would just discard this waste heat into the air for free [12]. The firms that use heat as an input resource are called receiver companies. Normally they would use natural gas, but this can be exchanged for the waste heat from the producer companies. Also, the producer and receiver companies can exchange the condensate of the usage of the waste heat. This condensate can be returned from the receiver to the producer. So, if a producer and a receiver were to exchange waste heat and condensate, they would have two material flows between them. One from the producer to the receiver with the waste heat and one from the receiver to the producer with the condensate.

We consider three producer companies in this case. They are called A, B, and C. The waste heat that each company can exchange per year is given per scenario, since this will differ. There are three receiver companies. Just like in [12], each receiver company is able to receive the same amount of waste heat. An overview of the receiver companies can be found in Table 5.1.

Receiver	Gas demand (per year)
D	12.50 GWh/y
E	12.50 GWh/y
F	12.50 GWh/y

Table 5.1: The waste heat receiving companies and their gas demand per year [12].

5.2 Defining the Game

To define the game based on the waste heat case, we need two things, the set of players and the characteristic function. Defining the set of players is simple. The six companies are the players. We call the waste producing companies A, B, and C. The waste receiving companies are called D, E, and F.

The characteristic function of an industrial symbiosis game consists of two parts. It is defined as $v(S) = T(S) - O(S)$ for each coalition $S \subseteq N$. $T(S)$ are the traditional operating costs.

For the waste producing companies, the traditional costs are zero, since they can dispose their waste heat into the air for free using a flue gas [12]. For the waste receiving companies, the traditional costs consist of purchasing natural gas, which can be substituted by the waste heat. So, to determine the traditional costs per year, we multiply the gas demand of the receiver companies by the gas price. Using the gas price for medium industries in Europe [1], the gas price per gigawatt hour is €25618.32.

For the industrial symbiosis relation operating costs $O(S)$, there are three kinds of costs relevant here. The first are the investment costs of a pipeline between the participating companies. These are estimated using a general pipeline equation [15] in the same manner as in [12]. The other important costs are the annual maintenance cost for the pipeline. These can be estimated at 2% of the investment costs per year [10]. Also, it is assumed that the pipelines have a life of 25 years. Therefore, the investment costs are divided equally over these 25 years. The last kind of costs relevant here, are the costs which occur when there is more demand than supply. In that case, the receiver companies need to purchase natural gas.

The last parameter we need is the distance between the companies. It is assumed that all the companies have the same distance between them, just as in [12]. This distance differs between the case scenarios.

An overview of the different kinds of costs for the case study can be found in Table 5.2.

P_{gas}	Gas price per GWh	€25618.32
$C_{\text{investment}}$	Investment costs per km pipeline	€820185.24

Table 5.2: Parameters for the case study.

Now the characteristic function can be used to calculate the value of all the coalitions. For this, the following formula is used:

$$v(S) = D(S) \cdot P_{\text{gas}} - 3 \cdot C_{\text{investment}} \cdot d \cdot n_{\text{pipes}}(S)/50 - P_{\text{gas}} \cdot \max(0; D(S) - D_{\text{supply}}(S)) \quad (5.1)$$

Here, $D(S)$ is the total demand of coalition S. $n_{\text{pipes}}(S)$ is the number of pipes necessary for the industrial symbiosis network. $D_{\text{supply}}(S)$ is the supply that the producing companies can supply.

The first term of this formula represents the traditional costs. The second term represents the investment and maintenance costs. The last term represents the possible purchasing costs in case of a demand/supply mismatch.

5.3 Scenario 1

In the first scenario, the waste heat per producer company is as in Table 5.3. The distance between companies is set at 1 kilometre for this scenario.

Producer	Produced material	Waste heat (per year)
A	Rockwool	7.50 GWh/y
B	Enzymes	6.00 GWh/y
C	Oil Refinery	32.00 GWh/y

Table 5.3: The waste heat producing companies and their waste heat per year [12].

Using Equation 5.2, the characteristic function for this case scenario can be defined. The values for every possible coalition can be found in Table 5.4.

