Spare part stocking with service differentiation and compounded arrivals

Master thesis

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OPRA TURBINES
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Management summary

OPRA is delivering advanced gas turbine technology that have the purpose to generate electrical power and their corresponding heat from various fuel decomposition. Considering that physical failures cannot be fully avoided, spare parts are essential to minimize downtime of these gas turbines. Nevertheless, the low demand predictability, the partial compounded arrivals and the different service performance preferences leads to the continuous challenge of stocking the right spares. Keeping high stock levels is inherent to financial risks, as too less spares might directly influence the gas turbine performance. This is summarized in the following main research question:

How can the current inventory management and control policy for the spare parts be improved considering the trade-off between the inventory risks and the end-product downtime?

Considering that spares required for corrective maintenance have a higher impact when they are not available, these spares are prioritized within this research. These spare parts have the tendency of being extreme slow moving and when they cannot be delivered, they are back-ordered at OPRA's suppliers. Whereas is seems logical that spare parts are demanded in single order-sizes, approximately 30% of OPRA's spares are compound demanded. This is caused by customers who hold their own stock and by combining preventive and corrective maintenance. To apply service differentiation, OPRA is providing the alternative to compile a Long Time Service Agreement (LTSA) among with the gas turbine. Roughly 25% of the spare part demand is originated by these 'premium customers'. Since, OPRA's is expecting to sell more LTSA's in the near future, service-differentiation is inevitable.

We introduce the fill-rate, the fraction of demand that is met through immediate stock availability, to describe the service performance of the (compound) Poisson demanded spares. Subsequently, we conclude that OPRA is maintaining unnecessary high service levels for some of their spares. In contrast, various spares have no stock, thus are directly back-ordered. Hence, the current inventory investment is €162.622 and corresponds with a Weighted Average Fill-Rate(WAFR) of 70%.

To improve the spare part management at OPRA, a literature study is conducted based on the context. Due to the commonality of spares in different gas turbine setups, we focus on a multi-item approach. From the literature findings we conclude that applying service differentiation to back-ordered demand results in irrelevant programming complexity. Therefore, a critical level policy, based on lost-sales, is presented as conceptual model to solve the core problem at OPRA.

Next, the conceptual model is designed to manage single and compounded demand. To differentiate the spare parts between customers classes, a continuous time Markov chain

is applied to compute the fill-rates. Furthermore, the column generation technique is used to minimize a set of finite stocking policies combined with a local search method to find interesting new policies. The conceptual model is verified and validated by using various corresponding techniques. This guarantees that the model is correctly implemented and that the outcomes are sufficient. We analyze the models behaviour by conducting numerical experiments. The demand, the premium customer demand ratio, the Target Fill-Rates (TFR) and the customer group responsible for the compounded demand are chosen as experimental factors. From these results we derive the following conclusions:

- From the current inventory investments roughly €60.000-€100.000 is redundant since these investments do not directly contribute to the service performance.
- To increase the current service performance, spare part investments are inevitable. Initially, it is worthwhile to invest in the lower fill-rated and valued spares due to the lower financial risks.
- Inventory savings up to 10%, dependent on the degree of differentiation, can be prevented by applying service differentiation with critical levels.
- Approximately 22% of the inventory investments are related to the compounded behaviour of the demand of spares. Serving compound demand results in a demand higher variety, thus more spare parts levels are required to manage this.

Hence, a model was proposed that considers the trade-off between the desired service performance of two customer classes and the related inventory investments. Despite that the optimal model parameters for OPRA are unknown yet and a current stock levels are maintained already, we draw the following general recommendations:

- It is suggested to decrease the spare parts levels which are assigned with irrelevant high fill-rates. Subsequently, considering that the current WAFR of 70% is low, we recommend to invest in the spare parts which are currently back-ordered and have a lower value than €1.000. This will raise the current WAFR with 20% to 90% and costs €4.200.
- We recommend to reconsider the stocking purpose of spares with a higher value than €2.500. Spare parts stocked for premium customer exclusively, must be assigned with a critical level to prevent future misconceptions. Furthermore, due to the additional investments of serving customer with compounded demand, we recommend to reassess the compounded high valued spares.
- Our final recommendation is to apply the proposed model for new stocking decisions and projects. The model emphasizes which spare parts require attention, thus contributes in the inventory decision making process. The collection of the actual demand data, the back-orders and their origin, will result in more justified stocking decisions.

Acknowledgements

This thesis has been written to finish the Master Industrial Engineering Management at the University of Twente. After studying for 4 years in Groningen and the last three years in Enschede, my era as student has come to an end. When I started this Master thesis, at the end of February, nobody would even think of the draconian impact of the COVID-19 virus to our society. Fortunately, most of my work could continue from home. Nevertheless, I cannot neglect that this also affected the duration my thesis.

First of all, I would like to thank Elwin for making this assignment possible. We had various interesting discussions and you provided me with an enthusiastic working environment at OPRA. Furthermore, I would like to thank Hilbert for his information and feedback contributions.

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Julius Jansink
Oldenzaal, December 2020

Contents

List of Figures

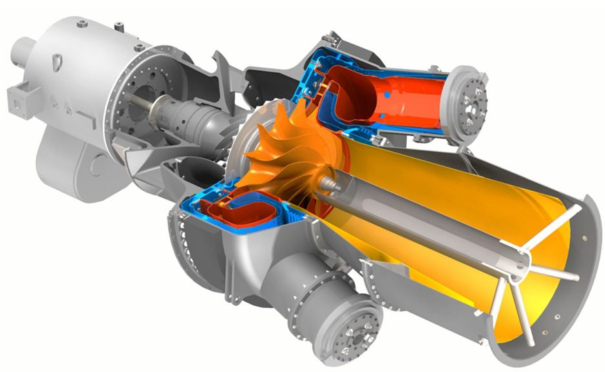
List of Tables

1 Introduction

This chapter introduces this research and is organized as follows: In Section 1.1 the problem owner, OPRA Turbines, is introduced. Section 1.2 explains the motivation of this research project. The problem description is described in Section 1.3, while Section 1.4 is scoped to the core problem. Section 1.5 introduces the research approach and the corresponding derivables. Finally, Section 1.6 describes the structure of the report.

1.1 OPRA Turbines

OPRA Turbines in Hengelo is delivering advanced gas turbine technology. The company was founded by the family Mowill in 1991. Some regional investments firms invited OPRA to locate in Hengelo. OPRA needed 14 years to develop their first commercial gas turbine package, the OP16 (Figure 1.1b) in which the gas turbine engine (Figure 1.1a) is placed. Ever since, the company transitioned from a developing to a more production oriented firm. Currently, there are roughly 80 people working at OPRA. An overview of the organizational structure at OPRA is provided in Appendix A. After the family Mowill sold the majority of their shares in 2017, the Chinese Energas group became the new owner of the company.



(a) The gas turbine engine

(b) The OP16 package

Figure 1.1: Gas turbine setup

The name OPRA is derived from the words Optimal Radial. The customer can choose to customize the OP16 with one of the three combustors, dependent on the available fuel compositions at the customer. A single OP16 can transform the fuel to approximately 2 MW power along with corresponding released heat. Due to its efficient, compact and reliable design the OP16 is often used by globally located customers in the oil and gas extraction industries. OPRA believes that clean natural gas will have a key role in providing the world's energy during the transition from fossil fuels towards renewables.

1.2 Research motivation

Gas turbine packages require preventive scheduled maintenance and corrective maintenance when a failure occurs over their life-time. To prevent the downtime of these gas turbines, OPRA is stocking spare parts at their central warehouse in Hengelo or locally at customers. Nevertheless, stocking too much spares causes financial risks, thus stocking the right spares is a challenging task.

1.3 Problem description

Currently, different problems arise concerning the spare part inventory management at OPRA. OPRA is managing one central service warehouse to cover their spare part demand. Hereby, the transportation time from the suppliers to OPRA's central warehouse can be eliminated. When a spare part is not available in the central warehouse, it is back-ordered, which causes additional transportation time.

To serve each customer, OPRA is providing the alternative to engage a service contract among with the delivered gas turbine package. In these service contracts, as we call a Long Time Service Agreement(LTSA) within this project, various service differentiation options are included. Currently, around 25% of the gas turbine packages are delivered with an LTSA. Nevertheless, OPRA has the goal to increase this ratio by procuring gas turbine packages with LTSA's only in the near future. Since these 'premium' LTSA customers are paying more to receive additional service, stocking the right spares becomes more important.

At the moment, the spare part levels, considering their related demand, are perceived as high. The service warehouse, where the spares are stocked, has an inventory value of €547.124. Approximately, 30% of this inventory value(€162.662) is assigned to spares which were demanded over the last five years. If stationary demand arises, it takes on average 23 years to market the current stock of these 'active spares'. Given that the spare parts have an average lead-time of 30 days, we conclude that the current spare part levels are significantly high. In contradiction, there are 22 recently demanded spares while they are not stocked. Hence, these spares are directly back-ordered if demand occurs, so inherent with low service levels.

We label the high inventory costs and the low service levels as the observed problems and discuss their underlying causes. To analyse the context of the problems, the book Geen Probleem [Heerkens & van Winden, 2012] is used. In Figure 1.2 the causal relationships are represented. The root problems, illustrated in the grey nodes of the figure, are discussed in the subsections below.

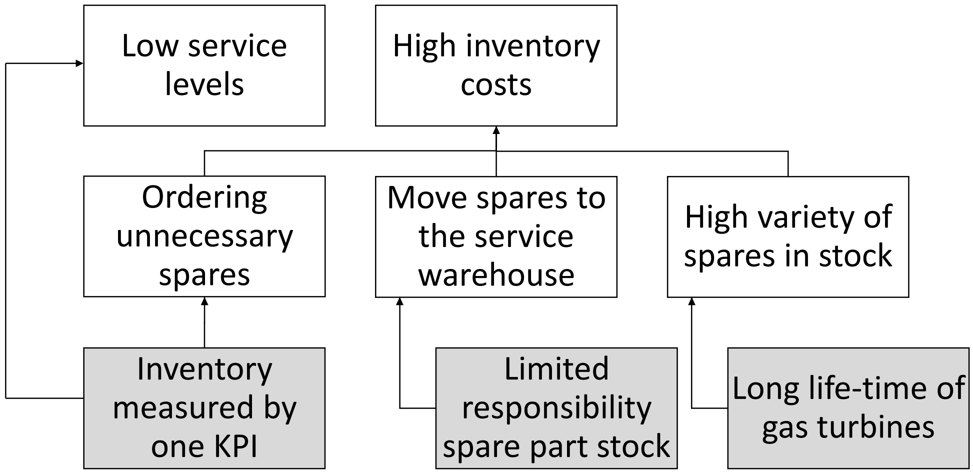


Figure 1.2: Problem cluster

Long life-time

One cause of these high inventory levels is the presence of old gas turbines. When a gas turbine is fully written off, it might be kept as a back-up option. This results in a high variety of gas turbines, thus also a high variety of spares required to cover these gas turbines. Considering that no agreements are made according the length of spare part stocking, a high variety of spares is represented in the warehouse. We assume that these 'old' spares have a high relation with the 70% of inventory value caused by inactive spares.

Limited responsibility

Another cause was the limited responsibility for spares in the past. When other departments decided to discard their obsolete SKU's, the service warehouse adopted these SKU's as spares in their warehouse. Recently, the Key Performance Indicator (KPI) 'inventory investment' of the service warehouse is assigned to the Aftermarket department to increase the responsibility.

Measuring spares by one KPI

When a gas turbine package is delivered, the Aftermarket determines, based on expert knowledge, which spare parts and what quantities are required over the life-time of the gas turbine package. These decisions are made without considering the current stock, the demand or their corresponding service level. Also, it is unknown which parts are installed at the customer site. Considering the spares are measured by one KPI only, the related service level information is lacking. This causes high inventory levels due to unnecessary stocking and low service levels because of delayed or lacking stock replenishments.

Conclusion

Based on the findings from the causal problem analysis, this project is focused on the inventory measurement. The long life-time of gas turbines and the limited responsibility for spares on stock are problems that we cannot solve or problems that are already partially solved. Therefore, the measurement by one KPI is selected as the most urgent problem in this project. Consequently, we define the following core problem:

It is unknown how the spare parts are affecting the service levels and how these service levels can be assigned with their corresponding financial risks

Thus, the goal of this research is to develop a method that considers the trade-off between the service level and the inventory risks.

1.4 Research scope

The research is scoped to describe the boundaries of the research project. At first, only the Stock Keeping Units (SKU's) which are located in the service warehouse, are considered. We label them as spare parts since they have the main purpose to serve customers when a failure occurs. Spares which are needed for corrective maintenance have a higher impact when they are back-ordered compared to the parts needed for preventive maintenance [Basten & Ryan, 2015].

To prevent the pollution from old data, a time-interval of the last five years was analyzed. This time-interval is determined in consultation with the After-market department. The main data is retrieved via the ERP-system. Additional expert interviews are conducted to clarify process steps. Finally, the inventory investment is used to represent the financial inventory risks within this research. No additional KPI's are introduced due to the lack of information.

1.5 Research approach

Given the research scope and the problem context, the following main research question is stated.
How can the current inventory management and control policy for the spare parts be improved considering the trade-off between the inventory risks and the end-product downtime?
To answer this main research question, five research questions, with the corresponding sub-questions, are formulated. These research questions represent the main chapters of this report.

Research question 1: How is OPRA managing their spare parts stock?
1.1 How is the spare part demand forecasted?
1.2 How are the inventories controlled?
1.3 How often a stock-out of a spare occurs?
1.4 Which effects do these stock-outs have to the service level of a gas turbine package?
1.5 Which spare parts are compound demanded?
1.6 Which service differentiation techniques are applied?

The first research question increases the understanding of the current situation. We gather the information by executing expert interviews, consulting the EPR-system and reviewing internal documents. The expert interviews are applied to determine which spare parts process steps are taken. The ERP-system will be consulted to identify the spare part characteristics. By reviewing the internal documents additional information is gained.

Research question 2: What literature is available that relates to our main research question?
2.1 Which demand spare part forecasting methods are available in literature?
2.2 Which spare part inventory control policies are available in literature for single and compounded demand?
2.3 Which service differentiation techniques are available in literature?
2.4 What are the benefits and limitations for each promising forecasting, inventory control and service differentiation methods?
2.5 Which of the found literature is the most promising for OPRA?

A literature study will be conducted to find potential interesting methods which could contribute to our main research question. We briefly review the advantages and limitations of the methods and select some potential interesting methodologies.

Research question 3: Which method suits the best for solving the problem at OPRA?
3.1 Which methodologies are proposed based on the literature study?
3.2 How do the methods perform, given the context at OPRA?
3.3 Which method is proposed to solve the problem?

Based on the findings of the literature study, research question 3 formulates a general model to solve the problem at OPRA. The results of the different methods, given the context at OPRA, are analyzed, discussed and if necessary extended. It closes with a final model proposition.

Research question 4: How can the performance of the proposed model be validated?
4.1 Which validation and verification techniques can be applied?
4.2 How sensitive are the parameters to the model performance?
4.3 What is the effect of the control policy based on numerical experiments?

At this stage the proposed model is validated and verified to guarantee the corresponding outcome. Next, we test different numerical experiments which are briefly analyzed and translated to the consequences for OPRA.

Question 5: How should the proposed method be implemented at OPRA?
5.1 What activities should be carried out for a successful implementation of the proposed model?
5.2 How do these activities have to be carried out?
5.3 Which further improvements are suggested?
This last phase guides the implementation of the chosen model. An action plan will be drafted to identify all the steps should be taken. Furthermore, future suggestions are provided to strengthen the results.

Answering all main research question leads to the following deliverables of this project:

- A computer model which determines a stock level by considering the trade-off between the service level and the linked financial impact.

- The findings of numerical experiments discussed in a report.

1.6 Report structure

The report is structured as follows. In Chapter 2 the context of spare part management at OPRA is discussed. The literature study can be found in Chapter 3. Chapter 4 is applying the findings of the literature study to a solution approach. Chapter 5 is covering the validation, verification and the numerical analysis of the model. Finally, in Chapter 6 the conclusions, recommendations and limitations of the research are discussed.

2 Context

In this chapter the current situation, concerning spare part management, is described. In section 2.1, the spare part groups are analyzed, while in Section 2.2 the current forecasting procedure is discussed. The most important spare part characteristics are described in Section 2.4. The sequential Section is focusing on the stocking policies at OPRA. Considering that OPRA is not treating every customer equally due to service agreements, these agreements and their corresponding service differentiation methods are explained in Section 2.5. To measure the service performance of the stock, we consult a short literature study in Section 2.6 which results in the introduction of an additional Key Performance Indicator. Finally, the chapter is closed with conclusions in Section 2.7.

