

# A Framework to Model Core Deposits and their Influence on the Risk Management of Non-Maturing Deposits

Master Thesis by:

Ilse Schepers

Mail address: [ilseschepers@hotmail.com](mailto:ilseschepers@hotmail.com)

Phone number: +316 23704666

Master of Industrial Engineering & Management  
Financial Engineering & Management

Faculty of Behavioural, Management and Social Sciences  
of the University of Twente

Supervisors:

Dr. B. Roorda (University of Twente), Prof. dr. ir. A. Bruggink (University of  
Twente), J. Tijink MSc. (KPMG)

November 2020

**UNIVERSITY  
OF TWENTE.**



## Management summary

Non-maturing deposits (NMD) are a significant portion of a bank's balance sheet. Therefore, it is important for the bank to understand the liquidity risk and interest rate risk arising from NMDs. As a result, the objective of this research is to gain more insight into modelling NMDs and to help to identify the liquidity risk and interest rate risk a bank faces. NMDs have two characteristics. First, the bank is always allowed to adjust the deposit rate. Second, the customer is always allowed to withdraw his or her money. Because of the second characteristic of NMDs, the deposit volume could drop significantly in a short period of time, and this induces liquidity risk. Furthermore, interest rates influence the deposit rates and behavior of customers. As a result, significant changes in interest rates can cause changes in the deposit volumes and therefore, changes in the cash flows from NMDs. This is also known as interest rate risk.

There are various ways to manage the liquidity risk and interest rate risk arising from NMDs. During this research, we are focusing on the replicating portfolio and the Economic Value of Equity (EVE). However, to be able to measure the expected cash outflow from NMDs, first, the proportion of core deposits needs to be determined. The volume of NMDs can be divided into core and non-core deposits. Core deposits are deposits that are stable and unlikely to reprice even under significant changes in interest rate environment. The proportion of core deposits can influence the outcome of the replicating portfolio and the EVE. A replicating portfolio assigns a fixed maturity profile to NMDs using standard trading instruments. And because core deposits are a stable funding source, a longer maturity is assigned to core deposits. The EVE measures the difference between the net present value of all cash inflows and outflows. If the proportion of core deposits increases then the cash outflows will probably decrease. As a result, the proportion of core deposits affects the replicating portfolio, the EVE, and thus also the interest rate risk and liquidity risk of a bank. With this research, we hope to identify this effect. Therefore, we have come up with the following main research question: "Which influence does the proportion of core and non-core deposits have on the risk management of non-maturing deposits of a bank?"

The determination of the ratio of core and non-core deposits is still not addressed extensively in the literature. During this research, we developed a framework for the determination of

core and non-core deposits. This framework consists of two steps. First, stable deposits have to be determined. Stable deposits stay undrawn with a high degree of likelihood. To measure the proportion of stable deposits, we use a deposit volume model to calculate the sensitivity under significant changes in interest rate environment. Deposit volumes are affected by interest rates. As a result, by applying various interest rate shock scenarios, the proportion of stable deposits can be determined. For every scenario, there will remain a certain deposit volume. The minimum volume occurring during these scenarios is defined as stable. Second, from the proportion of stable deposits, the core deposits can be determined. Therefore, we need to determine which deposits are unlikely to reprice even under significant changes in interest rate environment. The deposits with a fixed rate are automatically core deposits. To determine the proportion of core deposits with a floating rate, we have to elaborate on the behavior of the bank concerning repricing NMDs. Elaborating on the behavior of the bank is done by measuring the upward and downward adjustment of the deposit rate. The upward adjustment of the deposit rate is represented by  $\lambda^+$ , and the downward adjustment by  $\lambda^-$ . The height of  $\lambda^+$  and  $\lambda^-$  depends on the behavior of the bank. For example, when  $\lambda^+$  is equal to 0.4 then 40 percent of the deposits are repriced upwards. As a result, the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment is  $\min[(1 - \lambda^+), (1 - \lambda^-)]$ . After we determined the proportion of core deposits, we have to elaborate on the cap on the proportion of core deposits set by the Basel Committee of Banking Supervision (BCBS). The proportion of core deposits cannot be higher than 70 percent. If it is higher, then the proportion of core deposits is simply set back at 70 percent.

Subsequently, we elaborated on how we can apply the proposed framework to determine the proportion of core and non-core deposits to NMD models. To determine the proportion of core deposits, we need to develop a market rate model, deposit rate model, and deposit volume model. We developed a market rate model by using Principal Component Analysis (PCA). The obtained set of yield curves by using a stochastic model is often limited by the structure of the model, they are based on a risk-neutral world. If we want to measure unhedged risk, it is important to predict the true expectations about the uncertainty surrounding the future evolution of interest rates. As a result, developing a real-world model by using PCA will provide better results than a risk-neutral model. For the deposit rates, we used an asymmetric partial adjustment model. The asymmetric partial adjustment is an

interesting feature of a deposit rate model because it incorporates the behavior of the bank in terms of repricing deposits. However, with the current low interest rates, we have seen that banks are eager to keep the deposit rates positive and therefore there is not a clear asymmetric adjustment in the deposit rate. As a result, we assumed that the upward and downward adjustment is equal. Lastly, we have developed a deposit volume model. In this model, we have incorporated some behavior of customers of the bank. To be specific, if the deposit rate is lower than the market rate and the spread between these rates is high, then more depositors are willing to invest their money in the market instead of keeping it in a savings deposit. This means that the deposit volume could decrease. Furthermore, because deposit volumes may not become negative, we used a log-normally distributed model.

With the market rate model, the deposit rate model, and the deposit volume model, we could determine the proportion of core deposits. After applying the interest rate shock scenarios, we simulated 10000 possible outcomes of the deposit volume. The stable volume is defined as €215,053.4 million. This is 67.8 percent of the current deposit volume. Furthermore, only 8.5 percent of the changes in the interest rates are immediately passed to the customer. As a result, the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment is 91.5 percent. This is logical because banks are eager to keep the deposit rates above zero, despite the negative interest rates. As a result, banks are barely passing the changes in the interest rates to the customer. Because the core deposits cannot be higher than the proportion of stable deposits, the proportion of core deposits is defined as 67.8 percent.

After determining the core and the non-core, the deposits have to be placed in the correct time buckets. This can be done with a replicating portfolio approach. The non-core deposits are invested in the overnight bucket and it serves as protection against volume fluctuations. The core deposits are invested in various time bands. The BCBS set a cap on the average maturity of core deposits, namely 4.5 years. Various studies show that a dynamic replicating portfolio approach is superior to static replication, a lot of the shortcomings of a static replicating portfolio approach can be overcome with a dynamic approach. Therefore, we use a dynamic replicating portfolio approach during this research, specifically, the net present value Monte Carlo simulation model. To research the effect of the proportion of core and non-core deposits on the risk management of NMDs of a bank, we also construct

a replicating portfolio without the restriction that the non-core deposits are invested in the overnight bucket. The results show that the proportion of core deposits can have a great influence on the outcome of the replicating portfolio. For the portfolio without the restriction that the non-core deposits are invested in the overnight bucket, we can see that the percentage which is invested in the overnight bucket is much less than the proportion of non-core deposits. In conclusion, we recommend that the core deposits are determined before replicating a portfolio and that the proportion of non-core deposits is invested in the overnight bucket. In this way, the bank is save against volume fluctuations. Implementing this restricting in a dynamic or static replicating portfolio approach is not difficult. As a result, already existing approaches can be easily altered. Furthermore, we have seen that there is a dominance of the ten-year bucket. Which is as expected because of the expected outflow after 10 years.

We calculated the change in EVE based on the standardized method. We measured the cash flows under various interest rate scenarios with the help of a market rate model, deposit rate model, and two deposit volume models. The second deposit volume model is normally distributed. Thereafter, the cash flows are discounted using a risk-free rate. We have seen that the bank does not need the proportion of core and non-core deposits to determine the change in EVE. As a result, the outcome of the change in EVE is not based directly on the proportion of core and non-core deposits. However, we can see that there is a relationship between the proportion of core deposits and the change in EVE. The change in EVE is lower when the proportion of core deposits is higher because a smaller proportion of the NMDs is affected by changes in the interest rate environment. As a result, the change in the net present value of all cash flows originating from NMDs resulting from a change in interest rates is smaller. In conclusion, we can see that there is a negative relation between the proportion of core deposits and the change in EVE. Furthermore, the behavioral assumptions made to model deposit volumes of NMDs is a determining factor for the exposure of IRRBB under the EVE. However, the behavioral assumptions made to model deposit volumes also affects the proportion of core and non-core deposits. In conclusion, it is not the proportion of core deposits that influence the change in EVE, it is only the behavioral assumptions made to model deposit volumes, and these behavioral assumptions also influence the proportion of core and non-core deposits.

## Acknowledgments

This document is the final result for my master Industrial Engineering & Management with the specialization Financial Engineering & Management at the University of Twente. It describes the result of my research about modelling core and non-core deposits and the magnitude of the impact of the proportion of core and non-core deposits on the risk management of NMDs.

Finishing my thesis was not possible without the help of my supervisors and family. Therefore, I would like to express my gratitude to my supervisor Berend Roorda from the University of Twente for his useful comments, feedback, and engagement through the learning process of my master's thesis. I would also like to thank my second supervisor Bert Bruggink from the University of Twente for his feedback in the last stages of writing my thesis.

I am thankful that I had the opportunity to write my thesis at KPMG. Therefore, I would like to thank KPMG and my colleagues at KPMG for this opportunity and the amazing time there. Especially, I would like to express my gratitude to my supervisor Jan Tijink from KPMG for his feedback, support, and many helpful meetings.

Lastly, I would like to thank my father, mother, brother, sister, and boyfriend for the loving support throughout my entire study and helping me to get the best out of myself.

I hope you will enjoy reading my master's thesis. If you have any questions, please feel free to contact me.

Ilse Schepers

Enschede, 12 November 2020

# Contents

<b>Management summary</b>	<b>ii</b>
<b>Acknowledgments</b>	<b>vi</b>
<b>List of Tables</b>	<b>3</b>
<b>List of Figures</b>	<b>4</b>
<b>1 Introduction</b>	<b>5</b>
1.1 The research . . . . .	5
1.2 Research motivation . . . . .	6
1.3 Company description . . . . .	8
1.4 Research questions . . . . .	8
1.5 Research design . . . . .	11
1.6 Thesis outline . . . . .	11
<b>2 Modelling Non-Maturing Deposits</b>	<b>12</b>
2.1 Implications of Non-Maturing Deposits . . . . .	12
2.2 Modelling market rates . . . . .	13
2.3 Modelling deposit rates . . . . .	16
2.4 Modelling deposit volume . . . . .	19
2.5 Replicating portfolios . . . . .	21
2.5.1 Static replicating portfolio approach . . . . .	22
2.5.2 Dynamic replicating portfolio approach . . . . .	24
2.6 Measuring the Economic Value of Equity . . . . .	25
<b>3 Framework for determining the core and non-core deposits</b>	<b>28</b>
3.1 Determining stable deposits . . . . .	29
3.1.1 Interest rate sensitivity . . . . .	30
3.2 Determining core deposits . . . . .	32
3.2.1 Repricing NMDs . . . . .	32

---

<b>4</b>	<b>Application to savings deposits</b>	<b>35</b>
4.1	Market rate model . . . . .	36
4.1.1	Calibration and validation . . . . .	37
4.1.2	Historical market rates . . . . .	38
4.2	Deposit rate model . . . . .	40
4.2.1	Calibration and validation . . . . .	42
4.2.2	Historical deposit rates . . . . .	43
4.3	Deposit volume model . . . . .	43
4.3.1	Calibration and validation . . . . .	45
4.3.2	Historical deposit volumes . . . . .	46
4.4	Replicating portfolio . . . . .	47
4.5	Economic Value of Equity . . . . .	49
<b>5</b>	<b>Results</b>	<b>52</b>
5.1	Modelling market rates . . . . .	52
5.2	Modelling deposit rates . . . . .	58
5.3	Modelling deposit volumes . . . . .	60
5.4	Modelling core deposits . . . . .	63
5.5	Replicating portfolio . . . . .	64
5.6	Economic Value of Equity . . . . .	66
<b>6</b>	<b>Conclusion</b>	<b>68</b>
6.1	Conclusion to the research questions . . . . .	68
6.2	Conclusion to the main research question . . . . .	71
<b>7</b>	<b>Discussion and Future work</b>	<b>73</b>
7.1	Discussion . . . . .	73
7.2	Future Work . . . . .	74
	<b>Bibliography</b>	<b>76</b>

## List of Tables

2.1	Specified size of interest rate shocks [1]. . . . .	26
3.1	Caps on core deposits and average maturity per category [2] . . . . .	29
5.1	Factor loadings for the market zero rates. . . . .	53
5.2	Standard deviation, proportion of variance, and cumulative proportion of Factor Scores. . . . .	54
5.3	Market rate simulation model parameters. . . . .	56
5.4	Deposit rate model parameters. . . . .	58
5.5	Deposit volume model parameters. . . . .	60
5.6	Replicating portfolio with restriction non-core is invested in the overnight bucket. . . . .	64
5.7	Replicating portfolio without restriction non-core is invested in the overnight bucket. . . . .	65
5.8	Normally distributed deposit volume model parameters. . . . .	66
5.9	The change in EVE using the log-normally distributed deposit volume model.	67
5.10	The change in EVE using the normally distributed deposit volume model. .	67

# List of Figures

3.1	Division of NMDs in core and non-core deposits. . . . .	29
3.2	Overview of the core and non-core deposits. . . . .	30
4.1	Model design NMDs. . . . .	35
4.2	Historical data of the Euribor. . . . .	38
4.3	Historical data of swap rates. . . . .	39
4.4	Observed yield curves in June 2018, 2019, and 2020. . . . .	40
4.5	Historical deposit rates [3]. . . . .	43
4.6	Historical deposit volumes [4]. . . . .	46
5.1	First three principal components. . . . .	54
5.2	Two numerical solutions . . . . .	55
5.3	Simulated outcomes of the one-month Euribor rate over a ten-year period. .	57
5.4	Simulated outcomes of the one-month Euribor rate over a ten-year period. .	57
5.5	Actual time series of the deposit rates vs. fitted values. . . . .	59
5.6	Simulated outcomes of the deposit rate over a ten-year period. . . . .	59
5.7	Mean value of the deposit rate over a ten-year period. . . . .	60
5.8	Actual time series of the deposit volumes vs. fitted values. . . . .	61
5.9	Simulated outcomes of the deposit volume over a ten-year period. . . . .	62
5.10	Mean value of the deposit volume over a ten-year period. . . . .	62
5.11	Division of the deposit volume into a stable and non-stable part. . . . .	63

# Chapter 1

## Introduction

### 1.1 The research

This research represents our effort to gain more insight into the liquidity risk and interest rate risk faced by a bank. Liquidity risk is defined as the risk that someone is not able to meet his obligations as they come due. This plays an important role in the banking sector and is still difficult to measure because of the many different aspects of liquidity, which we discuss in the next sub-section. Each aspect of liquidity needs to be identified to eventually be able to measure liquidity risk. As a result, during this research, we elaborate on funding from non-maturing deposits (NMDs). NMDs have two characteristics; the bank is always allowed to adjust the deposit rate, and the customer is always allowed to withdraw his or her money. Because of the second characteristic of NMDs, the deposit volume could drop significantly in a short period of time, and this induces liquidity risk. Furthermore, interest rates influence the deposit rates and customers. As a result, significant changes in interest rates can cause changes in the deposit volume and the cash flows from NMDs. This is also known as interest rate risk. To be able to measure the expected cash outflow from NMDs, the proportion of core deposits need to be determined. The volume of NMDs can be divided into core and non-core deposits. Core deposits are deposits that are unlikely to change even under significant changes in the market. As a result, core deposits are a stable funding source for the bank and affect the expected cash outflow. A replicating portfolio assigns a fixed maturity profile to NMDs using standard trading instruments. And because core deposits are a stable funding source, a longer maturity is assigned to core deposits. The Economic Value of Equity (EVE) measures the difference between the net present value

of all cash inflows and outflows. If the proportion of core deposits increases then the cash outflows will probably decrease. This means that the proportion of core deposits affects the replicating portfolio, the EVE, and thus also the interest rate risk and liquidity risk of a bank. With this research, we hope to identify this effect.

## 1.2 Research motivation

The liquidity of banks has played and still plays an important role in the financial sector, especially during a financial crisis. As we have seen during the 2007-2009 credit crisis, bank failures occurred because of liquidity problems, for example, Lehman Brothers and Northern Rock [5]. Bank failures are costly for society and need to be prevented [6]. Therefore, liquidity problems in the banking sector must be identified and mitigated. After the credit crisis, the Basel Committee on Banking Supervision (BCBS) realized that regulations were needed to address liquidity risk [5].

The BCBS [7] defines liquidity as the ability of a bank to fund increases in assets and meet obligations as they come due. If a bank is not able to do this, it is exposed to liquidity risk. The BCBS introduced Basel III in 2010 to reduce liquidity mismatches, among other things [8]. Two liquidity risk measurements were introduced, namely the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). The LCR and NSFR measure liquidity in a different way. The LCR ensures that banks can meet their short-term obligations [9]. The NSFR ensures that banks maintain a stable funding profile concerning the composition of their off-balance-sheet assets and activities [10]. These measurements are used to determine the minimum requirements for banks and give a good start in regulating the liquidity risk of banks. However, there is no consensus on how to measure liquidity risk [11]. Measuring liquidity is hard because there are many different aspects of liquidity [12]. These aspects are ranging from a bank's overall funding liquidity, the market liquidity of its assets, to the reliance on short-term debt [12]. Each aspect of liquidity needs to be identified to eventually be able to measure liquidity risk. During this research, we are only focusing on funding from NMDs. The main reason for this decision is that NMDs are an important funding source for a bank. They are a significant portion of a bank's balance sheet [13] [14] that provides liquidity.

NMDs have two characteristics. First, a bank is allowed to adjust the deposit rate at any time as a matter of policy [14]. Second, customers are allowed to withdraw their money at any time. On the liability side of the balance sheet, NMDs include savings deposits and sight deposits<sup>1</sup>. On the asset side of the balance sheet, they include credit card loans and variable-rate mortgages. For simplicity, we restrict ourselves to the management of savings deposits. The second characteristic from NMDs, customers are allowed to withdraw their money at any time, implies that the funding source from NMDs can quickly disappear. This induces liquidity risk. One big reason why customers would withdraw their money is because of interest rates changes. When interest rates rise but deposit rates stay unchanged, the customer could withdraw their money from his savings deposit to invest it somewhere else to get a higher interest. This induces interest rate risk. As a result, it is important that a bank models the expected cash outflow from NMDs to be able to manage the liquidity risk and interest rate risk arising from these deposits. A bank must adhere to regulations when the expected cash outflow for NMDs is modelled. One of those regulations is that the core deposits need to be determined. According to the European Banking Authority (EBA), these core deposits are deposits that are stable and unlikely to reprice even under significant changes in interest rate environment, and/or other deposits whose limited elasticity to interest rate changes could be modeled by banks [1]. The determination of the ratio of core and non-core deposits is still not addressed extensively in the literature.

As mentioned before, a bank must manage his liquidity risk and interest rate risk arising from NMDs. There are various ways to do this. During this research, we are focusing on the replicating portfolio and the Economic Value of Equity (EVE). A replicating portfolio assigns a fixed maturity profile to NMDs using standard trading instruments. In this way, uncertain cash flows are transformed into certain ones. Consequently, one can forecast the cash flows which are required to manage liquidity risk [15] and interest rate risk. The treatment of balances from NMDs influences the replicating portfolio. The proportion of core and non-core deposits influences the replicating portfolio because non-core deposits have to be invested in the overnight bucket and serve as protection against volume fluctuations. Consequently, a longer maturity is assigned to core deposits because core deposits are a

---

<sup>1</sup>Sight deposits are deposits from which customers can withdraw money from at any time, or after a very short notice period. They can be seen as private accounts or business accounts. Sight deposits are primarily used as a means of payment. Unlike savings deposits, which are used as savings vehicles.

stable funding source.