S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	0	{AB}	0	{ABC}	0	{ABCD}	24962.31	{ABCDE}	49924.62	{ABCDEF}	74886.94
{B}	0	{AC}	0	{ABD}	123384.54	{ABCE}	24962.31	{ABCDF}	49924.62		
{C}	0	{AD}	93715.17	{ABE}	123384.54	{ABCF}	24962.31	{ABCEF}	49924.62	Avg	74886.94
{D}	0	{AE}	93715.17	{ABF}	123384.54	{ABDE}	-47841.60	{ABDEF}	-244686.06	Max	74886.94
{E}	0	{AF}	93715.17	{ACD}	123384.54	{ABDF}	-47841.60	{ACDEF}	370153.62		
{F}	0	{BC}	0	{ACE}	123384.54	{ABEF}	-47841.60	{BCDEF}	370153.62		
		{BD}	55287.69	{ACF}	123384.54	{ACDE}	246769.08				
Avg	0	{BE}	55287.69	{ADE}	-4707.06	{ACDF}	246769.08	Avg	107565.84		
Max	0	{BF}	55287.69	{ADF}	-4707.06	{ACEF}	246769.08	Max	370153.62		
		{CD}	221806.77	{AEF}	-4707.06	{ADEF}	-103129.29				
		{CE}	221806.77	{BCD}	123384.54	{BCDE}	246769.08				
		{CF}	221806.77	{BCE}	123384.54	{BCDF}	246769.08				
		{DE}	0	{BCF}	123384.54	{BCEF}	246769.08				
		{DF}	0	{BDE}	-43134.54	{BDEF}	178672.23				
		{EF}	0	{BDF}	-43134.54	{CDEF}	524519.55				
				{BEF}	-43134.54						
		Avg	221806.77	{CDE}	443613.54	Avg	134135.94				
		Max	74161.93	{CDF}	443613.54	Max	524519.55				
				{CEF}	443613.54						
				{DEF}	0						
				Avg	114888.84						
				Max	443613.54						

Table 5.4: The characteristic function for scenario 1 of the case.

Now, the algorithm from Section 3.3 can be used. This results in the following optimal coalition structure: {{ABD},{CEF}} with a total value of €566998.08. Coalition {ABD} earns €123384.54 and coalition {CEF} earns €443613.54.

Computing the Shapley values for these coalition results in the following payoff distributions. For coalition {ABD}, the payoff distribution is {€38318.15, €19104.41, €65961.99}. For coalition {CEF}, the payoff distribution is {€221806.77, €110903.39, €110903.39}. See Table 5.5 and Table 5.6.

Company	Allocation
A	€38318.15
B	€19104.41
D	€65961.99

Table 5.5: The Shapley values for the coalition {ABD}.

Company	Allocation
C	€221806.77
E	€110903.39
F	€110903.39

Table 5.6: The Shapley values for the coalition {CEF}.

5.4 Scenario 2

In this scenario, the waste heat per producer company is as in Table 5.7. The distance between companies is set at 2 kilometres for this scenario.

Producer	Produced material	Waste heat (per year)
A	Rockwool	7.50 GWh/y
B	Enzymes	6.00 GWh/y
C	Oil Refinery	32.00 GWh/y

Table 5.7: The waste heat producing companies and their waste heat per year [12].

Using Equation 5.2, the characteristic function for this case scenario can be defined. The values for every possible coalition can be found in Table 5.9.

Now, the algorithm from Section 3.3 can be applied. This results in the following optimal coalition structures: {{CDE},{A},{B},{F}}, {{CDF},{A},{B},{E}}, or {{CEF},{A},{B},{D}}. Since all the receiver companies are interchangeable, there are three optimal coalition structures. If the decision maker decided to go for {{CDE},{A},{B},{F}}, the payoff distribution can be found in Table 5.8.

Company	Allocation
C	€123384.54
D	€61692.27
E	€61692.27

Table 5.8: The Shapley values for the coalition {CDE}.