2.1 Spare part grouping

We divide the spares located in the service warehouse into the groups: the active spares (last 5 years demanded) and the inactive spares. Due to data limitations, the spare part sales data is used to determine the demand. Within this research, we focus on the active spares only. Approximately 30% of the active spare parts are assigned with compounded demand. We find two causes of this compounded behaviour:

- Batching by customers. It occurs that customers order multiple spares to maintain their own stock on site. LTSA customers spares are replenished by OPRA's central warehouse, while regular customers maintain their own stock. However, no clear data concerning this subject is provided.
- The second cause of the compound arrivals is combining preventive and corrective maintenance. For example: when a failure occurs, the broken spare is replaced among with a commonality of spares required for preventive maintenance. OPRA is not tracking this data either.

Additionally, we split the active spare parts into the single and the compound demanded groups. We assume that the single demanded spares are required for corrective maintenance exclusively. On the other hand, the compounded demanded spares can be required for preventive and corrective maintenance simultaneously. In Figure 2.1 we describe the spare part hierarchy used within this research. The planned spare part demand creates a deterministic demand, while unplanned maintenance creates a random spare part demand [Basten & Ryan, 2015]. Considering that the explicit urgency of the compounded spares is unknown, we are enforced to model these compounded. There is chosen, in conformity with the Aftermarket manager, that all active spares are treated equally based on their criticality.

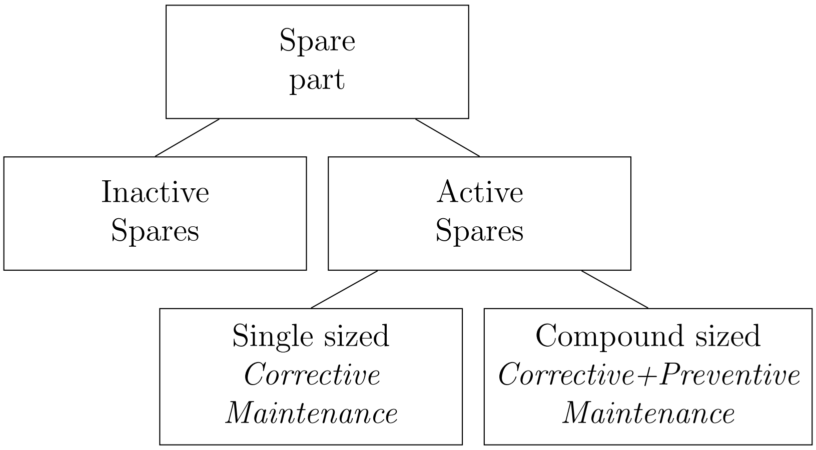


Figure 2.1: Spare part hierarchy and the relation with maintenance

2.2 Forecasting

The availability of spare parts has a high impact on the performance of the gas turbines delivered by OPRA. When a gas turbine fails, the downtime can significantly be reduced when the right spare part is available on site. But stocking too many spare parts can lead to huge investment cost losses. Therefore, an accurate spare part forecast is an essential starting point for spare parts management. [Van Jaarsveld & Dekker, 2011].

Currently, OPRA has five gas turbine packages where unplanned and planned spares are stocked on customer site. This is agreed within the corresponding LTSA. The decision to stock these items and in which quantities is based on analyzing historical data and consulting expert knowledge. Unfortunately, no clear replenishment data available is available of the 'forecasted' spares, which makes it hard to measure the performance. Hence, since these forecasted spares cover a small fraction of the spares which we labelled as active, the practical usage of the current 'forecast' is limited.

2.3 Spare part characteristics

Now that we identified the interesting spares for this project, their corresponding characteristics are discussed in this section. We label the following three spare part characteristics are the most important: demand, price and their lead-time. In the subsections below they are further analyzed.

Demand

We identify an extreme slow moving spare part demand at OPRA, which is shown in a box-plot of the demand of all active spares in Figure 2.2. The median demand value of 0.2 per/year, which is also the minimum in the box-plot, shows that most of the spares are demanded once in the time-interval of five years. Due to the higher variety of the compounded spares, these spares have a higher correlation to the higher demand. Unfortunately, this slow moving demand leads to a limited set of data-points which consequently restricts the appliance of a forecast. The data-set is too small to perform a useful forecast outcome. Therefore, it is chosen to model the demand stationary within this project. This means that we assume that that events which occurred in the past, also occur in the future.

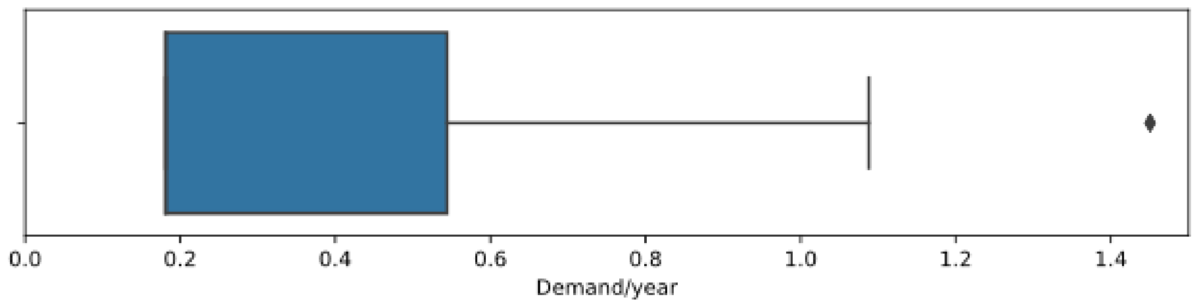


Figure 2.2: Box-plot demand of spares

Due to the high intermittency of demand, a discrete distribution is expected to perform better than a continuous distribution [Syntetos *et al.*, 2011]. At this stage of the research, the Poisson distribution is expected to perform the best with our extremely slow moving spare part demand. The literature study in the next chapter will review alternative distributions.

Prices

When we look to the spare part prices, a high variety between the spares is observed. Stocking higher valued spares is often coherent with more financial risks. Thus, these risks have to be aligned with service level to the customers. In Figure 2.3 a box-plot of the prices is provided. There are some high-value outliers which were cut off to emphasize the rest of the plot. The green interquartile range indicates that most spares have a price range between €50 and €800.

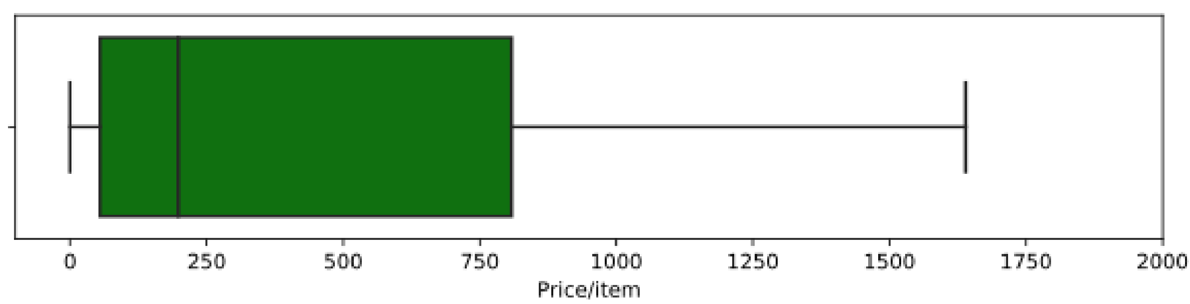


Figure 2.3: Box-plot price of spares (€)

Lead-times

The inventory at OPRA is used to overcome the lead-time between the suppliers and OPRA itself. Thus, spares with high lead-times require more items on stock to maintain the same fill-rate compared with lower lead-time items. Figure 2.4 shows the distribution of the spare part lead-times at OPRA. The lead-times of OPRA are consistent enough to model them as a fixed values. Subsequently, the correlation between these three spare parts characteristics (demand, price and lead-time) are computed. We identify a small correlation between spare part prices and their lead-times. This seems like a logical cause since expensive spares often have a higher level of complexity.

Figure 2.4: Box-plot lead-time of spares (days)

After analyzing the spare part characteristics we come to the following findings. The spares at OPRA are extremely slow movers which limits the appliance of a forecast. We also identified the compound demand of spares, which is caused by customer batching and combining corrective and preventive maintenance.

2.4 Spare part stocking

To fill the items on stock OPRA uses an (s,Q) inventory control policy. Because 89 of the 91 active spares have an order quantity which equals one, the (s, Q) inventory control policy equal to the (S-1, S) inventory control policy. This implies that every time an order is fulfilled, OPRA's initial stock levels will be restored. There are 28 spares which are assigned with a safety stock. These stock levels are determined by shortages that occurred in the past. The stock levels are continuous reviewed by the ERP system, which called SAP Business. When the inventory positions drops below the safety stock, the system advises to re-order the stock. When a spare part order cannot be fulfilled due to stock limitations, other warehouse sections might be replenished. Since this stock is initially reserved for other projects, these actions might interfere them. Therefore, OPRA prefers to give the Aftermarket department the authority to replenish from their own warehouse only. If the spare cannot be delivered by OPRA from stock, OPRA will place an order at their supplier. Hence, the demand of the OPRA's customers will be back-ordered if it cannot be fulfilled from OPRA's stock.

2.5 Long Term Service Agreement

OPRA is offering additional service options if preferred by the customer. An agreement between OPRA and the customer is drafted where a variety of service conditions are harmonized. We denote this agreement as an LTSA (Long Term Service Agreement) and their corresponding customers as LTSA or premium customers. At the moment 25% of the gas turbines packages are bounded to an LTSA. One of OPRA's goals is to shift their current production oriented revenue model towards a more service oriented revenue model. Therefore, in the future OPRA prefers to sell gas turbines with an LTSA only. This will lead to the intensification of these service differentiation methods. OPRA is applying the following three service differentiation techniques to these packages:

Two-echelon system

When we stock spares at ones single location, we have a single-echelon system. One of OPRA's differentiation options is the stocking of spare parts at customer. This is called a two-echelon system, which is shown in Figure 2.5, because spares are placed at two locations.

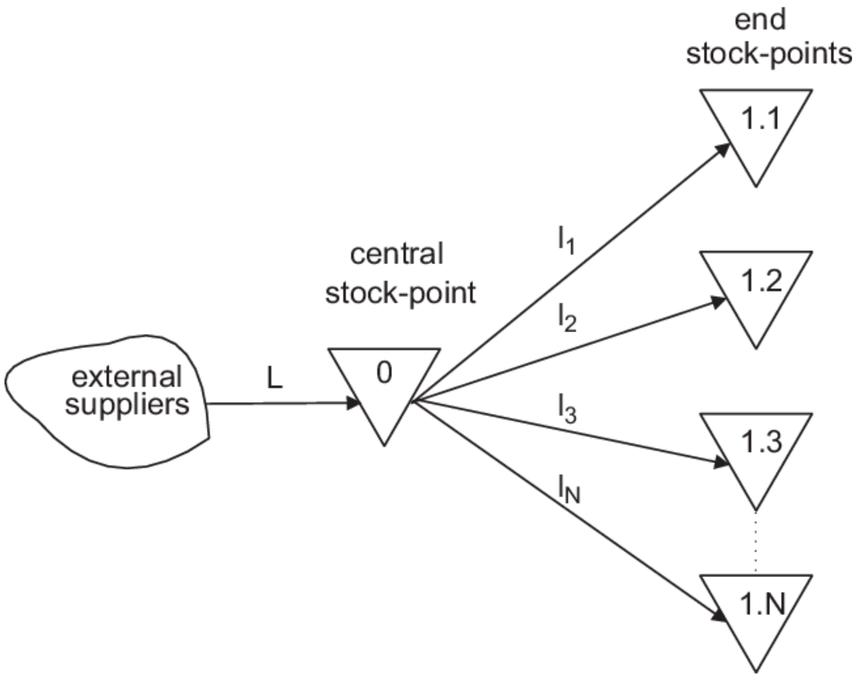


Figure 2.5: A two-echelon inventory system [Lagodimos & Koukoumialos, 2008]

This approach has a major advantage: the transportation time from OPRA to the customer can be eliminated. OPRA will remain the owner of the spare during the LTSA duration. When the LTSA is expired and the part is not used, it will return to the central warehouse in Hengelo. There are five gas turbine packages, at three different customers, which are applying this two-echelon system. The current value of the spares at customers varies between €10.000 and 60.000 €per customer. As we mentioned in the previous section already, the local demand data is limited, which makes it hard to use. Given that we cannot serve all customers by a two-echelon system, we approach a single-echelon stocking procedure within this research.

Back-up gas turbine

Another service differentiation technique that is applied by OPRA is using a back-up gas turbine. There are two gas turbines which have the purpose to serve as back-up. The one located in the central warehouse in Hengelo is covering the European LTSA customers, while the other gas turbine is located and used in North-America.

Stock differentiation

The last service differential method we discuss is stock differentiation. When OPRA sells all stock from an SKU to a customer without an LTSA, the situation might occur that an LTSA customer cannot be served with this spare. To prevent this situation, OPRA is not always selling their full on-hand inventory. These decisions are based on expert knowledge which makes it hard to measure the impact.

2.6 Performance Indicators

Currently, the spare parts are measured by one Key Performance Indicator (KPI), the inventory investment. Nevertheless, a second KPI is required to measure the service level of these spares. Therefore, we conduct a short literature study in this section. There are two often used measurement methods in spare part literature: the fill rate and the ready rate. The fill rate is calculating the amount of demand that can be replenished directly from stock. The ready rate is the fraction of time that the on-hand stock is positive. Considering that most spares are demanded with an order size of one, the fill-rate is equal to the ready rate if these spares arrive according to the Poisson distribution [Axsäter, 2015]. Since the fill-rate contains replenishment information and we deal with compounded arrivals, the fill-rate gives a better representation of the corresponding service level. Thus, the fill-rate is used as second KPI to describe the service performance of the stock.

To analyze the service performance of the current spare part stock at OPRA, the fill-rates are computed for all spares. This implies that we have to conduct literature to determine the fill-rates. The demand is modelled stationary, which means that events that occurred in the past will also occur in the future. For the compounded demand, the Compound Poisson distribution is used, while the single demanded spares are modelled with the Pure Poisson distribution. These approaches require a different fill-rate calculation which is explained in the subsections below.

Fill-rate Pure Poisson

[Palm, 1938] queueing theorem states that if demand is Poisson distributed, and a (S-1,S) policy is used, we can compute the steady state probability distribution of the number of units in resupply at a random point of time. When the lead time of an item and the average arrivals per time unit for that corresponding item are known, the number of items in a queue can be computed with Little's Law (L= λ*W). Here λ represents the arrival rate and W the average time spend in a system. We are using notation T to define the average time spend in a system. The following formula is used to compute the number of units in resupply, where h(x) is the steady state probability that x units are in resupply.

$$h(x) = p(x|\lambda T) = \frac{(\lambda T)^x e^{-\lambda T}}{x!} \qquad\qquad 0 \leq x \leq \infty. \tag{2.1}$$

Consequently, it can be computed what the cumulative sum of probabilities over all the SKUS's in stock is. This represents the probability that the stock on hand is positive (the ready rate). Considering that the demands sizes are one, this is identical to the fill-rate β. So the fill-rate, given that S items are stocked is computed with:

$$\beta(S) = \sum_{x=0}^{S-1} p(x|\lambda T) \tag{2.2}$$

Fill-rate Compound Poisson

Since formula 2.2 only holds for items with demand sizes of one, an alternative fill rate approach is required for the SKU's compounded spares. The (S-1,S) is not applicable anymore, due to the compounded arrivals. Hence, it is chosen to apply a (s,Q) policy for these compounded SKU's. Hereby Q represents the replenishment order size and s the re-order level. Given that the inventory position (IP) follows a uniform distribution, the steady state distribution of the net stock (NS), can be computed by:

$$P(NS = j) = \frac{1}{Q} \sum_{k=max(s+1,j)}^{s+Q} P(D(L) = k - j) \tag{2.3}$$

[Axsäter, 2015] computed the fill-rate for compounded demand as the ratio between the expected satisfied quantity and the expected total demand quantity. This implies that orders can be fulfilled partially. For example: when a customers needs two items of a part and only one is available, the customers is 50% satisfied. The following formula is applied to calculate fill rate β:

$$\beta(s) = \frac{\sum_{k=1}^{K} \sum_{j=1}^{s+Q} min(s,k) f_k P(NS = j)}{\sum_{k=1}^{K} k f_k} \tag{2.4}$$

where, f_k describes the empirical probability distribution that an order of size k occurs, S is the order-up-to level and capital K are all the orders that occurred over a predefined time-interval.