Furthermore, the treatment of balances from NMDs also influences the EVE [2]. The EVE is namely a cash flow calculation that takes the present value of all liability cash flows and subtracts that from the present value of all asset cash flows [2]. The EVE analyzes long-term interest rate risk by using stress-tests and shows what could happen in a period of interest rate volatility causing significant deposit withdrawals and loss of earnings [2]. This, of course, is related to the liquidity risk of a bank.

In short, there is not much research about the determination of the proportion of core and non-core deposits. Besides that, the magnitude of the impact of the proportion of core and non-core deposits on the risk management of NMDs is unclear. As a result, the problem is that there is not a framework for the determination of the proportion of core and non-core deposits, and it is unclear which effect it has on the risk management of NMDs. As a result, during this research, we examine how the core and non-core deposits can be determined. Furthermore, we also examine how the proportion of core and non-core deposits affect the replicating portfolio and the EVE.

### **1.3 Company description**

This research is executed at KPMG. KPMG is a global network of independent member firms. It offers audit, tax, and advisory services. Our research is executed at the department of Financial Risk Management (FRM), which is part of advisory. The FRM team offers banks, pensions, insurance companies, and asset managers advice about how to mitigate risks and create opportunities. This research helps KPMG, specifically the FRM team, to gain knowledge about modelling NMDs and the identification of liquidity risk and interest rate risk a bank faces. Consequently, this knowledge can help KPMG with projects about the risk management of NMDs in a bank.

### **1.4 Research questions**

The objective of this research is to gain more insight into modelling NMDs and to help to identify the liquidity risk and interest rate risk a bank faces. To do this, we want to

clarify which influence core and non-core deposits have on the risk management of NMDs. With this information, we have come up with the following main research question: “*Which influence does the proportion of core and non-core deposits have on the risk management of non-maturing deposits of a bank?*” To answer the main research question; we need certain knowledge, to perform certain activities, and make certain decisions [16]. We discuss this further in Section 1.5. The sub-questions that need to be answered are listed below.

1. How are non-maturing deposits modelled?
  - (a) Which market rate models are there in the literature?
  - (b) Which deposit rate models are there in the literature?
  - (c) Which deposit volume models are there in the literature?

To be able to model NMDs, the development of a market rate model, deposit rate model, and deposit volume model is necessary. Therefore, we provide an overview of the various market rates models, deposit rates models, and deposit volumes models and their advantages and disadvantages.

2. How to construct a replicating portfolio?
  - (a) Which replicating portfolio approaches are there in the literature?

One subject of the risk management of NMDs is to replicate a portfolio to be able to calculate the expected cash outflows of NMDs. To be able to know which effect core and non-core deposits have on the outcome of the replicating portfolio, we need to know how a replicating portfolio can be constructed. In the banking sector, a static replicating portfolio approach or dynamic replicating portfolio approach is often used. As a result, we provide an overview of these approaches, discuss the differences, their deficiencies, and good qualities.

3. How to measure the Economic Value of Equity?

Another subject of the risk management of NMDs is the calculation of the EVE. As a result, to be able to know which effect core and non-core deposits have on the EVE we need to know how to measure the EVE.

4. How can a framework be built to determine which non-maturing deposits are stable and unlikely to reprice even under significant changes in interest rate environment?
  - (a) Which different approaches are there in the literature for measuring interest rate sensitivity?
  - (b) How to reprice a non-maturing deposit?
  - (c) Which data from customers is required?

As mentioned before, the core deposits are defined as NMDs that are stable and unlikely to reprice even under significant changes in interest rate environment. By answering this question, we explain how core and non-core deposits can be determined. To be able to do this we have to give an overview of how interest rate sensitivity can be measured, how NMDs are repriced, and which data from the customer is needed.

5. How can the proposed framework for determining core and non-core deposits be applied to non-maturing deposits models?

After answering sub-question one and four, we can answer sub-question five. We explain our working method, and we set out our methodology. As a result, we have an NMD model in which core deposits are determined.

6. How to use the proposed non-maturing deposits model to construct a replicating portfolio?
7. How to use the proposed non-maturing deposits model to calculate the Economic Value of Equity?

By answering the last two sub-questions, we come back to answer our main research question. After constructing the replicating portfolio and calculating the EVE, we can see which impact the proportion of core and non-core deposits have on the outcome of the replicating portfolio and EVE.

## 1.5 Research design

In this section, we discuss the framework of research methods and techniques chosen to perform our research. There are various types of research designs and our study can be classified as quantitative research. Based on numerical data, we want to gather insights about the influence of core deposits on the risk management of NMDs. As a result, this research could also be seen as correlational research because we want to establish the relationship between the proportion of core deposits and the risk management of NMDs.

During our research, we are first going to gather knowledge. This is done by performing a literature study to answer sub-question one, two, and three. After we have collected all the knowledge needed, we develop a framework for determining the proportion of core and non-core deposits, i.e. answering sub-question four. For the development of the framework, it is important to take into account that the framework is not too large because then the process may become long and expensive. Furthermore, we are elaborating on already existing solutions to develop the framework. We are, for example, studying which solutions the literature gives for measuring interest rate sensitivity. The developed framework is combined with our NMD model, i.e. answering sub-question five. The gathering of data is done by elaborating on public databases. These databases are tested based on their reliability to ensure that we gather reliable data. With the gathered data the parameters of our NMD model are calibrated and validated. The method for calibration and validation is discussed in Section 4. Lastly, we measure the influence of the proportion of core and non-core deposits on the outcome of the replicating portfolio and the EVE. Thereby we answer our main research question.

## 1.6 Thesis outline

This thesis is organized as follows: In Section 2, we perform the literature study which is mentioned in the previous sub-section. Following, in Section 3, we propose our framework for determining the proportion of core and non-core deposits. Section 4 explains our working method and the application to savings deposits. In Section 5, we discuss the results of our research. The last sections include a conclusion and discussion in which the main research question is answered. Furthermore, further research is suggested.

## Chapter 2

# Modelling Non-Maturing Deposits

In this chapter, we perform a literature review to get a theoretical background in modelling NMDs. We discuss the implications of NMDs in Section 2.1. Furthermore, to be able to model NMDs, a market rate model, deposit rate model, and deposit volume model have to be developed. The existing models in the literature and their advantages and disadvantages are discussed in Section 2.2, 2.3, and 2.4. Consequently, we discuss the deficiencies and good qualities of static and dynamic replicating portfolio approaches in Section 2.5. Finally, we discuss how to measure the Economic Value of Equity in Section 2.6.

### 2.1 Implications of Non-Maturing Deposits

The management of interest rate risk and liquidity risk for NMDs is difficult. However, it is also from great importance [17] [15]. According to the BCBS [2], NMDs can be an important determining factor for the exposure of interest rate risk in the banking book (IRRBB) under the economic value. Moreover, managing the exposure to IRRBB is important because IRRBB plays a central role in the regulations of Basel II [13].

Furthermore, there is no contractual maturity on NMDs [18]. A customer can withdraw his money from his savings deposit rapidly and without having to incur any costs. Besides that, customers react to changes in the market. For example, customers react to the rising or falling of interest rates or the attractiveness of an alternative investment [13]. As a consequence, the timing and amount of future cash flows are uncertain [15], and the volume of NMDs may fluctuate heavily [13]. This makes the management of interest rate risk and

liquidity risk difficult and induces volume risk. Volume risk, in this case, is the uncertainty of the quantity of the funding source from NMDs. The volume risk cannot be hedged directly because the volume is not traded in the market. Therefore, conventional hedging techniques can only be used with additional assumptions about the cash flows. Uncertain cash flows need to be transformed into certain ones. In this way, one can forecast the cash flows which are required to manage liquidity risk [15]. The transformation of uncertain cash flows into certain ones can be achieved by assigning a fixed maturity profile to the NMDs, i.e. constructing a replicating portfolio using standard trading instruments. The construction of the replicating portfolio has to be done correctly. Otherwise, it could result in inefficient hedging positions [13]. After constructing the replicating portfolio, the bank can mitigate the exposure to interest rate risk. There are different approaches to construct such a portfolio. In Section 2.5, an overview is given of various replicating portfolio approaches.

The current low interest rates are also an implication for the management of NMDs. The low interest rates affect the profitability of a bank and its risk-taking [19]. To be able to maintain a competitive position, banks are facing a smaller margin. If banks would charge customers with a negative deposit rate, it could experience a contraction in their deposit volumes because customers can switch to a bank that offers a positive deposit rate [13]. However, many banks already charge a negative deposit rate for customers with a high financial capacity. The research of Weistroffer [20] showed that banks are indeed changing their business models as a result of the low interest rates. Manganelli & Wolswijk [21] found that banks are less risk-averse because of the lower interest rate, i.e. a bank is willing to take more risk for a possible higher return. As a result, a bank has to make a trade-off between risk and return. Proper management of interest rate risk and liquidity risk arising from NMDs gives a clear overview of the possible level of the margin and helps the bank to make the trade-off between risk and return.

## 2.2 Modelling market rates

Market rates are important variables for modelling deposit rates and deposit volumes. It can form a framework for valuing the cash flows of NMDs that are specified by deposit rates and deposit volumes [17]. Furthermore, market rates can be used to simulate short rate

paths by generating yield curve scenarios and can be fitted to the prices of plain vanilla instruments, e.g. a zero-coupon bond. According to Kalkbrenner & Willing [17], the market rates should not be exactly fitted to the current prices of the plain vanilla instruments. There should be a focus on the realistic development of the market rates over a long period of time. As a result, Kalkbrenner & Willing [17] use historical time-series for fitting the market rates to plain vanilla instruments.

There are various possibilities to model market rates. Frauendorfer & Schürle [13] use the two-factor Vasicek model for modelling market rates. In their earlier work, they found that this model gives better results than the one-factor Vasicek model. The one-factor Vasicek model has several shortcomings. One shortcoming is that this model assumes that the interest rates are perfectly correlated between different maturities [5]. The two-Factor Vasicek model overcomes this problem.

The Vasicek model incorporates mean reversion [22]. Mean reversion means that the market rate will move to its mean over time. Moreover, this method is easy to understand and widely used. With the one-factor Vasicek model, the market rate follows the following stochastic differential equation that is a risk-neutral process:

$$dr_t = a(b - r_t)dt + \sigma dW_t \quad (2.1)$$

Where the parameters  $a$ ,  $b$ , and  $\sigma$  are nonnegative constants, the short rate,  $r_t$ , is pulled to a level  $b$  at rate  $a$ , and  $W_t$  is a wiener process. According to [22], the price of a zero-coupon bond is given by:

$$P(t, T) = A(t, T)e^{-B(t, T)r_t} \quad (2.2)$$

Where

$$A(t, T) = \exp\left(\frac{(B(t, T) - T + t)(ab - \frac{\sigma^2}{2})}{a^2} - \frac{\sigma^2 B(t, T)^2}{4a}\right) \quad (2.3)$$

and

$$B(t, T) = \frac{1 - e^{-a(T-t)}}{a} \quad (2.4)$$

For the two-factor Vasicek model, the factors are described by the following stochastic differential equations [13]:

$$\begin{aligned} dr_{1t} &= a_1(b - r_{1t})dt + \sigma_1 dW_{1t} \\ dr_{2t} &= -a_2 r_{2t}dt + \sigma_2 dW_{2t} \end{aligned} \quad (2.5)$$

Where, the parameters  $a_1$ ,  $a_2$ ,  $b$ ,  $\sigma_1$ , and  $\sigma_2$  are non-negative constants. Furthermore,  $W_{1t}$  and  $W_{2t}$  are Wiener processes. It is assumed that the two Wiener processes are uncorrelated. The short rate is given by:

$$r_t = r_{1t} + r_{2t} \quad (2.6)$$

According to [13], the price of a zero-coupon bond is given by:

$$P(t, T; r_{1t}, r_{2t}) = \sum_{i=1}^2 A_i(t, T) e^{-B_i(t, T)r_{it}} \quad (2.7)$$

Where

$$A_i(t, T) = \exp\left(\frac{(B_i(t, T) - T + t)(a_i b - \frac{\sigma_i^2}{2})}{a_i^2} - \frac{\sigma_i^2 B_i^2(t, T)}{4a_i}\right) \quad (2.8)$$

and

$$B_i(t, T) = \frac{1 - e^{a_i(T-t)}}{a_i} \quad (2.9)$$

Kalkbrener & Willing [17] base the calibration of the two-factor Vasicek model on a principal component analysis (PCA).

Besides Vasicek's model, Cox, Ingersoll, and Ross proposed the CIR model. O'Brien [23] uses the one-factor CIR model to model market rates. The difference between Vasicek's model and the CIR model is that under the CIR model the standard deviation of the change in the short rate in a short period of time is proportional to  $\sqrt{r_t}$  [5]. As a result, if the short-term interest rate increases, the standard deviation increases. However, under Vasicek's model, the short rate may become negative. Simulating negative short rates under the CIR model is not possible. This is a disadvantage of the CIR model because we have

seen that interest rates can become negative. Therefore, we are not using the CIR model to model market rates.

Kalkbrener & Willing [17] use the non-parametric model, i.e. a Gaussian Heath-Jarrow-Morton (HJM) model with piecewise constant volatility functions. They found better calibration results with this model than with the two-factor Vasicek model. Kalkbrener & Willing [17] base the calibration of the non-parametric HJM model on a PCA.

In conclusion, the main goal of modelling market rates is to produce realistic yield curves. However, using a stochastic yield curve model, for example, as mentioned above, the one-factor/two-factor Vasicek model, the non-parametric model, or the CIR model, to obtain a sample set of yield curves has its limitations. The obtained set of yield curves by using a stochastic model is often limited by the structure of the model. Besides that, such a set of yield curves obey no-arbitrage conditions.

The models mentioned above are based on a risk-neutral world<sup>2</sup>. If we want to measure unhedged risk, it is important to predict the true expectations about the uncertainty surrounding the future evolution of interest rates [24]. As a result, a real-world model will provide better results than a risk-neutral model. Consequently, for predicting market rates, we want to use a real-world model for modelling market rates. As an alternative to stochastic models for obtaining a sample set of yield curves, PCA can also be applied to a set of historical yield curves, i.e. a real-world model.

As a result, during this research, we are using PCA in combination with a historical sample set to model market rates. We discuss this further in Section 4.1.

## 2.3 Modelling deposit rates

A characteristic of NMDs is that banks are allowed to adjust the deposit rates at any time. However, there are some reasons why the bank should align its deposit rates to other banks. One, if the bank offers a lower deposit rate than other banks, the bank could experience

---

<sup>2</sup>A model is calibrated to risk-neutral probabilities if the parameters can be inferred from traded security prices and if there is some anti-arbitrage assumption. In this case, a model is based on a risk-neutral world.

a contraction in their deposit volumes because customers will switch to a bank that offers a higher deposit rate. Two, if the bank offers a higher deposit rate than other banks, the bank will experience a lower margin than other banks. This makes it difficult to compete with other banks.

In the literature, it is a consensus that banks adjust the deposit rates rapidly if the market rates are decreasing. However, when the market rates are rising, banks react slower to adjust the deposit rate because a bank is less willing to pay higher interest rates. As a result, there is an asymmetry in the adjustment of the deposit rates. Paraschiv & Schürle [15] found that the greater the disequilibrium between the deposit rate and market rate, the greater the speed of adjustment of the deposit rate. Furthermore, deposit rates are often given discretely than continuously because the deposit rates are generally not changed every day. As a result, they are little affected by small changes in the market. However, they are heavily influenced by market rates [17].

The rates of different kinds of deposits differ significantly because of the various sensitivities to interest rate changes [17]. As a result, Kalkbrenner & Willing [17] suggest that deposit rates should be modelled by using a deterministic function with only the market rates as stochastic arguments. An example of such a model is that of Jarrow & van Deventer [25]. The model in discrete time is given as:

$$d_t = d_0 + \beta_0 t + \beta_1 \sum_{j=0}^{t-1} r_{t-j} + \beta_2 (r_t - r_0) \quad (2.10)$$

The model in continuous time is given as:

$$d_t = d_0 + \beta_0 t + \beta_1 \int_0^t r_s \delta s + \beta_2 (r_t - r_0) \quad (2.11)$$

Where  $d_t$  is the deposit rate at time  $t$ ,  $r_t$  is the short rate at time  $t$ , and  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  are parameters that have to be estimated.

There are also more simplistic models, e.g. linear models. Nyström [26] proposes a framework for modelling non-maturing deposits. In this framework, the deposit rates need to be

modelled by using a function of the market rate or deposit volume. As a result, he proposes to use the following function to model the deposit rates:

$$d_t = \beta_1 r_t \quad (2.12)$$

Where  $d_t$  is the deposit rate,  $r_t$  is a short market rate, and  $\beta_1$  is a constant that should be fitted. Notice that with the current negative market rates, the deposit rates will also be negative. As already mentioned in Section 2.1, in reality, if the deposit rates are negative, the bank could experience a contraction in their deposit volumes because customers will switch to a bank that offers a positive deposit rate. As a result, this model is not realistic in the current market. However, it is not unrealistic that in the future the deposit rate will become negative if the market rates stay negative or even further declines. For some deposits, there already is a negative deposit rate. For example, the Rabobank offers a deposit rate of -0.50 percent if the customer has more than one million euros in his deposit [27]. However, at this moment there are still positive deposit rates while the market rates are negative. Consequently, this model is not the right fit at this moment, and we are not going to use it.

Elkenbracht & Nauta [28] similarly do not explicitly give a model for deposit rates. However, they also use a linear model, namely the following function:

$$d_t = \beta_0 + \beta_1 r_t \quad (2.13)$$

Where  $d_t$  is the deposit rate,  $r_t$  is a short market rate, and  $\beta_0$  and  $\beta_1$  are constants that should be fitted.

Instead of using a linear model, O'Brien [23] uses an asymmetric partial adjustment model for modelling deposit rates because of the asymmetry in the adjustment of the deposit rates mentioned at the beginning of this section. The asymmetric partial adjustment model is based on auto-regressive processes. The one-period change in the deposit rate is modelled as follows:

$$\Delta d_t = (\lambda^+ I_t + \lambda^-(1 - I_t))(d_t^e - d_{t-1}) + \xi_t \quad (2.14)$$

Or

$$d_t = d_{t-1} + (\lambda^+ I_t + \lambda^-(1 - I_t))(d_t^e - d_{t-1}) + \xi_t \quad (2.15)$$

Where

$$d_t^e = br_t - g \quad (2.16)$$

$$I_t = I_{(d_t^e - d_{t-1} > 0)} \quad (2.17)$$

Here,  $r_t$  is the market rate<sup>3</sup>,  $d_t$  is the deposit rate, and  $\xi_t$  is assumed to be an i.i.d. zero-mean process.  $I_t$  is an indicator function, i.e. if  $d_t^e - d_{t-1}$  is bigger than zero then  $I_t$  is one, otherwise, it is zero. The parameter  $\lambda^+$  represents the upward adjustment of the deposit rate and  $\lambda^-$  the downward adjustment. As a result, if  $\lambda^+ < \lambda^-$ , then the upward adjustment is slower than the downward adjustment. In this model, the current value is based on the immediately preceding value.

During this research, we are also going to use the asymmetric partial adjustment model for modelling deposit rates. We discuss this further in Section 4.2.

## 2.4 Modelling deposit volume

Kalkbrener & Willing [17] found that market rates and deposit volumes do not have a high correlation. Therefore, they introduced an additional stochastic factor to model deposit volumes. They use a normally distributed model and a log-normally distributed model. The normally distributed model is the sum of a deterministic linear function  $f(t)$  and an Ornstein-Uhlenbeck process  $X(t)$ . As a result, the deposit volume is defined as follows:

$$v_t = f(t) + X(t) \quad (2.18)$$

Where

$$f(t) = a + b * t \quad (2.19)$$

---

<sup>3</sup>O'Brien [23] uses the month-end 3-month Treasury bill yield.