S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	0	{AB}	0	{ABC}	0	{ABCD}	-270304.38	{ABCDE}	-540508.75	{ABCDEF}	-810913.13
{B}	0	{AC}	0	{ABD}	-73459.92	{ABCE}	-270304.38	{ABCDF}	-540508.75		
{C}	0	{AD}	-4707.06	{ABE}	-73459.92	{ABCF}	-270304.38	{ABCEF}	-540508.75	Avg	-810913.13
{D}	0	{AE}	-4707.06	{ABF}	-73459.92	{ABDE}	-441530.51	{ABDEF}	-835219.43	Max	-810913.13
{E}	0	{AF}	-4707.06	{ACD}	-73459.92	{ABDF}	-441530.51	{ACDEF}	-220379.75		
{F}	0	{BC}	0	{ACE}	-73459.92	{ABEF}	-441530.51	{BCDEF}	-220379.75		
		{BD}	-43134.54	{ACF}	123384.54	{ACDE}	-146919.83				
Avg	0	{BE}	-43134.54	{ADE}	-201551.52	{ACDF}	-146919.83	Avg	-482967.53		
Max	0	{BF}	-43134.54	{ADF}	-201551.52	{ACEF}	-146919.83	Max	-220379.75		
		{CD}	123384.54	{AEF}	-201551.52	{ADEF}	-398395.98				
		{CE}	123384.54	{BCD}	-73459.92	{BCDE}	-146919.83				
		{CF}	123384.54	{BCE}	-73459.92	{BCDF}	-146919.83				
		{DE}	0	{BCF}	-73459.92	{BCEF}	-146919.83				
		{DF}	0	{BDE}	-239979.00	{BDEF}	-116594.46				
		{EF}	0	{BDF}	-239979.00	{CDEF}	229252.86				
				{BEF}	-239979.00						
		Avg	15108.59	{CDE}	246769.08	Avg	-220184.08				
		Max	123384.54	{CDF}	246769.08	Max	229252.86				
				{CEF}	246769.08						
				{DEF}	0						
				Avg	-62271.18						
				Max	246769.08						

Table 5.9: The characteristic function for scenario 2 of the case.

5.5 Scenario 3

In this last scenario, the waste heat per producer company is as in Table 5.10. The distance between companies is set at 2 kilometres for this scenario.

Producer	Produced material	Waste heat (per year)
A	Rockwool	7.50 GWh/y
B	Enzymes	6.00 GWh/y
C	Oil Refinery	175.00 GWh/y

Table 5.10: The waste heat producing companies and their waste heat per year [12].

Using Equation 5.2, the characteristic function for this case scenario can be defined. The values for every possible coalition can be found in Table 5.12.

To solve the industrial symbiosis game from this case scenario, again the algorithm from Section 3.3 will be used. With this algorithm, we will be able to form the best possible coalitions from the players in the game. The best solution for this scenario has a value of 370153.62. In this case, the coalition structure is {{A}, {B}, {CDEF}}. In this scenario this is also the expected result for this scenario, since C has enough waste heat exchange to supply all the receiver companies on its own.

For the coalition {CDEF}, we will now calculate the Shapley values so the companies know how to divide the coalition benefits. The Shapley values can be found in Table 5.11.

Company	Allocation
C	€185076.81
D	€61692.27
E	€61692.27
F	€61692.27

Table 5.11: The Shapley values for coalition {CDEF}.