Fill-rate results

When computing the fill-rates for all active spares we directly indicate that most of the spares have a fill-rate of 100% or 0%. There are only 22 spares which have a fill-rate between 1% and 99%, which is visible in Figure 2.6. This indicates that when a spare is stocked, the fill-rate is significantly high. For example: the stock investment of one spare part could be reduced with €9.698 and still maintain a fill-rate of 99.5%. By investing this into spares with lower fill-rates, their service levels can be improved. Thus, a distribution of the investment based on their fill-rates could improve the service level.

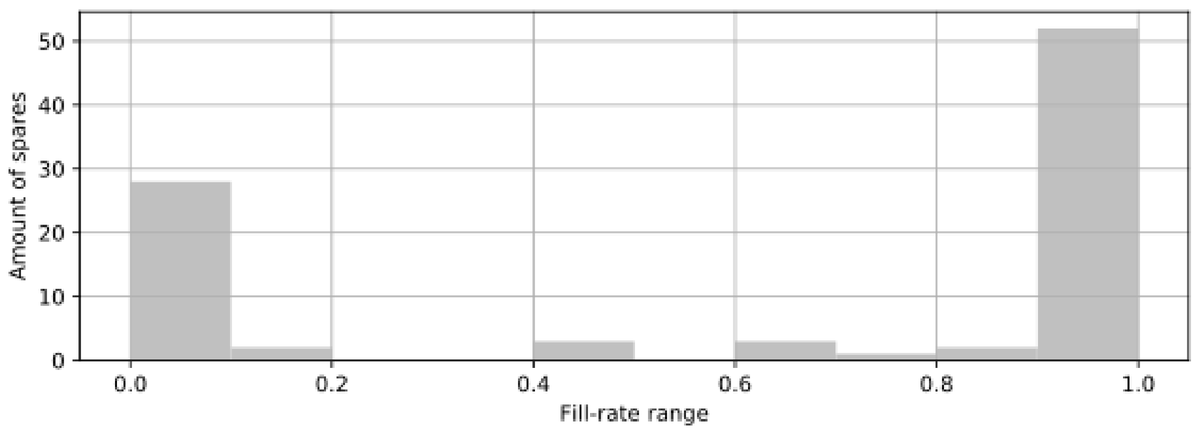


Figure 2.6: Histogram of the current fill-rates

To identify which spare characteristics have high fill-rates, we chose to divide the spares in two equal sized groups based on their fill-rate value. In Figure 2.7 the three spare part characteristics are plotted in their corresponding fill-rate group. It shows that OPRA currently is maintaining higher demanded, lower valued spares on stock. The lead-time effect to the fill-rate seems less interesting.

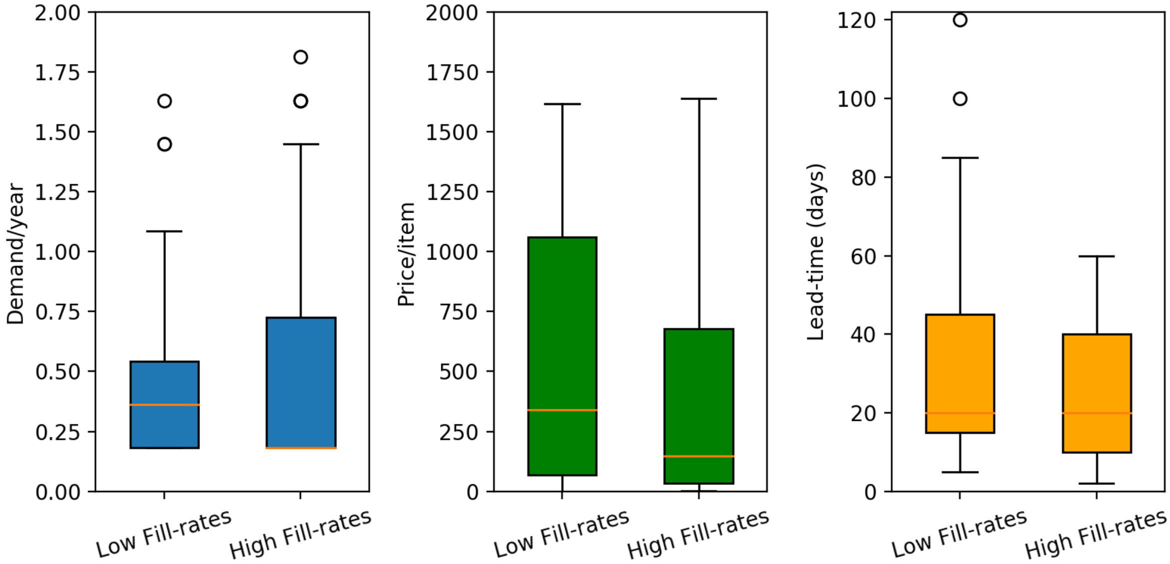


Figure 2.7: Fill-rates effect to spares

2.7 Conclusions

Hereby, we can say something about the spare part inventory management situation at OPRA. We summarize the key findings of this chapter in the list below. These findings serve as input for the literature study in Chapter 3.

- Extreme slow moving spares restricts the appliance of a forecast.
- Urgency of the 30% compound spare part demand unknown, thus enforced to model it compounded as well.
- Spares are back-ordered, if they cannot be replenished.
- Single echelon approach required to cover all spares.
- The 25% premium demand ratio tends to increase.
- Stock service differentiation is applied, based on expert knowledge.

3 Literature study

In this chapter the literature related to our main question is discussed. It is organized as follows: in Section 3.1, spare part modelling literature is considered. Section 3.2 is focusing on spare part inventory management systems and in Section 3.3 literature concerning a solution approach of these systems are analyzed sequentially. Finally, we close the chapter with a conclusion is Section 3.4.

3.1 Demand modelling

To start this literature review, the demand modelling literature is consulted first. A short subsection is spend on forecasting literature which might be used in the future, but not within this project. Also, various distributions are discussed in the spare part distribution subsection.

Forecast demand

In the previous chapter the applicability of a forecast was already discussed. Considering the sales growth and the importance of spare part stocking, the demand data-set might be expanded thus a forecast might be interesting then. Therefore, we introduce the following four potential forecast methods for spare parts:

- The simple moving average
- Wrights modified Holt's method
- Crontons method
- Bootstrapping method

Further on in this project, the spare part demand will be modelled stationary. This means that the statistical demand properties will be the same in the future as they have been in the past.

Spare part distributions

Spare parts are often modelled with different distributions compared to 'normal' Stock Keeping Units (SKU's). Due to the high intermittency of demand arrivals discrete distributions are often used. Therefore we first discuss the most applied spare part demand distribution: the Poisson distribution. The demand for spare parts follows the Poisson distribution when the event occurs at a constant average rate and in any interval independent of the number of events occurring [Archibald & Silver, 1978]. Although the essence of intermittent demand makes it natural to use discrete distribution, there are some non-compound continuous distributions that have been used in literature to model demand arrivals. For SKU's with long lead-times, the demand will sum up over this period and therefore the central limit theorem is applicable. This makes the normal distribution a decent distribution to model the lead-time demand. Therefore, [Silver *et al.*, 2016] stated the following rule of thumb: If the mean lead-time demand is at least 10, a continuous demand distribution can be used. Otherwise, a discrete distribution model like the pure Poisson, the Gamma, compounded Poisson or an Erlang-distribution is recommended. [Syntetos *et al.*, 2011] developed a classification scheme to identify the best fit to the data dependent on the average inter-demand interval (p) and the squared co-efficient of variation of demand sizes (CV^2), which is shown in Figure 3.1. Applying this approach to OPRA's spares leads to classifying each single sized demanded spares as Poisson and each compound demanded spare as Stuttering Poisson/Negative Binomial Distributions.

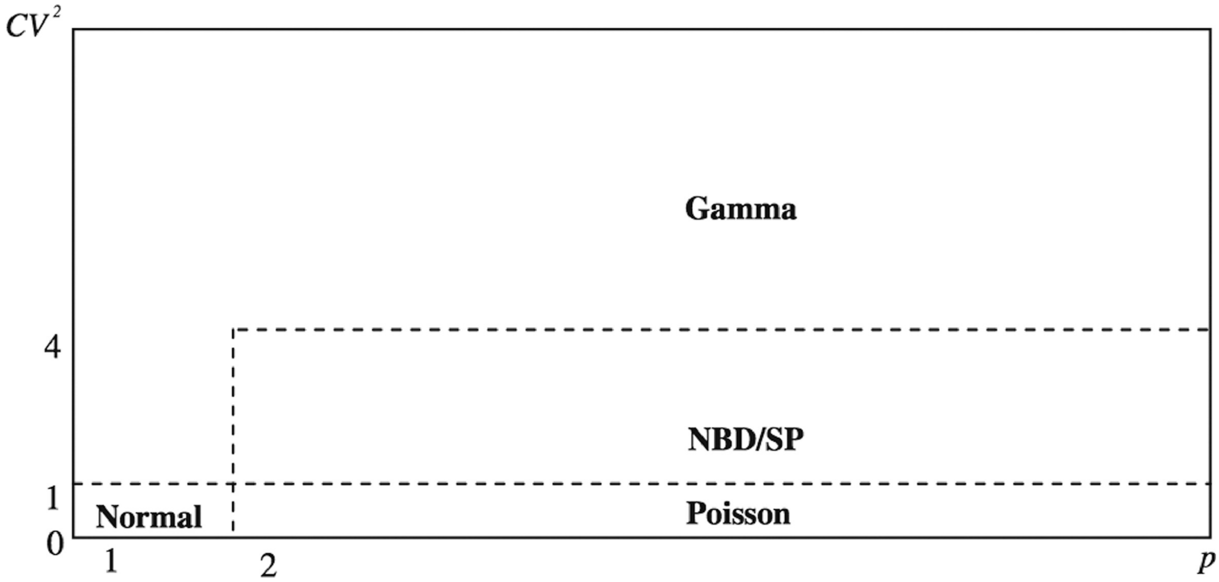


Figure 3.1: Distribution classification scheme. Source: [Syntetos *et al.*, 2011]

A limitation of the pure Poisson approach is that order sizes are always one. As the previous chapter already suggested, this is not necessarily realistic. Therefore, an alternative approach is preferred to model the compounded spare parts. The compound Poisson demand process is an extension of the Pure Poisson process which assumes that order sizes are arriving compounded. This approach was already applied in the previous chapter to compute the fill-rate of the spares. Whereas a Poisson failure process implies a constant failure rate, [Saidane *et al.*, 2010] assumes an Erlang-distribution because the failure rate is dependent on time. Over time, a spare part is assigned with a higher probability of failure. The demand sizes are determined by a Gamma distribution. This prevents the overfitting of spares which are assigned to stock based on a Poisson based approach.

3.2 Spare parts inventory management

The last decade much research was devoted to spare part inventory management systems. The complex problems like the presence of intermittent or lumpy demand patterns [Boylan & Syntetos, 2010], the high responsiveness required due to downtime [Murthy *et al.*, 2004] and the risk of stock obsolescence [Cohen *et al.*, 2006] make spare spart inventory systems an interesting subject. We mainly focus literature based on single echelon, service differentiated inventory models for single and compounded arrivals. In the subsection below various inventory systems are discussed.

Poisson based inventory models

By consulting the literature, it became directly clear that most of the spare part literature is based on Poisson input. If demand is Poisson distributed and replenishment lead times follow an exponential distribution, the equilibrium probabilities can be derived. The number of items in stock can be modelled as a Markov process. Since Poisson arrivals see time average (PASTA), the equilibrium probabilities can be used to determine the fill rate. The theory is that when the probability distribution is known, we can estimate the number of units in the pipeline at a random point in time based on Palm's theorem [Palm, 1938]. In 1966 [Feeney & Sherbrooke, 1966] developed one of the first (compound) Poisson based, single-site operating inventory policies. The demand can be modelled via a back-order case or the lost-sales approach.

Multi-item approach

By applying a single-item analysis, the inventory levels of each individual item are analyzed independently. However, customers are not interested in item specific performance. In 2006 [Sherbrooke, 2006] proofs that analyzing by a multi-item approach, also known as a system approach, gives significant savings in the inventory investments compared to a single-approach. Under a multi-item approach all the parts in a system are considered when making stock decisions. The main purpose of this methodology was to minimize the impact of critical items in a system that result in down-time of the whole system.

Single-item service differentiation

There are multiple options to differentiate service levels among customers available in literature. Considering that differentiated service levels can be offered by assigning stock to specific customers, literature covering so called *critical level policies* is consulted. The problem of multiple demand streams from different customer classes was first introduced by [Veinott Jr, 1965], who introduced the concept of critical level policies (CLP). With a CLP, items are only delivered to a certain customer class as long as their on-hand stock is above the critical level of the customer class. By withholding items, they are reserved for customers with higher service level requirements. Since then, many extensions and modifications of this theory are proposed. In figure 3.2 an example of a (S,c) policy for a single-item can be found, where event 1 represents premium demand occurrence, event 2 is regular customer demand and events labelled with R are replenishments of the inventory.

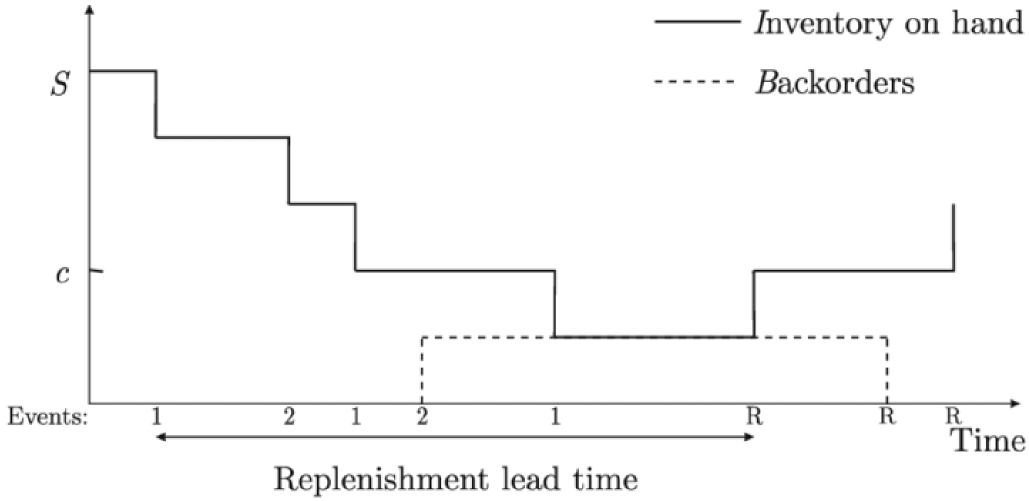


Figure 3.2: An example of the critical level policy. Source: [Enders *et al.*, 2014].

In 1997, [Ha, 1997] derived the optimality of the critical level policy for a continuously reviewed, Poisson demanded on lost sales based model. While [De Véricourt *et al.*, 2002] adjusted this method to a back-ordering based case, it obtained the same results. He also emphasized the need for a clear heuristic if the set of demand classes is higher than one. This is because for each class, an extra dimensional sub-problem is added and therefore applying complete enumeration becomes computational intensive. [Möllering & Thonemann, 2010] analyzed a periodic review inventory system with back-ordering demand where low and high priority customers are served. The inventory is controlled by a (S,c) policy, where S is the order up to base stock level and c is the critical level.
The fill-rate is the most commonly used key performance indicator to represent the service level of the corresponding stock. By using a continuous-time Markov Chain, the steady state distributions can be computed. Considering, that we are interested in back-ordering based models, a two-dimensional state space is required to model the number of back-orders per state [Alvarez *et al.*, 2013]. Since the state probabilities are hard to compute due to the additional dimension, an upper bound is computed to approach the state probabilities. Thus, back-ordering demand leads to additional complexity to the model. Though, no literature which considers critical level policy for compounded arrivals is found.

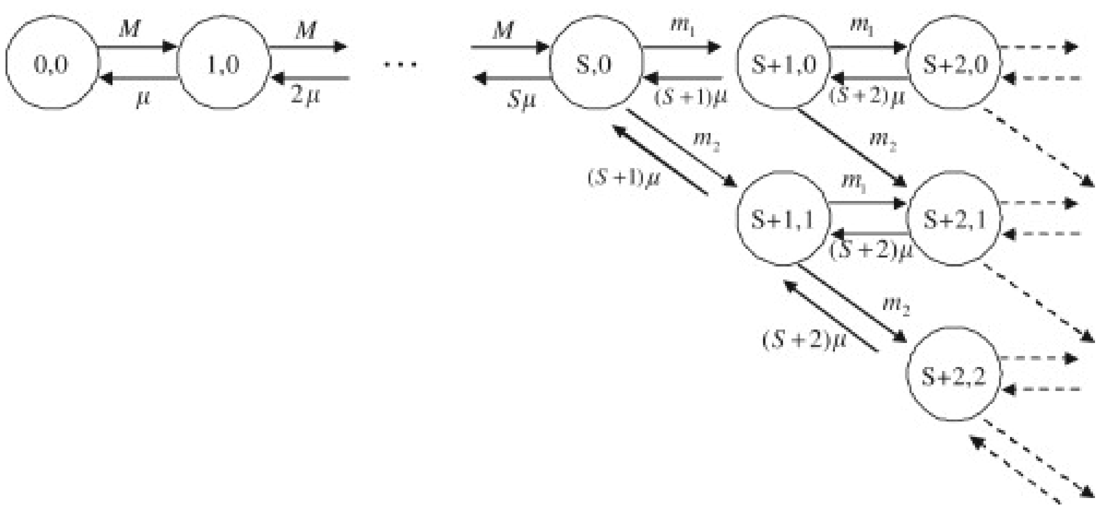


Figure 3.3: Markovian transition diagram for a back-ordering model . Source: [Alvarez *et al.*, 2013].