And

$$dX(t) = \mu_V X(t)dt + \sigma_V dW(t) \quad (2.20)$$

Where  $v_t$  is the deposit volume at time  $t$ ,  $\mu_V$  and  $\sigma_V$  are constants and  $W(t)$  is a Brownian motion. The deterministic linear function  $f(t)$  identifies the trend of the deposit volume. The Ornstein-Uhlenbeck process incorporates mean reversion. As a result, the deposit volume fluctuates around the trend with mean reversion  $\mu_V$  and volatility  $\sigma_V$ .

Kalkbrenner & Willing [17] mention that the deposit volumes may become negative with a normally distributed model. This is important to take into account. On the contrary, with the log-normally distributed model the deposit volumes never become negative. The log-normally distributed model is similar to the normal model. The difference is that the deposit volume is defined as follows:

$$\ln v_t = f(t) + X(t) \quad (2.21)$$

Or

$$v_t = e^{f(t)+X(t)} \quad (2.22)$$

Frauendorfer & Schürle [13] use a (log-) normally distributed diffusion model to model deposit volumes. The relative changes in the deposit volume are modelled as follows:

$$\ln v_t = \ln v_{t-1} + e_0 + e_1 t + e_2 r_{1t} + e_3 r_{2t} + \xi_t \quad (2.23)$$

Where  $v_t$  is the deposit volume at time  $t$ ,  $r_{1t}$  and  $r_{2t}$  are market rates (a short rate and a long rate),  $e_1$ ,  $e_2$  and  $e_3$  are constants and  $\xi_t$  is an additional stochastic factor uncorrelated with the market rates. According to Frauendorfer & Schürle [13], the additional stochastic factor takes into account the fact that the deposit volume is not fully explained by market rates. After modelling deposit volumes, Frauendorfer & Schürle [13] compute the values and sensitivities of the NMDs by applying Monte Carlo simulation. By shifting the yield curve and parts of the yield curve, they obtained the delta profile. They use this profile to construct the replicating portfolio.

Both Frauendorfer & Schürle [13] and Kalkbrenner & Willing [17] do not include deposit rates in their model because there is not a high correlation between deposit rates and deposit volumes. However, Paraschiv & Schürle [15] do incorporate deposit rates. The deposit volume is modelled as follows:

$$v_t = e_1 + e_2 t + e_3 v_{t-1} + e_4 [R_{t-3} - (\delta r_{t-3}^{short} + (1 - \delta) r_{t-3}^{long})] + \xi_t \quad (2.24)$$

Where  $r_t^{short}$  and  $r_t^{long}$  are market rates at time  $t$ ,  $R_t$  the deposit rate at time  $t$ , and  $\xi_t$  a stochastic factor. The fourth explanatory variable is the spread between the market rates and the deposit rates at time  $t - 3$  because the depositors will probably not react immediately to changes in the deposit rates and market rates.

Nyström [26] uses behavioral models to model deposit volumes. He adds a component that takes into account the possibility that one customer has multiple deposits at the bank. It is common sense that the customer will transfer its money to the deposit which gives the highest return. Nyström [26] did not incorporate components that take into account the possibility of the arrival of new customers or that current customers close their deposits. Maes & Timmermans [29] mention that both market conditions and personal events could cause customers to withdraw a part or all of their money. Personal events are, for example, divorce, relocation, or death. According to Maes & Timmermans [29], these events are diversifiable across all customers. As a result, deposit volumes can be modelled with only general market conditions.

During our research, we are using a log-normally distributed model, based on the model of Frauendorfer & Schürle [13] and Paraschiv & Schürle [15], to model deposit volumes. We discuss this further in Section 4.3.

## 2.5 Replicating portfolios

As mentioned in Section 2.1, a replicating portfolio is constructed to be able to assign a fixed maturity profile to the NMDs using standard trading instruments. There are different approaches besides constructing a replicating portfolio for the risk management of savings deposits. However, because of time restriction, we focus on the replicating portfolio

approach, a method used by most banks in the Netherlands for the risk management of savings deposits [19]. Besides that, the BCBS [2] claims that a replicating portfolio is a common technique for achieving a constant maturity profile of matching assets that produces a moving average fixed return in line with the risk appetite of the bank.

There are two different replicating portfolio approaches. The static replicating portfolio approach and the dynamic replicating portfolio approach. Both approaches require the knowledge of the historical development of market rates used to create deposit rates, deposit volumes, and yield curves. To create a replicating portfolio, a bank needs to model the market rates, deposit rates, and deposit volume, which is discussed in Section 2.2, 2.3, and 2.4, respectively.

It is important to mention that both static and dynamic replication have advantages and deficiencies. A common advantage of the two replicating portfolio approaches is that the transaction costs remain low because the standard trading instruments are held until maturity to avoid rebalancing [18]. A common deficiency is that the bank only reinvests its funds into zero-coupon instruments with known maturities, which is a necessary assumption for the optimization exercise. See for example the models from Kalkbrener & Willing [17], Maes & Timmermans [29], Frauendorfer & Schürle [13], and Dewachter et al. [30]. In reality, a bank also reinvests its funds into mortgages and loans. We mention other not common advantages and deficiencies of the replicating portfolio approaches in the following sub-sections.

### 2.5.1 Static replicating portfolio approach

We notice that the most used approach in the banking industry is the static replicating portfolio approach. The composition of a static replicating portfolio is constructed by analyzing historical data of the NMDs. Then the portfolio has to meet the condition that the cash flows of the standard trading instruments have to match those of the NMDs as close as possible [13]. The weights of the portfolio can be determined in several ways. For example, maximizing the margin, or minimizing the variance of the margin [30]. These methods result in a portfolio where the risk is minimized. However, Feilitzen [31] suggests maximizing the Sharpe ratio to determine the weights of the portfolio. She found that

maximizing the Sharpe ratio results in a portfolio where the average margin is significantly higher than using different methods to determine the weights of the portfolio. The drawback of maximizing the Sharpe ratio is that it increases the variance of the margin. However, this increase is very small and thus not significant. The eventually resulting portfolio weights remain constant over time [32]. Therefore, this approach can be characterized as a static approach.

In the literature, it is consensus that static replication has many deficiencies. Frauendorfer & Schürle [13] found that the weights depend heavily on the sample period. Therefore, the margins are unstable, this implies a substantial model risk. Maes & Timmermans [29] and Feilitzen [31] argue that static replication provides ambiguous results. Furthermore, the static replicating portfolio approach is backward-looking [19]. It uses one single optimization based on the last 5 to 10 years of data, without considering the possibility that market rates, deposit rates, and deposit volumes are moving differently in the future. Despite these deficiencies, the advantage of static replication is its transparency, and it often provides a decent fit [33]. Bardenhewer [34] states that if the complexity of the model is high, the probability of failures rises. Consequently, a good model should also be transparent.

There are different ways to construct a static replicating portfolio. First, to renew maturing instruments at the maturity. If a volume change occurs, tranches of the specified instruments are sold, or additional tranches are bought. Second, to determine a core balance and a volatile component. This approach is used by many banks. The determination of the core is often done in a rather arbitrary way without theoretical justification [17]. When the core is determined it is subdivided into components and invested in various time bands [17]. The volatile component is invested in the overnight bucket and it serves as protection against volume fluctuations [13]. An example from the latter is the static replicating portfolio approach from Maes & Timmermans [29]. The approach replicates the dynamics and characteristics of deposit balances. The total deposit volume is divided into three parts, namely the core deposits, volatile deposits, and remaining balance. The remaining balance is the difference between the core and the volatile part. It is unclear how Maes & Timmermans [29] determine the ratio of the different parts. The core is invested into a long-term asset, the volatile deposits into a risk-free short-horizon asset, and the remaining balance is replicated with the portfolio. To determine the weights of the portfolio, Maes &

Timmermans [29] minimize the standard deviation of the margin.

The static replicating portfolio with a moving average by Bardenhewer [34] is similar to the static replicating portfolio of Maes & Timmermans [29]. The weights of the portfolio are also determined by minimizing the standard deviation of the margin. The difference is that the total deposit volume is divided into an expected trend component and an unexpected trend component. The expected trend component is estimated from historical data with either a linear, quadratic, or exponential model. Furthermore, Bardenhewer [34] measures the moving average return of every asset. The moving average is used to measure the return of the replicating portfolio and consequently the standard deviation of the margin.

### 2.5.2 Dynamic replicating portfolio approach

The dynamic replicating portfolio approach includes simulations. These simulations obtain an average margin for several scenarios of the market rate. Each market rate results in different deposit rates and deposit volumes. As a result, each market rate scenario results in a different reinvestment, i.e. deposits are reinvested with different weights. That makes this approach dynamic.

Dynamic replication aims to actively react to changes in the market environment. However, the question is if dynamic replication provides a better result than static replication. The results from the research of Frauendorfer & Schürle [18] and the research of Cambou & Filipovic [35] show that a dynamic replicating portfolio approach is superior to static replication. The margin is bigger, and the variance of the margin is smaller. Moreover, Feilitzen [31] claims that the results of the dynamic replicating portfolio are more reliable because the portfolio weights are reevaluated at regular intervals. Furthermore, a lot of the shortcomings of a static replicating portfolio approach can be overcome with a dynamic approach. Specifically, the deficiency that the static replicating portfolio approach is only looking backward can be overcome by using multistage stochastic programming techniques such as dynamic replication [19]. However, Derman & Taleb [36] claim that financial valuation does not require the complexity of dynamic replication. Similar results can often be obtained with a more simplistic model. Besides that, Pelsser [37] thinks that dynamic replication can be difficult in practice, particularly over a long period of time. Subsequently,

we discuss an example of a dynamic replication portfolio approach.

Frauendorfer & Schürle [13] suggest a multistage stochastic programming model. As mentioned in Section 2.2.1, the composition of a static replicating portfolio is constructed by analyzing historical data of the NMDs. However, for dynamic replication, the construction is based on each scenario for future market rates, deposit rates, and deposit volumes at all points in time. The objective of the model is minimizing the expected downside deviation of not meeting the overall costs. Frauendorfer & Schürle [13] state that minimizing the standard deviation of the margin may result in a portfolio that is not able to cover the overall costs, and therefore, it does not minimize the risk. In conclusion, using the objective to minimize the expected downside deviation of not meeting the overall costs is according to Frauendorfer & Schürle [13] better.

Henningsson & Skoglund [38] suggest using a net present value Monte Carlo simulation model. The replicating portfolio exists out of market securities. These securities replicate the net present cash flows of the NMDs. Because the cash flows of the market securities are taking place on different maturity dates than the cash flows of the NMDs, Henningsson & Skoglund [38] create time buckets in which the cash flows of the NMDs are placed.

During our research, we use a dynamic replicating portfolio approach, namely a net present value Monte Carlo simulation model. We discuss this further in Section 4.4.

## 2.6 Measuring the Economic Value of Equity

As we have mentioned in Section 1.2, the EVE is a cash flow calculation that takes the present value of all liability cash flows and subtracts that from the present value of all asset cash flows [2]. The EVE analyzes long-term interest rate risk by using stress-tests and shows what could happen in a period of interest rate volatility causing significant deposit withdrawals and loss of earnings [2]. As a result, a bank must measure the EVE. In this section, we explain how the EVE can be measured.

The interest rate risk can be assessed by measuring the Economic Value at Risk (EVaR) for a given confidence interval [1]. The EVaR measures the maximum equity value change.

To be able to measure the EVaR, a distribution of the change in EVE has to be made using historical simulations or Monte Carlo simulations. However, the BCBS [2] mentions that the EVaR based on historical simulations does not adequately assess tail risk, and calculating the EVaR using Monte Carlo simulations demands a lot of computational power and technology. The interest rate risk can also be assessed by the change in EVE for specific interest rate scenarios. The measurement of the total Economic Value (EV) is often complex. Therefore, banks calculate the (change in) EVE often instead. The change in EVE is the change in the net present value of all cash flows originating from NMDs, among other things, resulting from a change in interest rates, assuming that all banking book positions run-off [1]. According to the EBA [1], banks are obliged to calculate the impact on their EVE, i.e. the change in EVE, at least quarterly. The EBA [1] considers six standard interest rate shock scenarios that need to be considered to measure the change in EVE:

1. Parallel shock up.
2. Parallel shock down.
3. Short rates shock down, and long rates shock up.
4. Short rates shock up, and long rates shock down.
5. Shock rates shock up; and
6. Short rates shock down.

The size for the six interest rate shock scenarios is based on historical interest rates, see Table 2.1. The institution should self-consider which other interest rate shock scenarios are important to take into account. Besides that, the calculation of the change in EVE should also be based on historical and hypothetical interest rate stress scenarios.

Table 2.1: Specified size of interest rate shocks [1].

Shock	Basis points
Parallel	+/- 200
Short rates	+/- 250
Long rates	+/- 100

According to the EBA [1], banks should apply an appropriate risk-free yield curve when the change in EVE is measured. For example, swap rate curves. We have to take this into

account when we collect historical market rates.

To calculate the EVE and the change in EVE, the bank can use the standardized method or internal methods [2]. With the standardized EVE risk measure, all cash flows should be determined per interest rate shock scenario and the current interest rate term structure. According to EBA [1], the cash flows should be modelled partially or fully conditional on the interest rate scenario. In this way, the bank captures not only gap risk<sup>4</sup>, but also basis risk<sup>5</sup>. If all cash flows are modelled fully on the interest rate scenario, the bank is also capturing option risk<sup>6</sup>. During our research, we are modelling the cash flows partially conditional on the interest rate scenario. In this way, we can capture gap risk and basis risk.

During our research, we use the standardized EVE risk measure. We discuss this further in Section 4.5.

---

<sup>4</sup>Gap risk is the risk that arises from the timing of interest rate changes from banking book instruments [2].

<sup>5</sup>Basis risk is the risk of relative changes in interest rates for financial instruments. These financial instruments have similar tenors. However, they are priced using different interest rate indices. [2]

<sup>6</sup>Option risk arises from positions in option derivatives or from optional elements placed in the assets, liabilities, or off-balance sheet items of a bank, where the customer or the bank is allowed to change the timing and level of the cash flows. [2]

## Chapter 3

# Framework for determining the core and non-core deposits

According to the guidelines from the BCBS [2] and EBA [1], banks should determine the appropriate assumptions for its NMDs. Consequently, a bank should divide the NMDs into a core and non-core by analyzing its depositor base. As we already mentioned, core deposits are defined as stable deposits and deposits that are unlikely to reprice even under significant changes in interest rate environment, and/or other deposits whose limited elasticity to interest rate changes could be modelled by banks. In this section, a framework is developed that determines the proportion of core and non-core deposits.

There are certain guidelines regarding the determination of core deposits. First, a bank needs to divide NMDs into stable deposits and non-stable deposits, see Figure 3.1. Stable deposits are defined as deposits with a high probability that it stays undrawn. The non-stable deposits are then defined as non-core deposits. For the stable deposits, the bank needs to divide them into core and non-core deposits. To make this division the bank needs to determine which deposits are unlikely to reprice even under significant changes in interest rate environment, i.e. core deposits.

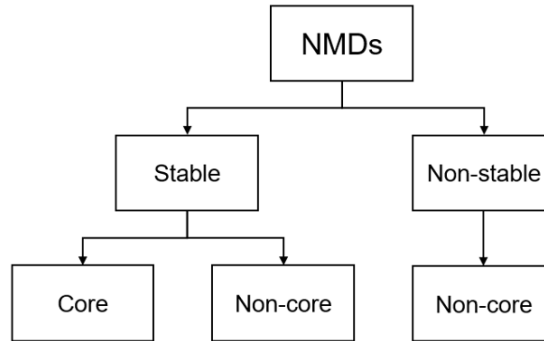


Figure 3.1: Division of NMDs in core and non-core deposits.

With the determination of the core, the bank needs to take into account that there is a cap on the proportion of core deposits, see Table 3.1. There is a division between wholesale, retail/non-transactional, and retail/transactional. Savings deposits are covered by retail/non-transactional deposits. As a result, we have to take into account that the cap on the proportion of core deposits is 70 percent. Furthermore, there is also a cap on the average maturity of core deposits. For savings deposits, the cap on the average maturity of core deposits is 4.5 years. This is important to take into account when we construct a replicating portfolio.

Table 3.1: Caps on core deposits and average maturity per category [2]

	Cap on the proportion of core deposits	Cap on the average maturity of core deposits
Wholesale	50 percent	4 years
Retail/non-transactional	70 percent	4.5 years
Retail/transactional	90 percent	5 years

The EVE needs to be calculated at least quarterly according to the guidelines of the BCBS [2]. Because we expect that the proportion of core deposits affect the EVE, we think that the proportion of core and non-core deposits also have to be determined quarterly.

### 3.1 Determining stable deposits

Stable deposits stay undrawn with a high degree of likelihood. To determine the proportion of stable deposits, we use the observed volume changes over the last ten years. This is according to the guidelines of the BCBS [2]. See Figure 3.2 for a clear overview of the

division of the deposit volume into stable and non-stable deposits. We explain our method for determining the proportion of stable deposits in Section 3.1.1.

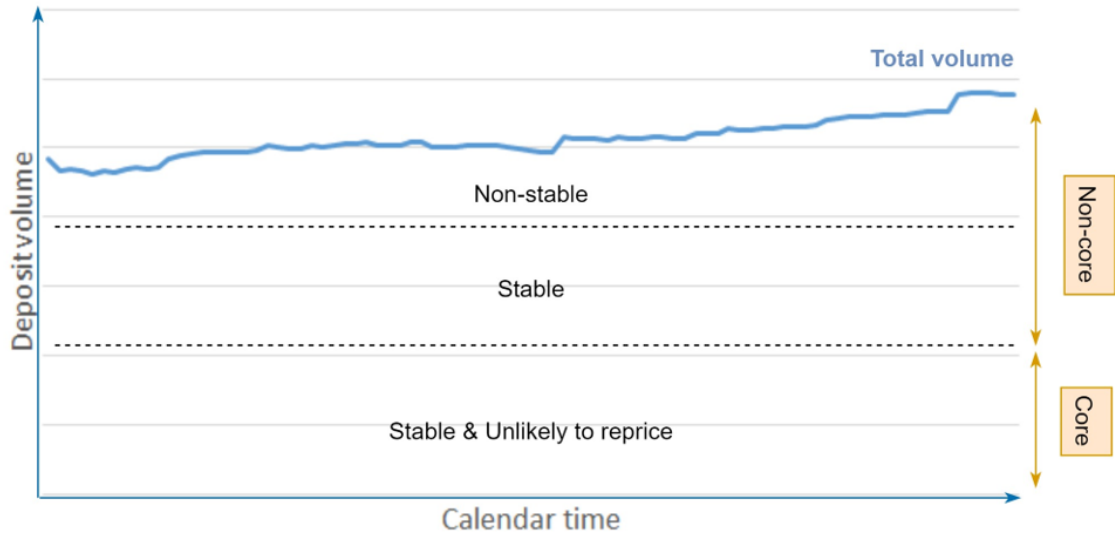


Figure 3.2: Overview of the core and non-core deposits.

### 3.1.1 Interest rate sensitivity

Banks can calculate the split in stable and non-stable deposits in different ways. For example, the bank can calculate the moving average based on a certain lookback period. The moving average is defined as stable deposits. Although this method is easy, it also has deficiencies. First, the lookback period influences the outcome. Second, it is not a statistical method. Therefore, we do not recommend using this method to determine the proportion of stable deposits. Instead, we use the deposit volume model to calculate the sensitivity under significant changes in interest rate environment. Deposit volumes are affected by interest rates. As a result, by applying various interest rate shock scenarios, the proportion of stable deposits can be determined. For every scenario, there will remain a certain deposit volume. We define the minimum volume occurring during these scenarios as the proportion of stable deposits. This method is statistically more viable and prospective.