S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	0	{AB}	0	{ABC}	0	{ABCD}	-270304.38	{ABCDE}	-540608.75	{ABCDEF}	-810913.13
{B}	0	{AC}	0	{ABD}	-73459.92	{ABCE}	-270304.38	{ABCDF}	-540608.75		
{C}	0	{AD}	-4707.06	{ABE}	-73459.92	{ABCF}	-270304.38	{ABCEF}	-540608.75	Avg	-810913.13
{D}	0	{AE}	-4707.06	{ABF}	-73459.92	{ABDE}	-441530.51	{ABDEF}	-835219.43	Max	-810913.13
{E}	0	{AF}	-4707.06	{ACD}	-73459.92	{ABDF}	-441530.51	{ACDEF}	-220379.75		
{F}	0	{BC}	0	{ACE}	-73459.92	{ABEF}	-441530.51	{BCDEF}	-220379.75		
		{BD}	-43134.55	{ACF}	-73459.92	{ACDE}	-146919.83				
Avg	0	{BE}	-43134.55	{ADE}	-201551.52	{ACDF}	-146919.83	Avg	-482967.53		
Max	0	{BF}	-43134.55	{ADF}	-201551.52	{ACEF}	-146919.83	Max	-220379.75		
		{CD}	123384.54	{AEF}	-201551.52	{ADEF}	-398395.98				
		{CE}	123384.54	{BCD}	-73459.92	{BCDE}	-146919.83				
		{CF}	123384.54	{BCE}	-73459.92	{BCDF}	-146919.83				
		{DE}	0	{BCF}	-73459.92	{BCEF}	-146919.83				
		{DF}	0	{BDE}	-239979.00	{BDEF}	-116594.46				
		{EF}	0	{BDF}	-239979.00	{CDEF}	370153.62				
				{BEF}	-239979.00						
		Avg	15108.59	{CDE}	246769.08	Avg	-210790.70				
		Max	123384.54	{CDF}	246769.08	Max	370153.62				
				{CEF}	246769.08						
				{DEF}	0						
				Avg	-62271.18						
				Max	246769.08						

Table 5.12: The characteristic function for scenario 3 of the case.

5.6 Advice for the Decision Maker

In this final section of the chapter, the results from the case study are summarised for the decision maker. In this case, the decision maker is a cluster manager who is responsible for the six companies considered here. He/She makes the decision about which companies should cooperate and gives advice on how they should divide the costs and benefits of their cooperation.

For this case, we looked at three different scenarios to see the influence of certain operational factors, such as distance and having one player with a lot of influence. In all the scenarios we saw that the producer with the most influence was in a coalition. This is no surprise, since this player can supply at least two receivers on his own in all the scenarios. And since the investment and maintenance costs are quite high, it is better to have one producer supply two receivers than to have two producers for two receivers.

If the companies are geographically closer together, more smaller coalitions are formed. This makes sense, since the investment costs are linearly dependent on the distance between companies. If they are closer together, it will be more profitable to cooperate.

Another operational aspect is the price of natural gas. In this case study, the gas price was the same for all scenarios. However, if the gas price increases, the profit that the companies gain will also increase, since the traditional operational costs are linearly dependent on the gas price. So, cooperation is more beneficial if the gas price is higher.

The proposed coalitions for each scenario are the coalitions that together maximise the total payoff of all the players. The proposed payoff distributions allocate the payoff in each coalition in a way that is fair and rational for all the coalition partners.

Even though the algorithm returns only the coalitions which are in the optimal coalition structure, there are also possibilities for the sub-optimal coalitions. For example, in scenario 2, the optimal coalitions structure is a coalition of C, D, and E, and the other companies operating on their own. A and B can supply 13.50 GWh and F has a demand of 12.50 GWh. So, in terms of supply and demand, A, B, and F would make a good coalition. Since the distance between the companies is 2 kilometres, the investment and maintenance costs are too high to make an industrial symbiotic relation between these companies profitable. However, if governmental authorities wanted to give an incentive to these companies to use waste heat instead of natural gas, they could raise taxes on natural gas for companies or give a subsidy for the investment costs for such a project. With this model, we cannot only get the most profitable coalition structure for an industrial symbiosis networks, but we can also see where there are opportunities in sub-optimal coalitions.

For this case, the decision maker can use the model to see which industrial symbiosis networks will be profitable. He can also see how the different operational parameters influence the characteristic function of the underlying industrial symbiosis game. The algorithm proposed in Section 3.3 can be used to determine which coalitions are the most profitable. When this is determined, the decision maker can compute the Shapley values for the coalitions proposed by the algorithm and use that payoff distribution for the coalition.