Multi-item service differentiation

Considering that customers are not interested in the single-item performance, all the items in a system are approached. Consequently, [Cohen *et al.*, 1992] proposed an extension of his earlier single-item model to a multi-item service differentiation model for a (s,S)-policy. Nevertheless, this is approach is not using a critical level policy. Consequently, [Kranenburg & Van Houtum, 2004] presented a multi-item model, single location, with critical level policies based on lost sales. By meeting a target service performance for all items together, savings up to 50% could be achieved compared to a single-item approach where the the target has to fulfill each item.

Conclusion

Apparently, the compounded based spare part inventory literature is limited. Thus, an alternative approach is required to model this demand within our project. To differentiate between customer classes, various critical level policy literature is consulted. All the corresponding proposed models assume that demand is lost when it cannot be delivered (lost-sales). Furthermore, no back-ordering based models with a critical level policy are found in literature, which might be caused by the inherent additional complexity. Thus, to apply a critical level policy we are limited with two options: modify a lost-sales model to a back-ordering model or apply the lost-sales model to model back-ordered demand. Considering the time investment, the extra complexity and the potential added value, we chose to apply the lost sales model for our demand. Moreover, because of the benefits a multi-item approach offer, the decision was made to apply the model proposed by [Kranenburg & Van Houtum, 2004] and modify it for compounded arrivals when needed.

3.3 Solution approach

The approach of an optimal or near-optimal solution is highly dependent on the complexity of the model. For most of the single-item approaches the main problem is divided into several sub-problems (item individually). Considering it is known in an earlier stage, compared to a multi-item approach, what the optimal solution will be, the solution space is smaller. To find the optimal solution of a method with integer-valued and non-linear constraints (multi-item approach), complete enumeration seems the only possibility. From a computational view this is not very efficient. Therefore, heuristics are often used for spare part inventory control methods, particularly for multi-item methods. In the remainder of this subsection various heuristics are described.

Greedy heuristic

An efficient way to apply the multi-item approach is a technique called marginal analysis/greedy. It iteratively selects the alternative that provides the biggest bang for the buck. The algorithm stops when a certain stopping criterion is met: maximum budget or minimum service level. [Sherbrooke, 2006] used this method develop an optimal back-order versus inventory value curve. The optimality of this method can be proven by the convexity of the function, see Figure 3.4 for an example of a convex and non-convex function. The limitation of this approach is that the convexity of a function cannot be proven when an extra dimension is added due to a critical level. Thus we need an alternative approach for this problem.

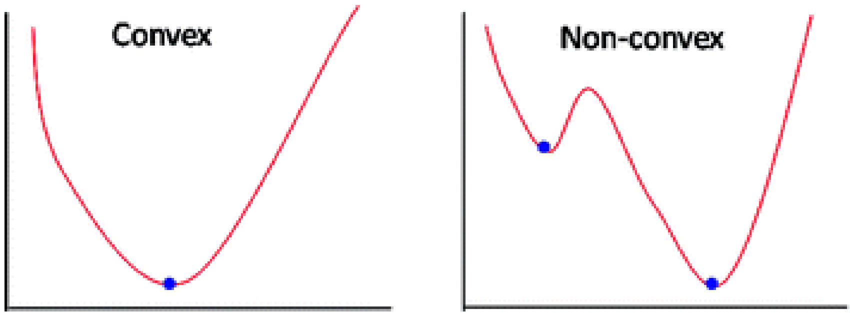


Figure 3.4: Example of an convex and a non-convex function. Source: [He *et al.*, 2010]

Dantzig-Wolfe decomposition

Using Dantzig-Wolfe decomposition, a multi-item problem with differentiated service level constraints can be decomposed into a single-item cost minimization problems with multiple demand classes. [Kranenburg & Van Houtum, 2004] used this approach by adding all policies as column and restricting that at least one policy is chosen. First, it implies that policies can be chosen partly, when no further improvements can be made, a restriction is added to prevent this.

Lagrangian relaxation

Another technique that is often used for multi-item problems with aggregated service level constraints is the Lagrangian relaxation. The solution procedure lead to a lower bound for the optimal costs and a heuristic solution. This lower bound can be used to evaluate the accuracy of the heuristic.

Conclusion

Considering that a greedy-heuristic could be applied if there is no service differentiation, this would take the least amount of computation time to approach a near-optimal solution. Nevertheless, an alternative approach is required for the critical level policy. Since the Lagrangian approach is very similar to the Dantzig-Wolfe decomposition, we also chose to apply this Dantzig-Wolfe decomposition for our critical level problem. Since [Lübbecke & Desrosiers, 2005] also applied this solution approach, which could serve as guideline in our approach, we made this decision for simplicity reasoning.

3.4 Conclusions

There are multiple approaches introduced for solving the problem which is discussed in Chapter 1. The most important findings of this literature review will be adopted in the next research stage. The findings of this chapter are summarized as follows:

- Majority of spare part literature is Pure Poisson based.
- Additional extensions required to model the compounded spare parts.
- Multi-item approach expected to outperform a single-item approach.
- Greedy heuristic chosen to approach a multi-item problem without service differentiation.

- Critical level policy with back-ordering demand significant complex, thus lost-sales model proposed.
- Dantzig-wolfe decomposition chosen to approach the more complex CLP problem.

4 Solution approach

In this chapter the found literature is applied to the current situation at OPRA. The organization is as follows; in Section 4.1 it is described how the demand is modelled within this project. In the next subsection the demand and other spare part characteristics will be used within various spare part inventory model which we found in Chapter 3. The results from the models are compared with the current situation and with each-other. Various modifications are applied and explained and in the last subsection conclusions are composed.

4.1 Equal items, equal customers inventory policies

This subsection focuses on inventory policies that do not differentiate between customers or spares. To analyze the current service performance of the spares, we already introduced two fill-rate approaches in Chapter 2. Considering that no service differentiation is applied, equations 2.2 and 2.4 can be applied to compute the fill-rates for Pure Poisson and Compound Poisson demanded spares. In this section the spares are treated equally, so one target fill-rate is set. This implies that that each spare is restricted to exceed the predefined target fill-rate. So, the problem can be written as follows, where c represents the value of a spare and s the stock level:

$$\text{Min } \ c * S \tag{4.1}$$

$$\text{subject to} \qquad \beta(S) \geq \beta_{Target} \tag{4.2}$$

To give a representation of the results of this single-item approach, all the spare part inventory values are initially assigned with no stock. Consequently, the item with the lowest fill-rate is increased by one unit until the target fill-rate is met. Since the lowest fill-rate of a spare is representing the target fill-rate of all spares, plot these minimum fill-rates with their corresponding investment in Figure 4.1. Thus, iteratively the lowest fill-rate of is assigned with more stock.

Considering that items have to be stocked to get a positive fill-rate, an initial investment of at least €68.429 is required in the first iteration. To compare this approach with the initial solution, the current inventory value of €162.622 is plotted as a reference line. Because currently not every SKU is stocked, the minimum fill-rate of all SKU's, which is also the target fill-rate, will become zero. Therefore, the target fill-rate of the current state is not shown in Figure 4.1. Hence, applying a single-item approach leads to multiple independent problems to be solved. Therefore, a multi-item approach might be an interesting alternative for OPRA.

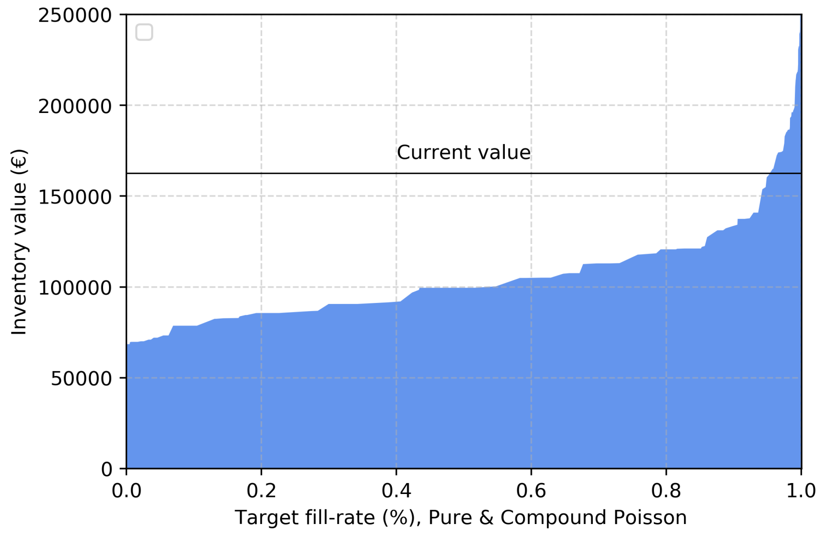


Figure 4.1: The equal fill-rate effect to inventory investment

4.2 Multi-item, equal customers inventory policies

This section focuses on inventory policies that differentiate between items, as no customer differentiation is applied. Due to the fuel variations of OPRA's customers it does not often occur that systems/gas turbine packages are exactly the same. Nevertheless, the gas turbines have a significant amount of commonality of spares, which makes an multi-item approach interesting. In accordance with the Aftermarket department, the spares are not differentiated based on their criticality.

Weighted Average Fill-Rate

To compute the fill-rates of all spares, equations 2.2 and 2.4 are used again for the corresponding spares (Pure & Compound Poisson demand fill-rate with back-ordering). For the overall performance of the inventory we take the average of the by the demand scaled fill-rates, which we label as the Weighted Average Fill-Rates (WAFR). This scaling is applied to prevent assigning higher demand spares with higher fill-rates. Hence, the weighted average fill rate is defined as the probability that arbitrary demand for a group of SKU's is fulfilled immediately [Van Houtum & Kranenburg, 2015]. Therefore, the following formula is used to compute the weighted average fill rate, where λ represents the sum of all arrivals rates, λ_i the arrival rate of a individual item and $\beta_i(S_i)$ the fill-rate of an item.

$$\beta^{obj} = \sum_{i=1}^{I} \frac{\lambda_i}{\lambda} \beta_i(S_i) \qquad i = 1, ..., I. \tag{4.3}$$

Greedy heuristic

According to [Van Houtum & Kranenburg, 2015], for each SKU, the item fill rate $\beta_i(S_i)$ is increasing on its whole domain and concave for $S_i \geq max([\lambda_i T_i - 1], 0)$. This allows us to generate a set of efficient solutions by a greedy algorithm. Whereas [Van Houtum & Kranenburg, 2015] applied his greedy approach considering the reduction in back-orders, we apply the incremental fill-rate approach for simplicity reasoning. For each SKU, the increase in $\beta(S)$ relative to the objective function that minimizes the inventory investment when S_i would increase by one unit. When an item is added to the stock, the objective function increases with c_i, while the $\beta(S)$ equals:

$$\Delta_i\beta(S) = \frac{\lambda_i}{\lambda}\Delta\beta_i(S_i) = \frac{\lambda_i}{\lambda}(\beta_i(S_i+1) - \beta_i(S_i)) = \frac{\lambda_i}{\lambda}P(x_i = S_i) \tag{4.4}$$

So, the increase in $\beta(S)$, divided by the objective function increase, can be written as:

$$\Gamma_i = \frac{\lambda_i P(x_i = S_i)}{\lambda c_i} \tag{4.5}$$

For the item with the highest value of Γ_i, the corresponding stock is increased by one unit and a new efficient solution is provided. However, this theory only holds for items which are arriving in demand sizes of one. Considering the compounded arrivals at OPRA, an alternative approach is required to handle these spares since concavity cannot be proven anymore. In Figure 4.2 the fill-rate a compounded SKU is shown. The function of an SKU transforms from convex to concave (hollowed inward). Hence, for compounded demand the transition point is required to ensure the concave function.

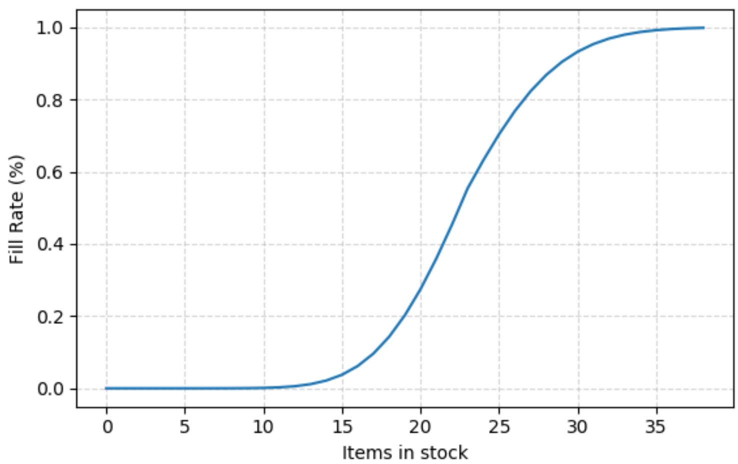


Figure 4.2: Non concave behaviour of an compounded SKU (demand size 24)

To find this transition point, for each iteration the obtained fill-rate can be subtracted from the previous fill-rate one. The maximum difference between two fill-rates describes the begin of the concave behaviour, which we denote as α_i. It is chosen to start with an empty solution for the greedy heuristic. To solve the concave problem, the heuristic considers adding α_i of items to the solution in the first iteration. The gamma calculation is changed as follows:

$$\Gamma_i = \frac{\lambda_i(\beta_i(S_i + \alpha_i) - \beta_i(S_i))}{\lambda c_i \alpha_i} \tag{4.6}$$

After at least α_i is for SKU i is added to the solution, it can increase with one unit again since concavity can be proven.

Results Greedy heuristic

The outcomes of the greedy algorithm considering Pure Poisson and Compound Poisson demand can be found in Figure 4.3. Due to the iterative behaviour of the heuristic, we can plot the investments versus weighted average fill-rate in the Figure. The results indicate that, just like the previous approach, significant improvements can be made aiming on reducing the investments or improving the fill-rate.

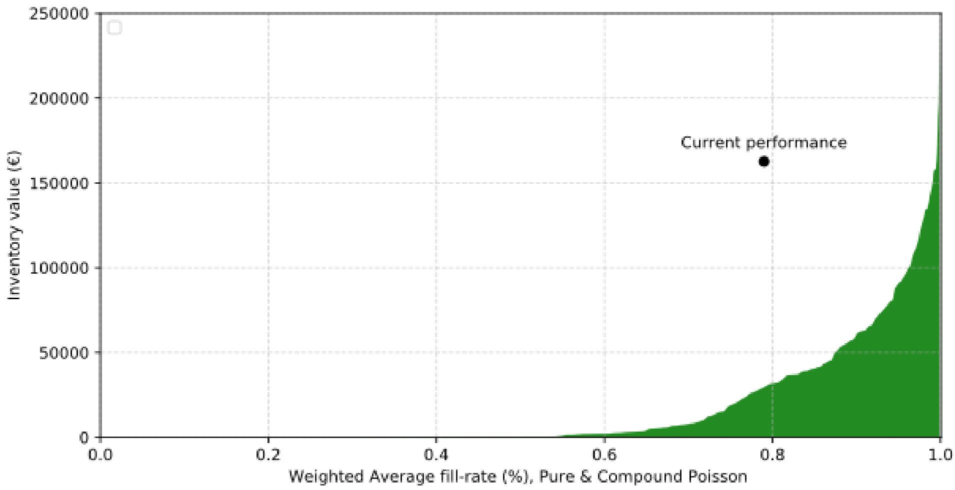


Figure 4.3: Greedy heuristic for Pure and Compound demand

To see which SKU's are added in each iteration, the spare part characteristics are plotted in the Figure 4.4. It shows that the SKU demand and their corresponding value are affecting the heuristic the most. The high demanded, low-value items are added firstly and later on, more expensive spares are assigned to the solution. When the model approaches a fill-rate of 0.8, previous selected spares are assigned again. Furthermore, the exponential behaviour of the fill-rate versus the investments results in more iterations. Hence, low value, high demanded spares are the most beneficial to maintain on stock. Nevertheless, for higher service performance more varying spare part stock is required. Also, approaching our defined core problem by a multi-item approach appears to outperform a single-item approach since spares can be treated differently.