To be fully able to capture the interest rate sensitivity, the magnitude of the interest rate shock scenarios should be significant enough. The magnitude of the interest rate shock scenarios is based on Table 1. For simplicity, we use the same interest rate shock scenarios

as for the calculation of the impact on the EVE:

1. Parallel shock up.
2. Parallel shock down.
3. Steeper shock (short rates shock down, and long rates shock up).
4. Flattener shock (short rates shock up, and long rates shock down).
5. Short rates shock up; and
6. Short rates shock down [1].

Before we can apply these shocks, we have to calibrate the parameters of the deposit rate model, and deposit volume model based on data over the last ten years. We explain this further in Section 4.

It is important to take into account that not all customers are sensitive to interest rate changes. For example, some customers are poorly informed about the changes in conditions of their deposits, or some customers think it cost too much effort to switch to a different bank that offers a higher deposit rate. Consequently, the interest rate sensitivity of customers of NMDs cannot only be calculated based on quantitative methods. Besides that, according to the EBA [1], the bank should not only rely on quantitative and statistical methods to determine the cash flow profile of NMDs. An example of a non-quantitative model is a behavioral model. In these models, the expected behavior of the customers and or banks are incorporated. As a result, it is important to incorporate the behavior of customers into the model for deposit volumes.

As mentioned before, there are various kinds of NMDs. And according to the regulations, the bank should make different frameworks for the determination of core and non-core deposits for every kind of NMD. We focus on savings deposits. As a result, our deposit volume model has to incorporate the behavior of customers concerning their savings. The consensus is that customers are willing to withdraw their money from their savings deposits if they can invest their money somewhere else with a much higher interest without being more at risk. Consequently, we are incorporating this behavior into our deposit volume model. We discuss this further in Section 4.3.

## 3.2 Determining core deposits

After the determination of stable deposits, we have to determine the core deposits. The stable deposits can be further divided into a core and a non-core. The determination of the core and non-core of NMDs is important because they are defined as important risk parameters for NMDs by the BCBS [2]. As a result, in this section, we need to determine which deposits are unlikely to reprice even under significant changes in interest rate environment.

First, we explain what is meant by repricing a deposit. After a significant change in interest rate environment, the bank will often alter the deposit rates, i.e. repricing a deposit. For example, lower interest rates result in lower deposit rates. In this way, the bank can lower their opportunity costs for holding deposits. NMDs are a funding source for banks. If the deposit rates increase, then the funding costs increase, and this could reduce the margin of the banks. This is called repricing risk.

Knowing how NMDs are repriced is important for determining the core and non-core. Deposit rates affect the margin of a bank and can also affect the deposit volumes. A higher deposit rate can increase the deposit volume, and this leads to a bigger funding source from savings deposits for the bank. However, a higher deposit rate decreases the margin of a bank. As a result, the extent to which NMDs are repriced affects the risk of a bank and thus the proportion of core deposits of a bank.

### 3.2.1 Repricing NMDs

Some NMDs do not have a deposit rate. Deposits without a deposit rate are not sensitive to significant changes in interest rate environment. As a result, all stable deposits without a deposit rate are automatic core deposits. However, with the current low interest rates, the bank may be required to change these deposits without a deposit rate to deposits with a negative deposit rate. The bank needs to take this into account when determining the core deposits and non-core deposits. Nonetheless, savings deposits intend to have a deposit rate even though the deposit rate could be zero percent. During this research, we focus on savings deposits. As a result, we do not have to take deposits into account without a deposit rate.

There are savings deposits with a fixed rate and a floating rate. The repricing date of a deposit with a fixed rate is fixed. Outside this fixed date, the deposit is not repriced even under significant changes in interest rate environment. It depends on how often a deposit with a fixed rate is repriced if it can be automatically defined as a core deposit. We stated at the beginning of this section that the proportion of core and non-core deposits needs to be determined quarterly. As a result, if a deposit with a fixed rate is not repriced more often than once per quarter it is automatically defined as a core deposit. In conclusion, it is important to know the conditions of every deposit. Therefore, the following data need to be collected from the bank:

- Has the deposit a fixed rate or a floating rate.
  - How often are the deposits with a fixed rate repriced; and
- The amount of money on the deposit.

If the proportion of savings deposits with a fixed rate, is higher than 70 percent, then the proportion of core deposits is simply set back at 70 percent. If the proportion is lower than 70 percent, then we have to elaborate on the savings deposits with a floating rate and determine which proportion of these deposits are core deposits. Furthermore, the proportion of core deposits can never be higher than the proportion of stable deposits.

We mentioned before that the bank should not only rely on quantitative and statistical methods to determine the cash flow profile of NMDs. As a result, to determine the proportion of core deposits of savings deposits with a floating rate, we have to elaborate on the behavior of the bank concerning repricing savings deposits. The behavior of the bank plays an important role in the determination of core deposits. Interest rates affect the deposit rates. Despite the negative interest rates, banks are eager to keep the deposit rates positive to maintain their competitive position. If interest rates stay negative, the deposit rates may become negative. Moreover, banks react slower to adjust the deposit rate when the market rates are rising because a bank is less willing to pay higher interest rates. However, when interest rates are falling, the bank reacts much faster to adjust the deposit rate to lower their costs. We have to take this behavior of banks into account when we determine which deposits are unlikely to reprice even under significant changes in interest rate environment. This behavior is incorporated into the deposit rate model. We discuss this further in Section

## 4.2.

So, when are deposits repriced? As we have mentioned in Section 2.4, we use an asymmetric partial adjustment model to model deposit rates, see Equation 2.15. The parameter  $\lambda^+$  represents the upward adjustment of the deposit rate and  $\lambda^-$  the downward adjustment. For example, when the market rates go up and if  $\lambda^+$  is equal to 0.4 then 40 percent of the deposits are repriced. In the same way, when the market rates go down and if  $\lambda^-$  is equal to 0.5 then 50 percent of the deposits are repriced. As a result, the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment is  $\min[(1 - \lambda^+), (1 - \lambda^-)]$ .

After we determined the proportion of core deposits, we have to elaborate on the cap on the proportion of core deposits set by the BCBS. Savings deposits are part of retail/non-transactional deposits. As a result, the proportion of core deposits cannot be higher than 70 percent. If it is higher, then the proportion of core deposits is simply set back at 70 percent.

## Chapter 4

# Application to savings deposits

Now that we have developed the framework to determine the core and non-core deposits, we can discuss the methodology of the complete model. See Figure 4.1, for an overview of the model. In this chapter, we discuss the various model components. The data we need to gather over the last ten years are market rates, deposit rates, and deposit volumes. The data is public information and thus can be easily gathered from year reports of banks and open market rate sources. With this data, the market rate, deposit rate, and deposit volume model can be calibrated. We discuss this in Section 4.1, 4.2, and 4.3 respectively including the validation of the models.

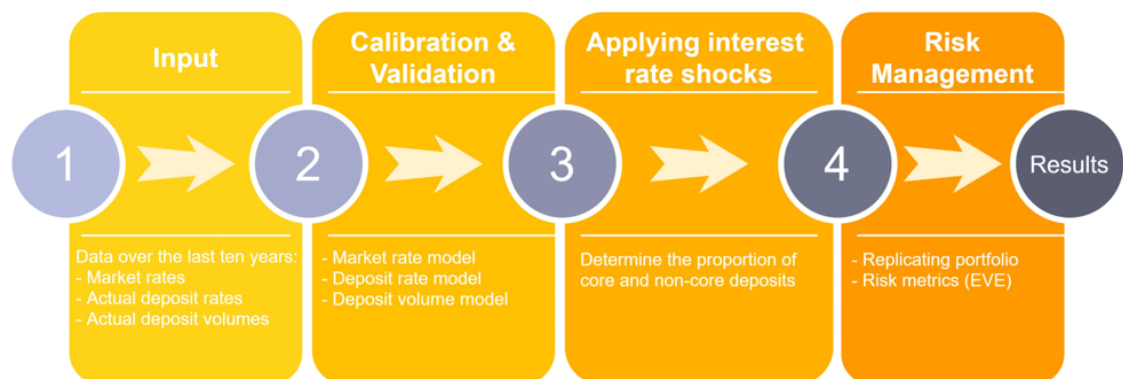


Figure 4.1: Model design NMDs.

After these models are calibrated and tested, we simulate various interest rate shock scenarios. The interest rate shock scenarios are needed to determine the proportion of core and non-core deposits. We already discussed the exact method in Section 3. Finally, to be

able to manage the emerging risk from NMDs, a replicating portfolio is constructed, and the EVE is measured. The approach we use to construct a replicating portfolio is discussed in Section 4.4 and the method to measure the EVE is discussed in Section 4.5.

## 4.1 Market rate model

As we mentioned in Section 2.3, if we want to measure unhedged risk, it is important to predict the true expectations about the uncertainty surrounding the future evolution of interest rates. Using a stochastic yield curve model obtains a sample set of yield curves with limitations because it is based on the risk-neutral world. For this reason, we are using PCA in combination with a historical sample set to model market rates. Our data set of market rates is discussed in Section 4.1.2.

PCA can be used to process a set of yield curves with standard mathematical methods to define a model. This model can be represented as:

$$R_t = \sum_{i=1}^n \alpha_{i,t} PC_i \quad (4.1)$$

Where,  $R_t$  represents the entire vector of 13 zero rates making up the yield curve at time  $t$ ,  $\alpha_{i,t}$  is a real value to scale the  $i^{th}$  principal component to reproduce the yield curve at time  $t$ , and  $PC_i$  represents the vector that is the  $i^{th}$  principal component. Furthermore,  $n = 13$  is the number of zero rates and therefore also the number of principal components.

From this model, a reduced form model of yield curves can be defined [5]. This reduced form model has a smaller number of principal components that can reproduce the vast majority of the yield curves from the sample set. Such a reduced model can be represented as:

$$R_t = \sum_{i=1}^k \alpha_{i,t} PC_i \quad (4.2)$$

As we have mentioned, there are 13 principal components in the complete model. As a result, in the reduced model  $k < 13$ .

To simulate future yield curves, we use the following model:

$$R_{t,i} = \sum_{j=1}^k \theta_{j,i}^t PC_j \quad (4.3)$$

Where

$$\theta_{j,i}^t = \beta_1 + \beta_2 \theta_{j,i}^{t-1} + \sigma Z_i^t, \quad Z_i^t \sim N(0, 1) \quad (4.4)$$

Here  $R_{t,i}$  is the yield curve at time  $t$  for scenario  $i$ ,  $\theta_{j,i}^{t-1}$  is a real value to scale the  $j^{\text{th}}$  principal component to reproduce the yield curve at time  $t-1$  for scenario  $i$ ,  $\beta_1$  and  $\beta_2$  are constants, and  $\sigma$  is the volatility. Furthermore,  $i = 1, 2, \dots, N$ .

#### 4.1.1 Calibration and validation

The first step of PCA is to compute a covariance matrix from the data. A covariance matrix is a  $n \times n$  matrix where the  $(i, j)$  entry (where  $i \neq j$ ) is the covariance between variable  $i$  and  $j$  and the entries on the diagonal are variances (where  $i = j$ ).

The second step is to determine the eigenvalues and eigenvectors. The eigenvector corresponding to the highest eigenvalue is the first principal component. The eigenvector corresponding to the second-highest eigenvalue is the second principal component, etcetera. To know the percentage of the overall variance explained by the  $i^{\text{th}}$  principal component we take the eigenvalue of the  $i^{\text{th}}$  principal component as a percentage of the sum of all the eigenvalues.

The third step is to determine how many principal components are included in the reduced model. In the literature, it is common to use at least three principal components. Furthermore, we want that the model can explain at least 95 percent of the variance. To finally decide how many principal components we include in the model, we are elaborating on the fit to more extreme yield curves.

After determining the number of principal components, the constants used to scale the principal components have to be determined. Therefore, we use least squares regression. The least squares regression line is the best line of fit that minimizes the variance, i.e. the sum of squares of the errors.

### 4.1.2 Historical market rates

The sample set has to be a good representation of the Dutch economy. Otherwise, the reproduction of the yield curves is incorrect [39]. Therefore, our historical sample sets exist out of the one-month, three-month, and six-month Euribor rates and the one-year, two-year, three-year, four-year, five-year, six-year, seven-year, eight-year, nine-year, and ten-year European swap rates.

Each subset of the Euribor rates contains 124 data points between January 2010 and April 2020 of monthly market rates. The historical market rates are shown in Figure 4.2. The graph shows that in the year 2015, the market rates became negative and this is still the case.

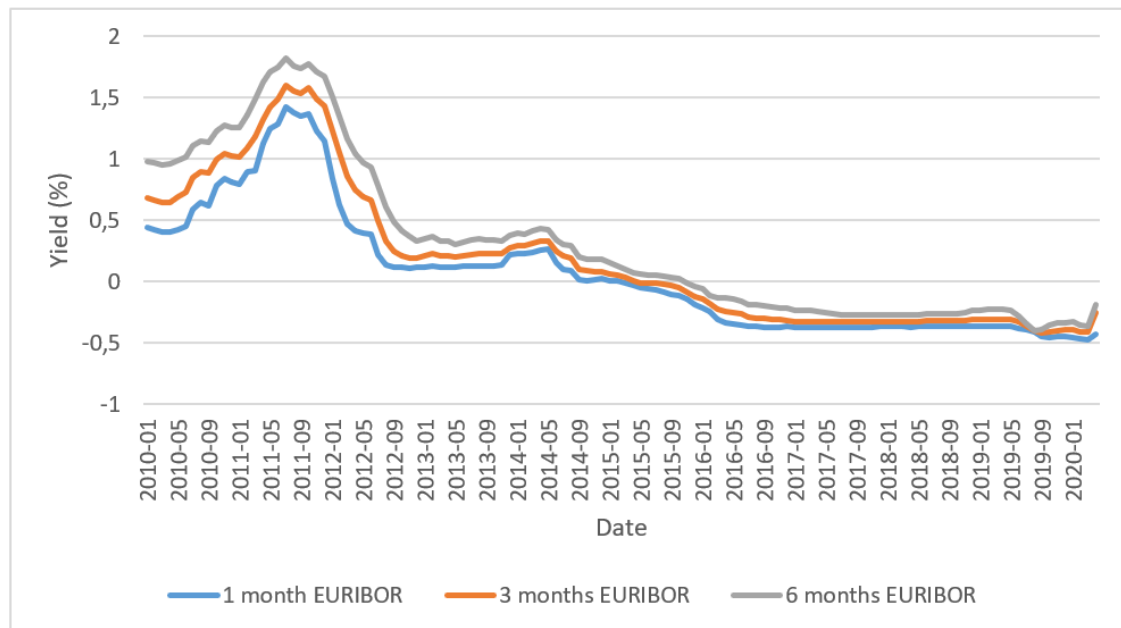


Figure 4.2: Historical data of the Euribor.

Each subset of the European swap rates contains 124 data points between January 2010 and April 2020 of monthly rates. The historical swap rates are shown in Figure 4.3. The graph shows that in the year 2015, the swap rates with a short maturity became negative. In the year 2019, the rates became negative for every maturity. Currently, all the rates are still negative.

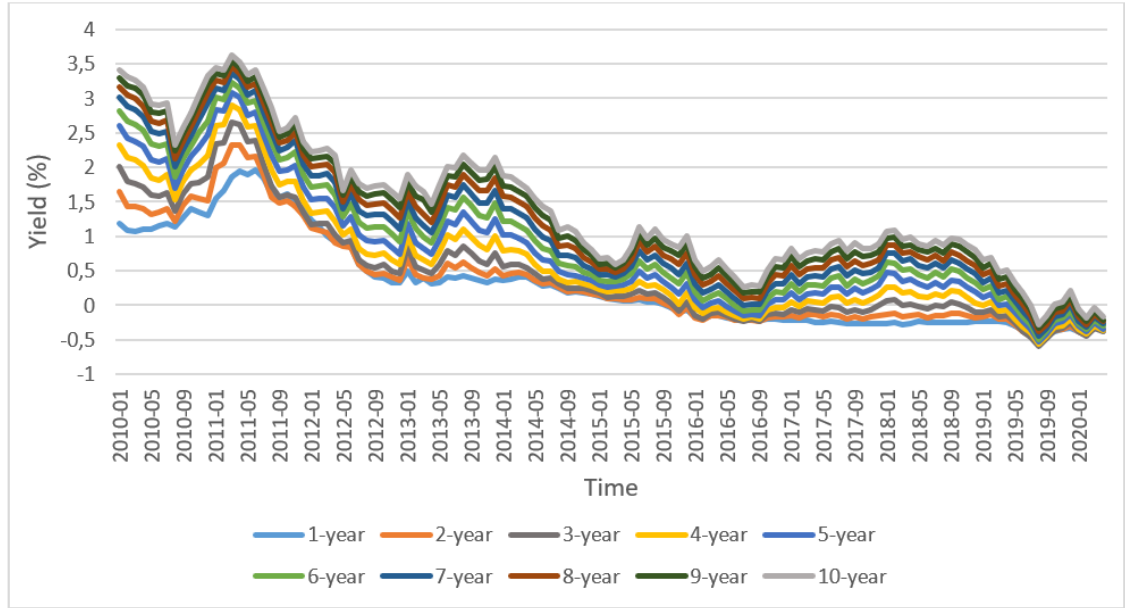


Figure 4.3: Historical data of swap rates.

The Euribor rates are identified as market zero rates. However, the European swap rates can be used to derive market zero rates with a longer maturity. According to Henningsson & Skoglund [38], the following equation can be used to derive market zero rates from swap rates:

$$R_n * \Delta t \sum_{i=1}^n \frac{1}{(1 + z(t_i))^{t_i}} = 1 - \frac{1}{(1 + z(t_n))^{t_n}} \quad (4.5)$$

Where,  $R_n$  is the current swap rate with maturity  $t_n$  years and  $z(t_i)$  is the zero rate. For  $n = 1$ , Equation 4.5 can be converted to:

$$z(t_n) = (1 + R_n)^{\frac{1}{t_n}} - 1 \quad (4.6)$$

For  $n > 1$ , Equation 4.5 can be converted to:

$$z(t_n) = \left( \frac{1 + R_n}{1 - \sum_{i=1}^{n-1} \frac{R_n}{(1+z(t_i))^{t_i}}} \right)^{\frac{1}{t_n}} - 1 \quad (4.7)$$

Equations 4.6 and 4.7 are used to derive the market zero rates. As a result, Figure 4.4 shows the observed yield curve in June 2018, June 2019, and June 2020. Normally, a longer maturity is awarded with a higher yield. This can be seen for the observed yield curve in

June 2018. However, for the observed yield curve in June 2019 and June 2020 we see a big kink in the yield curve. This is most likely due to current negative interest rates.



Figure 4.4: Observed yield curves in June 2018, 2019, and 2020.

## 4.2 Deposit rate model

It is consensus that deposit rates are heavily influenced by market rates. In conclusion, it is important to incorporate market rates into the model for deposit rates. As we have already discussed in Section 2.4, in history we have seen that banks adjust the deposit rates rapidly if the market rates are decreasing. When the market rates are rising, banks react slower to adjust the deposit rates. Consequently, there is an asymmetry in the adjustment of the deposit rates. However, this asymmetry in the adjustment of the deposit rates is observed when market rates were positive. At this moment, market rates are negative, and banks are eager to keep the deposit rates positive, i.e. the bank does not rapidly adjust the deposit rates if the market rates are decreasing. As a result, we expect that the upward and downward adjustment of the deposit rate will be low.

For modelling deposit rates, we use a deposit rate model which is based on the deposit rate model of O'Brien [23]. This model can incorporate the asymmetry in the adjustment of

deposit rates. The model O'Brien [23] uses, is an asymmetric partial adjustment model, see Equation 2.15. Our model is as follows:

$$d_t = \beta_1 + \beta_2 d_{t-1} + \lambda^+ \Delta R_t^+ + \lambda^- \Delta R_t^- + \sigma Z_t, \quad Z_t \sim N(0, 1) \quad (4.8)$$

Where

$$\Delta R_t^+ = \max(0, r_t - d_{t-1}) \quad (4.9)$$

$$\Delta R_t^- = \min(0, r_t - d_{t-1}) \quad (4.10)$$

Here,  $d_t$  is the deposit rate at time  $t$ ,  $\beta_1$  and  $\beta_2$  are constants,  $r_t$  is the market rate at time  $t$ , and  $\sigma$  is the volatility. Furthermore,  $\lambda^+$  is the upward adjustment of the deposit rate and  $\lambda^-$  the downward adjustment, both are constants. The market rate in this model is represented by the three-month Euribor rate. The method for the calibration of the parameters is discussed in Section 4.2.1.