Chapter 6

Conclusions & Recommendations

6.1 Conclusion

The aim of this thesis is to model the emergence of planned and self-organised industrial symbiosis networks to give decision support to the decision makers of these industrial symbiosis networks.

For planned or top-down industrial symbiosis networks, decision support is provided to, for example, a cluster manager. He has the relevant information of the possible industrial symbiosis partners and can use this to make a cooperative game model, the industrial symbiosis game. For the cluster manager, the most important question is how to form the industrial symbiosis networks from the companies. For this purpose, the algorithm from Chapter 3 can be used. This algorithm will tell the cluster manager which industrial symbiosis networks will be the most profitable and which will be stable. The cluster manager can also help the companies within an industrial symbiosis network with the question on how to divide the profit from their cooperation. The Shapley values can be used for this. This will divide the benefits in a fair manner and will also be stable if it is in the core. The Shapley values are always in the core if the game is convex. This can be easily checked if the values of the characteristic function are known.

The case study was about waste heat and condensate exchange between six hypothetical companies. It was performed to test the applicability of this algorithm for top-down coalition forming. With this case, we saw the influence of different operational aspects on the profitability of the industrial symbiosis networks. If the distance between the companies increased, less industrial symbiosis networks were profitable, since the investment costs were very high. If one of the producer companies was able to supply all the receiver companies, the most profitable industrial symbiosis network was one consisting of just this producer and all the receivers. By making an industrial symbiosis game out of this case, the decision maker is able to quickly see which coalitions are profitable and which are not. Also, the influence of the distance between the companies or the price of natural gas can be seen by adjusting the input parameters and seeing how the game model behaves. For this case, applying the algorithm to the game model will return the most profitable coalitions.

For self-organised or bottom-up industrial symbiosis networks, a bargaining game model was used to model the emergence of the industrial symbiosis network. Here, the decision makers are the companies themselves. To provide them with decision support, the algorithm in Chapter 4 was proposed. This algorithm generates the best strategies for all the players. With these strategies the players ensure that they get the best possible payoff while ensuring that their industrial symbiosis network is stable, meaning that no firms in the industrial symbiosis network prefer to be in another industrial symbiosis network.

6.2 Recommendations for Further Research

For the industrial symbiosis bargaining game, an assumption was made that all the companies have perfect information. They all know how much other companies can gain from the different coalition. For application to a real life situation, this may not always be realistic. One can imagine that companies are not willing to share with everyone how much they pay for their waste disposal, input purchasing, or how much of a certain resource they use. To see how the model behaves in such a case with imperfect information, it could be simulated with an agent-based model. Each agent represents a company which tries to make the best coalition proposal given the information it has. With imperfect information, it is not likely that a coalition is formed with no delay of agreement. Maybe there will be no coalition formed at all. This is a very interesting point for further research.

For both the models, the top-down and bottom-up it might be interesting to see what they would propose when applied to real life industrial symbiosis networks. The models could propose the same industrial symbiosis networks, but they might also give something completely different. It will be very interesting to see if their industrial symbiosis network is very different from the one already implemented.

Bibliography

[1] Eurostat - Tables, Graphs and Maps Interface (TGM) table. <https://ec.europa.eu/eurostat/tgm/refreshTableAction.do;jsessionid=qgKJCtZAdL8tc7NkEa7wdmh35e-BYdryRe7hl1yzm322swfmiv8q!-357443795?tab=table&plugin=1&pcode=ten00118&language=en>, 2019.

[2] Abreu, M. C. S. d., and Ceglia, D. On the implementation of a circular economy: The role of institutional capacity-building through industrial symbiosis. *Resources, Conservation and Recycling 138* (Nov. 2018), 99–109.

[3] Andreu Mas-Colell, Michael Dennis Whinston, Jerry R. Green, et al. *Microeconomic theory.* Oxford University Press, New York, 1995.

[4] Chertow, M. R. "Uncovering" Industrial Symbiosis. *Journal of Industrial Ecology 11*, 1 (2007), 11–30.