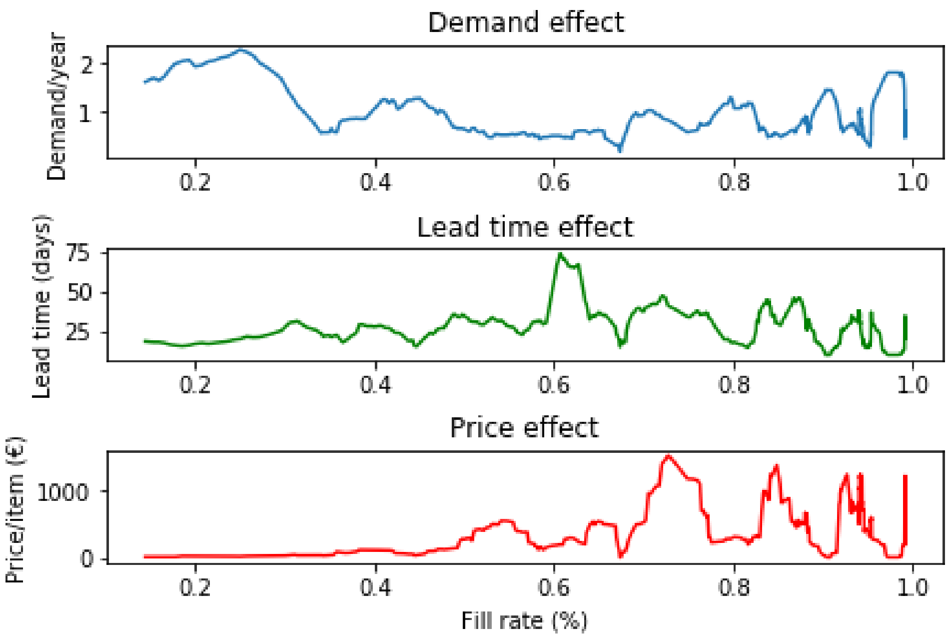


Figure 4.4: Spare part characteristics assigned to the solution by the Greedy heuristic

4.3 Multi-item, customers differentiation inventory policies

In this subsection we apply the differentiation between spares and between the two customer classes, namely: LTSA-customers and regular customers. There was chosen to apply a critical level policy that is based on the lost-sales model. Applying a critical level policy for back-ordering will lead to an expansion of the problem by adding an extra dimension. The limited back-ordering information, the extra complexity and the questionable time investment, affected the decision to apply a lost-sales model for demand which is being back-ordered.

Pure Poisson fill-rate

The models in the previous subsections imply that there is no service differentiation between customers. While the current 25% of demand is via LTSA customers is expected to increase, the interest on customer based service differentiation has grown. The critical level policy is a method that differentiates the stock levels between customers. The demand from regular customers can order items from stock until the inventory position of S-c is met, while premium demand can order all items. A one dimensional continuous Markov chain is used to compute the steady state probabilities of the pipeline which is shown in Figure 4.5. This implies that we use a BCMP network with a First Come First Serve discipline. Therefore, our exponential lead-time assumptions is no prerequisite.

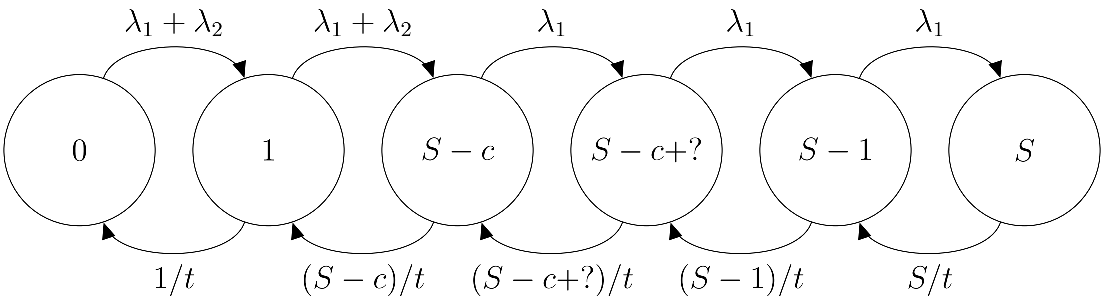


Figure 4.5: Critical level policy in a one dimensional continuous time Markov chain

To compute the exact fill-rates of the demand classes, the cumulative sum of probabilities over the stocked items must be computed again. Considering that a critical level policy deals with multiple demand streams, which vary if a critical level is used, equation cannot be used due to the limited input parameters and the back-ordering approach. Therefore, the theory of [Baskett *et al.*, 1975] is applied to calculate the steady state probability q_x for having x amount of items in the pipeline. This theory implies that the demand sizes of SKU's arrive Pure Poisson, so one by one. In the formula below, the state probabilities are computed where h represents the states of the queuing network, the lead-time is denoted by t, S is describing the maximum inventory state, while λ_h is used for the demand per class.

$$q(x) = \left\{ \prod_{h=0}^{x-1} \lambda_h \right\} \frac{t^x}{x!} q_0 \qquad x \in \{0, ..., S\} \tag{4.7}$$

$$q(0) = \left(\sum_{i=0}^{S} \left\{ \prod_{h=0}^{x-1} \lambda_h \right\} \frac{t^x}{x!} \right)^{-1} \tag{4.8}$$

Using these state probabilities we obtain the fill-rate probabilities for the first customer class represented in equation 4.9. The regular customers are not eligible on the critical stock, therefore the fill-rate for these customers is calculated by equation 4.10.

$$\beta_1(S, c) = \sum_{x=0}^{S-1} q_x \tag{4.9}$$

$$\beta_2(S, c) = \sum_{x=0}^{S-c-1} q_x \tag{4.10}$$

Compound Poisson fill-rate

To model the compounded arrivals with a continuous time Markov chain, certain states have to be differentiated, since only the minimum of the demand size and the inventory level can be replenished. There are two different compound arrivals at OPRA: demand that is arriving with the same order size or the arrival of different order sizes from the same SKU. The arrivals of the same order size are quite easy to model, considering that intermediate states can be skipped. The complexity increases when a spare is ordered in multiple sizes. We denote the probability that a customer j is ordering a spare of size k by $f_j(k)$. In Figure 4.6 an example is shown of an SKU with order sizes {1,2}, S=3 and c=2.

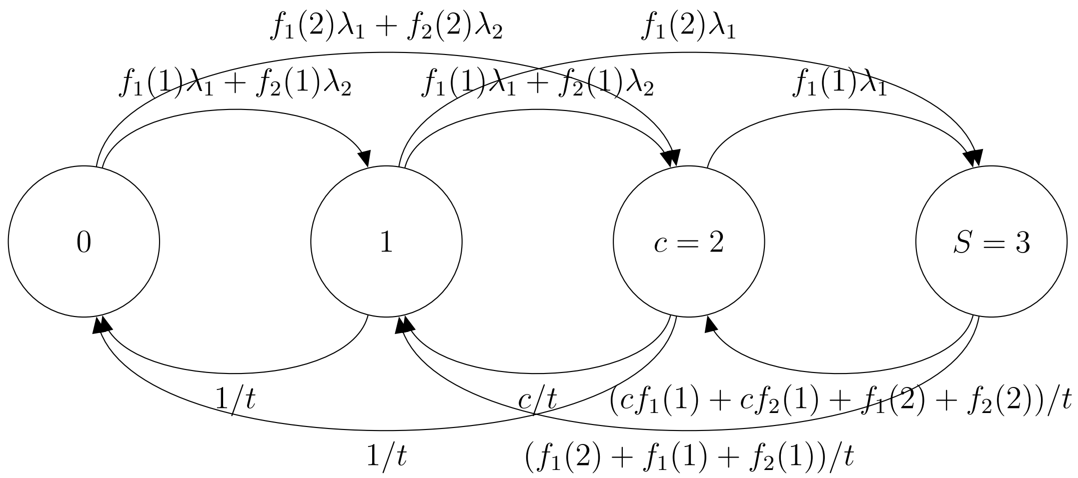


Figure 4.6: One dimensional Markov chain with different order sizes

Consequently, the exact fill-rate of both customer classes can be computed by summing up the state probabilities q_x of the Markov chain which represents the items in the pipeline at an arbitrary point in time. The difference with the Pure Poisson approach is that the state probabilities are multiplied with the demand size and their corresponding probability, and divided by the mean delivery size, to compute the fill-rate. The fill-rates for both customer classes, can be found in equations 4.11 and 4.12 respectively.

$$\beta_1(S,c) = \frac{\sum\limits_{x=0}^{S-1}\sum\limits_{k=1}^{S-x} kf_1(k)q_x}{\sum\limits_{k=1}^{K} kf_1(k)} \tag{4.11}$$

$$\beta_2(S,c) = \frac{\sum\limits_{x=0}^{S-c-1}\sum\limits_{k=1}^{S-c-x} kf_2(k)q_x}{\sum\limits_{k=1}^{K} kf_2(k)} \tag{4.12}$$

To compute the state probabilities, we first derived our own approach due to limited literature availability. Just when we derived a solution for the state probabilities, we noticed the publication by [Kouki & Larsen, 2020]. In this approach the state probabilities for compound Poisson demand are computed. Considering the compact design, we adopted this approach and used our own derivation to verify the results.
The state probabilities q_x are determined by equation 4.13 to compute the state probabilities of compounded arrivals. Since lost-sales are assumed, it iterates until state S is reached. The consecutively step is applying equation 4.14 which is the normalizing procedure. The Compound Poisson steady state equations (4.14) gives the same results as the Pure Poisson steady state equation when modelling single sized spares.

$$p(x) = \frac{t}{x}\sum_{h=0}^{x-1}(\sum_{j|x\leq S-c}^{J} \lambda_j f_j(x-h))(x-h)p(h) \qquad n = 1, ..., S \tag{4.13}$$

$$q(x) = (\sum_{y=0}^{S} p(y))^{-1} p(x) \tag{4.14}$$

Solution approach CLP

To approach a solution for the CLP problem, there was chosen to apply the Dantzig-Wolfe decomposition. Firstly, the Master Problem is introduced where the original problem is described as convex combinations of columns that contain all possible values for the decision variables of the original problem [Van Houtum & Kranenburg, 2015]. We denote K as the set of finite critical level policies, I represents all the SKU's in a system and J are the amount of customers classes. A new binary variable x_i^k is introduced which determines whether a policy k is chosen or not. Since at least one decision must be made, constraint 4.16 and 4.17 are introduced as service level constraints which hold for each SKU. Equation 4.18 implies that we have an LP-problem since policies can be chosen partly.

$$\text{Min} \sum_{i\in I}\sum_{k\in K} c_i(S_i^k, CL_i^k)x_i^k \tag{4.15}$$

$$\text{subject to} \qquad \sum_{i\in I}\sum_{k\in K} \frac{\lambda_{i,j}}{\lambda_j}\beta_{i,j}(S_i^k, CL_i^k)x_i^k \geq \beta_{j,obj} \qquad j \in J. \tag{4.16}$$

$$\sum_{k\in K} x_i^k = 1 \qquad i \in I, \tag{4.17}$$

$$x_i^k \geq 0 \qquad i \in I, k \in K. \tag{4.18}$$

To differentiate between the customers classes j $\beta_{j,obj}$ is representing the objectives that the weighted average fill-rate has to fulfill. Just like the multi-item approach without customer differentiation, the fill-rates are scaled by their corresponding demand. Considering that we only 91 spares and two customers, the problem is not restricted to increase the computation time. To determine which policies are interesting, the column generation technique is applied.
To start, an initial set of policies are required that guarantee feasibility of the defined problem. By adding SKU's to the stock for each spare until they meet the target fill rate for the highest customer class ($\beta_{1,obj}$), an initial feasible solution is guaranteed. Hence, no service differentiation is applied for the first policy set, see Table 4.1. There are at least two policies sets required as input for the LP-model. Subsequently, a second policy set is defined by adopting the first set of stock levels, but now service differentiation is applied by assigning CL_i with the minimum order size of the corresponding spare.

Table 4.1: Feasible solution heuristic

```
For i = 1 To length(spare_part_list)
    S_i = 0;
    CL_i = 0;
    Do while β_{i,j}(S_i, CL_i) ≤β_{1,obj}
        S_i=S_i+min_order_size
```

Now an LP-solver is required to solve our LP-problem given the two initial policy sets. To make the model comprehensible, Microsoft Excel with the COIN-OR CBC(Linear solver) package is used to compute the optimal LP-solution, given the two policies. From the generated solution, we obtain the dual prices (shadow prices) of all constraints in equations 4.16 and 4.17 which we denote with u and v_i respectively. Consequently, we search for new policies to improve the results of the LP-problem. To find these policies, the reduced cost coefficient is required. Considering the scaling that was applied for the target fill-rate, we adopt this scaling to determine the reduced cost coefficient.

$$C_i^{red}(S_i, CL_i) = c_i(S_i, CL_i) - v_i + \sum_{j=1}^{J} u_j \frac{\lambda_{i,j}}{\lambda_j} \beta_{i,j}(S_i^k, CL_i^k) \qquad i \in I \tag{4.19}$$

With equation 4.16 new policies are added as columns to the solution space until the reduced cost coefficient remains negative. So the sub-problem for SKU i can be written as follows:

$$\min C_i^{red}(S_i, CL_i) \tag{4.20}$$

$$\text{subject to: } 0 = CL_i \leq S_i \qquad i \in I \tag{4.21}$$

Considering that minimizing the reduced costs (equation 4.20) is not a linear function, there was chosen write a local search heuristic. The heuristic analyzes the neighbor solution space and stops if the reduced costs cannot be further reduced. Variables c and S are in- or decreased in steps of their minimal order sizes in this local search heuristic. The policies with the lowest reduced costs coefficient are added to the solution space and a new solution is computed by the LP-solver. This process continues until no new policies are found. Thus, the reduced costs coefficient cannot be further reduced. Lastly, the binary constraint is added and subsequently solved to ensure that no policies are fractional. The flowchart of the solution approach can be found in Figure 4.7 below.

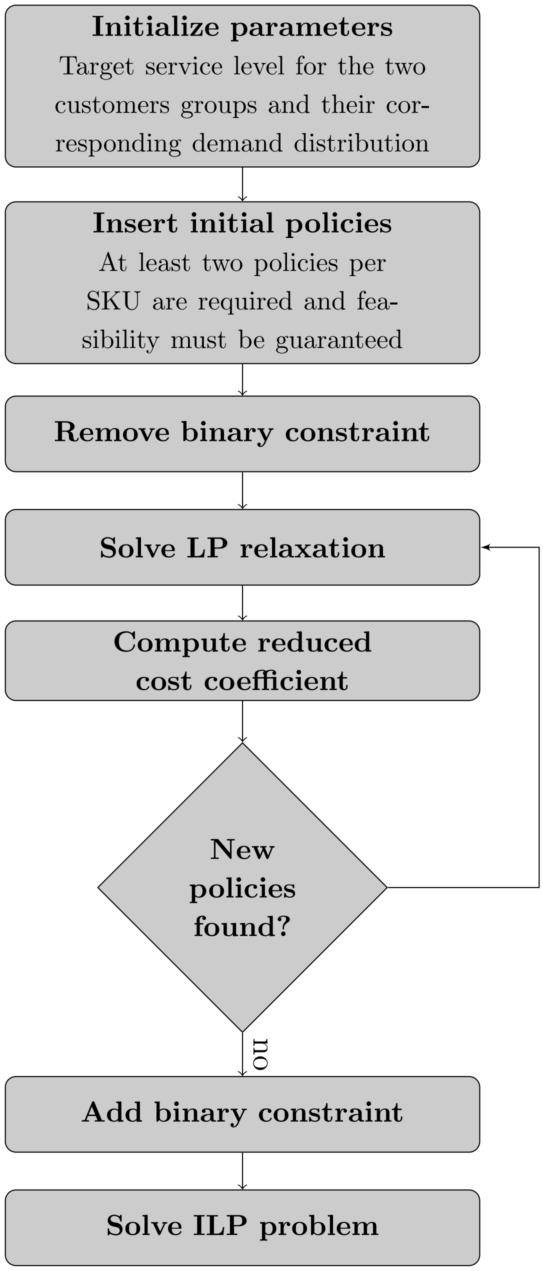


Figure 4.7: Flowchart CLP solution approach

The following assumption are applied in our model, for simplicity purposes:

- Pure or Compound Poisson demand input required
- If demand cannot be replenished it is lost in the system
- The multi-item approach links spares to each-other
- Maximum of two demand classes
- Demand is partially satisfied when the on-stock is not sufficient

To test how much iterations and how much time the model requires to approach a solution, we set up an initial experiment. Because the service performance of the spares are not tracked currently, the target fill-rate for LTSA customers is to 99% and 90% for regular customers. We expect that 25% of the demand comes from LTSA customers, which is also expected as the current premium demand ratio. The solutions of this input can be found in Table 4.2.