The deposit rate scenarios are simulated by replacing the market rates in Equation 4.8 with the market rate scenarios obtained from Equation 4.3 and 4.4. This results in:

$$d_{t,i} = \beta_1 + \beta_2 d_{t-1,i} + \lambda^+ \Delta R_{t,i}^+ + \lambda^- \Delta R_{t,i}^- + \sigma Z_{t,i}, \quad Z_{t,i} \sim N(0, 1) \quad (4.11)$$

Where

$$\Delta R_{t,i}^+ = \max(0, r_{t,i} - d_{t-1,i}) \quad (4.12)$$

$$\Delta R_{t,i}^- = \min(0, r_{t,i} - d_{t-1,i}) \quad (4.13)$$

Here,  $d_{t,i}$  is the deposit rate at time  $t$  in scenario  $i$ ,  $\beta_1$  and  $\beta_2$  are constants,  $r_{t,i}$  is the market rate at time  $t$  in scenario  $i$ , and  $\sigma$  is the volatility. Furthermore,  $\lambda^+$  is the upward adjustment of the deposit rate and  $\lambda^-$  the downward adjustment, both are constants.

Dutch banks often distinguish between deposits with a low, medium, and high volume. For example, the Rabobank [27] divides the deposits into three categories:

- Deposits with a volume up to and including €100,000.

- Deposits with a volume above €100,000 and up to and including €1,000,000; and
- Deposits with a volume above €1,000,000.

Each category has a different deposit rate. This has to be taken into account for the deposit rate model. For the deposit rate model, the historic data of the deposit rates of the various categories must be gathered. Consequently, for every category the deposit rate model is calibrated and validated. Furthermore, if the deposit rate is incorporated into the deposit volume model then for every category the deposit volume model should be calibrated and validated.

In short, if the bank uses various deposit rates for various non-maturing deposits, it is important that for every type of NMD the deposit rate model is calibrated and validated. In our case, we assume that every deposit is included in one category.

#### 4.2.1 Calibration and validation

First, we are separating the data set into a training data set and a test data set. The training data set contains 80 percent of the data, beginning at the oldest dates. With the training data set, we are calibrating the parameters. However, before we calibrate the parameters it is important to select the correct model, i.e. test which variables have a significant relation to the deposit rates. Therefore, we are elaborating on the P-values of the variables. A variable with a P-value higher than 0.1 does not have a significant relation to the deposit rates and will not be included in the model.

After selecting the correct variables, we are calibrating the parameters. The parameters of the deposit rate model are calibrated using least squares regression. The least squares regression line is the best line of fit that minimizes the variance, i.e. the sum of squares of the errors.

When the parameters of the deposit rate model are calibrated based on our training data set, we need to validate the model. As we have mentioned, 80 percent of our data set is used as a training data set. Consequently, 20 percent is used to validate the models. This is

approximately the data from the last 2 years. For validating the model, we are forecasting the deposit rates of the last 2 years and compare this with the actual deposit rates.

### 4.2.2 Historical deposit rates

The data we are using for the calibration of the parameters are historical deposit rates. These deposit rates represent the average deposit rates of savings deposits of Dutch households. Our data set contains 124 data points between January 2010 and April 2020 of monthly deposit rates corresponding to the deposit volumes in Figure 4.6. The historical deposit rates are shown in Figure 4.5. The graph shows that the deposit rates decreased considerably in recent years. Furthermore, the deposit rates did not drop below zero.

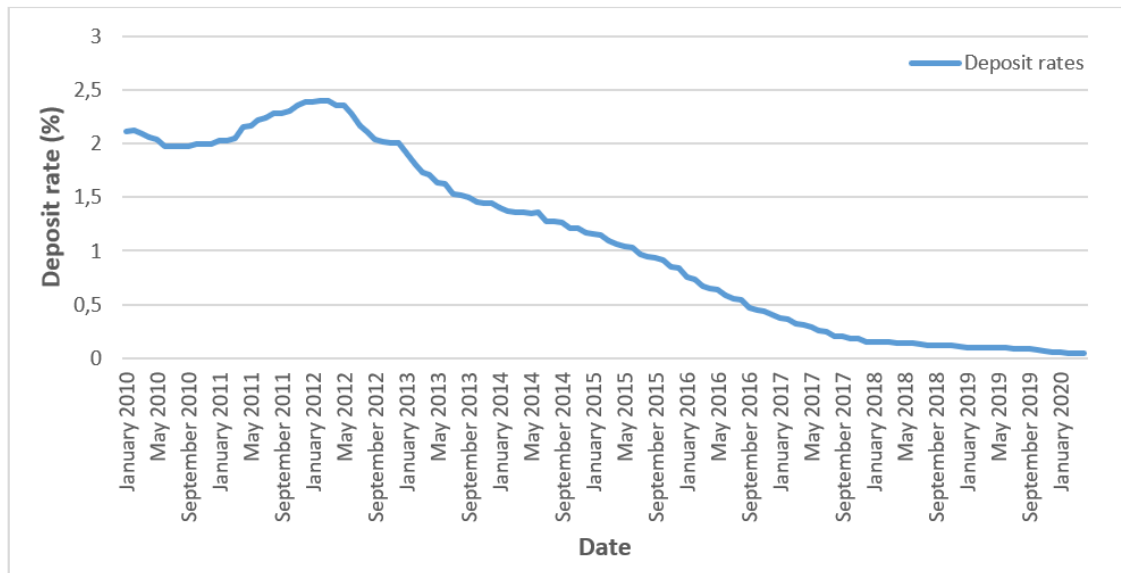


Figure 4.5: Historical deposit rates [3].

## 4.3 Deposit volume model

During our literature study, we found that not only market conditions influence the deposit volumes. Personal events could cause customers to withdraw a part or all of their money. According to Maes & Timmermans [29], these events are diversifiable across all customers. Consequently, we are not taking personal events of customers into account for the deposit volume model. However, the incorporation of behavioral components is interesting and also recommended by the BCBS [2]. Next, we are discussing the behavior of depositors of a

savings deposit.

If the deposit rate is lower than the market rate and the spread between these rates is high, then more depositors are willing to invest their money in the market instead of keeping it in a savings deposit. This means that the deposit volume could decrease. Also, if the deposit rate is higher than the market rate and the spread between these rates is high, then more people are willing to put their money on a savings deposit instead of investing their money in the market. This could lead to an increase in deposit volumes. We are incorporating this behavior of customers in our deposit volume model by adding an explanatory variable which is the spread between the market rates and the deposit rates. As a result, we are including the deposit rate in the deposit volume model. However, some researchers do not include the deposit rate in the deposit volume model because there is not a high correlation between deposit rates and deposit volumes. See Figures 7 and 8, deposit rates have decreased, but deposit volumes have increased in recent years. However, we think that the explanatory variable, which is the spread between the market rates and the deposit rates, has a significant relation to the deposit volumes.

Furthermore, as we have already mentioned in Section 2.5, deposit volumes may become negative with a normally distributed model although a bank cannot have negative deposit volumes. Consequently, we are not using a normally distributed model but a log-normally distributed model. The latter results in a model where the deposit volumes never become negative.

As a result, during this research, we are using a log-normally distributed model, based on the model of Frauendorfer & Schürle [13] and Paraschiv & Schürle [15], to model deposit volumes. See Equation 2.23 and 2.24, the combination of these models results in the following model:

$$\ln v_t = \ln v_{t-1} + \beta_1 + \beta_2 t + \beta_3 [d_{t-1} - (\delta r_{t-1}^{short} + (1 - \delta) r_{t-1}^{long})] + \sigma Z_t, \quad Z_t \sim N(0, 1) \quad (4.14)$$

Where,  $v_t$  is the deposit volume at time  $t$ ,  $\beta_i$  is a constant where  $i = 1, 2, 3$ ,  $d_t$  is the deposit rate at time  $t$ ,  $r_t^{short}$  is the short rate at time  $t$ ,  $r_t^{long}$  is the long rate at time  $t$ , and  $\sigma$  is the volatility. For the market rates, we consider a short and long maturity. According to

Paraschiv & Schürle [15], a longer maturity has more influence on the volume of savings deposits because of the lower volatility and it has a bigger influence on the behavior of customers. Therefore, they give the long rate a higher weight than the short, they use  $\delta = 0.33$ . During our research, the long rate also gets a higher weight than the short rate. We use  $\delta = 0.35$ . Furthermore, if the deposit rate is lower than the market rate, then the spread is negative and can result in lower deposit volumes. As a result, we expect that the constant  $\beta_3$  is positive. For the deposit volume at time  $t$ , we elaborate on the deposit rates, short rates, and long rates at time  $t - 1$  because we believe that it takes time for the depositors to react on the changes in deposit rates, short rates, and long rates. The short rate in this model is represented by the three-month Euribor rate and the long rate is represented by the five-year swap rate. We discuss the method for the calibration of the parameters of this model in Section 4.3.1.

The deposit volume scenarios are simulated by replacing the deposit rates and market rates in Equation 4.14 with the deposit rate scenarios and market rate scenarios obtained from Equation 4.11, and Equations 4.3 and 4.4, respectively. This results in:

$$\ln v_{t,i} = \ln v_{t-1,i} + \beta_1 + \beta_2 t + \beta_3 [d_{t-1,i} - (\delta r_{t-1,i}^{short} + (1-\delta)r_{t-1,i}^{long})] + \sigma Z_{t,i}, \quad Z_{t,i} \sim N(0, 1) \quad (4.15)$$

Where,  $v_{t,i}$  is the deposit volume at time  $t$  in scenario  $i$ ,  $d_{t,i}$  is the deposit rate at time  $t$  in scenario  $i$ ,  $r_{t,i}^{short}$  is the short rate at time  $t$  in scenario  $i$ , and  $r_{t,i}^{long}$  is the long rate at time  $t$  in scenario  $i$ .

### 4.3.1 Calibration and validation

First, we are separating the data set into a training data set and a test data set. The training data set contains 80 percent of the data, beginning at the oldest dates. With the training data set, we are calibrating the parameters. However, before we calibrate the parameters it is important to select the correct model, i.e. test which variables have a significant relation to the deposit volumes. Therefore, we are elaborating on the P-values of the variables. A variable with a P-value higher than 0.1 does not have a significant relation to the deposit volumes and will not be included in the model.

After selecting the correct variables, we are calibrating the parameters. The parameters of the deposit volume model are calibrated using least squares regression. The least squares regression line is the best line of fit that minimizes the variance, i.e. the sum of squares of the errors.

When the parameters of the deposit volumes model are calibrated based on our training data set, we need to validate the model. As we have mentioned, 80 percent of our data set is used as a training data set. Consequently, 20 percent is used to validate the models. This is approximately the data from the last 2 years. For validating the model, we are forecasting the deposit volumes of the last 2 years and compare this with the actual deposit volumes.

### 4.3.2 Historical deposit volumes

The data we are using for the calibration of the parameters are historical deposit volumes. These deposit volumes represent the savings deposits of Dutch households with a notice period. Our data set contains 124 data points between January 2010 and April 2020 of monthly deposit volumes corresponding to the deposit rates in Figure 4.5. The historical deposit volumes are shown in Figure 4.6. The graph shows that the deposit volumes have increased. It appears that there is a seasonal effect. At the beginning of each year, the deposit volumes increase, and at the end of each year, the deposit volumes drop.

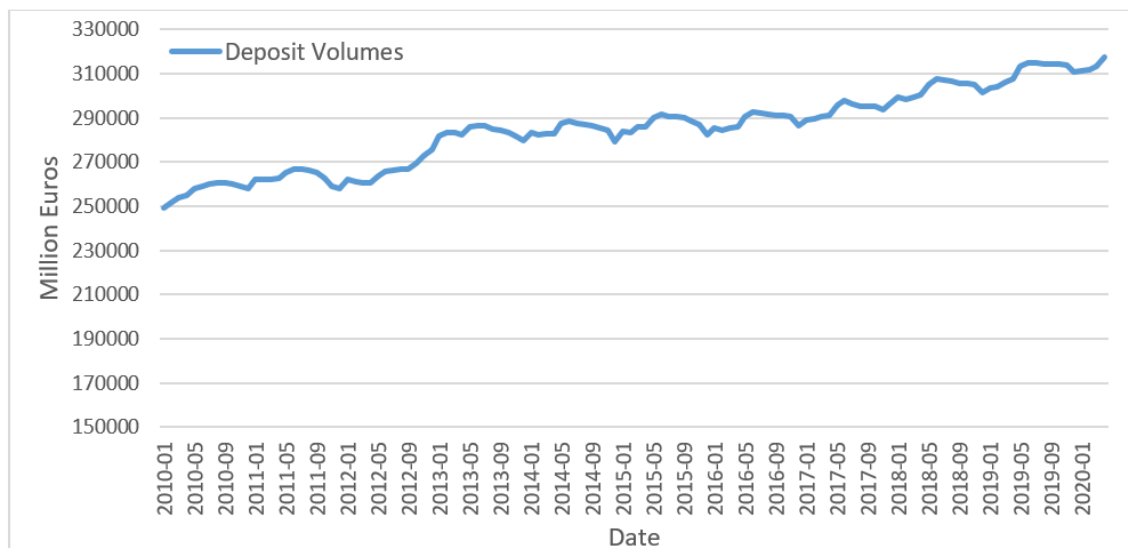


Figure 4.6: Historical deposit volumes [4].

## 4.4 Replicating portfolio

After determining the core and the non-core, the deposits have to be placed in the correct time buckets. According to the regulations of the BCBS [2], the placement of the deposits in the correct time bucket can be done with a replicating portfolio approach.

Maes & Timmermans [29] divide the total deposit volume into three parts, namely the core deposits, volatile deposits, and remaining balance. In our case, the total deposit volume is divided into two parts, namely the core deposits and the non-core deposits. The non-core deposits are invested in the overnight bucket and it serves as protection against volume fluctuations. The core deposits are invested in various time bands. The BCBS [2] set a cap on the average maturity of core deposits. For savings deposits, the cap on the average maturity of core deposits is 4.5 years. This is important to take into account when we construct a replicating portfolio.

Various studies show that a dynamic replicating portfolio approach is superior to static replication, a lot of the shortcomings of a static replicating portfolio approach can be overcome with a dynamic approach. As we have discussed in Section 2.2.2, the results of dynamic replication are more reliable because the portfolio weights are reevaluated at regular intervals. Moreover, the margin is bigger, and the variance of the margin is smaller. For these reasons, we use a dynamic replicating portfolio approach during this research based on the net present value Monte Carlo simulation model of Henningsson & Skoglund [38].

To construct a replicating portfolio, we need to examine what the minimum deposit volume is for the investment over the period  $[0, t]$ . Therefore, we use the following equation:

$$M^i(t) := \min_{0 \leq s \leq t} v_{s,i}, \quad i = 1, 2, \dots, N \quad (4.16)$$

Where,  $N$  is the number of simulations and  $v_{s,i}$  is the deposit volume at time  $s$  in scenario  $i$ . Following:

$$M(t) = [M^{(1)}(t), \dots, M^{(N)}(t)] \quad (4.17)$$

Here  $M(t)$  is a vector consisting of minimal deposit volumes over the period  $[0, t]$ . It is ordered such that  $M^{(1)}(1) \leq \dots \leq M^{(N)}(t)$ . Sequentially, to calculate the amount that is

available for investment over the period  $[0, t]$ , we use the following equation:

$$m^p(t) := M_{[Np]}(t) \quad (4.18)$$

Where  $M_{[Np]}(t)$  is the  $Np^{th}$  element in the vector  $M(t)$ .

To calculate the cash flows of the savings deposits we use the following equation:

$$CF_t^i = \begin{cases} m^p(t) - m^p(t-1) - d_{t-1,i} * m^p(t-1), & \text{if } t = 1, 2, \dots, T-1 \\ -m^p(t-1) - d_{t-1,i} * m^p(t-1), & \text{if } t = T \end{cases} \quad (4.19)$$

And the present value of the savings deposits can be formulated as:

$$PVCF_t^i = \frac{CF_t^i}{B_t^i} \quad (4.20)$$

Where,

$$B_t^i = B_{t-1}^i * e^{r_{t-1}^i * dt}, \quad B_0^i = 1 \quad (4.21)$$

Here,  $B_t^i$  is a numeraire,  $r_{t-1}^i$  is the short rate at time  $t-1$  in scenario  $i$ , and  $dt = \frac{1}{12}$ .

The present values of the savings deposits are summed into different time buckets that match the maturities of the assets. The time buckets are defined as:

$$TB_1^i = PVCF_1^i \quad (4.22)$$

$$TB_2^i = PVCF_{t_1+1}^i + \dots + PVCF_{t_2}^i$$

⋮

$$TB_k^i = PVCF_{t_k+1}^i + \dots + PVCF_{t_k}^i$$

Where,  $t_k$  is the maturity of the assets.

The optimization problem to replicate a portfolio is defined as:

$$\min_{w_1, \dots, w_n} \sum_{i=1}^N \sum_{k=1}^n \left( TB_k^i - \sum_{j=1}^n w_j \frac{CF_j(t_k)}{B_{t_k}^i} \right)^2 \quad (4.23)$$

s.t.

$$w_j \geq 0 \quad (4.24)$$

$$\sum_{j=1}^n w_j \leq v_0 \quad (4.25)$$

$$w_1 \geq NC * v_0 \quad (4.26)$$

Where,  $CF_j(t_k)$  is the cash flow of the replicating asset  $j$  at time  $t_k$ . Furthermore,  $NC$  is the proportion of non-core deposits. The last restriction is due to the fact that the non-core deposits have to be invested in the overnight bucket.

The duration of the replicating portfolio is defined as:

$$D_{RP} = \sum_{j=1}^n \frac{w_j}{v_0} * t_j \quad (4.27)$$

The cap on the average maturity of core deposits is 4.5 years. As a result,  $D_{RP} \leq 4.5$  is also a restriction of the optimization problem.

We want to research the effect of the proportion of core and non-core deposits on the risk management of NMDs of a bank. Therefore, we are replicating two portfolios one portfolio with the restriction that the non-core deposits are invested in the overnight bucket and one without this restriction. In this way, we could see if there is a significant difference between the two replicating portfolios.

## 4.5 Economic Value of Equity

The interest rate risk can be assessed by measuring the EVaR. The EVaR measures the maximum equity value change. To be able to measure the EVaR, a distribution of the change in EVE has to be made using historical simulations or Monte Carlo simulations. However, the EVaR does not adequately assess tail risk, or it demands a lot of computational power and technology. For this reason, we are assessing the interest rate risk by the change in EVE for specific interest rate scenarios. For simplicity, we are using the standardized EVE risk measure by the BCBS [2].

Under the standardized method, the cash flows per interest rate shock scenario have to be measured and placed in their respective time bucket. This will be done according to the method described in Section 4.4. Subsequently, the net notional repricing cash flows are weighted in each time bucket by a continuously compounded discount factor. The discount factor is defined as:

$$DF_i(t_k) = e^{-R_i(t_k)*t_k} \quad (4.28)$$

Where,  $DF_i(t_k)$  is the discount factor for shock scenario  $i$ ,  $t_k$  is the midpoint of time bucket  $k$ , and  $R_i(t_k)$  is the shocked interest rate for shock scenario  $i$ .