[5] Chertow, M. R., Ashton, W. S., and Espinosa, J. C. Industrial Symbiosis in Puerto Rico: Environmentally Related Agglomeration Economies. *Regional Studies 42*, 10 (Dec. 2008), 1299–1312.

[6] Driessen, T. S. H. *Cooperative Games, Solutions and Applications.* Springer Science & Business Media, 1988.

[7] Ehrenfeld, J. R., and Chertow, M. R. Industrial symbiosis: the legacy of Kalundborg. In *A handbook of industrial ecology*, R. U. Ayres and L. Ayres, Eds. Edward Elgar Pub, Cheltenham, UK ; Northampton, MA, 2002.

[8] Ellen MacArthur Foundation. Effective industrial symbiosis. <https://www.ellenmacarthurfoundation.org/case-studies/effective-industrial-symbiosis>, 2019.

[9] European Comission. *Closing the Loop - An EU Action Plan for the Circular Economy.* COM, Bruxelles, 2015.

[10] Hackl, R., and Harvey, S. Identification, cost estimation and economic performance of common heat recovery systems for the chemical cluster in Stenungsund. *Chalmers University Of Technology* (2013).

[11] Hein, A. M., Jankovic, M., Farel, R., and Yannou, B. A Conceptual Framework for Eco-Industrial Parks. American Society of Mechanical Engineers, pp. V004T05A024–V004T05A024.

[12] Lok, E. Identifying the Economic Performance of Heat-Based Industrial Symbiosis Networks: A Simulation Study. *University of Twente* (2019).

[13] Nash, J. Non-Cooperative Games. *Annals of Mathematics 54*, 2 (1951), 286–295.

[14] Okada, A. A Noncooperative Coalitional Bargaining Game with Random Proposers. *Games and Economic Behavior 16*, 1 (Sept. 1996), 97–108.

[15] Parker, N. Using Natural Gas Transmission Pipeline Costs to Estimate Hydrogen Pipeline Costs. *UC Davis: Institute of Transportation Studies (UCD)* (2004).

[16] Peters, H. *Game Theory: A Multi-Leveled Approach.* Springer Texts in Business and Economics. Springer Berlin Heidelberg, Berlin, Heidelberg, 2015.

[17] Rahwan, T., Ramchurn, S. D., Jennings, N. R., and Giovannucci, A. An Anytime Algorithm for Optimal Coalition Structure Generation. *Journal of Artificial Intelligence Research 34* (Apr. 2009), 521–567.

[18] Shapley, L. S. A value for n-person games. In *Contributions to the Theory of Games II*, H. W. Kuhn and A. W. Tucker, Eds. Princeton University Press, Princeton, New Jersey, 1953, pp. 307–317.

[19] Symbiosis Institute. Kalundborg Symbiosis. <www.symbiosis.dk>, 2019. Accessed June 2019.

[20] United Nations. Transforming our world: the 2030 Agenda for Sustainable Development .:. Sustainable Development Knowledge Platform, 2015.

[21] Yazdanpanah, V., Yazan, D., and Zijm, H. Industrial Symbiotic Networks as Coordinated Games. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10-15, 2018* (2018), IFAAMAS, 3 pages.

[22] Yazdanpanah, V., and Yazan, D. M. Industrial Symbiotic Relations as Cooperative Games. In *arXiv:1802.01167 [cs]* (Feb. 2018).