	TFR Class 1	TFR Class 2	Inventory Investment
Iteration 1	0.9952	0.9952	118.059
Iteration 2	0.9900	0.9000	99.174
Iteration 3	0.9900	0.9000	74.174
Iteration 4	0.9900	0.9000	65.599
Iteration 5	0.9900	0.9000	65.596
Final solution (ILP)	0.9901	0.9003	66.117

Table 4.2: Iteration results CLP (TFR Class 1=99%, TFR Class 2=90%)

The results indicate that six iterations are required to approach a near-optimal solution. The customer differentiation is obtained by assigning 17 spares with a critical level. When we compare the €66.117 inventory investment with the multi-item approach without service differentiation, the added value of the CLP becomes clear. The near-optimal solution is found in approximately 4 minutes, which we consider as an adequate computation time of the model. The results also imply that a significant small integrality (ILP solution-LP solution) gap is achieved. Thus we can conclude that the ILP solution is close to optimal.

Lost-sales effect

For simplicity reasoning a lost-sales critical level policy was applied for demand which is back-ordered. In the lost-sales approach, orders can be lost within the system, where the back-ordering approach reserves SKU's to replenish later on. Hence, the lost-sales approach tends to maintain higher service levels, because higher inventory levels are sustained, see Figure 4.8.

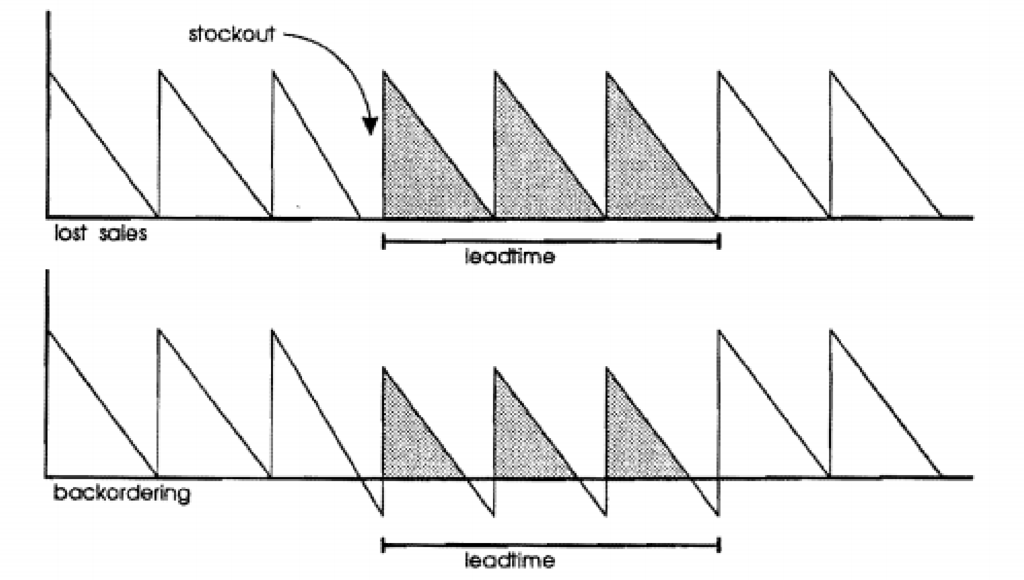
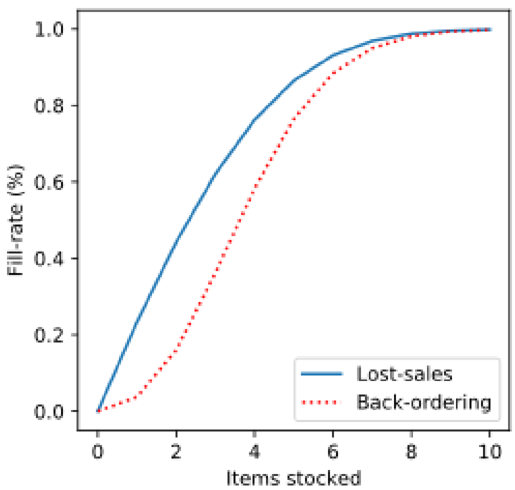
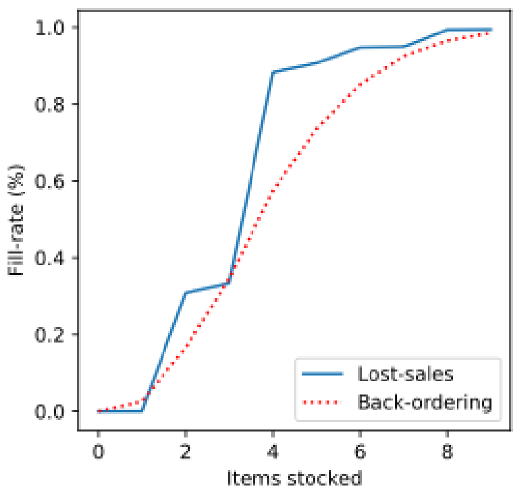


Figure 4.8: Differences in stock levels for lost-sales and back-ordering approach. Source: [Rutten *et al.*, n.d.]

To determine the effect of this simplification, the fill-rates from multi-item approach without service differentiation where demand is back-ordered, are compared with the critical level policy with the lost-sales approach. This implies that the service differentiation cannot be applied within this comparison. In Figure 4.9 the fill-rate effect of both approaches are provided. Figure 4.9a indicates that for Pure Poisson demand, maintaining high fill-rates, results in lower deviation effects. By increasing the demand or the lead-time the deviation becomes higher, which is caused by a higher probability of back-ordering. In Figure 4.9b an SKU with demand sizes 2 and 4 is shown, both with 50% probability of occurrence. These demand sizes explain the fill-rate behaviour of the lost-sales model, since some states of the Markov chain are never visited. Again, the deviation between the lost sales and the back-ordering approach decrease by higher fill-rates, lower demand and lower lead-times.



(a) Pure Poisson example



(b) Compound Poisson example

Figure 4.9: Fill-rate deviation between the Lost Sales and the Back-order approach

Given the finding that the deviation is significantly small when high fill-rates are maintained, the premium customer classes fill-rate is expected to be more accurate by sustaining service differentiation. For the regular demand, more back-orders are expected so the fill-rate is expected to deviate more.

Conclusions

Hence, service differentiation for compounded spares can be applied via the proposed critical level policy. More spares are assigned to the premium customers which results in higher fill-rates for this class, while the contradiction applies for the regular customers. The back-ordered demand, modelled via the lost-sales approach, becomes more sensitive when maintaining low-service levels. By considering this deviation within the solution approach, we assume to approach a near-optimal solution to our problem. Thus, the combination of the service differentiated, multi-item approach for Poisson and Compound Poisson demand is the best fitting model for this research.

4.4 Conclusions

Different inventory models are analysed in this chapter. We propose the modified CLP model to solve our problem. In the next chapter the results are further analysed and discussed. The most important findings of this chapter are denoted in the list below:

- Multi-item approach outperforms a single-item approach.
- Greedy-heuristic can approach a solution fast, nevertheless no service customer differentiation can be applied.
- Critical Level Policy proposed to apply service differentiation. Modifications required to model the compounded arrivals.
- Difference between lost sales and back-ordering approach are lower by sustaining high fill-rates.

5 Numerical analysis

This chapter describes the numerical experiments and the analysis of the corresponding results. We hereby use the proposed model from the previous chapter. In the first section the numerical objectives and the numerical experiments are described. In Section 5.2 the results of the numerical experiments are analysed and discussed. The last section contains the conclusions of the chapter.

5.1 Verification and validation

Now that the model design is defined, we can start the validation and verification phase. The verification checks whether a system meets the design specifications. While the validation is the process that checks whether a model gives a representative presentation of the system, which in this case is the spare part inventory system.

Verification techniques

To verify our proposed model, we use the following three of the eight verification techniques which [Law *et al.*, 2000] proposes. These techniques assist to secure that the proposed computer model is correctly implemented.

- The first technique that is applied is writing and debugging the model in sub-modules. The implementation of the main model is divided in the following consecutive sub-modules: fill-rate computation, feasible solution composition, LP-model, reduced price computation, local search of new solutions and the ILP-model. All these sub-modules are implemented and debugged separately to ensure that they provide the required outcomes. In the final step of this verification technique the entire main model is debugged to guarantee that it is correctly consolidated.

- Another commonly used verification technique is running the model under simplified assumptions. We test if all the critical levels are zero if no service differentiation is applied, thus the target service levels are set equal. Moreover, the target fill-rates are set on zero to see whether errors occur. In both situations, the model performs as expected.

- The last verification technique that will be applied is input parameter variation. The input parameters are checked whether a reasonable model output occurs. For example, if the amount of premium customer demand increases, an additional inventory investment is expected to maintain the service levels. By applying this for all input parameters, no strange behaviour was observed.

Validation techniques

To ensure that a model is useful for the purpose of this research, various validation steps are taken. These validation steps check if the model represents the actual current system. As the current data from practice is limited, the validation process is applied by logic reasoning. Again, the theory from [Law *et al.*, 2000] is used to determine which validation techniques to apply.

- Initially, the quality of the input data for our model must be guaranteed. Primary, the ERP-system is conducted to retrieve the input data. Consequently, expert interviews with the supply chain manager and the service manager are conducted to verify these outcomes. In consolidation with these experts, divergent and missing data is complemented or adjusted.

- The second validation technique that we applied is: maintaining a written assumption list which is found in Section 4.3 and establish structured walk-through found in Appendix B.

- The last validation step is considering the models output. Since the Pure Poisson fill-rate equation was defined in Section 2.6, and we applied an approach which could deal with Pure and Compound Poisson demand, these two equations can be compared to verify the results. The outcomes of both approaches matches when Pure Poisson demand is modelled, thus the model performs as expected. Furthermore, the models outcomes are analysed and discussed by two experts of OPRA. No strange output was noticed within this session.

Conclusion

Given all the verification and validation results, we consider our model as appropriate to use. All the sub-modules seem to work within the model and no significant difference in the results is found. Hence, experiments can be conducted to analyse the results of the model.

5.2 Experimental settings

In this section we discuss the system performance analysis based on the experimental design. Various experiments are conducted to provide insight in the models performance and addressing critical points. We classify the following variables in this experimental setup: target fill-rates, demand uncertainty, premium demand ratio and the customer class responsible for the compounded demand. The choices and the experiment setup for these variables are explained in the subsections below.

Target fill-rates

Currently, no data is available about service differentiation at OPRA. Therefore, it is chosen to measure the fill-rates for the spares currently assigned with a minimum stock level, determined by expert knowledge. Thus we measure service performance of the current minimal stock levels to give a representative indication which TFR to maintain within the experimental setting. In Table 5.1 this performance can be found, the median of 99% fill-rate gave us a representative first experimental settings for the Target Fill-Rate (TFR) of Class 1/premium customers. Because the current spare part levels are experienced as high and maintaining 99% a TFR leads to assigning stock to each spare, the second experimental factor for the TFR of Class 1 is set lower to 95%. Consequently, the TFR for the regular customers is set on 95% and 90% due to their lower priority. Thus in the experimental setup, one combination is made without service differentiation.

	Min	25‰	Median	75‰	Max
Fill-rate	0%	96.9%	99.0%	99.6%	100%

Table 5.1: Service performance of the spares assigned with a minimal stock level

Demand in- and decrease

Initially, the demand characteristics were modelled stationary, which means that events that occurred in the past also occur in the future. The spare part demand is affected by the amount of gas turbines OPRA is sustaining. Considering the applied multi-item approach, we expect that the spare part demand is influenced simultaneously. At the start of this project, a sales growth was expected, which would effect the spare part demand. However, through the current COVID-19 pandemic, the end-product sales seem to stagnate. Therefore, three experiments are defined which represent a demand increase, decreases and the initial scenarios. This implies that all the SKU's are effected equally.

Demand ratio

To determine the ratio between the regular and the premium demand, the end-products of LTSA/premium customers are divided by all the end-products. This implies that currently 25% of the demand is caused by premium customers. Considering that OPRA's goal is selling products with LTSA's only, this ratio should increase in the near future. Hence, three different experimental factors are set for the demand ratio: 25%, 37,5% and 50%. A decrease of the premium demand ratio is unlikely to happen because OPRA prefers to sell LTSA's only. Therefore, no reduction of the current ratio is expected.

Order size distribution between classes

The last variable that we analyse is the order size distribution. At the moment, it is unknown if a specific customer class is responsible for compounded demand. Thus, the compounded demand cannot be assigned to a specific customer class. In the context Chapter, two causes of the compounded demand were described, namely; batching at customers and combining preventive and corrective maintenance. Considering that our model can assign these compounded arrivals to specific customer classes, the related results are discussed based on three experimental settings. In the first experiment set, the premium customers are responsible for the compounded arrivals, Premium Compound Responsibility(PCR). The second experiment both customer classes are responsible, Both Compound Responsible (BCR), the last experiment set consists of regular customers ordering compounded demand, Regular Compound Responsibility (RCR).

Variables	Experiments (range)	Increase	Experiments
TFR Class 1	99% & 95%	-	2
TFR Class 2	95% & 90%	-	2
Demand	[80%, 120%]	20%	3
Demand ratio	[25%, 50%]	12.5%	3
Order size distribution	PCR & BCR & RCR	-	3

Table 5.2: Experimental design

Experimental reduction

By applying a full-factorial design of the experimental setup, 108 experiments have to be conducted, see Table 5.2 for all the settings. Considering the computation time per experiment, we chose to exclude the following experiments which are assumed to be highly unlikely to happen.

- Demand in- or decrease with same demand ratio. As we already mentioned earlier, OPRA has the goal to sells gas turbine packages with LTSA only in the future. Therefore, we exclude the experiments where demand increases, while the ratio between premium and regular customers remains the same. Subsequently, a demand reduction with the same demand ratio is also unlikely to happen. Regular customers have a higher correlation with older gas turbines packages, thus if the total spare part demand decreases, it is caused by this customer group firstly.
- When more premium customers are expected, thus a higher premium demand ratio, we determine the TFR of 95% for this corresponding group as too low. A significant amount of spares are not stocked, which we consider as unsuitable when maintaining higher demand ratios. Hence, these variable combinations are removed from the experimental setup
- Finally, we exclude the experiments with the higher demand ratios while the regular customers order all the compounded demand. Since in the current situation both customer classes are responsible for the compounded demand, we assume that it is doubtful that the regular customer class is demanding compounded spares when they are less represented.

Conclusion

By applying these exclusion procedures, there are 36 experiments left. Hereby, the experimental results are reduced, thus the more realistic experiments are compared with eachother. The full experimental setup can be found in Appendix C.

5.3 Numerical experimental analysis

In the following subsections we show and discuss the most interesting findings from the performed experiments. Within this analysis we initially analyze the most representative experiments, where demand is 100%, premium ratio 25% and both customer classes order compounded. Subsequently, these experiments are compared with corresponding other experiments to identify the models behaviour.

Inventory investment

The lowest inventory costs can be achieved by maintaining a low TFR for the first customer class. This is the effect of the exponential behaviour of the fill-rate related to the costs. By comparing all the experiments with the same settings, but with different target fill-rates, the additional service costs of the experiments are identified. By decreasing the TFR for regular demand from [0,99;0.95] to [0.99;0.90] approximately 3.2% of spare part investments can be prevented, see Table 5.3. The table also shows the high impact of the target fill-rate for the first customer class. For example, increasing TFR for Class 1 from 95% to 99% causes approximately 60% more investments. This is the effect of the exponential behaviour of the TFR for class 1 to the inventory investments.

	TFRC2:95%	TFRC2:90%	Difference (%)
TFRC1:99%	€105.010	€101.805	-3.2%
TFRC1:95%	€65.737	€61.711	-6.5%
Difference (%)	-59.7%	-64.97%	-

Table 5.3: Average inventory costs based on their target fill-rates

To indicate which spares are required to maintain higher fill-rates we compare two experiments with a different TFR for class 1. In Figure 5.1 it is shown that when a higher TFR for the first class is set, more expensive spares are added to stock. Due to the high variety of spare parts values, we ordered the spares based on their value/item in a descending order. The less valuable spares are not varying because the added value is limited to the final solution. We verified this outcome with more experiments, nevertheless, the model behaves practically the same. Hence, we can conclude that lowering the TFR for Class 1 allows higher valued spares to maintain a lower service level. From a financial perspective it is not worth it to maintain lower service levels for low valued spares, the higher ones are responsible for the main outcome.