Then we are summing the risk-weighted cash flows per interest rate shock scenario up. This determines the EVE under every interest rate shock scenario. The full change in EVE per interest rate shock scenario can be determined by subtracting the EVE under the interest rate shock scenario from the EVE of the current interest rate term structure:

$$\Delta EVE_i = \sum_{k=1}^K CF_0(k) * DF_0(t_k) - \sum_{k=1}^K CF_i(k) * DF_i(t_k) \quad (4.29)$$

Where,  $\Delta EVE_i$  is the change in EVE for interest rate shock scenario  $i$ ,  $CF_0(k)$  is the cash flow for time bucket  $k$  under the base scenario, and  $CF_i(k)$  is the cash flow for time bucket  $k$  for shock scenario  $i$ .

Finally, the maximum loss across all interest rate shock scenarios is defined as the standardized EVE risk measure:

$$\text{Standardized EVE risk measure} = \max_{i \in \{1, \dots, 6\}} \{max(0, \Delta EVE_i)\} \quad (4.30)$$

As we have mentioned in Section 2.6, the change in EVE is the change in the net present value of all cash flows originating from NMDs, among other things, resulting from a change in interest rates, assuming that all banking book positions run-off [1]. We are only elaborating on the change in the net present value of all cash flows originating from NMDs.

We want to research the effect of the proportion of core and non-core deposits on the risk management of NMDs of a bank. Therefore, we are calculating the change in EVE with two different deposit volume models. It is important that the two deposit volume models

divide the NMDs into different proportions of core and non-core deposits. In this way, we could see if the proportion of core and non-core deposits affect the outcome of the change in EVE. We use the following normally distributed deposit volume model:

$$v_t = \beta_1 + \beta_2 v_{t-1} + \beta_3 r_t + \beta_4 d_t + \sigma Z_{t,i}, \quad Z_{t,i} \sim N(0, 1) \quad (4.31)$$

Where,  $v_t$  is the deposit volume at time  $t$ ,  $\beta_i$  is a constant where  $i = 1, 2, 3, 4$ ,  $d_t$  is the deposit rate at time  $t$ ,  $r_t$  is the short rate at time  $t$ , and  $\sigma$  is the volatility. The short rate in this model is represented by the three-month Euribor rate. With a normally distributed model, the deposit volumes could become negative. However, by adding the following restriction  $v_t \geq 0$ , the deposit volumes will never become negative. The method for the calibration of the parameters of this model is the same as for the log-normally distributed deposit volume model, see Section 4.3.1.

## Chapter 5

# Results

In this section, we present the results of this thesis. We calibrate and validate the market rate model, the deposit rate model, and the deposit volume model. Furthermore, we present the results of the division of the deposit volume into core and non-core deposits. Lastly, we show the outcome of the replicating portfolio and the EVE.

### 5.1 Modelling market rates

Table 3 shows the factor loadings for the market zero rates. The first column in Table 5.1 shows the maturities of the rates that were considered. The first principal component shows a roughly parallel shift in the yield curve. The second principal corresponds to a change of slope of the yield curve. The rates with a maturity between one month and five years move in one direction and the rates with a maturity between six and ten years move in on direction. The third principal component corresponds to a bowing in the yield curve. The rates with a maturity between two years and seven years move in one direction and the other rates move in one direction. The first three principal components can be seen in Figure 5.1.

Table 5.1: Factor loadings for the market zero rates.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7
1-month	-0.08753	-0.40032	0.25696	0.72750	0.33315	-0.10301	-0.33297
3-month	-0.11846	-0.38225	0.37495	0.11994	-0.20190	0.35016	0.69708
6-month	-0.15344	-0.34819	0.45198	-0.42658	-0.39183	0.12831	-0.53153
1-year	-0.16080	-0.35062	0.07659	-0.34986	0.28781	-0.67956	0.26464
2-year	-0.18535	-0.30763	-0.2576	-0.26926	0.37561	0.11169	-0.03918
3-year	-0.21668	-0.25245	-0.38969	-0.09202	0.17808	0.34381	-0.09779
4-year	-0.25307	-0.17278	-0.36232	0.05525	-0.08495	0.21086	-0.05138
5-year	-0.28851	-0.08436	-0.28137	0.14213	-0.28096	-0.01289	0.07316
6-year	-0.32127	0.00680	-0.16395	0.14644	-0.31986	-0.22564	0.10606
7-year	-0.35018	0.09787	-0.04462	0.11758	-0.26289	-0.27940	-0.05994
8-year	-0.37556	0.18955	0.07518	0.05051	-0.07871	-0.11184	-0.11364
9-year	-0.39813	0.27855	0.18784	-0.01901	0.16818	0.03395	0.03954
10-year	-0.41824	0.36192	0.29112	-0.09788	0.38345	0.26563	0.04726
	PC8	PC9	PC10	PC11	PC12	PC13	
1-month	-0.03090	-0.0442	-0.01402	-0.02693	0.02227	-0.00012	
3-month	0.14113	0.09557	0.02576	0.04999	-0.05663	0.01260	
6-month	-0.07469	-0.06974	-0.03472	-0.03629	0.05820	0.00962	
1-year	-0.29460	0.09978	-0.01707	0.03355	-0.06273	-0.07851	
2-year	0.60474	-0.34881	0.25372	-0.02942	0.09324	0.12546	
3-year	-0.07310	0.53512	-0.45638	-0.25057	-0.05259	-0.06311	
4-year	-0.38990	-0.15561	0.15783	0.70260	-0.14757	0.02965	
5-year	-0.30893	-0.08522	0.50035	-0.55470	0.23641	-0.09579	
6-year	0.14442	-0.49138	-0.62373	-0.03858	0.07927	-0.15755	
7-year	0.22399	0.28537	0.06165	-0.00148	-0.28078	0.69784	
8-year	0.34956	0.2828	0.23100	0.13758	-0.27077	-0.65987	
9-year	-0.01336	0.19797	-0.03719	0.23880	0.76864	0.09308	
10-year	-0.27359	-0.30420	-0.04625	-0.22779	-0.38807	0.08902	



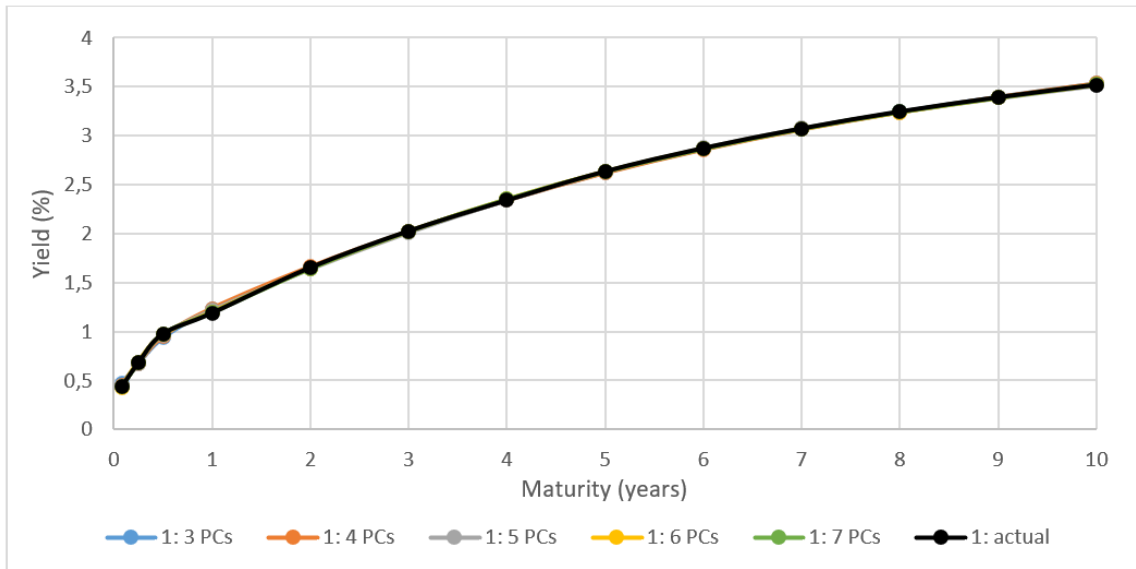
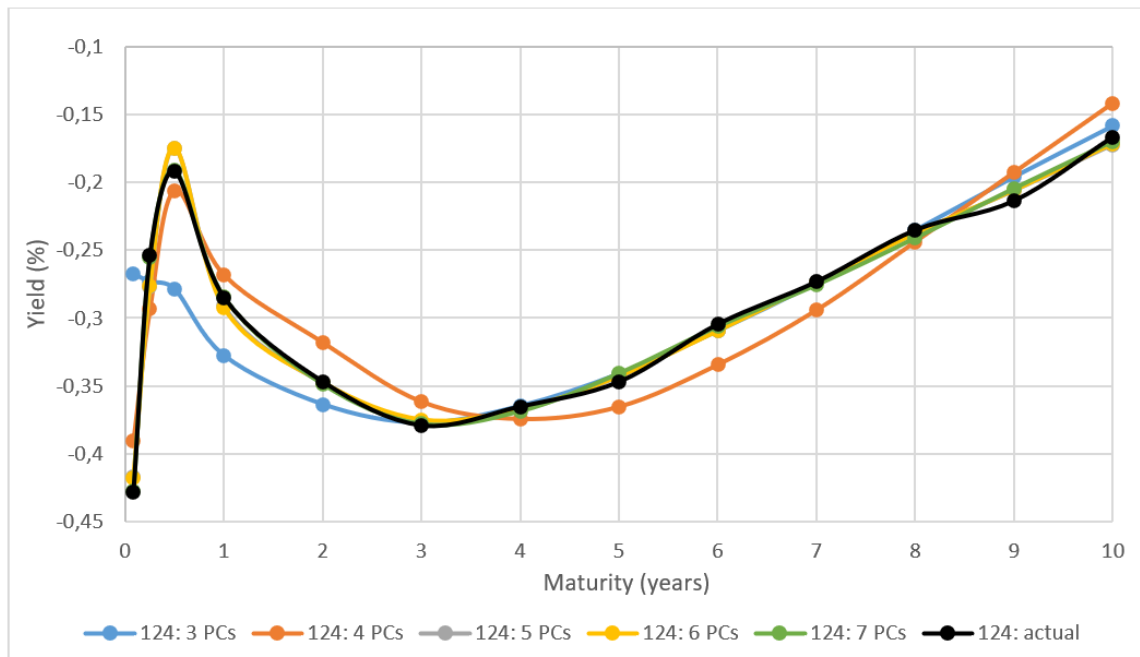
(a) Yield curve at time  $t = 1$ .(b) Yield curve at time  $t = 124$ .

Figure 5.2: Reproduction of yield curves using a reduced model with three, four, five, six, and seven principal components. Subfigure (a) represent a more typical yield curve. Subfigure (b) represent a more extreme yield curve.

See Figure 5.2a, this graph represents the reproduction of a more typical yield curve. We see that the yield curve at time  $t = 1$  is already perfectly fitted with three principal components. See Figure 5.2b, this graph represents the reproduction of a more extreme yield curve. We see that the yield curve at time  $t = 124$  is roughly reproduced when at least five principal components are included in the reduced model. It is clear that the reduced model cannot reproduce more extreme yield curves if only the first three or four principal components are included in the reduced model. As a result, in the reduced model we use five principal components. In this way, we are also able to reproduce more extreme yield curves.

Per principal component, we calculated the constants to scale every principal component. To simulate future yield curves, we need to calibrate the parameters of the market rate simulation model. This can be found in Table 5.3. Table 5.3 shows that  $\beta_1$  does not have a significant relation with  $\theta_{j,i}^t$ . As a result, the model to simulate real values to scale the  $j^{\text{th}}$  principal component is defined as:

$$\theta_{j,i}^t = \beta_2 \theta_{j,i}^{t-1} + \sigma Z_i^t, \quad Z_i^t \sim N(0, 1) \quad (5.1)$$

Table 5.3: Market rate simulation model parameters.

	$\beta_1$	$\beta_2$	$\sigma$
PC 1	0.0002261 (0.677)	0.9817383 (<2e-16)	0.004182
PC 2	0.0001102 (0.441)	0.9819477 (<2e-16)	0.001253
PC 3	0.0001364 (0.16)	0.8458749 (<2e-16)	0.0011
PC 4	-0.00002396 (0.504)	0.8878 (<2e-16)	0.0003929
PC 5	0.00003036 (0.0.9)	0.8510 (<2e-16)	0.000268

The parameters that are stated in Table 5.3, are used to carry out a Monte Carlo simulation for  $N = 10000$  number of future possible outcomes of the market rates for ten years ahead. The result, for the 10000 simulations for the one-month Euribor rate is shown in Figure 5.3.

The simulation shows monthly time steps. The period from 0 to 124 represents the period from January 2010 till April 2020. The simulation stops in April 2030.

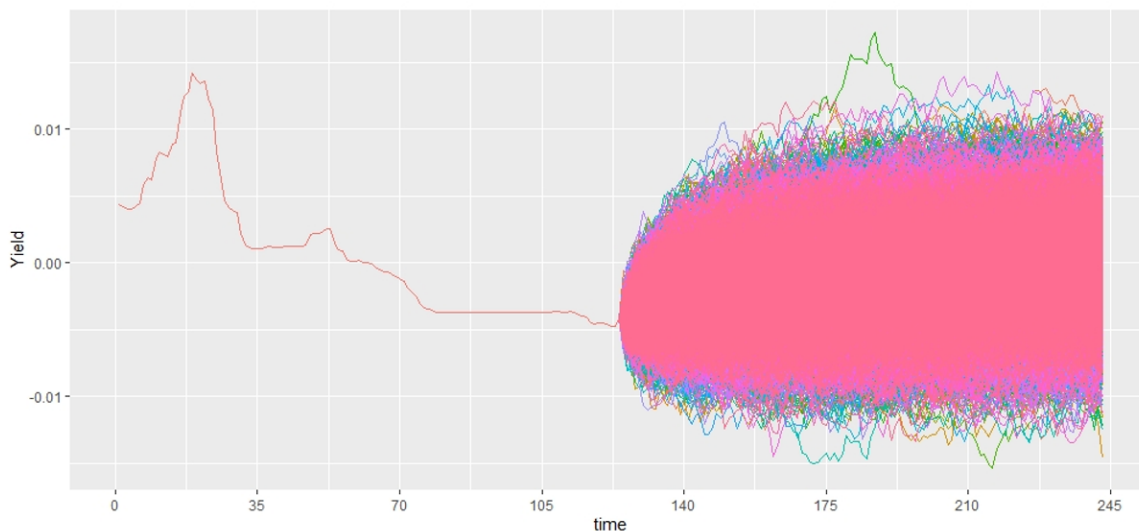


Figure 5.3: Simulated outcomes of the one-month Euribor rate over a ten-year period.

The mean value of the one-month Euribor over ten years is presented in Figure 5.4. From this graph, we can see that the one-month Euribor rate is expected to increase in the coming ten years.

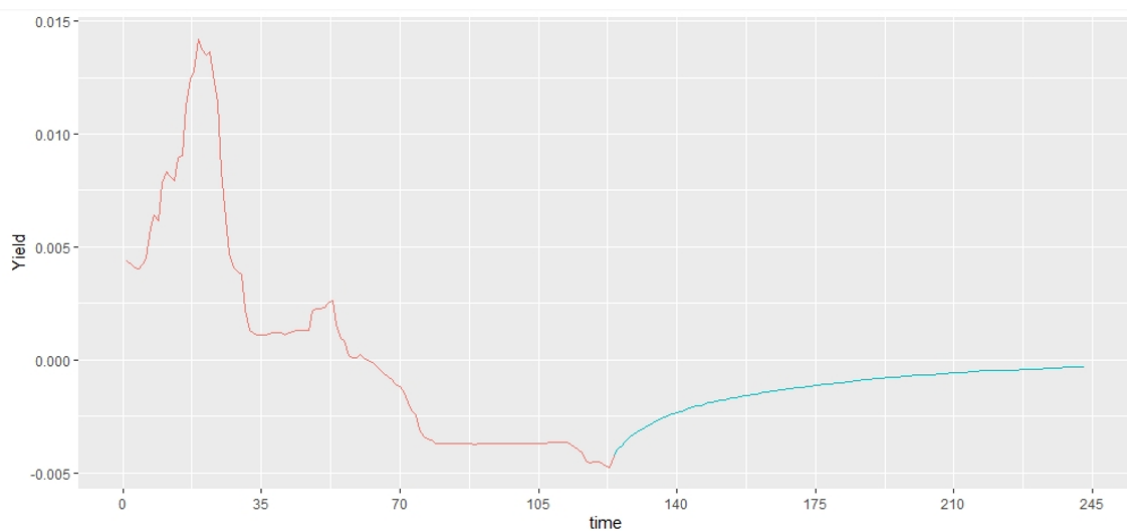


Figure 5.4: Simulated outcomes of the one-month Euribor rate over a ten-year period.

## 5.2 Modelling deposit rates

The deposit rate model requires the estimation of five parameters. A least squares regression is performed on Equation 4.8, in order to obtain the parameters. Furthermore, we use data from the last ten years. However, during this time, the deposit rate always has been higher than the three-month Euribor rate. As a result, the variable  $\Delta R_t^+$  was the past ten years zero and thus cannot be defined because of singularities. However, we assume that the upward adjustment rate is not zero. For this reason, we assume that  $\lambda^+ = \lambda^-$ . The outcome of the calibration of the deposit rate model is presented in Table 5.4.

Table 5.4: Deposit rate model parameters.

Parameter	Parameter value	P-value
$\beta_1$	0.00023943	0.00382
$\beta_2$	1.034071	<2e-16
$\lambda^+$	0.085007	-
$\lambda^-$	0.085007	6.56e-15
$\sigma$	0.0002571	-

From Table 5.4 it becomes clear that  $\beta_1$ ,  $\beta_2$ , and  $\lambda^-$  are significant at a 99 percent level. We calibrated the parameters based on the training data set. To validate the model, we use the test data. The test data exist out of data points between April 2018 and April 2020 of monthly deposit rates. See Figure 5.5, for the actual deposit rates vs. the fitted values. As we can see, the model is for a fair proportion able to reproduce the deposit rates.

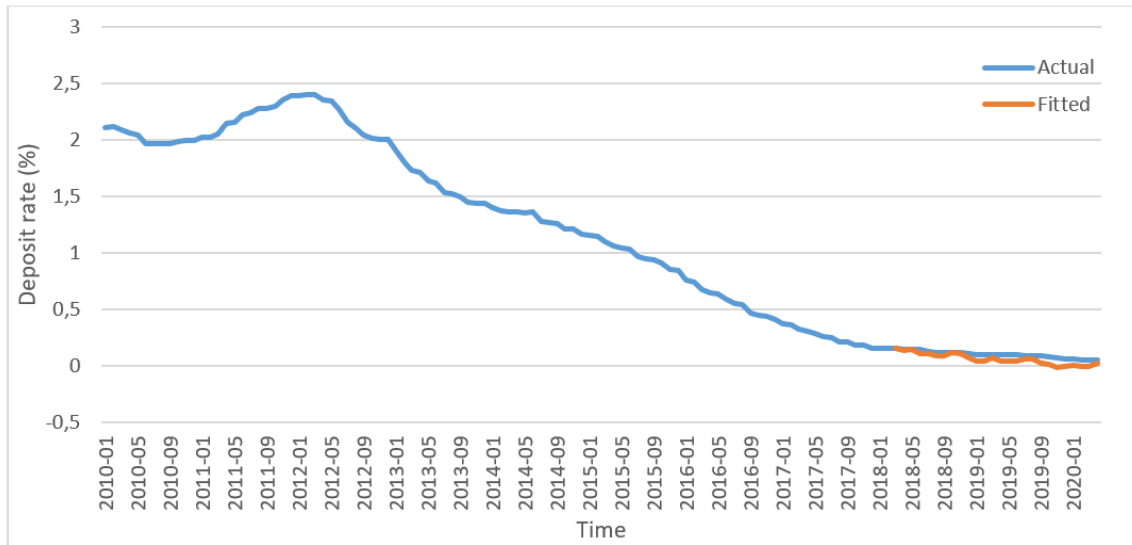


Figure 5.5: Actual time series of the deposit rates vs. fitted values.