Appendix A

ISBG Strategy Generation — Python Implementation

```
"""
Industrial Symbiosis Bargaining Game Strategy Generation

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Date: 22-06-2019

Python 3.7
"""

# define coalition values
coalition_values = {(1, 0, 0, 0, 0): 0, (0, 1, 0, 0, 0): 0, (0, 0, 1, 0, 0):
↪  0, (0, 0, 0, 1, 0): 0, (0, 0, 0, 0, 1): 0,
                    (1, 1, 0, 0, 0): 16, (1, 0, 1, 0, 0): 14, (1, 0, 0, 1, 0):
                    ↪  11, (1, 0, 0, 0, 1): 16,
                    (0, 1, 1, 0, 0): 18, (0, 1, 0, 1, 0): 10, (0, 1, 0, 0, 1):
                    ↪  11, (0, 0, 1, 1, 0): 16,
                    (0, 0, 1, 0, 1): 15, (0, 0, 0, 1, 1): 16, (1, 1, 1, 0, 0):
                    ↪  19, (1, 1, 0, 1, 0): 22,
                    (1, 1, 0, 0, 1): 22, (1, 0, 1, 1, 0): 20, (1, 0, 1, 0, 1):
                    ↪  20, (1, 0, 0, 1, 1): 16,
                    (0, 1, 1, 1, 0): 18, (0, 1, 1, 0, 1): 18, (0, 1, 0, 1, 1):
                    ↪  21, (0, 0, 1, 1, 1): 19,
                    (1, 1, 1, 1, 0): 32, (1, 1, 1, 0, 1): 35, (1, 1, 0, 1, 1):
                    ↪  34, (1, 0, 1, 1, 1): 33,
                    (0, 1, 1, 1, 1): 36, (1, 1, 1, 1, 1): 38
                    }
n = 5           # number of companies
delta = 0.9     # discount factor


proposal_values = [[0 for _ in range(n)] for __ in range(n)]    # initialise
↪  the proposal values
proposals = [[0 for _ in range(n)] for __ in range(n)]          # initialise
↪  the proposals
for i in range(n):
    proposals[i][i] = 1

expected_payoffs = [1 for _ in range(n)]        # initialise the expected
↪  payoff
new_expected_payoffs = [0 for _ in range(n)]    # initialise the new expected
↪  payoffs
```

```
def norm(old_vector, new_vector):
    """
    calculates the 2-norm between two vectors
    :param old_vector: first vector
    :param new_vector: second vector
    :return: 2-norm
    """
    difference = 0
    for i in range(len(old_vector)):
        difference += (old_vector[i] - new_vector[i]) ** 2
    return difference


# while the difference between the expected and new expected payoffs is not
↪  small enough
while norm(expected_payoffs, new_expected_payoffs) > 0.4:
    expected_payoffs = new_expected_payoffs[:]      # update the expected
    ↪  payoffs

    for i in range(n):      # loop over all companies and solve the LP for all
    ↪  companies
        minimum_proposal_values = [0 for _ in range(n)]
        for j in range(n):  # loop over all possible coalition candidates and
        ↪  calculate the minimum proposal value
            if j != i:
                temp_sum = 0
                for k in range(n):
                    if k != i:
                        temp_sum += proposal_values[k][j]
                minimum_proposal_values[j] = delta / n * temp_sum / (1 - delta
                ↪  / n)

        max_value = -1
        max_coalition = None
        for s in range(16):     # loop over all possible coalitions
            string_s = format(s, '04b')
            string_s = string_s[:i] + '1' + string_s[i:]
            tuple_s = tuple(map(lambda x: int(x), string_s))

            # calculate the value of the current coalition s
            current_value = coalition_values[tuple_s]
            for j in range(n):
                if j != i and tuple_s[j] == 1:
                    current_value -= minimum_proposal_values[j]

            # check if the current value is better than the maximum up until
            ↪  now, update if necessary
            if current_value > max_value:

                max_value = current_value
                max_coalition = tuple_s

        # update the proposals and proposal values
        proposals[i] = max_coalition
        proposal_values[i][i] = max_value
```

```
        for j in range(n):
            if j != i:
                if max_coalition[j] == 1:
                    proposal_values[i][j] = minimum_proposal_values[j]
                else:
                    proposal_values[i][j] = 0

    # update the new expected payoffs
    for i in range(n):
        temp_expected_payoff = 0
        for j in range(n):
            temp_expected_payoff += proposal_values[j][i]
        new_expected_payoffs[i] = temp_expected_payoff / n


# print the results
print(proposals)
print(proposal_values)
```