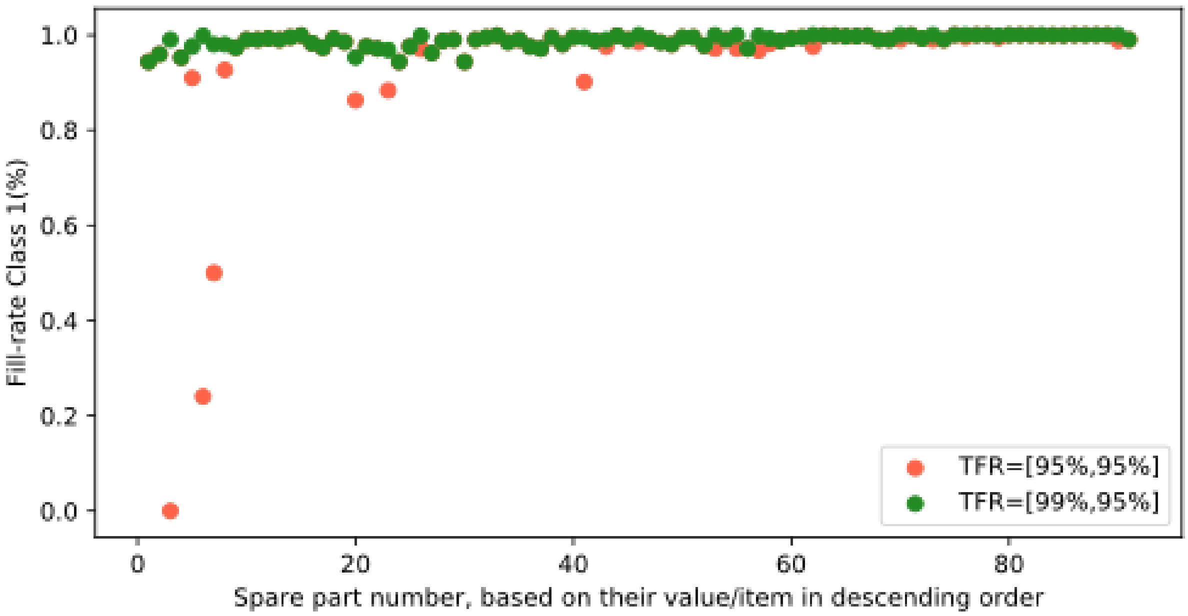


Figure 5.1: Fill-rate difference between differen TFR Class 1

Critical levels

In this subsection we discuss the spares which are assigned with a critical level. By varying the TFR of the second customer class, we can analyze the models behaviour to the critical levels. In Figure 5.2 the variety of fill-rates is shown when more service differentiation is applied. The solution in Figure 5.2a requires an inventory investment of €105.988 where 28% of the value is critical stock. For the second solution a spare part investment of €111.196 recommended with 8.2% of critical stock. The red dots in the Figure indicate that the critical levels have a higher attendance in the higher spectrum of spare part values. To conclude, by increasing the service differentiation between the two customer classes, it becomes worthwhile to apply differentiation to higher valued spares. Savings up-to 10% can be gained by applying service differentiation via a critical level policy.

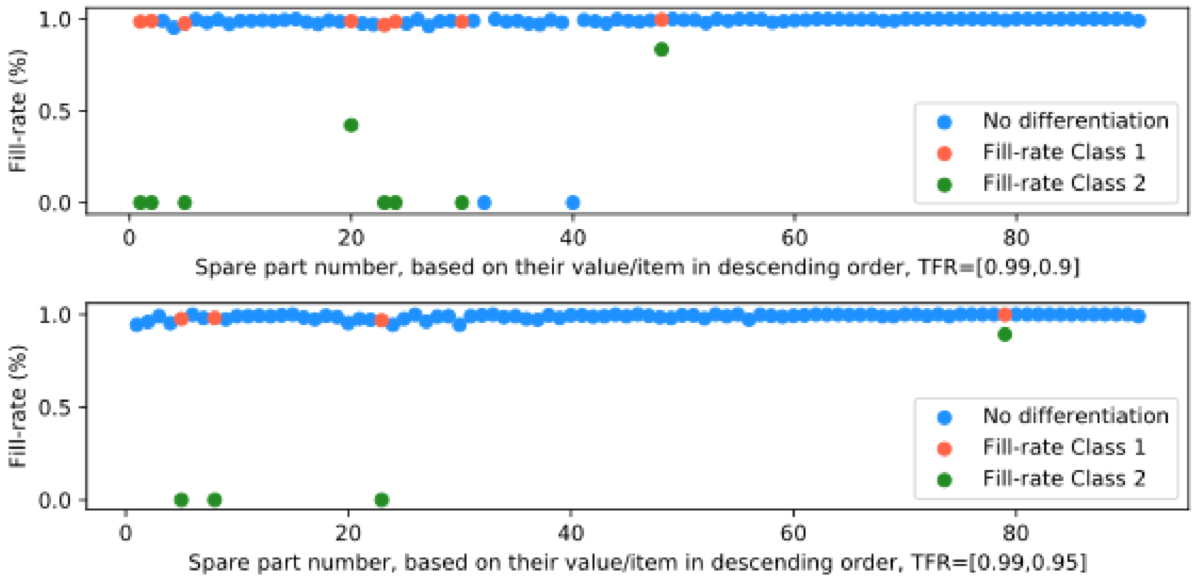


Figure 5.2: Fill-rate effect by a higher degree of service differentiation

Demand effect

Within the experimental set-up it was chosen to analyze the effect of 20% in- and decrease. By increasing the demand with 20% for all spares approximately [2%-3%] additional investments are required. A demand decrease of 20% results in roughly [4%-5%] less investment in the inventory. We expected that the demand effect was more sensitive to the models performance. Subsequently, we also experimented with the premium demand ratio. By increasing this variable higher investments are inherent, nevertheless, these additional investments are minimal. If the premium customer class covers 50% of the total demand, instead of the current 25%, approximately 2% more inventory investment is required. There is no clear trend of spare part characteristics which differ in comparable experiments with these different demand variables. We conclude that this limited demand effect is caused by the exponential behaviour of the fill-rate versus the investments.

Compound orders

The last experimental factor that we discuss is the compounded demand responsibility. The results indicate that significant investments can be prevented if the compounded demand can be linked to the lower customer class. Obviously, this only holds when there is differentiated between customers. Fully assigning the compounded spares to the lower customer group correlates with higher fill-rates for the premium customers. Approximately, 10% of the spare parts require less SKU's on stock when the compounded spares are assigned to the lowest customer class. In Figure 5.3 the investments related to the single and the compound demanded items, from three experiments (experiments 7, 23 and 35), is shown. It indicates that the first two experiments are not highly varying. Nevertheless the third experiment, where the regular customer is ordering compounded, shows that significantly fewer investments are required for the compounded spares. Roughly 22% compared to the BCR approach, and 24% to the PCR approach, investments can be prevented by assigning the compounded spares to these related customer classes. To maintain the TFR of Class 1 additional single sized spares are required.

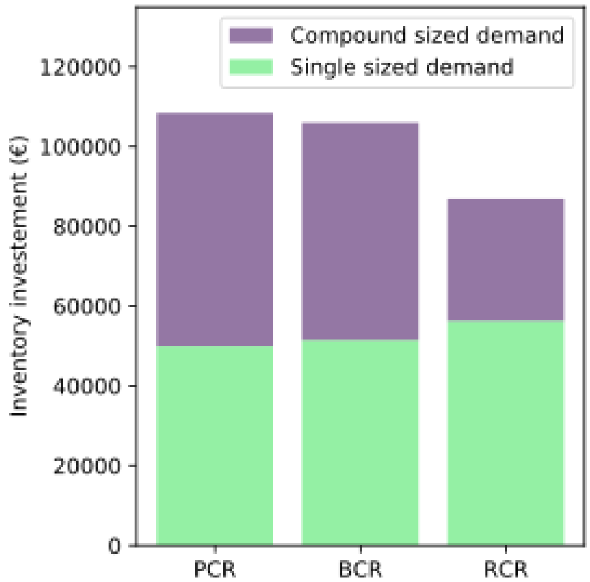


Figure 5.3: Compound demand assigned to a specific customer class

Proposed model versus the current situation

Currently, a Weighted Average Fill-Rate (WAFR) of 70% with a related investment of €162.622 for both classes is maintained. This current WAFR is perceived as too low, thus higher a TFR is maintained within the experiments. The experiments indicates that a WAFR of 95% can be sustained with an approximately inventory value of €65.000. This implies that for almost every spare, items are held on stock, see Figure 5.4. Knowing that the less valuable spares are selected first by the model to increase the WAFR, we see that an additional investment of €5.000 to the current investment can already improve the WAFR from 70% to 90%. Approximately, a supplementary of €22.000 is required to sustain the TFR of 95% (see Figure 5.4).

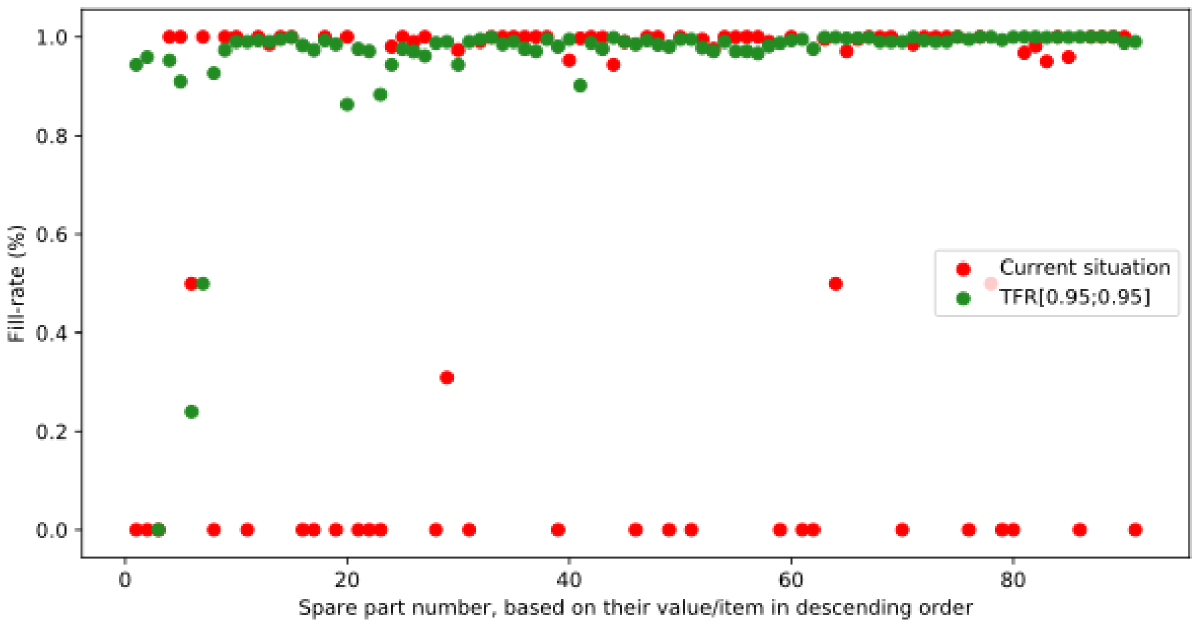


Figure 5.4: Current situation vs. TFR[0.95;0.95]

Conclusions

Given the experimental analysis results, we draw the following conclusions. For higher TFR's additional investments are required, thus more stock. Additional savings can be achieved by applying service differentiation between customers. Because the current service performance of spares is not measured, it is difficult to recommend a final parameter setting of the model. To maintain higher service levels, more expensive spares are required. This also holds for the service differentiation, a higher degree of differentiation relates to more higher valued spares set as critical levels. Due to the exponential behaviour of the fill-rate, demand in-or decreases and premium demand ratio are not highly sensitive to the outcome of the model. On the other hand, assigning compounded arrivals to the lower customer class results in noteworthy cost reductions. Thus, serving all customer with compounded demand causes proportional extra stock with the related costs.

5.4 Conclusions

In this chapter the model is verified and validated with various techniques first. Subsequently, numerical experiments are conducted to identify the performance of the proposed model. We summarize the following findings from this chapter:

- Approximately €60.000-€100.000 of the current spare part investments are not contributing to the service performance.
- Low valued spares essential to keep on stock, higher valued only when maintaining high service levels.
- Higher degree of service differentiation leads to more expensive spares to differentiate. Investment savings up to 10% can be acquired by applying a critical level policy.
- Serving all customer with compounded demand requires 22% of the investments.

6 Conclusions and recommendations

This chapter completes this research project. First, the main conclusions of this research are given. Subsequently, the results are discussed. In the third section the limitations of this research are analyzed. Next, the theoretical and the practical contributions are given. Conclusively, we present our recommendations for OPRA.

6.1 Conclusions

The following research question was addressed in this research:

How can the current inventory management and control policy for the spare parts be improved considering the trade-off between the inventory risks and the end-product downtime?

OPRA's spare parts are characterized by their extreme slow moving demand and the combination of single and compounded demand. Currently, the spares at OPRA are measured based by their inventory investment. Thus, no service performance indicator analyzed. Therefore, the fill-rate is introduced to describe the service performance of a spare part. Considering this fill-rate with the corresponding investment, it is concluded that the current stock levels are significantly high for some SKU's, while others are directly back-ordered because their stock levels are zero. Moreover, the interest of applying service differentiation between LTSA customers and the regular customers tends to growth in the near future.

A critical level policy with partial lost sales was proposed to solve the defined core problem. A near-optimal solution is found by applying the column generation technique. We discuss the key findings of our research in the points below:

- The current stock levels are too high considering their service performance (€162.622 and roughly 70% weigthed average fill-rate). From the currently active spare part investments, approximately €60.000-€100.000 (dependent on preferred TFR) is redundant since it does not directly influence the service performance to the customers when maintaining lower inventory levels.

- To increase the current service performance of 70%, low valued spares are the most interesting to maintain on stock due to their low financial risks. It is not worthwhile to maintain lower service performance for these spares. Higher valued spares are inevitable to leave out of stock when maintaining a relatively high TFR.

- Investment savings up to 10% can be gained by assigning critical levels to differentiate between the two customer classes. A higher degree of differentiation leads to more prevention of inventory investments.

- Currently supplying all customers with compounded orders causes roughly 22% of the total investments. The compounded demand, which is inherit with a higher demand variability, causes more investments to maintain the same TFR's.

To conclude, our proposed model is able to improve the current service levels and simultaneously can lower the investments. Nevertheless, given that OPRA is already sustaining spares on stock it is not realistic to adopt the proposed inventory levels instantly. Therefore we give OPRA various suggestions in the subsection Recommendations.

6.2 Discussion

Spare part management remains a challenging problem due to the high demand uncertainty. In this research, we used the recently active data from the last five years. Considering that approximately 50% of the currently active spares were demand once in this time-frame, there exists a close line between being in- or excluded within the proposed multi-item model.

To determine spare part demand, the sales data was we used. Unfortunately, this implies that we cannot determine the amount of back-orders. This sales and demand data increasingly diverges when stock-level are kept zero for long time-intervals. Thus, sustaining our model with the actual demand would improve the accuracy of the outcome.

Finally, we assumed that demand could arrive compounded to meet customer preferences. This implies that compound demand which can consist of stock replenishment or preventive maintenance, are equally treated as spares required for corrective maintenance. Hence, theoretically the situation might occur that a spare part required for corrective maintenance cannot be delivered since a relatively less important compounded order was replenished upfront. Moreover, the results indicate that significant additional costs arise to serve all customers with compounded orders. These additional investments and probability of prioritizing less urgent spares over urgent corrective maintenance required spares, make the current compounded approach debatable.

6.3 Limitations

This researcher consists of various limitations. The first limitation that we discuss is the appliance of a lost-sales based critical level for demand which is being back-ordered. We discuss this limitation within the research, yet clear approach is provided to solve this. Hence, sustaining low fill-rates within our model overestimates the actual fill-rate. This overestimation becomes smaller by maintaining high fill-rates for spares.

Next, the proposed model is limited by two customer classes. The LTSA customers are classified to premium customer group the rest to the regular customer. For simplicity reasoning and the lack of service level data, we chose to assign these two customer groups. However, as the ratio of premium customers tends to increase, more service differentiation, between this corresponding group is expected. Accordingly, an extension of our model is required if OPRA prefers to differentiate their stock to more than two customer classes.

Furthermore, we discuss is the assumption of equal criticality of spares. Within our model it is assumed that when a spare is demanded, this directly effects the performance of a system. However, a breakdown might have a partial influence on the systems performance. For example a gas turbine might perform on 50% of their capacity, due to the a broken spare. No distinguishing based on this critcality is applied.