The parameters that are stated in Table 5.4 and the results from the Monte Carlo simulation of the three-month Euribor rate are used to carry out a Monte Carlo simulation for  $N = 10000$  number of future possible outcomes of the deposit rate for ten years ahead. The result, for the 10000 simulations for the deposit rate is shown in Figure 5.6. The simulation shows monthly time steps. The period from 0 to 124 represents the period from January 2010 till April 2020. The simulation stops in April 2030.

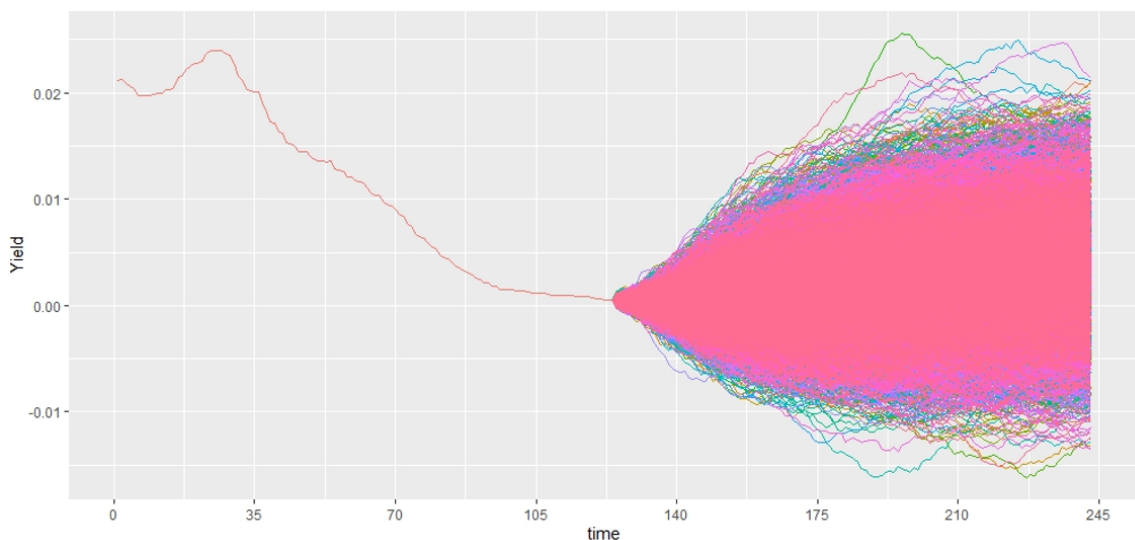


Figure 5.6: Simulated outcomes of the deposit rate over a ten-year period.

The mean value of the deposit rate over ten years is presented in Figure 5.7. From this graph, we can see that the deposit rate is expected to increase in the coming ten years and that the deposit rate stays positive.

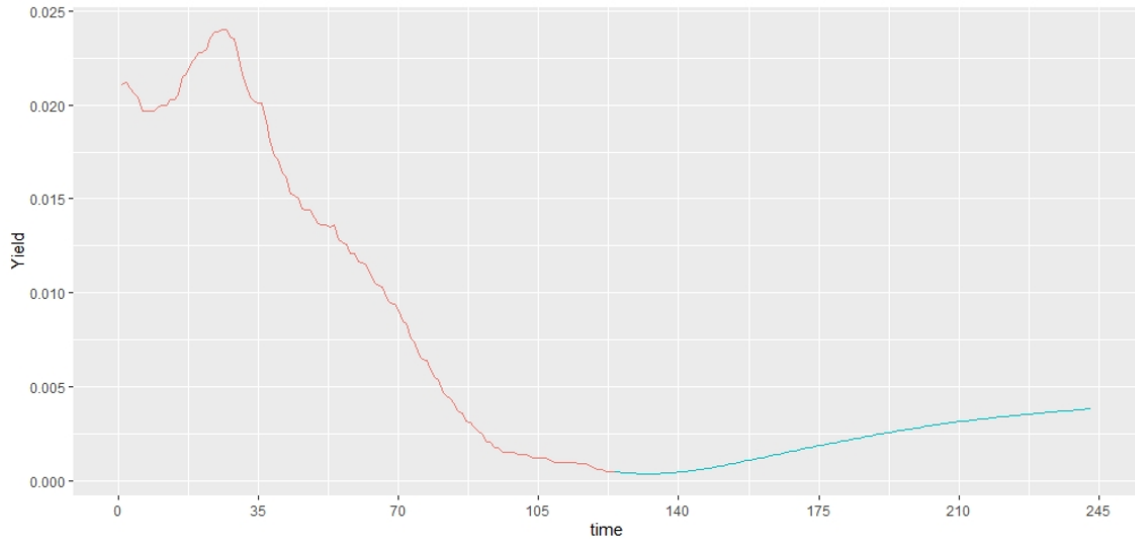


Figure 5.7: Mean value of the deposit rate over a ten-year period.

### 5.3 Modelling deposit volumes

The deposit volume model requires the estimation of three parameters. A least squares regression is performed on Equation 4.14, in order to obtain the parameters. The outcome of the calibration of the deposit volume model is presented in Table 5.5.

Table 5.5: Deposit volume model parameters.

Parameter	Parameter value	P-value
$\beta_1$	0.01183	4.43e-10
$\beta_2$	-0.00005353	7.07e-07
$\beta_3$	0.6860	9.76e-09
$\sigma$	0.01148	-

From Table 5.5 it becomes clear that  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are significant at a 99 percent level. We calibrated the parameters based on the training data set. To validate the model, we use the test data. The test data exist out of data points between April 2018 and April 2020 of

monthly deposit rates. See Figure 5.8, for the actual deposit volumes vs. the fitted values. As we can see, the model can reproduce the deposit volumes for a fair proportion.

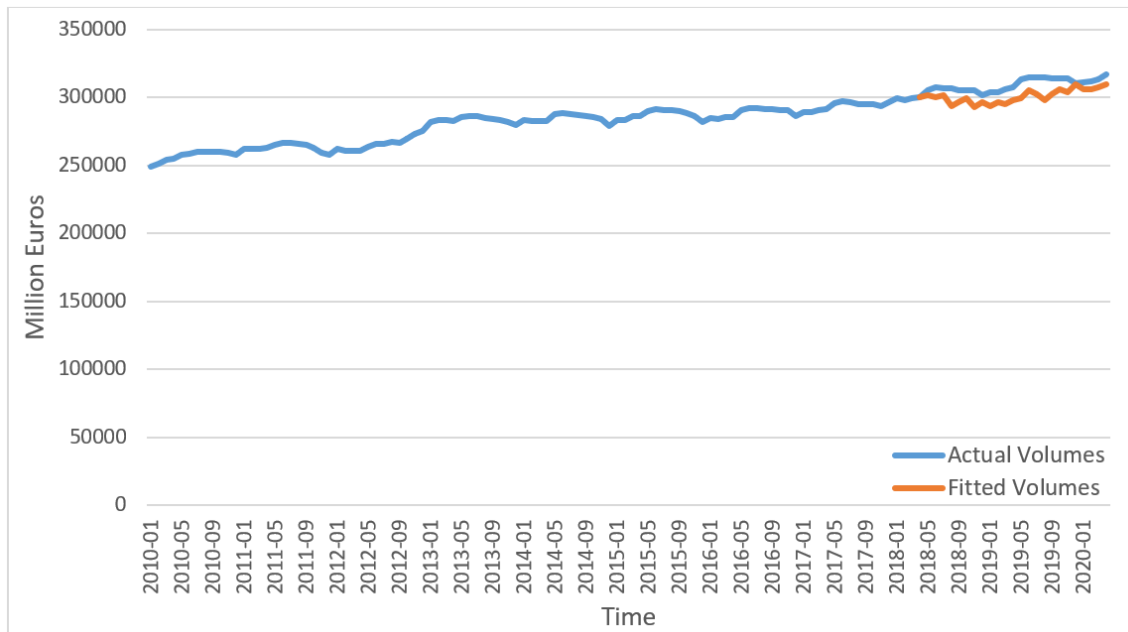


Figure 5.8: Actual time series of the deposit volumes vs. fitted values.

The parameters that are stated in Table 5.5, the results from the Monte Carlo simulation of the market rates, and the results from the Monte Carlo simulation of the deposit rates are used to carry out a Monte Carlo simulation for  $N = 10000$  number of future possible outcomes of the deposit volume for ten years ahead. The result, for the 10000 simulations for deposit volumes is shown in Figure 5.9. The simulation shows monthly time steps. The period from 0 to 124 represents the period from January 2010 till April 2020. The simulation stops in April 2030.

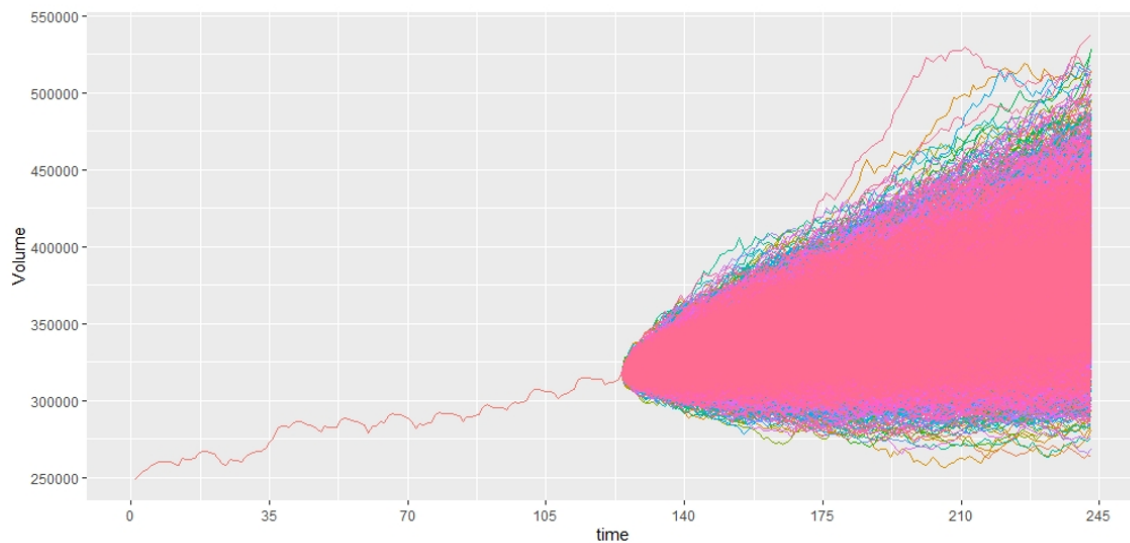


Figure 5.9: Simulated outcomes of the deposit volume over a ten-year period.

The mean value of the deposit volumes over ten years is presented in Figure 5.10. From this graph, we can see that the deposit volume is expected to increase in the coming ten years.

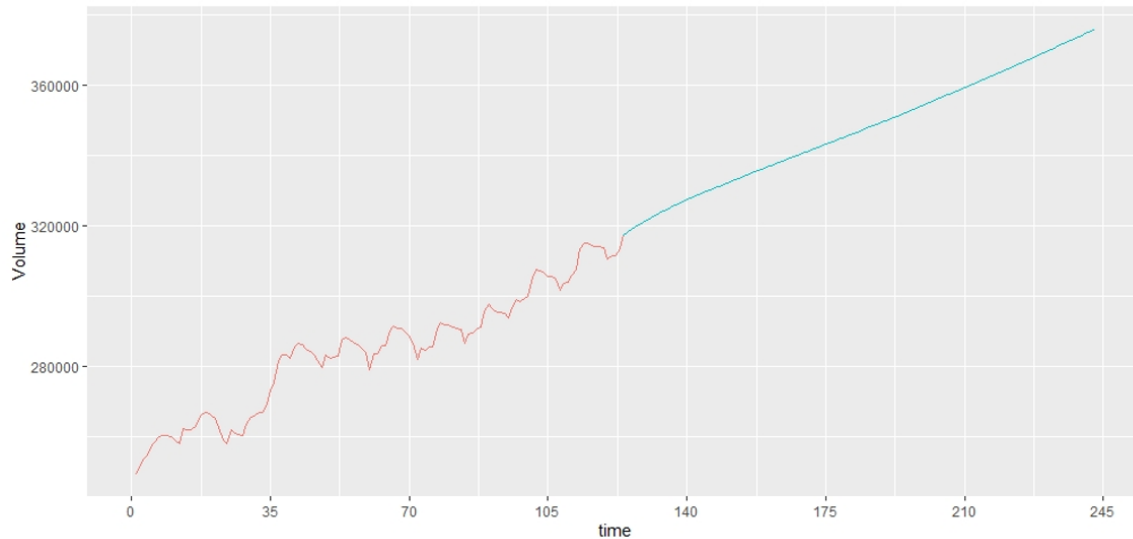


Figure 5.10: Mean value of the deposit volume over a ten-year period.

## 5.4 Modelling core deposits

For the parameterization of the standardized interest rate shock scenarios, we used the standard method of the EBA [1]. After applying the interest rate shock scenarios, we simulated 10000 possible outcomes of the deposit volume. The minimum volume occurring during these scenarios is defined as stable. The results can be found in Figure 5.11. The stable volume is defined as €215,053 million. This is 67.8 percent of the current deposit volume.

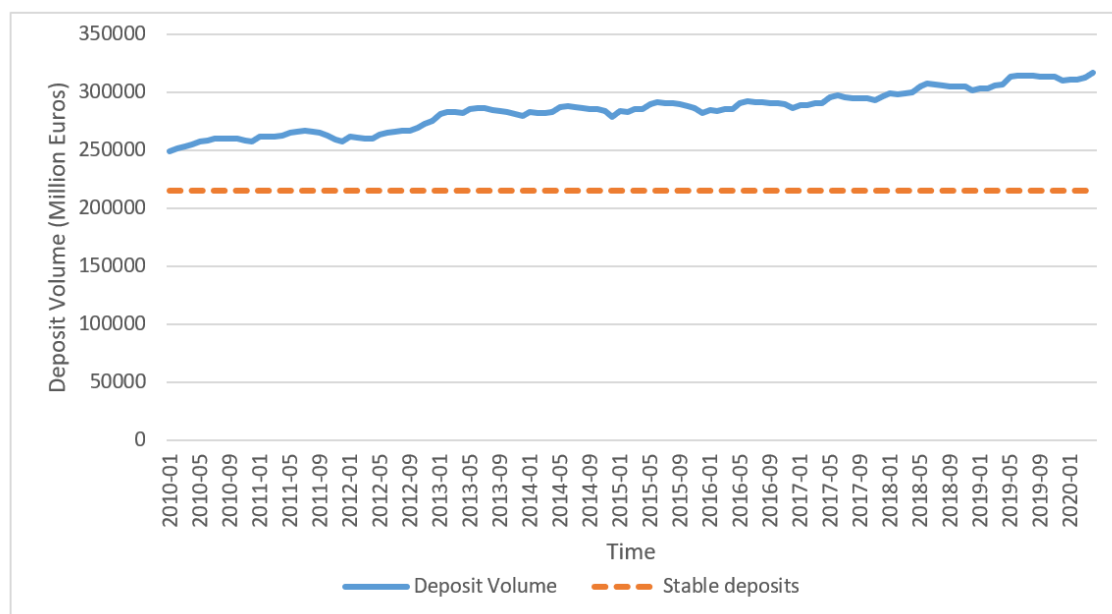


Figure 5.11: Division of the deposit volume into a stable and non-stable part.

To determine the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment, we elaborate on two parameters of the deposit rate model. To be precise,  $\lambda^+$  which represents the upward adjustment of the deposit rate, and  $\lambda^-$  which represents the downward adjustment. Based on the results in Table 5.4,  $\lambda^+ = \lambda^- = 0.085007$ . Therefore, only 8.5 percent of the changes in the interest rates are immediately passed to the customer. As a result, the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment is 91.5 percent. This is logical, because banks are eager to keep the deposit rates above zero, despite the negative interest rates, and are therefore barely passing the changes in the interest rates to

the customer.

To determine the proportion of core deposits we have to keep in mind that it cannot be higher than the proportion of stable deposits. Regardless of the proportion of 91.5 percent of deposits that are unlikely to reprice even under significant changes in interest rate environment, the proportion of core deposits is defined as 67.8 percent. The cap on the proportion of core deposits is 70 percent. The proportion of core deposits we determined is lower. As a result, we do not have to alter it.

## 5.5 Replicating portfolio

The dynamic replicating portfolio is based on the market rate scenarios and the deposit rate scenarios. The obtained replicating portfolio can be seen in Table 5.6 in percentages of the deposit volume in April 2020.

Table 5.6: Replicating portfolio with restriction non-core is invested in the overnight bucket.

Assets	Percentage
O/N	32.20
1 month	8.91
3 months	6.66
6 months	5.26
1 year	2.75
2 years	0
3 years	0
4 years	0
5 years	0
6 years	0
7 years	0
8 years	0
9 years	0
10 years	44.22

As we can see, 32.2 percent of the deposit volume is invested in the overnight bucket and serves as protection against volume fluctuations. This percentage of the deposit volume is equal to the proportion of non-core deposits determined in Section 5.5. Furthermore, we can see that 44.22 percent is invested in the ten-year bucket. The dominance of the ten-year bucket is due to the assumption that all deposit volumes will be paid back in ten years.

To see the influence of the proportion of core deposits on the replicating portfolio, we also replicate a portfolio without the restriction that the proportion of non-core deposits has to be invested in the overnight bucket. The obtained replicating portfolio can be seen in Table 5.7 in percentages of the deposit volume in April 2020.

Table 5.7: Replicating portfolio without restriction non-core is invested in the overnight bucket.

Assets	Percentage
O/N	13.13
1 month	13.84
3 months	11.5
6 months	9.93
1 year	7.13
2 years	1.35
3 years	0
4 years	0
5 years	0
6 years	0
7 years	0
8 years	0
9 years	0
10 years	43.12

As we can see, much less is invested in the overnight bucket with this replicating portfolio. As a result, adding the restriction that the proportion of non-core deposits has to be invested in the overnight bucket, can have a great influence on the outcome of the replicating portfolio. Therefore, the proportion of core and non-core deposits can also have a

great influence on the outcome of the replicating portfolio. Furthermore, there is again a dominance of the ten-year bucket and there is nothing invested in the three-year bucket till the nine-year bucket in either portfolio. This is because of the cap on the average maturity of core deposits, which is 4.5 years.

## 5.6 Economic Value of Equity

To research the effect of the proportion of core deposits on the change in EVE, we used two different deposit volume models. The normally distributed deposit volume model requires the estimation of four parameters. A least squares regression is performed on Equation 4.31, in order to obtain the parameters. The outcome of the calibration of the normally distributed deposit volume model is presented in Table 5.8.

Table 5.8: Normally distributed deposit volume model parameters.

Parameter	Parameter value	P-value
$\beta_1$	40800	2.13e-05
$\beta_2$	0.865	2.00e-16
$\beta_3$	-13020	0.0089
$\beta_4$	-17890	0.0393
$\sigma$	2355	-

With the log-normally distributed deposit volume model, the proportion of core deposits is 67.8 percent. With the normally distributed deposit volume model, the proportion of core deposits is 74.2 percent. However, because of the regulations, this has to be set back to 70 percent.

The change in EVE is calculated by applying six interest rate shock scenarios mentioned in Section 2.6. Furthermore, we used the market rate model, the deposit rate model, and two different deposit volume models. The obtained results can be seen in Tables 5.9 and 5.10.

Table 5.9: The change in EVE using the log-normally distributed deposit volume model.

Current volume (in million €)	Change in EVE (in million €)	Change in EVE (in percent of the current volume)
317322	5643.826	1.779

Table 5.10: The change in EVE using the normally distributed deposit volume model.