Finally, a multi-item approach was initially designed to maximize the overall performance of one single system. Since the current data from OPRA limits the modelling of gas turbine as the system, all the active spares are selected to represent a system. Within the current state our model represents a quite accurate system, nevertheless, the system approach has to be guaranteed to approximate good results.

6.4 Contributions

Our research contributes to the theory by introducing a critical level policy for compound Poisson demand. To our knowledge, this critical level approach has not been applied before in such a manner. The research emphasizes the financial impact which the compounded demand related to service differentiated customer classes, has.

Practically this research contributes to an understanding of the spare part management system at OPRA. The proposed model considers the trade-off between the service performance and the inherent investments of spare parts. Although the currently demand data is limited, the attendance points of the spare part management at OPRA are highlighted. Also, the aimed users of the model, the Aftermarket department, saw the potential of the model to assist in the decision making process concerning spare part management.

6.5 Recommendations

In this subsection, we present our recommendations based on our research. Throughout this research different further improvements regarding the spare part management at OPRA are identified.

The first recommendation considers the reduction of the current spare parts levels. Unnecessary high service levels are maintained which have no purpose since the related additional service performance is limited. At the moment, no procedure to determine the potential usage of a spare is applied. We scoped this subject out of our research, nevertheless, actions like offering discounted spares to customers or even discarding are recommended for these spares.

In contrast, we recommend to increase the stock levels for the low-valued spares which are currently directly back-ordered. Although, due to data limitations, it is not clear which explicit TFR value to sustain yet, we recommend to use the following rule of thumb: when a spare parts has a lower value than €1.000 is worthwhile to maintain them on stock. By applying this rule the current WAFR increases from ±70% to ±90% with an investment of €4.200.

Third, we recommend to collect more reliable spare part data to strengthen the results from the model. By tracking the actual demand, the back-orders and their origin can be identified, thus more justified stocking decisions can be made. Additionally, it is also recommended to identify the purpose of the compounded demand. Additional investments are required to compensate the high variability of the compounded demand, thus it can be worthwhile for OPRA to check if this compounded customer preference is essential. Moreover, we also suggest to audit the active spares. Currently, it is unknown if the considered active spares are all installed at a gas turbine package. Tracking the spare parts per project might accelerate the discarding process of unnecessary spares.

Subsequently, we suggest to clarify which Target Fill-rates to sustain for the customer classes. We provided various numerical experiments with the related solutions, nevertheless, it currently not clear to what extend OPRA prefers their service performance. Additionally, because OPRA was already struggling with assigning spares to specific customer classes, we suggest to reconsider the stocking purpose of the spares with a higher value than €2.500. When a spare is stocked for premium customers only, a critical level must be assigned to the ERP-system to prevent future debate.

We finally recommend to apply the model for new projects. Given the system approach we can identify which spares are required to maintain a fixed TFR. Next, the results can be compared with OPRA's current spares and if required, stocking decisions can be applied and justified.

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A Organization chart

Currently, there are 80 people working at OPRA. The New Build department is the biggest departments of the company, since almost 30 people work here. The supply chain manager, who is responsible for the warehouse, is delivering to the New Build director. See Figure A.1 for an organization chart. Considering the Aftermarket department has no own supply chain manager, the supply chain manager from New Build is also responsible for the spare part warehouse. This research project will take place at the Aftermarket department, but the supply chain manager will be the project leader during the project.

Figure A.1: The Organization structure of OPRA

B Structured model Walk-through

This model was developed by Julius Jansink in 2020 and was requested by OPRA Turbines B.V. The purpose of the model was to simulate all the spare parts in a gas turbine package. The model aims at minimizing the spare parts investments given service level to maintain. Thus the model is not build to considering highly specialized spares. It is optional to differentiate between two customer classes by assigning different target fill-rates.

B.1 Model input

The model was programmed Microsoft Visual Basic among with the Excel package. To solve the linear problem the OpenSolver package was used which is a prerequisite to run the model. In the first tab, the spare part input parameters must be assigned (see Figure B.1).

Article number	Leadtime	Demand/year	Price/piece	Current stock	Critical level	Fill rate	Scaled FR	Investment	Orders	Order size1	Order size2
5010216	10	2,176	€ 236	0	0	0,0000	0,0000	€ -	1	1	1
5010925	10	1,451	€ 3	72	0	1,0000	0,0217	€ 200	8	8	8
5020027	10	0,181	€ 547	0	0	0,0000	0,0000	€ -	1	1	1
5020080	10	0,181	€ 308	0	0	0,0000	0,0000	€ -	1	1	1
6060005	130	0,544	€ 182	133	0	1,0000	0,0082	€ 24.271	3	3	3
6080103	50	0,181	€ 104	0	0	0,0000	0,0000	€ -	1	1	1
6200014	15	0,363	€ 198	0	0	0,0000	0,0000	€ -	1	1	1
6200192	55	0,725	€ 308	1	0	0,5000	0,0054	€ 308	4	4	4
6200275	20	0,181	€ 55	0	0	0,0000	0,0000	€ -	1	1	1
6200895	15	0,181	€ 1.129	0	0	0,0000	0,0000	€ -	1	1	1
6200962	15	0,181	€ 198	9	0	1,0000	0,0027	€ 1.782	1	1	1

Investment	WAFR
€ 162.622	75,7%

Input variables
Output variables
Formulas
Headers

Figure B.1: Example of the spare part input parameters

B.2 Model parameters

When the input is clear, we can determine the prefered parameters of the model. To apply service differentiation the TFRC1 has to be greater than the TFRC2. Subsequently, the demand ratio of the two customer classes have to be determined. Afterwards, the defined problem can be solved by clicking the button (see Figure B.2).

Article number	Demand/year	Price/piece	Leadtime	S	c	Fill-rate class 1	Fill-rate class 2	Costs	Shadow Prices	Shadow Prices	Reduced Costs
5010216	2,176	€ 236	10	2	0	99,83%	99,83%	€ 472,000	€ 382.062	€ -17.579	€ 4.636
5010925	1,451	€ 3	10	16	0	99,92%	99,92%	€ 44,444	€ 27.780	€ -12.001	€ 3.094
5020027	0,181	€ 547	10	1	0	99,51%	99,51%	€ 547,000		€ -952	€ 385
5020080	0,181	€ 308	10	1	0	99,51%	99,51%	€ 308,260		€ -1.191	€ 385
6060005	0,544	€ 182	130	3	3	95,38%	0,00%	€ 547,466		€ -3.472	€ 1.032
6080103	0,181	€ 104	50	1	1	99,38%	0,00%	€ 104,000		€ -1.366	€ 433
6200014	0,363	€ 198	15	1	0	98,53%	98,53%	€ 198,000		€ -2.771	€ 763
6200192	0,725	€ 308	55	4	0	90,15%	90,15%	€ 1.232,000		€ -4.237	€ 1.431
6200275	0,181	€ 55	20	1	0	99,02%	99,02%	€ 54,890		€ -1.437	€ 383
6200895	0,181	€ 1.129	15	1	0	99,26%	99,26%	€ 1.128,750		€ -367	€ 384
6200962	0,181	€ 198	15	1	0	99,26%	99,26%	€ 198,000		€ -1.298	€ 384
7001080	0,181	€ 1.223	55	1	0	97,34%	97,34%	€ 1.223,000		€ -244	€ 377
7001126	0,181	€ 55	20	1	0	99,02%	99,02%	€ 54,667		€ -1.437	€ 383
7001207	0,181	€ 1.128	30	1	0	0,00%	0,00%	€ 1.128,000		€ -357	€ 1.485
7001228	0,181	€ 3.580	100	0	0	0,00%	0,00%	€ -		€ -	€ -
7001231	0,181	€ 481	50	1	1	99,38%	0,00%	€ 481,000		€ -989	€ 433
7001232	0,181	€ 561	120	1	0	0,00%	0,00%	€ 561,400		€ -861	€ 1.422
7001485	0,181	€ 6.690	85	0	0	0,00%	0,00%	€ -		€ -	€ -
7001726	0,544	€ 39	20	2	0	99,96%	99,96%	€ 78,000		€ -4.440	€ 1.160
7001731	0,181	€ 649	25	1	0	98,77%	98,77%	€ 649,000		€ -839	€ 382
7001775	0,181	€ 561	20	1	0	99,02%	99,02%	€ 561,000		€ -931	€ 383
7001901	0,181	€ 1.502	20	1	0	99,02%	99,02%	€ 1.502,000		€ -	€ 393
7001955	0,181	€ 649	20	1	0	99,02%	99,02%	€ 649,000		€ -843	€ 383

Solve LP-problem

Variables	
TFR Premium	92,0%
TFR Regular	85,0%
Premium demand (%)	25%
Demand in/decrease	100%

Results iterations	
Iteration 1	€ 101.943
Iteration 2	€ 57.706
Iteration 3	€ 48.409
Iteration 4	€ 48.239
Iteration 5	€ 48.953

Results current settings	
WAFR Premium	89,67%
WAFR Regular	83,97%
Inventory investments	€ 48.953

Input variables
Model variables
Output variables
Formulas
Headers

Figure B.2: Example of the model parameters

B.3 Recommendations

When a solution is found the results can be compared with the initial settings in the tab:"Recommendations". Per spare-part and recommendation is given and the related costs/savings, see Figure B.3.

Article number	Leadtime	mand/ye	Price/pie	urrent sto	Current FR	Recon	Recor	Recom. FRC1	Recom. FRC2	Recom. Stock	Recom. Inv. change	Recom. Decision
5010216	10	2,18	236	0	0,00%	2	0	99,83%	99,83%	2	€ 472,00	Increase or maintain stock
5010925	10	1,45	3	72	100,00%	16	0	99,92%	99,92%	-56	€ -155,56	Reduce the stock
5020027	10	0,18	547	0	0,00%	1	0	99,51%	99,51%	1	€ 547,00	Increase or maintain stock
5020080	10	0,18	308	0	0,00%	1	0	99,51%	99,51%	1	€ 308,26	Increase or maintain stock
6060005	130	0,54	182	133	100,00%	3	3	95,38%	0,00%	-130	€ -23.723,53	Reduce the stock
6080103	50	0,18	104	0	0,00%	1	1	99,38%	0,00%	1	€ 104,00	Increase or maintain stock
6200014	15	0,36	198	0	0,00%	1	0	98,53%	98,53%	1	€ 198,00	Increase or maintain stock
6200192	55	0,73	308	1	50,00%	4	0	90,15%	90,15%	3	€ 924,00	Increase or maintain stock
6200275	20	0,18	55	0	0,00%	1	0	99,02%	99,02%	1	€ 54,89	Increase or maintain stock
6200895	15	0,18	1129	0	0,00%	1	0	99,26%	99,26%	1	€ 1.128,75	Increase or maintain stock
6200962	15	0,18	198	9	100,00%	1	0	99,26%	99,26%	-8	€ -1.584,00	Reduce the stock
7001080	55	0,18	1223	1	97,34%	1	0	97,34%	97,34%	0	€ -	Reduce the stock
7001126	20	0,18	55	3	100,00%	1	0	99,02%	99,02%	-2	€ -109,33	Reduce the stock
7001207	30	0,18	1128	1	98,53%	1	0	0,00%	0,00%	0	€ -	Reduce the stock
7001228	100	0,18	3580	1	95,27%	0	0	0,00%	0,00%	-1	€ -3.580,00	Reduce the stock
7001231	50	0,18	481	1	97,58%	1	1	99,38%	0,00%	0	€ -	Reduce the stock
7001232	120	0,18	561	0	0,00%	1	0	0,00%	0,00%	1	€ 561,40	Increase or maintain stock
7001485	85	0,18	6690	1	95,95%	0	0	0,00%	0,00%	-1	€ -6.690,00	Reduce the stock
7001726	20	0,54	39	7	100,00%	2	0	99,96%	99,96%	-5	€ -195,00	Reduce the stock
7001731	25	0,18	649	0	0,00%	1	0	98,77%	98,77%	1	€ 649,00	Increase or maintain stock
7001775	20	0,18	561	1	99,02%	1	0	99,02%	99,02%	0	€ -	Reduce the stock
7001901	20	0,18	1502	0	0,00%	1	0	99,02%	99,02%	1	€ 1.502,00	Increase or maintain stock
7001955	20	0,18	649	13	100,00%	1	0	99,02%	99,02%	-12	€ -7.788,00	Reduce the stock
7001976	40	0,73	2966	0	0,00%	0	0	0,00%	0,00%	0	€ -	Reduce the stock
7002012	10	0,18	1502	3	100,00%	1	1	99,88%	0,00%	-2	€ -3.004,00	Reduce the stock
7002013	15	0,18	33	0	0,00%	1	0	99,26%	99,26%	1	€ 32,96	Increase or maintain stock

	Current state	Model recommendation
WAFRC1	75,69%	92,00%
WAFRC2	75,69%	85,19%
Investment	€ 162.621,59	€ 48.952,54

Output variables
Formulas
Headers

Figure B.3: Example of recommendation per spare

C Experimental settings

Exp. number	TFR Class 1	TFR Class 2	Demand	Demand ratio	Order size dist.
1	99%	95%	80%	37.5%	PCR
2	95%	95%	80%	37.5%	PCR
3	99%	90%	80%	37.5%	PCR
4	95%	90%	80%	37.5%	PCR
5	99%	95%	100%	25%	PCR
6	95%	95%	100%	25%	PCR
7	99%	90%	100%	25%	PCR
8	95%	90%	100%	25%	PCR
9	99%	95%	100%	37.5%	PCR
10	99%	90%	100%	37.5%	PCR
11	99%	95%	120%	37.5%	PCR
12	99%	90%	120%	37.5%	PCR
13	99%	95%	100%	50%	PCR
14	99%	90%	100%	50%	PCR
15	99%	95%	120%	50%	PCR
16	99%	90%	120%	50%	PCR
17	99%	95%	80%	37.5%	BCR
18	95%	95%	80%	37.5%	BCR
19	99%	90%	80%	37.5%	BCR
20	95%	90%	80%	37.5%	BCR
21	99%	95%	100%	25%	BCR
22	95%	95%	100%	25%	BCR
23	99%	90%	100%	25%	BCR
24	95%	90%	100%	25%	BCR
25	99%	95%	100%	37.5%	BCR
26	99%	90%	100%	37.5%	BCR
27	99%	95%	120%	37.5%	BCR
28	99%	90%	120%	37.5%	BCR
29	99%	95%	100%	50%	BCR
30	99%	90%	100%	50%	BCR
31	99%	95%	120%	50%	BCR
32	99%	90%	120%	50%	BCR
33	99%	95%	100%	25%	RCR
34	95%	95%	100%	25%	RCR
35	99%	90%	100%	25%	RCR
36	95%	90%	100%	25%	RCR

Table C.1: Experimental settings

D Summary experimental results

Exp. #	Inventory (€)	WAFRC1(%)	WAFRC2(%)	CL(€)	Avg. value/spare(€)
1	104.103	0.99	0.9	30.497	181
2	64.092	0.95	0.95	0	120
3	103.153	0.99	0.90	32.523	181
4	63.965	0.95	0.90	7.072	120
5	111.842	0.99	0.95	15.580	172
6	66.810	0.95	0.95	0	125
7	108.440	0.99	0.90	28.549	161
8	64.628	0.95	0.90	7.460	121
9	114.083	0.99	0.95	19.432	161
10	109.046	0.99	0.90	31.106	121
11	115.440	0.99	0.95	9.749	166
12	112.391	0.99	0.90	34.138	160
13	114.973	0.99	0.95	12.142	172
14	112.715	0.99	0.90	26.657	160
15	117.398	0.99	0.95	7.198	170
16	113.754	0.99	0.90	33.450	163
17	106.798	0.99	0.95	13.336	169
18	63.403	0.95	0.95	0	169
19	103.611	0.99	0.90	0	158
20	61.149	0.95	0.90	9.085	117
21	111.196	0.99	0.95	9.152	191
22	65.428	0.95	0.95	0	116
23	105.988	0.99	0.90	30.304	167
24	61.983	0.95	0.90	9.791	120
25	111.543	0.99	0.95	17.499	161
26	106.846	0.99	0.90	29.019	157
27	113.246	0.99	0.95	11.336	162
28	108.970	0.99	0.90	29.422	156
29	112.773	0.99	0.95	9.564	166
30	108.474	0.99	0.90	42.921	158
31	114.369	0.99	0.95	23.426	162
32	111.647	0.99	0.90	31.883	164
33	91.114	0.99	0.95	597	138
34	68.952	0.95	0.95	0	130
35	86.887	0.99	0.90	13.512	136
36	56.831	0.95	0.90	1.001	108

Table D.1: Experimental results