Current volume (in million €)	Change in EVE (in million €)	Change in EVE (in percent of the current volume)
317322	746.872	0.235

As we can see, the change in EVE is lower when the normally distributed deposit volume model is used. This is logical because the proportion of core deposits with the normally distributed deposit volume model is higher. A smaller proportion of the NMDs is affected by changes in the interest rate environment. As a result, the change in the net present value of all cash flows originating from NMDs resulting from a change in interest rates is smaller. However, the proportion of core and non-core deposits is not needed to calculate the change in EVE. This means that the proportion of core and non-core deposits does not affect the outcome of the change in EVE directly. We can see that there is a relationship between the proportion of core deposits and the change in EVE rather than that the proportion of core deposits affects the change in EVE. When the proportion of core deposits is higher the change in EVE will be lower, and when the proportion of core deposits is lower, the change in EVE will be higher.

Furthermore, according to the BCBS [2], the behavioral assumptions made to model deposit volumes of NMDs is a determining factor for the exposure of IRRBB under the EVE. However, the behavioral assumptions made to model deposit volumes also affects the proportion of core and non-core deposits. In conclusion, it is not the proportion of core deposits that influence the change in EVE, it is only the behavioral assumptions made to model deposit volumes and these behavioral assumptions also influence the proportion of core and non-core deposits.

## Chapter 6

# Conclusion

This thesis provides a framework for determining core and non-core deposits, gives an insight into modelling NMDs, and gives an insight into the risk management of NMDs. In this section, we conclude the research. We discuss the conclusion to the research questions in Section 6.1 and the conclusion to the main research question in Section 6.2.

### 6.1 Conclusion to the research questions

We developed the first research question to get more knowledge about modeling NMDs: *"How are non-maturing deposits modelled?"* To be able to model NMDs, the development of a market rate model, deposit rate model, and deposit volume model is necessary. First, we developed a market rate model by using PCA. The obtained set of yield curves by using a stochastic model is often limited by the structure of the model, they are based on a risk-neutral world. If we want to measure unhedged risk, we need to predict the true expectations about the uncertainty surrounding the future evolution of interest rates. As a result, developing a real-world model by using PCA will provide better results than a risk-neutral model. However, during our research, we did not have the time to develop a risk-neutral model. Therefore, we were not able to compare our real-world model with a risk-neutral model. Elaborating on the simulated outcomes of the one-month Euribor rate, we see that the one-month Euribor rate could become -1.5 percent. These low interest rates are not impossible. However, they are unrealistic. The occurrence of the very low interest rates is possibly due to the constant decrease in interest rates in recent years. The unrealistic low interest rates will no longer occur during a simulation if a floor is added

to the market rate model. Furthermore, elaborating on different risk-neutral models could also produce different results. Although our market rate model is not perfect, the results from the simulation are still acceptable.

Second, we developed a model for the deposit rates, specifically an asymmetric partial adjustment model. The asymmetric partial adjustment is an interesting feature of a deposit rate model because it incorporates the behavior of the bank. However, with the current low interest rates, we have seen that banks are eager to keep the deposit rates positive, and therefore, there is not a clear asymmetric adjustment in the deposit rate. Therefore, a bank must recalibrate the deposit rate model regularly, for example, every quarter. Or when there is a significant change in the economy. For example, when deposit rates become negative. Recalibrating regularly is also important for the market rate model and deposit volume model.

Lastly, we have developed a deposit volume model. In this model, we have incorporated some behavior of customers of the bank. However, we think that there is still a lot of room for improvement in this model. We discuss this in Section 7. These models are required to generate various scenarios, apply interest rate shocks, determine the proportion of core deposits, construct a replicating portfolio, and measure the change in EVE.

We developed the second research question to get more knowledge about various approaches to construct a replicating portfolio: *"How to construct a replicating portfolio?"* One subject of the risk management of NMDs is to replicate a portfolio to be able to calculate the expected cash outflows of NMDs. We have noticed that the most used approach, in the banking industry, is the static replicating portfolio approach. However, the static replication has a lot of deficiencies. We have discussed these deficiencies in Section 2.5. Subsequently, we have seen that various researches show that a dynamic replicating portfolio approach is superior to static replication. The results of dynamic replication are more reliable because the portfolio weights are reevaluated at regular intervals. Furthermore, a lot of the shortcomings of a static replicating portfolio approach can be overcome with a dynamic approach. As a result, we recommend using a dynamic replicating portfolio approach.

We developed the third research question to get more knowledge about how the EVE can be measured: *"How to measure the Economic Value of Equity?"* The interest rate risk can be assessed by measuring the EVaR. The EVaR measures the maximum equity value change.

To be able to measure the EVaR, a distribution of the change in EVE has to be made using historical simulations or Monte Carlo simulations. However, the EVaR does not adequately assess tail risk and it can demand a lot of computational power and technology. For this reason, we are recommending to assess the interest rate risk by the change in EVE for specific interest rate scenarios. Measuring the change in EVE can be done by using the standardized method or an internal model. During our research, we used the standardized method.

Our aim for the fourth research question was to get more insight into how core and non-core deposits can be determined: *"How can a framework be built to determine which non-maturing deposits are stable and unlikely to reprice even under significant changes in interest rate environment?"* We developed a framework for the determination of core and non-core deposits. This framework consists of two steps. First, stable deposits have to be determined. To measure the proportion of stable deposits, we use a deposit volume model to calculate the sensitivity under significant changes in interest rate environment. Deposit volumes are affected by interest rates. As a result, by applying various interest rate shock scenarios, the proportion of stable deposits can be determined. For every scenario, there will remain a certain deposit volume. The minimum volume occurring during these scenarios is defined as stable. Second, from the proportion of stable deposits, the core deposits can be determined. Therefore, we need to determine which deposits are unlikely to reprice even under significant changes in interest rate environment. The deposits with a fixed rate are automatically core deposits. To determine the proportion of core deposits with a floating rate, we have to elaborate on the behavior of the bank concerning repricing NMDs. Elaborating on the behavior of the bank is done by measuring the upward and downward adjustment of the deposit rate. With the upward and downward adjustment of the deposit rate, we can determine the proportion of core deposits, i.e. determine which deposits are unlikely to reprice even under significant changes in interest rate environment. Our framework is a good starting point. However, to implement this framework in a bank changes should be made in such a way that the framework matches the risk profile of the bank. This is further discussed in Section 7.

Our aim for the fifth, sixth, and seventh research question was to get more insight into how our framework can be combined with NMDs models, a replicating portfolio approach, and an approach to calculate the EVE:

- *”How can the proposed framework for determining core and non-core deposits be applied to non-maturing deposits models?”*
- *”How to use the proposed non-maturing deposits model to construct a replicating portfolio?”*
- *”How to use the proposed non-maturing deposits model to calculate the Economic Value of Equity?”*

Our framework can be easily used within various models for modelling NMDs because the bank can easily use different market rate models, deposit rate models, and deposit volume models within the framework. We discuss this further in Section 7. To replicate a portfolio, the bank should add one additional restriction to the optimization problem such that the proportion of non-core deposits is invested in the overnight bucket. This is a simple additional restriction and can be added to many replicating portfolio approaches. Furthermore, a bank can easily use various NMD models to measure the (change in) EVE. However, the bank should consider if the cash flows should be modelled partially or fully conditional on the interest rate scenario. In this way, the bank captures not only gap risk, but also basis risk. If all cash flows are modelled fully on the interest rate scenario, the bank captures not only gap risk and basis risk, but also option risk.

## 6.2 Conclusion to the main research question

The objective of this research is to gain more insight into modelling NMDs and to help to identify the liquidity risk and interest rate risk a bank faces. To do this, we want to clarify which influence core and non-core deposits have on the risk management of NMDs. With this information, we have come up with the following main research question: *”Which influence does the proportion of core and non-core deposits have on the risk management of non-maturing deposits of a bank?”*

We have seen that the proportion of core deposits, can have a great influence on the outcome of the replicating portfolio. We recommend that the core deposits are determined

before replicating a portfolio and that the proportion of non-core deposits is invested in the overnight bucket. In this way, the bank is save against volume fluctuations. Implementing this restricting in a dynamic or static replicating portfolio approach is not difficult. As a result, already existing approaches can be easily altered. Furthermore, we have seen that there is a dominance of the ten-year bucket. Which is as expected because of the expected outflow after 10 years.

We calculated the change in EVE based on the standardized method. We calculated the cash flows under various interest rate scenarios with the help of a market rate model, deposit rate model, and two deposit volume models. Thereafter, we discounted the cash flows using a risk-free rate. We have seen that the bank does not need the proportion of core and non-core deposits to determine the change in EVE. As a result, the outcome of the change in EVE is not based directly on the proportion of core and non-core deposits. However, we can see that there is a relationship between the proportion of core deposits and the change in EVE. The change in EVE is lower when the proportion of core deposits is higher because a smaller proportion of the NMDs is affected by changes in the interest rate environment. As a result, the change in the net present value of all cash flows originating from NMDs resulting from a change in interest rates is smaller. In conclusion, we can see that there is a negative relation between the proportion of core deposits and the change in EVE. However, we only elaborated on two different deposit volume models. More research needs to be done to support our conclusion.

Furthermore, as we have mentioned in Section 5.6, the behavioral assumptions made to model deposit volumes of NMDs is a determining factor for the exposure of IRRBB under the EVE. However, the behavioral assumptions made to model deposit volumes also affects the proportion of core and non-core deposits. In conclusion, it is not the proportion of core deposits that influence the change in EVE, it is only the behavioral assumptions made to model deposit volumes, and these behavioral assumptions also influence the proportion of core and non-core deposits.

## Chapter 7

# Discussion and Future work

### 7.1 Discussion

As we have mentioned in the previous section, to implement the framework for determining core and non-core deposits in a bank, changes should be made. However, this framework can be easily adjusted for various banks. For determining the proportion of stable deposits, the bank has to apply various interest rate shock scenarios and calculate the possible size of the deposit volume. For calculating the possible size of the deposit volume, the bank needs to use a deposit volume model. A general deposit rate model and deposit volume model is not possible because of the various behaviors of banks and customers. As a result, we recommend that each bank develops a deposit rate model and deposit volume model based on their behavior and the behavior of the customers. The framework allows using any kind of deposit volume model. Furthermore, it is understandable that a bank would want to add more interest rate shock scenarios to address their risk profile. The framework gives the opportunity to easily add more interest rate shock scenarios or even interest rate stress scenarios. For determining the proportion of deposits that are unlikely to reprice even under significant changes in interest rate environment, the bank needs the upward and downward adjustment of the deposit rate. This can be determined by different kinds of deposit rate models. However, the bank can also use the opinion of a professional that knows how the bank alters its deposit rates. As a result, our developed framework for determining core deposits is a good starting point for a bank and can be easily altered to match the risk profile of the bank. Furthermore, the framework can be easily used within various frameworks for modelling NMDs.

During our research, we have seen that banks are eager to keep the deposit rates positive, despite the negative interest rates. However, the Rabobank, for example, already offers a deposit rate of -0.50 percent if the customer has more than one million euros in his deposit. From January 1, 2021, this amount will drop to €250,000. Banks are under increasing pressure to lower the deposit rates due to the persistent negative interest rates. But how are customers going to react to negative deposit rates? Because of COVID-19, people are afraid to make investments. As a result, we expect that the deposit volumes will rise, despite the low deposit rates. The effect of COVID-19 on the deposit volumes was out of scope for this research. However, banks should think about the effect of COVID-19 on the deposit volumes and incorporate that in the deposit volume model.

## 7.2 Future Work

For further research, it is interesting to extend the framework to all NMD categories. In this research, we only elaborated on savings deposits, i.e. the category retail/non-transactional. The framework should be extended to the categories of retail/transactional and wholesale.

We also mentioned in Section 6 that there is still a lot of room for improving the deposit volume model. From the data of historical deposit volumes, it appears that there is a seasonal effect. At the beginning of each year, the deposit volume increases, and at the end of each year, the deposit volume drops. It could be interesting to include this seasonal effect in the model. Furthermore, it is important to elaborate on the inclusion of more behavioral components into the deposit volume model. For example, we could assess the potential migration between deposits. For future research, it is also interesting to elaborate on the possibility to incorporate macro-economic variables into the deposit volume model. For example, the Gross Domestic Product (GDP), inflation, and unemployment. Logically, these variables could have an impact on the deposit volume of a bank. However, it is important to keep in mind that the deposit volume model does not become too complicated. A simpler model often gives more transparency.

The incorporation of more behavioral components and/or macro-economic variables into the deposit volume model could also affect the proportion of the core deposits because the proportion of core deposits is determined with the deposit volume model. For further re-

search, it could be interesting to elaborate on this effect.

As we have concluded in the previous section, it is not the proportion of core deposits that influence the (change in) EVE, it is the behavioral assumptions made to model deposit volumes. As we have already mentioned, for future research it is important to elaborate on the incorporation of more behavioral components and/or macro-economic variables into the deposit volume model. This of course will also affect the (change in) EVE. It could be interesting to research this effect. Furthermore, we concluded that there is a negative relationship between the proportion of core deposits and the change in EVE. However, to see this relationship, we only elaborated on two different deposit volume models. More research needs to be done to support our conclusion. For future research, more various deposit volume models need to be used to better substantiate our conclusion.

Because of time restriction, we only elaborated on the replicating portfolio approach and the change in EVE for the risk management of NMDs. For future work, we could look at more various risk measures to get a clear overview of the risk induced by NMDs. For example, it would be interesting to continue this research by also modelling the net interest income (NII). NII is the short-term earnings risk of interest rate risk (IRR). It is known that if a bank minimizes its EVE, the bank also runs the risk of bigger volatility of the earnings. Therefore, there is a relationship between the EVE and NII. As a result, it would be interesting to research the risk relation between EVE and NII.

# Bibliography

- [1] EBA, “Guidelines on the management of interest rate risk arising from non-trading book activities,” *European Banking Authority*, 2018.
- [2] BASEL, “Interest rate risk in the banking book,” *Basel Committee on Banking Supervision*, 2016.
- [3] De Nederlandsche Bank, “Rente,” <https://statistiek.dnb.nl/dashboards/rente/index.aspx>, note = Accessed: 2020-08-01.
- [4] DNB, “Spaargeld huishoudens,” <https://statistiek.dnb.nl/dashboards/spaargeld/index.aspx>, note = Accessed: 2020-08-01.
- [5] J. C. Hull, *Risk Management and Financial Institutions (5 ed.)*. Hoboken: John Wiley & Sons, 2018.
- [6] J. Cooke, C. Koch, and A. Murphy, “Liquidity mismatch helps predict bank failure and distress,” *Economic Letter*, vol. 10, no. 6, pp. 1–4, 2015.
- [7] BASEL, “Principles for sound liquidity risk management and supervision,” *Basel Committee on Banking Supervision*, 2008.
- [8] Basel, “Basel iii: A global regulatory framework for more resilient banks and banking systems,” *Basel Committee on Banking Supervision*, 2010.
- [9] BASEL, “Liquidity coverage ratio: Calculation,” *Basel Committee on Banking Supervision*, 2019.
- [10] Basel, “Net stable funding ratio: Calculation and reporting,” *Basel Committee on Banking Supervision*, 2019.

- 
- [11] J. Bai, A. Krishnamurthy, and C.-H. Weymuller, “Measuring liquidity mismatch in the banking sector,” *The Journal of Finance*, vol. 73, no. 1, pp. 51–93, 2018.
- [12] M. Brunnermeier, G. Gorton, and A. Krishnamurthy, “Liquidity mismatch measurement,” in *Risk topography: Systemic risk and macro modeling*. University of Chicago Press, 2013, pp. 99–112.
- [13] K. Frauendorfer and M. Schürle, “Dynamic modeling and optimization of non-maturing accounts,” *Liquidity risk: Measurement and management*. Singapore: John Wiley and Sons, pp. 327–359, 2007.
- [14] F. Paraschiv, “Adjustment policy of deposit rates in the case of swiss non-maturing savings accounts,” *Journal of Applied Finance & Banking*, vol. 3, no. 3, pp. 271–323, 2013.
- [15] F. Paraschiv and M. Schürle, “Modeling client rate and volumes of non-maturing accounts,” 2010.
- [16] H. Heerkens and A. v. Winden, *Geen probleem*. Nieuwegein: Van Winden Communicatie, 2012.
- [17] M. Kalkbrenner and J. Willing, “Risk management of non-maturing liabilities,” *Journal of Banking & Finance*, vol. 28, no. 7, pp. 1547–1568, 2004.
- [18] K. Frauendorfer and M. Schürle, “Stochastic optimization in asset & liability management: A model for non-maturing accounts,” in *Probabilistic Constrained Optimization*. Springer, 2000, pp. 67–101.
- [19] A. Liviello and F. Ducuroir, “Assessing the consequences of low rates environment on alm and non-maturing liabilities modelling,” 2018.
- [20] C. Weistroffer, “Ultra-low interest rates how japanese banks have coped,” *Deutsche Bank Research*, 2013.
- [21] S. Manganelli and G. Wolswijk, “What drives spreads in the euro area government bond market?” *Economic Policy*, vol. 24, no. 58, pp. 191–240, 2009.
- [22] J. C. Hull, *Options futures and other derivatives*. Pearson Education, 2015.

- [23] J. M. O'Brien, "Estimating the value and interest rate risk of interest-bearing transactions deposits," 2000.
- [24] N. Song, T. K. Siu, F. A. Fard, W.-K. Ching, and E. S. Fung, "Risk measures and behaviors for bonds under stochastic interest rate models," *Mathematical and Computer Modelling*, vol. 56, no. 9-10, pp. 204–217, 2012.
- [25] R. A. Jarrow and D. R. Van Deventer, "The arbitrage-free valuation and hedging of demand deposits and credit card loans," *Journal of Banking & Finance*, vol. 22, no. 3, pp. 249–272, 1998.
- [26] K. Nyström, "On deposit volumes and the valuation of non-maturing liabilities," *Journal of Economic Dynamics and Control*, vol. 32, no. 3, pp. 709–756, 2008.
- [27] Rabobank, "Onze rentes en voorwaarden wijzigen," <https://www.rabobank.nl/particulieren/sparen/rentewijziging/>, note = Accessed: 2020-08-01.
- [28] M. Elkenbrach and B. Nauta, "Managing interest rate risk for non-maturity deposits," *RISK-LONDON-RISK MAGAZINE LIMITED-*, vol. 19, no. 11, p. 82, 2006.
- [29] K. Maes, T. Timmermans *et al.*, "Measuring the interest rate risk of belgian regulated savings deposits," *National Bank of Belgium Financial Stability Review*, vol. 2005, pp. 137–151, 2005.
- [30] H. Dewachter, M. Lyrio, and S. Maes, "A multi-factor model for the valuation and risk management of demand deposits," *National Bank of Belgium working paper*, no. 83, 2006.
- [31] H. von Feilitzen, "Modeling non-maturing liabilities," 2011.
- [32] E. Beutner, A. Pelsser, and J. Schweizer, "Theory and validation of replicating portfolios in insurance risk management," *Available at SSRN 2557368*, 2016.
- [33] F. Stavrén and N. Domin, "Modeling of non-maturing deposits," 2019.
- [34] M. M. Bardenhewer, "Modeling non-maturing products," *Liquidity risk: Measurement and management. Singapore: John Wiley and Sons*, pp. 220–256, 2007.

- 
- [35] M. Cambou and D. Filipović, “Replicating portfolio approach to capital calculation,” *Finance and Stochastics*, vol. 22, no. 1, pp. 181–203, 2018.
- [36] E. Derman\* and N. N. Taleb, “The illusions of dynamic replication,” *Quantitative finance*, vol. 5, no. 4, pp. 323–326, 2005.
- [37] A. Pelsser, “Pricing and hedging guaranteed annuity options via static option replication,” *Insurance: Mathematics and Economics*, vol. 33, no. 2, pp. 283–296, 2003.
- [38] P. Henningsson and C. Skoglund, “A framework for modeling the liquidity and interest rate risk of demand deposits,” 2016.
- [39] D. Redfern and D. McLean, “Principal component analysis for yield curve modelling,” *Enterprise Risk Solutions*, 2014.